Research on Collision Prediction & Avoidance Strategy of Formation Flying

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***Abstract*—In the satellite formation task, due to a satellite failure or formation task change, it is necessary to reconstruct the formation. In the process of reconfiguration of satellites, satellites anti-collision research is one of the most important links. In this paper, by collapsing the initial covariance matrix of the satellite, the collision probability density function is integrated in the region containing the satellite. The collision probability of formation satellites is obtained. When the collision probability is greater than the security threshold, two avoidance strategies (increase the long axis of the formation satellites and increase the track drift distance of the formation satellites) are proposed based on the Gaussian perturbation equation. The simulation experiments show that the method has great significance for the collision between satellites.**

Keywords-collision probability; formation flying; avoidance strategy; Halo orbit

I. INTRODUCTION

Satellites formation flying is defined by the several satellites consisted of certain configuration to complete some space missions together. Satellites in formation flying have many advantages. It is convenient, small amount of energy, strong performance maintain, er al. But it is easy to collide when some satellites fly in the close orbit. The problem to be considered in this paper is based the collision probability for satellites. Thus the collision probability will be reduced to an acceptable safety level. With the development of the small satellites technology, the research on the function of a certain function has become one of the hotspots in the field of space research. Compared with the traditional single large satellite [1], the small satellite has the advantages of low quality, low cost and high reliability. When a satellite group of a satellite failure, you can run orbital reconstruction, thus extending the life of the entire system. As a result, it has received wide attention. Since the beginning of 1990s, the United States has launched "ION-F" [2], "TechSat-21" [3] and other research programs. Satellite formation flying technology is one of the major characteristics of a small satellite in space to form a specific configuration collaborative work, close contact, in order to form a large distribution of virtual satellite"(also called "distributed satellite system"). Thus, the phenomenon of "Emergence" in the system theory is produced, and the performance of the system is beyond the single satellite system.

In formation flying, due to the influence of various perturbations, the formation of the formation will drift, and due to the various hardware and software problems [4], [5], it will increase the probability of collision in the formation process. How to avoid the collision between formation satellites is an important problem to be considered in the design of satellite formation flying? For the formation of satellites collision avoidance problem, the commonly used method is the way of maneuvering orbit. It is an instantaneous speed increase to the satellite, making the satellite deviate from the original orbit to reduce the possibility of collision.

In this paper, the integral of collision probability density function in the specific area of the satellite by the initial state covariance matrix of the recursive satellite. The collision probability of formation satellites is calculated. When the collision probability is greater than the security threshold, two avoidance strategies (increase the long axis of the formation satellites and increase the track drift distance of the formation satellites) are proposed based on the Gaussian perturbation equation. Finally, numerical simulation is carried out.

II. COLLISION PROBABILITY CALCULATION OF SATELLITE FORMATION FLYING

A. C-W equation of satellite formation flying

When consideration is given to the reference satellite in Halo orbit (the orbital eccentricity "e=0") [6], Fig. 1 shows the Halo orbit in the x-y plane map.

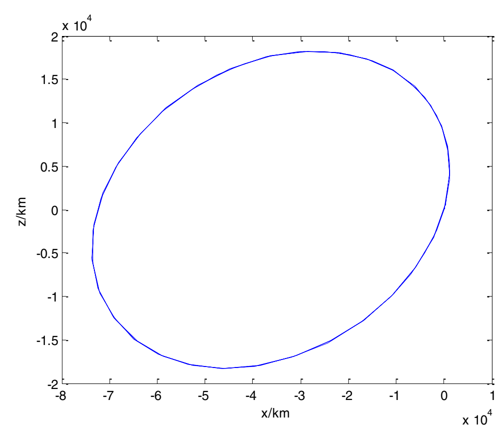

Figure 1. Halo orbit in the x-y plane map.

