Energy & Potential

Energy Expended in moving a point charge in an electric field

→ In an electric field exerts force on a unit test charge. To move this test charge in electric field external force has to be exerted in opposite direction.

Suppose we wish to move a charge Q a distance dL in an electric field $\vec{E}$. The force on Q arising from the electric field is.

$$\vec{F_E} = Q\vec{E}$$

The Component of this force in the direction of $d\vec{L}$ is.

$$F_{EL} = \vec{F} \cdot \hat{a}_L = Q\vec{E} \cdot \hat{a}_L$$

$\hat{a}_L$ is the unit vector in direction of $d\vec{L}$

The force that we must apply is the equal & opposite force associated with the field.

$$F_{appl} = -Q\vec{E} \cdot \hat{a}_L$$

The differential amount of work done to move Q is.

$$dw = \left[-Q\vec{E} \cdot \hat{a}_L\right] dL$$

$$dw = -Q(\vec{E} \cdot d\vec{L})$$

The work done to move the charge a finite distance must be determined by integration

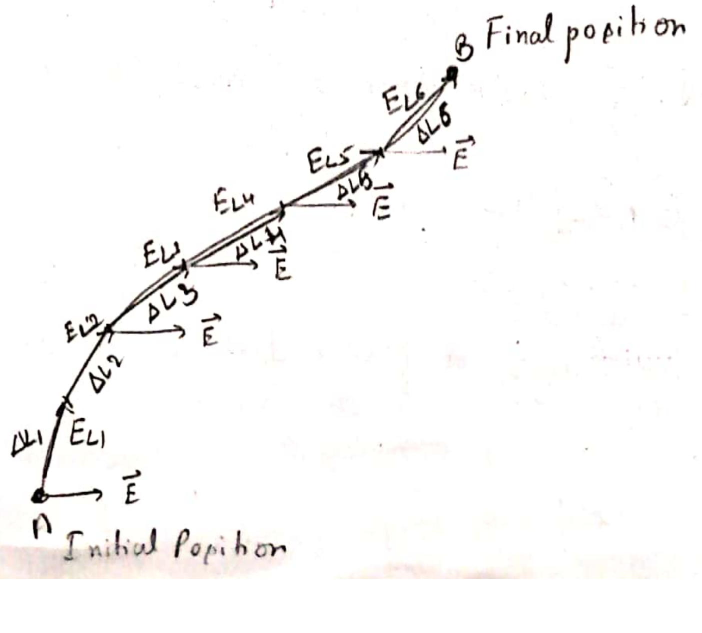
$$\boxed{w = -Q\int_{final}^{Init} \vec{E} \cdot d\vec{L}}$$

The Line Integral

The integral expression for the work done in moving a point charge Q from one position to another. The line integral tells us to choose a path, break it up into a large number of small segments, multiply the component of the field along each segment by the length of segment and then add the results for all segments. This result is summation, and the integral is obtained exactly when the number of segments become infinity.

Let us consider a charge at initial position B placed in unifor electric field $\vec{E}$. A path is chosen from initial position B to final position A. The path is divided into 6 segments, $\Delta L1, \Delta L2, \ldots \Delta L6$ and component of $\vec{E}$ along each segment is $E_{L1}, E_{L2}, \ldots E_{L6}$. The work involved in moving a charge Q from B to A is then approximately

$$W = -Q\left[E_{L1}\Delta L_1 + E_{L2}\Delta L_2 + \ldots + E_{L3}\Delta L_3\right]$$



using vector notation

$$W = -Q\left[\vec{E_1}\cdot\Delta\vec{L_1} + \vec{E_2}\cdot\Delta\vec{L_2} + \ldots + \vec{E_6}\cdot\Delta\vec{L_6}\right]$$

Since we have assumed uniform field $\vec{E_1} = \vec{E_2} = \vec{E_3} = \ldots \vec{E}$

$$W = -Q\vec{E}\cdot\left[\Delta\vec{L_1} + \Delta\vec{L_2} + \ldots \Delta\vec{L_6}\right]$$

$$W = -Q\,\vec{E}\cdot\vec{L}_{BA}$$

Using integral expression

$$W = -Q\int_B^A \vec{E}\cdot d\vec{L}$$

The work involved in moving the charge depends only on $Q, \vec{E}$ and $\vec{L}_{BA}$ a vector drawn from the initial point to the final point of the path chosen. It does not depend on the particular path along with to carry charge. We may proceed from B to A on a straight line or via an arc ; the answer is same.

$d\vec{L}$ in different Coordinate systems

$$d\vec{L} = dx\,\hat{a}_x + dy\,\hat{a}_y + dz\,\hat{a}_z \rightarrow \text{Rectangular}$$

$$d\vec{L} = d\rho\,\hat{a}_\rho + \rho\, d\phi\,\hat{a}_\phi + dz\,\hat{a}_z \rightarrow \text{Cylindrical}$$

$$d\vec{L} = dr\,\hat{a}_r + r\,d\theta\,\hat{a}_\theta + r\sin\theta\, d\phi\,\hat{a}_\phi \rightarrow \text{Spherical.}$$

Potential Difference & Potential

Work done in moving a charge Q from one position to other is

$$W = -Q\int_{Init}^{Final} \vec{E}\cdot d\vec{L}$$

The potential difference V is the work done in moving a unit positive charge from one point to another in an electric field.

$$\text{Potential difference} = V = -\int_{Init}^{Final} \vec{E}\cdot d\vec{L}$$

If V_{AB} is the potential difference between points A & B it is the work done in moving a unit charge from B to A. Hence B is the initial point & A is the final point. It is measured in J/c for which the 'volt' is defined as a more common unit

$$\boxed{V_{AB} = -\int_B^A \vec{E}\cdot d\vec{L}\ \text{V.}}$$

Eg: For point charge located at the origin of a sphere the potential difference between points A & B located at radial distances r_A & r_B from charge Q. is

$\vec{E} = E_r \hat{a}_r$ & $d\vec{L} = dr\,\hat{a}_r$

$$V_{AB} = -\int_B^A \vec{E}\cdot d\vec{L} = -\int_{r_B}^{r_A} \frac{Q}{4\pi\varepsilon_0 r^2}\,dr = \frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{r_A} - \frac{1}{r_B}\right]$$

If $r_B > r_A$ the PD is V_{AB} is positive.

It is convinient to deal with potential or absolute potential. of a point. this means measurement of every potential difference with respect to a specified reference point, which is considered to have zero potential.

The most universal zero reference point in experimental or physical potential measurements is ground. i.e., the potential of the surface region of the earth itself.

If the potential at A is V_A & at B is V_B then

$$V_{AB} = V_A - V_B$$

Let us consider potential difference between two point charges

$$V_{AB} = -\frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{r_A} - \frac{1}{r_B}\right] = V_A - V_B$$

Let $V=0$ at infinity. If we let the point $r = r_B$ recede to infinity the potential at r_A becomes

$$V_A = \frac{Q}{4\pi\varepsilon_0 r_A}$$

At any general point around a point charge $\boxed{V = \frac{Q}{4\pi\varepsilon_0 r}}$

The potential field of a single point charge Q_1 located at $\vec{r}_1$ involves only the distance $(\vec{r}-\vec{r}_1)$ from Q_1 to the point at $\vec{r}$ where we desire value of potential. For zero reference at ∞ we have

$$V(r) = \frac{Q_1}{4\pi\varepsilon_0(\vec{r}-\vec{r}_1)}$$

The potential arising from two charges Q_1 at r_1 & Q_2 at r_2 is a function only of $(\vec{r}-\vec{r}_1)$ and $(\vec{r}-\vec{r}_2)$, the distances from Q_1 & Q_2 to the field point respectively.

$$V(r) = \frac{Q_1}{4\pi\varepsilon_0(\vec{r}-\vec{r}_1)} + \frac{Q}{4\pi\varepsilon_0|\vec{r}-\vec{r}_2|}$$

