Frequency discriminator or Simple Slope detector :- (Not in syllabus)

Principle of Slope detection:-

* Let us consider a Tuned Ckt shown in fig. A frequency modulated (FM) signal is applied to this tuned ckt. The centre frequency of the FM signal is 'f_c' & the frequency deviation is Δf. The resonant frequency of the tuned ckt depends on the frequency deviation of the I/p FM signal.

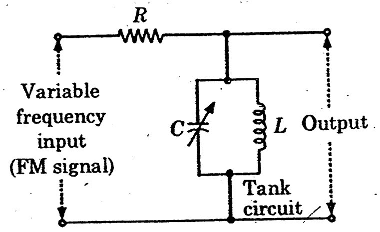


Fig ① *Tuned circuit.*

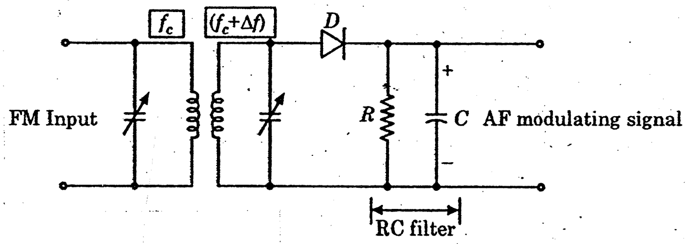


Fig ② *Simple slope detector.*

Fig ② shows the circuit diagram of a simple slope detector.

* When frequency of the I/p signal increases, it becomes more close to the resonant frequency, increasing the O/p voltage.

* When frequency of the I/p signal decreases, it moves away from the resonant frequency, decreasing the O/p voltage.

∴ Frequency variations in the I/p signal about the carrier center frequency produce proportional O/p voltage variations as shown in fig ③.

* The O/p voltage is applied to a diode detector with a RC load of suitable time constant to get the original modulating signal.

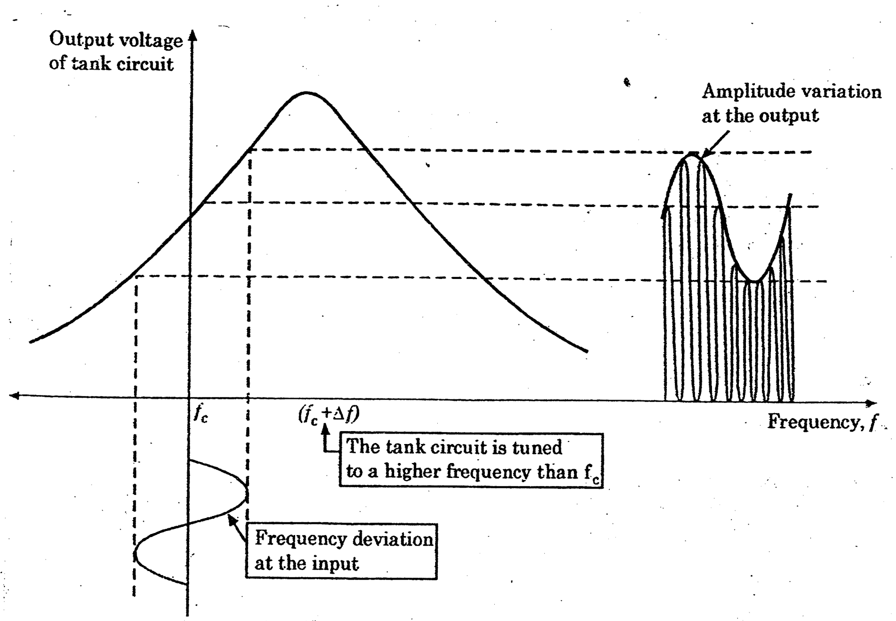


Fig ③ *Characteristics of a slope detector.*

Advantages :-

1) The circuit is Simple.

Disadvantages :-

1) The Ckt Sensitivity is poor i.e. Change in amplitude for a given Change of frequency is Small. (Inefficient)

2) It is linear only over a limited frequency range.

3) It is difficult to adjust as the primary & Secondary windings of the transformer must be tuned Slightly different frequencies

4) The Circuit non-linearity causes harmonic distortion.

Balanced Frequency discriminator or Balanced slope detector or Round – Travis Detector

- ❖ **Explain the detection process of FM signals using balanced frequency discriminator with relevant diagrams.** **July-09,6M(old)**
- ❖ **Draw the block diagram of balance frequency discriminator and explain it for demodulation of FM signal.** **Jan-09,8M**
- ❖ **Explain clearly how a balanced slope detector is used for FM demodulation.** **June-08,7M**
- ❖ **Explain the detection process of FM signals using balanced frequency discriminator with relevant diagrams.** **Jan-06,6M**
- ❖ **With associated diagrams and equations, explain how FM wave can be detected using ratio detector.** **July-05,7M June-09,6M**

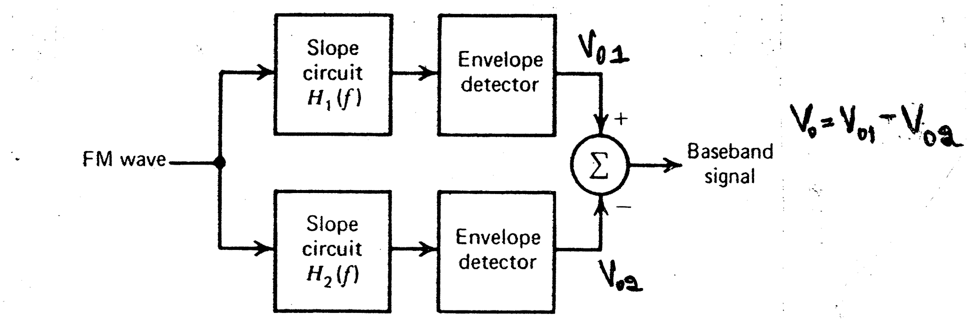


Fig: (a) Block Diagram

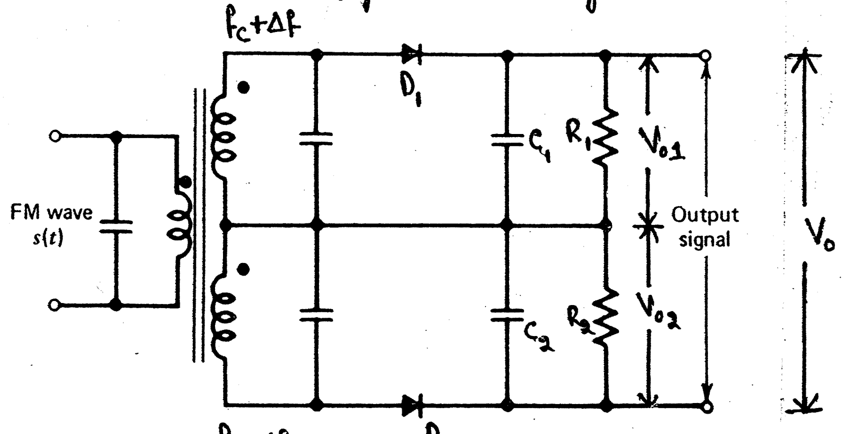


Fig (b): Ckt diagram

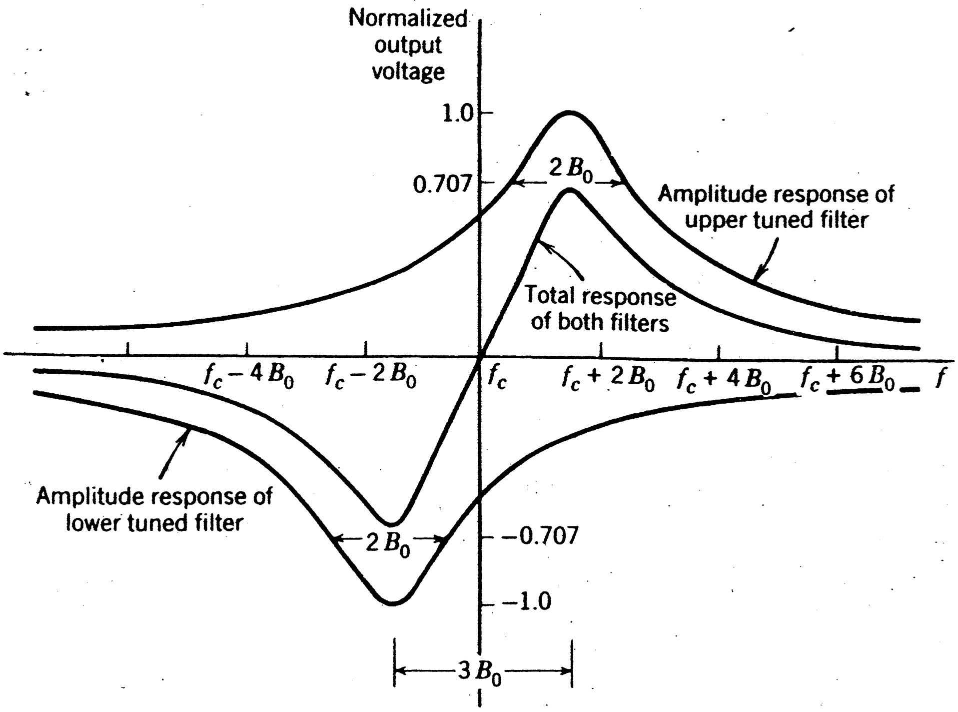


Fig ©: Frequency response

* The balanced slope detector consists of two slope detector ckts.
* The I/p transformer has a center tapped secondary. Hence the I/p voltages to the two slope detectors are 180° out of phase.
* There are 3 tuned ckts
  i) The primary is tuned to IF i.e. f_c.
  ii) The upper tuned ckt of the secondary (T_1) is <u>tuned above</u> f_c by Δf i.e. its resonant frequency is <u>$f_c + \Delta f$</u>.
  iii) The lower tuned ckt of the secondary (T_2) is <u>tuned below</u> f_c by Δf i.e. its resonant frequency is <u>$f_c - \Delta f$</u>
* R_1C_1 & R_2C_2 are the filter ckts.
* V_{01} & V_{02} are the o/p voltages of the two slope detectors.
* The final o/p voltage V_0 is obtained by taking the difference

of the Individual o/p voltages V_{01} & V_{02}.

i.e. $\boxed{V_0 = V_{01} - V_{02}}$

operation of the Ckt:-

We can understand the operation by dividing the I/p frequency into three ranges as follows:

i) $f_{in} = f_c$:-

When I/p frequency is equal to carrier freq 'f_c', the Induced voltage in the T_1 winding of Secondary is exactly equal to that Induced in the winding T_2.

Thus the I/p voltages to both the diodes D_1 & D_2 will be Same.

∴ The dc o/p voltages V_{01} & V_{02} will also be Identical but they have opposite polarities hence $V_0 = 0V$.

ii) $f_{in} > f_c$:-

$\boxed{f_{in} > f_c \; \uparrow (f_c + \Delta f)}$ i.e. $f_{in} \simeq f_c + \Delta f$

When I/p frequency is greater than 'f_c', the Induced voltage in 'T_1' winding is higher than that Induced in 'T_2'.

∴ The I/p to D_1 is higher than D_2. So +ve o/p V_{01} (of D_1) is higher than the -ve o/p V_{02} (of D_2).

Thus o/p voltage V_0 is positive. (The +ve o/p voltage increases as the I/p frequency increases towards $f_c + \Delta f$.)

iii) $f_{in} < f_c$:-

$\boxed{\text{i.e. } f_{in} \simeq f_c - \Delta f}$

When I/p frequency is less than 'f_c', the Induced voltage in 'T_2' winding is higher than in 'T_1', So I/p voltage to diode D_2 is higher than that of D_1.

Hence the -ve o/p 'V_{o2}' is greater than V_{o1}.

∴ The o/p voltage of the balanced Slope detector is -ve in this frequency range. { The -ve o/p voltage increases as f_{in} goes closer to '$f_c - \Delta f$' }

$$\therefore \boxed{V_o = \begin{matrix} 0 & , & f_{in} = f_c \\ +ve & , & f_{in} > f_c \\ -ve & , & f_{in} < f_c \end{matrix}}$$

Advantages :-

1) This ckt is more efficient than Simple Slope detector.

2) It has better linearity than the Simple Slope detector.

Disadvantages :-

1) This ckt is difficult to tune Since the three tuned ckts are to be tuned at different frequencies i.e. f_c, $(f_c + \Delta f)$, $(f_c - \Delta f)$.

2) Amplitude limiting is not provided.

Zero – Crossing Detector:-

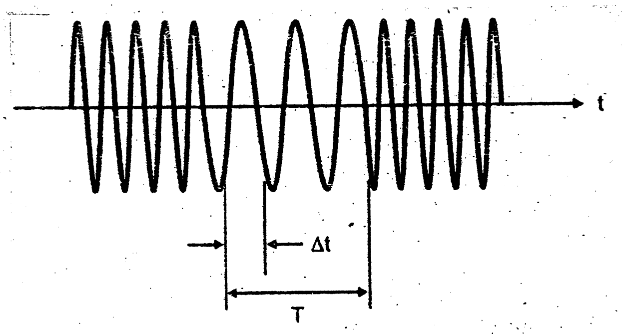
❖ **Explain FM demodulation using Zero crossing detector** **July-06,8M Jan-05,7M**

* The Zero Crossing detector operates on the principle that the instantaneous frequency of an FM wave is approximately given by

$$\boxed{f_i \simeq \frac{1}{2\Delta t}}$$

Where,

Δt is the time difference b/w adjacent Zero Crossing of the FM wave as shown in fig ①.



Definitions of T and Δt for an FM wave

* The time interval 'T' is chosen in accordance with the following two conditions:

1) The interval 'T' is small compared to the reciprocal of the message bandwidth 'W' i.e. $\left(\frac{1}{W}\right)$

2) The interval 'T' is large compared to the reciprocal of the carrier frequency 'f_c' of the FM wave i.e. $(1/f_c)$.

* Let 'n_0' denote the number of Zero Crossings inside the interval 'T'. Hence Δt is the time between the adjacent Zero Crossing points given by

$$\boxed{\Delta t = \frac{T}{n_0}}$$

∴ Instantaneous frequency is given by

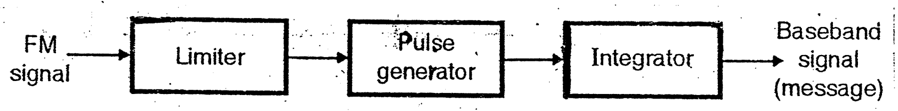
$$f_i \approx \frac{1}{2\Delta t}$$

$$f_i \approx \frac{1}{2\frac{T}{n_0}}$$

$$\boxed{f_i \approx \frac{n_0}{2T}}$$

* By the definition of Instantaneous frequency, W.K.T there is a linear relation b/w f_i & message signal $m(t)$. Hence we can recover $m(t)$ if n_0 is known.

* The simplified block diagram of the zero crossing detector based on this principle is shown below.



Block diagram of zero crossing detector

Phase Locked Loop:-

- **Starting from block diagram of PLL obtain its non-linear and linear model. Show that o/p of PLL is scaled version modulating signal** **June-10,12M**
- **With relevant analysis, explain the FM demodulation, using PLL** **Jan-10,10M**
- **Explain how first order PLL can be used for FM detection** **June-10,8M**
- **Explain with relevant mathematical expression the demodulation of a FM signal using PLL.** **June-09,10M**

Jan-09,6M July-08,5M Jan-08,10M June-07,5M Jan-07,7M June-05,5M

* PLL is a -ve Feedback System that Consists of three major - Components

i) A multiplier

ii) A loop filter

iii) A voltage controlled oscillator (VCO)

Connected in the form of a feedback loop as shown in fig ①.

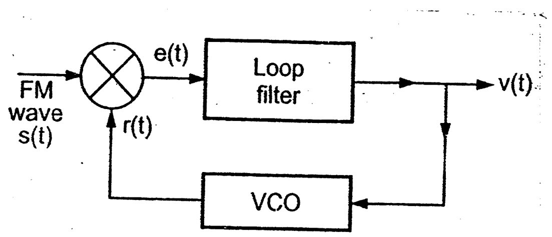


Fig ①: Block diagram of PLL

* The VCO is a Sine-wave generator whose frequency depends on the I/p Control voltage.

{ Any frequency modulator may serve as a VCO. }

* Initially assume that VCO is adjusted so that when the control voltage is zero, 2 conditions are satisfied:

1) The frequency of the VCO is precisely set at the unmodulated carrier frequency 'f_c' &

2) The VCO o/p has a 90° phase-shift w.r.to the unmodulated - carrier wave.

* Suppose that the I/p signal applied to the PLL is an FM wave defined by $s(t) = A_c \sin\left[2\pi f_c t + 2\pi k_f \int_0^t m(t) \cdot dt\right]$

$$\boxed{s(t) = A_c \sin\left[2\pi f_c t + \phi_1(t)\right]} \longrightarrow ①$$

Where A_c is the carrier amplitude with a modulating wave $m(t)$

We have $\phi_1(t) = 2\pi k_f \int_0^t m(t)\, dt$.

Where k_f is the frequency sensitivity of the frequency modulator.

Let the VCO o/p be defined as

$$\boxed{r(t) = A_v \cos[2\pi f_c t + \phi_2(t)]} \longrightarrow (2)$$

Where 'A_v' is the amplitude of VCO o/p. If the control voltage applied to VCO is $v(t)$ then

$$\boxed{\phi_2(t) = 2\pi K_v \int_0^t v(t) \cdot dt}$$

Where 'K_v' is the frequency sensitivity constant of the VCO having the unit Hz/V.

* When VCO I/p $v(t)$ equal to zero i.e. $v(t)=0$, then $\phi_2(t)=0$.

* The incoming FM wave $s(t)$ & the VCO o/p $r(t)$ are applied to the multiplier.

The o/p of the multiplier is

$$\boxed{e(t) = s(t)\, r(t)} \longrightarrow (3)$$

Substituting eq (1) & (2) in eq (3), we get

$$e(t) = \underbrace{A_c \sin[2\pi f_c t + \phi_1(t)]}_{s(t)} \cdot \underbrace{A_v \cos[2\pi f_c t + \phi_2(t)]}_{r(t)}$$

$$e(t) = A_c A_v \sin[2\pi f_c t + \phi_1(t)] \cdot \cos[2\pi f_c t + \phi_2(t)]$$

W.K.T $\boxed{\sin A \cdot \cos B = \frac{1}{2}\sin[A+B] + \frac{1}{2}\sin[A-B]}$

Put $A = [2\pi f_c t + \phi_1(t)]$ & $B = [2\pi f_c t + \phi_2(t)]$

$$e(t) = \frac{A_c A_v}{2} \sin[\underline{2\pi f_c t} + \phi_1(t) + \underline{2\pi f_c t} + \phi_2(t)] + \frac{A_c A_v}{2} \sin[\cancel{2\pi f_c t} + \phi_1(t) - \cancel{2\pi f_c t} - \phi_2(t)]$$

$$\boxed{e(t) = \frac{A_c A_v}{2} \sin[4\pi f_c t + \phi_1(t) + \phi_2(t)] + \frac{A_c A_v}{2} \sin[\phi_1(t) - \phi_2(t)]} \longrightarrow (4)$$

$$\boxed{e(t) = K_m A_c A_v \sin\left[4\pi f_c t + \phi_1(t) + \phi_2(t)\right] + K_m A_c A_v \sin\left[\phi_1(t) - \phi_2(t)\right]} \longrightarrow (5)$$

Where $K_m = \frac{1}{2}$ is the multiplier gain measured in volts.

* Equation (5) is the o/p of the product modulator & it has two components

1) A high-frequency component represented by

$$K_m A_c A_v \sin\left[4\pi f_c t + \phi_1(t) + \phi_2(t)\right]$$

2) A low frequency component represented by

$$K_m A_c A_v \sin\left[\phi_1(t) - \phi_2(t)\right]$$

* The high frequency component is eliminated by the LPF.

