Introduction

Bode Plot deals with the frequency response of a system separately in terms of magnitude and phase. Hence there are two plots

(i) Plot of the magnitude v/s frequency and
(ii) Plot of phase v/s frequency

A Bode plot is a (semilog) plot of the transfer function magnitude and phase angle as a function of frequency

The format is a log frequency scale on the horizontal axis and, on the vertical axis,
(i) magnitude in decibels and
(ii) phase in degrees

Bode plot-Applications

- For designing lead compensators
- For finding stability, gain and phase margins
- For system identification from the frequency response

Magnitude & Phase of Open loop TF

- The **magnitude** of the open loop transfer G(s)H(S)function in dB is

$M=20\log|G(j\omega)H(j\omega)|$

- The **phase angle** of the open loop transfer function in degrees is

$\phi=\angle G(j\omega)H(j\omega)$

Basic factors

Consider the following general transfer function

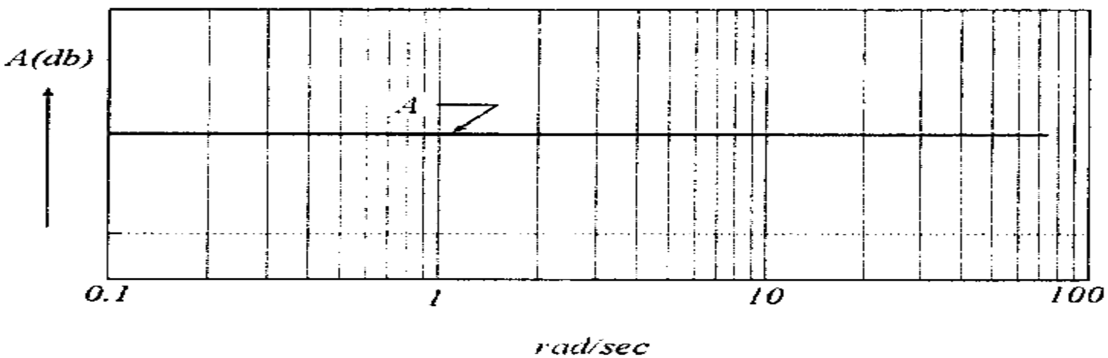
$$G(s) = \frac{k(1+T_a s)(1+T_b s)\ldots\ldots}{s^r(1+T_1 s)(1+T_2 s)\ldots\ldots(s^2+2\zeta\omega_n s+\omega_n{}^2)}$$

$$G(j\omega) = \frac{k(1+j\omega T_a)(1+j\omega T_b)\ldots\ldots}{(j\omega)^r(1+j\omega T_1)(1+j\omega T_2)\ldots\omega_n{}^2\left[1+j2\zeta\left(\frac{\omega}{\omega_n}\right)+\left(\frac{j}{\omega_n}\right)^2\right]\ldots}$$

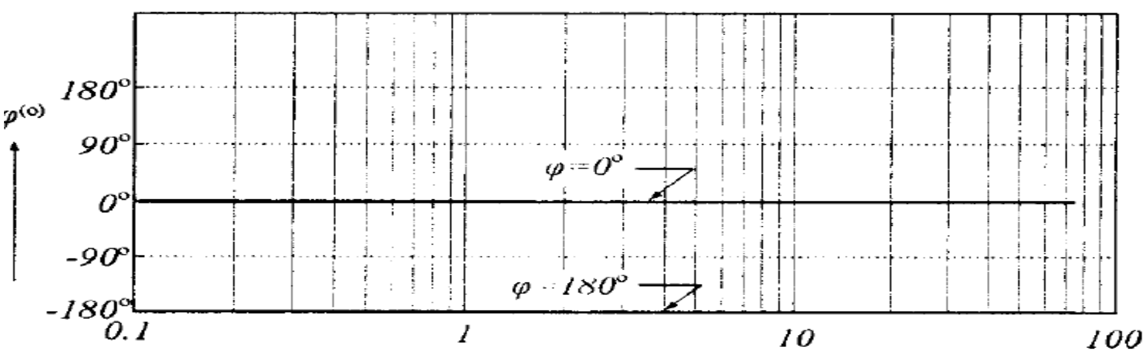
1. Gain k (Constant Term)
2. Integral or Derivative factors $(j\omega)^{\pm 1}$
   (Poles or zeros at origin)
3. First-order factors $(1+j\omega T)^{\pm 1}$
   (poles or zeros not at origin)
4. Quadratic factors $[1+2\zeta(j\omega/\omega_n)+(j\omega/\omega_n)^2]^{\pm 1}$
   (Complex poles or Complex zeros)

Gain Factor K

- Magnitude(A)=20log |K|



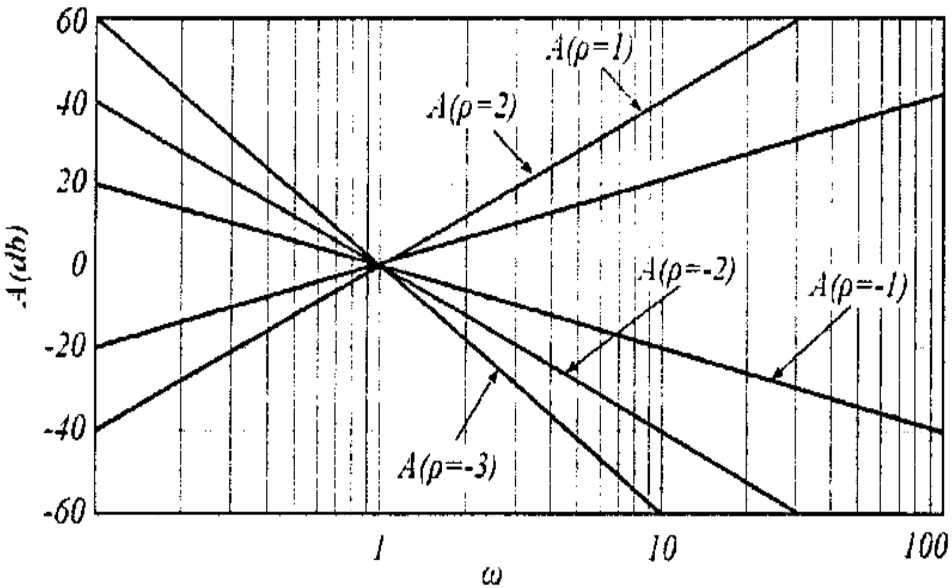
- Phase angle(ϕ)=K>0: 0° and K<0: ±180°

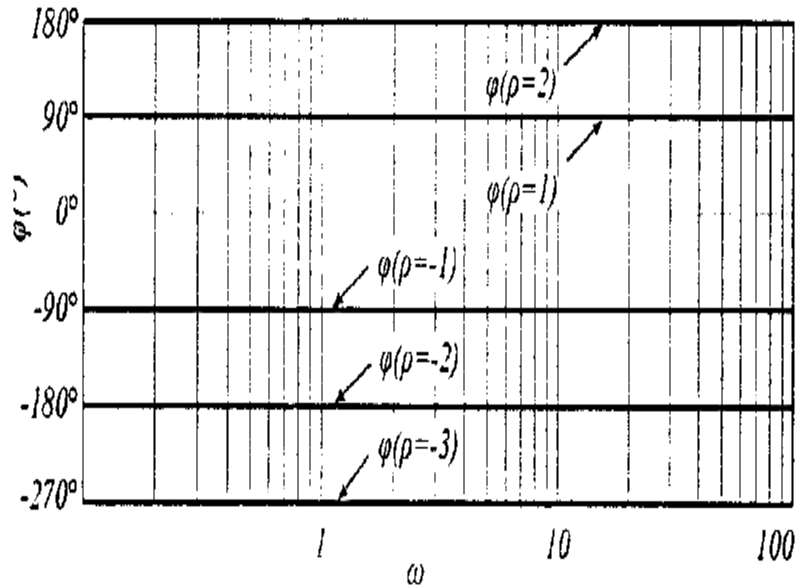


Pole /Zero at origin $(j\omega)^{\pm\rho}$

$$A = 20\log|(j\omega)^{\pm\rho}| = \pm 20\rho\log\omega$$

$$\varphi = \angle(j\omega)^{\pm\rho} = \pm 90^{\circ}\rho$$





Poles / Zeros of the Form $(j\omega T + 1)^{\pm\rho}$

$$A = \pm 20\rho \log |(j\omega T + 1)| = \pm 20\rho \log \sqrt{\omega^2 T^2 + 1}$$

