Study of Lattices

PHY412: Lab 2

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Aim

1. To study lattice properties via analogy of electrical oscillators.

Introduction

normal

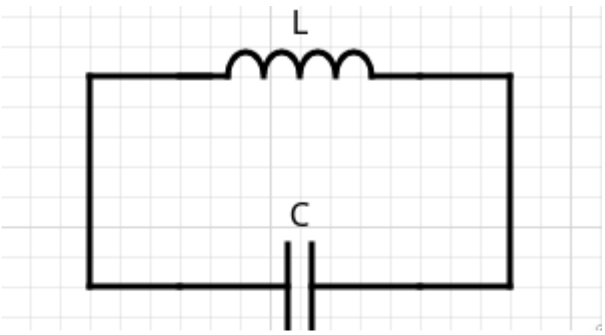
Most physical properties of any solid is given by its inter-molecular structure, since it forms the basis of a solid's existance. This structure often has a well-defined repeating structure known as a lattice, and this also heavily simplifies our analysis since the equations of dynamics are highly symmetric. Particularly, solids will have a state which is a minima at equilibrium. At this point, we can assume the intermolecular attraction as purely quadratic. Hence, the hamiltonian of this system is that of the Simple Harmonic Oscillator -

$$\mathcal{H} = \frac{1}{2} m \dot{x}^2 + \frac{1}{2} k x^2$$

Where x is displacement from the minima, p and m are the momentum and mass of the atom, and k is the "stiffness constant".

Electric Analogue

Now, consider the following LC circuit:



The hamiltonian of this system is:

$$\begin{aligned} L &\iff m \\ 1/C &\iff k \end{aligned}$$

Hence, this is exactly the same Hamiltonian as the previous SHO system, and hence will have the same equations of "motion". Hence, we can draw the analogy in the following way:

This analogy now allows us to easily measure the responses of an electrical system and gain knowledge about a mechanical system which is often harder to study directly.

Monoatomic Chain

In a Monoatomic chain, all atoms are identical and hence thier masses and interatomic forces are equal. By our analogy, we can study the following LC Chain

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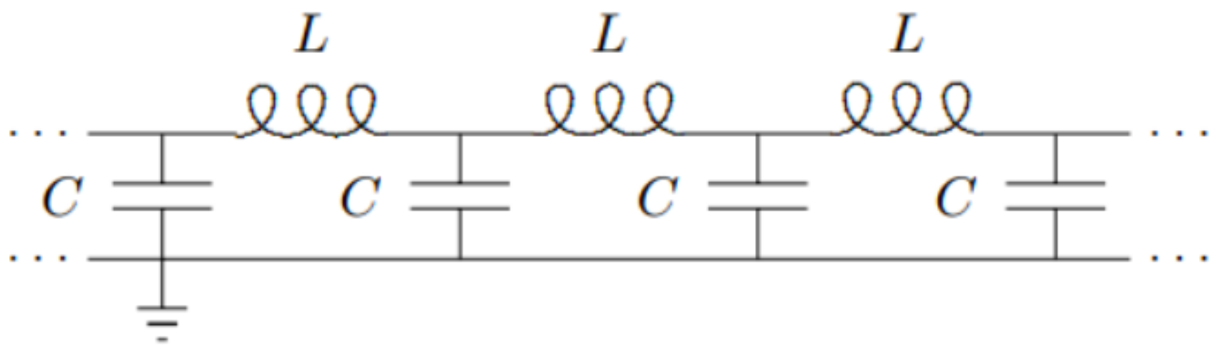

Solving for

Assuming a charge dissipating through the th inductor, we get

An ansatz which satisfies this differeential equation is

Dispersion Relations

$$\implies \omega^2 = \frac{4}{LC}\sin^2\frac{\theta}{2}$$

From the ansatz, we see

Also, the charge on capacitor , , and the potential across capacitor is and across inductor is .

