Unit-5
AC Fundamentals

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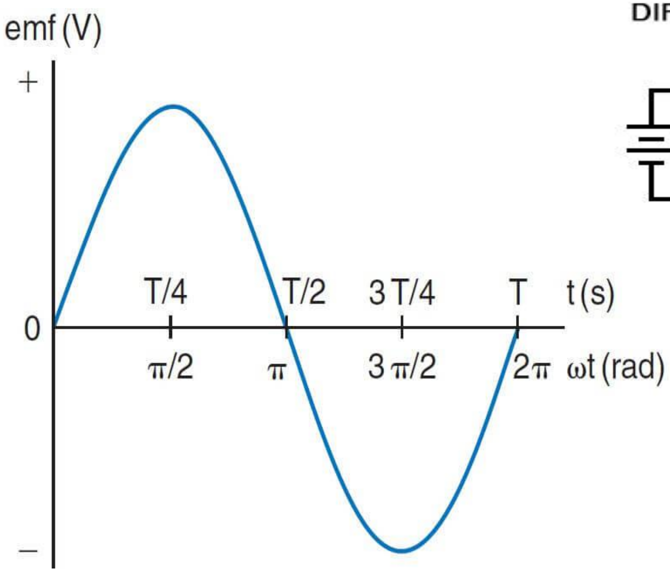
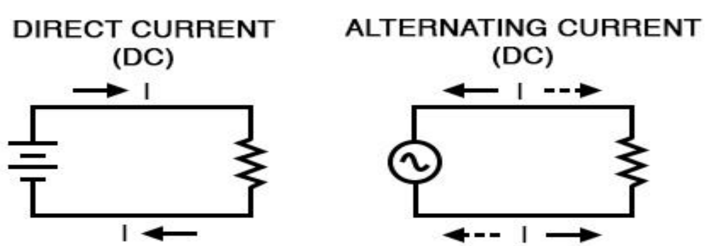
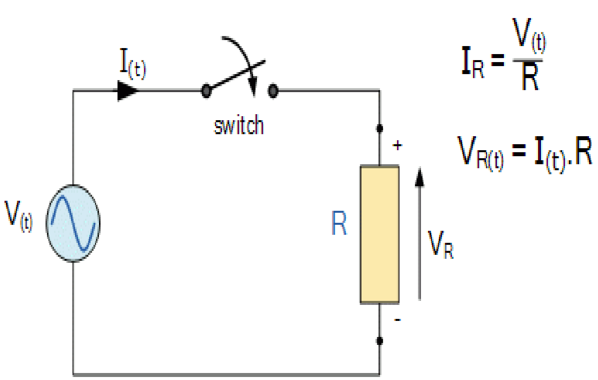






Content

- Introduction and definition
- Relation b/w speed and frequency
- Average and RMS value
- Sinusoidal wave shapes
- Examples

Introduction

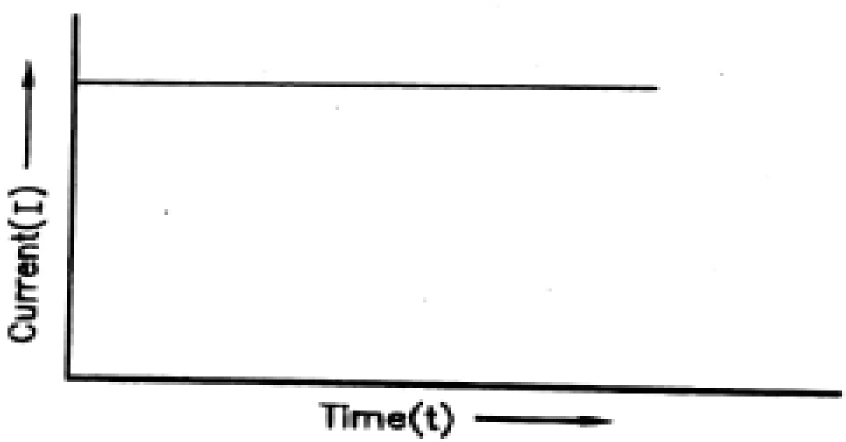
- Electrical energy used in our homes, offices, shops factories is in the form of a.c. (alternating current).

- There are 3 types of current that flows in the electrical circuits.

I. Direct Current or DC
II. Fluctuating Current
III. Alternating Current or AC

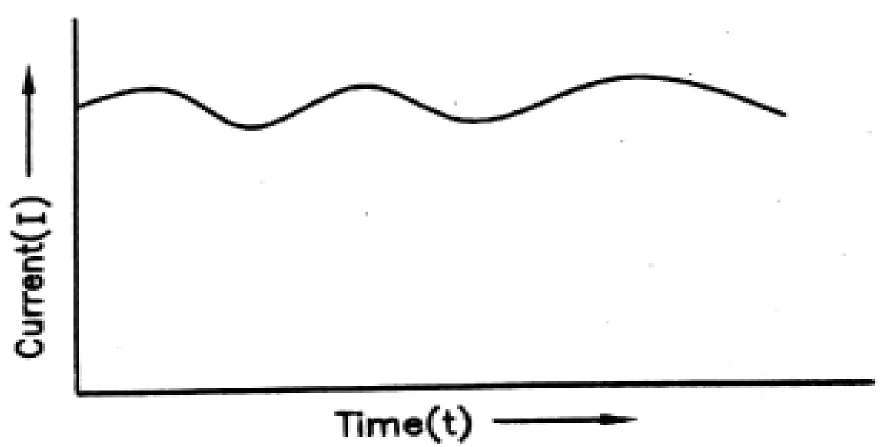
1) Direct Current or DC:-

- The current which always flow in one direction in a circuit is called as direct current.
- Thus the current whose magnitude remains constant with time and flows continuously in a definite direction is called direct current.



