Example 2.6.1 *A uniform line charge of linear charge density 25 nC/m lies on the line $x = -3$ m and $z = 4$ m in free space. Find the electric field intensity at a point (2, 15, 3) m.* **Jan.-05, Marks 6**

Solution : The line has $x = -3$ constant and $z = 4$ constant and only y co-ordinate is variable hence it is parallel to y axis as shown in the Fig. 2.6.3. The charge is infinite hence $\bar{E} = \dfrac{\rho_L}{2\pi\varepsilon_0 r}\bar{a}_r$.

While finding $\bar{r}$, consider a point on the line as (–3, y, 4) and P (2, 15, 3).

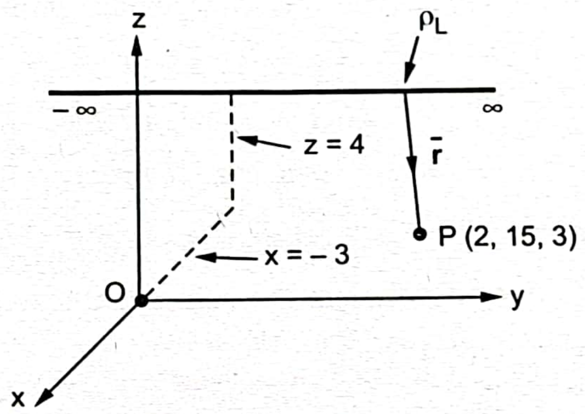


Fig. 2.6.3

Key Point *Do not consider y co-ordinate while finding $\bar{r}$ as line charge is parallel to y axis and can not have any component in $\bar{a}_y$ direction.*

$$\therefore \quad \bar{r} = [2-(-3)]\bar{a}_x + [3-4]\bar{a}_z = 5\bar{a}_x - \bar{a}_z, \ |\bar{r}| = \sqrt{26}, \ \bar{a}_r = \frac{\bar{r}}{|\bar{r}|}$$

$$\therefore \quad \bar{E} = \frac{25\times10^{-9}}{2\pi\times8.854\times10^{-12}\times\sqrt{26}}\left[\frac{5\bar{a}_x - \bar{a}_z}{\sqrt{26}}\right] = 86.42\bar{a}_x - 17.284\,\bar{a}_z \ \text{V/m}$$

Example 2.6.2 *A uniform line charge, infinite in extent, with $\rho_L = 20$ nC/m, lies along the z-axis. Find $\vec{E}$ at (6, 8, 3) m.* **Feb.-08, Marks 6**

Solution : The charge is shown in the Fig. 2.6.4.

Key Point *As line charge is along z-axis there cannot be any component of $\bar{E}$ along z direction. Thus z co-ordinate need not be considered for calculating $\bar{r}$.*

Any point on line charge is (0, 0, z).

$$\therefore \bar{r} = (6-0)\,\bar{a}_x + (8-0)\,\bar{a}_y$$

$$= 6\,\bar{a}_x + 8\,\bar{a}_y,\ |\bar{r}| = \sqrt{6^2+8^2} = 10$$

$$\therefore \bar{E} = \frac{\rho_L}{2\pi\varepsilon_0 r}\,\bar{a}_r$$

$$= \frac{20\times10^{-9}}{2\pi\times 8.854\times10^{-12}\times10}\left[\frac{6\,\bar{a}_x+8\,\bar{a}_y}{10}\right]$$

$$= \mathbf{21.5705\,\bar{a}_x + 28.76\,\bar{a}_y\ \ V/m}$$

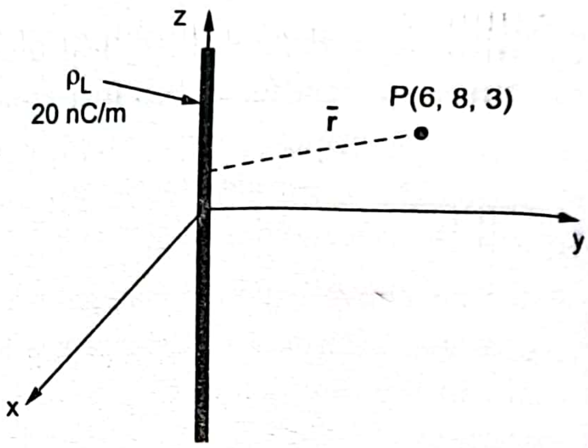


Fig. 2.6.4

Example 2.6.3 *Two uniform line charges of density 4 nC/m and 6 nC/m lie in x = 0 plane at y = + 5 m and – 6 m respectively. Find $\bar{E}$ at (4, 0, 5) m.* **May-10, Marks 6**