$$\varphi = \pm\rho \tan^{-1}(\omega T)$$

a. When $\omega \ll 1/T$,

$$A = \pm 20\rho \log \sqrt{\omega^2 T^2 + 1} \simeq 20\rho \log 1 = 0$$

& $\Phi \approx 0^o$

b. When $\omega = 1/T$,

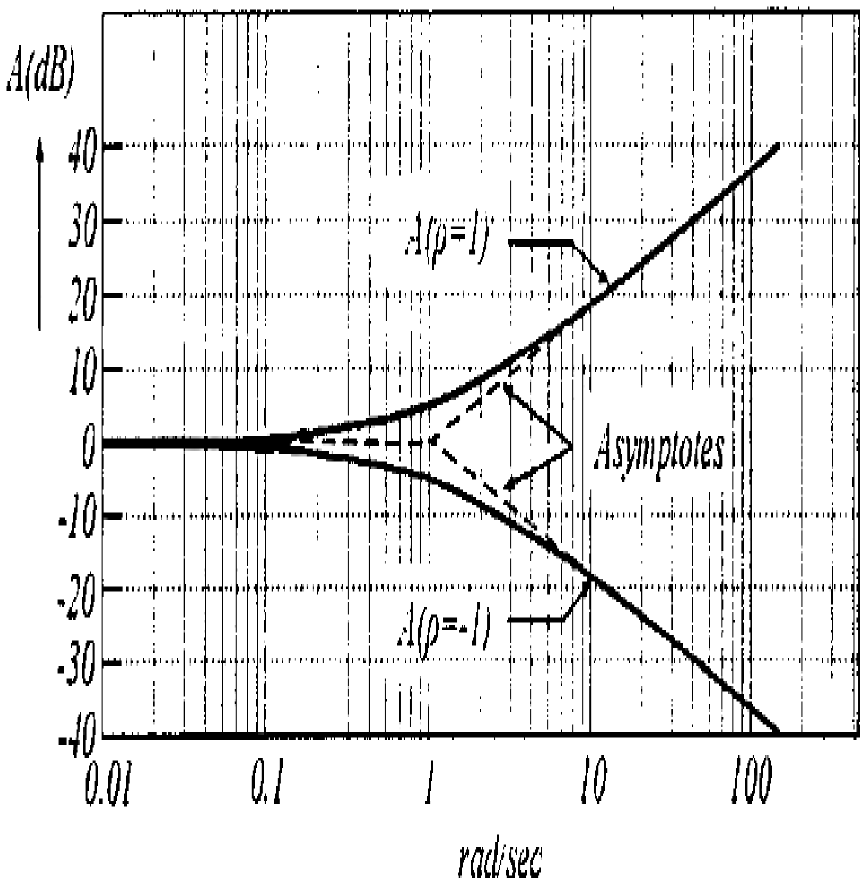
$$A = \pm 20\rho \log \sqrt{2} \simeq \pm 3\rho$$ & $\Phi \approx \pm 45^o \rho$

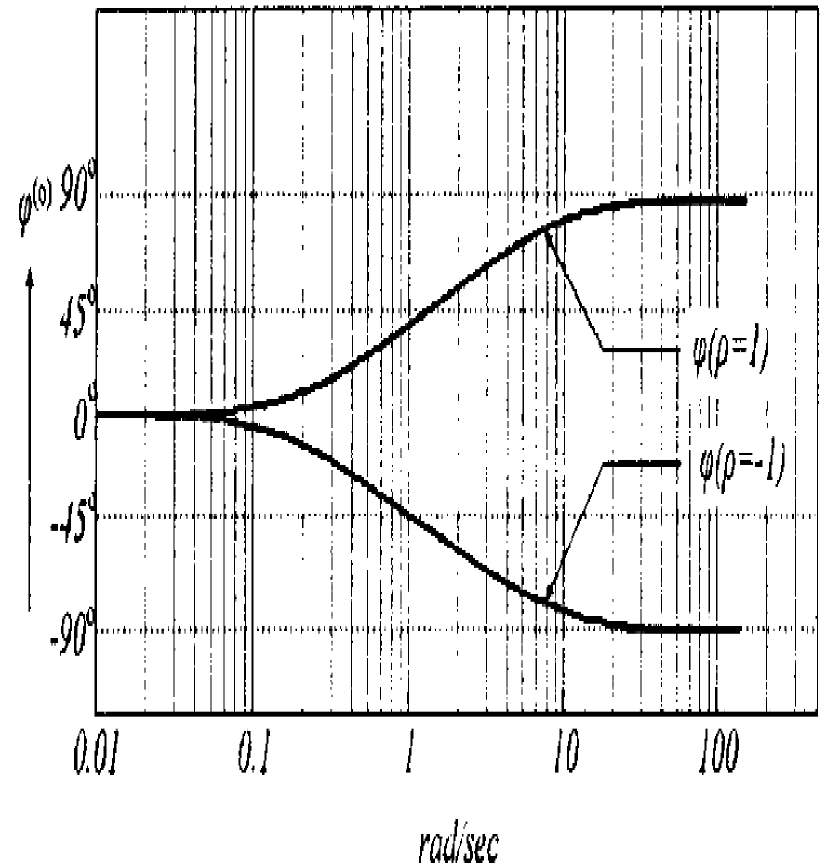
The frequency $\omega = 1/T$ is called the *corner frequency*.cy.

c. When $\omega \gg 1/T$,

$$A \simeq \pm 20\rho \log \omega T$$ & $\Phi \approx \pm 90^o \rho$

Poles / Zeros of the Form $(j\omega T + 1)^{\pm\rho}$





Quadratic factor

$$[\omega_n^{-2}(j\omega)^2 + 2\zeta\omega_n^{-1}(j\omega) + 1]^{\pm\rho}$$

$$A = \pm 20\rho \log \sqrt{(1 - u^2)^2 + 4\zeta^2 u^2}$$

$$\varphi = \pm\rho \tan^{-1}\left[\frac{2\zeta u}{1 - u^2}\right]$$

where $u = \omega/\omega_n$.

a. When $u \ll 1$, then $A \simeq \pm 20\rho \log 1 = 0$

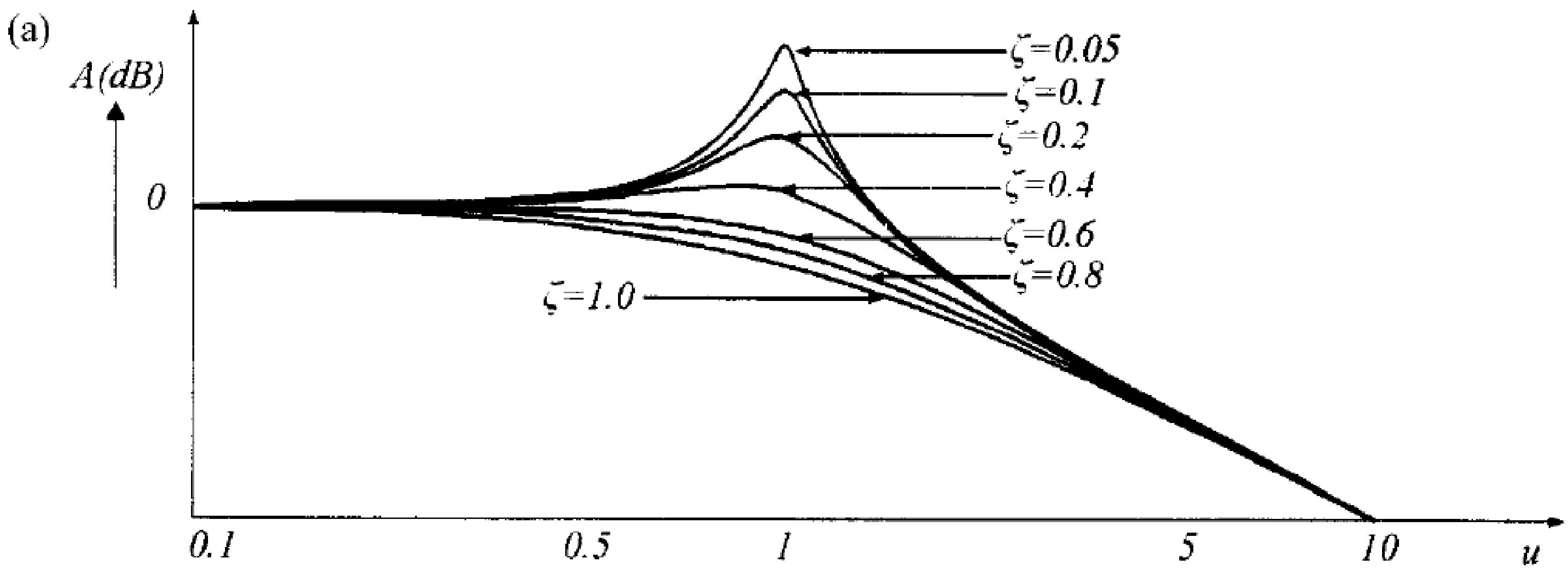
b. When $u \gg 1$, then $A \simeq \pm 40\rho \log u$

c. When $\zeta = 1$, then $A = \pm 20\rho \log |1 + u^2|$

d. When $\zeta = 0$, then $A = \pm 20\rho \log |1 - u^2|$.

