Unit-4
Electromagnetism

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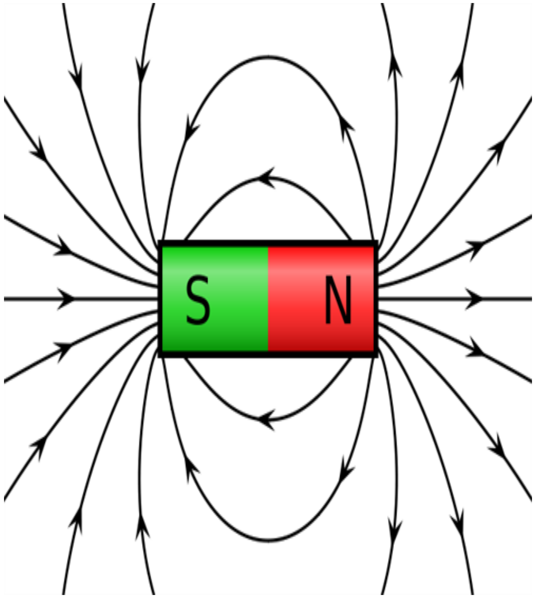
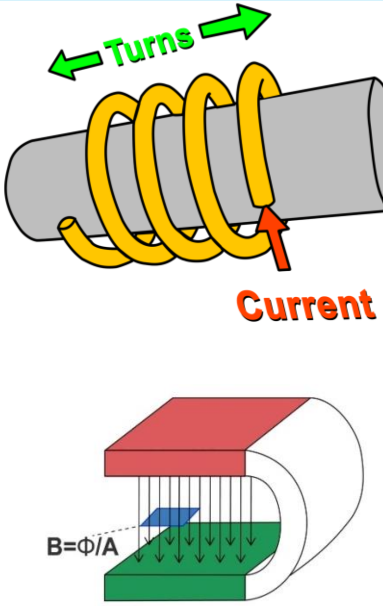
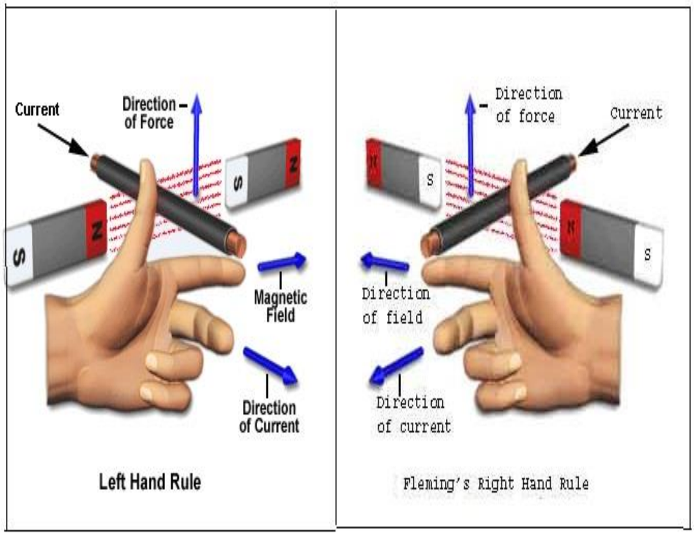




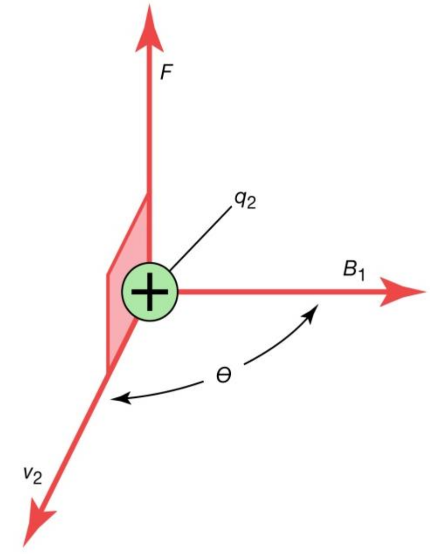


Content

Magnetic Force

- The magnetic force between two moving charges may be described as the effect exerted upon either charge by a magnetic field created by the other.
- Electric forces exist among stationary electric charges; both electric and magnetic forces exist among moving electric charges.

- Magnetic force F is perpendicular to the plane of velocity v2 of charge q2 and magnetic field B1



Coulomb's Law Of Magnetism

1. *Like poles repel each other while unlike poles attract each other.*

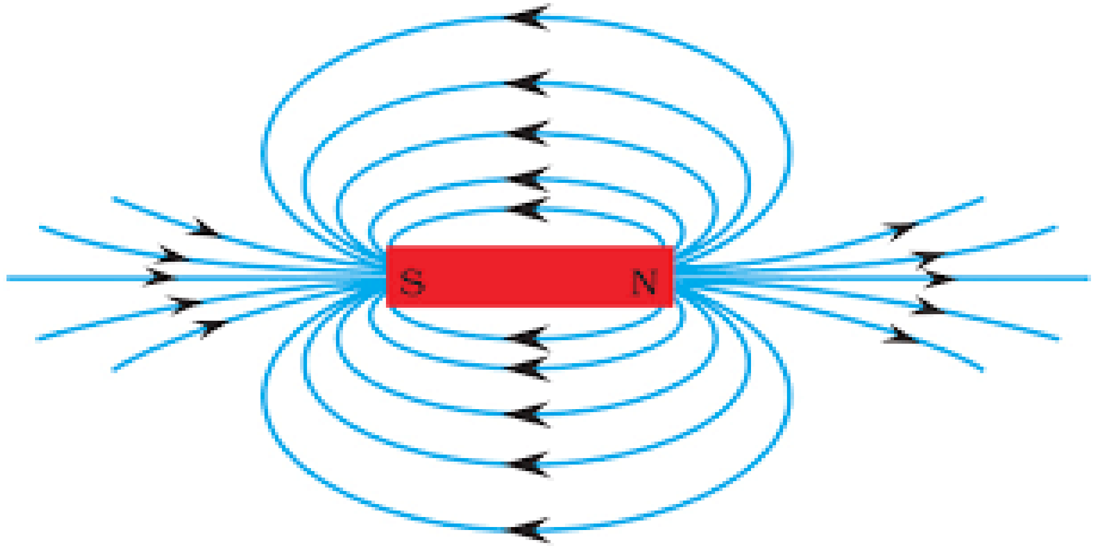
2. *The force between two magnetic poles is directly proportional to the product of their pole strengths and inversely proportional to the square of distance between their centers.*

$$F \propto \frac{m_1 m_2}{d^2}$$

$$= K \frac{m_1 m_2}{d^2}$$

Magnetic Field

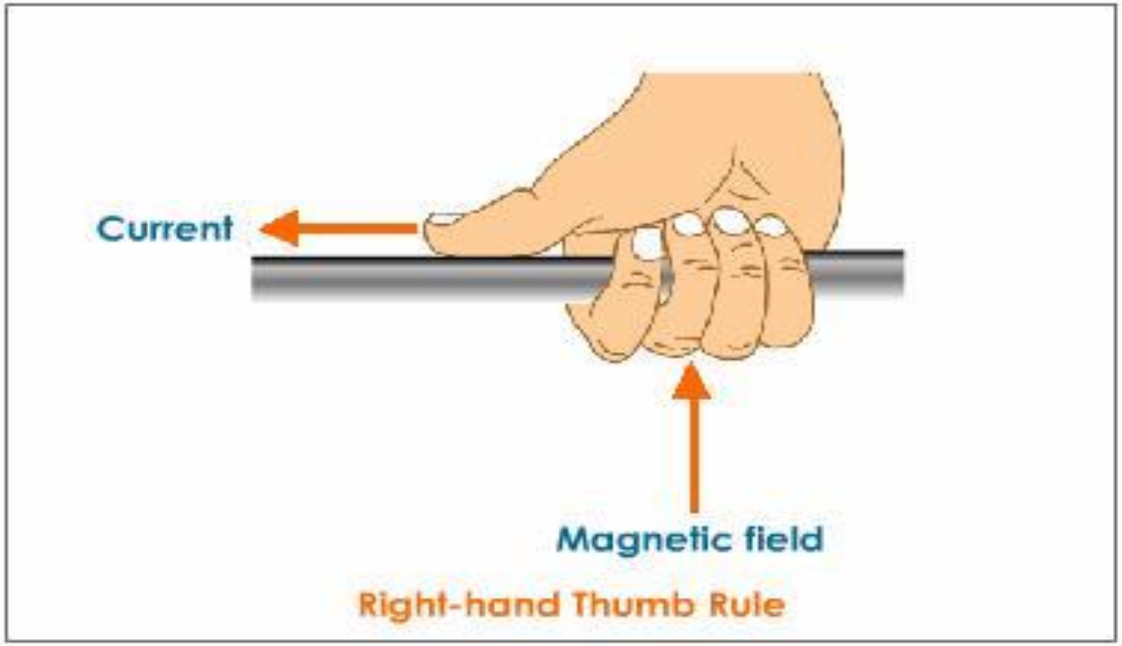
- The area around a magnet where the effect of the magnetic force produced by the magnet can be detected is called magnetic field.
- The magnetic field are shown in figure as lines in space.



