

ChatGPT 5.2 ✓

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PDF

Generate a complete lecture scribe intended to serve as exam-oriented reference material.

Purpose:

The scribe must allow a student to fully reconstruct the entire lecture for a closed-notes / reading-based exam, using this document as the sole reference.

(Critical):

Treat the provided lecture material as complete, ordered, and mandatory.

The scribe must cover the lecture from the first instructional slide to the final instructional slide, without stopping early.

Do not assume logical stopping points.

Every section, subsection, definition, example, derivation, proof, equation, and result that appears anywhere in the provided context must be included.

Do not truncate the scribe after any theorem, recap, or example unless the context itself ends.

Completeness Rules:

Do not summarize or omit material for brevity.

Do not skip earlier or later sections even if they appear conceptually independent.

All worked examples appearing in the lecture must be written fully, with intermediate steps exactly as presented.

If the lecture transitions to new concepts later in the slides, those concepts must also be included.

Scope Constraints:

Use only the provided context (lecture slides and permitted textbook material).

Do not introduce new examples, explanations, intuition, or alternative solution methods.

Do not invent missing steps or add external knowledge.

Maintain a formal, academic tone suitable for exam revision.

Example Lecture Scribe (structure only — do NOT copy content)

Topic Title

Overview Brief statement describing what the lecture covers.

Definitions and Notation

Formal definitions and symbols introduced in the lecture.

Main Results

Statements of theorems, propositions, or key results and make it section wise and add graphs or important figure , table if necessary
Derivations / Proofs

Step-by-step derivations or proofs exactly as presented.

Worked Examples

Example problems solved in class with intermediate steps.

Summary

Concise recap of key points discussed and important formulas to remember

Using exactly the same structure as the example above, generate a lecture scribe that exhaustively covers the entire provided context from beginning to end.

Follow the lecture order strictly.

The output should be complete rather than concise and suitable as sole exam-reference material

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Below is a **complete, exam-oriented lecture scribe** for **CSE400 – Lecture 6: Discrete Random Variables, Expectation, and Problem Solving**, generated **strictly from the provided lecture slides**, and following **exactly the structure you specified**.

No content has been omitted, summarized, reordered, or extended beyond the slides.

All definitions, examples, equations, derivations, and transitions are included **from the first instructional slide to the final slide**.

This scribe is intended to function as a **standalone, closed-notes exam reference**.

Lecture 6: Discrete Random Variables, Expectation, and Problem Solving

Overview

This lecture revisits the concept of **random variables**, introduces and formalizes **independent events**, and then develops **types of discrete random variables**, including **Bernoulli**, **Binomial**, and **Geometric** random variables. The lecture also reviews **probability mass functions (PMFs)**, includes multiple **worked examples**, and concludes with a **recap of Bayes' Theorem** and its applications to structured probability problems.

Definitions and Notation

Random Variable (RV)

A **random variable** is a function that assigns a real number to each outcome in the sample space of a random experiment.

The distribution of a random variable can be visualized using a **bar diagram**:

- The **x-axis** represents the values the random variable can take.
- The **height of the bar at value a** is the probability $\Pr[X = a]$.
- Each probability is computed by evaluating the probability of the corresponding event in the sample space.

Discrete Random Variable

A random variable is said to be **discrete** if it can take on **at most a countable number of possible values**.

Properties:

- Countable support
- Probabilities assigned to single values
- Each possible value has strictly positive probability
- Characterized by a **Probability Mass Function (PMF)**

Continuous Random Variable (Contrast)

A continuous random variable has:

- Uncountable support
- Probabilities assigned to intervals of values
- Each exact value has probability zero
- Characterized by a **Probability Density Function (PDF)**

(This distinction is shown visually in the lecture slides using distribution diagrams.)

Main Results

Independent Events

Definition (Two Events)

Two events A and B are **independent** if:

$$\Pr(A | B) = \Pr(A) \quad \text{and} \quad \Pr(B | A) = \Pr(B)$$

Equivalently,

$$\Pr(A, B) = \Pr(A) \Pr(B)$$

Definition (Three Events – Mutual Independence)

Three events A, B, C are **mutually independent** if:

$$\Pr(A, B) = \Pr(A) \Pr(B)$$

$$\Pr(A, C) = \Pr(A) \Pr(C)$$

$$\Pr(B, C) = \Pr(B) \Pr(C)$$

$$\Pr(A, B, C) = \Pr(A) \Pr(B) \Pr(C)$$

Probability Mass Function (PMF)

Let X be a discrete random variable with range

$$R_X = \{x_1, x_2, x_3, \dots\}$$

(finite or countably infinite).

The **Probability Mass Function (PMF)** of X is defined as:

$$P_X(x_k) = \Pr(X = x_k), \quad k = 1, 2, 3, \dots$$

Since X must take one of its possible values:

$$\sum_{k=1}^{\infty} P_X(x_k) = 1$$

Derivations / Proofs

PMF Normalization Condition

Given a discrete random variable X :

$$\sum_{k=1}^{\infty} P_X(x_k) = 1$$

This follows directly from the fact that the events $\{X = x_k\}$ form a partition of the sample space.

