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Random Var

If in n trials there are exhaustive, mutually exclusive & equally likely events, m out of the n are the favourable cases of happening the event E is given by

$$P(E) = \frac{\text{Total no. of fav. cases of } (E)}{\text{Total no. of cases is the S.S.}}$$

$$P(E) = \frac{m}{n} = \frac{n(E)}{n(S)}$$

(1) $0 \leq P(E) \leq 1$

eg- Tossing of two fair dice

S= { (1,1), (1,2), (1,3), (1,4), (1,5), (1,6),
(2,1), (2,2), (2,3), (2,4), (2,5), (2,6)
(3,1), (3,2), (3,3), (3,4), (3,5), (3,6)
(4,1) — — — ,
(5,1) — —
(6,1) — — — — (6,6)

Q. Two fair dice are tossing simultaneously. Find the probability of the events.

i) The sum of the no. on two dice is 7
ii) First die getting prime no. & second die is getting even no.
iii) Atleast one die getting odd no.

(i) $F(E) = \{(1,6), (2,5), (3,4), (4,3), (5,2), (6,1)\}$

$P(E) = \frac{6}{36} = \frac{1}{6}$

ii) $F(E_2) = \{(2,4), (2,6), (2,2), (3,2), (3,4), (3,6), (5,2), (5,4), (5,6)\}$

$P(E_2) = \frac{9}{36} = \frac{1}{4}$

iii) $F(E_3) = \{(1,1), (1,2), (1,3), (1,4), (1,5), (1,6), (2,1), (2,3), (2,5), (3,1), (3,2), (3,3), (3,4), (3,5), (3,6), (4,1), (4,3), (4,5), (5,1), (5,2), (5,3), (5,4), (5,5), (5,6), (6,1), (6,3), (6,5)\}$

$P(E_3) = \frac{27}{36} = \frac{3}{4}$

Some Std formula.

1. $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Prob. of happ. either A or B or both
OR
at least one of them happened.

$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(A \cap C) + P(A \cap B \cap C)$

Q. A card is drawn from an ordinary pack of cards & a gambler bets that it is a spade or an ace. What is the prob of his winning the bet

$P(A) = \frac{^{13}C_1}{^{52}C_1} = \frac{13}{52} = \frac{1}{4}$

$P(B) = \frac{^{4}C_1}{^{52}C_1} = \frac{4}{52} = \frac{1}{13}$

$A \to$ spade
$B \to$ ace.
$n(S) = 52$

$P(A \cap B) = \frac{^{1}C_1}{^{52}C_1} = \frac{1}{52}$

$n(E) = 13 + 4 - 1 = 16$

$P(E) = \frac{16}{52} = \frac{4}{13}$

Random variable is a var. whose possible values are determined by the outcome of a random exp.
It assigns a numerical value to each outcome of that random exp.

Q. Tossing of two coins:

$S = \{HH, HT, TH, TT\}$

Suppose X is a random variable which defines as "no. of heads)

$X = \{2, 1, 1, 0\}$

$X \in \{2, 1, 0\} = \{0, 1, 2\}$

Q- If two cards are drawn from a well shuffled 52 cards. Write the set of random var. which denotes "no. of queens".

$\{0, 1, 2\}$

Q. If a fair coin is tossing three times. Write the, all poss. values of random var. which denotes "no. of heads − (no. of tails)".

Total outcomes = 8

$S = \{HHH, HHT, HTH, THH, HTT, THT, TTH, TTT\}$

$= \{3, 1, -3, -9\}$

$P(X=3) = \frac{1}{8}$

$P(X=1) = \frac{3}{8}$

$P(X=-3) = \frac{3}{8}$

$P(X=-9) = \frac{1}{8}$

Probability dist. is

$10k^2 + 9k = 1$

$10k^2 + 9k - 1 = 0$

$10k^2 + 10k - k - 1 = 0$

$10k(k+1) - 1(k+1) = 0$

$(k+1)(10k-1) = 0$ $k = -1, \frac{1}{10}$

Possible value of $k = \frac{1}{10}$

ii) $P(X < 6) = P(0) + P(1) + P(2) + P(3) + P(4) + P(5)$

iii) The value of x corresponding to min val of the probability.

iv) The min. value of x such that − $P(X < x) > \frac{1}{2}$

$P(X \leq 0) = P(0) = 0$

$P(X \leq 1) = P(0) + P(1) = \frac{1}{10} = 0.1$

$P(X \leq 2) =$

$P(0) + P(1) + P(2) = 0 + \frac{1}{10} + \frac{2}{10} = 0.3$

$P(X \leq 3) = 0.3 + P(3) = 0.3 + \frac{2}{10} = 0.5$

$P(X \leq 4) = 0.5 + P(4)$

$= 0.5 + \frac{3}{10}$

$x = 4$

Q- Age (X) − 25 30 35 40 45 50 55 6

Y/No · 0 0 0 1 1 1 1 1

Q- 2 bad eggs are mixed accidentally with 10 good eggs. Find the pmf of the no. of bad egg in 3, drawn at random w/o replacement from this lot. Also determine the mean & variance of the dist.

2 bad eggs + 10 good eggs = 12

3 eggs are drawn at random w/o repl.