Similary potential arising from n point charges is

$$V(r) = \frac{Q_1}{4\pi\varepsilon_0|\vec{r}-\vec{r}_1|} + \frac{Q_2}{4\pi\varepsilon_0|\vec{r}-\vec{r}_2|} + \cdots + \frac{Q}{4\pi\varepsilon_0|\vec{r}-\vec{r}_n|}$$

$$V(r) = \sum_{m=1}^{n} \frac{Q_m}{4\pi\varepsilon_0|\vec{r}-\vec{r}_m|}$$

If the charge distribution takes the form of a line charge, surface charge & volume charge

$$V(r) = \int \frac{\rho_L \, dL'}{4\pi\varepsilon_0|\vec{r}-\vec{r}'|}$$

$$V(r) = \int_S \frac{\rho_S \, dS'}{4\pi\varepsilon_0|\vec{r}-\vec{r}'|}$$

$$V(r) = \int_{vol} \frac{\rho_v(\vec{r}') \, dv'}{4\pi\varepsilon_0|\vec{r}-\vec{r}'|}$$

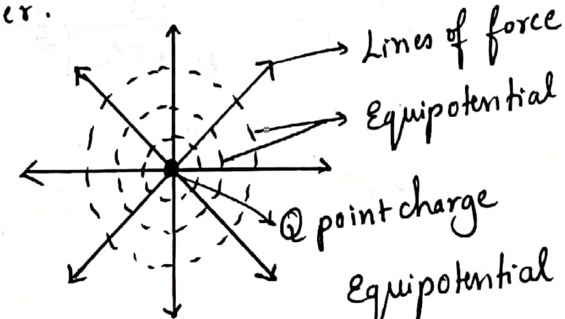
No work is done in carrying the unit charge around any closed path or $\oint \vec{E} \cdot d\vec{L} = 0$

Potential Gradient

→ From previous discussion we learnt to find the potential directly from $\vec{E}$ by means of line integral and another from basic charge distribution. But in most practical applications neither $\vec{E}$ nor charge distribution are often known.

→ In practical applications consists of description of two equipotential surfaces such as parallel conductors at potentials +100 & −100V & hence finding capacitance between them or charge and current distribution from which losses may be calculated.

→ Equipotential surface is an imaginary surface in an electric field, in which all points on the surface are at same potential. Since all points on equipotential surface are at same potential the potential difference between any two points will be equal to zero. The potential change is highest in a direction perpendicular to the equipotential surface which is direction of field intensity (Lines of force) Thus equipotential surfaces and field lines will be $\perp^{lar}$ to each other.



Equipotential surface for a point charge

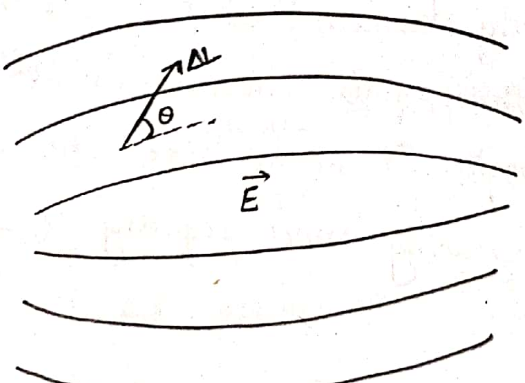
We need to find electric field intensity from the potential we have general line integral relationship as

$$V = -\int \vec{E} \cdot d\vec{L}$$

When this line integral is applied to a very short element of length ΔL leading to an incremental potential difference ΔV

$$\Delta V = -\vec{E} \cdot \Delta \vec{L}$$

Consider a general region of space in which both $\vec{E}$ & V both change as we move from point to point. The expression for ΔV tells us to choose an incremental vector element of length $\Delta \vec{L} = \Delta L \hat{a}_L$ and multiply its magnitude by the component of $\vec{E}$ in direction of $\hat{a}_L$ to obtain small potential difference ΔV



Let the angle between $\Delta \vec{L}$ & $\vec{E}$ is θ then

$$\Delta V = -(E_L \hat{a}_L) \cdot (\Delta L \hat{a}_L)$$

$$\Delta V = -(E \cos\theta)(\Delta L)$$

$$\frac{\Delta V}{\Delta L} = -E\cos\theta$$

Since V may be interpreted as $V(x, y, z)$ function, We may pass to the limit and consider $\frac{dv}{dL}$

$$\frac{dv}{dL} = -E\cos\theta$$

The maximum positive increment of potential ΔV_{max} will occur when $\cos\theta = -1$ or ΔL points in the direction opposite to $\vec{E}$.

Hence

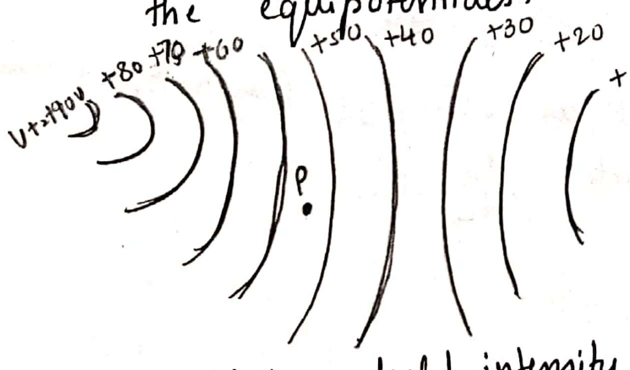
$$\left.\frac{dV}{dL}\right|_{max} = E$$

This shows us two characteristics of the relationship between $\vec{E}$ & V at any point.

① The magnitude of $\vec{E}$ is given by the maximum value of the rate of change of potential with distance

② This maximum value is obtained when direction of distance increment is opposite to $\vec{E}$ · or the direction of $\vec{E}$ is opposite to the direction in which the potential is increasing the most rapidly.

Consider the equipotential surfaces (shown as lines in 2D-sketch) We desire information about the $\vec{E}$ at point P. If we allow $\Delta\vec{L}$ to lay off in different directions, the most rapid change (increasing) in potential is appears to be left & slightly upward. The $\vec{E}$ is therefore oppositely directed. (To the right and slightly downward at P). Its magnitude is given by dividing the small increase in potential by the small element of length. It seems like the direction in which potential is increasing most rapidly is $\perp^{lar}$ to the equipotentials.



Let $\hat{a}_N$ is the direction of unit vector normal to the equipotential surface. & directed towards higher potentials.

The electric field intensity is then expressed in terms of potential as

$$\vec{E} = -\left.\frac{dV}{dL}\right|_{max} \hat{a}_N$$

Energy Density in the Electrostatic Field.

Bringing a positive charge from infinity into the field of another positive charge requires work.

In order to find the potential energy present in a system of charges, we must find the work done in positioning the charges.

Let us start by visualizing empty universe. Bringing a charge Q_1 from infinity to any position requires no work. The positioning of Q_2 at a point in the field of Q_1 requires an amount of work given by

$$\text{Work to position } Q_2 = Q_2 V_{2,1}$$

$V_{2,1}$ is the potential at location of Q_2 due to Q_1

Similarly

$$\text{Work to position } Q_3 = Q_3 V_{3,1} + Q_3 V_{3,2}$$

$$\text{Work to position } Q_4 = Q_4 V_{4,1} + Q_4 V_{4,2} + Q_4 V_{4,3}$$

Total positioning work = Potential energy of field

$$W_E = \left[Q_2 V_{2,1} + Q_3 V_{3,1} + Q_3 V_{3,2} + Q_4 V_{4,1} + Q_4 V_{4,2} + Q_4 V_{4,3} + \cdots\right]$$

Let us Consider $Q_3 V_{3,1} = Q_3 \dfrac{Q_1}{4\pi\varepsilon_0 R_{13}} = Q_1 \dfrac{Q_3}{4\pi\varepsilon_0 R_{31}} = Q_1 V_{1,3}$

$R_{13} = R_{31}$ is scalar distance between Q_1 & Q_3.

