NIET Greater Noida
Autonomous Institute

Noida Institute of Engineering and Technology, Greater Noida

Statistical Technique-I

Module 1

Subject Name: Statistics and Probability

Subject Code: BAS0303N

Dr. Lokesh Chaudhary
Department of Mathematics

B Tech 3rd Sem

Faculty Introduction

Name: Dr. Lokesh Chaudhary
Qualification: M.Sc., Ph.D.
Total Experience: 18 years of teaching experience.
Member of educational societies: 2
National and international research papers:02

Evaluation Scheme

NOIDA INSTITUTE OF ENGINEERING & TECHNOLOGY, GREATER NOIDA, GAUTAM BUDDH NAGAR
(AN AUTONOMOUS INSTITUTE)

Bachelor of Technology
Computer Science and Engineering (Artificial Intelligence & Machine Learning)

Evaluation Scheme
SEMESTER-III

Sl. No.	Subject Codes	Subject	Types of Subjects	Periods		Evaluation Schemes					End Semester		Total	Credit
				L	T	P	CT	TA	TOTAL	PS	TE	PE		
1	BCSCC0301	Employability Skill Development - I	Mandatory	2	0	0	60	40	100				100	2
2	BAS0303N	Statistics and Probability	Mandatory	3	1	0	30	20	50		100		150	4
3	BCSE0303	Operating Systems	Mandatory	2	0	0	30	20	50		50		100	2
4	BCSE0301	Data Structures and Algorithms-I	Mandatory	3	0	0	30	20	50		100		150	3
5	BCSAI0301N	Artificial Intelligence and Machine Learning	Mandatory	2	0	0	30	20	50		50		100	2
6	BCSAI0304	High Performance Computing	Mandatory	3	0	0	30	20	50		100		150	3
7	BCSE0353N	Operating Systems Lab	Mandatory	0	0	4				50		50	100	2
8	BCSE0351	Data Structures and Algorithms-I Lab	Mandatory	0	0	4				50		50	100	2
9	BCSAI0351N	Artificial Intelligence and Machine Learning Lab	Mandatory	0	0	2				25		25	50	1
10	BCSE0352	Object Oriented Techniques using Java	Mandatory	0	0	6				50		100	150	3
11	BCSE0359X	Social Internship	Mandatory	0	0	2				50			50	1
12	BNC0302/ BNC0301	Environmental Science/Artificial Intelligence and Cyber Ethics	Compulsory Audit	2	0	0	30	20	50				50	NA
		Massive Open Online Courses (For B.Tech. Hons. Degree)	*MOOCs											
		TOTAL		17	1	18	210	140	350	225	400	225	1200	25

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Syllabus

Module 1: (Statistical Techniques-I)

Introduction: Measures of central tendency: Mean, Median, Mode, Standard deviation, Quartile deviation, Moment, Skewness, Kurtosis.

Module 2: (Statistical Techniques-II)

Curve Fitting, Method of least squares, fitting of straight lines, Fitting of second-degree parabola, Exponential curves, Correlation and Rank correlation, Linear regression, nonlinear regression and multiple linear regression.

Module 3: (Probability and Random Variable)

Random Variable: Definition of a Random Variable, Discrete Random Variable, Continuous Random Variable, Probability mass function, Probability Density Function, Distribution functions.

Multiple Random Variables: Joint density and distribution Function, Properties of Joint Distribution function, Marginal density Functions, Conditional Distribution and Density, Statistical Independence, Central Limit Theorem (Proof not expected).

Syllabus

Module 4: (Expectations and Probability Distribution)

Expectations of single Random Variable, Mean, Variance, Moment Generating Function, Binomial, Poisson, Normal, Exponential distribution.

Module 5: (Hypothesis Tests and Control Charts)

Testing a Hypothesis, Null hypothesis, Alternative hypothesis, Level of significance, Confidence limits, Test of significance of difference of means, Z-test, t-test and Chi-square test, F-test, One way ANOVA.

Statistical Quality Control (SQC), Control Charts, Control Charts for variables (Mean and Range Charts), Control Charts for Variables (p, np and C charts).

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Branch Wise Application

- Data Analysis
- Artificial intelligence
- Machine Learning
- Network and Traffic modeling

Course Objectives

- The objective of this course is to familiarize the students with concepts of Probability and statistical techniques. It aims to equip the students with adequate Knowledge of statistics that will enable them in formulating Problems and solving problems analytically.

The students will learn:

- Understand the concept of correlation, moments, skewness and kurtosis and curve fitting.
- Apply the concept of hypothesis testing and statistical quality control to create control charts.
- Remember the concept of probability to evaluate probability distributions.
- Understand the concept of Mathematical Expectations and Probability Distribution.

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Course Outcomes

CO1: Apply the concept of moments, skewness and kurtosis in relevant field.

CO2: Apply the concept of correlation, regression and curve fitting with real world problems.

CO3: Apply the concept of probability and random variable.

CO4: Apply the concept of Mathematical Expectations and Probability Distribution in real life problems.

CO5: Apply the concept of hypothesis testing and statistical quality control to create control charts.

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Program Outcomes

Sr. No.	Program Outcomes (POs)
PO1	Engineering Knowledge
PO2	Problem Analysis
PO3	Design/Development of Solutions
PO4	Conduct Investigation of Complex Problems
PO5	Modern Tool Usage
PO6	The Engineer & Society
PO7	Environment and Sustainability
PO8	Ethics
PO9	Individuals & Team Work
PO10	Communication
PO11	Project Management & Finance

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PSOs

PSO	Program Specific Outcomes(PSOs)
PSO1	The ability to identify, analyze real world problems and design their ethical solutions using artificial intelligence, robotics, virtual/augmented reality, data analytics, block chain technology, and cloud computing
PSO2	The ability to design and develop the hardware sensor devices and related interfacing software systems for solving complex engineering problems.

Program Educational Objectives(PEOs)

PEO-1: To have an excellent scientific and engineering breadth so as to comprehend, analyze, design and provide sustainable solutions for real-life problems using state-of-the-art technologies.

PEO-2: To have a successful career in industries, to pursue higher studies or to support entrepreneurial endeavors and to face the global challenges.

PEO-3: To have an effective communication skills, professional attitude, ethical values and a desire to learn specific knowledge in emerging trends, technologies for research, innovation and product development and contribution to society.

PEO-4: To have life-long learning for up-skilling and re-skilling for successful professional career as engineer, scientist, entrepreneur and bureaucrat for betterment of society.

CO-PO Mapping

S. No	Course Outcome	PO1	PO2	PO3	PO4	PO5	PO6	PO7	PO8	PO9	PO10	PO11
1	CO1	3	2	2	3	1	1	-	1	2	2	2
2	CO2	3	2	2	3	1	1	-	1	2	2	2
3	CO3	3	2	1	2	-	-	-	-	1	2	2
4	CO4	3	2	2	3	1	1	-	1	2	2	2
5	CO5	3	2	2	3	1	1	-	1	2	2	2
6	**Average**	**3.0**	**2.0**	**1.8**	**2.8**	**0.8**	**0.8**	**0.0**	**0.8**	**1.8**	**2.0**	**2.0**

CO-PSO Mapping

COs	PSO1	PSO2
CO1	3	1
CO2	1	2
CO3	2	2
CO4	3	2
CO5	3	2
Average	2.4	1.8

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Result Analysis

Branch	Semester	Sections	No. of enrolled Students	No. Passed Students	% Passed
CS	IV	A	67	65	97%
IOT	IV	A	49	45	91.83%

End Semester Question Paper Template

Printed page: .... **Subject Code:**

Roll No:

NOIDA INSTITUTE OF ENGINEERING AND TECHNOLOGY ,GREATER NOIDA

(An Autonomous Institute Affiliated to AKTU, Lucknow)

B.Tech/B.Voc./MBA/MCA/M.Tech (Integrated)

(SEM: ..... THEORY EXAMINATION (2024-2025)

Subject .....

Time: 3 Hours **Max. Marks:100**

General Instructions:

- All questions are compulsory. Answers should be brief and to the point.
- This Question paper consists of .....pages & ...8.....questions.
- It comprises of three Sections, A, B, and C. You are to attempt all the sections.
- **Section A** -Question No- 1 is objective type questions carrying 1 mark each, Question No- 2 is very short answer type carrying 2 mark each. You are expected to answer them as directed.
- **Section B** - Question No-3 is Long answer type -I questions with external choice carrying 6 marks each. You need to attempt any five out of seven questions given.
- Section C - Question No. 4-8 are Long answer type –II (within unit choice) questions carrying 10 marks each. You need to attempt any one part *a* or *b*.
- Students are instructed to cross the blank sheets before handing over the answer sheet to the invigilator.
- No sheet should be left blank. Any written material after a blank sheet will not be evaluated/checked.

End Semester Question Paper Template

		SECTION – A		CO
	Attempt all parts-		[10×1=10]	
	1-a.	Question-	(1)	
	1-b.	Question-	(1)	
	1-c.	Question-	(1)	
	1-d.	Question-	(1)	
	1-e.	Question-	(1)	
	1-f.	Question-	(1)	
	1-g.	Question-	(1)	
	1-h.	Question-	(1)	
	1-i.	Question-	(1)	
	1-j.	Question-	(1)	

End Semester Question Paper Template

	2-a.	Question-	(2)	
	2-b.	Question-	(2)	
	2-c.	Question-	(2)	
	2-d.	Question-	(2)	
	2-e.	Question-	(2)	
	SECTION – B			CO
3.	Answer any five of the following-		[5×6=30]	
	3-a.	Question-	(6)	
	3-b.	Question-	(6)	
	3-c.	Question-	(6)	
	3-d.	Question-	(6)	

End Semester Question Paper Template

	3-d.	Question-	(6)	
	3-e.	Question-	(6)	
	3-f.	Question-	(6)	
	3-g.	Question-	(6)	
		SECTION - C		CO

End Semester Question Paper Template

4.		**Answer any one of the following-**	**[5×10=50]**	
	4-a.	Question-	(10)	
	4-b.	Question-	(10)	
5.		Answer any one of the following-		
	5-a.	Question-	(10)	
	5-b.	Question-	(10)	
6.		Answer any one of the following-		
	6-a.	Question-	(10)	
	6-b.	Question-	(10)	
7.		Answer any one of the following-		
	7-a.	Question-	(10)	
	7-b.	Question-	(10)	
8.		Answer any one of the following-		
	8-a.	Question-	(10)	
	8-b.	Question-	(10)	

Prerequisite and Recap (CO1)

- Knowledge of Maths 1 B.Tech.
- Knowledge of Maths 2 B.Tech.
- Knowledge of Permutation and Combination.