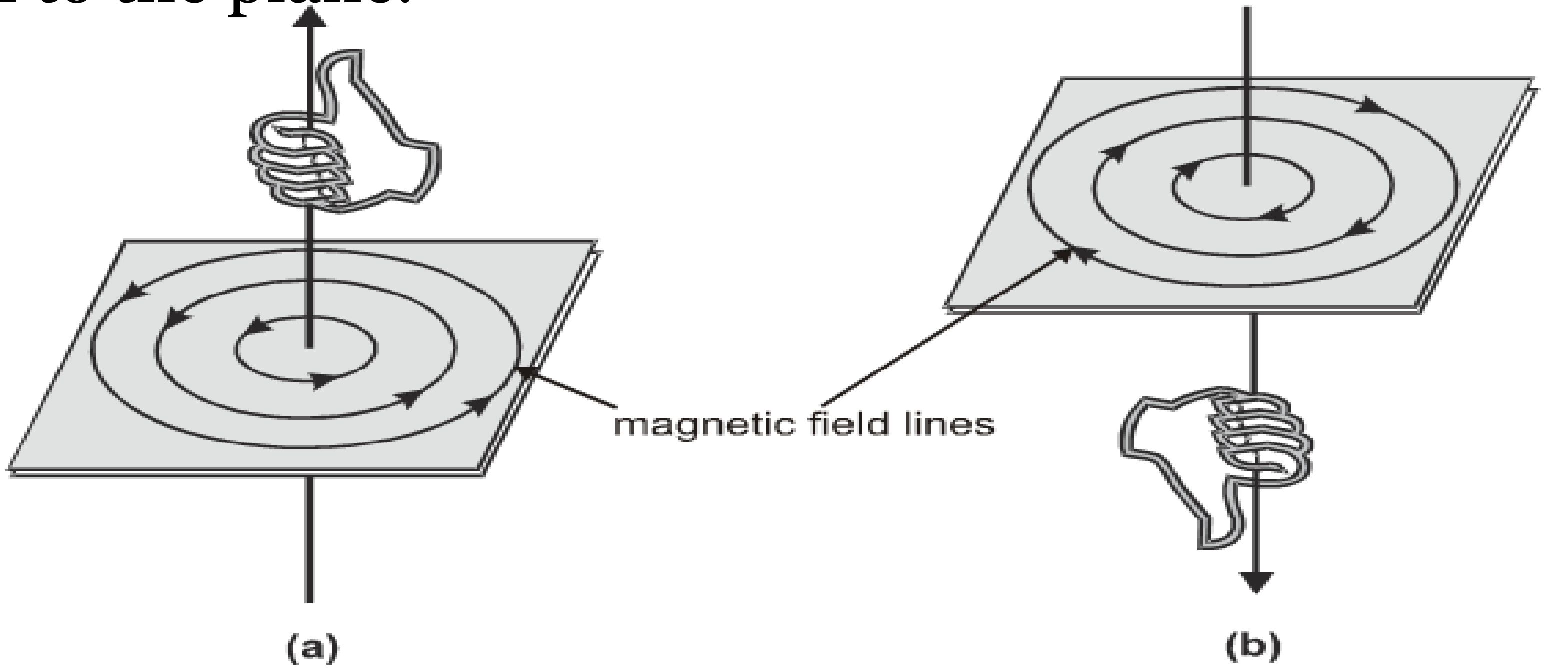
- Video: https://youtu.be/yXCeuSiTOug

Direction of Magnetic Field

- RIGHT HAND THUMB RULE:-
- Hold the conductor in right hand such that the thumb points the direction of current flow, then the bent fingers show the direction of magnetic field.



- When the conductor is placed perpendicular to the plane of the paper then fig (a) shows that the current is coming out and fig (b) shows that the current is flowing in to the plane.



In figure, the current comes out of the plane of the paper, the direction of the field is anti-clockwise

In figure, the current flows into the plane of paper and the direction of the field is clockwise

Magnetic Flux

- The total number of magnetic field lines of force in magnetic field is called magnetic flux.

- It is denoted by ø

- The unit of magnetic flux is **Weber (Wb)**.

$$
\begin{aligned}
1 \text{ Weber} &= 10^8 \text{ magnetic lines} \\
&= 10^8 \text{ Maxwells}
\end{aligned}
$$

- Characteristic of magnetic flux lines:-

1. They have no physical existence.
2. They form closed paths.
3. They never intersect each-other.
4. Lines of magnetic flux closer to each-other and having the same direction repel each-other.
5. Lines of magnetic flux closer to each-other and having the opposite direction attract each-other.
6. They exert lateral pressure.

Magnetic Flux Density

- The magnetic flux density is defined as the magnetic flux passing normally per unit area.
- It is denoted by '**B**'
- The unit of magnetic flux density is Wb/m^2 or Tesla.
- Mathematically,

$$\boxed{B = \frac{\phi}{A}}$$

Where, ϕ = flux in Wb

A = area of cross-section in m^2

Magneto Motive Force

- Magneto motive force is defined as the force that tends to establish the flux through a magnetic circuit.
- It is equal to the product of the current (I) flowing the coil and the number of turns (N) of the coil.
- Thus,

$$\boxed{m.m.f. = NI}$$

- Its unit is **ampere-turns** or **AT** but it is taken as **ampere (A)** since N is dimensionless.
- The work done in carrying a unit magnetic pole (1 Wb) once through the entire magnetic circuit is called magneto motive force.

Magnetic Intensity

- Magnetic field intensity is defined as a the magneto motive force per unit length of the magnetic flux path.
- It is denoted by ‘**H**’
- Its unit is **AT/m**

$$\boxed{\begin{aligned} H &= mmf/L \\ &= NI/L \end{aligned}}$$

Permeability

- It is the ability of a magnetic material to create the magnetic flux through it.
- The greater the permeability of a material, the greater is its conductivity for magnetic flux and vice-versa.
- Mathematically, it is the product of permeability of a vacuum and relative permeability of a magnetic material.