If each term of the total energy expression is replaced by its equal $W_E = \left[Q_1 V_{1,2} + Q_1 V_{1,3} + Q_2 V_{2,3} + Q_1 V_{1,4} + \cdots\right]$

This shows that the magnitude of $\vec{E}$ is given by the maximum space rate of change of V and the direction of $\vec{E}$ is normal to the equipotential surface (in the direction of decreasing potentials) & hence

$$\boxed{\vec{E} = -\frac{dV}{dN}\hat{a}_N}$$

Since $\left.\frac{dV}{dL}\right|_{max}$ occurs when $\Delta\vec{L}$ is in direction of $\hat{a}_N$

The vector operator on V by which $-\vec{E}$ is obtained is known as the gradient

$$\vec{E} = -\text{grad } V$$

Since V is a unique function of (x, y, z)

$$\vec{E} = -\left[\frac{\partial V}{\partial x}\hat{a}_x + \frac{\partial V}{\partial y}\hat{a}_y + \frac{\partial V}{\partial z}\hat{a}_z\right] = (-\text{grad } V)$$

The vector operator $\nabla = \frac{\partial}{\partial x}\hat{a}_x + \frac{\partial}{\partial y}\hat{a}_y + \frac{\partial}{\partial z}\hat{a}_z$ can be used as an operator on scalar V to produce vector $\vec{E}$

$$\boxed{\vec{E} = -\nabla V}$$

The gradient in different coordinate systems

$$\nabla V = \frac{\partial V}{\partial x}\hat{a}_x + \frac{\partial V}{\partial y}\hat{a}_y + \frac{\partial V}{\partial z}\hat{a}_z \rightarrow \text{Rectangular}$$

$$\nabla V = \frac{\partial V}{\partial \rho}\hat{a}_\rho + \frac{1}{\rho}\frac{\partial V}{\partial \phi}\hat{a}_\phi + \frac{\partial V}{\partial z}\hat{a}_z \rightarrow \text{Cylindrical}$$

$$\nabla V = \frac{\partial V}{\partial r}\hat{a}_r + \frac{1}{r}\frac{\partial V}{\partial \theta}\hat{a}_\theta + \frac{1}{r\sin\theta}\frac{\partial V}{\partial \phi}\hat{a}_\phi \rightarrow \text{Spherical.}$$

Adding the two energy expressions

$$2W_E = Q_1(V_{1,2} + V_{1,3} + V_{1,4} + \cdots) + Q_2(V_{2,1} + V_{2,2} + V_{2,3} + \cdots) + Q_3(V_{3,1} + V_{3,2} + V_{3,4} + \cdots) + \cdots$$

$$V_{1,2} + V_{1,3} + V_{1,4} + \cdots = V_1$$

hence

$$W_E = \frac{1}{2}\left[Q_1V_1 + Q_2V_2 + \cdots\right] = \frac{1}{2}\sum_{m=1}^{m=N} Q_m V_m$$

The expression for energy stored in a region of continous charge distribution can be obtained by integral of $\rho_v V$.

$$W_E = \frac{1}{2}\int_{vol} \rho_v V \, dv$$

Replace ρ_v by its equivalent $\nabla\cdot\vec{D}$ & using vector identity

$$\nabla\cdot(V\vec{D}) = V(\nabla\cdot\vec{D}) + \vec{D}\cdot(\nabla V)$$ applying in above

expression

$$W_E = \frac{1}{2}\int_{vol} (\nabla\cdot\vec{D})V\,dv$$

$$= \frac{1}{2}\int_{vol}\left[V(\nabla\cdot\vec{D}) + \vec{D}\cdot(\nabla V)\right]dv.$$

Using divergence theorem

$$W_E = \frac{1}{2}\oint_S (V\vec{D})\cdot d\vec{s} - \frac{1}{2}\int_{vol}(\vec{D}\cdot(\nabla V))\,dv$$

The surface integral equal to zero, for over this closed surface surrounding the universe

As we see that $V \propto 1/r$, $\vec{D} \propto 1/r^2$ while the differential element of surface $d\vec{S} \propto r^2$ hence the resultant of product $(\propto 1/r)$ tends to zero as $r \to \infty$ hence both integrand and integral approach zero. Substituting $\vec{E} = -\nabla V$ in the remaining volume integral

$$W_E = \frac{1}{2} \int_{vol} \vec{D} \cdot \vec{E} \, dv = \frac{1}{2} \int_{vol} \epsilon_0 E^2 dv$$

$$\boxed{W_E = \frac{1}{2} \int_{vol} \epsilon_0 E^2 dv}$$

Current & Current Density

Electric charges in motion constitute current. The unit is Ampere (A). One ampere of current is defined as rate of movement of charge passing a given reference point of one coulomb per second.

$$I = \frac{dQ}{dt}$$

We shall find the concept of current density measured in A/m^2 more useful. Current density is a vector represented by $\vec{J}$

The increment of current ΔI crossing an incremental surface ΔS normal to current density is

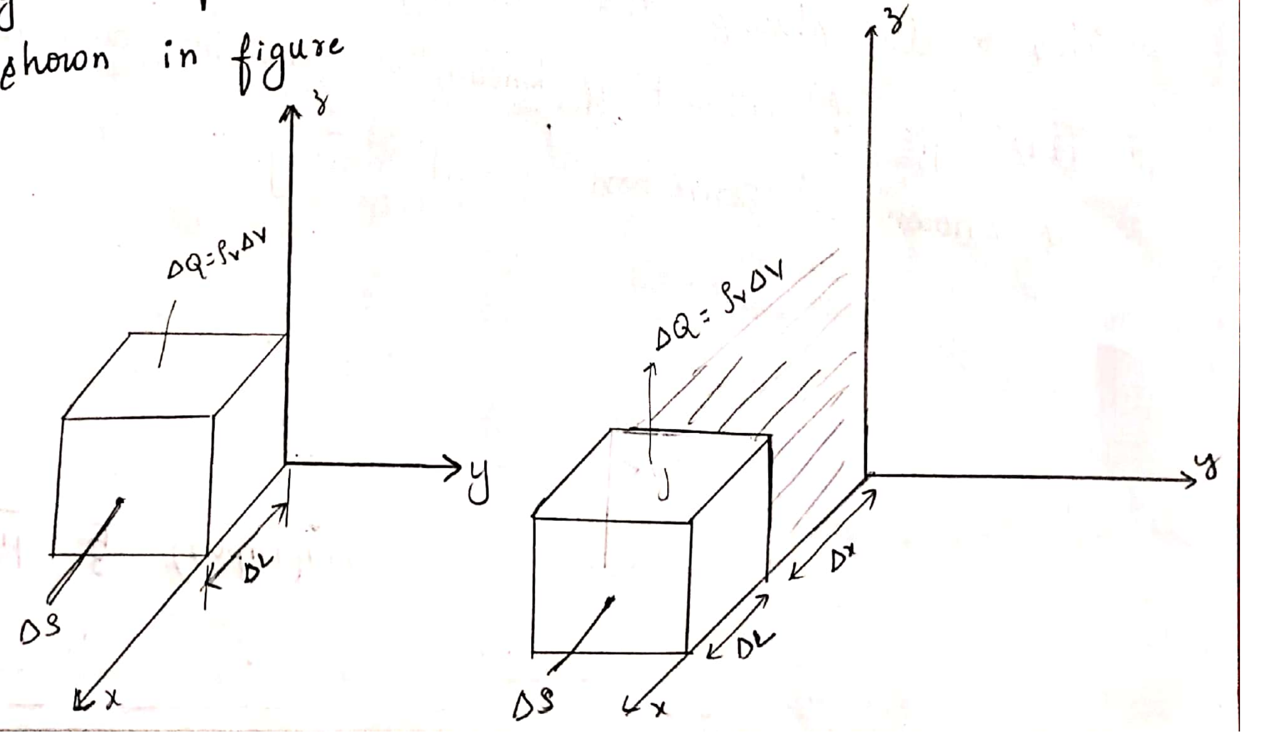
$$\Delta I = J_N \Delta S$$

In case if $\vec{J}$ is not perpendicular to $\Delta \vec{S}$

$$\Delta I = \vec{J} \cdot \Delta \vec{S}$$

Total current is obtained by $I = \int_S \vec{J} \cdot d\vec{S}$

Current density may be related to the velocity of volume charge density at a point. Consider the element of charge $\Delta Q = \rho_v \Delta V$ as shown in figure



