CREDIT DERIVATIVES

A Project Report Submitted

for the Course

MA498 Project I

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to the

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CERTIFICATE

This is to certify that the work contained in this project report entitled "Credit Derivatives" submitted by Dhruvesh Bhure (Roll No.: 200123018) and Dev Shah (Roll No.: 200123074) to the Department of Mathematics, Indian Institute of Technology Guwahati towards partial requirement of Bachelor of Technology in Mathematics and Computing has been carried out by him/her under my supervision.

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ABSTRACT

This project explores the dynamic landscape of credit derivatives, a crucial component in modern financial markets. Credit derivatives play a pivotal role in managing and transferring credit risk, influencing risk management strategies of financial institutions and investors globally. The study delves into innovative credit derivative products, ranging from credit default swaps to collateralized debt obligations, analyzing their structures, applications, and the intricacies of risk associated with these instruments. Additionally, the project investigates market dynamics, regulatory frameworks, and emerging trends in the credit derivatives market. By blending theoretical insights with practical considerations, the project aims to provide a comprehensive understanding of credit derivatives, their evolving role in financial ecosystems, and the challenges and opportunities they present to market participants. Through this exploration, the project contributes to the broader discourse on financial innovation and risk management strategies in the contemporary financial landscape.

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Chapter 1

Introduction

Derivatives are financial instruments designed to efficiently transfer some form of risk between two or more parties. They are classified based on the form of risk that is being transferred. Different types of derivatives are as follows -

- Interest rate derivatives
- Credit Derivatives
- Foreign exchange derivatives
- Commodity Derivatives
- Equity Derivatives
- Real Estate Derivatives

Credit Derivatives (CDs) are derivative contracts that enable a lender to transfer a debt instrument's credit risk to a third party in exchange for a payment. However, there is no actual transfer of ownership of the instrument. They protect the lender against the loss associated with the risk of default by the borrower.

1.1 Types of credit derivatives

1. **Funded Credit Derivatives**

   The investor in the note is the credit protection seller and is making an upfront payment to the protection buyer when buying the note. Here the protection buyer (lender) is not exposed to the credit risk from the protection seller (counter party). This is because the protection seller pays an appropriate sum to the protection buyer to cover any loan default in the future.

   Examples :

2. **Unfunded Credit Derivatives**

   The protection seller does not make an upfront payment to the protection buyer. Under such a contract, the protection seller pays only when the reference entity (borrower) defaults. Therefore, unfunded Credit Derivatives expose the protection buyer to the risk of default from the protection seller.

   Examples :

Example : Suppose a company XYZ is issuing a bond with $1 million par value and 7% coupon rate. ABC Bank has excess funds at its disposal and is willing to invest in XYZ bond. However, XYZ is rated low by the credit rating agency. Therefore, ABC bank seeks a credit default swap (CDS) from MNM to mitigate its exposure to credit risk. ABC will pay 1% of the face value of the bond (fees) to MNM in return for its insurance against XYZ's default. If XYZ defaults, ABC (CDS buyer) will get a payment from MNM (CDS seller). However, if XYZ doesn't default, MNM stands to benefit as it gains the fees without covering for any default.

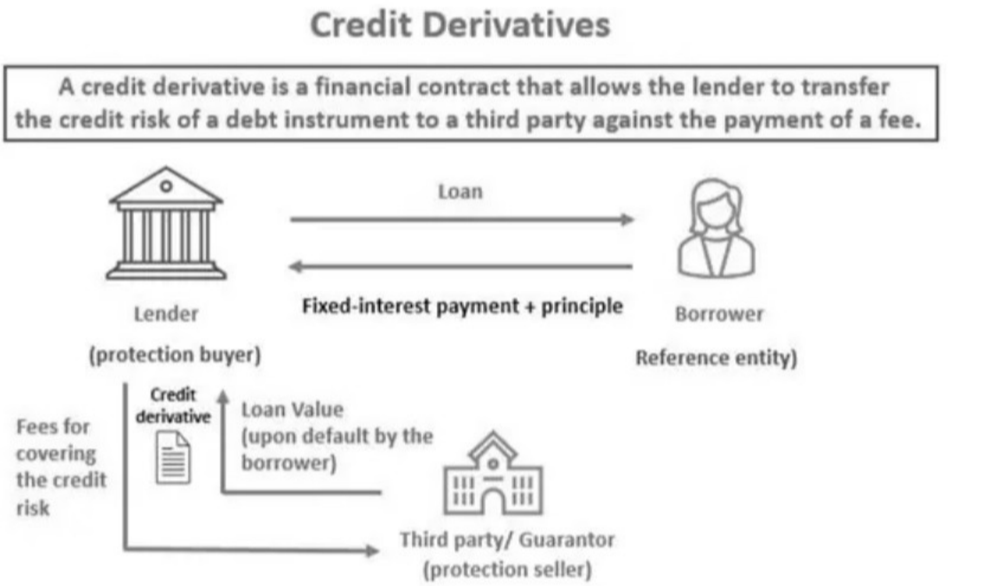

Figure 1.1: Credit Derivatives

Chapter 2

Credit risk

2.1 Types of credit risk

1. **Default risk**

   It is the risk that the issuer of a bond or the debtor (borrower) will not repay the outstanding debt in full. Either no amount of the bond or loan is repaid, or some portion of the original debt will be recovered.