$$\boxed{\mu = \mu_o \mu_r}$$

Where, $\mu_0 = 4\pi \times 10^{-7}$ H/m

Relative Permeability

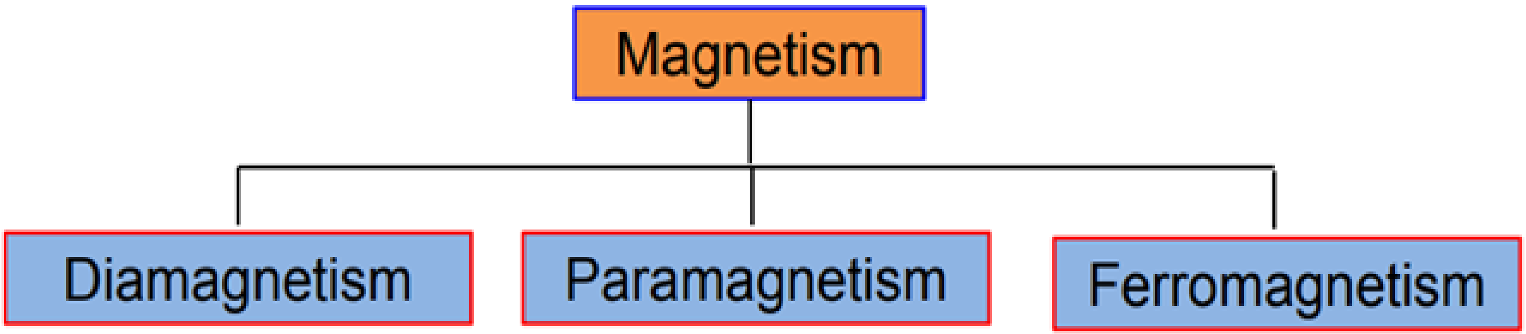
- The relative permeability of a medium is defied as the ratio of flux density produced in the material to the flux density produced in vacuum(or air) by the same magnetic field intensity.
- It is denoted by μ_r
- Mathematically,

$$\boxed{\mu_r = \frac{\text{Flux density in the medium}}{\text{Flux density in vacuum}}}$$

For air or non-magnetic material $\mu_r = 1$

Magnetism

- Magnetism is a property of atoms that produces a field which causes a force that attracts or repels other objects.
- Depending on the existence and alignment of magnetic moments with or without application of magnetic field, three types of magnetism can be defined.



Classification of Magnetic Material

- Magnetic materials are classified according to their relative permeability

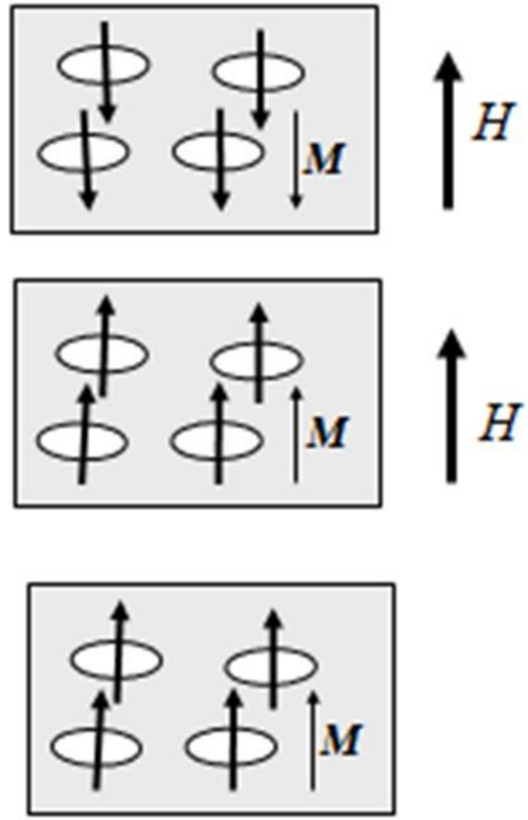
FERROMAGNETIC MATERIAL	PARAMAGNETIC MATERIAL	DIAMAGNETIC MATERIAL
Relative permeability is greater than unity and dependent on field strength.	Relative permeability is slightly greater than unity	Relative permeability is slightly less than unity
They are easily magnetized.	They are slightly magnetized.	Have the opposite effect of paramagnetic materials.
Ex: Iron, Cobalt, Nickel	Ex: Aluminum, Platinum	Ex: Silver, copper, hydrogen

Summary of magnetic responses:

diamagnetic
(opposes H)

paramagnetic
(aligns with H)

ferromagnetic
(even without H!)



$\chi \ll 1$, **negative**

$\chi \ll 1$, **positive**

$\chi > 1$, **positive**

Reluctance

- It is defined as the opposition offered by a magnetic circuit to the establishment of magnetic flux.

- Mathematically, it is directly proportional to the length of the magnetic circuit and inversely proportional to the area of cross-section of the magnetic path.

$$S = \frac{1}{\mu_0 \mu r} \frac{l}{A}$$

- Its unit is AT/Wb

Permeance

- Permeance is defined as the reciprocal of reluctance.
- Thus it is a measure of ease with which flux can be set up in the magnetic material.
- It is denoted by Δ
- Its unit is Wb/AT.

$$\boxed{\text{Permeance} = \frac{1}{\text{reluctance}}}$$

- It is analogous to conductance in an electric circuit.

Reluctivity

- Reluctivity or specific reluctance is defined as the reluctance offered by a magnetic circuit of a unit length and unit cross-section.

We know that the reluctance is given by

$$S = \frac{1}{\mu} \times \frac{l}{A}$$

When $l = 1$ m and $A = 1$ m^2 we have

$$\therefore \quad S = \frac{1}{\mu} \times \frac{1}{1} = \frac{1}{\mu}$$

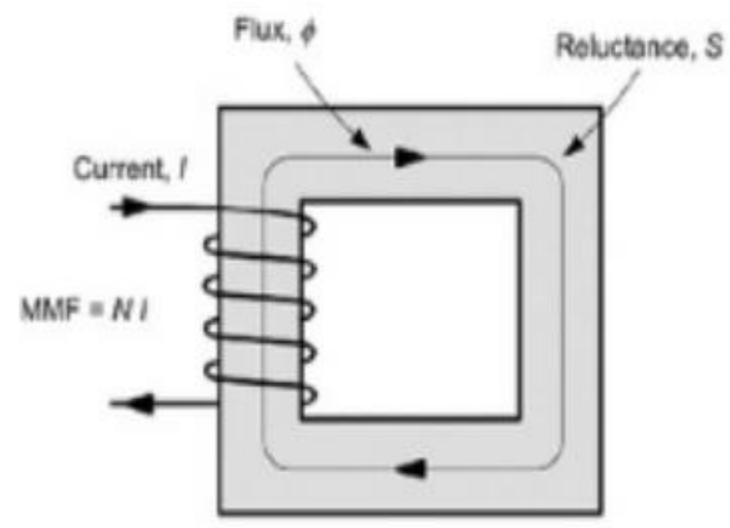
$$\therefore \text{ Specific reluctance or reluctivity } = \frac{1}{\mu} = \frac{1}{\text{absolute permeability}}$$

- Thus, reluctivity is equal to the reciprocal of the absolute permeability. Its unit is meter/Henry.
- It is analogous to resistivity in an electric circuit.

Magnetic Circuit

- A **magnetic circuit** is considered as the path in space through which magnetic flux passes.
- The figure below shows an iron-cored solenoid.
- When we supply DC through the solenoid, it will produce flux. Each flux line, will be initiated from N pole, passing through the air surrounding the magnet and finally reached to S pole and then from S pole it will come to the N pole through the iron core as shown.
- As each of the flux lines pass through the air as well as iron, this is called composite magnetic circuit.

- The lines of force inside the iron core are represented by numbers of uniformly spaced, paralleled lines.
- As a result the magnetic field within the iron core is uniform.