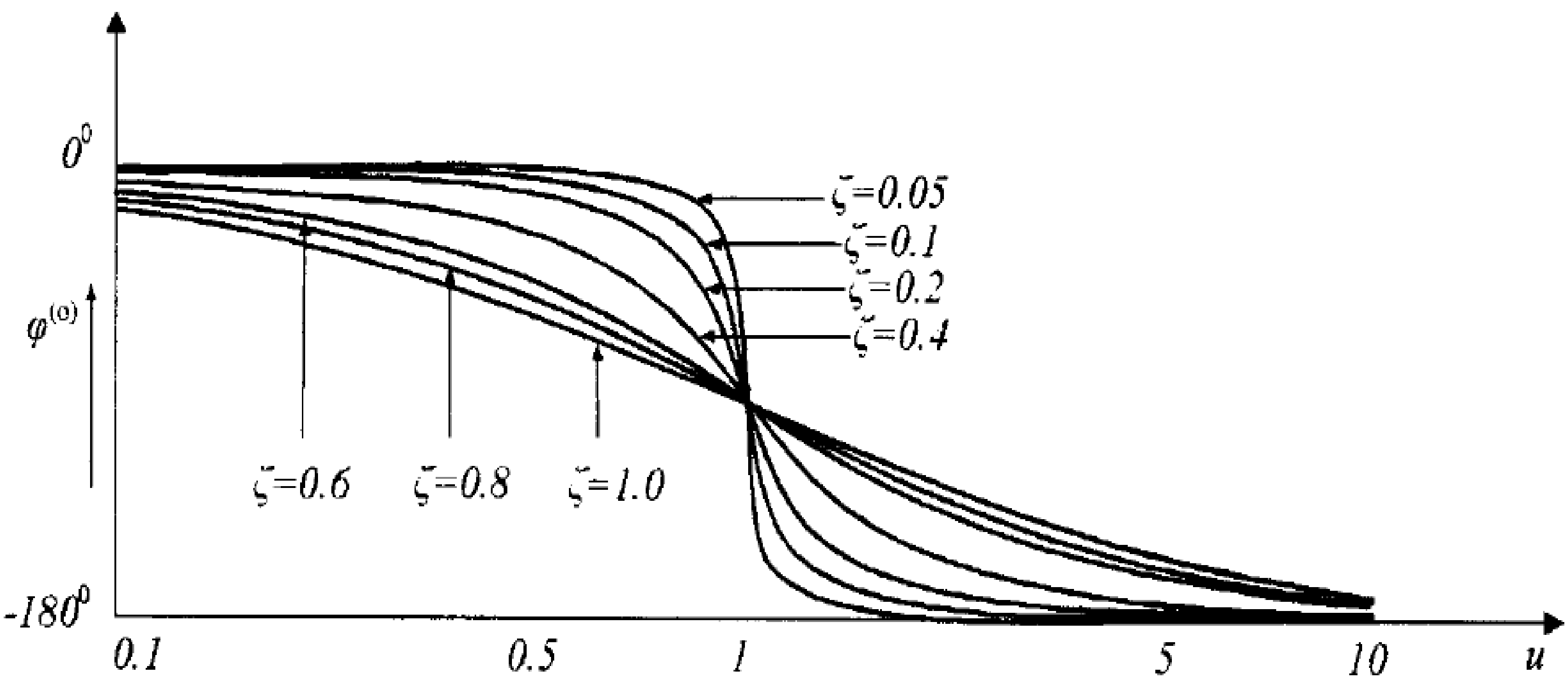
first asymptote coincides with the 0 dB-axis and the second has a slope of $\pm 40\rho$ dB and crosses over the u-axis at the point $u = 1$. In the vicinity of the point of intersection of the two asymptotes, the form of the curve of A is decisively influenced by the damping ratio ζ.

Magnitude plot



Phase plot

a. When $u = 0$, then $\varphi = 0^\circ$
b. When $u = 1$, then $\varphi = \pm 90^\circ \rho$
c. When $u = \infty$, then $\varphi = \pm 180^\circ \rho$.



Bode Plot based on Asymptotic Approximation

Term	**Magnitude**	**Phase**
Constant: K	$20\log_{10}(\|K\|)$	K>0: 0° K<0: ±180°
Pole at Origin (Integrator) 1/s	-20 dB/decade passing through 0 dB at ω=1	-90°
Zero at Origin (Differentiator) s	+20 dB/decade passing through 0 dB at ω=1 *(Mirror image, around x axis,of Integrator)*	+90° *(Mirror image, around x axis, of Integrator about)*
Real Pole $\dfrac{1}{\dfrac{s}{\omega_0}+1}$	1. Draw low frequency asymptote at 0 dB 2. Draw high frequency asymptote at -20 dB/decade 3. Connect lines at ω_0.	1. Draw low frequency asymptote at 0° 2. Draw high frequency asymptote at -90° 3. Connect with a straight line from $0.1\cdot\omega_0$ to $10\cdot\omega_0$
Real Zero $\dfrac{s}{\omega_0}+1$	1. Draw low frequency asymptote at 0 dB 2. Draw high frequency asymptote at +20 dB/decade 3. Connect lines at ω_0. *(Mirror image, around x-axis, of Real Pole)*	1. Draw low frequency asymptote at 0° 2. Draw high frequency asymptote at +90° 3. Connect with a straight line from $0.1\cdot\omega_0$ to $10\cdot\omega_0$ *(Mirror image, around x-axis, of Real Pole about 0°)*

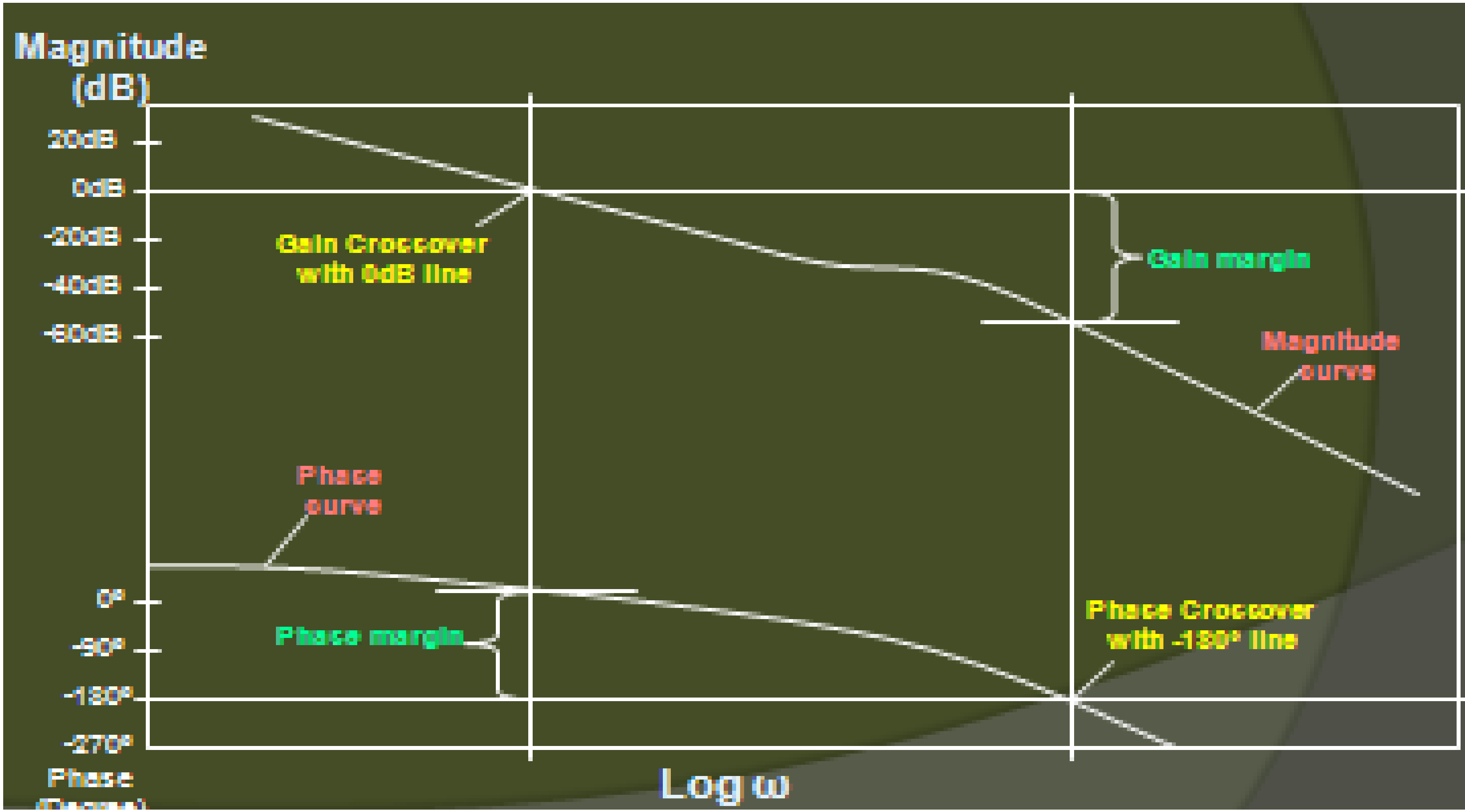
Contd..

