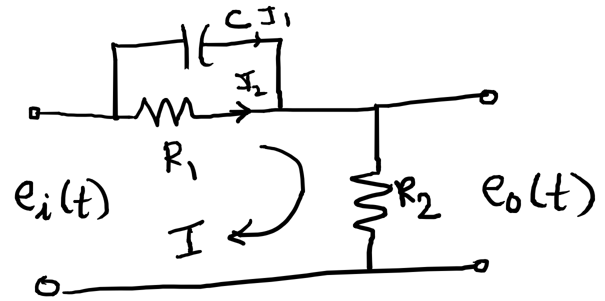
Transfer Functions of Lead and Lag Compensators:

Lead compensator:



$$I_1 + I_2 = I$$

$$C\frac{d\left(e_i(t) - e_o(t)\right)}{dt} + \frac{e_i(t) - e_o(t)}{R_1} = \frac{e_o(t)}{R_2}$$

Taking L.T, $SC\,E_i(S) - SC\,E_o(S) + \frac{E_i(S)}{R_1} - \frac{E_o(S)}{R_1} = \frac{E_o(S)}{R_2}$

$$E_i(S)\left[SC + \frac{1}{R_1}\right] = E_o(S)\left[SC + \frac{1}{R_1} + \frac{1}{R_2}\right]$$

$$\therefore\ T.F = \frac{E_o(S)}{E_i(S)} = \frac{SC + \frac{1}{R_1}}{SC + \frac{1}{R_1} + \frac{1}{R_2}} = \frac{C\left[S + \frac{1}{R_1C}\right]}{C\left[S + \frac{1}{R_1C} + \frac{1}{R_2C}\right]}$$

$$T.F = \frac{S + \frac{1}{R_1C}}{S + \frac{1}{R_1C} + \frac{1}{R_2C}} = \frac{S + \frac{1}{R_1C}}{S + \frac{R_2 + R_1}{R_1R_2C}} = \frac{S + \frac{1}{R_1C}}{S + \frac{1}{\left(\frac{R_2}{R_1+R_2}\right)R_1C}}$$

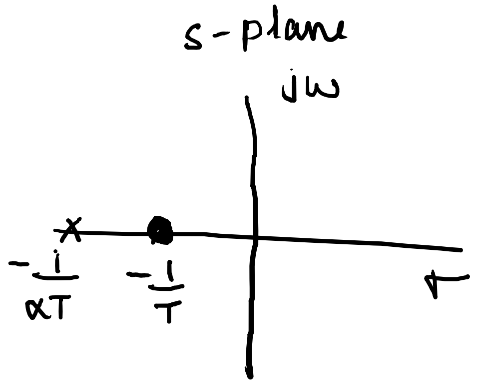
let $T = R_1C$ and $\frac{R_2}{R_1+R_2} = \alpha$

$\rightarrow 0 < \alpha < 1$

$$T.F = \frac{S + \frac{1}{T}}{S + \frac{1}{\alpha T}}$$

$\rightarrow$ This T.F is having a zero at $S = -\frac{1}{T}$ and a pole at $S = \frac{1}{\alpha T}$

$\because$ value of α lies b/n 0 & 1, pole always lies to the left of zero.



Effects of lead compensation

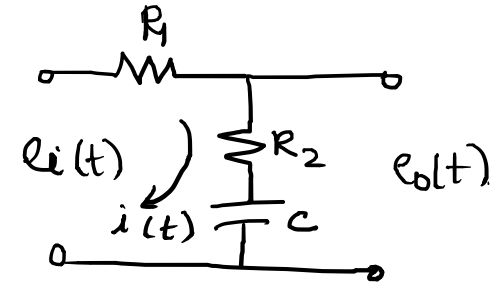
→ It adds a dominant zero and a pole. Thus ↑ the damping which results in lesser overshoot, smaller rise time and smaller settling time, i.e., there is improvement in transient response.

→ It also increases the bandwidth of the system.

Limitation

→ larger gain requirement

Lag compensator



$$e_i(t) = i(t)R_1 + i(t)R_2 + \frac{1}{C}\int i\,dt$$

Taking L.T

$$E_i(s) = I(s)R_1 + I(s)R_2 + \frac{I(s)}{Cs}$$

$$E_i(s) = I(s)\left[R_1 + R_2 + \frac{1}{Cs}\right]$$

The o/p equation is

$$e_o(t) = i(t)R_2 + \frac{1}{C}\int i\,dt$$

Taking L.T, $E_o(s) = I(s)\left[R_2 + \frac{1}{Cs}\right]$

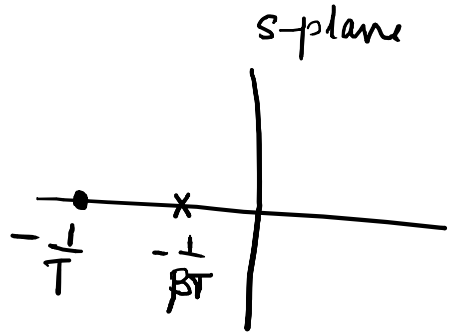
$$\therefore \ T.F = \frac{E_o(s)}{E_i(s)} = \frac{R_2 + \frac{1}{Cs}}{R_1 + R_2 + \frac{1}{Cs}} = \frac{R_2Cs + 1}{(R_1+R_2)Cs + 1}$$

$$T.F = \frac{R_2}{R_1+R_2} \cdot \frac{s + \frac{1}{R_2C}}{s + \frac{1}{(R_1+R_2)C}}$$

let $T = R_2 C$ and $\beta = \frac{R_1 + R_2}{R_2}$, β is > 1

$$T.F = \frac{1}{\beta} \frac{s + \frac{1}{T}}{s + \frac{1}{\beta T}}$$

Pole-zero diagram



Effects of lag Compensator

→ it improves the steady state performance
→ Bandwidth gets reduced
→ System response becomes slower