Random Variable

Types of random var.

Discrete random var. is random var. which takes only random value or discret isolated value is k/a discret random var.

eg Tossing of two coins in which a random variable which define "As no. of heads" can take values 0, 1, 2, is a discrete random var.

Continuous random variable is a var. which can take all possible values in the given interval is called cont. rand. var.
weight of individual groups,
height of ind. groups

I Probability mass function (pmf)-

Let we have-

$$x \rightarrow x_1, \quad x_2, \quad x_3, \; \dots \; x_n$$

$$P(x) \rightarrow p_1, \quad p_2, \quad p_3, \; \dots \; p_n$$

i.e. $P(x) = f(x) = \begin{cases} p_i, & \text{for } i = 1, 2, 3, \dots \\ 0, & \text{otherwise} \end{cases}$

is k/a pmf or probability fn or prob. dist. or prob. fn dist. etc.

Mean & Variance of pmf-

Mean $= \dfrac{\Sigma x\, p(x)}{\Sigma p(x)}$, $\because \Sigma p(x) = 1$

Mean $\mu(\bar{x}) = \Sigma x\, P(x)$

Now,

Variance $= \dfrac{\Sigma (x-\bar{x})^2 p(x)}{\Sigma p(x)}$

$= \Sigma (x-\bar{x})^2 p(x)$

$= \Sigma (x^2 + \bar{x}^2 - 2x\bar{x}) p(x)$

$= \Sigma x^2 p(x) + \Sigma \bar{x}^2 p(x) - \Sigma 2x\bar{x}\, p(x)$

$= \Sigma x^2 P(x) + \bar{x}^2 \Sigma p(x) - 2\bar{x}\, \Sigma x\, P(x)$

$= \Sigma x^2 P(x) + \bar{x}^2 - 2x\bar{x}$

$V = \Sigma x^2\, p(x) - \bar{x}^2$

$S.D = \sqrt{\Sigma x^2 p(x) - \bar{x}^2}$

A bag contains 4 white & 3 red balls, 3 balls are drawn with replacement (one by one) from this bag. Find the pmf for the no. of red balls drawn. Also find the mean & var. of the probability dist.

$n(W) = 4$ Let X be a random var.

$n(R) = 3$

3 balls are drawn with replacement.

4W 3R $\boxed{0 \quad 0 \quad 0}$ $= \{0, 1, 2, 3\}$

$X =$	0	1	2	3
$P(X) =$	$P(0)$	$P(1)$	$P(2)$	$(P3)$

$P(0)$:

$\frac{4}{7} \quad \frac{4}{7} \quad \frac{4}{7}$

$\cancel{P(1)} = P(0) = \left(\frac{4}{7}\right)^3 = \frac{64}{343}$ $\quad {}^3C_0$

$P(1) = \frac{3}{7} \times \frac{4}{7} \times \frac{4}{7} + \frac{4}{7} \times \frac{3}{7} \times \frac{4}{7} + \frac{4}{7} \times \frac{4}{7} \times \frac{3}{7}$

$= {}^3C_1 \left(\frac{3}{7} \times \frac{4}{7} \times \frac{4}{7}\right)$

$P(2) = {}^3C_2 \times \frac{3}{7} \times \frac{3}{7} \times \frac{4}{7} = 3 \times \frac{36}{343} = \frac{108}{343}$

$P(3) = {}^3C_3 \left(\frac{3}{7} \times \frac{3}{7} \times \frac{3}{7}\right) = \frac{27}{343}$

Hence pmf is

x	0	1	2	3
$P(x)$	$\frac{64}{343}$	$\frac{144}{343}$	$\frac{108}{343}$	$\frac{27}{343}$

Mean $= \mu(\bar{x}) = \Sigma x\, P(x)$

$\bar{x} = 0 \times \frac{64}{343} + 1 \times \frac{144}{343} + 2 \times \frac{108}{343} + 3 \times \frac{27}{343}$

$\bar{x} = 9/7 = 1.28$

Variance $= \Sigma x^2 P(x) - \bar{x}^2$

$= 0^2 \times \frac{64}{343} + 1^2 \times \frac{144}{343} + 2^2 \times \frac{108}{343}$

$+ 3^2 \times \frac{27}{343}$

$= \frac{36}{49} = 0.73$

Properties of pmf

1) $f(x)$ or $p(x) \geq 0$
ii) $\Sigma P(x) = 1$

Q.2 A random variable x has the following probability f^n (x → discrete value)

X →	0	1	2	3	4	5	6	7
P(X) →	0	K	2k	2k	3k	k^2	$2k^2$	$(7k^2+k)$

Find : i) The value of k.
ii) $P(X<6)$, $P(X \geq 6)$, $P(3<X\leq 6)$

We know that $\Sigma P(x) = 1$

$0 + k + 2k + 3k + k^2 + 2k^2 + 7k^2 + 1 = 1$

Joint Probability dist. for two rand. var.

Let X and Y both are discrete random var. of sample space then joint probability distribution or joint probability f^n or joint probability mass f^n is defined as :

$X \rightarrow x_1, x_2, x_3, \ldots\ldots x_n$
$Y \rightarrow y_1, y_2, y_3, \ldots\ldots y_n$

then the f^n $f(x,y) = P(x = x_i, Y = y_j) = P_{xy}$
$= P_{ij} = P(x_i, y_j)$
is called joint pmf.

Q. Two dice are tossing simultaneously. Let X & Y be two random var denote the no. appearing both dice. Obtain the joint probability mass f^n of the same.