Hence,

Plotting the Dispersion Graph

L = 0.5

C = 0.4

C_1 = 0.8

normal

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monoatomic (generic function with 1 method)
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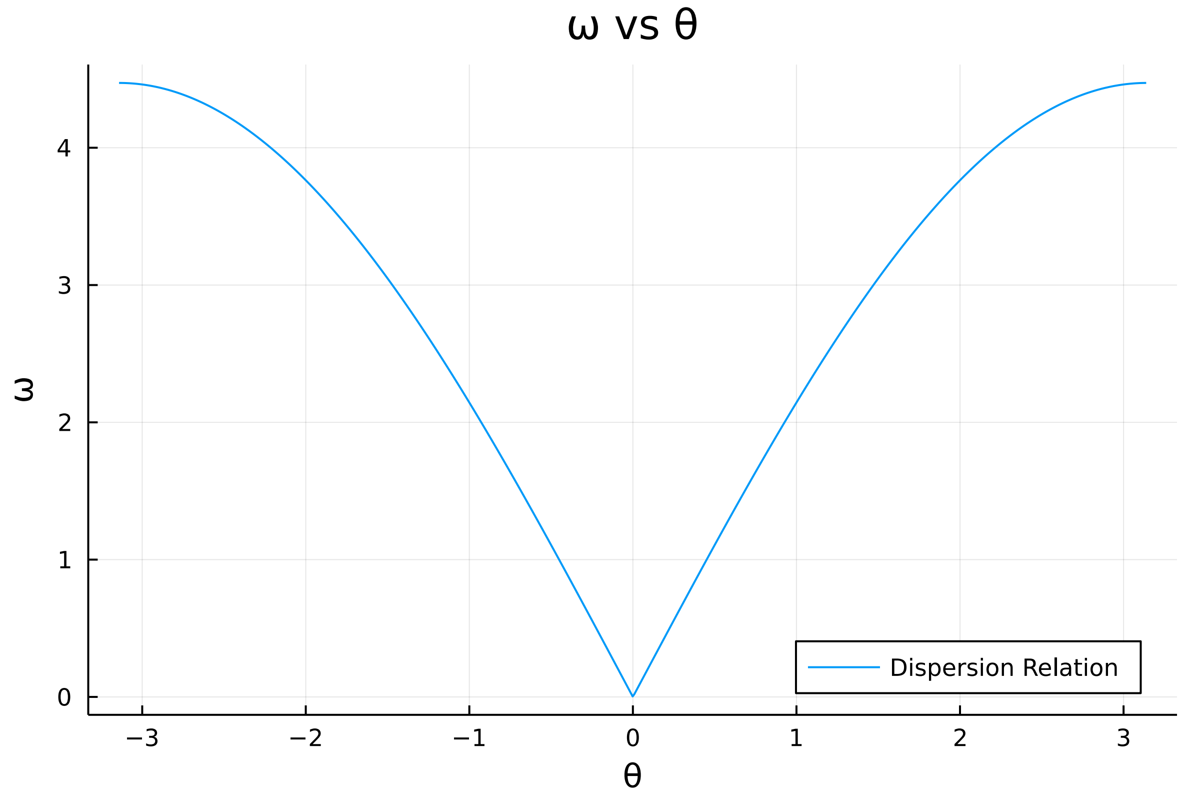
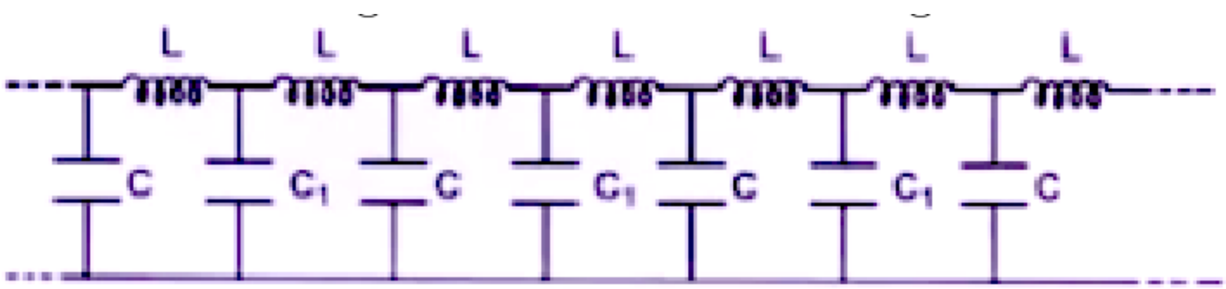


Analogues with the Mechanical System

Diatomic Chain

In such a system, technically both and change, however, we can assume that is held constant. By our analogy, we can study the following LC Chain



