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(HW) AI maths

① orthogonal vectors:

Two vectors are orthogonal if they are perpendicular to each other.

→ The dot product (inner product) of two orthogonal vectors is 0.

Now, $\bar{a} \cdot \bar{b} = |\bar{a}||\bar{b}| \cdot \cos\theta$

but as $\bar{a} \cdot \bar{b} = 0$

$\therefore \; |\bar{a}||\bar{b}| \cos\theta = 0$

$\therefore \; \cos\theta = 0$

$\therefore \; \theta = 90^\circ$

② Orthonormal vectors:

→ Two vectors having unit magnitude (magnitude = 1) and having 90° angle between them are orthonormal.

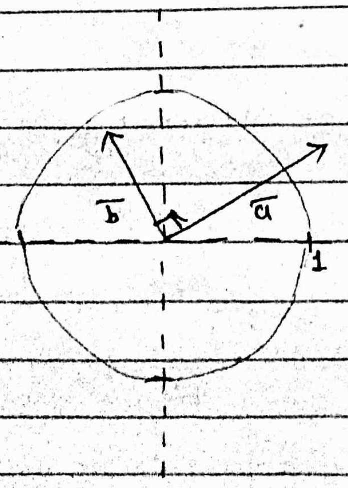
→ orthonormal vectors have many convinient characterstics, hence those are used in developing more advanced methods.

Applications:

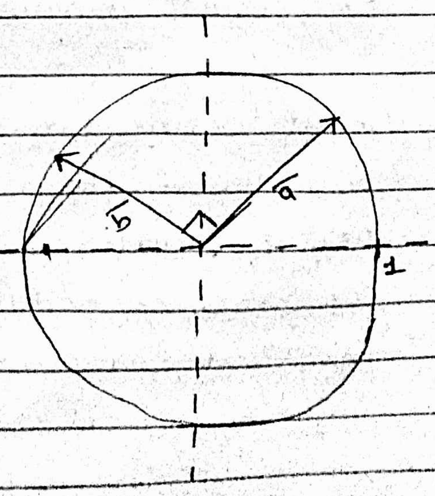
① SVD (method for dimensionality reduction)

② Regularization of convolutional layer in CNN

③ Haar Transformer (method for image compression)



orthogonal vectors



orthonormal vectors.

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HW1

⑨ orthogonal matrix:
→ A square matrix who's rows and columns are orthogonal to each other.
→ It's transpose is it's actual Inverse.
i.e. $A^T = A^{-1}$ ... if A is orthogonal
→ Product of any orthogonal matrix and its transpose produces Identity matrix.
i.e. $A \cdot A^T = I$

Ex.

* cosine similarity:

→ cosine similarity = measure of similarity between two data points in a plane.

→ It is used for determining distance between two data points in ML algorithms.

→ Also used to find similarity between two objects in Recommendation systems. Also used to find similarity between two textual data instences.

⇒ cosine similarity is cosine of angle between two vectors. cosine similarity operates entirely on cosine principles.

→ use of cosine similarity in machine learning:
① classification of data
② Evaluation metric in KNN.
③ Recommendation system
④ Text compareson in NLP.

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HW3.

→ Cosine similarity has the ability to handle data of variable length.
ex. cosine similarity considers frequency of each word while calculating similarity between textual documents.

To calculate similarity between two data vectors $\bar{A}$ & $\bar{B}$

$$\cos\theta = \frac{\bar{A}\cdot\bar{B}}{|\bar{A}||\bar{B}|}$$

- similarity varies between -1 and 1.
- if similarity = 1, $\theta = 0°$, similar vectors
- if similarity = 0, $\theta = 90°$, orthogonal vectors.
- if similarity = -1, $\theta = 180°$, opposite vectors.

4. Singular Value Decomposition (SVD).

⇒ SVD of a matrix is factorization of matrix into three matrices. It conveys important theoritical and geometrical insights about linear transformation.

⇒ It has important applications in Data Science.

⇒ The SVD of the matrix $A_{m\times n}$ is given by following formula:

$$A = U \Sigma V^T$$

- U = $m \times m$ matrix of orthonormal eigenvectors of AA^T
- V^T = Transpose of $n\times n$ matrix containing orthonormal eigenvectors of A^TA.
- Σ = Diagonal matrix with r elements equal to root of positive ~~eigenvector~~ eigenvalues of AA^T or A^TA

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(HW)

*. Principal Component Analysis (PCA).

→ Principal Component Analysis is a dimensionality reduction and machine learning method that simplifies large and complex datasets into smaller ones while preserving important patterns and trends.

⇒ Calculation of principal component analysis.

1. Standardize range of continuous initial variables.
2. Compute covariance matrix.
3. Compute eigenvectors and eigenvalues of covariance matrix.
4. Create feature vector to decide which principal component to keep.
5. Recast data along principal component axes.

→ Reasons of using PCA.

1. Simplification : Data becomes easy to manipulate and analyze.
2. Noise Reduction : PCA helps in reducing noise from data / Redundand information from data
3. Performance Improvement : After applying PCA to Data, it becomes easy to manage and analyze it.

⇒ Applications :

1. Data visualization: Data simplification for visualization
2. Image compression : maintains important features while reducing size of the image.
3. Pattern Recognition: Used for identification tasks.

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HW

* Eigenvector:

Given a square matrix (A), A non-zero vector $\vec{x}$ is eigen vector of (A) if multiplying (A) by $\vec{x}$ results in vector which is scaler multiple of $\vec{x}$

i.e. $A \cdot \vec{x} = \lambda \cdot \vec{x}$

* Eigenvalue:

A scaler λ is called eigenvalue of (A) with respect to $\vec{x}$

• computation of Eigenvalue and Eigenvector:

Equation to find them is

$$A \cdot \vec{x} = \lambda \cdot \vec{x}$$
$$\therefore A \cdot \vec{x} - \lambda \cdot \vec{x} = 0$$
$$\therefore A \cdot \vec{x} - \lambda \cdot I \cdot \vec{x} = 0$$
$$\therefore (A - \lambda \cdot I) \cdot \vec{x} = 0$$
$$\therefore (A - \lambda \cdot I) = 0$$

Ex.

$$A = \begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}.$$

by $(A - \lambda \cdot I) = 0$, we get.

$$\begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = 0$$

$$\therefore \begin{bmatrix} 2-\lambda & 1 \\ 1 & 3-\lambda \end{bmatrix} = 0$$

$$\therefore (2-\lambda)(3-\lambda) - 1 = 0$$
$$\therefore 6 - 2\lambda - 3\lambda + \lambda^2 - 1 = 0$$
$$\therefore \lambda^2 - 5\lambda + 5 = 0$$
$$\therefore \lambda = \frac{+5 \pm \sqrt{25 - 4(1)(+5)}}{2}$$
$$\therefore \cancel{\lambda = \frac{5 \pm \sqrt{45}}{2}} \quad \lambda = \frac{5 \pm \sqrt{5}}{2}.$$