2. **Downgrade risk**

   It is the risk that a nationally recognized statistical rating organizations such as Standard & Poor's, Moody's Investors Services, or Fitch Ratings reduces its outstanding credit rating for an issuer based on an evaluation of that issuer's current earning power versus its capacity to pay its debt obligations as they become due.

3. **Credit spread risk**

   It is the risk that the spread over a reference rate will increase for an outstanding debt obligation.

Downgrade risk pertains to a specific, formal credit review by an independent rating agency, while the credit spread risk is the financial market's reaction to perceived credit deterioration.A downgrade by a rating agency can lead to an automatic widening of the credit spread as investors adjust their expectations and pricing based on the revised credit rating.

2.2 Credit default risk

Credit default risk is the risk that the issuer will fail to satisfy the terms of the obligation with respect to the timely payment of interest and repayment of the amount borrowed.

2.2.1 Credit ratings

The bond market can be divided into two sectors:

- **Investment Grade Bonds:** These are bonds with higher ratings, typically rated 4 or above.
- **Non-Investment Grade Bonds or High-Yield Bonds or Junk Bonds:** This category includes bonds rated below 4, representing higher risk. These bonds are often referred to as high-yield or junk bonds.
    - **Distressed Debt:** Bonds in this category may be undergoing bankruptcy proceedings, in default of coupon payments, or experiencing some other form of financial distress.

Factors Considered in Rating Corporate Bond Issues

- consider the four Cs of credit:

- Character of management : the foundation of sound credit

  - Strategic directions
  - Financial philosophy
  - Conservatism
  - Track record
  - succession planning
  - Control systems

- **Capacity** : the ability of an issuer to repay its obligations

  - Analysis of financial statement

- Collateral : assets pledged to secure the debt
- Covenants : the terms and conditions of the lending agreement.

2.2.2 Default rates

- **Altman** : default rate is the par value of all high-yield bonds that defaulted in a given calendar year, divided by the total par value outstanding during the year.
- **Drexel Burnham Lambert (DLB)** : Default rate is the cumulative dollar value of all defaulted high-yield bonds, divided by the cumulative dollar value of all high-yield issuance, and further divided by the weighted average number of years outstanding to obtain an average annual default rate.
- **Asquith, Mullins, and Wolff** : Default rate is the total par value of defaulted issues as of the date of their study, divided by the total par amount originally issued to obtain a cumulative default rate.

2.2.3 Recovery rates

- **Default loss of principal** : the default rate for the year multiplied by the average loss of principal.

- **Average loss of principal** : the difference between par value of \$100 and the recovery of principal.
- **Default loss of coupon** : the default rate multiplied by the weighted average coupon rate divided by two (because the coupon payments are semiannual)
- **Default loss Rate** : default loss of principal + Default loss of coupon

2.3 Credit spread risk

It is the excess premium over the government or risk free rate required by the market for taking on a certain assumed credit exposure. The higher the credit rating, the smaller the credit spread.

- **Duration** : is a measure of the change in the value of a bond when interest rate changes. Duration is that it is the approximate percentage change in the value of a bond for a 100 bp change in *interest rates.* (interest rate is same as treasury rate)
- **spread duration** is a measure of how a credit-risky bond's price will change if the credit spread sought by the market changes. Spread duration is that it is the approximate percentage change in the value of a bond for a 100 bp change in *credit spread* holding interest rate constant.
- **Yield of Credit Risky Bonds** = Treasury Yield + Credit Spread

Chapter 3

Credit risk modelling

The pricing of the credit derivatives should aim to provide a fair value for the credit derivative instrument. To value credit derivatives it is necessary to be able to model credit risk. 2 used approaches to model credit risk are -

1. **Structural models or firm value models** - company defaults on its debt if the value of the assets of the company falls below a certain default point.

    - Fisher black and myron scholes - how equity owners hold a call option on the firm.
    - Black scholes merton model - robert merton extended the framework and analyzed risk debt behavior with the model.
    - Robert Geske model - extended the BSM model to include multiple debts.

2. **Reduced form models** - do not look inside the firm, they model directly the likelihood of default or downgrade

    - Jarrow-Turnbull

- Duffie-singleton

3.1 Complexities in credit risk modelling

Default risk - It is a result of an inability to pay for corporate debtors. It is a very rare event. It is not an universal concept.

Default data are considerably less in comparison to the data available for the modelling of interest rate risk where time series data of U.S treasury prices are available on a daily basis. Moreover the data collected collected regarding the default rates are not necessarily consistent with the definition of credit events for determining payout trigger for a credit default swap. These things makes it difficult to model credit risk.

