Unit-7
Polyphase circuits

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Introduction

- The generator producing a single-phase supply has only one armature winding.
- But if the generator is arranged to have three separate but identical winding displaced 120 degree apart and rotate in a common magnetic field, it produces three voltages of same magnitude and frequency but displaced by 120 degree electrical from one another. This is called a three-phase system.

Advantages of 3-phase system over single-phase system

1. **Constant Power:-** In a single phase system, output power varies sinusoidally at a frequency twice the supply frequency.

- This pulsating nature of current is harmful to some applications whereas the balanced 3-phase system supplies constant current at all instants of time.

2. **Self start:-** The 3-phase systems are self starting as they do not require any starting device.

- However, single phase systems require starting device.

Advantages of 3-phase system over single-phase system

3. **Greater output:-** The power generated by a 3-phase system is greater than that of a single phase system for a given volume and weight of the generator.

- This is the distinct advantages over the single phase generator

4. **More economical:-** The 3-phase system is much smaller and less expensive than single phase system because less material is required for a given output power at a given voltage.

Advantages of 3-phase system over single-phase system

5. **Less voltage drop:-** The voltage drop from the generator to the load is less in a 3-phase system in comparison to the single-phase system
6. **Power transmission economics:-** The conductor material required to transmit a given power at a given voltage of material required in single phase system.

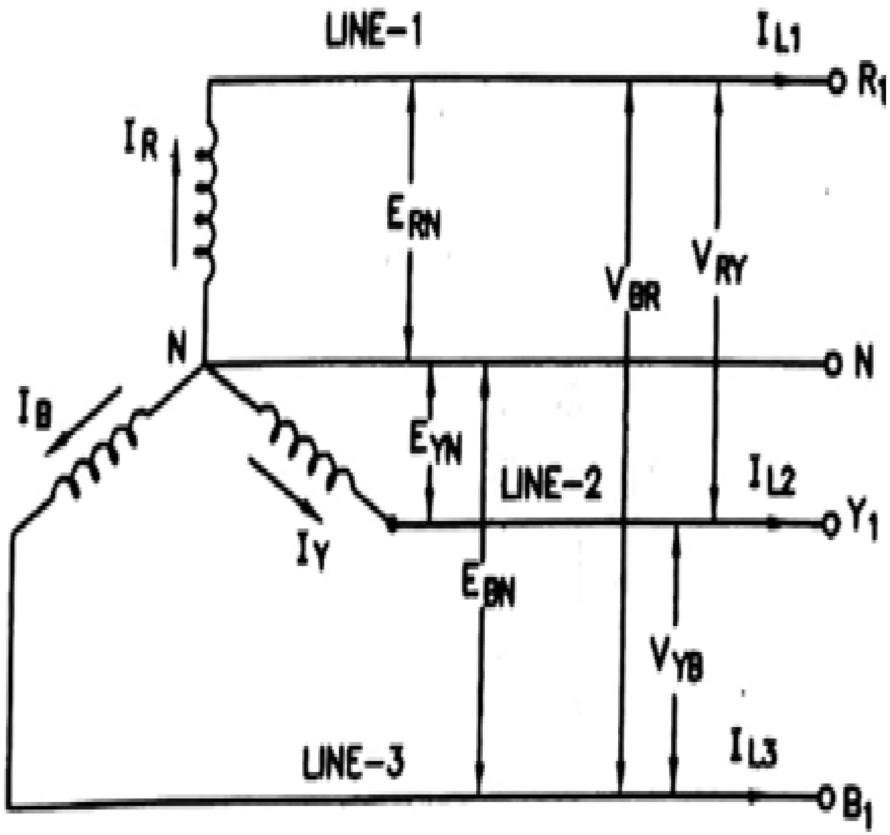
- This means a saving in material and strength of transmitting towers.

7. **High Efficiency:-** The 3-phase motors are efficient and have a higher power factor than single phase motors of the same capacity.

Phase sequence

- In three phase system, there are three voltages having same magnitude and frequency displaced by an electrical angle of 120 degree.
- They are attaining their positive maximum value in a particular order.
- The order in which voltages in the three-phase attain their maximum positive value is known as phase sequence.

Voltage and current relation in star connected system



- The emf across each winding is called phase voltage.
- They are denoted by E_{RN}, E_{YN} and E_{BN}.
- The voltage between any two lines is called line voltage.
- They are represented by V_{RY}, V_{YB} and V_{BR} respectively.
- Similarly currents flowing in the each winding is known as the phase current and current flowing in each line is called the line current.

