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Thrust Parameter Identification for Orbital Climbing of Continuous Thrust Spacecraft

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***Abstract*—Based on the analysis of the dynamic characteristics of a continuous thrust spacecraft, the variation of its orbit radius during orbital climb is studied, and the process of orbital climb is described in a formulaic manner, followed by identification of its thrust parameters. Firstly, describe the basic problem of orbital climb, analyze the spatial coordinate system and radar visibility characteristics; Secondly, formula derivation was carried out, and the transformation between spatial coordinate systems and the McLaughlin expansion of orbit radius were carried out; Finally, experimental simulations were designed to verify the accuracy of the proposed thrust parameter identification method. Using the LM algorithm, the solution accuracy can reach 1.5%.**

Keywords- Continuous Thrust; Orbital Climbing; Parameter Identification; LM algorithm

I. INTRODUCTION

With the continuous deepening of human exploration of the universe, research on continuous thrust spacecraft is also receiving increasing attention. Equipped with an electric thruster, it can carry out long-term propulsion on orbit, greatly improving the efficiency of task completion. Compared with traditional pulse maneuvering spacecraft, continuous thrust spacecraft has better flexibility and controllability, and can adapt to more complex task requirements. SpaceX plans to deploy approximately 12000 "StarLink" satellites in near-Earth space by 2024, and approximately 42000 "StarLink" satellites by 2027. According to Starlink's deployment strategy [1], after sending satellites into space using launch vehicles, all satellites will undergo orbital climb to enter the target orbit. The propulsion system is a crucial component in the orbital climb process of spacecraft. Continuous thrust spacecraft, as a common launch method, needs to continuously provide thrust during orbital climb to achieve control of orbital height and speed.

Reference [2] provides a satellite orbit determination algorithm with continuous thrust control, which can estimate continuous thrust acceleration. However, it generally calculates larger thrust and does not identify thrust in the millinewton order; Reference [3] can perform maneuver detection on non cooperative space target orbits under different thrusts, which can meet the majority of non cooperative target orbit maneuver detection needs. However, it can only estimate the maneuver size and direction by determining the range of thrust magnitude, and cannot provide accurate thrust parameter values; The propulsion system of continuous thrust spacecraft is influenced by various factors, such as fuel consumption, thruster wear, etc., which cause changes in thrust parameters, thereby affecting the motion status and control accuracy of the spacecraft. Therefore, it is necessary to identify thrust parameters during the flight of spacecraft. This article will introduce the thrust parameter identification method for continuous thrust spacecraft based on the basic principle of orbital climb of thrust spacecraft. Firstly, analyze the thrust required for orbital climb and obtain its dynamic equation and control method. Secondly, the Levenberg Marquardt method is used to identify thrust parameters. Finally, the effectiveness and feasibility of the proposed method were verified through experiments, and future research was prospected.