$X \rightarrow 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6$
$Y \rightarrow 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6$

Y \ X	1	2	3	4	5	6
1	1/36	"				
2	1/36	"				
3	"					
4	"					
5	"					
6	"					

$P(X=3, Y=4)$ ii) $P(X+Y=7)$

$P(X\leq 3, Y\leq 2)$

i) $P(3,4) = \frac{1}{36}$

ii) $P(X+Y=7) = \frac{6}{36} = \frac{1}{6}$

iii) $P(X\leq 3, Y\leq 2) = P(1,1) + P(1,2) + P(2,1) + P(2,2) + P(3,1) + P(3,2)$

$= \frac{1}{36} + \frac{1}{36} + \frac{1}{36} + \frac{1}{36} + \frac{1}{36} + \frac{1}{36} + \frac{1}{36} = \frac{6}{36} = \frac{1}{6}$

Q. A company test two comp. from a batch let X be the no. of defective comp. of type A. & Y be the no. of defective components of type B. The Joint pmf is given as   ① Marginal prob.

X \ Y	0	1	2
0	0.50	0.10	0.05
1	0.15	0.05	0.02
2	0.03	0.02	0.01

i) What is the probability that there is exactly 1 defective type A comp.?

ii) What is the probability that there are no defective type B comp.?

(M.P)

i) $P(X=1) = P(1,0) + P(1,1) + P(1,2)$

$= 0.15 + 0.05 + 0.02 = 0.22$

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Marginal Probability - Let X and Y are two random discrete random variable of same sample space. Then marginal probability of $X = x$ is

$$p_i = P(X = x) = P(x_i, y_1) + P(x_i, y_2) + P(x_i, y_3) + \dots + P(x_i, y_n)$$

$$P(X = x_i) = \text{Marginal Probability of } X = x$$
$$= \sum_{j=1}^{m} P(x_i, y_j)$$

Similarly, the marginal probability $y = y_j$

$$= p_j = P(y = y_j) = \sum_{i=1}^{n} P(x_i, y_j)$$

The marginal probability is the probability of an event occurring for just one variable, ignoring the other var.
The key idea is ignoring the other var. We marginalise them out.
It is an unconditional probability. It does not depend on any specific condn related to the other var.

Suppose we survey 100 people abt. their gender & their fav. sport.

	Soccer	Basketball	Tennis	
Male	20	15	15	50
Female	10	25	15	50
	30	40	30	100

Find: i) Marginal probability of being Male (ignoring fav sport)
ii) MP of preferring Soccer (ignoring gender).
iii) What is the prob. that a randomly selected person is a male & prefers soccer.

i) $P(\text{male}) = P(\text{male, soccer}) + P(m, B) + P(m, T)$
$$= \frac{20}{100} + \frac{15}{100} + \frac{15}{100} = \frac{50}{100} = \frac{1}{2}$$

ii) $P(\text{soccer}) = P(m, s) + P(F, s) = \frac{20}{100} + \frac{10}{100} = 0.3$

iii) $P(m, s) = \frac{20}{100} = 0.20$.

Q- The foll. table is the jt. prob. dist. of the two rand. var. X & Y as

X \ Y	1	2	3	4
1	4/36	3/36	2/36	1/36
2	1/36	3/36	3/36	2/36
3	5/36	1/36	1/36	1/36
4	1/36	2/36	1/36	5/36

Q. The marginal probability dist of X

X	1	2	3	4
P(X)	10/36	9/36	8/36	9/36

$$P(X=1) = \frac{4}{36} + \frac{3}{36} + \frac{2}{36} + \frac{1}{36} = \frac{10}{36}$$

$$P(X=2) = \frac{1}{36} + \frac{3}{36} + \frac{3}{36} + \frac{2}{36} = \frac{9}{36}$$

$$P(X=3) = \frac{5}{36} + \frac{1}{36} + \frac{1}{36} + \frac{1}{36} = \frac{8}{36}$$

$$P(X=4) = \frac{1}{36} + \frac{2}{36} + \frac{1}{36} + \frac{5}{36} = \frac{9}{36}$$

Q. Marginal Probability for Cont. rand. var.

i) Ma

ii) Marginal probability fn of Y

$$h(y) = \int_{-\infty}^{\infty} f(x,y)\, dx.$$

$f(x,y) \to$ joint pdf

(10m) The joint density function of the two random variables is given by-

$$f(x,y) = \begin{cases} \frac{1}{8}(6-x-y), & 0 \le x < 2,\ 2 \le y < 4 \\ 0, & \text{otherwise} \end{cases}$$

Find i) the marginal prob of X
ii) the " " of Y

$$g(x) = \int_{y=-\infty}^{\infty} f(x,y)\, dy$$

$$= \int_2^4 \frac{1}{8}(6-x-y)\,dy$$

$$= \frac{1}{8}\left[6y - xy - \frac{y^2}{2}\right]_2^4$$

$$= \frac{3-x}{4} \quad ; 0 < x < 2$$

ii) $h(y) = \int_{x=-\infty}^{\infty} f(x,y) = \int$

Conditional Prob. for two rand. var.