Let us assume that the charges element is oriented with its edges parallel to the coordinate axes and that it possesses only an x component of velocity. In time interval Δt the element of charge has moved a distance Δx. Hence a charge $\Delta Q = \rho_v \Delta S \Delta x$ has moved through a reference plane perpendicular to the direction of motion in a time increment Δt, & the resultant current is

$$\Delta I = \frac{\Delta Q}{\Delta t} = \rho_v \Delta S \frac{\Delta x}{\Delta t}$$

As we take the limit with respect to time

$$\Delta I = \rho_v \Delta S V_x$$

where $V_x \rightarrow$ x component of velocity $\vec{V}$ In terms of current density

$$J_x = \rho_v V_x$$

& in general $\boxed{\vec{J} = \rho_v \vec{V}}$

This results shows that charge in motion constitutes current. This type of current is known as convection current. $\vec{J}$ or $\rho_v \vec{V}$ is known as convection current density.

Continuity of Current

The principle of Conservation of charge states simply that charges can be neither created nor destroyed, although equal amounts of positive and negative charge may be simultaneously created through different methods.

When we consider any region bounded by a closed surface the current through the closed surface is

$$I = \oint_S \vec{J} \cdot d\vec{s}$$

This outward flow of positive charge must be balanced by a decrease of positive charge within the closed surface

If the charge inside the closed surface is denoted by Q_i then rate of decrease is $-dQ_i/dt$

$$I = \oint \vec{J} \cdot d\vec{s} = \frac{-dQ_i}{dt}$$

The presence or absence of negative sign depends on what current and charge we consider.

The above expression is integral form of continuity equation

The point form is obtained by divergence theorem.

$$\oint_S \vec{J} \cdot d\vec{s} = \int_V (\nabla \cdot \vec{J})\, d\vec{v}$$

$$\int_V (\nabla \cdot \vec{J})\, dv = -\frac{d}{dt}\int_V \rho_v \, dv$$

If surface is constant this derivative becomes partial derivative

$$\int_V (\nabla \cdot \vec{J})\, dv = \int_V \frac{-\partial \rho_v}{\partial t}\, dv$$

Since the expression is true for any volume

$$(\nabla \cdot \vec{J})\Delta V = -\frac{\partial \rho_v}{\partial t}\Delta V$$

$$\boxed{\nabla \cdot \vec{J} = -\frac{\partial \rho_v}{\partial t}}$$

Point form of continuity equation. This equation indicates that the current or charge per second, diverging from a small volume per unit volume is equal to the time rate of decrease of charge per unit volume at every point.

Metallic Conductors & Properties

In conductors the valence electrons (free) move under the influence of an electric field. With a field $\vec{E}$, an electron having a charge $Q = -e$ will experience a force

$$\vec{F} = -e\vec{E}$$

In free space, the electron would accelarate and continously increase its velocity. The velocity of electron is termed as drift velocity & it is related to $\vec{E}$ by the mobility of the electron in the given material.

$$\vec{V}_d = -\mu_e \vec{E}$$

μ_e is mobility of an electron & is positive by definition. Hence from above expression $\vec{E}$ will be opposite to electron velocity.

We know that current density $\vec{J} = \rho_v \vec{V}$

Substituting for velocity

$$\vec{J} = -\rho_e \mu_e \vec{E}$$

$\rho_e \rightarrow$ Electron charge density

From above expression the conductivity measured in Siemens per meter (S/m) can be expressed as

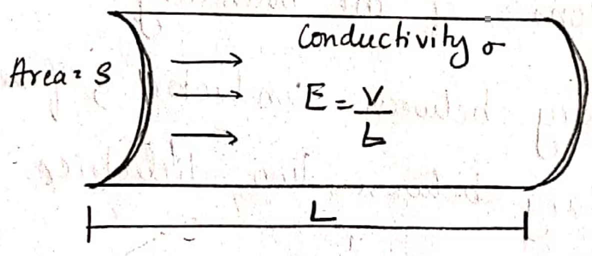
$$\boxed{\sigma = -\rho_e \mu_e}$$

The relationship between $\vec{J}$ & $\vec{E}$ for a metallic conductor is specified by the conductivity (σ)

$$\boxed{\vec{J} = \sigma \vec{E}}$$

The Ohm's law in point form to a macroscopic region leads to a more familiar form. Let us assume J & are uniform.

$I = JS$



$$I = \int_S \vec{J} \cdot d\vec{S} = JS \quad \&$$

$$V_{ab} = -\int_b^a \vec{E} \cdot d\vec{L} = -\vec{E} \cdot \int_b^a d\vec{L} = -\vec{E} \cdot \vec{L}_{ba} = \vec{E} \cdot \vec{L}_{ab}$$

or

$$V = EL$$

Thus $\vec{J} = \dfrac{I}{S} = \sigma E = \sigma \dfrac{V}{L}$

$$\boxed{V = \frac{L}{\sigma S} I}$$

The ratio of potential difference between two ends of the cylinder to the current entering to more positive end is recognised as resistance $V = IR$. Wher $R = \dfrac{L}{\sigma S}$

Above expression is known as Ohm's law & enables us to compute the resistance R, measured in ohms, of conducting objects which possess uniform fields.

Boundary Conditions between Conductor & free space

→ Consider a boundary between conductor & free space. The conductor is ideal having infinite conductivity.

→ When an electric field passes from one medium to another medium it is important to study the conditions at the boundary. Depending upon the nature of the media, there are two situations of the boundary conditions.

1. Boundary between conductor & free space
2. Boundary between two dielectrics with different properties.

For studying boundary conditions, the Maxwell's equations for electrostatics are required.

$$\oint \vec{E} \cdot d\vec{L} = 0 \quad \& \quad \oint_S \vec{D} \cdot d\vec{S} = Q$$

The field intensity $\vec{E}$ is required to be decomposed into two components namely Tangential & Normal to the boundary.

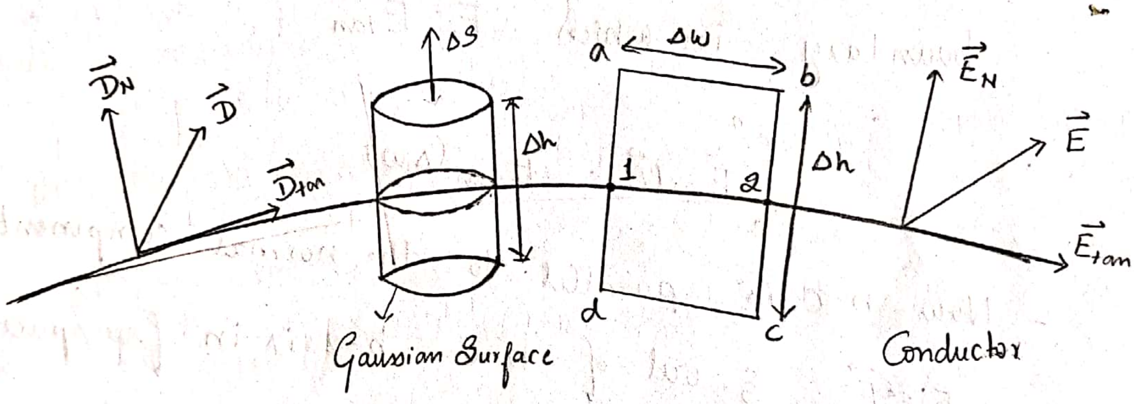
$$\vec{E} = \vec{E}_{tan} + \vec{E}_N$$

For ideal Conductors

1. The field intensity inside a conductor is zero and the flux density inside a conductor is zero.
2. No charge can exist within a conductor. The charge appears on the surface as surface charge density.
3. The charge within the conductor is zero.