- **Causes of defaulting**
  - *Microeconomic factor* : poor management
  - *Macroeconomic factors* : high interest rates and recession

3.2 Binomial default process

Branches leading to default result in contract termination and trigger a recovery payment. Conversely, branches leading to survival allow the contract to persist, facing potential defaults in the future. This serves as a broad framework for explaining the occurrence of defaults and contract termination. Different models vary in their definitions of default probabilities and the methods used to model recovery.

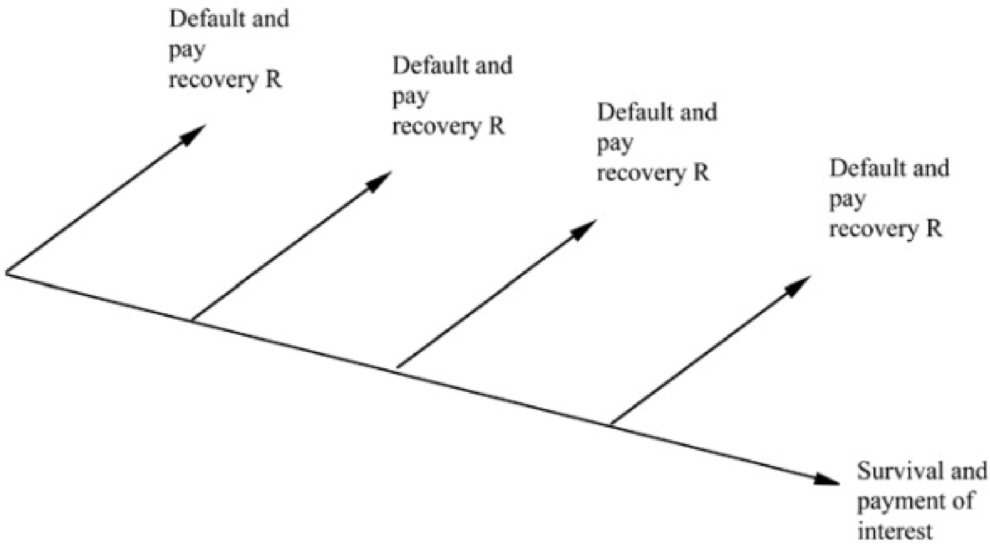

Figure 3.1: Binomial defaulting process for a Debt Instrument

Chapter 4

Structural modelling

The basic idea, common to all the structural models, is that a company defaults on its debt if the value of the assets of the company falls below a certain default point.For this reason they are also known as firm value models. They are characterised by modelling the firm's value in order to provide the probability of corporate defaults. In these models it has been demonstrated that default can be modeled as an option and, as a result, researchers were able to apply the same principles used for option pricing to the valuation of risky corporate securities. The application of option pricing theory avoids the use of risk premium and tries to use other marketable securities. In these models, default is endogenous. The computation of debt values of different maturities are dependent. Defaults of the later maturity debts are contingent claim on defaults of earlier maturity debts.

4.1 The Black-Scholes-Merton model

Default event occurs predictably when a firm has insufficient assets to pay its debt. Corporate liabilities can be viewed as a *covered call*. It can be viewed in a way where company has only one zero-coupon debt, at the maturity of the debt, the *debt holder* either gets paid the face value of the debt - in such a case, the ownership of the company is transferred to the equity holders or debt holder takes control of the company - in such a case, the equity holders receives nothing. The *debt holder* of the company is therefore a subject to a default risk for he/she may not be able to receive the face value of his/her investment.Black Scholes Merton (BSM model) effectively turned a risky debt evaluation into a covered call evaluation whereby the option pricing formulas can readily apply. The notation used in modelling is described below.

E(t) : market value of the issued equity at time t.
D(t,T) : market value of the debt at time t Maturing at T.
T : maturity time of the debt.
K : face value of the debt maturing at time T.
A(t) : asset value of the firm at time t.

At time T, the market value of the issued equity of the company is the amount remaining after the debts have been paid out of the firm's asset; i.e.

$$E(T) = \max\{A(T) - K, 0\} \tag{4.1}$$

This payoff is similar to that of a call option on the value of the firm's asset struck at the face value of the debt. The holders of the debt gets paid either the face value = K, under no default or take over the firm = A, in case of default.

Hence the value of the debt on the maturity date is given by -

$$D(T,T) = min(A(T), K) \tag{4.2}$$
$$= A(T) - max(A(T) - K, 0) \tag{4.3}$$
$$= K - max(K - A(T), 0) \tag{4.4}$$

Equation 4.3 decomposes the risky debt into asset and a short call. This interpretation shows that the equity owners owns the call option of the company. If the company performs well, then equity holders should call the company.
Equation 4.4 decomposes the risky debt into a risk-free debt and a short put. The issuer(equity owners) can put the company back to the debt owner when the performance is bad. Hence the default risk is the put option.

