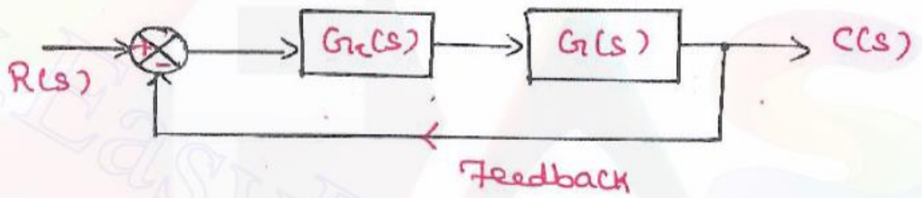
Controller:-

A controller is a device which when introduced in feedback or forward path system, controls the steady state and transient response as per the requirement.

P-Controller:- [Proportional Controller]



The proportional controller is a device that produces a control signal u(t) which is proportional to the error signal.

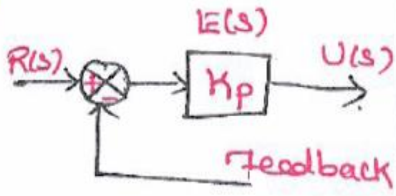
$$u(t) \propto e(t)$$

$$u(t) = K_p e(t)$$

Taking laplace transform,

$$U(s) = K_p E(s)$$

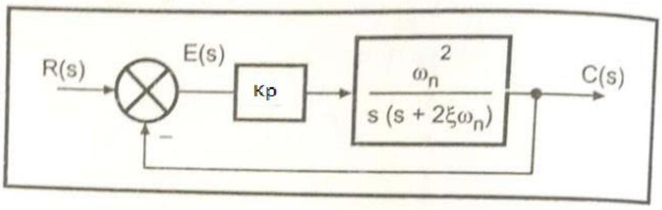
$$\boxed{\frac{U(s)}{E(s)} = K_p.}$$



Consider such second order system where controller input is error itself and proportional constant is $K_p = 1$ as shown in the Fig. 7.40.

$$G(s)\,H(s) = \frac{\omega_n^2}{s(s + 2\xi\,\omega_n)}$$

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\xi\,\omega_n\,s + \omega_n^2}$$



For this system damping ratio is ξ and natural frequency ω_n.

And for steady state error

$$K_p = \lim_{s \to 0} G(s)\,H(s) = \infty$$

and

$$K_v = \lim_{s \to 0} s\,G(s)\,H(s) = \frac{\omega_n}{2\xi}$$

Now if transient response is to be improved, damping ratio must be changed.

In general good time response demands,

i) Less settling time ii) Less overshoot

iii) Less rise time iv) Smallest s.s. error

By increase in K_v i.e. increase in system gain, s.s. error can be reduced but due to high gain settling time and peak overshoot increases. This may lead to instability of system.

So compromise is made to keep steady state error and overshoot within acceptable limits by providing following different types of controllers.

i) PD → Proportional + Derivative Action.

ii) PI → Proportional + Integral Action .

iii) PID → Proportional + Derivative + Integral Action.

→ The main drawback of P-Controller is, it produces constant steady state error.

PI-Controller:- [Proportional Plus Integral Controller]

The PI Controller produces an output signal consisting of two terms.

a) proportional to error signal

b) Proportional to integral of error signal

In PI-controller,

$$u(t) \propto \left[e(t) + \int e(t)\,dt\right]$$

$$u(t) = K_p e(t) + \frac{K_p}{T_i}\int e(t)\cdot dt.$$

Taking laplace transform,

$$U(s) = K_p E(s) + \frac{K_p}{T_i s} E(s)$$

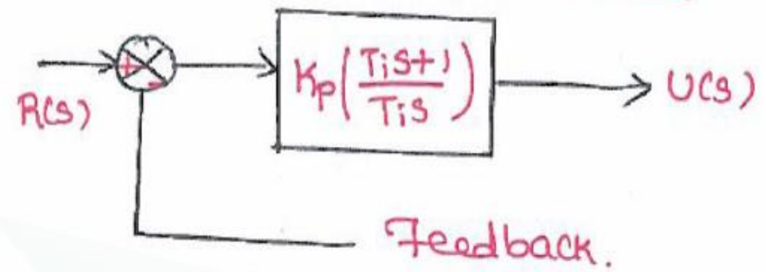
$$U(s) = E(s)\left[K_p + \frac{K_p}{T_i s}\right]$$

$$\frac{U(s)}{E(s)} = K_p\left[1 + \frac{1}{T_i s}\right]$$

$$\boxed{\frac{U(s)}{E(s)} = K_p\left[\frac{T_i s + 1}{T_i s}\right]}$$

$T_i \rightarrow$ Time constant

$K_p \rightarrow$ Proportional gain.



