Amplitude Modulation

❖ **Define modulation? Explain different types of modulation.**

Modulation is the process of changing some characteristics (amplitude, frequency & phase) of a carrier wave in accordance with the instantaneous value of the modulating signal.

There are 3 types of modulations:

i) Amplitude modulation
ii) Frequency modulation and
iii) phase Modulation.

i) Amplitude modulation :-

Amplitude modulation is defined as the modulation in which the amplitude of the carrier wave is varied in accordance with the instantaneous amplitude of the modulating signal, keeping its (carrier) frequency & phase constant.

ii) Frequency modulation :-

Frequency modulation is defined as the modulation in which the frequency of the carrier wave is varied in accordance with the instantaneous amplitude of the modulating signal, keeping its (carrier) amplitude & phase constant.

iii) phase modulation :-

Phase modulation is defined as the modulation in which the phase of the carrier wave is varied in accordance with the instantaneous amplitude of the modulating Signal, Keeping its (carrier) amplitude and frequency Constant.

- **Explain the NEED for modulation?**
- **Explain the advantages of modulation?**

The advantages of modulation are

1) Reduces the height of antenna:-

Height of antenna is a function of Wavelength 'λ'. The minimum height of antenna is given by $\lambda/4$.

i.e. height of antenna $= \frac{\lambda}{4} = \frac{c}{4f}$ $\boxed{\because \lambda = \frac{c}{f}}$

Where, $\lambda = \frac{c}{f}$,

$c = 3\times10^8$, velocity of light

f = Transmitting frequency.

ex:- i) $f = 15\,KHz$,

height of antenna $= \frac{\lambda}{4} = \frac{c}{4f} = \frac{3\times10^8}{4\times15\times10^3} = 5000$ meters

ii) $f = 1\,MHz$,

height of antenna $= \frac{\lambda}{4} = \frac{c}{4f} = \frac{3\times10^8}{4\times1\times10^6} = 75$ meters.

From above two examples it is clear that as the transmitting frequency is increased, height of the antenna is decreased.

2) Avoids mixing of Signals:-

All audio (message) Signals ranges from 20 Hz to 20 KHz.

The transmission of message signals from various sources causes the mixing of signals and then it is difficult to separate these signals at the receiver end.

3) Increases the range of communication :-

* Low frequency signals have poor radiation and they get highly attenuated. Therefore baseband signals cannot be transmitted directly over long distances.
* Modulation increases the frequency of the signal and thus they can be transmitted over long distances.

4) Allows multiplexing of signals :-

* Modulation allows the multiplexing to be used. Multiplexing means transmission of two or more signals simultaneously over the same communication channel.

eg:-

1) Number of TV channels operating simultaneously.
2) Number of radio stations broadcasting the signals in MW & SW band simultaneously.

5) Allows adjustments in the bandwidth :-

Bandwidth of a modulated signal may be made smaller or larger.

6) Improves quality of reception :-

Modulation techniques like frequency modulation, pulse

Code modulation reduces the effect of noise to great extent. Reduction of noise improves quality of reception.

- **Define standard form of amplitude modulation and explain the time and frequency domain expression of AM wave**

July 09- 6M Jan 05 - 4M

- **Define amplitude modulation. Derive the expression on AM by both time domain and frequency domain representation with necessary waveforms.**

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- Amplitude modulation is defined as the modulation in which the amplitude of the carrier wave is varied in accordance with the instantaneous amplitude of the modulating signal keeping its (carrier) frequency & phase constant.

Time-Domain Description :-

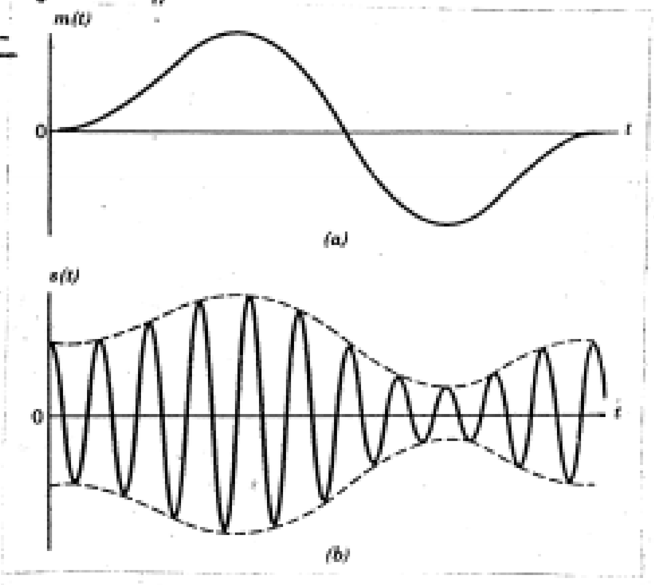


Fig (a) Message signal, (b) AM wave s(t).

* The instantaneous value of modulating signal is given by

$$\boxed{m(t) = A_m \cos(2\pi f_m t)} \longrightarrow (1)$$

Where, $A_m \rightarrow$ maximum amplitude of the modulating Signal
$f_m \rightarrow$ frequency of modulating Signal.

* The Instantaneous value of carrier Signal is given by

$$\boxed{c(t) = A_c \cos(2\pi f_c t)} \longrightarrow (2)$$

Where,
$A_c \rightarrow$ Maximum amplitude of the carrier Signal.
$f_c \rightarrow$ frequency of carrier Signal.

The Standard equation for AM wave is given by

$$\boxed{s(t) = A_c\left[1+K_a m(t)\right]\cos(2\pi f_c t)} \longrightarrow (3)$$

Where,
K_a is a constant called the amplitude Sensitivity of the modulator.

Substituting eq (1) in eq (3), we get

$$s(t) = A_c\left[1+\underline{K_a A_m}\cos(2\pi f_m t)\right]\cos(2\pi f_c t)$$

$$s(t) = A_c\left[1+\mu\cos(2\pi f_m t)\right]\cos(2\pi f_c t)$$

Where, $\boxed{\mu = K_a A_m}$ is called the modulation Index or modulation factor.

$$s(t) = A_c\cos(2\pi f_c t) + \mu A_c\cos(2\pi f_c t)\cdot\cos(2\pi f_m t) \longrightarrow (4)$$

equation (4) can be further expanded, by means of the trigonometric -al relation:

$$\boxed{\cos a \cdot \cos b = \frac{1}{2}\cos(a-b) + \frac{1}{2}\cos(a+b)}$$

$$s(t) = A_c\cos(2\pi f_c t) + \frac{\mu A_c}{2}\cos\left[2\pi f_c - 2\pi f_m\right]t + \frac{\mu A_c}{2}\cos\left[2\pi f_c + 2\pi f_m\right]t$$

equation ⑤ is the amplitude modulated Signal, consist of three frequency Component

1) The first term is the carrier itself. It has a frequency 'f_c' and amplitude 'A_c'.

2) The 2nd Component is $\frac{\mu A_c}{2} \cos 2\pi(f_c - f_m)t$. It has frequency $(f_c - f_m)$ Called Lower Sideband (F_{LSB}) and having amplitude $\frac{\mu A_c}{2}$

3) Similarly 3rd component is $\frac{\mu A_c}{2} \cos 2\pi(f_c + f_m)t$. It has frequency $(f_c + f_m)$ called upper Sideband (F_{USB}) and having amplitude $\frac{\mu A_c}{2}$.

Frequency-Domain Description :-

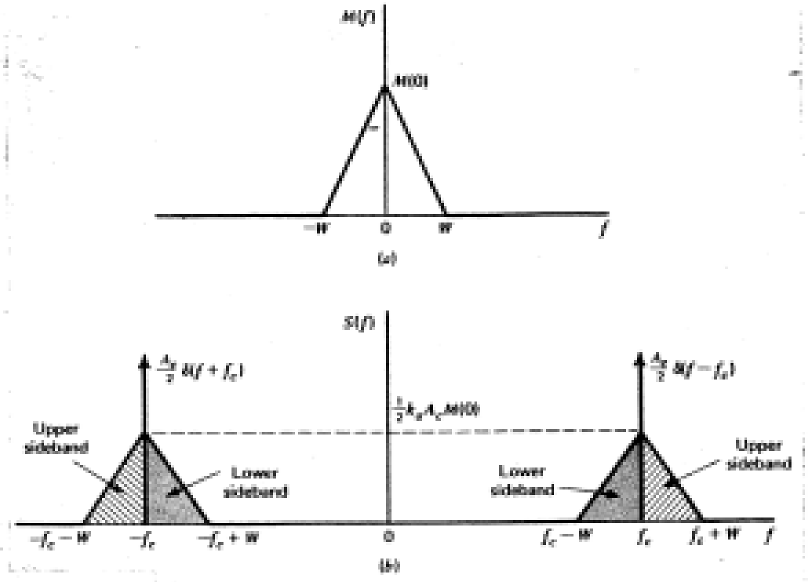
The time domain description of a conventional AM wave is given below:

$$S(t) = A_c[1 + K_a m(t)] \cos 2\pi f_c t.$$

$$S(t) = A_c \cos 2\pi f_c t + A_c K_a m(t) \cos 2\pi f_c t \longrightarrow ①$$

Taking fourier transforms on both the Sides of eq ①, we get

$$S(f) = \frac{A_c}{2}[\delta(f - f_c) + \delta(f + f_c)] + \frac{A_c K_a}{2}[M(f - f_c) + M(f + f_c)]$$



(a) Spectrum of baseband signal. (b) Spectrum of AM wave.

* The amplitude spectrum of the AM wave has 2 sidebands on either sides of $\pm f_c$.
* For +ve frequencies, the highest frequency component of the AM wave equals $f_c + W$, called UPPER sideband f_{USB} and the lowest frequency component equals $f_c - W$, called LOWER sideband f_{LSB}.

Transmission Bandwidth (B_T):-

The difference between upper sideband and lower sideband frequencies defines the transmission bandwidth 'B_T'.

$$B_T = f_{USB} - f_{LSB}$$
$$= (f_c + f_m) - (f_c - f_m)$$
$$= \cancel{f_c} + f_m - \cancel{f_c} + f_m$$
$$\boxed{B_T = 2f_m}$$

$\therefore$ Bandwidth required for transmission of an AM wave is twice the modulating signal frequency i.e. $2f_m$.

❖ Define modulation index and percentage modulation index.

The ratio of change in amplitude of modulating Signal to the amplitude of carrier wave is Known as modulation Index or modulation factor or modulation Co-efficient or depth of - modulation or degree of modulation 'μ'.

$$\boxed{\mu = \frac{A_m}{A_c}} \quad \text{or} \quad \boxed{\mu = K_a A_m}$$

percentage modulation index

$$\boxed{\% \mu = \left(\frac{A_m}{A_c}\right) \times 100}$$

NOTE :-

* If A_m is greater than A_c then distortion is introduced into the System.
* The modulating Signal voltage 'A_m' must be less than carrier signal voltage 'A_c' for proper amplitude modulation.

❖ Explain transmission efficiency of an AM wave.

Transmission efficiency is defined as the ratio of the power carried by the Sidebands to the total transmitted power is called transmission efficiency 'η' and is given by

$$\boxed{\eta = \frac{P_{USB} + P_{LSB}}{P_T}}$$

WKT

$$P_T = P_c\left(1+\frac{\mu^2}{2}\right) \text{ and}$$

$$P_{USB} = P_{LSB} = \frac{\mu^2 A_c^2}{8R}$$

$$\eta = \frac{\frac{\mu^2 A_c^2}{8R} + \frac{\mu^2 A_c^2}{8R}}{P_c(1+\mu^2/2)}$$

$$= \frac{\cancel{2}\frac{\mu^2 A_c^2}{_4 \cancel{8}R}}{P_c\left[\frac{2+\mu^2}{2}\right]} = \frac{\frac{\mu^2}{2}\left[\frac{A_c^2}{2R}\right]}{P_c\left[\frac{2+\mu^2}{2}\right]}$$

$$= \frac{\frac{\mu^2}{2}\cancel{P_c}}{\cancel{P_c}\left[\frac{2+\mu^2}{2}\right]} = \frac{\frac{\mu^2}{\cancel{2}}}{\frac{2+\mu^2}{\cancel{2}}}$$

$$\boxed{\eta = \frac{\mu^2}{\mu^2+2}}$$

❖ **Obtain the expression for total transmitted power of AM wave.**

W.K.T

$$S(t) = A_c \cos 2\pi f_c t + \frac{\mu A_c}{2}\cos 2\pi[f_c - f_m]t + \frac{\mu A_c}{2}\cos 2\pi[f_c+f_m]t$$

The AM wave has three components : unmodulated carrier, lower sideband and upper sideband.

∴ The total power of AM wave is the sum of the carrier power 'P_c' and powers in the two sidebands i.e. P_{USB} & P_{LSB}

$$P_T = P_c + P_{USB} + P_{LSB}$$

* The average carrier power

$$P_c = \frac{(A_c/\sqrt{2})^2}{R}$$

$$\boxed{P_c = \frac{A_c^2}{2R}}$$

W.K.T.
RMS value $V_{rms} = \frac{V_m}{\sqrt{2}}$

$||^{ly}$ $A_{c_{rms}} = \frac{A_c}{\sqrt{2}}$

W.K.T

Power 'p' $= \frac{V_{rms}^2}{R}$

$$P_c = \frac{A_{c_{rms}}^2}{R}$$

$$P_c = \frac{(A_c/\sqrt{2})^2}{R}$$

* The average Sideband power:

$$P_{USB} = P_{LSB} = \frac{\left(\mu A_c/2\sqrt{2}\right)^2}{R}$$

$$= \frac{\frac{\mu^2 A_c^2}{4\times 2}}{R}$$

$$\boxed{P_{USB} = P_{LSB} = \frac{\mu^2 A_c^2}{8R}}$$

$$P_{USB} = P_{LSB} = \frac{\left(\frac{\mu A_c}{2\sqrt{2}}\right)^2}{R}$$

$$= \frac{\frac{\mu^2 A_c^2}{4\times 2}}{R}$$

$$P_{USB} = P_{LSB} = \frac{\mu^2 A_c^2}{8R}$$

∴ The average total power,

$$P_T = P_c + P_{USB} + P_{LSB}$$

$$= \frac{A_c^2}{2R} + \frac{\mu^2 A_c^2}{8R} + \frac{\mu^2 A_c^2}{8R}$$

$$= \frac{A_c^2}{2R}\left[1 + \frac{\mu^2}{4} + \frac{\mu^2}{4}\right]$$

$$\boxed{\frac{\mu^2}{4} + \frac{\mu^2}{4} = \frac{\mu^2}{2}}$$

$$\boxed{P_T = P_c\left[1 + \frac{\mu^2}{2}\right]}$$

For 100% modulation $\mu = 1$, we have

$$P_T = P_c\left[1 + \frac{1^2}{2}\right]$$

$$P_T = P_c\left[\frac{2+1}{2}\right]$$

$$= P_c\left[\frac{3}{2}\right]$$

$$\boxed{P_T = 1.5\,P_c}$$

<u>NOTE :-</u>

$$P_T = 1.5\,P_c$$

$$P_c = \frac{1}{1.5}\,P_T$$

$$\boxed{P_c = 0.666\,P_T}$$

In Amplitude modulated wave, the 66.66% of the transmitted power is used by the <u>carrier Signal</u> and remaining 33.33% of the power is used by the <u>sidebands</u> (P_{USB} & P_{LSB}).

❖ **Derive the followings:**

i. Modulation index interms of P_T and P_C.

ii. Current relation of AM wave.

iii. Modulating index interms of I_T and I_C.

iv. Voltage relation of AM wave.

v. Modulation index interms of V_T and V_C.

<u>i) Modulation Index interms of P_T & P_c :-</u>

W.K.T $\quad P_T = P_c\left[1+\frac{\mu^2}{2}\right]$

$$\frac{P_T}{P_c} = 1+\frac{\mu^2}{2}$$

$$\frac{\mu^2}{2} = \frac{P_T}{P_c} - 1$$

$$\mu^2 = 2\left[\frac{P_T}{P_c} - 1\right]$$

$$\boxed{\mu = \sqrt{2\left[\frac{P_T}{P_c} - 1\right]}}$$

ii) Current relation of AM Wave :-

Let $I_T \rightarrow$ Total current

$I_c \rightarrow$ Carrier current

W.K.T $\quad P = I^2 R$

$||^{ly}$ $\quad P_T = I_T^2 R$ and

$P_c = I_c^2 R.$

$$P_T = P_c\left[1 + \frac{\mu^2}{2}\right]$$

$$I_T^2 \cancel{R} = I_c^2 \cancel{R}\left[1 + \frac{\mu^2}{2}\right]$$

$$I_T^2 = I_c^2\left[1 + \frac{\mu^2}{2}\right]$$

$$I_T = \sqrt{I_c^2\left[1 + \frac{\mu^2}{2}\right]}$$

$$\boxed{I_T = I_c\sqrt{1 + \frac{\mu^2}{2}}}$$

iii) Modulation Index interms of I_T & I_c :-

W.K.T

$$I_T = I_c \sqrt{1+\mu^2/2}$$

$$I_T^2 = I_c^2 \left(1+\frac{\mu^2}{2}\right)$$

$$1+\frac{\mu^2}{2} = \left(\frac{I_T}{I_c}\right)^2$$

$$\frac{\mu^2}{2} = \left(\frac{I_T}{I_c}\right)^2 - 1$$

$$\mu^2 = 2\left(\frac{I_T}{I_c}\right)^2 - 1$$

$$\boxed{\mu^2 = \sqrt{2\left(\frac{I_T}{I_c}\right)^2 - 1}}$$

iv) Voltage relation of AM Wave :-

Let $A_T = V_T$ = Total voltage &

$A_c = V_c$ = Carrier voltage.

$\boxed{V = A}$

$A_T = V_T$

$A_c = V_c$

W.K.T $\quad P = \frac{V^2}{R}$

$\parallel^{ly}$ $\quad P_T = \frac{A_T^2}{R}$ &

$$P_c = \frac{A_c^2}{R}.$$

$$P_T = P_c\left[1+\frac{\mu^2}{2}\right]$$

$$\frac{A_T^2}{\cancel{R}} = \frac{A_c^2}{\cancel{R}}\left[1+\frac{\mu^2}{2}\right]$$

$$A_T = \sqrt{A_c^2\left[1+\frac{\mu^2}{2}\right]}$$

$$\boxed{A_T = A_c\sqrt{1+\left[\mu^2/2\right]}}$$

v) Modulation Index interms of A_T & A_c.

W.K.T $\quad A_T = \sqrt{A_c^2\left[1+\mu^2/2\right]}$

$$A_T^2 = A_c^2\left[1+\mu^2/2\right]$$

$$1+\frac{\mu^2}{2} = \left(\frac{A_T}{A_c}\right)^2$$

$$\frac{\mu^2}{2} = \left[\frac{A_T}{A_c}\right]^2 - 1$$

$$\mu^2 = 2\left[\frac{A_T}{A_c}\right]^2 - 1$$

$$\boxed{\mu = \sqrt{2\left[A_T/A_c\right]^2 - 1}}$$

❖ **Derive modulation index using AM wave.**

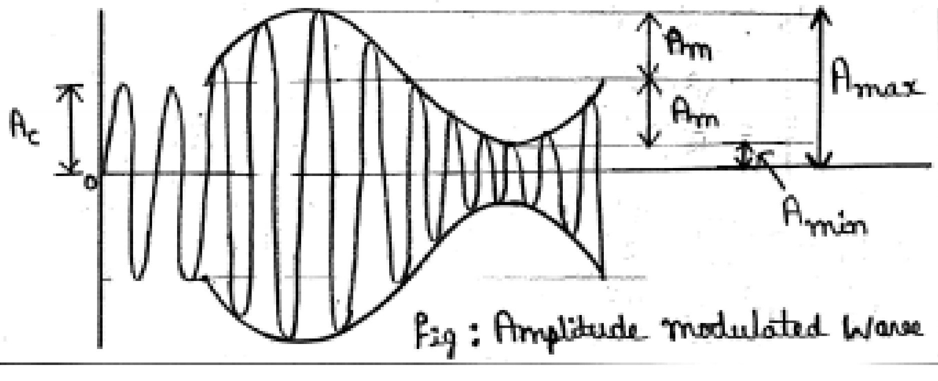


Fig : Amplitude modulated Wave

We can calculate the modulation Index from the amplitude modulate wave.

W.K.T $\boxed{\mu = \frac{A_m}{A_c}}$

From figure,

$$\boxed{A_m = \frac{A_{max} - A_{min}}{2}} \longrightarrow (1)$$

$$A_c = A_{max} - A_m \longrightarrow (2)$$

Substituting equation (1) in equation (2)

$$A_c = A_{max} - \left[\frac{A_{max} - A_{min}}{2}\right]$$

$$A_c = \frac{2A_{max} - A_{max} + A_{min}}{2}$$

$$\boxed{A_c = \frac{A_{max} + A_{min}}{2}}$$

$$\therefore \mu = \frac{A_m}{A_c} = \frac{A_{max} - A_{min}/\cancel{2}}{A_{max} + A_{min}/\cancel{2}}$$

$$\boxed{\mu = \frac{A_{max} - A_{min}}{A_{max} + A_{min}}}$$

❖ Explain amplitude modulation for single tone information.

A Single-tone modulating Signal m(t) has a Single (tone) Frequency Component 'f_m' and is defined as follows:

$$\boxed{m(t) = A_m \cos(2\pi f_m t)} \rightarrow (1)$$

Where A_m is the amplitude of the modulating wave and f_m is the Frequency of the modulating wave.

Let $$\boxed{c(t) = A_c \cos(2\pi f_c t)} \rightarrow (2)$$

Where A_c is the amplitude of the carrier wave and f_c is the frequency of the carrier wave.

* The time-domain expression for the Standard AM wave is

$$\boxed{S(t) = A_c[1 + K_a m(t)] \cos 2\pi f_c t} \rightarrow (3)$$

Substituting eq (1) in eq (3), we get

$$S(t) = A_c[1 + K_a A_m \cos 2\pi f_m t] \cos 2\pi f_c t$$

Since, the modulation Index $\boxed{\mu = K_a A_m}$

We get

$$S(t) = A_c[1 + \mu \cos 2\pi f_m t] \cos 2\pi f_c t. \rightarrow (4)$$

equation (4) can be further expanded, by means of the trigonometric-al relation

$$\boxed{\cos a \cdot \cos b = \frac{1}{2}\cos(a-b) + \frac{1}{2}\cos(a+b)}$$

$$S(t) = A_c \cos(2\pi f_c t) + \mu A_c \underbrace{\cos(2\pi f_c t)}_{\cos a} \cdot \underbrace{\cos(2\pi f_m t)}_{\cos b}$$

$$S(t) = A_c \cos(2\pi f_c t) + \frac{\mu A_c}{2} \cos[2\pi f_c - 2\pi f_m]t + \frac{\mu A_c}{2} \cos[2\pi f_c + 2\pi f_m]t \rightarrow (5)$$

Taking Fourier transform on both sides of eq (5), we get

$$\boxed{S(f) = \frac{A_c}{2}\left[\delta(f-f_c) + \delta(f+f_c)\right] + \frac{\mu A_c}{4}\left\{\delta[f-(f_c-f_m)] + \delta[f+(f_c-f_m)]\right\} + \frac{\mu A_c}{4}\left\{\delta[f-(f_c+f_m)] + \delta[f+(f_c+f_m)]\right\}}$$

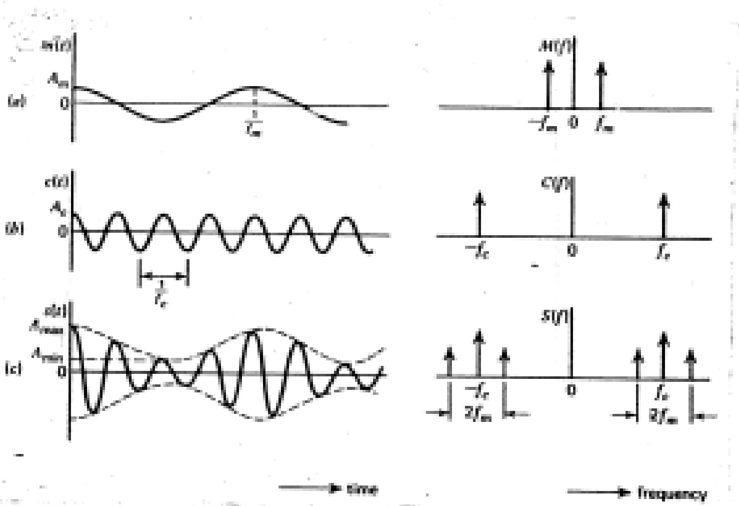


fig (1) Illustrating the time-domain (on the left) and frequency domain (on the right) characteristics of a standard amplitude modulation produced by a single tone.
(a) Modulating wave (b) Carrier wave (c) AM wave.

* In practice, the AM wave $s(t)$ is a voltage or current wave. The average power delivered by an AM wave to a 1-ohm resistor is calculated as follows:

Average carrier power 'P_c' $= \frac{A_c^2}{2}$

'P_{USB}', Upper Side-Frequency power $= \frac{\mu^2 A_c^2}{8}$

'P_{LSB}', Lower Side-Frequency power $= \frac{\mu^2 A_c^2}{8}$

The transmission efficiency 'η' is the ratio of the total Sideband power to the total power in the modulated wave

$$\eta = \frac{\text{Power in Sidebands}}{\text{Total power } (P_T)} = \frac{P_{USB} + P_{LSB}}{P_c\left[1 + \frac{\mu^2}{2}\right]}$$

$$= \frac{\frac{\mu^2 A_c^2}{8} + \frac{\mu A_c^2}{8}}{P_c\left[1 + \mu^2/2\right]} = \frac{\frac{\cancel{2}\mu^2 A_c^2}{_4\cancel{8}}}{P_c\left[\frac{2+\mu^2}{2}\right]}$$

$$= \frac{\frac{\mu^2 A_c^2}{4}}{P_c\left[\frac{2+\mu^2}{2}\right]} = \frac{\frac{\mu^2}{2}\left[\frac{A_c^2}{2}\right]}{P_c\left[\frac{2+\mu^2}{2}\right]}$$

$$= \frac{\frac{\mu^2}{2}\cancel{P_c}}{\cancel{P_c}\left[\frac{2+\mu^2}{2}\right]} = \frac{\frac{\mu^2}{\cancel{2}}}{\frac{2+\mu^2}{\cancel{2}}}$$

$$\boxed{\eta = \frac{\mu^2}{\mu^2 + 2}}$$

If $\mu = 1$, that is, 100 percent modulation is used, the total power in the two Side Frequencies of the resulting AM wave is only $1/3^{rd}$ of the total power in the modulated wave as Shown in fig ③.

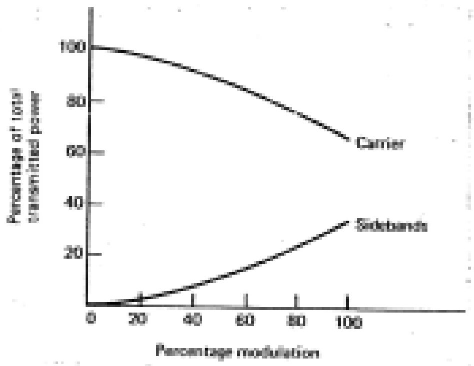


Fig ② variations of carrier power and total sideband power with percentage modulation.

❖ **A multitone modulating signal has the following time-domain form:**

$m(t)=E_1\cos2\pi f_1t + E_2\cos2\pi f_2t + E_3\cos2\pi f_3t$ volts   **where $E_1 > E_2 > E_3$**
$f_3 > f_2 > f_1$

i. **Give the time – domain expression for the conventional AM wave.**

ii. **Draw the amplitude spectrum for the AM wave obtained in part i. Also find the minimum transmission bandwidth.**

Sol:-

ⓐ The time-domain expression for the conventional AM wave is

$$s(t) = A_c[1+K_a m(t)]\cos 2\pi f_c t \rightarrow ①$$

Substituting the value of $m(t)$ in eq ①, we get

$$s(t) = A_c[1+\underline{K_a E_1}\cos 2\pi f_1 t + \underline{K_a E_2}\cos 2\pi f_2 t + \underline{K_a E_3}\cos 2\pi f_3 t] \times \cos 2\pi f_c t$$

W.K.T, $\mu_1 = K_a E_1$, $\mu_2 = K_a E_2$ & $\mu_3 = K_a E_3$

$$s(t) = A_c[1+\mu_1\cos 2\pi f_1 t + \mu_2\cos 2\pi f_2 t + \mu_3\cos 2\pi f_3 t]\cos 2\pi f_c t.$$

$$s(t) = A_c\cos 2\pi f_c t + \mu_1 A_c\cos 2\pi f_c t.\cos 2\pi f_1 t + \mu_2 A_c\cos 2\pi f_c t.\cos 2\pi f_2 t + \mu_3 A_c\cos 2\pi f_c t.\cos 2\pi f_3 t$$

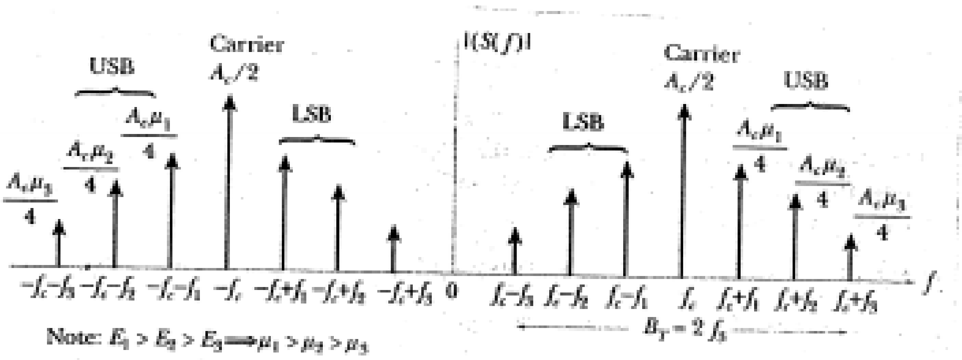
W.K.T $\boxed{\cos A.\cos B = \frac{1}{2}\cos(A+B) + \frac{1}{2}\cos(A-B)}$

$$s(t) = A_c \cos 2\pi f_c t + \frac{\mu_1 A_c}{2} \cos 2\pi (f_c - f_1)t + \frac{\mu_1 A_c}{2} \cos 2\pi (f_c + f_1)t$$
$$+ \frac{\mu_2 A_c}{2} \cos 2\pi (f_c - f_2)t + \frac{\mu_2 A_c}{2} \cos 2\pi (f_c + f_2)t$$
$$+ \frac{\mu_3 A_c}{2} \cos 2\pi (f_c - f_3)t + \frac{\mu_3 A_c}{2} \cos 2\pi (f_c + f_3)t \rightarrow (2)$$

b) Taking Fourier transform on both sides of equation (2), we get

$$S(f) = \frac{A_c}{2}\left[\delta(f - f_c) + \delta(f + f_c)\right] + \frac{\mu_1 A_c}{4}\left\{\delta[f - (f_c - f_1)] + \delta[f + (f_c - f_1)]\right\}$$
$$+ \frac{\mu_1 A_c}{4}\left\{\delta[f - (f_c + f_1)] + \delta[f + (f_c + f_1)]\right\} + \frac{\mu_2 A_c}{4}\left\{\delta[f - (f_c - f_2)] + \delta[f + (f_c - f_2)]\right\}$$
$$+ \frac{\mu_2 A_c}{4}\left\{\delta[f - (f_c + f_2)] + \delta[f + (f_c + f_2)]\right\} + \frac{\mu_3 A_c}{4}\left\{\delta[f - (f_c - f_3)] + \delta[f + (f_c - f_3)]\right\}$$
$$+ \frac{\mu_3 A_c}{4}\left\{\delta[f - (f_c + f_3)] + \delta[f + (f_c + f_3)]\right\}$$

The amplitude spectrum is shown below



The maximum frequency is f_3.

$\therefore$ The transmission bandwidth $\boxed{B_T = 2f_3}$

❖ **Derive an expression for multitone amplitude modulation, total transmitted power and total modulation index.**

W.K.T an amplitude modulated wave is expressed as:

$$S(t) = A_c\left[1+K_a m(t)\right]\cos 2\pi f_c t.$$

For simplicity consider two modulating signal:

$$m_1(t) = A_{m1}\cos 2\pi f_{m1} t$$

$$m_2(t) = A_{m2}\cos 2\pi f_{m2} t.$$

$$\therefore\quad S(t) = A_c\left[1+K_a\left(m_1(t)+m_2(t)\right)\right]\cos 2\pi f_c t.$$

$$= A_c\left[1+K_a\left(A_{m1}\cos 2\pi f_{m1} t + A_{m2}\cos 2\pi f_{m2} t\right)\right]\cos 2\pi f_c t.$$

$$= A_c\left[1+\underbrace{K_a A_{m1}}_{\mu_1}\cos 2\pi f_{m1} t + \underbrace{K_a A_{m2}}_{\mu_2}\cos 2\pi f_{m2} t\right]\cos 2\pi f_c t.$$

$$S(t) = A_c\left[1+\mu_1\cos 2\pi f_{m1} t + \mu_2\cos 2\pi f_{m2} t\right]\cos 2\pi f_c t.$$

$$S(t) = A_c\cos 2\pi f_c t + \mu_1 A_c\cos 2\pi f_c t\cdot\cos 2\pi f_{m1} t + \mu_2 A_c\cos 2\pi f_c t\cdot\cos 2\pi f_{m2} t$$

W.K.T

$$\boxed{\cos A\cdot\cos B = \frac{1}{2}\cos(A-B) + \frac{1}{2}\cos(A+B)}$$

$$S(t) = A_c\cos 2\pi f_c t + \frac{\mu_1 A_c}{2}\cos 2\pi\left[f_c - f_{m1}\right]t + \frac{\mu_1 A_c}{2}\cos 2\pi\left[f_c + f_{m1}\right]t$$
$$+ \frac{\mu_2 A_c}{2}\cos 2\pi\left[f_c - f_{m2}\right]t + \frac{\mu_2 A_c}{2}\cos 2\pi\left[f_c + f_{m2}\right]t \rightarrow (5)$$

From equation (5) it is clear that, when we have two modulating frequencies, we get four additional frequencies, two upper - sidebands (USB) f_c+f_{m1}, f_c+f_{m2} and two lower sidebands 'LSB' f_c-f_{m1}, f_c-f_{m2}.