II. BASIC PROBLEM DESCRIPTION

A. Track Climbing

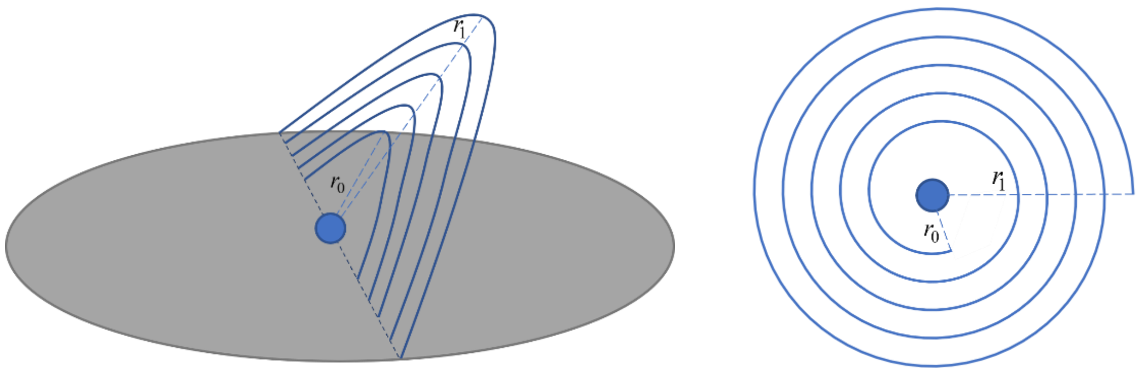
Continuous thrust spacecraft orbital climb refers to the process of elevating a spacecraft from its initial orbital altitude to the target orbital altitude, using only continuous thrust for spacecraft orbital maneuvers, which is an extremely important step in many space missions. Thrust parameter identification refers to the determination of the magnitude and variation patterns of thrust parameters through modeling and data analysis of the thrust system. This is crucial for the design, control, and optimization of thrust spacecraft. However, due to the complexity of the thrust system itself and the interference of environmental factors, accurate identification of thrust parameters is a difficult problem that requires the use of advanced mathematical models and algorithms for analysis and calculation. Starting from the perturbation equation, analyze the thrust required for spacecraft orbital climb. For Johannes Kepler orbital parameters $[a, e, i, \Omega, \omega, M]^{\mathrm{T}}$, denoted as the control force in radial, heading, and normal directions $[F_R, F_T, F_N]^{\mathrm{T}}$, and the control force per unit mass is $\bar{F}_R = \frac{F_R}{m}, \bar{F}_T = \frac{F_T}{m}, \bar{F}_N = \frac{F_N}{m}$. Under the action of $[\bar{F}_R, \bar{F}_T, \bar{F}_N]^{\mathrm{T}}$, the Gaussian perturbation equation of orbital motion [4] is

$$
\begin{cases}
\dot{a} = \dfrac{2}{n\sqrt{1-e^2}}\left[\bar{F}_R \cdot e \cdot \sin f + \bar{F}_T \cdot (1+e\cdot\cos f)\right] \\
\dot{e} = \dfrac{\sqrt{1-e^2}}{na}\left[\bar{F}_R \cdot \sin f + \bar{F}_T \cdot \left(\cos f + \dfrac{e+\cos f}{1+e\cdot\cos f}\right)\right] \\
\dot{i} = \dfrac{r\cos u}{na^2\sqrt{1-e^2}}\bar{F}_N \\
\dot{\Omega} = \dfrac{r\sin u}{na^2\sqrt{1-e^2}\sin i}\bar{F}_N \\
\dot{\omega} = \dfrac{\sqrt{1-e^2}}{nae}\left[-\bar{F}_R \cdot \cos f + \bar{F}_T \cdot \left(1+\dfrac{r}{p}\right)\sin f\right] - \dot{\Omega}\cdot\cos i \\
\dot{M} = n - \dfrac{1-e^2}{nae}\left[-\bar{F}_R \cdot \left(\cos f - 2e\dfrac{r}{p}\right) + \bar{F}_T \cdot \left(1+\dfrac{r}{p}\right)\sin f\right]
\end{cases}
\tag{1}
$$

For the orbit with small eccentricity, the orbit equation of continuous thrust is simplified as [5]

$$
\begin{cases}
\dot{a} = \dfrac{2\bar{F}_T}{n} \\
\dot{e} = \dfrac{1}{na}\left[\bar{F}_R \cdot \sin f + 2\bar{F}_T \cdot \cos f\right] \\
\dot{i} = \dfrac{r\cos u}{na^2}\bar{F}_N \\
\dot{\Omega} = \dfrac{r\sin u}{na^2\sin i}\bar{F}_N \\
\dot{e}_x = \dfrac{\sqrt{1-e^2}}{na}\left[\bar{F}_R \cdot \sin u + \bar{F}_T \cdot \left(\left(1+\dfrac{r}{p}\right)\cos u + \dfrac{r}{p}\cdot e_x\right)\right] + e_y \cdot \dot{\Omega}\cdot\cos i \\
\dot{e}_y = \dfrac{\sqrt{1-e^2}}{na}\left[\bar{F}_R \cdot \cos u + \bar{F}_T \cdot \left(\left(1+\dfrac{r}{p}\right)\sin u + \dfrac{r}{p}\cdot e_y\right)\right] - e_x \cdot \dot{\Omega}\cdot\cos i
\end{cases}
\tag{2}
$$

