

1. Find the unit vector extending from the origin toward the point $G(2, -2, -1)$

$$\vec{G} = 2\hat{a}_x + (-2)\hat{a}_y + (-1)\hat{a}_z$$

$$|\vec{G}| = \sqrt{(2)^2 + (-2)^2 + (-1)^2} = \sqrt{9} = 3$$

$$\hat{a}_G = \frac{\vec{G}}{|\vec{G}|} = \frac{2\hat{a}_x - 2\hat{a}_y - \hat{a}_z}{3} = \underline{\underline{0.667\hat{a}_x - 0.667\hat{a}_y - 0.333\hat{a}_z}}$$

2. Given points $M(-1, 2, 1)$, $N(3, -3, 0)$ and $P(-2, -3, -4)$
find a) \vec{R}_{MN} b) $\vec{R}_{MN} + \vec{R}_{MP}$ c) $|\vec{R}_M|$ d) \hat{a}_{MP} e) $|2\vec{R}_P - 3\vec{R}_N|$

$$a) \vec{R}_M = -\hat{a}_x + 2\hat{a}_y + \hat{a}_z$$

$$\vec{R}_N = 3\hat{a}_x - 3\hat{a}_y$$

$$\vec{R}_{MN} = \vec{R}_N - \vec{R}_M = \underline{\underline{4\hat{a}_x - 5\hat{a}_y - \hat{a}_z}}$$

$$b) \vec{R}_P = -2\hat{a}_x - 3\hat{a}_y - 4\hat{a}_z$$

$$\vec{R}_{MP} = \vec{R}_P - \vec{R}_M = -\hat{a}_x - 5\hat{a}_y - 5\hat{a}_z$$

$$\vec{R}_{MN} + \vec{R}_{MP} = \underline{\underline{3\hat{a}_x - 10\hat{a}_y - 6\hat{a}_z}}$$

$$c) |\vec{R}_M| = \sqrt{(-1)^2 + (2)^2 + (1)^2} = \sqrt{6} = \underline{\underline{2.5}}$$

$$d) \hat{a}_{MP} = \frac{\vec{R}_{MP}}{|\vec{R}_{MP}|} = \frac{-\hat{a}_x - 5\hat{a}_y - 5\hat{a}_z}{\sqrt{(-1)^2 + (5)^2 + (5)^2}} = \underline{\underline{-0.14\hat{a}_x - 0.7\hat{a}_y - 0.7\hat{a}_z}}$$

$$e) 2\vec{R}_P - 3\vec{R}_N = (-4\hat{a}_x - 6\hat{a}_y - 8\hat{a}_z) - (9\hat{a}_x - 9\hat{a}_y) \\ = -13\hat{a}_x + 3\hat{a}_y - 8\hat{a}_z$$

$$|2\vec{R}_P - 3\vec{R}_N| = \sqrt{13^2 + 3^2 + 8^2} = \underline{\underline{15.56}}$$

3) A vector field \vec{s} is represented by rectangular coordinates as

$$\vec{s} = \frac{125}{[(x-1)^2 + (y-2)^2 + (z+1)^2]} [(x-1)\hat{a}_x + (y-2)\hat{a}_y + (z+1)\hat{a}_z]$$

- a) Evaluate \vec{s} at $P(2, 4, 3)$
- b) Determine a unit vector that gives the direction of \vec{s} at P
- c) Specify the surface $f(x, y, z)$ on which $|\vec{s}| = 1$.

a) $\vec{s} = \frac{125}{[1 + 4 + 16]} [\hat{a}_x + 2\hat{a}_y + 4\hat{a}_z]$

$$\vec{s} = 5.95\hat{a}_x + 11.904\hat{a}_y + 23.8\hat{a}_z$$

b) $\hat{a}_s = \frac{\vec{s}}{|\vec{s}|} = 0.218\hat{a}_x + 0.4365\hat{a}_y + 0.872\hat{a}_z$

c) $|\vec{s}| = \frac{125}{\sqrt{[(x-1)^2 + (y-2)^2 + (z+1)^2]}} = 1$

$$\sqrt{(x-1)^2 + (y-2)^2 + (z+1)^2} = 125$$

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a) Give the rectangular coordinates of the point

$$C(s = 4.4, \phi = -115^\circ, z = 2)$$

b) Give the cylindrical coordinates of the point

$$D(x = -3.1, y = 2.6, z = -3)$$

c) Specify the distance from C to D.

a) $x = s \cos \phi = -1.86$

$$y = s \sin \phi = -3.99$$

$$z = z = 2$$

$$C(-1.86, -3.99, 2)$$

b) $s = \sqrt{x^2 + y^2} = 4.05$

$$\phi = \tan^{-1}(y/x) = -39.98^\circ$$

$$z = z = -3$$

$$D(s = 4.05, \phi = -39.98^\circ, z = -3)$$

c) Distance from C to D is

$$= \sqrt{(x_C - x_D)^2 + (y_C - y_D)^2 + (z_C - z_D)^2}$$

$$= \sqrt{1.53 + 43.42 + 25}$$

$$= \underline{\underline{8.36 \text{ units}}}$$

Transform to cylindrical coordinates

a) $\vec{F} = 10\hat{a}_x - 8\hat{a}_y + 6\hat{a}_z$ at point $P(10, -8, 6)$

b) $\vec{G} = (2x+y)\hat{a}_x - (y-4x)\hat{a}_y$ at point $Q(s, \phi, z)$

c) Give the rectangular components of the vector

$\vec{H} = 20\hat{a}_y - 10\hat{a}_\phi + 3\hat{a}_z$ at $P(x=5, y=2, z=-1)$

a) $\vec{F} = 10\hat{a}_x - 8\hat{a}_y + 6\hat{a}_z$

$$\vec{F} = A_s \hat{a}_s + A_\phi \hat{a}_\phi + A_z \hat{a}_z$$

$$A_s = \vec{F} \cdot \hat{a}_s = 10(\hat{a}_x \cdot \hat{a}_s) - 8(\hat{a}_y \cdot \hat{a}_s) + 6(\hat{a}_z \cdot \hat{a}_s) = 10 \cos \phi - 8 \sin \phi = 12.8$$

$$A_\phi = \vec{F} \cdot \hat{a}_\phi = 10(\hat{a}_x \cdot \hat{a}_\phi) - 8(\hat{a}_y \cdot \hat{a}_\phi) + 6(\hat{a}_z \cdot \hat{a}_\phi) = -8 \sin \phi - 8 \cos \phi = -2.19 \times 10^3$$

$$A_z = \vec{F} \cdot \hat{a}_z = 6$$

$$\boxed{\vec{F} = 12.8\hat{a}_s + 6\hat{a}_z}$$

b) $\vec{G} = (2x+y)\hat{a}_x - (y-4x)\hat{a}_y$

$$\vec{G} = G_s \hat{a}_s + G_\phi \hat{a}_\phi + G_z \hat{a}_z$$

$$G_s = \vec{G} \cdot \hat{a}_s = [2s \cos \phi + s \sin \phi] \cos \phi - [s \sin \phi - 4s \cos \phi] \sin \phi$$

$$G_\phi = \vec{G} \cdot \hat{a}_\phi = [2s \cos \phi + s \sin \phi](\hat{a}_x \cdot \hat{a}_\phi) - [s \sin \phi - 4s \cos \phi](\hat{a}_y \cdot \hat{a}_\phi)$$

