

Generate a Lecture Scribe as chat from the given Tutorial "Tutorial_2" using the structure from the PDF "Example_Scribe". Follow

Tutorial_2



Example_Scribe



Generate a Lecture Scribe as chat from the given Tutorial "Tutorial_2" using the structure from the PDF "Example_Scribe".

Follow the below two sets of instructions properly to generate the Lecture Scribe :

Instructions : 1) The lecture scribe is to be generated as per the structure given in Example_Scribe 2) Use the content from Tutorial_2 only and DO NOT generate or add new content 3) The lecture scribe should be created as exam reference material

Instructions for following the example scribe : 1) For List of topics covered, you need to analyse the tutorial properly 2) For Explanation of Topics Covered, explain using the questions given in tutorial only 3) For List of Definitions and Theorems, use content of tutorial only but explain in detailed way 4) For Important Examples, select important questions from the tutorial_2 attached in such a way that it covers all the concepts discussed. *Write step by step solution to questions*. INCLUDE EXAMPLES FROM Tutorial ONLY, DO NOT GENERATE BY YOURSELF 5) For the List of Important Formulas,

write the list of all the important formulas used in the Tutorial Questions



Lecture Scribe: Fundamentals of Probability in Computing

1) List of Topics Covered

- **Continuous Uniform Distribution:** Analysis of points chosen on line segments and roads.
- **Bayes' Theorem and Normal Distribution:** Calculating posterior probabilities and minimizing error in region classification.
- **Exponential Distribution:** Modeling repair times, fire station placement, and battery lifetimes.
- **Law of Total Probability and Conditional Probability:** Determining probabilities for sequential events and multi-type components.
- **Poisson Distribution and Binomial Structure:** Finding conditional probabilities of independent counts.
- **Transformation of Random Variables:** Deriving Cumulative Distribution Functions (CDF) and Probability Density Functions (PDF) for piecewise and non-linear functions.
- **Independent Trials and Binomial Distribution:** Approximating stock price movements over multiple periods.
- **Joint Distributions and Order Statistics:** Analyzing the relationship between successive maximums of i.i.d. variables.

2) Explanation of Topics Covered

Uniform and Exponential Spatial Distributions

The tutorial explores how to model events occurring "at random" over a defined space. For a road of length A , a fire station is placed to minimize the expected distance $\mathbb{E}[|X - a|]$. In a uniform setting, the optimal location is the midpoint, whereas, for an exponentially distributed fire distance, the location is determined by the rate λ .

Bayesian Classification and Normal Density

Classification problems are addressed by comparing posterior probabilities. In the case of an image with black and white regions, we use the Normal density formula to determine the likelihood of a specific reading given the region. The goal is to find an image fraction α where the probability of error is identical for both region conclusions.

Reliability and Memoryless Properties

The tutorial examines lifetimes of components using the exponential distribution. It specifically looks at conditional survival: the probability a battery operates for an additional s hours given it has already lasted t hours.

Transformation and Joint CDFs

When a random variable Y is a function of X ($Y = g(X)$), the distribution of Y is found piecewise. This includes identifying "point masses" where a range of X values maps to a single Y value. For joint distributions, such as M_n and M_{n+1} , the tutorial demonstrates how to define the CDF based on whether the evaluation point x is less than or greater than y .

3) List of Definitions and Theorems

1) Bayes' Theorem (Posterior Probability)

Used to update the probability of a hypothesis (region type) given observed evidence (a reading $X = 5$):

$$P(B|X = 5) = \frac{\alpha f(5|B)}{\alpha f(5|B) + (1 - \alpha) f(5|W)}$$

where α is the prior probability $P(B)$.

2) Normal Density Formula

The probability density function for a normally distributed random variable with mean μ and variance σ^2 :

$$f(x) = \frac{1}{\sigma \sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

3) Law of Total Probability

Used to find the overall probability of an event by considering all mutually exclusive scenarios:

$$P(X > t) = p_1 P(X > t|T = 1) + p_2 P(X > t|T = 2)$$

4) Leibniz's Rule

Applied when differentiating an integral whose limits depend on the variable of differentiation (z):

$$\frac{d}{dz} \int_{-z}^z g(x, z) dx$$

This is crucial for finding the PDF of a transformed variable $Z = X^2 + Y^2$.