Underdamped Poles (Complex conjugate poles) $\dfrac{1}{\left(\dfrac{s}{\omega_0}\right)^2 + 2\zeta\dfrac{s}{\omega_0} + 1}$ $0 < \zeta < 1$	1. Draw low frequency asymptote at 0 dB 2. Draw high frequency asymptote at -40 dB/decade 3. If $\zeta<0.5$, then draw peak at ω_0 with amplitude $\lvert H(j\omega_0)\rvert=-20\cdot\log_{10}(2\zeta)$, else don't draw peak 4. Connect lines	1. Draw low frequency asymptote at 0° 2. Draw high frequency asymptote at -180° 3. Connect with straight line from $\omega=\omega_0/10^{\zeta}$ to $\omega_0\cdot 10^{\zeta}$
Underdamped zeros (Complex conjugate zeros) $\left(\dfrac{s}{\omega_0}\right)^2 + 2\zeta\dfrac{s}{\omega_0} + 1$ $0 < \zeta < 1$	1. Draw low frequency asymptote at 0 dB 2. Draw high frequency asymptote at +40 dB/decade 3. If $\zeta<0.5$, then draw peak at ω_0 with amplitude$\lvert H(j\omega_0)\rvert=+20\cdot\log_{10}(2\zeta)$, else don't draw peak 4. Connect lines	1. Draw low frequency asymptote at 0° 2. Draw high frequency asymptote at +180° 3. Connect with straight line from $\omega=\omega_0/10^{\zeta}$ to $\omega_0\cdot 10^{\zeta}$

Gain Margin and Phase Margin

- GM: is the factor by which magnitude of $G(j\omega)H(j\omega)$ at the phase crossover frequency is to be multiplied to make it unity
- The gain margin from Bode plot is the number of dB that is below 0 dB at the phase crossover frequency ($\phi=-180^\circ$).
- PM: is the amount of phase to be added to the phase angle at gain cross over frequency to make it -180°
- The phase margin from Bode plot is the number of degrees the phase that is above -180° at the gain crossover frequency

GM and PM



Magnitude (dB)
20dB
0dB
-20dB
-40dB
-60dB
Gain Crossover with 0dB line
Gain margin
Magnitude curve
Phase curve
0°
-90°
-180°
-270°
Phase margin
Phase Crossover with -180° line
Phase
Log ω

- For closed loop system to be stable, Magnitude should be less than 0dB at phase cross over frequency and the phase angle must be greater than -180º at gain cross over frequency
- GM (in dB)=0-gain in dB at ω_{pc}
- PM=phase angle at ω_{gc}+180º

Bode Plot: Example 1

Draw the Bode Diagram for the transfer function:

$$H(s) = \frac{100}{s+30}$$

Step 1: Rewrite the transfer function in proper form.

Make both the lowest order term in the numerator and denominator unity. The numerator is an order 0 polynomial, the denominator is order 1.

$$H(s) = \frac{100}{30}\frac{1}{\frac{s}{30}+1} = 3.3\frac{1}{\frac{s}{30}+1}$$

- **Step 2: Separate the transfer function into its constituent parts.**

The transfer function has 2 components:

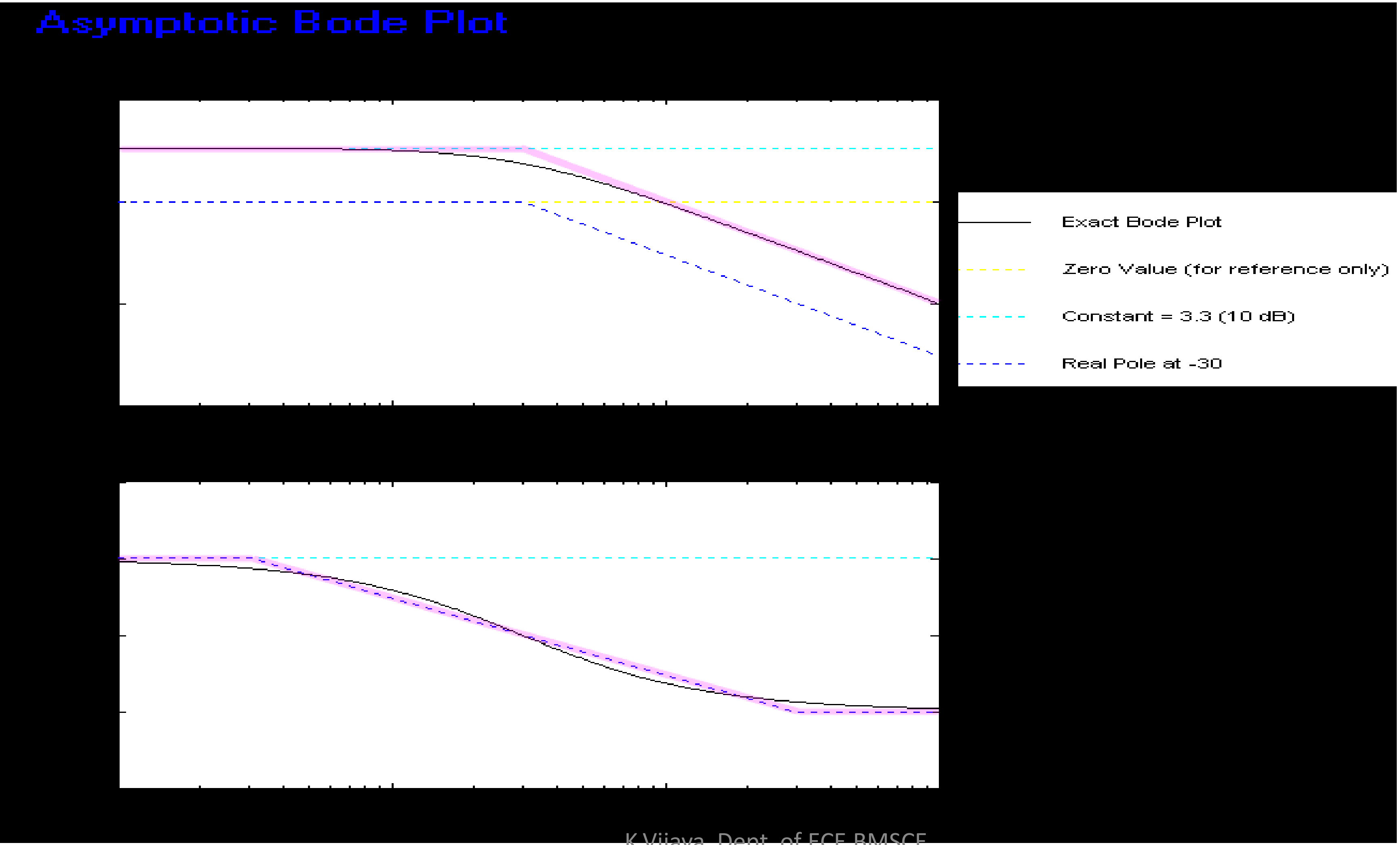
- A constant of 3.3
- A pole at s=-30

- **Step 3: Draw the Bode diagram for each part.**

This is done in the diagram below.