The relative motion equations are simplified as:

$$
\begin{aligned}
&\ddot{x}-2n\dot{y}-3n^2x=0\\
&\ddot{y}+2n\dot{x}=0\\
&\ddot{z}+n^2z=0
\end{aligned} \tag{1}
$$

Equation (1) is also known as C-W equation or Hill equation[7]-[9]. The analytical solution of this equation is:

$$
\begin{cases}
x(t)=\left(4x_0+\dfrac{2\dot{y}_0}{n}\right)+\dfrac{\dot{x}_0}{n}\sin(nt)-\left(3x_0+\dfrac{2\dot{y}_0}{n}\right)\cos(nt)\\
y(t)=-\left(6nx_0+3\dot{y}_0\right)t+\left(y_0-\dfrac{2\dot{x}_0}{n}\right)+\left(6x_0+\dfrac{4\dot{y}_0}{n}\right)\sin(nt)+\dfrac{2\dot{x}_0}{n}\cos(nt)\\
\mathrm{z(t)}=\dfrac{\dot{z}_0}{n}\sin(nt)+z_0\cos(nt)\\
\dot{x}(t)=\dot{x}_0\cos(nt)+\left(3x_0n+2\dot{y}_0\right)\sin(nt)\\
\dot{y}(t)=-\left(6nx_0+3\dot{y}_0\right)+\left(6x_0+4\dot{y}_0\right)\cos(nt)-2\dot{x}_0\sin(nt)\\
\dot{z}(t)=\dot{z}_0\cos(nt)-z_0n\sin(nt)
\end{cases} \tag{2}
$$

where, x_0, $\dot{x}_0$, y_0, $\dot{y}_0$, z_0, $\dot{z}_0$ are the initial values of x, $\dot{x}$, y, $\dot{y}$, z, $\dot{z}$ at t = 0, We can also transform the C-W equation into a standardized form:

$$
\dot{X} = AX + Dw + \Gamma u \tag{3}
$$

Relative state quantity X is:

$$
X=\begin{bmatrix} x & y & z & \dot{x} & \dot{y} & \dot{z} \end{bmatrix}^T \tag{4}
$$

where:

x, y, z: Coordinate in the coordinate system of the center of mass;

$\dot{x}$, $\dot{y}$, $\dot{z}$: The relative speed of the three directions;

w: Interference vector of the three directions;

u: Control vector of the three directions.

In the equation,

$$
A=\begin{bmatrix}
0 & 0 & 0 & 1 & 0 & 0\\
0 & 0 & 0 & 0 & 1 & 0\\
0 & 0 & 0 & 0 & 0 & 1\\
3n^2 & 0 & 0 & 0 & 2n & 0\\
0 & 0 & 0 & -2n & 0 & 0\\
0 & 0 & -n^2 & 0 & 0 & 0
\end{bmatrix},
D=\Gamma=\begin{bmatrix}
0 & 0 & 0\\
0 & 0 & 0\\
0 & 0 & 0\\
1 & 0 & 0\\
0 & 1 & 0\\
0 & 0 & 1
\end{bmatrix} \tag{5}
$$

where: ω as reference satellite orbit angular velocity.

The solution Equation 3 is obtained:

$$
X(t)=\phi(t,t_0)X(t_0)+\int_0^t \phi(t)[Dw(\tau)+\Gamma u(\tau)]d\tau \tag{6}
$$

where: $\phi(t)$ as state transition matrix.

$$
\phi(t)=\begin{bmatrix}
4-3\cos(nt) & 0 & 0 & \dfrac{\sin(nt)}{n} & \dfrac{(2-2\cos(nt))}{n} & 0\\
-6nt+6\sin(nt) & 1 & 0 & \dfrac{(2\cos(nt)-n)}{n} & \dfrac{4\sin(nt)}{n}-3t & 0\\
0 & 0 & \cos(nt) & 0 & 0 & \dfrac{\sin(nt)}{n}\\
3n\sin(nt) & 0 & 0 & \cos(nt) & 2\sin(nt) & 0\\
-6nt+6\cos(nt) & 0 & 0 & -2\sin(\omega t) & -3+4\cos(nt) & 0\\
0 & 0 & -n\sin(nt) & 0 & 0 & \cos(nt)
\end{bmatrix} \tag{7}
$$

B. Calculation of Collision Probability

If the relative state error is considered as random error, Gauss distribution is unbiased distribution. Assume that ω follows as the N(0, P) distribution. Then, $X(0)=N(0,P_0)$; $X(t)=N(0,P(t))$. The covariance matrix of the relative state at any time is expressed as:

$$
\overset{\bullet}{P}(t)=AP_0+P_0A^T+DWD^T \tag{8}
$$

where: W as covariance matrix of perturbation acceleration; P as covariance matrix of initial state.