- Since the system is balanced, $I_R = I_Y = I_B = I_{Ph}$

$$I_{L1} = I_{L2} = I_{L3} = I_L$$

$$E_{RN} = E_{YN} = E_{BN} = E_{Ph}$$

$$V_{RY} = V_{YB} = V_{BR} = V_L$$

- Relation between line current and phase current:

From Fig. 1, it is clear that

$$I_R = I_{L1} \rightarrow I_{Ph} = I_L$$

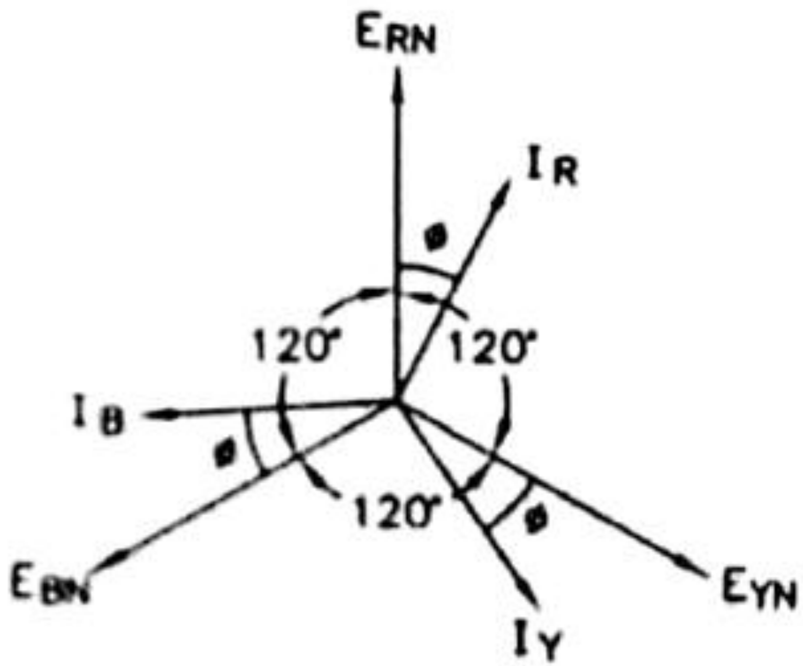
$$I_Y = I_{L2} \rightarrow I_{Ph} = I_L$$

$$I_B = I_{L3} \rightarrow I_{Ph} = I_L$$

Thus in star connection,

$$\boxed{\text{Line current, } I_L = \text{Phase current, } I_{Ph}}$$

- **Relation between line voltage and phase voltage**:
- It is seen from figure that in star connection, there are two phase windings between each pair of line terminals.
- Since similar ends of these two winding are connected together, the emfs across them oppose each other and their instantaneous values will have opposite polarities.
- Therefore the rms value of line voltage between any two lines will be obtained by the vector difference of the two phase voltages.
- The phasor diagram of the phase emfs and currents in a star connected system is shown below:



- Line voltage between terminals R and Y, $V_{RY} = E_{RN} + E_{NY}$

$$= E_{RN} + (-E_{YN})$$

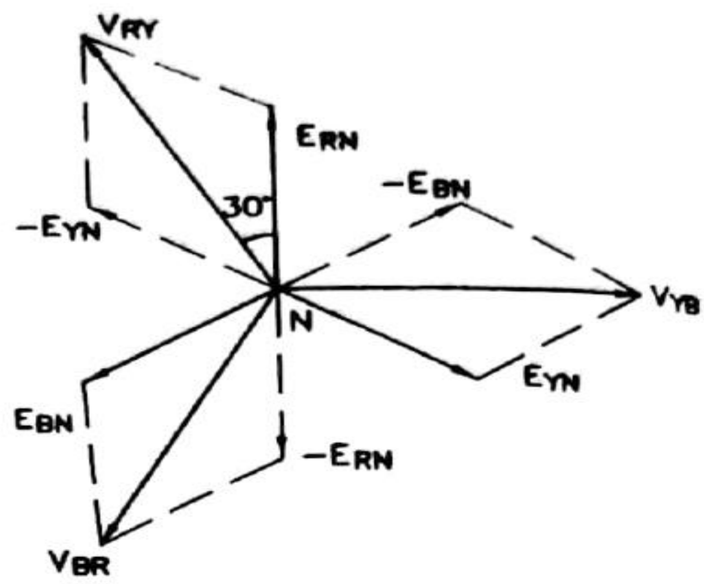
$$= E_{RN} - E_{YN}$$

$$= \text{phasor difference}$$

Similarly

$$E_{YB} = E_{YN} - E_{BN} \text{ and } \qquad E_{BR} = E_{BN} - E_{RN}$$

- Hence it is clear that in a star connected system, the line voltage is obtained as the vector difference of the two corresponding phase voltages.
- This is shown in fig below, for examples V_{RY} is found by adding V_{RN} and V_{YN} reversed and its magnitude is given by the diagonal of the parallelogram.