Brief Introduction about the Subject with Videos

- We will discuss the concept of measures of central tendency, moments, skewness and kurtosis.
- In module 2, student understand the concept of curve fitting, correlation and regression.
- In module 3,we will discuss probability to evaluate probability distributions.
- In module 4,we will discuss Mathematical Expectations and Probability Distribution.
- In module 5, student understand the concepts of hypothesis testing and statistical quality control to create control charts.

https://youtu.be/iUhwCfz18os

https://youtu.be/1y4S0oi3Yz8

https://youtu.be/f8XzF9_2ijs

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Video links(module-wise)

Module 1: https://archive.nptel.ac.in/courses/111/105/111105042/

Module 2: https://archive.nptel.ac.in/courses/110/107/110107114/

Module 3: https://archive.nptel.ac.in/courses/117/105/117105085/

Module 4: https://archive.nptel.ac.in/courses/111/104/111104032/

Module 5: https://archive.nptel.ac.in/courses/103/106/103106120/

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Module Content

- Introduction
- Measures of central tendency: Mean, Median, Mode.
- Measures of dispersion – mean deviation, standard deviation, quartile deviation, variance
- Moment
- Skewness
- Kurtosis

Module Objectives(CO1)

- The objective of this course is to familiarize the engineers with concept of Statistical techniques.
- It aims to show case the students with standard concepts and tools from B. Tech to deal with advanced level of mathematics and applications that would be essential for their disciplines.

Topic objectives (CO1)

Measures of central tendency

- **To present a brief picture of data-** It helps in giving a brief description of the main feature of the entire data.
- **Essential for comparison-** It helps in reducing the data to a single value which is used for doing comparative studies.
- **Helps in decision making-** Most of the companies use measuring central tendency to plan and develop their businesses economy.
- **Formulation of policies-** Many governments rely on this medium while forming any policies.

Measures of Central Tendency (CO1)

❑ Measures of Central Tendency or Averages:

Definition : According to Prof. Bowley: Averages are "statistical constants which enable us to comprehend in a single effort the significance of the whole."

Types of Measures of Central Tendency: There are five types of measures of central tendency

- Arithmetic Mean or Simple Mean
- Median
- Mode
- Geometric Mean
- Harmonic Mean

Arithmetic Mean (CO1)

➢ Arithmetic Mean

Definition

Arithmetic mean of a set of observations is their *sum divided by the number of observations, e.g.,* the arithmetic mean $\bar{x}$ of n observations $x_1, x_2, \ldots, x_n$ is given by:

$$\bar{x} = \frac{x_1 + x_2 + \ldots + x_n}{n} = \frac{1}{n}\sum_{i=1}^{n} x_n$$

❖ **In case of the frequency distribution** $x_i | f_i, i = 1, 2, \ldots, n,$ where f_i is the frequency of the variable x_i,

$$\bar{x} = \frac{f_1 x_1 + f_2 x_2 + \cdots + f_n x_n}{f_1 + f_2 + \cdots + f_n} = \frac{\sum_{i=1}^{n} f_i x_i}{\sum_{i=1}^{n} f_i} = \frac{1}{N}\sum_{i=1}^{n} f_i x_i \text{ , where } \sum_{i=1}^{n} f_i = N$$

Arithmetic Mean(CO1)

In case of grouped or continuous frequency distribution, x is taken as the mid-value of the corresponding class.

Example: Find the arithmetic mean of the following frequency distribution:

x:	1	2	3	4	5	6	7
f:	5	9	12	17	14	10	6

Solution: Computation of mean

$$\bar{x} = \frac{f_1x_1+f_2x_2+\cdots+f_nx_n}{f_1+f_2+\cdots+f_n} = \frac{\sum_{i=1}^{n} f_ix_i}{\sum_{i=1}^{n} f_i} = \frac{1}{N}\sum_{i=1}^{n} f_ix_i$$

where $\sum_{i=1}^{n} f_i = N$

Arithmetic Mean(CO1)

X	f	fx
1	5	5
2	9	18
3	12	36
4	17	68
5	14	70
6	10	60
7	6	42
Total	73	299

By using formula $\sum_{i=1}^{n} f_i = N = 73, \sum_{i=1}^{n} f_i x_i = 299$

$$Mean = \frac{1}{N}\sum_{i=1}^{n} f_i x_i = \frac{299}{73} = 4.0989$$

Daily Quiz (CO1)

Example: Calculate the mean for the following frequency distribution:

Class interval	**0-8**	**8-16**	**16-24**	**24-32**	**32-40**	**40-48**
Frequency	8	7	16	24	15	7

Solution: Arithmetic mean =25.4026

Example: The average salary of male employees in a farm was Rs. 5,200 and that of females was Rs. 4,200. The mean salary of all the employees was Rs. 5,000.Find the percentage of male and female employees.

Solution: 80% male and 20% female employees.

Median(CO1)

➢ Median:

Definition: Median of a distribution is the value of the variable which divides it into two equal parts.

It is the value such that the number of observations above it is equal to the number of observations below it. The median is thus a *positional average.*

❖ Ungrouped Data:

- If the number of observations is odd then median is the middle value after the values have been arranged in ascending or descending order of magnitude.
- In case of even number of observations, there are two middle terms and median is obtained by taking the arithmetic mean of middle terms.

Median(CO1)

Example

1. Median of Values 25, 20, 15, 35, 18. Median: 20
2. Median of Values 8, 20, 50, 25, 15, 30. Median: 22.5

❖ Discrete Frequency Distribution

In this case median is obtained by considering the cumulative frequencies. The steps involved

i. Find $\frac{N}{2}$, where $N = \sum_{i=1}^{n} f_i$

ii. See the cumulative frequency (c.f.) just greater than $\frac{N}{2}$.

iii. corresponding value of x is median.

Median(CO1)

Example: Obtain the median for the following frequency distribution:

x:	1	2	3	4	5	6	7	8	9
f:	8	10	11	16	20	25	15	9	6

Solution:

i. Find $\frac{N}{2} = \frac{8+10+11+16+20+25+15+9+6}{2} = \frac{120}{2} = 60$,

where $N = \sum_{i=1}^{n} f_i$

i. See the cumulative frequency (c.f.) just greater than $\frac{N}{2}$.

ii. corresponding value of x is median.

Median(CO1)

x	f	c.f.
1	8	8
2	10	18
3	11	29
4	16	45
5	20	65
6	25	90
7	15	105
8	9	114
9	6	120
Total	N=120	

Here N = 120, The cumulative frequency just greater than $\frac{N}{2}$ is 65 and the value of x corresponding to 65 is 5. Therefore, median is 5.

Median(CO1)

❖ Continuous Frequency Distribution

In this case, the class corresponding to the c.f. just greater $\frac{N}{2}$ is called the median class and the value of median is obtained by the formula:

$$\text{Median} = l + \frac{h}{f}\left(\frac{N}{2} - c\right)$$

where

- l is the lower limit of the class,
- f is the frequency of the median class,
- h is the magnitude of the median class,
- c is the c.f. of the class preceding the median class,
- $N = \sum_{i=1}^{n} f_i$

Daily Quiz(CO1)

Example: Find the median wages of the following distribution.

Wages	No. of workers
2000-3000	3
3000-4000	5
4000-5000	20
5000-6000	10
6000-7000	5

Solution: The median wage is Rs. 4,675.

Mode(CO1)

➢ **Mode:**

- Mode is the value which occurs most frequently in a set of observations and around which the other items of the set cluster densely.
- It is the point of maximum frequency or the point of greatest density.
- In other words the mode or modal value of the distribution is that value of the variate for which frequency is maximum.

Calculation of Mode

❖ **In case of discrete distribution:** Mode is the value of x corresponding to maximum frequency but in any one (or more) of the following cases.

Mode(CO1)

i. If the maximum frequency is repeated.

ii. If the maximum frequency occurs in the very beginning or at the end of distribution .

iii. If there are irregularities in the distribution, the value of mode is determined by the method of grouping.

❖ **In case of continuous frequency distribution:** mode is given by the formula

$$\text{Mode} = l + \frac{f_m - f_1}{2f_m - f_1 - f_2} \times h$$

where l is the lower limit, h is the width and f_m is the frequency of the model class $f_1 and\ f_2$ are the frequencies of the classes preceding and succeeding the modal class respectively. While applying the above formula it is necessary to see that the class intervals are of the same size.

Mode(CO1)

❖ **For a symmetrical distribution, mean, median and mode coincide.**

When mode is ill defined ,where the method of grouping also fails its value can be ascertained by the formula

Mode=3Median-2Mean

This measure is called the empirical mode.

Q. Calculate the mode from the following frequency distribution.

Size(x)	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**
Freqency (f)	2	5	8	9	12	14	14	15	11	13

Solution: Method of Grouping :

Mode(CO1)

$Size(x)$	1	2	3	4	5	6
4	2	7				
5	5		13	15		
6	8	17			22	
7	9		21			29
8	12	26		35		
9	14		28		40	
10	14	29				43
11	15		26	40		
12	11	24			39	
13	13					

Mode(CO1)

Columns	Size of item having max. frequency
1 max.15	11
2 max 29	10, 11
3 max 28	9, 10
4 max 40	10, 11, 12
5 max 40	8, 9, 10
6 max 43	9, 10, 11

Since the item 10 occurs maximum number of times i.e. 5times, hence the mode is 10.