Thus the I/p to the loop filter is given by:

$$e(t) = K_m A_c A_v \sin\left[\phi_1(t) - \phi_2(t)\right]$$

$$\boxed{e(t) = K_m A_c A_v \sin\left[\phi_e(t)\right]} \longrightarrow (6)$$

Where, $\phi_e(t)$ is the phase ~~error~~ defined by

$$\boxed{\phi_e(t) = \phi_1(t) - \phi_2(t)} \longrightarrow (7)$$

$$\phi_e(t) = \phi_1(t) - 2\pi K_v \int_0^t v(t).\,dt$$

* The loop filter operates on its I/p $e(t)$ to produce the o/p

$$v(t) = e(t) * h(t).$$

$$\boxed{v(t) = \int_{-\infty}^{\infty} e(\tau).\,h(t-\tau)\,d\tau}$$

* Differentiating eq (7) w.r.to t, we get

$$\frac{d\phi_e(t)}{dt} = \frac{d\phi_1(t)}{dt} - \frac{d\phi_2(t)}{dt}$$

$$= \frac{d\phi_1(t)}{dt} - \left[\cancel{\frac{d}{dt}}\left(2\pi K_v \cancel{\int_0^t} v(t)\cdot \cancel{dt}\right)\right]$$

$$= \frac{d\phi_1(t)}{dt} - \left[2\pi K_v v(t)\right] \qquad \because v(t) = e(t) * h(t)$$

$$\frac{d\phi_e(t)}{dt} = \frac{d\phi_1(t)}{dt} - \left[2\pi K_v \left(e(t) * h(t)\right)\right] \longrightarrow (8)$$

Substituting eq (6) in eq (8), we get

$$= \frac{d\phi_1(t)}{dt} - \left[2\pi K_v \left(K_m A_c A_v \sin\phi_e(t) * h(t)\right)\right]$$

$$= \frac{d\phi_1(t)}{dt} - 2\pi K_v K_m A_c A_v \left[\sin\phi_e(t) * h(t)\right]$$

$$\boxed{\frac{d\phi_e(t)}{dt} = \frac{d\phi_1(t)}{dt} - 2\pi K_0 \int_{-\infty}^{\infty} \sin\left[\phi_e(\tau)\cdot h(t-\tau)\, d\tau\right]}$$

Where 'K_0' is a loop parameter defined by

$$K_0 = K_m K_v A_c A_v$$

Equation necessary for developing the block diagram of PLL:-

W.K.T $\quad \phi_e(t) = \phi_1(t) - \phi_2(t) \rightarrow (1)$

Where,

$$\phi_1(t) = 2\pi K_f \int_0^t m(t)\cdot dt$$

$$\phi_2(t) = 2\pi K_v \int_0^t V(t)\cdot dt$$

Differentiating $\phi_2(t)$ w.r.to 't' we get

$$\frac{d\phi_2(t)}{dt} = 2\pi K_v \cancel{\frac{d}{dt}} \cancel{\int_0^t} V(t)\cdot \cancel{dt}$$

W.K.T

$$\boxed{V(t) = e(t) * h(t)}$$

W.K.T

$$\boxed{e(t) = K_m A_c A_v \sin\phi_e(t)}$$

$$\frac{d\phi_2(t)}{dt} = 2\pi K_v \cdot V(t) \longrightarrow (2)$$

$$\frac{d\phi_2(t)}{dt} = 2\pi K_v \left[e(t) * h(t) \right] \longrightarrow (3)$$

Substituting $e(t)$ in eq (3), we get

$$\frac{d\phi_2(t)}{dt} = 2\pi K_v \left[K_m A_c A_v \sin\phi_e(t) * h(t) \right]$$

$$\boxed{\frac{d\phi_2(t)}{dt} = 2\pi K_0 \sin\phi_e(t) * h(t)} \longrightarrow (4)$$

Where, $K_0 = K_v K_m A_c A_v$

From equation (2), we can write

$$V(t) = \frac{1}{2\pi K_v} \frac{d\phi_2(t)}{dt} \longrightarrow (5)$$

Substituting eq (5) in eq (4) we get

$$\boxed{V(t) = \frac{1}{2\pi K_v} 2\pi K_0 \sin\phi_e(t) * h(t)}$$

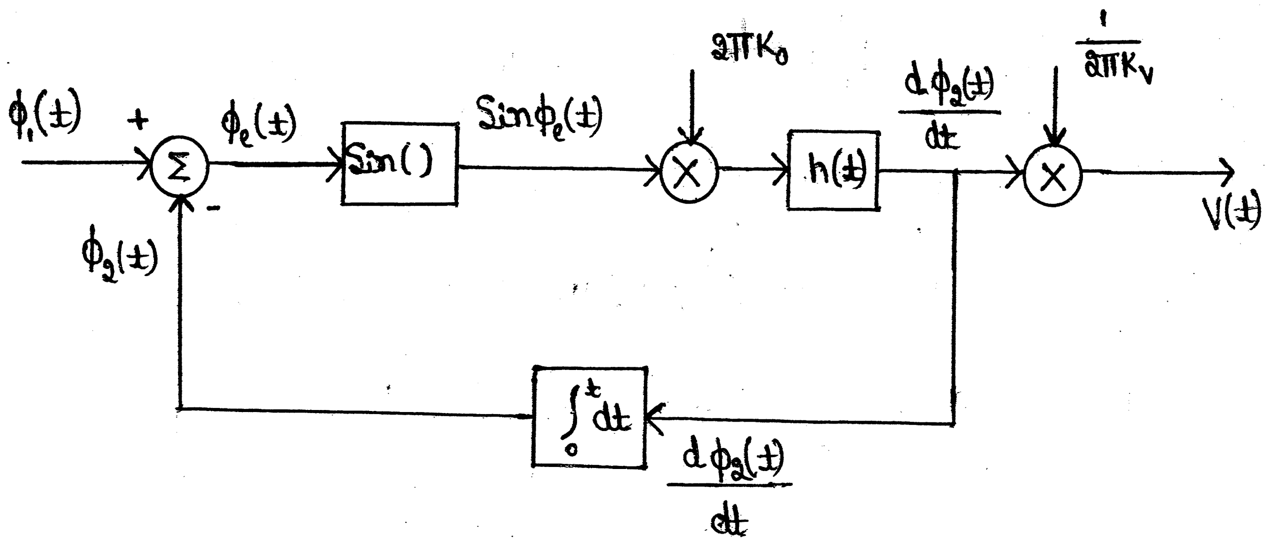


Fig (2): Non-linear model of PLL

* We can observe that the multiplier of Fig ① is replaced by a Sinusoidal non linearity & the VCO by an Integrator because of the Sinusoidal Non linearity, the above representation is known as the non-linearity representation of PLL.

Linearized PLL:- June-07,5M

Linearized Model :-

* When the phase error $\phi_e(t)$ is Zero, the PLL is said to be in phase locked.

* When $\phi_e(t)$ is very small compared to 0.5 radian, at all times, we may use the following approximation.

$$\boxed{\sin\phi_e(t) \approx \phi_e(t)}$$

Thus Fig ② reduces to Fig ③.

Fig ③ is known as the Linearized model of PLL.

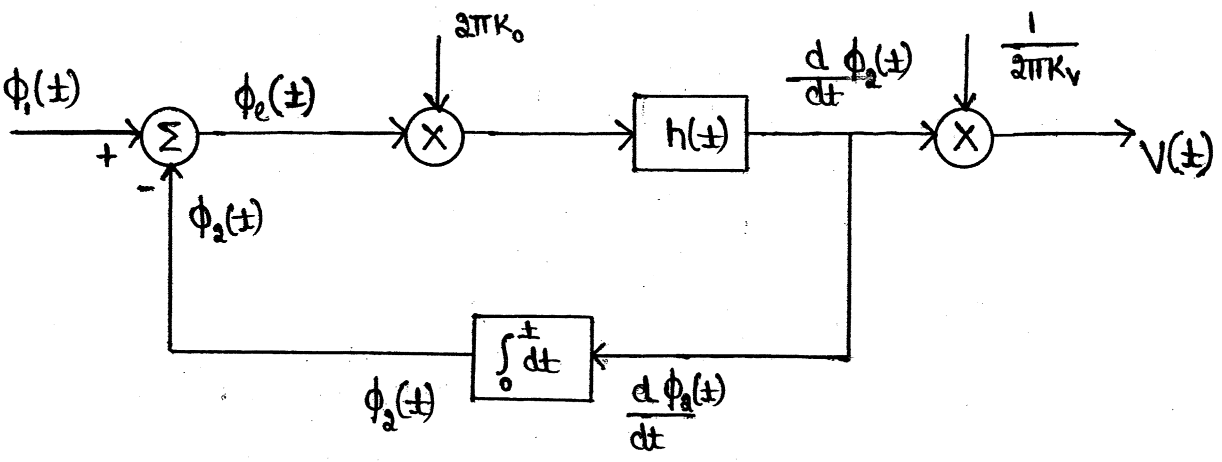


Fig ③: Linear model of PLL.

From Fig ③, we have

$$\phi_e(t) = \phi_1(t) - \phi_2(t) \longrightarrow ①$$

$$\phi_1(t) = \phi_e(t) + \phi_2(t) \longrightarrow (2)$$

differentiating both Sides of eq (2), we get

$$\frac{d}{dt}\phi_1(t) = \frac{d}{dt}\phi_e(t) + \underline{\frac{d}{dt}\phi_2(t)}$$

$$\frac{d}{dt}\phi_1(t) = \frac{d}{dt}\phi_e(t) + 2\pi K_0 \underline{\sin \phi_e(t)} * h(t)$$

$\because \frac{d}{dt}\phi_2(t) = 2\pi K_0 \sin \phi_e(t) * h(t)$ & W.K.T, $\sin \phi_e(t) \approx \phi_e(t)$

$$\boxed{\frac{d}{dt}\phi_1(t) = \frac{d}{dt}\phi_e(t) + 2\pi K_0 \phi_e(t) * h(t)} \longrightarrow (3)$$

Taking Fourier transform on both Sides of the eq (3), we get

$$j2\pi f \phi_1(f) = j2\pi f \phi_e(f) + 2\pi K_0 \phi_e(f) \cdot H(f)$$

{ NOTE :-

$$\frac{d}{dt}\phi_1(t) \xrightarrow{FT} j2\pi f \phi_1(f)$$

$$\frac{d}{dt}\phi_e(t) \xrightarrow{FT} j2\pi f \phi_2(f)$$

$$\phi_e(t) * h(t) \xrightarrow{FT} \phi_e(f) \cdot H(f)$$

}

$$\cancel{j2\pi f}\, \phi_1(f) = \cancel{j2\pi f}\left[\phi_e(f) + \frac{1}{jf} K_0 \phi_e(f) \cdot H(f)\right]$$

$$\phi_1(f) = \phi_e(f) + \frac{K_0 H(f)}{jf} \cdot \phi_e(f)$$

Let $L(f) = \dfrac{K_0 H(f)}{jf}$

Then,

$$\phi_1(f) = \phi_e(f) + L(f) \cdot \phi_e(f)$$

$$\phi_1(f) = \phi_e(f)\left[1 + L(f)\right]$$

$$\boxed{\phi_e(f) = \frac{\phi_1(f)}{1 + L(f)}} \longrightarrow (4)$$

Where $L(f)$ is called the open loop transfer function of the PLL.

W.K.T

$$V(t) = \frac{1}{2\pi K_v} \cdot 2\pi K_o \underline{\sin\phi_e(t)} * h(t)$$

$\because$ $\boxed{\sin\phi_e(t) \approx \phi_e(t)}$

$$\boxed{V(t) = \frac{1}{2\pi K_v} 2\pi K_o \, \phi_e(t) * h(t)} \longrightarrow 5$$

Eq 5 is the o/p of the PLL.

* In frequency domain the o/p of the PLL is given

$$V(f) = \frac{1}{\cancel{2\pi} K_v} \cancel{2\pi} K_o \, \phi_e(f) \cdot H(f)$$

$$V(f) = \frac{K_o}{K_v} \cdot \underline{\phi_e(f)} \cdot H(f) \longrightarrow 6$$

Substituting eq 4 in eq 6, we get

$$V(f) = \frac{K_o}{K_v} \cdot \frac{\phi_1(f)}{[1+L(f)]} \cdot H(f)$$

$\because$ $\boxed{\phi_e(f) = \frac{\phi_1(f)}{1+L(f)}}$

Since $L(f) \gg 1$, we can write $1+L(f) \approx L(f)$

hence, $V(f) = \frac{K_o}{K_v} \frac{\phi_1(f)}{L(f)} H(f)$

$\because$ $\boxed{L(f) = \frac{K_o H(f)}{jf}}$

$$V(f) = \frac{\cancel{K_o}}{K_v} \frac{\phi_1(f)}{\frac{\cancel{K_o H(f)}}{jf}} \cdot \cancel{H(f)}$$

$$\boxed{V(f) = \frac{1}{K_v} \cdot jf \, \phi_1(f)} \longrightarrow 7$$

x-lying & ÷ing RHS of eq 7 by 2π

$$V(f) = \frac{1}{2\pi K_v} \cdot j2\pi f \phi_1(f)$$

W.K.T $\frac{d}{dt} \phi_1(t) \xrightarrow{FT} j2\pi f \phi_1(f)$

hence,
$$V(f) = \frac{1}{2\pi K_v} \frac{d}{dt}\phi_1(t) \longrightarrow (8)$$

Substituting $\phi_1(t) = 2\pi K_f \int_0^t m(t).\,dt$ in eq (8), we get

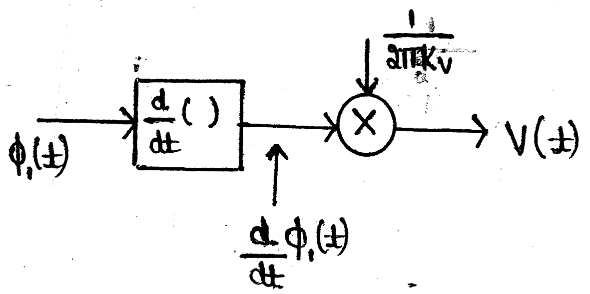
$$V(f) = \frac{1}{\cancel{2\pi} K_v} \cancel{\frac{d}{dt}}\left[\cancel{2\pi} K_f \cancel{\int_0^t} m(t).\,\cancel{dt}\right]$$

$$\boxed{V(f) = \frac{K_f}{K_v} m(t)}$$

The corresponding time-domain relation of eq (8) is

$$\boxed{V(t) = \frac{1}{2\pi K_v} \frac{d}{dt}\phi_1(t)} \longrightarrow (9)$$

From eq (9), we can write



Linearized Model :-

From Godse - Book

* When the phase error $\phi_e(t)$ is zero, the PLL is said to be in phase - locked.

* When $\phi_e(t)$ is very small compared to 0.5 radians, at all times, we may use the following approximation.

$$\boxed{\sin\phi_e(t) \approx \phi_e(t)}$$

Thus fig (2) reduces to fig (3).

Fig ③ is Known as the Linearized model of PLL.

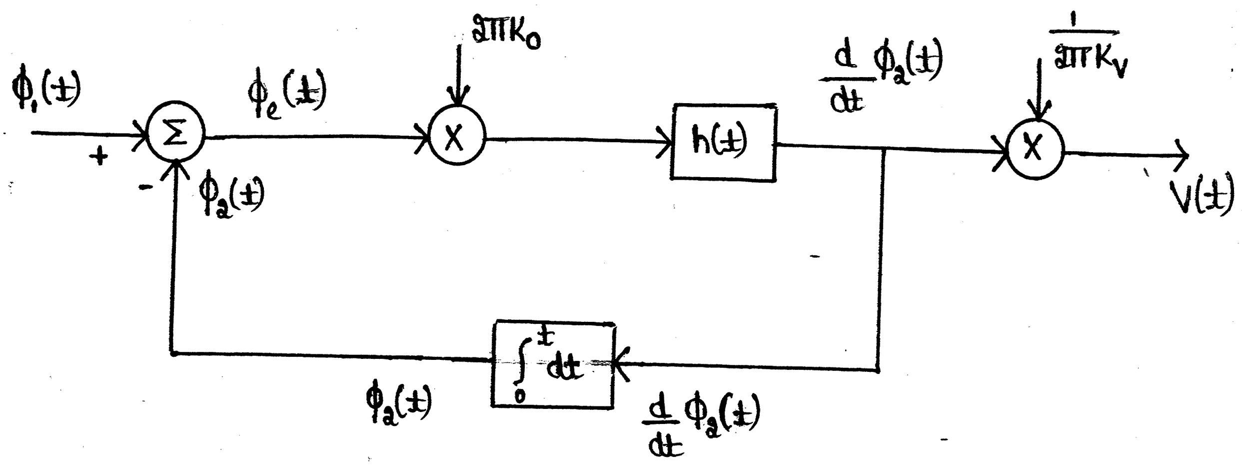


Fig ③: Linear model of PLL

W.K.T

$$\phi_1(t) = 2\pi K_f \int_0^t m(t) \cdot dt \quad \&$$

$$\phi_2(t) = 2\pi K_v \int_0^t V(t) \cdot dt.$$

From fig ③, We have

$$\phi_e(t) = \phi_1(t) - \phi_2(t). \longrightarrow ①$$

Assuming Small error So $\phi_e(t) \approx 0$

$$0 = \phi_1(t) - \phi_2(t)$$

$$\phi_2(t) = \phi_1(t) \longrightarrow ②$$

$$\cancel{2\pi} K_v \int_0^t V(t) \cdot dt = \cancel{2\pi} K_f \int_0^t m(t)\, dt \longrightarrow ③$$

Differentiating both Sides of eq ③ w.r.to time 't'

$$K_v V(t) = K_f m(t)$$

$$\boxed{V(t) = \frac{K_f}{K_v} m(t)}$$

Non- Linear effects in FM:-

❖ **Explain the non linearity and its effect in FM systems** | **Jan-09,6M** | **July-06,6M**

* Non-linearities are present in all electrical networks. Consider a Communication Channel having a non-linear transfer Characteristic is given by

$$\boxed{V_0(t) = a_1 V_i(t) + a_2 V_i^2(t) + a_3 V_i^3(t)} \longrightarrow (1)$$

Where,

$V_i(t) \rightarrow$ I/p Signal

$V_0(t) \rightarrow$ O/p Signal

a_1, a_2 & $a_3 \rightarrow$ Constants

* Let the I/p to the Channel be an FM wave given by

$$V_i(t) = A_c \cos\left[2\pi f_c t + \phi(t)\right] \longrightarrow (2)$$

Where, $\phi(t) = 2\pi K_f \int_0^t m(t).dt$

Substituting eq (2) in eq (1), we get

$$V_0(t) = a_1 A_c \cos[2\pi f_c t + \phi(t)] + a_2 A_c^2 \cos^2[2\pi f_c t + \phi(t)] + a_3 A_c^3 \cos^3[2\pi f_c t + \phi(t)]$$

W.K.T

$$\boxed{\begin{aligned} \cos^2\theta &= \frac{1}{2} + \frac{\cos 2\theta}{2} \\ \cos^3\theta &= \frac{3\cos\theta}{4} + \frac{\cos(3\theta)}{4} \end{aligned}} \qquad \boxed{\theta = [2\pi f_c t + \phi(t)]}$$

$$\Big\{ \quad a_2 A_c^2 \cos^2[2\pi f_c t + \phi(t)] = \frac{a_2 A_c^2}{2} + \frac{a_2 A_c^2}{2} \cos\left[4\pi f_c t + 2\phi(t)\right]$$

$$a_3 A_c^3 \cos^3[2\pi f_c t + \phi(t)] = \frac{3a_3 A_c^3}{4} \cos[2\pi f_c t + \phi(t)] + \frac{a_3 A_c^3}{4} \cos[6\pi f_c t + 3\phi(t)]$$

}

$$V_0(t) = a_1 A_c \cos[2\pi f_c t + \phi(t)] + \frac{a_2 A_c^2}{2} + \frac{a_2 A_c^2}{2} \cos[4\pi f_c t + 2\phi(t)]$$
$$+ \frac{a_3 A_c^3}{4} 3\cos[2\pi f_c t + \phi(t)] + \frac{a_3 A_c^3}{4} \cos[6\pi f_c t + 3\phi(t)] \longrightarrow (3)$$

* The equation (3) indicates that the channel o/p consists of a DC component & three frequency modulated signals with carrier frequencies f_c, $2f_c$ & $3f_c$.

* The required FM wave centered at f_c is obtained by passing '$V_0(t)$' through a BPF.

* The o/p of the BPF is

$$V_0(t) = a_1 A_c \cos[2\pi f_c t + \phi(t)] + \frac{a_3 A_c^3}{4} 3\cos[2\pi f_c t + \phi(t)]$$

$$\boxed{V_0(t) = \cos[2\pi f_c t + \phi(t)]\left(a_1 A_c + \frac{3}{4} a_3 A_c^3\right)} \longrightarrow (4)$$

* Equation (4) reveals that $V_0(t)$ is the original FM signal except for the change in amplitude. Thus, amplitude non-linearities of the channel does not affect an FM signal (unlike in amplitude modulation).

* For this reason, FM is widely used in Microwave & Satellite Communication.

Compare FM with AM and PM **June-06,6M**

Sl No	FM	AM
1)	The equation for FM wave is: $S(t)_{FM} = A_c \sin[\omega_c t + \beta \sin \omega_m t]$	The equation for AM wave is $S(t)_{AM} = A_c[1+\mu \sin\omega_m t]\sin\omega_c t$.
2)	The modulation Index can have any value i.e. less than 1 or more than 1.	The modulation Index is always in between 0 and 1.
3)	All the transmitted power is useful	Carrier power and one Sideband power are useless.
4)	$P = \frac{A_c^2}{2R}$	$P_T = P_c[1+\mu^2/2]$
5)	The modulation Index determines the number of Sidebands in an FM Signal	In AM, only two Sidebands are produced, irrespective of the modulation Index.
6)	$BW = 2[\Delta f + f_m]$. The BW depends on modulation Index	$BW = 2f_m$. The BW does not depends on modulation Index
7)	For FM, $\% \text{Modulation} = \frac{\text{Actual freq deviation}}{\text{Max allowed freq deviation}} \times 100$	For AM, $\% \text{modulation} = \frac{A_m}{A_c} \times 100$
8)	The main advantage of FM over AM is its noise Immunity.	The AM System is more Susceptible to Noise & more affected by Noise than FM.

Sl No	FM	AM
9)	The BW required to transmit FM Signal is much larger than the BW of AM (i.e ≈200KHz)	The BW required to transmit AM Signal is much less than that of FM (i.e. ≈10KHz)
10)	FM transmission & reception equipment are more Complex	AM equipments are less Complex.
11)	FM transmission is expensive than AM transmission	AM transmission is Cheaper than FM transmission.
12)	Used for Short distance Comm	Used for long distance Comm

Sl No	FM	PM
1)	The equation for FM Wave is $S(t)_{FM} = A_c \cos[W_c t + 2\pi K_f m(t)]$	The equation for PM Wave is $S(t)_{PM} = A_c \cos[W_c t + K_p m(t)]$
2)	Amplitude of FM Wave is Constant	Amplitude of PM Wave is Constant
3)	Frequency deviation is proportional to modulating voltage.	phase deviation is proportional to the modulating voltage.
4)	The modulation Index of an FM Signal is the ratio of the frequency deviation to the modulating frequency.	The modulation Index is proportional to the maximum amplitude of the modulating Signal.
5)	Noise Immunity is better than AM & PM	Noise Immunity is better than AM but worse than FM.