$$G_z = \vec{G} \cdot \hat{a}_z = 0$$

$$\vec{G} = [2s \cos^2 \phi + s \sin \phi \cos \phi - s \sin^2 \phi + 4s \cos \phi \sin \phi] \hat{a}_s +$$

$$[2s \cos \phi \sin \phi - s \sin^2 \phi - s \sin \phi \cos \phi + 4s \cos^2 \phi] \hat{a}_\phi$$

$$\vec{G} = [2s \cos^2 \phi + 5s \sin \phi \cos \phi - s \sin^2 \phi] \hat{a}_s +$$

$$[4s^2 \cos \phi - 3s \sin \phi \cos \phi - s \sin^2 \phi] \hat{a}_\phi$$

c) $\vec{H} = A_x \hat{a}_x + A_y \hat{a}_y + A_z \hat{a}_z$

$$A_x = 20 \cos \phi - 10(-\sin \phi) = 21.283 \quad A_y = -1.857 \quad A_z = 3 \quad \vec{H} = 21.283 \hat{a}_x - 1.857 \hat{a}_y + 3 \hat{a}_z$$

The three vertices of a triangle are located at A(6, -1, 2) B(-2, 3, -4) and C(-3, 1, 5). Find a) \vec{R}_{AB} b) \vec{R}_{AC} c) the angle θ_{BAC} at vertex A; d) the vector projection of \vec{R}_{AB} on \vec{R}_{AC}

$$a) \vec{R}_{AB} = \vec{R}_B - \vec{R}_A = -8\hat{a}_x + 4\hat{a}_y - 6\hat{a}_z$$

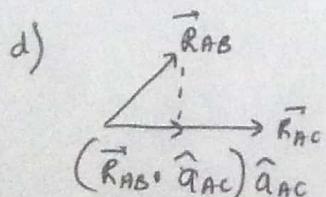
$$b) \vec{R}_{AC} = \vec{R}_C - \vec{R}_A = -9\hat{a}_x + 2\hat{a}_y + 3\hat{a}_z$$

$$c) \vec{R}_{BA} \cdot \vec{R}_{AC} = |\vec{R}_{BA}| |\vec{R}_{AC}| \cos \theta_{BAC}$$

$$(-8\hat{a}_x + 4\hat{a}_y - 6\hat{a}_z) \cdot (-9\hat{a}_x + 2\hat{a}_y + 3\hat{a}_z) = [\sqrt{116}] [\sqrt{94}] \cos \theta_{BAC}$$

$$62 = 104.42 \cos \theta_{BAC}$$

$$\underline{\theta_{BAC} = 53.57^\circ}$$



$$\hat{a}_{AC} = \frac{\vec{R}_{AC}}{|\vec{R}_{AC}|} = \frac{-9\hat{a}_x + 2\hat{a}_y + 3\hat{a}_z}{\sqrt{94}}$$

$$= \left[(-8\hat{a}_x + 4\hat{a}_y - 6\hat{a}_z) \cdot \left(\frac{-9\hat{a}_x + 2\hat{a}_y + 3\hat{a}_z}{\sqrt{94}} \right) \right] \left[\frac{-9\hat{a}_x + 2\hat{a}_y + 3\hat{a}_z}{\sqrt{94}} \right]$$

$$= \frac{1}{(\sqrt{94})} \left[\frac{(78 + 8 - 18)}{\sqrt{94}} \right] \left[-9\hat{a}_x + 2\hat{a}_y + 3\hat{a}_z \right]$$

Projection of \vec{R}_{AB}
on \vec{R}_{AC} = $-5.94\hat{a}_x + 1.319\hat{a}_y + 1.97\hat{a}_z$

The three vertices of a triangle are located at $A(6, -1, 2)$, $B(-2, 3, -4)$ and $C(-3, 1, 5)$ find

- a) $\vec{R}_{AB} \times \vec{R}_{AC}$ b) The area of the triangle c) A unit vector perpendicular to the plane in which the triangle is located.

$$a) \vec{R}_{AB} = -8\hat{a}_x + 4\hat{a}_y - 6\hat{a}_z$$

$$\vec{R}_{AC} = -9\hat{a}_x + 2\hat{a}_y + 3\hat{a}_z$$

$$\vec{R}_{AB} \times \vec{R}_{AC} = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ -8 & 4 & -6 \\ -9 & 2 & 3 \end{vmatrix} = \hat{a}_x(12+12) - \hat{a}_y(-24-54) + \hat{a}_z(-16+36)$$

$$\vec{R}_{AB} \times \vec{R}_{AC} = 24\hat{a}_x + 78\hat{a}_y + 20\hat{a}_z$$

$$b) |\vec{R}_{AB} \times \vec{R}_{AC}| = \text{Area of } \square \text{ formed by } \vec{R}_{AB} \text{ & } \vec{R}_{AC}$$

$$\text{The area of } \triangle = \frac{1}{2} |\vec{R}_{AB} \times \vec{R}_{AC}| = \underline{\underline{42.0}} \text{ unit}$$

$$c) \vec{R}_{AB} \times \vec{R}_{AC} = |\vec{R}_{AB}| |\vec{R}_{AC}| \sin \theta \hat{a}_n$$

$$\hat{a}_n = \left[\frac{(24\hat{a}_x + 78\hat{a}_y + 20\hat{a}_z)}{\sqrt{7060}} \right] = \frac{\vec{R}_{AB} \times \vec{R}_{AC}}{|\vec{R}_{AB} \times \vec{R}_{AC}|}$$

$$\hat{a}_n = 0.286\hat{a}_x + 0.928\hat{a}_y + 0.238\hat{a}_z$$

Given the two points $C(-3, 2, 1)$ and $D(r=5, \theta=20^\circ, \phi=-70^\circ)$, find

- The spherical coordinates of C
- The rectangular coordinates of D
- The distance from C to D

$$a) r = \sqrt{x^2 + y^2 + z^2} = 3.74$$

$$\theta = \cot^{-1} \frac{z}{\sqrt{x^2 + y^2}} = 74.5^\circ$$

$$\phi = \tan^{-1}(y/r) = -33.69^\circ$$

$$(r, \theta, \phi) = (3.74, 74.5^\circ, -33.69^\circ)$$

$$b) x = r \sin \theta \cos \phi = 0.585$$

$$y = r \sin \theta \sin \phi = -1.607$$

$$z = r \cos \theta = 4.7$$

$$(x, y, z) = (0.585, -1.607, 4.7)$$

$$c) \text{Distance from } C \text{ to } D = \sqrt{0.585^2 + (-1.607)^2 + 4.7^2} = \underline{\underline{6.9 \text{ units}}}$$