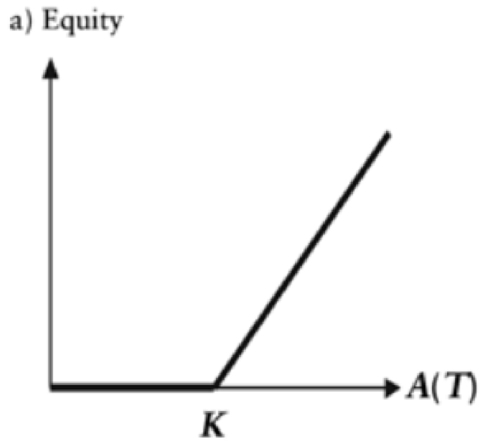

Figure 4.1: Payoff diagram at maturity for Equity

The value of the equity and debt when added together must equal the assets of the firm at all times, i.e

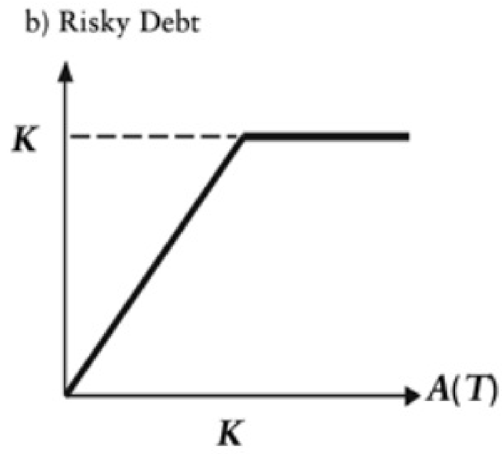

Figure 4.2: Payoff diagram at maturity for risky debt

Figure 4.3: Payoff diagram at maturity for risk free debt

$$A(T) = E(t) + D(t,T) \tag{4.5}$$

Clearly, at maturity, this is true as we have

$$E(T) + D(T,T) = \max\{A(T) - K, 0\} + \min\{A(T), K\} \tag{4.6}$$
$$= A(T) \tag{4.7}$$

Since any corporate debt is contingent (dependent) claim on the firm's future asset value at the time the debt matures, this is what we must model in order to capture the default. Firm's asset value evolve randomly. BSM assumed that the dynamics of the asset value follow a lognormal stochastic process of the form

$$\frac{dA(t)}{dt} = rdt + \sigma dW(t) \tag{4.8}$$

where :

$$r = \text{instantaneous risk-free rate (assumed constant)}$$
$$\sigma = \text{percentage volatility}$$
$$W(t) = \text{Wiener process under risk neutral measure}$$

This is same process as is generally assumed within equity markets for the evolution of stock prices.

- $A(t) \geq 0$
- $dA(t) \propto A(t)$

It is possible to use the option pricing equation developed by BSM to price risky corporate liabilities.

At t $= T$:

$$A(T) \geq K \implies \text{No default}$$

$$A(T) < K \implies \text{Default}$$

The probability of defaulting at maturity is

$$p = \int_{-\infty}^{K} \phi(A(T))\, dA(T) = 1 - N(d_2) \tag{4.9}$$

where :

$$\phi(.) = \text{log normal density function}$$
$$N(.) = \text{cumulative normal probability}$$
$$d_2 = \frac{\ln \frac{A(t)}{K} + (r - \frac{\sigma^2}{2})(T-t)}{\sigma\sqrt{T-t}}$$

$N(d_2)$ is the **survival probability**. Using BSM to find the current value of the equity. We have,

$$E(t) = A(t)N(d_1) - e^{-r(T-t)}KN(d_2) \tag{4.10}$$

where

$$d_1 = d_2 + \sigma\sqrt{T-t}$$

The current value of the debt is a covered call value :

$$
\begin{aligned}
D(t,T) &= A(t) - E(t) && (4.11)\\
&= A(t) - [A(t)N(d_1) - e^{-r(T-t)}KN(d_2)] && (4.12)\\
&= A(t)[1 - N(d_1)] + e^{-r(T-t)}KN(d_2) && (4.13)
\end{aligned}
$$

NOTE : first term in 4.13 is the ***recovery value*** whereas the second term is the ***present value of probability weighted face value of the debt***, it means that if default does not occur with probability $N(d_2)$. the debt owner receives the face value K. Since the probability is risk neutral, the probability-weighted value is discounted by the risk-free rate. The two values together make up the value of the debt.

Yield of the debt(y) is calculated by solving for y

$$D(t,T) = Ke^{-y(T-t})\tag{4.14}$$

$$y = \frac{\ln K - \ln D(t,T)}{T-t}\tag{4.15}$$

4.2 Advantages and drawbacks

Structural models, model default on the very reasonable assumption that it is a result of the value of the firm's asset falling below the value of its debt. In the case of the BSM model, the outputs of the model show how the credit risk of a corporate debt is a function of the leverage and the asset volatility of the issuer. However, they are difficult to calibrate and so are not suitable for the frequent marking to market of credit contingent securities.

