# UNIT 3-Part 2

## ROOT LOCUS TECHNIQUE

# Content

- Introduction to root locus concepts
- Construction Rules
- Analysis of stability by Root Locus Plots

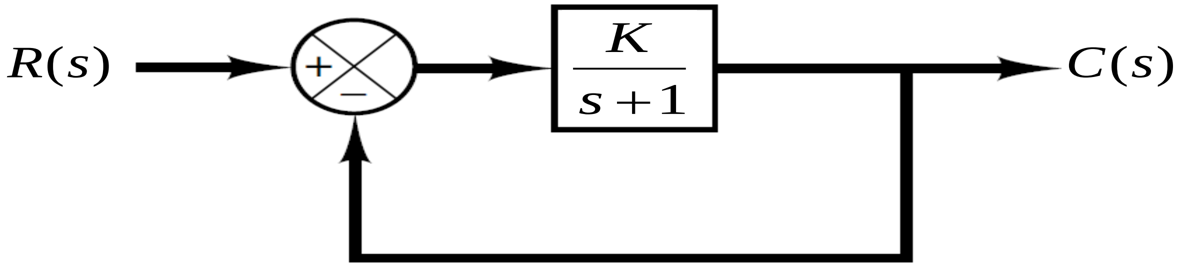
# Introduction

# Root Locus

- is a graphical approach to the Analysis and Design of Feedback Control Systems.
- is the locus of the closed loop poles as a system parameter is varied.
- Can be used to study and analyze the transient performance of a system & its relative stability

# Basic concepts

- Consider a unity feedback control system shown below.




- The open loop transfer function G(s) of the system is $G(s) = \frac{K}{s+1}$, Poles does not depend on K

- And the closed transfer function is

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)} = \frac{K}{s+1+K}, \quad \text{Poles depend on K}$$

- The open loop stability does not depend upon gain K.

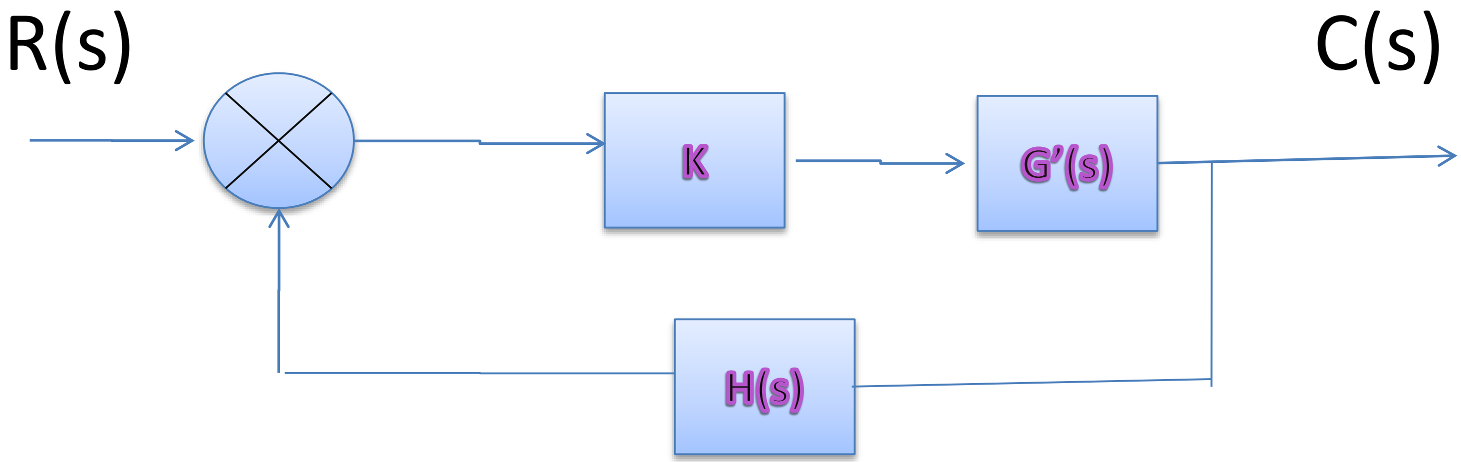
$$G(s)=\frac{K}{s+1}$$

- Whereas, the location of closed loop poles vary with the variation in gain.

$$\frac{C(s)}{R(s)}=\frac{K}{s+1+K}$$

# Feedback System

The poles of closed loop TF are obtained as solution Characteristic equation, which is complicated and depends on K, where K is system gain




Charcterstic equation of closed loop TF is

1+G(s)H(s)=0,   G(s)=KG'(s)

1+G(s)H(s)=0   i.e.,   1+KG'(s)H(s)=0

KG'(s)H(s) is the open loop TF, Poles does not depend on K & it is simpler to obtain the poles

- Finding the roots of the characteristic equation of degree higher than 3 is laborious and will need computer solution.
- A simple method for finding the roots of the characteristic equation has been developed by W. R. Evans and used extensively in control engineering.
- This method, called the *root-locus method,* is one in which the roots of the characteristic equation are plotted for all values of a system parameter.
- The root locus is the path of the roots of the characteristic equation traced out in the s-plane as a system parameter (usually K)varies

- The roots corresponding to a particular value of this parameter can then be located on the resulting graph.