From the above equation, it can be seen that changing the orbit radius only requires applying heading thrust to the spacecraft, and its orbit changes are as follows.



a Schematic diagram of Earth's inertial space　　b Schematic diagram of track plan

Fig.1 Orbital Climbing Process

B. *Analysis of radar characteristics*

The radar observation schematic is shown in the following figure. The blue dashed line represents the orbit of the satellite, and the yellow part represents the visible orbit segment. From the figure, it can be seen that the satellite enters the visible arc segment after reaching a certain angle above the ground.

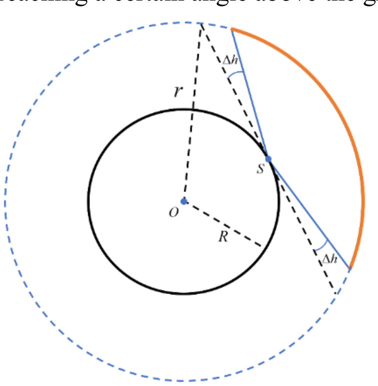


Fig.2 Schematic diagram of radar observation

After analyzing geometric relationships, it is also necessary to consider the errors present in the data. There are many sources of radar observation errors, including technical limitations of the radar itself, weather conditions, target characteristics, and other factors [6]. When using radar to observe and simulate in orbit spacecraft, a certain observation error needs to be added to the initial data as simulation data. Common errors include distance error, azimuth error, and pitch angle error. Distance error is caused by the transmission and reception mechanism of radar, as well as the loss of signal propagation in the air. Azimuth and elevation errors may be caused by inaccurate antenna pointing or radar scanning mode issues. Adding errors on the basis of simulation data will increase the difficulty of theoretical verification and reduce the accuracy of theoretical values. However, considering errors that are more in line with engineering practice is more meaningful for solving practical problems.

III. FORMULA DERIVATION

A. *Spatial Coordinate System Conversion*

The elementary transformation matrix between the three coordinate systems is the first elementary transformation matrix, the second elementary transformation matrix, and the third elementary transformation matrix:

$$\mathbf{M_1}[\theta]=\begin{bmatrix}1 & 0 & 0\\ 0 & \cos\theta & \sin\theta\\ 0 & -\sin\theta & \cos\theta\end{bmatrix}$$

$$\mathbf{M_2}[\theta]=\begin{bmatrix}\cos\theta & 0 & -\sin\theta\\ 0 & 1 & 0\\ \sin\theta & 0 & \cos\theta\end{bmatrix} \tag{3}$$

$$\mathbf{M_3}[\theta]=\begin{bmatrix}\cos\theta & \sin\theta & 0\\ -\sin\theta & \cos\theta & 0\\ 0 & 0 & 1\end{bmatrix}$$

The spatial relationship between geocenter O, station S, and spacecraft P in the inertial system is shown in the figure