Chapter 5

Reduced form modelling

These models are more recent. they do not look inside the firm. Instead they model the likelihood of default or downgrade. they are not limited to modelling current probability of default, some researchers have attempted to model a forward curve of default probabilities that can be used to price instruments of varying maturities. Modelling a probability has the effect that the default event is a random event which can suddenly occur at any instance making default a surprise event. They are mainly represented by **Jarrow-Turnbull** and **Duffie-Singleton**

Properties of these models :

- arbitrage free
- employ risk neutral measure to price securities
- default is exogenous
- The computations of debt values of different maturities are independent

5.1 The Poisson process

It is the theoretical framework for reduced form models.
Let the value of this process be N_t at time t and λ be the intensity parameter. The values taken by N_t are an increasing set of integers $0, 1, 2, ...$ and the the probability of a jump from one integer to the next occurring over a small interval dt is given by -

$$P(N_{t+dt} - N_t = 1) = \lambda dt$$

As dt is small, there is negligible probability of two jumps occurring in the same time interval. So we can write the probability of no event occurring in the same time interval can be given by -

$$P(N_{t+dt} - N_t = 0) = 1 - \lambda dt$$

When the Poisson process jump from 0 to 1 it is viewed as default in reduced form models. Time taken till first default event will give **default time distribution**. Let T_0 be the time until the first event occurs , then T_0 follows an exponential distribution with the same parameter as the original Poisson process. So the probability that there is no default event from current time t to some future time T_0 is given by

$$P(T_0 > t) = e^{-\lambda(T_0 - t)}$$

It can also be viewed as survival probability at time t till T_0 :

$$Q(t, T_0) = P(T_0 > t) = e^{-\lambda(T_0 - t)}$$

5.2 The Jarrow-Turnbull model

This model is a simple model of default and recovery based on the Poisson default process.

Assumptions :

- The recovery payment will be paid at maturity T, no matter when the default occurs in case of defaults.
- An arbitrary constant recovery value is assumed

The coupon bond value is given as :

$$\begin{aligned} B(t) &= p(t,T)R(T)\int_t^T -dQ(t,u)du + \sum_{j=1}^{n} P(t,T_j)c_j e^{-\lambda(T_j-t)} \\ &= P(t,T)R(T)(1-e^{-\lambda(T-t)}) + \sum_{j=1}^{n} P(t,T_j)c_j e^{-\lambda(T_j-t)} \end{aligned}$$

where :

$$\begin{aligned} P(t,T) &= \text{risk-free discount factor} \\ c_j &= \text{j-th coupon} \\ Q(t,u) &= \text{survival probability at time t till time u} \\ R &= \text{recovery ratio} \end{aligned}$$

if R = 0 , zero coupon bond with face value = \$1 then :

$$B(t) = P(t,T)e^{-\lambda(T-t)}$$

this equation is comparable with one-period binomial model with bond's forward yield spread y

$$D(t,T) = P(t,T)e^{-y(T-t)}$$

Therefore intensity parameter λ is the bond's forward yield spread in this case. D(t,T) is known as the **risky discount factor**, which is the present value of \$1 if there is no default.

If λ is a function of time and the recovery is paid at the time of default then

$$\begin{aligned} B(t) &= \int_t^T p(t,u)R(u)(-dQ(u)) + \sum_{j=1}^{n} P(t,T_j)c_j Q(t,T_j) \\ &= \int_t^T P(t,u)R(u)\lambda(u)e^{-\int_t^u \lambda(w)dw} + \sum_{j=1}^{n} P(t,T_j)c_j e^{-\int_t^{T_j} \lambda(w)dw} \end{aligned}$$

To actually implement this equation, we take λ to be constant in adjacent time points. If we further assume that default can occur only at coupon times then

$$B(t) = \sum_{j=1}^{n} P(t,T_j)R(T_j)\lambda(T_j)e^{-\sum_{k=1}^{j} \lambda(T_k)} + \sum_{j=1}^{n} P(t,T_j)c_j e^{-\sum_{k=1}^{n} \lambda(T_k)}$$

Since a debt contract pays interest under survival and pays recovery upon default, the expected payment is naturally the weighted average of the two payoffs.

From now on let the survival probability till any future time t be $Q(0,t)$. Therefore $Q(0,s) - Q(0,t)$ where $s > t$ is the default probability between two future time points t and s. Let $Q(t,T)$ represent the survival probability from the current time t until some future time T. The difference $Q(t,T) - Q(t,T+\tau)$ corresponds to the default probability between T and $T+\tau$—that is, the likelihood of surviving until T but defaulting at $T+\tau$. Assuming defaults can only occur at discrete time points $T_1, T_2, \ldots, T_n$, the cumulative probability of default over the lifespan of the credit default swap is the sum of all individual per-period default probabilities.

$$\sum_{j=0}^{n} Q(t,T_j) - Q(t,T_{j+1}) = 1 - Q(T_n) = 1 - Q(T)$$

where $t = T_0 < T_1 < T_2 < ... < T_n = T$ and T is the maturity time of the credit default swap.

A \$1 *risky* cash flow received at time T has a risk-neutral expected value of Q(t,T) and a present value of P(t,T)Q(t,T) where P is the risk-free discount factor. A "risky" annuity of \$1 can therefore be written a

$$\sum_{j=1}^{n} P(t,T_j)Q(t,T_j)$$

This value represents the expected receipt of \$1 until default. A *risky* bond with no recovery upon default and a maturity of n can thus be written as

$$B(t) = \sum_{j=1}^{n} P(t,T_j)Q(t,T_j) + P(t,T_n)Q(t,T_n)$$

This outcome bears resemblance to the risk-free coupon bond, wherein solely risk-free discount factors are employed. Because of this similarity, we can consider PQ as the discount factor associated with risk.