4) Important Examples

Example 1: Finding Probability on a Line Segment

Question: A point is chosen at random on a line segment of length L . Find the probability that the ratio of the shorter to the longer segment is less than $1/4$.

Solution:

1. Let the point be at distance x from one end, where $0 < x < L$.
2. The ratio $R = \frac{\min(x, L-x)}{\max(x, L-x)}$.
3. **Case 1** ($x \leq L/2$): $\frac{x}{L-x} < \frac{1}{4} \Rightarrow 4x < L - x \Rightarrow x < L/5$.
4. **Case 2** ($x > L/2$): $\frac{L-x}{x} < \frac{1}{4} \Rightarrow 4L - 4x < x \Rightarrow x > 4L/5$.
5. **Valid region:** $(0, L/5) \cup (4L/5, L)$. Total length = $2L/5$.
6. **Probability:** $P = \frac{2L/5}{L} = \frac{2}{5}$.

Example 2: Conditional Poisson Distribution

Question: Given $X \sim \text{Poisson}(\lambda_1)$ and $Y \sim \text{Poisson}(\lambda_2)$ are independent, find $P(X = k | X + Y = n)$.

Solution:

1. **Formula:** $P(X = k | X + Y = n) = \frac{P(X=k)P(Y=n-k)}{P(X+Y=n)}$.
2. **Substitute PMFs:**

$$\frac{\left(\frac{e^{-\lambda_1} \lambda_1^k}{k!} \right) \left(\frac{e^{-\lambda_2} \lambda_2^{n-k}}{(n-k)!} \right)}{\frac{e^{-(\lambda_1+\lambda_2)} (\lambda_1+\lambda_2)^n}{n!}}$$

3. **Simplify:** This gathers into a binomial structure:

$$= \binom{n}{k} \left(\frac{\lambda_1}{\lambda_1 + \lambda_2} \right)^k \left(\frac{\lambda_2}{\lambda_1 + \lambda_2} \right)^{n-k}$$

Example 3: Joint CDF of Successive Maximums

Question: Find the joint distribution of $M_n = \max(X_1, \dots, X_n)$ and $M_{n+1} = \max(M_n, X_{n+1})$.

Solution:

1. **Case 1 ($x \leq y$):** Both $M_n \leq x$ and $X_{n+1} \leq y$ must occur.

$$P(M_n \leq x, M_{n+1} \leq y) = F(x)^n F(y)$$

2. **Case 2 ($x > y$):** If $M_{n+1} \leq y$, then M_n is naturally $\leq y$, which satisfies $M_n \leq x$.


$$P(M_n \leq x, M_{n+1} \leq y) = P(M_{n+1} \leq y) = F(y)^{n+1}$$

3. **Final Joint CDF:**

$$P(M_n \leq x, M_{n+1} \leq y) = \begin{cases} F(x)^n F(y), & x \leq y \\ F(y)^{n+1}, & x > y \end{cases}$$

5) List of Important Formulas

Distribution/Concept	Formula/Result
Uniform PDF	$f_X(x) = \frac{1}{A}$ for $0 \leq x \leq A$

Distribution/Concept	Formula/Result
Exponential PDF	$f_X(x) = \lambda e^{-\lambda x}$
Poisson PMF	$P(X = k) = \frac{e^{-\lambda} \lambda^k}{k!}$
Binomial PMF	$P(k) = \binom{n}{k} p^k (1 - p)^{n-k}$
Exponential Survival	$P(X > t) = e^{-\lambda t}$
Conditional Prob.	$P(A B) = \frac{P(A \cap B)}{P(B)}$
Transformed PDF	$f_Y(y) = f_X(x) \left \frac{dx}{dy} \right $
Optimal Station (U)	$a = \frac{A}{2}$ (Midpoint)
Optimal Station (Exp)	$a = \frac{\ln 2}{\lambda}$
	

Would you like me to create a summary of the transformation methods used in Question 6 and Question 9?