Total transmitted power:-

The total power in the amplitude modulated wave is calculated as follows:

$$P_T = P_c + P_{USB1} + P_{USB2} + P_{LSB1} + P_{LSB2}$$

$$= \frac{(A_c/\sqrt{2})^2}{R} + \frac{\mu_1 A_c^2}{8R} + \frac{\mu_1^2 A_c^2}{8R} + \frac{\mu_2^2 A_c^2}{8R} + \frac{\mu_2^2 A_c^2}{8R}$$

$$= \frac{A_c^2}{2R} + \cancel{2}\cdot\frac{\mu_1 A_c^2}{4\cancel{8}R} + \cancel{2}\cdot\frac{\mu_2^2 A_c}{4\cancel{8}R}$$

$$= \frac{A_c^2}{2R} + \frac{\mu_1^2 A_c^2}{4R} + \frac{\mu_2^2 A_c}{4R}$$

$$= \frac{A_c^2}{2R}\left[1 + \frac{\mu_1^2}{2} + \frac{\mu_2^2}{2}\right]$$

$$\boxed{P_T = P_c\left[1 + \frac{\mu_t^2}{2}\right]}$$

$$\boxed{P_c = \frac{A_c^2}{2R}}$$

Where,

$$\frac{\mu_t^2}{\cancel{2}} = \frac{\mu_1^2}{\cancel{2}} + \frac{\mu_2^2}{\cancel{2}}$$

$$\mu_t^2 = \mu_1^2 + \mu_2^2$$

$$\boxed{\mu_t = \sqrt{\mu_1^2 + \mu_2^2}}$$

In general, Total modulation index is given by

$$\boxed{\mu_t = \sqrt{\mu_1^2 + \mu_2^2 + \mu_3^2 + \dots + \mu_n^2}}$$

Generation of AM wave:-

There are two important methods of AM generation for low power applications:

1.Square Law Modulator

2.Switching Modulator.

- **Explain generation of AM wave using SQUARE- LAW modulator helps to produce AM wave. Derive the related equations and draw the waveforms** **July-05,8M**
- **Explain the generation of AM wave using SQUARE- LAW modulator along with relevant diagram & analysis.** **July-08,10M**

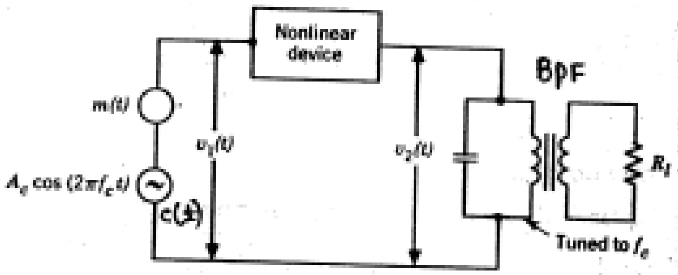


Fig ①: Square-law modulator.

The Square-law modulator consists of three elements:

1) Summer: It adds the carrier and modulating Signal.

2) Non-linear device: A device with non-linear I/p - o/p relation.

3) Band pass filter (BPF): It extract desired Signal (term) from modulator product.

* The Semiconductor diodes or transistor can be used as non-linear element and Single or double tuned circuit can be

used as the filter.

* When a non-linear element such as diode is suitably biased and the signal applied is relatively weak, it is possible to approximate the transfer characteristics as:

$$\boxed{V_2(t) = a_1 V_1(t) + a_2 V_1^2(t)} \longrightarrow ①$$

Where a_1 and a_2 are constants.

* The I/p voltage '$V_1(t)$' is the sum of carrier signal and modulating signal.

ie. $$\boxed{V_1(t) = A_c \cos 2\pi f_c t + m(t)} \longrightarrow ②$$

Substituting equation ② in equation ①

$$V_2(t) = a_1\left[A_c \cos 2\pi f_c t + m(t)\right] + a_2\left[A_c \cos 2\pi f_c t + m(t)\right]^2$$

W.K.T $\boxed{(a+b)^2 = a^2 + b^2 + 2ab}$

$$V_2(t) = a_1 A_c \cos 2\pi f_c t + a_1 m(t) + a_2\left[A_c^2 \cos^2 2\pi f_c t + m^2(t) + 2m(t).A_c \cos 2\pi f_c t\right]$$

$$V_2(t) = \underline{a_1 A_c \cos 2\pi f_c t} + a_1 m(t) + a_2 A_c^2 \cos^2 2\pi f_c t + a_2 m^2(t) + \underline{2a_2 m(t).A_c \cos 2\pi f_c t}.$$

$$V_2(t) = a_1 A_c \cos 2\pi f_c t + 2a_2 m(t) A_c \cos 2\pi f_c t + a_1 m(t) + a_2 A_c^2 \cos^2 2\pi f_c t + a_2 m^2(t).$$

$$= \underbrace{a_1 A_c\left[1 + \frac{2a_2}{a_1} m(t)\right] \cos 2\pi f_c t}_{\text{AM Wave}} + \underbrace{a_1 m(t) + a_2 A_c^2 \cos^2 2\pi f_c t + a_2 m^2(t)}_{\text{unwanted terms}} \rightarrow ③$$

* The first term of equation ③ is the desired AM wave with $K_a = \frac{2a_2}{a_1}$, amplitude sensitivity of the AM wave.

* The remaining three terms are unwanted and are removed by appropriate filtering.

$$\therefore\ S(t) = a_1 A_c \left[1 + K_a m(t)\right] \cos 2\pi f_c t.$$

❖ **With a neat block diagram, relevant waveforms and expressions explain generation of AM wave using SWITCHING MODULATOR**

Jan-08,10M

❖ **Explain the generation of AM wave using SWITCHING MODULATOR with relevant equations waveforms and spectrum before and after filtering process.** **Jan-07,10M Jan-05,6M July-07,10M July-08,6M July-09,8M Jan-10,10M June-107M July-09,8M**

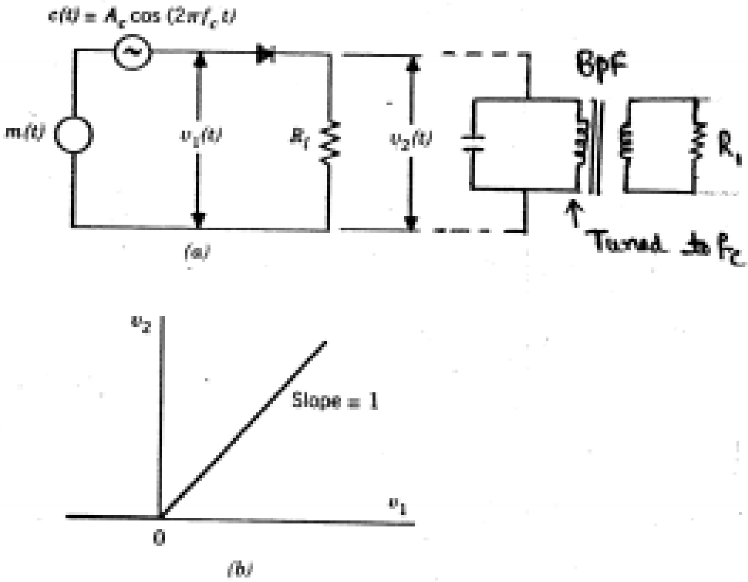


Fig ①

Switching modulator. (a) Circuit diagram. (b) Idealized input–output relation.

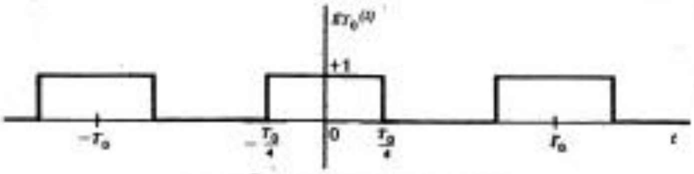


Fig ② Periodic pulse train.

* Consider a Semiconductor diode used as an ideal Switch to which a Carrier wave $c(t)$ and an message Signal $m(t)$ are Simultaneously applied as Shown in Fig ①.

* It is assumed that the Carrier wave $c(t)$ applied to the diode is large in amplitude.

The total I/p '$V_1(t)$' to the diode is given by

$$V_1(t) = m(t) + c(t)$$

$$\boxed{V_1(t) = m(t) + A_c \cos 2\pi f_c t} \rightarrow ①$$

Where $|m(t)| \ll A_c$.

* The o/p of the diode is

$$V_2(t) = \begin{cases} V_1(t), & c(t) > 0 \\ 0, & c(t) < 0 \end{cases}$$

i.e. the o/p of the diode varies between 0 & V_1 at a rate equal to carrier frequency $T_0 = \frac{1}{f_c}$.

* The non-linear behavior of the diode can be replaced by assuming the weak modulating Signal compared with the carrier wave. Thus the o/p of the diode is approximately equivalent to linear-time varying operation.

Mathematically, the o/p of the diode can be written as:

$$V_2(t) = V_1(t) \cdot g_p(t) \rightarrow ②$$

$$V_2(t) = [m(t) + A_c \cos 2\pi f_c t]\, g_p(t) \rightarrow ③$$

Where $g_p(t)$ (fig ⑤) is a rectangular pulse train with a period equal to $T_0 = 1/f_c$.

Representing $g_p(t)$ by its Fourier Series, we have

$$g_p(t) = \frac{1}{2} + \frac{2}{\pi}\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{2n-1} \cos[2\pi f_c (2n-1) t]$$

$$g_p(t) = \frac{1}{2} + \underbrace{\frac{2}{\pi} \cos 2\pi f_c t}_{n=1} + \text{odd harmonic components} \rightarrow ④$$

Substituting equation ④ in equation ③

$$V_2(t) = [m(t) + A_c \cos 2\pi f_c t]\left[\frac{1}{2} + \frac{2}{\pi}\cos 2\pi f_c t + \cdots\right]$$

$$V_2(t) = \frac{1}{2} m(t) + \frac{2m(t)}{\pi}\cos 2\pi f_c t + \frac{A_c}{2}\cos 2\pi f_c t + \frac{2A_c}{\pi}\underline{\cos^2 2\pi f_c t} + \cdots$$

W.K.T $\boxed{\cos^2\theta = \frac{1}{2} + \frac{\cos 2\theta}{2}}$

$$V_2(t) = \frac{m(t)}{2} + \frac{2m(t)}{\pi}\cos 2\pi f_c t + \frac{A_c}{2}\cos 2\pi f_c t + \frac{2A_c}{\pi}\left[\frac{1}{2} + \frac{\cos 2[2\pi f_c t]}{2}\right]$$

$$V_2(t) = \frac{m(t)}{2} + \frac{2m(t)}{\pi}\cos 2\pi f_c t + \frac{A_c}{2}\cos 2\pi f_c t + \frac{\cancel{2}A_c}{\cancel{2}\pi} + \frac{\cancel{2}A_c \cos 4\pi f_c t}{\cancel{2}}$$

$$V_2(t) = \frac{m(t)}{2} + \frac{2m(t)}{\pi}\cos 2\pi f_c t + \frac{A_c}{2}\cos 2\pi f_c t + \frac{A_c}{\pi} + A_c \cos 4\pi f_c t + \cdots \rightarrow ⑤$$

* The required AM wave centred at f_c is obtained by passing '$V_2(t)$' through an ideal 'BPF' having a centre frequency 'f_c' and bandwidth $B_T = 2W$ Hz.

* The o/p of the BPF is

$$V_2'(t) = \frac{2}{\pi} m(t) \cos 2\pi f_c t + \frac{A_c}{2} \cos 2\pi f_c t.$$

$$V_2'(t) = \frac{A_c}{2} \cos 2\pi f_c t \left[1 + \frac{2.2}{\pi A_c} m(t) \right]$$

$$= \frac{A_c}{2} \cos 2\pi f_c t \left[1 + \frac{4}{\pi A_c} m(t) \right]$$

Where $K_a = \frac{4}{\pi A_c}$ amplitude Sensitivity

$$\boxed{V_2'(t) = \frac{A_c}{2} \cos 2\pi f_c t \left[1 + K_a m(t) \right]}$$

Define Demodulation (or Detection)? Mention different types of AM demodulation

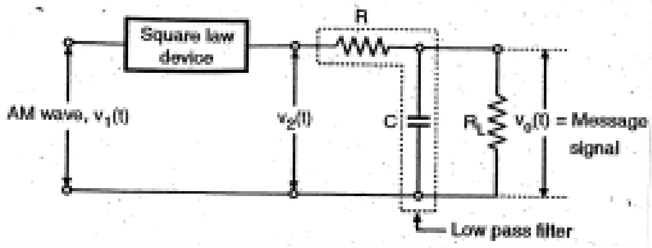
Demodulation or detection is the process of recovering the original message signal from the modulated wave at the receiver. Demodulation is the inverse of the modulation process.

There are two types of detectors:

1) Square-law demodulator
2) Envelope detector.

❖ **Show that a SQUARE LAW device can be used for the detection of an AM wave.** **Jan-07,6M**

❖ **Show that a SQUARE LAW can be used for the detection of an AM wave.** **June-10,6M**



A square law detector

* A Square-law detector is essentially obtained by using a Square-law modulator for the purpose of detection.

* An AM Signal can be demodulated by Squaring it and then passing the Squared Signal through a Low pass filter (LPF)

The transfer Characteristics of a non-linear device is given by:

$$\boxed{V_2(t) = a_1 V_1(t) + a_2 V_1^2(t)} \longrightarrow (1)$$

Where,

$V_1(t) \rightarrow$ I/p voltage

$V_2(t) \rightarrow$ O/p voltage

a_1 and $a_2 \rightarrow$ are Constants.

* The I/p voltage of the AM wave is given by

$$\boxed{V_1(t) = A_c\left[1 + K_a m(t)\right] \cos 2\pi f_c t} \longrightarrow (2)$$

Substituting equation ② in equation ①, we get

$$V_2(t) = a_1\left\{A_c[1+K_a m(t)]\cos 2\pi f_c t\right\} + a_2\left\{A_c[1+K_a m(t)]\cos 2\pi f_c t\right\}^2$$

$$V_2(t) = a_1 A_c[1+K_a m(t)]\cos 2\pi f_c t + a_2\left\{A_c^2[1+K_a m(t)]^2\cos^2 2\pi f_c t\right\}$$

W.K.T

$$\boxed{(a+b)^2 = a^2+b^2+2ab} \quad \text{and} \quad \boxed{\cos^2\theta = \frac{1+\cos 2\theta}{2}}$$

$$V_2(t) = a_1 A_c[1+K_a m(t)]\cos 2\pi f_c t + a_2 A_c^2 \cos^2 2\pi f_c t\left[1^2+K_a^2 m^2(t)+2K_a m(t)\right]$$

$$V_2(t) = a_1 A_c[1+K_a m(t)]\cos 2\pi f_c t + a_2 A_c^2\left[\frac{1+\cos 2(2\pi f_c t)}{2}\right]\left[1+K_a^2 m^2(t) + 2K_a m(t)\right]$$

$$V_2(t) = a_1 A_c[1+K_a m(t)]\cos 2\pi f_c t + \frac{a_2 A_c^2}{2}\left[1+K_a^2 m^2(t)+2K_a m(t)\right](1+\cos 4\pi f_c t) \rightarrow ③$$

* In eq ③ $\frac{a_2 A_c^2}{\cancel{2}}\cancel{2}K_a m(t)$ is the desired term which is due to the $a_2 V_1^2$ term. Hence the name of this detector is - Square law detector.

* The desired term is extracted by using a LPF (Fig.). Thus the o/p of LPF is $\boxed{V_o(t) = a_2 A_c^2 K_a m(t)}$

Thus the message signal $m(t)$ is recovered at the o/p of the message signal.

Distortion in the detector o/p :-

* The other term which passes through the LPF to the load resistance R_L is as follows : $\frac{1}{2} a_2 A_c^2 K_a^2 m^2(t)$.

* This is an unwanted Signal & gives rise to a Signal distortion.

The ratio of desired Signal to the undesired one is given by:

$$D = \frac{\cancel{a_2} \cancel{A_c^2} \cancel{k_a} \cancel{m(t)}}{\frac{1}{2} \cancel{a_2} \cancel{A_c^2} k_a^{\cancel{2}} m^{\cancel{2}}(t)} = \frac{1}{\frac{1}{2} k_a m(t)} = \frac{2}{k_a m(t)}$$

{

We Should maximize this ratio in order to minimize the distortion. To achieve this we Should Choose $|k_a m(t)|$ Small as Compared to unity for all values of t. If K_a is Small then the AM wave is weak.

}

Envelope Detector:

❖ **How a modulating signal can be detected using a AM detector? Use a envelope detector and explain.** **July-05,8M**

❖ **Explain the detection of message signal from amplitude modulated signal using an envelope detector & bring out the significance of RC time constant**

July-09,6M July-07,5M June-09,6M July-06,5M

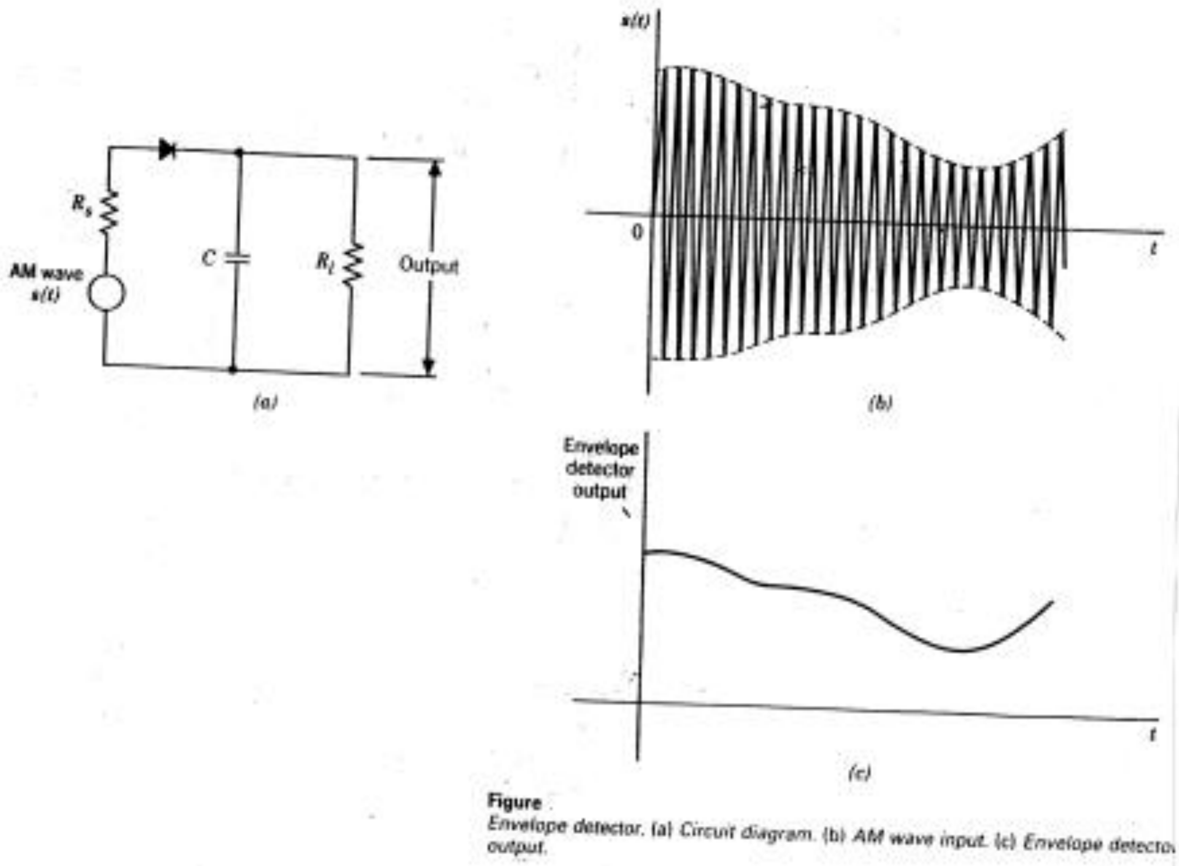


Figure
Envelope detector. (a) Circuit diagram. (b) AM wave input. (c) Envelope detector output.

* Envelope detector is a Simple and highly effective device used to demodulate AM Wave. It Consists of a diode and a resistor capacitor (RC) filter.

Operation :-

During positive half cycle of the I/p Signal, diode is forward-biased and the capacitor 'C' charges upto the peak-value of the I/p Signal. When the I/p voltage falls below this value the diode becomes reverse biased and capacitor 'C' discharges slowly through the load resistor R_L. As a result only positive half cycle of AM wave appears across R_L.

The discharging process continues until the next positive half cycle. When the I/p Signal becomes greater than the voltage across the capacitor, the diode conducts again and the process is repeated.

Selection of the RC time Constant :-

* The Capacitor charges through 'D' & R_S when the diode is 'ON' & it discharges through 'R_L' when diode is OFF.
* The charging time constant R_SC should be short as compared to the carrier period $1/f_c$ $\therefore$ $\boxed{R_SC \ll \frac{1}{f_c}}$ So capacitor 'C' charges rapidly.
* On the other hand the discharging time constant R_LC should be long enough to ensure that the capacitor discharges slowly through the load resistance 'R_L' b/w +ve peak of the carrier wave i.e. $\boxed{\frac{1}{f_c} \ll R_LC \ll \frac{1}{W}}$, Where W = Maximum modulating frequency.

Result is that the capacitor voltage or detector o/p is very nearly the same as the envelope of AM wave. The detector o/p usually has a small ripple at the carrier frequency. This ripple is easily removed by Low pass filter.

Advantages of AM:-

1) AM Tx's are less complex
2) AM receivers are simple, detection is easy.
3) AM receivers are cost efficient.
4) AM waves can travel a longer bandwidth.
5) Low bandwidth.

* Mention the disadvantages of AM wave.
OR
* Mention the limitation of DSB-FC wave (AM)

The disadvantages of AM waves are:

1) power is wasted in the transmitted signal.
2) AM needs larger bandwidth
3) AM waves gets affected due to noise.

* Applications of AM:-

1) Radio broadcasting
2) Picture transmission in a TV system.

Explain the disadvantages & Limitation of AM Wave (DSB-FC)

Amplitude modulation has Several disadvantages:

1) Power is wasted in the transmitted Signal

* Most of the transmitted power is in the carrier, which does not carry any information.
* For 100% modulation i.e. $\mu=1$, only 33.33% ($1/3^{rd}$) of the total power will be in Sidebands which carries the information and 66.67% ($2/3^{rd}$) of the total power will be in the carrier, which does not contain any information.

2) The DSB-FC System is Bandwidth inefficient System. The transmitted Signal requires twice the bandwidth of the message Signal i.e. $B_T = 2f_m$. This is due to the transmission of both the Sidebands, out of which only one Sideband is Sufficient to convey all the information. Thus the bandwidth of DSB-FC is double than actually required.

3) AM wave gets affected due to noise:-

When the AM wave travels from the transmitter to receiver over a Communication Channel, noise gets added to it. The noise will change the amplitude of the envelope of AM in a random manner. As the information is contained in the amplitude variations of the AM wave, the noise will contaminate the information contents in the AM. Hence the performance of AM is very poor in presence of noise.

Power Wattage in AM (DSB-FC):

* power wattage due to DSB-FC (AM) transmission.

W.K.T, the total power transmitted by an AM wave is given by

$$P_T = P_c + P_{USB} + P_{LSB} \rightarrow (1)$$

$$P_T = P_c + \frac{\mu^2}{4}P_c + \frac{\mu^2}{4}P_c \rightarrow (2)$$

In equation (2), carrier component does not contain any information & one sideband is redundant. So out of the total power, $P_T = P_c\left[1+\frac{\mu^2}{2}\right]$, the wasted power is given by:

$$\text{Power wastage} = P_c + \frac{\mu^2}{4}P_c$$

Other types of Amplitude Modulation :-

Amplitude Modulation (AM)

- DSB-FC (Double Sideband Full Carrier System)
- DSB-SC (Double Sideband Suppressed Carrier System)
- SSB-SC (Single Sideband Suppressed Carrier System)
- ISB (Independent Sideband System)
- VSB (Vestigial Sideband System)

❖ What is DSB-SC modulation? Explain the time and frequency domain expression of DSB-SC wave.

To overcome the drawback of power wattage in AM wave (DSB-FC) an DSB-SC method is used.

* DSB-SC is a method of transmission where only the TWO Sidebands are transmitted without the Carrier (Suppressing Carrier)

OR

The Conventional AM wave in which the carrier is Suppressed is called DSB-SC modulation.

Time domain representation of DSB-SC Wave:-

* Let $m(t)$ be the message Signal having a bandwidth equal to 'W' Hz and

$c(t) = A_c \cos 2\pi f_c t$ represents the carrier, then the time-domain expression for DSB-SC wave is

$$S(t) = m(t)\, c(t)$$

$$\boxed{S(t) = A_c \cos(2\pi f_c t)\, m(t)} \rightarrow (1)$$

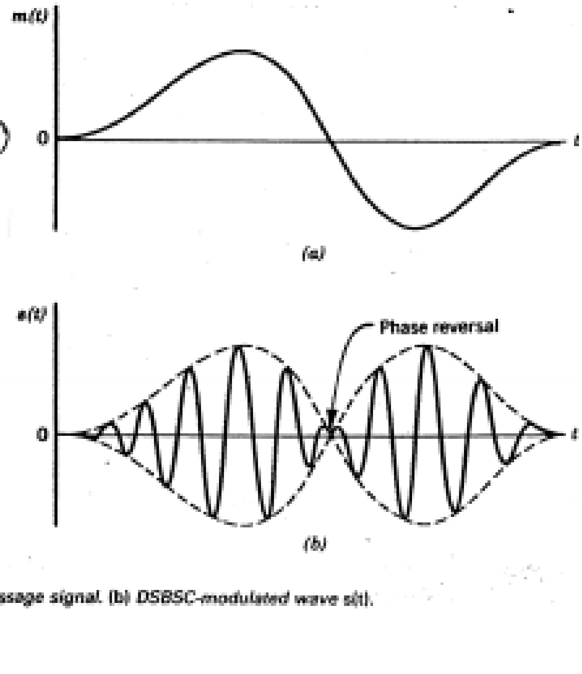


Figure
(a) Message signal. (b) DSBSC-modulated wave s(t).

* The $S(t)$ Signal undergoes a phase reversal whenever the message Signal crosses Zero.

Frequency - Domain Description :-

Taking Fourier transform on both Sides of equation ②, we get

$$\boxed{S(f) = \frac{A_c}{2}\left[M(f-f_c)+M(f+f_c)\right]} \longrightarrow ③$$

Where $S(f)$ is the Fourier transform of the modulated wave $S(t)$

$M(f)$ is the Fourier transform of the message Signal $m(t)$.

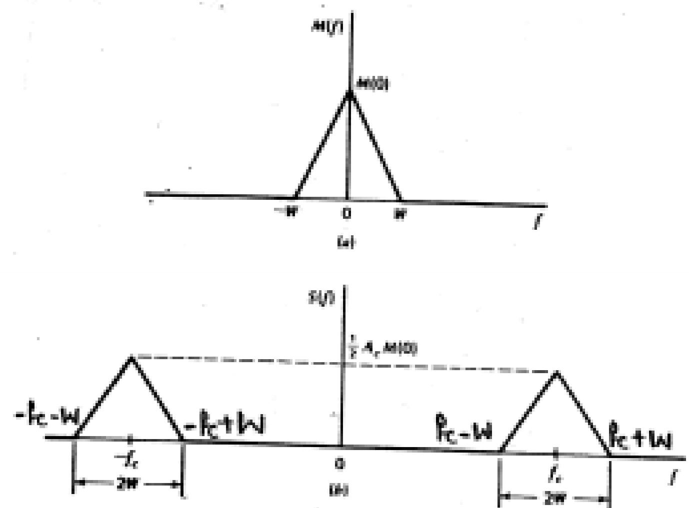


Figure
(a) Spectrum of message signal. (b) Spectrum of DSBSC modulated wave.

The amplitude Spectrum drawn above exhibits the following facts

i) on either Sides of $\pm f_c$, we have two Sidebands designated as Lower and upper Sidebands.

ii) The Impulse are absent at $\pm f_c$ in the amplitude Spectrum, Signifying the fact that the carrier term is Suppressed in the transmitted Wave.

iii) The minimum transmission bandwidth required is 2W i.e. twice the message bandwidth.

NOTE :- A DSB-SC Signal can be generated by a multiplier. A carrier Signal can be Suppressed by adding a carrier Signal opposite in phase but equal in magnitude to the amplitude modulated wave, So the carrier get cancelled. Finally double Sidebands are available in the DSB-SC Wave.

❖ **Explain DSB-SC modulation for single tone information.**

* Let $m(t) = A_m \cos 2\pi f_m t$ be the Single tone modulating Signal and

$$\boxed{c(t) = A_c \cos 2\pi f_c t}$$

be the carrier Signal.

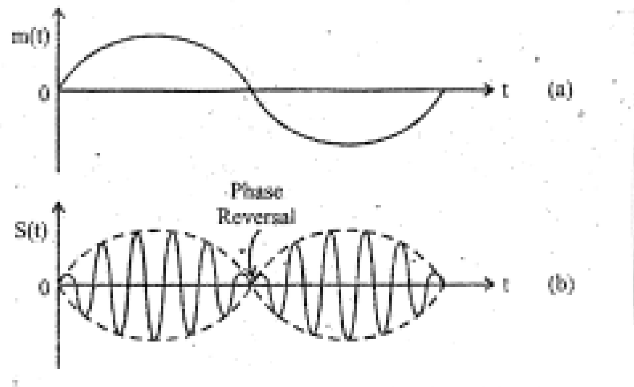


Fig. : (a) Modulating signal m(t); (b) DSBSC modulated wave s(t)

Then the time domain expression for the DSB-SC Wave is

$$S(t) = m(t) \cdot c(t)$$

$$S(t) = A_m \cos 2\pi f_m t \cdot A_c \cos 2\pi f_c t$$

W.K.T.

$$\boxed{\cos A \cdot \cos B = \frac{1}{2}\cos(A-B) + \frac{1}{2}\cos(A+B)}$$

$$\boxed{S(t) = \frac{A_m A_c}{2}\cos 2\pi(f_c - f_m)t + \frac{A_m A_c}{2}\cos 2\pi(f_c + f_m)t} \rightarrow ①$$

Taking Fourier transform on both sides of the equation ①

$$S(f) = \frac{A_m A_c}{4}\left\{\delta[f-(f_c-f_m)] + \delta[f+(f_c-f_m)]\right\}$$
$$+ \frac{A_m A_c}{4}\left\{\delta[f-(f_c+f_m)] + \delta[f+(f_c+f_m)]\right\}.$$

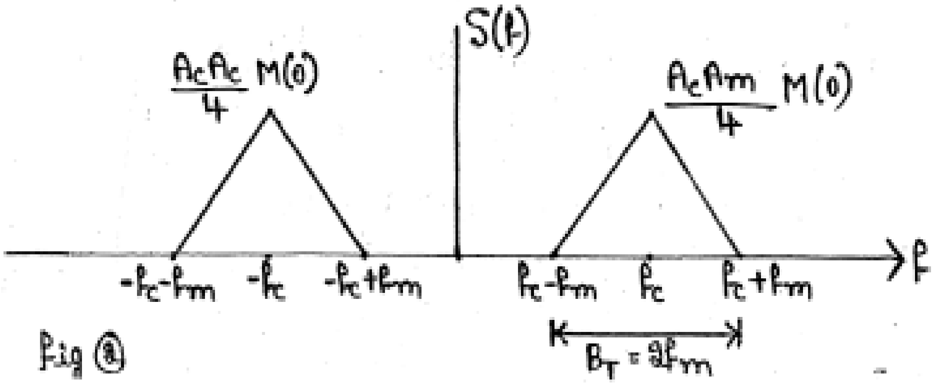
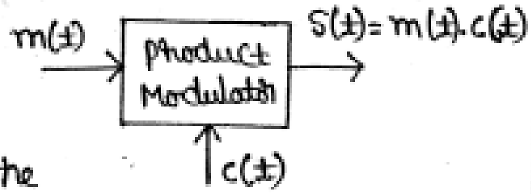


fig ②

fig ② shows amplitude spectrum of a DSB-SC signal. We observe that either side of $\pm f_c$, we have lower and upper sideband also the carrier term is suppressed in the spectrum as there are no impulses at $\pm f_c$.

* The minimum transmission bandwidth in DSB-SC is '$2f_m$'.

Generation of DSB-SC Wave:



* A DSB-SC Wave Simply Consists of the product of the modulating Signal and the carrier Signal.
* The devices used to generate DSB-SC Waves are Known as the product modulators.

There are two types of modulators:

1) Balanced Modulator
2) Ring Modulator.

Balanced Modulator:

❖ **With a neat block diagram, explain the balanced modulator method of generating DSB-SC wave.** **June-10,6M**

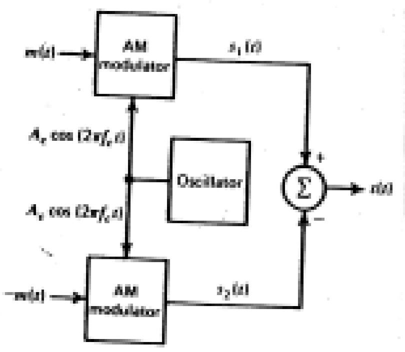


Figure
Balanced modulator.

