NAME:- DHRUVIL DAVE     Reg No: 19BIT0065
Course Code:- MAT 1014     slot:- A1+TA1+TAA1+U1

.Q1) without using truth table show

(a) $(\sim P \wedge (\sim Q \wedge R)) \vee ((Q \wedge R) \vee (P \wedge R)) \Rightarrow R$

$= (\sim P \wedge \sim Q \wedge R) \vee (Q \wedge R \wedge (P \vee \sim P)) \vee (P \wedge R \wedge (Q \vee \sim Q))$

$= (\sim P \wedge \sim Q \wedge R) \vee (P \wedge Q \wedge R) \vee (\sim P \wedge Q \wedge R) \vee (P \wedge Q \wedge R)$
$\qquad \vee (P \wedge \sim Q \wedge R)$

$= (\sim P \wedge R)(Q \vee \sim Q) \vee (P \wedge Q \wedge R) \vee (P \wedge \sim Q \wedge R)$

$\qquad = (\sim P \wedge R) \vee (P \wedge R)$

$\qquad \qquad = R \wedge (\sim P \vee P)$

$\qquad \qquad \qquad = \underline{\underline{R}}$

(b) $\sim (P \Leftrightarrow Q) \Leftrightarrow (P \vee Q) \wedge \sim (P \wedge Q)$

$\qquad \sim \{(P \rightarrow Q) \wedge (Q \rightarrow P)\}$

$\qquad \sim (P \rightarrow Q) \vee \sim (Q \rightarrow P)$

$\qquad = \sim (\sim P \vee Q) \vee \sim (\sim Q \vee P)$

$\qquad = (P \wedge \sim Q) \vee (Q \wedge \sim P)$

$\qquad \qquad = (P \vee Q) \wedge (\sim P \vee \sim Q)$

$\qquad \qquad = P \wedge Q$

$\qquad \qquad = (P \vee Q) \wedge \sim (P \wedge Q)$

Q2) Obtain DNF of :-

(a) $P \rightarrow ((P \rightarrow Q) \land \sim(\sim Q \lor \sim P))$

$\Rightarrow \sim P \lor ((P \rightarrow Q) \land (Q \land P))$

$\sim P \lor ((\sim P \lor Q) \land (Q \land P))$

$\sim P \lor ((\sim P \land (Q \land P)) \lor (Q \land (Q \land P)))$

$$\boxed{\sim P \lor (\sim P \land P \land Q) \lor (Q \land Q \land P)}$$

(b) $\sim(P \lor Q) \longleftrightarrow (P \land Q)$

$\Rightarrow$

$(\sim(P \lor Q) \rightarrow (P \land Q)) \land [(P \land Q) \rightarrow \sim(P \lor Q)]$

$[(P \lor Q) \lor (P \land Q)] \land [(\sim P \lor \sim Q) \lor (\sim P \land \sim Q)]$

$[(P \lor Q \lor P) \land (P \lor Q \lor Q)] \land [(\sim P \lor \sim Q \lor \sim P) \land$

$(\sim P \lor \sim Q \lor \sim Q)]$

$(P \lor Q) \land (\sim P \lor \sim Q)$

$$\boxed{(P \land \sim P) \lor (P \land \sim Q) \lor (Q \land \sim P) \lor (Q \land \sim Q)}$$

3) Obtain CNF of :-

) $P \rightarrow ((P \rightarrow Q) \wedge \sim(\sim Q \vee \sim P))$

$\sim P \vee ((P \rightarrow Q) \wedge \sim(\sim Q \vee \sim P))$

$\sim P \vee ((\sim P \vee Q) \wedge (Q \wedge P))$

$(\sim P \vee \sim P \vee Q) \wedge (\sim P \vee (Q \wedge P))$

$(\sim P \vee Q) \wedge (\sim P \vee Q) \wedge (\sim P \vee P)$

b) $\sim(P \vee Q) \longleftrightarrow (P \wedge Q)$

$(\sim(P \vee Q) \rightarrow (P \wedge Q)) \wedge [(P \wedge Q) \rightarrow \sim(P \vee Q)]$