6)	Signal to Noise ratio is better than that of PM	Signal to Noise ratio is Inferior to that of FM.
7)	FM is widely used	PM is used in Some mobile - Systems
8)	It is possible to receive FM on a PM receiver.	It is possible to receive PM on a FM receiver.
9)	FM is direct method of producing FM Signal	PM is Indirect method of producing FM.
10)	Noise is better Suppressed in FM Systems as Compared to PM System	Noise Immunity is Inferior to that of FM.
11)	To have better quality of - transmission & reception of higher audio frequencies, pre-emphasis & de-emphasis Circuits are used.	The amount of frequency Shift produced by a phase modulator increases with the modulating frequency. Hence an audio equalizer is required to - Compensate this.
12)	FM is mainly used for FM broadcasting. i.e. Entertainment purpose	PM is used in mobile Comm System

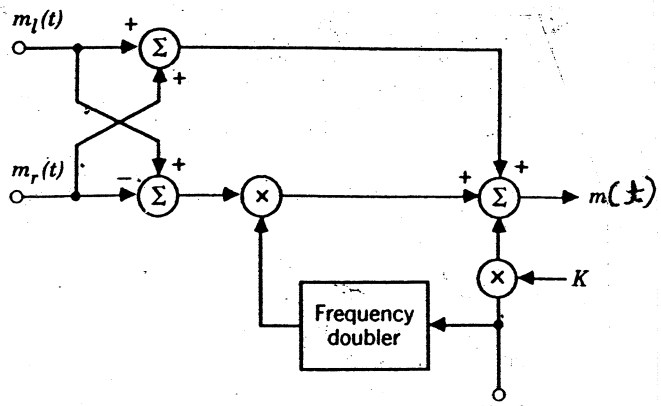
FM Stereo multiplexing:-

- Explain with relevant block diagram FM stereo multiplexing **June-10,8M**
- FM stereo multiplexing **Jan-10,5M**
- With a neat block diagram, explain the operation of FM stereo multiplexing and demultiplexing **June-10,10M(OLD)**
- Stereo FM **June-06,3M**

* Stereo multiplexing is a form of frequency-division multiplexing (FDM) designed to transmit two separate signals via the same carrier.

* It is widely used in FM broadcasting to send two different elements of a program (eg: two different sections of an orchestra, a vocalist & an accompanist) so as to give a spatial dimension to its perception by a listener at the receiving end.

FM Stereo Transmitter:-



(a)

* Let $m_l(t)$ & $m_r(t)$ denotes the signals picked up by left-hand & right-hand microphones at the transmitting end of the system.

* These signals are then applied to a simple matrixer that generates the sum signal i.e. $m_l(t)+m_r(t)$, & the difference

Signal i.e. $m_l(t) - m_r(t)$.

* The Sum Signal is left unprocessed in its baseband form: it is available for monophonic reception.

* The difference Signal & a 38KHz Subcarrier are applied to a product modulator, producing a DSB-SC modulated wave.

* The Sum Signal, DSB-SC modulated wave & a 19KHz pilot Signal are combined to form a multiplexed Signal $s(t)$.

{ Here pilot Signal is included to provide a reference for the Coherent detection of the difference Signal at the Stereo receiver.}

Thus multiplexed Signal $m(t)$ is

$$\boxed{m(t) = [m_l(t) + m_r(t)] + [m_l(t) - m_r(t)] \cos 4\pi f_c t + K \cos 2\pi f_c t}$$

Where $f_c = 19KHz$.

FM Stereo Receiver:-

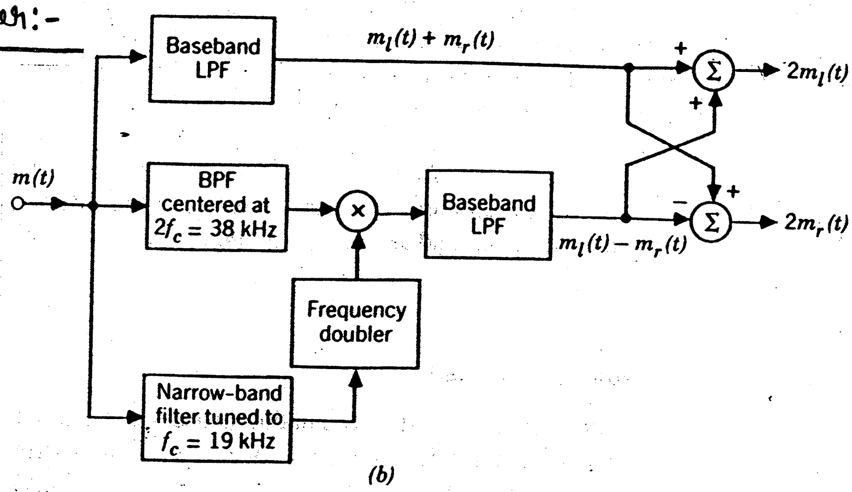


Figure 7.58
(a) Multiplexer in transmitter of FM stereo. (b) Demultiplexer in receiver of FM stereo.

* At a Stereo receiver, the multiplexed Signal $m(t)$ is recovered from the incoming FM wave.

* The $m(t)$ is then applied to the demultiplexing as shown in fig ⓑ.
* The individual components of the multiplexed signal $m(t)$ are separated by the use of three appropriate filters.
* The recovered pilot signal is frequency - doubled to produce the desired 38KHz subcarrier which enables the coherent detection of the DSB-SC modulated wave, thereby recovering the difference signal $m_l(t) - m_r(t)$.
* The baseband LPF in the top path of fig ② is designed to pass the sum signal $m_l(t) + m_r(t)$.
* Finally, the simple matrixer reconstruct the left-hand signal $m_l(t)$ & right hand signal $m_r(t)$ & applies them to their respective speaker.

FM FORMULAE

1.	**Carrier Frequency**	$W_c = 2\pi f_c$, $f_c = W_c/2\pi$
2.	**Modulating Frequency**	$W_m = 2\pi f_m$, $f_m = W_m/2\pi$
3.	**Modulation Index (β or m_f)**	$\beta = \Delta f / f_m$
4.	**Power dissipation**	$P = A^2{}_c / 2R$
5.	**Frequency deviation**	$\Delta f = K_f A_m$ $\Delta f = \beta f_m$
6.	**Frequency sensitivity**	$K_f = \Delta f / A_m$
7.	**Deviation ratio**	$D = \Delta f_{max} / f_{max}$
8.	**Highest frequency reached**	$(f_i)_{max} = f_c + \Delta f$
9.	**Lowest frequency reached**	$(f_i)_{min} = f_c - \Delta f$
10.	**Carrier Swing**	$(f_i)_{max} - (f_i)_{min}$
11.	**Carrier Swing**	$2x\Delta f$
12.	**Frequency deviation**	Δf = Carrier Swing / 2 $\Delta f = (f_i)_{max} - f_C$
13.	**Bandwidth (Carson rule)**	$BW = 2(\Delta f + f_m)$ or $BW = 2\Delta f\,(1 + 1/\beta)$
14.	**Message signal**	$S(t) = A_c \cos[W_{ct} + \beta \sin W_m t]$ $S(t) = A_c \sin[W_{ct} + \beta \sin W_m t]$ $S(t) = A_c \cos[2\pi f_{ct} + 2\pi k_f \int m(t)\,dt]$ $S(t) = A_c \cos[W_{ct} + 2\pi k_f \int m(t)\,dt]$ $S(t) = A_c \cos[W_{ct} + 2\pi k_f\, m(t)]$

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PM FORMULAE

1. **Phase deviation** — $\Delta P = K_p A_m f_m$
2. **Bandwidth** — $BW = 2(\Delta f + f_m)$ or
   (Carson rule) — $BW = 2\Delta f\,(1+1/\beta)$
3. **Message signal** — $S(t) = A_c \cos[W_c t + K_p m(t)]$

   $S(t) = A_c \cos[2\pi f_c t + K_p m(t)]$

1. The equation for a FM wave is $S(t) = 10\cos[5.7\times10^{8}t + 5\sin(12\times10^{3})t]$.
Calculate. i. Carrier frequency ii. Modulating frequency
iii. Modulation index iv. Frequency deviation
v. Power dissipated in a 100Ω resistor load.

June-10,6M July-09,5M (old) June-08,10M June-06,6M

Sol:-

$$S(t) = 10\cos[5.7\times10^{8}t + 5\sin(12\times10^{3})t] \rightarrow ①$$

Compare eq ① with Standard equation for FM

$$S(t) = A_c\cos[\omega_c t + \beta\sin\omega_m t] \longrightarrow ②$$

$A_c = 10V$, $\omega_c = 5.7\times10^{8}$, $\beta = 5$ & $\omega_m = 12\times10^{3}$

i) Carrier frequency $f_c = \dfrac{\omega_c}{2\pi} = \dfrac{5.7\times10^{8}}{2\pi}$

$$\boxed{f_c = 90.7183\ MHz}$$

ii) Modulating frequency $f_m = \dfrac{\omega_m}{2\pi} = \dfrac{12\times10^{3}}{2\pi}$

$$\boxed{f_m = 1.909\ KHz}$$

iii) Modulation Index

$$\boxed{\beta = 5}$$

iv) Frequency deviation $\Delta f = \beta f_m = 5\times1.909\ KHz$

$$\boxed{\Delta f = 9.545\ KHz}$$

v) power dissipated in a 100Ω resistor load

$$P = \frac{A_c^2}{2R} = \frac{10^2}{2\times100}$$

$$\boxed{P = 0.5\ W}$$

2. A FM signal has sinusoidal modulation with f_m=15KHz and modulation index β=2. Using carson's rule, find the transmission bandwidth and deviation ratio. Assume Δf =75 KHz. **June-10,6M**

Given :-

$f_m = 15KHz$, $\beta = 2$, $\Delta f = 75 KHz$

BW = ? & Deviation ratio 'D' = ?

* $BW = 2(\Delta f + f_m) = 2(75KHz + 15KHz) = \underline{180KHz.}$

* $D = \dfrac{\Delta f}{f_m} = \dfrac{75KHz}{15KHz} = \underline{5}$

---

3. A sinusoidal modulating voltage of amplitude 5V and frequency 1 KHz is applied to frequency modulator. The frequency sensitivity of modulator is 40 Hz/V. The carrier frequency is 100KHz. Calculate i. Frequency deviator ii. Modulation index **June-10,5M**

Given :- $A_m = 5V$, $f_m = 1KHz$, $K_f = 40 Hz/V$ & $f_c = 100KHz.$

i) Frequency deviation $\Delta f = K_f A_m = 40 \times 5 = \underline{200Hz}$

ii) Modulation Index 'β' $= \dfrac{\Delta f}{f_m} = \dfrac{200}{1000} = \underline{0.2}$

---

4. A sinusoidal modulating waveform of amplitude 10V and a frequency of 1 KHz is applied to FM generator that has a frequency sensitivity constant of is 40 Hz/V. Determine the i. Frequency deviation and ii. Modulation index **Jan-10,4M**

Given : $A_m = 10V$, $f_m = 1KHz$, $K_f = 40Hz/V.$

i) Frequency deviation $\Delta f = K_f A_m = 40 \times 10 = 400\,Hz$

ii) Modulation Index $\beta = \frac{\Delta f}{f_m} = \frac{400\,Hz}{1\,KHz} = 0.4$

5. A carrier wave of 100 MHz is frequency modulated by a 100 KHz sinewave of amplitude 20V, the sensitivity of the modulator is 25 KHz/V.

i. Determine the frequency deviation and bandwidth of the modulated signal using Carson's rule.

ii. Repeat your calculation for PM wave, assume k_p=k_f **June-10,6M(IT)**

Given: $f_c = 100\,MHz$, $f_m = 100\,KHz$, $A_m = 20V$, $K_f = 25\,KHz/V$.

i) $BW = 2[\Delta f + f_m]$

$\Delta f = K_f A_m = 25\,KHz \times 20 = 500\,KHz$

$BW = 2[500\,KHz + 100\,KHz]$

$\boxed{BW = 1200\,KHz}$

OR

$BW = 2f_m(1+\beta)$

$\beta = \frac{500\,KHz}{100\,KHz} = 5$

$BW = 2 \times 100\,KHz(1+5)$

$\boxed{BW = 1200\,KHz}$

ii) Assuming that $K_p = K_f$ for PM wave

$\Delta f = K_p A_m f_m = 25\,KHz \times 20 \times 100\,KHz$

$\boxed{\Delta f = 50000\,KHz}$

6. A single tone FM signal is given by: $S(t) = 10 \sin[16\pi \times 10^6 t + 20 \sin 2\pi \times 10^3)t]$.

Calculate. i. Modulation index **ii. Modulation Frequency**

iii. Frequency deviation **iv. Carrier frequency**

v. Power of the FM signal. **Jan-09,8M**

Sol :- $S(t) = 10 \sin[16\pi \times 10^6 t + 20 \sin 2\pi \times 10^3 t] \longrightarrow (1)$

Comparing eq (1) with Standard equation for FM

$$S(t) = A_c \sin[\omega_c t + \beta \sin \omega_m t] \longrightarrow (2)$$

we get, $A_c = 10V$, $\omega_c = 16\pi \times 10^6$, $\omega_m = 2\pi \times 10^3$, $\beta = 20$

i) Modulation Index $\beta = 20$.

ii) Modulating Frequency $f_m = \frac{\omega_m}{2\pi} = \frac{2\pi \times 10^3}{2\pi} = 1KHz$

iii) Frequency deviation $\Delta f = \beta f_m = 20 \times 1 \times 10^3 = 20KHz$

iv) Carrier Frequency $f_c = \frac{\omega_c}{2\pi} = \frac{16\pi \times 10^6}{2\pi} = 8MHz$

v) Power 'p' $= \frac{A_c^2}{2R} = \frac{10^2}{2R} = \frac{50}{R}$ W

---

7. An angle modulated signal is defined by $S(t) = 10 \sin[2\pi x10^6 t + 0.2 \sin(2000\pi)t]$ volts. Find the following:

i. Power in the modulated signal **ii. Frequency deviation**

iii. Phase deviation **iv. Approximate transmission bandwidth.**

Jan-08,10M

Given :- $S(t) = 10 \cos[2\pi \times 10^6 t + 0.2 \sin(2000\pi t)] \longrightarrow (1)$

Comparing eq (1) with Standard equation for FM

$$S(t) = A_c \cos[\omega_c t + \beta \sin \omega_m t] \longrightarrow (2)$$

We get, $A_c = 10V$, $\beta = 0.2$, $\omega_m = 2000\pi$, $\omega_c = 2\pi \times 10^6$

* $f_m = \frac{\omega_m}{2\pi} = \frac{2000\pi}{2\pi} = 1KHz$

* $f_c = \frac{\omega_c}{2\pi} = \frac{2\pi \times 10^6}{2\pi} = 1MHz$

i) $P = \frac{A_c^2}{2R} = \frac{10^2}{2 \cdot R} = \frac{50}{R}$ W

ii) $\Delta f = \beta f_m = 0.2 \times 1000 = \underline{200Hz}$

iii) Phase deviation $\Delta\theta = \beta = \frac{\Delta f}{f_m} = \frac{200Hz}{1KHz} = \underline{0.2}$

iv) $BW = 2(\Delta f + f_m) = 2(200 + 1000) = \underline{2400Hz}$

OR

$BW = 2\Delta f\left(1 + \frac{1}{\beta}\right) = 2 \times 200\left(1 + \frac{1}{0.2}\right) = \underline{2400Hz}$

8. A given angle modulated signal is S(t) given by the equation:
$S(t) = 12\cos(12\pi 10^8 t + 200\cos 2\pi 10^3 t)$. Find its bandwidth.

June-07,5M

Given :-

$$S(t) = 12\cos(12\pi 10^8 t + 200\cos 2\pi 10^3 t) \rightarrow (1)$$

Compare eq (1) with Standard equation for FM

$$S(t) = A_c\cos(\omega_c t + \beta\sin\omega_m t) \longrightarrow (2)$$

We get, $A_c = 12V$, $\omega_c = 12\pi 10^8$, $\beta = 200$, $\omega_m = 2\pi 10^3$

* $f_m = \frac{\omega_m}{2\pi} = \frac{2\pi 10^3}{2\pi} = 1KHz$

* $\Delta f = \beta f_m = 200 \times 1KHz = 200KHz$

* $BW = 2(\Delta f + f_m) = 2(200KHz + 1KHz) = 402KHz.$

9. A modulated signal $5\cos 2\pi\, 15\times10^3 t$, angle modulates a carrier $A\cos w_c t$. Find the modulation index and the bandwidth for the FM system. Determine the change in the bandwidth and modulation index if f_m is reduced to 5 KHz. What is the conclusion of the two results?
Assume $k_p = k_f = 15KHz/Volt$.

Jan-07,13M

Given :- $A_m = 5V$, $f_m = 15KHz$, $K_p = K_f = 15KHz/V$.

For FM System:

i) Frequency deviation $\Delta f = K_f A_m = 15KHz \times 5 = 75KHz$

ii) Modulation Index $\beta = \dfrac{\Delta f}{f_m} = \dfrac{75KHz}{15KHz} = 5$

iii) $BW = 2(\Delta f + f_m) = 2(\Delta f + f_m) = 2(75KHz + 15KHz) = \underline{180KHz.}$

When f_m is reduced to 5KHz i.e. now $\boxed{f_m = 5KHz}$

i) $\Delta f = K_f A_m = 15KHz \times 5 = 75KHz$

ii) $\beta = \dfrac{\Delta f}{f_m} = \dfrac{75KHz}{5KHz} = 15$

iii) $BW = 2(\Delta f + f_m) = 2(75KHz + 5KHz) = \underline{160KHz}$

Conclusion:

Bandwidth changes only slightly with modulating frequency 'f_m'.

---

10. Find the carrier and modulating frequencies along with modulation index and maximum deviation of the FM wave represented by deviation of the FM wave represented by the voltage equation: V = 12 sin (6x10^{8}t + 5 sin 1250t). What power will the FM wave dissipated in a 10Ω resistor? **July-05,5M**

Given :- $S(t) = 12 \sin(6\times10^8 t + 5 \sin 1250t) \longrightarrow ①$

Comparing eq① with standard equation for FM

$S(t) = A_c \sin(\omega_c t + \beta \sin \omega_m t) \longrightarrow ②$

We get, $A_c = 12V$, $\omega_c = 6\times10^8$, $\beta = 5$, $\omega_m = 1250$

i) $f_c = \dfrac{\omega_c}{2\pi} = \dfrac{6\times10^8}{2\pi} = \underline{95.5MHz}$

ii) $f_m = \frac{\omega_m}{2\pi} = \frac{1250}{2\pi} = \underline{199 Hz}$

iii) $\beta = \underline{5}$

iv) $\Delta f = \beta f_m = 5 \times 199 = \underline{995 Hz}$

v) $P = \frac{A_c^2}{2R} = \frac{12^2}{2 \times 100} = \underline{7.2W}$

---

11. An angle modulated signal is described by $S(t) = 10\cos[2\pi(10^6)t + 0.1\sin(10^3)t$. Find the message signal m(t).

i. Considering S(t) is PM with k_p=10.

ii. Considering S(t) is FM with k_f=5. **Jan-05,5M**

Sol:- The equation for PM wave is given by:

$$S(t) = A_c \cos[\omega_c t + K_p m(t)]$$

Comparing this equation with given equation, we have

$$K_p m(t) = 0.1 \sin(10^3)\pi t$$

$$m(t) = \frac{0.1}{K_p} \sin(10^3)\pi t$$

$$m(t) = \frac{0.1}{10} \sin(10^3)\pi t$$

$$\boxed{m(t) = 0.01 \sin(10^3)\pi t}$$

ii) The equation for FM wave is given by

$$S(t) = A_c \cos[\omega_c t + 2\pi K_f m(t)]$$

Comparing this equation with given equation, we have

$$2\pi K_f m(t) = 0.1 \sin(10^3)\pi t$$

$$m(t) = \frac{0.1}{2\pi K_f} \sin(10^3)\pi t$$

$$m(t) = \frac{0.1}{\cancel{2\pi} \times 5} \sin(10^3)\cancel{\pi}t$$

$$\boxed{m(t) = 0.01 \sin(10^3)\, t}$$

An angle modulated Signal is described by $\boxed{\text{Aug-2000}}$

$x_c(t) = 10 \cos[2\pi(10^6)t + 0.1 \sin(10^3)\pi t]$ Considering $x_c(t)$ as a PM Signal With $K_p = 10$. Find $m(t)$.

Sol:- The equation For PM wave is given by

$$s(t) = A_c \cos[\omega_c t + K_p m(t)]$$

Comparing this equation with the given equation, we have

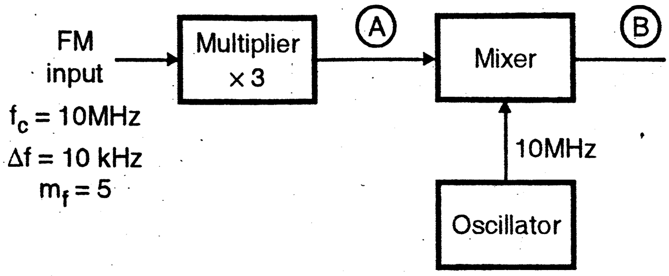
$$K_p m(t) = 0.1 \sin(10^3)\pi t$$

$$m(t) = \frac{0.1}{K_p} \sin(10^3)\pi t$$

$$= \frac{0.1}{10} \sin(10^3)\pi t$$

$$\boxed{m(t) = 0.01 \sin(10^3)\, t}$$

* In the block diagram shown in Fig. find out the carrier frequency, frequency deviation and modulation index at the points A and B. Assume that at the output of the mixer, the additive frequency component is being selected.



June-09, 5M

Fig.

Soln. :

(i) At point (A) :

The carrier $f_c = 3 \times 10\,\text{MHz} = 30$ MHz.