Solving for

normal

Following a similar process as before,

with ansatz

Finding the dispersion relations

Inserting the ansatz back into the equations, we get

Which has solutions only if determinant is 0. This condition reduces to

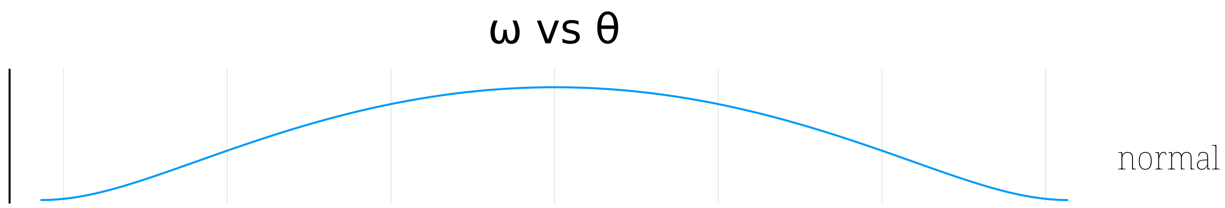
which has solution

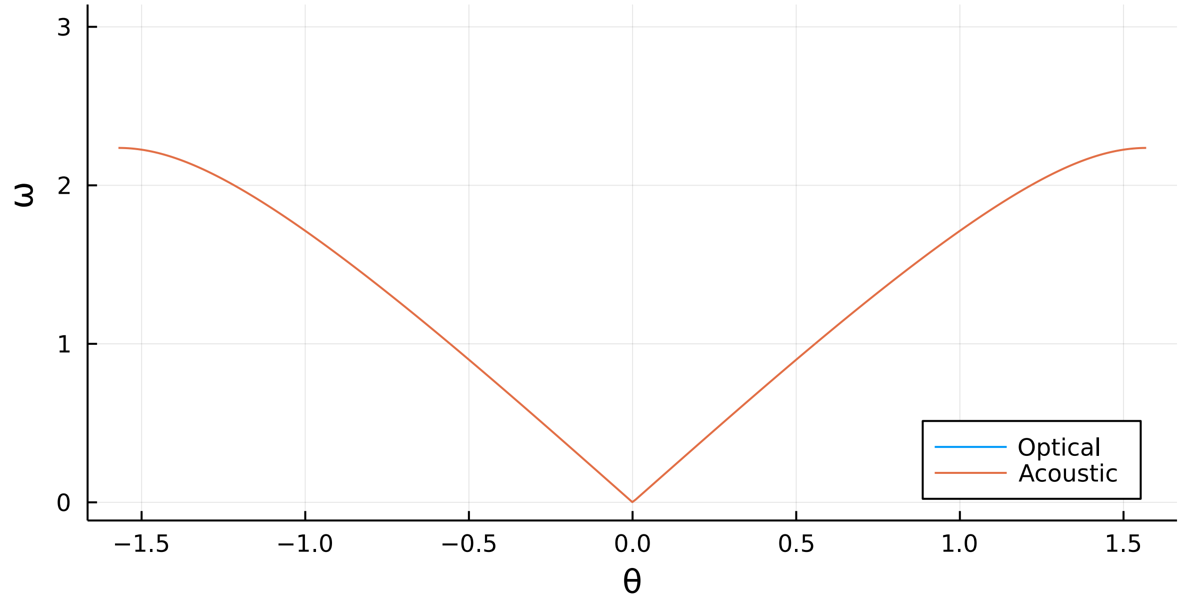
Plotting the dispersion relations

L = 0.5

C = 0.4

C_1 = 0.8



