

Research on Redundancy Solution of Satellite Transponders Based on Reliability Analysis

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ABSTRACT

With the lifespan of most modern navigation and communication satellites greater than 10 years, it is very important to find an optimal redundancy solution for satellite transponders. In this paper, we propose redundancy solutions of satellite transponders with six signal paths. Firstly, we introduce the mathematical models of the reliability of typical schemes. Specifically, we propose a method to simplify the Boolean truth table and RBD (Reliability Block Diagrams) model by combining the full probability formula, to obtain the reliability analytical solution for the complex redundant system such as ring backup (hot/cold backup). We also provide the boundary conditions of different redundancy configuration tradeoffs based on reliability analytical solution. Monte Carlo simulation is used to verify the correctness of the analytical models and the analysis results. Finally, we verified the common redundancy solutions, and obtain the optimal solution based on analysis results. In satellite product design, the conclusion based on the method proposed in this paper can be considered in addition to the complexity and performance impact of the space borne product, which conform the basis to choose the redundancy solution for the designers.

KEYWORDS: redundancy solution, reliability analysis, transponder, ring backup, Monte Carlo simulation

1 INTRODUCTION

The design lifespan of modern navigation or communication satellites is usually more than 10 years [1-2]. In order to meet the reliability requirement in such a long mission time and ensure the operation of satellite payload, hardware redundancy design is required [3]. Redundant design is one way for a system or device to achieve high reliability, high security, and survivability.

Travelling Wave Tube Amplifier (TWTA) is an important component for downlink broadcast channel of navigation and communication satellites. It amplifies and transmits radio frequency signals. For requirements with a small number of downlink broadcast channels, it is usually possible to configure a backup TWTA for each channel (i.e. 2:1 backup). However, for satellites with a large number of downlink broadcast channels, a more economical redundancy scheme is needed due to the limitation of on-board resource. Therefore, optimizing the redundancy scheme to obtain higher reliability becomes a practical problem in engineering design.

Usually, there are many ways to obtain redundancy, such as k/n (G), hot backup, cold backup, ring backup and so on [4]. The optimal redundancy scheme not only relates to the redundancy mode, but also depends on the application constraints such as the failure rate of functional units and mission time, which are easily neglected factors in engineering.

In this paper, we take the redundancy design of a 6-channel TWTA subsystem as an example and introduce the mathematical models of reliability for typical schemes. Specifically, we propose a method to simplify the Boolean truth table and RBD (Reliability Block Diagrams) model by combining the full probability formula to obtain the reliability analytical solution for the complex redundant system such as ring backup (hot/cold backup). We provide boundary conditions of different redundancy configuration tradeoff based on reliability analytical solution. Finally, we use Monte Carlo simulation to verify the correctness of the analytical models and the analysis results.

2 RELIABILITY MATHEMATICAL MODELS

2.1 Engineering Background

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A certain satellite needs to provide 6-channel forwarding capability. The system is implemented with the TWTAs as a redundant configuration, and switching the path through the switch.

Due to on-board resource limitations, up to three additional backups are allowed. According to the engineering needs, common redundancy configuration schemes are: 6/8(G) cold standby model, 2/3(G) cold standby model (3 sets in series), conventional 6/9(G) voting model, 6/9(G) ring backup (hot standby) and 6/9(G) ring backup (cold standby) model.

2.2 6/8(G) Cold Standby Model

In this paper, we assume the same failure rate λ and exponential distribution for each unit. Let the switching switches of each redundant scheme to be completely reliable, the reliability mathematical model of k/n(G) cold standby model is:

$$R_s(t) = \sum_{i=0}^{n-k} \frac{(k\lambda t)^i}{i!} \cdot e^{-k\lambda t} \quad (1)$$

where n denotes the total number of units making up the system, k is the number of normal units, and t is the working time.