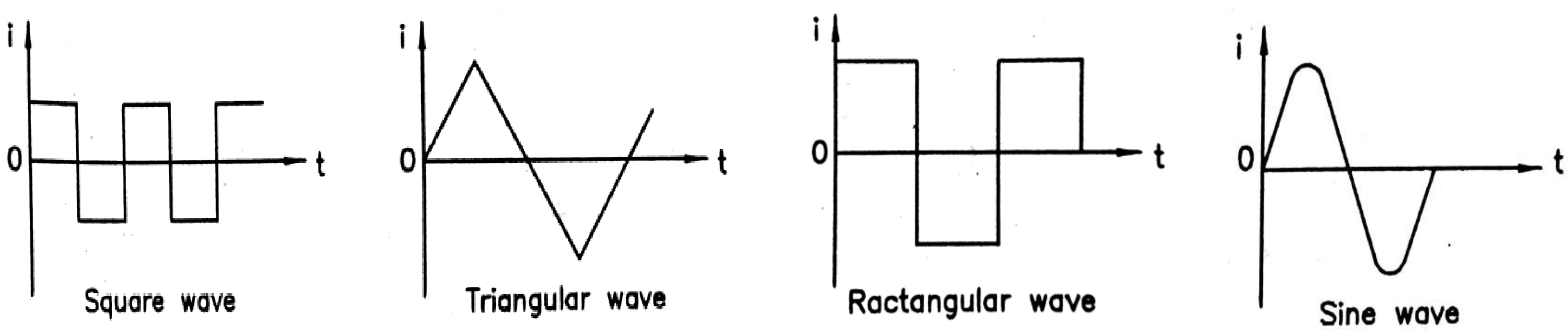
2) Fluctuating Current:-

- If the current generated by devices like rectifier, it will be observed that the direction of current remains constant but the magnitude has small periodic variations with time. Such type of current is called fluctuating current.



3) Alternating Current:-

- The current which changes its direction and magnitude periodically at regular intervals of time in a circuit is called alternating current.
- For alternating current or voltage the only necessary conditions is the periodic variation with time.
- Some of the wave forms are shown as:



Advantages of AC over DC

1. The alternating voltage or currents can be increased or decreased by means of a transformer without any appreciable loss of energy whereas direct current is varied by resistance alone resulting loss of energy due to heating.
2. In AC a wide range of voltage or current is available with the help of transformer.
3. The generation of AC is cheaper than that of DC.
4. Line losses in AC power transmission is negligible in comparison to DC power transmission.
5. AC can be easily converted into DC by rectifier when so required but conversion of DC into AC is costlier.
6. AC motors are cheaper and simpler in construction.

Disadvantages of Ac over DC

1. AC is comparatively more dangerous to use during faulty insulation as it attracts a person who touches it unlike DC which gives a repelling shock.
2. For certain purpose such as electric traction, electronic circuits, electroplating, electro-refining, computers etc…, where only dc is required, ac can not be used directly.

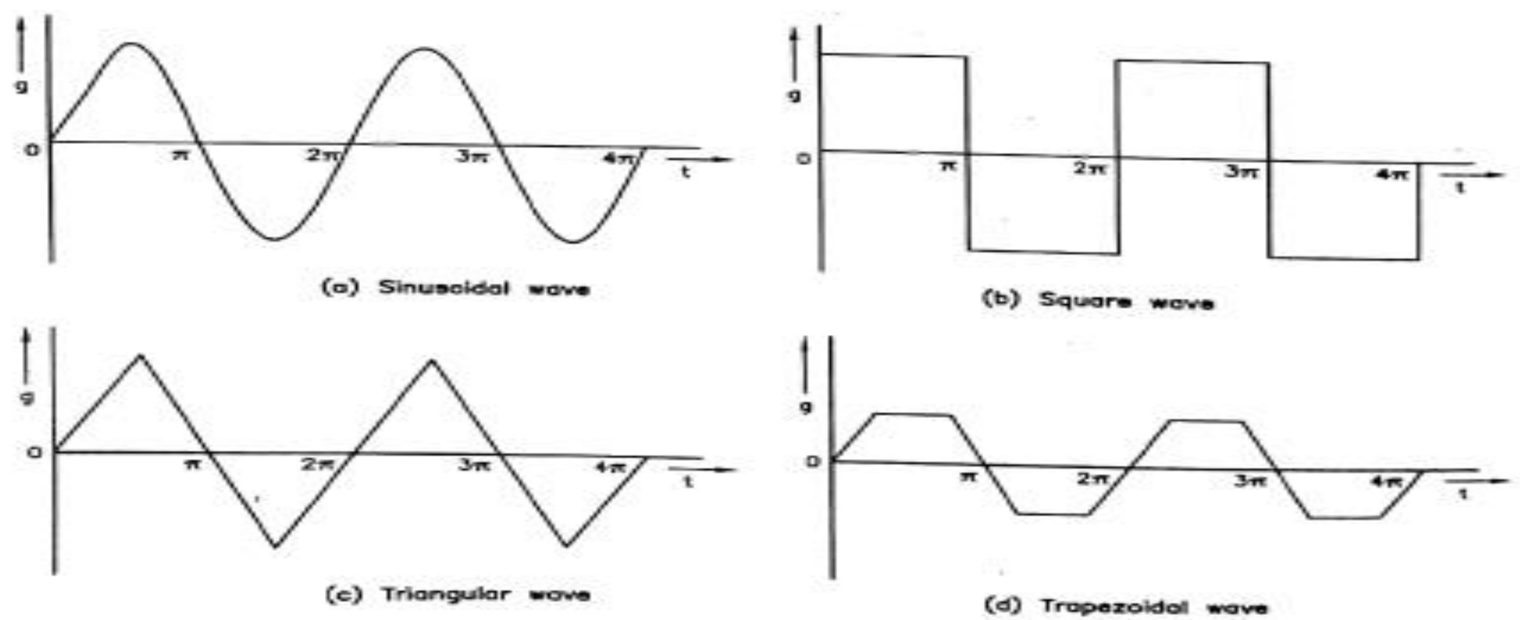
Comparison of Ac with DC

Alternating Current	Direct Current
1. In AC voltage and current reverses periodically.	1. In DC voltage and current remains constant.
2. Low cost of power generation.	2. Higher cost of power generation.
3. Cost of transmitting AC power can be reduced by using step-up transformers.	3. No such provision can be made.
4. AC can be converted into DC by using a device, called convertor (rectifier).	4. DC can be converted to AC by using choppers (inverter).
5. AC cannot be used directly for electroplating, electrotyping, etc...	5. DC can be used directly for carrying out such operations.
6. AC motors and other appliances are more robust, and durable.	6. DC motors, and appliances are less durable.
7. AC attracts a person, so faulty insulations of AC are more dangerous.	7. DC gives a repelling shock to a person, so faulty insulations of DC are less dangerous.