Transform the following vectors to spherical coordinates at the points given

$$a) 10\hat{a}_x \text{ at } P(x=-3, y=2, z=4)$$

$$b) 10\hat{a}_y \text{ at } Q(r=5, \theta=30^\circ, \phi=45^\circ)$$

$$c) 10\hat{a}_z \text{ at } M(r=4, \theta=110^\circ, \phi=120^\circ)$$

$$r = \sqrt{x^2 + y^2 + z^2} = 5.385$$

$$\theta = \cos^{-1}(\frac{z}{r}) = 42.029^\circ$$

$$\phi = \tan^{-1}(\frac{y}{x}) = -33.69^\circ$$

a) $10\hat{a}_x$

$$\vec{A} = A_n \hat{a}_n + A_\theta \hat{a}_\theta + A_\phi \hat{a}_\phi$$

$$A_n = (\vec{A}) \cdot (\hat{a}_n) = (10\hat{a}_x) \cdot (\hat{a}_n) = 10 \sin\theta \cos\phi = 5.57 \hat{a}_n$$

$$A_\theta = (\vec{A}) \cdot (\hat{a}_\theta) = (10\hat{a}_x) \cdot (\hat{a}_\theta) = 10 \cos\theta \cos\phi = 6.18 \hat{a}_\theta$$

$$A_\phi = (\vec{A}) \cdot (\hat{a}_\phi) = (10\hat{a}_x) \cdot (\hat{a}_\phi) = -10 \sin\phi = 5.55 \hat{a}_\phi$$

$$\vec{A} = 5.57 \hat{a}_n + 6.18 \hat{a}_\theta + 5.55 \hat{a}_\phi$$

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b) $10\hat{a}_y$

$$x = r \cos\phi = 4.33 \quad r = 6.403$$

$$y = r \sin\phi = 2.5 \quad \theta = 51.33^\circ$$

$$\vec{A} = A_n \hat{a}_n + A_\theta \hat{a}_\theta + A_\phi \hat{a}_\phi \quad \beta = \delta = 4 \quad \phi = 30^\circ$$

$$A_n = (\vec{A} \cdot \hat{a}_n) = (10\hat{a}_y) \cdot (\hat{a}_n) = 10 \sin\theta \sin\phi = 3.9$$

$$A_\theta = (\vec{A} \cdot \hat{a}_\theta) = (10\hat{a}_y) \cdot (\hat{a}_\theta) = 10 \cos\theta \sin\phi = 3.12$$

$$A_\phi = (\vec{A} \cdot \hat{a}_\phi) = (10\hat{a}_y) \cdot (\hat{a}_\phi) = 10 \cos\phi = 8.66$$

$$\vec{A} = 3.9 \hat{a}_n + 3.12 \hat{a}_\theta + 8.66 \hat{a}_\phi$$

c)

$10\hat{a}_z$

$$\vec{A} = A_n \hat{a}_n + A_\theta \hat{a}_\theta + A_\phi \hat{a}_\phi$$

$$A_n = \vec{A} (10\hat{a}_z) \cdot \hat{a}_n = 10 \cos\theta = -3.42$$

$$A_\theta = (10\hat{a}_z) \cdot \hat{a}_\theta = -10 \sin\theta = -9.4$$

$$A_\phi = (10\hat{a}_z) \cdot \hat{a}_\phi = 0$$

$$\vec{A} = -3.42 \hat{a}_n - 9.4 \hat{a}_\theta$$

A charge of $Q_1 = 3 \times 10^{-4} C$ is located at $M(1, 2, 3)$ and a charge of $Q_2 = -10^{-4} C$ at $N(2, 0, 5)$ in a vacuum. Find the force exerted on Q_2 by Q_1 .

$$\vec{r}_1 = \hat{a}_x + 2\hat{a}_y + 3\hat{a}_z$$

$$\vec{r}_2 = 2\hat{a}_x + 5\hat{a}_z$$

$$\vec{R}_{12} = \vec{r}_2 - \vec{r}_1 = \hat{a}_x - 2\hat{a}_y + 2\hat{a}_z$$

$$\vec{F}_{12} = \frac{(3 \times 10^{-4})(-10^{-4})}{4\pi\epsilon_0 (1+4+4)^{3/2}} [\hat{a}_x - 2\hat{a}_y + 2\hat{a}_z]$$

$$\vec{F}_{12} = \frac{-3 \times 10^{-8} [\hat{a}_x - 2\hat{a}_y + 2\hat{a}_z]}{3 \times 10^{-9}}$$

$$\vec{F}_{12} = -10\hat{a}_x + 20\hat{a}_y - 20\hat{a}_z$$

A charge $Q_A = -20 \mu C$ is located at $A(-6, 4, 7)$ and a charge $Q_B = 50 \mu C$ is at $B(5, 8, -2)$ in free space. If distances are given in meters, find.

- a) \vec{R}_{AB} b) R_{AB} . Determine the vector forces exerted on Q_A by Q_B if $\epsilon_0 =$ c) $10^9 / 36\pi F/m$ d) $8.854 \times 10^{-12} F/m$

a) $\vec{R}_{AB} = 11\hat{a}_x + 4\hat{a}_y - 9\hat{a}_z$

b) $R_{AB} = 14.76 m$

c) $F_{BA} = \frac{(-20 \times 10^{-6})(50 \times 10^{-6})}{4\pi\epsilon_0 (14.76)^3} [-11\hat{a}_x - 4\hat{a}_y + 9\hat{a}_z]$

$$\vec{F}_{BA} = 30.72\hat{a}_x + 11.18\hat{a}_y - 25.16\hat{a}_z \text{ mN.}$$

With $\epsilon_0 = 8.854 \times 10^{-12} F/m$

$$\vec{F}_{BA} = 30.72\hat{a}_x + 11.169\hat{a}_y - 25.13\hat{a}_z \text{ mN}$$

Find \vec{E} at $P(1, 1, 1)$ caused by four identical 3nc charges located at $P_1(1, 1, 0)$, $P_2(-1, 1, 0)$, $P_3(-1, -1, 0)$ and $P_4(1, -1, 0)$

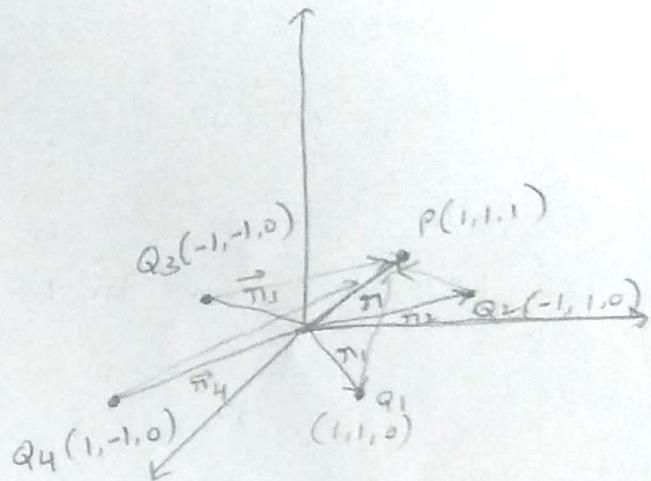
$$\vec{r} = \hat{a}_x + \hat{a}_y + \hat{a}_z$$

$$\vec{r}_1 = \hat{a}_x + \hat{a}_y$$

$$\vec{r}_2 = -\hat{a}_x + \hat{a}_y$$

$$\vec{r}_3 = -\hat{a}_x - \hat{a}_y$$

$$\vec{r}_4 = \hat{a}_x - \hat{a}_y$$



$$\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + \vec{E}_4$$

$$= \frac{Q_1}{4\pi\epsilon_0} \frac{(\vec{r} - \vec{r}_1)}{|\vec{r} - \vec{r}_1|^3} + \frac{Q_2}{4\pi\epsilon_0} \frac{(\vec{r} - \vec{r}_2)}{|\vec{r} - \vec{r}_2|^3} + \frac{Q_3}{4\pi\epsilon_0} \frac{(\vec{r} - \vec{r}_3)}{|\vec{r} - \vec{r}_3|^3} + \frac{Q_4}{4\pi\epsilon_0} \frac{(\vec{r} - \vec{r}_4)}{|\vec{r} - \vec{r}_4|^3}$$