Fig ① Shows the block diagram of a balanced modulator used for generating a DSB-SC Signal.

* It Consist of two amplitude modulators that are interconnected in Such a way as to Suppress the carrier.

* one I/p to the amplitude modulator is from an oscillator that generates a carrier wave. The Second I/p to the amplitude modulator in the top path is the modulating Signal $m(t)$ while in the bottom path is $-m(t)$.

. The o/p of the two AM modulator are as follows:

$$S_1(t) = A_c[1+K_a m(t)]\cos 2\pi f_c t \quad \text{and}$$

$$S_2(t) = A_c[1-K_a m(t)]\cos 2\pi f_c t.$$

The o/p of the Summer is

$$S(t) = S_1(t) - S_2(t)$$

$$S(t) = A_c[1+K_a m(t)]\cos 2\pi f_c t - \left[A_c\left(1-K_a m(t)\right)\cos 2\pi f_c t\right]$$

$$= A_c\cos 2\pi f_c t + A_c K_a m(t)\cos 2\pi f_c t - \left[A_c\cos 2\pi f_c t - A_c K_a m(t)\cos 2\pi f_c t\right]$$

$$= \cancel{A_c\cos 2\pi f_c t} + A_c K_a m(t)\cos 2\pi f_c t - \cancel{A_c\cos 2\pi f_c t} + A_c K_a m(t)\cos 2\pi f_c t.$$

$$\boxed{S(t) = 2A_c K_a m(t)\cos 2\pi f_c t} \rightarrow ①$$

* The balanced modulator o/p is equal to the product of the modulating Signal $m(t)$ & carrier $c(t)$ except the Scaling factor $2K_a$.

Taking Fourier Transform on both Side of equation ①, we get

$$S(f) = \frac{\cancel{2}A_c K_a}{\cancel{2}}\left[M(f-f_c) + M(f+f_c)\right]$$

$$\boxed{S(f) = A_c K_a\left[M(f-f_c) + M(f+f_c)\right]}$$

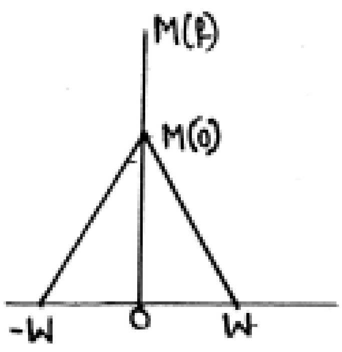


Fig (a): Message Spectrum

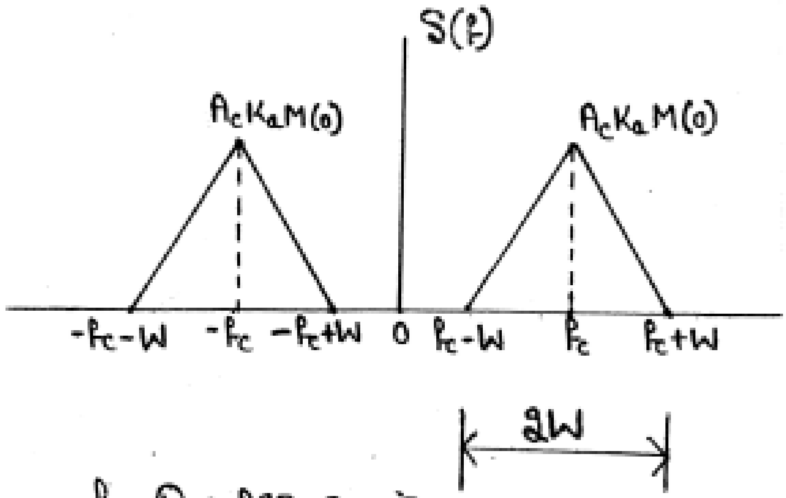


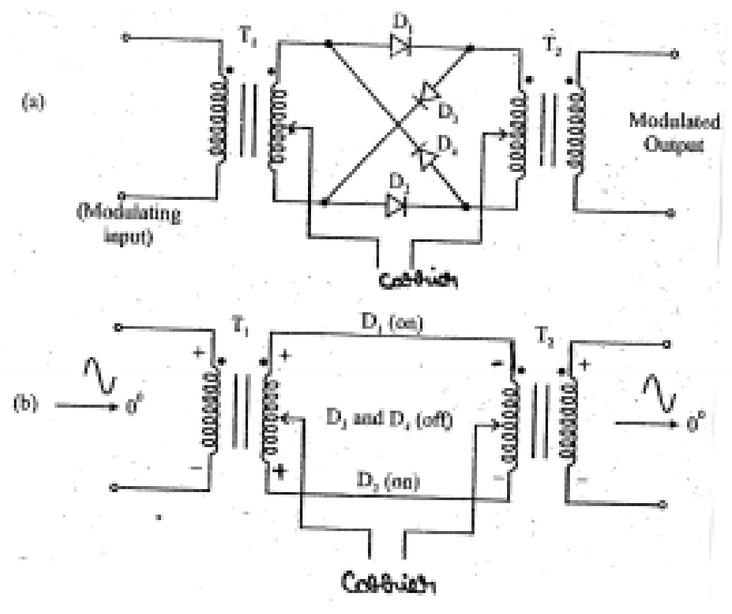
Fig (b): DSB-SC Spectrum

Since the carrier component is eliminated, the o/p is called DSB-SC Signal.

- **Explain how RING modulator can be used to generate DSB-SC modulation** **Jan-05,9M**

- **Briefly explain generation of DSB-SC modulated wave using RING modulator. Give relevant mathematical expressions and waveforms.**

Jan-08,10M Jan-07,8M Jan-09,6M July-09,10M June-10,10m



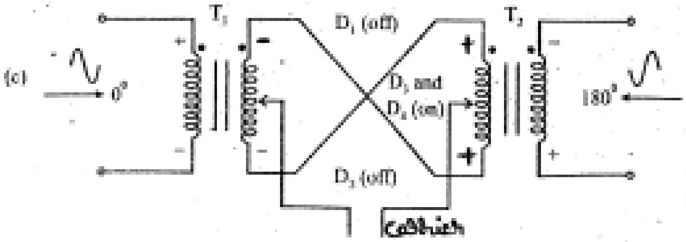


Fig. : (a) Balanced Ring Modulator
(b) Equivalent Circuit when square wave carrier positive
(c) Equivalence circuit when square wave carrier negative

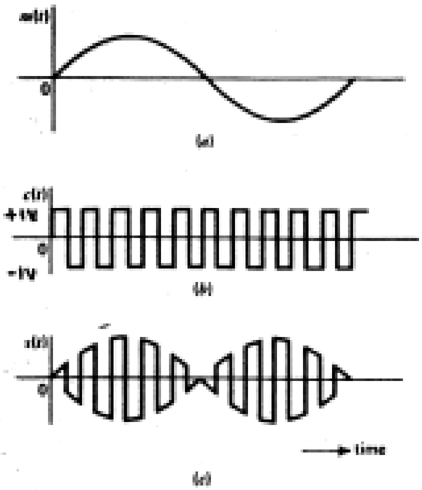


Figure
Waveforms illustrating the operation of the ring modulator for a sinusoidal modulating wave. (a) Modulating wave. (b) Square-wave carrier. (c) Modulated wave.

Ring modulator is a product modulator used for generating DSB-SC modulated wave. The ring modulator consists of :-

1) I/p transformer 'T_1'
2) O/p transformer 'T_2'
3) Four diodes connected in a bridge circuit (Ring)

The carrier amplitude 'A_c' is greater than the modulating signal amplitude 'A_m' i.e. $A_c > A_m$ and carrier frequency 'f_c' is greater than modulating signal 'f_m' $= W$ i.e. $f_c > W$. These conditions ensures that the diode operation is controlled by $c(t)$ only.

* The diodes are controlled by a square wave carrier $c(t)$

of frequency 'f_c' which is applied by means of two center-tapped transformer.

* The modulating signal $m(t)$ is applied to the I/p transformer 'T_1'. The o/p appears across the secondary of the transformer 'T_2'.

Operation :-

i) When the carrier is +ve, the diodes D_1 & D_2 are forward-biased and diodes D_3 & D_4 are reverse biased. Hence the modulator multiplies the message signal $m(t)$ by +1 i.e.

$V_0(t) = m(t)$.

ii) When the carrier is -ve, the diodes D_3 & D_4 are forward biased whereas D_1 & D_2 are reverse biased. Thus the modulator multiplies the message signal $m(t)$ by -1 i.e. $V_0(t) = -m(t)$.

* The square wave carrier $c(t)$ can be represented by a Fourier series as:

$$c(t) = \frac{4}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{2n-1}\cos\left[2\pi f_c t(2n-1)\right]$$

$$c(t) = \frac{4}{\pi}\Big[\underbrace{\cos 2\pi f_c t}_{n=1} - \underbrace{\frac{1}{3}\cos 6\pi f_c t}_{n=2} + \cdots\Big] \longrightarrow (1)$$

The ring modulator o/p is

$$S(t) = c(t)\cdot m(t) \longrightarrow (2)$$

Substituting equation (1) in equation (2), we get

$$S(t) = \left[\frac{4}{\pi}\cos 2\pi f_c t - \frac{4}{3\pi}\cos 6\pi f_c t + \cdots\right] m(t)$$

$$S(t) = \frac{4}{\pi}m(t)\cos 2\pi f_c t - \frac{4}{3\pi}m(t)\cos 6\pi f_c t + \cdots \Big] \longrightarrow (3)$$

Taking Fourier transform on both sides of equation ③, we get

$$S(f) = \frac{\cancel{4}^{2}}{\cancel{2}\pi}\left[M(f-f_c)+M(f+f_c)\right] - \frac{\cancel{4}^{2}}{\cancel{2}\times 3\pi}\left[M(f-3f_c)+M(f+3f_c)\right]$$

$$\boxed{S(f) = \frac{2}{\pi}\left[M(f-f_c)+M(f+f_c)\right] - \frac{2}{3\pi}\left[M(f-3f_c)+M(f+3f_c)\right]}$$

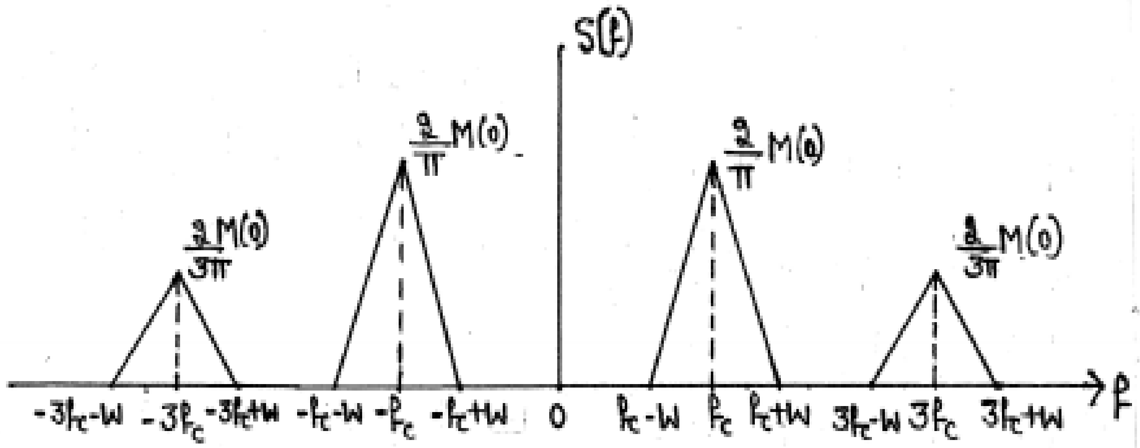


Fig : Amplitude Spectrum of S(f).

* The DSB-SC wave is extracted from s(t) by passing equation ③ (s(t)) through an Ideal BPF having centre frequency 'f_c' and bandwidth equal to 2W Hz.

The o/p of the BPF is

$$\boxed{s(t) = \frac{4}{\pi} m(t) \cos 2\pi f_c t}$$

COHERENT Detection of DSB-SC wave:-

❖ **With block diagram and related equations explain coherent detection of a DSB-SC wave. What are its disadvantages? Explain the synchronous receiving system(COSTAS Loop)** **June-10,8M July-08,10M**

❖ **Write a note on how coherent detection is used in DSB-SC receiver** **July-06,7M**

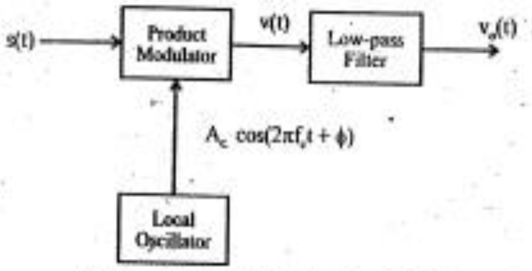


Fig. : Coherent detective for DSBSC

* The modulating Signal $m(t)$ is recovered from a DSB-SC wave $S(t)$ by first multiplying $S(t)$ with a locally generated carrier wave and then low pass filtering the product as shown in fig ①.

* For faithful recovery of modulating Signal $m(t)$, the local oscillator o/p Should be exactly Coherent or Synchronized in both frequency and phase with the carrier wave $C(t)$ used in the product modulator to generate $V_o(t)$ with the local oscillator o/p equal to $\cos(2\pi f_c t+\phi)$.

The product modulator o/p can be given as:

$$V(t) = S(t)\cdot \cos(2\pi f_c t+\phi) \longrightarrow ①$$

W.K.T $\quad S(t) = A_c \cos 2\pi f_c t \cdot m(t) \longrightarrow ②$

Substituting equation ② in equation ①, we get

$$V(t) = A_c \cos(2\pi f_c t+\phi)\cos(2\pi f_c t)\cdot m(t)$$

W.K.T
$$\boxed{\cos A \cdot \cos B = \frac{1}{2}\cos(A-B) + \frac{1}{2}\cos(A+B)}$$

$$V(t) = \frac{A_c m(t)}{2}\left[\cos(\cancel{2\pi f_c t} + \phi - \cancel{2\pi f_c t})\right] + \frac{A_c m(t)}{2}\left[\cos(2\pi f_c t + \phi + 2\pi f_c t)\right]$$

$$V(t) = \frac{A_c m(t)}{2}\cos\phi + \frac{A_c m(t)}{2}\cos(4\pi f_c t + \phi) \longrightarrow (2)$$

{ Taking Fourier transform on both Sides of equation (2), we get

$$V(f) = \frac{A_c}{2}M(f)\cos\phi + \frac{A_c}{4}\left[M(f-2f_c) + M(f+2f_c)\right]$$

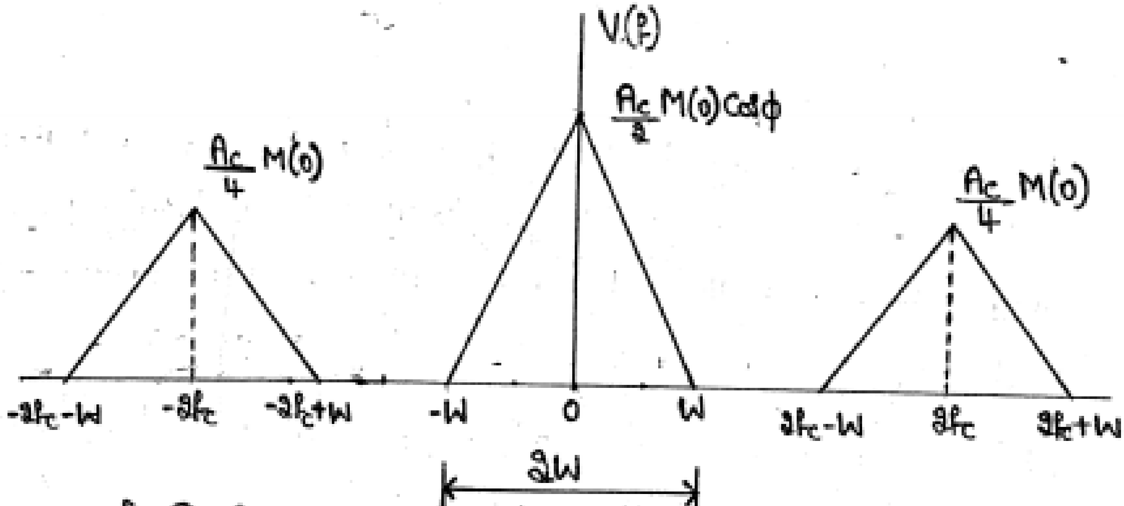


Fig (2) Amplitude Spectrum of V(f).

}

* The desired message Signal is obtained by passing V(t) through a LPF having the bandwidth greater than 'W' Hz but less than '$2f_c - W$' Hz.

* The o/p of the LPF is

$$\boxed{V_0(t) = \frac{A_c}{2}\cos\phi\, m(t)}$$

The demodulated Signal $V_0(t)$ is therefore proportional to m(t).

Where, $\phi \longrightarrow$ phase error.

When ϕ = Constant, $V_0(t)$ is proportional to $m(t)$

When $\phi = 0$, Amplitude of $V_0(t)$ is maximum.

When $\phi = \pm\pi/2$, Amplitude of $V_0(t)$ is minimum (represents the quadrature Null effect of the Coherent detector)

COSTAS LOOP:-

- **Explain the method of obtaining a practical synchronous receiving system with DSB-SC modulated waves using COSTAS loop** **June-10,8M July-08,10M**
- **With a neat block diagram explain the synchronous receiving system for receiving DSB-SC modulated waves.** **July-07,6M**
- **With neat block diagram of DSB-SC, the detection using COSTAS receiver.** **Jan-09,6M July-09, 5M**

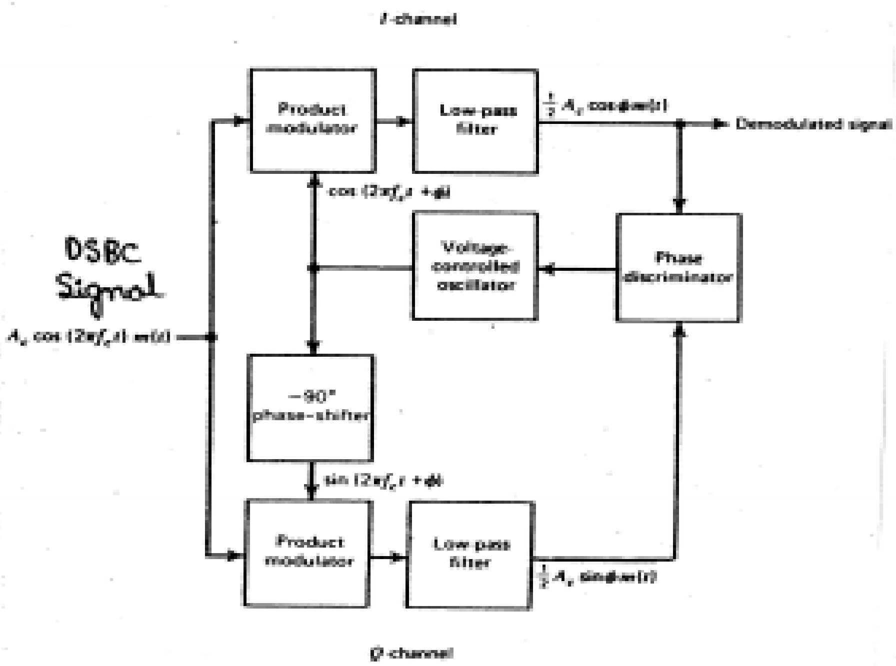


Fig : Costas loop

* The Costas loop is a method of obtaining a practical synchronous receiver system, suitable for demodulating DSB-SC waves.

* The receiver consists of two coherent detectors supplied with the same I/p signal (DSB-SC wave) $A_c \cos(2\pi f_c t) m(t)$, but with individual local oscillator signals that are in-phase quadrature with respect to each other. (i.e. the local oscillator signal supplied to the product modulators are 90° out of phase).

* The frequency of the local oscillator is adjusted to be the same as the carrier frequency 'f_c'.

* The detector in the upper path is referred to as the In-phase coherent detector or I-Channel and that in the lower path is referred to as the Quadrature-phase coherent detector or Q-Channel.

* These two detectors are coupled together to form a negative feedback system designed in such a way as to maintain the local oscillator synchronous with the carrier wave.

operation:-

1) When local oscillator signal is of the same phase as the carrier wave $A_c \cos(2\pi f_c t)$ used to generate the incoming DSB-SC wave under these conditions, the I-Channel o/p contains the desired demodulated signal $m(t)$, where as Q-Channel o/p is zero.

$$V_{OI} = \frac{1}{2} A_c m(t) \cos\phi$$

i.e. whenever the carrier is synchronized

$\phi = 0$ and $\cos\phi = \cos(0) = 1$

$$\boxed{V_{OI} = \frac{1}{2} A_c m(t)}$$ and

$$\sin\phi = \sin(0) = 0$$

$$\boxed{V_{OQ} = 0}$$

ii) When local oscillator phase changes by a small angle 'ϕ' radians, the I-channel o/p will remain unchanged, but Q-channel produces some o/p which is proportion to Sinϕ.

The o/p of I and Q-channels are combined in phase-discriminator (which consists of a multiplier followed by a LPF), a dc control signal is obtained that automatically corrects for local phase errors in the voltage controlled-oscillator (VCO).

Disadvantages of DSB-SC Coherent detection :-

Amplitude of the demodulated signal is maximum when $\phi = 0$ & minimum when $\phi = \pm\pi/2$ So, perfect synchronization has to be achieved for detection which inturn increases the cost of the receiver.

Quadrature Carrier Multiplexing or Quadrature Amplitude Modulation:-

- **Explain the principle of QAM and with a functional block diagram describe the salient features of QAM transmitter and receiver.** **Jan-06,8M**

- **What is Quadrature null effect? How it can be eliminated?** **Jan-07,8M**

- **With neat block diagram, explain the operation of Quadrature carrier multiplexing.** **July-09,5M Jan-10,6M July-09,8M July-07,5M**

* QAM is a technique in which we can transmit more number of signals (DSB-SC Wave) within the same channel bandwidth.

∴ QAM is a bandwidth-conservation scheme.

Principle of QAM Scheme:-

The QAM enables two DSB-SC modulated waves to occupy the same transmission channel bandwidth and allows the separation of the two message signals at the receiver o/p.

∴ QAM is called Bandwidth-Conservation Scheme.

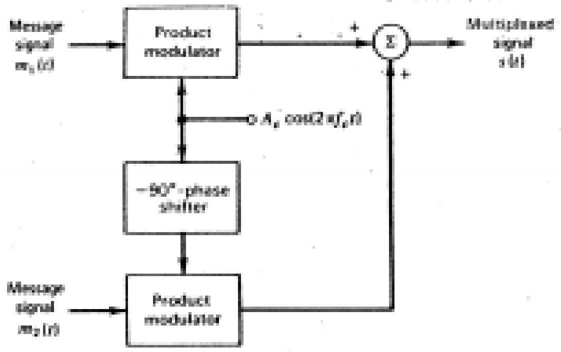


Fig a: QAM Transmitter

Fig ⓐ Shows QAM transmitter. It consist of two product modulators that are supplied with two carrier waves of the same frequency but differing in phase by $-90°$.

* The o/p of the two product modulators are summed to produce multiplexed signal $S(t)$.

i.e.

$$\boxed{S(t) = A_c m_1(t)\cos 2\pi f_c t + A_c m_2(t)\sin 2\pi f_c t}$$

Where $m_1(t)$ and $m_2(t)$ denotes the two different message signals applied to the product modulators.

* Thus $S(t)$ occupies a channel bandwidth of '$2W$' centered at the carrier frequency 'f_c', where 'W' is the message bandwidth of $m_1(t)$ & $m_2(t)$.

QAM Receiver :

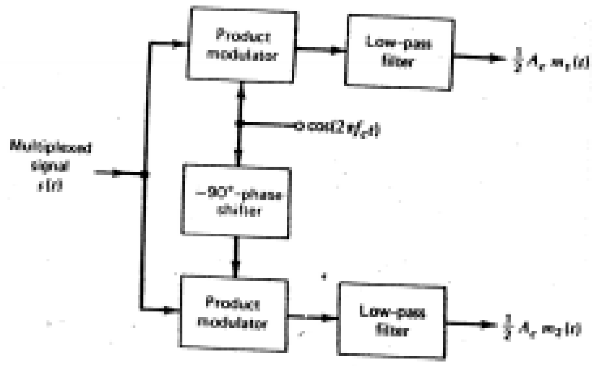


Fig ⓑ QAM receiver

* Fig ⓑ Shows the QAM receiver, which consists of two coherent detectors which are fed by locally generated carrier signals having same frequency but out of phase by $90°$.

The received multiplexed Signal S(t) is applied to the two product modulator. The o/p of the Top product modulator is given by $\boxed{S_1(t) = S(t)\cos 2\pi f_c t}$

The top LPF removes the high frequency term and allows only $\frac{A_c m_1(t)}{2}$.

$$\therefore \quad \boxed{S_1(t) = \frac{A_c}{2} m_1(t)}$$

* The o/p of the bottom product modulator is given by

$$S_2(t) = S(t) \cdot \sin 2\pi f_c t.$$

* The bottom LPF removes the high frequency term & allows only $\frac{A_c m_2(t)}{2}$. Thus the o/p of LPF is

$$\boxed{S_2(t) = \frac{A_c}{2} m_2(t)}$$

* For correct operation of the Quadrature carrier multiplexing-System it is necessary to maintain the correct phase and frequency relationship between the local oscillator used in transmitter and receiver of the System.

Salient features of QAM:-

1) We can transmit more number of DSB-SC Wave within the Same Channel bandwidth.

2) QAM is a bandwidth-Conservation Scheme.

3) QAM finds application in Colour Television (CTV)

NOTE : (Dont write in the exam)

* The o/p of the top product modulator is given by :

$$\boxed{S_1(t) = S(t)\cos 2\pi f_c t}$$

$$S_1(t) = [A_c m_1(t)\cos 2\pi f_c t + A_c m_2(t)\sin 2\pi f_c t]\cos 2\pi f_c t.$$

$$S_1(t) = A_c m_1(t)\underline{\cos} 2\pi f_c t.\underline{\cos} 2\pi f_c t + A_c m_2(t)\sin 2\pi f_c t.\cos 2\pi f_c t.$$

$$S_1(t) = A_c m_1(t)\cos^2 2\pi f_c t + A_c m_2(t)\sin 2\pi f_c t.\cos 2\pi f_c t.$$

W.K.T

$$\boxed{\cos^2\theta = \frac{1}{2} + \frac{\cos 2\theta}{2}} \quad \text{and} \quad \boxed{\sin A\cos B = \frac{1}{2}\sin(A-B) + \sin(A+B)}$$

$$S_1(t) = A_c m_1(t)\left[\frac{1}{2} + \frac{\cos 2(2\pi f_c t)}{2}\right] + \frac{A_c m_2(t)}{2}\sin[\cancel{2\pi f_c t - 2\pi f_c t}]^{0} + \frac{A_c m_2(t)}{2}\sin[2\pi f_c t + 2\pi f_c t]$$

$$\boxed{S_1(t) = \frac{A_c m_1(t)}{2} + \frac{A_c m_1(t)}{2}\cos 4\pi f_c t + \frac{A_c m_2(t)}{2}\sin[4\pi f_c t]}$$

* The top LPF removes the high frequency terms & allows only $\frac{A_c m_1(t)}{2}$

$$\therefore \quad \boxed{S_1(t) = \frac{A_c}{2} m_1(t)}$$

* The o/p of the bottom product modulator is given by :

$$S_2(t) = S(t)\sin 2\pi f_c t$$

$$S_2(t) = [A_c m_1(t)\cos 2\pi f_c t + A_c m_2(t)\sin 2\pi f_c t]\sin 2\pi f_c t$$

$$S_2(t) = A_c m_1(t)\sin 2\pi f_c t.\cos 2\pi f_c t + A_c m_2(t).\sin^2 2\pi f_c t.$$

W.K.T

$$\boxed{\sin^2\theta = \left[\frac{1}{2} - \frac{\cos 2\theta}{2}\right]} \quad \& \quad \boxed{\sin A.\cos B = \frac{1}{2}\sin(A-B) + \frac{1}{2}\sin(A+B)}$$

$$S_2(t) = \frac{A_c m_1(t)}{2}\sin[2\pi f_c t - 2\pi f_c t] + \frac{A_c m_1(t)}{2}\sin[2\pi f_c t + 2\pi f_c t] + \frac{A_c m_2(t)}{2} - \frac{A_c m_2(t)}{2}\cos 4\pi f_c t.$$

* The bottom LPF removes the high frequency term & allows only $\frac{A_c}{2} m_2(t)$.

Thus the o/p of LPF is

$$\boxed{S_2(t) = \frac{A_c}{2} m_2(t)}$$

Distortion in Envelope detector:-

❖ Discuss the drawbacks of envelope detector **Jan-10,4m**

There are two types of distortions which can occur in the detector output. They are:

1. **Diagonal Clipping and**
2. **Negative peak clipping.**

Diagonal Clipping:

This type of distortion occurs when the **RC time constant** of the load current is **too long**. Due to this the RC circuit cannot follow the fast change in the modulating envelope and is as shown in fig1.

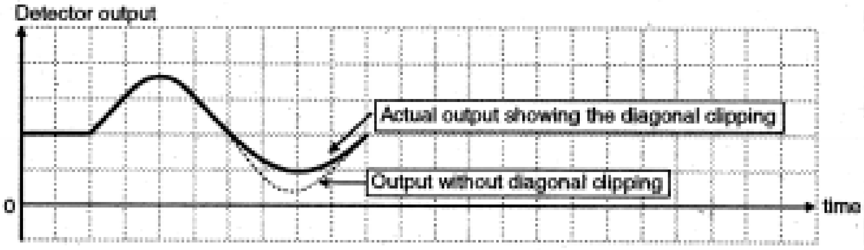


Fig. 1 : Diagonal clipping

Negative Peak Clipping:

This distortion occurs due to a fact that the modulation index on the output side of the detector is higher than that on its input side. So at higher depths of modulation of the transmitted signal, the **over modulation** may take place at the output of the detector. As a result negative peak clipping take place as shown in fig2.

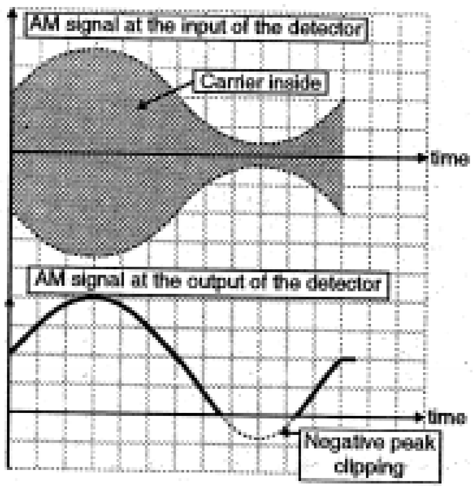


Fig. 2 : Negative peak clipping

Remedy: The distortions in detector output is reduced or eliminated by **properly choosing RC time constant.**

❖ **What is DSB-SC modulation? What are the advantages and limitations of DSB-SC as compared to standard AM?** June-09,6M Jan-05,5M

Advantages:

1. Low Power consumption or power saving.
2. The modulation system is simple.
3. Efficiency is more than AM
4. Carrier wave is suppressed
5. Linear modulation type is required
6. It can be used for point to point communication

Disadvantages:

1. Design of receiver is complex
2. Bandwidth required is same as that of AM

Application:

1. Analogue TV systems to transmit color information.

FORMULAE

1. Equation for AM wave : $s(t)=A_c(1+\mu\cos2\pi f_m t)\cos2\pi f_c t$
2. Modulation Index : $\mu = Am/Ac$
3. Amplitude of each sideband : $\mu Ac/2$
4. Upper Sideband Freq. : $f_{USB} = (f_c+f_m)$
5. Lower Sideband Freq. : $f_{LSB} = (f_c-f_m)$
6. Bandwidth of AM : $BW= 2f_m$
7. Total Txed Power : $P_t=P_c+P_{USB}+P_{LSB}$

   $P_t= Pc\ (1+ \mu^2/2)$

   $Pt=I_t^2R$
8. Power in each sideband : $P_{USB} = P_{LSB} = Pc\ (\mu/4)$
9. Total Sideband power P_{SB} : $P_{USB} + P_{LSB} = Pc\ (\mu^2/2)$

   $P_{SB} = P_T - P_C$
10. Power Wastage : $Pc + Pc\ (\mu^2/4)$
11. Transmission efficiency : $\eta = \mu^2/(2+ \mu^2)$
12. Carrier Power : $Pc=A_c^2/2R$

    $Pc=I_c^2/R$
13. Maximum Freq. in AM wave : $f_{max} = f_c+f_m$
14. Minimum Freq. in AM wave : $f_{min} = f_c-f_m$
15. Modulation index from AM Wave :

$$\mu = (Amax-Amin)/Amax+Amin)$$

Modulation by Several sinewaves

16. AM Wave with two Modulating signals :

$$s(t)=A_c(1+\mu_1\cos 2\pi f_{m1}t+\mu_2\cos 2\pi f_{m2}t)\cos 2\pi f_c t$$

17. Transmitted power : $Pt=Pc(1+ \mu^2{}_1/2 + \mu^2{}_2/2)$

18. Total Modulation index or Effective modulation index

$$\mu = \sqrt{\mu_1^2 + \mu_2^2}$$

19. Total Power : $Pt=Pc(1 + \mu^2{}_t / 2)$

20. Amplitudes of AM Wave : $Amax = Ac (1 + \mu^2)$

$Amin = Ac (1 - \mu^2)$

21. Peak Amplitude of carrier : $Ac = (Amax + Amin) / 2$

22. Peak Amplitude of message signal : $Ac = (Amax - Amin) / 2$

23. Modulation index from AM wave :

$$\mu = (Amax + Amin) /(Amax - Amin)$$

An amplitude modulated signal is given by

$s(t)=[10\cos(2\pi\times10^{6}t) + 5\cos(2\pi\times10^{3}t) + 2\cos(2\pi\times10^{6}t) + \cos(4\pi\times10^{3}t)]$ volts.

