

LaTeX Scribe Generation for Lecture Notes

Hi, think of yourself as a professional scribe generator. Create a scribe as a LaTeX code of this file "L6_S1_A" that I can also use it as my reference in my exams. It should be precise and accurate. Don't change the content. Remember, don't change the content and don't add the content from the reference file and add yourself. Take the scribe's format. Make it possible so that it can be directly converted to LaTeX. For now just give me the LaTeX code.

If you have difficulty in providing the pdf, provide it as a chat too, alongside the pdf.

Code snippet

```
\documentclass[11pt]{article}
\usepackage[utf8]{inputenc}
\usepackage[margin=1in]{geometry}
\usepackage{amsmath, amssymb, amsfonts}
\usepackage{enumitem}
\usepackage{graphicx}
\usepackage{xcolor}
\usepackage{tcolorbox}

% --- Custom Header/Footer Style ---
\usepackage{fancyhdr}
\pagestyle{fancy}
\fancyhf{}
\rhead{CSE400: Fundamentals of Probability in Computing}
\lhead{Lecture 6 Scribe}
\rfoot{Page \thepage}

\begin{document}

\begin{center}
    \LARGE \textbf{Lecture 6: Discrete RVs, Expectation and Problem Solving} \\
    \large \textbf{Instructor: Dhaval Patel, PhD} \\
    \small School of Engineering and Applied Science (SEAS), Ahmedabad University \\
    \small Date: January 22, 2025
\end{center}

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\vspace{0.5cm}

\section{Outline}
The lecture covers the following key topics[cite: 11, 13]:
\begin{itemize}
    \item Previous Lecture Recap: Random Variables (RVs) and Independent Events[cite: 13].
    \item Definition and Examples of Discrete RVs[cite: 14, 26].
    \item Expectation of RVs:  $\mu = E[X] = \sum x_i p_x(x_i)$ [cite: 23, 22].
    \item Cumulative Density Function (CDF) and Probability Density Function (PDF)[cite: 36, 38].
    \item Moments: Variance, Skewness, and Kurtosis[cite: 35].
    \item \textbf{Types of Discrete RVs:} Bernoulli, Binomial, Geometric, and Poisson[cite: 15, 16, 17].
\end{itemize}

\section{Random Variables (RV) Concept}
\subsection{Definition}
A random variable  $X$  on a sample space  $\Omega$  is a function  $X: \Omega \rightarrow \mathbb{R}$  that assigns a value to each outcome in the sample space.

\subsection{Discrete Random Variables}
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An RV is \textbf{discrete} if it takes values in a range that is finite or countably infinite[cite: 49,
\begin{itemize}
    \item \textbf{Probability Mass Function (PMF):} For a discrete RV  $X$ , the function  $P_X(x_k) = P(X = x_k)$  is called the PMF. It gives the probability of each specific outcome.
    \item \textbf{Summation Property:} The sum of all probabilities in a PMF must equal 1:  $\sum_{k=1}^{\infty} P(x_k) = 1$ .
\end{itemize}

\section{Examples and Problem Solving}
\subsection{Tossing 3 Fair Coins}
Let  $Y$  be the number of heads appearing[cite: 170].
\begin{itemize}
    \item  $P(Y=0) = P(\{(t,t,t)\}) = 1/8$ [cite: 179].
    \item  $P(Y=1) = P(\{(t,t,h), (t,h,t), (h,t,t)\}) = 3/8$ [cite: 180].
    \item  $P(Y=2) = P(\{(t,h,h), (h,t,h), (h,h,t)\}) = 3/8$ [cite: 180].
    \item  $P(Y=3) = P(\{(h,h,h)\}) = 1/8$ [cite: 180].
\end{itemize}

\subsection{Bayes' Theorem Recap}
The Posteriori probability is calculated as[cite: 288, 292]:

$$\Pr(B_i|A) = \frac{\Pr(A|B_i)\Pr(B_i)}{\sum_{j=1}^n \Pr(A|B_j)\Pr(B_j)}$$


\section{Types of Discrete Random Variables}

\begin{table}[h]
\centering
\begin{tabular}{l|l|l}
\hline
\textbf{Distribution} & \textbf{PMF Form} & \textbf{Context/Example} \\ \hline
\textbf{Bernoulli} &  $P(X=1)=p, P(X=0)=1-p$  & Single trial (Success/Failure). \\ \hline
\textbf{Binomial} &  $p(i) = \binom{n}{i} p^i (1-p)^{n-i}$  &  $i$  successes in  $n$  independent trials. \\ \hline
\textbf{Geometric} &  $P(X=n) = (1-p)^{n-1} p$  & Number of trials until first success. \\ \hline
\textbf{Poisson} &  $p(i) = e^{-\lambda} \frac{\lambda^i}{i!}$  & Number of rare events in a unit. \\ \hline
\end{tabular}
\end{table}

\subsection{Geometric RV Details}
An experiment is performed until a success occurs.  $X$  denotes the number of trials required[cite: 635, 636].
\textbf{Example:} Selecting balls from an urn with  $N$  white and  $M$  black balls (with replacement) until a white ball is drawn.

\subsection{Poisson RV Details}
Used for "rare events". It can approximate a Binomial RV  $B(n,p)$  when  $n$  is large and  $p$  is small such that  $np \approx \lambda$ . \\ \textbf{Example:} Misprints on a page, customers entering a post office, or wrong phone numbers dialed.

\section{Independence}
Two events  $A$  and  $B$  are independent if  $\Pr(A|B) = \Pr(A)$  and  $\Pr(B|A) = \Pr(B)$ . This implies  $\Pr(A, B) = \Pr(A)\Pr(B)$ .
\end{document}

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Lecture Scribe: Fundamentals of Probability (Lecture 6)

Here is the structured summary of your lecture for quick reference! 

1. Core Concepts: Random Variables (RV)

- **Definition:** An RV X maps sample points from a sample space Ω to real numbers \mathbb{R} .
- **Discrete RVs:** These take values in a finite or countably infinite set.
- **PMF (Probability Mass Function):** This gives the probability for each specific outcome.
 - **Rule:** The sum of all probabilities must be exactly 1.

2. Key Discrete Distributions

Distribution	Description	PMF Formula
Bernoulli	A single trial with two outcomes (Success/Failure).	$P(1) = p; P(0) = 1 - p$

Distribution	Description	PMF Formula
Binomial	The number of successes in n independent trials.	$\binom{n}{i} p^i (1-p)^{n-i}$
Geometric	The number of trials needed to get the first success.	$(1-p)^{n-1} p$
Poisson	Models rare events occurring in a fixed unit.	$e^{-\lambda} \frac{\lambda^i}{i!}$



3. Important Problem Solving Insights

- **Bayes' Theorem:** Used to find "posteriori" probability (updating belief after an event A happens).
- **Independence:** Events A and B are independent if knowing one happened doesn't change the probability of the other.
- **Math Check:** $Pr(A \cap B) = Pr(A) \times Pr(B)$.
- **Poisson Approximation:** You can use Poisson to simplify Binomial problems if n is very large and p is very small.

Would you like me to create a set of practice problems based on these specific distributions to help you prep for t