Case-I Let X & Y two discrete random variables of same sample space S. Then the conditional probability of X given Y:

Jt. Prob.

$$P(X/Y) = P(X=x / Y=y) = \frac{P(X=x, Y=y)}{P(Y=y)}$$

marg. Prob. of Y

& the conditional probability of Y given X:

$$P(Y=y / X=x) = P(Y/X) = \frac{P(X=x, Y=y)}{P(X=x)}$$

Case-II X & Y both are continuous rand. var.
cond. Prob. f^n of X given Y.

$$f(X/Y) = \frac{f(X=x, Y=y)}{f(Y=y)} \xrightarrow{} \text{Jt. pdf}$$

$f(Y=y) \rightarrow$ marg. prob. function

This is also k/a conditional density f^n of X.

iii) Conditional Prob. f^n of Y given X:

$$f(Y/X) = \frac{f(x,y)}{g(x)} = \frac{f(x,y)}{f(x)}$$

$f(x,y) \rightarrow$ Joint pd

$f(x) \rightarrow$ marg. prob. f^n of X.

Q- $P(x, y) = \frac{(x^2+y)}{32}$ for $x = 0, 1, 2, 3$, $y = 0, 1$.

Find (VI) $P\left(X = 2 / Y = 1\right)$.

$P(2,1) = \frac{4+1}{32} = \frac{5}{32}$

	0	1	2	3
0	0	1/36		
1				

$= \frac{P(2,1)}{P(Y=1)}$

$= \frac{5/32}{P(0,1) + P(1,1) + P(2,1) + P(3,1)}$

$= \frac{5/32}{1/32 + 2/32 + 5/32 + 10/32} = \frac{5}{18}$

Q- Three balls are drawn at random from a box containing 2 whites (X), 3 red (Y) and 4 black balls. If X denotes the no. of white balls & Y denotes the no. of red balls drawn. Find:

1) The jt. prob. distr. of (X & Y.)

11) The marginal probability of X & Y.

$X = 0, 1, 2$, $Y = 0, 1, 2, 3$

$P(0,0) = P(0 \text{ white}, 0 \text{ Red})$
$= P(3 \text{ black balls})$
$= \frac{^4C_3}{^9C_3} = \frac{1}{21}$

	X \ Y	0	1	2	3	MP of X
5/12	0	1/21	3/14	1/7	1/84	
1/2 = 7/14	1	1/7	2/7	1/14	0	
1/12	2	1/21	1/28	0	0	
	MP of Y					

(0,1) P(0 white 1 Red) : = P(1 red 2 blacks)

$= \frac{{}^3C_1 \times {}^4C_2}{{}^9C_3} = \frac{3}{14}$

(0,2) P(0 white 2 Red) = P(2 red ~~0~~ black)

$= \frac{{}^3C_2 \times {}^4C_1}{{}^9C_3} = \frac{1}{7}$

(0,3) P(0 white 3 Red) = P(3 red 0 black)

$= \frac{{}^3C_3}{{}^9C_3} = \frac{1}{\frac{9\times8\times7}{3\times2\times1}} = \frac{1}{84}$

(1,0) P(1 white 0 red) = P(1 white 2 black)

$= \frac{{}^2C_1 \times {}^4C_2}{{}^9C_3} = \frac{1}{7}$

(1,1) P(1 white 1 Red) = P(1 white 2

$= {}^2C_1 \times$

II)

X	0	1	2
P(X)	5/12	1/2	1/12

Y	0	1	2	3
P(Y)	5/21	15/28	3/14	1/84

III) The conditional prob. of X given Y=0 & cond. prob of Y given X=0.

Conditional Prob. of X given by -

$P(X/Y) =$

X	0	1	2
P(X/Y)	5/12	1/2	1/12

P(X =

Y	0	1	2	3
	5/21	15/28	3/14	1/84

$P\left(\frac{X}{Y}\right) = P(X=1/Y=y) = P\left(\frac{X=1, Y=y}{Y=y}\right)$

Conditional Prob. of X given Y = 0:

X	0	1	2
P(X/Y)	1/5	3/5	1/5

Y	0	1	2	3
P(Y/X=0)	P(Y=0/X=0)	P(Y=1/X=1)	P(Y=2/X=2)	P(Y=3/X=0)

$\frac{P(X=0, Y=0)}{P(X=0)} = 4/35.$

$P\left(\frac{Y=1}{X=0}\right) = \frac{P(X=0, Y=1)}{P(X=0)} = \frac{\frac{3}{14}}{5/12} = \frac{18}{35}$

Q- The joint pdf is given by:

$$f(x,y) = \begin{cases} e^{-(x+y)}, & \text{for } 0 \le y \le x < \infty \\ 0, & \text{elsewhere} \end{cases}$$

Find (i) marginal pdf of X
(ii) " " " Y
(iii) conditional pdf of X given Y.

1) $g(x) = \int_{-\infty}^{\infty} f(x,y)\, dy = \int_{y=0}^{x} e^{-(x+y)}\, dy$

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NOTE- If X & Y both are independent random variables then

$$P(X, Y) = P(X) \cdot P(Y).$$

Here X & Y both are discrete or continuous random variables

Q- The joint pdf of the two-dimensional random variables is given by:

$$f(x,y) = \begin{cases} 2, & 0 < x < 1, \quad 0 < y < x \\ 0, & \text{otherwise.} \end{cases}$$