For circular reference orbits, analytic solution of P is obtained [10].

$$
P(t)=\phi(t,t_0)P_0\phi^T(t,t_0)+(\omega+1)\int_0^t \phi(t)DWD^T\phi^T(t)d\tau \tag{9}
$$

Assuming that the covariance matrix P of the relative state is calculated at a certain time, the probability density function of X is:

$$
pdf(X)=\frac{1}{\sqrt{(2\pi)^n|P|}}\exp\left(-\frac{1}{2}X^TP^{-1}X\right) \tag{10}
$$

In order to simplify the calculation, assume that the satellite's danger zone is a circle with a radius of 5m. If two satellites form a formation flying, the integral region is transformed into a circular region with a radius of 10m. Fig. 1 is a description of satellites encounter.

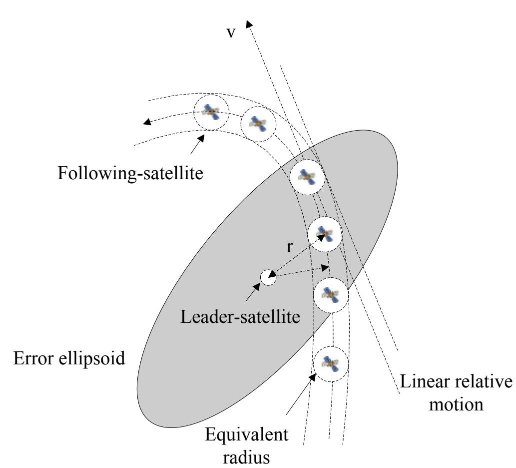

Figure 2. Description of satellites encounter.

Because we only care about the relative position error covariance matrix of the satellite collision probability, the upper left corner matrix 3*3 of the relative state covariance matrix $P(t)$ is obtained [11], [12]. Marked as: P_X . Then, the probability density function is:

$$pdf(X) = \frac{1}{\sqrt{(2\pi)^3 |P_X|}} \exp\left(-\frac{1}{2} X^T P_X^{-1} X\right) \tag{11}$$

The probability density function is integrated in the region where possible collision is possible. Then, collision probability is calculated.

$$P_c = \frac{1}{\sqrt{(2\pi)^n |P_X|}} \iiint \exp\left(-\frac{1}{2} X^T P_X^{-1} X\right) dxdydz \tag{12}$$

III. COLLISION AVOIDANCE STRATEGY

Once the collision probability is greater than the safety threshold, it is necessary to carry out the collision avoidance strategy, in order to reducing the risk of collision.

The Gaussian change equation of the near-circular orbit is simplified as:

$$\begin{bmatrix} Da \\ De_x \\ De_y \\ Di_x \\ Di_y \\ Du \end{bmatrix} = \frac{1}{v} \begin{bmatrix} 0 & 2a & 0 \\ \sin u & 2\cos u & 0 \\ -\cos u & 2\sin u & 0 \\ 0 & 0 & \cos u \\ 0 & 0 & \sin u \\ 0 & (-3v/a)\Delta t & 0 \end{bmatrix} \begin{bmatrix} \Delta v_R \\ \Delta_\Gamma \\ \Delta_N \end{bmatrix} \tag{13}$$

where: Da , De , Di , Du are the correction amount of a , e , i , u ; Δv_R , Δ_Γ , Δ_N are the velocity increment in the radial direction, velocity direction, vertical direction of the satellite.