Definitions

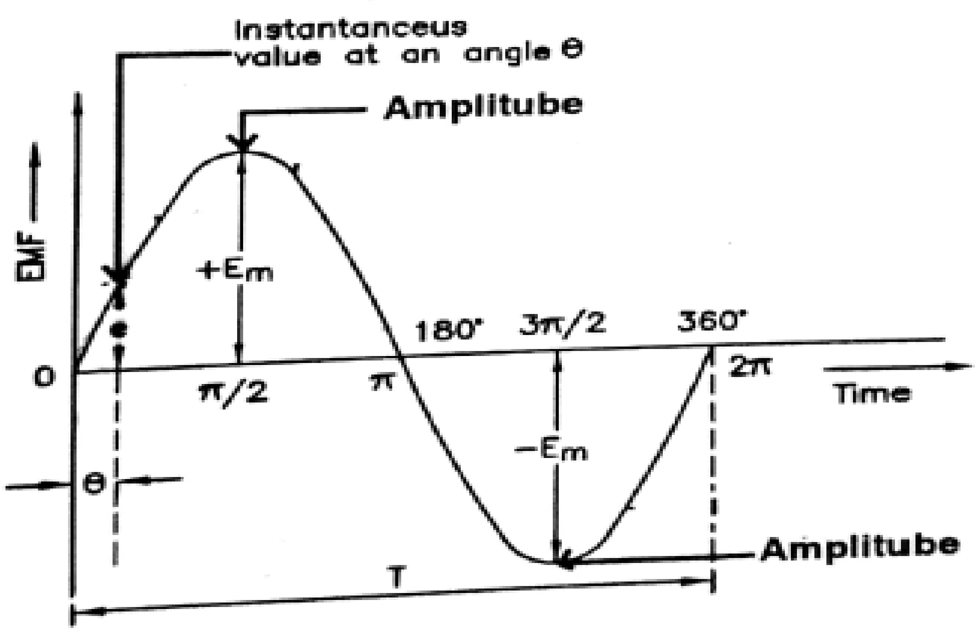
1. Waveform:

- The shape of the curve obtained by plotting the instantaneous values of alternating quantity along y-axis and time or angle along x-axis is called waveform.



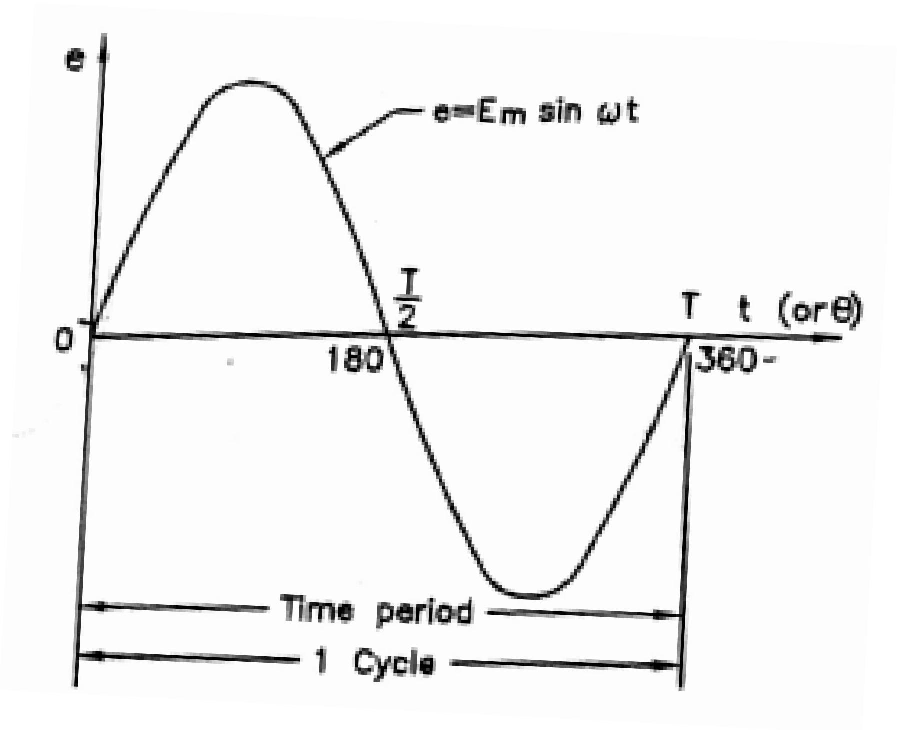
2. Instantaneous Value:-

- The value of an alternating quantity (voltage current and power etc…) at any instant is called its instantaneous value.
- It is represented by small English letters i.e. v, i, p, e respectively.



3. Cycle:-

- One complete set of positive and negative values of an alternating quantity is known as cycle.



4. Amplitude:-

- The maximum value (positive or negative) of an alternating quantity is known as its amplitude.

5. Time Period:-

- The time taken by an alternating quantity to complete one cycle is called its time period.
- It is denoted by **T**.
- It is expressed in **seconds**.
- The relationship b/w frequency and periodic time(T) is given by:

$$\boxed{T = \frac{1}{f}}$$

6. Frequency:-

- The number of cycles completed by an alternating quantity per second is known as frequency.
- It is denoted by **f**
- It is expressed in **hertz (Hz)** or **cycle/second**.
- The frequency of alternating voltage or current is given by:

$$\boxed{f = \frac{PN}{120}}$$

where, f = frequency,

P = no. of poles of the alternator,

N = speed of the alternator in rpm

7. Angular Frequency:-

- Angular frequency of an alternating quantity is defined as

$$\omega = 2\pi f = \frac{2\pi}{T}$$

where, f is the frequency and T is a time period.

8. Phase:-

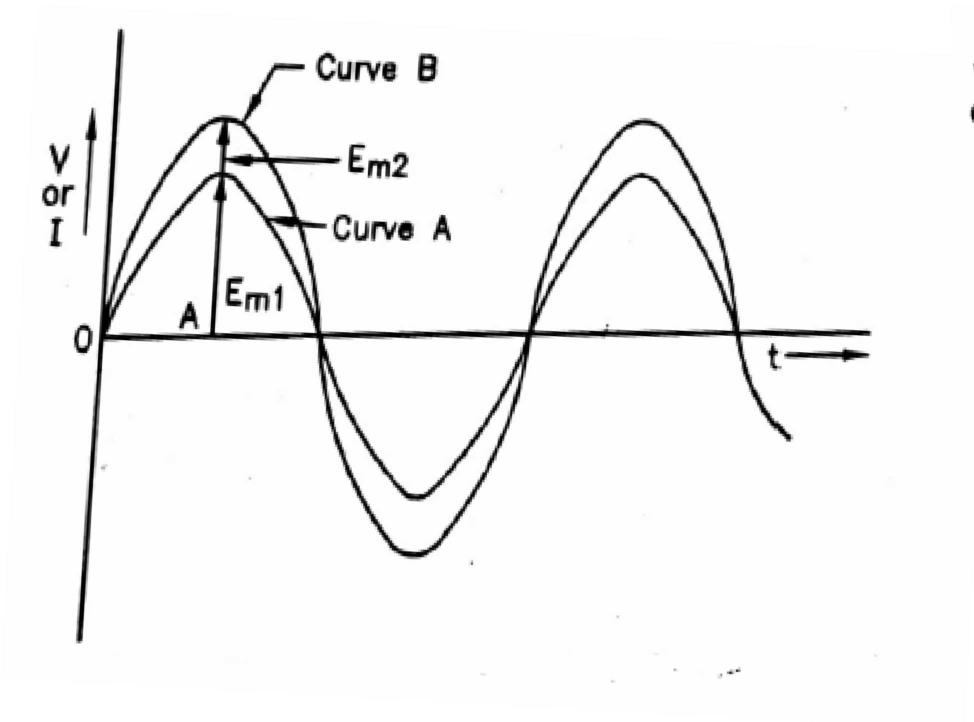
- It is defined as the fractional part of the cycle through which the alternating quantity has advanced from the origin(reference point).
- The phase at any instant t from that instant where time is zero, is given by 1/T, where T be the time period of alternating current.