Find i) total modulated power ii) Sideband power and iii) net modulation index.

Jan-10,6M

Sol :-

WKT

$$\boxed{S(t) = A_c\left[1 + \mu_1 \cos 2\pi f_1 t + \mu_2 \cos 2\pi f_2 t\right] \cos 2\pi f_c t} \longrightarrow (1)$$

Given

$$S(t) = \left[10 \cos(2\pi \times 10^6 t) + 5 \cos(2\pi \times 10^6 t) \cos(2\pi \times 10^3 t) + 2 \cos(2\pi \times 10^6 t) \cos(4\pi \times 10^3 t)\right]$$

$$S(t) = 10 \cos(2\pi \times 10^6 t)\left[1 + \frac{5}{10} \cos(2\pi \times 10^3 t) + \frac{2}{10} \cos(4\pi \times 10^3 t)\right]$$

$$\boxed{S(t) = 10 \cos(2\pi \times 10^6 t)\left[1 + 0.5 \cos(2\pi \times 10^3 t) + 0.2 \cos(4\pi \times 10^3) t\right]} \longrightarrow (2)$$

Comparing eq (1) & (2), we get

$A_c = 10\text{V}$, $\mu_1 = 0.5$, $\mu_2 = 0.2$, $f_1 = 1\times10^3$ Hz, $f_2 = 2\times10^3$ Hz, $f_c = 1\times10^6$ Hz

* Net modulation index $\mu_t = \sqrt{\mu_1^2 + \mu_2^2} = \sqrt{(0.5)^2 + (0.2)^2}$

$\boxed{\mu_t = 0.538}$ $\longrightarrow$ $\boxed{\text{2 Marks}}$

$R = 1\Omega$

* Carrier power $P_c = \frac{A_c^2}{2R} = \frac{(10)^2}{2\times1}$

$\boxed{P_c = 50\text{W}}$

* Sideband power $P_{SB} = P_{USB} + P_{LSB} = \frac{\mu_t^2}{2} P_c = \frac{(0.538)^2}{2} 50$

$\boxed{P_{SB} = 7.25\text{W}}$ $\longrightarrow$ $\boxed{\text{2 Marks}}$

* Total modulated power

$$P_T = P_c\left[1+\frac{\mu_t^2}{2}\right]$$

$$= 50\left[1+\frac{0.538^2}{2}\right]$$

$$\boxed{P_T = 57.25W} \longrightarrow \boxed{2\text{ Marks}}$$

OR

$$P_T = P_c + P_{SB}$$

$$= 50W + 7.25W$$

$$\boxed{P_T = 57.25W}$$

NOTE :-

* Sideband power $P_{SB} = P_T - P_c = 57.25W - 50W$

$$\boxed{P_{SB} = 7.25W}$$

Consider a message signal m(t)=20cos(2πt)volts and a carrier signal c(t)=50cos(100πt)volts.

i. Sketch to scale resulting AM wave for 75% modulation.

ii. Find the power delivered across a load of 100Ω due to this AM wave. June-10,6M

Given :- $A_m = 20V$, $f_m = 1Hz$, $A_c = 50V$, $f_c = 50Hz$, $\mu = 0.75$ & $R = 100\Omega$

WKT AM wave is given by

$$S(t) = A_c[1+\mu\cos 2\pi f_m t]\cos 2\pi f_c t$$

$$\therefore \boxed{S(t) = 50[1+0.75\cos 2\pi(1)t]\cos 2\pi(50)t} \longrightarrow \boxed{2\text{ Marks}}$$

i) $A_{max} = A_c(1+\mu) = 50(1+0.75) = \underline{87.5V}$

$A_{min} = A_c(1-\mu) = 50(1-0.75) = \underline{12.5V}$

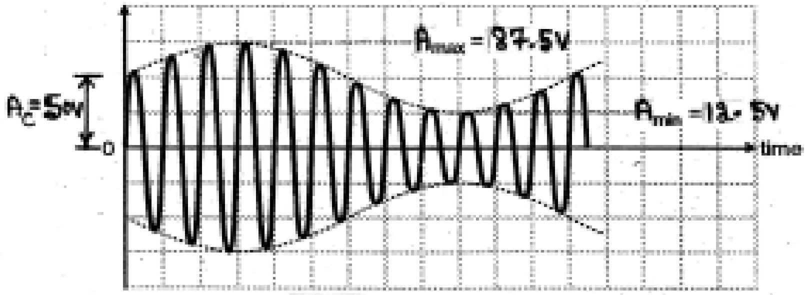


Fig. : AM wave for m = 0.75

→ $\boxed{2 \text{ Marks}}$

ii) $P_T = P_c\left[1 + \frac{\mu^2}{2}\right]$

$* \; P_c = \frac{A_c^2}{2R} = \frac{50^2}{2 \times 100} = \underline{12.5\,W}$ → $\boxed{1 \text{ Mark}}$

$P_T = 12.5\left[1 + \frac{0.75^2}{2}\right]$

$\boxed{P_T = 16.015\,W}$. → $\boxed{1 \text{ Mark}}$

A carrier wave with amplitude 12V and frequency 10 MHz is amplitude modulated to 50% level with a modulated frequency of 1 KHz. Write down the equation for the above wave and sketch the modulated signal in frequency domain. June-10,7M

Given : $A_c = 12V$, $f_c = 10\,MHz$, $\mu = 0.5$, $f_m = 1\,KHz$

Sol :-

WKT AM wave is given by :

$S(t) = A_c[1 + \mu \cos 2\pi f_m t] \cos 2\pi f_c t$

$\boxed{S(t) = 12[1 + 0.5 \cos 2\pi(1 \times 10^3) t] \cos 2\pi(10 \times 10^6) t}$ → $\boxed{3 \text{ Marks}}$

WKT for Single tone modulation is given by

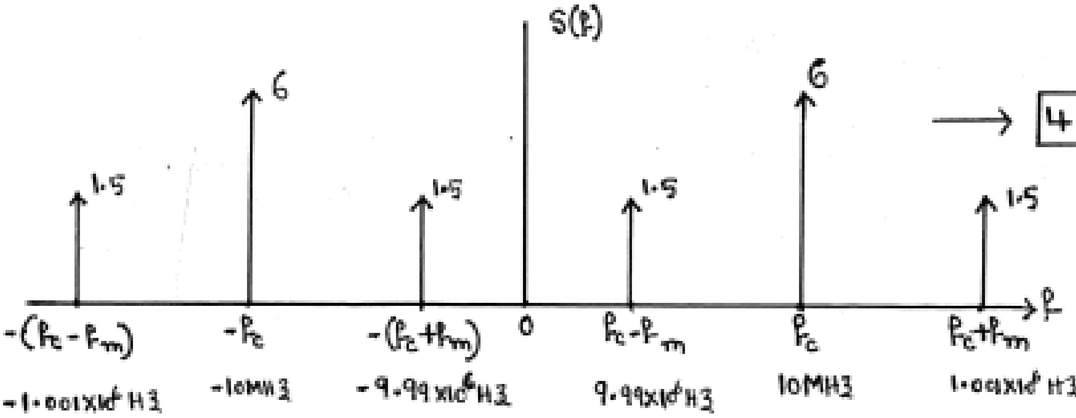
$$S(t) = A_c \cos 2\pi f_c t + \frac{\mu A_c}{2} \cos 2\pi(f_c+f_m)t + \frac{\mu A_c}{2} \cos 2\pi(f_c-f_m)t$$

$$S(t) = 12 \cos 2\pi(10\times10^6 t) + \frac{0.5\times12}{2} \cos 2\pi\left(10\times10^6 + 1\times10^3\right)t + \frac{0.5\times12}{2} \cos 2\pi\left(10\times10^6 - 1\times10^3\right)t$$

$$S(t) = 12 \cos 2\pi(10\times10^6 t) + 3 \cos 2\pi(1.001\times10^6 t) + 3 \cos 2\pi(9.99\times10^6 t) \longrightarrow (1)$$

Taking FT on both Side of eq (1), we get

$$S(f) = \frac{12}{2}\left[\delta(f-10\times10^6) + \delta(f+10\times10^6)\right] + \frac{3}{2}\left[\delta(f-1.001\times10^6) + \delta(f+1.001\times10^6)\right] + \frac{3}{2}\left[\delta(f-9.99\times10^6) + \delta(f+9.99\times10^6)\right]$$



$\longrightarrow$ $\boxed{\text{4 Marks}}$

Consider a message signal m(t)=20cos(2πt)volts and a carrier signal c(t)=50cos(100πt)volts.

i. The resulting AM wave for 75% modulation.

ii. Sketch the Spectrum of this AM wave

iii. Find the power developed across the load of 100Ω. Jan-08, 10M June-10,10M

Given : $m(t) = 20 \cos 2\pi t\ V$, $c(t) = 50 \cos 100\pi t$ & $\mu = 0.75$

$\omega_m = 2\pi$ $2\pi f_m = 2\pi$ $f_m = 1Hz$	$\omega_c = 100\pi$ $2\pi f_c = 100\pi$ 50 $f_c = 50Hz$

i) WKT

$$s(t) = A_c\left[1+\mu \cos 2\pi f_m t\right] \cos 2\pi f_c t$$

$$s(t) = 50\left[1+0.75 \cos 2\pi(1) t\right] \cos 2\pi(50) t$$

$$s(t) = 50 \cos 2\pi(50)t + \underline{37.5 \cos 2\pi(50)t \cdot \cos 2\pi(1)t}$$

$$\boxed{\cos A \cdot \cos B = \frac{1}{2}\cos(A-B) + \frac{1}{2}\cos(A+B)}$$

$$s(t) = 50\cos 2\pi(50)t + \frac{37.5}{2}\cos 2\pi(50-1)t + \frac{37.5}{2}\cos 2\pi(50+1)t$$

$$\boxed{s(t) = 50\cos 2\pi(50)t + 18.75 \cos 2\pi(49)t + 18.75 \cos 2\pi(51)t} \longrightarrow (1)$$

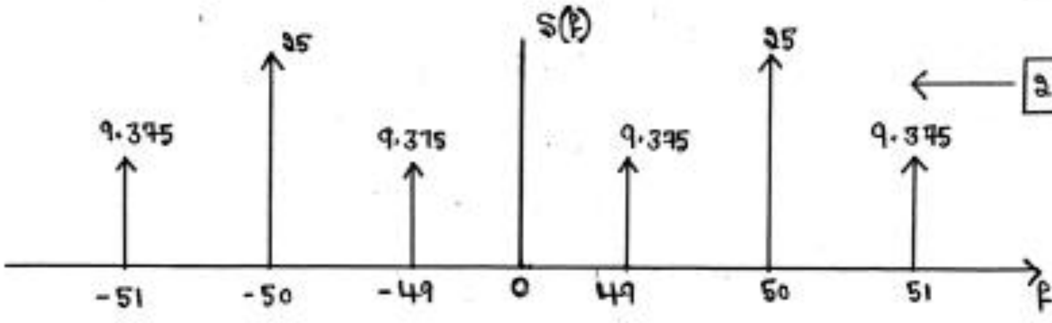
3 Marks

ii) Taking FT of eq (1), we get

$$S(f) = \frac{50}{2}\left[\delta(f-50)+\delta(f+50)\right] + \frac{18.75}{2}\left[\delta(f-49)+\delta(f+49)\right] + \frac{18.75}{2}\left[\delta(f-51)+\delta(f+51)\right]$$

$$\boxed{S(f) = 25\left[\delta(f-50)+\delta(f+50)\right] + 9.375\left[\delta(f-49)+\delta(f+49)\right] + 9.375\left[\delta(f-51)+\delta(f+51)\right]}$$

2 Marks



2 Marks

iii) $P_T = P_c\left(1+\frac{\mu^2}{2}\right)$

$$P_c = \frac{A_c^2}{2R} = \frac{(50)^2}{2\times 100} = \underline{12.5\,W}$$

$$P_T = 12.5\left(1+\frac{0.75^2}{2}\right) = \underline{16\,W}$$

3 Marks

The antenna current of an AM broadcast transmitter modulated to a depth of 40% by an audio sine wave is 11A. It increases to 12A as a result of sinusoidal modulation by another audio sine wave. What is the modulation index due to second wave?

OLD June-10,6M

Given: i) $\mu_1 = 0.4$, $I_{t1} = 11A$, $I_c = ?$

ii) $I_{t2} = 12A$, $\mu_2 = ?$

Sol:

i) $I_{t1} = I_c\sqrt{1+\frac{\mu_1^2}{2}}$

$$I_c = \frac{I_{t1}}{\sqrt{1+\frac{\mu_1^2}{2}}} = \frac{11}{\sqrt{1+\frac{0.4^2}{2}}}$$

$$\boxed{I_c = 10.58A}$$

ii) $I_{t2} = I_c\sqrt{1+\frac{\mu_t^2}{2}}$

$$I_{t2}^2 = I_c^2\left(1+\frac{\mu_t^2}{2}\right)$$

$$\frac{I_{t2}^2}{I_c^2} = 1+\frac{\mu_t^2}{2}$$

$$\frac{\mu_t^2}{2} = \left(\frac{I_{t2}^2}{I_c^2}\right) - 1$$

$$\frac{\mu_t^2}{2} = \left(\frac{12^2}{10.58^2}\right) - 1$$

$$\frac{\mu_t^2}{2} = 1.286 - 1$$

$$\frac{\mu_t^2}{2} = 0.286$$

$$\mu_t^2 = 0.572$$

$$\boxed{\mu_t = 0.757}$$

* WKT $\mu_t = \sqrt{\mu_1^2 + \mu_2^2}$

$$\mu_t^2 = \mu_1^2 + \mu_2^2$$

$$\mu_2^2 = \mu_t^2 - \mu_1^2$$

$$= 0.757^2 - 0.4^2$$

$$\mu_2^2 = 0.4130$$

$$\boxed{\mu_2 = 0.642}$$

Find the ratio of maximum average power to unmodulated carrier power in AM

Sol :- Jan-07,4M

WKT $P_T = P_c\left(1+\frac{\mu^2}{2}\right)$

When $\mu = 1$ i.e. for 100% modulation

$$P_{T(max)} = P_c\left[1+\frac{(1)^2}{2}\right]$$

$$P_{T(max)} = 1.5 P_c$$

$$\frac{P_{T(max)}}{P_c} = 1.5$$

$$\frac{P_{T(max)}}{P_c} = \frac{1.5}{1}$$

$$\therefore \boxed{P_{T(max)} : P_c \text{ is } 1.5 : 1}$$

An audio frequency signal 5sin2π(1000)t is used to amplitude modulate a carrier of 100sin2π(10⁶)t. Assume modulation index of 0.4. Find

i. Sideband frequencies **iii. Amplitude of each sideband**

ii. Bandwidth required **iv. Total power delivered to a load of 100Ω**

Jan-05,10M

Sol:- Given:

$A_m = 5$, $A_c = 100$, $\mu = 0.4$, $f_m = 1000\,Hz$, $f_c = 1\times10^6\,Hz$.

i) Sideband frequencies:

$$f_{USB} = f_c + f_m = 1MHz + 1000Hz = 1.001\,MHz$$

$$f_{LSB} = f_c - f_m = 1MHz - 1000Hz = 999\,000\,Hz = 0.999\,MHz$$

ii) Amplitude of each Sideband frequencies:

$$\frac{\mu A_c}{2} = \frac{0.4 \times 100}{2} = 20V.$$

∴ Amplitude of upper & lower Sideband is 20V.

iii) Bandwidth required:

$$BW = 2f_m = 2 \times 1KHz = 2KHz$$

or

$$BW = f_{USB} - f_{LSB} = 1.001MHz - 999000\,Hz = 2KHz$$

iv) Total power delivered to a load of 100 Ω:

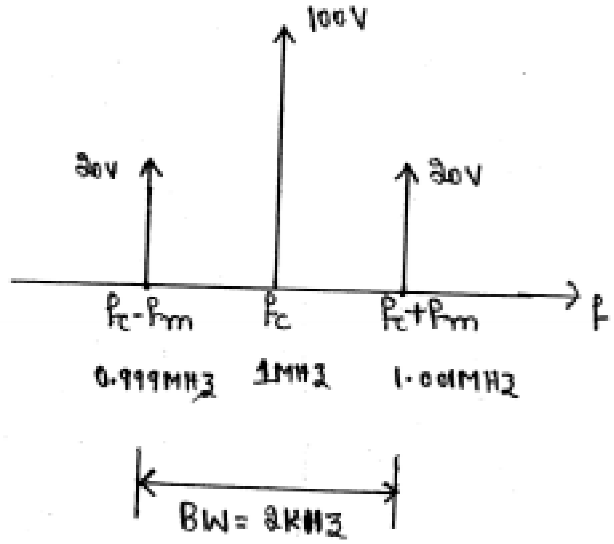
W.K.T

$$P_T = P_c\left[1+\frac{\mu^2}{2}\right] = \frac{A_c^2}{2R}\left[1+\frac{\mu^2}{2}\right]$$

$$= \frac{(100)^2}{2\times100}\left[1+\frac{(0.4)^2}{2}\right]$$

$$\boxed{P_T = 54W}$$

v) Spectrum of AM wave:



A 1000KHz carrier is simultaneously modulated by 300Hz, 800Hz and 2KHz audio sine waves. What will be the frequency content of AM signals. **July-05,6M**

Sol:

Given: $f_c = 1000KHz$

$f_{m1} = 300Hz$, $f_{m2} = 800Hz$ & $f_{m3} = 2000Hz$

* $f_{USB1} = f_c + f_{m1} = 1000KHz + 300Hz = 1000.3KHz$
* $f_{LSB1} = f_c - f_{m1} = 1000KHz - 300Hz = 999.7KHz$.
* $f_{USB2} = f_c + f_{m2} = 1000KHz + 800Hz = 1000.8KHz$
* $f_{LSB2} = f_c - f_{m2} = 1000KHz - 800Hz = 999.2KHz$
* $f_{USB3} = f_c + f_{m3} = 1000KHz + 2KHz = 1002KHz$
* $f_{LSB3} = f_c - f_{m3} = 1000KHz - 2KHz = 998KHz$

A carrier wave $4\sin(2\pi x 500x10^3 t)$volts is amplitude modulated by an audio wave [0.2 sin 3(2πx500t) + 0.1 sin 5(2πx500t)] volts. Determine the upper and lower sideband and sketch the complete spectrum of the modulated wave. Estimate the total power in the sideband. June-09,6M

Sol: Given : $c(t) = 4\sin(2\pi \times 500\times10^3 t) \rightarrow A_c = 4V,\ f_c = 500KHz$

$$m(t) = \underset{A_{m1}}{0.2}\sin 2\pi(\underset{f_{m1}}{1500})t + \underset{A_{m2}}{0.1}\sin 2\pi(\underset{f_{m2}}{2500})t$$

The message signal consists of two sinewaves.

$A_{m1} = 0.2V$, $f_{m1} = 1500Hz$

$A_{m2} = 0.1V$, $f_{m2} = 2500Hz$

* USB & LSB :-

i) $USB_1 = (f_c + f_{m1}) = 500KHz + 1.5KHz = 501.5KHz$

$LSB_1 = (f_c - f_{m1}) = 500KHz - 1.5KHz = 498.5KHz$

ii) $USB_2 = (f_c + f_{m2}) = 500KHz + 2.5KHz = 502.5KHz$

$LSB_2 = (f_c - f_{m2}) = 500KHz - 2.5KHz = 497.5KHz.$

* Modulation Index of individual modulating Signals :

i) Modulation Index for 1st Signal $\mu_1 = \frac{A_{m1}}{A_c} = \frac{0.2}{4} = 0.05$

ii) Modulation Index for 2nd Signal $\mu_2 = \frac{A_{m2}}{A_c} = \frac{0.1}{4} = 0.025$

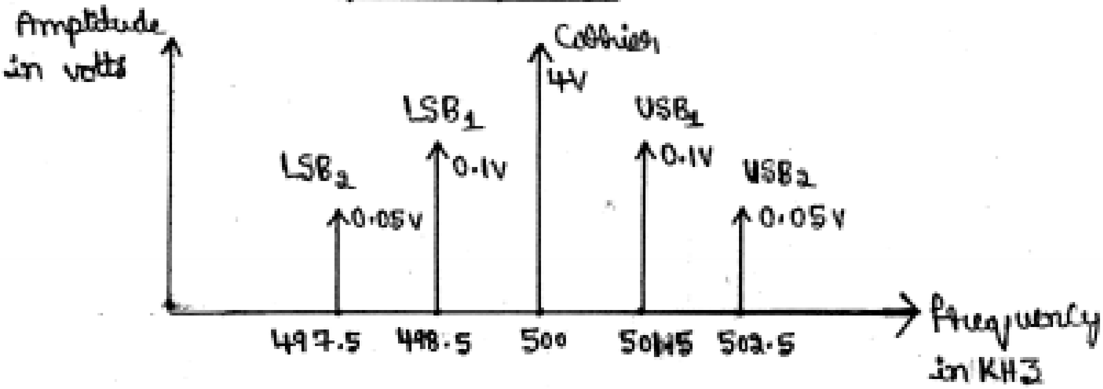
* Sideband amplitudes:

In general, amplitude of each Sideband is given by $\frac{\mu A_c}{2}$

i) Amplitude of USB_1 & LSB_1 will be: $\frac{\mu_1 A_c}{2} = \frac{0.05 \times 4}{2} = 0.1V$

ii) Amplitude of USB_2 & LSB_2 will be: $\frac{\mu_2 A_c}{2} = \frac{0.025 \times 4}{2} = 0.05V$

* Complete Spectrum of AM Signal:



* Total power in the Sidebands:-

WKT, the total power in the Sidebands is given by

$$P_{SB} = P_{USB} + P_{LSB} = P_c\left(\frac{\mu^2}{2}\right)$$

for two Signals,

$$P_{SB} = P_c\left(\frac{\mu_t^2}{2}\right)$$

Where,

μ_t = total modulation Index =

i.e. $\mu_t = \sqrt{\mu_1^2 + \mu_2^2} = \sqrt{(0.05)^2 + (0.025)^2} = 0.0559$

$$\therefore \quad P_{SB} = P_c\left[\frac{\mu_t^2}{2}\right]$$

W.K.T. $P_c = \frac{A_c^2}{2R}$

$$= \frac{A_c^2}{2R}\left[\frac{\mu_t^2}{2}\right]$$

$$= \frac{(4)^2}{2R}\left[\frac{(0.0559)^2}{2}\right]$$

$$= \frac{10^8}{8R}\left[1.56 \times 10^{-3}\right]$$

$$= \frac{8}{R}\left[1.56 \times 10^{-3}\right]$$

$$\boxed{P_{SB} = \frac{0.0125}{R}}$$

A broadcast AM transmitter radiates 50Kw of carrier power. What will be the radiated power at 85% modulation? June-08,2M

Sol:-

Given : $P_c = 50KW$ & $\mu = 0.85$

$$P_T = P_c\left[1 + \frac{\mu^2}{2}\right]$$

$$= 50 \times 10^3\left[1 + \frac{(0.85)^2}{2}\right]$$

$$\boxed{P_T = 68.0625\ KW}$$

---

* Consider the message signal $m(t) = 20 \cos(2\pi t)$ volts & carrier wave $c(t) = 50 \cos(100\pi t)$ volts. Derive an expression for the resulting AM wave for 75% modulation. August - 2002

Sol: Given : $A_m = 20V$, $f_m = 1Hz$

$A_c = 50V$, $f_c = 50Hz$ & $\mu = 0.75$

WKT $s(t) = A_c\left[1 + \mu \cos 2\pi f_m t\right] \cos 2\pi f_c t.$

$$\boxed{s(t) = 50\left[1 + 0.75 \cos 2\pi t\right] \cos(100\pi t)}$$

An audio frequency signal $10\sin 2\pi(500)t$ is used to amplitude modulate a carrier of $50\sin 2\pi(10^5)t$. Assume modulation index = 0.2. Find

i. Sideband frequencies

ii. Amplitude of each sideband

iii. Bandwidth required OLD June-09,6M

Given : $\mu = 0.2$, $f_m = 500\,Hz$, $f_c = 10^5\,Hz$, $A_m = 10V$, $A_c = 50V$

Sol :-

i) Sideband frequencies :

$$f_{USB} = f_c + f_m = 10^5 + 500 = 1.0005 \times 10^6\ Hz$$

$$f_{LSB} = f_c - f_m = 10^5 - 500 = 0.9995 \times 10^6\ Hz$$

ii) Amplitude of each Sideband :

$$\frac{\mu A_c}{2} = \frac{0.2 \times 50}{2} = 5V$$

iii) Bandwidth 'B' $= 2f_m = 2 \times 500 = 1000\ Hz$

An amplitude modulated signal is given by

$s(t)=10\cos 2\pi 10^6 t + 5\cos 2\pi \times 10^6 t \cos 2\pi 10^3 t + 2\cos 2\pi 10^6 t \cos 4\pi 10^3 t$ volts.

Find various frequency components present and the corresponding modulation indices. Draw the line spectrum and find the bandwidth. Jan-07,12M

Given :-

$$S(t) = 10\cos 2\pi 10^6 t + 5\cos 2\pi 10^6 t \cos 2\pi 10^3 t + 2\cos 2\pi 10^6 t \cos 4\pi 10^3 t$$

$$S(t) = 10\cos 2\pi(10^6)t \left[1 + \frac{5}{10}\cos 2\pi(10^3)t + \frac{2}{10}\cos 2\pi(2\times 10^3)t\right]$$

$$S(t) = 10\cos 2\pi(10^6)t \left[1 + 0.5\cos 2\pi(10^3)t + 0.2\cos 2\pi(2\times 10^3)t\right] \longrightarrow ①$$

WKT

$$s(t) = A_c \cos 2\pi f_c t \left[1 + \mu_1 \cos 2\pi f_1 t + \mu_2 \cos 2\pi f_2 t \right] \longrightarrow ②$$

Comparing eq ① & eq ②, we get

$A_c = 10V$, $\mu_1 = 0.5$, $\mu_2 = 0.2$, $f_1 = 10^3 Hz$, $f_2 = 2\times10^3 Hz$ & $f_c = 10^6 Hz$

Equation ① can be rewritten as

$$s(t) = 10 \cos 2\pi(10^6) t + 5 \cos 2\pi (10^6) t \cdot \cos 2\pi (10^3) t + 2 \cos 2\pi (10^6) t \cdot \cos 2\pi (2\times10^3) t$$

By using trigonometric identity

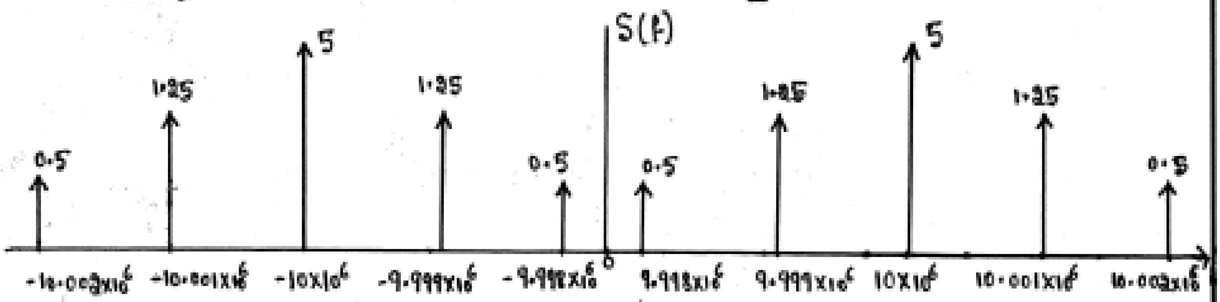
$$\boxed{\cos A \cdot \cos B = \frac{1}{2} \cos (A-B) + \frac{1}{2} \cos (A+B)}$$

$$s(t) = 10 \cos 2\pi(10^6) t + \frac{5}{2} \cos 2\pi (10^6 - 10^3) t + \frac{5}{2} \cos 2\pi (10^6 + 10^3) t + \frac{2}{2} \cos 2\pi (10^6 - 2\times10^3) t + \frac{2}{2} \cos 2\pi (10^6 + 2\times10^3) t$$

$$s(t) = 10 \cos 2\pi(10^6) t + 2.5 \cos 2\pi (9.99\times10^6) t + 2.5 \cos 2\pi (10.001\times10^6) t + \cos 2\pi (9.998\times10^6) t + \cos 2\pi (10.002\times10^6) t \longrightarrow ③$$

Taking FT of eq ③, we get

$$S(f) = \frac{10}{2}\left[\delta(f-10^6) + \delta(f+10^6) \right] + \frac{2.5}{2}\left[\delta(f - 9.999\times10^6) + \delta(f + 9.999\times10^6) \right] + \frac{2.5}{2}\left[\delta(f - 10.001\times10^6) + \delta(f + 10.001\times10^6) \right] + \frac{1}{2}\left[\delta(f - 9.998\times10^6) + \delta(f + 9.998\times10^6) \right] + \frac{1}{2}\left[\delta(f - 10.002\times10^6) + \delta(f + 10.002\times10^6) \right]$$



BW = 4KHz

$* \; BW = 2[\max(f_1, f_2)]$

$BW = 2f_2 = 2\times2KHz$

$\boxed{BW = 4KHz}$

iii) W.K.T.

$$A_{max} = A_c(1+\mu) = 50(1+0.75) = 87.5V$$

$$A_{min} = A_c(1-\mu) = 50(1-0.75) = 12.5V$$

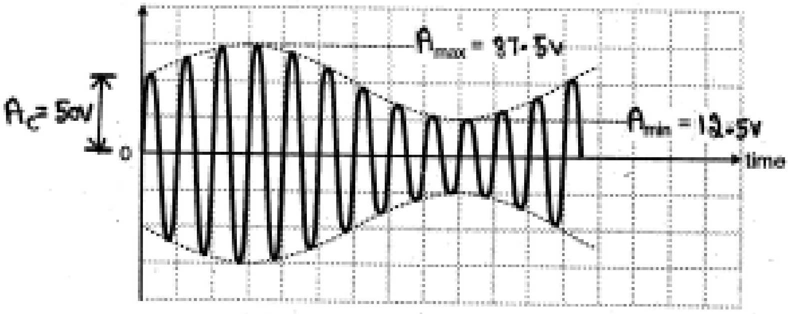


Fig. : AM wave for m = 0.75

An amplitude modulated waveform has the form

$$x_c(t) = 10[1 + 0.5\cos 2000\pi t + 0.5\cos 4000\pi t]\cos(20000\pi t).$$

i) Sketch the amplitude Spectrum of $x_c(t)$

ii) Find the average power Content of each Spectral Component including the Carrier

iii) Modulation Index

August - 2002.

Sol: Given : $A_c = 10V$, $\mu_1 = 0.5$, $\mu_2 = 0.5$

$2\pi f_{m1} = 2000\pi$, $f_{m1} = \frac{2000\pi}{2\pi} = 1000Hz$

$2\pi f_{m2} = 4000\pi$, $f_{m2} = \frac{4000\pi}{2\pi} = 2000Hz$

$2\pi f_c = 20000\pi$, $f_c = \frac{20000\pi}{2\pi} = 10KHz.$

$\therefore$ $f_{m1} = 1000Hz$, $f_{m2} = 2000Hz$, $f_c = 10KHz.$

* Consider a message signal $m(t) = 20\cos(2\pi t)$ V & the carrier wave $c(t) = 50\cos(100\pi t)$ V.

i) Write an expression for the resulting AM wave for 75% modulation in time domain.

ii) Draw the spectrum of AM wave.

iii) Sketch the resulting wave for 75% modulation.