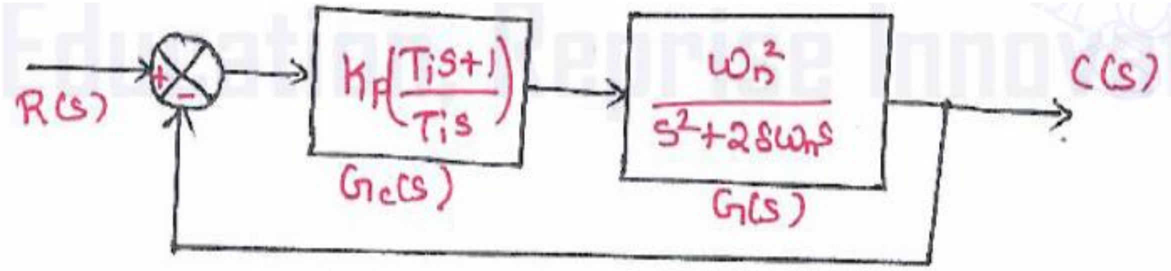
Advantages of PI - Controller:-

- $\rightarrow$ Increases the loop gain
- $\rightarrow$ Reduces the steady state error.

Effect of PI - Controller:-

The transfer function of PI - controller is,

$$G_c(s) = K_p\left(\frac{T_i s + 1}{T_i s}\right).$$



$$G(s)_{new} = G_c(s) \cdot G(s)$$

$$G(s)_{new} = K_p\left(\frac{T_is+1}{T_is}\right)\left(\frac{\omega_n^2}{s^2+2\delta\omega_n s}\right)$$

$$= \frac{\omega_n^2 K_p(T_is+1)}{T_i s(s^2+2\delta\omega_n s)}$$

The closed loop transfer function,

$$\frac{C(s)}{R(s)} = \frac{G(s)_{new}}{1+G(s)H(s)}$$

$$= \frac{\dfrac{\omega_n^2 K_p(1+T_is)}{s(s^2+2\delta\omega_n s)T_i}}{1+\dfrac{\omega_n^2 K_p(1+T_is)}{s(s^2+2\delta\omega_n s)T_i}\times 1}$$

$$= \frac{\dfrac{\omega_n^2 K_p(1+T_i s)}{s(s^2+2\delta\omega_n s)T_i}}{\dfrac{s(s^2+2\delta\omega_n s)T_i + \omega_n^2 K_p(1+T_i s)}{s(s^2+2\delta\omega_n s)T_i}}$$

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2 K_p(1+T_i s)}{s^3 T_i + 2\delta\omega_n s^2 T_i + \omega_n^2 K_p T_i s + \omega_n^2 K_p}$$

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2 K_p + \omega_n^2 K_p T_i s}{s^3 T_i + 2\delta\omega_n s^2 T_i + \omega_n^2 K_p T_i s + \omega_n^2 K_p}$$

$$\frac{C(s)}{R(s)} = \frac{(K_p/T_i)\omega_n^2 + \omega_n^2 K_p s}{s^3 + 2\delta\omega_n s^2 + \omega_n^2 K_p s + \omega_n^2 (K_p/T_i)}$$

Assume, $K_p/T_i = K_i$

$$\boxed{\frac{C(s)}{R(s)} = \frac{K_i\omega_n^2 + \omega_n^2 K_p s}{s^3 + 2\delta\omega_n s^2 + \omega_n^2 K_p s + \omega_n^2 K_i}}$$

→ It is observed that, the PI-controller introduces zero in the system and increase the order by one.

→ To increase the type number, results in reduce the steady-state error.

Now as order increases by one, system relatively becomes less stable as K_i must be designed in such a way that system will remain in stable condition. Second order system is always stable. Hence transient response gets affected badly if controller is not designed properly.

While $$K_p = \lim_{s \to 0} G(s)\,H(s) = \infty,\ e_{ss} = 0$$

$$K_v = \lim_{s \to 0} s\,G(s)\,H(s) = \infty,\ e_{ss} = 0$$

Hence as type is increased by one, error becomes zero for ramp type of inputs i.e. steady state of system gets improved and system becomes more accurate in nature.

Hence PI controller has following effects :

i) It increases order of the system.

ii) It increases TYPE of the system.

iii) Design of K_i must be proper to maintain stability of system. So it makes system relatively less stable.

iv) Steady state error reduces tremendously for same type of inputs. i.e. in general this controller improves steady state part affecting the transient part.