Analysis of simple magnetic circuit

- Consider a circular solenoid or a toroidal ring having N turns wound on an iron core as shown in figure. When current I amperes is flowing, magnetic flux is set up in the core

Let

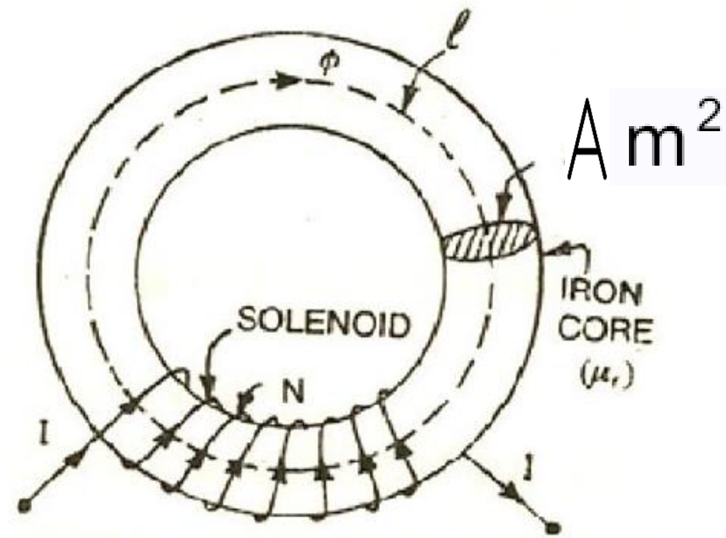
l = mean length of magnetic path in m

A = area of cross section of core in m^2

μ_r = relative permeability of core material

According to the definition of field strength.

$$H = \frac{NI}{l}$$



- Flux density in the core material is given by:

$$B = \frac{\phi}{A} \text{ webers/m}^2$$

$$B = (\mu_r \mu_0) H$$
$$= (\mu_r \mu_0) \frac{NI}{l}$$

Total flux produced

$$\Phi = B \times A = (\mu_r \mu_0) \frac{NI}{l} A$$
$$= \frac{NI}{l / (\mu_r \mu_0) A}$$
$$\Phi = \frac{NI}{S}$$

$$\text{Flux} = \frac{\text{mmf}}{\text{reluctance}}$$

It may be noted that the above expression has a strong resemblance to Ohm's law for electric circuit given below.

$$\text{Current} = \frac{\text{e.m.f.}}{\text{resistance}}$$

The m.m.f. is analogous to e.m.f. in electric circuit, reluctance is analogous to resistance and flux is analogous to current. Because of this similarity, the above expression i.e.

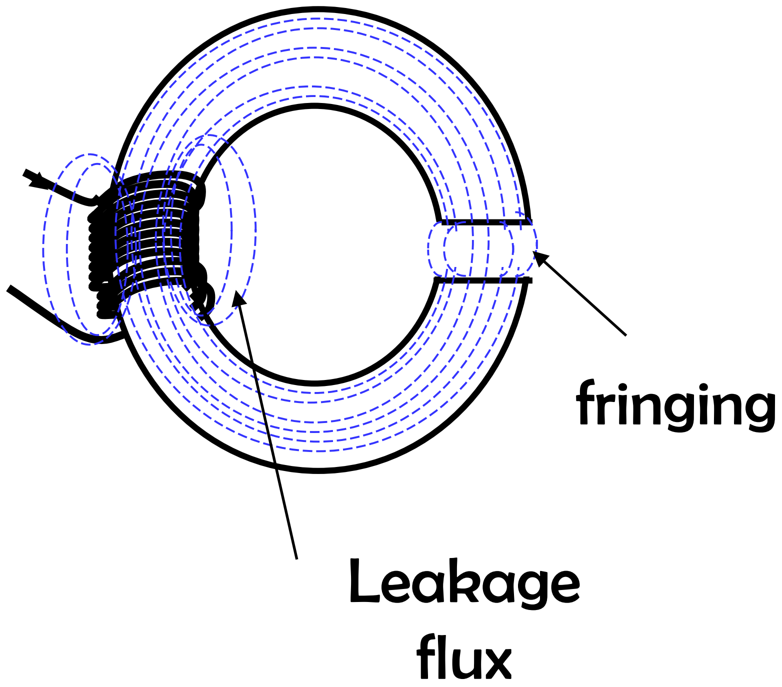
$$\boxed{\text{flux} = \frac{\text{m.m.f.}}{\text{reluctance}}}$$

is also known as "Ohm's law of magnetic circuit."

Leakage Flux

- **Leakage flux** is defined as the magnetic flux which does not follow the particularly intended path in a magnetic circuit.

- In other words, the type of flux which is not used for any work is called **Leakage Flux** and is denoted by φ_l.



Leakage Coefficient

- The total flux Φ produced by the solenoid in the magnetic circuit is the sum of the leakage flux and the useful flux and is given by the equation shown below:

$$\boxed{\varphi = \varphi_u + \varphi_l}$$

- The ratio of the total flux produced to the useful flux set up in the air gap of the magnetic circuit is called a **leakage coefficient or leakage factor**. It is denoted by (λ).

$$\boxed{\lambda = \frac{\varphi}{\varphi_u}}$$

Fringing Effect

- The useful flux when sets up in the air gap, it tends to bulge outward as shown in above figure, because of this bulging, the effective area of the air gap increases and the flux density of the air gap decreases. This effect is known as **Fringing.**

- Fringing is directly proportional to the length of the air gap that means if the length increases the fringing effect will also be more and vice versa.

Similarities b/w Magnetic And Electric Circuit

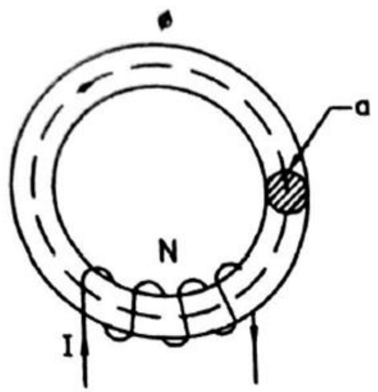


FIG. 3.18

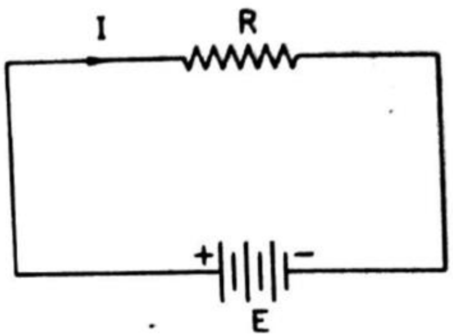


FIG. 3.19

Similarities

Magnetic circuit	Electric circuit
1. The closed path for magnetic flux is called magnetic circuit.	1. The closed path for electric current is called electric circuit.
2. $\text{Flux} = \dfrac{\text{m.m.f}}{\text{Reluctance}}$	2. $\text{Current} = \dfrac{\text{e.m.f}}{\text{resistance}}$

3.	Flux, ϕ in Weber	3.	Current, I in ampere
4.	m.m.f. in Amp Turn (AT)	4.	e.m.f. in volts (V)
5.	Reluctance $S = \dfrac{l}{\mu_o \mu_r A}$ AT / Wb	5.	Resistance $R = \varrho \dfrac{l}{a}$ ohms
6.	Permeance $= \dfrac{1}{\text{Reluctance}}$	6.	Conductance $= \dfrac{1}{\text{resistance}}$
7.	Permeability μ	7.	Conductivity $\sigma = \dfrac{1}{\varrho}$
8.	Reluctivity	8.	Resistivity
9.	Flux–density, $B = \dfrac{\phi}{A}$ Wb / m^2	9.	Current density, $J = \dfrac{I}{A}$ A/m^2
10.	Magnetic intensity $H = \dfrac{NI}{l}$ AT / m	10.	Electric intensity, $E = \dfrac{V}{d}$

Dissimilarities b/w Magnetic And Electric Circuit

Dissimilarities	
Magnetic circuit	**Electric circuit**
1. In fact, magnetic flux does not flow but is set up in the magnetic circuit.	1. The electric current actually flows in an electric circuit.
2. For magnetic flux, there is no perfect insulator.	2. For electric circuit, there are large number of perfect insulators like glass, air, rubber, mica etc.
3. At constant temp., the reluctance of a magnetic circuit is not constant but varies with μ_r.	3. At constant temp. the resistance of an electric circuit is constant as its value depends on resistivity which is almost constant.
4. Once magnetic flux is setup in a magnetic circuit, no energy is needed.	4. Energy is needed as long as current flows through electric circuit.