$[(P \vee Q) \vee (P \wedge Q)] \wedge [(\sim P \vee \sim Q) \vee (\sim P \wedge \sim Q)]$

$[(P \vee Q \vee P) \wedge (P \vee Q \vee Q)] \wedge [(\sim P \vee \sim Q \vee \sim P) \wedge$
$\qquad (\sim P \vee \sim Q \vee \sim Q)]$

$[(P \vee Q) \wedge (P \vee Q)] \wedge [(\sim P \vee \sim Q) \wedge (\sim P \vee \sim Q)]$

$[P \vee Q] \wedge [\sim P \vee \sim Q]$

04) PDNF:-

a) $(P \wedge Q) \vee (\sim P \wedge R) \vee (Q \wedge R)$

$$(P \wedge Q \wedge (R \vee {\sim}R)) \vee ({\sim}P \wedge R \wedge (Q \vee {\sim}Q)) \vee (|P \vee \ldots$$
$$Q \wedge R)?$$

$$= (P \wedge Q \wedge R) \vee (P \wedge Q \wedge {\sim}R) \vee ({\sim}P \wedge Q \wedge R) \vee ({\sim}P \wedge {\sim}Q$$
$$\vee (P \wedge Q \wedge R) \vee ({\sim}P \wedge Q \wedge R)$$

$$= \boxed{(P \wedge Q \wedge R) \vee (P \wedge Q \wedge {\sim}R) \vee ({\sim}P \wedge Q \wedge R) \vee ({\sim}P \wedge {\sim}Q \wedge R)}$$

(b) $({\sim}P \rightarrow R) \wedge (Q \leftrightarrow P) = S$

$({\sim}P \rightarrow R \Leftrightarrow P \vee R)$

| P | Q | R | P∨R | P→Q | Q→P | P↔Q | S |
|---|---|---|---|---|---|---|---|
| T | T | T | T | T | T | T | T |
| T | T | F | T | T | T | T | T |
| T | F | T | T | F | T | F | F |
| T | F | F | T | F | T | F | F |
| F | T | T | T | T | F | F | F |
| F | T | F | F | T | F | F | F |
| F | F | T | T | T | T | T | T |
| F | F | F | F | T | T | T | F |

$$= (P \wedge Q \wedge R) \vee (P \wedge Q \wedge {\sim}R) \vee ({\sim}P \wedge {\sim}Q \wedge R)$$

5) Obtain PCNF

a) $(\sim P \to R) \cap (Q \leftrightarrow P)$

$= (\sim \sim P \vee R) \cap ((\sim Q \vee P) \cap (\sim P \vee Q))$

$= (P \vee R) \cap ((\sim Q \vee P) \cap (\sim P \vee Q))$

$= (P \vee R \vee (Q \wedge \sim Q)) \cap (\sim Q \vee P \vee (R \wedge \sim R)) \cap (\sim P \vee Q \vee (R \wedge \sim R))$

$\Rightarrow (P \vee R \vee Q) \wedge (P \vee \sim Q \vee R) \wedge (P \vee \sim Q \vee \sim R)$
$\wedge (\sim P \vee Q \vee R) \wedge (\sim P \vee Q \vee \sim R)$

b) $Q \cap (P \vee \sim Q)$

$= (Q \vee (P \wedge \sim P)) \cap (P \vee \sim Q)$

$= (P \vee Q) \cap (\sim P \vee Q) \cap (P \vee \sim Q)$

Q6) Obtain PCNF & PDNF without using truth table

a) $(Q \cap \sim R \cap \sim S) \vee (R \cap S)$

(i) PDNF

$= (Q \cap \sim R \cap \sim S) \vee (R \cap S \cap (Q \vee \sim Q))$

$= (Q \cap \sim R \cap \sim S) \vee (Q \cap R \cap S) \vee (\sim Q \cap R \cap S) = A$