Find:
i) $P(X<1)$ ii) Check whether X & Y are independent iii) $P(Y)$ iv) $P(Y/X)$

i) $P(X<1) = \int 2\, dy = 2y\Big|_0^x = 2x - 0 = 2x$

iii) $P(Y) = \int_0^1 2\, dx = 2x\Big|_0^1 = 2 - 0 = 2$

iv) $P(Y/X) = \dfrac{P(Y=y, X=x)}{P(X=x)} = \dfrac{\cancel{2}}{\cancel{2}x} = \dfrac{1}{x}$, $0<x<1$

ii) To check dependency: $P(X, Y) = P(X) \cdot P(Y)$

$\cancel{2x} \cdot 2 \neq 2 \cdot 2x$

$P(\cancel{X}) =$

$P(Y) = \int_{x=0}^{1} 2\, dx = 2[x]_0^1 = 2y^0$, $0<y<x$

$P(X, Y) = P(X) \cdot P(Y)$, for Independency of X & Y

Now $P(X) \cdot P(Y) = 2 \times 2x = 4x$, for $0 < x < 1, 0 \le y < 2$

But

$P(X, Y) = 2$, $0 < x < 1$, $0 \le y < x$

Hence $P(X) \cdot P(Y) \neq P(X, Y)$.

Hence both X & Y are not Independent.

v) $P\left(\frac{1}{2} < x < 1\right) = 2x$, for $\frac{1}{2} < x < 1$

$P(X \le 1) = P(X < 1) = 2x$, $0 < x < 1$

A coin is tossed 4 times

Mathematical Expectation of a random variable is a weighted average of all possible values of the RV, where the wts are the probabilities associated with the corresponding values.

$X \rightarrow 0, 1, 2, 3, 4$

$P(x)$	$\left(\frac{1}{2}\right)^4$	$^4C_1\left(\frac{1}{2}\right)^4$	$^4C_2\left(\frac{1}{2}\right)^4$	$^4C_3\left(\frac{1}{2}\right)^4$	$^4C_4\left(\frac{1}{2}\right)^4$
	$\frac{1}{16}$	$\frac{1}{4}$	$\frac{3}{8}$	$\frac{1}{4}$	$\frac{1}{16}$

Expectation $= \sum x P(x) = E(x)$

$E(x) = 0 \times \frac{1}{16} + \frac{4}{16} \times 1 + \frac{6}{16} \times 2 + 4 \times \frac{1}{16} \times 3 + \frac{1}{16} \times 4$

$= \frac{1}{16}\left[4 + 12 + 12 + 4\right] = \frac{32}{16} = 2$

Variance$(x) = \sum x^2 P(x) - \left[\sum x P(x)\right]^2$

$= \sum x^2 P(x) - (2)^2$

$= 0 + \frac{1^2 \times 4}{16} + \frac{2^2 \times 6}{16} + \frac{3^2 \times 4}{16} + \frac{4^2 \times 1}{16} - 4$

$$V(x) = \frac{4+24+36+16}{16} - 4$$
$$= \frac{80}{16} - 4$$
$$= 5 - 4 = 1$$

Laws of Expectation -

i) $E(\alpha) = \alpha$

* ii) $E(ax) = \sum ax\, P(ax)$
$= a \sum x\, P(ax)$

$E(ax) = aE(x)$

iii) $E(x+y) = \sum (x+y)\, P(x+y)$
$= \sum x\, P(x+y) + \sum y\, P(x+y)$

$E(x+y) = E(x) + E(y)$

iv) $E(x-y) = E(x) - E(y)$

v) $E(ax+b) = E(ax) + E(b)$

$E(ax+b) = aE(x) + b$

vi) If X and Y both are independent random variable then.
($P(x, y) = P(X) \cdot P(Y) \rightarrow P$) X, Y - indep. same o/p
$E(XY) = E(X) \cdot E(Y) \rightarrow E$

Q. What is the mathematical expectation of the sum of the pts. appeared on a tossing of the 2 dice ?

Event - Sum of no. appear

$X \rightarrow 2, 3, 4, \ldots, 12$

X:	2	3	4	5	6	7	8	9	10	11	12
P(Y):	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{4}{36}$	$\frac{5}{36}$	$\frac{6}{36}$	$\frac{5}{36}$	$\frac{4}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$

$$E(x) = 2 \times \frac{1}{36} + 3 \times \frac{2}{36} + \cdots \frac{12 \times 1}{36}$$

$$= \frac{1}{36}\left[2 + 3 + 4 + \cdots 12\right]$$

$E(x) = 7$

Q.2 $E(x+y) = E(x) + E(y)$
$= \frac{7}{2} + \frac{7}{2}$

n tossing $= \frac{7}{2} n$

Q. Find the math. exp. of the sum of pts. appeared on tossing n dice.

Q. ... of the product of the points that appeared tossing on n dice.

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Binomial Probability distribution :

Let x be a discrete random var. then binomial probability is given by -

$$P(x) = {}^nC_x \, p^x \, q^{n-x}$$

where $n \rightarrow$ Total no of independent trials

$p = 1 - q$ $\boxed{p+q=1}$

$p \rightarrow$ probability of success of event in the single trial

$q \rightarrow$ Prob. of failure of event in the single trial.