* Proportional Derivative (PD) Controller *

It produces an output, which is the combination of the outputs of proportional and derivative controllers.

$$u(t) \propto e(t) + \frac{d}{dt} e(t)$$

$$u(t) = K_p\, e(t) + K_D \frac{d}{dt} e(t)$$

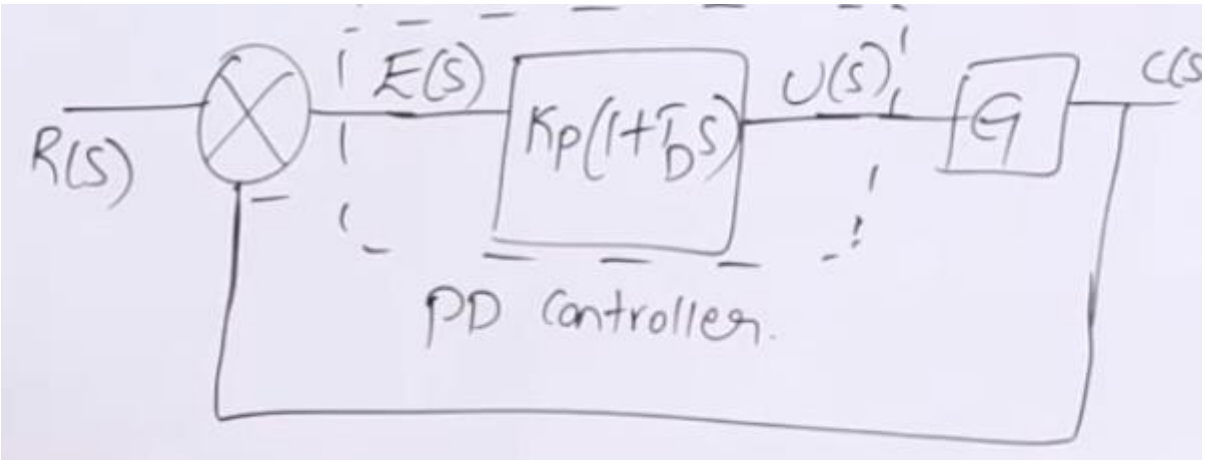
Take L.T on B.S

$$U(s) = K_p E(s) + K_D\, s\, E(s) = E(s)\left(K_p + K_D s\right)$$

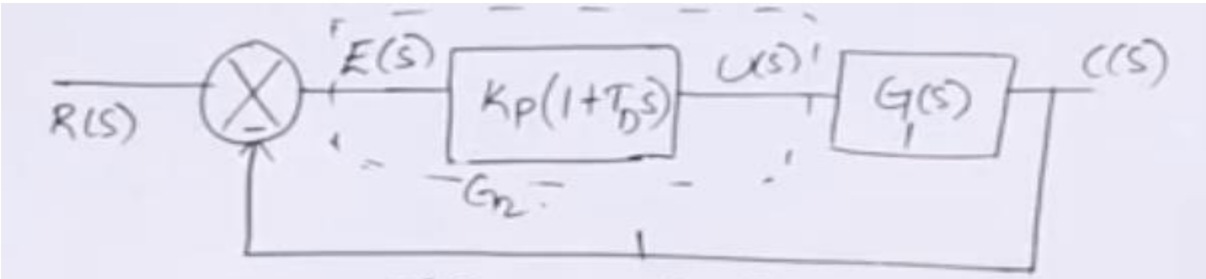
$$\frac{U(s)}{E(s)} = K_p + K_D s$$

$$\Rightarrow K_P\left(1+\frac{K_D}{K_P}S\right)$$

$$K_P(1+T_D S).$$



Effect of PD controller:



$$\frac{U(S)}{E(S)} = K_P(1+T_D(S)) \qquad T_D = \frac{K_D}{K_P}$$

$$\Rightarrow K_D = K_P T_D$$

$$G_1(S) = \frac{\omega_n^2}{S(S+2\zeta\omega_n)}.$$

$$G(S) = G_1(S)G_2(S) = \frac{K_P\omega_n^2(1+T_D S)}{S^2+2\zeta\omega_n S}$$

$$\frac{C(S)}{R(S)} = \frac{G(S)}{1+G(S)H(S)} = \frac{G(S)}{1+G(S)}.$$

$$= \frac{K_P \omega_n^2 (1+T_D S)}{S^2 + 2\zeta\omega_n S + K_P \omega_n^2 (1+T_D S)}.$$

$$\frac{C(S)}{R(S)} = \frac{K_P \omega_n^2 + K_P T_D S \omega_n^2}{S^2 + 2\zeta\omega_n S + K_P \omega_n^2 + K_P T_D S \omega_n^2}$$

$$= \frac{K_P \omega_n^2 + K_P T_D S \omega_n^2}{S^2 + (2\zeta\omega_n + K_P T_D \omega_n^2) S + K_P \omega_n^2}$$

$$\frac{C(S)}{R(S)} = \frac{K_P \omega_n^2 + K_D S \omega_n^2}{S^2 + (2\zeta\omega_n + K_D \omega_n^2) S + K_P \omega_n^2}$$

As there is no change in coefficients, error also will remain same. Hence PD controller has following effects on system.

i) It increases damping ratio.

ii) 'ω_n' for system remains unchanged.

iii) 'TYPE' of the system remains unchanged.

iv) It reduces peak overshoot.

v) It reduces settling time.

vi) Steady state error remains unchanged.