Solution : The line charges are shown in the Fig. 2.6.5. The line charges are parallel to z-axis.

Key Point *As charges are parallel to z-axis, $\bar{E}$ cannot have any component in $\bar{a}_z$ direction.*

Do not consider z co-ordinate while calculating $\bar{r}_1$ and $\bar{r}_2$.

$$\therefore\ \bar{E} = \bar{E}_1 + \bar{E}_2$$

$$= \frac{\rho_{L1}}{2\pi\varepsilon_0 r_1}\bar{a}_{r1} + \frac{\rho_{L2}}{2\pi\varepsilon_0 r_2}\bar{a}_{r2}$$

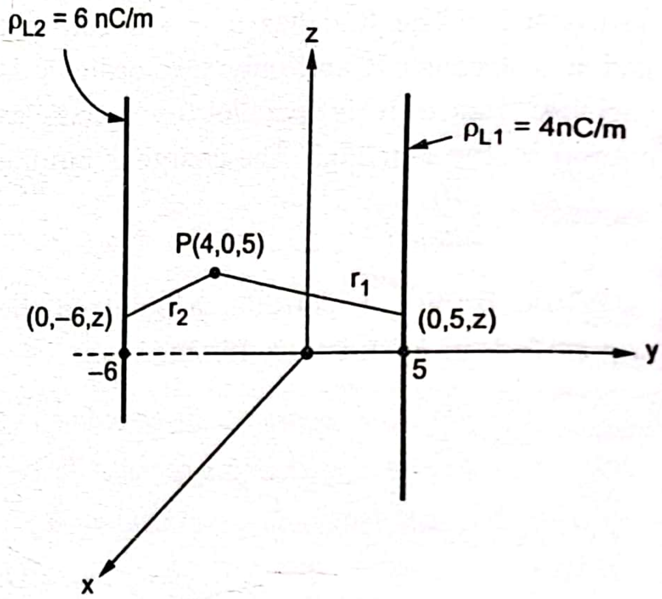


Fig. 2.6.5

$$\bar{r}_1 = 4\,\bar{a}_x - 5\,\bar{a}_y,\ |\bar{r}_1| = \sqrt{16+25} = \sqrt{41}]$$

$$\bar{a}_{r1} = \frac{4\,\bar{a}_x - 5\,\bar{a}_y}{\sqrt{41}}$$

$$\bar{r}_2 = 4\,\bar{a}_x + 6\,\bar{a}_y\ ,\ |\bar{r}_2| = \sqrt{16+36} = \sqrt{52}\ ,\ \bar{a}_{r2} = \frac{4\,\bar{a}_x - 6\,\bar{a}_y}{\sqrt{52}}$$

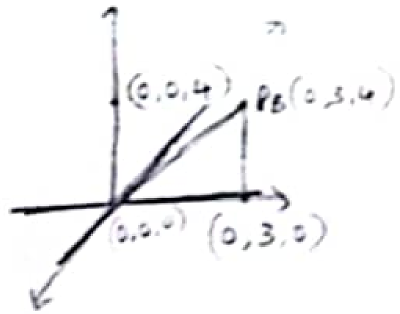
$$\therefore\ \bar{E} = \frac{4\times10^{-9}}{2\pi\times 8.854\times10^{-12}\times\sqrt{41}}\left[\frac{4\bar{a}_x - 5\,\bar{a}_y}{\sqrt{41}}\right] + \frac{6\times10^{-9}}{2\pi\times 8.854\times10^{-12}\times\sqrt{52}}\left[\frac{4\bar{a}_z + 6\bar{a}_y}{\sqrt{52}}\right]$$

$$\therefore\ \bar{E} = \mathbf{15.311\,\bar{a}_x + 3.676\,\bar{a}_y\ \ V/m} \qquad \ldots\bar{E}\ \text{at P}$$