Mode(CO1)

Q. Find the mode of the following:

Marks	**0-5**	**6-10**	**11-15**	**16-20**	**21-25**
No.of candidates	7	10	16	32	24
Marks	26-30	31-35	36-40	41-45	
No.of candidates	18	10	5	1	

Solution: Here the greatest frequency 32 lies in the class 16-20. Hence modal class is 16-20. But the actual limits of this class are 15.5-20.5.

$$l = 15.5, f_m = 32, f_1 = 16, f_2 = 24, h = 5$$

Mode(CO1)

$$\text{Mode} = l + \frac{f_m - f_1}{2f_m - f_1 - f_2} \times h$$

$$= 15.5 + \frac{32 - 16}{64 - 16 - 24} \times 5$$

$$= 15.5 + \frac{16}{24} \times 5$$

$$= 15.5 + \frac{10}{3}$$

$$= 18.83\ marks$$

Daily Quiz (CO1)

Q.1 Calculate the mean, median and mode of the following data-

Wages (in Rs)	**0-20**	**20-40**	**40-60**	**60-80**	**80-100**	**100-120**	**120-140**
No. of Workers	6	8	10	12	6	5	3

Answers: Mean = 62.4, Median = 61.67 and Mode = 65.

Q.2 Compute for the following data:

Wages (Rs.)	**0-25**	**25-50**	**50-75**	**75-100**	**100-125**	**125-above**
No. of persons	10	30	40	25	20	15

Answer: Rs. 30

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Recap(CO1)

- ✓ Measures of central tendency
- ✓ Mean
- ✓ Mode
- ✓ Median

OBJECTIVES OF MEASURING DISPERSION (CO-1)

- To determine the reliability of an average
- To compare the variability of two or more series
- For facilitating the use of other statistical measures
- Basis of Statistical Quality Control

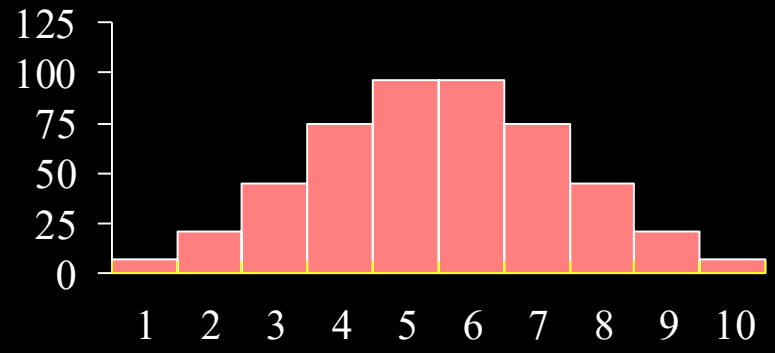
Measures of Dispersion(CO1)

Measures of dispersion are descriptive statistics that describe how similar a set of scores are to each other

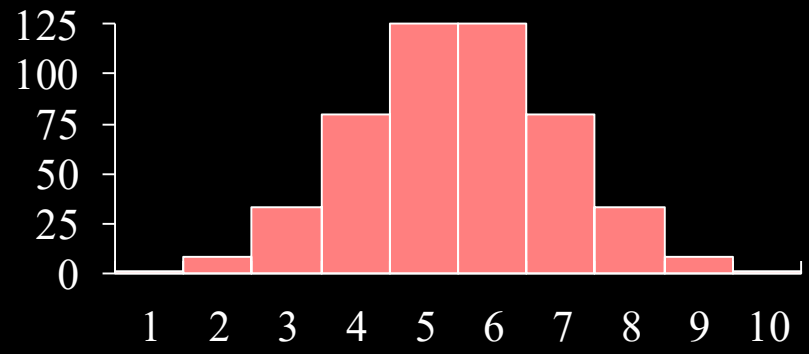
- The more similar the scores are to each other, the lower the measure of dispersion will be
- The less similar the scores are to each other, the higher the measure of dispersion will be
- In general, the more spread out a distribution is, the larger the measure of dispersion will be

Measures of Dispersion(CO1)

- Which of the distributions of scores has the larger dispersion?



- The upper distribution has more dispersion because the scores are more spread out
- That is, they are less similar to each other



Measures of Dispersion(CO1)

Absolute

- Expressed in the same units in which data is expressed
- Ex: Rupees, Kgs, Ltr, Km etc.

Relative

- In the form of ratio or percentage, so is independent of units
- It is also called **Coefficient of Dispersion**

Measures of Dispersion(CO1)

There are some measures of dispersion

- Range
- Quartile deviation
- Mean deviation
- Standard deviation
- Variance
- Coefficient of Variation

RANGE (R) (CO1)

RANGE:-

- It is the simplest measures of dispersion
- It is defined as the difference between the largest and smallest values in the series

- R = L – S
- R = Range
- L = Largest Value
- S = Smallest Value
- Coefficient of Range= $\frac{L-S}{L+S}$

RANGE (R) (CO1)

❖ Individual Series:-

Q1: Find the range & Coefficient of Range for the following data:

20, 35, 25, 30, 15

Solution:- L = Largest Value=35, S = Smallest Value=15,

(Range)R = L – S=35-15=20

Coefficient of Range $=\frac{L-S}{L+S}=\frac{35-1}{35+15}=\frac{20}{50}=0.4$

❖ Discrete Frequency Distribution:

Q2: Find the range & Coefficient of

X	10	20	30	40	50	60	70
F	15	18	25	30	16	10	9

Solution:- L = Largest Value=70, S = Smallest Value=10

RANGE (R) (CO1)

(Range)R = L – S=70-10=60

Coefficient of Range $=\frac{L-S}{L+S}=\frac{70-10}{70+10}=\frac{60}{80}=0.75$

Continuous Frequency Distribution

Q3: Find the range & Coefficient of Range:

Size	5-10	10-15	15-20	20-25	25-30
f	4	9	15	30	40

Solution:- L = Upper limit of Largest class=30

S =Lower limit of Smallest Value=5

(Range)R = L – S=30-5=25

Coefficient of Range $=\frac{L-S}{L+S}=\frac{30-}{30+5}=\frac{25}{35}=\frac{5}{7}=0.714$

Daily Quiz

Q1: Find the range & Coefficient of Range for the following data: 25, 38, 45, 30, 15

Answer: 30, 2

Q2: Find the range & Coefficient of Range:

X:	**1**	**2**	**3**	**4**	**5**	**6**	**7**
F:	5	9	12	17	14	10	6

Answer: 6, 0.75

Q3: Find the range & Coefficient of Range:

X:	**0-10**	**10-20**	**20-30**	**30-40**	**40-50**	**50-60**
F:	12	18	27	20	17	6

Answer: 60, 1

INTERQUARTILE RANGE & QUARTILE DEVIATION(CO1)

- ***Interquartile Range*** is the difference between the upper quartile (Q_3) and the lower quartile (Q_1)
- It covers dispersion of middle 50% of the items of the series
- Symbolically, Interquartile Range = $Q_3 - Q_1$
- Symbolically, Quartile Deviation = $\frac{Q_3 - Q_1}{2}$
- ***Quartile Deviation*** is half of the interquartile range. It is also called Semi Interquartile Range
- ***Coefficient of Quartile Deviation***: It is the relative measure of quartile deviation.
- Coefficient of Q.D. = $\frac{Q_3 - Q_1}{Q_3 + Q_1}$

INTERQUARTILE RANGE & QUARTILE DEVIATION(CO1)

Example: Find interquartile range, quartile deviation and coefficient of quartile deviation: 28, 18, 20, 24, 27, 30, 15.

Solution: Arranging data in ascending order 15,18,20,24,27,28,30

$$Q_1 = Size\ of\ \left(\frac{n+1}{4}\right)^{th}\ item = Size\ of\ \left(\frac{7+1}{4}\right)^{th}\ item = 18$$

$$Q_3 = Size\ of\ 3\left(\frac{n+1}{4}\right)^{th}\ item = Size\ of\ 3\left(\frac{n+1}{4}\right)^{th}\ item = 28$$

Symbolically, Interquartile Range $= Q_3 - Q_1 = 28\text{-}18 = 10$

$$\text{Quartile Deviation} = \frac{Q_3 - Q_1}{2} = \frac{28-18}{2} = 5$$

$$\text{Coefficient of Q.D.} = \frac{Q_3 - Q_1}{Q_3 + Q_1} = \frac{28-18}{28+1} = 0.217$$

INTERQUARTILE RANGE & QUARTILE DEVIATION(CO1)

Example: Find interquartile range, quartile deviation and coefficient of quartile deviation:

X:	10	20	30	40	50	60
F:	2	8	20	35	42	20

Solution:

X	F	C.F.
10	2	2
20	8	10
30	20	30
40	35	65
50	42	107
60	20	127
	N=127	

INTERQUARTILE RANGE & QUARTILE DEVIATION(CO1)

Solution:

$$Q_1 = Size\ of \left(\frac{N+1}{4}\right)^{th} item = Size\ of \left(\frac{127+1}{4}\right)^{th} item = 40$$

$$Q_3 = Size\ of\ 3\left(\frac{N+1}{4}\right)^{th} item = Size\ of\ 3\left(\frac{127+1}{4}\right)^{th} item = 50$$

Symbolically, Interquartile Range = $Q_3 - Q_1$ =50-40 =10

$$\text{Quartile Deviation} = \frac{Q_3 - Q_1}{2} = \frac{50-40}{2} = 5$$

$$\text{Coefficient of Q.D.} = \frac{Q_3 - Q_1}{Q_3 + Q_1} = \frac{50-40}{50+40} = 0.11$$

Daily Quiz

Find quartile deviation and coefficient of quartile deviation:

Q1: 4,8,10,7,15,11,18,14,12,16.
Answer: 3.75, 0.32

Q2:

X	20	40	60	80	100
f	4	10	15	20	11

Answer: 10, 0.14

MEAN DEVIATION(M.D.) (CO1)

- It is also called Average Deviation
- It is defined as the arithmetic average of the deviation of the various items of a series computed from measures of central tendency like mean or median.

There are some formulas to calculate mean deviation.