July-06, 8M

Sol:- Given: $A_m = 20V$, $f_m = 1Hz$

$A_c = 50V$, $f_c = 50Hz$

$\mu = 0.75$

$\because 2\pi f_c = 100\pi$

$f_c = \frac{100\pi}{2\pi} = 50$

i) The AM signal is given by:

$$S(t) = A_c[1 + \mu \cos 2\pi f_m t] \cos 2\pi f_c t.$$

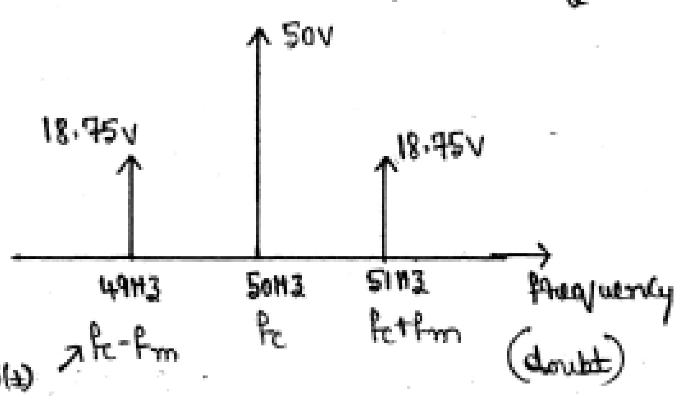
$$\boxed{S(t) = 50[1 + 0.75 \cos 2\pi t] \cos 2\pi(50)t.}$$

ii) Spectrum of AM wave:-

$f_{USB} = f_c + f_m = 50 + 1 = 51Hz$

$f_{LSB} = f_c - f_m = 50 - 1 = 49Hz.$

* Amplitude of each sideband is $\frac{\mu A_c}{2} = \frac{(0.75) \times 50}{2} = 18.75V$



(doubt)

NOTE:- By taking FT of S(t) plot the spectrum

* Spectrum of AM wave :-

i) $f_{USB1} = f_c + f_{m1} = 10KHz + 1KHz = 11KHz$

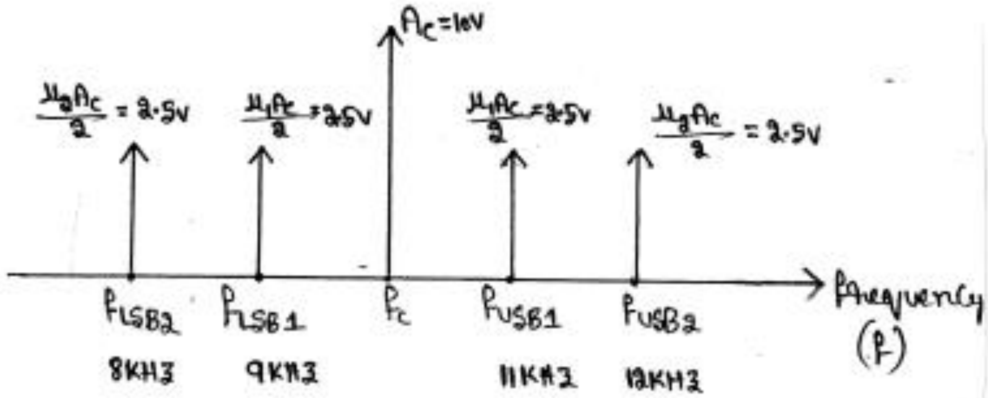
$f_{LSB1} = f_c - f_{m1} = 10KHz - 1KHz = 9KHz$

ii) $f_{USB2} = f_c + f_{m2} = 10KHz + 2KHz = 12KHz$

$f_{LSB2} = f_c - f_{m2} = 10KHz - 2KHz = 8KHz$

* Amplitude of each Side band is i) $\frac{\mu_1 A_c}{2} = \frac{0.5 \times 10}{2} = 2.5V$

ii) $\frac{\mu_2 A_c}{2} = \frac{0.5 \times 10}{2} = 2.5V$



ii) Average power (P_T) :-

$$P_T = P_c + P_{USB1} + P_{USB2} + P_{LSB1} + P_{LSB2}$$

$$P_T = \frac{A_c^2}{2R} + \frac{\mu_1^2 A_c^2}{8R} + \frac{\mu_2^2 A_c^2}{8R} + \frac{\mu_1^2 A_c^2}{8R} + \frac{\mu_2^2 A_c^2}{8R}$$

$$= \frac{(10)^2}{2R} + \frac{(0.5)^2 (10)^2}{8R} + \frac{(0.5)^2 \times (10)^2}{8R} + \frac{(0.5)^2 \times (10)^2}{8R} + \frac{(0.5)^2 \times (10)^2}{8R}$$

$$\boxed{P_T = \frac{50}{R} + \frac{3.125}{R} + \frac{3.125}{R} + \frac{3.125}{R} + \frac{3.125}{R}}$$

iii) Modulation Index :-

$$\mu_t = \sqrt{\mu_1^2 + \mu_2^2} = \sqrt{(0.5)^2 + (0.5)^2} = \underline{0.707}$$

17

Using message Signal $m(t) = \frac{t}{1+t^2}$, determine & Sketch the modulated wave for AM whose percentage modulation is equal to the following values.

i) $\mu = 50\%$ ii) $\mu = 100\%$ iii) $\mu = 125\%$

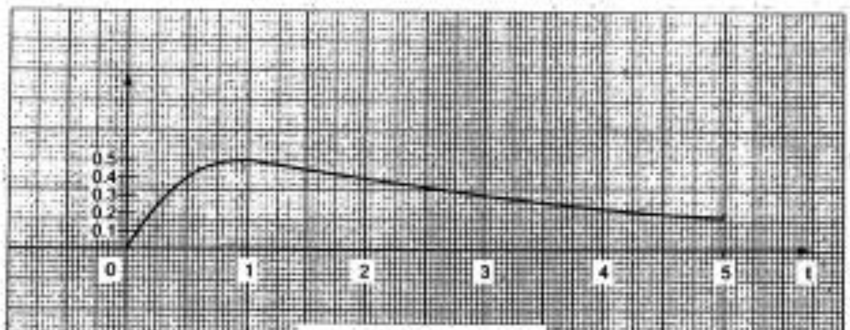
Feb, 2002, 6M

Sol: Message Signal m(t) :-

The modulating Signal is determined from the following table & is shown in fig ①.

t	0	0.2	0.4	0.6	0.8	1.0	2.0	3.0	4.0	5.0
m(t)	0	0.192	0.345	0.44	0.49	0.5	0.4	0.3	0.235	0.192

↑ Peak or maximum value of m(t)



(a) Modulating Signal

From above figure, the maximum amplitude of $m(t) = \underline{0.5V}$

* Calculate A_c for different values of μ :-

Given : $A_m = 0.5V$

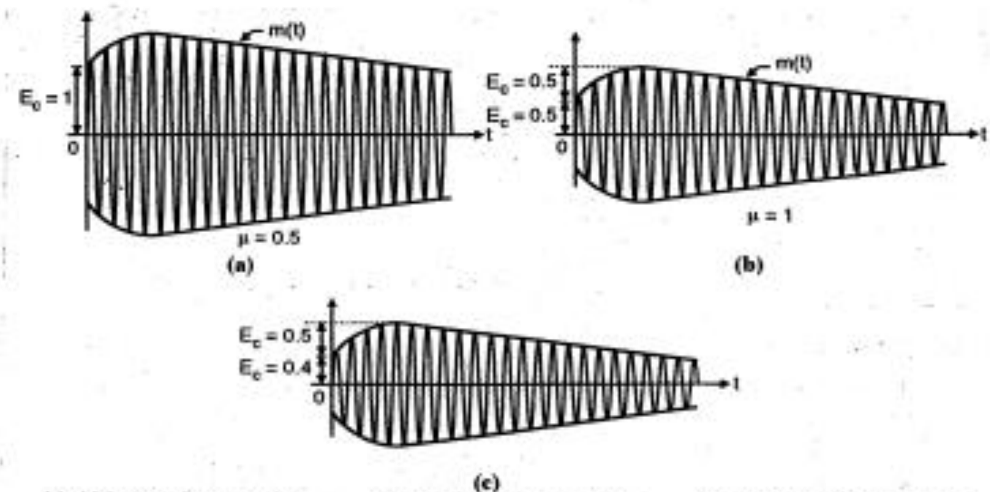
W.K.T $\boxed{\mu = \frac{A_m}{A_c}}$ & $\boxed{A_c = \frac{A_m}{\mu}}$

i) $\mu = 0.5$, $A_c = \frac{A_m}{\mu} = \frac{0.5V}{0.5} = 1V$

ii) $\mu = 1$, $A_c = \frac{A_m}{\mu} = \frac{0.5V}{1} = 0.5V$

iii) $\mu = 1.25$, $A_c = \frac{A_m}{\mu} = \frac{0.5V}{1.25} = 0.4V$

The Waveforms of the AM wave for different values of μ are Shown below



(a) AM wave for $\mu = 0.5$ (b) AM wave for $\mu = 1.0$ (c) AM wave for $\mu = 1.25$

Draw the Spectrum of an AM Signal with $c(t) = A_c \cos^2(\pi f_c)t$ & $m(t) = A_m \cos^2(\pi f_m t)$.

Sol: Given : $c(t) = A_c \cos^2(\pi f_c t)$ June-02, 6M

W.K.T $\boxed{\cos^2\theta = \frac{1}{2} + \frac{\cos 2\theta}{2}}$

$$c(t) = A_c\left[\frac{1}{2} + \frac{\cos 2\pi f_c t}{2}\right]$$

$$\therefore \quad c(t) = \frac{A_c}{2} + \frac{A_c}{2}\cos 2\pi f_c t. \longrightarrow (1)$$

Similarly

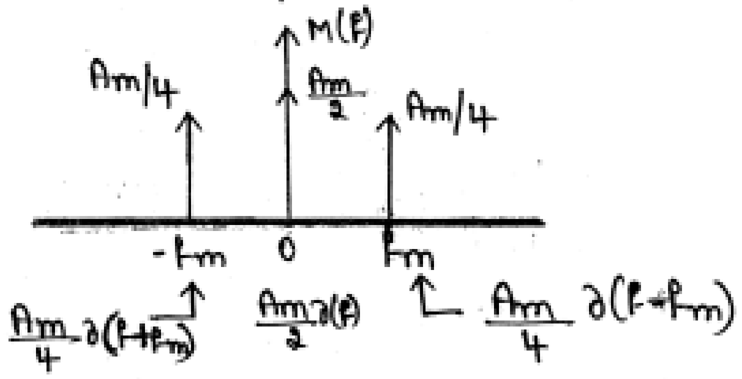
$$m(t) = A_m \cos^2(\pi f_m t)$$

$$m(t) = \frac{A_m}{2} + \frac{A_m}{2}\cos 2\pi f_m t. \rightarrow (2)$$

* Spectrum of m(t) :-

We can get Spectrum of m(t) by taking it's Fourier transform i.e. eq (2).

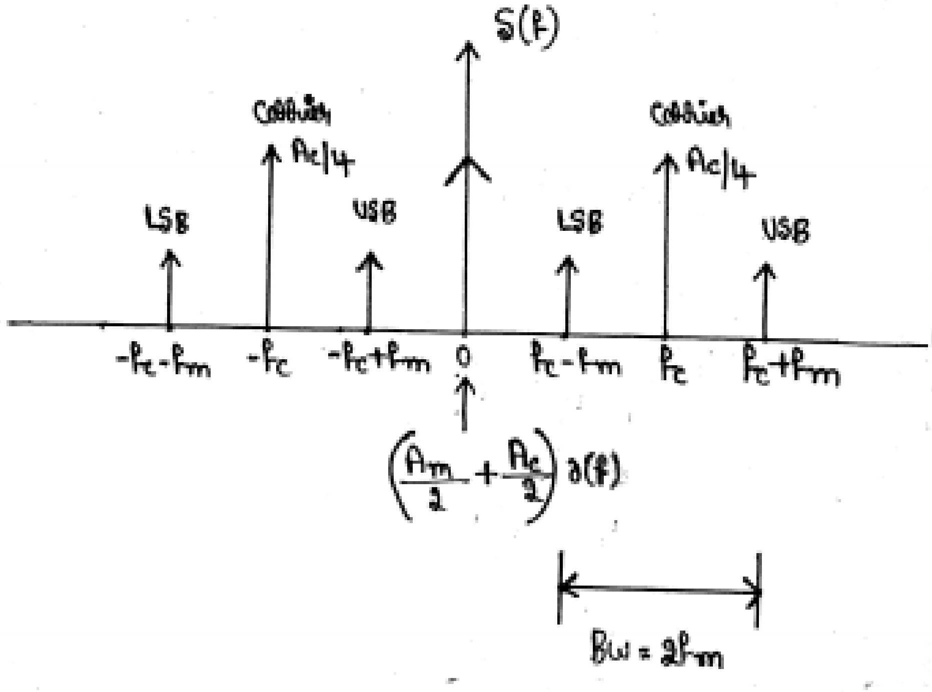
$$M(f) = \frac{A_m}{2}\delta(t) + \frac{A_m}{4}\left[\delta(f+f_m) + \delta(f-f_m)\right]$$



* Spectrum of AM Signal :-

Taking Fourier transform of c(t) i.e. eq (1), we get

$$C(f) = \frac{A_c}{2}\delta(t) + \frac{A_c}{4}\left[\delta(f+f_c) + \delta(f-f_c)\right]$$



For a PN junction diode, the current 'i' through the diode and the voltage 'v' across it are related by

$$i = I_0\left[e^{-v/V_T} - 1\right]$$

Where 'I_0' is the reverse saturation current and 'V_T' is the thermal voltage. At room temperature, $V_T = 0.026$ volts.

ⓐ Expand 'i' as a power series in v, retaining terms upto v^2.

ⓑ Let $v = 0.01\cos 2\pi f_m t + 0.01\cos 2\pi f_c t$ volts, where $f_m = 1KHz$ & $f_c = 100KHz$. Sketch the spectrum of the diode current 'i'.

c) Specify the required bandpass filter to extract from 'i', an AM wave with carrier frequency 'f_c'.

d) What is the percentage modulation index.

Jan-06, 12M

Solution :- ⓐ Given $v = 0.01\cos 2\pi f_m t + 0.01\cos 2\pi f_c t$ & $V_T = 0.026\,V$

$$i(t) = I_0\left[e^{-v/V_T} - 1\right] \longrightarrow ①$$

$$\frac{i(t)}{I_0} = \left[e^{-v/V_T} - 1\right] \longrightarrow ②$$

We can write $\boxed{e^{-x} = 1 - x + \frac{1}{2}x^2}$ Put $x = +v/V_T$

$$\boxed{e^{-v/V_T} = 1 - \frac{v}{V_T} + \frac{1}{2}\left(\frac{v}{V_T}\right)^2} \longrightarrow ③$$

Substituting equation ③ in eq ②, we get

$$\frac{i(t)}{I_0} = \cancel{1} - \frac{v}{V_T} + \frac{1}{2}\left(\frac{v}{V_T}\right)^2 - \cancel{1}$$

$$\boxed{\frac{i(t)}{I_0} = -\frac{v}{V_T} + \frac{1}{2}\left(\frac{v}{V_T}\right)^2} \longrightarrow (4)$$

b) Let $v = 0.01 \cos 2\pi f_m t + 0.01 \cos 2\pi f_c t$ volts. $\rightarrow (5)$

divide both RHS and LHS of equation (5) by V_T

$$\frac{v}{V_T} = \frac{0.01}{V_T} \cos 2\pi f_m t + \frac{0.01}{V_T} \cos 2\pi f_c t.$$

given, $V_T = 0.026V$

$$\frac{v}{V_T} = \frac{0.01}{0.026} \cos 2\pi f_m t + \frac{0.01}{0.026} \cos 2\pi f_c t$$

$$\frac{v}{V_T} = 0.384 \cos 2\pi f_m t + 0.384 \cos 2\pi f_c t \longrightarrow (6)$$

Substituting equation (6) in equation (4), we get

$$\frac{i(t)}{I_0} = -0.384 \cos 2\pi f_m t - 0.384 \cos 2\pi f_c t + \frac{1}{2}\left[0.384 \cos 2\pi f_m t + 0.384 \cos 2\pi f_c t\right]^2$$

W.K.T. $(a+b)^2 = a^2 + b^2 + 2ab$

$$\frac{i(t)}{I_0} = -0.384 \cos 2\pi f_m t - 0.384 \cos 2\pi f_c t + \frac{1}{2}\left[(0.384)^2 \cos^2 2\pi f_m t + (0.384)^2 \cos^2 2\pi f_c t + 2(0.384) \cos 2\pi f_m t \cdot (0.384) \cos 2\pi f_c t\right]$$

$$\frac{i(t)}{I_0} = -0.384 \cos 2\pi f_m t - 0.384 \cos 2\pi f_c t + \frac{(0.384)^2}{2} \cos^2 2\pi f_m t + \frac{(0.384)^2}{2} \cos^2 2\pi f_c t + \frac{1 \cdot \cancel{2}}{\cancel{2}} (0.384)^2 \cdot \cos 2\pi f_c t \cdot \cos 2\pi f_m t.$$

$$\frac{i(t)}{I_0} = -0.384 \cos 2\pi f_m t - 0.384 \cos 2\pi f_c t + 0.073 \cos^2 2\pi f_m t + 0.073 \cos^2 2\pi f_c t + 0.147 \cos 2\pi f_c t \cdot \cos 2\pi f_m t.$$

W.K.T

$$\boxed{\cos^2\theta = \frac{1}{2} + \frac{\cos 2\theta}{2}} \qquad \boxed{\cos A \cdot \cos B = \frac{1}{2}\cos(A-B) + \frac{1}{2}\cos(A+B)}$$

$$\frac{i(t)}{I_0} = -0.384\cos 2\pi f_m t - 0.384\cos 2\pi f_c t + \frac{0.073}{2} + \frac{0.073}{2}\cos 4\pi f_m t$$

$$+ \frac{0.073}{2} + \frac{0.073}{2}\cos 4\pi f_c t + \frac{0.147}{2}\cos[2\pi(f_c - f_m)t]$$

$$+ \frac{0.147}{2}\cos[2\pi(f_c + f_m)t] \longrightarrow (7)$$

$$\boxed{\text{NOTE:- } \frac{0.073}{2} + \frac{0.073}{2} = 0.073}$$

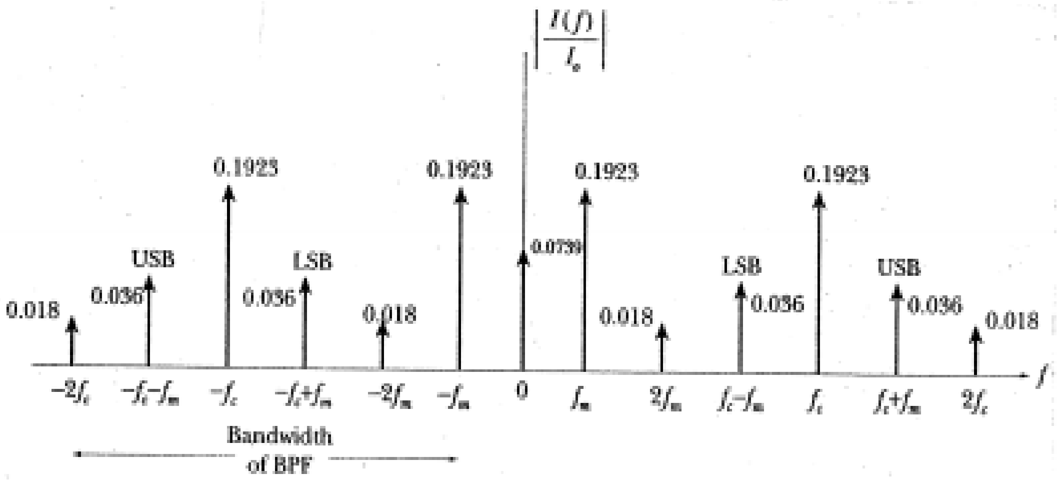
Taking Fourier transform on both side of equation (7), we get

$$\frac{I(f)}{I_0} = -\frac{0.384}{2}\left[\delta(f - f_m) + \delta(f + f_m)\right] - \frac{0.384}{2}\left[\delta(f - f_c) + \delta(f + f_c)\right]$$

$$+ 0.073\,\delta(f) + \frac{0.073}{4}\left[\delta(f - 2f_c) + \delta(f + 2f_c)\right]$$

$$+ \frac{0.147}{4}\left\{\delta[f - (f_c - f_m)t] + \delta[f + (f_c + f_m)]\right\}$$

$$+ \frac{0.147}{4}\left\{\delta[f - (f_c + f_m)t] + \delta[f + (f_c + f_m)]\right\}$$



Magnitude spectrum of $\frac{I(f)}{I_o}$

c) The required AM wave centred at f_c is obtained by passing the diode current through an ideal BPF having center-frequency, $f_c = 100KHz$ and $BW = 2f_m$. $f_m = 1KHz$

$$\boxed{BW = 2KHz}$$

d)

W.K.T

$$\boxed{S(t) = A_c \cos 2\pi f_c t + \frac{\mu A_c}{2} \cos 2\pi (f_c - f_m) t + \frac{\mu A_c}{2} \cos 2\pi (f_c + f_m) t}$$

∴ The time-domain expression from the o/p of the BPF is

$$S(t) = -0.3846 \cos 2\pi f_c t + \frac{0.1479}{2} \cos 2\pi (f_c + f_m) t + \frac{0.1479}{2} \cos 2\pi (f_c - f_m) t.$$

$$S(t) = -0.3846 \cos 2\pi f_c t + 0.0739 \cos 2\pi (f_c + f_m) t + 0.0739 \cos 2\pi (f_c - f_m) t \rightarrow (8)$$

From eq (8), we get

$$|A_c| = 0.3846 \quad , \quad \frac{\mu A_c}{2} = 0.0739$$

$$\frac{\mu A_c}{2} = 0.0739$$

$$\mu A_c = 0.1479$$

$$\mu = \frac{0.1479}{A_c} = \frac{0.1479}{0.3846}$$

$$\mu = 0.384$$

$$\boxed{\%\ \mu = 38.4\%}$$

10) An audio frequency Signal $10 \sin 2\pi \times 500 t$ is used to amplitude modulate a Carrier of $50 \sin 2\pi \times 10^5 t$. Calculate

Calculate

i) Modulation Index

ii) Sideband frequencies

iii) Amplitude of each Sideband frequencies

iv) Bandwidth required

v) Total power delivered to the load of 600Ω.

vi) plot frequency spectrum.

Sol: W.K.T modulating Signal $m(t)$ is given by

$$m(t) = A_m \cos 2\pi f_m t \text{ or } m(t) = A_m \sin 2\pi f_m t. \text{ and}$$

$$c(t) = A_c \cos 2\pi f_c t \text{ or } c(t) = A_c \sin 2\pi f_c t.$$

Given:

$$m(t) = 10 \sin 2\pi \times 500 t$$

$$c(t) = 50 \sin 2\pi \times 10^5 t.$$

$$\therefore \quad A_m = 10V, \quad A_c = 50V$$

i) Modulation Index:

$$\mu = \frac{A_m}{A_c} = \frac{10}{50} = 0.2$$

$$\% \; \mu = 0.2 \times 100 = 20$$

ii) Sideband frequencies:

W.K.T $\quad W_m = 2\pi f_m = 2\pi \times 500$

$$\therefore \boxed{f_m = 500Hz}$$

$$W_c = 2\pi f_c = 2\pi \times 10^5$$

$$\therefore \boxed{f_c = 100KHz}$$

$$f_{USB} = f_c + f_m = 100\,KHz + 500\,Hz = 100.5\,KHz$$

$$f_{LSB} = f_c - f_m = 100\,KHz - 500\,Hz = 99.5\,KHz$$

iii) Amplitude of each Sideband frequencies

$$\frac{\mu A_c}{2} = \frac{0.2 \times 50}{2} = 5V.$$

iv) Bandwidth required.

$$BW = 2f_m = 2 \times 500 = 1000\,Hz$$

(or)

$$BW = f_{USB} - f_{LSB} = 100.5\,KHz - 99.5\,KHz = 1000\,Hz$$

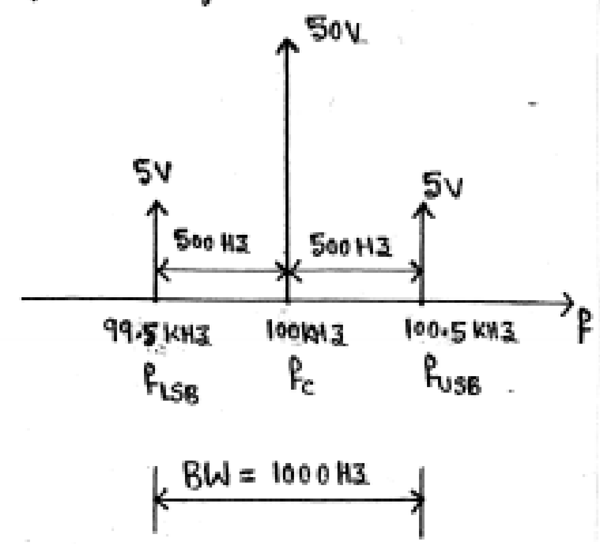
v) Total power delivered into a load of 600 Ω.

$$P_T = P_c\left[1 + \frac{\mu^2}{2}\right] = \frac{A_c^2}{2R}\left[1 + \frac{\mu^2}{2}\right] \qquad \because P_c = \frac{A_c^2}{2R}$$

$$= \frac{(50)^2}{2 \times 600}\left[1 + \frac{(0.2)^2}{2}\right]$$

$$\boxed{P_T = 2.125 \text{ Watts}}$$

vi) Frequency Spectrum of AM Wave:-



A Carrier Wave with the amplitude 12V and frequency 10MHz is amplitude modulated to 50%. with modulating frequency 1KHz. Write down equations of the above wave and sketch the waveform in frequency domain and also find its bandwidth.

Sol:- Given: $A_c = 12V$, $f_c = 10\times10^6 Hz$, $\mu = 50\% = 0.5$

* Equation of carrier wave is

$$c(t) = A_c \cos 2\pi f_c t$$

$$\therefore \boxed{c(t) = 12 \cos[2\pi\times10\times10^6]t}$$

* W.K.T $\mu = \frac{A_m}{A_c}$

$$A_m = \mu \times A_c = 0.5\times12$$

$$\boxed{A_m = 6V}$$

The Amplitude of modulating wave is 6V with the frequency 1KHz

∴ Modulating wave is

$$m(t) = A_m \cos 2\pi f_m t$$

$$\boxed{m(t) = 6 \cos[2\pi\times1\times10^3]t}$$

∴ The amplitude modulated wave is given by

$$S(t) = A_c[1+\mu\cos(2\pi f_m)t].\cos(2\pi f_c)t$$

$$\boxed{S(t) = 12[1+0.5\cos(2\pi\times1\times10^3)t]\cos(2\pi\times10\times10^6)t}$$

To sketch the frequency spectrum we need f_{USB} & f_{LSB}.

$$f_{USB} = f_c + f_m = 10\times10^6 + 1\times10^3$$

$$\boxed{f_{USB} = 10.001\times10^6\ Hz}$$

$$f_{LSB} = f_c - f_m = 10\times10^6 - 1\times10^3$$

$$\boxed{f_{LSB} = 9.999\times10^6}$$

Sideband amplitude is $= \dfrac{\mu A_c}{2} \rightarrow ①$

W.K.T $\mu = \dfrac{A_m}{A_c} \rightarrow ②$

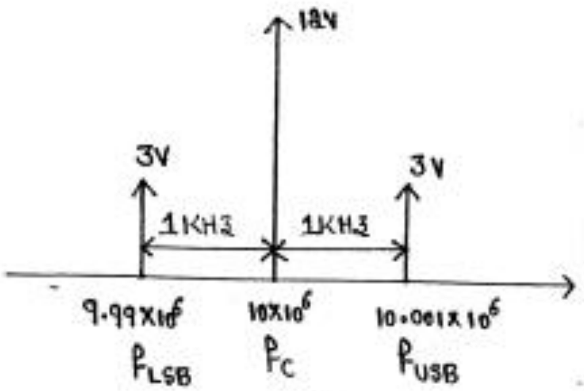
Substituting 'μ' value in eq ①

$$= \frac{A_m}{\cancel{A_c}} \cdot \frac{\cancel{A_c}}{2}$$

$$= \frac{A_m}{2}$$

$$= \frac{6}{2}$$

$$\boxed{\text{Sideband amplitude} = 3V}$$



Bandwidth :-

$$BW = 2f_m = 2\times1\times10^3$$

$$\boxed{BW = 2KHz}$$

Prove that efficiency of ordinary AM is given by:

$$\eta = \frac{P_S}{P_T} \times 100 = \frac{\mu^2}{2+\mu^2} \times 100 \quad \text{for } \mu \leq 1.$$

Where,

P_S; Power carried by the sidebands

P_T; Power (total) in the AM signal.

Further i) Find η for $\mu = 0.5$ or 50% modulation

ii) Show that for a single tone AM η_{max} is 33.3% at $\mu = 1$.

* Efficiency is given by,

$$\eta = \frac{P_S}{P_T} = \frac{P_{USB} + P_{LSB}}{P_T} \rightarrow (1)$$

W.K.T

$$P_S = P_{USB} + P_{LSB}$$

i.e. $P_{USB} = \frac{\mu^2 A_c^2}{8R}$

$$P_{LSB} = \frac{\mu^2 A_c^2}{8R} \quad \&$$

$$P_T = P_c\left(1 + \frac{\mu^2}{2}\right)$$

Substituting the value of P_T, P_{USB} & P_{LSB} in eq (1), we get

$$\eta = \frac{\frac{\mu^2 A_c^2}{8R} + \frac{\mu^2 A_c^2}{8R}}{P_c(1+\mu^2/2)}$$

$$= \frac{\frac{\cancel{2}\mu^2 A_c^2}{4\cancel{8}R}}{P_c\left[\frac{2+\mu^2}{2}\right]} = \frac{\frac{\mu^2 A_c^2}{4R}}{P_c\left[\frac{2+\mu^2}{2}\right]}$$

$$= \frac{\frac{\mu^2}{2}\left[\frac{A_c^2}{2R}\right]}{P_c\left[\frac{2+\mu^2}{2}\right]} \qquad \because P_c = \frac{A_c^2}{2R}$$

$$= \frac{\cancel{P_c}\frac{\mu^2}{2}}{\cancel{P_c}\left[\frac{2+\mu^2}{2}\right]}$$

$$= \frac{\frac{\mu^2}{\cancel{2}}}{\frac{2+\mu^2}{\cancel{2}}}$$

$$\boxed{\eta = \frac{\mu^2}{2+\mu^2}}$$

i) For $\mu = 0.5$

$$\eta = \frac{\mu^2}{2+\mu^2} = \frac{(0.5)^2}{2+(0.5)^2} = 0.111$$

$$\boxed{\%\,\eta = 11.11\,\%}$$

ii) For $\mu = 1$

$$\eta = \frac{\mu^2}{2+\mu^2} = \frac{(1)^2}{2+(1)^2} = 0.33$$

$$\boxed{\%\,\eta = 33.33\,\%}$$

Draw the AM waveforms for less than 100%, with 100%, more than 100% & with 0% percentage modulation.
Assume that the modulating Signal is a pure Sine wave.

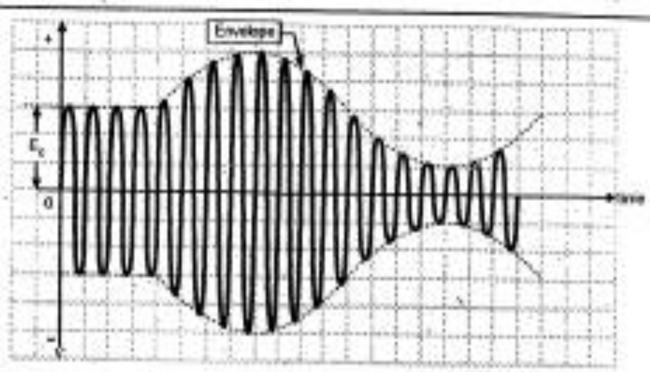
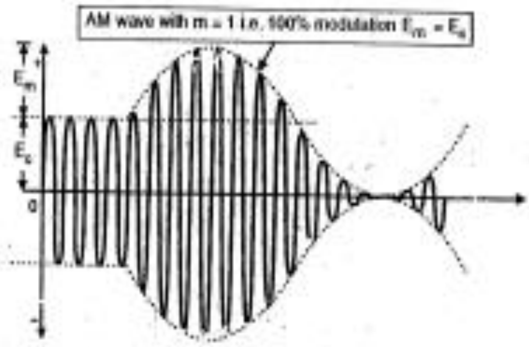
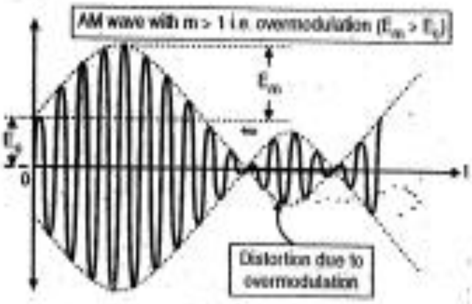


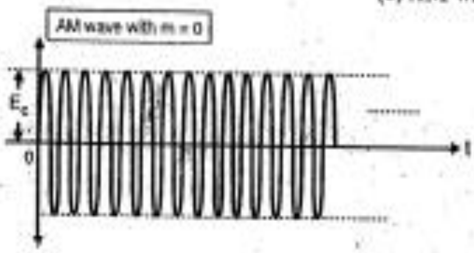
Fig. P. 2.2.1(a) : AM wave for percentage modulation less than 100 %



(b) AM wave with 100% modulation



(c) AM with over modulation



(d) AM wave with m = 0

Envelope Detector

* For minimum clippings, the time Constant $R_L C_S$ Should be in between the time period of I/p & O/p Signal. If O/p Signal m(t) ranges from 0 to W Hz & I/p is at frequency f_c,

$$\boxed{\frac{1}{f_c} < R_L C_S < \frac{1}{W}}$$

Formula:

* The Condition for minimum distortion is :

$$\boxed{R_L C_S \leqslant \frac{1}{2\pi f_m} \frac{\sqrt{1-\mu^2}}{\mu}}$$

Show that in an envelope detector circuit the demodulator is to follow the envelope of m(t), it is required that at any time

$$\frac{1}{R_L C_S} \geqslant \frac{\omega_m \mu \sin\omega_m t}{1+\mu \cos\omega_m t}$$

Sol:- Let us assume that the Capacitor discharges from the peak value 'E' at some arbitrary instant t=0. Then the voltage across the Capacitor 'V_c' is given by

$$V_c = E e^{-t/R_L C_S} \rightarrow ①$$

Using Taylor Series

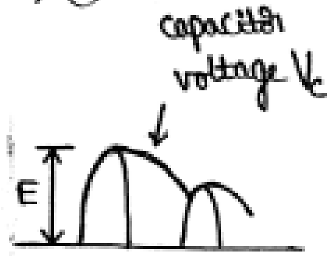
$$V_c \cong E\left[1 - \frac{t}{R_L C_S}\right] \rightarrow ②$$

(differentiating equation ② w.r.to dt)

* The Slope of discharge is

$$\frac{dV_c}{dt} = -\frac{E}{R_L C_s} \rightarrow ③$$

* The amplitude 'E' at any instant is



$$E = A_c[1 + \mu \cos 2\pi f_m t] \rightarrow ④$$

* The Slope of this envelope is (differentiating eq ④)

$$\frac{dE}{dt} = -\mu A_c \sin 2\pi f_m t \cdot (2\pi f_m) \rightarrow ⑤$$

* In order for the Capacitor to follow the envelope 'E(t)', the magnitude of the Slope of the $R_L C_s$ discharge must be greater than the magnitude of the Slope of the envelope E(t). Hence

$$\left|\frac{dV_c}{dt}\right| \geq \left|\frac{dE}{dt}\right| \rightarrow ⑥$$

Substituting eq ③ & ⑤ in eq ⑥, we get

$$\frac{E}{R_L C_s} \geq \mu A_c \sin 2\pi f_m t \cdot (2\pi f_m) \rightarrow ⑦$$

Substituting eq ④ in eq ⑦, we get

$$\frac{A_c[1 + \mu \cos 2\pi f_m t]}{R_L C_s} \geq \mu A_c \sin 2\pi f_m t \cdot (2\pi f_m)$$

$$\frac{1}{R_L C_s} \geq \frac{\mu \cancel{A_c} \sin 2\pi f_m t \cdot (2\pi f_m)}{\cancel{A_c}[1 + \mu \cos 2\pi f_m t]}$$

$$\boxed{\frac{1}{R_L C_s} \geq \frac{\omega_m \mu \sin \omega_m t}{1 + \mu \cos \omega_m t}}$$

W.K.T
$\omega_m = 2\pi f_m$

* The AM wave $10[1+0.5\cos(2\pi 500t)]\cdot\cos(2\pi 10^6 t)$ is demodulated by an envelope detector. Find the time constant τ and the resistor if capacitor used is 100 pF.