Infinite uniform line charges of 5nC/m lie along the x and y axes in free space find $\vec{E}$ at

a) $P_A(0,0,4)$ b) $P_B(0,3,4)$

a) $$\vec{E}_A = \frac{5\times10^{-9}}{2\pi\varepsilon_0(4)}\hat{a}_z + \frac{5\times10^{-9}}{2\pi\varepsilon_0(4)}\hat{a}_z$$



$$\vec{E}_A = 45\,\hat{a}_z \text{ V/m}$$

b) $$\vec{E}_B = \frac{5\times10^{-9}\,(4\hat{a}_z)}{2\pi\varepsilon_0\,(4)^2} + \frac{5\times10^{-9}\,(3\hat{a}_y+4\hat{a}_z)}{2\pi\varepsilon_0\,(\sqrt{9+16})^2}$$

$$\vec{E}_B = 10{\cdot}78\,\hat{a}_y + 14{\cdot}38\,a_z + 22{\cdot}46\,\hat{a}_z$$

$$\vec{E}_B = 10{\cdot}8\,\hat{a}_y + 36{\cdot}9\,\hat{a}_z \text{ V/m}$$

where $\bar{a}_n$ = Direction normal to the surface charge

Thus for the points below xy plane, $\bar{a}_n = -\bar{a}_z$ hence,

$$\therefore \quad \bar{E} = -\frac{\rho_S}{2\varepsilon_0}\bar{a}_z \ \text{V/m} \quad \text{... For points below xy plane.}$$

Note The equation (2.8.6) is standard result and can be used directly to solve the problems.

Key Point *Thus electric field due to infinite sheet of charge is everywhere normal to the surface and its magnitude is independent of the distance of a point from the plane containing the sheet of charge.*

Important observations :

1. $\bar{E}$ due to infinite sheet of charge at a point is not dependent on the distance of that point from the plane containing the charge.
2. The direction of $\bar{E}$ is perpendicular to the infinite charge plane.
3. The magnitude of $\bar{E}$ is constant every where and given by $|\bar{E}| = \rho_S / 2\varepsilon_0$.

Example 2.8.1 *Two infinite sheets each of charge density ρ_S are located at $x = \pm 1$. Determine $\bar{E}$ in all regions.*

Solution : The sheets are shown in the Fig. 2.8.4.

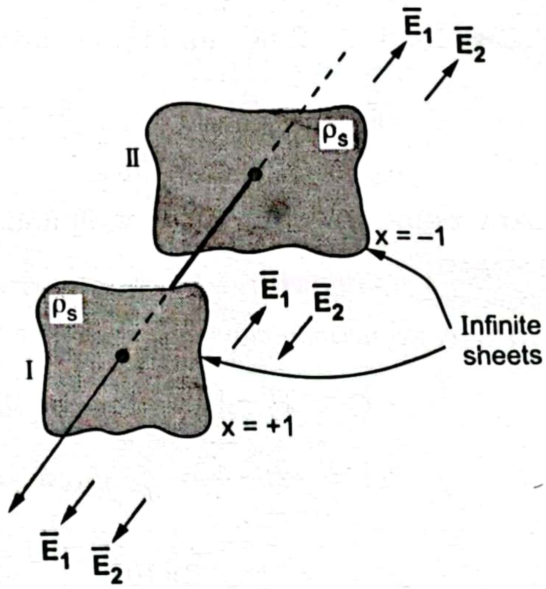


Fig. 2.8.4

For the infinite sheets,

$$\bar{E} = \frac{\rho_S}{2\varepsilon_0}\bar{a}_N$$

Here, $\bar{a}_N = \pm\bar{a}_x$

a) Region x > 1

$$\bar{E}_1 = \bar{E}_2 = \frac{\rho_S}{2\varepsilon_0}\bar{a}_x$$

$$\therefore \quad \bar{E} = 2\,\bar{E}_1 = \frac{\rho_S}{\varepsilon_0}\bar{a}_x \ \text{V/m}$$