- M.D from Mean$(M.D._{\bar{x}}) = \frac{\sum|\boldsymbol{d_{\bar{x}}}|}{n}$
- Coefficient of $M.D._{\bar{x}} = \frac{M.D._{\bar{x}}}{\bar{x}}$
- M.D from Median$(M.D._{M}) = \frac{\sum|\boldsymbol{d_{m}}|}{n}$
- Coefficient of $M.D._{M} = \frac{M.D._{M}}{M}$

MEAN DEVIATION(M.D.) (CO1)

Q1: Calculate M.D. from Mean & Median & coefficient of Mean Deviation from the following data: 20, 22, 25, 38, 40, 50, 65, 70, 75

Solution: Mean $\bar{x} = \frac{\sum x}{n} = \frac{20+22+25+38+40+50+65+70+75}{9} = \frac{405}{9} = 45$

Median=Size of $\left(\frac{n+1}{2}\right)^{th}$ term

=Size of $\left(\frac{9+1}{2}\right)^{th}$ term = 40

M.D from Mean$(M.D._{\bar{x}}) = \frac{\sum|\boldsymbol{d_{\bar{x}}}|}{n} = \frac{160}{9} = 17.78$

Coefficient of $M.D._{\bar{x}} = \frac{M.D._{\bar{x}}}{\bar{x}} = \frac{17.78}{45} = 0.39$

M.D from Median$(M.D._{M}) = \frac{\sum|\boldsymbol{d_m}|}{n} = \frac{155}{9} = 17.22$

Coefficient of $M.D._{M} = \frac{M.D._{M}}{M} = \frac{17.22}{40} = 0.43$

MEAN DEVIATION(M.D.) (CO1)

Marks X	Deviation from mean $45\|d_{\bar{x}}\| = \|X - 45\|$	Deviation from median $40\|d_m\| = \|X - 40\|$
20	25	20
22	23	18
25	20	15
38	7	2
40	5	0
50	5	10
65	20	25
70	25	30
75	30	35
N=9, $\sum X = 405$	$\sum \|d_{\bar{x}}\|$=160	$\sum \|d_m\|$=155

MEAN DEVIATION(M.D.) (CO1)

Example: Calculate M.D. from Mean & Median & coefficient of Mean Deviation from the following data:

X:	20	30	40	50	60	70
F:	8	12	20	10	6	4

Solution:

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MEAN DEVIATION(M.D.) (CO1)

x	F	c.f	$\|d_m\| = \|X - 40\|$	f$\|d_m\|$	Fx	$\|d_{\bar{x}}\| = \|X - 41\|$	f$\|d_{\bar{x}}\|$
20	8	8	20	160	160	21	168
30	12	20	10	120	360	11	132
40	20	40	0	0	800	1	20
50	10	50	10	100	500	9	90
60	6	56	20	120	360	19	114
70	4	60	30	120	280	29	116
	N=60			$\sum$ f$\|d_m\|$ = 620	2460		$\sum$ f$\|d_{\bar{x}}\|$ = 640

MEAN DEVIATION(M.D.) (CO1)

M=Size of $\left(\frac{N+1}{2}\right)^{th}$ item = Size of $\left(\frac{60+1}{2}\right)^{th}$ item = 40

M.D from Median= $\frac{\sum f|\boldsymbol{d_m}|}{N} = \frac{620}{60} = 10.33$

Coefficient of $M.D._M = \frac{M.D._M}{M} = \frac{10.33}{40} = 0.258$

Mean$\bar{x} = \frac{\sum fx}{N} = \frac{2460}{60} = 41$

M.D. from Mean= $\frac{\sum f|\boldsymbol{d_{\bar{x}}}|}{N} = \frac{640}{60} = 10.67$

Coefficient of $M.D._{\bar{x}} = \frac{M.D._{\bar{x}}}{\bar{x}} = \frac{10.67}{41} = 0.26$

Daily Quiz

Q1. Calculate the Mean Deviation & coefficient of Mean Deviation from the following data:

Marks	**5**	**15**	**25**	**35**	**45**	**55**
No. of students	10	20	30	50	40	30

Answer: 11.11, 0.3175

Q2. Calculate the Mean Deviation & coefficient of Mean Deviation from the following data:

Marks	**0-10**	**10-20**	**20-30**	**30-40**	**40-50**	**50-60**
No. of students	10	20	30	50	40	30

Answer: 11.33, 0.3148

Variance (CO1)

- **For an Individual Series :**If $x_1, x_{2,\ldots..} x_n$ are the values of the variable under consideration , $\bar{x}$ is defined as

$$\sigma^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n};$$

- **For a frequency Distribution:** If $x_1, x_{2,\ldots..,} x_n$ are the values of a variable x with the corresponding frequencies $f_1, f_2, \ldots., f_n$ respectively $\bar{x}$ is defined as

$$\mu = \bar{x} = \frac{\sum fx}{\sum f}$$

$$N = \sum f$$

Variance (CO1)

$$\sigma^2 = \frac{\sum_{i=1}^{n} f_i (x_i - \bar{x})^2}{N};$$

where $N = \sum_{i=1}^{n} f_i$

Note. In case of a frequency distribution with class intervals, the values of x are the midpoints of the intervals.

Example 1. Find the Variance and standard deviation for the following individual series.

x	3	6	8	10	18

Solution:

Variance (CO1)

x	$x - \bar{x}$	$(x - \bar{x})^2$
3	-6	36
6	-3	9
8	-1	1
10	1	1
18	9	81
$\sum x = 45$		$\sum (x - \bar{x})^2 = \mathbf{128}$

$n=5, \sum x = 45, \ \bar{x} = \frac{\sum x}{n} = \frac{45}{5} = 9$

$\sigma^2 = \frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2 = \frac{128}{5} = 25.6,$

Standard deviation = $\sqrt{\text{variance}} = \sqrt{25.6} = 5.05$

Variance (CO1)

Example Find the variance and standard deviation for the following frequency distribution.

Marks	**5-15**	**15-25**	**25-35**	**35-45**	**45-55**	**55-65**
No.of students	10	20	25	20	15	10

Sol.

Marks	**No. of Std's (f)**	**Mid-Pt. (x)**	fx	$x-\bar{x}=x-34$	$f(x-\bar{x})^2$
5-15	10	10	100	-24	5760
15-25	20	20	400	-14	3920
25-35	25	30	750	-4	400
35-45	20	40	800	6	720
45-55	15	50	750	16	3840
55-65	10	60	600	26	6760
	N=100		$\sum fx$= 3400		$\sum f(x-\bar{x})^2$=21400

Variance (CO1)

$$\bar{x} = \frac{\sum fx}{N} = \frac{3400}{100} = 34$$

$$\sigma^2 = \frac{\sum f(x - \bar{x})^2}{N} = \frac{21400}{100} = 214$$

Standard deviation (σ) = $\sqrt{\text{variance}} = \sqrt{214} = 14.62$

Daily Quiz

Q1: Calculate standard deviation and variance of the given data:

Size	**6**	**7**	**8**	**9**	**10**	**11**	**12**
Frequency	3	6	9	13	8	5	4

Answer: 1.6, 2.56

Q2: Calculate the standard deviation of 100 students.

Mass (kg)	**60-62**	**63-65**	**66-68**	**69-71**	**72-74**
No. of students	10	20	30	50	40

Answer: 2.9202

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Recap (CO1)

- Measures of Central tendency
- Measures of dispersions

Topic Objective (CO1)

Moments

- In mathematical statistics it involve a basic calculation. These calculations can be used to find a probability distribution's mean, variance and skewness.

Moments (CO1)

❑ Moments:

The moment of a distribution are the arithmetic means of the various powers of the deviations of items from some given number.

- ➢ Moments about mean (central moment)
- ➢ Moments about any arbitrary number (Raw Moment)
- ➢ Moments about origin

Central Moments (CO1)

- **Moment about mean (central moment):**

❖ **For an Individual Series :**

If $x_1, x_2, \ldots . x_n$ are the values of the variable under consideration , the r^{th} moment μ_r about mean $\bar{x}$ is defined as

$$\text{Moment about mean } \mu_r = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^r}{n}; \text{r} = 0,1,2, \ldots .$$

❖ **For a frequency Distribution:**

If $x_1, x_2, \ldots, x_n$ are the values of a variable x with the corresponding frequencies $f_1, f_2, \ldots . , f_n$ respectively then r^{th} moment μ_r about the mean $\bar{x}$ is defined as

Central Moments (CO1)

$$\mu_r = \frac{\sum_{i=1}^{n} f_i (x_i - \bar{x})^r}{N}; \text{r} = 0,1,2 \ldots .$$

where $N = \sum_{i=1}^{n} f_i$

in particular $\mu_0 = \frac{1}{N}\sum_{i=1}^{n} f_i (x_i - \bar{x})^0 = \frac{1}{N}\sum_{i=1}^{n} f_i = \frac{N}{N} = 1$

Note. In case of a frequency distribution with class intervals, the values of x are the midpoints of the intervals.

Example 1. Find the first four moments for the following individual series.

Solution: Calculation of Moments

x	3	6	8	10	18

Central Moments (CO1)

S. No.	x	$x - \bar{x}$	$(x - \bar{x})^2$	$(x - \bar{x})^3$	$(x - \bar{x})^4$
1	3	-6	36	-216	1296
2	6	-3	9	-27	81
3	8	-1	1	-1	1
4	10	1	1	1	1
5	18	9	81	729	6561
n=5	$\sum x = 45$	$\sum x - \bar{x} = 0$	$\sum (x - \bar{x})^2 = 128$	$\sum (x - \bar{x})^3 = 486$	$\sum (x - \bar{x})^4 = 7940$

Central Moments (CO1)

For any distribution, $\mu_0 = 1$

$$\mu_1 = \frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x}) = 0$$

For any distribution, $\mu_1 = 0$, for r=2,

$$\mu_2 = \frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2 = \frac{128}{5} = 25.6$$

Therefore for any distribution ,μ_2 coincides with the variance of the distribution.

Similarly, $\mu_3 = \frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^3 = \frac{486}{5} = 97.2$

$$\mu_4 = \frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^4 = \frac{7940}{5} = 1588$$

Central Moments (CO1)

Now $\bar{x} = \frac{\sum x}{n} = \frac{45}{5} = 9$

$\mu_1 = \frac{\sum(x-\bar{x})}{n} = \frac{0}{5} = 0,$

$\mu_2 = \frac{\sum(x-\bar{x})^2}{n} = \frac{128}{5} = 25.6,$

$\mu_3 = \frac{\sum(x-\bar{x})^3}{n} = \frac{486}{5} = 97.2,$

$\mu_4 = \frac{\sum(x-\bar{x})^4}{n} = \frac{7940}{5} = 1588,$

Central Moments for Frequency Distribution (CO1)

For any distribution, $\mu_0 = 1$ for r=1

$$\mu_1 = \frac{1}{N}\sum_{i=1}^{n} f_i(x_i - \bar{x}) = \frac{1}{N}\sum_{i=1}^{n} f_i x_i - \bar{x}\left[\frac{1}{N}\sum_{i=1}^{n} f_i\right] = \bar{x} - \bar{x} = 0$$

For any distribution, $\mu_1 = 0$, for r=2,

$$\mu_2 = \frac{1}{N}\sum_{i=1}^{n} f_i(x_i - \bar{x})^2 = (S.D)^2 = Variance$$

Therefore for any distribution ,μ_2 coincides with the variance of the distribution.

