

Unit - 2

Curve fitting

Q.1 fit a straight a line to the foll. data

x:	0	1	2	3	4
y:	1	1.8	3.3	4.5	6.3

Sol. Normal equations

$$y = a + bx \quad \text{--- (1)} \quad \text{or} \quad y = ax + b$$

Normal equation

Since coeff. of a is 1 so multiply eqⁿ (1) by 1

$$\sum y = a \sum 1 + b \sum x$$

Since coeff. of b is n so multiply by n in eqⁿ (1)

$$\sum xy = a \sum x + b \sum x^2$$

Now we can write eqⁿs as

$$\begin{cases} \sum y = na + b \sum x \\ \sum xy = a \sum x + b \sum x^2 \end{cases}$$

— (2)

— (3)

[Here $\sum 1 = n = \text{no. of observations}$]

Solving Equations

x	y	x^2	xy	
0	1	0	0	
1	1.8	1	1.8	
2	3.3	4	6.6	
3	4.5	9	13.5	
4	6.3	16	25.2	
$\sum x = 10$	$\sum y = 16.9$	$\sum x^2 = 30$	$\sum xy = 47.1$	

$$\begin{aligned} 16.9 &= 5a + 10b \\ 47.1 &= 10a + 30b \end{aligned}$$

$$\begin{aligned} a &= 0.72 \\ b &= 1.33 \end{aligned}$$

$$y = 0.72 + 1.33x$$

Ans

Q2. fit $y = \frac{c_0}{x} + c_1 \sqrt{x}$

x:	0.1	0.2	0.4	0.5	1	2
y:	21	11	7	6	5	6

Sol. $y = \frac{c_0}{x} + c_1 \sqrt{x} \quad \text{--- (1)}$

$$\sum \frac{y}{x} = c_0 \sum \frac{1}{x^2} + c_1 \sum \frac{1}{\sqrt{x}} \quad \text{--- (2)}$$

$$\sum \sqrt{x}y = c_0 \sum \frac{1}{\sqrt{x}} + c_1 \sum x \quad \text{--- (3)}$$

x	y	$\frac{1}{x}$	$y\sqrt{x}$	$\frac{1}{x^2}$	$\frac{1}{\sqrt{x}}$	\sqrt{x}
0.1	21	210	6.64	100	3.16	0.31
0.2	11	55	4.91	25	2.23	0.44
0.4	7	12.5	4.042	6.25	1.58	0.63
0.5	6	12	4.24	4	1.41	0.71
1	5	5	5	1	1	1
2	6	2.5	8.48	0.25	0.70	1.41
		302.5	33.71	136.5	10.10	

$$(136.5)c_0 + c_1(10.10) = 33.71$$

--- (1)

$$(10.10)c_0 + c_1(4.2) = 33.71$$

$$c_0 = 1.9733$$

$$c_1 = 3.2803$$

$$y = \frac{1.9733}{x} + 3.2803 \sqrt{x}$$

A

Note:

1. fit a st. line

$$y = ax + b \text{ or } y = a + bx$$

2. fit a second degree parabola

$$y = a + bx + cx^2 \text{ or } y = ax^2 + bx + c$$

$$\sum y = a n + b \sum x + c \sum x^2$$

$$\sum xy = a \sum x + b \sum x^2 + c \sum x^3$$

$$\sum x^2 y = a \sum x^2 + b \sum x^3 + c \sum x^4$$

3. fit $y = c_0 x + \frac{c_1}{\sqrt{x}}$

x:	0.2	0.3	0.5	1	2
y:	16	14	11	6	3

Sol. $y = c_0 x + \frac{c_1}{\sqrt{x}}$

$$\sum xy = c_0 \sum x^2 + c_1 \sum \sqrt{x}$$

$$\sum \frac{y}{\sqrt{x}} = c_0 \sum \sqrt{x} + c_1 \sum \frac{1}{x}$$

x	y	xy	y/\sqrt{x}	\sqrt{x}	x^2	$1/x$
0.2	16	3.2	35.7	0.44	0.04	5
0.3	14	4.2	25.5	0.54	0.09	3.33
0.5	11	5.5	15.5	0.70	0.25	2
1	6	6	10.41	1	1	1
2	3	6	2.12	1	4	0.5
			85.015	4.116	5.38	11.833