$$= \frac{Q}{4\pi\epsilon_0} \left[\frac{\hat{a}_z}{1} + \frac{2\hat{a}_x + \hat{a}_y}{(5)^{3/2}} + \frac{2\hat{a}_x + 2\hat{a}_y + \hat{a}_z}{(9)^{3/2}} + \frac{2\hat{a}_y + \hat{a}_z}{(5)^{3/2}} \right]$$

$$= 26.96 \left[\hat{a}_z + \frac{2\hat{a}_x + \hat{a}_y}{5^{3/2}} + \frac{2\hat{a}_x + 2\hat{a}_y + \hat{a}_z}{9^3} + \frac{2\hat{a}_y + \hat{a}_z}{5^{3/2}} \right]$$

$$\vec{E} = \underline{6.82 \hat{a}_x + 6.82 \hat{a}_y + 32.8 \hat{a}_z} \text{ V/m}$$

A charge of $-0.3 \mu C$ is located at $A(25, -30, 15)$ cm and a second charge of $0.5 \mu C$ is at $B(-10, 8, 12)$ cm ifind \vec{E} at a) The origin b) $P(15, 20, 50)$ cm

$$a) \vec{E} = \frac{-0.3 \times 10^{-6} (-25\hat{a}_x + 30\hat{a}_y - 15\hat{a}_z) \times 10^2}{4\pi\epsilon_0 [0.25^2 + 0.3^2 + 0.15^2]^{3/2}} + \frac{0.5 \times 10^{-6} (10\hat{a}_x - 8\hat{a}_y - 12\hat{a}_z) \times 10^2}{4\pi\epsilon_0 [0.1^2 + 0.08^2 + 0.12^2]^{3/2}}$$

$$\vec{E} = (9207.77\hat{a}_x - 11049.33\hat{a}_y + 5524.66\hat{a}_z) + (83137\hat{a}_x - 66509.6\hat{a}_y - 99764.4\hat{a}_z)$$

$$\vec{E} = 92.3\hat{a}_x - 77.55\hat{a}_y - 94.23\hat{a}_z \text{ kV/m}$$

$$b) \vec{E} = \frac{-0.3 \times 10^{-6} (-10\hat{a}_x + 50\hat{a}_y + 35\hat{a}_z) \times 10^2}{4\pi\epsilon_0 [0.1^2 + 0.5^2 + 0.35^2]^{3/2}} + \frac{0.5 \times 10^{-6}}{4\pi\epsilon_0} \frac{(25\hat{a}_x + 12\hat{a}_y + 38\hat{a}_z)}{[0.25^2 + 0.12^2 + 0.38^2]^{3/2}}$$

$$\vec{E} = (-1138.78\hat{a}_x - 5693.94\hat{a}_y - 3985.76\hat{a}_z) + (10791.66\hat{a}_x + 5179.999\hat{a}_y + 16403.33\hat{a}_z)$$

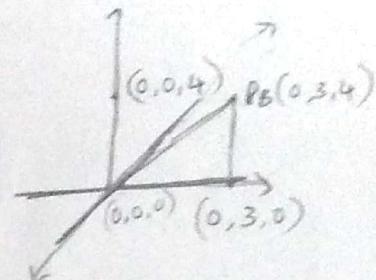
$$\vec{E} = 11.9\hat{a}_x - 0.519\hat{a}_y + 12.4\hat{a}_z \text{ kV/m}$$

Infinite uniform line charges of 5nC/m lie along the x and y axes in free space find \vec{E} at

- a) $P_A(0,0,4)$ b) $P_B(0,3,4)$

a) $\vec{E}_A = \frac{5 \times 10^{-9}}{2\pi\epsilon_0(4)} \hat{a}_x + \frac{5 \times 10^{-9}}{2\pi\epsilon_0(4)} \hat{a}_y$

$$\vec{E}_A = 45 \hat{a}_x \text{ V/m}$$



b) $\vec{E}_B = \frac{5 \times 10^{-9}}{2\pi\epsilon_0 (4)^2} (4\hat{a}_x) + \frac{5 \times 10^{-9}}{2\pi\epsilon_0 (\sqrt{9+16})^2} (3\hat{a}_y + 4\hat{a}_z)$

$$\vec{E}_B = 10.78 \hat{a}_x + 14.38 \hat{a}_y + 22.46 \hat{a}_z$$

$$\vec{E}_B = 10.8 \hat{a}_x + 36.9 \hat{a}_y \text{ V/m}$$

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Three infinite uniform sheets of charge are located in free space as follows: 3nC/m^2 at $z=-4$, 6nC/m^2 at $z=1$ and -8nC/m^2 at $z=4$. Find \vec{E} at the point

- a) $P_A(2,5,-5)$ b) $P_B(4,2,-3)$ c) $P_C(-1,-5,2)$ d) $P_D(-2,4,5)$

- a) $P_A(2,5,-5)$

$$\vec{E}_A = \vec{E}_1 + \vec{E}_2 + \vec{E}_3$$

$$= \frac{3 \times 10^{-9}}{2\epsilon_0} (-\hat{a}_z) + \frac{6 \times 10^{-9}}{2\epsilon_0} (\hat{a}_z) + \frac{-8 \times 10^{-9}}{2\epsilon_0} (\hat{a}_z)$$

$$\vec{E}_A = -56.46 \hat{a}_z$$

$$\Rightarrow \vec{E}_B = \frac{3 \times 10^{-9}}{2\epsilon_0} (\hat{a}_3) - \frac{6 \times 10^{-9}}{2\epsilon_0} (\hat{a}_3) - \left(\frac{-8 \times 10^{-9}}{2\epsilon_0} \right) \hat{a}_3$$

$$\underline{\underline{\vec{E}_B = 283 \hat{a}_3}}$$

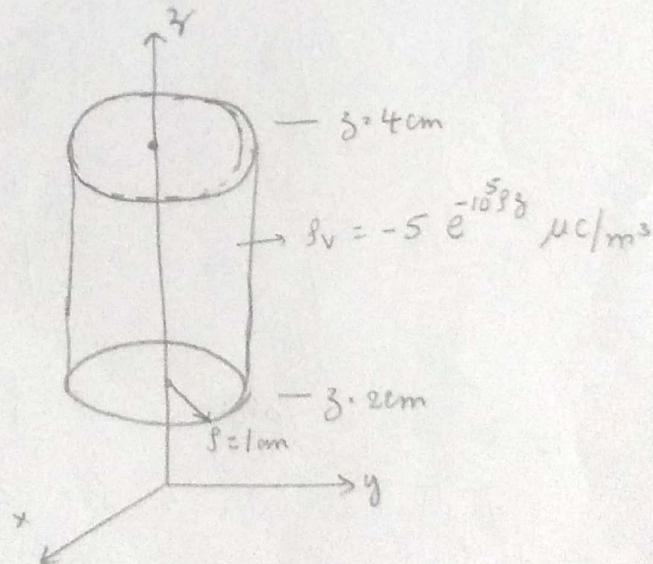
$$c) \vec{E} = \frac{3 \times 10^{-9}}{2\epsilon_0} \hat{a}_3 + \frac{6 \times 10^{-9}}{2\epsilon_0} \hat{a}_3 - \left(\frac{-8 \times 10^{-9}}{2\epsilon_0} \right) \hat{a}_3$$

$$\underline{\underline{\vec{E} = 961 \hat{a}_3}}$$

$$d) \vec{E}_D = \frac{3 \times 10^{-9}}{2\epsilon_0} \hat{a}_3 + \frac{(6 \times 10^{-9})}{2\epsilon_0} \hat{a}_3 + \left(\frac{-8 \times 10^{-9}}{2\epsilon_0} \right) \hat{a}_3$$

$$\underline{\underline{\vec{E} = 56.5 \hat{a}_3}}$$

Find the total charge contained in a electron beam shown in figure.