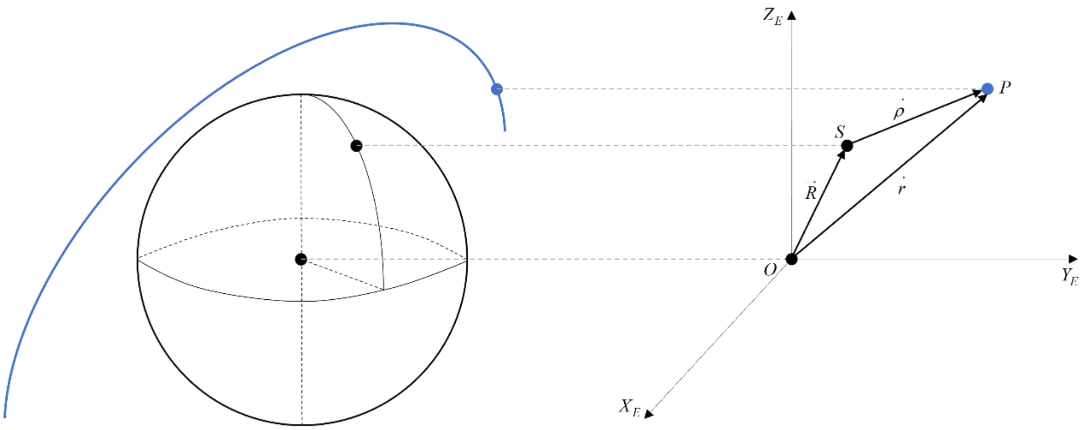


Fig.3 Position of the Earth's center, station, and spacecraft in I

In the figure, represents the position of the spacecraft in the terrestrial inertial system, represents the position of the measuring station in the terrestrial inertial system, and represents the position of the spacecraft relative to the measuring station in the terrestrial inertial system

$$\vec{r}=\vec{R}+\vec{\rho} \tag{4}$$

The transformation matrix from E to I is as follows:

$$\mathbf{I_E}=\mathbf{M}_3[-\Omega_G] \tag{5}$$

The transformation matrix from S to I is as follows:

$$\mathbf{I_S}=\mathbf{M}_3[-\lambda_s-\Omega_G]\cdot\mathbf{M}_2[\varphi_s] \tag{6}$$

The coordinates of the measuring station S in the geocentric system can be expressed as

$$\overrightarrow{I_s}=(R\sin\varphi_s, R\cos\varphi_s\sin\lambda_s, R\cos\varphi_s\cos\lambda_s) \tag{7}$$

In the formula, R is the Earth radius, and the position $\vec{R}$ of the station in the ground inertial system can be calculated by using the transformation matrix formula (5),

$$\vec{R}=\overrightarrow{I_s}\cdot\mathbf{M}_3[-\Omega_G] \tag{8}$$

The radar observation data is (ρ, Az, El), where ρ is the observation distance, Az is the azimuth angle, and El is the elevation angle; Represent radar data as $\mathbf{S}$ in station system

$$\boldsymbol{\rho}_\mathbf{s}=(\rho\sin El, \rho\cos El\sin Az, \rho\cos El\cos Az) \tag{9}$$

The transformation matrix equation (6) can be used to calculate the position $\vec{\rho}$ of the spacecraft relative to the measuring station in the terrestrial inertial system,

$$\vec{\rho}=\boldsymbol{\rho}_\mathbf{s}\cdot\mathbf{M}_3[-\lambda_s-\Omega_G]\cdot\mathbf{M}_2[\varphi_s] \tag{10}$$

Thus, according to equation (4), determine the position $\vec{r}$ of the spacecraft in the terrestrial inertial system.

$$\vec{r}=\begin{bmatrix} R\sin\varphi_s\cos(-\Omega_G)-R\cos\varphi_s\sin\lambda_s\sin(-\Omega_G)+\rho\sin El\cos\varphi_s\cos(-\lambda_s-\Omega_G)-\rho\cos El\sin Az\cos\varphi_s\sin(-\lambda_s-\Omega_G)+\rho\cos El\cos Az\sin\varphi_s\\ R\sin\varphi_s\sin(-\Omega_G)+R\cos\varphi_s\sin\lambda_s\cos(-\Omega_G)+\rho\sin El\sin(-\lambda_s-\Omega_G)+\rho\cos El\sin Az\cos(-\lambda_s-\Omega_G)\\ R\cos\varphi_s\cos\lambda_s-\rho\sin El\sin\varphi_s\cos(-\lambda_s-\Omega_G)+\rho\cos El\sin Az\sin\varphi_s\sin(-\lambda_s-\Omega_G)+\rho\cos El\cos Az\cos\varphi_s \end{bmatrix} \tag{11}$$