$$c_0 = -1.1832$$

$$c_1 = 7.5961$$

$$y = -1.1832x + \frac{7.5961}{\sqrt{x}}$$

$$5.38 c_0 + 4.116 c_1 = 24.9$$

$$4.116 c_0 + 11.833 c_1 = 85.015$$

A

Q4. fit a curve $y = ab^x$

x: 2 3 4 5 6

y: 144 172.8 207.4 248.8 298.5

Sol $y = ab^x$

$$\log y = \log a + \log b^x$$

$$\log y = \log a + x \log b$$

$$Y = A + xB \quad \text{--- (1)}$$

Normal eqⁿ

$$\sum Y = A n + B \sum x \quad \text{--- (2)}$$

$$\sum xy = A \sum x + B \sum x^2 \quad \text{--- (3)}$$

$$a = \text{Anti log } A$$

$$b = \text{Anti log } B$$

x	y	$\log y = Y$	$x \cdot Y$	x^2
2	144	$\log(144)$	6.7126	4
3	172.8	$\log(172.8)$	9.2672 6.7126	9
4	207.4	$\log(207.4)$	11.9792 9.2672	16
5	248.8	$\log(248.8)$	11.9792	25
6	298.5	$\log(298.5)$	14.8496	36
20	1021.5	11.58350969	47.1255091	90

$$11.58350969 = A5 + B20 \quad \text{--- (1)}$$

$$47.1255091 = A20 + B90 \quad \text{--- (2)}$$

$$A = 2.000113802$$

$$B = 0.079147034$$

$$\left. \begin{array}{l} \\ \end{array} \right\} \begin{array}{l} A = 100.0262073 \\ B = 1.199905472 \end{array}$$

$$y = (100.0262073 \times 1.199905472)^x$$

$$0.5. \text{ fit } y = ax^b$$

$x:$	1	2	3	4	5
$y:$	7.1	27.8	62.1	110	161

Sol $\log y = \log a + b \log x$

$$y = A + bx$$

$$\begin{aligned} \sum y &= An + b \sum x \\ \sum xy &= A \sum x + b \sum x^2 \end{aligned} \quad \left. \begin{array}{l} \\ \end{array} \right\} \quad A = \text{anti-log } a$$

x	y	$x = \log x$	$y = \log y$	xy	x^2
1	7.1	0	0.85125	0	0
2	27.8	0.30	1.440	0.4347	0.6020
3	62.1	0.477	1.7930	0.855	0.9542
4	110	2.04139	2.0413	1.2290	1.2041
5	161	0.698	2.2068	1.5425	1.3979
		2.079181246	11.85512725	4.0617688	4.158362492

$$11.85512725 = 5A + b \times 2.079181246$$

$$4.061768867 = A \times 2.079181246 + b \times 4.158362492$$

$$A = 2.480612957 \quad = 302.4217046$$

$$b = -0.263535243$$

$$y = 302.4217046 x^{-0.263535243}$$

Fit $PV^{\alpha} = C$ to the foll. data

$P :$	0.5	1	1.5	2	2.5	3	$\rightarrow x$
$V :$	1620	1000	750	620	520	460	$\rightarrow y$

Sol. $PV^{\alpha} = C$

$$\log P + \alpha \log V = \log C$$

$$x + \alpha y = \log C$$

$$y = \log C - x$$

$$y = \frac{1}{\alpha} \log C - \frac{1}{\alpha} x$$

$$y = A + BX \quad \text{--- (1)}$$

$$A = \frac{1}{n} \log C, \quad B = -\frac{1}{n} \alpha$$

$$\sum y = An + B \sum x \quad \text{--- (2)}$$

$$\sum xy = A \sum x + B \sum x^2 \quad \text{--- (3)}$$

$$A = \frac{1}{n} \log C \Rightarrow \log C = A n \Rightarrow C = 10^{An}$$

$$\mu = -\frac{1}{B}$$

P	V	$x = \log P$	$y = \log V$	xy	x^2
0.5	1620	-0.301	3.209	-0.965	-0.0906
1	1000	0	3	0	0
1.5	750	0.176	2.875	0.506	0.0309
2	620	0.301	2.792	0.840	0.0906
2.5	520	0.397	2.716	1.078	0.1576
3	460	0.477	2.662	1.269	0.2275
		1.051152522	17.25572	2.73196	0.5982
		22	914		

minor
def.

$$17.25572914 = 5A + B (1.051152522)$$

$$2.73196 = A (1.051152522) + B (0.5982)$$

$$A = 2.99912, \quad B = -0.7030$$

$$C = 10^{\frac{A}{B}}, \quad A = -\frac{1}{B}$$

$$A = -\frac{1}{0.7030} = 1.42247$$

$$C = \frac{1}{10} (2.99912 \times 1.42247) = 18462.48$$

$$\boxed{PV^{1.42247} = 18462.48}$$

min. diff.