$$Q = \int_V \rho_v dv$$

$$Q = \int_{0.02}^{0.04} \int_0^{2\pi} \int_0^{0.01} -5 \times 10^{-6} e^{-10^5 \rho} \rho d\phi dz$$

$$= \int_{0.02}^{0.04} \int_0^{0.01} (-5 \times 10^{-6}) e^{-10^5 \rho} (2\pi) \rho d\rho d\phi dz$$

$$= \int_0^{0.04} \int_0^{0.01} (-10^{-5}\pi) e^{-10^5 \rho z} \rho dz d\rho$$

$$= \int_{\rho=0}^{0.01} \left[\frac{-10^{-5} \pi e^{-10^5 \rho z}}{-10^5 \rho} \right]_{z=0.02}^{0.04}$$

$$= \int_{\rho=0}^{0.01} \left[-10^{-10} \pi e^{-10^5 \rho z} dz \right]_{z=0.02}^{0.04}$$

$$= -10^{-10} \pi \left[\int_{\rho=0}^{0.01} \left(\frac{e^{-2000\rho}}{-2000} - \frac{e^{-4000\rho}}{4000} \right) dz \right]$$

$$= -10^{-10} \pi \left[\frac{e^{-2000\rho}}{-2000} - \frac{e^{-4000\rho}}{4000} \right]_0^{0.01}$$

$$Q = -10^{-10} \pi \left[\frac{1}{2000} - \frac{1}{4000} \right] = -\frac{\pi}{40} = \underline{\underline{0.0785 pC}}$$

Calculate the total charge within each of indicated volumes.

a) $0.1 \leq |x|, |y|, |z| \leq 0.2$ & $\rho_v = \frac{1}{x^3 y^3 z^3}$

b) $0 \leq \rho \leq 0.1$, $0 \leq \phi \leq \pi$, $2 \leq z \leq 4$ & $\rho_v = \rho^2 z^2 \sin(\phi)$

c) Universe $\rho_v = \frac{e^{-2r}}{r^2}$

a) $Q = \iiint_{x=0.1, y=0.1, z=0.1}^{0.2, 0.2, 0.2} \frac{1}{x^3 y^3 z^3} dx dy dz = \int_{x=0.1}^{0.2} \int_{y=0.1}^{0.2} \int_{z=0.1}^{0.2} \frac{1}{x^3 y^3} \frac{z^{-2}}{-2} dz dx dy$

$$= 37.5 \int_{\rho=0.1}^{0.2} \left[\frac{1}{\rho^3} \frac{y^{-2}}{-2} \right]_{0.1}^{0.2} = 1406.25 \left[\frac{\bar{x}^2}{-2} \right]_{0.1}^{0.2}$$

$$\underline{Q} = 52.73 \times 10^3 C$$

b) $Q = \int_{\rho=0}^{0.1} \int_{\phi=0}^{\pi} \int_{z=2}^4 \rho^2 z^2 \sin(0.6\phi) \rho dz d\phi dz$

$$= \int_{\rho=0}^{0.1} \int_{\phi=0}^{\pi} \left[\rho^3 \sin(0.6\phi) \frac{z^3}{3} \right]_2^4 d\rho d\phi$$

$$= 18.66 \int_{\phi=0}^{\pi} \sin(0.6\phi) \left[\frac{\rho^4}{4} \right]_0^{0.1} d\phi$$

$$= 4.665 \times 10^{-4} \left[-\frac{\cos(0.6\phi)}{0.6} \right]_0^{\pi}$$

$$\underline{Q} = 1.077 mC$$

c) $Q = \int_{\pi=0}^{\infty} \int_{\theta=0}^{\pi} \int_{\phi=0}^{2\pi} \frac{e^{-an}}{2\pi} \rho^2 \sin\theta d\pi d\theta d\phi$

$$= 2\pi \int_{\pi=0}^{\infty} \int_{\theta=0}^{2\pi} e^{-an} \rho \sin\theta d\pi d\theta$$

$$= 2\pi \int_{\pi=0}^{\infty} e^{-an} d\pi = 2\pi(2) \left[\frac{e^{-an}}{-a} \right]_{\pi=0}^{\infty}$$

$$\underline{Q} = -2\pi [0-1] = 2\pi = 6.28 C$$

Given a $60\mu C$ point charge located at the origin, find the total electric flux passing through a) That portion of sphere $r=26\text{cm}$ bounded by $0 < \theta < \pi/2$ & $0 < \phi < \pi/2$
 b) The closed surface defined by $r=26\text{cm}$ and $z = \pm 26\text{cm}$
 c) The plane $z = 26\text{cm}$

$$a) A' = \int_0^{\pi/2} \int_0^{\pi/2} r^2 \sin\theta \, d\theta \, d\phi = r^2 (\pi/2) \quad (1) = (26)^2 \frac{\pi}{2}$$

$$\text{Portion of area} = A' = 1061.858 \text{ cm}^2$$

$$\text{Total area} = A = 4\pi r^2 = 8494.86 \text{ cm}^2.$$

The flux passing through portion of area A is

$$\Psi' = \frac{Q}{A} \times A' = \frac{60 \times 10^{-6}}{8494.86} \times 1061.858 = \underline{\underline{7.5 \mu C}}$$

b) The closed surface defined by $r = 26\text{cm}$ & $z = \pm 26\text{cm}$ is closed cylinder hence total flux is equal to total charge enclosed = $60 \mu C$

c) Approximately half of the flux pass through plane $= 26\text{cm}$ hence Flux crossing this plane is $\underline{\underline{30 \text{ cm}^2 \mu C}}$

Calculate \vec{D} in rectangular coordinates at point $P(2, -3, 6)$ produced by
 a) a point charge $Q_p = 55 \mu C$ at $Q(-2, 3, -6)$
 b) a uniform line charge $S_{LB} = 20 \mu C/m$ on the x-axis.
 c) a uniform surface charge density $S_{SC} = 120 \mu C/m^2$ on the plane $z = -5\text{m}$.

a) $\vec{D} \cdot \epsilon_0 \vec{E}$

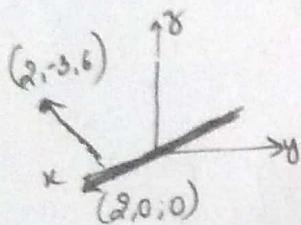
$Q(-2, 3, -6)$ $(2, -3, 6)$

$$\vec{D} = \frac{Q}{4\pi R^2} \hat{a}_n$$

$$= \frac{55 \times 10^{-3}}{4\pi \left[\sqrt{4^2 + 6^2 + 12^2} \right]^2} \left[4\hat{a}_x - 6\hat{a}_y + 12\hat{a}_z \right]$$

$$\vec{D} = \underline{\underline{6.38\hat{a}_x - 9.57\hat{a}_y + 19.14\hat{a}_z \text{ } \mu C/m^2}}$$