- The constant is the cyan line (A quantity of 3.3 is equal to 10.4 dB). The phase is constant at 0 degrees.
- The pole at 30 rad/sec is the blue line. It is 0 dB up to the break frequency, then drops off with a slope of -20 dB/dec. The phase is 0 degrees up to 1/10 the break frequency (3 rad/sec) then drops linearly down to -90 degrees at 10 times the break frequency (300 rad/sec)

Bode plot1


Asymptotic Bode Plot
Exact Bode Plot
Zero Value (for reference only)
Constant = 3.3 (10 dB)
Real Pole at -30

Example2 :GM and PM

Bode Plot – Example

For the following T.F draw the Bode plot and obtain Gain cross over frequency

Phase cross over frequency , Gain Margin and Phase Margin.

$$G(s) = 20 / [s\ (1+3s)\ (1+4s)]$$

Term	Corner Frequency	Slope db/dB	Change in slope
20/jw	---	-20	
1/(1+j4w)	wc_1=1/4=0.25	-20	-20-20=-40
1/(1+j3w)	wc_2=1/3=0.33	-20	-40-20=-60

The sinusoidal T.F of G(s) is obtained by replacing s by jw in the given T.F

G(jw) = 20 / [jw (1+j3w) (1+j4w)]

Corner frequencies: wc_1= 1/4 = 0.25 rad /sec ;

wc_2 = 1/3 = 0.33 rad /sec

Calculation of Gain (A) (MAGNITUDE PLOT)

A @ w_1 ; $A = 20 \log [20 / 0.025] = 58 .06$ dB

A @ wc_1 ; $A = [\text{Slope from } w_1 \text{ to } wc_1 \times \log (wc_1 / w_1] + \text{Gain } (A)@w_1$

$= - 20 \log [0.25 / 0.025] + 58.06$

$= 38.06$ dB

A @ wc_2 ; $A = [\text{Slope from } wc_1 \text{ to } wc_2 \times \log (wc_2 / wc_1] + \text{Gain } (A)@ wc_1$

$= - 40 \log [0.33 / 0.25] + 38$

$= 33$ dB

Choose W_h as $10W_{c2}$

A @ w_h ; A = [Slope from wc_2 to w_h x log (w_h / wc_2] + Gain (A) @ wc_2

= - 60 log [3.3 / 0.33] + 33

= - 27 dB

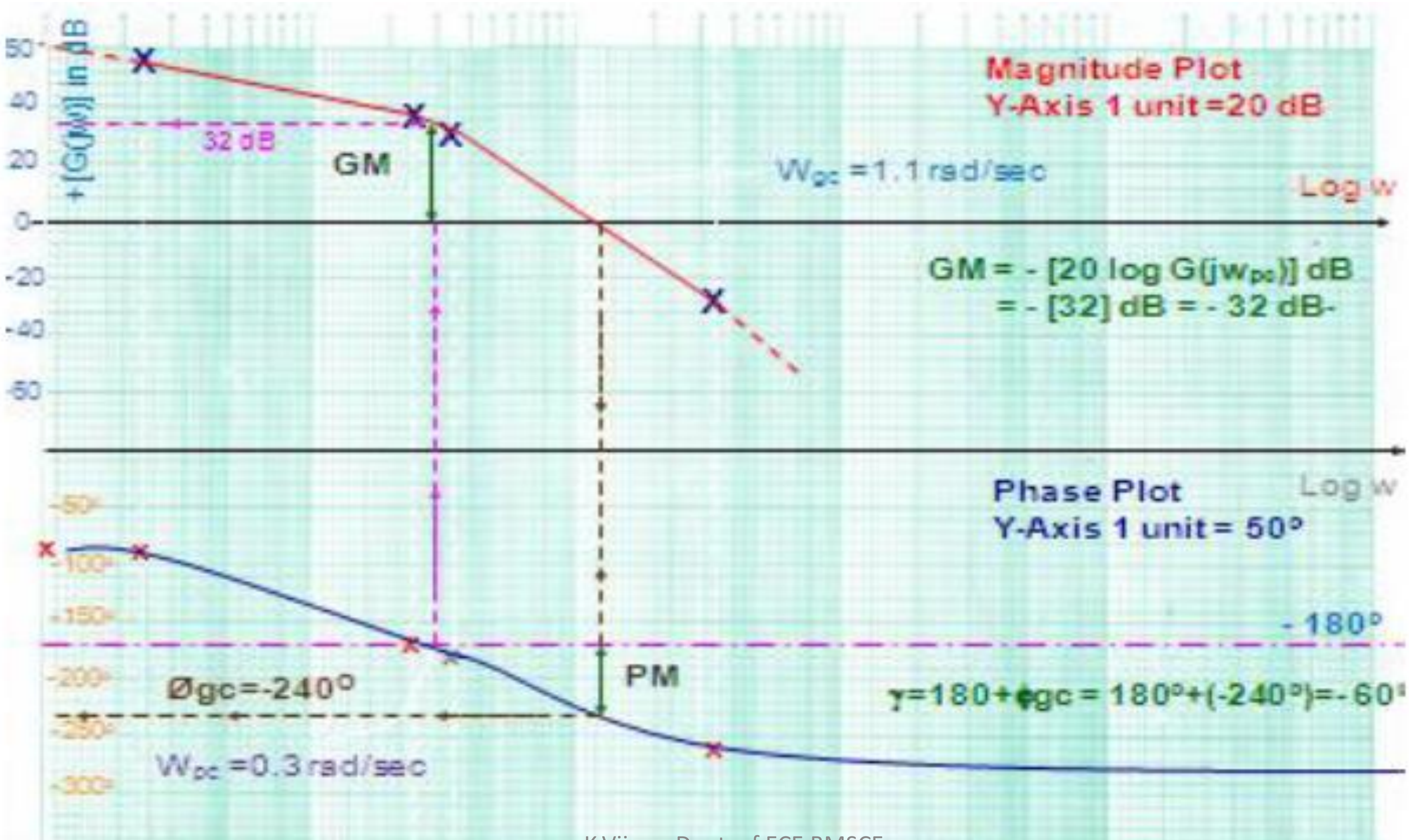
Phase plot

$\varnothing = -90^{o} - \tan^{-1} 3w - \tan^{-1} 4w$

When

Frequency in rad / sec	Phase Angle in degrees
w=0	$\varnothing= -90^{\circ}$
w=0.025	$\varnothing= -99^{\circ}$
w=0.25	$\varnothing= -172^{\circ}$
w=0.33	$\varnothing= -188^{\circ}$
w=3.3	$\varnothing= -259^{\circ}$
w=∞	$\varnothing= -270^{\circ}$

Bode plot



50
40
20
0
-20
-40
-60
+[G(jw)] in dB
32 dB
GM
Magnitude Plot
Y-Axis 1 unit =20 dB
Wgc =1.1 rad/sec
Log w
GM = - [20 log G(jwpc)] dB
= - [32] dB = - 32 dB-
Log w
Phase Plot
Y-Axis 1 unit = 50°
-50°
-100°
-150°
-200°
-250°
-300°
-180°
PM
Øgc=-240°
γ=180+φgc = 180°+(-240°)=-60°
Wpc =0.3 rad/sec