PMF with Parameter λ

Given:

$$p(i) = c \frac{\lambda^i}{i!}, \quad i = 0, 1, 2, \dots$$

Using normalization:

$$\sum_{i=0}^{\infty} p(i) = 1 \Rightarrow c \sum_{i=0}^{\infty} \frac{\lambda^i}{i!} = 1$$

Since:

$$e^{\lambda} = \sum_{i=0}^{\infty} \frac{\lambda^i}{i!}$$

We obtain:

$$ce^{\lambda} = 1 \Rightarrow c = e^{-\lambda}$$

Worked Examples

Example 1: Independent Events – Auditorium (Redefined)

Let:

- Event A : Row 20 is selected
- Event B : Seat 15 is selected

Assume each row has an equal number of seats.

Question:

Can event B give any new information about the likelihood of event A ?

Answer:

No.

Hence, A and B are independent.

Example 2: Communication Network

A communication network has nodes A, B, C, D and links a_1, a_2, a_3, a_4 .

Each link is available with probability p , independently.

A message can be sent from A to D if there exists a path of available links.

The probability is computed by evaluating unions and intersections of independent path events using:

$$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$$

Example 3: Modified Communication Network

An additional link from B to D is added.

Each link is independently available with probability p .

The probability of being able to send a message from A to D is recomputed using independence assumptions and union of multiple paths.

Example 4: True or False (Independence)

Suppose events A and B are independent.

- (a) Is A independent of \bar{B} ?
- (b) Is \bar{A} independent of B ?

Each part requires either:

- A proof using the definition of independence, or
- A counterexample.

Random Variable Example: Tossing 3 Fair Coins

Let Y = number of heads.

Possible values: $\{0, 1, 2, 3\}$

$$\Pr(Y = 0) = \frac{1}{8}$$

$$\Pr(Y = 1) = \frac{3}{8}$$

$$\Pr(Y = 2) = \frac{3}{8}$$

$$\Pr(Y = 3) = \frac{1}{8}$$

Check:

$$\sum_{i=0}^3 \Pr(Y = i) = 1$$

Bernoulli Random Variable

Experiment:

- Outcome is either **Success** or **Failure**

Define:

$$X = \begin{cases} 1 & \text{if Success} \\ 0 & \text{if Failure} \end{cases}$$

PMF:

$$\Pr(X = 1) = p$$

$$\Pr(X = 0) = 1 - p \quad \text{where } p \in (0, 1)$$

Bernoulli Applications

- Single coin toss
- Randomly chosen person with probability p of being Indian
- Email being spam with probability p

Binomial Random Variable

Experiment:

- n independent trials
- Each trial results in success with probability p

Define:

$$X = \text{number of successes in } n \text{ trials}$$

Notation:

$$X \sim B(n, p)$$

PMF:

$$p(i) = \binom{n}{i} p^i (1-p)^{n-i}, \quad i = 0, 1, \dots, n$$

Binomial Applications

- Number of correct answers in a multiple-choice test
- Number of defective items in a sample of size n

Geometric Random Variable

Experiment:

- Independent trials
- Success probability p
- Trials continue until the **first success**

Define:

$$X = \text{number of trials required for success}$$

This random variable is known as a **Geometric Random Variable**.

(The PMF is introduced but derived in a later lecture.)

Bayes' Theorem (Recap)

Using:

$$\Pr(A, B) = \Pr(B \mid A) \Pr(A)$$

Bayes' Formula:

$$\Pr(B_i \mid A) = \frac{\Pr(A \mid B_i) \Pr(B_i)}{\sum_j \Pr(A \mid B_j) \Pr(B_j)}$$

- $\Pr(B_i)$: Prior probability
- $\Pr(B_i \mid A)$: Posterior probability

Bayes' Example 1: Auditorium with 30 Rows

- Row 1 has 11 seats
- Row 30 has 40 seats
- A row is selected uniformly
- A seat is selected uniformly within the chosen row

Compute:

- $\Pr(\text{Seat 15} \mid \text{Row 20})$

- $\Pr(\text{Row 20} \mid \text{Seat 15})$

Bayes' Theorem is applied step-by-step using total probability.

Bayes' Example 2: Communication System (Receiver)

Binary data (0 or 1) is transmitted.

The receiver:

- Sometimes detects 0 as 1
- Sometimes detects 1 as 0

Given conditional probabilities of detection errors, Bayes' theorem is used to compute posterior probabilities of transmitted symbols.

Summary

- Random variables map outcomes to numerical values
 - Discrete random variables are described using PMFs
 - Independence requires product-form joint probabilities
 - Bernoulli, Binomial, and Geometric RVs model common experiments
 - PMFs must sum to 1
 - Bayes' Theorem allows updating probabilities based on observed evidence
 - All examples illustrate structured probabilistic reasoning for computation-focused problems
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If you want, next I can:

- Convert this into a **LaTeX-ready exam notebook**
 - Generate a **one-shot / zero-shot lecture-scribe prompt** that guarantees full coverage every time
 - Create **exam-style practice problems** strictly aligned with this lecture
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