The frequency deviation $\delta = 3 \times 10\,\text{kHz} = 30$ kHz and modulation index $m_f = 3 \times 5 = 15$.

The minimum frequency $f_{min} = 30\,\text{MHz} - 30\,\text{kHz} = 29.970$ MHz

The maximum frequency $f_{max} = 30\,\text{MHz} + 30\,\text{kHz} = 30.030$ MHz.

(ii) At point (B) :

Carrier frequency $f_c = 30$ MHz $+ 10$ MHz $= 40$ MHz.

Maximum frequency $f_{max} = 30.03 + 10 = 40.03$ MHz

Minimum frequency $f_{min} = 29.970 + 10 = 39.970$ MHz.

As there is no change in deviation due to mixing, the modulation index will remain same i.e. $m_f = 15$.

Determine the bandwidth of FM Signal, if the maximum value of frequency deviation Δf is fixed at 75KHZ for Commercial FM broadcasting by radio & modulation frequency is W = 15KHZ.

Aug-2001

Sol:- Given : $\Delta f = 75KHZ$

$W = 15KHZ$ or f_m

i) Deviation ratio : $D = \frac{\Delta f}{W} = \frac{75KHZ}{15KHZ} = 5$

ii) Using Carson rule

$$B_T = 2[1+D]W$$

$$= 2[1+5]\,15\times10^3$$

$$B_T = 180KHZ$$

OR

$$* \quad \beta = D = \frac{\Delta f}{W} = \frac{75KHZ}{15KHZ} = 5$$

$$* \quad B_T = 2[1+\beta]f_m$$

$$= 2[1+5]\,15KHZ$$

$$B_T = 180KHZ.$$

OR

$$* \quad B_T = 2\Delta f + 2f_m$$

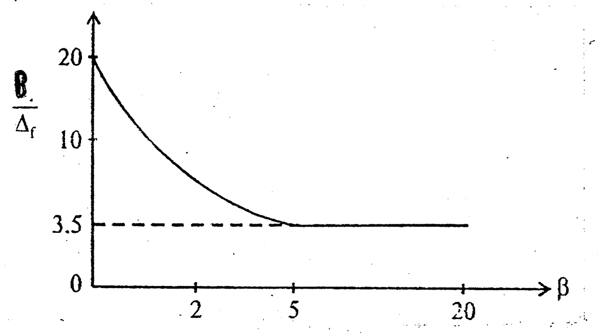
$$= 2(75KHZ) + 2(15KHZ)$$

$$B_T = 180KHZ$$

A carrier wave of 100MHz is frequency modulated by a sine wave of amplitude 20 volts & frequency 100KHz. The frequency sensitivity of modulator is 25 KHz/v. Determine

i) Transmission bandwidth using Carson's rule.

ii) Transmission bandwidth using universal rule (The universal graph is as shown below).



Sol:- Given: $f_c = 100MHz$, $A_m = 20V$, $f_m = 100KHz$, $K_f = 25KHz/volt$.

ii) Modulation Index

$$\beta = \frac{\Delta f}{f_m} = \frac{K_f A_m}{f_m} = \frac{25\times10^3 \times 20V}{100\times10^3} = 5$$

From the given universal graph the value of $\frac{B}{\Delta f}$ for $\beta = 5$ is

$$\frac{B}{\Delta f} = 3.5 \;\&$$

$$\Delta f = K_f A_m = (25\times10^3)(20) = 500KHz$$

$$B = \Delta f \times 3.5$$

$$= 500KHz \times 3.5$$

$$\therefore \boxed{B = 1750KHz}$$

i) The transmission BW using Carson's rule is

$$B_T = 2[\Delta f + f_m] = 2[500KHz + 100KHz]$$

$$\boxed{B_T = 1200KHz}$$

Sol:- Given: $f_c = 100MHz$, $f_m = 100KHz$, $A_m = 20V$, $K_f = 25KHz/V$

i) $B_T = 2[\Delta f + f_m]$

$\Delta f = K_f A_m = (25KHz/\cancel{V}) \times 20\cancel{V} = 500KHz$

$B_T = 2[500KHz + 100KHz]$

$\boxed{B_T = 1.2MHz}$

ii) $\beta = \frac{\Delta f}{f_m} = \frac{500KHz}{100KHz} = \underline{5}$

From universal curve, for $\underline{\beta = 5}$, we have $\boxed{\frac{B}{\Delta f} = 3.2}$

$B = \Delta f \times 3.2$

$B = 500KHz \times 3.2$

$\boxed{B = 1.6MHz}$

iii) Modulating voltage is doubled $= 2 \times 20V = 40V$.

* $\Delta f = K_f A_m = (25KHz/\cancel{V})(40\cancel{V}) = \underline{1MHz.}$

* BW using Carson's rule

$B_T = 2[\Delta f + f_m]$

$= 2[1MHz + 100KHz]$

$\boxed{B_T = 2.2 MHz}$

* $\beta = \frac{\Delta f}{f_m} = \frac{1 MHz}{100 KHz} = \underline{10}$

From universal curve, for $\beta = 10$, we have $\frac{B}{\Delta f} = 3$

A carrier wave frequency 100MHz is frequency modulated by a sinusoidal wave of amplitude 20V & frequency 100KHz. The frequency sensitivity of the modulator is 25KHz per volt.

i) Determine the approximate bandwidth of the FM signal, using Carson's rule.

ii) Determine the bandwidth by transmitting only those side frequencies whose amplitude exceed 1 percent of the unmodulated carrier amplitude. Use the universal curve of fig ① for this calculation.

iii) Repeat the calculation, assuming that the amplitude of the modulating signal is doubled.

iv) Repeat the calculations, assuming the modulation freq is doubled

July - 2008, 8M

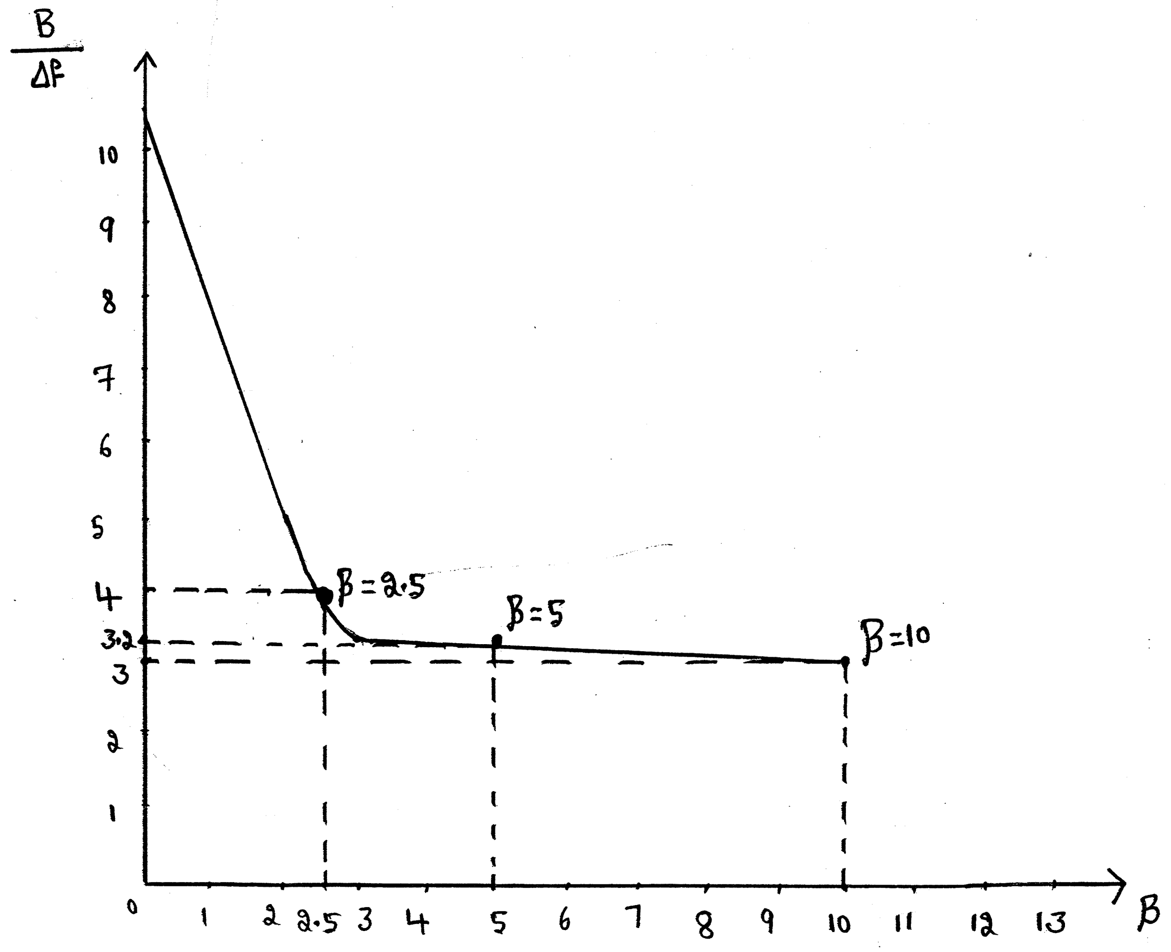


Figure ①: universal curve

$$B = \Delta f \times 3 = 1\,MHz \times 3$$

$$\boxed{B = 3\,MHz}$$

iv) Given : $f_m = 200\,KHz$, $A_m = 20V$

* Frequency deviation $\Delta f = K_f A_m$

$$= (25\,KHz/\cancel{V}) \times 20\cancel{V}$$

$$\boxed{\Delta f = 500\,KHz}$$

* Bandwidth using Carson's rule

$$B_T = 2[\Delta f + f_m]$$

$$= 2[500\,KHz + 200\,KHz]$$

$$\boxed{B_T = 1.4\,MHz}$$

$$\beta = \frac{\Delta f}{f_m} = \frac{500\,KHz}{200\,KHz} = 2.5$$

* From universal curve, for $\beta = 2.5$,

We have, $\dfrac{B}{\Delta f} = 4.0$

$$B = \Delta f \times 4.0$$

$$= 500\,KHz \times 4$$

$$\boxed{B = 2\,MHz}$$

Sketch the variations of the frequency of the resulting FM & PM Signal as a function of time where a carrier signal is modulated by a modulating signal $m(t) = \frac{A}{T_0} t \quad 0 \leq t \leq T_0$, which is periodic with period T_0. Assume the following:

Carrier frequency $f_c = 100$ KHz, $A = 5$ volts, $T_0 = 1$ msec, $K_p = 0.2\pi$ rad/sec & $K_f = 2$KHz/V.

Derive the equation for the PM & FM signals & draw the relevant block diagram.

Jan-2006, 11M

Sol:-

i) FM Wave:-

W.K.T the FM Wave is given by:

$$S(t) = A_c \cos\left[\omega_c t + 2\pi K_f \int_0^t m(t).\,dt\right] \longrightarrow ①$$

$$= A_c \cos\left[2\pi f_c + 2\pi K_f \int_0^t \frac{A}{T_0} t.dt\right]$$

$$= 5\cos\left[2\pi \times 10^5 t + (2\pi \times 2\times10^3)\left\{\frac{A}{T_0}\,\frac{t^2}{2}\right\}\right]$$

$$= 5\cos\left[2\pi \times 10^5 t + \frac{4\pi\times10^3}{2}\left(\frac{5}{1\times10^{-3}}\cdot t^2\right)\right]$$

$$\boxed{S(t) = 5\cos\left[2\pi\times10^5 t + 10\pi\times10^6 t^2\right]} \longrightarrow ②$$

Eq ② is the FM modulated wave for the given values.

* The Instantaneous frequency is given by

$$f_i = f_c + K_f m(t)$$

$$= 100\times10^3 + 2\times10^3\left(\frac{A}{T_0} t\right)$$

$$= 10^5 + 2\times10^3 \times \frac{5}{1\times10^{-3}}\, t$$

$$f_i = 10^5 + 2\times10^3 (5000\, t)$$

$$\boxed{f_i = 10^5 + 10\times10^6\, t}$$

at $\underline{t = 1\, msec}$

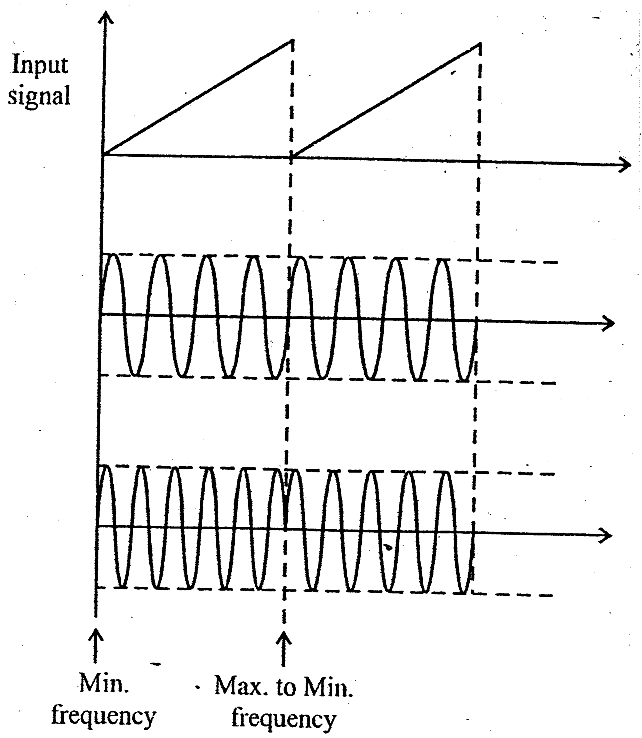
$$f_i = 10^5 + 10\times10^6 (1\times10^{-3})$$

$$\boxed{f_i = 110\, KHz}$$

t in msec	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
$m(t) = \left(\frac{A}{T_0} t\right)$ in V	0	0.5	1	1.5	2	2.5	3	3.5	4	4.5	5

Where, $T_0 = 1\, msec$

$A = 5V$



For PM Wave :-

WKT the PM Wave is given by

$$S(t) = A_c \cos[\theta(t)]$$

$$S(t) = A_c \cos[2\pi f_c t + K_p m(t)]$$

$$S(t) = 5 \cos\left[2\pi \times 10^5 t + 0.2\pi \left(\frac{A}{T_0} t\right)\right]$$

$$S(t) = 5 \cos\left[2\pi \times 10^5 t + 0.2\pi \frac{5}{1\times10^{-3}} t\right]$$

$$\boxed{S(t) = 5 \cos[2\pi \times 10^5 t + \pi(1\times10^3) t]} \longrightarrow (3)$$

Equation (3) is the modulated equation of PM Wave.

* The Instantaneous frequency of the phase modulated wave is given by

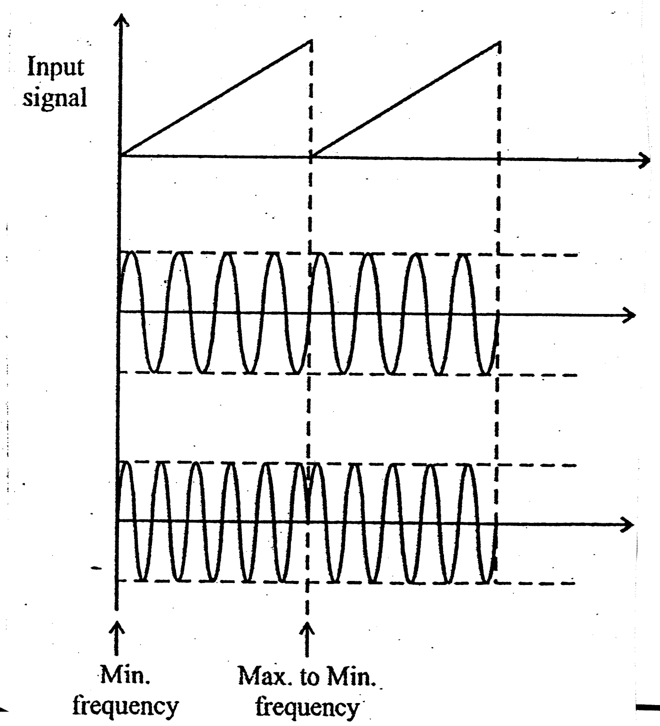
$$\omega_i(t) = \frac{d}{dt}\theta(t) \rightarrow (4)$$

From eq (3), $\theta = [2\pi \times 10^5 t + \pi(1\times10^3) t]$

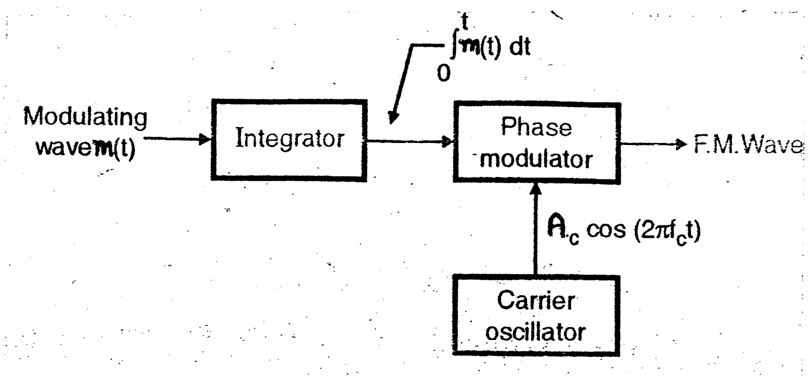
$$2\pi f_i(t) = \frac{d}{dt}[2\pi \times 10^5 t + \pi(1\times10^3) t]$$

$$f_i(t) = \frac{1}{\cancel{2\pi}}\left[\cancel{2\pi} \times 10^5 + \frac{\cancel{1000\pi}}{500}\right]$$

$$\boxed{f_i(t) = [1\times10^5 + 500]\ Hz}$$



i) Generation of FM using PM (phase Modulator) :-

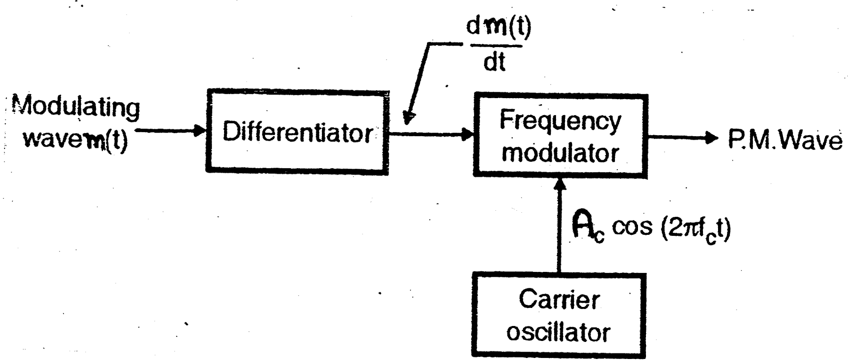


Generation of FM from phase modulator

* FM can be generated by 1st Integrating m(t) & then using the result as the I/p to a phase modulator as shown in above figure.

$$\boxed{S(t) = A_c \cos\left[2\pi f_c t + \underbrace{2\pi K_p}_{K_p} \int_0^t m(t) \cdot dt\right]}$$

ii) Generation of PM using a FM (Frequency Modulator) :-



Generation of P.M. wave using frequency modulator

* The PM signal can be generated by 1st differentiating m(t) & then using the result as the I/p to a frequency-modulator as shown in fig above.

$$\therefore\ S(t) = A_c \cos\left[2\pi f_c t + 2\pi K_f \cancel{\int_0^t} \frac{\cancel{d}}{\cancel{dt}} m(t) \cdot \cancel{dt}\right]$$

Substituting $2\pi K_f = K_p$

$$S(t) = A_c \cos\left[2\pi f_c t + \underline{2\pi K_f}\, m(t)\right]$$

$$\boxed{S(t) = A_c \cos\left[2\pi f_c t + K_p\, m(t)\right]}$$

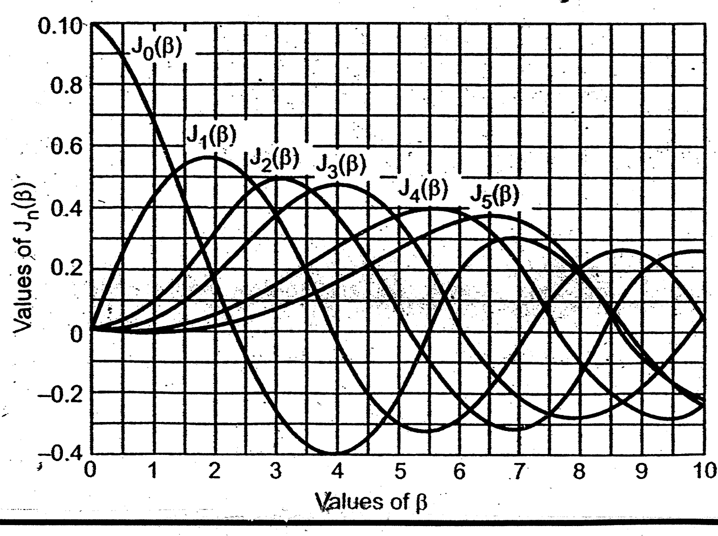
Bessels Functions Table:-

Bessel functions

x	n or Order																
(β)	J_0	J_1	J_2	J_3	J_4	J_5	J_6	J_7	J_8	J_9	J_{10}	J_{11}	J_{12}	J_{13}	J_{14}	J_{15}	J_{16}
0.00	1.00	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
0.25	0.98	0.12	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
0.5	0.94	0.24	0.03	—	—	—	—	—	—	—	—	—	—	—	—	—	—
1.0	0.77	0.44	0.11	0.02	—	—	—	—	—	—	—	—	—	—	—	—	—
1.5	0.51	0.56	0.23	0.06	0.01	—	—	—	—	—	—	—	—	—	—	—	—
2.0	0.22	0.58	0.35	0.13	0.03	—	—	—	—	—	—	—	—	—	—	—	—
2.5	–0.05	0.50	0.45	0.22	0.07	0.02	—	—	—	—	—	—	—	—	—	—	—
3.0	–0.26	0.34	0.49	0.31	0.13	0.04	0.01	—	—	—	—	—	—	—	—	—	—
4.0	–0.40	–0.07	0.36	0.43	0.28	0.13	0.05	0.02	—	—	—	—	—	—	—	—	—
5.0	–0.18	–0.33	0.05	0.36	0.39	0.26	0.13	0.05	0.02	—	—	—	—	—	—	—	—
6.0	0.15	–0.28	–0.24	0.11	0.36	0.36	0.25	0.13	0.06	0.02	—	—	—	—	—	—	—
7.0	0.30	0.00	–0.30	–0.17	0.16	0.35	0.34	0.23	0.13	0.06	0.02	—	—	—	—	—	—
8.0	0.17	0.23	–0.11	–0.29	–0.10	0.19	0.34	0.32	0.22	0.13	0.06	0.03	0.01	—	—	—	—
9.0	–0.09	0.24	0.14	–0.18	–0.27	–0.06	0.20	0.33	0.30	0.21	0.12	0.06	0.03		—	—	—
10.0	–0.25	0.04	0.25	0.06	–0.22	–0.23	–0.01	0.22	0.31	0.29	0.20	0.12	0.06	0.03	0.01	—	—
12.0	0.05	–0.22	–0.08	0.20	0.18	–0.07	–0.24	–0.17	0.05	0.23	0.30	0.27	0.20	0.12	0.07	0.03	0.01
15.0	–0.01	0.21	0.04	–0.19	–0.12	0.13	0.21	0.03	–0.17	–0.22	–0.09	0.10	0.24	0.28	0.25	0.18	0.12

Figure

Bessel functions



A Carrier wave of amplitude 5V & frequency 90MHz is frequency modulated by a Sinusoidal voltage of amplitude 5V & frequency - 15KHz. The frequency deviation constant is 1KHz/V. Sketch the Spectrum of the modulated FM wave.