The *forward default probability* refers to the likelihood of default during a forward interval, contingent upon surviving until the commencement of that interval. This probability can be expressed as

$$p(T_j) = \frac{Q(t, T_{j-1} - Q(t, T_j)}{Q(t, T_{j-1)}}$$

5.2.1 Advantages and drawbacks

The primary strength of the Jarrow-Turnbull model lies in its calibration capability. By externally defining default probabilities and recovery rates, it becomes possible to employ a set of risky zero-coupon bonds for the calibration process. Through this approach, it becomes feasible to derive a default probability curve and, consequently, a spread curve. In this model as we assume the recovery rate, being an exogenously specified percentage of the default-free payoff, may actually exceed the price of the bond at the moment of default.

5.3 The Duffie-Singleton model

This model is also called as fractional recovery model

Assumptions :

- This model allows the payment of recovery to occur at any time.
- the amount of recovery is restricted to be the proportional of the bond price at default time.

$$R(t) = \delta D(t, T)$$

where :

$$R(t) = \text{recovery ratio}$$
$$\delta = \text{fixed ratio}$$
$$D(t,T) = \text{market value of value if default did not occur}$$

The underlying reasoning for this methodology lies in the observation that as the creditworthiness of a bond diminishes, its price tends to decrease. In the event of default, the recovery price is determined as a fraction of the final price just before default. This approach is adopted to circumvent a potential inconsistency seen in the Jarrow-Turnbull model, where the recovery rate, defined as an externally specified percentage of the default-free payoff, could paradoxically surpass the bond's price at the time of default.

The debt value at time t is given by

$$D(t,T) = \frac{1}{1+r\Delta t}\{p\delta E[D(t+\Delta t,T)] + (1-p)E[D(t+\Delta t,T)]\}$$

By recursive substitution, current value of the debt as its terminal payoff if no default occurs :

$$D(t,T) = \left[\frac{1-p\Delta t(1-\delta)}{1+r\Delta t}\right]^n X(T)$$

The instantaneous default probability being $p\Delta t$ is consistent with Poisson distribution,

$$-\frac{dQ}{Q} = p\Delta t$$
$$\Delta t = T/n$$

$$D(t,T) = \frac{exp(-p(1-\delta)T)}{exp(rT)}X(T) = exp(-(r+s)T)X(T) \tag{5.1}$$

when r and s are not constant, 5.1 can be written as follows,

$$D(t,T) = E_t\left[exp\left(\int_t^T [r(u)+s(u)]du\right)\right]X(T)$$

where $s(u) = p_u(1-\delta)$

Intuitive interpretation : $p(1-\delta)$ serves as spread over the risk-free discount rate.

- p is small $\implies$ credit spread is small
- R(t) is high (i.e 1-δ is small) $\implies$ credit spread is small

5.3.1 Advantages and drawbacks

- If contract has no payoff at maturity such as CDS then it implies zero value today which is not true.
- While we can calculate the spreads, we cannot separate the recovery from the default probability.

5.4 Advantages and drawbacks

The rapid calibration capability is the primary factor that makes reduced form models highly preferred by practitioners in the real-world credit derivatives markets. These models face the limitation that default is assumed to always be an unexpected event. Although this holds true in rare instances, data from both Moody's and Standard & Poor's indicates that defaults directly from investment-grade quality bonds are quite infrequent. Typically, default is a culmination of a series of downgrades and spread widenings, allowing for a certain level of anticipation.

Chapter 6

Credit default swaps

Credit default swaps (CDS) are financial derivatives that allow an investor to swap or offset their credit risk with that of another investor. These instruments provide a form of insurance against the default of a borrower or issuer of debt.

Parties Involved :

- **Protection Buyer:** This party pays a periodic fee (premium) to the protection seller.
- **Protection Seller :** This party receives the premium and, in return, provides insurance against the default of a particular borrower or issuer.

Reference Entity or Obligation:

The agreement specifies a reference entity, which is usually the entity whose default could trigger the CDS. Alternatively, a specific debt obligation (reference obligation) can be designated.

Premiums and Payments :

- The protection buyer pays regular premiums to the protection seller over the life of the contract.
- If a credit event, such as a default, occurs with the reference entity or obligation, the protection seller is obligated to make a payment to the protection buyer.

Credit Events :

Credit events triggering payments may include default, bankruptcy, or other specified events.

Termination and Settlement :

- The CDS terminates when the contract expires, the reference entity undergoes a credit event, or other predefined conditions are met.
- Settlement may involve a physical delivery of the defaulted debt or a cash settlement based on the market value of the defaulted debt.