Examples

1. A coil is uniformly wound with 300 turns over a steel ring of relative permeability 900 and having a mean dia. of 20 cm. the steel ring is made of a bar having cross section of dia 2 cm. If coil has a resistance of 50 ohm and connected to 250 Volt d.c. calculate (1) M.M.F. (2) Field Intensity (3) Reluctance (4) Total flux (5) Permeance of the ring.

$N = 300 \quad D = 20 \text{ cm} \quad d = 2 \text{ cm}$

$\mu_r = 900, \quad V = 250 \text{ V}, \quad R = 50\ \Omega$

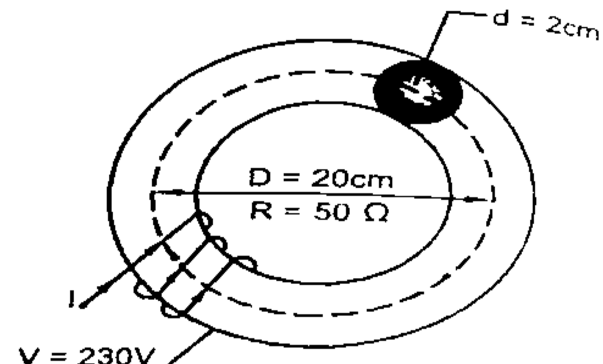


Fig.- E.3.1

(i) Current through the coil, $I = \dfrac{V}{R}$

$$= \frac{250}{50}$$

$$= 5 \text{ A}$$

$$\text{m.m.f.} = N I$$

$$= 300 \times 5 = 1500 \text{ AT}$$

(ii) Mean length, $l = \pi D$

$$= \pi \times \frac{20}{100}\, m$$

$$= 0.2\pi \text{ m.}$$

Field intensity, $H = \dfrac{N I}{l}$

$$= \frac{300 \times 5}{0.2\pi} \text{ AT/m}$$

$$= 2387.3 \text{ AT/m}$$

(iii) area of cross–section A $= \frac{\pi}{4} d^2$

$$= \frac{\pi}{4} \times (2 \times 10^{-2})^2$$

$$= \pi \times 10^{-4}\ m^2$$

Reluctance, $S = \frac{l}{\mu_0 \mu_r A}$

$$= \frac{0.2\pi}{4\pi \times 10^{7} \times 900 \times \pi \times 10^{-4}}$$

$$= 17.684 \times 10^5 \text{ AT/Wb}$$

(iv) Permeance $= \frac{1}{\text{reluctance}}$

$$= \frac{1}{17.684 \times 10^5}$$

$$= 5.655 \times 10^{-5} \text{ Wb/AT}$$

(v) Flux $= \frac{\text{m.m.f.}}{\text{reluctance}}$

$$= \frac{1500}{17.684 \times 10^5}$$

$$\mathbf{= 0.848\ mWb}$$

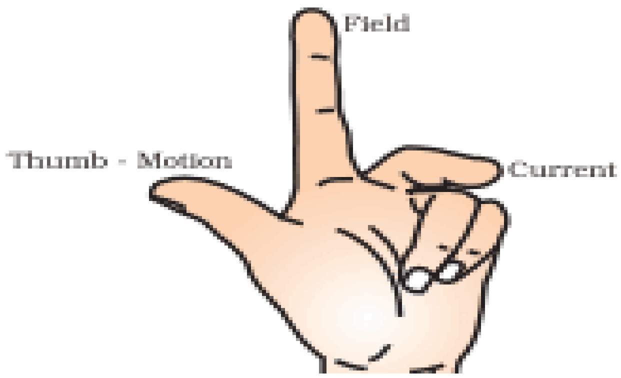
Force on a current carrying conductor placed in a magnetic field

- Video: https://www.youtube.com/watch?v=F1PWnu01IQg

Fleming's Left Hand Rule

- It states that "Whenever a current-carrying conductor is placed in a magnetic field, the conductor experiences a force which is perpendicular to both the magnetic field and the direction of the current. "
- In other words,

If the first three fingers(Thumb, forefinger and middle finger) of the left hand are held mutually at right angles to each other and if the forefinger indicates the direction of the original field, and if the middle finger indicates the direction of current flowing through the conductor, then the thumb indicates the direction of the force exerted on the conductor.



Video:
https://www.youtube.com/watch?v=JOIzcGQKIk8

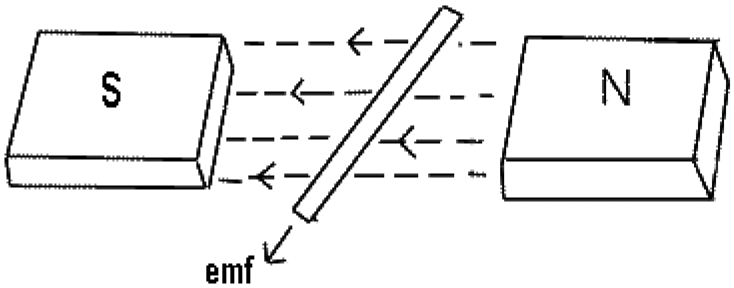
Electromagnetic Induction

- Electromagnetic Induction or Induction is a process in which a conductor is put in a particular position and magnetic field keeps varying or magnetic field is stationary and a conductor is moving. This produces a Voltage or EMF (Electromotive Force) across the electrical conductor.

- Video:

https://www.youtube.com/watch?v=yU--8Zk57-Y

Faraday's law of Electromagnetic Induction



- **First law:** Whenever a conductor is placed in a varying magnetic field, EMF induces and this emf is called an induced emf and if the conductor is a closed circuit than the induced current flows through it.

- **Second law:** The magnitude of the induced EMF is equal to the rate of change of flux linkages.

Consider, a magnet is approaching towards a coil. Here we consider two instants at time T_1 and time T_2.

Flux linkage with the coil at time,

$$T_1 = N\phi_1 \ wb$$

Flux linkage with the coil at time,

$$T_2 = N\phi_2 \ wb$$

Change in flux linkage,

$$N(\phi_2 - \phi_1)$$

Let this change in flux linkage be,

$$\phi = (\phi_2 - \phi_1)$$

So, the Change in flux linkage

$$N\phi$$

Now the rate of change of flux linkage

$$\frac{N\phi}{t}$$

Take derivative on right hand side we will get

$$N\frac{\mathrm{d}\phi}{\mathrm{d}t}$$

The rate of change of flux linkage

$$E = N\frac{d\phi}{dt}$$

But according to Faraday's law of electromagnetic induction, the rate of change of flux linkage is equal to induced emf.

$$E = -N\frac{d\phi}{dt}$$

Direction of Induced EMF

1) FLEMING'S RIGHT HAND RULE:-

- As per Faraday's law of electromagnetic induction, whenever a conductor is forcefully moved in an electromagnetic field, an emf gets induced across the conductor. If the conductor is provided a closed path, then the induced emf causes a current to flow.

- According to the **Fleming's right hand rule**, the thumb, fore finger and middle finger of the right hand are stretched to be perpendicular to each other, and if the thumb represents the direction of the movement of conductor, fore-finger represents direction of the magnetic field, then the middle finger represents direction of the induced current.

Remember

- Fleming’s left hand rule is mainly applicable to electric motors and Fleming's right hand rule is mainly applicable to electric generators.

- These rules do not determine the magnitude but instead show the direction of any of the three parameters (magnetic field, current, force) when he direction of the other two parameters are known.

Direction of Induced EMF

2) LENZ'S LAW:-

- Lenz's law states that the direction of an e.m.f. induced by changing magnetic field through a loop or coil of wire will be such that if it were to cause a current to flow in a conductor in an external circuit, then that current would generate a field that would oppose the change that created it.