(ii) PCNF

In part (i)

$$A = (Q \wedge R \wedge S) \vee (Q \wedge \sim R \wedge \sim S) \vee (\sim Q \wedge R \wedge S)$$

$\sim A$ (remaining terms)

$$= (Q \wedge R \wedge \sim S) \vee (Q \wedge \sim R \wedge S) \vee (\sim Q \wedge R \wedge \sim S) \vee (\sim Q \wedge \sim R \wedge S)$$
$$\vee (\sim Q \wedge \sim R \wedge \sim S)$$

$$\sim (\sim A) = A$$

$$\sim A = (\sim Q \vee \sim R \vee S) \wedge (\sim Q \vee R \vee \sim S) \wedge (Q \vee \sim R \vee S)$$
$$\wedge (Q \vee R \vee \sim S) \wedge (Q \vee R \vee S)$$

$$\hookrightarrow PCNF$$

(b) $(P \leftrightarrow Q) \leftrightarrow ((P \leftrightarrow Q) \wedge (\sim Q \vee P))$

$$((P \to Q) \wedge (Q \to P)) \leftrightarrow ((\sim P \vee Q) \wedge (P \vee \sim Q))$$

$$= ((\sim P \vee Q) \wedge (P \vee \sim Q)) \leftrightarrow ((\sim P \vee Q) \wedge (P \vee \sim Q))$$

$$= ((\sim P \vee Q) \wedge (P \vee \sim Q)) \to ((\sim P \vee Q) \wedge (P \vee \sim Q)) \wedge$$
$$((\sim P \vee Q) \wedge (P \vee \sim Q)) \to ((\sim P \vee Q) \wedge (P \vee \sim Q))$$

$\not\equiv$

$$(P \wedge \sim Q) \vee (\sim P \wedge Q)) \vee ((\sim P \vee Q) \wedge (P \vee \sim Q)) \wedge$$

$$[((P \wedge \sim Q) \vee (\sim P \wedge Q) \vee ((\sim P \vee Q) \wedge (P \vee \sim Q))])$$

$$((P \wedge \sim Q) \vee (\sim P \wedge Q)) \vee ((\sim P \vee Q) \wedge (P \vee \sim Q)))$$

$$= (P \wedge \sim Q) \vee (\sim P \wedge Q) \vee (P \wedge \sim P) \vee (\sim P \wedge \sim Q)$$
$$\vee (P \wedge Q) \vee (Q \wedge \sim Q)$$

$$= (P \wedge Q) \vee (P \wedge \sim Q) \vee (\sim P \wedge Q) \vee (\sim P \wedge \sim Q)$$

$$\quad \quad \quad \quad \llcorner \!\!\!\rightarrow PDNF$$

$$= Tautology$$

PCNF => Does not exist

Q7) Using rule CP, prove that

(a) $P \to Q$, $Q \to R$, $R \to S \Rightarrow P \to S$

| 1 | $P \to Q$ | Rule P |
|---|---|---|
| 2 | $Q \to R$ | Rule P |
| 3 | $P \to R$ | Rule T (1),(2) |
| 4 | $R \to S$ | Rule P |
| 5 | $P \to S$ | Rule T (3),(4) |

(b) $P \lor Q$, $P \to R$, $Q \to S \Rightarrow \sim S \to R$

Given premises are $\to P \lor Q$, $P \to R$, $Q \to S$

Assumed premise $\to \sim S$

| 1 | $P \lor Q$ | rule P |
|---|---|---|
| 2 | $\sim P \to Q$ | rule T |
| 3 | $Q \to S$ | rule P |
| 4 | $\sim P \to S$ | Hypothetical Syllogism |
| 5 | $\sim S$ | rule P |
| 6 | $P$ | Modus Tollens (4)(5) |
| 7 | $P \to R$ | rule P |
| 8 | $R$ | Modus Pollens (6),(7) |
| 9 | $\sim S \to R$ | rule CP (5),(8) |