Q. $x \rightarrow$ event that x times happens out of n trials.

Q. A coin is tossing 10 times. Find the probability

i) getting 7 heads.

ii) getting atleast 7 heads 7, 8, 9, 10

iii) atmost 7 heads 1, 2, ... 7

$n = 10$, $x = 7$

We know that

$$P(x) = {}^nC_x \, p^x \, q^{n-x}$$

${}^nC_x = {}^nC_{n-x}$

$p = 1/2$, $q = 1 - p = 1/2$

i) $P(7) = {}^{10}C_7 \left(\frac{1}{2}\right)^7 \left(\frac{1}{2}\right)^{10-7}$

$= {}^{10}C_7 \left(\frac{1}{2}\right)^7 \left(\frac{1}{2}\right)^3$

$= {}^{10}C_3 \left(\frac{1}{2}\right)^{10}$

$\left\{ {}^nC_x = {}^nC_{n-x} \quad {}^{10}C_7 = {}^{10}C_{10-7} = {}^{10}C_3 \right.$

$P(7) = \frac{\cancel{10}^{5} \times \cancel{9}^{3} \times \cancel{8}^{4}}{\cancel{3} \times \cancel{2} \times 1} \times \frac{1}{1024} = \frac{15}{128}$

ii) P(atleast 7 heads)

$= P(7) + P(8) + P(9) + P(10)$

$= {}^{10}C_7 \left(\frac{1}{2}\right)^7 \left(\frac{1}{2}\right)^3 + {}^{10}C_8 \left(\frac{1}{2}\right)^8 \left(\frac{1}{2}\right)^2 + {}^{10}C_9 \left(\frac{1}{2}\right)^9 \left(\frac{1}{2}\right)^1$
$+ {}^{10}C_{10} \left(\frac{1}{2}\right)^{10} \left(\frac{1}{2}\right)^0$

$= \left(\frac{1}{2}\right)^{10} \left[{}^{10}C_7 + {}^{10}C_8 + {}^{10}C_9 + {}^{10}C_{10} \right]$

$= \frac{1}{1024} \left[\frac{\cancel{10}^{5} \times \cancel{9}^{3} \times 8}{\cancel{3} \times \cancel{2} \times 1} + \frac{\cancel{10}^{5} \times 9}{\cancel{2} \times 1} + 10 + 1 \right]$

$= \frac{1}{1024} \left[120 + 45 + 10 + 1 \right] = \frac{176}{1024} = \frac{44}{256} = \frac{11}{64}$

iii) $P(\text{atmost heads}) = P(0) + P(1) + P(2) + P(3) + P(4) + P(5) + P(6) + P(7)$

$$= \left(1 - \left[P(8) + P(9) + P(10)\right]\right) \frac{1}{1024}$$

$$= \left(1 - \frac{[45 + 10 + 1]}{1024}\right)$$

$$= 1 - \frac{\cancel{56}^{7}}{\cancel{1024}_{128}} = \frac{121}{128}$$

Q- Probability of bomb dropped from the plane will strike the target is 4/5. If 6 bom are dropped

i) exactly two bombs will strike the target.
ii) atleast 2

$n = 6, \quad x = 2$

$$p = \frac{1}{5}$$

$$q = 1 - p = 1 - \frac{1}{5} = \frac{5-1}{5} = \frac{4}{5}$$

$$P(x) = {}^nC_x \, p^x \, q^{n-x}$$

$$= {}^6C_2 \left(\frac{1}{5}\right)^{\cancel{6}} \left(\frac{4}{5}\right)^4$$

i) P (exact 2 bomb.)

$$= {}^6C_2 \left(\frac{1}{5}\right)^2 \left(\frac{4}{5}\right)^4 = {}^6C_2 \left(\frac{4}{5}\right)^6$$

$$= \frac{\cancel{6}^{3} \times \cancel{5}}{\cancel{2} \times 1} \times \frac{4^6}{5^{\cancel{6}}5} = \frac{768}{3125}$$

ii) P (atleast 2 bombs will strike target).

$$= P(2) + P(3) + P(4) + P(5) + P(6)$$

$$1 - \left[P(0) + P(1)\right]$$

$$1 - \left[{}^6C_0 \left(\frac{1}{5}\right)^0 \left(\frac{4}{5}\right)^6 + {}^6C_1 \left(\frac{1}{5}\right)^1 \left(\frac{4}{5}\right)^5\right]$$

$$1 - \left[\frac{4^6}{5^6} + \frac{6 \times 4^5}{5^6}\right]$$

$$1 - \frac{4^5}{5^6}(4 + 6) = 1 - \frac{4^5 \times \cancel{10}^{2}}{5^6 \; 5^5}$$

$$\frac{3125 - 2048}{3125} = \frac{1077}{3125}$$

Mean of Binomial Probability Distr.

Mean = np

Variance of binomial prob. distrib.

variance = npq, S.D = $\sqrt{npq}$

Q. If the mean of binomial dist is 3 & the variance is 3/2. T

i) Total no. of trials
ii) Prob. of getting success in single trial

iii) Probability of getting success atleast in 4 successes.

Mean : $np = 3$ —— (I)

Variance = $npq = 3/2$ —— (II)

(2)/(1) : $\frac{npq}{np} = \frac{3/2}{3}$

$q = 1/2$

But $p = 1 - q = 1 - \frac{1}{2} = \frac{1}{2}$

from (I). $n \times \frac{1}{2} = 3 \Rightarrow \boxed{n = 6}$

1) Total no. of trials = 6

ii) $p = 1/2$

iii) $P(\text{at least 4 success}) = P(4) + P(5) + P(6)$

$= {}^6C_4\left(\frac{1}{2}\right)^4\left(\frac{1}{2}\right)^2 + {}^6C_5\left(\frac{1}{2}\right)^5\left(\frac{1}{2}\right)^1 + {}^6C_6\left(\frac{1}{2}\right)^6\left(\frac{1}{2}\right)^0$