Hence the reliability of 6/8(G) cold standby model is:

$$R_{6/8} = (1 + 6\lambda t + 18\lambda^2 t^2) e^{-6\lambda t} \quad (2)$$

2.3 2/3(G) Cold Standby Model (3 sets in series)

According to Eq. (1), the reliability of 2/3(G) cold standby model is:

$$R_{2/3} = (1 + 2\lambda t) e^{-2\lambda t} \quad (3)$$

To ensure the proper working conditions of the six-path payload, and to consider the influence of the satellite assembly layout on the performance parameters, the satellite may adopt 3 sets of 2/3(G) cold standby models in series. Its total reliability $R_{2/3}^3$ becomes:

$$R_{2/3}^3 = (1 + 6\lambda t + 12\lambda^2 t^2 + 8\lambda^3 t^3) e^{-6\lambda t} \quad (4)$$

2.4 6/9(G) Voting Model

The voting system consists of n units and one voting device, when the voting device is normal and the number of normal units is not less than k ($1 \leq k \leq n$), the system will not fail. Such a system is collectively known as the voting system, which is a form of the working storage model. The reliability analysis model of the k/n (G) is:

$$R_s(t) = R_m \sum_{i=k}^n C_n^i p^i q^{n-i} \quad (5)$$

Where p denotes the reliability of the unit (same for all units) and q denotes the unreliability of the unit (all units are the same), i.e. $p = e^{-\lambda t}$.

Assuming the same failure rate for each unit and exponential distribution, and the reliability of the voting system is 1, then the reliability of 6/9(G) voting system becomes:

$$R_{6/9} = 84p^6q^3 + 36p^7q^2 + 9p^8q + p^9 \quad (6)$$

Substitute $q = 1 - p$ into Eq. (6), we have:

$$R_{6/9} = 84p^6 - 216p^7 + 189p^8 - 56p^9 \quad (7)$$

2.5 6/9(G) Ring Backup (hot standby) Model

As shown in Figure 1, the 6/9(G) ring backup system works as follows: normally, there are 6 modules (A1~A6) working, constituting 6 pathways, and the remaining 3 modules (B1~B3) are backup parts. B1 can back up the paths of A1~A3, but can not backup A4~A6. B2 can back up all the paths from A1 to A6. B3 can back up any path in A4~A6, rather than A1~A3.

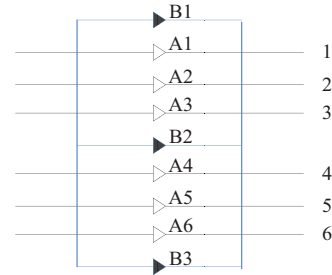


Figure 1 Schematic diagram of 6/9(G) ring backup system

Since the spare parts in the ring backup system can only replace the adjacent parts in the system, it is different from the general parallel, cold standby or voting system.

It is possible to obtain that mathematical model of the system through the Boolean method. However, the Boolean truth table is very complicated as there are 512 types of logic combination relations among 9 modules in the 6/9(G) ring backup system (B1~B3 hot standby). The analysis process can be simplified by combining full probability formula. Group the channels A1~A3 and A4~A6 respectively. The backup modules B1, B2, B3 are taken as one class, which are analyzed to correspond to the substitution relationship of the two groups. Since 3 channels of the same group fail, the system must be ineffective, and the combination of system failures can be discarded. The reliability analysis of the system is shown in Table 1, suggesting that the probability that the system can work normally.

In Table 1, “0” denotes failure, and the occurrence probability is q . “1” denotes no failure, and the probability is p . “any” means either 0 or 1, with a probability of 1. “ ≥ 1 work” indicates that at least one of the two modules is not invalid.

The reliability of the system can be obtained by summing the probability of all systems' non-failure combinations.

$$R_{6/9-ring(hot)} = p^6 + 6p^7q + 18p^6q^2 + 45p^6q^3 + 9p^7q^2 \quad (8)$$

Similarly, substitute $q = 1 - p$ into Eq. (8) to obtain:

$$R_{6/9-ring(hot)} = 64p^6 - 156p^7 + 129p^8 - 36p^9 \quad (9)$$

Table 1 6/9(G) ring backup (hot standby) Boolean truth analysis table

A1~A3	Pro.	A4~A6	Pro.	B1	B2	B3	Pro.	System	P _{Si}
0Failure	p ³	0Failure	p ³	any	any	any	1	1	p ⁶
0Failure	p ³	1Failure	3p ² q ¹	any	≥1		p ² +2pq	1	3p ⁷ q ¹ +6p ⁶ q ²
0Failure	p ³	2Failure	3p ¹ q ²	any	1	1	p ²	1	3p ⁶ q ²
1Failure	3p ² q ¹	0Failure	p ³	≥1 work		any	p ² +2pq	1	3p ⁷ q ¹ +6p ⁶ q ²
1Failure	3p ² q ¹	1Failure	3p ² q ¹	0	1	1	p ² q ¹	1	9p ⁶ q ³
1Failure	3p ² q ¹	1Failure	3p ² q ¹	1	≥1 work		p ³ +2p ² q	1	9p ⁷ q ² +18p ⁶ q ³
1Failure	3p ² q ¹	2Failure	3p ¹ q ²	1	1	1	p ³	1	9p ⁶ q ³
2Failure	3p ¹ q ²	0Failure	p ³	1	1	any	p ²	1	3p ⁶ q ²
2Failure	3p ¹ q ²	1Failure	3p ² q ¹	1	1	1	p ³	1	9p ⁶ q ³
others								0	/

