CREDIT DERIVATIVES

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Outline

- Introduction
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- Modelling
- Structural Modelling
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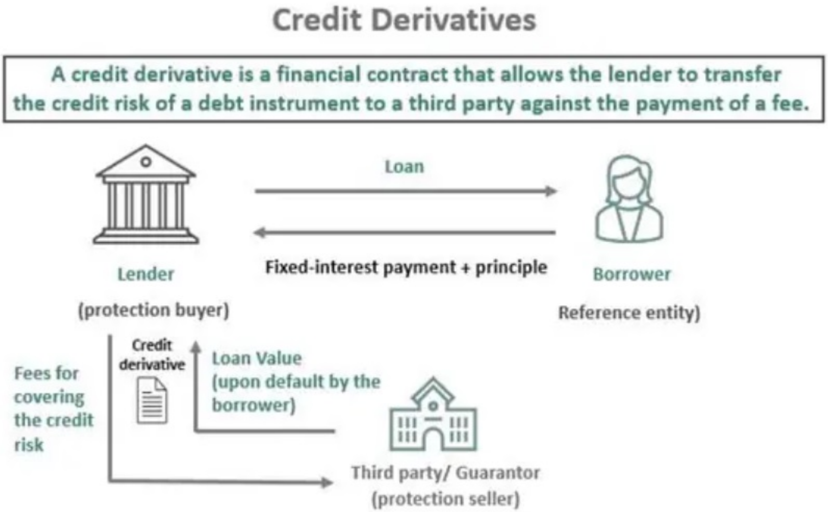
- There are three parties to a credit derivative contract: borrower (reference entity), lender (protection buyer), and third party (protection seller).

Functioning



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    - Risk Management
    - Enhanced Yield
    - Liquidity and Market Access
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- The banks use them to mitigate credit risk exposure and expand their credit portfolio.
- Insurance companies also use them to improve returns on their asset portfolio.

Types of Credit Derivatives

- Funded Credit Derivatives
  - the investor in the note is the credit protection seller and is making an upfront payment to the protection buyer when buying the note.
  - the lender is not exposed to the credit risk from the counterparty.
  - because the counterparty pays an appropriate sum to the lender to cover any loan default in the future.
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- Unfunded Credit Derivatives
  - the protection seller does not make an upfront payment to the protection buyer.
  - the protection buyer (lender) does not receive any initial payment from the protection seller (counterparty) for covering the credit risk.
  - payment is made on termination of the trade (if there is a credit event).
  - unfunded CDs expose the lender to the risk of default from the counterparty.
  - Examples : Credit Default Swap (CDS)

Credit Risk

- Credit risk refers to the potential loss that can occur as a result of the failure of a borrower to meet their debt obligations. It is a fundamental component of financial risk and arises from the possibility that borrowers may default on their loans or fail to make timely interest payments. Credit risk is prevalent in various financial transactions, including loans, bonds, credit derivatives, and other debt instruments.

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- Types of Credit Risk
    - Default Risk
    - Downgrade Risk
    - Credit Spread Risk

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- To value credit derivatives it is necessary to be able to model credit risk.
- Their price depends on the borrower's credit rating.
    - Formal credit ratings : Standard and poor's , Moody's Investor service and Fitch Ratings
    - In all rating systems the term "high grade" means low credit risk or, conversely, high probability of future payments.
    - AAA - Prime, AA - High quality, A - upper medium grade etc.

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$$E(T) = \max\{A(T) - K, 0\} \tag{1}$$

$$\begin{aligned} D(T,T) &= min(A(T), K) && (2)\\ &= A(T) - max(A(T) - K, 0) && (3)\\ &= K - max(K - A(T), 0) && (4) \end{aligned}$$

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Clearly, at maturity, this is true as we have

$$\begin{aligned} E(T) + D(T,T) &= \max\{A(T) - K, 0\} + \min\{A(T), K\} && (6) \\ &= A(T) && (7) \end{aligned}$$

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- probability of default at maturity is

$$p = \int_{-\infty}^{K} \phi(A(T))\, dA(T) = 1 - N(d_2) \tag{9}$$

$N(d_2)$ is the **survival probability**.