GM and PM

- Calculations of Gain cross over frequency

  The frequency at which the dB magnitude is Zero $w_{gc} = 1.1$ rad / sec

- Calculations of Phase cross over frequency

  The frequency at which the Phase of the system is -180°

  wpc = 0.3 rad / sec

- Gain Margin

  The gain margin in dB is given by the negative of dB magnitude of G(jw) at

  $$GM = -\{20 \log [G(jw_{pc})] = -\{32\} = -32 \text{ dB}$$

- Phase Margin

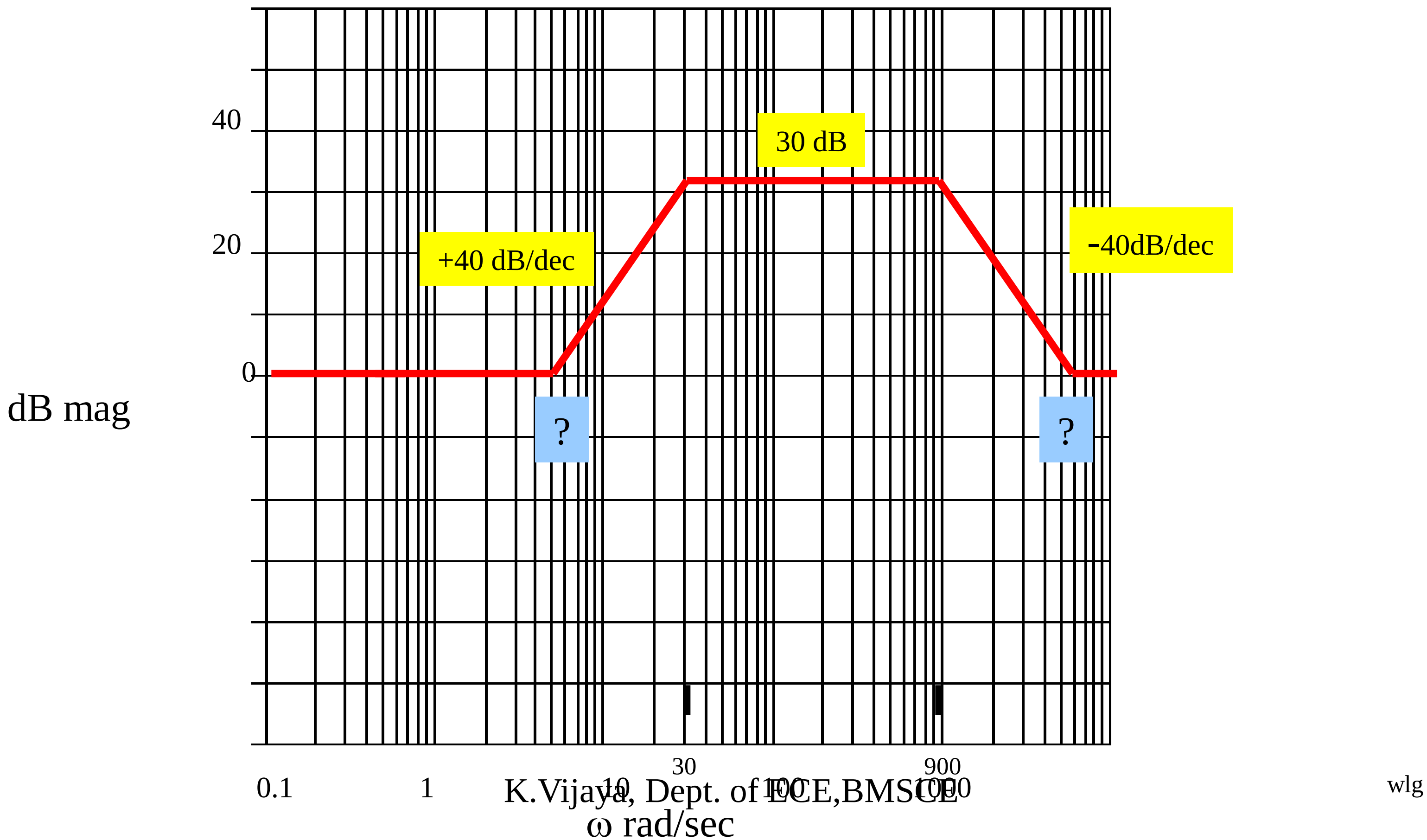
  $\Gamma = 180^{\circ} + \O_{gc} = 180^{\circ} + (-240^{\circ}) = -60^{\circ}$

- Conclusion

  For this system GM and PM are negative in Values. Therefore the system is unstable in nature.

Example3: TF from Bode plot

Obtain the TF from the given Bode Plot



Procedure: The two break frequencies need to be found. Recall:

$$\#dec = \log_{10}[w_2/w_1]$$

Then we have:

$$(\#dec)(40dB/dec) = 30\ dB$$

$$\log_{10}[w_1/30] = 0.75 \longrightarrow \underline{w_1 = 5.33\ rad/sec}$$

Also

$$\log_{10}[w_2/900]\ (-40dB/dec) = -30dB$$

This gives $\underline{w_2 = 5060\ rad/sec}$

$$G(s)=\frac{(1+s/5.3)^2(1+s/5060)^2}{(1+s/30)^2(1+s/900)^2}$$

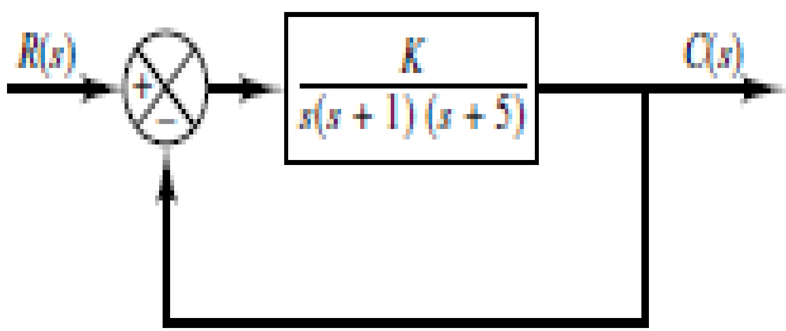
Clearing:

$$G(s)=\frac{(s+5.3)^2(s+5060)^2}{(s+30)^2(s+900)^2}$$

Tutorials

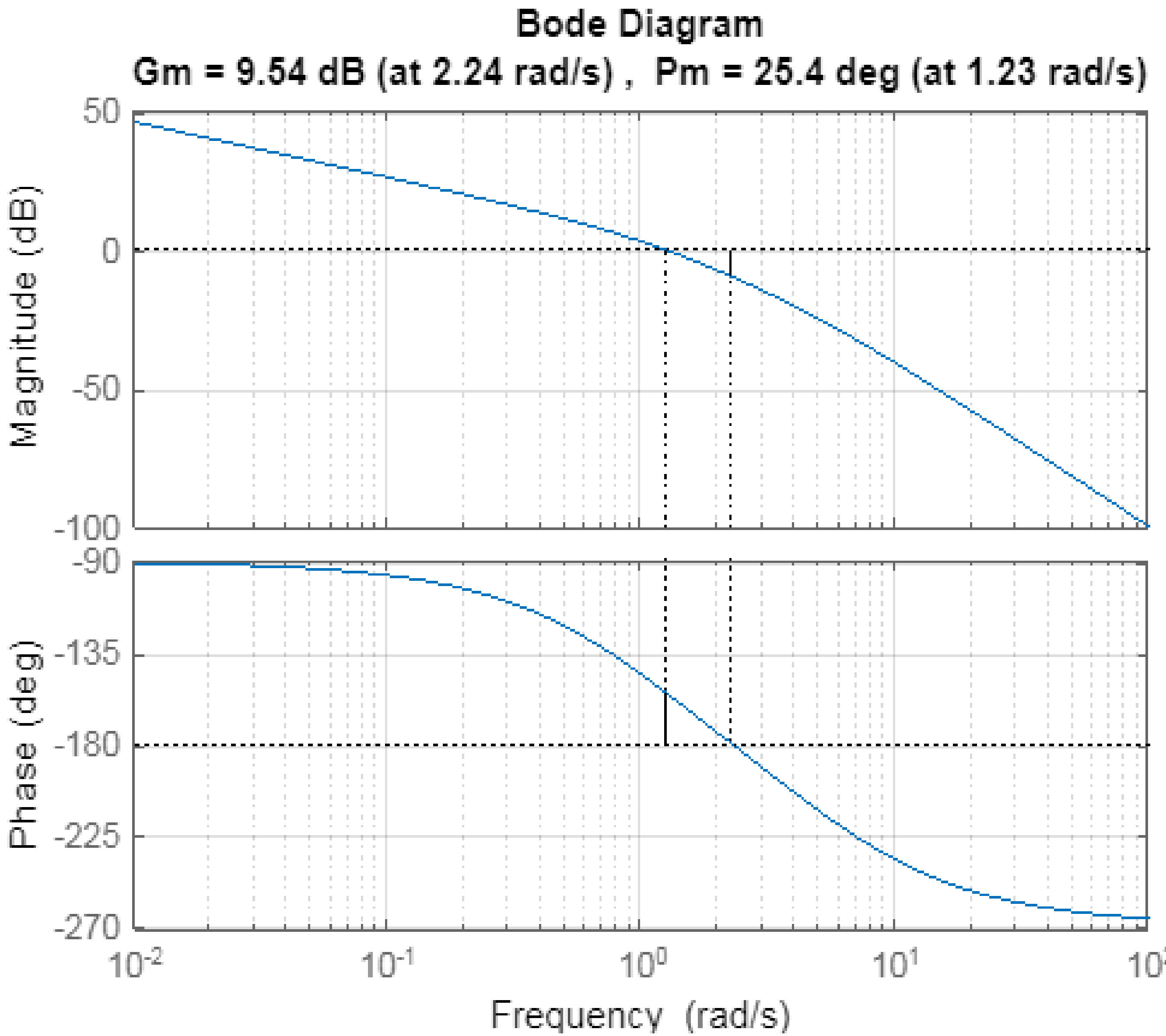
Problem 1

Obtain the phase and gain margins of the system shown in Figure for the two cases where K=10 and K=100