In general it improves transient part without affecting steady state.

PID-Controller:-

The PID-controller Produces an output signal consist of three terms,

a) one Proportional to error signal

b) Proportional to integral of error signal

c) Proportional to derivative of error signal

$$u(t) \propto [e(t) + \int e(t)dt + \frac{d}{dt} e(t)]$$

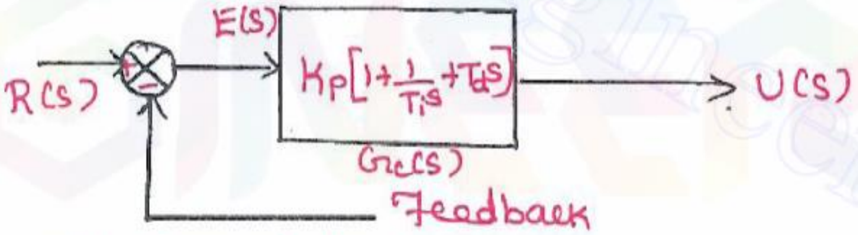
$$u(t) \propto K_p e(t) + \frac{K_p}{T_i} \int e(t)dt + K_p T_d \frac{d}{dt} e(t)$$

Taking Laplace Transform,

$$U(s) = K_p E(s) + \frac{K_p}{T_i s} E(s) + K_p T_d s E(s)$$

$$U(s) = E(s) \cdot K_p \left[1 + \frac{1}{T_i s} + T_d s\right]$$

$$\boxed{\frac{U(s)}{E(s)} = K_p\left[1 + \frac{1}{T_i s} + T_d s\right]}$$



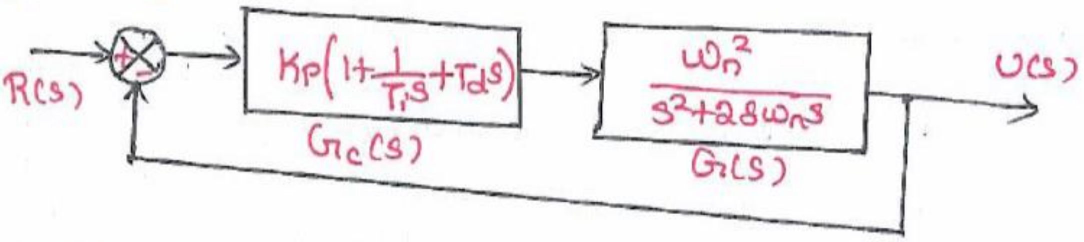
Effect of PID-Controller:-

→ A suitable condition of three basic modes P, I, D can improve all the aspect of the system performance.

→ The P-Controller increase the loop gain and stabilize the gain, but Produce steady-state error.

→ The I-Controller eliminate the steady state error.

→ The D-Controller reduces the rate of change of error.



The new transfer function is,

$$G(s)_{new} = G_c(s)\cdot G(s)$$

$$= K_p\left(1+\frac{1}{T_i s}+T_d s\right)\cdot\frac{\omega_n^2}{s^2+2\delta\omega_n s}$$

$$G(s)_{new} = \frac{K_p\omega_n^2\left(1+\frac{1}{T_i s}+T_d s\right)}{s^2+2\delta\omega_n s}.$$

closed loop transfer function,

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)H(s)}$$

$$= \frac{\dfrac{K_p \omega_n^2 \left(1+\frac{1}{T_i s}+T_d s\right)}{s^2+2\delta\omega_n s}}{1+\dfrac{K_p \omega_n^2 \left(1+\frac{1}{T_i s}+T_d s\right)}{s^2+2\delta\omega_n s}(1)}$$

$$\boxed{\frac{C(s)}{R(s)} = \frac{K_p \omega_n^2 \left(1+\frac{1}{T_i s}+T_d s\right)}{s^2+2\delta\omega_n s+K_p \omega_n^2\left(1+\frac{1}{T_i s}+T_d s\right)}}$$

This introduces:

Two zeros (improves the stability)

One pole at origin (Type of the system increases this intern reduces steady state error)