BBG (2), BGG (1), G,G,G (0)

X:	0	1	2
P(X)	P(0)	P(1)	P(2)

$P(0) = \frac{10}{12} \times \frac{9}{11} \times \frac{8}{10}$ | $^{10}C_3 \times {}^{2}C_0$

$P(1) = \left(\frac{2}{12} \times \frac{10}{11} \times \frac{9}{11}\right) + \left(\frac{10}{12} \times \frac{2}{11} \times \frac{9}{10}\right) + \left(\frac{10}{12} \times \frac{9}{11} \times \frac{2}{10}\right)$

$= {}^{3}C_1 \left(\frac{3}{22}\right) = \frac{9}{22}$

$P(2) = {}^{3}C_2 \left(\frac{2}{12} \times \frac{1}{11} \times \frac{10}{10}\right) = \frac{3}{66} = \frac{1}{22}$

Mean $= \Sigma x\, P(x) = 0.5$, $V = 0.343$

Hence PMF :

X :	0	1	2
P(X) :	$\frac{12}{22}$	$\frac{9}{22}$	$\frac{1}{22}$

CDF (Cumulative dist. f^n) - The CDF of random var. (X) denoted by F(x). If the probability that X will take a value less than or equal to a specific no. x.

i.e. $F(x) = P(X \le x)$.

For discrete random var. the cdf is a step f^n. It jumps upward at each value that the random var. can take & is constant between those values.

How to find CDF from PMF →

S-I List all poss. values of random var. X in increasing order.

S-II Calculate the cumulative probability for each value of random var.

S-III Write the piecewise all the above prob. in the S-II.

Q- If a fair coin is tossing two time. Find the cumulative dist. f^n for the no. of heads.

X → no. of heads S = {HH, HT, TT, TH}

X →	0	1	2
P(X) →	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$

CDF: $x < 0$, $F(x) = P(X < 0) = 0$

for $x = 0$, $F(x) = P(X \leq 0) = P(0) = \frac{1}{4}$

for $x = 1$, $F(x) = P(X \leq 1) = P(0) + P(1)$

$= \frac{1}{4} + \frac{1}{2} = \frac{3}{4}$

for $x = 2$, $F(x) = P(X \leq 2) = P(0) + P(1) + P(2)$

$= \frac{1}{4} + \frac{1}{2} + \frac{1}{4} = 1$

for $x > 2$, $F(x) = P(X \leq 2) = 1$

★ Also,

$$F(x) = \begin{cases} 0 & \text{for } x < 0 \\ 1/4 & \text{for } 0 \leq x < 1 \\ 3/4 & \text{for } 1 \leq x < 2 \\ 1 & \text{for } x \geq 2 \end{cases}$$

Q. The pmf of a discrete random var Y is given by $P(Y = y) = \begin{cases} K(y+1) & \text{for } y = 0, 1, 2, 3 \\ 0 & \text{, otherwise} \end{cases}$

i) Find the constant K that makes this as valid PMF

ii) Find its CDF.

$y =$	0	1	2	3
$P(y) =$	K	2K	3K	4K

$\sum P(y) = 1$

$K + 2K + 3K + 4K = 1$

$10K = 1 \Rightarrow K = \frac{1}{10}$

$y < 0$, $F(x) = P(Y < 0) = 0$

0.1 for $0 \leq y < 1$

0.3 for $1 \leq y < 2$

0.6 for $2 \leq y < 3$

1 for $y \geq 3$

Probability density f^n (pdf) - If X is a cont. random var. then the corresponding f^n of X is called the pdf of density f^n or prob. density distribution or PF.

Properties of Pdf:

i) $f(x) = P(x) \geq 0$

ii) $\int_{-\infty}^{\infty} f(x)\,dx = 1$

Note: Let $f(x)$ is pdf defined in the $a \leq x \leq b$.

$P(a \leq x \leq b) = \int_a^b f(x)\,dx = P(a < x < b)$

$= P(a < x \leq b) = P(a \leq x < b)$

Q-1. If a f^n is defined $f(x) = ce^{-x}$, for $0 \le x < \infty$ find:

i) The value of c so that $f(x)$ is a valid pdf

ii) $P(1 < x < 3)$

iii) $P(x < 2)$

i) for pdf : $\int_{-\infty}^{\infty} f(x)\,dx = 1$

$$= \int_0^{\infty} c e^{-x}\,dx = 1$$

$$= c\int_0^{\infty} e^{-x}\,dx = 1$$

$$= c\left[-e^{-x}\right]_0^{\infty} = 1$$

$$= c\left[-e^{-\infty} + e^{-0}\right] = 1$$

$$= c[0+1] = 1 \qquad \boxed{c = 1}$$

Hence pdf is : $f(x) = e^{-x}, \quad 0 \le x < \infty$

ii) $P(1 < x < 3) = \int_1^3 e^{-x}\,dx = \left[-e^{-x}\right]_1^3$

$$= -e^{-3} + e^{-1} = \left(\frac{1}{e} - \frac{1}{e^3}\right)$$

iii) $P(x < 2) = \int_0^2 e^{-x}\,dx = \left[-e^{-x}\right]_0^2$

$$= -e^{-2} + e^{-0} = \left(1 - \frac{1}{e^2}\right)$$