$= \frac{1}{2^{10}}\left[{}^6C_4 + {}^6C_5 + {}^6C_6\right)$

$= \frac{1}{1024}\left(\frac{6\times5}{2\times1} + 6 + 1\right)$

Q. The sum of the product of mean & var. of a binomial distr. are 25/3 & 50/3 resp. Find the binomial prob. distr.

Note: Binomial distrib. $= (n, p)$
$= (q+p)^n$

$np + npq = \frac{25}{3} = np(1+q) = \frac{25}{3}$ —— (I)

$(np)(npq) = \frac{50}{3}$. $n^2p^2q = \frac{50}{3}$ —— (II)

Apply (I)^2/(II) :

$\frac{n^2p^2(1+q)^2}{n^2p^2q} = \frac{\frac{625}{9}}{\frac{50}{3}}$

$\frac{(1+q)^2}{q} = \frac{625}{3} \times \frac{1}{50}$

$\frac{(1+q^2+2q)}{q} = \frac{25}{6}$

$6 + 6q^2 + 12q = 25q$
$6q^2 - 13q + 6 = 0$
$6q^2 - 9q - 4q + 6 = 0$
$3q(2q-3) - 2(2q-3) = 0$
$(3q-2)(2q-3) = 0$
$q = \frac{2}{3}$, $q = \frac{3}{2}$ ✗ 1.5

But $p = 1 - \frac{2}{3} = \frac{1}{3}$

from (I) : $np(1+q) = \frac{25}{3}$

$n \times \frac{1}{3}\left(1 + \frac{2}{3}\right) = \frac{25}{3}$

$n \times \frac{1}{3} \times \frac{5}{3} = \frac{25}{3}$ $\Rightarrow n = 15$

Hence the bin. dist. is $(n,p) = (15, \frac{1}{3})$

$= (\frac{2}{3} + \frac{1}{3})^{15}$

4 are (graduated)

Q- In a gr. of 20 are graduate. If 4 persons are selected at random from 20

i) all are graduates.

$n = 4, \quad p = \frac{4}{20} = \frac{1}{5}, \quad q = 1 - \frac{1}{5} = \frac{4}{5}$

i) $P(X=4) = {}^4C_4 \left(\frac{1}{5}\right)^4 \left(\frac{4}{5}\right)^0$

$= 1 \times \frac{1}{5^4} \times 1 = \frac{1}{625}$

ii) atleast one is graduate. $= 0.59$

Note- If n independent trials constitute one experiment & this experiment is repeated N times then the frequency of r successes is $N \cdot {}^nC_r\, p^r q^{n-r}$

Q- 6 dice are thrown 729 times. How many times do you expect that -

i) the 3 dice to show 5 or 6

ii) atleast three dice to show 5 or 6

iii) $N = 729, \quad n = 6, \quad p = \frac{2}{6} = \frac{1}{3}, \quad q = \frac{2}{3}$

i) $r = 3$

freq. of 3 dice to show (5 or 6) $= N \times {}^nC_r\, p^r q^{n-r}$

$= 729 \times {}^6C_3 \left(\frac{1}{3}\right)^3 \left(\frac{2}{3}\right)^{6-3}$

$= 729 \times \frac{6 \times 5 \times 4}{3 \times 2 \times 1} \times \frac{8}{3^6}$

$= 20 \times 8 = 160$

Poisson Prob. distr. : is a limiting case of binomial Prob. distr. as $n \to \infty$ & $p \to 0$, then the poisson prob. is given by :

$p(r) = \frac{e^{-\lambda} \lambda^r}{\lfloor r}, \quad r = 0, 1, 2, \ldots$

Where, λ = Parameter $= np$ = mean

Q- Suppose that a book of 600 pages contains 40 printing mistakes. Assume that these errors are randomly distributed throughout the book. The r represents the no. of errors per page. What is the probability that 10 pages selected at random will be free from errors?

$n_1 = 600, \quad p = \frac{40}{600} = \frac{1}{15} \longrightarrow$ Poisson Dist.

np .

$P(x) = \frac{e^{-\lambda} \lambda^x}{\lfloor x}$ where $\lambda = np = \frac{10 \times 1}{15} = \frac{2}{3}$

$P(X=0) = \frac{e^{-2/3} \left(\frac{2}{3}\right)^0}{\lfloor 0} = e^{-2/3}$

Q If the probability of bad rxn from a certain injection is 0.0002. Determine the chance that out of 1000 individuals more than 2 will get a bad rxn.

$p = 0.0002, \quad n = 1000$

$P(r) = \frac{e^{-\lambda} \lambda^r}{\lfloor r}, \quad \lambda = np$

$= 1000 \times 0.0002$

$= 0.2$

$P(r>2) = P(3) + P(4) + P(5) + \dots + P(1000)$

$= 1 - [P(0) + P(1) + P(2)]$

$= 1 - \left[\frac{e^{-0.2}(0.2)^0}{\lfloor 0} + \frac{e^{-0.2}(0.2)^1}{\lfloor 1} + \frac{e^{-0.2}(0.2)^2}{\lfloor 2} \right]$

$= 0.0012$

Q. In a certain factory turning out razor blade there is a small chance of 0.002 for any blade to be defective. The blades are supplied in a packet of 10. Calculate the approx. no. of packets containing

i) No defective blade

ii) One defective blade

iii) two " " in a consignment of 10,000 packets

$N = 10,000$, $n = 10$, $p = 0.002$

$\lambda = np = 10 \times 0.002 = 0.02$

i) $P(r) = \dfrac{e^{-\lambda} \lambda^r}{\lfloor r}$

$P(0) = \dfrac{e^{-0.02}(0.02)^0}{\lfloor 0} = e^{-0.02} = 0.9802$

The no. of packets out of 1000 having no defective blades

$= 104 \times P(0)$

$= 10^4 \times 0.9802$

≈ 9802

ii) $P(1) = \dfrac{e^{-0.02}(0.02)^1}{1}$

(196)

$= 0.02 \times 0.98 = 0.0196$

iii) $P(2) = \dfrac{e^{-0.02}(0.02)^2}{\lfloor 2}$

$= 10^4 \times 0.00019$

≈ 2

Mean & Variance of Poisson Prob.