- Since side of the parallelogram are of equal length and angle between two phase voltages is 60 degree.
- The line voltage is given by,

$$V_{RY} = V_{RN} - V_{YN}$$

$$= 2\ V_{Ph}\ \cos\frac{60°}{2}$$

$$= 2\ V_{Ph}\ \cos 30°$$

$$= 2\ E_{Ph} \times \frac{\sqrt{3}}{2}$$

$$= \sqrt{3}\ E_{Ph}$$

Similarly $V_{YB} = V_{BR} = \sqrt{3}\ E_{Ph} = V_L$

Thus in balanced star connected system,

$$\boxed{V_L = \sqrt{3}\ E_{Ph}}$$

i.e. Line voltage = $\sqrt{3}$ × phase voltage

- The total power dissipated in the 3-phase star connected system is the arithmetic sum of the powers dissipated in the three phases.

$$\therefore \quad \text{Total power} = 3 \times \text{power per phase}$$

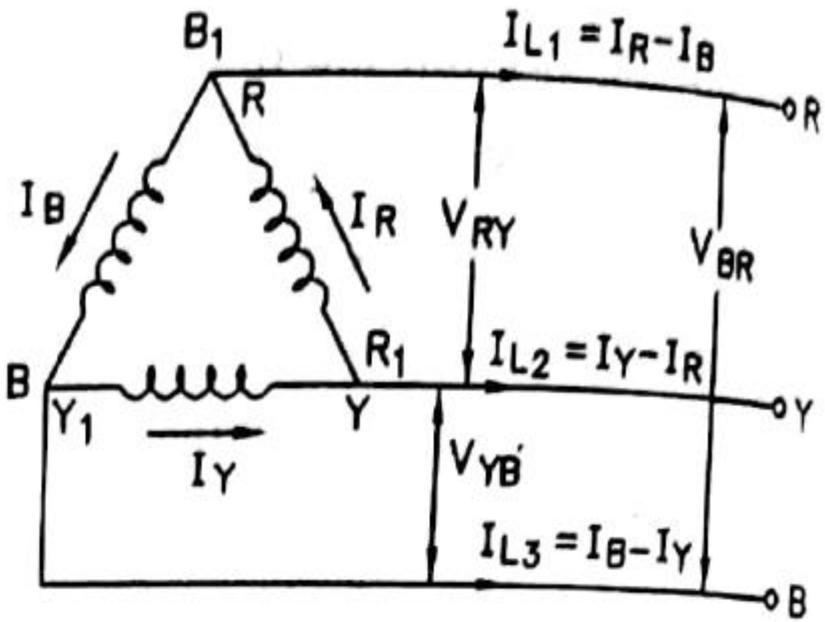
$$= 3 \times V_{ph} \times I_{ph} \times \cos\phi$$

$$= 3 \times \frac{V_L}{\sqrt{3}} \times I_L \times \cos\phi$$

$$= \boxed{\sqrt{3}\ V_L\ I_L \cos\phi}$$

It should be noted that ϕ is the angle between phase voltage and phase current and not between the line voltage and line current.

Voltage and current relation in delta connected system



- The emf across each winding is called phase voltage.
- They are denoted by E_R, E_Y and E_B.
- The voltage between any two lines is called line voltage.
- They are represented by V_{RY}, V_{YB} and V_{BR} respectively.
- Similarly currents flowing in the each winding is known as the phase current and current flowing in each line is called the line current.