Similarly, $\mu_3 = \frac{1}{N}\sum_{i=1}^{n} f_i(x_i - \bar{x})^3$

$\mu_4 = \frac{1}{N}\sum_{i=1}^{n} f_i(x_i - \bar{x})^4$ and so on.

Central Moments (CO1)

Example $\mu_1, \mu_2, \mu_3, \mu_4$ for the following frequency distribution.

Marks	**5-15**	**15-25**	**25-35**	**35-45**	**45-55**	**55-65**
No.of students	10	20	25	20	15	10

Sol. Calculation of Moments

$$\bar{x} = \frac{\sum fx}{N} = \frac{3400}{100} = 34$$

Central Moments (CO1)

Marks	No.of Students(f)	Mid-Point (x)	fx	$x-\bar{x}=x-34$	$f(x-\bar{x})$	$f(x-\bar{x})^2$	$f(x-\bar{x})^3$	$f(x-\bar{x})^4$
5-15	10	10	100	-24	-240	5760	-138240	3317760
15-25	20	20	400	-14	-280	3920	-54880	768320
25-35	25	30	750	-4	-100	400	-1600	6400
35-45	20	40	800	6	120	720	4320	25920
45-55	15	50	750	16	240	3840	61440	983040
55-65	10	60	600	26	260	6760	175760	4569760
	N=100		$\sum fx$ =3400		$\sum f(x-\bar{x})$=0	$\sum f(x-\bar{x})^2$=21400	$f(x-\bar{x})^3$=46800	$f(x-\bar{x})^4$=9671200

Central Moments (CO1)

$$\mu_1 = \frac{\sum f(x-\bar{x})}{N} = \frac{0}{100} = 0$$

$$\mu_2 = \frac{\sum f(x-\bar{x})^2}{N} = \frac{21400}{100} = 214$$

$$\mu_3 = \frac{\sum f(x-\bar{x})^3}{N} = \frac{46800}{100} = 468$$

$$\mu_4 = \frac{\sum f(x-\bar{x})^4}{N} = \frac{9671200}{100} = 96712$$

Raw Moments (CO1)

➢ Moments about an arbitrary number(Raw Moments):

If $x_1, x_2, x_3, \ldots\ldots, x_n$ are the values of a variable x with the corresponding frequencies $f_1, f_2, f_{3,\ldots\ldots} f_n$ respectively then r^{th} moment $\mu_r{}'$ about the number $x = A$ is defined as

$$\mu'_r = \frac{1}{N}\sum_{i=1}^{n} f_i(x_i - A)^r; r = 0,1,2/, \ldots$$

Where, $N = \sum_{i=1}^{n} fi$

For $r = 0, \mu'_0 = \frac{1}{N}\sum_{i=1}^{n} f_i(x_i - A)^0 = 1$

Raw Moments (CO1)

For $r = 1, \mu'_1 = \frac{1}{N}\sum_{i=1}^{n} f_i(x_i - A) = \frac{1}{N}\sum_{i=1}^{n} f_i x_i - \frac{A}{N}\sum_{i=1}^{n} fi = \bar{x} - A$

For $r = 2, \mu'_2 = \frac{1}{N}\sum_{i=1}^{n} f_i(x_i - A)^2$

For $r = 3, \mu'_3 = \frac{1}{N}\sum_{i=1}^{n} f_i(x_i - A)^3$ and so on.

In Calculation work, if we find that there is some common factor $h(>1)$ in values of $x - A$,we can ease our calculation work by defining

$$u = \frac{x - A}{h}.$$

In that case , we have

$$\mu'_r = \frac{1}{N}\left(\sum_{i=1}^{n} f_i {u_i}^r\right) h^r; r = 0,1,2, \ldots$$

Moments about the origin (CO1)

➢ Moments about the Origin:

If $x_1, x_2, \ldots\ldots, x_n$ be the values of a variable x with corresponding frequencies $f_1, f_2, \ldots\ldots, f_n$ respectively then r^{th} moment about the origin v_r is defined as

$$v_r = \frac{1}{N}\sum_{i=1}^{n} f_i {x_i}^r \text{ ; r} = 0,1,2, \ldots.$$

Where, $N = \sum_{i=1}^{n} f_i$

For $r = 0, v_0 = \frac{1}{N}\sum_{i=1}^{n} f_i {x_i}^0 = \frac{N}{N} = 1$

For $r = 1, v_1 = \frac{1}{N}\sum_{i=1}^{n} f_i x_i = \bar{x}$

For $r = 2, v_2 = \frac{1}{N}\sum_{i=1}^{n} f_i {x_i}^2$ and so on.

Relations (CO1)

- **Relation Between μ_r and μ'_r :**

$$\mu_1 = 0$$
$$\mu_2 = \mu_2{}' - \mu_1{}'^2$$
$$\mu_3 = \mu_3{}' - 3\mu_2{}'\mu_1{}' + 2\mu_1{}'^3$$
$$\mu_4 = \mu'_4 - 4\mu'_3\mu_1{}' + 6\mu_2{}'\mu_1{}'^2 - 3\mu_1{}'^4$$

- **Relation Between v_r and μ_r:**

$$v_1 = \bar{x}$$
$$v_2 = \mu_2 + \bar{x}^2$$
$$v_3 = \mu_3 + 3\mu_2\bar{x} + \bar{x}^3$$
$$v_4 = \mu_4 + 4\mu_3\bar{x} + 6\mu_2\bar{x}^2 + \bar{x}^4$$

Relation Between Moments(CO1)

Example1 : The first three moments of a distribution about the value "2" of the variable are 1,16 and −40.Show that the mean is 3,variance is 15 and $\mu_3 = -86$.

Solution: We have A=2,$\mu'_1 = 1$, $\mu'_2 = 16$ and $\mu'_3 = -40$

We have that $\mu'_1 = \bar{x} - A \Longrightarrow \bar{x} = \mu'_1 + A = 1 + 2 = 3$

Variance=$\mu_2 = \mu'_2 - {\mu'_1}^2 = 16 - (1)^2 = 15$

$\mu_3 = \mu'_3 - 3\mu'_2\mu'_1 + 2{\mu'_1}^3 = -40 - 3(16)(1) + 2(1)^3 = -40 - 48 + 2 = -86.$

Relation Between Moments(CO1)

Example 2: The first moments of a distribution about the value "35" are$-1.8,240,-1020\ and\ 144000$.Find the values of $\mu_1, \mu_2, \mu_3, \mu_4$.

Solution: $\mu_1 = 0$

$\mu_2 = \mu'_2 - {\mu_1}'^2 = 240 - (-1.8)^2 = 236.76$

$\mu_3 = \mu'_3 - 3\mu'_2\mu'_1 + 2{\mu'_1}^3$

$= -1020 - 3(240)(-1.8) + 2(-1.8)^3 = 264.36$

$\mu_4 = \mu'_4 - 4\mu'_3\mu'_1 + 6\mu'_2{\mu'}^2_1 - 3{\mu'}^4_1$

$= 144000 - 4(-1020)(-1.8) + 6(240)(-1.8)^{2-}3(-1.84)^4 = 141290.11.$

Relation Between Moments(CO1)

Example 3: Calculate the variance and third central moment from the following data.

x_i	0	1	2	3	4	5	6	7	8
F_i	1	9	26	59	72	52	29	7	1

Solution: Calculation of Moments

x	f	$u = \frac{x-A}{h}, A = 4, h = 1$	fu	fu^2	fu^3
0	1	-4	-4	16	-64
1	9	-3	-27	81	-243
2	26	-2	-52	104	-208
3	59	-1	-59	59	-59
4	72	0	0	0	0

Relation Between Moments(CO1)

5	52	1	52	52	52
6	29	2	58	116	232
7	7	3	21	63	189
8	1	4	4	16	64
			$\sum fu = -7$	$\sum fu^2 = 507$	$\sum fu^3 = -37$

$$\mu'_1 = \left(\frac{\sum fu}{N}\right) \mathrm{h} = \frac{-7}{256} = -0.02734$$

$$\mu'_2 = \left(\frac{\sum fu^2}{N}\right) h^2 = \frac{507}{256} = 1.9805$$

$$\mu'_3 = \left(\frac{\sum fu^3}{N}\right) h^3 = \frac{-37}{256} = -0.1445$$

Relation Between Moments(CO1)

Moments about Mean:

$\mu_1 = 0$

$\mu_2 = \mu'_2 - \mu'^{\,2}_1 = 1.9805 - (-.02734)^2 = 1.97975$

Variance=1.97975

Also $\mu_3 = \mu'_3 - 3\mu'_2\mu'_1 + 2\mu_1'^3$

$= (-0.1445) - 3(1.9805)(-0.02734) + 2(-0.02734)^3$

=0.0178997

Third central moment= 0.0178997.

Relation Between Moments(CO1)

Example 4: The first four moments of a distribution about the value '4'of the variable are -1.5,17,−30 and 108.Find the moments about mean, about origin.