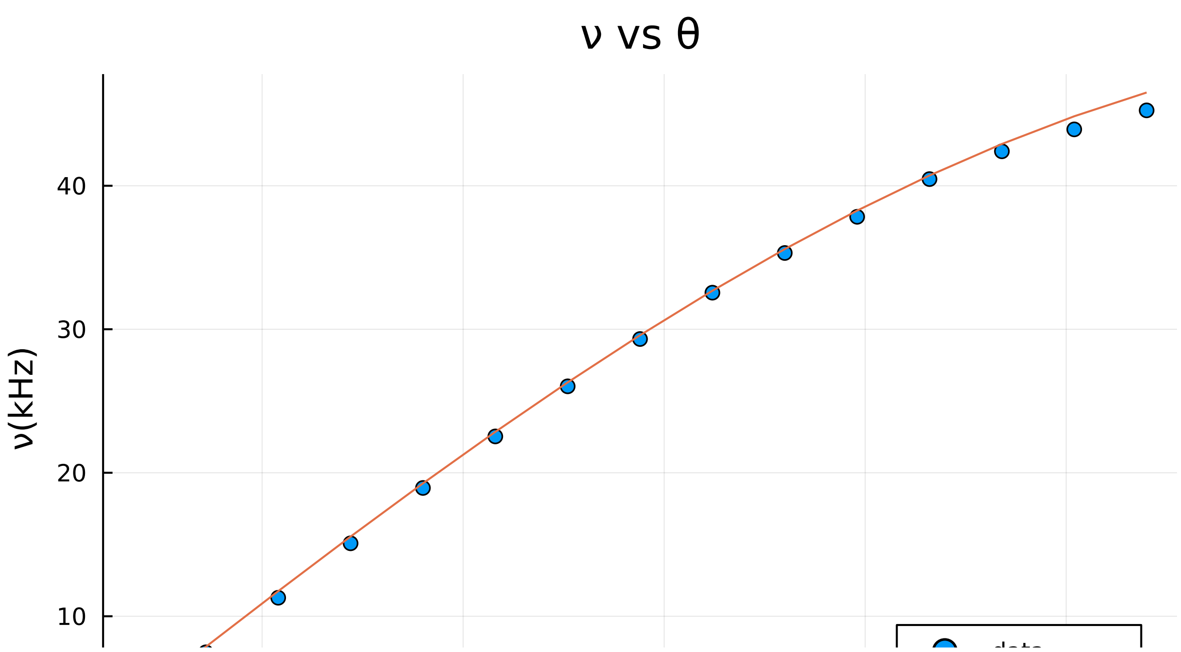


Analogues with the Mechanical System

Observations

From the data provided, we can plot the experimental and expected dipersion graphs.

Monoatomic chain



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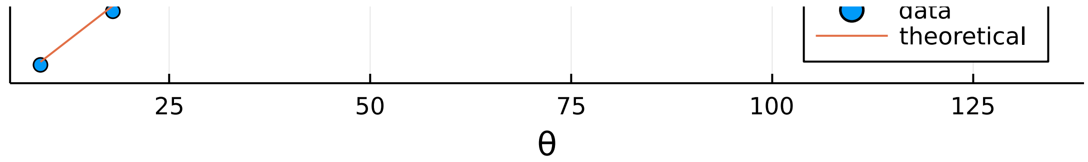
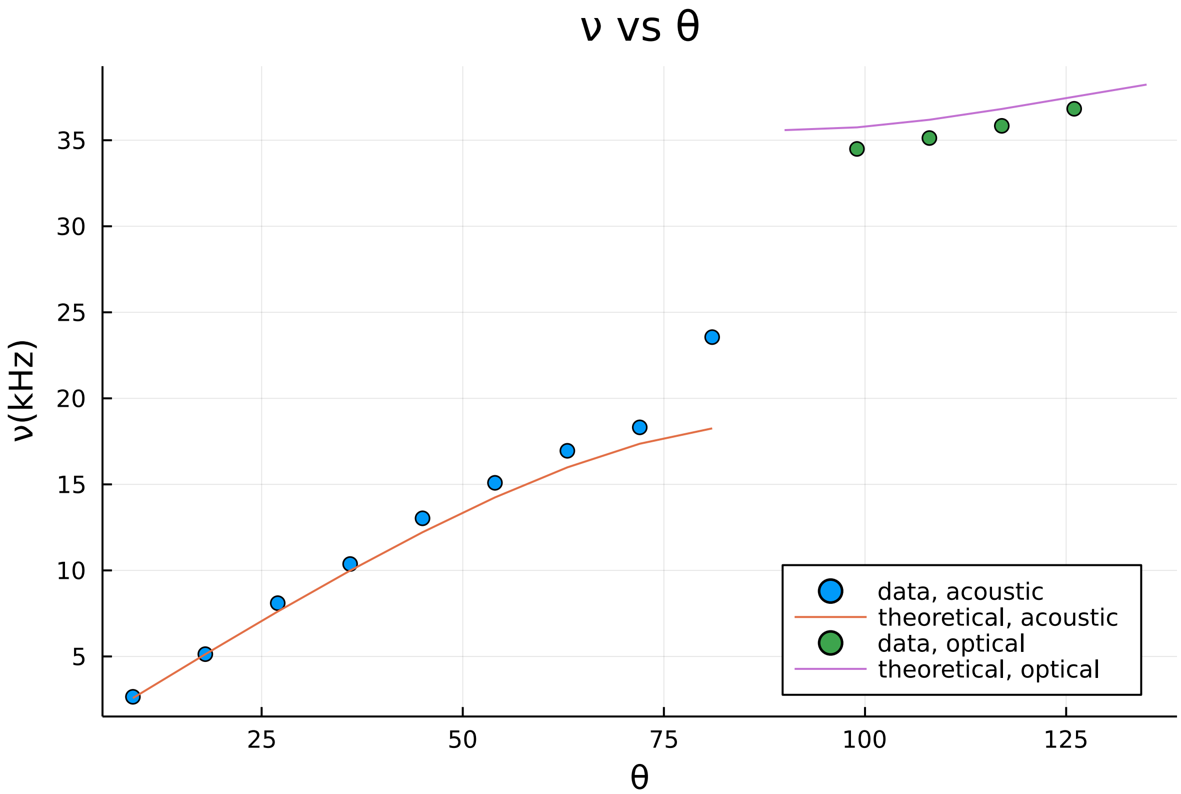


Diatomic Chain



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