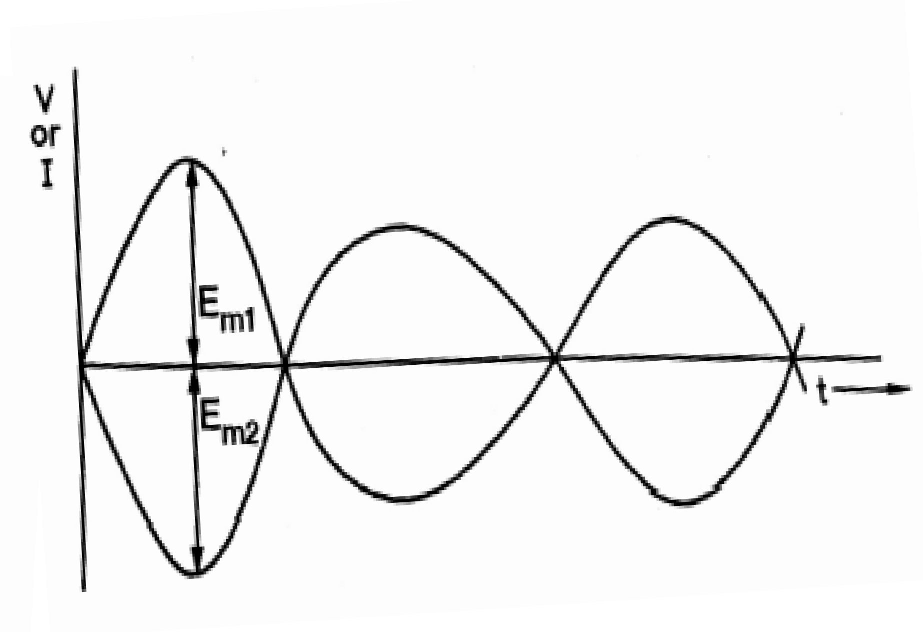
9. Phase Angle:-

- It is defined as the phase measured in terms of angle.
- The phase angle at any instant t is given by $2\pi/T=\omega t$.
- It is measured in terms of electrical degrees.

10.Phase Difference:-

- The phase difference between two alternating quantities having same frequency is the difference of planes at the maximum or zero or minimum value of the two alternating quantities.



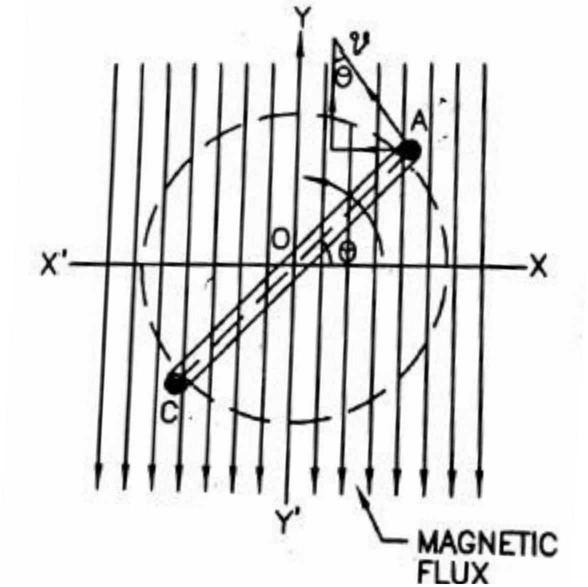


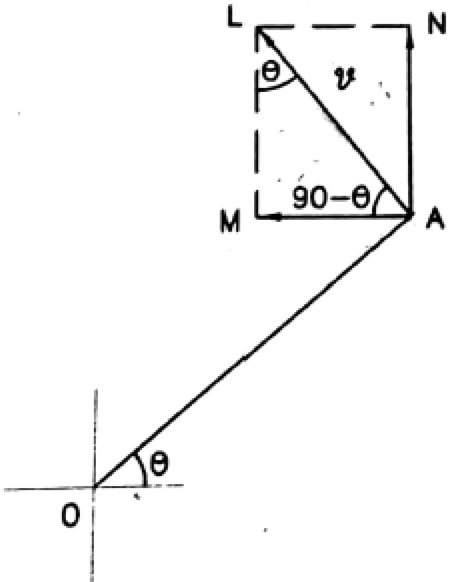
(a) Two alternating quantities are called in phase, when they attain maximum values or zero values at the same instant.

(b) Two alternating quantities are called out of phase when one alternating quantity attains maximum an another minimum at the same instant.

Equation of alternating voltage(e.m.f)

- A rectangular coil AC having N turns and rotating in a uniform magnetic field with a constant angular velocity in anti-clockwise direction is shown in fig.





Let B = flux density of the magnetic field (tesla or Wb/m^2)

v = peripheral velocity of the coil (m/sec)

l = length of the coil (m)

b = breadth of the coil (m)

f = speed of rotation of coil (frequency) in revolutions/sec.