Consider the conductor free space boundary as shown in figure

$\vec{E}$ at the boundary



Consider a rectangular closed path abcda as shown in figure making some angle with boundary.

$\vec{E}$ has two components $\vec{E}_{tan}$ & $\vec{E}_N$

It is known that $\oint \vec{E} \cdot d\vec{L} = 0$

Considering path abcda in clockwise direction

$$\oint \vec{E} \cdot d\vec{L} = \int_a^b \vec{E} \cdot d\vec{L} + \int_b^c \vec{E} \cdot d\vec{L} + \int_c^d \vec{E} \cdot d\vec{L} + \int_d^a \vec{E} \cdot d\vec{L}$$

The closed contour is placed in such a way that its two sides a-b & c-d are parallel to the surface and other two are normal to the surface. at the boundary.

The rectangle has elementary Δh & elementary width Δw.

It is placed such that half of the rectangle is in free space and other half is in conductor.

The portion c-d is inside conductor hence $\int_c^d \vec{E} \cdot d\vec{L} = 0$

As width Δw is very small, $\vec{E}$ over it can be assumed constant & Δw is along tangential direction to the boundary in which $\vec{E} = \vec{E}_{tan}$

$$\int_a^b \vec{E} \cdot d\vec{L} = E_{tan}(\Delta w)$$

Now b-c is parallel to the normal component so we have $\vec{E} = \vec{E}_N$ & out of b-c. b-2 is in free space and 2-c is in conductor where $\vec{E} = 0$.

$$\int_b^c d\vec{L} = \int_b^2 d\vec{L} + \int_2^c d\vec{L} = \frac{\Delta h}{2} + 0 = \frac{\Delta h}{2}$$

$$\int_b^c \vec{E} \cdot d\vec{L} = E_N\left(\frac{\Delta h}{2}\right)$$

Similarly for d-a the condition is same as for the path b-c only direction is opposite.

$$\int_d^a \vec{E} \cdot d\vec{L} = -E_N\left(\frac{\Delta h}{2}\right)$$

Finally we get

$$E_{tan}\,\Delta w + E_N\left(\frac{\Delta h}{2}\right) - E_N\left(\frac{\Delta h}{2}\right) = 0$$

$$E_{tan}\,\Delta w = 0$$

$$\boxed{E_{tan} = 0}$$

Hence the tangential component of electric field intensity is zero at the boundary between conductor & free space.

$$\boxed{D_{tan} = \varepsilon_0 E_{tan} = 0}$$

The tangential component of electric flux density is zero at the boundary between conductor & free space.

To find the normal component of E & D, select a closed Gaussian surface in the form of right circular cylinder. Its height is Δh & placed in such a way that $\Delta h/2$ is in the conductor and remaining $\Delta h/2$ is in the free space. Its axis is in the normal direction to the surface.

According to Gauss's law $\oint \vec{D}\cdot d\vec{s} = Q$

The surface integral must be evaluated over three surfaces.

$$\int_{Top} \vec{D}\cdot d\vec{s} + \int_{Bottom} \vec{D}\cdot d\vec{s} + \int_{Lateral} \vec{D}\cdot d\vec{s} = Q$$

The bottom surface is in the conductor where $\vec{D} = 0$

The lateral surface direction is normal to D_N hence $\vec{D}_N \cdot d\vec{S} = 0$ over the lateral surface.

Only component of $\vec{D}$ present is the normal component having magnitude D_N. The bottom surface is in the conductor where $\vec{D} = 0$ hence corresponding integral reduces to

$$\int_{top} \vec{D} \cdot d\vec{S} = D_N \int_{top} d\vec{S} = D_N \Delta S$$

From Gauss's Law

$$D_N \Delta S = Q$$

At the boundary charge exists as surface charge density.

$$D_N \Delta S = Q = \rho_S \Delta S$$

$$D_N \Delta S = \rho_S \Delta S$$

$$\boxed{D_N = \rho_S}$$

Hence the flux leaves normally & the normal component of flux density is equal to surface charge density.

$$D_N = \varepsilon_0 E_N = \rho_s$$

$$\boxed{E_N = \frac{\rho_s}{\varepsilon_0}}$$

Hence at the boundary of dielectric (free space) & Conductor

$E_{tan} = 0$	$E_N = \dfrac{\rho_s}{\varepsilon_0}$
$D_{tan} = 0$	$D_N = \rho_s$

1) Given the electric field $\vec{E} = \frac{1}{z^2}(8xyz\,\hat{a}_x + 4x^2z\,\hat{a}_y - 4x^2y\,\hat{a}_z)$ V/m find the differential amount of work done in moving a 6nc charge a distance of 2μm, starting at P(2,-2,3) and proceeding in the direction $\hat{a}_L =$

a) $-\frac{6}{7}\hat{a}_x + \frac{3}{7}\hat{a}_y + \frac{2}{7}\hat{a}_z$

b) $\frac{6}{7}\hat{a}_x - \frac{3}{7}\hat{a}_y - \frac{2}{7}\hat{a}_z$

c) $\frac{3}{7}\hat{a}_x + \frac{6}{7}\hat{a}_y$

$$\vec{E}_{P=(2,-2,3)} = \left[-\frac{96}{9}\hat{a}_x + \frac{48}{9}\hat{a}_y + \frac{32}{9}\hat{a}_z\right] \text{ V/m}$$

differential amount of work done $dw = -Q\,\vec{E}\cdot d\vec{L}$

$$= -Q\,dL\,(\vec{E}\cdot\hat{a}_L)$$

a) $\hat{a}_L = -\frac{6}{7}\hat{a}_x + \frac{3}{7}\hat{a}_y + \frac{2}{7}\hat{a}_z$

$$dw = (6\times10^{-9})(2\times10^{-6})\left(-\frac{96}{9}\hat{a}_x + \frac{48}{9}\hat{a}_y + \frac{32}{9}\hat{a}_z\right)\cdot\left(-\frac{6}{7}\hat{a}_x + \frac{3}{7}\hat{a}_y + \frac{2}{7}\hat{a}_z\right)$$

$$dw = -12\times10^{-15}(-12.44) = 149.33\times10^{-15}\text{ J}$$

$$\boxed{dw = -149.33\text{ fJ}}$$

b) $dw = -12\times10^{-15}\left[-\frac{96}{9}\hat{a}_x + \frac{48}{9}\hat{a}_y + \frac{32}{9}\hat{a}_z\right]\cdot\left[\frac{6}{7}\hat{a}_x - \frac{3}{7}\hat{a}_y - \frac{2}{7}\hat{a}_z\right]$

$$\boxed{dw = +149.33\text{ fJ}}$$

c) $dw = -12\times10^{-15}\left[-\frac{96}{9}\hat{a}_x + \frac{48}{9}\hat{a}_y + \frac{32}{9}\hat{a}_z\right]\cdot\left[\frac{3}{7}\hat{a}_x + \frac{6}{7}\hat{a}_y\right]$

$$\boxed{dw = \cdot 0}$$

(2) Given a non uniform field $\vec{E} = y\,\hat{a}_x + x\,\hat{a}_y + 2\hat{a}_z$. Determine the work expended in carrying 2C from B(1,0,1) to A(0.8, 0.6, 1) along the shorter arc of the circle, $x^2+y^2=1$ $z=1$

Work done is $W = -Q\int_{Init}^{Final} \vec{E}\cdot d\vec{L}$

$$W = -2\int_B^A (y\hat{a}_x + x\hat{a}_y + 2\hat{a}_z)\cdot(dx\,\hat{a}_x + dy\,\hat{a}_y + dz\,\hat{a}_z)$$

$$= -2\int_B^A (y\,dx + x\,dy + 2dz) = -2\left[\int_B^A \sqrt{1-x^2}\,dx + \int_B^A \sqrt{1-y^2}\,dy + \int_B^A 2dz\right]$$

$$= -2\left\{\left[\frac{1}{2}\left(x\sqrt{1-x^2} + \sin^{-1}x\right)\right]_{x=1}^{0.8} + \left[\frac{1}{2}\left(y\sqrt{1-y^2} + \sin^{-1}y\right)\right]_{y=0.6}^{0} + 2z\Big]_1^1\right\}$$

$$W = -2\left(\frac{1}{2}\right)(0.96) = \underline{\underline{-0.96\ \text{J}}}$$

(3) Find the work required to carry 2C from B to A in the same field as in above problem using straight line path from B to A.