Sol :- Given : $A_c = 5V$, $f_c = 90MHz$, $A_m = 5V$, $f_m = 15KHz$. $K_f = 1KHz/V$.

* Frequency deviation $\Delta f = K_f A_m = 1KHz/V \times 5V = 5KHz$.

* $\beta = \frac{\Delta f}{f_m} = \frac{5KHz}{15KHz} = 0.333$

From the table of Bessel functions, for $\beta = 0.333$

Use approximate values for J_0, J_1 & J_2.

i) For carrier : $J_0 = 0.96$

ii) 1st Side frequency : $J_1 = 0.18$

iii) 2nd Side frequency : $J_2 = 0.02$

Higher order Side frequencies are negligible since β is Small.

i) Amplitude Spectrum of the Carrier : $A_c J_0(\beta) = 5V \times 0.96 = 4.8V$

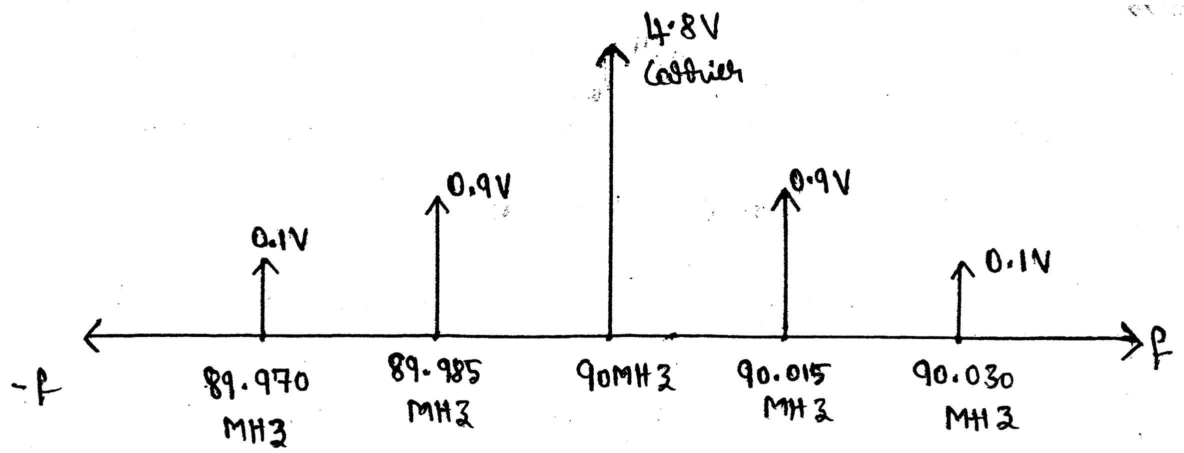
Carrier frequency $f_c = 90MHz$

ii) Amplitude Spectrum of the 1st Side frequency : $A_c J_1(\beta) = 5 \times 0.18 V = 0.9V$

1st Side frequency : $f_c + f_m = 90MHz + 15KHz = 90.015 MHz$

iii) Amplitude Spectrum of the 2nd Side frequency : $A_c J_2(\beta) = 5 \times 0.02V = 0.1V$

2nd Side frequency : $f_c + 2f_m = 90MHz + 2(15KHz) = 90.030MHz$



$f_c - f_m = 90MHz - 15KHz = 89.985\ MHz$

$f_c - 2f_m = 90MHz - 2(15\ KHz) = 89.970MHz.$

NOTE :-

1) Carrier Signal $\rightarrow A_c J_0(\beta) \cos 2\pi f_c t$
2) 1^{st} pair of Side frequencies $\rightarrow A_c J_1(\beta) \cos 2\pi(f_c \pm f_m)t$
3) 2^{nd} pair of Side frequencies $\rightarrow A_c J_2(\beta) \cos 2\pi(f_c \pm 2f_m)t$

$\vdots$

n) n^{th} pair of Side frequencies $\rightarrow A_c J_n(\beta) \cos 2\pi(f_c \pm nf_m)t$

A Carrier wave is frequency modulated using a Sinusoidal Signal of frequency f_m & amplitude A_m.

i) Determine the values of modulation Index β for which the carrier component of the FM Wave is reduced to Zero.

ii) In a certain experiment conducted with $f_m = 1KHz$ and increasing A_m from Zero, it is found that the carrier component of FM wave is reduced to Zero for the 1st time when $A_m = 2.0V$. Find the frequency Sensitivity of the modulator.

iii) What is the value of A_m for which the carrier component becomes Zero for the Second time?

Jan-2006, 6M

Sol:- Given: $f_m = 1KHz$, $A_m = 2V$, $K_f = ?$

i) From Bessel function Table & plot of Bessel function of the 1st Kind, the carrier disappears for the modulation Index

$$\boxed{\beta = 2.408,\ 5.52,\ 8.6,\ 11.8 \text{ and So on.}}$$

ii)
$$\beta = \frac{\Delta f}{f_m} = \frac{K_f A_m}{f_m}$$

$J_0(\beta) = 0$ i.e. First time carrier is 0 at $\beta = 2.405$

$$K_f = \frac{\beta f_m}{A_m} = \frac{(2.405) \times 1 \times 10^3}{2V} = 1.2025\ KHz/V$$

iii) Now $J_0(\beta) = 0$ i.e. Second time carrier is 0 at $\beta = 5.52$, $A_m = ?$

$$\beta = \frac{\Delta f}{f_m} = \frac{K_f A_m}{f_m}$$

$$A_m = \frac{\beta f_m}{K_f} = \frac{5.52 \times 1 \times 10^3}{1.2025 \times 10^3} = 4.9V$$

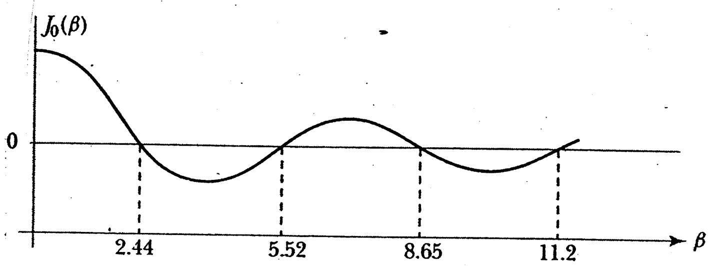
A Carrier wave is frequency modulated using $m(t) = A_m \cos 2\pi f_m t$.

a) List the values of Index β for which the Carrier Component of FM wave is reduced to Zero.

b) In a certain experiment Conducted using Spectrum analyzer With $f_m = 1KHz$ & increasing A_m Starting from Zero volts, it is found that Carrier Component of FM Wave is reduced to Zero for the 1^{st} time with $A_m = 2V$. What is the frequency Sensitivity of the modulator? What is the value of A_m for which the Carrier Component is reduced to Zero for the Second time?

Sol :- Given : $A_m = 2V$, $f_m = 1KHz$

$\beta = 2.44$, Since the 1^{st} time $J_0(\beta)$ is Zero



■ Plot of $J_0(\beta)$ v/s β.

a) The amplitude of the Carrier in FM Wave is $A_c J_0(\beta)$. This means that if we can make $J_0(\beta) = 0$, the carrier gets Suppressed in the FM waveform. The typical values of β for which $J_0(\beta) = 0$ are 2.44, 5.52, 8.65, 11.2 etc.

b) W.K.T. $\beta = \dfrac{\Delta f}{f_m} = \dfrac{K_f A_m}{f_m}$

$$K_f = \frac{\beta f_m}{A_m} = \frac{(2.44) \times 1 \times 10^3}{2} = 1.22 \times 10^3 \ Hz/V$$

Also, $\beta = 5.52$, Since for the Second time $J_0(\beta)$ is Zero

Hence, $$A_m = \frac{\beta f_m}{K_f} = \frac{(5.52) \times 1 \times 10^3}{1.22 \times 10^3}$$

$$\boxed{A_m = 4.52V}$$

A Carrier wave of amplitude 10V & frequency 100MHz is frequency modulated by a Sinusoidal voltage. The modulating-voltage has an amplitude of 5V & frequency $f_m = 20KHz$. The frequency deviation Constant is 2KHz/V. Draw the frequency Spectrum of FM wave.

Sol :- Given : $A_c = 10V$, $f_c = 100MHz$, $K_f = 2KHz/V$.
$A_m = 5V$, $f_m = 20KHz$.

* $\Delta f = K_f A_m = 2KHz/\cancel{V} \times 5\cancel{V} = 10KHz$

* $\beta = \frac{\Delta f}{f_m} = \frac{10KHz}{20KHz} = 0.5$

* From the table of Bessel functions, for $\beta = 0.5$ use approximate values of J-Coefficients are:

$$\boxed{J_0 = 0.94, \quad J_1 = 0.24, \quad J_2 = 0.03}$$

* The amplitude, frequencies of the Carrier & Sidebands are as follows:

i) Carrier amplitude → $A_c J_0(\beta) = 10V \times 0.94 = 9.4V$

Carrier frequency → 100MHz

ii) Frequency of 1st Sideband → $f_c + f_m = 100.2\,MHz$

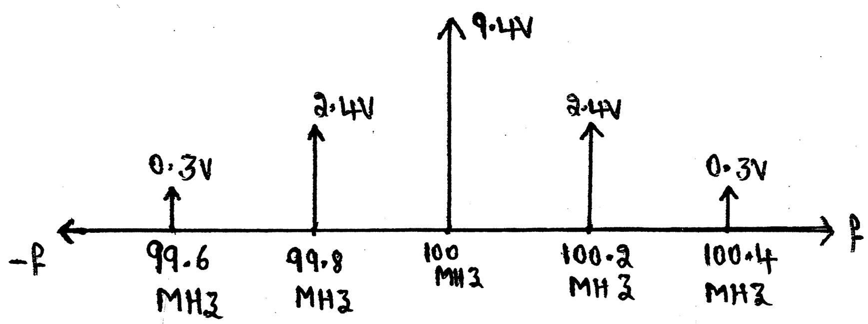
$f_c - f_m = 99.8\,MHz$

Amplitude of 1st Sideband → $A_c J_1(\beta) = 10V \times 0.24 = 2.4V$

iii) Frequency of 2nd Sideband → $f_c + 2f_m = 100.4\,MHz$

$f_c - 2f_m = 99.6\,MHz$

Amplitude of 2nd Sideband → $A_c J_2(\beta) = 10V \times 0.03 = 0.3V$.



Spectrum of FM wave.

An unmodulated carrier has amplitude 10V & frequency 100MHz. A Sinusoidal Waveform of frequency 1KHz, frequency-modulates this carrier Such that the frequency deviation is 75KHz. The modulated waveform passes through Zero & is increasing at time $t=0$. Write the time-domain expression for the modulated carrier Waveform.

Sol :- $A_c = 10V$, $f_c = 100MHz$, $f_m = 1KHz$, $\Delta f = 75KHz$.

W.K.T the time-domain expression of FM wave is

$$S(t) = A_c \cos\left[2\pi f_c t + \beta \sin 2\pi f_m t\right]$$

$$* \quad \beta = \frac{\Delta f}{f_m} = \frac{75KHz}{1KHz} = 75$$

$$\boxed{S(t) = 10\cos\left[2\pi \times 100\times10^6 t + 75 \sin 2\pi \times 1\times10^3 t\right]}$$

An angle modulated Signal is represented by

$$S(t) = 10 \cos[2\pi \times 10^6 t + 5 \sin 2000\pi t + 10 \sin 3000\pi t] \text{ volts.}$$

Find the following:

a) The power in the modulated Signal

b) The frequency deviation Δf

c) The deviation ratio

d) The phase deviation $\Delta\theta$

e) The approximate transmission bandwidth B_T.

Sol:- Given: $A_c = 10V$.

a) $$P = \frac{A_c^2}{2R} = \frac{(10)^2}{2R} = \frac{50}{R}$$

Assume $R = 1\Omega$

$$P = \frac{50}{1} = 50 \text{ Watts}$$

b) $$\Delta f = \frac{|\omega_i(t) - \omega_c|_{max}}{2\pi} \quad \text{OR} \quad \Delta\omega = |\omega_i(t) - \omega_c|_{max}$$

$$\because 2\pi\Delta f = \Delta\omega$$

W.K.T $S(t) = A_c \cos[\theta_i(t)] \rightarrow (1)$

Comparing eq (1) with given equation, we get

$$\theta_i(t) = 2\pi \times 10^6 t + 5 \sin 2000\pi t + 10 \sin 3000\pi t$$

* The instantaneous angular frequency is

$$\omega_i(t) = \frac{d}{dt}\theta_i(t)$$

$$\omega_i(t) = \frac{d}{dt}[2\pi \times 10^6 t + 5 \sin 2000\pi t + 10 \sin 3000\pi t]$$

$$W_i(t) = 2\pi \times 10^6 + 5\cos 2000\pi t\,(2000\pi) + 10\cos 3000\pi t\,(3000\pi)$$

$$\boxed{W_i(t) = 2\pi \times 10^6 + 5 \times 2000\pi \cdot \cos 2000\pi t + 10 \times 3000\pi \cdot \cos 3000\pi t}$$

Hence,

$$\Delta W = \left| W_i(t) - W_c \right|_{max}$$

Where,

$$\boxed{W_c = 2\pi \times 10^6}$$

$$\Delta W = \left| \cancel{2\pi \times 10^6} + 5 \times 2000\pi \cdot \cos 2000\pi t + 10 \times 3000\pi \cdot \cos 3000\pi t - \cancel{2\pi \times 10^6} \right|_{max}$$

$$\Delta W = \left| \underline{5 \times 2000\pi} \cdot \cos 2000\pi t + \underline{10 \times 3000\pi} \cdot \cos 3000\pi t \right|_{max}$$

$$\Delta W = \left| \underline{10,000\pi} \cos 2000\pi t + \underline{30,000\pi} \cos 3000\pi t \right|_{max}$$

$$\Delta W = 10000\pi + 30000\pi \text{ rad/Sec}$$

example

$$\boxed{\begin{aligned} \Delta f &= \left| \underline{5} \cos\theta \right|_{max} \\ \Delta f &= 5 \end{aligned}}$$

$$\boxed{\Delta W = 40,000\pi \text{ rad/Sec}}$$

$$2\pi \Delta f = 40,000\pi \text{ rad/Sec}$$

$$\Delta f = \frac{40,\cancel{000\pi}}{\cancel{2\pi}} = 20\,KHz$$

c) Deviation ratio,
$$D = \frac{\Delta f}{f_m} = \frac{20\,KHz}{1500\,Hz} = 13.33$$

* In given equation, f_m is the highest Significant frequency present in the modulating Signal i.e. $\underline{3000\pi}$

$$W_m = 3000\pi$$

$$2\pi f_m = 3000\pi$$

$$f_m = \frac{3\cancel{000\pi}}{\cancel{2\pi}} = 1500\,Hz$$

d) $\Delta\theta = \left| \theta_i(t) - \theta_c \right|_{max}$ Where $\theta_c = 2\pi \times 10^6 t$

$\Delta\theta = \left| \cancel{2\pi \times 10^6 t} + 5 \sin 2000\pi t + 10 \sin 3000\pi t - \cancel{2\pi \times 10^6 t} \right|_{max}$

$\Delta\theta = \left| \underline{5} \sin 2000\pi t + \underline{10} \sin 3000\pi t \right|_{max}$

$\Delta\theta = 5 + 10$ radians

$\boxed{\Delta\theta = 15 \text{ radians}}$

e) Approximate bandwidth using Carson's formula is

$B_T = 2[D+1] f_m$

$= 2[13.33+1]\ 1500$

$\boxed{B_T = 43 \text{ KHz}}$

26) An angle modulated Signal is defined by

$S(t) = 10 \cos[2\pi \times 10^6 t + 0.2 \sin 2000\pi t]$ volts.

Find the following:

a) The power in the modulated Signal

b) The frequency deviation

c) phase deviation $\Delta\theta$

d) The approximate transmission bandwidth.

Sol:-

a) $P = \frac{A_c^2}{2R} = \frac{(10)^2}{2R} = \frac{50}{R}$

Assume $R = 1\,\Omega$, $P = \frac{50}{1} = \underline{50 \text{ Watts}}$.

b) The instantaneous phase of the angle modulated Signal is

$$\theta_i(t) = 2\pi \times 10^6 t + 0.2 \sin 2000\pi t$$

$$\Delta W = \left| W_i(t) - W_c \right|_{max}$$

* WKT

$$W_i(t) = \frac{d}{dt}\theta_i(t)$$

$$W_i(t) = \frac{d}{dt}\left[2\pi \times 10^6 t + 0.2 \sin 2000\pi t\right]$$

$$W_i(t) = 2\pi \times 10^6 + 0.2 \cos(2000\pi t) \cdot 2000\pi$$

$$\boxed{W_i(t) = 2\pi \times 10^6 + 0.2 \times 2000\pi \cos(2000\pi t)}$$

$$\Delta W = \left| \cancel{2\pi \times 10^6} + \underline{0.2 \times 2000\pi} \cos(2000\pi t) - \cancel{2\pi \times 10^6} \right|_{max}$$

Where $W_c = 2\pi \times 10^6$

$$\Delta W = 0.2 \times 2000\pi$$

$$2\pi \Delta f = 0.2 \times 2000\pi$$

$$\Delta f = \frac{0.2 \times \cancel{2000\pi}^{1000}}{\cancel{2\pi}}$$

$$\boxed{\Delta f = 200 Hz}$$

c) $\Delta\theta = \left| \theta_i(t) - \theta_c \right|_{max}$

Where $\theta_c = 2\pi \times 10^6 t$

$$\Delta\theta = \left| 2\pi \cancel{\times 10^6} t + 0.2 \sin 2000\pi t - 2\pi \cancel{\times 10^6} t \right|$$

$$\Delta\theta = \left| \underline{0.2} \sin 2000\pi t \right|$$

$$\boxed{\Delta\theta = 0.2 \text{ radians}}$$

d) Transmission bandwidth

$$B_T = 2[\Delta f + f_m]$$

W.K.T $\quad W_m = 2000\pi$

$$2\pi f_m = 2000\pi$$

$$f_m = \frac{\cancel{2000}\cancel{\pi}\ 1000}{\cancel{2\pi}}$$

$$\boxed{f_m = 1KHz}$$

$$B_T = 2[200 + 1000]$$

$$\boxed{B_T = 2400Hz}$$

A Carrier is frequency modulated by a Sinusoidal modulating Signal of frequency 2KHz, resulting in a frequency deviation of 5KHz.

a) What is the bandwidth occupied by the modulated waveform?

b) If the amplitude of the modulating Signal is increased by a factor of 2 & its frequency lowered to 1KHz, what is the new bandwidth?

Sol:- Given: $f_m = 2KHz$, $\Delta f = 5KHz$.

a) $B_T = 2[\Delta f + f_m] = 2[5KHz + 2KHz]$

$$\boxed{B_T = 14KHz}$$

b) Now A_m is increased by a factor 2 & f_m is decreased to 1KHz, BW=?

W.K.T $\boxed{\Delta f = K_f A_m}$

Since the amplitude of the modulating signal is increased by a factor of 2, the frequency deviation also increases by the same factor. Hence, the new frequency deviation is

$$\Delta f = 2 \times 5\,\text{KHz} = 10\,\text{KHz} \quad \& \quad \text{Now } f_m = 1\,\text{KHz}$$

$$\therefore \quad B_T = 2[\Delta f + f_m] = 2[10\,\text{KHz} + 1\,\text{KHz}]$$

$$\boxed{B_T = 22\,\text{KHz}}$$

A sinusoidal modulating wave $m(t) = A_m \cos 2\pi f_m t$ is applied to a phase modulator with phase sensitivity 'K_p'. The unmodulated carrier wave has frequency 'f_c' & amplitude 'A_c'. Determine the spectrum of the resulting phase modulated wave, assuming that maximum phase deviation $\beta_p = K_p A_m$ does not exceed 0.3 radian.