Sol:- Given: $f_m = 500\,Hz$, $f_c = 10^6\,Hz$, $\mu = 0.5$ & $C_s = 100\,pF$

The time constant $\tau = R_L C_s$ should satisfy the condition

$$\frac{1}{f_c} < R_L C_s < \frac{1}{f_m} \quad \text{and}$$

$$R_L C_s \leq \frac{1}{2\pi f_m} \frac{\sqrt{1-\mu^2}}{\mu}$$

$$\leq \frac{1}{2\pi \times 500} \cdot \frac{\sqrt{1-(0.5)^2}}{0.5} \leq \frac{\sqrt{0.75}}{500\pi}$$

$$R_L C_s \leq 5.51 \times 10^{-4}$$

Time constant $\tau = 5.51 \times 10^{-4}$ Sec

for $C_s = 100\,pF$

$$R_L \leq \frac{5.51 \times 10^{-4}}{C_s} \leq \frac{5.51 \times 10^{-4}}{100 \times 10^{-12}}$$

$$\boxed{R_L = 5.51 M\Omega}$$

Explain the detection of message signal from the amplitude modulated signal using an envelope detector and bringout the significance of the RC time constant of the circuit in detection of the message signal without distortion. Estimate this for f_m=3KHz and f_c=100KHz. **Jan-06,6M**

Sol:- i) Explain envelope detector

ii) Given : $f_m = 3KHz$, $f_c = 100KHz$.

W.K.T for correct demodulation, it is required that,

$$\frac{1}{f_c} \ll RC \ll \frac{1}{f_m}$$

$$\frac{1}{100\times10^3} \ll RC \ll \frac{1}{3\times10^3}$$

$$\boxed{0.01\,msec \ll RC \ll 0.33\,msec.}$$

* The time constant $\tau = RC$ Should Satisfy the Condition

$$RC \leq \frac{1}{2\pi f_m} \quad \frac{\sqrt{1-\mu^2}}{\mu}$$

assuming $\mu = 0.5$

$$RC \leq \frac{1}{2\pi \times 3\times10^3} \cdot \frac{\sqrt{1-(0.5)^2}}{0.5}$$

$$\boxed{RC \leq 0.27\,msec}$$

* In the absence of modulation Index,

$$RC \leq \frac{1}{2\pi f_m}$$

$$RC \leq \frac{1}{2\pi \times 3\times10^3}$$

$$\boxed{RC = 0.05\,msec}$$

DSB-SC Modulator Problems

1. A message signal m(t) with spectrum shown in fig.1 is applied to a product modulator with a carrier wave $A_c\cos(2\pi f_c t)$ producing the DSB-SC modulated wave s(t). This modulated wave is next applied to a coherent detector. Assuming a perfect coherence between the transmitter and the receiver, determine the spectrum of the detector output when

   i. fc=1.25KHz — Jan-06,6M

   ii. fc=.75KHz and sketch the same

   iii. The lowest fc so that m(t) is uniquely determined from s(t)

2. Consider a message signal m(t) with a spectrum shown in fig.2. The message bandwidth w=1KHz. This signal is applied to a product modulator, together with a carrier wave $A_c\cos(2\pi f_c t)$, producing the DSB-SC modulated signal s(t). The modulated signal is next applied to a coherent detector. Assuming perfect synchronism between the carrier waves in the modulator and detector. Determine the spectrum of the detector output when:

   i. fc = 1.25KHz   ii. fc= 0.75KHz.

   What is the lowest carrier frequency for which each component of the modulated signal s(t) is uniquely determined by m(t).

Jan-09,6M

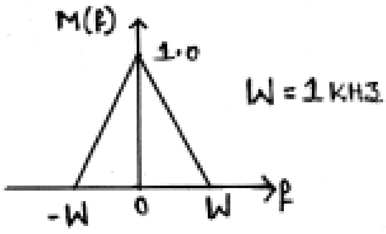


fig ①

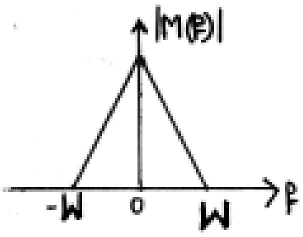


fig ②

Given :- $W = f_m = 1KHz$

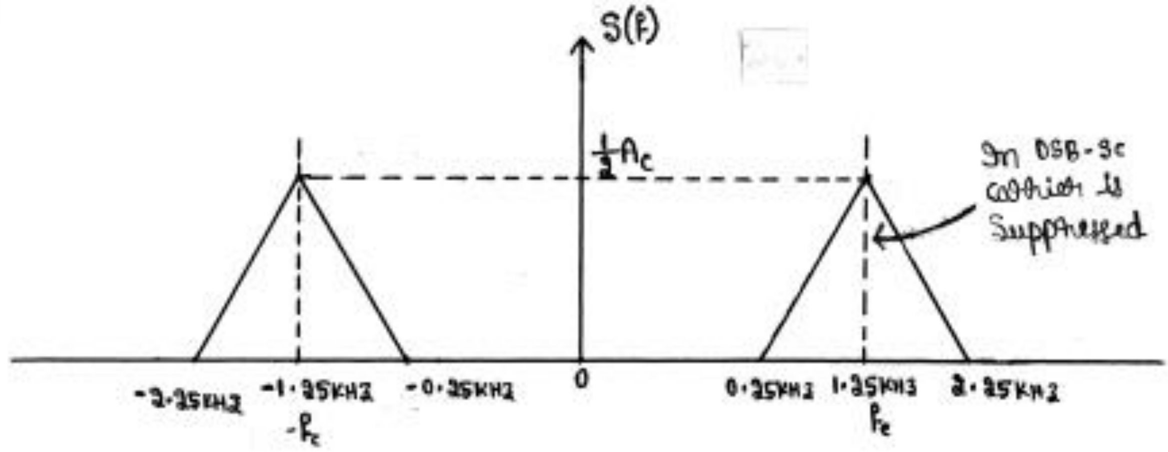
i) When $f_c = 1.25$ KHz

$f_c + W = 1.25KHz + 1KHz = 2.25KHz$

$f_c - W = 1.25KHz - 1KHz = 0.25KHz$

$-f_c + W = -1.25KHz + 1KHz = -0.25KHz$

$-f_c - W = -1.25KHz - 1KHz = -2.25KHz$



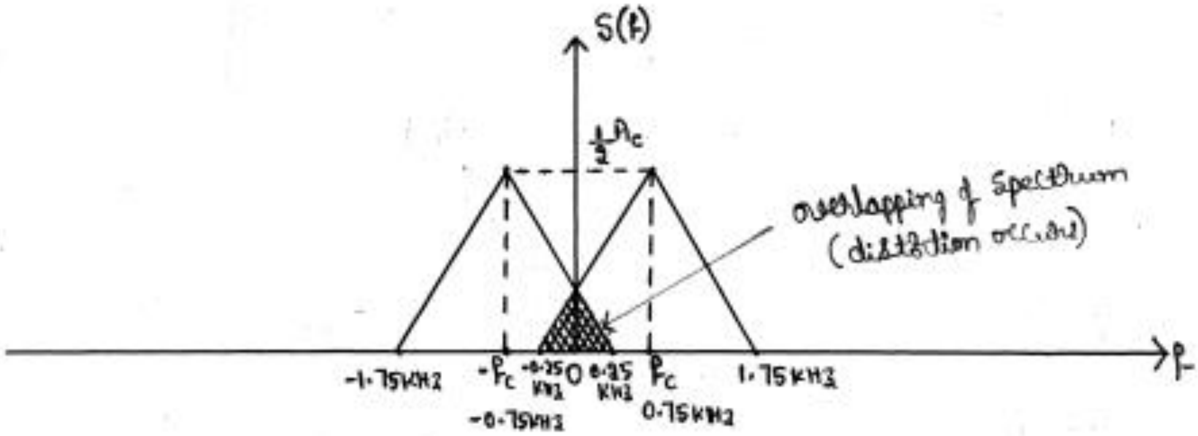
ii) When $f_c = 0.75KHz$:

$f_c + W = 0.75KHz + 1KHz = 1.75KHz$

$f_c - W = 0.75KHz - 1KHz = -0.25KHz$

$-f_c + W = -0.75KHz + 1KHz = 0.25KHz$

$-f_c - W = -0.75KHz - 1KHz = -1.75KHz$



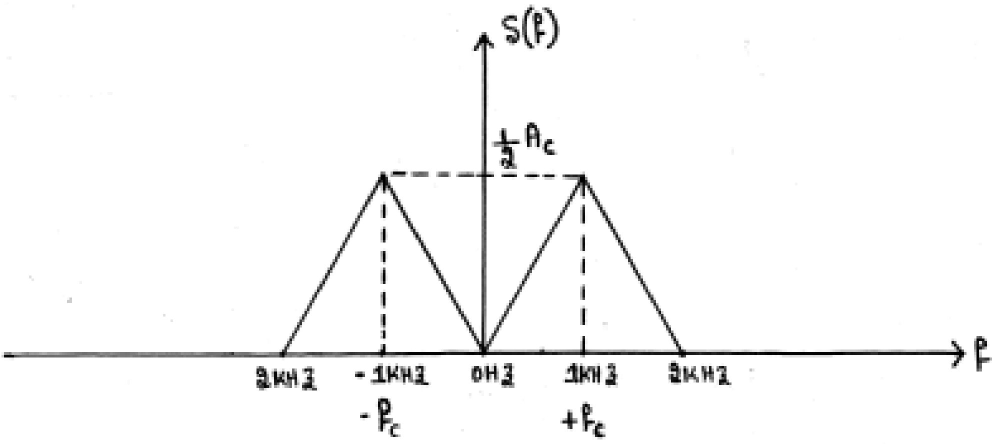
iii) The lowest f_c so that $m(t)$ is uniquely determined from $S(t)$ is 1KHz i.e. $\boxed{f_c = 1KHz}$

$f_c + W = 1KHz + 1KHz = 2KHz$

$f_c - W = 1KHz - 1KHz = 0Hz$

$-f_c + W = -1KHz + 1KHz = 0Hz$

$-f_c - W = -1KHz - 1KHz = -2KHz$



$\therefore$ Lowest f_c is $\underline{1KHz}$ so that $m(t)$ is uniquely determined from $S(t)$.

- **Consider a resultant wave obtained by adding a non-coherent carrier $A_c\cos(2\pi f_c t + \phi)$ to a DSB-SC wave $\cos(2\pi f_c t)\ m(t)$. This composite wave is applied to an ideal envelope detector. Find the resulting detector output. Evaluate this output for $\phi=0$.**

Jan-10,8M

Sol:-

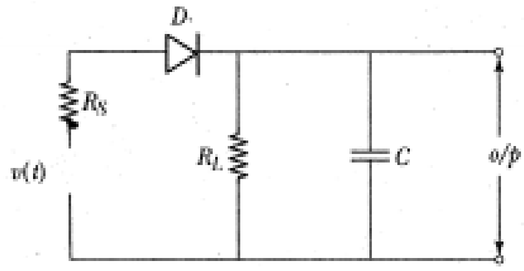


Figure P2.6 ■ Envelope detector

The I/p to the envelope detector is the sum of non-coherent carrier and a DSBSC wave:

$$V(t) = A_c \underline{\cos(2\pi f_c t + \phi)} + m(t)\cos 2\pi f_c t$$

WKT $\cos(A+B) = \cos A \cdot \cos B - \sin A \sin B$

$$V(t) = A_c \cos 2\pi f_c t \cos\phi - A_c \sin 2\pi f_c t \cdot \sin\phi + m(t) \cos 2\pi f_c t$$

$$V(t) = \cos 2\pi f_c t \left[A_c \cos\phi + m(t) \right] - A_c \sin 2\pi f_c t \cdot \sin\phi$$

The o/p of the envelope detector is

$$a(t) = \sqrt{(\text{Inphase Components})^2 + (\text{Quadrature Components})^2}$$

$$= \sqrt{\left[A_c \cos\phi + m(t)\right]^2 + \left[A_c \sin\phi\right]^2}$$

When $\phi = 0$, We get

$$= \sqrt{\left[A_c \cos(0) + m(t)\right]^2 + \left[A_c \sin(0)\right]^2}$$

$$= \sqrt{\left[A_c(1) + m(t)\right]^2 + \left[A_c(0)\right]^2}$$

$$= \sqrt{\left[A_c + m(t)\right]^2}$$

$$\boxed{a(t) = A_c + m(t)}$$

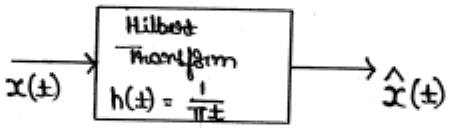
Hence, the o/p of the envelope detector contains the message signal $m(t)$.

Hilbert Transform:-

* The device which produces a phase shift of $-90°$ for all +ve frequencies & a phase shift of $+90°$ for all -ve frequencies.

The amplitude of all frequency components of the I/p signal are unaffected by transmission through device.

Such an ideal device is called a Hilbert transform.



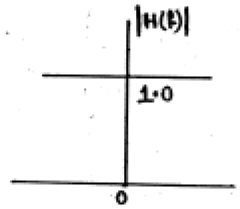


Fig ⓑ: Amplitude response

$$\hat{x}(t) = x(t) * h(t)$$

$$\hat{x}(t) = x(t) * \frac{1}{\pi t}$$

$$\boxed{\hat{x}(t) = \frac{1}{\pi}\int_{-\infty}^{+\infty} \frac{x(\tau)}{(t-\tau)}\, d\tau}$$

Where, $\hat{x}(t)$ is the hilbert transform of $x(t)$

INVERSE Hilbert transform :-

We can recover back the original signal $x(t)$ back from $\hat{x}(t)$ by taking the inverse hilbert transform as follows:

$$\boxed{x(t) = -\frac{1}{\pi}\int_{-\infty}^{\infty} \frac{\hat{x}(\tau)}{(t-\tau)}\, d\tau}$$

Interpretation of Hilbert Transform :-

The Fourier transform of $x(t)$ & $\frac{1}{\pi t}$ are as follows:

$$\boxed{\begin{array}{l} x(t) \xrightarrow{FT} X(f) \\ \frac{1}{\pi t} \xrightarrow{FT} -j\,\mathrm{Sgn}(f) \end{array}}$$

Where Sgn is the Signum function defined as

$$\mathrm{Sgn} = \begin{cases} 1 & , f>0 \\ 0 & , f=0 \\ -1 & , f<0 \end{cases}$$

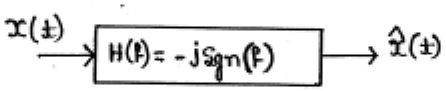
$$\therefore \boxed{\hat{x}(t) = x(t) * \frac{1}{(\pi t)}} \longrightarrow ①$$

Taking Fourier Transform on both Side of eq①, we get

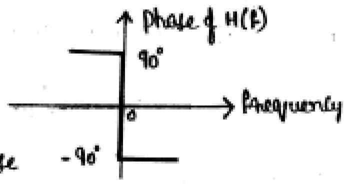
$$\hat{X}(f) = X(f)\left[-j\,\mathrm{Sgn}(f)\right]$$

$$\therefore \boxed{\hat{X}(f) = -j\,\mathrm{Sgn}(f) \cdot X(f)} \longrightarrow ②$$

Thus the hilbert transform $\hat{x}(t)$ of Signal $x(t)$ is obtained by passing $x(t)$ through a linear two port device whose transfer function is equal to $-j\,\mathrm{Sgn}(f)$ as shown below.



Fig①: Two port device



Fig② Phase response

Properties of Hilbert Transform:-

❖ **Define Hilbert transform. State and prove the properties of Hilbert transform.**

July-09,5M	July-07,4M	June-10,4M	Jan-10,4M	Jan-09,6M

Property 1:-

It states that the signal $x(t)$ & its hilbert transform $\hat{x}(t)$ have the same amplitude spectrum.

Proof:-

* Fourier transform of $\hat{x}(t) = \hat{X}(f) = -j\,\mathrm{sgn}(f) \cdot X(f)$
* The magnitude of $-j\,\mathrm{sgn}(f)$ is equal to 1 for all values of 'f'.

$$\therefore \quad |\hat{X}(f)| = (1)\,|X(f)| \qquad \because \boxed{|-j\,\mathrm{sgn}(f)| = 1}$$

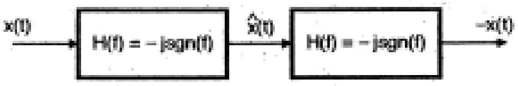
$$\boxed{|\hat{X}(f)| = |X(f)|}$$

$\therefore$ The amplitude spectrum of $\hat{x}(t)$ is equal to $x(t)$.

Property 2:-

This property states that if $\hat{x}(t)$ is the hilbert transform of $x(t)$ then the hilbert transform of $\hat{x}(t)$ is $-x(t)$.

Proof:-



Cascading the ideal two port devices to obtain the double Hilbert transform

The property-2 suggests that the hilbert transform is taken twice as shown in above figure.

$$H(f) * H(f) = -j\,\mathrm{sgn}(f) \times -j\,\mathrm{sgn}(f)$$
$$= +j^2\,\mathrm{sgn}^2(f)$$

but $\mathrm{sgn}^2(f) = 1$ & $j^2 = -1$

$$H'(f) = H(f) * H(f) = -1$$

$\therefore$ $H'(f) = -1$ for all the values of 'f'.

Hence the FT of o/p is

$$X(f) \times H'(f) = -X(f)$$

$$-X(f) \xrightarrow{IFT} -x(t)$$

* Thus the hilbert transform of $\hat{x}(t)$ is equal to $-x(t)$.

Property - 3 :-

This property states that the signal $x(t)$ & its Hilbert transform $\hat{x}(t)$ are orthogonal.

Proof :- We have to prove that $\int_{-\infty}^{\infty} x(t) \cdot \hat{x}(t)\, dt = 0$

W.K.T $x(t) \xrightarrow{FT} X(f)$

$\hat{x}(t) \xrightarrow{FT} \hat{X}(-f)$

$$\therefore \int_{-\infty}^{\infty} x(t) \cdot \hat{x}(t) \cdot dt = \int_{-\infty}^{\infty} X(f) \cdot \hat{X}(f) \cdot df$$

W.K.T
$$\boxed{\begin{aligned} \hat{X}(f) &= -j\,sgn(f) \cdot X(f) \\ \hat{X}(-f) &= -j\,sgn(-f) \cdot X(-f) \\ \hat{X}(-f) &= +j\,sgn(f)\, X(-f) \end{aligned}}$$

$\|^{ly}$ $\qquad -j\,sgn(-f) = +j\,sgn(f)$

$$\int_{-\infty}^{\infty} x(t) \cdot \hat{x}(t) \cdot dt = \int_{-\infty}^{\infty} j\,sgn(f)\, X(f) \cdot X(-f) \cdot df$$

but $\boxed{X(f) \cdot X(-f) = |X(f)|^2}$

$$\boxed{\int_{-\infty}^{\infty} x(t).\hat{x}(t).\,dt = \int_{-\infty}^{\infty} j\,\mathrm{Sgn}(f)\,|X(f)|^2\,df} \longrightarrow (1)$$

Eq (1) is an product of odd & even function.

Where, $\mathrm{Sgn}(f)$ is an odd function &

$|X(f)|^2$ is an even function.

* The Integration of an odd function over the range $-\infty$ to $+\infty$ will yield to a Zero value.

$$\therefore \boxed{\int_{-\infty}^{\infty} x(t).\,\hat{x}(t)\,dt = 0}$$ Hence proved.

Additional properties of Hilbert transform :-

1) The magnitude Spectra of a Signal $x(t)$ & its hilbert transform $\hat{x}(t)$ are Identical.

2) The hilbert transform of an even function is odd and vice-versa.

3) The hilbert transform of a real Signal is also real.

---

pre-envelope :-

For a real valued Signal $x(t)$, its pre-envelope is defined as the Complex valued function Such that

$$\boxed{x_+(t) = x(t) + j\hat{x}(t)} \longrightarrow (1)$$

Where $x(t)$ is the real part of the pre-envelope & $\hat{x}(t)$ - represents the imaginary part of the pre-envelope.

* Let $X_+(f)$ represents the FT of $x_+(t)$ & is given by

$$X_+(f) = F[x(t) + j\hat{x}(t)]$$

$$X_+(f) = X(f) + j\hat{X}(f) \longrightarrow ②$$

W.K.T

$$\hat{X}(f) = -j\,Sgn(f) \cdot X(f) \longrightarrow ③$$

Substituting eq ③ in eq ②, we get

$$X_+(f) = X(f) + j[-j\,Sgn(f) \cdot X(f)]$$

$$= X(f) - j^2 Sgn(f) \cdot X(f) \qquad j^2 = -1$$

$$= X(f) + Sgn(f) X(f) \qquad -j^2 = 1$$

$$\boxed{X_+(f) = X(f)[1 + Sgn(f)]} \longrightarrow ④$$

Where

$$Sgn(f) = \begin{cases} 1, & \text{for } f > 0 \\ 0, & \text{for } f = 0 \\ -1, & \text{for } f < 0 \end{cases}$$

Substituting the values of $Sgn(f)$ in eq ④, we get

i) $Sgn(f) = 1$, for $f > 0$

$$X_+(f) = X(f)[1+1]$$

$$\boxed{X_+(f) = 2X(f)}$$

ii) $Sgn(f) = 0$, for $f = 0$

$$X_+(f) = X(0)[1+0]$$

$$\boxed{X_+(f) = X(0)}$$

iii) $Sgn(f) = -1$, for $f < 0$

$$X_+(f) = X(f)[1-1]$$

$$\boxed{X_+(f) = 0}$$

$$\boxed{X_+(f) = \begin{cases} 2X(f), & \text{for } f > 0 \\ X(0), & \text{for } f = 0 \\ 0, & \text{for } f < 0 \end{cases}}$$

Where $X(0)$ is the value of $X(f)$ at $f = 0$.

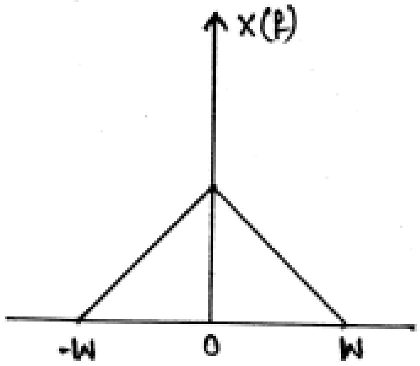


Fig ⓐ: Amplitude Spectrum of Low pass Signal $x(t)$

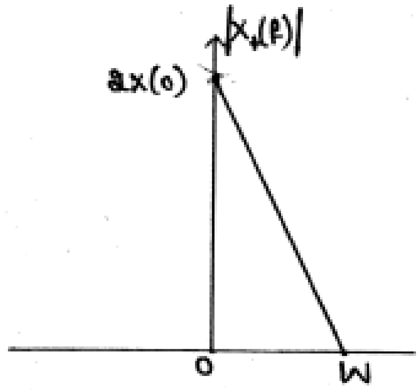


Fig ⓑ: Amplitude Spectrum of the pre-envelope $x_+(t)$.

Pre-envelope for -ve frequencies :-

* The pre-envelope for -ve frequencies is defined as:

$$\boxed{x_-(t) = x(t) - j\hat{x}(t)}$$

* The two pre-envelopes $x_+(t)$ & $x_-(t)$ are simply the complex conjugates of each other:

$$\boxed{x_-(t) = x_+^*(t)}$$

i.e. $X_-(f) = 0$, for $f>0$

$= X(0)$, for $f=0$

$= 2X(f)$, for $f<0$

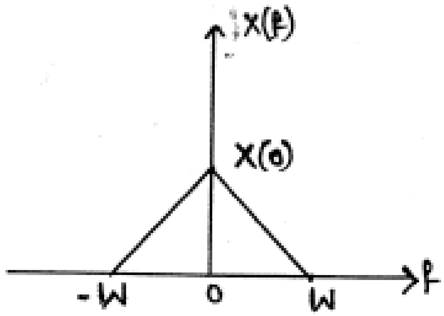


Fig ⓐ: Amplitude Spectrum of Low pass Signal $x(t)$

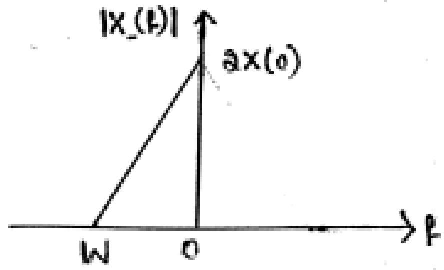


Fig ⓑ: Amplitude Spectrum of the pre-envelope $X_-(f)$.

NOTE :-

* The Sum of $x_+(t)$ & $x_-(t)$ is given by

$$x_+(t) + x_-(t) = [x(t) + j\hat{x}(t)] + [x(t) - j\hat{x}(t)]$$

$$= x(t) + \cancel{j\hat{x}(t)} + x(t) - \cancel{j\hat{x}(t)}$$

$$\boxed{x_+(t) + x_-(t) = 2x(t)}$$

* What is the Indication of $(\pm)$ Sign in the Subscript of the pre-envelope?

+ Sign:-

The $(+)$ Sign in the Subscript of pre-envelope '$x_+(t)$' indicates that the Spectrum of pre-envelope $x_+(t)$ is Non-Zero, only for the +ve frequencies.

− Sign :-

The $(-)$ Sign in the Subscript of pre-envelope '$x_-(t)$' indicates that the Spectrum of pre-envelope $x_-(t)$ is Non-Zero, only for the −ve frequencies.

Canonical representation of Band pass Signals:-

* Let the pre-envelope of a Narrow band Signal $x(t)$, with its FT $X(f)$ centered about Some frequency $\pm f_c$, be expressed in the form

$$x_+(t) = \tilde{x}(t) \exp(j2\pi f_c t)$$

$$\boxed{x_+(t) = \tilde{x}(t)\, e^{j2\pi f_c t}} \longrightarrow (1)$$

Where $\tilde{x}(t)$ is the complex envelope of the Signal.

Eq ① is the definition for the Complex envelope $\tilde{x}(t)$ in terms of the pre-envelope $x_+(t)$.

* The Spectrum of $x_+(t)$ is limited to the frequency band $f_c - W \leq f \leq f_c + W$ as Shown in fig ⓑ.

* Applying frequency - Shifting property of the Fourier transform to eq ①, then the Spectrum of the Complex envelope $\tilde{x}(t)$ is limited to the band $-W \leq f \leq W$ & Centered at the origin as Shown in fig ⓒ.

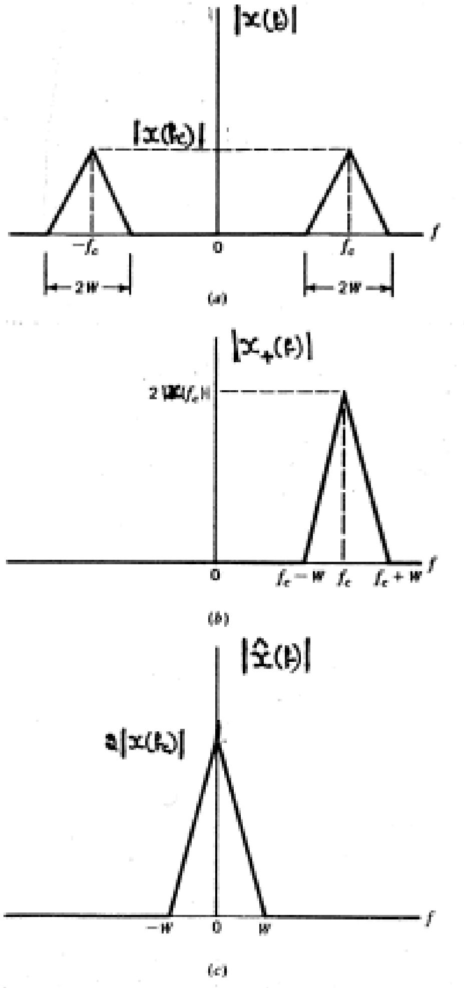


Figure . (a) Amplitude spectrum of band-pass signal $x(t)$. (b) Amplitude spectrum of pre-envelope $x_+(t)$. (c) Amplitude spectrum of complex envelope $\tilde{x}(t)$.

* The Signal $x(t)$ is the real part of the pre-envelope $x_+(t)$. Hence the given bandpass Signal $x(t)$ can be expressed interms of the Complex envelope as:

$$\boxed{x(t) = R_e\left[\hat{x}(t)\, e^{j2\pi f_c t}\right]} \longrightarrow (1)$$

* In general, $\hat{x}(t)$ is a Complex quantity, we can express it as:

$$\boxed{\hat{x}(t) = x_I(t) + j x_Q(t)} \longrightarrow (2)$$

Substituting eq (2) in eq (1), we get

$$x(t) = R_e\left[x_I(t) + j x_Q(t)\, e^{j2\pi f_c t}\right] \longrightarrow (3)$$

W.K.T $\quad e^{j\theta} = \cos\theta + j\sin\theta \qquad \theta = 2\pi f_c t$

$$\boxed{e^{j2\pi f_c t} = \cos(2\pi f_c t) + j\sin(2\pi f_c t)} \longrightarrow (4)$$

Substituting eq (4) in eq (3), we get

$$x(t) = R_e\left\{\left[x_I(t) + j x_Q(t)\right]\left(\cos(2\pi f_c t) + j\sin(2\pi f_c t)\right)\right\}$$

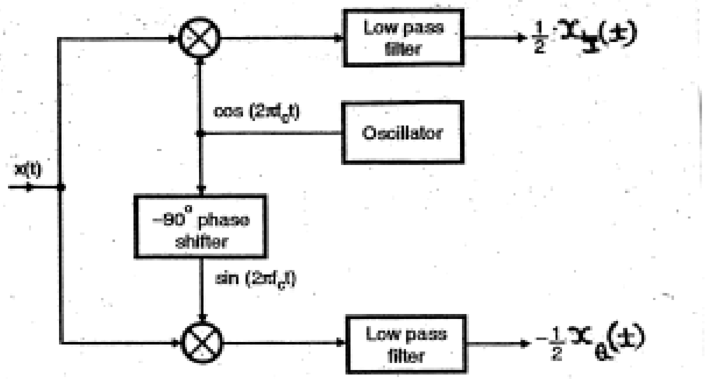
$$x(t) = R_e\left\{x_I(t)\cos(2\pi f_c t) + j x_I(t)\sin(2\pi f_c t) + j x_Q(t)\cos(2\pi f_c t) + j^2 x_Q(t)\sin(2\pi f_c t)\right\}$$

$$x(t) = R_e\left\{x_I(t)\cos(2\pi f_c t) + j x_I(t)\sin(2\pi f_c t) + j x_Q(t)\cos(2\pi f_c t) + (-1) x_Q(t)\sin(2\pi f_c t)\right\}$$

$$\boxed{x(t) = x_I(t)\cos 2\pi f_c t - x_Q(t)\sin(2\pi f_c t)} \longrightarrow (5)$$

* In eq (5) $x_I(t)$ is the in-phase component of the band pass Signal & $x_Q(t)$ is the Quadrature of the Signal $x(t)$.

Generation of In-Phase and Quadrature phase components:-

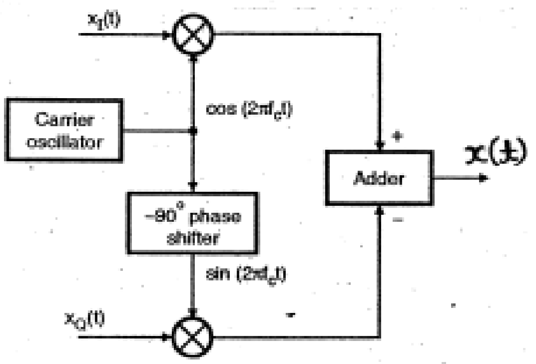


Scheme to generate the in phase and quadrature components of bandpass signal x (t)

* The $x_I(t)$ & $x_Q(t)$ are low pass Signals limited to the band $-W \le f \le W$. The bandwidth of each filter is 'W'.

* The in-phase Component $x_I(t)$ is produced by multiplying $x(t)$ with $\cos(2\pi f_c t)$ & passing the product through a LPF.

* The Quadrature Component $x_Q(t)$ is obtained by multiplying $x(t)$ with $\sin(2\pi f_c t)$ & passing the product through an Identical LPF.



Reconstruction of x (t) from x_I (t) and x_Q (t)

* The in-phase low pass Signal $x_I(t)$ & $x_Q(t)$ are multiplied with the $\cos(2\pi f_c t)$ & $\sin(2\pi f_c t)$ respectively.

* The resultant product terms are then Subtracted to get the bandpass Signal $x(t)$.