Using any of planes through the line equations

$$(y - y_B) = \left[\frac{y_A - y_B}{x_A - x_B}\right](x - x_B) \qquad (x - x_B) = \left[\frac{x_A - x_B}{z_A - z_B}\right](z - z_B)$$

$$(z - z_B) = \left[\frac{z_A - z_B}{y_A - y_B}\right](y - y_B)$$

$$(y-0) = \left[\frac{0.6-0}{0.8-1}\right](x-1)$$

$$y = -3(x-1)$$

$$x = 1 - y/3$$

$$W = -Q\int_B^A \vec{E}\cdot d\vec{L} = -2\int_B^A (y\hat{a}_x + x\hat{a}_y + 2\hat{a}_z)\cdot(dx\,\hat{a}_x + dy\,\hat{a}_y + dz\,\hat{a}_z)$$

$$W = -2\int_B^A (y\,dx + x\,dy + z\,dz) = -2\int_B^A -3(x-1)\,dx - 2\int_B^A (1-y/3)\,dy - 2\int_B^A 2\,dz$$

$$W = -2\int_{x=1}^{0.8} (-3)(x-1)\,dx - 2\int_{y=0}^{0.6} (1-y/3)\,dy - 4\int_{z=1}^{1} dz$$

$$W = 6\left[\frac{x^2}{2} - x\right]_1^{0.8} - 2\left[y - \frac{y^2}{6}\right]_0^{0.6}$$

$$W = 0.12 - 1.08 = -0.96\,J$$

$$\boxed{W = -0.96\,J}$$

(4) Calculate the work done in moving a 4C charge from B (1,0,0) to A (0,2,0) along the path $y = 2-2x$, $z=0$ in the field $\vec{E}$ = a) $5\hat{a}_x$ V/m b) $5x\,\hat{a}_x$ V/m c) $5x\hat{a}_x + 5y\hat{a}_y$ V/m

a) $$W = -Q\int_{Init}^{Final} (\vec{E}\cdot d\vec{L}) = -4\int_B^A (5\,\hat{a}_x)\cdot(dx\,\hat{a}_x + dy\,\hat{a}_y + dz\,\hat{a}_z)$$

$$W = -4\int_B^A 5\,dx = -20\,x\Big]_1^0 = -20(-1) = \underline{20J}$$

b) $$W = -4\int_B^A (5x\,\hat{a}_x)\cdot(dx\,\hat{a}_x + dy\,\hat{a}_y + dz\,\hat{a}_z)$$

$$W = -4\int_B^A 5x\,dx = -20\,\frac{x^2}{2}\Big]_1^0 = \underline{10\,J}$$

c) $$W = -4 \int_B^A (5x\,\hat{a}_x + 5y\,\hat{a}_y) \cdot (dx\,\hat{a}_x + dy\,\hat{a}_y + dz\,\hat{a}_z)$$

$$= -4 \int_B^A (5x\,dx + 5y\,dy)$$

$$= -20 \left[\int_B^A x\,dx + \int_B^A y\,dy \right]$$

$$= -20 \left\{ \left[\frac{x^2}{2}\right]_{01}^{0} + \left[\frac{y^2}{2}\right]_{y=0}^{2} \right\} = \frac{-20}{2}\left\{(-1) + (4-0)\right\}$$

$$\boxed{W = -30\text{ J}}$$

(5) An electric field is expressed in rectangular coordinates by $\vec{E} = 6x^2\,\hat{a}_x + 6y\,\hat{a}_y + \hat{a}_z$ V/m find

a) V_{MN} if points M & N are specified by $M(2,6,-1)$ and $N(-3,-3,2)$

b) V_M if $V=0$ at $Q(4,-2,-35)$

c) V_N if $V=2$ at $P(1,2,-4)$

a) $$V_{MN} = -\int_N^M \vec{E} \cdot d\vec{L} = -\int_N^M (6x^2\,\hat{a}_x + 6y\,\hat{a}_y + \hat{a}_z) \cdot (dx\,\hat{a}_x + dy\,\hat{a}_y + dz\,\hat{a}_z)$$

$$= -\left[\int_{x=-3}^{2} 6x^2\,dx + \int_{y=-3}^{6} 6y\,dy + \int_{z=2}^{-1} dz \right]$$

$$V_{MN} = -\left[\left[\frac{6x^3}{3}\right]_{-3}^{2} + \left[\frac{6y^2}{2}\right]_{-3}^{6} + \left[z\right]_{2}^{-1} \right]$$

$$\boxed{V_{MN} = -139\text{ V}}$$

b) $V=0$ at $Q(4,-2,-35) \Rightarrow V_Q = 0$

$$V_{MQ} = -\int_Q^M \vec{E}\cdot d\vec{L} = -\left\{ \int_{x=4}^{2} 6x^2\,dx + \int_{y=-2}^{6} 6y\,dy + \int_{z=-35}^{-1} 4\,dz \right\}$$

$$= \left\{ -\frac{6x^3}{3}\Big]_4^2 + \frac{6y^2}{2}\Big]_{-2}^{6} + 4z\Big]_{-35}^{1} \right\}$$

$V_{MQ} = V_M - V_Q = -120V$

Since $V_Q = 0 \Rightarrow \boxed{V_M = -120V}$

c) $$V_{NP} = -\int_P^N \vec{E}\cdot d\vec{L} = -\left\{ \int_{x=1}^{-3} 6x^2\,dx + \int_{y=2}^{-3} 6y\,dy + \int_{z=-4}^{2} 4\,dz \right\}$$

$$V_{NP} = -\left\{ \frac{6x^3}{3}\Big]_1^{-3} + \frac{6y^2}{2}\Big]_2^{-3} + 4z\Big]_{-4}^{2} \right\}$$

$V_{NP} = V_N - V_P = 17$

$V_N = 17 + V_P = 19V$

$\boxed{V_N = 19V}$

6) A 15nC point charge is at the origin in free space. Calculate V_1 if point P_1 is located at $P_1(-2,3,-1)$ and

a) $V=0$ at $(6,5,4)$

b) $V=0$ at infinity

c) $V=5V$ at $(2,0,4)$

a) $V_{P1,A} = \frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{r_{P1}} - \frac{1}{r_A}\right]$

$= 15\times10^{-9} \times 9\times10^{9}\left[\frac{1}{\sqrt{4+9+1}} - \frac{1}{\sqrt{36+25+16}}\right]$

$V_{P1,A} = 20.695V = V_{P1} - V_A$

Since $V_A = 0$

$\boxed{V_{P1,A} = 20.695V}$

b) Let $B(\infty,\infty,\infty)$ & Given $V_B = 0$

$V_{P1B} = \frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{r_{P1}} - \frac{1}{r_B}\right]$

$V_{P1B} = 15\times10^{-9}\times9\times10^{9}\left[\frac{1}{\sqrt{14}} - \frac{1}{\infty}\right]$

$\boxed{V_{P1B} = 36.08V}$

c) Let $c(2,0,4)$ & given $V_c = 5V$

$V_{P1C} = \frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{r_{P1}} - \frac{1}{r_c}\right]$

