

School of Engineering and Applied Science (SEAS)
Ahmedabad University

CSE 400: Fundamentals of Probability in Computing

Tutorial-2 Scribe

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Q1. Random Point on Line Segment

Let a point be chosen uniformly on $(0, L)$. Let the point be at distance x from one end.

Two segments: x and $L - x$.

$$R = \frac{\min(x, L - x)}{\max(x, L - x)}$$

We want:

$$P\left(R < \frac{1}{4}\right)$$

Case 1: $x \leq \frac{L}{2}$

$$\frac{x}{L - x} < \frac{1}{4} \Rightarrow 4x < L - x \Rightarrow 5x < L \Rightarrow x < \frac{L}{5}$$

Case 2: $x > \frac{L}{2}$

$$\frac{L - x}{x} < \frac{1}{4} \Rightarrow 4(L - x) < x \Rightarrow 5x > 4L \Rightarrow x > \frac{4L}{5}$$

Valid region:

$$(0, \frac{L}{5}) \cup (\frac{4L}{5}, L)$$

Total valid length:

$$\frac{2L}{5}$$

Since uniform over length L :

$$P = \frac{2L/5}{L} = \frac{2}{5}$$

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Q2. Bayesian Classification with Normal Densities

Given:

$$X|W \sim N(4, 4), \quad X|B \sim N(6, 9)$$

$$P(B) = \alpha, \quad P(W) = 1 - \alpha$$

Observed: $X = 5$

Equal error condition:

$$P(B|X = 5) = P(W|X = 5) = \frac{1}{2}$$

Using Bayes' theorem:

$$\frac{\alpha f(5|B)}{\alpha f(5|B) + (1 - \alpha) f(5|W)} = \frac{1}{2}$$

Normal density:

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

$$f(5|B) = \frac{1}{3\sqrt{2\pi}} e^{-1/18}$$

$$f(5|W) = \frac{1}{2\sqrt{2\pi}} e^{-1/8}$$

Solving:

$$3\alpha = 2(1 - \alpha)e^{-5/72}$$

$$\alpha \approx 0.3827$$

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Q3. Minimizing Expected Distance

(a) Uniform Case

$X \sim \text{Uniform}(0, A)$

$$E[|X - a|] = \int_0^a (a - x) \frac{1}{A} dx + \int_a^A (x - a) \frac{1}{A} dx$$

Differentiating and solving:

$$a = \frac{A}{2}$$

(b) Exponential Case $X \sim \text{Exponential}(\lambda)$

$$E[|X - a|] = \int_0^a (a - x)\lambda e^{-\lambda x} dx + \int_a^\infty (x - a)\lambda e^{-\lambda x} dx$$

Derivative:

$$1 - 2e^{-\lambda a} = 0$$

$$a = \frac{\ln 2}{\lambda}$$

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Q4. Mixture of Exponentials

Lifetime:

$$P(X > t) = p_1 e^{-\lambda_1 t} + p_2 e^{-\lambda_2 t}$$

$$P(X > t + s) = p_1 e^{-\lambda_1(t+s)} + p_2 e^{-\lambda_2(t+s)}$$

Conditional probability:

$$P(X > t + s | X > t) = \frac{p_1 e^{-\lambda_1(t+s)} + p_2 e^{-\lambda_2(t+s)}}{p_1 e^{-\lambda_1 t} + p_2 e^{-\lambda_2 t}}$$

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Q5. Conditional Poisson Distribution

$$X \sim \text{Poisson}(\lambda_1)$$

$$Y \sim \text{Poisson}(\lambda_2)$$

$$P(X = k | X + Y = n) = \binom{n}{k} \left(\frac{\lambda_1}{\lambda_1 + \lambda_2} \right)^k \left(\frac{\lambda_2}{\lambda_1 + \lambda_2} \right)^{n-k}$$

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Q6. Transformation of Uniform Random Variable

Given:

$$X \sim \text{Uniform}[-2, 2]$$

$$g(x) = \begin{cases} x, & x \in [-2, -1] \\ 0, & x \in (-1, 1) \\ x, & x \in [1, 2] \end{cases}$$

CDF

Piecewise:

$$F_Y(y) = \begin{cases} 0, & y < -2 \\ \frac{y+2}{4}, & -2 \leq y < -1 \\ \frac{1}{4}, & -1 \leq y < 0 \\ \frac{3}{4}, & 0 \leq y < 1 \\ \frac{y+2}{4}, & 1 \leq y \leq 2 \\ 1, & y > 2 \end{cases}$$

Point mass:

$$P(Y = 0) = \frac{1}{2}$$

PDF

$$f_Y(y) = \begin{cases} \frac{1}{4}, & -2 < y < -1 \\ \frac{1}{4}, & 1 < y < 2 \\ 0, & \text{otherwise} \end{cases}$$

with $P(Y = 0) = \frac{1}{2}$.

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Q7. Stock Model Approximation

$$S_{1000} = S_0 u^k d^{1000-k}$$

Condition:

$$u^k d^{1000-k} \geq 1.30$$

Taking logs:

$$k \geq 470$$

$$k \sim \text{Binomial}(1000, 0.52)$$

$$P(k \geq 470) = \sum_{k=470}^{1000} \binom{1000}{k} (0.52)^k (0.48)^{1000-k}$$

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Q8. Exponential Repair Time

$$\lambda = \frac{1}{2}$$

(a)

$$P(X > 2) = e^{-1}$$

(b)

$$P(X \geq 10 | X > 9) = \frac{e^{-5}}{e^{-9/2}} = e^{-1/2}$$

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Q9. Distribution of $Z = \sqrt{X^2 + Y^2}$

$$F_Z(z) = \iint_{x^2+y^2 \leq z^2} f_X(x) f_Y(y) dx dy$$

PDF:

$$f_Z(z) = z \int_{-z}^z f_X(x) \frac{f_Y(\sqrt{z^2 - x^2}) + f_Y(-\sqrt{z^2 - x^2})}{\sqrt{z^2 - x^2}} dx$$

for $z > 0$.

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Q10. Joint Distribution of M_n and M_{n+1}

$$M_n = \max(X_1, \dots, X_n)$$

$$P(M_n \leq x) = F(x)^n$$

Joint CDF:

$$P(M_n \leq x, M_{n+1} \leq y) = \begin{cases} F(x)^n F(y), & x \leq y \\ F(y)^{n+1}, & x > y \end{cases}$$