6.1 Introduction

In the context of a credit default swap, the documentation specifies either the reference entity or the reference obligation. The reference entity refers to the issuer of the debt instrument, which can be a corporation, a sovereign government, or a bank loan. When a reference entity is identified, the party involved in the credit default swap holds the option to deliver one of the issuer's obligations, adhering to predetermined constraints. For instance, if the reference entity is Ford Motor Credit Company, any acceptable senior bond issue from that issuer can be delivered. On the other hand, a reference obligation is a specific obligation for which protection is being sought.

Within a credit default swap arrangement, the protection buyer remits a fee, known as the swap premium, to the protection seller. This payment grants the protection buyer the entitlement to receive a payment contingent upon the default of either the reference obligation or the reference entity. The sum of payments made by the protection buyer is collectively referred to as the *premium leg*, while the potential contingent payment that the protection seller might have to make is termed the *protection leg*.

- **premium leg** : Collectively, the payments made by the protection buyer
- **protection leg** : the contingent payment that might have to be made by the protection seller

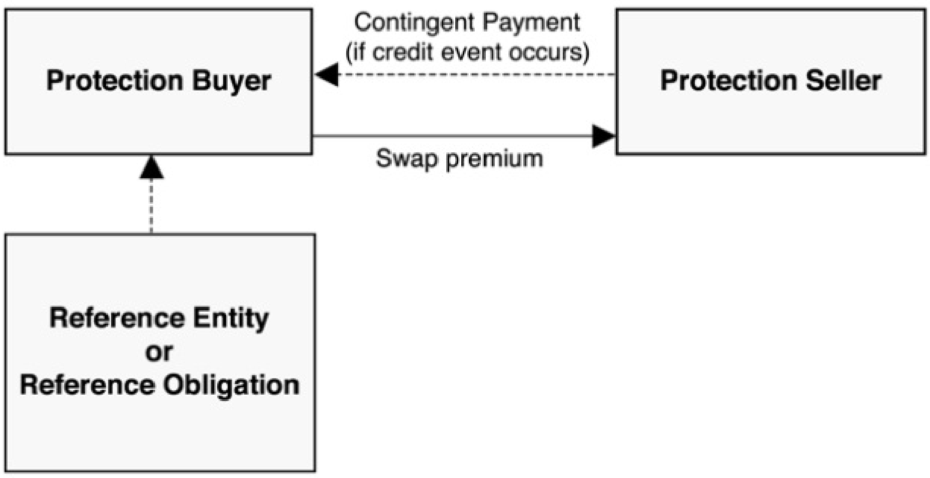

Figure 6.1: Credit Default Swap

Credit default swaps can be classified as follows :

- Single name credit default swaps

- Basket swaps

6.2 Single name credit default swaps

Single-Name Credit Default Swaps (CDS) are financial derivatives that provide a means for investors to manage and transfer credit risk associated with a specific individual or entity. These instruments are distinct from standard (multi-name) CDS as they focus on the credit risk of a single reference entity.

Key Features :

- **Reference Entity:** In a Single-Name CDS, the reference entity is the sole focus, and the contract is tailored to cover the credit events associated with that specific entity.
- **Credit Events:** Similar to standard CDS, credit events triggering payments may include default, bankruptcy, or other predefined events related to the specified reference entity.
- **Premiums and Payments:** The protection buyer pays premiums to the protection seller over the contract's duration. If a credit event occurs, the protection seller is obliged to make a payment to the protection buyer.
- **Termination and Settlement:** The Single-Name CDS terminates based on predefined conditions, and settlement may involve either physical delivery of the defaulted debt or a cash settlement.

Single-Name CDS provide a focused and justifiable approach to managing credit risk for investors, allowing them to specifically hedge or speculate on the creditworthiness of individual entities.

The tenor of a swap is referred to as the **scheduled term** because a credit event will result in a payment by the protection seller, resulting in the credit default swap being terminated.

Terminologies used :

- **trade date** is the date the parties enter into the credit derivative transaction.
- **effective date** is the date when contractual payments begin to accrue.
- **scheduled termination date** is a date specified by the parties in the contract.
- **termination date** under the contract is the earlier of the scheduled termination date or a date upon which a credit event occurs and notice is provided.

Calculation of the swap premium

$$\text{Quarterly swap premium payment} = \frac{\text{Notional amount} \times \text{Annual rate} \times \text{no. of days in quarter}}{360}$$

6.3 Basket default swaps

A Basket Default Swap (BDS) is a financial derivative that allows investors to manage and transfer credit risk associated with a portfolio, or basket, of reference entities. Unlike single-name credit default swaps, which focus on the credit risk of a specific entity, a Basket Default Swap aggregates multiple reference entities into a single contract.

Key Features:

- **Reference Basket:** The BDS covers a predefined portfolio of reference entities. Credit events triggering payments may include defaults, bankruptcies, or other predefined events related to any entity within the specified basket.
- **Diversification:** By including multiple entities in the basket, BDS provides a level of diversification, spreading the credit risk across different issuers. This can help mitigate the impact of a single entity's default on the overall portfolio.
- **Premiums and Payments:** Similar to single-name CDS, the protection buyer pays premiums to the protection seller over the contract's duration. If a credit event occurs for any entity in the basket, the protection seller is obliged to make a payment to the protection buyer.
- **Termination and Settlement:** The BDS terminates based on predefined conditions. Settlement may involve either physical delivery of the defaulted debt from entities in the basket or a cash settlement based on the overall creditworthiness of the basket.