- The vertical lines represent the lines of magnitude flux. Let the time be measured from the instant the coil lies in the horizontal position. The angle θ swept by the rotating coil in a time t is given by

$$\theta = \omega t$$

- When the coil has rotated through an angle θ in time t seconds as shown in fig, its peripheral velocity can be represented by

$$\frac{AM}{AL} = \sin\theta \qquad \frac{AN}{AL} = \cos\theta$$

$$\therefore\ AM = AL\sin\theta \qquad AN = AL\cos\theta$$

$$= v\sin\theta \qquad = v\cos\theta$$

- The emf induced in the side A of the coil is entirely due to the component of velocity perpendicular to the magnetic field i.e. Vsinθ
- Hence, the emf induced in one side of the coil at time t is given by,

$$e = Bl\, v \sin\theta \text{ volts}$$

- Total emf generated in both sides of the coil is

$$e = 2\, B\, l\, v \sin\theta \text{ volts}$$

- If the coil has N turns in series, the total emf induced in the coil at any instant t is given by

$$e = N \times 2\, B\, l\, v \sin\theta \text{ volts}$$

- The coil makes f revolutions per second. So, the linear distance travelled per second or peripheral velocity v is given by

$$v = f \times 2\pi \times \frac{b}{2}$$

$$= \pi\, b\, f \text{ metre/sec}$$

- Substituting the value of v,

$$\begin{aligned} e &= N \times 2\, B\, l\, (\pi\, b\, f) \sin\theta \text{ volts} \\ &= 2\pi\, N\, B\, l\, b\, f \sin\theta \\ &= 2\pi\, f\, N\, B\, (l\, b) \sin\theta \\ &= 2\pi\, f\, N\, B\, A \sin\theta \text{ volts} \end{aligned}$$

where $A = lb$ = area of the coil.

- When θ=90, sinθ=1, hence e has maximum value, say E_m,

$$E_m = 2\pi f N B A$$

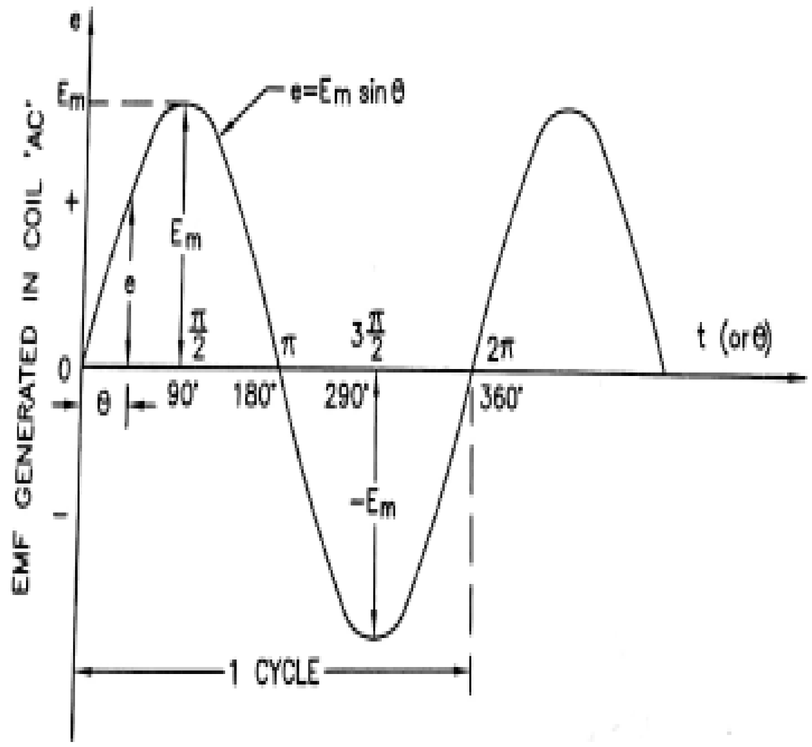
$$= N B A \omega \; (\because \omega = 2\pi f)$$

- Hence,

$$e = E_m \sin\theta \text{ volts} \quad \text{or}$$

$$\boxed{= E_m \sin\omega t \text{ volts} \quad (\because \theta = \omega t)}$$

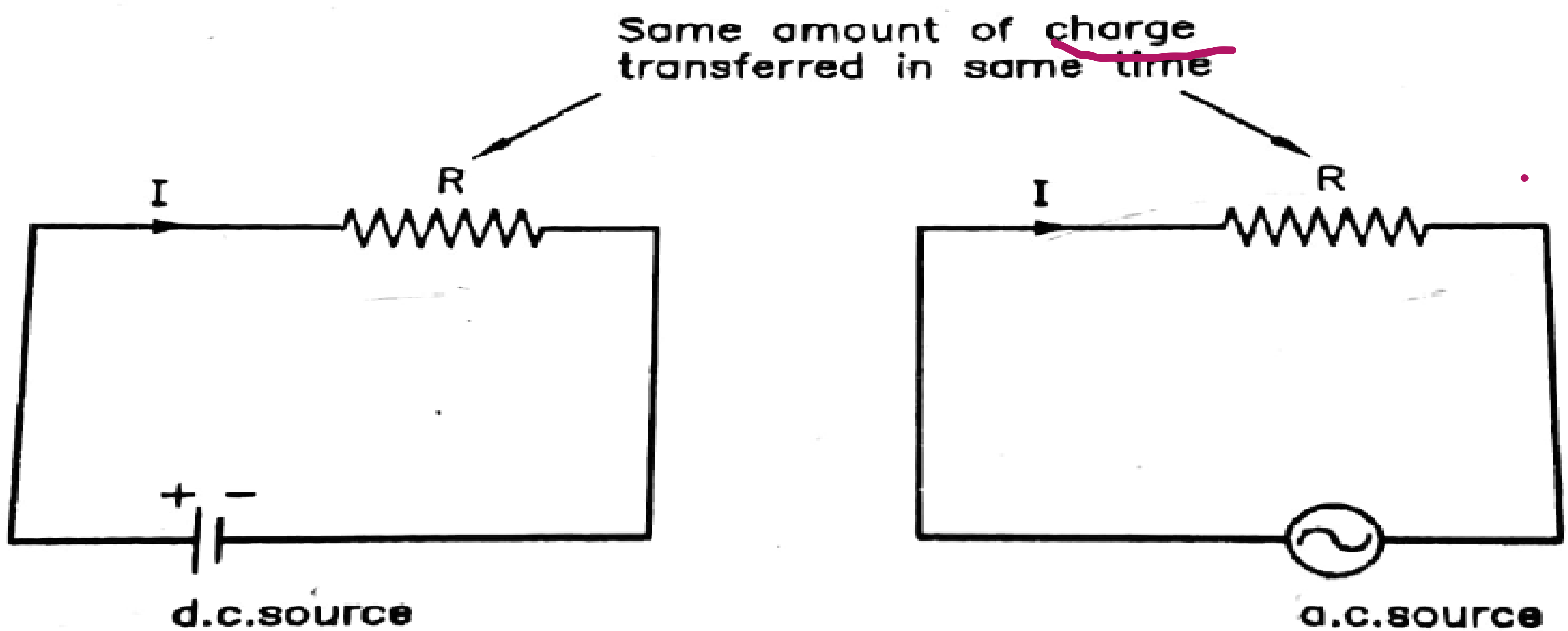
- This emf can be represented by a sine wave as shown in fig. in which E_m is the maximum value of emf and e is the emf when the coil has rotated through an angle θ from the position of zero emf.