* The multiplication process of $x_I(t)$ & $x_Q(t)$ with the carriers is a linear modulation process.

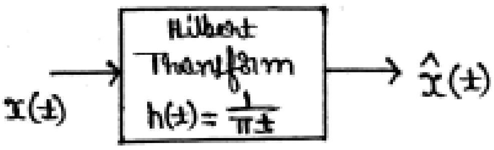
❖ **Define:**

i. **Hilbert transform**
ii. **Pre-envelope**
iii. **Complex envelope**

July-09,6M July-05,6M

i) Hilbert transform :-

When phase angles of all components of a given signal are shifted by $\pm 90^\circ$, the resulting function of time is known as Hilbert transform.



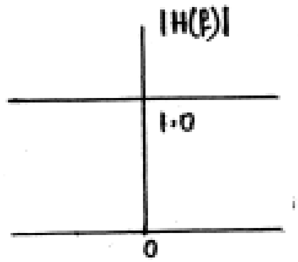


fig (a): Amplitude response.

$$\hat{x}(t) = x(t) * h(t)$$

$$\hat{x}(t) = x(t) * \frac{1}{\pi t}$$

$$\boxed{\hat{x}(t) = \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{x(\tau)}{(t-\tau)} \cdot d\tau} \longrightarrow (1)$$

Where, $\hat{x}(t)$ is the hilbert transform of $x(t)$

* Taking Fourier transform on both sides of eq (1), we get

$$\boxed{\hat{X}(f) = -j\,\mathrm{Sgn}(f) \cdot X(f)} \longrightarrow (2)$$

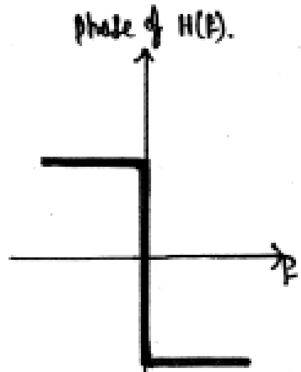


fig (b): phase response of H(f)

ii) Pre-envelope :-

The pre-envelope of the Signal $x(t)$, is defined as the Complex-valued function can be given as:

$$\boxed{x_+(t) = x(t) + j\hat{x}(t)} \longrightarrow ①$$

Where,

$x(t)$ is the real part of the pre-envelope &

$j\hat{x}(t)$ represents the Imaginary part of the pre-envelope.

$\hat{x}(t)$ is the hilbert transform of $x(t)$.

Taking FT of $x_+(t)$ & is given by

$$X_+(f) = X(f)\,[1 + \text{Sgn}(f)] \longrightarrow ②$$

Substituting Sgn(f) value in eq ②, we get

$$X_+(f) = \begin{cases} 2X(f), & \text{for } f > 0 \\ X(0), & \text{for } f = 0 \\ 0, & \text{for } f < 0 \end{cases}$$

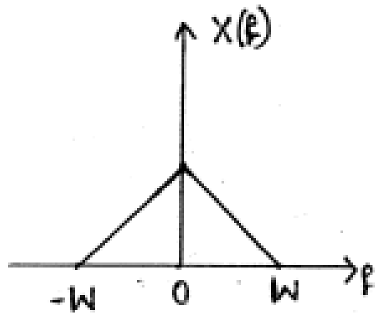


Fig ⓐ: Amplitude Spectrum of Low pass Signal

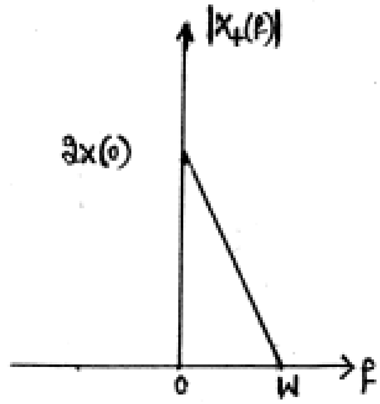


Fig ⓑ: Amplitude Spectrum of the pre-envelope $X_+(t)$.

iii) Complex envelope :-

* The pre-envelope of a Narrow band Signal $x(t)$ expressed in the form

$$\boxed{x_+(t) = \tilde{x}(t)\, e^{j2\pi f_c t}} \longrightarrow ①$$

Where $\tilde{x}(t)$ is the Complex envelope of the Signal.

Eq ① is the definition for the Complex envelope $\tilde{x}(t)$ interms of the pre-envelope $x_+(t)$.

* The bandpass Signal $x(t)$ can be expressed interms of the Complex envelope as:

$$\boxed{x(t) = R_e\left[\hat{x}(t)\, e^{j2\pi f_c t}\right]} \longrightarrow ②$$

In general, $\hat{x}(t)$ is a Complex quantity, we can express it as:

$$\boxed{\hat{x}(t) = x_I(t) + j x_Q(t)} \longrightarrow ③$$

Substituting eq ③ in eq ②, we get

$$\boxed{x(t) = R_e\left[x_I(t) + j x_Q(t)\, e^{j2\pi f_c t}\right]}$$

Single Sideband Modulation (SSB Modulation) :-

* Standard Amplitude modulation & DSB-SC modulation are - wasteful of bandwidth because they both require a transmission bandwidth equal to twice the message bandwidth i.e. $BW = 2f_m$.

* In both case (AM & DSB-SC) half of the transmission bandwidth is occupied by the upper sideband of the modulated wave, where as the other half of the transmission bandwidth is occupied by the lower sideband of the modulated wave.

* The upper & lower sidebands are uniquely related to each other by virtue of their symmetry about the carrier frequency 'f_c'. Thus only one sideband is necessary for transmission of information - & if both the carrier & the other sideband are suppressed at the transmitter, no information is lost.

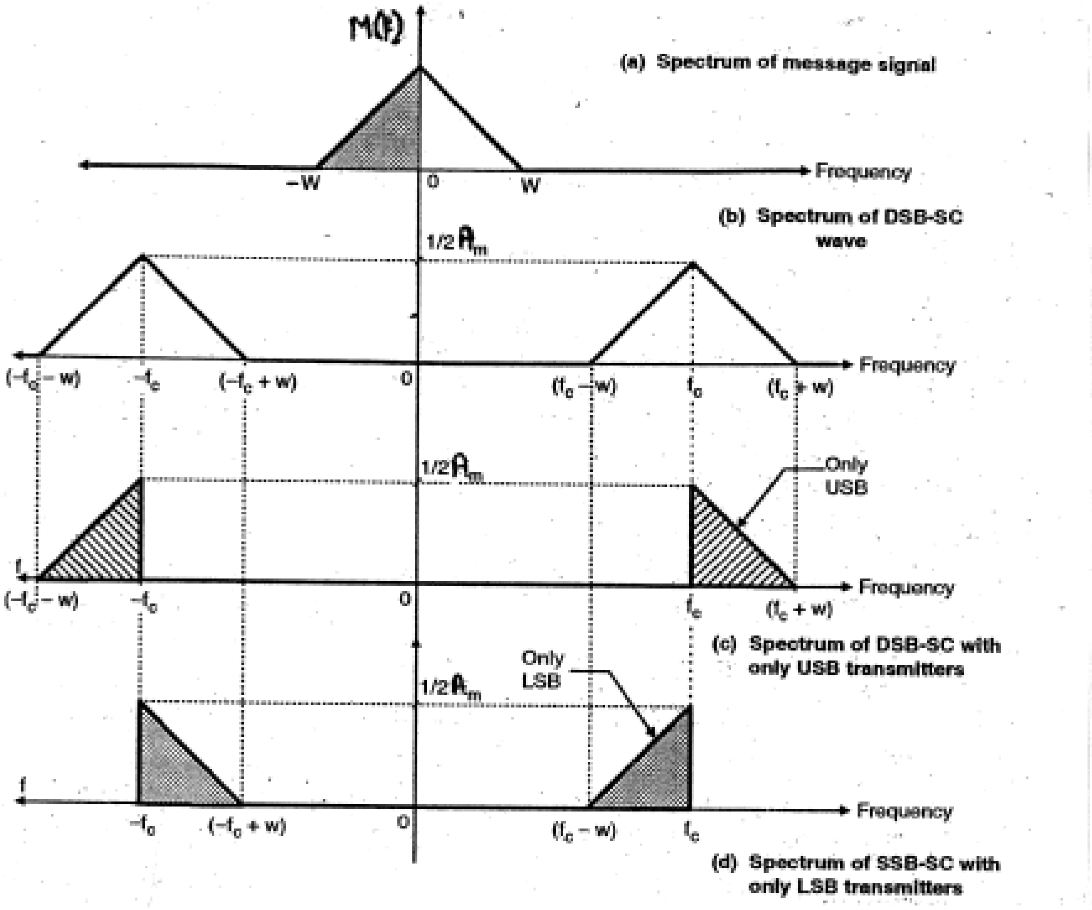
∴ channel required the same bandwidth as the message signal.

* When only one sideband is transmitted, the modulation referred to as Single-Sideband Modulation.

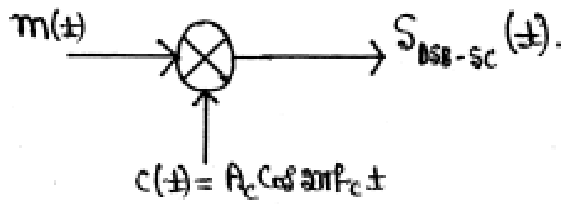
Frequency-domain description of SSB :-

* The frequency-domain description of a SSB modulated wave - depends on which sideband is transmitted.

fig (a) shows spectrum of modulating signal '$M(f)$'. The spectrum is limited to the band $-W \leq f \leq W$.



* The DSB-SC Wave can be obtained by multiplying $m(t)$ by the carrier wave $A_c \cos(2\pi f_c t)$ as shown below.



* Fig (b) Shows the DSB-SC modulated Wave.

In SSB, When only upper Sideband is transmitted, we get frequency Spectrum as Shown in fig (c).

* When only the lower Sideband is transmitted, we get frequency Spectrum as Shown in fig (d).

Advantages of SSB :-

1) SSB required half the bandwidth required of AM & DSB-SC Signals.

2) Due to Suppression of carrier and one Sideband, power is Saved.

3) Reduced Interference of noise. This is due to the reduced bandwidth. As the bandwidth increases, the amount of noise added to the Signal will Increase.

4) Fading does not occur in SSB transmission. (Fading means that a Signal alternately increases & decreases in Strength as it is picked up by the receiver. It occurs because the carrier & Sideband may reach the receiver Shifted in time & phase w.r.to each other.)

Disadvantages of SSB :-

1) The generation & reception of SSB Signal is a Complex process.

2) Since carrier is absent, the SSB transmitter & Receiver need to have an excellent frequency Stability.

3) The SSB modulation is expensive & highly Complex to Implement

Applications of SSB :-

1) SSB transmission is used in the applications where the power Saving is required in mobile Systems.

2) SSB is also used in applications in which bandwidth - requirements are low.

ex:- Point to point Communication, Land, air, maritime mobile communications, TV, Telemetry, military Communications, radio navigation & amateur radio.

FORMULAE

1) $S_u(t) = R_e\left[\tilde{S}_u(t)\, e^{j2\pi f_c t}\right]$

2) $\tilde{S}_u(t) = \text{IFT}\left\{\tilde{H}_u(f) \cdot \tilde{S}_{DSBSC}(f)\right\}$

<u>IFT :-</u>

$$M(f) \xrightarrow{\text{IFT}} m(t)$$

$$-j\,\text{Sgn}(f) M(f) \xrightarrow{\text{IFT}} \hat{m}(t)$$

$$\text{Sgn}(f)\, M(f) \longrightarrow j\hat{m}(t)$$

---

i) $-j\,\text{Sgn}(f) \cdot M(f) \xrightarrow{\text{IFT}} \hat{m}(t)$

ii) $\text{Sgn}(f)\, M(f) \xrightarrow{\text{IFT}} j\,\hat{m}(t)$

<u>Proof :-</u>

$$j\hat{m}(t) = j\left[-j\,\text{Sgn}(f) M(f)\right]$$

$$= -j^2\, \text{Sgn}(f) M(f)$$

$$= -(-1)\, \text{Sgn}(f)\, M(f)$$

$$\boxed{j\hat{m}(t) = \text{Sgn}(f) M(f)}$$

iii) $\text{Sgn}(-f)\, M(f) \xrightarrow{\text{IFT}} -j\,\hat{m}(t)$

<u>Proof :-</u> $-j\,\hat{m}(t) = -j\left[-j\,\text{Sgn}(f)\, M(f)\right]$

$$= +j^2\, \text{Sgn}(f) \cdot M(f)$$

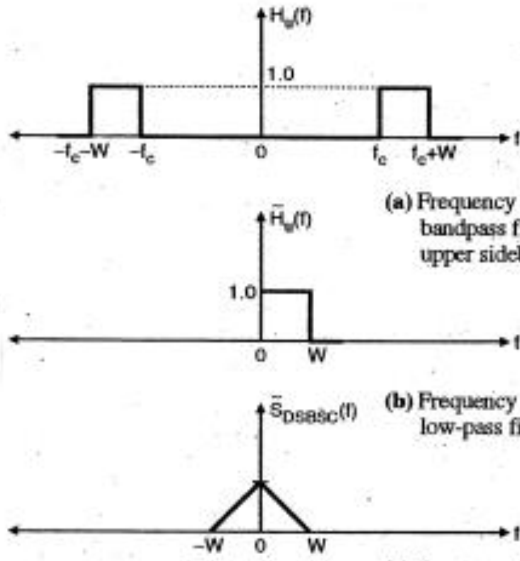
$$\boxed{-j\,\hat{m}(t) = -\text{Sgn}(f)\, M(f)}$$

Time-domain description of SSB wave :-

- **Using Hilbert transform, derive the equations for SSB signals. Specify the advantages of SSB over DSB-SC.** **July-05,12M**
- **Derive an expression for SSB modulated wave for which upper sideband is retained.** **Jan-09,8M Jan-05,8M**

Sol:-

The SSB Signal may be generated by passing a DSB-SC modulated wave through a BPF of transfer function $H_u(f)$.



(a) Frequency response of ideal bandpass filter to select the upper sideband.

(b) Frequency response of equivalent low-pass filter.

(c) Spectrum of complex envelope of DSBSC.

* The DSB-SC modulated wave is defined mathematically as

$$\boxed{S_{DSBSC}(t) = A_c m(t) \cos(2\pi f_c t)}$$

Where, $m(t) \rightarrow$ Message Signal

$A_c \cos 2\pi f_c t \rightarrow$ Carrier Signal

* The Low pass Complex envelope of the DSB-SC modulated wave is expressed as:

$$\boxed{\tilde{S}_{DSBSC}(t) = A_c m(t)}$$

* Consider the SSB modulated wave $S_u(t)$, in which only the USB is retained. It has quadrature as well as in-phase component.

Then $\tilde{S}_u(t)$ is the complex envelope of $S_u(t)$ & we can write

$$S_u(t) = R_e\left[\tilde{S}_u(t)\exp(j2\pi f_c t)\right]$$

$$\boxed{S_u(t) = R_e\left[\tilde{S}_u(t)\, e^{j2\pi f_c t}\right]} \longrightarrow (1)$$

Where, $R_e \longrightarrow$ real part.

* To determine $\tilde{S}_u(t)$, we proceed as follows:

i) The BPF transfer function $H_u(f)$ is replaced by an equivalent LPF of transfer function $\tilde{H}_u(f)$ as shown in fig (b).

We can express $\tilde{H}_u(f)$ as follows:

$$\tilde{H}_u(f) = \begin{cases} \frac{1}{2}[1+\mathrm{Sgn}(f)], & 0<f<W \\ 0, & \text{otherwise} \end{cases} \longrightarrow (2)$$

Where, $\mathrm{Sgn}(f)$ is the Signum function.

ii) The DSB-SC modulated wave is replaced by its complex-envelope. The spectrum of this envelope is as shown in fig (c). i.e.

$$\boxed{\tilde{S}_{DSBSC}(f) = A_c M(f)} \longrightarrow (3)$$

iii) The desired complex envelope $\tilde{S}_u(t)$ is determined by evaluating the IFT of the product $\tilde{H}_u(f) \cdot \tilde{S}_{DSBSC}(f)$

i.e. $$\tilde{S}_u(t) = \mathrm{IFT}\left[\tilde{H}_u(f) \cdot \tilde{S}_{DSBSC}(f)\right] \longrightarrow (4)$$

Substituting eq ② & eq ③ in eq ④, we get

$$\tilde{S}_u(t) = IFT\left[\frac{1}{2}\left[1+Sgn(f)\right]\cdot A_c M(f)\right]$$

$$= IFT\left\{ \frac{A_c}{2}\left[M(f)+Sgn(f)M(f)\right]\right\}$$

$$\boxed{\tilde{S}_u(t) = \frac{A_c}{2}\left[m(t)+j\hat{m}(t)\right]} \longrightarrow ⑤$$

Substituting eq ⑤ in eq ①, we get

$$\text{i.e } S_u(t) = R_e\left[\underline{\tilde{S}_u(t)}\, e^{j2\pi f_c t}\right] \longrightarrow ①$$

$$S_u(t) = R_e\left\{\frac{A_c}{2}\left[m(t)+j\hat{m}(t)\right]\cdot e^{j2\pi f_c t}\right\}$$

$$S_u(t) = R_e\left\{\frac{A_c}{2}\left[m(t)+j\hat{m}(t)\right] Cos(2\pi f_c t) + jSin(2\pi f_c t)\right\}$$

$$= R_e\left\{\frac{A_c}{2}\left[m(t)Cos(2\pi f_c t) + jm(t)Sin(2\pi f_c t) + j\hat{m}(t)Cos(2\pi f_c t) + \underline{j^2}\hat{m}(t)Sin(2\pi f_c t)\right]\right\}$$

$$= R_e\left\{\frac{A_c}{2}\left[m(t)Cos(2\pi f_c t) + jm(t)Sin(2\pi f_c t) + j\hat{m}(t)Cos(2\pi f_c t) - \hat{m}(t)Sin(2\pi f_c t)\right]\right\}$$

$$\boxed{S_u(t) = \frac{A_c}{2}\left[\underbrace{m(t)Cos(2\pi f_c t)}_{\text{In-phase Component}} - \underbrace{\hat{m}(t)Sin(2\pi f_c t)}_{\text{Quadrature Component}}\right]} \longrightarrow ⑥$$

Equation ⑥ Shows that the SSB modulated wave Contains only USB With an In-phase Component & a quadrature Component.

Single Tone SSB Modulation

Explain single tone modulation for transmitting only upper side (USB) frequency of SSB modulation.

* Let the modulating signal $m(t)$ is represented as

$$\boxed{m(t) = A_m \cos(2\pi f_m t)} \longrightarrow (1)$$

* The hilbert transform of the modulating signal $m(t)$ is obtained by passing it through a -90° phase shifter. So the hilbert transform is given by

$$\boxed{\hat{m}(t) = A_m \sin(2\pi f_c t)} \longrightarrow (2)$$

* WKT the SSB wave with only USB is given by

$$S_u(t) = \frac{A_c}{2}\left[\underline{m(t)} \cos(2\pi f_c t) - \underline{\hat{m}(t)} \sin(2\pi f_c t)\right] \longrightarrow (3)$$

Substituting eq (1) & eq (2) in eq (3), we get

$$S_u(t) = \frac{A_c}{2}\left[A_m \cos(2\pi f_m t) \cdot \cos(2\pi f_c t) - A_m \sin(2\pi f_m t) \cdot \sin(2\pi f_c t)\right]$$

W.K.T

$$\boxed{\cos(A+B) = \cos(A) \cdot \cos(B) - \sin(A) \cdot \sin(B)}$$

$$S_u(t) = \frac{A_c A_m}{2}\left[\underset{\cos(A)}{\cos(2\pi f_c t)} \cdot \underset{\cos(B)}{\cos(2\pi f_m t)} \underset{-}{-} \underset{\sin(A)}{\sin(2\pi f_c t)} \cdot \underset{\sin(B)}{\sin(2\pi f_m t)}\right]$$

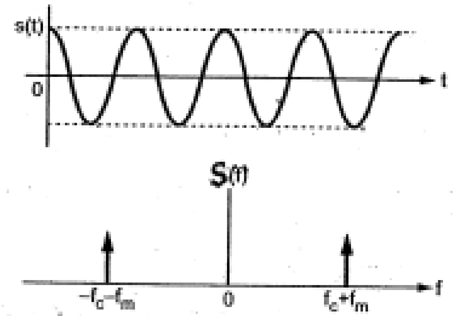
$$S_u(t) = \frac{A_c A_m}{2}\left[\cos(2\pi f_c t + 2\pi f_m t)\right]$$

$$\boxed{S_u(t) = \frac{A_c A_m}{2} \cos 2\pi\left[f_c + f_m\right]t} \longrightarrow (4)$$

Equation (4) Shows that the SSB Wave Consists of only the upper Sideband of frequency (f_c+f_m).

* This is exactly Same as the result obtained by Suppressing the Lower Side-frequency (f_c-f_m) of the corresponding DSB-SC Wave.

Spectrum of SSB with lower sideband supressed



Explain single tone modulation for transmitting only lower side (LSB) frequency of SSB modulation.

* Let the modulating Signal m(t) is represented as

$$\boxed{m(t) = A_m \cos(2\pi f_m t)} \longrightarrow (1)$$

* The hilbert transform of the modulating Signal m(t) is obtained by passing it through a -90° phase Shifter. So the hilbert transform is given by:

$$\boxed{\hat{m}(t) = A_m \sin(2\pi f_m t)} \longrightarrow (2)$$

W.K.T the SSB Wave with only LSB is given by:

$$\boxed{S_L(t) = \frac{A_c}{2}\left[m(t)\cos(2\pi f_c t) + \hat{m}(t)\sin(2\pi f_c t)\right]} \longrightarrow (3)$$

Substituting eq ① & eq ② in eq ③, we get

$$S_L(t) = \frac{A_c}{2}\left[A_m \cos(2\pi f_m t)\cos(2\pi f_c t) + A_m \sin(2\pi f_m t)\cdot \sin(2\pi f_c t)\right]$$

$$S_L(t) = \frac{A_c A_m}{2}\Big[\underbrace{\cos(2\pi f_c t)}_{\cos(A)} \cdot \underbrace{\cos(2\pi f_m t)}_{\cos(B)} + \underbrace{\sin(2\pi f_c t)}_{\sin(A)} \cdot \underbrace{\sin(2\pi f_m t)}_{\sin(B)}\Big]$$

W.K.T

$$\boxed{\cos(A-B) = \cos A \cdot \cos B + \sin A \cdot \sin B}$$

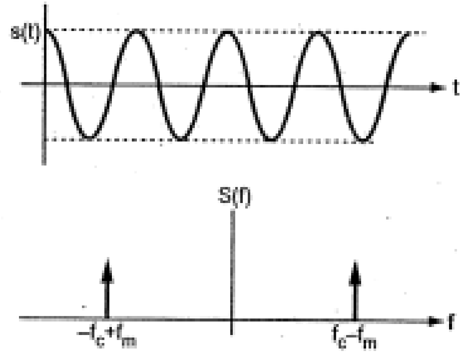
$$S_L(t) = \frac{A_c A_m}{2}\left[\cos(2\pi f_c t - 2\pi f_m t)\right]$$

$$\boxed{S_L(t) = \frac{A_c A_m}{2} \cos 2\pi\left[f_c - f_m\right] t} \longrightarrow ④$$

Equation ④ shows that the SSB wave consists of only the Lower Side Frequency $(f_c - f_m)$.

* This is exactly same as the result obtained by suppressing the upper side frequency $(f_c + f_m)$ of the corresponding DSB-SC wave.

Spectrum of SSB with upper sideband supressed



Generation of SSB Wave:-

1. **Frequency discrimination method**
2. **Phase discrimination method or Hartley modulator**

Phase discriminator method or Hartley Modulator :-

- **Explain the generation of SSB-SC wave using Phase discrimination method with the help of a neat functional diagram. Bring out the merits and demerits of this.** **Jan-06,8M**

With a neat diagram, explain how SSB wave is generated using Phase shift method.

June-10,8M June-10,6M(IT) Jan-10,7M Jan-07,5M July-06,7M

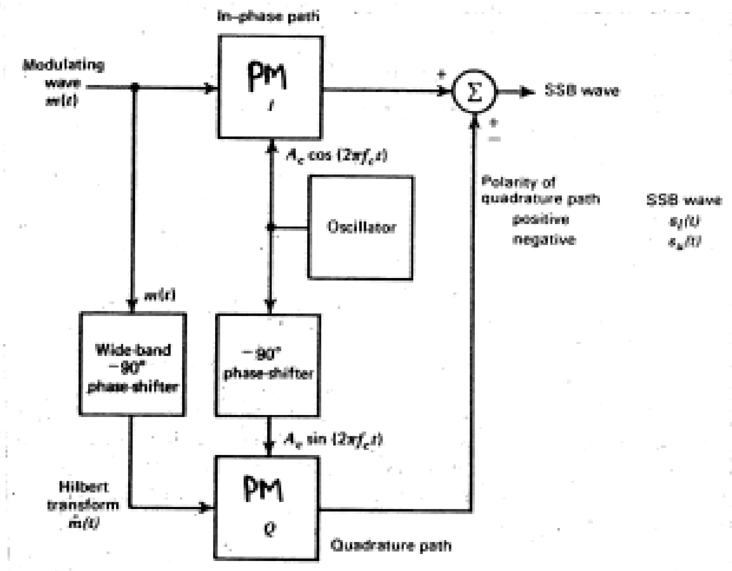


Figure
Block diagram of the phase discrimination method for generating SSB modulated waves.

Fig ① Shows the block diagram of phase discriminator method of generating SSB.

* The SSB modulator uses two product modulators I & Q, Supplied with carrier waves in phase quadrature to each other.

* The message Signal $m(t)$ & a carrier Signal $A_c \cos(2\pi f_c t)$ is directly applied to the product modulator I, producing a DSB-SC wave.

* The hilbert transform $\hat{m}(t)$ (-90° phase Shift) of $m(t)$ & carrier Signal Shifted by 90° are applied to the product modulator Q, producing DSB-SC Wave.

* The o/p of product modulator 'I' is

$$\boxed{S_I(t) = m(t) A_c \cos(2\pi f_c t)}$$

* The o/p of product modulator 'Q' is

$$\boxed{S_Q(t) = \hat{m}(t) \cdot A_c \sin(2\pi f_c t)}$$

These Signals $S_I(t)$ & $S_Q(t)$ are fed to a Summer.

* The o/p of the Summer is

$$S(t) = S_I(t) \pm S_Q(t)$$

$$\boxed{S(t) = A_c m(t) \cos 2\pi f_c t \pm A_c \hat{m}(t) \sin 2\pi f_c t}$$

* The plus Sign at the Summing junction yields an SSB with only the LSB i.e.

$$\boxed{S_L(t) = A_c m(t) \cos 2\pi f_c t + A_c \hat{m}(t) \sin 2\pi f_c t}$$

* Similarly the minus Sign at the Summing junction yields an SSB with only the USB i.e.

$$\boxed{S_u(t) = A_c m(t)\cos 2\pi f_c t - A_c \hat{m}(t)\sin 2\pi f_c t}$$

This SSB modulator is also known as the Hartley modulator.

Frequency discrimination method or Filtering method :-

Frequency discriminator method can be used for generating the SSB modulated wave if the message signal satisfies the following conditions:

1) The message signal should not have any low frequency content. (i.e the message spectrum $M(f)$ has "holes" at zero frequency)

The audio signals posses this property.

eg:- The telephone signals will have a frequency range - extending from 300Hz to 3.4kHz. The frequencies in the range 0 - 300Hz are absent, thereby creating an energy gap from 0 to 300Hz

2) The highest frequency component 'W' of the message signal $m(t)$ is much less than the carrier frequency 'f_c'.

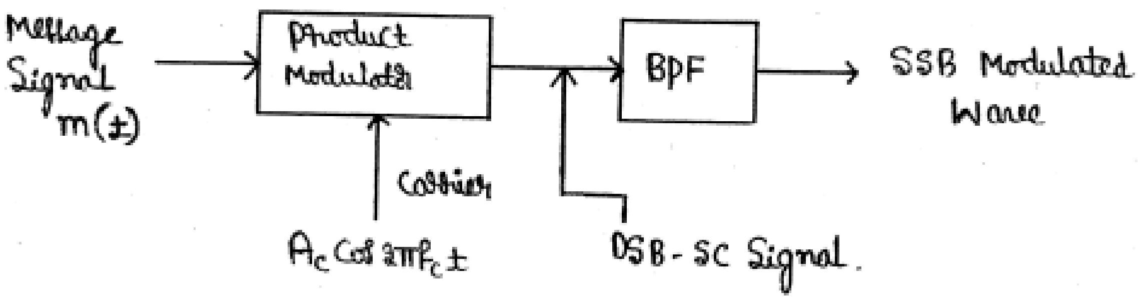


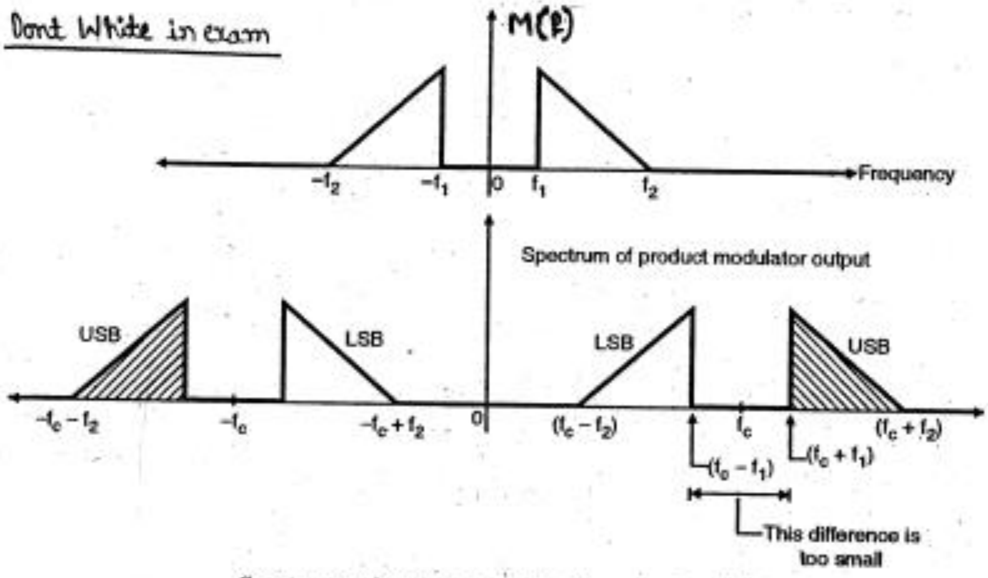
Fig ①: Block diagram of SSB modulator using frequency discriminator method

* This modulator consists of a product modulator, carrier oscillator & BPF designed to pass the desired sideband.

* At the o/p of the product modulator, we get the DSB-SC modulated wave which contains only two sidebands.

* The BPF will pass only one sideband & produce the SSB modulated wave & its o/p.

{ Dont Write in exam



Spectrums of message signal and product modulator output

}

Demerits:-

1) The frequency difference between the highest frequency in LSB & the lowest frequency in USB is too small as shown in fig (a):(b)

This makes the design of BPF extremely difficult, because its frequency response need to have very sharp change over from attenuation to pass band & vice-versa.

Design of BPF :-

The design of BPF must Satisfy two basic requirements.

1) The pass band of the filter occupies the Same frequency range as the Spectrum of the desired SSB modulated wave.

2) The Width of the guard band Which Separates the passband from Stop band be twice the lowest frequency Component of the message Signal.

i.e. $\boxed{\text{Guard band} = 2f_1 \text{ Hz}}$

The Conditions mentioned above are Satisfied only by the highly Selective filters using Crystal resonators with a high Q-factor typically in the range 1000 to 2000.

Two Stage SSB Modulator :-

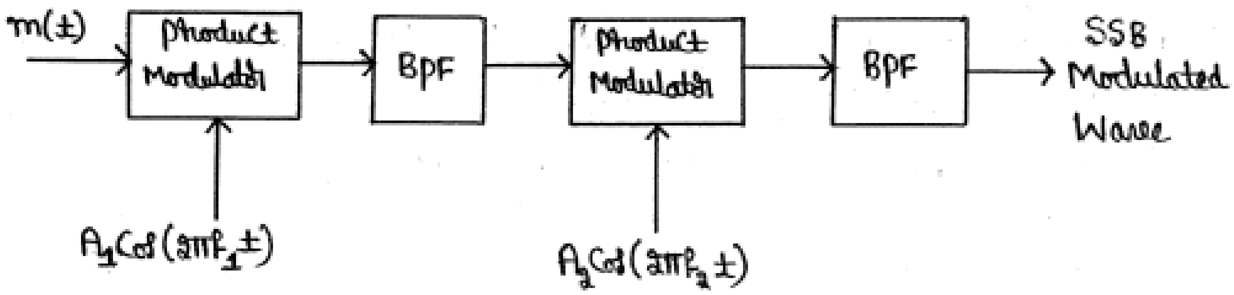


Fig 3 : A Two-Stage SSB modulator.

{ Dont Write:

* When the carrier frequency is very high as Compared to the message frequency, the SSB modulated wave occupies the frequency band which is much higher than that of the message Signal.

* under such operating conditions it becomes extremely difficult to design a BPF that passes the desired sideband & attenuates the unwanted sideband.

}

* The message signal m(t) modulates the carrier f_1 to produce a DSB-SC signal. This signal is passed through the 1st BPF to produce an SSB modulated signal.

* The o/p of the 1st BPF is then used to modulate another carrier 'f_2'' which is higher than 'f_1''. Then the o/p of the 2nd product modulator we get another DSB-SC signal.

* Thus increases the Guard band between sideband frequency, which will make the filter design easy.