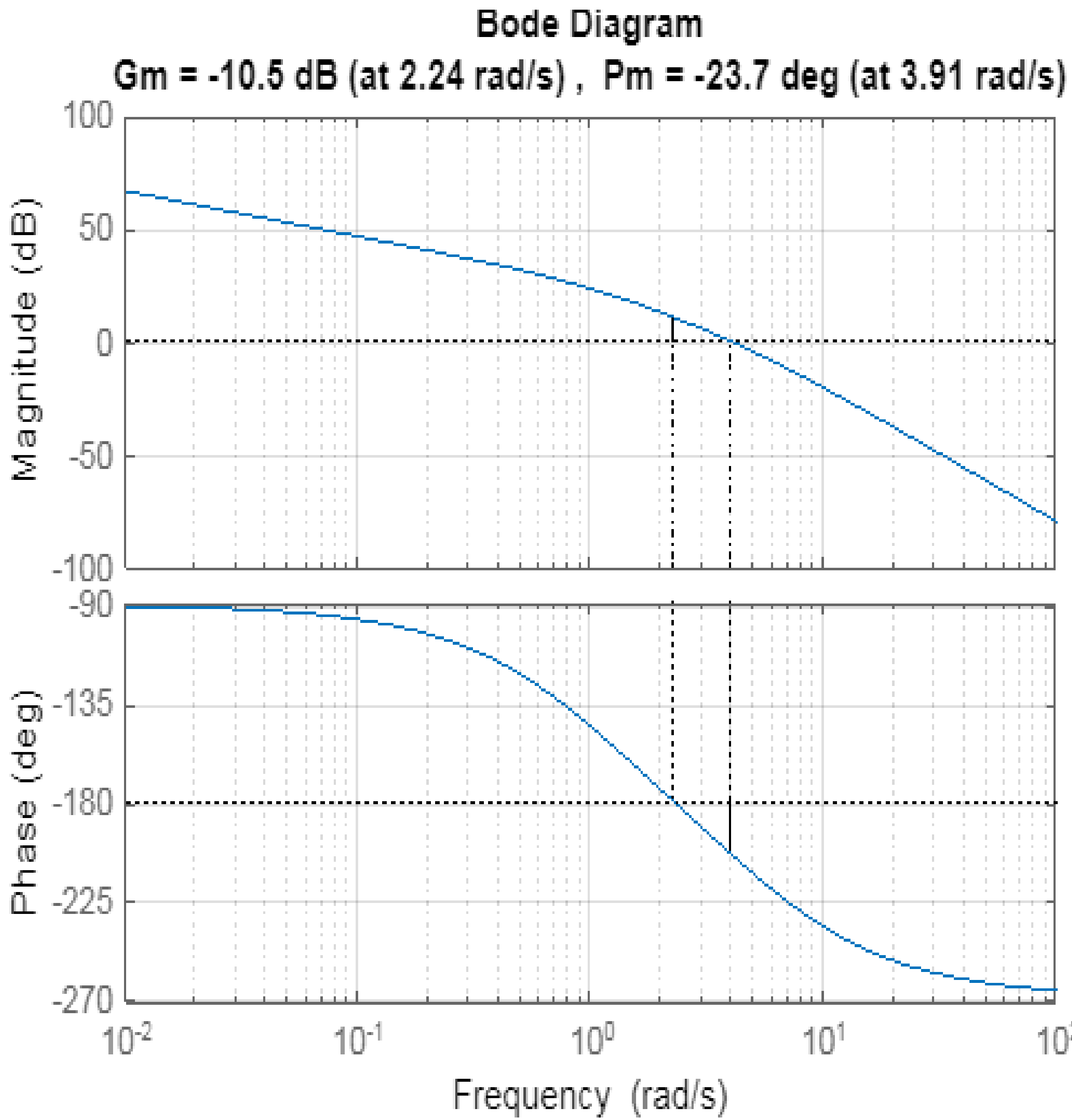
ANS:

K=10



Bode Diagram
Gm = 9.54 dB (at 2.24 rad/s) , Pm = 25.4 deg (at 1.23 rad/s)
50
0
-50
-100
Magnitude (dB)
-90
-135
-180
-225
-270
Phase (deg)
10^-2
10^-1
10^0
10^1
10^2
Frequency (rad/s)

K=100



Bode Diagram
Gm = -10.5 dB (at 2.24 rad/s) , Pm = -23.7 deg (at 3.91 rad/s)
100
50
0
-50
-100
Magnitude (dB)
-90
-135
-180
-225
-270
Phase (deg)
10-2
10-1
100
101
102
Frequency (rad/s)

Problem 2

The system has an open loop transfer function $G(s) = \frac{k}{s(1+s)(1+0.1s)}$. Find the gain k such that

(i) GM = 10dB

(ii) GM = 30dB

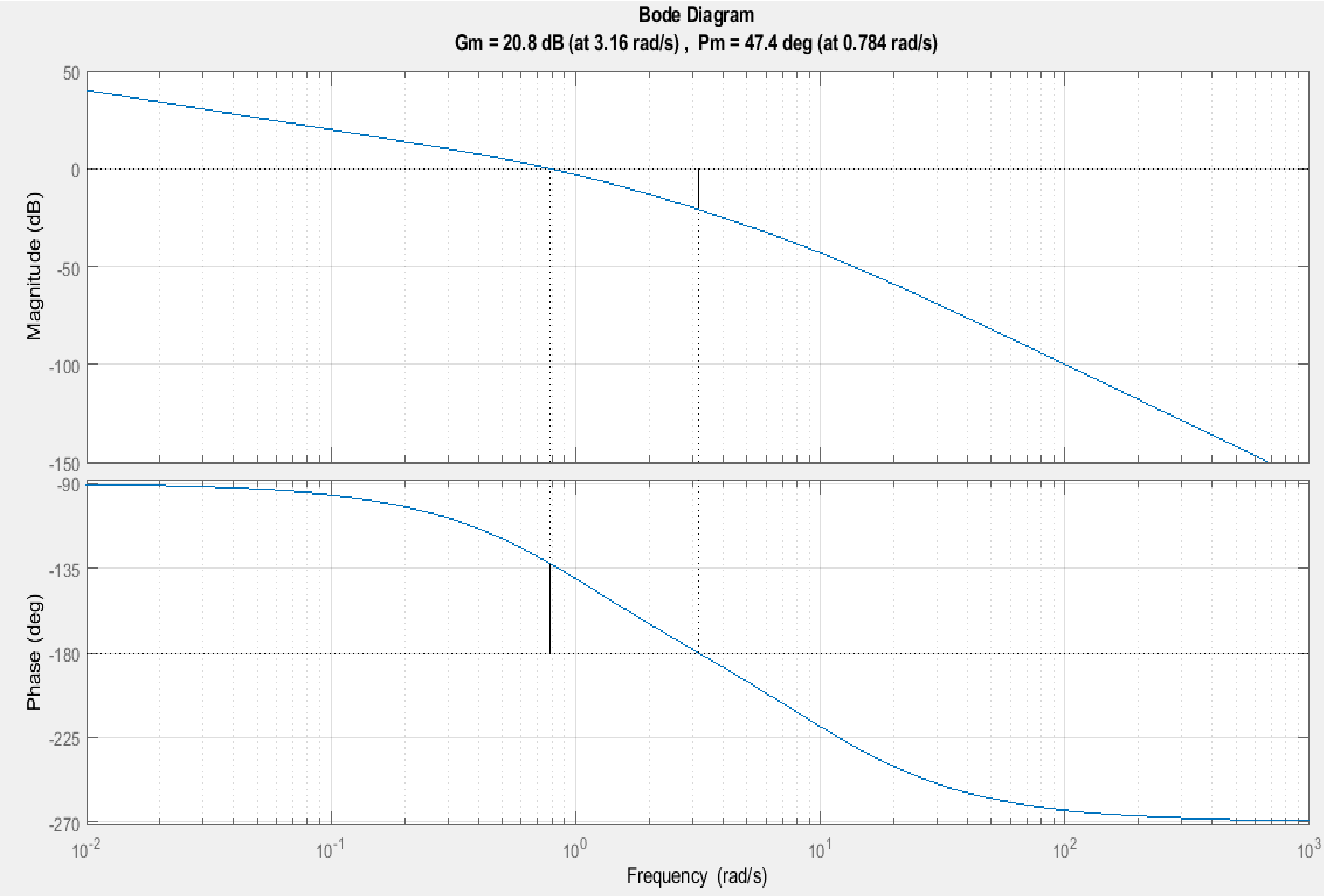
(iii) PM = 24^o

Assume K=1,

Draw the Bode plot

$$M_{db} = 20\log 1 - 20\log \omega - 20\log \sqrt{1+\omega^2} - 20\log \sqrt{1+(0.1\omega)^2}$$