b) Region −1< x< +1

$$\bar{E}_1 = \frac{\rho_S}{2\varepsilon_0}(-\bar{a}_x), \quad \bar{E}_2 = \frac{\rho_S}{2\varepsilon_0}\bar{a}_x$$

$$\therefore \quad \bar{E} = \bar{E}_1 + \bar{E}_2 = 0 \ \text{V/m}$$

c) Region x < – 1

$$\bar{E}_1 = \bar{E}_2 = \frac{\rho_S}{2\varepsilon_0}(-\bar{a}_x)$$

$$\therefore \quad \bar{E} = \bar{E}_1 + \bar{E}_1 = -\frac{\rho_S}{\varepsilon_0}\bar{a}_x$$

Example 2.8.2 *A line charge of 2 nC/m lies along y-axis while surface charge densities of 0.1 and – 0.1 nC/m² exist on the plane Z = 3 and Z = – 4 m respectively. Find the electric field intensity at a point (1, –7, 2).* **Aug.-04, Marks 8**

Solution : The arrangement is shown in the Fig. 2.8.5

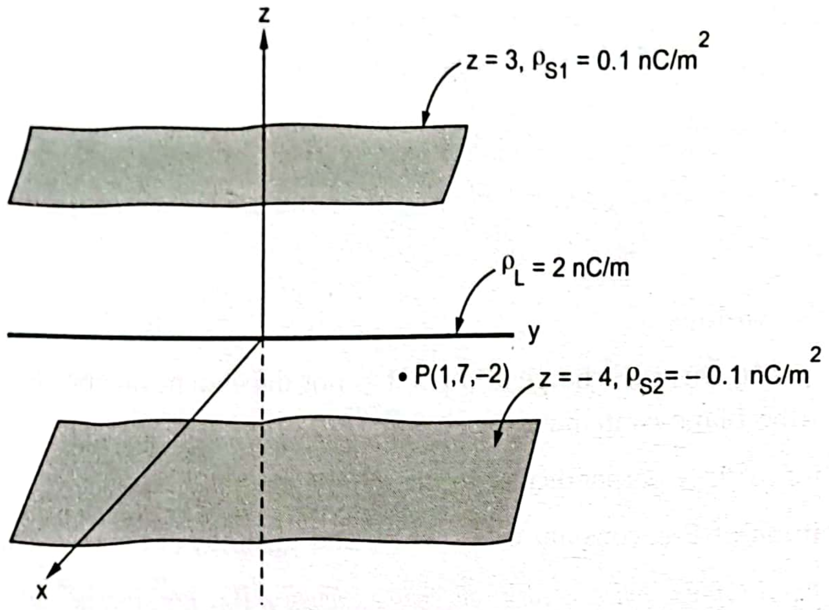


Fig. 2.8.5

Case 1 : ρ_L = 2 nC/m, infinite along y axis.

$$\bar{E}_1 = \frac{\rho_L}{2\pi\varepsilon_0 r}\bar{a}_r \qquad \text{... As charge is inifinite}$$

Any point on charge is (0, y, 0) and P(1, 7, –2).

Key Point *As line charge is along y axis, $\bar{E}_1$ can not have any component along y direction. So there is no need to consider y co-ordinate while calculating $\bar{a}_r$.*

$$\therefore \quad \bar{r} = (1-0)\,\bar{a}_x + (-2-0)\,\bar{a}_z = \bar{a}_x - 2\bar{a}_z$$

$$\therefore \quad |\bar{r}| = \sqrt{1+4} = \sqrt{5} \text{ and } \bar{a}_r = \frac{\bar{r}}{|\bar{r}|}$$

$$\therefore \quad \bar{E}_1 = \frac{2\times10^{-9}}{2\pi\times 8.854\times10^{-12}\times\sqrt{5}}\left[\frac{\bar{a}_x - 2\bar{a}_z}{\sqrt{5}}\right] = 7.19\,\bar{a}_x - 14.38\,\bar{a}_z \text{ V/m}$$

Case 2 : ρ_{S1} = 0.1 nC/m² along z = 3.