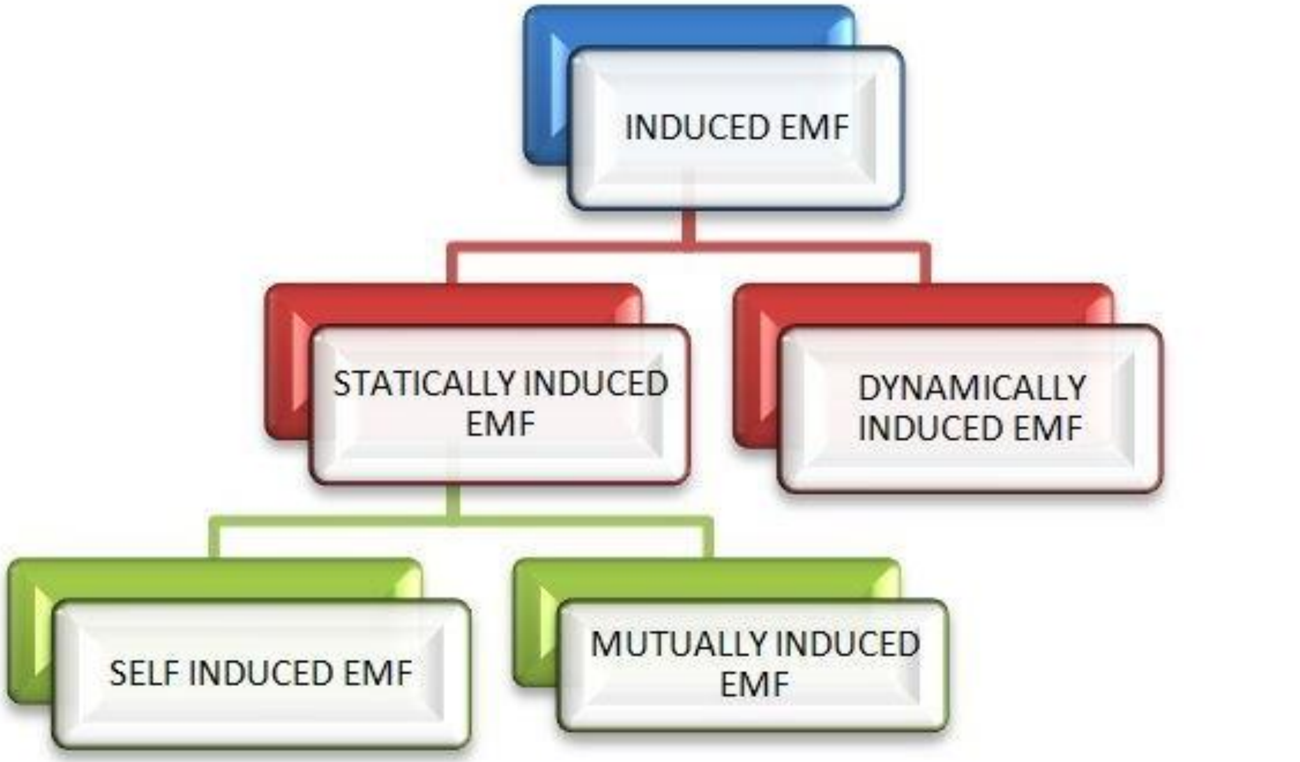
- Lenz's Law combined with Faraday's Law gives:

$$\boxed{\text{induced e.m.f.} = -N\frac{\Delta\phi}{\Delta t}}$$

where the e.m.f. is induced in a coil of N turns by a change of magnetic flux $\Delta\phi$ through the coil in a time Δt.

Induced EMF

- An Electromotive Force or EMF is said to be induced when the flux linking with a conductor or coil changes.



Statically Induced EMF

- This type of **EMF** is generated by keeping the coil and the magnetic field system, stationary at the same time; that means the change in flux linking with the coil takes place without either moving the conductor (coil) or the field system.
- This change of flux produced by the field system linking with the coil is obtained by changing the electric current in the field system.

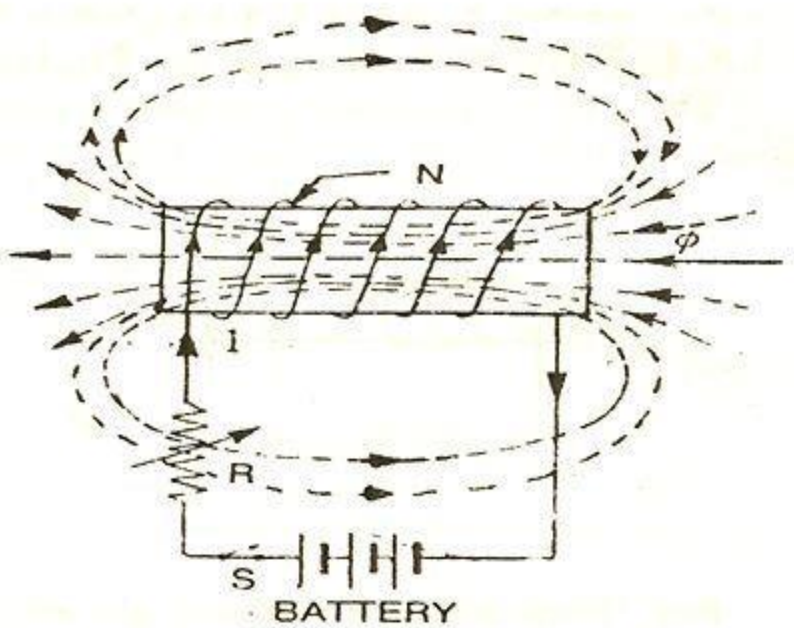
- It is further divided in two ways:

1) **Self-induced electromotive force**
2) **Mutually induced electromotive force**

Self-Induced EMF

- **Self-induced emf** is the e.m.f induced in the coil due to the change of flux produced by linking it with its own turns.

- Consider a coil having N number of turns as shown in the above figure. When the switch S is closed and current I flows through the coil, it produces flux (φ) linking with its own turns. If the current flowing through the coil is changed by changing the value of variable resistance (R), the flux linking with it, changes and hence emf is induced in the coil. This induced emf is called Self Induced emf.



- The direction of this induced emf is such that it opposes its very own cause which produces it, that means it opposes the change of current in the coil. This effect is because of Lenz's Law.
- Since the rate of change of flux linking with the coil depends upon the rate of current in the coil.

$$e \propto \frac{dI}{dt}$$

OR

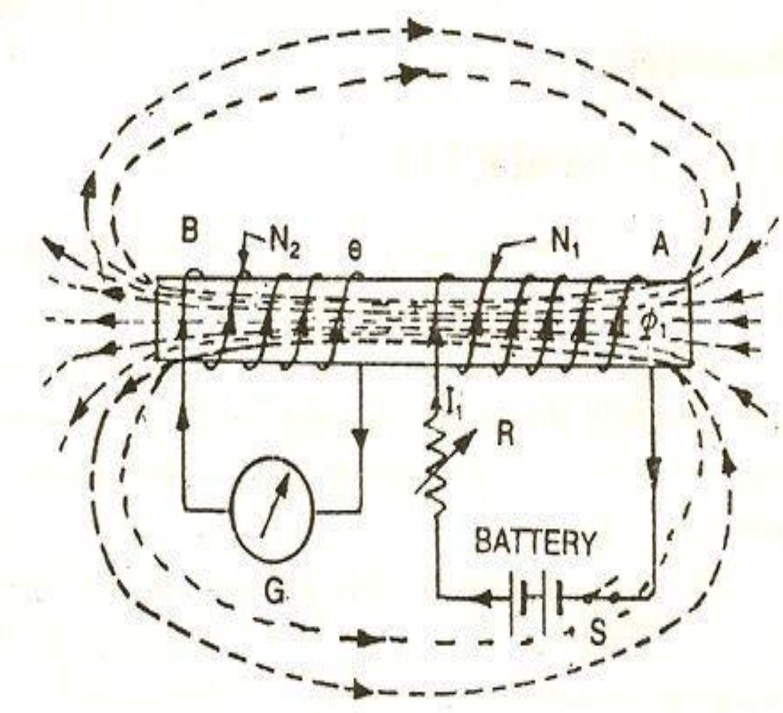
$$e = L\frac{dI}{dt}$$

- The magnitude of self-induced emf is directly proportional to the rate of change of current in the coil. L is constant of proportionality and called as **Self Inductance or the Coefficient of Self Inductance or Inductance of the coil.**

Mutually Induced EMF

- The emf induced in a coil due to the change of flux produced by another neighboring coil linking to it, is called **Mutually Induced emf.**

- Consider a coil AB. Coil B is having N2 number of turns and is placed near another coil A having N1 number of turns, as shown in the figure
- When the switch (S) is closed in the circuit shown, current I_1 flows through the coil A, and it produces the flux φ_1. Most of the flux says φ_{12} links with the other coil B.