B. Calculation of radius

Expanding the orbit radius by McLaughlin [7] can solve the problem where it cannot be directly integrated, as can be seen from the first equation in equation (2)

$$\frac{dr}{dt}=\frac{2\overline{F}_T}{n}=\frac{2r\overline{F}_T}{v} \tag{12}$$

Expand the radius of time t into a McLaughlin series, with

$$r=r_0+\left(\frac{dr}{dt}\right)_0\tau+\frac{1}{2!}\left(\frac{d^2r}{dt^2}\right)_0\tau^2+\frac{1}{3!}\left(\frac{d^3r}{dt^3}\right)_0\tau^3+\frac{1}{4!}\left(\frac{d^4r}{dt^4}\right)_0\tau^4+\cdots \tag{13}$$

For the first derivative, there is

$$\frac{dr}{dt}=\frac{2r\overline{F}_T}{v} \tag{14}$$

According to the vitality formula $v=\sqrt{\mu/r}$ under circular orbit, there are

$$\frac{dr}{dt}=\frac{2r\overline{F}_T}{v}=\frac{2r\overline{F}_T}{\sqrt{\mu/r}}=2\overline{F}_T\mu^{-\frac{1}{2}}r^{\frac{3}{2}} \tag{15}$$

Continue to derive its higher-order derivative

$$\frac{d^2r}{dt^2}=-3v^2\dot{v}\frac{2\mu\overline{F}_T}{v^6}=\frac{-6\mu\overline{F}_T{}^2}{\mu^2/r^2}=-6\overline{F}_T{}^2\mu^{-1}r^2 \tag{16}$$

$$\begin{aligned}\frac{d^3r}{dt^3}&=2r\dot{r}\frac{-6\overline{F}_T{}^2}{\mu}\\&=2r\cdot 2\overline{F}_T\mu^{-\frac{1}{2}}r^{\frac{3}{2}}\cdot\frac{-6\overline{F}_T{}^2}{\mu}\\&=-24\overline{F}_T{}^3\mu^{-\frac{3}{2}}r^{\frac{5}{2}}\end{aligned} \tag{17}$$

$$\frac{d^mr}{dt^m}=-(m+1)!\overline{F}_T{}^m\mu^{-\frac{m}{2}}r^{\left(\frac{m}{2}+1\right)} \qquad \mathrm{m}\geq 2 \tag{18}$$

$$R_m=\frac{d^mr}{dt^m}=\begin{cases} r & ,m=0\\ 2\overline{F}_T\mu^{-\frac{1}{2}}r^{\frac{3}{2}} & ,m=1\\ -(m+1)!A^m\cos^m\beta\mu^{-\frac{m}{2}}r^{\left(\frac{m}{2}+1\right)} & ,m\geq 2\end{cases} \tag{19}$$

Equation (13) can be expressed as

$$r=r_0+R_1\tau+\frac{1}{2!}R_2\tau^2+\frac{1}{3!}R_3\tau^3+\frac{1}{4!}R_4\tau^4+\cdots \tag{20}$$

$$r=\sum_{n=0}R_m\tau^m=F(r_0,A,\beta,t) \tag{21}$$

IV. SIMULATION VERIFICATION

The simulation sets the spacecraft mass to 750kg, with a continuous thrust of 170mN, and an initial orbit height of 500km; The measurement station is located at point S, with latitude and longitude coordinates of (36°N,124°E). The spacecraft undergoing orbital climb is observed from 2020.04.02 00:00:00 to 2020.04.05 24:00:00. In subsequent simulations, an error of ±30m was added to the distance measurement, and an observation error of ±0.1 ° was added to both azimuth and elevation data. The increased errors were Gaussian distributed within their respective intervals. Conduct visibility analysis on radar observation satellites. After reaching a certain angle above the ground, the satellite is considered visible. Set the elevation angle to be greater than 3 °, and the radar observation window is as follows:

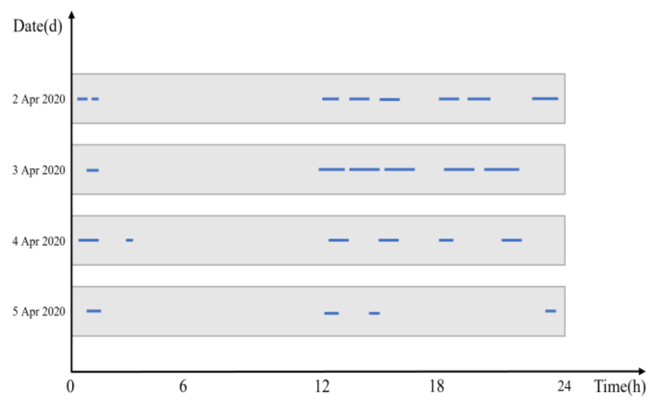


Fig.4 Radar visibility analysis

According to equation (11), convert the radar observation data into the coordinates of the spacecraft in the Earth's inertial coordinate system. Calculate the modulus of the coordinate vector to obtain the orbit radius data of the spacecraft. This study uses LM algorithm to solve the problem. LM(Levenberg-Marquardt) algorithm [8] is a kind of deformation of Newton's method, which is used to minimize the sum of squares of nonlinear functions. Its core idea is to use Jacobian matrix instead of Hessian matrix calculation, so as to improve the optimization efficiency. The thrust acceleration after adding error data is calculated as $2.2641\times10^{-4}m/s^2$ .Set up multiple sets of simulations, and the results are as follows:

Table.1 Solution Results

	Actual acceleration (m/s^2)	Solving acceleration (m/s^2)	Solving error (m/s^2)	Relative error
1	2.2667×10^{-4}	2.2641×10^{-4}	2.6×10^{-7}	1.1%
2	2.6667×10^{-4}	2.6635×10^{-4}	3.2×10^{-7}	1.2%
3	1.3000×10^{-3}	1.2937×10^{-3}	6.3×10^{-6}	4.8%
4	2.7000×10^{-3}	2.6984×10^{-3}	1.6×10^{-6}	0.6%
5	6.7000×10^{-3}	6.6976×10^{-3}	2.4×10^{-6}	0.4%
6	6.6670×10^{-2}	6.6664×10^{-2}	6.0×10^{-6}	0.1%

From the analysis of experimental results, it can be concluded that the accuracy of this method in solving the inference parameters for continuous thrust spacecraft orbital climb can be controlled at around 1.5%. The high-precision solution is thanks to multiple days of radar observation. If only two days of observation data are used, the accuracy of the solution will decrease to around 8%. In addition, the calculated accelerations are all smaller than the actual accelerations of the spacecraft. Based on the perturbation effect of the spacecraft in near-Earth space, it is speculated that the continuous thrust exerted by the spacecraft is partially offset by atmospheric resistance. The calculated acceleration is the combined acceleration of the spacecraft during climb.

V. SUMMARY ANALYSIS

This article is based on the dynamic characteristics of continuous thrust spacecraft orbital maneuvers, formulaic description of its orbital climb process, and analysis of the thrust required during this period. The research difficulty lies in the obvious orbital period characteristics of earth satellites and the arc effect of continuous thrust control. The core idea is to carry out McLaughlin expansion of orbital changes for the calculus system of orbital dynamics under the condition of continuous small thrust. The expanded formula is used to describe the orbit radius, and the LM algorithm is used to identify the thrust parameters during the process, combined with the idea of least squares. The solution accuracy is good.

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