# School of Engineering and Applied Science (SEAS), Ahmedabad University

**CSE 400: Fundamentals of Probability in Computing**
**Lecture 10: Randomized Min-Cut Algorithm**

---

**Lecturer:** Dhaval Patel, PhD | **Date:** February 5, 2026
**Topic:** Randomized Min-Cut Algorithm | **Scribe:** Lecture Summary

## 1 Outline

This lecture covers the fundamental concepts of the Min-Cut problem, comparing deterministic and randomized approaches. Key topics include:

- The Min-Cut Problem: Definition and Applications.
- Max-Flow Min-Cut Theorem.
- Deterministic Min-Cut (Stoer-Wagner Algorithm).
- Randomized Min-Cut (Karger's Algorithm).
- Performance Comparison and Success Probabilities.

## 2 The Min-Cut Problem

### 2.1 Why use Min-Cut?

Min-cut algorithms are utilized in various domains to solve problems related to network connectivity, reliability, and optimization:

- **Network Design:** Improves communication efficiency and optimizes network flow by finding the minimum capacity cut.
- **Communication Networks:** Helps understand network vulnerability to failures and aids in building robust, fault-tolerant systems.
- **VLSI Design:** Useful for partitioning circuits into smaller components to reduce interconnectivity complexity.

### 2.2 What is a Min-Cut?

- **Cut-set:** A set of edges whose removal breaks a graph into two or more connected components.
- **Min-cut Problem:** Given a graph $G = (V, E)$ with $n$ vertices, find a minimum cardinality cut-set in $G$.

### 2.3 Edge Contraction

The primary operation in randomized min-cut algorithms is **edge contraction**:

- It removes an edge $(u, v)$ and merges vertices $u$ and $v$ into a single vertex.
- All edges connecting $u$ and $v$ are eliminated.
- All other edges are retained; the resulting graph may have parallel edges but no self-loops.

# 3 Success and Failure in Min-Cut Runs

- **Successful Run:** An iteration of the algorithm that correctly identifies the minimum cut of the graph.
- **Unsuccessful Run:** An iteration where the algorithm fails to identify the minimum cut. This can happen if the algorithm happens to contract critical edges (edges belonging to the min-cut) early in the process.

# 4 Max-Flow Min-Cut Theorem

The theorem states: "In a flow network, the maximum amount of flow passing from the source ($S$) to the sink ($T$) is equal to the total weight of the edges in a minimum cut."

- **Capacity of a Cut:** The sum of the capacity of edges oriented from a vertex in set $X$ to a vertex in set $Y$.
- **Minimum Cut:** The cut in the network with the smallest possible capacity.
- **Maximum Flow:** The largest possible flow from source $S$ to sink $T$.

# 5 Deterministic Min-Cut Algorithm

## 5.1 Stoer-Wagner Algorithm

This algorithm provides an exact solution. The core theorem states that for two vertices $s$ and $t$, a minimum cut of $G$ is the smaller of:

1. A minimum $s-t$-cut of $G$.
2. A minimum cut of $G/\{s,t\}$ (the graph where $s$ and $t$ are merged).

**Complexity:** $O(V \cdot E + V^2 \log V)$.

# 6 Randomized Min-Cut Algorithm

## 6.1 Why Randomized Algorithms?

- Provide a probabilistic guarantee of success.
- Often provide accurate estimates with fewer iterations.
- Advantages: Efficiency, Parallelization, Approximation Guarantees, Robustness, and avoidance of worst-case instances.

## 6.2 Karger's Randomized Algorithm

Karger's algorithm uses random contraction steps to find a cut.

**Theorem for Min-Cut Set:** The algorithm outputs a min-cut set with probability at least:

$$P(\text{success}) \geq \frac{2}{n(n-1)}$$

**Complexity:** $O(V^2)$.

## 7 Comparison: Deterministic vs. Randomized

| **Feature** | **Deterministic (Stoer-Wagner)** | **Randomized (Karger's)** |
|---|---|---|
| **Guarantee** | Always finds exact minimum cut | Approximate min-cut with high probability |
| **Complexity** | $O(V \cdot E + V^2 \log V)$ | $O(V^2)$ |
| **Scalability** | Higher complexity for large graphs | Generally more efficient for large graphs |

## 8 Python Simulation

A simulation of the Randomized Min-Cut algorithm can be found in the lecture 10 Campuswire post (File: `L8_RandomizedMin...ipynb`).

**End of Scribe**