Solution: We have A=4,$\mu'_1 = -1.5, \mu'_2 = 17, \mu'_3 = -30, \mu'_4 = 108$

Moments about mean

$\mu_1 = 0$

$\mu_2 = \mu'_2 - {\mu_1}'^2 = 14.75$

$\mu_3 = \mu'_3 - 3\mu'_2\mu'_1 + 2{\mu_1}'^3 = 39.75$

$\mu_4 = \mu'_4 - 4\mu'_3\mu'_1 + 6\mu'_2{\mu_1}'^2 - 3{\mu_1}'^4 = 142.3125$

$\bar{x} = \mu'_1 + A = -1.5 + 4 = 2.5$

Relation Between Moments(CO1)

Moments about origin:

$v_1 = \bar{x} = 2.5$

$v_2 = \mu_2 + \bar{x}^2 = 14.75 + (2.5)^2 = 21$

$v_3 = \mu_3 + 3\mu_2\bar{x} + \bar{x}^3 = 166$

$v_4 = \mu_4 + 4\mu_3\bar{x} + 6\mu_2\bar{x}^2 + \bar{x}^4 = 1132$

Moments about the points$x = 2$

$\mu'_1 = \bar{x} - A = 2.5 - 2 = 0.5$

$\mu'_2 = \mu_2 + {\mu_1}'^2 = 14.75 + (.5)^2 = 15$

$\mu'_3 = \mu_3 + 3\mu'_2\mu'_1 - 2{\mu_1}'^3 = 39.75 + 3(15)(.5) - 2(.5)^3 = 62$

$\mu'_4 = \mu_4 + 4\mu'_3\mu'_1 - 6\mu'_2{\mu_1}'^2 + 3{\mu_1}'^4 = 244$

Daily Quiz(CO1)

Q1. The first four moments about mean of a distribution are 1, 3, 7, 9, 10.

Answer: $\mu_1 = 0, \mu_2 = 12, \mu_3 = -12, \mu_4 = 208.8$

Q2. Compute first four moments of the data 3, 5, 7, 9 about mean. Also, compute the first four moments about the point 4.

Answer: $\mu_1 = 0, \mu_2 = 5, \mu_3 = 0, \mu_4 = 41$

$\mu_1' = 2, \mu_2' = 9, \ \mu_3' = 38, \ \mu_4' = 177$

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Recap(CO1)

- ✓ Measures of central tendency
- ✓ Moment

Topic objective (CO1)

Skewness

- It tells us whether the distribution is normal or not
- It gives us an idea about the nature and degree of concentration of observations about the mean
- The empirical relation of mean, median and mode are based on a moderately skewed distribution

Skewness(CO1)

❑ Skewness:

- It means *lack of symmetry.*
- It *gives us an idea about the shape of the curve* which we can draw with the help of the given data.
- A distribution is said to be skewed if—

Mean, median and mode fall at different points, i.e.,

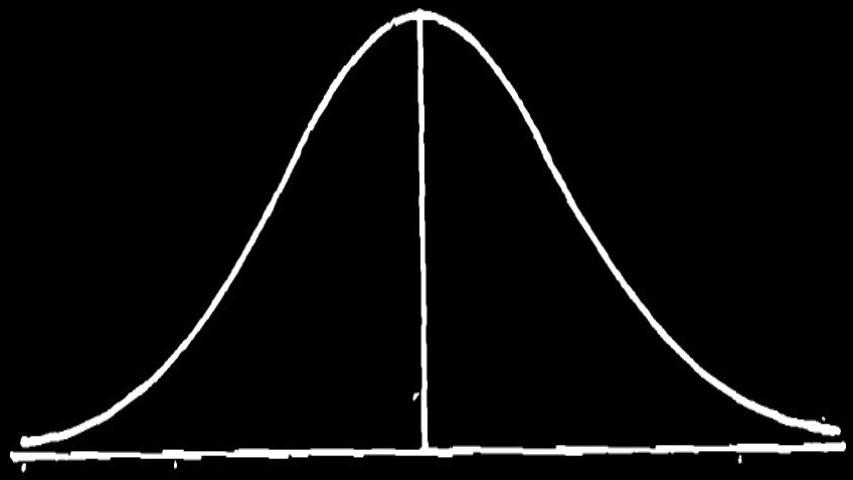
Mean f= Median f= Mode;

- Quartiles are not equidistant from median; and
- The curve drawn with the help of the given data is not symmetrical but stretched more to one side than to the other.

Skewness(CO1)

Symmetrical Distribution

A symmetric distribution is a type of distribution where the left side of the distribution mirrors the right side. In a symmetric distribution, the mean, mode and median all fall at the same point.



$\bar{x}$ (Mean) = M_0 = M_d

Skewness(CO1)

❖ Karl Pearson's β, γ Coefficients:

Karl Pearson defined the following four coefficients based upon the first four moments of a frequency distribution about it mean:

$$\beta_1 = \frac{{\mu_3}^2}{{\mu_2}^3} \qquad \beta_2 = \frac{\mu_4}{{\mu_2}^2} \qquad (\beta - \text{coefficients})$$

$$\gamma_1 = +\sqrt{\beta_1} \qquad \gamma_2 = \beta_2 - 3 \qquad (\gamma - \text{coefficients})$$

The practical use of this coefficients is to measure the skewness and kurtosis of a frequency distribution .These coefficients are pure numbers independent of units of measurement.

Skewness(CO1)

Measures of Skewness:

The measures of skewness are:

- $S_k = M - M_d$,
- $S_k = M - M_o$,
- $S_k = (Q_3 - M_d) - (M_d - Q_1)$,

where M is the mean, M_d, the median, M_o, the mode, Q_1, the first quartile deviation and Q_3, the third quartile deviation of the distribution.

These are the absolute measures of skewness.

Coefficients of Skewness:

For comparing two series we do not calculate these absolute measures but we calculate the relative measures called the *coefficients of skewness* which are pure numbers independent of units of measurement.

Skewness(CO1)

The following are the *coefficients of skewness:*

- Prof. Karl Pearson's Coefficient of Skewness,
- Prof. Bowley's Coefficient of Skewness,
- Coefficient of Skewness based upon Moments.

Prof. Karl Pearson's Coefficient of Skewness:

Definition: It is defined as:

$$SK_p = \frac{A.M. - Mode}{S.D} = \frac{3(M - M_d)}{\sigma}$$

where σ is the standard deviation of the distribution. If mode is ill-

$Mode$=3Median-2mean

defined, then using the empirical relation

Skewness(CO1)

$M_o = 3M_d - 2M$,

for a moderately asymmetrical distribution, we have

- From above two formulas, we observe that $S_k = 0$ if $M = M_o = M_d$.
- Hence for a symmetrical distribution, mean, median and mode coincide.
- Skewness is positive if $M > M_o$ or $M > M_d$, and negative if $M < M_o$ or $M < M_d$.
- Limits are: $|S_k| \leq 3$ or $-3 \leq S_k \leq 3$.
- However, in practice, these limits are rarely attained.

Skewness(CO1)

Coefficient of Skewness based upon Moments: Definition

It is defined as: $\gamma_1 = \frac{\mu_3}{\sqrt{\mu_2{}^3}}$

where γ_1 are Pearson's Coefficients and defined as:

$S_k = 0$, if either $\beta_1 = 0$ or $\beta_2 = -3$. Thus $S_k = 0$, if and only if $\beta_1 = 0$.

Thus for a symmetrical distribution $\beta_1 = 0$.

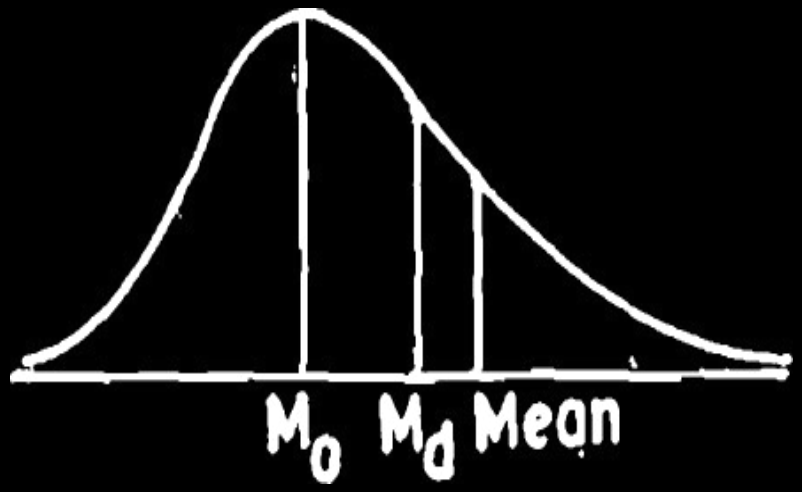
In this respect β_1 is taken as a *measure of skewness.*

Skewness(CO1)

- The coefficient of skewness based upon moments is to be regarded as without sign.
- The Pearson's and Bowley's coefficients of skewness can be positive as well as negative.

Positively Skewed Distribution:

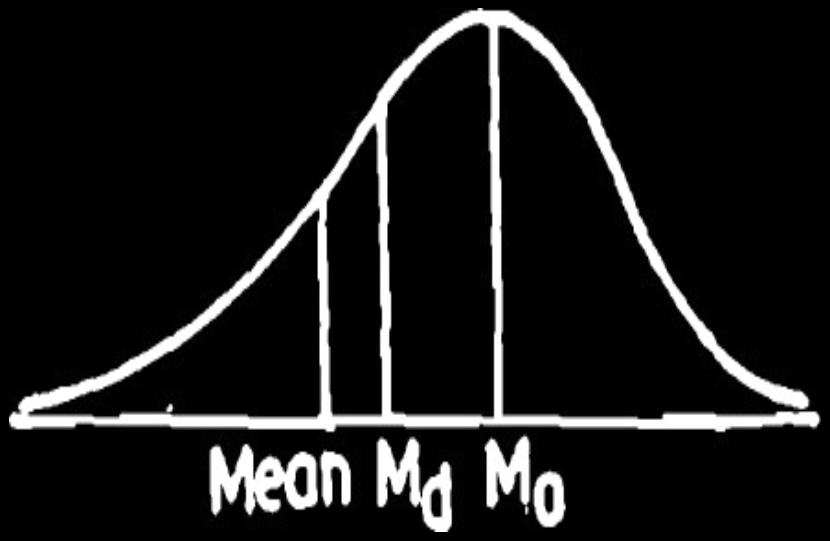
The skewness is positive if the larger tail of the distribution lies towards the higher values of the variate (the right),i.e., if the curve drawn with the help of the given data is stretched more to the right than to the left.



Skewness(CO1)

❖ **Negatively Skewed Distribution:**

The skewness is negative if the larger tail of the distribution lies towards the lower values of the variate (the left), i.e., if the curve drawn with the help of the given data is stretched more to the left than to the right.



Skewness(CO1)

Pearson's β_1 and γ_1 Coefficients:

$$\gamma_1 = \sqrt{\beta_1} = \pm \frac{\mu_3}{\sqrt{{\mu_2}^3}}$$

Example 1: Karl Pearson coefficient of skewness of a distribution is 0.32, its standard deviation is 6.5 and mean is 29.6. find the mode of the distribution.