- Since the system is balanced, $I_R = I_Y = I_B = I_{Ph}$

$$I_{L1} = I_{L2} = I_{L3} = I_L$$

$$E_R = E_Y = E_B = E_{Ph}$$

$$V_{RY} = V_{YB} = V_{BR} = V_L$$

- Relation between line voltage and phase voltage:

It is clear that

$$E_R = V_{RY} \rightarrow E_{Ph} = V_L$$

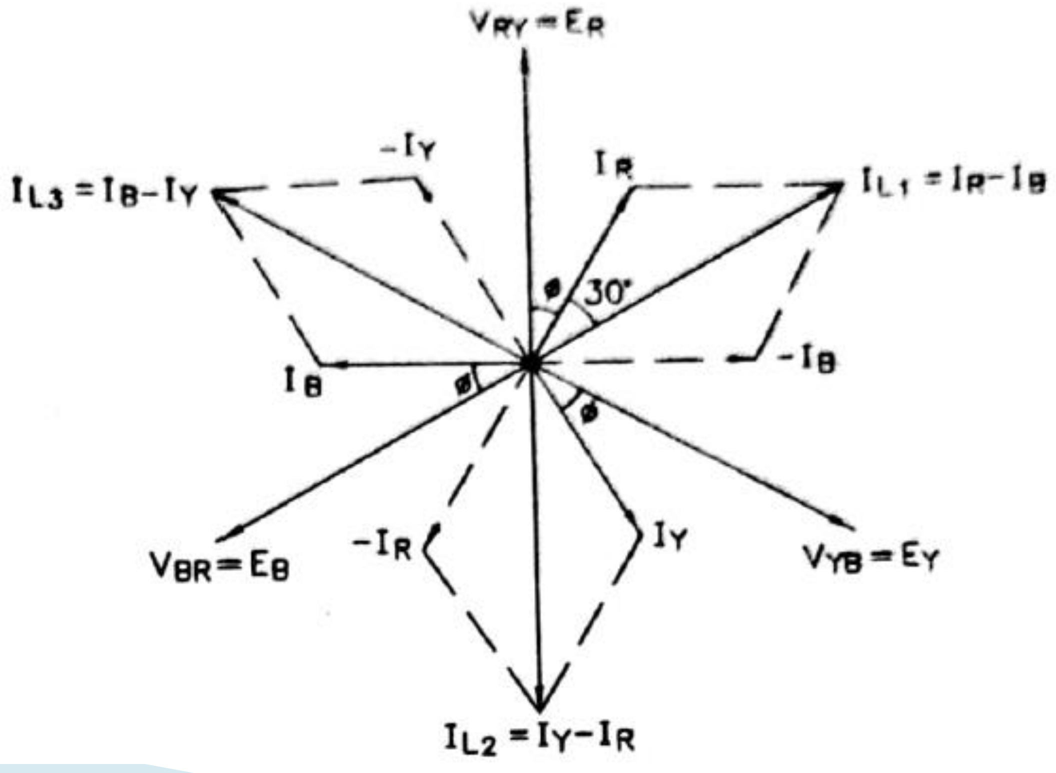
$$E_Y = V_{YB} \rightarrow E_{Ph} = V_L$$

$$E_B = V_{BR} \rightarrow E_{Ph} = V_L$$

Thus in delta connection,

$$\boxed{\textbf{Line voltage, } V_L = \textbf{Phase voltage, } E_{Ph}}$$

- **Relation between line current and phase current**:
- It is seen from figure that current flowing in each line is the vector difference of the two phase currents.
- Current in line 1, $I_{L1} = I_R - I_B$
- Current in line 2, $I_{L2} = I_Y - I_R$
- Current in line 3, $I_{L3} = I_B - I_Y$
- Current in line l can be found as the vector difference of the two corresponding phase currents. This is shown in fir below:



- I_{L1} can be obtained by adding I_R and I_B reversed and its value is given by the diagonal of the parallelogram as shown in vector diagram.
- Since the sides of parallelogram are equal in magnitude and the angle between them is 60 degree, the line current is given as:

$$I_{L1} = I_R - I_B \text{ (vector difference)}$$

$$= 2 \times I_{Ph} \times \cos\frac{60^\circ}{2}$$

$$= 2 \times I_{Ph} \times \cos 30^\circ$$

$$= 2 \times I_{Ph} \times \frac{\sqrt{3}}{2}$$

$$= \sqrt{3}\; I_{Ph}$$

Similarly $I_{L2} = I_{L3} = \sqrt{3}\; I_{Ph} = I_L$

Thus, in delta connection,

$$\boxed{\textbf{Line current, } I_L = \sqrt{3} \times \textbf{Phase current}}$$

- The total power in the 3-phase circuit is equal to the arithmetic sum of three phase power.
- Hence,

$$\text{Total power} = 3 \times \text{power per phase}$$

$$\boxed{= 3 \times V_{ph} I_{ph} \cos\phi}$$

1. A 415 V, 3-phase voltage is applied to a balanced star-connected 3-phase load of phase impedance (3+j4) ohms each. Calculate (i) line current and (ii) total power supplied in kW.