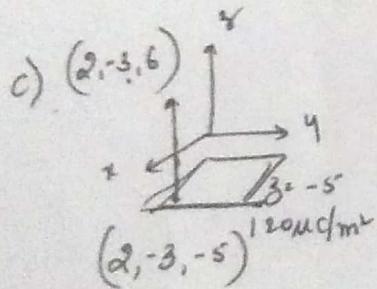
b) $\rho_{LB} = 20 \text{ mC/m}$



$$\vec{D} \cdot \frac{\rho_L}{2\pi r} \hat{a}_r$$

$$= \frac{20 \times 10^{-3}}{2\pi \left[\sqrt{+3^2 + 6^2} \right]^2} \left[-3\hat{a}_y + 6\hat{a}_z \right]$$

$$\vec{D} = \underline{\underline{-212.2\hat{a}_y + 424\hat{a}_z \text{ } \mu C/m^2}}$$



$$\vec{D} = \frac{\rho_s}{2} \hat{a}_n$$

$$= \frac{120 \times 10^{-6}}{2} (11\hat{a}_z)$$

$$\vec{D} = \underline{\underline{60\hat{a}_z \text{ } \mu C/m^2}}$$

Calculate the total electric flux leaving the cubical surface formed by the six planes $x, y, z = \pm 5$ if the charge distribution is

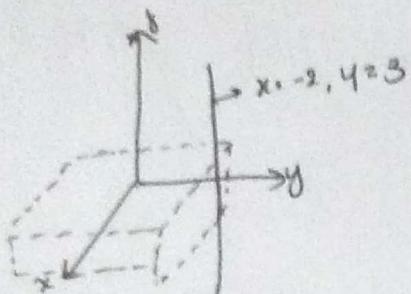
- a) Two point charges $0.1 \mu C$ at $(1, -2, 3)$ and $1 \mu C$ at $(-1, 2, -2)$
- b) A Uniform line charge of $\pi \mu C/m$ at $x = -2, y = 3$
- c) A uniform surface charge of $0.1 \mu C/m^2$ on the plane $y = 3x$.

a) Two point charges closed inside the cube of $x, y, z = \pm 5$

$$\text{Total flux} = \Psi = Q = (0.1 + \frac{1}{7}) 10^{-6} \text{C}$$

$$\underline{\underline{\Psi = 0.242 \mu\text{C}}}$$

b)

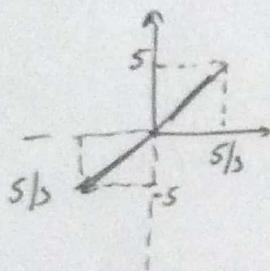


The total charge inside the cube of ± 5 ~~is~~ $\text{is } 1\text{A}$

$$Q \cdot \Psi = 10 \times \pi \mu\text{C}$$

$$\underline{\underline{Q = 31.4 \mu\text{C}}}$$

c)



$$y = 5 \Rightarrow x = 5/\sqrt{3} \quad \text{Length of sheet} = 2\sqrt{5^2 + (5/\sqrt{3})^2} = 10.5409 \text{ m}$$

$$y = -5 \Rightarrow x = -5/\sqrt{3}$$

Area of plane surface inside cube

$$A = 10 \times 10.5409 \text{ m} = 105.401 \text{ m}^2$$

$$Q = S_s A = 0.1 \times 10^{-6} \times 105.401 \text{ m}^2$$

$$\underline{\underline{Q = 10.54 \mu\text{C}}}$$

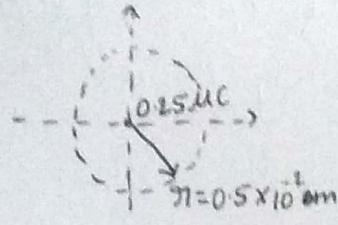
A point charge of $0.25 \mu\text{C}$ is located at $r = 0$, and uniform surface charge densities are located as follows.

Surface charge densities are located as follows.
 2 mC/m^2 at $r = 1 \text{ cm}$, and -0.6 mC/m^2 at $r = 1.8 \text{ cm}$ calculate

\vec{D} at

- a) $r = 0.5 \text{ cm}$, b) $r = 1.5 \text{ cm}$ c) $r = 2.5 \text{ cm}$ d) what uniform surface charge densities should establish at $r = 3 \text{ cm}$ to cause $\vec{D} = 0$ at $r = 3.5 \text{ cm}$?

a)

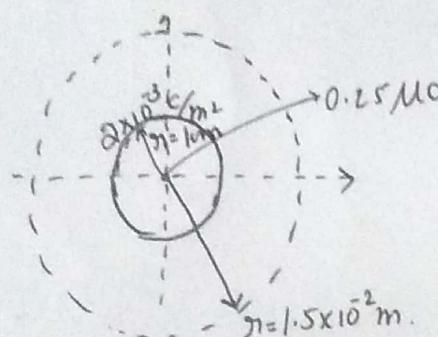


$$\vec{D} \cdot \frac{\text{charge enclosed}}{\text{Total surface area}}$$

$$\vec{D} = \frac{0.25 \times 10^{-6}}{4\pi (0.5 \times 10^{-2})^2}$$

$$\underline{\underline{\vec{D} = 796 \hat{a}_z \mu C/m^2}}$$

b)

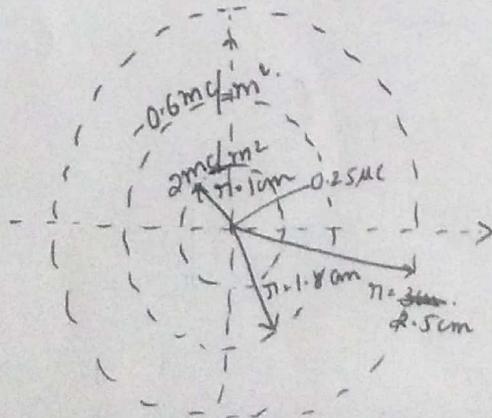


$$\vec{D} = \frac{Q_{\text{point}} + Q_{\text{physical surface}}}{4\pi r^2}$$

$$= \frac{(0.25 \times 10^{-6}) + 2 \times 10^{-3} \times 4\pi (1 \times 10^{-2})^2}{4\pi (1.5 \times 10^{-2})^2}$$

$$\underline{\underline{\vec{D} = 977.30 \hat{a}_z \mu C/m^2}}$$

c)



$$\vec{D} = \frac{Q_{\text{point}} + Q_{\text{surface of shell}} + Q_{\text{sphere g}}}{4\pi r^2}$$

$$= \frac{(0.25 \times 10^{-6}) + (2 \times 10^{-3})(4\pi (1 \times 10^{-2})^2) [4\pi (2.5 \times 10^{-2})^2 - 0.6 \times 10^{-3}]}{4\pi (2.5 \times 10^{-2})^2}$$

$$\underline{\underline{\vec{D} = 3.2037 \times 10^{-7} C/m^2}}$$

$$d) \quad \vec{D} = \frac{Q}{4\pi r^2} \Big|_{r=3.5 \text{ cm}} = 0 \Rightarrow Q=0$$

$$Q \text{ at } 3 \text{ cm} = -Q \text{ at } 5 \text{ cm} = -0.32037 \mu C$$

$$S_3 = \frac{-Q \text{ at } 3 \text{ cm}}{4\pi (3 \times 10^{-2})^2} = -28.32 \mu C/m^2$$