Solution: Given that $SK_p = 0.32$, σ=6.5 mean =29.6

$$SK_p = \frac{A.M. - Mode}{S.D} = \frac{3(M - M_d)}{\sigma}$$

$$0.32 = \frac{29.6 - Mode}{6.5} \Longrightarrow Mode = 27.52$$

Topic objective (CO1)

Kurtosis

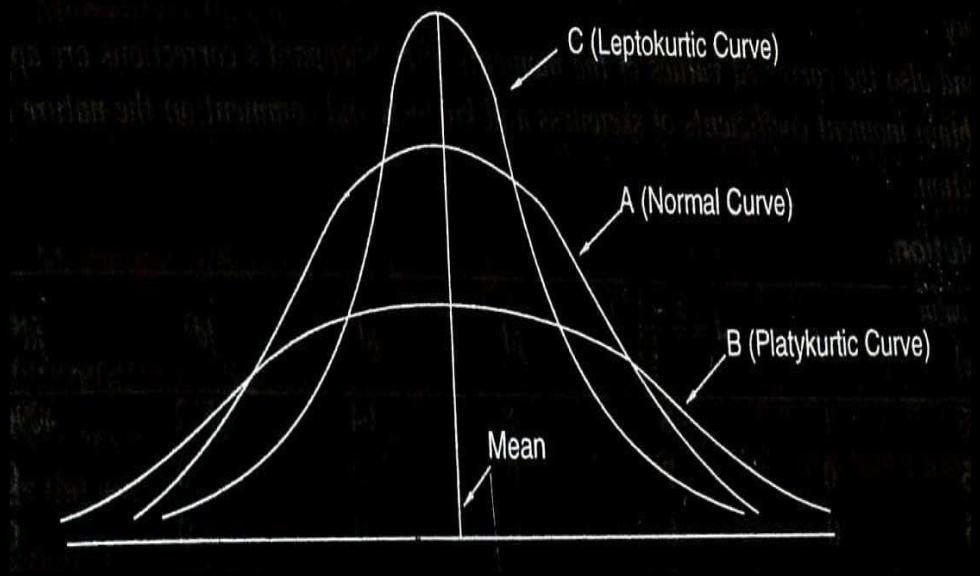
- Describe the concepts of kurtosis
- Explain the different measures of kurtosis
- Explain how kurtosis describe the shape of a distribution.

Kurtosis (CO1)

❑ Kurtosis

If we know the measures of central tendency, dispersion and skewness, we still cannot form a complete idea about the distribution. Let us consider the figure in which all the three curves.

A, B, and C are symmetrical about the mean and have the same range.



Kurtosis (CO1)

Definition:

Kurtosis is also known as *Convexity of the Frequency Curve* due to Prof. Karl Pearson.

- It *enables us to have an idea about the flatness or peakness* of the frequency curve.
- It is measure by the coefficient β_2 or its derivation γ_2 given as:

$$\beta_2 = \frac{\mu_4}{{\mu_2}^2}$$

- Curve of the type *A* which is *neither flat nor peaked* is called the *normal curve or mesokurtic curve* and for such curve $\beta_2 = 3$, i.e., $\gamma_2 = 0$.
- Curve of the type *B* which is *flatter than the normal curve* is known as *platycurtic curve* and for such curve $\beta_2 < 3$, i.e., $\gamma_2 < 0$.

Kurtosis (CO1)

Curve of the type *C* which is *more peaked than the normal curve* is called *leptokurtic curve* and for such curve $\beta_2 > 3$, i.e., $\gamma_2 > 0$.

Example 2: For a distribution, the mean is 10, variance is 16, γ_1 is +1 and β_2 is 4. Comment about the nature of distribution. Also find third central moment.

Solution: $1 = \pm\frac{\mu_3}{\sqrt{4096}} \Rightarrow \mu_3 = 64$, $\mu_2 = 16$,

$$4 = \frac{\mu_4}{256} \Rightarrow \mu_4 = 1024$$

Since $\gamma_1 = +1$, the distribution is moderately positively skewed, i.e,
if we draw the curve of the given distribution, it will have longer tail towards the right.
Further, since $\beta_2 = 4 > 3$, the distribution is leptokurtic, i.e.,
it will be sightly more peaked than the normal curve.

Kurtosis (CO1)

Example 3 The first four moment about the working mean 28.5 of a distribution are 0.294,7.144,42.409 and 454.98. Calculate the first four moment about mean. Also evaluate β_1 and β_2 and comment upon the skewness and kurtosis of the distribution.

Solution:$\mu'_1 = .294, \mu'_2 = 7.144, \mu'_3 = 42.409, \mu'_4 = 454.98$Moment about mean

$\mu_1 = 0,$

$\mu_2 = \mu'_2 - \mu_1{}'^2 = 7.0576.$

$\mu_3 = \mu'_3 - 3\mu'_2\mu_1' + 2\mu_1{}'^3 = 36.1588,$

$\mu_4 = \mu'_4 - 4\mu'_3\mu'_1 + 6\mu'_2\mu_1{}'^2 - 3\mu_1{}'^4 = 408.7896$

$\beta_1 = \frac{\mu^2{}_3}{\mu_2{}^3} = 3.7193, \qquad \beta_2 = \frac{\mu_4}{\mu_2{}^2} = 8.207$

Skewness: β_1 is positive, $\gamma_1 = 1.9285$ so distribution is positivley skewed.

Kurtosis: $\beta_2 = 8.207 > 3$ so distribution is leptokurtic.

Daily Quiz(CO1)

Q1. Calculate Karl Pearson's Coefficient of skewness from the table given below:

X	14.5	15.5	16.5	17.5	18.5	19.5	20.5	21.5
f	35	40	48	100	125	87	43	22

Answer: -0.245

Q2. The first four central moments of a distribution are 0, 2.3, 0.9, and 15.65. Test the skewness and kurtosis of the distribution. Also discuss the nature of the curve.

Answer: Skewness = 0.265, Kurtosis = 2.96. The distribution is platykurtic.

Weekly Assignment(CO1)

Q1. The First four moments of a distribution about $x = 4$ are $1, 4, 10, and\ 45$. Find the first four moments about mean. Discuss the Skewness and Kurtosis and also comment upon the nature of the distribution.

Q2. Define the Mode and calculate Mode for the distribution of monthly rent Paid by Libraries in Karnataka

Monthly rent	**500-1000**	**1000-1500**	**1500-2000**	**2000-2500**	**2500-3000**	**3000 & above**
No. of Library	5	10	8	16	14	12

Weekly Assignment(CO1)

Q3. Write Short Note on

i. Range

ii. Inter quartile range

iii. Mean deviation

iv. Standard deviation

v. Variance

Q 4. Explain the measures of dispersion and also find the range & Coefficient of Range for the following data: 20, 35, 25, 30, 15.

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Recap(CO1)

- ✓ Moments
- ✓ Relation between $v_r \; and \; \mu_r$
- ✓ Relation between $\mu_r \; and \; \mu'_r$
- ✓ Skewness
- ✓ Kurtosis

Faculty Video Links, Youtube & NPTEL Video Links and Online Courses Details

Suggested Youtube/other Video Links:

https://youtu.be/wWenULjri40

https://youtu.be/mL9-WX7wLAo

https://youtu.be/nPsfqz9EljY

https://youtu.be/nqPS29IvnHk

https://youtu.be/aaQXMbpbNKw

https://youtu.be/wDXMYRPup0Y

https://youtu.be/m9a6rg0tNSM

https://youtu.be/Qy1YAKZDA7k

https://youtu.be/Qy1YAKZDA7k

https://youtu.be/s94k4H6AE54

https://youtu.be/lBB4stn3exM

https://youtu.be/0WejW9MiTGg

https://youtu.be/QAEZOhE13Wg

MCQ (CO1)

Q1. Sum of squares of items 2430, mean is 7 N=12, find the variance.

i. 176.5

ii. 12.38

iii. 153.26

iv. 14

Q2. The mean of the range, mode and median of the data: 5, 10, 3, 6, 4, 8, 9, 3, 15, 2, 9, 4, 19, 11, 4

i. 10

ii. 12

iii. 8

iv. 9

MCQ (CO1)

Q3. The values of quartile and quartile deviation of the data: 17, 2, 7, 27, 15, 5, 14, 8

i. 2.5

ii. 3

iii. 5

iv. None of these

Q4. If a distribution has a high kurtosis, it is called:

i. Leptokurtic

ii. Platykurtic

iii. Mesokurtic

iv. Skewed

Glossary (CO1)

Q1: An in complete distribution is given below:

x	10-20	20-30	30-40	40-50	50-60	60-70	70-80
f	12	30	X	65	Y	25	18

Given that median value is 46 and N=229

i. X
ii. Y
iii. Mean
iv. Mode

Pick the correct option from glossary

a. 45.82
b. 33.5
c. 46.07
d. 45

Old Question Papers

First Sessional Set-1 (CSE,IT,CS,ECE,IOT).docx

Second Sessional Set-2 (CSE,IT,CS,ECE,IOT).docx

Maths IV PUT.docx

Maths IV final paper_2022.pdf

End Semester Question Paper

Printed Page:- 06 **Subject Code:- BAS0303**

Roll. No: | | | | | | | | | | | | | | | |

NOIDA INSTITUTE OF ENGINEERING AND TECHNOLOGY, GREATER NOIDA
(An Autonomous Institute Affiliated to AKTU, Lucknow)
B.Tech
SEM: III - THEORY EXAMINATION (2024- 2025)
Subject: Statistics & Probability

Time: 3 Hours **Max. Marks: 100**

General Instructions:

IMP: *Verify that you have received the question paper with the correct course, code, branch etc.*

1. *This Question paper comprises of* ***three Sections*** *-**A, B, & C.** It consists of Multiple Choice Questions (MCQ's) & Subjective type questions.*

2. *Maximum marks for each question are indicated on right -hand side of each question.*

3. *Illustrate your answers with neat sketches wherever necessary.*

4. *Assume suitable data if necessary.*

5. *Preferably, write the answers in sequential order.*

6. *No sheet should be left blank. Any written material after a blank sheet will not be evaluated/checked.*