$V_{P1C} = 5.8933 = V_{P1} - V_c$

$V_{P1} = V_{P1,C} + V_c = 5.8933 + 5V$

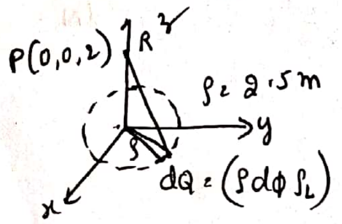
$\boxed{V_{P1} = 10.8933V}$

7) If we take zero reference for potential at infinity, find the potential at (0,0,2) caused by this charge configuration in free space a) 12 nC/m on the line $\rho = 2.5$ m, $z = 0$

b) point charge 18 nC at (1,2,-1)

c) 12 nC/m on the line $y = 2.5$, $z = 0$

a) It is a circular charged ring of charge density $\rho_L = 12$ nC/m of radius $\rho = 2.5$ m & at $z = 0$ $(-1 < x < +1)$



W.K.T potential due to a point charge Q

$$V = \frac{Q}{4\pi\varepsilon_0 R}$$

$$dV = \frac{dQ}{4\pi\varepsilon_0 R} = \frac{\rho\, d\phi\, \rho_L}{4\pi\varepsilon_0\sqrt{2.5^2+2^2}}$$

$$V = \int dV = \int_{\phi=0}^{2\pi} \frac{\rho_L\, \rho\, d\phi}{4\pi\varepsilon_0\sqrt{2.5^2+2^2}}$$

$$V = \frac{\rho_L\, \rho\, (2\pi)}{4\pi\varepsilon_0\sqrt{2.5^2+4}}$$

$\rho = 2.5$ cm

$\rho_L = 12$ nC/m

$$\boxed{V = 529.885\ V}$$

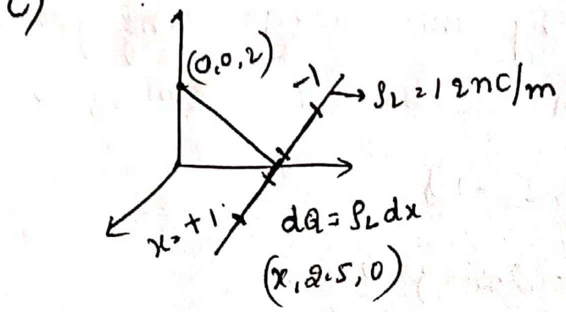
b) $$V = \frac{Q}{4\pi\varepsilon_0 R}$$

$$R = \sqrt{(0-1)^2+(0-2)^2+(2+1)^2} = \sqrt{14}$$

$$V = \frac{18\times10^{-9}\times9\times10^{9}}{\sqrt{14}} = 43.296\,V$$

$$dV = \frac{dQ}{4\pi\varepsilon_0 R} = \frac{\rho_L\, dx}{4\pi\varepsilon_0\sqrt{2.5^2+2^2}}$$

c)



$$V = \int_{x=-1}^{+} \frac{\rho_L\, dx}{4\pi\varepsilon_0\sqrt{2.5^2+2^2}}$$

$$V = \frac{\rho_L\, x(2)}{4\pi\varepsilon_0\sqrt{2.5^2+2^2}}$$

$$\boxed{V = 67.46V}$$

(8) Given the potential field in cylindrical coordinates $V = \frac{100}{z^2+1}\rho\cos\phi\ V$, and point P at $\rho = 3\,m$, $\phi = 60°$, $z = 2m$

find values at P for a) V b) $\vec{E}$ c) E d) $\frac{dV}{dN}$ e) $\hat{a}_N$

f) ρ_v in free space

a) $V_{P(3,60°,2)} = \frac{100}{2^2+1} 3\cos(60°) = 30V$

b) $\vec{E} = -\nabla V = -\left[\frac{\partial V}{\partial \rho}\hat{a}_\rho + \frac{1}{\rho}\frac{\partial V}{\partial \phi}\hat{a}_\phi + \frac{\partial V}{\partial z}\hat{a}_z\right]$

$$\vec{E} = -\left[\frac{100}{z^2+1}\cos\phi\,\hat{a}_\rho - \frac{100}{z^2+1}\sin\phi\,\hat{a}_\phi - \frac{100\rho\cos\phi(2z)\hat{a}_z}{(z^2+1)^2}\right]$$

$$\vec{E} = -\left[\frac{100\cos 60°}{4+1}\hat{a}_\rho - \frac{100}{5}\sin 60°\,\hat{a}_\phi - \frac{600\cos 60}{5}\hat{a}_z\right]$$

$$\vec{E} = -10\hat{a}_\rho + 17.32\hat{a}_\phi + 24\hat{a}_z\ V/m$$

c) $E = |\vec{E}| = 31.2407$

d) $\hat{a}_N \frac{dV}{dN} = -\vec{E}$

$$|\vec{E}| = \frac{dV}{dN} = 31.2407\ V/m$$

e) $\vec{E} = -\frac{dv}{dN}\hat{a}_N$

$$\hat{a}_N = \frac{\vec{E}}{(-dv/dN)} = \frac{1}{-31.24}\left[-10\hat{a}_\rho + 17.32\hat{a}_\phi + 24\hat{a}_z\right]$$

$$\hat{a}_N = 0.32\hat{a}_\rho - 0.554\hat{a}_\phi - 0.7682\hat{a}_z$$

f) $\nabla \cdot \vec{D} = \rho_v$

$$\rho_v = \nabla\cdot(\varepsilon_0 \vec{E}) = \varepsilon_0(\nabla\cdot\vec{E})$$

$$\rho_v = +\varepsilon_0\left[\frac{1}{\rho}\frac{\partial}{\partial\rho}(\rho E_\rho) + \frac{1}{\rho}\frac{\partial}{\partial\phi}(E_\phi) + \frac{\partial E_z}{\partial z}\right]$$

$$= +\varepsilon_0\left[\frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\frac{\rho 100\cos\phi}{z^2+1}\right) + \frac{1}{\rho}\frac{\partial}{\partial\phi}\left(\frac{-100\sin\phi}{z^2+1}\right) + \frac{\partial}{\partial z}\left(\frac{200\rho\cos\phi\, z}{(z^2+1)^2}\right)\right]$$

$$= \varepsilon_0\left[\frac{100\cos\phi}{\rho(z^2+1)} + \left(\frac{-100\cos\phi}{\rho(z^2+1)}\right) - \frac{200\rho\cos\phi}{(z^2+1)^4}\right]$$

$$\left((z^4+1^2) + 2z^2 - 4z^4 - 4z^2\right)$$

$$\rho_v = \varepsilon_0\left\{\frac{-200\rho\cos\phi}{(z^2+1)^4}\left(-3z^4+1-2z^2\right)\right\}$$

$$\boxed{\rho_v = -233.7456\ pc/m^3} \approx 234\ pc/m^3$$

(9) Find the energy stored in free space for the region $2mm < r < 3mm$, $0 < \theta < 90°$, $0 < \phi < 90°$ given the potential field $V =$ a) $\frac{200}{r}V$ b) $\frac{300\cos\theta}{r^2}V$

a) $V = \frac{200}{r} \; V$

$$\vec{E} = -\nabla V = -\left[\frac{\partial V}{\partial r}\hat{a}_r + \frac{1}{r}\frac{\partial V}{\partial \theta}\hat{a}_\theta + \frac{1}{r\sin\theta}\frac{\partial V}{\partial \phi}\hat{a}_\phi\right]$$

$$\vec{E} = -\left\{\frac{-200(1)}{r^2}\hat{a}_r\right\} = \frac{200}{r^2}\hat{a}_r \; V/m$$

$$|\vec{E}|^2 = E^2 = \frac{200^2}{r^4}$$

$$W_E = \frac{\varepsilon_0}{2}\int_{vol} E^2 dv = \frac{\varepsilon_0}{2}\int_{\theta=0}^{90^\circ}\int_{\phi=0}^{90^\circ}\int_{r=2mm}^{3mm} \frac{200^2}{r^4} r^2 \sin\theta \, dr \, d\theta \, d\phi$$

$$= \frac{\varepsilon_0}{2}(200^2)\left(\frac{\pi}{2}\right) \left.-\cos\theta\right]_0^{\pi/2} \int_r \frac{1}{r^2} dr$$

$$= \frac{\varepsilon_0}{2} 200^2 (\pi/2)(0+1)\left[\frac{-1}{r}\right]_{2mm}^{3mm}$$