Mean $= \lambda$

Variance $= \lambda$

Proof: $\text{Variance} = \lim\limits_{\substack{n\to\infty \\ p\to 0}} npq$

$= \lim\limits_{\substack{n\to\infty \\ p\to 0}} np(1-p)$ $\because q = 1-p$

$= \lim\limits_{p\to 0} \lambda(1-p) = \lambda(1-0) = \lambda.$

$\lim\limits_{n\to 0} \lambda(1-p)$

$\lim\limits_{n\to\infty} \lambda\left(1 - \dfrac{\lambda}{n}\right) = \lambda\left(1 - \dfrac{1}{\infty}\right)$

$= \lambda(1-0) = \lambda.$

Q. Fit a Poisson dist. to a following data & calculate theoretical frequencies

Deaths :	0	1	2	3	4
Frequencies :	122	60	15	2	1

Here $N = 200$, $\lambda = \dfrac{\Sigma f x}{\Sigma f} = 0.5$

r	NP(r)	Theor. freq.
0	$\dfrac{200 \times e^{-0.5} \times (0.5)^0}{\lfloor 0} = 121.306$	121
1	$\dfrac{200 \times e^{-0.5}(0.5)^1}{\lfloor 1} = 60.653$	61
2	$\dfrac{200 \times e^{-0.5}(0.5)^2}{\lfloor 2} = 15.163$	15

3. $\frac{200 \times e^{-0.5}(0.5)^3}{L3} = 2.527$ 3

4   $\frac{200 \times e^{-0.5}(0.5)^4}{L4} = 0.3159$ $= 0$

Normal Probability distribution :

Pdf of Normal Probability is given by :

$$f(x) = P(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}, \quad -\infty < x < \infty$$

where

$\mu \rightarrow$ mean of dist.
$\sigma \rightarrow$ S.D of "

Std. form of the Normal Probability is :

$$\frac{x-\mu}{\sigma} = Z \quad \text{—— (I)}$$

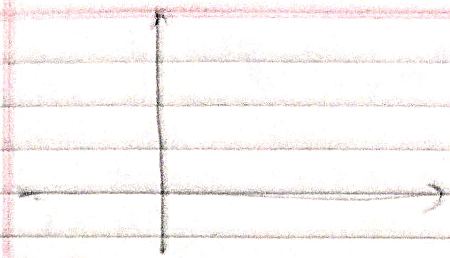
Then the Std. Normal Probability is

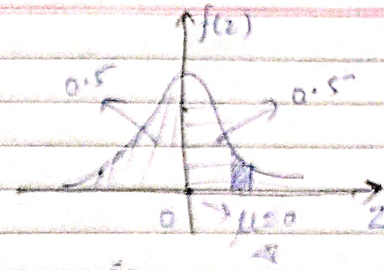
$$P(z) = f(z) = \frac{1}{\sqrt{2\pi}} e^{-z^2/2} \quad \text{—— (II)}$$

$$-\infty < z < \infty$$

The eqn (II) has properties :

$f(z) = P(z) \geq 0$ ii) $\int_{-\infty}^{\infty} f(z)\,dz = 1$





Note - 1) $P(z_1 > z > z_2) = \int_{z_1}^{z_2} f(z)\,dz$

Are of curve from $z = x_1$ to $z = x_2$

Area (from $z=0$ to $z = z_2$) $-$ Ar (from $z=0$ to $z=z_1$)

Q. If Z is a std. var, then find the probabilities of the following :

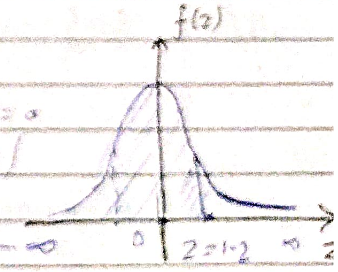
i) $P(z < 1.2)$ ii) $P(z > 1.2)$

iii) $P(-1.2 < Z < 1.3)$.

i) $P(z < 1.2) = 0.5 + \text{mean}$ (from $z=0$ to $z=1.2$)

$= 0.5 + 0.3849$

$= 0.8849$



ii) $P(z > 1.2) = 0.5 - \text{Mean}$ (from $z=0$ to $z=1.2$)

$= 0.5 - 0.3849 = 0.1151$

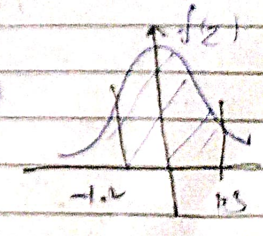
iii) $P(-1.2 < z < 1.3)$

$=$ Area (from $z=0$ to $z=-1.2$)

$+$ Ar (from $z=0$ to $z=1.3$)

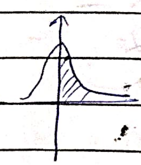
$= 0.3849 + 0.4032$

$= 0.7881$



Q. Students of a class were give a mechanical aptitude test. Their marks were found to be distributed abt mean = 60. Find the % of student scored ($S.D = 5$)

i) more than 60 marks,
ii) < 56 marks
iii) b/w 45 & 60
iv) exactly 55 marks



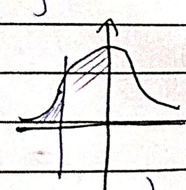
i) $x > 60$, $Z = \frac{x-\mu}{\sigma} = \frac{60-60}{5} = 0$

i.e. $Z > 0$ ∴ $P(Z>0) = 0.5$

$0.5 \times 100 = 50\ \%$

ii) <56 marks $Z = \frac{56-60}{5} = \frac{-4}{5} = -0.8$

80%, $P(Z<-0.8) =$



iii) ~~b/w 45 & 60~~.