SECTION-A 20

1. Attempt all parts:-

1-a. Wheat crops badly damaged on account of rains is: (CO1, K2) 1

(a) Cyclical movement
(b) Random movement
(c) Secular trend
(d) Seasonal movement

1-b. Let the average of three numbers be 16. If two of the numbers are 8 and 12, then the remaining number is..... (CO1, K3) 1

(a) 28
(b) 18
(c) 12
(d) 30

1-c. One card is drawn from a standard pack of 52 plying cards. Find the probability that it is either a king or a queen. (CO2, K3) 1

(a) $\frac{1}{13}$
(b) $\frac{2}{13}$
(c) $\frac{3}{13}$
(d) None of these

1-d. If two events A and B are mutually exclusive, then the probability $P(A \cap B)$ is: (CO2, K2) 1

(a) 0
(b) 1
(c) P(A).P(B)
(d) None of these

End Semester Question Paper

1-e. In binomial distribution probability of success in each trial remains _____.(CO3. K1) 1

(a) 0
(b) 1
(c) Constant
(d) Not defined

1-f. In Normal Distribution, Mean deviation about mean is (CO3, K1) 1

(a) σ
(b) $2\sigma/5$
(c) $4\sigma/5$
(d) $6\sigma/7$

1-g. The area of critical region depends on the size of.... (CO4, K1) 1

(a) Type I error
(b) Type II error
(c) Test statistics
(d) Sample

1-h. In conducting one way analysis of variance --- test statistics would be used. (CO4,K1) 1

(a) Z
(b) T
(c) Chi-Square
(d) F

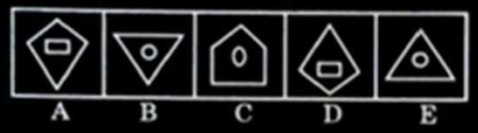
End Semester Question Paper

1-i. If $A=\{1,4,6\}$, $B=\{3,6\}$ and $C=\{3,4,6\}$ then $A \cap (B \cap C)$ is…. (CO5,K3) 1

(a) $\{3,4,5,6\}$
(b) $\{4,6\}$
(c) $\{1,4,6\}$
(d) $\{6\}$

1-j. Select a figure from amongst the answer figure which will continue the same series as stablished by the five-figure problem. (CO5,K2) 1

Problem Figure



Answer Figure

(a) 1
(b) 2
(c) 3
(d) 5

End Semester Question Paper

2. Attempt all parts:-

2.a.	Prove that if two variables are independent, their correlation is zero but vice versa is not true. (CO1,K2)	2
2.b.	Find the probability of getting 53 Sundays in a leap year. (CO2,K3)	2
2.c.	A Binomial random variable X satisfies the relation 9P(X=4) =P(X=2) ,When n=6 . Find value of P(X=1) . (CO3, K3)	2
2.d.	Define an estimator in statistics. (CO4, K1)	2
2.e.	Check whether the function f : N→N is defined by $f(x)=x^2+12$ is one-one or not.(CO5, K3)	2

SECTION-B 30

3. Answer any five of the following:-

3-a.	The mean and standard deviation of the marks of 100 candidates was found to be 40 and 5.1, respectively. Later, it was discovered that a score of 40 was wrongly read as 50. Find out the correct mean and standard deviation respectively. (CO1,K3)	6
3-b.	Calculate Spearman's rank correlation coefficient from the following data: (CO1, K3)	6

End Semester Question Paper

X	68	64	75	50	64	80	75	40	55	64
Y	62	58	68	45	81	60	68	48	50	70

3-c. A random variable X has the following distribution, Find the value of K. Also find Mean and variance (CO2, K3) 6

x	-2	-1	0	1	2	3
P(x)	0.1	K	0.2	2K	0.3	K

3-d. A bag X contains 2 white and 3 red balls and another bag Y contains 4 white and 5 red balls. One ball is drawn at random from one of the bags and is found to be red. Find the probability that it was drawn from bag Y. (CO2, K3) 6

3.e. If 10% of the bolts produced by a machine are defective, determine the probability that out of 10 bolts choosen at random 6
i) 1
ii) None
iii) at most 2 bolts will be defective. (CO3,K3)

3.f. 6
A sample of 20 items has mean 42 units and S.D. 5 units. Test the hypothesis that it is a random sample from a normal population with mean 45 units. (If the tabular value at 5% LOS for 19 d. f. is 2.09). (CO4, K3)

3.g. How many different words can be formed using all the letters of the world ALLAHABAD 6

1. *When the vowels occupy the even position.*
2. Both L do not occur together. (CO5,K3)

End Semester Question Paper

SECTION-C 50

4. Answer any one of the following:-

4-a. In a partially destroyed laboratory record of analysis of a correlation data, the following results only are legible: (CO1,K3) 10

Variance of x = 9; Regression equations: $8x - 10y + 66=0$, $40x - 18y = 214$.

What were (a) the mean values of x and y (b) the standard deviation of y (c) the coefficient correlation between x and y.

4-b. Fit a second degree parabola by the method of least squares to the following data . (CO1, K3) 10

X	0	1	2	3	4
Y	1	4	10	17	30

5. Answer any one of the following:-

5-a. State and prove Bayes theorem. 10

In a Neighbourhood, 90% children were falling sick due flu and 10% due to measles and no other disease. The probability of observing rashes for measles is 0.95 and for flu is 0.08. If a child develops rashes, find the child's probability of having flu. (CO2, K3)

5-b. Let the two dimensional continuous random variable(X,Y) has joint PDF given by 10

$$f(x,y) = \begin{cases} 6x^2y\,, & 0 < x < 1,\ 0 < y < 1 \\ 0\,, & \text{elsewhere} \end{cases}$$

Find (i) $P(0<x<3/4\ ,\ 1/3<y<2)$ (ii) $P(x + y < 1)$. (CO2, K3)

6. Answer any one of the following:-

6-a. A manufacturer of envelopes knowns that the weight of the envelopes is normally distributed with mean 1.9gm and variance 0.01 square gm. Find how many envelopes weighing (i)2gm or more (ii)2.1gm or more, can be expected in a given packet of 1000envelopes? Given that the area under the standard curve between $z = 0$ and $z = 1$ is 0.3413, between $z = 0$ and $z = 2$ is 0.4772. (CO3,K3) 10

6-b. 10

Four coins were tossed 200 times. The number of tosses showing 0,1,2,3 and 4 heads were found to be as under.

Fit a binomial distribution to these observed results. Find the expected frequencies. (CO3, K3)

No. of Heads	0	1	2	3	4
No. of Tosses	15	35	90	40	20

End Semester Question Paper

7. Answer any one of the following:-

7-a. 10

To test of significance of the variations of the retail prices in the commodity in three principal cities: Mumbai, Bangalore and Chennai. The four shops were chosen at random in each city and prices observed in INR were as follows:

Mumbai	16	8	12	14
Bangalore	14	10	10	6
Chennai	4	10	8	8

Do the data indicate that the prices in the three cities are significantly different? Given that the tabular value of F is 4.26 5% LOS with d.f. is (2,9). (CO4,K3)

7-b. From the following data, find whether hair color and gender are associated. 10

End Semester Question Paper

Gender↓	Colour					
	Fair	Red	Medium	Dark	Black	Total
Boys	529	849	504	119	36	2100
Girls	544	677	451	97	14	1783
Total	1136	1526	955	216	50	3883

Given that the tabular value of $\chi 2$ is 9.488 at 5%LOS with d.f. 4. (CO4, K3)

8. Answer any one of the following:-

8-a. Solve the following: (CO5,K3) 10

1. What is the sum of all five-digit numbers formed by 2, 3, 4, 5, 6 without repetition?
2. What is the sum of all five-digit numbers formed by 2, 3, 4, 5, 6 with repetition?

End Semester Question Paper

8-b. Study the following table and answer the questions based on it 10

Expenditures of a Company (in Lakh Rupees) per Annum Over the given Years.

Year	Item of Expenditure				
	Salary	Fuel and Transport	Bonus	Interest on Loans	Taxes
1998	288	98	3	23.4	83
1999	342	112	2.52	32.5	108
2000	324	101	3.84	41.6	74
2001	336	133	3.68	36.4	88
2002	420	142	3.96	49.4	98

i) Find the average amount of interest per year which the company had to pay during this period?

ii) The total amount of bonus paid by the company during the given period is approximately what percent of the total amount of salary paid during this period?
iii) Total expenditure on all these items in 1998 was approximately what percent of the total expenditure in 2002?
iv) The total expenditure of the company over these items during the year 2000 is?
v) The ratio between the total expenditure on Taxes for all the years and the total expenditure on Fuel and Transport for all the years respectively is approximately?
(CO5,K3)

Expected Questions for University Exam(CO1)

Q1: An incomplete distribution of families according to their expenditure per week is given below. The median and mode for the distribution are Rs 25 and Rs 24 respectively. Calculate the missing frequencies.

Expenditure	0-10	10-20	20-30	30-40	40-50
No. of families	14	?	27	?	15

Q2: The first four moments of a distribution about 2 are 1,2.5,5.5 and 16 respectively. Calculate the four moments about mean and about the origin.

Expected Questions for University Exam(CO1)

Q3: Calculate the mean and standard deviation for the following data:

Size of item	6	7	8	9	10	11	12
Frequency	3	6	9	13	8	5	4

Q4: The first four moments of the distribution about the value 4 of the variable are -1.5, 17, -30, and 108. Find the moments about mean, β_1 and β_2.

Recap (CO1)

We discussed the following topics:

- ✓ Measures of central tendency – mean, median, mode
- ✓ Measures of dispersion – mean deviation, standard deviation, quartile deviation, variance
- ✓ Moment
- ✓ Skewness
- ✓ Kurtosis

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- H. K. Dass: Introduction to Engineering Mathematics, S. Chand.
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- W. Feller, An Introduction to Probability Theory and its Applications, Vol. 1, 3rd Ed., Wiley, 1968.

References

Reference Books

- P. G. Hoel, S. C. Port and C. J. Stone, Introduction to Probability Theory, Universal Book Stall, 2003(Reprint).
- R.K. Jain and S.R.K. Iyenger: Advance Engineering Mathematics; Narosa Publishing House, New Delhi.
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- D.N.Elhance,V. Elhance & B.M. Aggarwal: Fundamentals of Statistics; Kitab Mahal Distributers, New Delhi.

Thank You