A

$$8. \text{ fit } PV^n = C$$

V	: 50	60	70	90	100	$\rightarrow n$
P	: 64.7	51.3	40.5	25.9	78	$\rightarrow y$

S.P.

$$n \log V + \log P = \log C$$

$$n X + Y = \log C$$

$$Y = \log C - n X$$

$$Y = A + BX \quad \text{--- (1)}$$

$$Y = \log P, \quad A = \log C$$

$$X = \log V, \quad B = -n$$

N.E.

$$\Sigma Y = An + B \Sigma X \quad \text{--- (2)}$$

$$\Sigma XY = A \Sigma X + B \Sigma X^2 \quad \text{--- (3)}$$

V	P	X	Y	XY	X^2
50	64.7	1.698970004	1.810904281		
60	51.3	1.77815125	1.710117365		
70	40.5	1.84509804	1.607455023		
90	25.9	1.95422509	1.413299764		
100	78	9	1.892094603	15.62954	17.27176
		9.276461804	8.433871036		

$$5A + B \times (9.276461804) = 8.433871036$$

$$(9.276461804)A + B (17.27176) = 15.62954$$

$$A = 2.22496763, \quad B = -0.290085505$$

$$y = 0.290085$$

(2.22496763)

$$c = 10$$

$$C = 167.8678894$$

$$PV \quad (0.290085) = 167.8678894$$

A

9. fit $y = a e^{bx}$

x	2	4	6	8	10
y	4.077	11.084	30.128	81.897	222.62

Sol.

$$y = a e^{bx}$$

$$\log_e y = \log_e a + bx$$

$$Y = A + bx \quad \text{--- } ①$$

with base (e)

$$Y = \log_e y$$

$$A = \log_e a \Rightarrow a = e^A$$

x	y	$\log_e y = Y$	x^2	xy
2	4.077	1.405361424	4	2.810722848
4	11.084	2.405502627	16	9.622010508
6	30.128	3.405454972	36	20.43272983
8	81.897	4.40546236	64	35.24369888
10	222.62	5.405466282	100	54.05466282
30		17.02724766	220	122.1638249

$$\sum Y = An + b \sum x \quad \text{--- } ②$$

$$\sum xy = A \sum x + b \sum x^2 \quad \text{--- } ③$$

$$\begin{aligned} 5A + b(30) &= 17.02724766 \\ 30A + b(220) &= 122.1638249 \end{aligned} \quad \left. \begin{array}{l} A = 0.405398691 \\ b = 0.50008473 \end{array} \right\}$$

$$a = 1.499900378$$

$$a = 1.49990$$

$$b = 0.50008$$

$$\boxed{y = (1.49990) e^{(0.50008)x}}$$

A

$$10. \quad f(t) = f(t) = a e^{-3t} + b e^{-2t}$$

$$t = 0.1 \quad 0.2 \quad 0.3 \quad 0.4$$

$$f(t) = 0.76 \quad 0.58 \quad 0.44 \quad 0.35$$

$$\text{Sof. } y = a e^{-3t} + b e^{-2t} \quad \text{--- (1)}$$

$$\sum e^{-3t} y = a \sum e^{-6t} + b \sum e^{-5t} \quad \text{--- (2)}$$

$$\sum e^{-2t} y = a \sum e^{-4t} + b \sum e^{-3t} \quad \text{--- (3)}$$

t	y	$y e^{-3t}$	$y e^{-2t}$	e^{-4t}	e^{-5t}	e^{-6t}
0.1	0.76	0.563021847	0.622235372	0.6703	0.6065	0.5488
0.2	0.58	0.318310748	0.388785626	0.4493	0.3678	0.3011
0.3	0.44	0.178890655	0.241477119	0.3011	0.2231	0.1652
0.4	0.35	0.105417974	0.157265137	0.2018	0.1353	0.0907
		1.165641221	1.409763256	1.6227	1.3328	1.106
		≈ 1.1656	≈ 1.4098			

$$a(1.106) + b(1.3328) = 1.1656 \quad \left. \begin{array}{l} \\ \end{array} \right\} a = 0.67778$$

$$a(1.3328) + b(1.6227) = 1.4098 \quad \left. \begin{array}{l} \\ \end{array} \right\} b = 0.312098$$

$$\boxed{f(t) = a e^{-3 \times (0.67778)} + b e^{-2 \times (0.312098)}}$$

A

Correlation

Correlation coefficient b/w variables $x + y$ is denoted by r_{xy} or r . $-1 \leq r \leq +1$

$$r = \frac{\sum (x - \bar{x})(y - \bar{y})}{\sqrt{\sum (x - \bar{x})^2} \times \sqrt{\sum (y - \bar{y})^2}}$$

$$r = \frac{\frac{1}{n} \sum (x - \bar{x})(y - \bar{y})}{\sqrt{\frac{\sum (x - \bar{x})^2}{n}} \times \sqrt{\frac{\sum (y - \bar{y})^2}{n}}}$$

$$r = \frac{\text{Cov}(x, y)}{\sigma_x \sigma_y}$$

Working formula

$$r = \frac{n \sum xy - \sum x \sum y}{\sqrt{n \sum x^2 - (\sum x)^2} \times \sqrt{n \sum y^2 - (\sum y)^2}}$$