- Note that the parameter is usually the gain, but any other variable of the open-loop transfer function may be used.

- By using the root-locus method the designer can predict the effects of varying the gain value, on the location of the closed-loop poles or adding open-loop poles and/or open-loop zeros.

- In General, The locus of the closed loop poles obtained when system gain 'K' is varied from

  **$-\infty$** to **$+\infty$** is called Root Locus.

1. Root loci: the portion of the root loci when K assumes positive values; that is, $0 < K < \infty$.
2. Complementary root loci: the portion of the root loci when K assumes negative values; that is, $-\infty < K < 0$.
3. Root contours: loci of roots when more than one parameter varies.

- The complete root loci refers to the combination of the root loci and the complementary root loci.

# ADVANTAGES OF ROOT LOCUS

- The absolute stability of the system can be predicted from the locations of the roots in the s-plane
- Limiting range of the values of the system gain ‘K’ can be decided for absolute stability of the system
- Using root locus, value of system gain ‘K’ which makes the system marginally stable can be determined and the corresponding value of the frequency of oscillations can be determined from intersection of root locus with imaginary axis
- For particular damping ratio of the system, gain ‘k’ can be determined which helps to design system more correctly
- Root locus analysis also helps in deciding the stability of the control system with time delay

# Angle and Magnitude Condition

- For a general closed loop system the characteristic equation is,

$$1+G(s)H(s)=0$$

$$G(s)H(s)=-1$$

As s-plane is complex,

$$G(s)H(s)=-1+j0$$

- Any value of ‘s’ if it has to be on the root locus, it must satisfy the above equation.

# Angle Condition

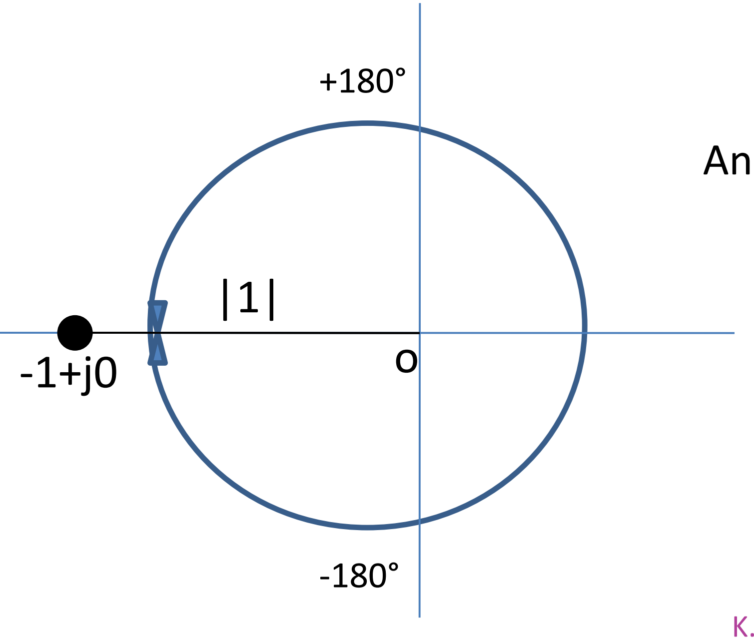
**At any point on the Root Locus, the angle of open loop TF is 180 degrees**

**G(s)H(s)=-1+j0**

- Equating angles of both sides,

$$\angle G(s)H(s) = \pm 180^\circ (2q+1) \qquad (q = 1,2,3...)$$

- Any point in s-plane which satisfies the angle condition has to be on the root locus of the corresponding system.




Magnitude=1

Angle=±180°,±540°,--------,±(2q+1)180°

# Use of angle condition

To check whether a given point lies on root locus

As all the points on the root locus must satisfy the angle condition ,to test any point in s-plane for its existence on the root locus of the given system.

# Magnitude condition

If magnitudes of the equation $G(s)H(s)=-1$ are equated then we get a magnitude condition.

$$|G(s)H(s)|=|-1+j0|=1$$

**At any point on the Root Locus, the Magnitude of open loop TF is Unity**

NOTE: Magnitude condition can be used only when a point in s-plane is confirmed for its existence on the root locus by use of angle condition.