$$\varnothing = 0 - 90 - \tan^{-1}\omega - \tan^{-1}(0.1\omega)$$


Bode Diagram
Gm = 20.8 dB (at 3.16 rad/s) , Pm = 47.4 deg (at 0.784 rad/s)
Magnitude (dB)
50
0
-50
-100
-150
Phase (deg)
-90
-135
-180
-225
-270
10^{-2}
10^{-1}
10^{0}
10^{1}
10^{2}
10^{3}
Frequency (rad/s)

(i) For k = 1, the GM = 20 dB

In order to get GM = 10dB the magnitude diagram at $\omega = \omega_p$ must be pushed by 10dB. This means that k must be increased by 10dB

Hence 20log k = 10

k = 3.162

(ii) To get GM = 30dB the magnitude diagram at $\omega = \omega_p$ must be pushed down by 10dB.

This means that k must be decreased by 10dB

Hence 20log k = -10

k = 0.3162

(iii) On the phase diagram locate margin of 24 deg .

In order to get the PM of 24^o ,the magnitude diagram must intercept the zero dB axis at ω = 1.55 rad/sec.

For this to happen, the magnitude must be pushed up by 8.2 dB. This means that gain must be increased by 8.2dB

$$20\log k = 8.2$$

$$k = 2.57$$

(iv) For the closed loop system to be stable the PM should be positive. For PM to be positive the gain cross over frequency should occur before ω_p = 3.16 rad/sec

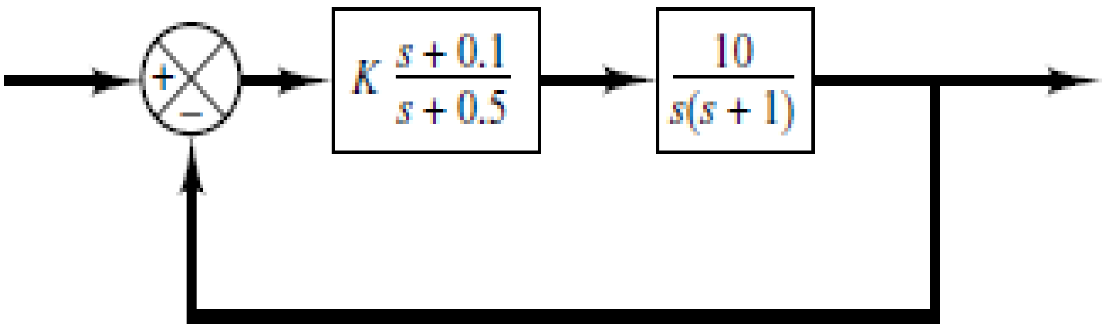
As a limiting case, for PM to be zero the gain cross over should be at $_p$ = 3.16 rad/sec. For this to happen, the magnitude plot should be pushed up by 20 dB

$$20 \log k = 20$$

$$k = 10$$

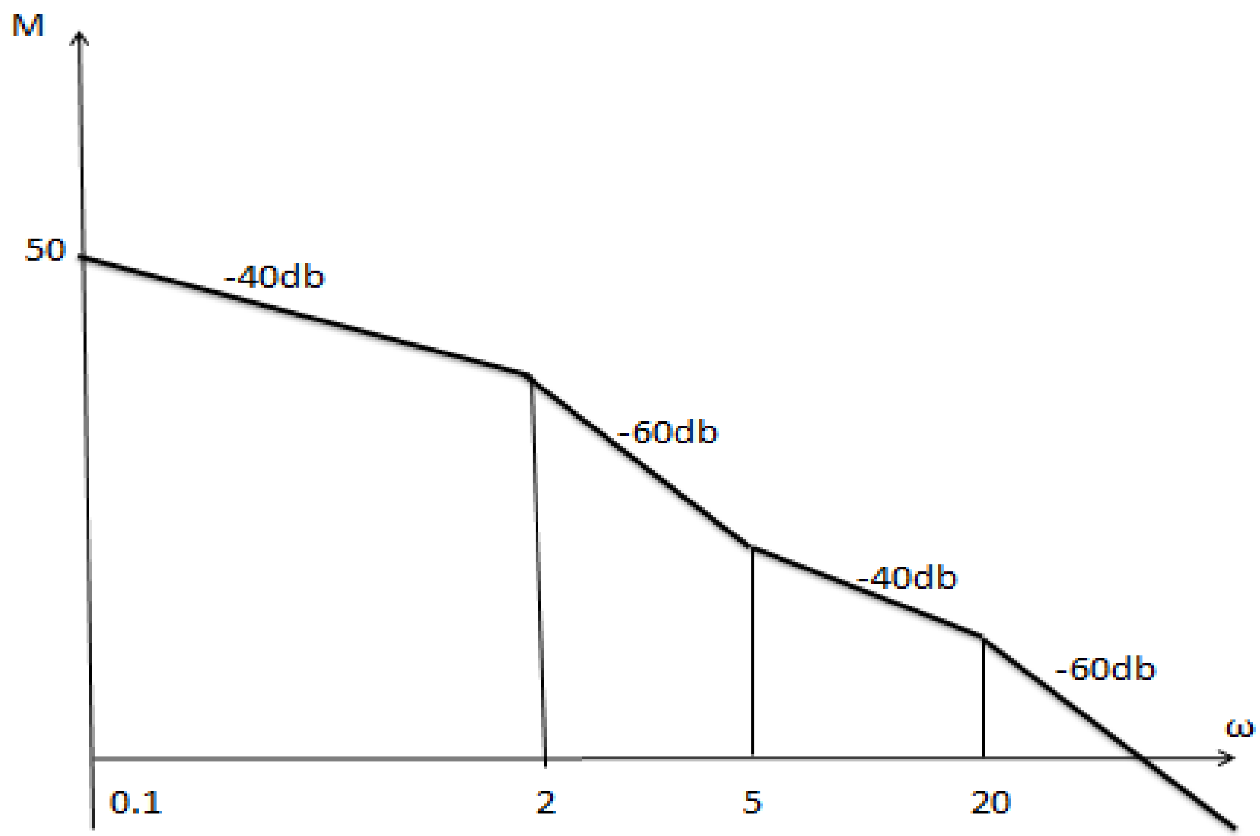
Assignment

Consider the system shown in Figure. Draw a Bode diagram of the open-loop transfer function, and determine the value of the gain K such that the phase margin is 50°. What is the gain margin of this system with this gain K?



Problem 3

Find the transfer function for the given Bode plot



$$G(s) = \frac{k(1+\frac{s}{5})}{s^2\left(1+\frac{s}{2}\right)(1+\frac{s}{20})}$$

At $\omega = 0.1$, M= 50db

$$20\ \log(\frac{k}{\omega^2})=50$$

$$20\log k + 20\ \log(\frac{1}{\omega^2}) = 50$$

$$20\log k - 40\log \omega = 50$$

$$20\log k - 40\log (0.1) = 50$$

$$20\log k + 40 = 50$$

$$20\log k= 10\ ,\ K = 10^{10/20} = 3.16$$

$$G(s) = \frac{3.16(\frac{5+s}{5})}{s^2\left(\frac{2+s}{2}\right)(\frac{20+s}{20})} = \frac{25.28(s+5)}{s^2(2+s)(20+s)}$$