In free space let $\vec{D} = 8xyz^4 \hat{a}_x + 4x^2z^4 \hat{a}_y + 16x^2yz^3 \hat{a}_z \text{ pC/m}^2$

- a) Find the total electric flux passing through the rectangular surface $z=2, 0 < x < 2, 1 < y < 3$ in \hat{a}_z direction
- b) Find \vec{E} at $P(2, -1, 3)$
- c) Find an approximate value for the total charge contained in an incremental sphere located at $P(2, -1, 3)$ and having a volume of 10^{-12} m^3 .

a) $z=2, 0 < x < 2, 1 < y < 3$

$$\begin{aligned}\Psi &= \int \vec{D} \cdot d\vec{s} = \iint_{x=0}^2 \int_{y=1}^3 16x^2yz^3 \times 10^{-12} dx dy = 16 \times 10^{-12} (2)^3 \iint_{x=0}^2 \int_{y=1}^3 xy^2 dz dx dy \\ &= 128 \times 10^{-12} \left[\frac{x^3}{3} \right]_0^2 \left[\frac{y^2}{2} \right]_1^3 = \frac{128 \times 10^{-12}}{6} (8-0)(9-1)\end{aligned}$$

$\Psi = 1365.33 \text{ pC}$

b) \vec{E} at $P(2, -1, 3)$

$$\vec{E} = \frac{\vec{D}}{\epsilon_0} = \left[\frac{8xyz^4 \hat{a}_x + 4x^2z^4 \hat{a}_y + 16x^2yz^3 \hat{a}_z}{\epsilon_0} \right] \times 10^{-12}$$

at $P(2, -1, 3)$

$$\vec{E} = [-146.37 \hat{a}_x + 146.37 \hat{a}_y - 195.16 \hat{a}_z] \text{ V/m}$$

c) Total charge enclosed in volume is

$$\begin{aligned}Q_{DV} &= \left[\frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z} \right] \Delta V \\ &= \left[\frac{\partial (8xyz^4)}{\partial x} + \frac{\partial (4x^2z^4)}{\partial y} + \frac{\partial (16x^2yz^3)}{\partial z} \right] 10^{-12} \text{ m}^3 \\ &= [(-64x \times 10^{-12}) + (0) + (-1728) \times 10^{-12}] 10^{-12} = -2.376 \times 10^{-21} \text{ C}\end{aligned}$$

In each of the following parts, find a numerical value for $\operatorname{div} \vec{D}$ at the point specified

a) $\vec{D} = (2xyz - y^2) \hat{a}_x + (x^2z - 2xy) \hat{a}_y + x^2y \hat{a}_z \text{ C/m}^2$ at $P_A(2,3,-1)$

b) $\vec{D} = 2\beta z^2 \sin^2 \phi \hat{a}_r + \beta z^2 \sin \alpha \phi \hat{a}_\theta + 2\beta^2 z \sin^2 \phi \hat{a}_\phi \text{ C/m}^2$ at $P_B(\beta=2, \phi=110^\circ, z=-1)$

c) $\vec{D} = 2n \sin \theta \cos \phi \hat{a}_r + n \cos \theta \cos \phi \hat{a}_\theta - n \sin \phi \hat{a}_\phi \text{ C/m}^2$ at $P_C(n=1.5, \theta=30^\circ, \phi=50^\circ)$

a) $\operatorname{div} \vec{D} = \left[\frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z} \right] = [2yz + (-2x) + 0]$

at $P_A(2,3,-1)$ $\operatorname{div} \vec{D} = -6 - 4 = \underline{-10}$

b) $\operatorname{div} \vec{D} = \frac{1}{r} \frac{\partial (\beta D_r)}{\partial r} + \frac{1}{r} \frac{\partial D_\theta}{\partial \theta} + \frac{\partial D_\phi}{\partial \phi}$
 $= \frac{1}{r} \frac{\partial (2\beta z^2 \sin^2 \phi)}{\partial r} + \frac{1}{r} \frac{\partial (\beta z^2 \sin \alpha \phi)}{\partial \theta} + \frac{\partial (2\beta^2 z \sin^2 \phi)}{\partial \phi}$
 $\operatorname{div} \vec{D} = \frac{1}{2} \frac{\partial (2\beta^3 z^2 \sin^2 \phi)}{\partial z} + \frac{1}{2} \frac{\beta z^2 \cos(\alpha \phi)}{2} + 2\beta^2 \sin^2 \phi$

$\operatorname{div} \vec{D}$ at $P_B = \underline{9.0358}$

c) $\operatorname{div} \vec{D} = \frac{1}{r^2} \frac{\partial (n^2 D_r)}{\partial r} + \frac{1}{r \sin \theta} \frac{\partial (\sin \theta D_\theta)}{\partial \theta} + \frac{1}{r \sin \theta} \frac{\partial D_\phi}{\partial \phi}$

$\vec{D} = 2n \sin \theta \cos \phi \hat{a}_r + n \cos \theta \cos \phi \hat{a}_\theta - n \sin \phi \hat{a}_\phi \text{ C/m}^2$

$\operatorname{div} \vec{D} = \frac{1}{r^2} [6n^2 \sin \theta \cos \phi] + \frac{1}{r \sin \theta} [n \cos^2 \theta \cos \phi] + \frac{1}{r \sin \theta} [-n \cos \phi]$

at $(P_C = n=1.5, \theta=30^\circ, \phi=50^\circ)$

$\operatorname{div} \vec{D} = 1.92836 + 0.64278 - 2.855$

$\operatorname{div} \vec{D} = 1.285 \text{ C/m}^3$

Determine an expression for the volume charge density associated with each \vec{D} field following

a) $\vec{D} = \frac{4xy}{z} \hat{a}_x + \frac{\alpha x^2}{z} \hat{a}_y - \frac{\alpha x^2 y}{z^2} \hat{a}_z$

b) $\vec{D} = z \sin \phi \hat{a}_x + z \cos \phi \hat{a}_y + z \sin \phi \hat{a}_z$

c) $\vec{D} = z \sin \theta \sin \phi \hat{a}_x + z \cos \theta \sin \phi \hat{a}_y + z \cos \theta \hat{a}_z$

a) $\operatorname{div} \vec{D} = \rho_v = \frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z}$

$$= \frac{4y}{z} - \alpha x^2 y [-\alpha z^{-3}] = \left[\frac{4y}{z} \right] + \left[\frac{4x^2 y}{z^3} \right] = \frac{4y z^2 + 4x^2 y}{z^3}$$

$$\rho_v = \frac{4y}{z^3} [x^2 + z^2]$$

b) $\rho_v = \operatorname{div} \vec{D} = \frac{1}{z} \frac{\partial (z D_x)}{\partial z} + \frac{1}{z} \frac{\partial D_\phi}{\partial \phi} + \frac{1}{z} \frac{\partial D_r}{\partial r}$

$$\rho_v = \frac{1}{z} \frac{\partial (z z \sin \phi)}{\partial z} + \frac{1}{z} \frac{\partial (z \cos \phi)}{\partial \phi} + \frac{1}{z} \frac{\partial (z \sin \phi)}{\partial r}$$