Q1. find the coeff. of correlation

x :	1	3	5	7	8	10
y :	8	12	15	17	18	20

<u>sol</u>	x	y	x^2	y^2	xy
	1	8	1	64	8
	3	12	9	144	36
	5	15	25	225	75
	7	17	49	289	119
	8	18	64	324	144
	10	20	100	400	200
	94	90	248	1448	582

$$r = \frac{n \sum xy - \sum x \sum y}{\sqrt{n \sum x^2 - (\sum x)^2} \times \sqrt{n \sum y^2 - (\sum y)^2}}$$

$$= \frac{3492 - 3060}{18 \cdot 22 \times 24} = \underline{\underline{+0.72}}$$

$$= 0.9879$$

Rank correlation Coeff.

$$r = 1 - \frac{6 \sum D^2}{n(n^2-1)}$$

if ranks are non repeated

$$r = 1 - \frac{6 [\sum D^2 + \frac{1}{12} m_1(m_1^2-1) + \frac{1}{12} m_2(m_2^2-1) + \dots]}{n(n^2-1)}$$

if ranks are repeated

X	Y	R _x	R _y	D = R _x - R _y	D ²
7	8	2	1	1	1
9	2	1	3	-2	4
2	3	3	2	1	1

$\sum D^2 = 6$

Q Calculate rank correlation Coeff.

X	Y	R _x	R _y	D	D ²
68	62	4	5	-1	1
64	58	6	7	-1	1
75	68	2.5	3.5	-1	1
50	45	9	10	-1	1
64	81	6	1	5	25
80	60	1	6	-5	25
75	68	2.5	3.5	-1	1
40	48	10	9	1	1
55	50	8	8	0	0
64	70	6	2	4	$\frac{16}{\sum D^2+2}$

$$m_1 = 2 \quad 75 \begin{cases} 2 \\ 3 \end{cases} \quad m_{\text{mean}} = 2.5$$

$$m_3 = 2 \quad 68 \begin{cases} 3 \\ 4 \end{cases}$$

$$m_2 = 3 \quad 64 \begin{cases} 5 \\ 6 \\ 7 \end{cases} \quad m = 6$$

$$M = 1 - \frac{6}{12} [72 - \frac{1}{12} \times 2(4-1) - \frac{1}{12} \times 3(9-1) - \frac{1}{12} \times 2(4-1)]$$

$$\frac{10(100-1)}{990}$$

$$= 1 - \frac{6}{990} [72 + \frac{1}{2} + 2 + \frac{1}{2}]$$

$$\frac{990}{990}$$

$$= 1 - \frac{6}{990} [72 + 3] = 1 - \frac{6 \times 75}{990} = 1 - \frac{450}{990}$$

$$= 1 - \frac{450}{990}$$

$$= 1 - \frac{4.515}{9.90} \\ = -0.2515$$

$$= 0.5454$$

~~✓~~

θ obtain rank correlatⁿ coeff.

X	Y	R _x	R _y	D	D ²
15	50	7	3	4	16
20	30	5.5	5	0.5	0.25
27	55	4	2	2	4
13	30	8	5	3	9
45	25	3	7	-4	16
60	10	2	8	-6	36
20	30	5.5	5	0.5	0.25
75	70	1	1	0	0
					$\sum D^2 = 81.5$

$$m_1 = 2, m_2 = 3$$

$$M = 1 - \frac{6}{8(64-1)} [81.5 + \frac{1}{12} \times 2(4-1) + \frac{1}{12} \times 3(9-1)] = 1 - \frac{6}{8(64-1)} (81.5 + 2)$$

$$= 1 - \frac{\frac{3}{8} \times 84^{21}}{8 \times 6821} \frac{8 \times 63}{8 \times 6821}$$

$$M = 1 - 1 = 0$$

Q Calculate rank correlat

X	Y	R _x	R _y	D	D ²
10	30	9	9	0	0
15	42	5	3	2	4
12	45	8	2	6	36
17	46	3	1	2	4
13	33	7	8	-1	1
16	34	4	7	-3	9
24	40	1	4	-3	9
14	35	6	6	0	0
22	39	2	5	-3	<u>$\frac{9}{72}$</u>

$$r_s = 1 - \frac{6 \times 24}{9(81-1)} = 1 - \frac{24}{40} = 1 - \frac{6}{10}$$

$$\boxed{r_s = 0.4}$$

A