- When θ varies from 0 to 180, the emf is considered positive and it is negative when θ varies b/w 180 to 360.
- Thus, in one cycle of the waveform, there is one positive half-cycle and one negative half-cycle.
- The number of such complete cycles that occur in one second is called the frequency of the emf.
- The duration of each cycle is called periodic time or time period.

Average value or mean value

- The steady current (d.c.) which flows through a circuit for a given time transfer same charge as transferred by the alternating current when flows through the same circuit for same time is called average value of the alternating current.



- The average or mean value of an a.c. quantity over a given interval is the sum of all instantaneous values divided by number of values taken over that interval.

$$\boxed{\text{Average Value} = \frac{\text{Area under the curve}}{\text{Length of the Base of the curve}}}$$

- **Analytical Method:-**

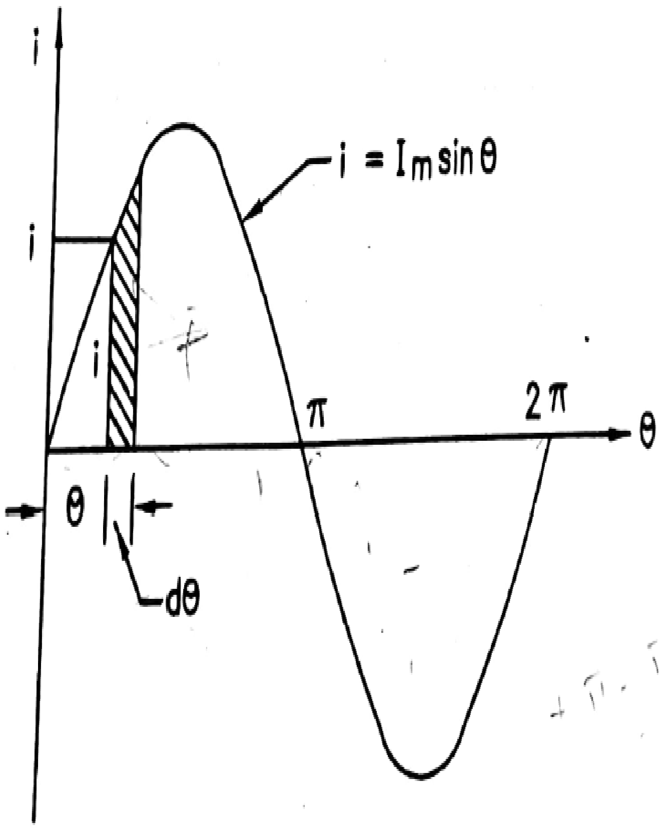
➢ This method is based on the definition of the average value of the alternating current.

➢ The average value can be obtained as:

$$I_{av} = \frac{1}{T}\int_0^T i \, dt$$

Average value of sinusoidal alternating current

- Since it is a symmetrical wave, we can consider only half- cycle.



Mathematically,

$$i = I_m \sin\theta \qquad 0 \le \theta \le \pi$$

By definition

$$I_{av} = \frac{\text{area under half}-\text{cycle}}{\text{length of the base over half}-\text{cycle}}$$

$$= \frac{\int_0^\pi i\, d\theta}{\pi}$$

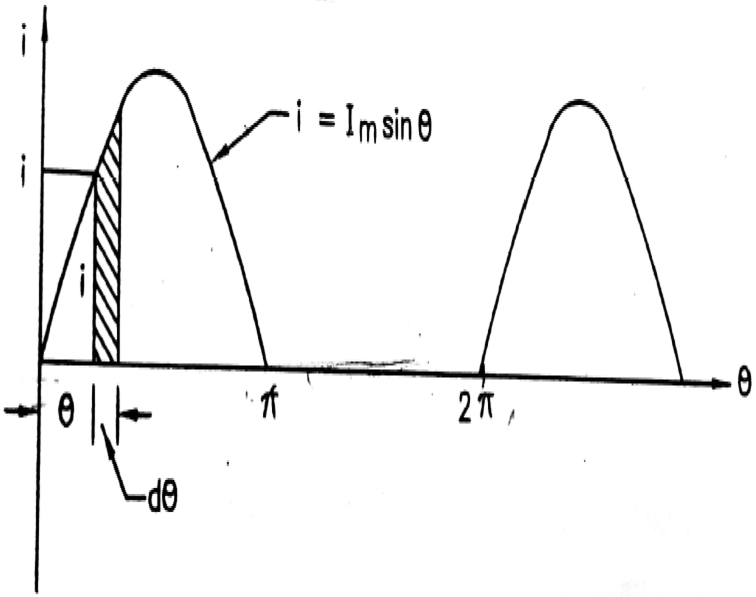
$$= \frac{1}{\pi}\int_0^\pi I_m \sin\theta\, d\theta \qquad [\because i = I_m \sin\theta]$$

$$= \frac{I_m}{\pi}[-\cos\theta]_0^{\pi}\ [\cos\pi - \cos 0] = \frac{-I_m}{\pi}[-1-1] = \frac{2I_m}{\pi}$$

$$= \frac{I_m}{\pi/2} = \boxed{0.637\ I_m}$$

Average value of half-wave rectified current

- Since it is a unsymmetrical, we can consider full- cycle.



$$i = I_m \sin\theta \qquad 0 \leq \theta \leq \pi$$

$$= 0 \qquad \pi \leq \theta \leq 2\pi$$

$$I_{av} = \frac{\text{area under full} - \text{cycle}}{\text{length of the base over full} - \text{cycle}}$$

$$= \frac{\int_0^{\pi} i\, d\theta + \int_{\pi}^{2\pi} 0 .\, d\theta}{2\pi}$$

$$= \frac{1}{2\pi}\int_0^{\pi} I_m \sin\theta\, d\theta = \frac{I_m}{2\pi}[-\cos\theta]_0^{\pi}$$

$$= \frac{-I_m}{2\pi}[\cos\pi - \cos 0] = \frac{-I_m}{2\pi}[-1-1] = \frac{I_m}{\pi}$$

