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Optimal Collision Avoidance Maneuver for Formation Flying Satellites Using Relative Orbital Elements

Hu Min, Zeng Guoqiang and Song Junling

***Abstract*—The safety of the formation flying satellites in the cross-track plane is investigated. It is important to ensure a minimum separation in the cross-track plane due to the uncertainty of the along-track drift. Once the satellite is within the avoidance region, the collision avoidance maneuver with optimization is planned to reach a safe ellipse, which remains out of the avoidance region. Firstly, the formation configuration is described based on the relative orbital elements. Secondly, formulas of the parameters of the ellipse in the cross-track plane are proposed and the formation control strategy based on the Gauss perturbation equations is put forward. Finally, the energy of the collision avoidance maneuver is expressed by the rotating angle of the safe ellipse in the cross-track plane, which is optimized by the Newton method. The simulation results indicate the simplicity and effectiveness of the presented method.**

***Keywords*—Collision avoidance maneuver; Formation flying satellites; Optimization; Relative orbital elements.**

I. INTRODUCTION

FORMATION flying is a key technology of distributed satellite system, which enables many future space missions. The idea of using several small, unconnected satellites to operate in a coordinated way may get better performance than a single monolithic satellite, such as increased instrument resolution, reduced cost, reconfigurability, and overall system robustness. An underlying requirement for the close formation flying is to ensure collision-free operations even in the presence of inaccuracy of relative states and possible failures of one of the satellites.

Several ambitious distributed satellite missions are currently being developed, such as the TanDEM-X mission by German Aerospace Center (DLR) and the PRISMA mission by Swedish Space Corporation (SSC). TanDEM-X comprises two nearly identical satellites (TSX and TDX) that will fly in close proximity and collect SAR interferograms for digital elevation model (DEM) generation[1]. The concept of e/i-vector separation has been adopted for the TanDEM-X mission and the “helix” orbit configuration has been preferred, which makes the distance between the two satellites in the cross-track plane always larger than 200m. The PRISMA mission will demonstrate Guidance, Navigation and Control strategies for advanced autonomous formation flying[2].

Research about collision avoidance maneuver control has made a lot of progress[3,4,5,6,7,8]. G.L. Slater[3]has calculated the collision probabilities of tight formation flight and velocity correction requirements to avoid collisions. J. B. Mueller[4,5,6] has discussed the relative orbit dynamics in circular and eccentric orbits and describes the methods of planning collision avoidance maneuvers by formulating the problem as a linear program(LP). A. Richards[7]has proposed a method for optimal spacecraft maneuver planning with collision avoidance constraints, and solved the problem by mixed integer linear program(MILP).

This paper focuses on the optimal collision avoidance maneuver by using the relative orbital elements. After a discussion of the relative motion model, the analytical relationship between the parameters of the formation configuration and the parameters of the ellipse in the cross-track plane are proposed, the target formation configuration is described by the rotating angle of the safe ellipse, and is optimized by the Newton method. A collision avoidance control strategy is therefore designed, and the method is validated by a numeric simulation.

II. FORMATION CONFIGURATION DESCRIPTION BASED ON THE RELATIVE ORBITAL ELEMENTS

The relative motion model has been extensively studied in the past. With respect to a near-circular reference orbit, and assuming the satellites are taken sufficiently close to each other, the relative motion given by several Keplerian elements differing can be treated to first order.

The Hill frame is defined as follows. The x axis is aligned in the radial direction, the z axis is aligned with the angular momentum vector and the y axis completes the right-handed system[9].

The relative dynamics discussed in this paper will make use of the orbital element sets:

$$[a, e, i, \Omega, \omega, M]$$

where a is the semi-major axis, e is the eccentricity, i is the

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inclination, Ω is the right ascension(longitude) of the ascending node, ω is the argument of perigee and M is the mean anomaly. The mean argument of latitude $u = \omega + M$.

Assuming that spacecraft-1 is the master and spacecraft-2 is the chaser, considering the reference orbit is circular or near-circular, and the relative motion is small, then the eccentricity vector $\Delta\boldsymbol{e}$ can be defined as follows:

$$\Delta\boldsymbol{e} = \begin{bmatrix} \Delta e_x \\ \Delta e_y \end{bmatrix} = \delta e \begin{bmatrix} \cos\varphi \\ \sin\varphi \end{bmatrix} = e_2 \begin{bmatrix} \cos\omega_2 \\ \sin\omega_2 \end{bmatrix} - e_1 \begin{bmatrix} \cos\omega_1 \\ \sin\omega_1 \end{bmatrix} \tag{1}$$

$$\varphi = arc\tan(\Delta e_y, \Delta e_x) \tag{2}$$

where δe is the module of the eccentricity vector $\Delta\boldsymbol{e}$, φ is the initial phase angle in the orbital plane.