$= 0.5 - Ar(\text{from } z=0 \text{ to } z=-0.8)$

$= 0.5 - 0.2881$

$= 0.2119$

∴ % students who have scored less than 56 marks

$= 0.2119 \times 100 = 21.19 \approx 21$

$x = 55$, $x_1 = 54.5$, $x_2 = 55.5$

Now,

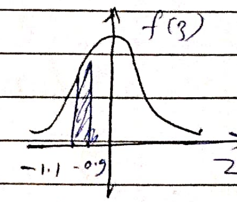
$Z_1 = \frac{x_1-\mu}{\sigma} = \frac{54.5-60}{5} = \frac{-5.5}{5} = -1.1$

$Z_2 = \frac{x_1-\mu}{\sigma} = \frac{55.5-60}{5} = \frac{-4.5}{9} = -0.9$

Now,

$P(-1.1<Z<-0.9) = Ar\ from(z=0\ to\ z=-1.1)$

$- Ar\ from(z=0\ to\ z=-0.9)$



$= 0.3643 - 0.3159 = 0.0484$

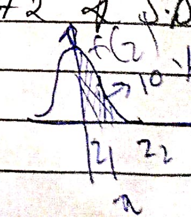
Q. Income dist. of workers in a certain factor was found normal with mean 500 & S.D = ₹ 50. There were 228 person getting abov ₹ 600. How many workers were the in all.

$\mu = 500$, $\sigma = 50$,

$x > 600$ $Z = \frac{x-\mu}{\sigma} = \frac{600-500}{50} = 2$

i.e. $Z > 2$

Q. Determine the min marks a student must get in order to receive get A grade if the f 10% of student were awarded A grade in exa where the mean marks is 72 & S.D is



$P(z) = 0.5 - 0.1 = 0.4$

$z = 1.28$

We have, $z = \frac{x-\mu}{\sigma}$

$1.28 = \frac{x-72}{9} \Rightarrow x = 1.28 \times 9 + 72 = 83.52$

Exponential Probability dist. –

The exponential probabilities is defined:

$$f(x) = \begin{cases} ke^{-kx}, & x \geq 0 \\ 0, & x < 0 \end{cases}$$

where $k \rightarrow$ parameter.

It is applicable for continuous random var. $-\infty < x < \infty$

Mean of exponential probability

$$\text{Mean} = \int_{-\infty}^{\infty} x f(x)\, dx = \int_{-\infty}^{\infty} x \times k e^{-kx}\, dx$$

$$= k \int_{-\infty}^{\infty} x e^{-kx}\, dx = \int_{0}^{\infty} x e^{-kx}\, dx$$

$$= k\left(\left. -xe^{-kx} \right|_0^{\infty} + \int_0^{\infty} e^{-kx}\, dx \right)$$

$$= k\left(0 - \left. \frac{e^{-kx}}{k} \right|^{\infty} \right) = -0 + \frac{1}{k} = \frac{1}{k}$$

$$\text{Mean} = \frac{1}{k} \qquad \text{Variance} = \frac{1}{k^2}$$

$S.D = \sqrt{\text{variance}}$, $S.D = \frac{1}{k}$

Q- A power supply unit for a computer component is assumed to follow exponential dist. with mean life 1200 hrs. What is the probability that comp. will:

fail in first 300 hrs

survives more than 1500 hrs

last b/w 1200 hrs. and 1500 hrs ?

$\frac{1}{k} = 1200$ hrs, $k = \frac{1}{1200}$ per hrs

1) $P(x \leq 300) = \int_0^{300} k e^{-kx}\, dx = \frac{1}{1200} \int_0^{300} e^{x/1200}\, dx$

$$= \frac{1}{1200} \left[\frac{e^{-x/1200}}{-1/1200} \right]_0^{300} = -\left[e^{-\frac{300}{1200}} - e^{-0} \right]$$

$$\left(e^{-0} = \frac{1}{e^0} = \frac{1}{1} = 1, \; e^0 = 1 \right)$$

$$= -\left[e^{-1/4} - 1 \right] = \left(1 - e^{-1/4} \right)$$

$\Rightarrow 0.2212$

Q- The income tax of a man is exponentially dist. with prob f^n given by.

$f(x)$

$$f(x) = \begin{cases} \frac{1}{3} e^{-x/3} ; & x \geq 0 \\ 0 , & x < 0 . \end{cases}$$

What is the prob that his income will exceed ₹17000 assuming that the income tax is levied @ of 15% on the income above 15,000?

Mean = 3

Req. prob $P(x > 300)$

$$\left\{ \frac{2000 \times}{10} = 300 \right.$$

$$= \int_0^{300} \frac{1}{3} e^{-x/3} = e^{-100}$$

$P(x > 300)$