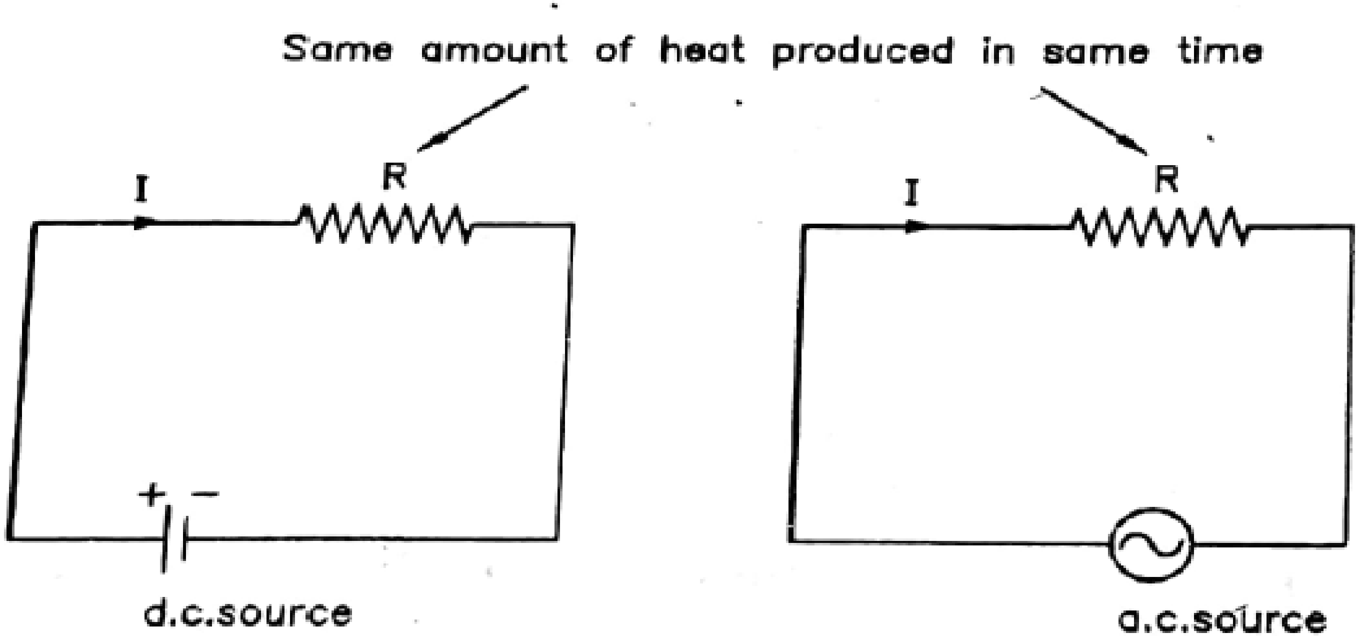
$$\boxed{= \frac{1}{\pi} \times I_m}$$

$$= \frac{-I_m}{2\pi}[\cos\pi - \cos\theta] = \frac{-I_m}{2\pi}[-1 - 1] = \frac{I_m}{\pi}$$

$$\boxed{= \frac{1}{\pi} \times I_m}$$

RMS value

- The steady current (d.c.) which flows through a circuit for a given time produces same amount of heat as produced by the alternating current when flows through the same circuit for the same time is called r.m.s. value of the alternating current.



R.M.S. value of sinusoidal alternating current

The expression of sinusoidal alternating current is

$$i = I_m \sin \omega t$$
$$= I_m \sin \theta$$

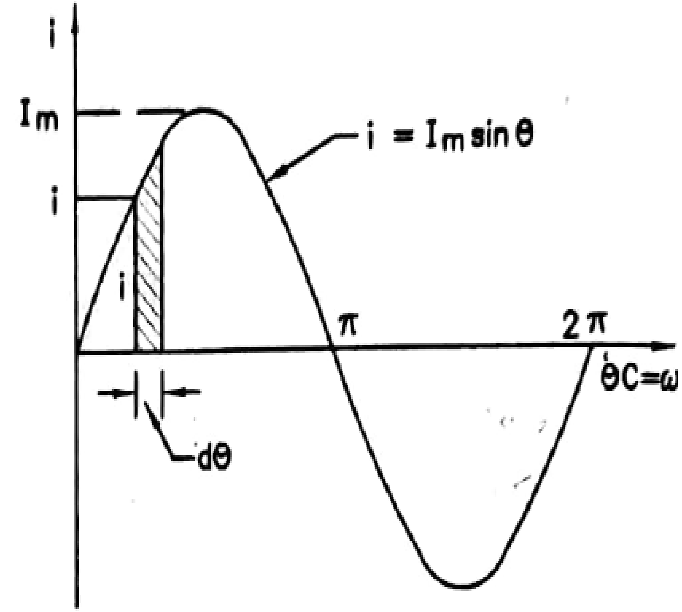
$$(\text{mean of } i^2) = \int_0^{2\pi} \frac{i^2\, d\theta}{2\pi}$$

$$= \int_0^{2\pi} \frac{(I_m \sin \theta)^2}{2\pi}\, d\theta$$

$$= \frac{I_m^2}{2\pi} \int_0^{2\pi} \sin^2 \theta\, d\theta$$

$$= \frac{I_m^2}{2\pi} \int_0^{2\pi} \left(\frac{1 - \cos 2\theta}{2}\right) d\theta \quad = \frac{I_m^2}{4\pi} \left[\theta - \frac{\sin 2\theta}{2}\right]_0^{2\pi} \quad = \frac{I_m^2}{4\pi} [2\pi - 0]$$

$$= \frac{I_m^2}{2}$$



The r.m.s. value of the alternating current $= \sqrt{\begin{array}{l}\text{mean of the squares of the} \\ \text{instantaneous values of currents}\end{array}}$

$$= \sqrt{\frac{I_m^2}{2}}$$

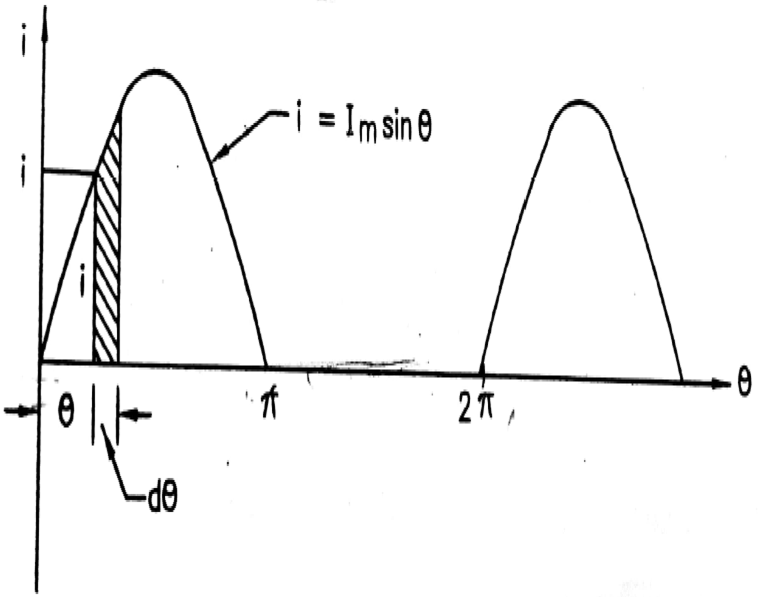
$$= \frac{I_m}{\sqrt{2}}$$

$$= \boxed{0.707\ I_m}$$

$$\boxed{\text{r.m.s. value of current} = 0.707 \times \text{maximum value of current}}$$