2.6 6/9(G) Ring Backup (cold standby) Model

As shown in Figure 1, the 6/9(G) ring backup system works as follows. Normally, there are 6 modules (A1~A6) working, constituting 6 pathways, and the remaining 3 modules (B1~B3) are backup parts. B1 can back up the paths of A1~A3, but cannot backup A4~A6.

According to the analysis of the implementation of ring backup function, the general RBD model of 6/9(G) ring backup (hot backup and cold backup) can be obtained, as shown in Figure 2.

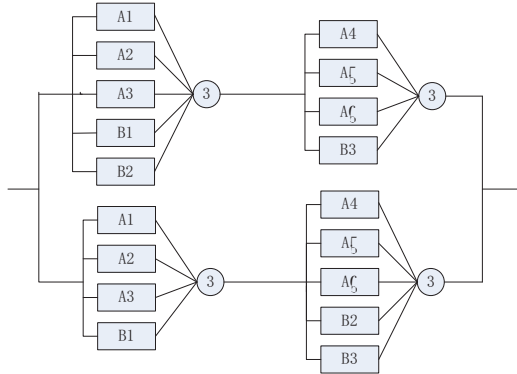


Figure 2 RBD of 6/9(G) ring backup system

The 6/9 (G) ring backup (cold standby) analytical model can be described as a 3/5 (G) cold standby model connected in series with the 3/4 (G) cold standby model and connected in parallel with another 3/5 (G) cold standby model in series with the 3/4 (G) cold standby model.

Eq. (10) and Eq. (11) describe the reliability of 3/5(G) cold standby model and 3/4(G) cold standby respectively.

$$R_{3/5} = \sum_{i=0}^2 \frac{(3\lambda t)^i}{i!} \cdot e^{-3\lambda t} = (1 + 3\lambda t + 4.5\lambda^2 t^2) e^{-3\lambda t} \quad (10)$$

$$R_{3/4} = \sum_{i=0}^1 \frac{(3\lambda t)^i}{i!} \cdot e^{-3\lambda t} = (1 + 3\lambda t) e^{-3\lambda t} \quad (11)$$

Thus, the reliability of 6/9(G) ring backup (cold backup) is:

$$R_{9/6-ring(cold)} = 1 - (1 - R_{3/5} \cdot R_{3/4}) \cdot (1 - R_{3/5} \cdot R_{3/4}) \quad (12)$$

Substitute Eq. (10) and (11) into the Eq. (12), and we calculate MATLAB symbols to obtain the mathematical model of the reliability of the 6/9 (G) ring backup (cold backup) model.

$$R_{9/6-ring(cold)} = (2 + 12\lambda t + 27\lambda^2 t^2 + 27\lambda^3 t^3) e^{-6\lambda t} a \dots \\ \dots - (1 + 12\lambda t + 63\lambda^2 t^2 + 189\lambda^3 t^3 + 344.25\lambda^4 t^4 \dots \\ \dots + 364.5\lambda^5 t^5 + 182.5\lambda^6 t^6) e^{-12\lambda t} \quad (13)$$

3 COMPARATIVE ANALYSES OF THE REDUNDANCY SCHEMES

3.1 “6/8(G) Cold Standby Model” and “2/3(G) Cold Standby Model (3 sets in series)”

According to Eq. (2) and (4), the mathematical model of the reliability of 6/8(G) cold standby system and 2/3(G) cold standby system (3 sets in series) can be determined. Assuming that:

$$R_{6/8} - R_{2/3}^3 = 2\lambda^2 t^2 (3 - 4\lambda t) e^{-6\lambda t} > 0 \quad (14)$$

Where $2\lambda^2 t^2 > 0$ and $e^{-6\lambda t} > 0$, when $(3 - 4\lambda t) > 0$, Eq. (14) is established. That is, when $\lambda t < 0.75$, $R_{6/8} > R_{2/3}^3$.