The inclination vector $\Delta\boldsymbol{i}$ can be defined as follows:

$$\Delta\boldsymbol{i} = \begin{bmatrix} \Delta i_x \\ \Delta i_y \end{bmatrix} = \delta i \begin{bmatrix} \cos\theta \\ \sin\theta \end{bmatrix} = \begin{bmatrix} \Delta i \\ \Delta\Omega\sin i_1 \end{bmatrix} \tag{3}$$

$$\theta = arc\tan(\Delta i_y, \Delta i_x) \tag{4}$$

where δi is the module of the inclination vector $\Delta\boldsymbol{i}$, θ is the initial phase angle in the cross-track plane.

In the case of near-circular reference orbit and $\Delta a = 0$, the relative motion of the formation flying satellites can be described by the following equations:

$$\begin{cases} x = -p\cos(u-\varphi) \\ y = 2p\sin(u-\varphi) + l \\ z = s\sin(u-\theta) \end{cases} \tag{5}$$

where p is the semi-minor axis of the relative in-plane ellipse, s is the cross-track amplitude, α is the difference between the in-plane initial phase angle and the cross-track initial plane angle ($\alpha = \varphi - \theta$), φ is the in-plane initial phase angle, and l is the along-track offset of the center of the in-plane motion. An example trajectory is shown in Fig. 1.

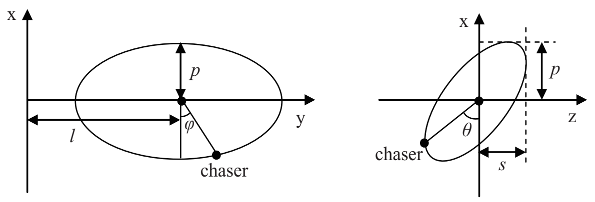

Fig. 1. Example of relative motion in near-circular reference orbit.

The parameters of formation configuration (p, s, α, φ, l) can be expressed by the relative orbital elements:

$$\begin{cases} p = a\delta e \\ s = a\delta i \\ \alpha = \varphi - \theta \\ \varphi = \varphi \\ l = a\Delta u \end{cases} \tag{6}$$

where $\Delta u = u_2 - u_1$.

III. The Geometry of Formation Configuration in the Cross-Track Plane

The projection of the formation configuration in the cross-plane is an ellipse, which can be defined by three parameters(a, b, ϕ). a is the semi-major axis of the ellipse, b is the semi-minor axis of the ellipse, and ϕ is the rotating angle of the ellipse. The ellipse in the cross-track plane is shown in Fig. 2.

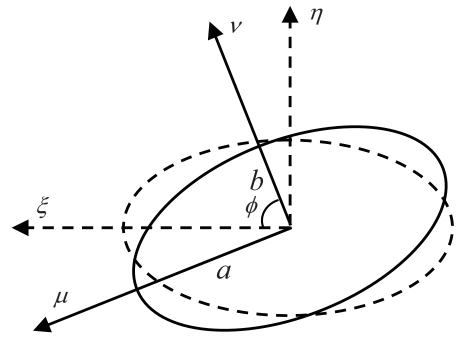

Fig. 2. The ellipse in the cross-track plane.

The distance in the cross-track plane can be expressed as follows:

$$r = \sqrt{x^2 + z^2} \tag{7}$$

By substituting (5) into (7):

$$\begin{aligned} r &= \sqrt{p^2\cos^2(u-\varphi) + s^2\sin^2(u-\theta)} \\ &= \sqrt{\frac{p^2 + s^2 + p^2\cos 2(u-\varphi) - s^2\cos 2(u-\theta)}{2}} \end{aligned} \tag{8}$$

where

$$\begin{aligned} &[p^2\cos 2(u-\varphi) - s^2\cos 2(u-\theta)]^2 + [p^2\sin 2(u-\varphi) - s^2\sin 2(u-\theta)]^2 = \\ &p^4 + s^4 - 2p^2s^2\cos 2(\theta-\varphi) = p^4 + s^4 - 2p^2s^2\cos 2\alpha \end{aligned} \tag{9}$$

so that

$$\left|p^2\cos 2(u-\varphi) - s^2\cos 2(u-\theta)\right| \le \sqrt{p^4 + s^4 - 2p^2s^2\cos 2\alpha} \tag{10}$$

By substituting (10) into (8), we can get the maximum and the minimum distance in the cross-track plane, then, the semi-major axis a and the semi-minor axis b can be expressed as follows:

$$a = \sqrt{\frac{p^2 + s^2 + \sqrt{p^4 + s^4 - 2p^2s^2\cos 2\alpha}}{2}} \tag{11}$$

$$b = \sqrt{\frac{p^2 + s^2 - \sqrt{p^4 + s^4 - 2p^2s^2\cos 2\alpha}}{2}} \tag{12}$$

