26/04/2021

Normal distribution → continuous random variable ↳ assumes values in an interval.

Continuous distribution

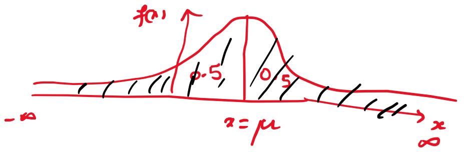
Probability density function (PDF)

$$f(x) = N(\mu, \sigma, x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

↳ normal variate $\left[\begin{matrix} -\infty < \mu < \infty \\ \sigma > 0 \end{matrix}\right]$

(i) $f(x) \geq 0$

(ii) $\int_{-\infty}^{\infty} f(x)\,dx = 1$



Note: (1) Bell shaped curve (Normal curve)

(2) symmetric about $x = \mu$

(3) Total area under the curve $= 1$

$$P(a \leq x \leq b) = \int_a^b f(x)\,dx = \frac{1}{\sigma\sqrt{2\pi}} \int_a^b e^{-\frac{(x-\mu)^2}{2\sigma^2}}\,dx$$

Standardization

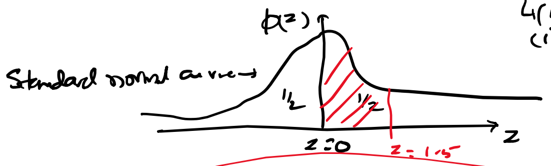
standard normal variate z at

$$z = \frac{x-\mu}{\sigma}$$

PDF $\quad N(0,1,z) = \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}} = \phi(z)$

↳ (i)

(ii) $\int_{-\infty}^{\infty} \phi(z)\,dz = 1$



$P(0 \leq z \leq 1.5)$

(1)

$$P(a \leq x \leq b) = \int_a^b f(x)\,dx = \frac{1}{\sqrt{2\pi}} \int_{z_1}^{z_2} e^{-z^2/2}\,dz = P(z_1 \leq z \leq z_2)$$

$z = \frac{x-\mu}{\sigma}$ $\qquad x_1 = a \rightarrow z = z_1$

$\qquad x_2 = b \rightarrow z = z_2$

$$\int_0^z \phi(z)\,dz = \frac{1}{\sqrt{2\pi}} \int_0^z e^{-z^2/2}\,dz$$

= Area under the standard normal curve from $z = 0$ to $z = z$

= $A(z)$ → tables of Normal Probability.

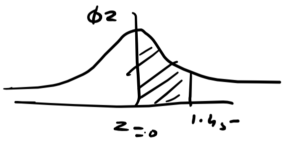
$A(1.5) =$

Illustrations: Evaluate :

(1) $P(0 \leq z \leq 1.45)$

$= A(1.45)$

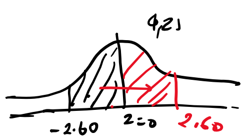
$= 0.4265$



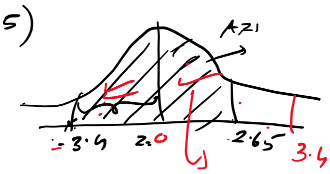
(2) $P(-2.60 \leq z \leq 0)$

$= P(0 \leq z \leq 2.60)$

$= A(2.60) = 0.4953$

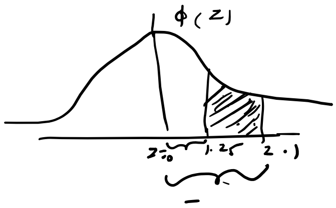


③ $P(-3.4 \le Z \le 2.65)$



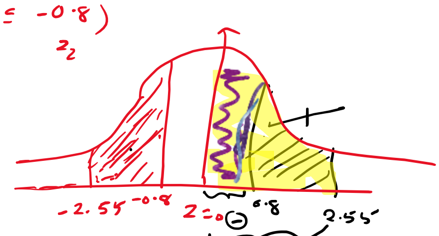
$= P(0 \le Z \le 2.65)$
$+ P(0 \le Z \le 3.4)$
$= A(2.65) + A(3.4)$
$= 0.49966 + 0.4960 = 0.99566$

④ $P(\underset{z_1}{1.25} \le Z \le \underset{z_2}{2.1})$



$= P(0 \le Z \le 2.1)$
$- P(0 \le Z \le 1.25)$
$= A(2.1) - A(1.25) = 0.4821 - 0.3914$
$= 0.0877$

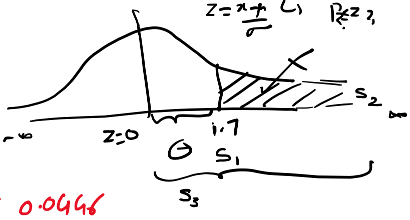
⑤ $P(\underset{z_1}{-2.55} \le Z \le \underset{z_2}{-0.8})$



$= P(0 \le Z \le 2.55)$
$- P(0 \le Z \le 0.8)$
$= A(2.55) - A(0.8) = 0.4946 - 0.2881$
$= 0.2065$

⑥ $P(Z \ge \underset{z_1}{1.7})$

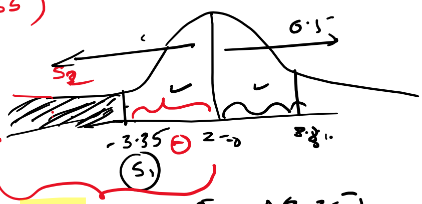
$\rightarrow P(a \le x \le b) = ?$
$z = \frac{x - \mu}{\sigma}$ $P(z \le z_1)$



$= P(0 \le Z \le \infty)$
$- P(0 \le Z \le 1.7)$
$= 0.5 - A(1.7)$ S4
$= 0.5 - 0.4554 = 0.0446$

$A(\infty) = 0.5$

⑦ $P(Z \le -3.35)$



$= P(-\infty \le Z \le 0)$
$- P(-3.35 \le Z \le 0)$ S3
$= P(0 \le Z \le \infty) - P(0 \le Z \le 3.35)$
$= 0.5 - A(3.35)$
$= 0.5 - 0.4996$
$= 0.0004$

⑧ $P(|Z| \le 1.85) = 2\,P(0 \le Z \le 1.85)$

$-1.85 \le Z \le 1.85$

$= 2\,A(1.85) = 2 \times 0.4678$
$= 0.9356.$