This indicates that $t < 0.75 \cdot \text{MTTF}$ is the boundary condition of “6/8(G) cold standby model” better than “2/3(G) cold standby model (3 sets in series)”, where MTTF represents mean time to fail. In this condition, the 6/8(G) cold standby model has higher reliability. Whereas, when $\lambda t > 0.75$, 2/3(G) cold standby model has higher reliability. When $\lambda t = 0.75$, both of them have the same reliability.

3.2 “6/8(G) Cold Standby Model” and “6/9(G) Voting Model”

According to Eq. (2) and (7), the mathematical model of the reliability of 6/8(G) cold standby system and 6/9(G) voting system can be determined, and $p = e^{-\lambda t}$, so:

$$R_{6/8} - R_{6/9} = (-83 + 6\lambda t + 18\lambda^2 t^2 + 216e^{-\lambda t} \dots \\ \dots - 189e^{-2\lambda t} + 56e^{-3\lambda t}) e^{-6\lambda t} \quad (15)$$

Let:

$$f(x) = -83 + 6\lambda t + 18\lambda^2 t^2 + 216e^{-\lambda t} \dots \dots - 189e^{-2\lambda t} + 56e^{-3\lambda t} \dots \quad (16)$$

Given $f(x) < 0$ and that it monotonically decreases, and $e^{-6\lambda t} > 0$, we have $R_{6/8} - R_{6/9} < 0$.

This concludes that "6/9(G) voting model" is better than "6/8(G) cold standby model".

3.3 "2/3(G) Cold Standby Model (3 sets in series)" and "6/9(G) Voting Model"

According to Eq. (4) and (7), the mathematical model of the reliability of 2/3(G) cold standby model (3 sets in series) and 6/9(G) voting model can be determined, and $p = e^{-\lambda t}$, so:

$$R_{2/3}^3 - R_{6/9} = (-83 + 6\lambda t + 12\lambda^2 t^2 + 8\lambda^3 t^3 \dots \dots + 216e^{-\lambda t} - 189e^{-2\lambda t} + 56e^{-3\lambda t})e^{-6\lambda t} \quad (17)$$

Let:

$$g(x) = -83 + 6\lambda t + 12\lambda^2 t^2 + 8\lambda^3 t^3 \dots \dots + 216e^{-\lambda t} - 189e^{-2\lambda t} + 56e^{-3\lambda t} \quad (18)$$

Given that $g(x) < 0$ and $e^{-6\lambda t} > 0$, so $R_{2/3}^3 - R_{6/9} < 0$. Therefore, the "6/9(G) voting model" is superior to the "2/3(G) cold standby model (3 sets in series)".

3.4 "6/9(G) Voting Model" and "6/9(G) Ring Backup (hot standby) Model"

According to Eq. (7) and (9), the mathematical model of the reliability of 6/9(G) voting system and 6/9(G) ring backup (hot standby) system can be determined. Subtracting the two equations, we have:

$$R_{6/9} - R_{6/9-ring(hot)} = 20p^6(1-p)^2 + 20p^8(1-p) \quad (19)$$

Where $p^6(1-p)^2 \geq 0$ and $p^8(1-p) \geq 0$. So when $p \neq 1$, $R_{6/9} > R_{6/9-ring(hot)}$.

Since spare parts in ring backup cannot replace all other units, the reliability of 6/9(G) ring backup (hot standby) is less than that of 6/9(G) voting system.

3.5 "6/9(G) Voting Model" and "6/9(G) Ring Backup (cold standby) Model"

Due to the existence of cold backup, the reliability of ring backup (cold backup) is higher than that of ring backup (hot backup), so there may be a certain boundary condition, which makes the reliability of ring backup (cold backup) model higher than 6/9(G) voting model.

According to Eq. (7) and (13), the mathematical model of the reliability of 6/9(G) voting system and 6/9(G) ring backup (cold standby) system can be determined. Subtracting the two equations, we have:

$$R_{6/9} - R_{6/9-ring(cold)} = (82 - 12\lambda t - 27\lambda^2 t^2 - 27\lambda^3 t^3)e^{-6\lambda t} - 216e^{-7\lambda t} \dots \dots + 189e^{-8\lambda t} - 56e^{-9\lambda t} + (1 + 12\lambda t + 63\lambda^2 t^2 + 189\lambda^3 t^3 \dots \dots + 344.25\lambda^4 t^4 + 364.5\lambda^5 t^5 + 182.5\lambda^6 t^6)e^{-12\lambda t} \quad (20)$$

When $\lambda t < 0.131$, we have $R_{6/9} - R_{6/9-ring(cold)} > 0$.