From Equation (13) we can get:

$$\begin{cases} Da = \dfrac{2a}{v} \Delta v_\Gamma = \dfrac{2}{n} \Delta v_\Gamma \\ Du = -\dfrac{3\Delta t}{a} \Delta v_\Gamma \Rightarrow l = 3Du = -3\Delta t \Delta v_\Gamma \end{cases} \tag{14}$$

where: n is the orbital angular velocity of the main star, and l is the track drift distance.

If a collision occurs in a short time, the velocity Δv is applied in the track direction by using the first formula of Equation (14), and to open a certain distance between the formation satellites in the height direction; If a collision occurs after a long period of time, the velocity Δv is applied in the track direction by using the second formula of Equation (14). So that the formation of satellites in the direction of the distance from the track to open a certain distance.

IV. NUMERICAL SIMULATION

Formation flying of two satellites is used as an example, and its orbital parameters [13]-[15] are shown in Table I.

TABLE I. THE LEADER-SATELLITE AND THE FOLLOWING-SATELLITE ORBIT PARAMETERS

Orbit parameter	Leader satellite	Following satellite
a	6892937.0018	6892937.0018
e	0.010117	0.001163
i	97.4433830	97.443830
Ω	90	89.99687
ω	0	357.888
M	0	2.1116

The initial relative position error is 0.1m; the initial relative velocity error is 0.0005m/s; the simulation time is 6 days. Fig. 3 is Collision probability of "xoy".

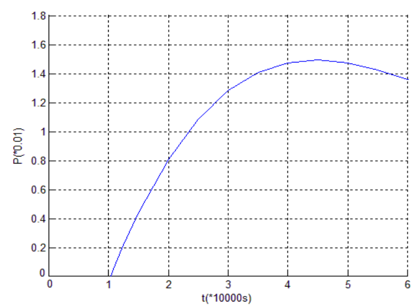

Figure 3. Collision probability of xoy.

For the reference orbit shown in Table I, when the orbital angular velocity $n = 0.0011 m/s^2$, From the first formula of Equation (14):

$$Da = \frac{2}{n} \Delta v_\Gamma \approx 1818 \Delta v_\Gamma \tag{15}$$

The length of the long axis of the orbit and the velocity along the track direction must satisfy the Equation (15).

If the second formula in Equation (14) is used to pull the formation satellites along the track direction by a distance of 2 km, the track curve is shown in Fig. 4:

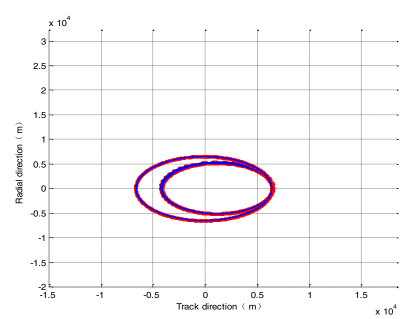

Figure 4. Formation of satellite track curve.

From Fig. 3, we can see that the collision probability increases gradually, then reaches a peak, then gradually decreases. At the beginning of the prediction, position covariance matrix values are very small, at this time, the distance between the center of the probability ellipse and the collision region is relatively large, so the collision probability is very small. As time goes on, the position covariance matrix increases rapidly, which lead to increase the value of collision probability. When the collision probability reaches the maximum value, two satellites are not necessarily collision. Two satellites may still be in a safe distance in the other direction. Collision avoidance is not required at this time. A common practice is: At first, the collision probability of the "xoz" projection surface is calculated. Then the probability value and the security threshold are compared. If the value of the collision probability is greater than the threshold value, the probability of the "xoy" projection surface is calculated. Only these two probability values are greater than the threshold value to avoid operation.

From Fig. 4, we can see that the use of increased trajectory drift distance method, the formation of the satellite orbit in the direction of the track is opened by 2000m distance. This shows that the use of the method is effective.

V. CONCLUSIONS

In this paper, the initial state covariance matrix of the formation satellite is derived. The collision risk between formation satellites is analyzed based on the method of collision probability. When the collision probability of the two satellites is larger than the threshold value, the appropriate avoidance strategy should be adopted. The research of evasion strategy will be the direction of future efforts.

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