- If the current flowing through the coil A is changed by changing the value of variable resistor R, it changes flux linking with the other coil B and hence emf is induced in the coil. This induced emf is called Mutually Induced emf.

- The direction of the induced emf is such that it opposes the cause which produces it, that means it opposes the change of current in the first coil. This effect of opposition caused by its own reason of production is called Lenz's Law. A galvanometer (G) is connected to coil B for measuring the induced emf.

- Since the rate of change of flux linking with the coil, B depends upon the rate of change of current in the coil A.

$$e_m \propto \frac{dI_1}{dt}$$

Or

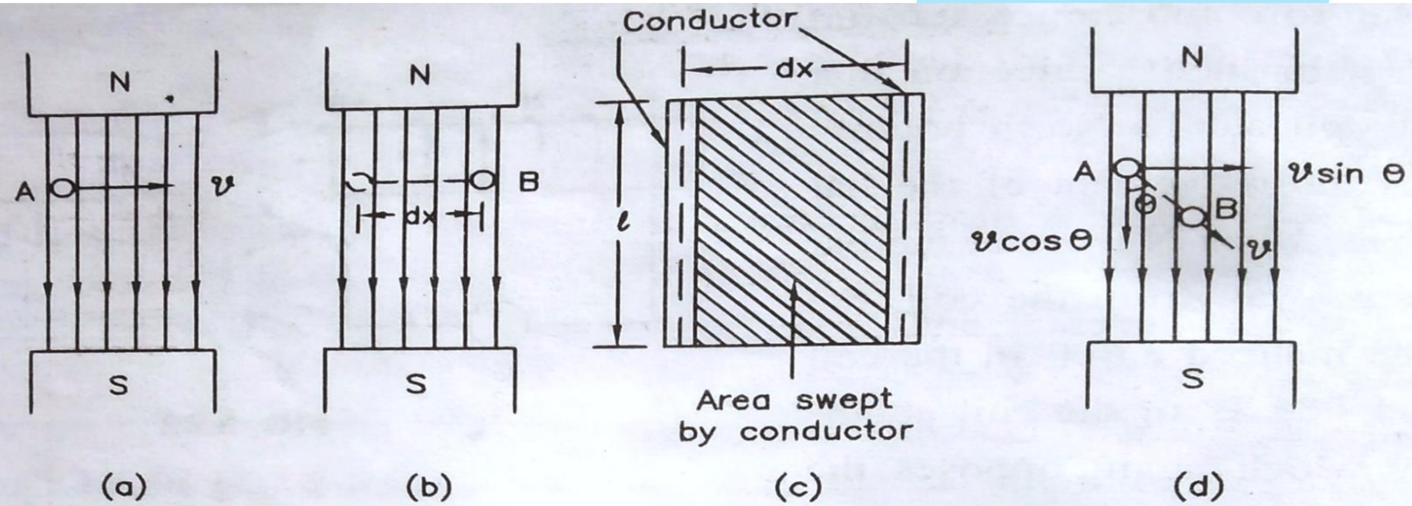
$$e_m = M \frac{dI_1}{dt}$$

- The rate of change of current in coil A is directly proportional to the magnitude of the mutually induced emf. M is called the Constant of Proportionality and is also called as Mutual Inductance or Coefficient of Mutual Inductance.

Dynamically Induced EMF

- In dynamically **induced electromotive force** the magnetic field system is kept stationary, and the conductor is moving, or the magnetic field system is moving, and the conductor is stationary. Thus by following either of the two process the conductor cuts across the magnetic field and the emf is induced in the coil.

- This phenomenon takes place in electric generators and back emf of motors and also in transformers.



[FIG. 3.43 DYNAMICALLY INDUCED E.M.F.]

Area swept by the moving conductor, $A = l\ dx\ \text{m}^2$

The flux cut by the conductor due to movement by dx metre

$= $ flux density $\times$ area swept

$= B \times l\,dx\ = B\,l\,dx$

$\therefore$ Rate of change of flux linkages $= \dfrac{d\phi}{dt}$

$$= \frac{B\ l\ dx}{dt}$$

According to Faraday's law, the e.m.f. induced in the conductor is equal to the rate of change of flux linkages

$$\therefore \text{ Induced e.m.f. } e = \frac{d\phi}{dt}$$

$$\therefore \text{ Induced e.m.f. } e = \frac{d\phi}{dt}$$

$$= B\,l\,\frac{dx}{dt}$$

$$= B\,l \cdot v$$

$$\text{where } \frac{dx}{dt} = v \text{ (velocity)}$$

Thus, dynamically induced e.m.f., $\boxed{e = B\,l\,v}$

Comparison between Dynamically & Statically induced emf

Dynamically induced e.m.f.	Statically induced e.m.f.
1. In this case, the magnetic field is stationary and the conductor moves across the field.	1. In this case, a conductor or a coil remains stationary and flux linking with it changes which can be changed by varying the current in the conductor or a coil.
2. The direction of the induced e.m.f. is determined by Fleming's Right Hand Rule.	2. The direction of the induced e.m.f. is determined by Lenz's law.
3. The magnitude of e.m.f. is given by $e = B\,l\,v \sin\theta$	3. The magnitude of e.m.f. is given by Faraday's law i.e. $e = -N\dfrac{d\phi}{dt}$
Generators work on the principle of dynamically induced e.m.f.	4. Transformers work on the principle of statically induced e.m.f.

Self-Inductance

- Self-inductance or in other words inductance of the coil is defined as the property of the coil due to which it opposes the change of current flowing through it. Inductance is attained by a coil due to the self-induced emf produced in the coil itself by changing the current flowing through it.

Mutual Inductance

- Mutual Inductance between the two coils is defined as the property of the coil due to which it opposes the change of current in the other coil, or you can say in the neighboring coil.

- When the current in the neighboring coil changes, the flux sets up in the coil and because of this, changing flux emf is induced in the coil called Mutually Induced emf and the phenomenon is known as Mutual Inductance.

Coefficient of Coupling

- The fraction of magnetic flux produced by the current in one coil that links with the other coil is called the **coefficient of coupling** between the two coils.

- It is denoted by k

Co-Efficient of coupling

When current flows through one coil, it produces flux ϕ_1. The whole of this flux may not be linking with the other coil magnetically coupled to it as shown in Fig. 3.46 It may be reduced because of leakage flux ϕ_ℓ.