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- The current value of the debt is a covered call value:

$$\begin{aligned} D(t,T) &= A(t) - E(t) && (12)\\ &= A(t) - [A(t)N(d_1) - e^{-r(T-t)}KN(d_2)] && (13)\\ &= A(t)[1 - N(d_1)] + e^{-r(T-t)}KN(d_2) && (14) \end{aligned}$$

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- the risky yield increases with the debt-to-asset leverage of the firm and its asset value volatility.
- they are difficult to calibrate and so are not suitable for the frequent marking to market of credit contingent securities.
- the main application of structural models is in the areas of credit risk analysis and corporate structure analysis.

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- the probability of defaulting in between time t and $T + \tau$ is given by $Q(t, T) - Q(t, T + \tau)$

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- The forward default probability is a conditional default probability for a forward interval conditional on surviving until the beginning of the interval.

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- A \$1 *risky* cash flow received at time T has a risk-neutral expected value of Q(t,T) and a present value of P(t,T)Q(t,T) where P is the risk free discount factor.

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- If λ is a function of time and the recovery is paid at the time of default then

$$B(t) = \int_t^T P(t, u)R(u)\lambda(u)e^{-\int_t^u \lambda(w)dw} + \sum_{j=1}^{n} P(t, T_j)c_j e^{-\int_t^{T_j} \lambda(w)dw} \tag{22}$$

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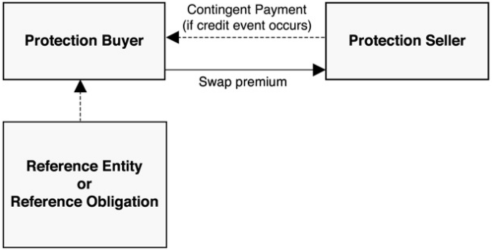
- **Intuitive interpretation** : $p(1-\delta)$ serves as spread over the risk-free discount rate.
  - p is small $\implies$ credit spread is small
  - R(t) is high (i.e 1-δ is small) $\implies$ credit spread is small

Credit Default Swap

- financial derivatives that allow an investor to swap or offset their credit risk with that of another investor.



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$$\text{Quarterly swap premium} = \frac{\text{Notional amount} \times \text{Annual rate} \times \text{no. of days in quarter}}{360}$$

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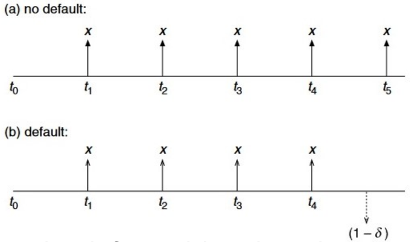
Loss resulting from default of first reference entity	=	\$6 million
Loss result from default of second reference entity	=	\$10 million
Loss result from default of third reference entity	=	\$16 million
Loss result from default of fourth reference entity	=	\$12 million
Loss result from default of fifth reference entity	=	\$15 million

Credit Default Swap Pricing



- A credit default swap takes the defaulted bond as the recovery value and pays par upon default and zero otherwise.

$$V = E\left[e^{-\int_0^u r(s)ds} 1_{u<T}[1 - R(u)]\right] \tag{26}$$

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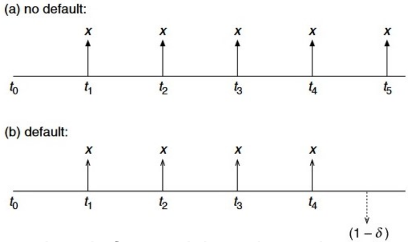

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$$V = E\left[e^{-\int_0^u r(s)ds} 1_{u<T}[1 - R(u)]\right] \tag{26}$$

- Therefore the value of the credit default swap (V) should be the recovery value upon default weighted by the default probability.

$$V = \sum_{j=1}^{n} P(t, T_j)\left[Q(t, T_{j-1}) - Q(t, T_j)\right]\left[1 - R(T_j)\right] \tag{27}$$

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Thank You!