Expressed in the $\xi - \eta$ plane, (5) becomes

$$\begin{cases}\eta = -p\cos(u-\varphi) = -p\cos\varphi\cos u - p\sin\varphi\sin u \\ \xi = s\sin(u-\theta) = -s\sin\theta\cos u + s\cos\theta\sin u\end{cases} \tag{13}$$

In the $\mu - \nu$ plane, the temporal equations of the ellipse are:

$$\begin{cases}\nu = -b\sin(u+\varphi_0) \\ \mu = a\cos(u+\varphi_0)\end{cases} \tag{14}$$

where φ_0 is the phase angle of the initial point.

We can define the transformation:

$$\begin{bmatrix} w \\ \eta \\ \xi \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & \sin(-\phi) \\ 0 & -\sin(-\phi) & \cos\phi \end{bmatrix} \begin{bmatrix} 0 \\ -b\sin(u+\varphi_0) \\ a\sin(u+\varphi_0) \end{bmatrix} \tag{15}$$

By expanding (15) and comparing to (13), the rotating angle ϕ can be expressed as follows:

$$\phi = \frac{1}{2}(\arctan\frac{p\cos\varphi + s\cos\theta}{p\sin\varphi + s\sin\theta} + \arctan\frac{p\cos\varphi - s\cos\theta}{s\sin\theta - p\sin\varphi}) \tag{16}$$

IV. Optimal Collision Avoidance Maneuver

A. Parameter of the Target Configuration

Because of the uncertainty of the along-track drift due to the perturbations, it is essential to ensure the minimum distance in the cross-track plane to guarantee the safety of the formation flying satellites. The region which the distance in the cross-track plane between the satellites is less than the pre defined minimum distance is called as the avoidance region.

When the satellite is already within the avoidance region, assuming that the cross-track amplitude s does not change, so there is no need of the cross-track control, and the along-track control is more efficient and practical. Assuming the radius of the avoidance circle in the cross-track plane is r, which equals the semi-minor axis of the safe ellipse b. Then, p of the target configuration can be calculated according to (12):

$$p = \sqrt{\left|\frac{r^2 - s^2}{r^2 - s^2\cos^2\alpha}\right|}\, r \tag{17}$$

If the formation flying satellites have the same inclination, then the phase angle of the inclination vector $\Delta\boldsymbol{i}$ equals $\pi/2$, so $\alpha = \varphi + \pi/2$. (16) can be simplified as:

$$\phi = \begin{cases} \dfrac{1}{2}\arctan\dfrac{2ps\cos\varphi}{s^2 - p^2} & p < s \\ \pm\dfrac{\pi}{2} + \dfrac{1}{2}\arctan\dfrac{2ps\cos\varphi}{s^2 - p^2} & p > s \end{cases} \tag{18}$$

α of the target configuration can be calculated according to (18):

$$\alpha = \frac{\pi}{2} \pm \arccos\frac{(s^2 - p^2)\tan(2\phi)}{2ps} \tag{19}$$

By combining (17) and (19), The analytic solution of p can be functioned as $p = f(r, s, \phi)$.

B. Formation Configuration Control

Assuming the nominal parameters of the safe configuration in the orbital plane are p_1 and φ_1, and the current parameters of configuration in the orbital plane are p_2 and φ_2. According to (5), the relative position of the chaser spacecraft to the master spacecraft in the orbital plane can be described as:

$$\begin{cases} x = -p_2\cos(u-\varphi_2) + p_1\cos(u-\varphi_1) \\ y = 2p_2\sin(u-\varphi_2) - 2p_1\sin(u-\varphi_1) \end{cases} \tag{20}$$

which equals

$$\begin{cases} x = -p_0\cos(u-\varphi_0) \\ y = 2p_0\sin(u-\varphi_0) \end{cases} \tag{21}$$

where

$$\begin{cases} p_0 = \sqrt{p_1^2 + p_2^2 - 2p_1p_2\cos(\varphi_2 - \varphi_1)} \\ \varphi_0 = \arctan\dfrac{p_2\sin\varphi_2 - p_1\sin\varphi_1}{p_2\cos\varphi_2 - p_1\cos\varphi_1} \end{cases} \tag{22}$$

The problem of controlling the current configuration to achieve the safe configuration equals the problem of controlling p_0 to zero. When the reference orbit is near-circular, according to the Gauss perturbation equation[10], the variances of the relative orbital elements can be expressed by the along-track Δv:

$$\begin{cases} \Delta\Delta a = (2a/V)\Delta v \\ \Delta\Delta l = -(3t)\Delta v \\ \Delta\Delta e_x = (2/V)\Delta v\cos u \\ \Delta\Delta e_y = (2/V)\Delta v\sin u \end{cases} \tag{23}$$

where V is the orbital velocity.