Sol :-

The time-domain expression for the PM wave is

$$S(t) = A_c \cos[2\pi f_c t + K_p m(t)] \longrightarrow (1)$$

Where K_p is the phase sensitivity constant in rad/volt.

Substituting $m(t) = A_m \cos 2\pi f_m t$ in eq (1), we get

$$S(t) = A_c \cos\left[2\pi f_c t + \underline{K_p A_m} \cos 2\pi f_m t\right]$$

$$S(t) = A_c \cos\left[2\pi f_c t + \beta_p \cos 2\pi f_m t\right]$$

W.K.T

$$\boxed{\cos(A+B) = \cos A \cdot \cos B - \sin A \cdot \sin B}$$

$$S(t) = A_c \cos(2\pi f_c t) \cdot \cos(\beta_p \cos 2\pi f_m t) - A_c \sin(2\pi f_c t) \cdot \sin(\beta_p \cos 2\pi f_m t) \rightarrow (2)$$

Since, $\beta_p \leq 0.3$ rad, we can make the following approximations

$$\cos(\beta_p \cos 2\pi f_m t) \approx 1 \quad \text{and}$$

$$\sin(\beta_p \cos 2\pi f_m t) \approx \beta_p \cos 2\pi f_m t$$

* Making use of above approximations in eq (2), we get expressions for NBFM Signal

$$S(t) = A_c \cos(2\pi f_c t) - A_c \sin(2\pi f_c t) \cdot \beta_p \cos(2\pi f_m t).$$

W.K.T

$$\boxed{\sin A \cdot \cos B = \frac{1}{2}\sin(A-B) + \frac{1}{2}\sin(A+B)}$$

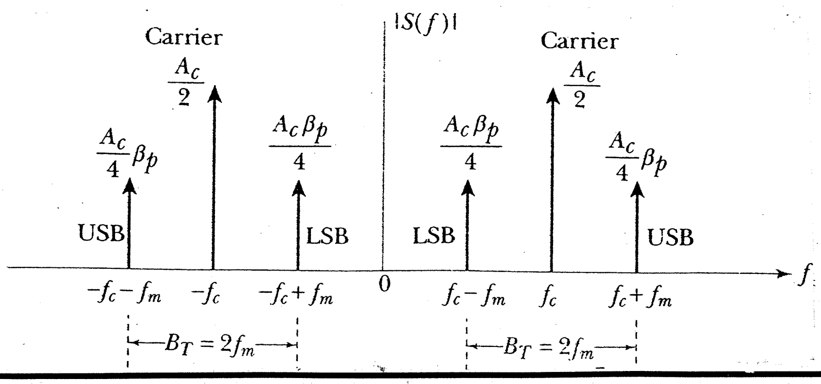
$$S(t) = A_c \cos(2\pi f_c t) - \left[\frac{A_c \beta_p}{2} \sin 2\pi(f_c - f_m)t + \frac{A_c \beta_p}{2} \sin 2\pi(f_c + f_m)t\right]$$

$$\boxed{S(t) = A_c \cos(2\pi f_c t) - \frac{\beta_p A_c}{2} \sin 2\pi(f_c - f_m)t - \frac{\beta_p A_c}{2} \sin 2\pi(f_c + f_m)t} \rightarrow (3)$$

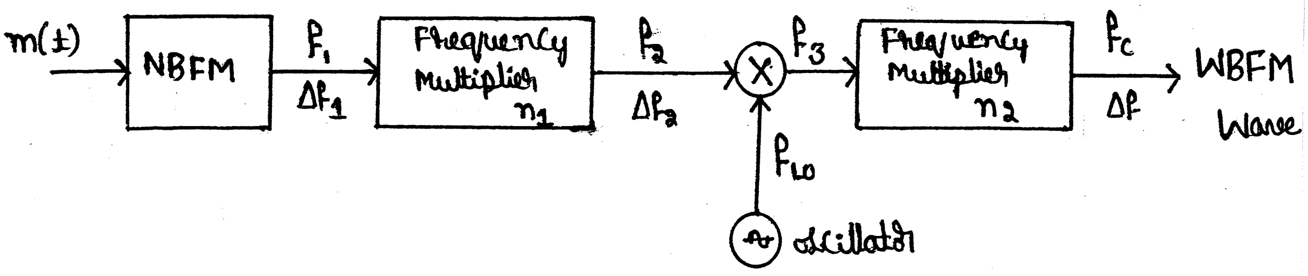
Taking FT on both sides of eq (3), we get

$$S(f) = \frac{A_c}{2}\left[\delta(f-f_c) + \delta(f+f_c)\right] - \frac{\beta_p A_c}{4j}\left[\delta(f-(f_c-f_m)) - \delta(f+(f_c-f_m))\right]$$

$$- \frac{\beta_p A_c}{4j}\left[\delta(f-(f_c+f_m)) - \delta(f+(f_c+f_m))\right]$$



The block diagram of an Armstrong AM transmitter is shown in Figure ①, Compute the maximum frequency deviation & o/p frequency of the transmitter. Take $f_1 = 200KHz$, $f_{LO} = 10.8MHz$, $\Delta f_1 = 25Hz$, $n_1 = 64$ & $n_2 = 48$.



Sol:-

1) The o/p of the 1st frequency multiplier

i) $f_2 = n_1 \times f_1 = 64 \times 200KHz = 12.8MHz$

ii) $\Delta f_2 = n_1 \times \Delta f_1 = 64 \times 25Hz = 1600Hz$

2) The o/p of the product modulator

$$\boxed{f_3 = f_2 \pm f_{LO}}$$

i) $f_3 = f_2 + f_{LO} = 23.6MHz$

ii) $f_3 = f_2 - f_{LO} = 2MHz$

* The o/p of the 2nd frequency multiplier i.e. Final o/p

i) $f_c = n_2 f_3 = 1.1328GHz$, When $f_3 = 23.6MHz$

$f_c = n_2 f_3 = 96MHz$, When $f_3 = 2MHz$

ii) $\Delta f = n_2 \times \Delta f_2 = 76.8MHz$

Calculate the Carrier Swing, Carrier frequency, frequency deviation & modulation Index for an FM Signal which reaches a maximum frequency of 99.047 MHz & a minimum freq of 99.023 MHz. The frequency of the modulating Signal is 7KHz.

Sol:- Given : $(f_i)_{max} = 99.047\ MHz$, $(f_i)_{min} = 99.023\ MHz$ &

$f_m = 7KHz$.

* Carrier Swing $= (f_i)_{max} - (f_i)_{min} = 99.047MHz - 99.023MHz$

$= 24KHz$.

* $\Delta f = \dfrac{\text{Carrier Swing}}{2} = \dfrac{24KHz}{2} = 12KHz$.

* W.K.T $(f_i)_{max} = f_c + \Delta f$

Carrier freq $\therefore$ $f_c = (f_i)_{max} - \Delta f$

$= 99.047MHz - 12KHz$

$f_c = 99.035\ MHz$.

* Modulation Index 'β' $= \dfrac{\Delta f}{f_m}$

$= \dfrac{12KHz}{7KHz}$

$\boxed{\beta = 1.714}$

A 93.2 MHz carrier is frequency modulated by a 5KHz sine wave. The resultant FM signal has a frequency deviation of 40KHz.

a) Find the carrier swing of the FM signal.

b) What are the highest & lowest frequencies attained by the frequency modulated signal.

c) Calculate the modulation index for the wave.

Sol:- Given : $f_c = 93.2\,MHz$, $f_m = 5KHz$, $\Delta f = 40KHz$.

a) Carrier Swing $= 2 \times \Delta f$

$= 2 \times 40KHz$

$= 80KHz$

b) $[f_i]_{max} = f_c + \Delta f$

$= 93.2MHz + 40KHz$

$= 93.24\,MHz$

$(f_i)_{min} = f_c - \Delta f$

$= 93.2MHz - 40KHz$

$= 93.16MHz$

c) Modulation Index $= \dfrac{\Delta f}{f_m} = \dfrac{40KHz}{5KHz} = 8$

NOTE:-

$\Delta f = (f_i)_{max} - (f_i)_{min}$

$= 93.24MHz - 93.16MHz$

$= 80KHz$

When a 50.4 MHz carrier is frequency modulated by a sinusoidal AF modulating signal, the highest frequency reached is 50.405 MHz. Calculate

a) The frequency deviation produced

b) Carrier swing of the wave

c) Lowest frequency reached.

Sol :-

a) $\Delta f = (f_i)_{max} - f_c = 50.405\ MHz - 50.4\ MHz = 5\ KHz$

b) $\text{Carrier Swing} = 2 \times \Delta f = 2 \times 5\ KHz = 10\ KHz$

c) $(f_i)_{min} = f_c - \Delta f = 50.4\ MHz - 5\ KHz = 50.395\ MHz$

What is the bandwidth required for a FM signal if the modulating frequency is 1KHz & the maximum deviation is 10KHz? What is bandwidth required for a DSBSC (AM) transmission? Comment on the result.

Sol :- Given : $\Delta f = 10\ KHz$, $f_m = 1\ KHz$.

* The Carson's rule to find out the bandwidth of FM :

$$BW = 2[\Delta f + f_m]$$

$$= 2[10 + 1]$$

$$\boxed{BW = 22\ KHz}$$

* The BW of DSBSC (AM) System $= 2f_m = 2(1\ KHz) = 2\ KHz$.

* This result shows that for the transmission of same signal the bandwidth required for FM is very much higher than that of an AM System.

In an FM System, When the audio frequency is 500Hz & modulating voltage 2.5 V, the deviation produced is 5KHz. If the modulating voltage is now increased to 7.5V, Calculate the new value of frequency deviation produced. If the AF voltage is raised to 10V while the modulating frequency dropped to 250 Hz, what is the frequency deviation? Calculate the modulation index in each case.

Sol :- Given : $f_m = 500 Hz$, $A_m = 2.5V$, $\Delta f = 5KHz$.

i) W.K.T $\Delta f = K_f A_m$

$$K_f = \frac{\Delta f}{A_m} = \frac{5KHz}{2.5V} = 2KHz/V.$$

$$\beta = \frac{\Delta f}{f_m} = \frac{5KHz}{500Hz} = 10$$

ii) When modulating voltage is raised to 7.5V i.e. $A_m = 7.5V$
$\Delta f = ?, \beta = ?$

$$\Delta f = K_f A_m = 2KHz/V \times 7.5V = 15KHz.$$

$$\beta = \frac{\Delta f}{f_m} = \frac{15KHz}{500Hz} = 30$$

iii) When modulating voltage is raised to 10V i.e. $A_m = 10V$ &
$f_m = 250Hz$.

$$\Delta f = K_f A_m = 2KHz/V \times 10V = 20KHz$$

$$\beta = \frac{\Delta f}{f_m} = \frac{20KHz}{250Hz} = 80$$

A 2KHz Sinusoidal Signal phase modulates a Carrier at 100MHz With a peak phase deviation of 45°. Using Carson's rule evaluate the approximate bandwidth of the PM Signal.

Sol:- The time-domain expression for a PM Wave is

$$S(t) = A_c \cos[2\pi f_c t + K_p \underline{m(t)}]$$

W.K.T $m(t) = A_m \cos 2\pi f_m t$, We get

$$S(t) = A_c \cos[2\pi f_c t + \underline{K_p A_m} \cos 2\pi f_m t]$$

$$S(t) = A_c \cos[2\pi f_c t + \beta_p \cos 2\pi f_m t]$$

$$S(t) = A_c \cos[\theta_c + \beta_p \cos 2\pi f_m t]$$

$$\boxed{S(t) = A_c \cos[\theta_i(t)]}$$

* The Instantaneous phase,

$$\theta_i(t) = \theta_c + \beta_p \cos 2\pi f_m t.$$

* Peak-phase deviation :-

$$\Delta\theta = |\theta_i(t) - \theta_c|_{max}$$

$$\boxed{\Delta\theta = \beta_p}$$

Given $\beta_p = \frac{\pi}{4}$ radians

* Using Carson's rule, the transmission bandwidth of the PM Wave is given by:

$$B_T = 2[\beta_p + 1] f_m$$

$$= 2\left[\frac{\pi}{4} + 1\right] 2\times10^3$$

$= 2[0.785 + 1]\, 2\times10^3$

$\boxed{B_T = 7.14\,KHz}$

In a FM System, the modulating frequency $f_m = 1KHz$, the modulating voltage $A_m = 2V$ & the deviation is $6KHz$. If the modulating voltage is raised to $4V$ then what is the new deviation? If the modulating voltage is further increased to $8V$ & modulating frequency is reduced to $500Hz$. what will be deviation? Calculate the modulation Index in each Case. Comment on the result.

Sol:- Given: $f_m = 1KHz$, $A_m = 2V$, $\Delta f = 6KHz$.

WKT $\Delta f = K_f A_m$

$$K_f = \frac{\Delta f}{A_m} = \frac{6KHz}{2V} = \underline{3KHz/V}$$

$$\beta = \frac{\Delta f}{f_m} = \frac{6KHz}{1KHz} = \underline{6}$$

* ∴ When modulating voltage is raised to 4V i.e. $A_m = 4V$.

$\Delta f = ?$

$$\Delta f = K_f A_m = (3KHz/V) \times 4V = \underline{12KHz}$$

$$\beta = \frac{\Delta f}{f_m} = \frac{12KHz}{1KHz} = \underline{12}$$

* For $A_m = 8V$ & $f_m = 500Hz$

$$\Delta f = K_f A_m = (3KHz/V) \times 8V = \underline{24KHz}$$

$$\beta = \frac{\Delta f}{f_m} = \frac{24KHz}{500Hz} = \underline{48}$$

Comment:-

The modulation index is dependent on the value of deviation as well as the modulating frequency.

The Carrier Swing of a FM Signal is 70KHz & the modulating Signal is a 7KHz Sine wave. Determine the modulation - Index of the FM Signal.

Sol:- Given Carrier Swing = 70 KHz, f_m = 7KHz. β = ?

$$\beta = \frac{\Delta f}{f_m}$$

$$\Delta f = \frac{\text{Carrier Swing}}{2} = \frac{70\,KHz}{2} = 35\,KHz$$

$$\beta = \frac{35\,KHz}{7\,KHz} = 5$$

NOISE

1. Define: Shot noise, Thermal noise, Noise figure. Jan 09 (6)
2. Derive the relation between noise figure and equivalent noise temperature.

   Jan 09 (6)
3. Explain the following terms: July 09 (10)
   a. Shot noise
   b. Thermal noise
   c. White noise
   d. Noise figure
   e. Transit time noise
4. Derive an expression for overall Equivalent noise temperature of the cascade connection of any number of noises for two port network. July 09 (5)
5. Define and explain the following and obtain the relation between them: July 09 (7)
   a. Noise figure
   b. Equivalent noise temperature.
6. What is noise equivalent band width? Derive an expression for noise equivalent bandwidth

   Jan 05 (8)
7. List and explain the types of noise which occur in an electronic circuit. July 05 (8)
8. Define noise figure and noise temperature along with related equations. July 05 (6)
9. Define and explain the following and obtain the relation between them: Jan 06 (6)
   a. Noise figure
   b. Equivalent noise temperature

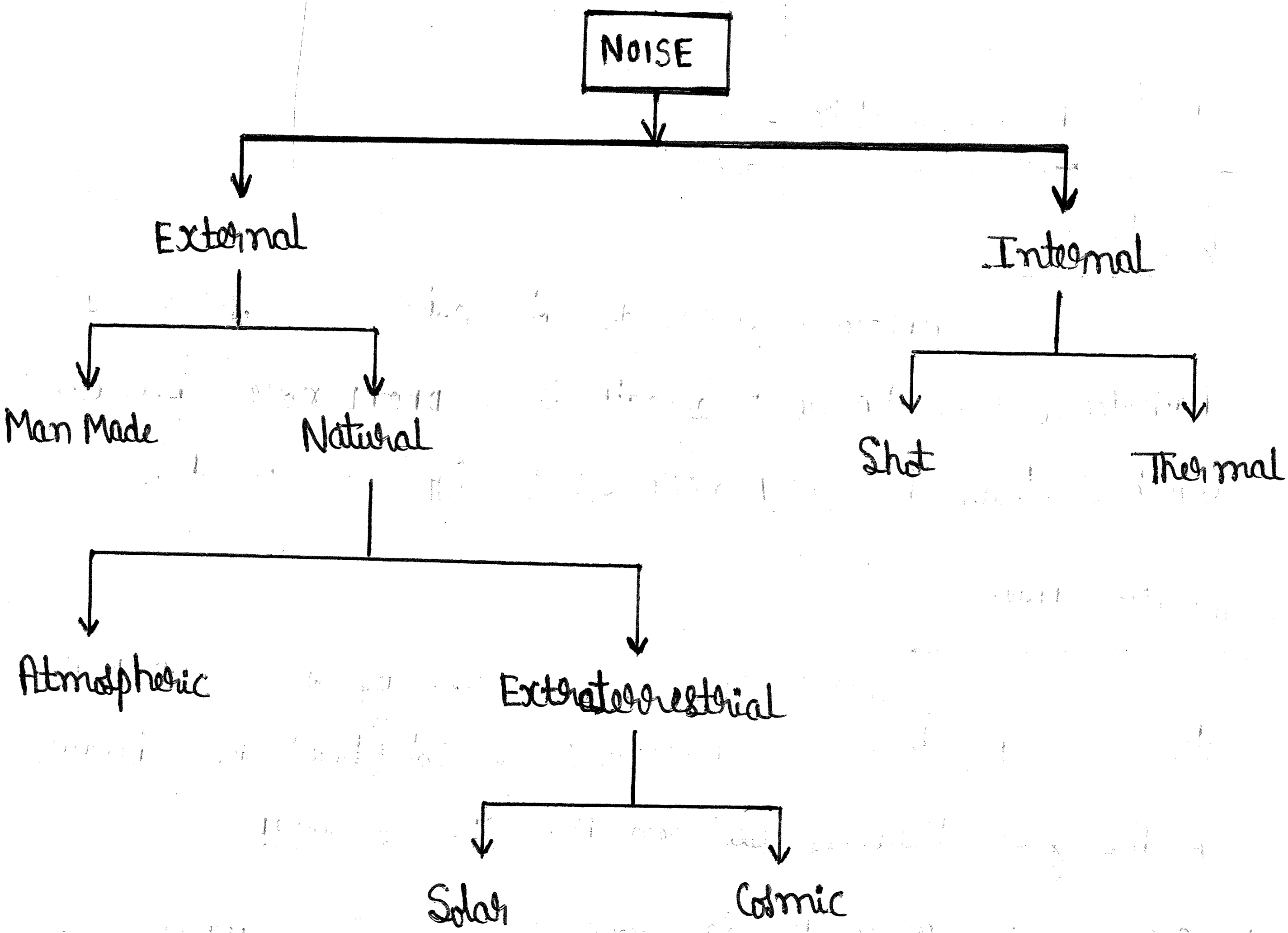
NOISE

NOISE :-

Noise is an unwanted Signal. Noise is random in nature & interferes with the desired Signal.

* Noise disturb the proper reception & reproduction of transmitted Signals.

Classification of Noise :-



I> External Noise:-

1> Atmospheric Noise :-

Atmospheric noise is also called as Static Noise. The atmospheric noise is mainly due to electrical disturbances, Such as lightning.

* Lightning refer to electric discharges that occur between clouds or between the earth & clouds.
  This results in a transient electrical signals that generates harmonic energy that can travel extremely long distances.

* Atmospheric noise effects the reception at frequencies less than 30MHZ & less effected for the frequencies above 30MHZ.

2> Extra terrestrial Noise :-

i> Solar Noise :-

Primary source of Solar noise is Sun. The Sun radiates a wide range of signals in a broad noise spectrum which includes the frequencies we use for communication.

ii> Cosmic Noise :-

Cosmic Noise is generated by stars, which are also Suns. The level of cosmic noise is not (high) great because of the great distances between the stars & earth.

* Cosmic noise effects the frequency ranging from 15KHZ to 150MHZ.

II) Internal Noise :-

Internal noise is generated internally in the circuit. Electronic components such as resistors, diodes and transistors etc produce this noise.

1) Shot Noise :-

* Shot Noise arises in electronic devices because of the discrete (pulse) nature of current flow in the device.
* Shot noise appears in the active devices due to random behaviour of charge carriers (electrons & holes).
* In vaccum tubes shot noise is generated due to random emission of electrons from the cathode.
* In semiconductor devices due to random diffusion of electrons & the random recombination of electrons with holes.
* In a photo diode, it is the random emission of photons.
* The nature of current variation w.r.to time in a vaccum diode is as shown in fig below.

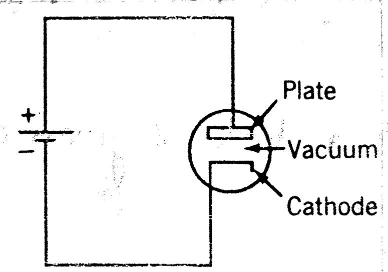


Fig ①: vaccum diode

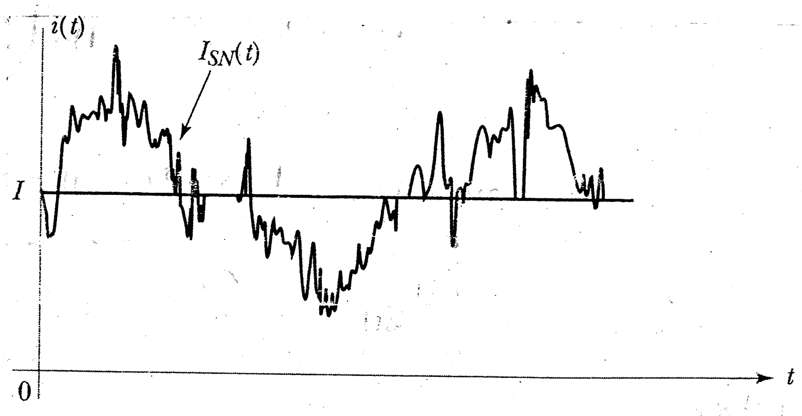


Figure 2 ■ Shot noise current.