Basket Default Swaps offer a flexible approach to managing credit risk for investors with exposure to multiple entities, providing a consolidated instrument for hedging or speculating on the creditworthiness of a portfolio.

They are classified as follows :

- N^{th} to default swaps
- Subordinate basket Default swaps
- Senior basket default swaps

N^{th} to default swap

An Nth to Default Swap is a financial derivative tailored for managing credit risk associated with a specific portfolio, where payment is triggered upon the occurrence of the Nth default within the reference entities. In contrast to standard credit derivatives, Nth to Default Swaps focus on a predefined number of defaults within the portfolio rather than individual entities. Investors use Nth to Default Swaps for targeted risk management, allowing them to specify the number of defaults that trigger payment. This derivative involves premiums paid by the protection buyer to the protection seller over the contract's term, with the protection seller obligated to make payments upon the Nth default event. Nth to Default Swaps provide a justifiable approach for hedging or speculating on the creditworthiness of a portfolio based on the occurrence of a specified number of defaults.

Subordinate basket Default swaps

A Subordinate Basket Default Swap (Sub BDS) is a financial derivative tailored to manage credit risk associated with a specified portfolio or basket of subordinate debt obligations issued by various entities. In contrast to Senior Basket Default Swaps (SBDS), Sub BDS focuses on the subordinate tranches within the capital structure of the reference entities. This derivative offers investors a targeted approach to diversify and mitigate credit risk specifically within the lower-ranking tiers of debt. Sub BDS involves premiums paid by the protection buyer to the protection seller, and in the event of a credit event impacting any subordinate debt in the basket, the protection seller is obliged to make payments to the protection buyer. Sub BDS terminates based on predefined conditions, providing a flexible tool for hedging or speculating on the creditworthiness of subordinate debt portfolios.

Senior basket default swaps

A Senior Basket Default Swap (SBDS) is a financial derivative designed for managing credit risk associated with a predefined portfolio of senior debt obligations from various entities. Unlike broader credit derivatives, SBDS specifically addresses credit events related to the senior tranches of debt within the capital structure of the reference entities. Investors utilize SBDS for diversification benefits, spreading credit risk across multiple issuers and mitigating the impact of a single issuer's default. The derivative involves premiums paid by the protection buyer to the protection seller over the contract's duration, with the protection seller obligated to make payments if a credit event occurs. SBDS terminates based on predefined conditions, offering a flexible tool for hedging or speculating on the creditworthiness of senior debt portfolios.

6.4 Prcing single name CDS

We examine the straightforward structure, where a protection buyer makes regular premium payments to a protection seller until the maturity date of the Credit Default Swap (CDS), unless a credit event prompts the termination of the CDS. In the event of termination, a contingent payment is made from the protection seller to the protection buyer. We assume that there is no counter party default risk. Default probabilities, interest rates and recovery rates are independent.

A Credit Default Swap (CDS) is comprised of two components: one associated with the regular premium payments, and the other linked to the contingent default payment. The present value of a default swap can be conceptualized as the algebraic sum of the present values of these two components.

Typically, the default payment on a Credit Default Swap (CDS) will be $1-\delta$ times its notional amount, where δ represents the recovery rate of the reference security. This payout rationale is evident—it enables the conversion of a risky asset into a risk-free asset by acquiring default protection associated with this credit. For instance, if the anticipated recovery rate for a specific reference asset is 30% of its face value, the protection seller would pay the remaining 70% in the event of default.

A credit default swap takes the defaulted bond as the recovery value and pays par upon default and zero otherwise.

$$V = E\left[e^{-\int_0^u r(s)ds} 1_{u<T}[1-R(u)]\right]$$

where u is the default time.

Therefore the value of the credit default swap (V) should be the recovery value upon default weighted by the default probability.

$$V = \sum_{j=1}^{n} P(t, T_j)\left[Q(t, T_{j-1}) - Qt, T_j)\right]\left[1 - R(T_j)\right]$$

where P(.) is the risk free discount factor and R(.) is the recovery rate.

In **??** it is implicitly assumed that the discount factor is independent of the survival probability.

spread(s) which is paid until default or maturity is given by

$$s = \frac{V}{\sum_{j=1}^{n} P(t, T_j) Q(t, T_j)}$$

Chapter 7

Jargons

- **Reference entity** : The borrower who needs credit / an issuer
- **Reference asset** : reference obligation
- **Protection buyer** : The lender who needs insurance for the loan
- **Protection seller/counter party** : The third party or the guarantor who wants to profit by covering the risk of a loan default.
- **Covered call** : own the asset but short a call option. is a combination of a selling call option and owning the same face value of the shares, which might have to be delivered should the option expire in the money. If the option expires in the money, a net profit equal to the strike is made. If the option expires worthless, then the position is worth the stock price.
- **Debt holder** : the investor/creditor who lends the money based on an agreement to be paid back more than what is lend today.

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