CSE400: Fundamentals of Probability in Computing
Lecture 10: Randomized Min-Cut Algorithm

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1 Introduction to the Min-Cut Problem

[cite$_s tart]Theminimumcut(min-cut)algorithmisutilizedinvariousapplicationstosolveproblemsrelatedtonetwor$
$57, 62]$.

1.1 Applications

- **Network Design:** Helps improve communication efficiency and optimize network flow by finding the minimum capacity cut[cite: 63, 64]. [cite$_s tart$]

- **Communication Networks:** Useful for understanding network vulnerability to failures and building robust, fault-tolerant systems[cite: 73, 74]. [cite$_s tart$]

- **VLSI Design:** Used for partitioning circuits into smaller components to reduce interconnectivity complexity[cite: 85].

1.2 Definitions

- **Cut-set:** A set of edges whose removal breaks a graph into two or more connected components[cite: 97, 103]. [cite$_s tart$]

- **Min-Cut Problem:** Given a graph $G = (V, E)$ with n vertices, the goal is to find a cut-set with minimum cardinality[cite: 104, 111].

- **Edge Contraction:** The primary operation in these algorithms. It removes an edge (u, v) while merging vertices u and v into one. [cite$_s tart]Alledgesconnecting$u$andvareeliminated, whileotheredgesc$
$loops)[cite : 119, 154, 155]$.

2 Max-Flow Min-Cut Theorem

[cite$_s tart]Thetheoremstates : "Inaflownetwork, themaximumamountofflowpassingfromthesourcetothesinkis$
$225, 231]$.

Capacity of a cut: The sum of capacities of edges oriented from a vertex in set X to a vertex in set Y[cite: 232]. [cite$_s tart$]
Max Flow: The largest possible flow from source S to sink T[cite: 235].

3 Deterministic vs. Randomized Algorithms

3.1 Deterministic: Stoer-Wagner Algorithm

[cite$_s tart]Thisapproachalwaysguaranteesanexactminimumcut[cite: 404]$.

Logic: A minimum cut is either the minimum s-t-cut of G, or the minimum cut of the graph $G/\{s, t\}$ obtained by merging s and t[cite: 263]. [cite$_s tart$]

Complexity: $O(VE + V^2 \log V)$[cite: 406].

3.2 Randomized: Karger's Algorithm

[cite$_s tart]Randomizedalgorithmsprovideaprobabilisticguaranteeofsuccessandmayprovideaccuratees$
$318, 324]$.

Efficiency: Karger's algorithm has a time complexity of $O(V^2)$[cite: 418]. [cite$_s tart$]

Probability of Success: The algorithm outputs a min-cut set with probability at least $\frac{2}{n(n-1)}$[cite: 423].

Sensitivity: It can be sensitive to the initial choice of edges. [cite$_s tart]Ifcriticaledgesarecontrac$
$112, 113]$.

4 Pseudocode

4.1 Deterministic Minimum Cut (Stoer-Wagner)

Algorithm 1: MinimumCutPhase(G, a)

1. $A \leftarrow \{a\}$
2. **while** $A \neq V$ **do** add to A the most tightly connected vertex.
3. **return** the cut weight as the "cut of the phase".

Algorithm 2: MinimumCut(G)

1. **while** $|V| \geq 1$ **do**
2. Choose any a from V.
3. Run *MinimumCutPhase(G, a)*.
4. **if** cut-of-the-phase ¡ current minimum cut **then** store it as current min-cut.
5. Shrink G by merging the two vertices added last.
6. **return** the minimum cut.

4.2 Recursive Randomized Min-Cut (Karger)

Algorithm 3: RECURSIVE-RANDOMIZED-MIN-CUT(G, α)

- **Input:** Undirected multigraph G with n vertices, integer constant $\alpha > 0$.
- **if** $n \leq \alpha^3$ **then** return min-cut via brute force.
- **else**
- **for** $i \leftarrow 1$ **to** a **do**
- $G' \leftarrow$ multigraph obtained by applying $n - \lceil \frac{n}{\sqrt{\alpha}} \rceil$ random contractions on G.
- $C' \leftarrow$ *RECURSIVE-RANDOMIZED-MIN-CUT*(G', α).
- **if** $i = 1$ or $|C'| < |C|$ **then** $C \leftarrow C'$.
- **return** C.

5 Comparison Summary

Feature	**Deterministic (Stoer-Wagner)**	**Randomized (Karger's)**
[cite$_{s}tart$]**Guarantee**	Exact minimum cut [cite: 404] [cite$_{s}tart$]	Approximate with high probability [cite:
[cite$_{s}tart$]**Complexity**	$O(VE + V^2 \log V)$ [cite: 406] [cite$_{s}tart$]	$O(V^2)$ [cite: 418]
[cite$_{s}tart$]**Efficiency**	Lower on large graphs [cite: 405] [cite$_{s}tart$]	Higher/Parallelizable [cite: 325, 326]