This indicates that $t < 0.131 \cdot \text{MTTF}$ is the boundary condition of "6/9(G) voting model" better than "6/9(G) ring backup (cold standby) model". Under this condition, the 6/9(G) voting model has higher reliability. Whereas, when $t > 0.131 \cdot \text{MTTF}$, 6/9(G) ring backup (cold standby) model has higher reliability. When $t = 0.131 \cdot \text{MTTF}$, both of them have the same reliability.

4 THE SIMULATION ANALYSIS

Due to the complex redundant structure of ring backup, it is difficult to build accurate models using straightforward methods. Therefore, we can use Monte-Carlo simulation method for modeling. The basic steps are as follows.

Establish a probabilistic model related to the solution, so that the solution is the probability distribution or mathematical expectation of the model. Then randomly sample observations on the model, i.e., random variables are generated. According to the logical substitution relation of ring backup, the use of backup parts is selected, then the success/failure status of the system is determined. Finally, the simulation value of system reliability is calculated.

In numerical simulation, the accuracy of the results increases with the increase of simulation times. In addition, the number of simulation times required to obtain the results with the same accuracy increases with the increase of the complexity of the system. Therefore, when the reliability of each unit is very high or when there are many units in the system, a lot of simulation iterations are needed to obtain the simulation results with sufficient accuracy.

Assuming the same failure efficiency for each unit following exponential distribution, $\lambda = 1.25 \times 10^{-6}$ (i.e. MTTF = 800000h), and the lifespan is 12 years (i.e. $t = 105120\text{h}$). It can be seen that $t = 0.1314 \cdot \text{MTTF}$.

Substituting the analysis conclusions in the sections of chapter 3, under this condition, we know that:

- 1) "6/8(G) cold standby model" is better than "2/3(G) cold standby model (3 sets in series)".
- 2) "6/9(G) voting model" is better than "6/8(G) cold standby model".
- 3) "6/9(G) voting model" is superior to "2/3(G) cold standby model (3 sets in series)".
- 4) "6/9(G) voting model" is superior to "6/9(G) ring backup (hot standby) model".
- 5) "6/9(G) ring backup (cold standby) model" is better than "6/9(G) voting model".

The reliability of each redundant model is calculated using the mathematical model given in chapter 2 and

Monte Carlo numerical simulation method, and the results are shown in Table 2.

The results suggest that the Monte Carlo simulation results are consistent with the theoretical analysis.

Table 2 The reliability of each redundant model

Redundant model	Mathematical model result	MC simulation results	simulation error
6/8(G) cold standby model	0.95423	0.95268	0.16%
2/3(G) cold standby model (3 sets in series)	0.91539	0.91664	0.14%
6/9(G) voting model	0.98267	0.98271	0.004%
6/9(G) ring backup (hot standby) model	0.96569	0.96588	0.02%
6/9(G) ring backup (cold standby) model	0.99548	0.98255	1.30%

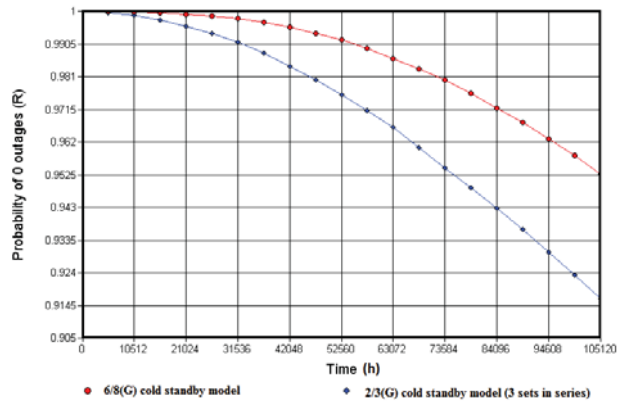


Figure 3 the reliability of “6/8(G) cold standby model” and “2/3(G) cold standby model (3 sets in series)”