The normal direction to z = 3 is $\bar{a}_n = -\bar{a}_z$ towards side where P is located.

$$\therefore \quad \bar{E}_2 = \frac{\rho_{S1}}{2\varepsilon_0}\bar{a}_n = \frac{0.1\times10^{-9}}{2\times8.854\times10^{-12}}(-\bar{a}_z) = -5.6471\,\bar{a}_z \text{ V/m}$$

Case 3 : $\rho_{S2} = -0.1 \text{ nC/m}^2$ along $z = -4$.

The normal direction to $z = -4$ is $\bar{a}_n = +\bar{a}_z$ towards side where P is located.

$$\therefore \quad \bar{E}_3 = \frac{\rho_{S2}}{2\varepsilon_0}\bar{a}_n = \frac{-0.1\times10^{-9}}{2\times8.854\times10^{-12}}(+\bar{a}_z) = -5.6471\ \bar{a}_z \text{ V/m}$$

Thus, $\bar{E}$ at P $= \bar{E}_1 + \bar{E}_2 + \bar{E}_3 = 7.19\ \bar{a}_x - 25.674\ \bar{a}_z$ **V/m.**

Example 2.8.3 *Find $\bar{E}$ at P (1, 5, 2) m in free space if a point charge of 6 μC is located at (0,0,1), the uniform line charge density $\rho_L = 180$ nC/m along x axis and uniform sheet of charge with $\rho_S = 25 \text{ nC/m}^2$ over the plane $z = -1$.*

Solution : Case 1 : Point charge $Q_1 = 6$ μC at A (0, 0, 1) and P (1, 5, 2)

$$\therefore \quad \bar{E}_1 = \frac{Q_1}{4\pi\varepsilon_0 R_{AP}^2}\bar{a}_{AP} = \frac{Q_1}{4\pi\varepsilon_0 R_{AP}^2}\left[\frac{\bar{R}_{AP}}{|\bar{R}_{AP}|}\right]$$

$$\bar{R}_{AP} = (1-0)\bar{a}_x + (5-0)\bar{a}_y + (2-1)\bar{a}_z = \bar{a}_x + 5\bar{a}_y + \bar{a}_z$$

$$\therefore \quad |\bar{R}_{AP}| = \sqrt{(1)^2 + (5)^2 + (1)^2} = \sqrt{27}$$

$$\therefore \quad \bar{E}_1 = \frac{6\times10^{-6}}{4\pi\times8.854\times10^{-12}\times\left(\sqrt{27}\right)^2}\left[\frac{\bar{a}_x + 5\bar{a}_y + \bar{a}_z}{\sqrt{27}}\right]$$

$$\therefore \quad \bar{E}_1 = 384.375\ \bar{a}_x + 1921.879\ \bar{a}_y + 384.375\ \bar{a}_z \text{ V/m}$$

Case 2 : Line charge ρ_L along x axis.

It is infinite hence using standard result,

$$\bar{E}_2 = \frac{\rho_L}{2\pi\varepsilon_0 r}\bar{a}_r = \frac{\rho_L}{2\pi\varepsilon_0 r}\frac{\bar{r}}{|\bar{r}|}$$

Consider any point on line charge i.e. (x, 0, 0) while P (1, 5, 2). But as line is along x axis, no component of $\bar{E}$ will be along $\bar{a}_x$ direction. Hence while calculating $\bar{r}$ and $\bar{a}_r$, do not consider x co-ordinates of the points.

$$\therefore \quad \bar{r} = (5-0)\bar{a}_y + (2-0)\bar{a}_z = 5\bar{a}_y + 2\bar{a}_z$$

$$\therefore \quad |\bar{r}| = \sqrt{(5)^2 + (2)^2} = \sqrt{29}$$

$$\therefore \quad \bar{E}_2 = \frac{\rho_L}{2\pi\varepsilon_0\times\sqrt{29}}\left[\frac{5\bar{a}_y + 2\bar{a}_z}{\sqrt{29}}\right] = \frac{180\times10^{-9}\left[5\bar{a}_y + 2\bar{a}_z\right]}{2\pi\times8.854\times10^{-12}\times29}$$

$$= 557.859\ \bar{a}_y + 223.144\ \bar{a}_z \text{ V/m}$$

Case 3 : Surface charge ρ_S over the plane z = – 1. The plane is parallel to xy plane and normal direction to the plane is $\bar{a}_n = \bar{a}_z$, as point P is above the plane. At all the points above z = – 1 plane the $\bar{E}$ is constant along $\bar{a}_z$ direction.