$$\rho_v = \frac{1}{z} z \sin \phi - \frac{1}{z} z \sin \phi$$

$$\underline{\underline{\rho_v = 0}}$$

c) $\operatorname{div} \vec{D} = \frac{1}{r^2 \sin \theta} \frac{\partial (r^2 D_r)}{\partial r} + \frac{1}{r \sin \theta} \frac{\partial (\sin \theta D_\theta)}{\partial \theta} + \frac{1}{r \sin \theta} \frac{\partial D_\phi}{\partial \phi}$

$$= \frac{1}{r^2} (2r \sin \theta \sin \phi) + \frac{1}{r \sin \theta} (\cos \theta \sin \phi) - \frac{\sin \phi}{r \sin \theta}$$

$$\operatorname{div} \vec{D} = \frac{\alpha \sin \theta \sin \phi}{r} + \frac{\cos \theta \sin \phi}{r \sin \theta} - \frac{\sin \phi}{r \sin \theta}$$

$$= \frac{\alpha \sin \theta \sin \phi}{r} - \frac{\alpha \sin \theta \sin \phi}{r}$$

$$\rho_v = \operatorname{div} \vec{D} = 0$$

Given the field $\vec{D} = 6s \sin(\frac{1}{2}\phi) \hat{a}_r + 1.5s \cos(\frac{1}{2}\phi) \hat{a}_\theta \text{ C/m}^2$,
 evaluate both sides of the divergence for region bounded
 by $s=2, \phi=0 \text{ to } \phi=\pi, z=0 \text{ to } z=5$

Divergence theorem can be written as $\oint \vec{D} \cdot d\vec{s} = \int_{V_{out}} \nabla \cdot \vec{D} dv$

$$\text{Consider } \oint \vec{D} \cdot d\vec{s} = \iint_{\substack{\phi=0 \\ s=0}}^{\substack{\pi \\ s=5}} (\vec{D})_{\phi=0} \cdot (s d\phi dz \hat{a}_\phi) + \int_{s=0}^2 \int_{\phi=0}^{\pi} (\vec{D})_{\phi=0} \cdot (-d s d\phi \hat{a}_\theta) + \\ \iint_{\substack{\phi=0 \\ s=0}}^{\substack{\pi \\ s=5}} (\vec{D})_{\phi=\pi} \cdot (d s d\phi \hat{a}_\phi) + \int_{\phi=0}^{\pi} \int_{s=0}^2 (\vec{D})_{\phi=\pi} \cdot (-s d\phi ds \hat{a}_z) + \int_{\phi=0}^{\pi} \int_{s=2}^5 (\vec{D})_{z=2} \cdot (s db ds \hat{a}_z)$$

$$= \iint_{\substack{\phi=0 \\ s=0}}^{\substack{\pi \\ s=5}} 6s^2 \sin(\phi/2) d\phi dz + \int_{s=0}^2 \int_{\phi=0}^{\pi} -1.5s \cos(\phi/2) ds d\phi + \\ \int_{\phi=0}^{\pi} \int_{s=0}^2 1.5s \cos(\phi/2) ds dz + 0 + 0$$

$$\oint \vec{D} \cdot d\vec{s} = 240 - 15 = 225 \text{ C}$$

$$\nabla \cdot \vec{D} = \frac{1}{s} \frac{\partial (s D_s)}{\partial s} + \frac{1}{s} \frac{\partial (D_\phi)}{\partial \phi} + \frac{\partial D_z}{\partial z} \\ = \frac{1}{s} \frac{\partial [6s^2 \sin(\phi/2)]}{\partial s} + \frac{1}{s} \frac{\partial [1.5s \cos(\phi/2)]}{\partial \phi} + 0 \\ = \frac{12s \sin(\phi/2)}{s} + \frac{1.5s(-\sin(\phi/2))}{s}$$

$$\nabla \cdot \vec{D} = 12 \sin(\phi/2) - 3/4 \sin(\phi/2) = 11.25 \sin(\phi/2) \text{ C/m}^3$$

$$\int_{V_{out}} \nabla \cdot \vec{D} = \int_{s=0}^5 \int_{\phi=0}^{\pi} \int_{z=0}^2 11.25 \sin(\phi/2) s d s d \phi d z = 225 \text{ C}$$

Verify the divergence theorem for the field $\vec{D} = 2xy \hat{a}_x + x^2 \hat{a}_y \text{ C/m}^2$ in the region bounded by plane $x=0 \& 1$; $y=0 \& 2$; $z=0 \& 3$.

From divergence theorem $\int \vec{D} \cdot d\vec{s} = \int_{Vol} \nabla \cdot \vec{D} dv$

$$\begin{aligned}
 \int \vec{D} \cdot d\vec{s} &= \iint_{\substack{x=0 \\ y=0 \\ z=0}} (\vec{D})_{x=0} \cdot (-dy dz \hat{a}_x) + \iint_{\substack{x=1 \\ y=0 \\ z=0}} (\vec{D})_{x=1} \cdot (dy dz \hat{a}_x) + \iint_{\substack{x=0 \\ y=0 \\ z=0}} (\vec{D})_{y=0} \cdot (-dz dx \hat{a}_y) + \\
 &\quad \iint_{\substack{x=0 \\ y=2 \\ z=0}} (\vec{D})_{y=2} \cdot (dz dx \hat{a}_y) + \iint_{\substack{x=0 \\ y=0 \\ z=0}} (\vec{D})_{z=0} \cdot (-dx dy \hat{a}_z) + \iint_{\substack{x=0 \\ y=0 \\ z=3}} (\vec{D})_{z=3} \cdot (dx dy \hat{a}_z) \\
 &= - \iint_{\substack{x=0 \\ y=0 \\ z=0}} (2xy dy dz) + \iint_{\substack{x=1 \\ y=0 \\ z=0}} (2xy dy dz) + \iint_{\substack{x=0 \\ y=0 \\ z=0}} (-x^2 dx dz) + \\
 &\quad \iint_{\substack{x=0 \\ y=2 \\ z=0}} (x^2 dx dz) + 0 + 0 \\
 &= \left\{ 2 \left[\frac{y^2}{2} \right]_0^2 \Big|_0^3 \right\} \left\{ \left[\frac{x^3}{3} \right]_0^1 \Big|_0^3 \right\} + \left\{ \left[\frac{x^3}{3} \right]_0^1 \Big|_0^3 \right\} \\
 &= (4)(3) - (2)(\frac{1}{3}) + (8)(\frac{1}{3}) = \underline{\underline{12C}}
 \end{aligned}$$

$$\nabla \cdot \vec{D} = \frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z} = \frac{\partial (2xy)}{\partial x} + \frac{\partial (x^2)}{\partial y} + \frac{\partial (0)}{\partial z} = 2y \text{ C/m}^2$$

$$\int_{Vol} \nabla \cdot \vec{D} dv = \iint_{\substack{x=0 \\ y=0 \\ z=0}}^1 \iint_{\substack{x=0 \\ y=0 \\ z=0}}^2 2y dx dy dz = 6 \left[\frac{y^2}{2} \right]_0^2 \Big|_0^1 = 3[4] = \underline{\underline{12C}}$$

Hence $\int \vec{D} \cdot d\vec{s} = \int_{Vol} \nabla \cdot \vec{D} dv$