2) Show that the following premises
were inconsistent
a) If Jack misses many classes through illness, then fails in high school
b) If Jack fails in high school, then he is uneducated
c) If Jack reads a lot of Books, then is not uneducated
d) Jack misses many classes through illness and reads a lots of books

$\Rightarrow$ $A(x)$ = Jack misses classes
$B(x)$ = Jack fails class in school
$C(x)$ = Jack is uneducated
$D(x)$ = Jack reads many books

Premises :- $A(x) \rightarrow B(x)$, $B(x) \rightarrow C(x)$, $D(x) \rightarrow \sim C(x)$

$A(x) \cap D(x)$

| | | |
|---|---|---|
| 1. | $A(x) \rightarrow B(x)$ | Rule P |
| 2 | $B(x) \rightarrow C(x)$ | Rule P |
| 3 | $A(x) \rightarrow C(x)$ | Hypothetical Syllogism |
| 4 | $A(x) \cap D(x)$ | rule P |
| 5 | $D(x)$ | |
| 6 | $A(x)$ | |
| 7 | $D(x) \rightarrow \sim C(x)$ | rule P |
| 8 | $\sim C(x)$ | modus Pollens (3), (6) |

9    $\sim D(x)$                            modus tollens

10   $D(x) \wedge \sim D(x) \Leftrightarrow F$

Q9) If A work hard, then either B or C
will enjoy themselves;

If B ~~work~~ enjoys himself, then A will
not work hard;

If D enjoys themselves, then C will not

Therefore If A works hard then D will
not enjoy himself.

$\Rightarrow$   A: A works hard
        B: B enjoys himself
        C: C enjoys himself
        D: D enjoys himself

Premises :- $A \to (B \vee C)$ ; $D \to \sim C$ ; $B \to \sim A$.

Assumed premise :- A

conclusion premise :-   $A \to \sim D$

| 1 | A | rule P |
|---|---|---|
| 2 | $A \to (B \vee C)$ | rule P |
| 3 | $B \vee C$ | modus Pollens (1),(2) |
| 4 | $B \to \sim A$ | rule P |
| 5 | $\sim B$ | Modus tollens |
| 6 | $\sim B \wedge (B \vee C)$ | rule T (5),(3) |
| 7 | $\sim B \wedge C$ | rule T (6) |

8.  C

9.  $D \to \sim C$     rule P

10    $\sim D$     Modus Tollens (8)

11    $A \to \sim D$     rule CP

10) Indirect method

a) $E \to S \,,\, S \to H \,,\, A \to \sim H \Rightarrow \sim (E \wedge A)$

    Assumed premises :- $E \wedge A$

| | | |
|---|---|---|
| 1. | $E \to S$ | Rule P |
| 2. | $S \to H$ | Rule P |
| 3. | $E \to H$ | Hypo. syllogism (1)(2) |
| 4. | $A \to \sim H$ | Rule P |
| 5. | $E \wedge A$ | Rule P |
| 6. | $A$ | simplification (5) |
| 7 | $E$ | simplification (5) |
| 8 | $H$ | Modus Pollen (3)(7) |
| 9 | $\sim H$ | Modus Pollen (4)(6) |
| 10 | $H \wedge \sim H \Leftrightarrow F$ | |

b) $P \to (Q \wedge R) \,,\, (Q \vee S) \to U \wedge (P \vee S) = U$

       Assumed premise $\Rightarrow \sim U$

| | | |
|---|---|---|
| 1 | $\sim U$ | rule P |
| 2 | $(Q \vee S) \to U$ | rule P |
| 3 | $\sim (Q \vee S)$ | M T (1)(2) |
| 4 | $\sim Q \wedge \sim S$ | rule T |
| 5 | $\sim S$ | Simplification |
| 6 | $\sim Q$ | Simplification |

7  $P \lor S$      rule P
8  $P$             D.S. (5)(7)
9  $P \to (Q \land R)$   Rule P
10 $Q \land R$     M.P. (8)(9)
11 $Q$             Simplification
12 $Q \land \sim Q = F$

$A = F$
$\sim A = T$