* Fig ② Shows the current fluctuation over the mean value 'I'.

* The fluctuating current is called Shot Noise & denoted as 'I_{SN}'. The fluctuating current $I_{SN}(t)$ is not observed by normal measuring instrument i.e. it look like constant current 'I'.

* The fluctuating nature of $I_{SN}(t)$ can be seen only in fast acting oscilloscopes.

∴ The total current flowing in the vaccum diode

$$\boxed{i(t) = I + I_{SN}(t)}$$

* For a vaccum diode, the mean square value of randomly - fluctuating component of current is given by :

$$\boxed{\overline{I^2_{SN}} = 2qIB_N \text{ amps}^2}$$

Where,

$q \rightarrow$ electron charge equal to 1.6×10^{-19} Coulombs.

$I \rightarrow$ The mean value of the current in amperes &

$B_N \rightarrow$ Noise bandwidth in Hz.

* Shot Noise has a uniform spectral density (like Thermal Noise)

* The mean square shot noise current for a diode is given as:

$$\boxed{\overline{I^2_{SN}} = 2q(I + 2I_S)B_N}$$

Where, $I \rightarrow$ dc current across the junction

$I_S \rightarrow$ Reverse Saturation current

$q \rightarrow$ electron charge $= 1.6 \times 10^{-19}$ C

$B_N \rightarrow$ Noise Bandwidth.

---

1) A noise generator using diode is required to produce 15 µV noise-voltage in a receiver which has an I/p impedance of 75Ω (purely resistive). The receiver has a noise power bandwidth of 200 KHz. Calculate the current through the diode.

Sol:- Given: $V_{SN} = 15\,\mu V$, $R = 75\Omega$, $B_N = 200\,KHz$, $I = ?$

W.K.T. $I_{SN}^2 = 2q(I + 2I_S)B_N$

$I \gg I_S$, neglecting I_S.

$$I_{SN}^2 = 2q(I)B_N. \rightarrow (1)$$

W.K.T. $I_{SN} = \frac{V_{SN}}{R} = \frac{15\,\mu V}{75\Omega} = 0.2\,\mu A$

* From eq (1),

$$I = \frac{I_{SN}^2}{2q B_N} = \frac{(0.2\,\mu A)^2}{2 \times 1.6 \times 10^{-9} \times 200\,KHz}$$

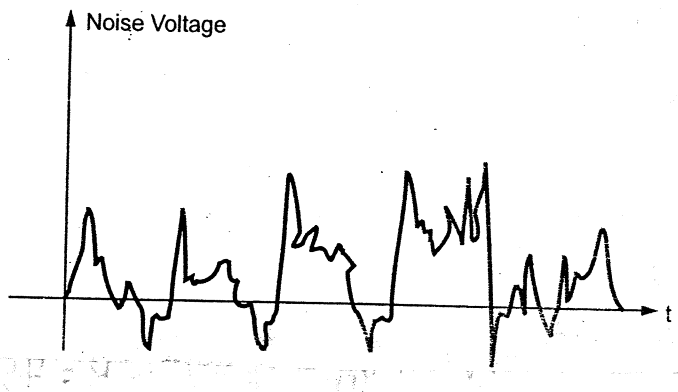
$$\therefore \boxed{I = 625\,mA}$$

2) Thermal Noise & Johnson's Noise :-

* The random movement of electrons inside the conductor - resulting in a randomly varying voltage across the conductor as shown in fig.

Figure

Thermal noise



* This randomly varying noise voltage produced across the conductor is called as thermal Noise. It is also known as Johnson Noise.

* The power spectral density of thermal noise produced by a resistor is given by:

$$\boxed{S_{TN}(f) = \frac{2h|f|}{\exp(h|f|/KT)-1}} \longrightarrow (1)$$

Where, $T \rightarrow$ absolute temperature in degrees Kelvin.

$K \rightarrow$ Boltzmann's constant i.e. 1.38×10^{-23} Joules/°K

$h \rightarrow$ Planck's constant i.e. 6.63×10^{-34} Joules/Sec.

* The power spectral density for low frequency is defined by $f \ll \frac{KT}{h}$, we may use the approximation

$$\boxed{\exp\left(\frac{h|f|}{KT}\right) = 1 + \frac{h|f|}{KT}} \longrightarrow (2)$$

Substituting eq ② in eq ①, we get

$$S_{TN}(f) = \frac{2h|f|}{\cancel{1} + \frac{h|f|}{KT} - \cancel{1}} = \frac{2h|f|}{\frac{h|f|}{KT}} = \frac{\cancel{2h|f|}\,KT}{\cancel{h|f|}}$$

$$\boxed{S_{TN}(f) = 2KT} \longrightarrow ③$$

* The mean Square value of the thermal noise voltage measured across the terminals of the resistor equals

$$\boxed{V_{TN}^2 = 2RB_N S_{TN}(f)} \longrightarrow ④$$

Substituting eq ③ in eq ④, we get

$$V_{TN}^2 = 2RB_N(2KT)$$

$$\boxed{V_{TN}^2 = 4KTB_NR \text{ volts}^2}$$

Where,

$V_{TN} \rightarrow$ root-mean Square noise voltage

$K \rightarrow$ Boltzmann's Constant

$T \rightarrow$ Temperature of the Conductor in Kelvins

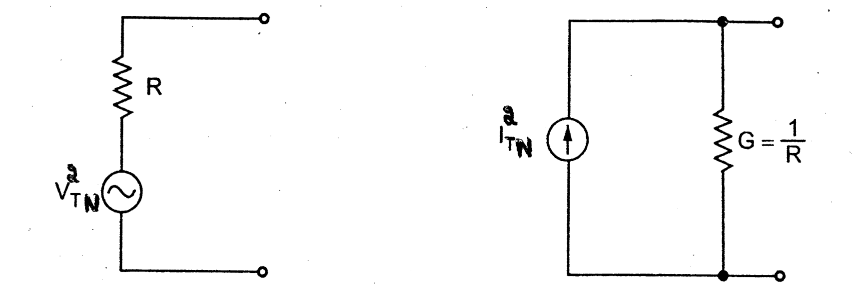
$B_N \rightarrow$ Noise bandwidth in Hz.

$R \rightarrow$ Resistance of the Conductor in ohms.

Equivalent Noise Sources for the thermal Noise :-

▶ **Figure**

Equivalent noise sources for thermal noise



(a) Thevenin equivalent circuit **(b) Norton equivalent circuit**

* Fig ⓐ Shows a model of a Noisy resistor.

The Thevenin equivalent CKt Consisting of a Noise voltage generator with a mean-Square values of V_{TN}^2 in Series with a noiseless resistor.

* Similarly Fig ⓑ Shows Norton equivalent CKt Consisting of a Noise current generator in parallel with a Noiseless Conductance.

The mean-Square value of the Noise current generator is :

$$I_{TN}^2 = \frac{V_{TN}^2}{R^2} = \frac{4KTB_N \cancel{R}}{R^{\cancel{2}}} = 4KTB_N \frac{1}{R}$$

$$\boxed{I_{TN}^2 = 4KTB_N G} \text{ amps}^2$$

Where, $G = \frac{1}{R}$ is the Conductance.

Available Noise power :-

* The Root-mean Square value of the voltage V_{RMS} across the matched load R_L is

$$\boxed{V_{RMS} = \frac{\sqrt{V_{TN}^2}}{2}}$$

* The maximum average Noise power delivered to the load is :

$$P_n = \frac{V_{RMS}^2}{R} = \frac{V_{TN}^2}{4R} = \frac{\cancel{4}KTB_N\cancel{R}}{\cancel{4R}}$$

$$\boxed{P_n = KTB_N}$$

Thus, the available Noise power 'P_a' is equal to 'KTB_N' & is independent of 'R'.

FORMULAE :

1) RMS Noise voltage : $V_{TN}^2 = 4KTB_NR$

$$\boxed{V_{TN} = \sqrt{4KTB_NR}}$$

2) Thermal Noise power

$$\boxed{P_n = KTB_N}$$

1) Calculate the rms noise voltage and thermal noise power appearing across a 20 kΩ resistor at 25°C temperature with an effective noise bandwidth of 10 KHz.

Sol :- Given : $R = 20k\Omega$, $T = 273 + 25 = 298K$, $B_N = 10\,KHz$, $K = 1.38\times10^{-23}$

* $V_{TN} = \sqrt{4KTB_NR} = \sqrt{4\times1.38\times10^{-23}\times298\times10\times10^{3}\times20\times10^{3}}$

$$\boxed{V_{TN} = 1.81\,\mu V}$$

* $P_n = KTB_N = 1.38\times10^{-23}\times298\times10\times10^{3}$

$$\boxed{P_n = 4.11\times10^{-17}}$$

2) A receiver has a noise power bandwidth of 12 KHZ. A resistor which matches with the receiver I/p Impedance is connected across the antenna terminals. What is the noise power contributed by this resistor in the receiver bandwidth? Assume temperature to be 30°c.

Sol :- Given : $B_N = 12KHZ$, $T = 30°c + 273 = 303°K$, $K = 1.38 \times 10^{-23}$

$$P_n = KTB_N = 1.38 \times 10^{-23} \times 303 \times 12 \times 10^3$$

$$\boxed{P_n = 5.01768 \times 10^{-17}\ W}$$

3) A 600 Ω resistor is connected across the 600 Ω antenna input of a radio receiver. The bandwidth of the radio receiver is 20KHZ & the resistor is at room temperature of 27°c. Calculate the noise power & the noise voltage applied at the I/p of the receiver.

Sol :- Given : $R_1 = 600\ \Omega$, $R_2 = 600\ \Omega$, $B_N = 20 \times 10^3\ HZ$, $T = 27 + 273 = 300°K$.

$P_n = ?$, $V_{TN} = ?$, $K = 1.38 \times 10^{-23}$

* Noise power : $P_n = KTB_N = 1.38 \times 10^{-23} \times 300 \times 20 \times 10^3$

$$\boxed{P_n = 8.28 \times 10^{-17}\ W}$$

* Noise voltage at the receiver I/p :

Since the two resistors are in parallel

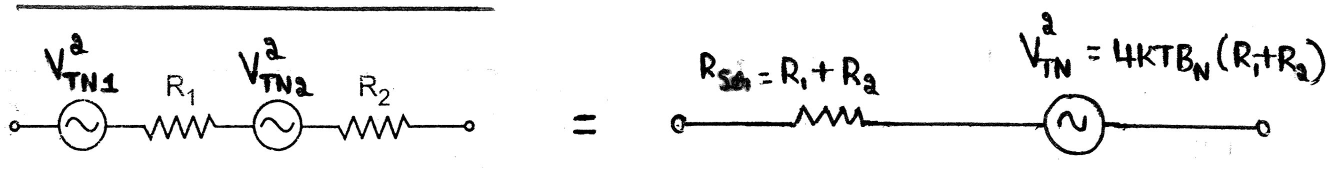
* $R = R_1 \| R_2 = 600\Omega \| 600\Omega = 300\ \Omega$.

$\therefore$ Noise voltage $V_{TN} = \sqrt{4KTB_N R} = \sqrt{4 \times 1.38 \times 10^{-23} \times 300 \times 20 \times 10^3 \times 300}$

$$\boxed{V_{TN} = 0.3152\ \mu V}$$

Thermal Noise Calculation :-

1) In Series resistors :-



(a) Series resistors

(b) Equivalent Ckt

* Fig (a) Shows two resistors R_1 & R_2 connected in Series.

∴ The total series resistance is given as

$$R_{ser} = R_1 + R_2$$

W.K.T

$$V_{TN}^2 = 4KTB_N R_{ser}$$

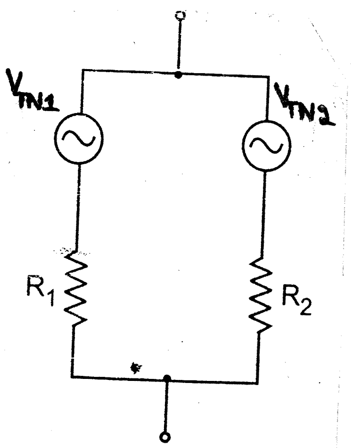
$$= 4KTB_N [R_1 + R_2]$$

$$V_{TN}^2 = 4KTB_N R_1 + 4KTB_N R_2$$

$$V_{TN}^2 = V_{TN1}^2 + V_{TN2}^2$$

$$\boxed{V_{TN} = \sqrt{V_{TN1}^2 + V_{TN2}^2}}$$

2) In parallel resistor :-



Parallel resistors

* When two resistors R_1 & R_2 are connected in parallel, then total Noise voltage V_{TN} is obtained.

$$V_{TN}^2 = 4KTB_N R_P$$

Where R_P = Equivalent resistance of parallel resistors R_1 & R_2 i.e.

$$R_P = R_1 \parallel R_2$$

1) Three 5 kΩ resistors are connected in series. For room temperature $KT = 4\times10^{-21}$ & an effective noise bandwidth of 1 MHz, determine

a) The noise voltage appearing across each resistor.

b) The noise voltage appearing across the series combination.

c) What is the rms noise voltage which appears across same three resistors connected in parallel under the same conditions?

Sol :- Given : $R_1 = R_2 = R_3 = 5K\Omega$, $KT = 4\times10^{-21}$, $B_N = 1MHz$.

a) $$V_{TN} = \sqrt{4KTB_NR} = \sqrt{4\times4\times10^{-21}\times1\times10^{6}\times5\times10^{3}}$$

$$\boxed{V_{TN} = 8.94\ \mu V}$$

b) $$R_{ser} = R_1+R_2+R_3 = 5K\Omega+5K\Omega+5K\Omega = 15K\Omega.$$

$$V_{TN} = \sqrt{4KTB_NR_{ser}} = \sqrt{4\times4\times10^{-21}\times1\times10^{6}\times5\times10^{3}}$$

$$\boxed{V_{TN} = 15.5\ \mu V}$$

c) $$R_p = R_1 \| R_2 \| R_3$$

$$\frac{1}{R_p} = \frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3} = \frac{1}{5K\Omega}+\frac{1}{5K\Omega}+\frac{1}{5K\Omega} = \frac{1}{6\times10^{-4}}$$

$$\boxed{R_p = 1.66\ K\Omega}$$

$$V_{TN} = \sqrt{4KTB_NR_p} = \sqrt{4\times4\times10^{-21}\times1\times10^{6}\times1.66\times10^{3}}$$

$$\boxed{V_{TN} = 5.15\ \mu V}$$

2) Two resistors 20 kΩ & 50 kΩ are at room temperature 290°K. Calculate for the bandwidth of 100 KHz, the thermal noise for the following Conditions:

i) For each resistor ii) For two resistors in Series

iii) For two resistors in parallel.

Sol: Given: $T = 290^\circ K$, $B_N = 100 KHz$, $R_1 = 20 k\Omega$, $R_2 = 50 k\Omega$,

$K = 1.38 \times 10^{-23}$

* $KT = 1.38 \times 10^{-23} \times 290 = 4 \times 10^{-21}$

i) For 20 kΩ resistor:

$$V_{TN} = \sqrt{4KTB_N R_1} = \sqrt{4 \times 4 \times 10^{-21} \times 100 \times 10^3 \times 20 \times 10^3}$$

$$\boxed{V_{TN} = 5.65 \mu V}$$

For 50 kΩ resistor

$$V_{TN} = \sqrt{4KTB_N R_2} = \sqrt{4 \times 4 \times 10^{-21} \times 100 \times 10^3 \times 50 \times 10^3}$$

$$\boxed{V_{TN} = 8.94 \mu V}$$

ii) $R_{ser} = R_1 + R_2 = 20 k\Omega + 50 k\Omega = 70 k\Omega$

$$V_{TN} = \sqrt{4KTB_N R_{ser}} = \sqrt{4 \times 4 \times 10^{-21} \times 100 \times 10^3 \times 70 \times 10^3}$$

$$\boxed{V_{TN} = 10.58 \mu V}$$

iii) $R_p = R_1 \parallel R_2 = 20 k\Omega \parallel 50 k\Omega = 14.28 k\Omega$

$$V_{TN} = \sqrt{4KTB_N R_p} = \sqrt{4 \times 4 \times 10^{-21} \times 100 \times 10^3 \times 14.28 \times 10^3}$$

$$\boxed{V_{TN} = 4.78 \mu V}$$

3) An amplifier operating over a frequency range from 17 to 19 MHz has a I/p resistance of 5 KΩ. What is the rms thermal noise voltage at the I/p of this amplifier? Assume the operating temperature to be 27°c.

Sol:- W.K.T. Bandwidth is given by: $f_2 - f_1$

$$B_N = 19\,MHz - 17\,MHz = \underline{2\,MHz.}$$

Given: $R = 5\,K\Omega$, $T = 27°c + 273 = 300°K$, $B_N = 2\,MHz$,

$K = 1.38 \times 10^{-23}$.

$$V_{TN} = \sqrt{4KTB_N R} = \sqrt{4 \times 1.38 \times 10^{-23} \times 300 \times 2 \times 10^{6} \times 5 \times 10^{3}}$$

$$\boxed{V_{TN} = 12.86\ \mu V}$$

Man made Noise & Industrial Noise :-

* Industrial noise is produced by aircraft ignition, electrical motors, welding machines, ignition System of the automobile etc.
* This noise effect the Signal having frequency range 1 MHz to 600 MHz.

Transit time Noise & High frequency Noise :-

* Transit time is the time taken by an electron to travel from cathode to anode & the time taken by the charge carriers to cross a p-n junction.

* The transit time occupies a small portion of the I/p period at lower frequencies. As the frequency is increased, the transit time occupies a considerable portion of the I/p period.

  In such a situation, the charge carriers may start diffuse back to the source i.e. emitter in the case of a transistor without reaching the collector.

* The diffusion of the carrier back to the source give rise to an I/p admittance in which the conductance increases with frequency.

* The noise current generator associated with this conductance increases with frequency.

* At very high frequencies it becomes a predominant noise component.

Flicker Noise or Low frequency Noise :-

* The flicker noise will appear at low frequency's. It is sometimes called as "$1/f$" noise.

* In the semiconductor devices, the flicker noise is generated - to the fluctuations in the carrier density.

Partition Noise :-

* partition Noise is generated when the current gets divided between two or more paths.

* It is generated due to the random fluctuations in the current divisions.

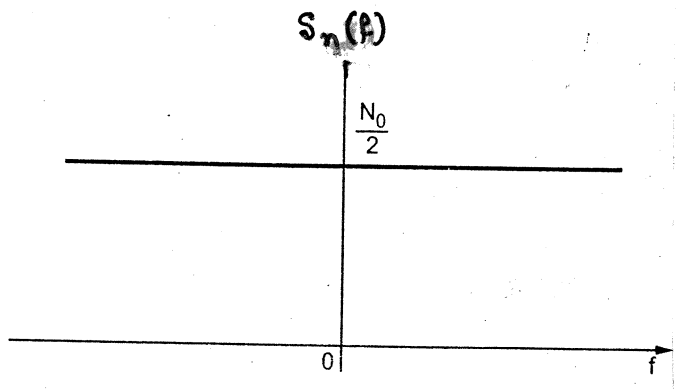
∴ The partition noise in a transistor will be higher than that in a diode.

White Noise :-

White Noise is the noise whose power spectral density is uniform over the entire frequency range as shown in fig.

▶ **Figure**

(a) Power spectral density of white noise



* The spectral density of white Noise is given by

$$\boxed{S_n(f) = \frac{N_0}{2}}$$

Where, $N_0 = KT_e$

$K \rightarrow$ Boltzmann's Constant

$T_e \rightarrow$ Equivalent noise temperature of the System.

Formulae :-

1) The Fourier transform of auto correlation function

$$R(\tau) \xrightarrow{FT} S(f)$$

2) The Inverse Fourier transform of $S(f)$ is $R(\tau)$ i.e.

$$R(\tau) = IFT|S(f)|$$

3) The IFT of

$$\frac{N_0}{2} \xrightarrow{IFT} \frac{N_0}{2}\,\delta(t)$$

1) Derive and plot the auto correlation function of a white gaussian noise which has a power spectral density of $N_0/2$.

Sol:- The auto correlation function is denoted by $R(\tau)$

* Taking FT of $R(\tau)$

$$FT[R(\tau)] \longrightarrow S(f)$$

or

$$IFT[S(f)] = R(\tau)$$

* Thus

$$R(\tau) = IFT[S(f)] \longrightarrow (1)$$

WKT $\boxed{S(f) = \frac{N_0}{2}} \longrightarrow (2)$

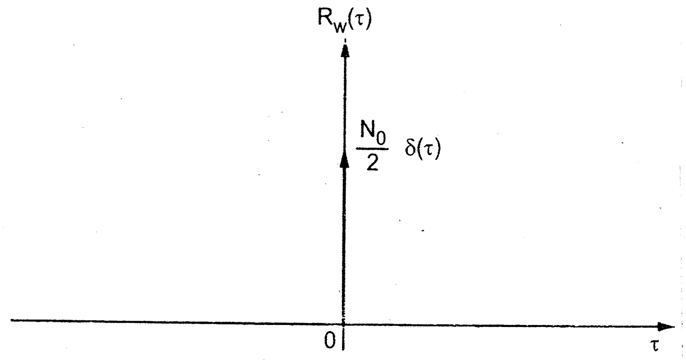
Substituting eq (2) in eq (1), we get

$$R(\tau) = IFT\left[\frac{N_0}{2}\right]$$

$$\boxed{R(\tau) = \frac{N_0}{2}\,\delta(t)}$$

Figure

Autocorrelation function of white noise



Noise Equivalent Bandwidth :-

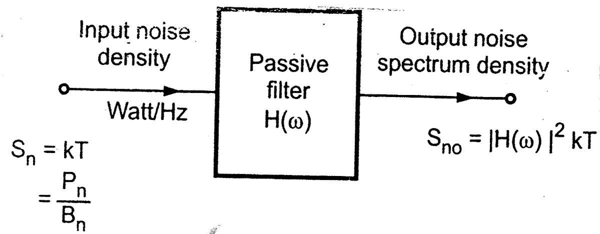
*) What is noise equivalent bandwidth? Derive an expression for noise equivalent bandwidth.