According to (1), the relative orbital element and the parameters of configuration have the following relationships:

$$\begin{bmatrix} \Delta e_{x0} \\ \Delta e_{y0} \end{bmatrix} = \frac{p_0}{a}\begin{bmatrix} \cos\varphi_0 \\ \sin\varphi_0 \end{bmatrix} \tag{24}$$

To control p_0 to zero equals the problem of controlling Δe_{x0} and Δe_{y0} to zero, so that

$$\begin{cases} (2/V)\Delta v\cos u = -(p_0/a)\cos\varphi_0 \\ (2/V)\Delta v\sin u = -(p_0/a)\sin\varphi_0 \end{cases} \Rightarrow \begin{cases} \Delta v = (p_0V/2a) \\ u = \varphi_0 + \pi \end{cases} \tag{25}$$

One possible solution is

$$\Delta v_1 = -\Delta v/2 \qquad \Delta v_2 = \Delta v/2 \tag{26}$$

at the following mean argument of latitude

$$u_1 = \varphi_0 \qquad u_2 = \varphi_0 + \pi \tag{27}$$

C. Optimal Collision Avoidance Maneuver

The next step is to choose the optimal p_1 and φ_1 to achieve the minimum fuel. As both p_1 and φ_1 can be parameterized by the rotating angle of the target safe ellipse as described in (17) and (19), p_0 can also be parameterized by the rotating angle of the target safe ellipse as described in (22).The fuel optimal problem turns to find the certain ϕ which makes p_0 minimum according to (25). This can be transformed into a pure mathematical problem as follows:

$$\min p_0(\phi), \phi \in (0, 2\pi) \tag{28}$$

By using the Taylor series expansion, $p_0(\phi)$ can be estimated by $\varphi(x)$ as follows:

$$\varphi(x) = p_0(\phi^{(k)}) + p_0'(\phi^{(k)})(\phi - \phi^{(k)}) + \frac{1}{2} p_0''(\phi^{(k)})(\phi - \phi^{(k)})^2 \tag{29}$$

so that

$$\varphi'(x) = p_0'(\phi^{(k)}) + p_0''(\phi^{(k)})(\phi - \phi^{(k)}) \tag{30}$$

and the stagnation point of $\varphi(x)$ can be achieved, denoted by $\phi^{(k+1)}$:

$$\phi^{(k+1)} = \phi^{(k)} - \frac{p_0'(\phi^{(k)})}{p_0''(\phi^{(k)})} \tag{31}$$

By using the iteration (31), the optimal ϕ can be achieved very quickly.

V. NUMERICAL SIMULATION

To illustrate the effectiveness of the optimal collision avoidance maneuver method, we consider the simulation shown in Fig. 3.

The reference orbit is { a =6892937.0m, e =0.00117, i = 97.4438°, Ω =90.0°, ω = 0.0°, M = 0.0°}. The initial formation configuration parameter is { p = 300m, s = 400m, φ = 23°, θ = -90°}. The radius of the safe ellipse is 200m.

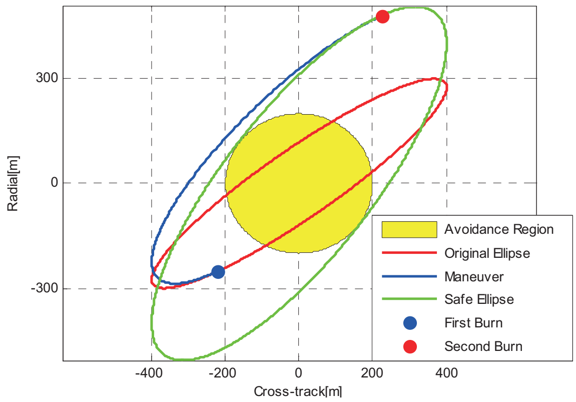

Fig. 3. Optimal maneuver to reach a safe ellipse.

From Fig. 3, we can see the original ellipse intersects the projected collision region, and the optimal collision avoidance maneuver method based on the relative orbital elements was used to reach the safe ellipse, the total Δv in this case is 0.126m/s.

VI. CONCLUSIONS

This paper has presented the optimal collision avoidance maneuver by using the relative orbital elements. The target formation configuration is deduced by the parameters of the safe ellipse which is intuitive, and is optimized by the Newton method, which has a linear relationship with the fuel needed. The simulation results indicate that the proposed method is easy for engineering implementation and efficient for formation collision avoidance control. The further research is needed to extend this method to elliptical orbits.

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