$$W_E = 46.359 \; \mu J$$

b) $V = \frac{300\cos\theta}{r^2} \; V$

$$\vec{E} = -\nabla V = -\left[\frac{\partial V}{\partial r}\hat{a}_r + \frac{1}{r}\frac{\partial V}{\partial \theta}\hat{a}_\theta + \frac{1}{r\sin\theta}\frac{\partial V}{\partial \phi}\hat{a}_\phi\right]$$

$$\vec{E} = -\left\{300\cos\theta\left(\frac{-2}{r^3}\right)\hat{a}_r + \frac{300}{r^3}(-\sin\theta)\hat{a}_\theta + 0\right\}.$$

$$\vec{E} = +\frac{300}{r^3}\left[2\cos\theta\,\hat{a}_r + \sin\theta\,\hat{a}_\theta\right]$$

$$\vec{E}^2 = |\vec{E}|^2 = \frac{300^2}{r^6}\left[4\cos^2\theta + \sin^2\theta\right]$$

$$E^2 dv = \frac{300^2}{r^4}\left(4\cos^2\theta + \sin^2\theta\right) r^2 \sin\theta \, dr \, d\theta \, d\phi$$

$$W_E = \frac{\varepsilon_0}{2}\int_{vol} E^2 dv = 36.65 \; J$$

(10) Given the Vector Current density $\vec{J} = 10\rho^2 z\,\hat{a}_\rho - 4\rho\cos^2\phi\,\hat{a}_\phi$ mA/m² a) Find the current density at $P(\rho=3, \phi=30^\circ, z=2)$

b) Determine the total current flowing outward through the circular band $\rho=3,\ 0<\phi<2\pi,\ 2<z<2.8$

a) $$\vec{J}_P = 10(3)^2(2)\,\hat{a}_\rho - 4(3)\cos^2(30^\circ)\,\hat{a}_\phi$$

$$\underline{\underline{\vec{J}_P = 180\,\hat{a}_\rho}} - 9\hat{a}_\phi \text{ mA/m}^2$$

b) $$I = \int_S \vec{J}\cdot d\vec{s} = \int_{\phi=0}^{2\pi}\int_{z=2}^{2.8}\left(10\rho^2 z\,\hat{a}_\rho - 4\rho\cos^2\phi\,\hat{a}_\phi\right)\cdot\left(\rho\,d\phi\,dz\,\hat{a}_\rho\right)$$

$$I = \int_{\phi=0}^{2\pi}\int_{z=2}^{2.8} 10\rho^3 z\,d\phi\,dz$$

$$I = 10(3)^3\left.\frac{z^2}{2}\right]_2^{2.8}(2\pi)\times10^{-3}$$

$$\underline{\underline{I = 3.26\text{ A}}}$$

(11) Current density is given in cylindrical coordinates as. $\vec{J} = -10^6 z^{1.5}\,\hat{a}_z$ A/m² in the region $0\le\rho\le20\,\mu m$; for $\rho\ge20\,\mu m$ $\vec{J}=0$. a) Find the total current crossing the surface $z=0.1$ m in the $\hat{a}_z$ direction b) If the charge velocity is 2×10^6 m/s at $z=0.1$ m, find ρ_v there c) If the volume charge density at $z=0.15$ m is -2000 C/m³, find the charge velocity there.

a) $I = \int \vec{J} \cdot d\vec{s} = \int (-10^6 z^{1.5} \hat{a}_s) \cdot (s\, d\phi\, ds\, \hat{a}_s)$

$$= -10^6 z^{1.5} \int_{s=0}^{20} \int_{\phi=0}^{2\pi} s\, d\phi\, ds$$

$\underline{I} = -39.73 \mu A$

b) $\vec{J} = \rho_v \vec{v}$

$$\rho_v = \frac{|\vec{J}|}{|\vec{v}|} = \frac{-31622.776}{2 \times 10^6} = -15.8\ mc/m^3$$

c) $$\vec{v} = \frac{|\vec{J}|}{\rho_v} = \frac{-58.094.75}{-2000} = 29\ m/s$$

(12) Find the magnitude of the current density in a sample of silver for which $\sigma = 6.17 \times 10^7$ S/m & $\mu_e = 0.0056\ m^2/Vs$ if a) The drift velocity is $1.5 \mu m/s$ b) The electric field intensity is $1 mV/m$ c) The sample is a cube $2.5 mm$ on a side having a voltage of $0.4 mV$ between opposite faces d) The sample is a cube $2.5 mm$ on a side carrying a total current of $0.5 A$.

a) Silver $\sigma = -\rho_e \mu_e = 6.17 \times 10^7$ S/m

$6.17 \times 10^7 = -\rho_e (0.0056)\ m^2/Vs$

$+\rho_e = \underline{-1.10178 \times 10^{10}}$

Not necessary

W.k.that $V_d = -\mu_e|\vec{E}|$

$$|\vec{E}| = \frac{-V_d}{\mu_e} = 2.6785 \times 10^{-4} \text{ V/m.}$$

$$|\vec{J}| = \sigma|\vec{E}|$$

$$|\vec{J}| = (6.17 \times 10^{7})(2.6785 \times 10^{-4}) \text{ A/m}^2$$

$$|\vec{J}| = 16.526 \text{ kA/m}^2$$

b)

$$|\vec{J}| = \sigma|\vec{E}|$$

$$|\vec{J}| = (6.17 \times 10^{7})(1 \times 10^{-3}) = 61.7 \text{ kA/m}^2$$

c)

$$|\vec{J}| = \frac{\sigma V}{L} = (6.17 \times 10^{7})\left(\frac{0.4 \times 10^{-3}}{(2.5 \times 10^{-3})}\right)$$

$$|\vec{J}| = 9.872 \text{ MA/m}^2$$

d)

$$|\vec{J}| = \frac{I}{S} = \frac{0.5}{(2 \times 10^{-3})^2} = 80.00 \text{ kA/m}^2$$

13 A Copper conductor has a diameter of 0.6 in. and it is 1200 ft long. Assume that it carries a total dc current of 50 A a) Find the total resistance of the conductor b) What current density exists in it? c) What is the dc voltage between the conductor ends? d) How much power dissipated in the wire?

a) For copper Conductivity $\sigma = 5.8 \times 10^{7}$ S/m

$$R = \frac{L}{\sigma S} = \frac{(1200 \times 0.3048)\,m}{(5.8 \times 10^{7})(\pi (0.3 \times 0.0254)^2)}$$

$$\underline{\underline{R = 0.0345\ \Omega}}$$

b) $J = \frac{I}{S} = \frac{50}{(0.3 \times 0.0254)^2 \pi} = \underline{\underline{2.74 \times 10^5\ A/m^2}}$

c) $R = \frac{V}{I}$

$V = (50)(0.0345) = \underline{\underline{1.725\ V}}$

d) $P = IV = (50)(1.725)$

$\underline{\underline{P = 86.25\ W}}$

(14) Given the potential field in free space $V = 100 \sinh 5x \sin 5y$ V, and a point $P(0.1, 0.2, 0.3)$ find at P
a) V b) $\vec{E}$ c) $|\vec{E}|$ d) $|\rho_s|$ if it is known that P lies on the conductor surface.

a) $V_p = (100 \sinh 5x \sin 5y)$ V.

$V_p = 43.811$

b) $\vec{E} = -\nabla V = -\left[\frac{\partial V}{\partial x}\hat{a}_x + \frac{\partial V}{\partial y}\hat{a}_y + \frac{\partial V}{\partial z}\hat{a}_z\right]$

$= -\left[100 \sin(5y)\, 5 \cosh(5x)\,\hat{a}_x + 500 \sinh(5x) \cos 5y\, \hat{a}_y\right]$

$$\vec{E} = -\left[474.43\,\hat{a}_x + 144.77\,\hat{a}_y\right]$$

d) $\rho_s = |\vec{D}_N| = |\varepsilon_0 \vec{E}_N|$

$$\rho_s = 4.39\ nC/m^2$$