CSE400: Fundamentals of Probability in Computing

Lecture 6: Discrete RVs, Expectation and Problem Solving

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1 Introduction to Random Variables (RVs)

[cite$_{s}tart$]A**random variable X**$onasamplespace\Omega$ is a function $X : \Omega \to \mathbb{R}$ that assigns a real number $X(\omega)$ to each sample point $\omega \in \Omega$[cite: 48].

- **Discrete RVs:** Take values in a range that is finite or countably infinite[cite: 49]. [cite$_{s}tart$]
- **Distribution Visualization:** Can be represented as a bar diagram where the x-axis shows values and the height shows $Pr[X = a]$[cite: 104, 105, 106].

2 Probability Mass Function (PMF)

[cite$_{s}tart$]$Arandomvariableisdiscreteifitcantakeonatmostacountablenumberofvalues[cite:204]$.

Definition: The function $P_X(x_k) = P(X = x_k)$ for $k = 1, 2, 3, \dots$ is the PMF of X[cite: 221, 222]. [cite$_{s}tart$]

Property: The sum of all probabilities must equal 1: $\sum_{k=1}^{\infty} P_X(x_k) = 1$[cite: 233].

3 Bayes' Theorem Recap

$$Pr(B_i|A) = \frac{Pr(A|B_i)Pr(B_i)}{\sum_{i=1}^{n} Pr(A|B_i)Pr(B_i)}$$

- **A priori probability:** $Pr(B_i)$ - formed from presupposed models[cite: 291]. [cite$_{s}tart$]
- **Posteriori probability:** $Pr(B_i|A)$ - calculated after observing event A[cite: 292].

4 Independence

- **Two Events:** A and B are independent if $Pr(A|B) = Pr(A)$ and $Pr(B|A) = Pr(B)$, implying $Pr(A, B) = Pr(A)Pr(B)$[cite: 448]. [cite$_{s}tart$]
- **Three Events:** A, B, and C are mutually independent if[cite: 460]:

- $Pr(A,B) = Pr(A)Pr(B)$, $Pr(B,C) = Pr(B)Pr(C)$, $Pr(A,C) = Pr(A)Pr(C)$[cite: 462, 463]. [cite$_s tart$]
- $Pr(A,B,C) = Pr(A)Pr(B)Pr(C)$[cite: 463].

5 Types of Discrete Random Variables

5.1 Bernoulli Random Variable

[cite$_s tart]Modelsanexperimentwithonlytwooutcomes : Success(1) or Failure(0)[cite : 535, 544]$.

PMF: $P_X(1) = p$ and $P_X(0) = 1 - p$, where $p \in (0,1)$[cite: 556, 557]. [cite$_s tart$]

Examples: Single coin toss, email spam classification[cite: 572, 574].

5.2 Binomial Random Variable $B(n,p)$

[cite$_s tart]Modelsthenumberofsuccessesinnindependenttrials, eachwithsuccessprobability$p$[cite : 597, 599]$.

PMF: $p(i) = \binom{n}{i} p^i (1-p)^{n-i}$ for $i = 0, 1, \ldots, n$[cite: 612]. [cite$_s tart$]

Examples: Correct answers on a test, number of defective items in a sample[cite: 626, 627].

5.3 Geometric Random Variable

[cite$_s tart]Modelsthenumberofindependenttrialsrequireduntilthefirstsuccessoccurs[cite : 644, 645]$.

PMF: $P_X(X = n) = (1-p)^{n-1} \times p$ for $n = 1, 2, \ldots$[cite: 659]. [cite$_s tart$]

Examples: Tosses until first head, attempts until first sale[cite: 669, 670].

5.4 Poisson Random Variable

[cite$_s tart]AdiscreteRVwithparameter\lambda > 0$[cite: 695].

- **PMF:** $p(i) = P\{X = i\} = e^{-\lambda} \frac{\lambda^i}{i!}$ for $i = 0, 1, 2, \ldots$[cite: 702, 703]. [cite$_s tart$]
- **Approximation:** Used for $B(n,p)$ when n is large and p is small such that $n \times p$ is moderate[cite: 725, 726]. [cite$_s tart$]
- **Examples:** Misprints on a page, customers entering a post office daily[cite: 740, 744].