RMS value of half-wave rectified current

- Since it is a unsymmetrical, we can consider full- cycle.



$$i = I_m \sin\theta \qquad 0 \leq \theta \leq \pi \qquad \ldots \text{(i)}$$

$$= 0 \qquad \pi \leq \theta \leq 2\pi \qquad \ldots \text{(ii)}$$

$$\text{Mean of } (i^2) = \frac{\int_0^{\pi} i^2 \, d\theta + \int_{\pi}^{2\pi} 0 \, d\theta}{2\pi}$$

$$= \frac{1}{2\pi}\int_0^{\pi} i^2 \, d\theta = \frac{1}{2\pi}\int_0^{\pi} (I_m \sin\theta)^2 \, d\theta$$

$$= \frac{I_m^2}{2\pi}\int_0^{\pi} \sin^2\theta \, d\theta = \frac{I_m^2}{2\pi}\int_0^{\pi}\left(\frac{1-\cos 2\theta}{2}\right) d\theta$$

$$= \frac{I_m^2}{4\pi}\left[\theta - \frac{\sin 2\theta}{2}\right]_0^\pi = \frac{I_m^2}{4\pi}[\pi - 0] = \frac{I_m^2}{4}$$

RMS value of current $= \sqrt{mean\ of\ (i^2)}$

$$= \sqrt{\frac{I_m^2}{4}} = \frac{I_m}{2}$$

$$\boxed{I = 0.5 \times I_m}$$

Form Factor

- It is defined as the ratio of r.m.s value of an alternating quantity.
- Mathematically,

$$\text{Form factor } (K_f) = \frac{\text{r.m.s. value of an alternating quantity}}{\text{average value of an alternating quantity}}$$

- In case of sinusoidal alternating current,

$$\text{r.m.s. value} = \frac{\text{Maximum value}}{\sqrt{2}} = 0.707 \times I_m$$

$$\text{and average value} = 0.637\ I_m$$

$$\therefore\ \text{Form factor} = \frac{0.707 \times I_m}{0.637 \times I_m}$$

$$= \boxed{1.11}$$

Peak Factor

- It is defined as the ratio of maximum value to r.m.s value of the alternating quantity.
- Mathematically,

$$\text{Peak factor (Kp)} = \frac{\text{Maximum value of alternating quantity}}{\text{r.m.s. value of alternating quantity}}$$

- For sinusoidal alternating current,

$$\text{Peak factor} = \frac{I_m}{I_m / \sqrt{2}} = \sqrt{2} = \boxed{1.414}$$

Examples:-

1. An alternating emf is represented by e=200sin 2π50t Find (i) Maximum value (ii) Frequency (iii) Time period (iv) Angular frequency.

Ans:-

$e = 200 \sin 2\pi\ 50\ t$. Comparing this equation with

$e = E_m \sin 2\pi f t$

The maximum value = $\boxed{200\ V}$ ... (i)

Frequency is f = 50 Hz ... (ii)

Time period $T = \frac{1}{f} = \frac{1}{50} = \boxed{0.02\ s}$ ... (iii)

Angular frequency $\omega = 2\pi f$

$= 2\pi 50 = \boxed{314.2\ \text{rad/sec.}}$ ... (iv)

2. A sinusoidal voltage has a value of 100 volts at 2.5 ms and it takes time of 20 ms to complete one cycle. Find the maximum value and time to reach it for the first time after zero.

Ans:-

$$T = 20 \text{ ms} \quad f = \frac{1}{T} = \frac{1}{20 \times 10^{-3}} = 50 \text{ Hz}$$

$$= 20 \times 10^{-3} \text{ sec}$$

$$e = 100 \text{ V When } t = 2.5 \text{ ms} = 2.5 \times 10^{-3} \text{ sec.}$$

(i) $e = E_m \sin 2\pi f t$

$$\therefore \quad 100 = E_m \sin (2 \times 180 \times 50 \times 2.5 \times 10^{-3})^0$$

$$= E_m \sin (45^\circ) = \frac{E_m}{\sqrt{2}}$$

$$\therefore \quad E_m = 100 \times \sqrt{2} = \boxed{141.4 \text{ volts}}$$

(ii) Now $e = 141.4 \sin (2\pi \times 50)\, t$

$e = 141.4 \sin (100\pi)\, t$

Now $e = E_m = 141.4$

$\therefore \quad 141.4 = 141.4 \sin (100 \times 180\, t)^0$

$\therefore \quad 1 = \sin (18000\, t)^0$

$\therefore \quad 18000\, t = 90°$

$$t = \frac{90}{18000} = \frac{1}{200}$$

$$= 5 \times 10^{-3} \text{ sec} = \boxed{5 \text{ ms}}$$

$\therefore \quad t = 5$ ms

3. RMS value of an alternating current is 30 A and its frequency is 25 Hz. Write its equation to find its instantaneous value. Also calculate (1) Its average value and (2) Time period.

Ans:- $I = 30$ A, $f = 25$ Hz

$$I_m = \sqrt{2} \times I = \sqrt{2} \times 30 = 42.43 \text{ A}$$

$$i = I_m \sin 2\pi f t$$

$$i = 42.43 \sin 2\pi \times 25 t$$

$$i = 42.43 \sin 50\pi t$$

(1) $I_{av} = 0.637 \, I_m = 0.637 \times 42.43$

$$I_{av} = \boxed{27 \text{ A}}$$

(2) $T = \dfrac{1}{f} = \dfrac{1}{25} = \boxed{0.01 \text{ s}}$

4. A sinusoidal alternating current is expressed by i=100 sin 377t. Calculate its (1) RMS value (2) Average value and (3) Frequency.

Ans:- $i = 100 \sin 377\,t$

comparing this with equation $i = I_m \sin 2\pi f t$

(1) $I_m = 100$ A

So $I_{RMS} = \frac{100}{\sqrt{2}} = \boxed{70.7\ A}$

(2) Average value $I_a = I_m \times 0.637$

$= 100 \times 0.637$

$I_a = \boxed{63.7\ A}$

(3) $2\pi f = 377$

$\therefore\ f = \frac{377}{2\pi} = \boxed{60\ Hz}$