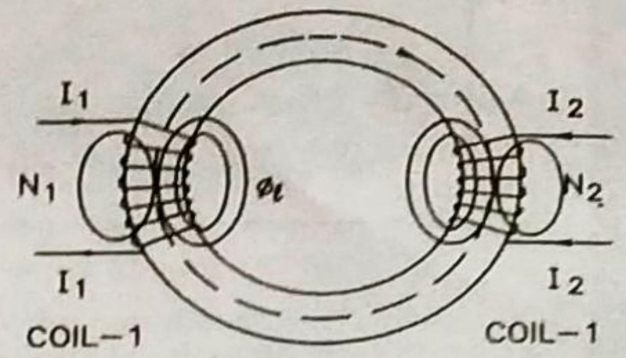


FIG. 3.46

Let N_1 = no. of turns on coil 1

N_2 = no. of turns on coil 2

I_1 = current flowing through coil 1

I_2 = current flowing through coil 2

S = reluctance of the magnetic circuit

ϕ_1 = flux produced by coil 1 due to I_1

ϕ_2 = flux produced by coil 2 due to I_2

The individual self–inductances of coil 1 and coil 2 are given by

$$L_1 = \frac{N_1^2}{S}, \quad L_2 = \frac{N_2^2}{S}, \quad \text{where } S = \frac{\ell}{\mu_0 \mu_r A} \qquad \ldots (1)$$

The flux ϕ_1 is produced by coil 1 due to the current I_1

$$\therefore \phi_1 = \frac{N_1 I_1}{S}$$

Suppose a fraction K_1 of this flux ϕ_1 links the coil 2,

$\therefore$ flux linking the coil 2 $= \phi_{12}$

$$= K_1\, \phi_1 \quad (\because K_1 \leq 1)$$

$\therefore$ Mutual inductance, $M = \dfrac{N_2\, \phi_{12}}{I_1}$

$$= \frac{N_2 \left(K_1\, \phi_1\right)}{I_1}$$

$$= \frac{N_2\, K_1}{I_1} \left(\frac{N_1\, I_1}{S}\right)$$

$$= \frac{K_1\, N_1\, N_2}{S} \qquad \ldots \ldots \ldots (2)$$

Similarly, the flux ϕ_2 is produced by coil 2 due to current I_2 in it

$$\therefore \phi_2 = \frac{N_2\, I_2}{S}$$

Suppose a fraction K_2 of this flux links the coil 1

$\therefore$ flux linking the coil 1 $= \phi_{21}$

$$= K_2 \phi_2$$

$\therefore$ Mutual inductance, $M = \dfrac{N_1 \phi_{21}}{I_2}$

$$= \frac{N_1 K_2 \phi_2}{I_2}$$

$$= \frac{N_1 K_2}{I_2} \left(\frac{N_2 I_2}{S} \right)$$

$$= \frac{K_2 N_1 N_2}{S} \qquad \ldots \quad \ldots \quad \ldots (3)$$

Multiplying eqn. (2) and eqn. (3), we get

$$M^2 = \left(\frac{K_1 N_1 N_2}{S} \right) \left(\frac{K_2 N_1 N_2}{S} \right)$$

$$= K_1 K_2 \frac{N_1^2}{S} \frac{N_2^2}{S}$$

$$= K_1 K_2 L_1 L_2 \qquad (\because \text{ from eqn. 1})$$

Putting $\sqrt{K_1 K_2} = K$

$\therefore \quad M^2 = K^2 L_1 L_2$

$M = K \sqrt{L_1 L_2}$

$$\therefore \quad \boxed{K = \frac{M}{\sqrt{L_1 L_2}}} \qquad \ldots \quad \ldots \quad \ldots (4)$$

If the flux due to one coil completely links the other coil, then the value of K is equal to unity. This is the maximum possible value of K.

Hence $$M = \sqrt{L_1 L_2} \qquad [\because K = 1]$$

$$= \sqrt{L_1 L_2} \qquad [\because K = 1]$$

The constant K is called the coefficient of coupling and may be defined as the ratio of mutual inductance actually present between the two coils to the maximum possible value

When $K = 1$ coils are said to be tightly coupled

and $K = 0$ coils are said magnetically isolated from each other.

- Video: https://www.youtube.com/watch?v=hoTInTKijoo

Examples

2. The self inductance of a coil of 500 turns is 0.25 H. If 60 percent of the flux is linked with a second coil of 10500 turns, calculate:

(1) The mutual inductance between the coils

(2) E.m.f induced in the second coil when current in the first coil changes at the rate of 100 A/Sec

Solution : $L_1 = 0.25$ H, $N_2 = 10{,}500$, $\frac{di_1}{dt} = 100$ A / S

$N_1 = 500$, $K = 0.6$,

$$\phi_{12} = k\,\phi_1$$
$$= 0.6\,\phi_1$$

We know that $L_1 = \frac{N_1\,\phi_1}{i_1}$

$$\therefore \quad \frac{\phi_1}{i_1} = \frac{L_1}{N_1} = \frac{0.25}{500} = 5 \times 10^{-4}$$

Now, $\phi_{12} = 0.6\,\phi_1$

$$\frac{\phi_{12}}{i_1} = 0.6 \times \frac{\phi_1}{i_1} = 0.6 \times 5 \times 10^{-4}$$
$$= 3 \times 10^{-4}$$

Mutual inductance, $M = \frac{N_2 \phi_{12}}{i_1}$

$$= 10{,}500 \times 3 \times 10^{-4} = 3.15 \text{ H}$$

E.m.f. induced in the second coil, $e_{m2} = M \frac{di_1}{dt}$

$$= 3.15 \times 100 = 315 \text{ V}$$

3. The number of turns in a coil is 250. when a current of 2A flows in this coil, the flux in the coil is 0.3 mWb. When this current is reduced to zero in 2 ms, the voltage induced in a coil lying in the vicinity of coil is 63.75 V. If the coefficient of coupling between the coils is 0.85, find self inductances of the two coils, mutual inductances and number of turns in the second coil.

Solution : $N_1 = 250,$ $\phi_1 = 0.3$ mWb $i_1 = 2A$

$dt = 2ms$ $e_{m2} = 63.75$ V $K = 0.85$

(i) Self inductance of coil 1, $L_1 = \frac{N_1 \phi_1}{i_1}$

$$= \frac{250 \times 0.3 \times 10^{-3}}{2} = 37.5 \text{ mH}$$

(ii) E.M.F. induced in second coil, $e_{m2} = M \frac{di_1}{dt}$

$$\therefore M = \frac{e_{m2}}{\frac{di_1}{dt}} = \frac{63.75}{\frac{(2-0)}{2 \times 10^{-3}}} = 63.75 \text{ mH}$$

(ii) We know that $K = \dfrac{M}{\sqrt{L_1 L_2}}$

$$\therefore \quad 0.85 = \frac{63.75 \times 10^{-3}}{\sqrt{L_1 L_2}}$$

$$\therefore \quad \sqrt{L_1 L_2} = 75 \times 10^{-3}$$

$$\therefore \quad L_1 L_2 = 5625 \times 10^{-6}$$

$$\therefore \quad L_2 = \frac{5625 \times 10^{-6}}{37.5 \times 10^{-3}}$$

$$= 150 \text{ mH}$$

(iv) Flux linking the second coil, $\phi_{12} = K\phi_1$

$$= 0.85 \times 0.3 \text{ m Wb}$$

$$= 0.255 \text{ mWb}$$

$$= 0.255 \times 10^{-3} \text{ Wb}$$

e.m.f. induced in second coil, $e_2 = N_2 \dfrac{d\phi_{12}}{dt}$

$$63.75 = N_2 \times \frac{0.255 \times 10^{-3}}{2 \times 10^{-3}}$$

$$\therefore \quad N_2 = 500$$

4. Two identical coils A and B consisting of 1500 turns each lie in parallel planes such that 70 percent of the flux produced by the current in coil A links the coil B. Current of 4A flowing in the coil A produces in it a flux of 0.04 mWb. Calculate:

   (i) Self inductance of each coil

   (ii) Mutual inductance between them

If the current in coil A changes from 4A to -4A in 0.02 sec, what will be the e.m.f. induced in coil B?

Given data: $N_A = 1500$ turns $\qquad \phi_A = 0.04$ mwb

$= 0.04 \times 10^{-3}$ wb

$I_A = 5A$

1) Self Inductance of coil A $L_A = \frac{N_A \phi_A}{I_A} = \frac{1500 \times 4 \times 10^{-5}}{4} = 15\, mH$

As coils are identical

$L_A = L_B = 15\, mH$

2) Mutual Inductance $M = \frac{N_2 \phi_{12}}{I_1} = 10.5\, mH$

Emf induced in coil B

$\frac{di_1}{dt} = \frac{(-4)-(4)}{0.02} = \frac{-8}{0.02} = -400\, A/sec$

$= -M \frac{di}{dt}$

$= -10.5 \times 10^{-3} \times (-400)$

$= 4.2$ Volt