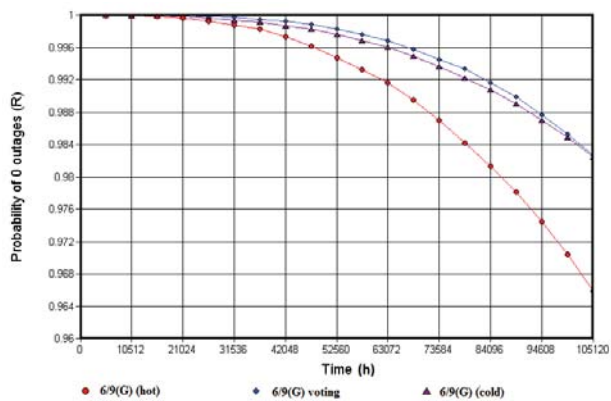


Figure 4 the reliability of “6/9(G) voting model” and “6/9(G) ring backup (hot standby/cold standby) model” (12years)

Figure 3 shows the variation of the reliability of “6/8(G) cold standby model” and “2/3(G) cold standby model (3 sets in series)” with the working time. Figure 4

shows the change of the reliability of the 6/9(G) ring backup (hot standby/cold standby) model with the working time.

Assuming that the MTTF remains unchanged and the lifespan becomes 15000 hours, the reliability of the “6/9(G) voting model” and the “6/9(G) ring backup (hot standby/cold standby) model” changes with the working time, as shown in Figure 5.

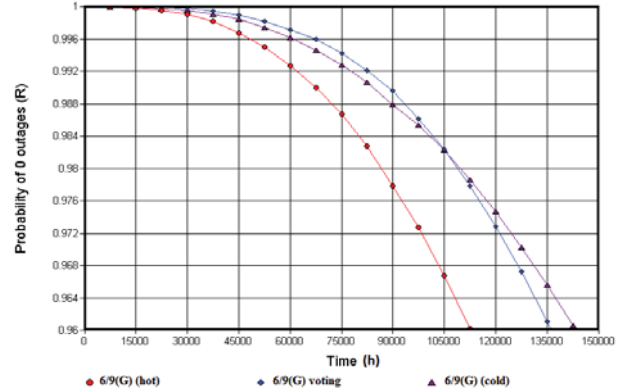


Figure 5 The reliability of “6/9(G) voting model” and “6/9(G) ring backup (hot standby/cold standby) model” (15years)

These analyses conclude that:

(1) The conclusions and boundary conditions of reliability analysis and comparison of each redundancy model based on the mathematical model are correct.

(2) In this case, the working time satisfies the boundary condition that $t < 0.75 \cdot \text{MTTF}$. Under this, the reliability of the “6/8(G) cold standby model” is higher than that of the “2/3(G) cold standby model (3 sets in series)”. From the perspective of reliability, it is recommended to select the 6/8(G) cold standby system.

(3) “6/9(G) voting model” is better than “6/9(G) ring backup (hot standby) model”.

(4) As shown in Figure 4, at the end of the satellite life, the reliability of the 6/9(G) ring backup (cold standby) is similar to that of the 6/9(G) voting model. According to Figure 5, the reliability of “6/9(G) ring backup (cold standby) model” and “6/9(G) voting model” changed before and after the boundary condition $t \approx 0.131 \cdot \text{MTTF}$. When $t < 0.131 \cdot \text{MTTF}$, the 6/9(G) voting model has A high reliability. When $t > 0.131 \cdot \text{MTTF}$, the 6/9(G) ring backup (cold standby) model has A high reliability.

(5) In this case, it is suggested to choose the 6/9(G) ring backup (cold backup) system only from the perspective of reliability analysis. From the perspective of reliability analysis and the number of parts used, it is recommended to choose 6/8(G) cold standby plan.

5 CONCLUSIONS

The existing satellite redundancy design primarily depends on the designer’s knowledge of product failure and redundant technology. To solve this problem, this

paper proposes the boundary condition of the common redundancy scheme of 6-channel signal paths, which is used to optimize the redundancy scheme in engineering design to obtain higher reliability.

1) When selecting the redundancy scheme, not only the redundancy mode but also the reliability level and mission time of the functional unit should be considered. In a particular application, accurate modeling and quantitative analysis are necessary to determine which scheme is the best one.

2) The case of TWTA subsystem which worked for 12 years in this paper shows that if the mission time of the same series of satellites is extended and the boundary condition is exactly crossed, different redundancy schemes can be selected.

3) In practice, Monte Carlo simulation is an effective tool to verify the reliability of various redundant schemes, especially complex redundant systems.

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