# Use of magnitude condition

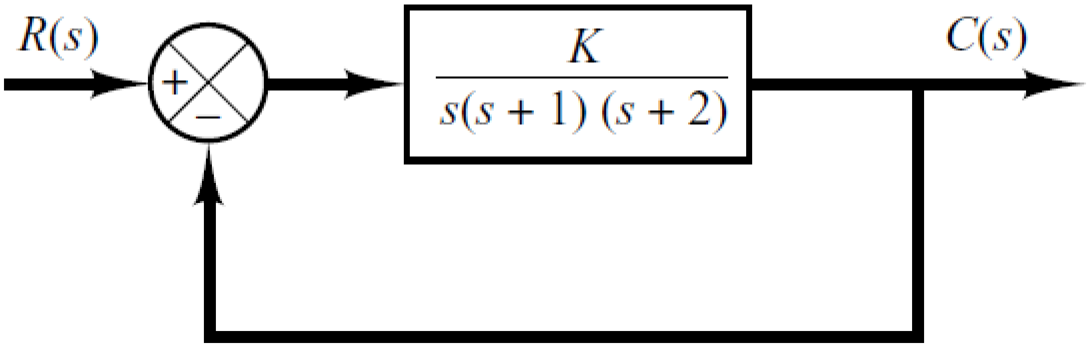
Once a point is known to be on the root locus by angle condition, we can use magnitude condition. This gives us a value of K for which a tested point is one of the roots of the characteristic equation.

- The values of ***s*** that fulfill both the angle and magnitude conditions are the roots of the characteristic equation, or the closed-loop poles.

- A locus of the points in the complex plane satisfying the angle condition alone is the root locus.

# Illustrative Example

Obtain angle and magnitude conditions on following unity feedback system & check whether $s=-0.25$ is on the root locus or not. If $s=-0.25$ is on the root locus determine the value of gain K at that point.



# Illustrative Example#1

- Here $$G(s)H(s) = \frac{K}{s(s+1)(s+2)}$$

- For the given system the angle condition becomes

$$\angle G(s)H(s) = \angle \frac{K}{s(s+1)(s+2)}$$

$$\angle G(s)H(s) = \angle K - \angle s - \angle(s+1) - \angle(s+2)$$

$$\angle K - \angle s - \angle(s+1) - \angle(s+2) = \pm 180^\circ(2k+1)$$

# Illustrative Example

- For example to check whether *s=-0.25* is on the root locus or not we can apply angle condition as follows.

$$\angle G(s)H(s)\Big|_{s=-0.25} = \angle K\Big|_{s=-0.25} - \angle s\Big|_{s=-0.25} - \angle(s+1)\Big|_{s=-0.25} - \angle(s+2)\Big|_{s=-0.25}$$

$$\angle G(s)H(s)\Big|_{s=-0.25} = -\angle(-0.25) - \angle(+0.75) - \angle(1.75)$$

$$\angle G(s)H(s)\Big|_{s=-0.25} = -180^\circ - 0^\circ - 0^\circ$$

$$\angle G(s)H(s)\Big|_{s=-0.25} = \pm 180^\circ(2k+1)$$

# Illustrative Example

- Here $$G(s)H(s)=\frac{K}{s(s+1)(s+2)}$$

- And the Magnitude condition becomes

$$\left|G(s)H(s)\right|=\left|\frac{K}{s(s+1)(s+2)}\right|=1$$

# Illustrative Example

- Now we know from angle condition that the point s=-0.25 is on the rot locus. But we do not know the value of gain K at that specific point.

- We can use magnitude condition to determine the value of gain at any point on the root locus.

$$\left|\frac{K}{s(s+1)(s+2)}\right|_{s=-0.25}=1$$

$$\left|\frac{K}{(-0.25)(-0.25+1)(-0.25+2)}\right|_{s=-0.25}=1$$

# Illustrative Example

$$\left|\frac{K}{(-0.25)(-0.25+1)(-0.25+2)}\right|_{s=-0.25}=1$$

$$\left|\frac{K}{(-0.25)(0.75)(1.75)}\right|=1$$

$$\left|\frac{K}{-0.3285}\right|=1$$

$$\frac{K}{0.328}=1$$

$$K=0.328$$

# Assignment

For the Unity FB given below,

(i)Check If $s=-0.2+j0.937$ is on the root locus, if yes, determine the value of gain K at that point.

(ii) Check If $s=-1+j2$ is on the root locus,if yes, determine the value of gain K at that point.