Jan - 2005, 8M

* Consider a passive filter having voltage-ratio transfer function $H(\omega)$. Let the I/p noise Spectrum density be

$$S_n = KT = \frac{P_n}{B_n}$$

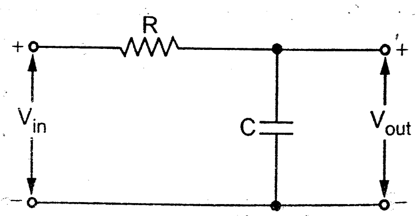
Where, P_n is noise power.



* The o/p Noise Spectrum density S_{no}, for an I/p density of $S_n = KT$ is

$$\boxed{S_{no} = |H(\omega)|^2 \cdot KT} \longrightarrow (1)$$

* Consider the passive R-C LPF Shown below.



* The transfer function is given by:

$$H(S) = \frac{V_{out}}{V_{in}} = \frac{1/SC}{R+\frac{1}{SC}} = \frac{1/\cancel{SC}}{\frac{RSC+1}{\cancel{SC}}}$$

$$H(S) = \frac{1}{1+SCR}$$

$$|H(w)| = \frac{1}{\sqrt{1+(WCR)^2}}$$

$$\boxed{|H(W)|^2 = \frac{1}{1+(WCR)^2}} \longrightarrow ②$$

Substituting eq ② in eq ①, we get

$$S_{no} = \frac{1}{1+(WCR)^2} \cdot KT$$

$$\boxed{S_{no} = \frac{KT}{1+(WCR)^2}} \longrightarrow ③$$

* The o/p Spectrum density, S_{no} decreases as the frequency increases. The total noise power is obtained by integrating S_{no} over the frequency range from 0 to ∞.

i.e. $P_{no} = \int_0^\infty S_{no}\, df \longrightarrow ④$

Substituting eq ③ in eq ④, we get

$$P_{no} = \int_0^\infty \frac{KT}{1+(WCR)^2} \cdot df$$

WKT $\boxed{W = 2\pi f}$

$$P_{no} = \int_0^{\pm} \frac{KT}{1+(2\pi f RC)} df$$

Let $2\pi f RC = x \rightarrow (a)$

differentiating eq (a) w.r.to 'f'

$$\frac{df}{dx} 2\pi RC = (1)$$

$$df = \frac{dx}{2\pi RC}$$

The limits remain unchanged

$$P_{no} = \int_0^\infty \frac{KT}{1+x^2} \cdot \frac{dx}{2\pi RC}$$

$$P_{no} = \frac{KT}{2\pi RC} \int_0^\infty \frac{dx}{1+x^2}$$

$$= \frac{KT}{2\pi RC} \int_0^\infty \frac{1}{1+x^2} dx$$

$$= \frac{KT}{2\pi RC} \left[\tan^{-1} x\right]_0^\infty$$

$$= \frac{KT}{2\pi RC} \left[\tan^{-1}(\infty) - \tan^{-1}(0)\right]$$

$$P_{no} = \frac{KT}{2\cancel{\pi} RC} \left[\frac{\cancel{\pi}}{2} - 0\right] = \frac{KT}{2RC} \cdot \frac{1}{2}$$

∴ The total noise power at the o/p is

$$\boxed{P_{no} = \frac{KT}{4RC}} \longrightarrow (5)$$

Comparing eq (5) with $P_n = KTB_N$, we get

Effective Noise BW as $\boxed{B_N = \frac{1}{4RC}} \longrightarrow (6)$

* The rms noise voltage, V_N will be given by

$$V_N^2 = 4KTB_N \cdot R \longrightarrow (7)$$

Substituting eq ⑥ in eq ⑦, we get

$$V_N^2 = \cancel{4}KT\left(\frac{1}{\cancel{4R}C}\right)\cdot\cancel{R}$$

$$\boxed{V_N^2 = \frac{KT}{C}}$$

* Although the capacitance 'C' does not contribute to the noise, it acts as a limiting factor to the rms noise voltage.

1) A signal circuit is equivalent to a parallel combination of $R = 1K\Omega$ & $C = 0.47\,\mu F$. Calculate the effective noise bandwidth.

Sol :- Effective bandwidth

$$B_N = \frac{1}{4RC} = \frac{1}{4\times 1\times 10^3 \times 0.47\times 10^{-6}} = \underline{531.915\ Hz}$$

Signal to Noise Ratio :- (SNR)

* Signal to Noise ratio is defined as the ratio of signal power to Noise power.

$$(SNR) = \frac{S}{N} = \frac{P_s}{P_n}$$

$$= \frac{V_s^2/\cancel{R}}{V_N^2/\cancel{R}}$$

$$\boxed{\frac{S}{N} = \left(\frac{V_S}{V_N}\right)^2}$$

* The signal to Noise ratio interms of decibels :

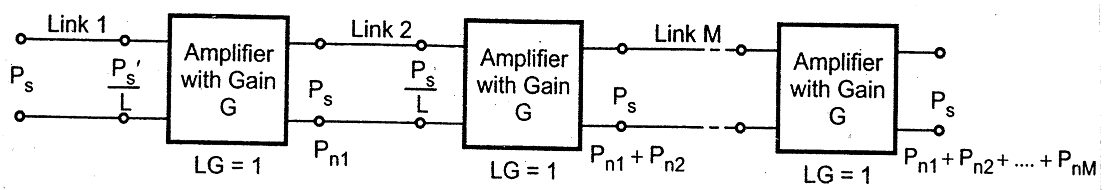
$$\left(\frac{S}{N}\right)_{dB} = 10\log\left(\frac{V_S}{V_N}\right)^2$$

$$\boxed{\left(\frac{S}{N}\right)_{dB} = 20 \log \left(\frac{V_S}{V_N}\right)}$$

* Signal to Noise ratio of a Tandem Connection :-

▶ **Figure**

Tandem connection



* In telephone Systems, telephone Cables are used as media to transmit Signals. The Signals gets attenuated as it travels through telephone Cables due to power loss in the telephone Cables. To make up this power loss the Signal is amplified Such that, if the power loss of a line Section is 'L', then the amplified power gain 'G' is Chosen So that $LG = 1$.

* A long telephone line is divided into equal Sections called links.

* As Signals travel through these links, each amplifier adds its own noise to the System.

Therefore at the receiving end we get the accumulated Noise power as Shown in Fig above.

* The total Noise power at the o/p of the M^{th} link is

$$P_n = P_{n1} + P_{n2} + P_{n3} + \dots + P_{nM}$$

Where,

P_{n1} = Noise power added at the end of 1^{st} link

P_{n2} = Noise power added at the end of 2^{nd} link.

P_{n3} = Noise power added at the end of 3^{rd} link.

⋮

P_{nM} = Noise power added at the end of M^{th} link.

* If links are identical such that each link adds Noise power 'P_n' then the total Noise power is given as:

$$\boxed{P_{n\,total} = M \times P_n}$$

∴ The o/p Signal to Noise ratio is:

$$\left(\frac{S}{N}\right)_{M\,dB} = 10\log\left(\frac{P_s}{P_{n\text{-}total}}\right)$$

$$= 10\log\left(\frac{P_s}{MP_n}\right)$$

$$= 10\log\left(\frac{P_s}{P_n}\right) - 10\log(M)$$

$$\boxed{\left(\frac{S}{N}\right)_{M\,dB} = \left(\frac{S}{N}\right)_{1\,dB} - (M)_{dB}}$$

Where,

$(M)_{dB}$ → Signal to Noise ratio at the end of M-links

$\left(\frac{S}{N}\right)_{1\,dB}$ → Signal to Noise ratio at the end of 1^{st} link.

1) Calculate the o/p Signal to Noise ratio in decibels for Four identical links. Assume that Signal to Noise of each link is 80 dB.

Sol :- Given : $\left(\frac{S}{N}\right)_{dB} = 80\,dB$, $M = 4$.

$(M)_{dB} = 10 \log (M)$

$(M)_{dB} = 10 \log (4) = 6.02\,dB$

$$\left(\frac{S}{N}\right)_{M\,dB} = \left(\frac{S}{N}\right)_{1\,dB} - (M)_{dB}$$

$$= 80\,dB - 6.02\,dB$$

$$\boxed{\left(\frac{S}{N}\right)_{M\,dB} = 73.98\,dB}$$

Noise Factor :-

* The Noise factor 'F' of an amplifier or any Network is defined interms of Signal to Noise ratio is defined as:

$$\text{Noise factor, } F = \frac{\text{available S/N power ratio at the I/p}}{\text{available S/N power ratio at the o/p}} = \frac{(SNR)_i}{(SNR)_o}$$

$$F = \frac{P_{si}/P_{ni}}{P_{so}/P_{no}}$$

$$\boxed{F = \frac{P_{si}}{P_{ni}} \times \frac{P_{no}}{P_{so}}} \longrightarrow (1)$$

{ * In general any amplifier will add Noise to the I/p Signal, therefore the SNR at the o/p of the amplifier is less than the SNR at the I/p. Hence the Noise factor is a measure of degradation of the Signal to Noise ratio or the amount of noise added by the S/M }

* The available power gain 'G' is given by

$$G = \frac{\text{Signal power at the o/p}}{\text{Signal power at the I/p}}$$

$$\boxed{G = \frac{P_{so}}{P_{si}}} \longrightarrow (2)$$

From eq (1), we can re-arrange

$$F = \left(\frac{P_{si}}{P_{so}}\right) \times \frac{P_{no}}{P_{ni}} \longrightarrow (3)$$

Substituting eq (2) in eq (3), we get

$$F = \frac{1}{G} \cdot \frac{P_{no}}{P_{ni}}$$

$$F = \frac{P_{no}}{G P_{ni}}$$

$$P_{no} = F G P_{ni}$$

W.K.T the Noise power at I/p, $P_{ni} = KTB_N$

$$\boxed{P_{no} = FGKTB_N}$$

Thus with increase in the Noise factor 'F', the noise power at the o/p will increase.

NOISE Figure :-

* When noise factor is expressed in decibels, it is called Noise Figure.

$$\text{Noise Figure} = 10 \log_{10} (F)$$

$$= 10 \log_{10} \left[\frac{\text{S/N at the I/p "}(S/N)_i\text{"}}{\text{S/N at the o/p "}(S/N)_o\text{"}} \right]$$

$$= 10 \log_{10} \left[\frac{(S/N)_i}{(S/N)_o} \right]$$

$$\boxed{\text{Noise Figure } (F)_{dB} = 10 \log_{10} (S/N)_i - 10 \log_{10} (S/N)_o}$$

* The Ideal value of Noise Figure is 0 dB.

---

1) The Signal power & Noise power measured at the I/p of an amplifier are 150 μw & 1.5 μw respectively. If the Signal power at the o/p 1.5W & Noise power is 40mW, calculate the amplifier noise factor & Noise figure.

Sol :- Given ; $P_{Si} = 150\,\mu w$, $P_{ni} = 1.5\,\mu w$, $P_{So} = 1.5W$, $P_{no} = 40mw$.

* Noise Factor 'F' $= \dfrac{P_{Si}}{P_{ni}} \times \dfrac{P_{no}}{P_{So}}$

$$= \frac{150 \times 10^{-6}}{1.5 \times 10^{-6}} \times \frac{40 \times 10^{-3}}{1.5}$$

$$\boxed{F = 2.666}$$

* Noise Figure $(F)_{dB} = 10 \log_{10} (F) = 10 \log_{10} (2.666)$

$$\boxed{(F)_{dB} = 4.26 \text{ dB}}$$

2) The Signal to Noise ratio at the I/p of an amplifier is 40 dB. If the Noise Figure of an amplifier is 20 dB, calculate the Signal to Noise ratio in dB at the amplifier o/p.

Sol :- Given : $(S/N)_i = 40\,dB$, $(S/N)_o = ?$, $(S/N)_o = 20\,dB$

W.K.T Noise Figure $(F)_{dB} = (S/N)_{i\,dB} - (S/N)_{o\,dB}$

$$(S/N)_{o\,dB} = (S/N)_{i\,dB} - (F)_{dB}$$

$$= 40\,dB - 20\,dB$$

$$\boxed{(S/N)_{o\,dB} = 20\,dB}$$

Amplifier I/p Noise interms of 'F' (P_{na}) :-

The total noise at the I/p of the amplifier is given by:

$$\boxed{\text{Total } P_{ni} = \frac{P_{no}}{G}} \longrightarrow (1)$$

W.K.T $P_{no} = FGKTB_N$.

Substituting 'P_{no}' value in eq (1), we get

$$\text{Total } P_{ni} = \frac{F\cancel{G}KTB_N}{\cancel{G}}$$

$$\therefore \boxed{\text{Total } P_{ni} = FKTB_N}$$

* out of this total I/p noise power, the I/p Source Contribution is only KTB_N & the remaining is contributed by the amplifier :

$$P_{ni(total)} = P_{ni} + P_{na}$$

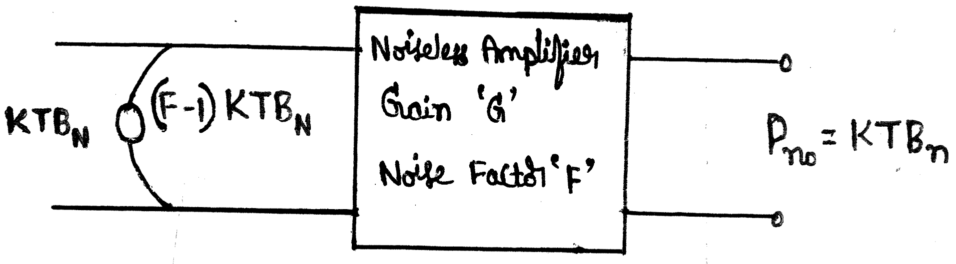
$$P_{na} = P_{ni(total)} - P_{ni}$$

$$P_{na} = FKTB_N - KTB_N$$

$$P_{na} = KTB_N (F-1)$$

$$\boxed{P_{na} = (F-1)\, KTB_N}$$

This can be Shown in below figure :



∴ The fractional of total available noise Contributed by the amplifier

$$\frac{(F-1)\cancel{KTB_N}}{(F)\cancel{KTB_N}} = \frac{(F-1)}{F}$$

Equivalent Noise Temperature at Amplifier I/P:-

Jan-06, 4M
July-09, 4M

* W.K.T, the noise power due to amplifier, having a noise factor 'F' is

$$\boxed{P_{na} = (F-1)KTB_N} \longrightarrow ①$$

* If 'T_e' represents the equivalent noise temperature representing noise Power, then

$$\boxed{P_{na} = KT_eB_N} \longrightarrow ②$$

Equating eq ① & ②, we get

$$\cancel{K}T_e\cancel{B_N} = (F-1)\cancel{K}T\cancel{B_N}$$

$$\boxed{T_e = (F-1)T} \longrightarrow ③$$

$$(F-1) = \frac{T_e}{T}$$

$$\boxed{F = \frac{T_e}{T} + 1}$$

Noise Temperature of Cascaded N/W:-

1) Derive an expression for overall Equivalent Noise temperature of the Cascade connection of any number of noises for two port N/w

July-09, 5M

* It is possible to develop an expression for the overall Noise temperature using Friis formula i.e.

$$F = F_1 + \frac{F_2-1}{G_1} + \frac{F_3-1}{G_1G_2} + \dots \longrightarrow ①$$

Subtract 1 from both Sides of eq ①, we get

$$F-1 = F_1 - 1 + \frac{F_2 - 1}{G_1} + \frac{F_3 - 1}{G_1 G_2} + \cdots$$

* If 'T_e' is overall equivalent Noise temperature of the Cascade, While T_{e1}, T_{e2}, ... are corresponding values for each amplifier in Cascade, then from eq ③ "$\frac{T_e}{T} = (F-1)$", we have

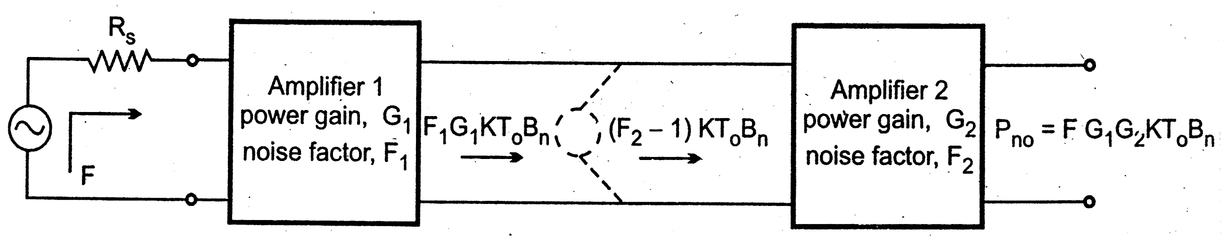
$$\frac{T_e}{T} = \frac{T_{e1}}{T} + \frac{T_{e2}/T}{G_1} + \frac{T_{e3}/T}{G_1 G_2} + \cdots$$

$$\boxed{T_e = T_{e1} + \frac{T_{e2}}{G_1} + \frac{T_{e3}}{G_1 G_2} + \cdots}$$

Noise Factor of amplifier in Cascade :-

▶ **Figure**

Noise factor of two amplifiers in cascade



* Consider two amplifier connected in Cascade as Shown above. The available Noise power at the o/p of 1st amplifier is

$$\boxed{P_{no1} = F_1 G_1 K T_o B_N} \longrightarrow ①$$

* This is available to the 2nd amplifier & 2nd amplifier has noise $(F_2-1)KTB_N$ of its own at its I/p of the 2nd amplifier is

$$\boxed{P_{ni2} = F_1 G_1 KTB_N + (F_2 - 1) KTB_N} \longrightarrow ②$$

* Consider 2nd amplifier as a Noiseless amplifier with amplifier gain 'G_2'

We have

$$\boxed{P_{no2} = G_2 P_{ni2}} \longrightarrow (3)$$

Substituting eq (2) in eq (3), We get

$$P_{no2} = G_2 \left[F_1 G_1 KTB_N + (F_2 - 1) KTB_N \right] \longrightarrow (4)$$

* WKT, the overall voltage gain of the two amplifiers in cascade is

$$G = G_1 G_2 \ \&$$

* From figure, the overall Noise power is

$$\boxed{P_{no} = F G_1 G_2 KTB_N} \longrightarrow (5)$$

* Equating eq (4) & (5), We get

$$P_{no} = P_{no2}$$

$$F G_1 G_2 KTB_N = G_2 \left[F_1 G_1 KTB_N + (F_2 - 1) KTB_N \right]$$

$$F = \frac{F_1 G_1 G_2 KTB_N + (F_2 - 1) G_2 KTB_N}{G_1 G_2 KTB_N}$$

$$F = \frac{\cancel{F_1 G_1 G_2 KTB_N}}{\cancel{G_1 G_2 KTB_N}} + \frac{(F_2 - 1) \cancel{G_2 KTB_N}}{G_1 \cancel{G_2 KTB_N}}$$

$$\boxed{F = F_1 + \frac{(F_2 - 1)}{G_1}}$$

By having G_1 large, the noise contribution of the 2nd stage can be made negligible.

* For Multistage amplifier

$$\boxed{F = F_1 + \frac{(F_2-1)}{G_1} + \frac{(F_3-1)}{G_1 G_2} + \cdots} \longrightarrow (7)$$

Equation (7) is Known as "Friis" Formula.

NOTE :-

For 4- Stage amplifier

$$\boxed{F = F_1 + \frac{(F_2-1)}{G_1} + \frac{(F_3-1)}{G_1 G_2} + \frac{(F_4-1)}{G_1 G_2 G_3}}$$

Important Formulae

Sr. No.			Expression
1.	Speed of light	:	$C = \lambda \times f$
2.	Thermal noise power	:	$P_n = kTB$
3.	Shot noise	:	$I_n^2 = 2(I + 2I_o)\,qB$
4.	Signal to noise ratio	:	$\frac{S}{N} = \left[\frac{V_s}{V_n}\right]^2$
5.	Noise factor	:	$F = \frac{S/N \text{ ratio at the input}}{S/N \text{ ratio at the output}}$
6.	Noise figure	:	$F_{dB} = 10\log_{10}$ (Noise factor)
7.	Noise temperature	:	$T_e = (F-1)\,T_o$
8.	Thermal noise voltage	:	$V_n = \sqrt{4\,k\,TBR}$
9.	Noise voltage for resistors in series	:	$V_n = \left[V_{n1}^2 + V_{n2}^2\right]^{1/2}$ and $R = R_1 + R_2$
10.	Total noise voltage for resistors in series	:	$V_n = 4\,k\,TB\,R_P$ where $R_P = R_1 \parallel R_2$
11.	Noise power contributed by an amplifier	:	$P_{na} = (F-1)\,k\,T_o\,B$
12.	Friiss formula	:	$F = F_1 + \frac{(F_2-1)}{G_1} + \frac{(F_3-1)}{G_1 G_2} + \ldots$
13.	Friiss formula for noise temperature	:	$T_{eq.} = T_{eq.1} + \frac{T_{eq2}}{G_1} + \frac{T_{eq3}}{G_1 G_2} \ldots$