Solution:- star connection $V_L = 415$ V $Z_{ph} = 3 + j\,4$

$$V_P = \frac{V_L}{\sqrt{3}} = \frac{415}{\sqrt{3}} = 239.6 \text{ Volt}$$

$$Z_{ph} = 3 + j\,4 = \sqrt{3^2 + 4^2} = 5\ \Omega$$

$$I_{Ph} = \frac{V_{Ph}}{Z_{Ph}} = \frac{239.6}{5} = 47.92 \text{ A}$$

$$\therefore I_L = I_{ph} = \boxed{47.92 \text{ A}}$$

$$\cos\phi = \frac{R_{Ph}}{Z_{Ph}} = \frac{3}{5} = 0.6$$

$$\text{Power} = \sqrt{3}\, V_L\, I_L \cos\phi = \sqrt{3} \times 415 \times 47.92 \times 0.6$$

$$\boxed{= 20.666 \text{ kW}}$$

2. A balanced mesh-connected load of 6+j8 ohms per phase is connected to a 3-phase, 230 V supply. Find the line current, p.f and power.

Solution:- Connection Delta

$$Z_{ph} = 6 + j\,8$$
$$= 10\ \Omega \text{ (magnitude)}$$

$$V_L = V_{ph} = 230\text{ V}$$

$$\therefore \quad I_{ph} = \frac{V_{ph}}{Z_{ph}} = \frac{230}{10} = 23\text{ A}$$

$$\therefore \quad I_L = \sqrt{3}\, I_{ph} = \sqrt{3} \times 23 = \boxed{39.83\text{ A}}$$

$$\text{Power factor} = \cos\phi = \frac{R_{ph}}{Z_{ph}}$$
$$= \frac{6}{10}$$
$$\boxed{= 0.6\text{ (lag)}}$$

$$\text{Power} = \sqrt{3}\, V_L\, I_L \cos\phi$$
$$= \sqrt{3} \times 230 \times 39.83 \times 0.6$$
$$\boxed{= 9.520\text{ kW}}$$

3. Three similar coils each of resistance 15 ohms and inductance of 0.25 H are connected (i) in star and (ii) in delta to a 3-phase, 400 V, 50 Hz supply. Calculate line and phase values of current and voltage in both the cases. Also calculate the power absorbed.

Solution:- $R_{Ph} = 15\,\Omega$, $L_{Ph} = 0.25$ H $\quad X_{Ph} = \omega L_{Ph} = 2 \times \pi \times 50 \times 0.25 = 78.54$

$$Z_{Ph} = \sqrt{R_{Ph}^2 + X_{Ph}^2} = \sqrt{15^2 + 78.54^2} = 79.96\,\Omega$$

Star connection :

$$V_L = \boxed{400 \text{ V}}$$

$$V_{Ph} = \frac{V_L}{\sqrt{3}} = \frac{400}{\sqrt{3}} = \boxed{230.94 \text{ V}}$$

$$I_{Ph} = \frac{V_{Ph}}{Z_{Ph}} = \frac{230.94}{79.96} = \boxed{2.888 \text{ A}}$$

$$I_L = I_{Ph} = \boxed{2.888 \text{ A}}$$

$$\text{Power factor } \cos\phi = \frac{R_{Ph}}{Z_{Ph}} = \frac{15}{79.96} = \boxed{0.1875 \text{ (lagging)}}$$

$$\text{Total power} = \sqrt{3}\, V_L I_L \cos\phi \quad (\text{or} = 3 \times V_{Ph} \times I_{Ph} \cos\phi)$$

$$= \sqrt{3} \times 400 \times 2.888 \times 0.1875$$

$$\boxed{= 375.17 \text{ W}}$$

Delta connection :

$$V_L = \boxed{400\ \text{V}} \qquad V_{Ph} = V_L = \boxed{400\ \text{V}}$$

$$I_{Ph} = \frac{V_{Ph}}{Z_{Ph}} = \frac{400}{79.96} = \boxed{5.0025\ \text{A}}$$

$$I_L = \sqrt{3}\ I_{Ph} = \sqrt{3} \times 5.0025 = \boxed{8.664\ \text{A}}$$

$$\cos\phi = \frac{R_{Ph}}{Z_{Ph}} = \frac{15}{79.96} = \boxed{0.1875\ \text{(lag)}}$$

$$\text{Total power} = \sqrt{3}\ V_L I_L \cos\phi = \sqrt{3} \times 400 \times 8.664 \times 0.1875$$

$$\boxed{= 1125.56\ \text{watt}}$$


Thank you