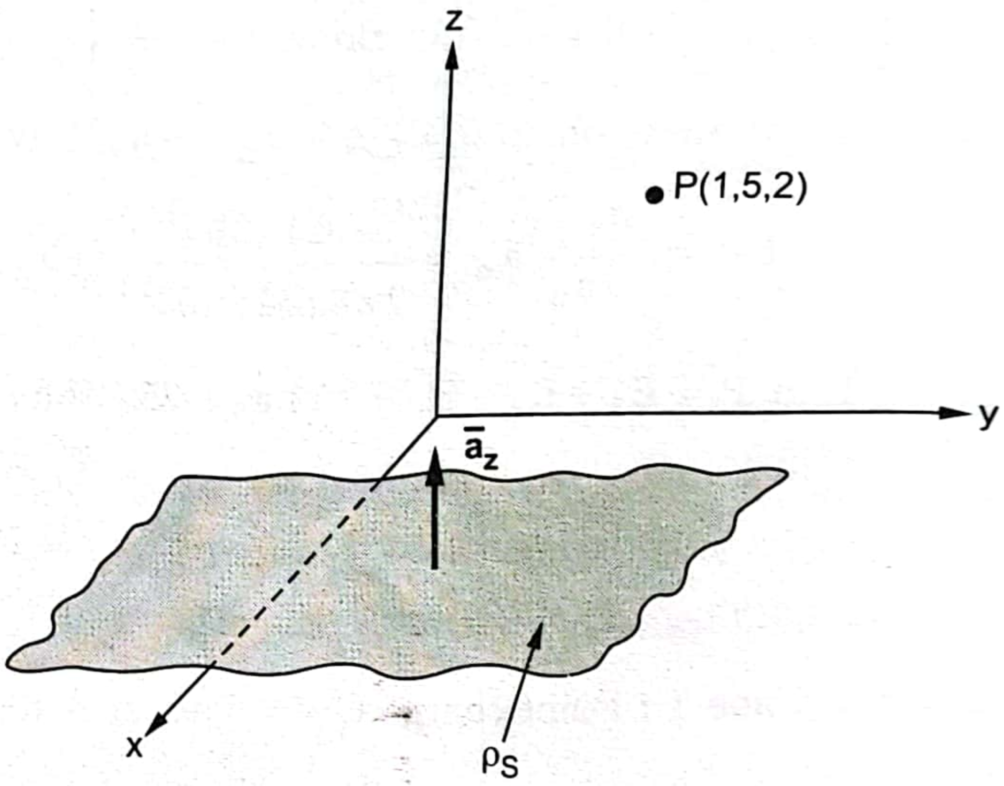


Fig. 2.8.6

$$\therefore \bar{E}_3 = \frac{\rho_S}{2\varepsilon_0}\bar{a}_n$$

$$= \frac{25\times10^{-9}}{2\times8.854\times10^{-12}}\bar{a}_z$$

$$= 1411.7913 \ \bar{a}_z \text{ V/m}$$

Hence the net $\bar{E}$ at point P is,

$$\bar{E} = \bar{E}_1 + \bar{E}_2 + \bar{E}_3$$

$$= 384.375\ \bar{a}_x + 1921.879\ \bar{a}_y + 384.375\ \bar{a}_z + 557.859\ \bar{a}_y + 223.144\ \bar{a}_z + 1411.7913\ \bar{a}_z$$

$$= \mathbf{384.375\ \bar{a}_x + 2479.738\ \bar{a}_y + 2019.3103\ \bar{a}_z \quad V/m}$$