Advantages of filter method (frequency discrimination method) :-

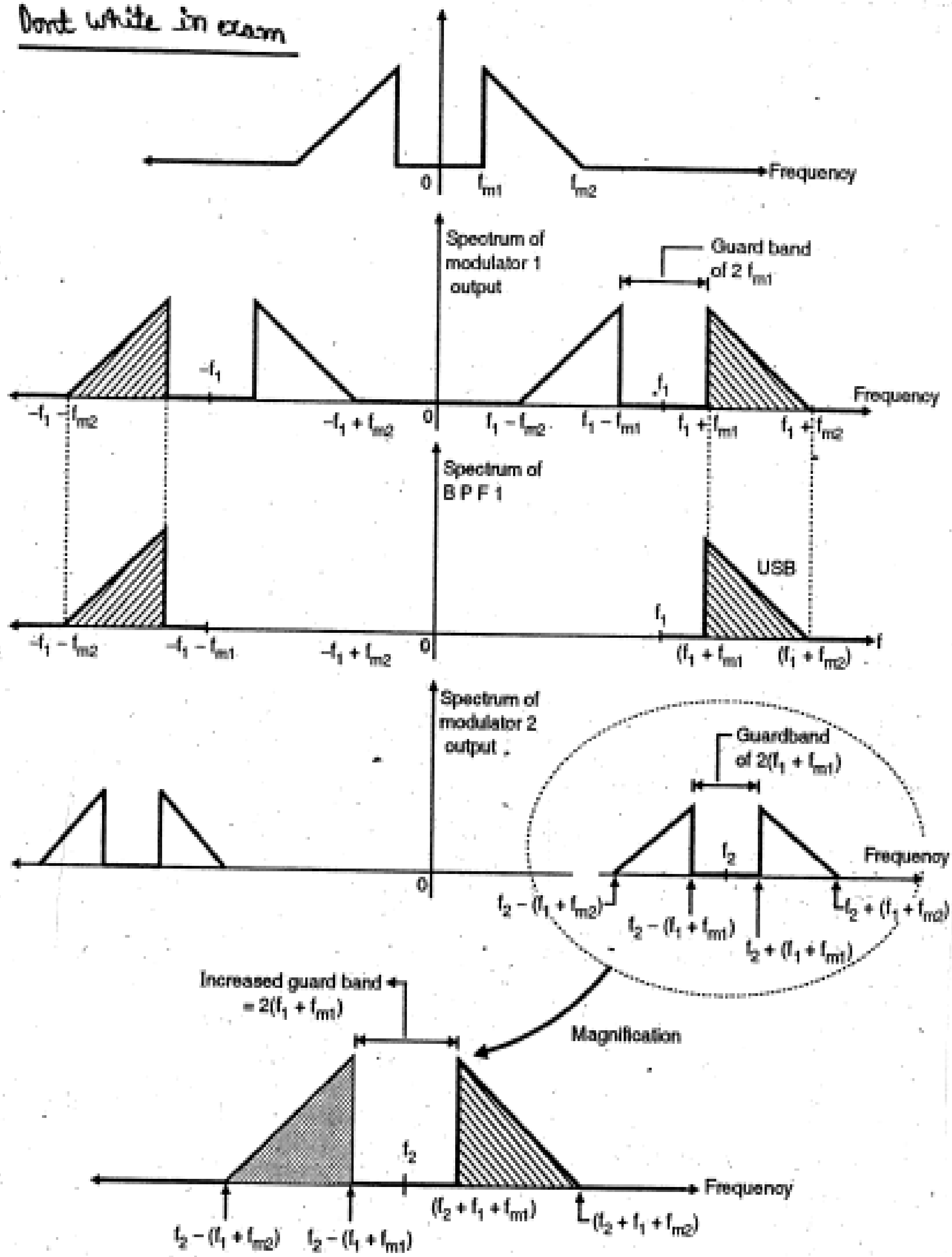
1) The filter method gives the adequate sideband suppression.
2) The sideband filters also helps to attenuate carrier if present in the o/p of balanced modulator.
3) The bandwidth is sufficiently flat & wide.

Disadvantages:-

1) They are bulky
2) Due to the inability of the system to generate SSB at high radio frequencies, the frequency up conversion is necessary.
3) Two expensive filters are to be used one for each sideband.

{ Spectrum of two Stage SSB-modulator :-

Dont write in exam



Spectrums of the two stage SSB modulator

}

Demodulation of SSB Wave:-

❖ **Show that the output of coherent detector of a SSB modulated wave is given by:** $V_o(t) = 1/4\, A_c m(t) \cos\phi + 1/4 A_c m(t) \sin\phi$

Where ϕ is the phase error.

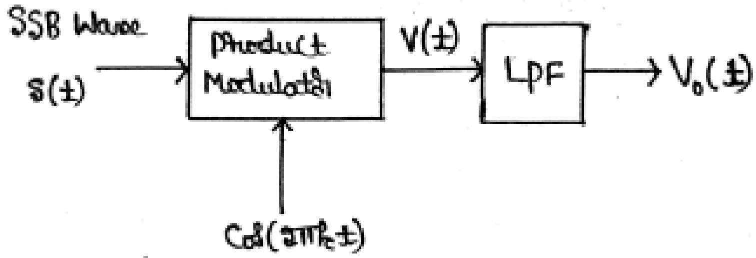


Fig ①: Coherent detection of an SSB modulated wave.

* The baseband signal m(t) can be recovered from the SSB wave S(t) by using coherent detection.

* The product modulator is having two I/p's. one I/p is the SSB modulated wave S(t) & another I/p is the locally generated carrier $\cos(2\pi f_c t)$ then low-pass filtering the modulator o/p as shown in above figure.

* Thus product modulator o/p is given by

$$\boxed{V(t) = S(t)\cos(2\pi f_c t)} \longrightarrow ①$$

WKT

$$S(t) = \frac{A_c}{2}\left[m(t)\cos 2\pi f_c t \pm \hat{m}(t)\sin 2\pi f_c t\right] \longrightarrow ②$$

Substituting eq ② in eq ①, we get

$$V(t) = \frac{A_c}{2}\left[m(t)\cos 2\pi f_c t \pm \hat{m}(t)\sin 2\pi f_c t\right]\cos 2\pi f_c t$$

$$V(t) = \frac{A_c}{2} m(t)\cos 2\pi f_c t \cdot \cos 2\pi f_c t \pm \frac{A_c}{2}\hat{m}(t)\cos 2\pi f_c t \cdot \sin 2\pi f_c t.$$

W.K.T

$$\boxed{\cos A \cdot \cos B = \frac{1}{2}\left[\cos(A+B) + \cos(A-B)\right]}$$

③a $$\boxed{\cos A \cdot \sin B = \frac{1}{2}\left[\sin(A+B) - \sin(A-B)\right]}$$

$$V(t) = \frac{A_c}{4} m(t)\left[\cos(2\pi f_c + 2\pi f_c)t + \cos(2\pi f_c - 2\pi f_c)t\right] \pm \frac{A_c}{4}\hat{m}(t)\left[\sin(2\pi f_c + 2\pi f_c)t + \sin(2\pi f_c - 2\pi f_c)t\right]$$

$$V(t) = \frac{A_c}{4} m(t)\left[\cos(4\pi f_c t) + \cos(0)\right] \pm \frac{A_c}{4}\hat{m}(t)\left[\sin(4\pi f_c t) + \sin(0)\right]$$

W.K.T, $\boxed{\cos(0) = 1,\ \sin(0) = 0}$

$$V(t) = \frac{A_c}{4} m(t)\left[\cos(4\pi f_c t + 1\right] \pm \frac{A_c}{4}\hat{m}(t)\left[\sin(4\pi f_c t) + 0\right]$$

$$V(t) = \frac{A_c}{4} m(t) + \frac{A_c}{4} m(t)\cos(4\pi f_c t) \pm \frac{A_c}{4}\hat{m}(t)\sin(4\pi f_c t)$$

$$V(t) = \underbrace{\frac{A_c}{4} m(t)}_{\text{Scaled message Signal}} + \frac{A_c}{4}\underbrace{\left[m(t)\cos(4\pi f_c t) \pm \hat{m}(t)\sin(4\pi f_c t)\right]}_{\text{Unwanted terms}}$$

* When $V(t)$ is passed through the filter, it will allow only the 1st term to pass through & will reject all other unwanted terms.

* Thus at the o/p of the filter we get the Scaled message Signal & the Coherent SSB demodulation is achieved.

$$\therefore \boxed{V_o(t) = \frac{A_c}{4} m(t)}$$

The detection of SSB modulated waves is based on the assumption that there is perfect Synchronization between local carrier & that in the transmitter both in frequency & phase.

* But in practice a phase error ϕ may arise in the locally - generated carrier wave. Thus the detector o/p is modified due to

phase error as follows:

$$\boxed{V_0(t) = \frac{A_c}{4} m(t)\cos\phi \pm \frac{A_c}{4}\hat{m}(t)\sin\phi}$$

NOTE :-

* The phase distortion is not serious with voice communication because the human ear is relative insensitivity to phase distortion. The presence of phase distortion gives rise to what is called the Donald Duck voice effect.

* The phase distortion cannot be tolerable in the transmission of music & video signal.

Formulae

1) $\sin A \cdot \cos B = \frac{1}{2}\sin(A-B) + \frac{1}{2}\sin(A+B)$

$$\sin(2\pi f_c t)\cdot\cos(2\pi f_c t) = \frac{1}{2}\sin[\cancel{2\pi f_c t - 2\pi f_c t}]^{0} + \frac{1}{2}\sin[2\pi f_c t + 2\pi f_c t]$$

$$\boxed{\sin(2\pi f_c t)\cdot\cos(2\pi f_c t) = \frac{\sin(4\pi f_c t)}{2}}$$

2) $\cos^2\theta = \frac{1+\cos 2\theta}{2}$

$$\cos^2(2\pi f_c t) = \frac{1+\cos 2(2\pi f_c t)}{2}$$

$$\boxed{\cos^2(2\pi f_c t) = \frac{1}{2} + \frac{\cos(4\pi f_c t)}{2}}$$

3) $\sin^2\theta = \frac{1-\cos 2\theta}{2}$

$$\sin^2(2\pi f_c t) = \frac{1-\cos 2(2\pi f_c t)}{2}$$

$$\boxed{\sin^2(2\pi f_c t) = \frac{1}{2} - \frac{\cos(4\pi f_c t)}{2}}$$

Time function	Hilbert Transform
$\cos(2\pi f_c t)$	$\sin(2\pi f_c t)$
$\sin(2\pi f_c t)$	$-\cos(2\pi f_c t)$

1) Let $S_u(t)$ denote the SSB Signal obtained by transmitting only upper Sideband & let $\hat{S}_u(t)$ denotes its Hilbert transform. Show that :

$$m(t) = \frac{2}{A_c}\left[S_u(t)\cos(2\pi f_c t) + \hat{S}_u(t)\sin(2\pi f_c t)\right] \quad \text{and}$$

$$\hat{m}(t) = \frac{2}{A_c}\left[\hat{S}_u(t)\cos(2\pi f_c t) - S_u(t)\sin(2\pi f_c t)\right]$$

Jan - 2009, 6M

Sol :-

W.K.T in SSB modulation $S_u(t)$ is given by

$$\boxed{S_u(t) = \frac{A_c}{2}\left[m(t)\cos(2\pi f_c t) - \hat{m}(t)\sin(2\pi f_c t)\right]} \longrightarrow (1)$$

Taking hilbert transform of eq (1), we get

$$\hat{S}_u(t) = \frac{A_c}{2}\left[m(t)\,\widehat{\cos}(2\pi f_c t) - \hat{m}(t)\,\widehat{\sin}(2\pi f_c t)\right]$$

$$\hat{S}_u(t) = \frac{A_c}{2}\left[m(t)\sin(2\pi f_c t) - \hat{m}(t)\left[-\cos(2\pi f_c t)\right]\right]$$

$$\boxed{\hat{S}_u(t) = \frac{A_c}{2}\left[m(t)\sin(2\pi f_c t) + \hat{m}(t)\cos(2\pi f_c t)\right]} \longrightarrow (2)$$

Multiply eq (1) by $\underline{\cos(2\pi f_c t)}$

$$S_u(t)\cos(2\pi f_c t) = \frac{A_c}{2}\left[m(t)\cos(2\pi f_c t) - \hat{m}(t)\sin(2\pi f_c t)\right]\cos(2\pi f_c t)$$

$$S_u(t)\cos(2\pi f_c t) = \frac{A_c}{2}m(t)\,\underline{\cos^2(2\pi f_c t)} - \hat{m}(t)\,\underline{\sin(2\pi f_c t)\cos(2\pi f_c t)}\,\frac{A_c}{2}$$

$$= \frac{A_c}{2}m(t)\left[\frac{1}{2} + \frac{\cos(4\pi f_c t)}{2}\right] - \hat{m}(t)\left[\frac{\sin(4\pi f_c t)}{2}\right]\frac{A_c}{2}$$

$$\boxed{S_u(t)\cos(2\pi f_c t) = \frac{A_c}{4}m(t) + \frac{A_c}{4}m(t)\cos(4\pi f_c t) - \frac{A_c}{4}\hat{m}(t)\sin(4\pi f_c t)} \longrightarrow (3)$$

multiply eq (2) by $\sin(2\pi f_c t)$

$$\hat{S}_u(t)\sin(2\pi f_c t) = \frac{A_c}{2}\left[m(t)\sin(2\pi f_c t) + \hat{m}(t)\cos(2\pi f_c t)\right]\sin(2\pi f_c t)$$

$$\hat{S}_u(t)\sin(2\pi f_c t) = \frac{A_c}{2} m(t)\sin^2(2\pi f_c t) + \frac{A_c}{2}\hat{m}(t)\sin(2\pi f_c t)\cdot\cos(2\pi f_c t)$$

$$= \frac{A_c}{2} m(t)\left[\frac{1}{2} - \frac{\cos(4\pi f_c t)}{2}\right] + \frac{A_c}{2}\hat{m}(t)\left[\frac{\sin(4\pi f_c t)}{2}\right]$$

$$\boxed{\hat{S}_u(t)\sin(2\pi f_c t) = \frac{A_c}{4} m(t) - \frac{A_c}{4} m(t)\cos(4\pi f_c t) + \frac{A_c}{4}\hat{m}(t)\sin(4\pi f_c t)} \rightarrow (4)$$

Add eq (3) & (4), We get

$$S_u(t)\cos(2\pi f_c t) + \hat{S}_u(t)\sin(2\pi f_c t) = \frac{A_c}{4} m(t) + \frac{A_c}{4} \cancel{m(t)\cos(4\pi f_c t)} - \frac{A_c}{4}\cancel{\hat{m}(t)\sin(4\pi f_c t)}$$
$$+ \frac{A_c}{4} m(t) - \frac{A_c}{4}\cancel{m(t)\cos(4\pi f_c t)} - \frac{A_c}{4}\cancel{\hat{m}(t)\sin(4\pi f_c t)}$$

$$S_u(t)\cos(2\pi f_c t) + \hat{S}_u(t)\sin(2\pi f_c t) = \frac{A_c}{2} m(t)$$

$$\boxed{m(t) = \frac{2}{A_c}\left[S_u(t)\cos(2\pi f_c t) + \hat{S}_u(t)\sin(2\pi f_c t)\right]}$$

Multiply eq (1) by $\sin(2\pi f_c t)$

$$S_u(t)\sin(2\pi f_c t) = \frac{A_c}{2}\left[m(t)\cos(2\pi f_c t) - \hat{m}(t)\sin(2\pi f_c t)\right]\sin(2\pi f_c t)$$

$$= \frac{A_c}{2} m(t)\,\underline{\sin(2\pi f_c t)\cdot\cos(2\pi f_c t)} - \frac{A_c}{2}\hat{m}(t)\,\underline{\sin^2(2\pi f_c t)}$$

$$= \frac{A_c}{2} m(t)\left[\frac{\sin(4\pi f_c t)}{2}\right] - \frac{A_c}{2}\hat{m}(t)\left[\frac{1}{2} - \frac{\cos(4\pi f_c t)}{2}\right]$$

$$\boxed{S_u(t)\sin(2\pi f_c t) = \frac{A_c}{4} m(t)\sin(4\pi f_c t) - \frac{A_c}{4}\hat{m}(t) + \frac{A_c}{4}\hat{m}(t)\cos(4\pi f_c t)} \rightarrow (5)$$

Multiply eq (2) by $\cos(2\pi f_c t)$

$$\hat{S}_u(t)\cos(2\pi f_c t) = \frac{A_c}{2}\left[m(t)\sin(2\pi f_c t) + \hat{m}(t)\cos(2\pi f_c t)\right]\cos(2\pi f_c t)$$

$$= \frac{A_c}{2} m(t)\,\underline{\sin(2\pi f_c t)\cdot\cos(2\pi f_c t)} + \frac{A_c}{2}\hat{m}(t)\,\underline{\cos^2(2\pi f_c t)}$$

$$= \frac{A_c}{2} m(t)\left[\frac{\sin(4\pi f_c t)}{2}\right] + \frac{A_c}{2}\hat{m}(t)\left[\frac{1}{2} + \frac{\cos(4\pi f_c t)}{2}\right]$$

$$\boxed{\hat{S}_u(t)\cos(2\pi f_c t) = \frac{A_c}{4} m(t)\sin(4\pi f_c t) + \frac{A_c}{4}\hat{m}(t) + \frac{A_c}{4}\hat{m}(t)\cos(4\pi f_c t)} \rightarrow ⑥$$

Subtract eq ⑥ - eq ⑤, we get

$$\hat{S}_u(t)\cos(2\pi f_c t) - S_u(t)\sin(2\pi f_c t) = \frac{A_c}{4} m(t)\cancel{\sin(4\pi f_c t)} + \frac{A_c}{4}\hat{m}(t)$$
$$+ \frac{A_c}{4}\hat{m}(t)\cancel{\cos(4\pi f_c t)} - \frac{A_c}{4} m(t)\cancel{\sin(4\pi f_c t)} + \frac{A_c}{4}\hat{m}(t)$$
$$- \frac{A_c}{4}\cancel{\hat{m}(t)\cos(4\pi f_c t)}$$

$$\hat{S}_u(t)\cdot\cos(2\pi f_c t) - S_u(t)\sin(2\pi f_c t) = \frac{A_c}{2}\hat{m}(t)$$

$$\boxed{\hat{m}(t) = \frac{2}{A_c}\left[\hat{S}_u(t)\cos(2\pi f_c t) - S_u(t)\sin(2\pi f_c t)\right]}$$

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*) Explain the generation of SSB-SC Wave using phase discriminator method with the help of a neat functional block diagram. Bring out the merits & demerits of this.

Jan- 2006, 8M

Merits :-

1) Bulkier filters are replaced by Small filters.
2) Low audio frequencies may be used for modulation
3) It can generate SSB at any frequency
4) Easy Switching from one Sideband to other Sideband is possible.

Demerits :-

1) The o/p of two balanced modulators must be exactly Same ; otherwise Cancellation will be incomplete.

2) If the phase shifter provides a phase change other than 90° at any audio frequency, that particular frequency will not be completely removed from the unwanted sideband.

Hence great care in adjustment is necessary.

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* In a coherent detection, if carrier applied is $\cos(2\pi f_c t+\phi)$, prove that there is a phase error in the o/p & o/p consists not only the message signal but also its hilbert transform.

Sol:-

W.K.T the SSB modulated wave is,

$$S(t) = \frac{A_c}{2}\left[m(t)\cos 2\pi f_c t \pm \hat{m}(t)\sin 2\pi f_c t\right] \longrightarrow ①$$

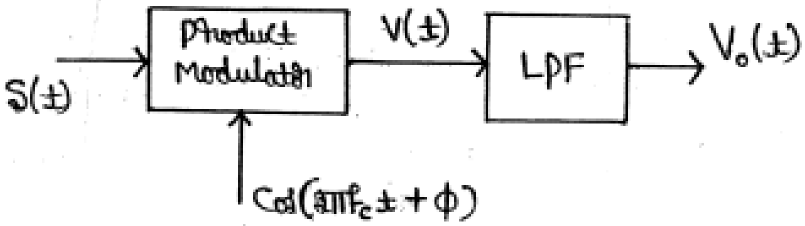


Fig ① Coherent detector

* The o/p of the product modulator is given by

$$\boxed{V(t) = S(t)\cdot\cos(2\pi f_c t+\phi)} \longrightarrow ②$$

Substituting eq ① in eq ②, we get

$$V(t) = \frac{A_c}{2}\left[m(t)\cos(2\pi f_c t) \pm \hat{m}(t)\sin 2\pi f_c t\right]\cos(2\pi f_c t+\phi)$$

$$V(t) = \frac{A_c}{2}\left[m(t)\,\underline{\cos(2\pi f_c t+\phi)\cdot\cos(2\pi f_c t)} \pm \hat{m}(t)\,\underline{\cos(2\pi f_c t+\phi)\cdot\sin(2\pi f_c t)}\right]$$

$\{$ W.K.T i) $\cos A\cdot\cos B = \frac{1}{2}\left[\cos(A-B)+\cos(A+B)\right]$

$$\cos(2\pi f_c t+\phi)\cdot\cos(2\pi f_c t) = \frac{1}{2}\left[\cos(\cancel{2\pi f_c t}+\phi-\cancel{2\pi f_c t}) + \cos(2\pi f_c t+\phi+2\pi f_c t)\right]$$

$$\boxed{\cos(2\pi f_c t+\phi)\cdot\cos(2\pi f_c t) = \frac{1}{2}\left[\cos\phi + \cos(4\pi f_c t+\phi)\right]}$$

ii) $\sin A\cdot\cos B = \frac{1}{2}\left[\sin(A-B)+\sin(A+B)\right]$

$$\sin(2\pi f_c t)\cos(2\pi f_c t+\phi) = \frac{1}{2}\left\{\sin(\cancel{2\pi f_c t}-[\cancel{2\pi f_c t}+\phi]) + \sin(2\pi f_c t+[2\pi f_c t+\phi])\right\}$$

$$= \frac{1}{2}\left\{\sin(-\phi)+\sin(4\pi f_c t+\phi)\right\}$$

$$\sin(2\pi f_c t)\cos(2\pi f_c t+\phi) = \frac{1}{2}\left[-\sin\phi+\sin(4\pi f_c t+\phi)\right]$$

$\boxed{\text{NOTE: } \pm(-\sin\phi) = \mp\sin\phi}$

}

$$\therefore V(t) = \frac{A_c}{2}\left\{\frac{m(t)}{2}\left[\cos\phi+\cos(4\pi f_c t+\phi)\right] \mp \frac{\hat{m}(t)}{2}\left[\sin\phi+\sin(4\pi f_c t+\phi)\right]\right\}$$

$$= \frac{A_c}{4}m(t)\cos\phi + \frac{A_c}{4}m(t)\cos(4\pi f_c t+\phi) \mp \frac{A_c}{4}\hat{m}(t)\sin\phi \mp \frac{A_c}{4}\hat{m}(t)\sin(4\pi f_c t+\phi)$$

$$V(t) = \underbrace{\frac{A_c}{4}\left[m(t)\cos\phi \mp \hat{m}(t)\sin\phi\right]}_{\text{Wanted Signal.}} + \underbrace{\frac{A_c}{4}\left[m(t)\cos(4\pi f_c t+\phi) \mp \hat{m}(t)\sin(4\pi f_c t+\phi)\right]}_{\text{unwanted Signal}} \rightarrow (3)$$

* The o/p of the product modulator 'V(t)' is passed through a LPF to get desired Signal.

Thus the o/p of the LPF is the desired Signal & is given by:

$$\boxed{V_o(t) = \frac{A_c}{4}\left[m(t)\cos\phi \mp \hat{m}(t)\sin\phi\right]} \rightarrow (4)$$

In eq (4), $\hat{m}(t)$ is hilbert transform of $m(t)$. This Shows that there is a phase distortion in the o/p, due to the phase error of ϕ in the local carrier Signal.

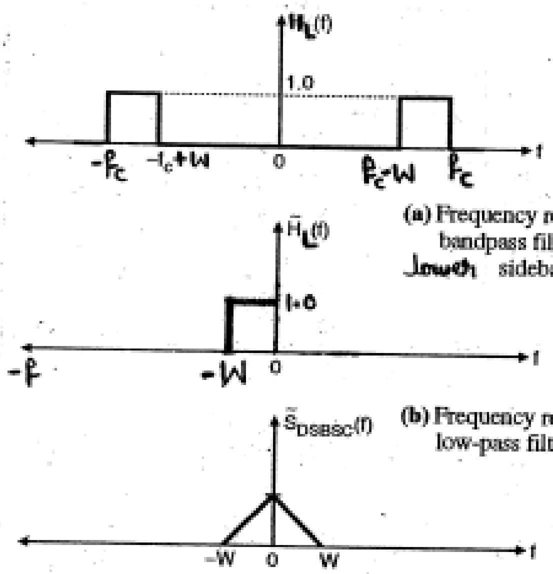
* Derive an expression for SSB modulated wave for which Lower Sideband is retained.

$\boxed{\text{Jan - 2009, 8M}}$

{ Let $S_L(t)$ denote an SSB modulated wave in which only the lower Sideband is retained.
}

Sol:-

The SSB Signal may be generated by passing a DSB-SC modulated wave through a BPF of transfer function $H_L(f)$



(a) Frequency response of ideal bandpass filter to select the lower sideband.

(b) Frequency response of equivalent low-pass filter.

(c) Spectrum of complex envelope of DSBSC.

* The DSB-SC modulated wave is defined mathematically as:

$$\boxed{S_{DSBSC}(t) = A_c m(t) \cos(2\pi f_c t)}$$

Where,

$m(t) \rightarrow$ Message Signal

$A_c \cos(2\pi f_c t) \rightarrow$ Carrier Signal

* The Low pass Complex envelope of the DSB-SC modulated wave is expressed as :

$$\boxed{\tilde{S}_{DSBSC}(t) = A_c m(t)}$$

* Consider the SSB modulated wave $S_L(t)$, in which only the LSB is retained. It has quadrature as well as In-phase Component.

Then $\tilde{S}_L(t)$ is the complex envelope of $S_L(t)$ & we can write

$$S_L(t) = R_e\left[\tilde{S}_L(t)\exp(j2\pi f_c t)\right]$$

$$\boxed{S_L(t) = R_e\left[\tilde{S}_L(t)\, e^{j2\pi f_c t}\right]}$$

Where, $R_e \rightarrow$ real part.

* To determine $\tilde{S}_L(t)$, We proceed as follows :

i) The BPF transfer function $H_L(f)$ is replaced by an equivalent LPF of transfer function $\tilde{H}_L(f)$ as shown in fig ⓑ.

We can express $\tilde{H}_L(f)$ as follows :

$$\tilde{H}_L(f) = \begin{cases} \frac{1}{2}\left[1 + \mathrm{sgn}(f)\right], & 0 < f < -W \\ 0, & \text{otherwise} \end{cases} \longrightarrow ②$$

Where, $\mathrm{Sgn}(-f)$ is the Signum function.

ii) The DSB-SC modulated wave is replaced by its Complex envelope. The Spectrum of this envelope is as Shown in fig ©

i.e.

$$\boxed{\tilde{S}_{DSBSC}(f) = A_c M(f)} \longrightarrow ③$$

iii) The desired Complex envelope $\tilde{S}_L(t)$ is determined by evaluating the IFT of the product $\tilde{H}_L(f) \cdot \tilde{S}_{DSBSC}(f)$

i.e. $$\tilde{S}_L(t) = IFT\left[\tilde{H}_L(f) \cdot \tilde{S}_{DSBSC}(f)\right] \longrightarrow (4)$$

Substituting eq (2) & eq (3) in eq (4), we get

$$\tilde{S}_L(t) = IFT\left\{\frac{1}{2}\left[1+\operatorname{Sgn}(-f)\right] \cdot A_c M(f)\right\}$$

$$\tilde{S}_L(t) = IFT\left\{\frac{A_c}{2}\left[M(f) + \operatorname{Sgn}(-f)\, M(f)\right]\right\}$$

$$\boxed{\tilde{S}_L(t) = \frac{A_c}{2}\left[m(t) - j\hat{m}(t)\right]} \longrightarrow (5)$$

Substituting eq (5) in eq (1), we get $\left(S_L(t) = R_e\left[\tilde{S}_L(t)\, e^{j2\pi f_c t}\right] \rightarrow (1)\right)$

$$S_L(t) = R_e\left\{\frac{A_c}{2}\left[m(t) - j\hat{m}(t)\right] e^{j2\pi f_c t}\right\}$$

$$S_L(t) = R_e\left\{\frac{A_c}{2}\left[m(t) - j\hat{m}(t)\right] \underline{\cos(2\pi f_c t) + j\sin(2\pi f_c t)}\right\}$$

$$= R_e\left\{\frac{A_c}{2}\left[m(t)\cos(2\pi f_c t) + jm(t)\sin(2\pi f_c t) - j\hat{m}(t)\cos(2\pi f_c t) - \underline{j^2}\,\hat{m}(t)\sin(2\pi f_c t)\right]\right\}$$

$$= R_e\left\{\frac{A_c}{2}\left[m(t)\cos(2\pi f_c t) + jm(t)\sin(2\pi f_c t) - j\hat{m}(t)\cos(2\pi f_c t) + \hat{m}(t)\sin(2\pi f_c t)\right]\right\}$$

$\because -j^2 = +1$

$$\boxed{S_L(t) = \frac{A_c}{2}\left[m(t)\cos(2\pi f_c t) + \hat{m}(t)\sin(2\pi f_c t)\right]} \longrightarrow (6)$$

In-phase Component   Quadrature Component.

Equation (6) Shows that the SSB modulated wave Contains only LSB With an Inphase Component & a Quadrature Component.

1. **Consider a 2-stage SSB modulator as shown in fig1. The i/p signal consists of a voice signal in a frequency range of 0.3 to 3.4 KHz. The two oscillator frequencies have values f1=100 KHz and f2=10 MHz. Specify the following:**

    i. **Sidebands of DSB-SC modulated waves appearing at the outputs of the product modulation (PM)**
    ii. **Sidebands of SSB modulated waves appearing at two BPF outputs.**
    iii. **The pass bands and guard bands of the two BPFs.**

June-10,8M

2. **Consider a two-stage product modulator with a BPF after each product modulator, where i/p signal consists of a voice signal occupying the frequency band 0.3 to 3.4 KHz. The two oscillator frequencies have values f1=100 KHz and f2=10 MHz. Specify the following:**

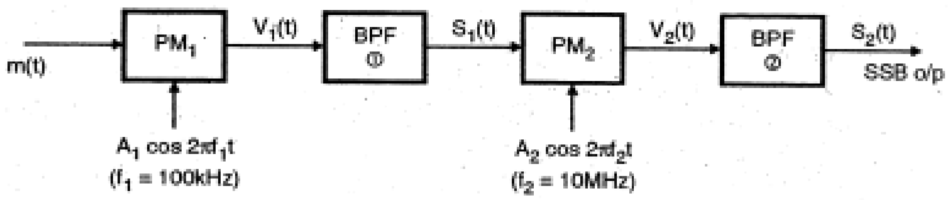
    i. **Sidebands of DSB-SC modulated waves appearing at the two product modulator output.**
    ii. **Sidebands of SSB modulated waves appearing at BPF outputs.**
    iii. **The pass bands of the two BPFs.**

June -10,9M (OLD)
March-2002, 8M

Given :-

$f_1 = 100KHz$, $f_2 = 10MHz$

$m(t) = 0.3KHz$ to $3.4KHz$



o/p of PM1 : $\boxed{f_c = f_1 = 100KHz,\ f_m = 0.3KHz \text{ to } 3.4KHz}$

The PM1 o/p consists of two sidebands as follows

$LSB = f_c - f_m = 100KHz - (0.3KHz \text{ to } 3.4KHz)$

$\boxed{LSB = 99.7KHz \text{ to } 99.6KHz}$

$USB = f_c + f_m = 100KHz + (0.3KHz \text{ to } 3.4KHz)$

$\boxed{USB = 100.3KHz \text{ to } 103.4KHz}$

o/p of BPF1 :- Assume that this BPF1 passes only the USB.

$$\boxed{S_1(t) = 100.3\,KHz \text{ to } 103.4\,KHz}$$

o/p of PM2 :- $\boxed{f_2 = f_c = 10\,MHz\ ,\ S_1(t) = f_m = 100.3\,KHz \text{ to } 103.4\,KHz}$

The PM2 o/p Consists of two Sidebands as follows:

$LSB = f_c - f_m = 10\,MHz - (100.3\,KHz \text{ to } 103.4\,KHz)$

$$\boxed{LSB = 9.899\,MHz \text{ to } 9.8966\,MHz}$$

$USB = f_c + f_m = 10\,MHz + (100.3\,KHz \text{ to } 103.4\,KHz)$

$$\boxed{USB = 10.1003\,MHz \text{ to } 10.1034\,MHz}$$

o/p of BPF2 :- Assume that this BPF2 passes only the USB

$$\boxed{S_2(t) = 10.1003\,MHz \text{ to } 10.1034\,MHz}$$

Guard band of BPF :-

Guard band is defined as the highest frequency Component of LSB to the lowest frequency Component of USB.

$$\boxed{\text{Guard band of BPF1} = 99.7\,KHz \text{ to } 100.3\,KHz}$$

$$\boxed{\text{Guard band of BPF2} = 9.8997\,MHz \text{ to } 10.1003\,MHz}$$

NOTE:-

Parameter	LSB	USB
O/P of PM1- V_1 (t)	99.7 KHz to 99.6 KHz	100.3 KHz to103.4 KHz
O/P 0f BPF1 – S_1 (t)	-	100.3 KHz to103.4 KHz
O/P of PM1- V_2 (t)	9.899 MHz to 9.8966 MHz	10.1003 MHz to 10.1034 MHz
O/P 0f BPF1 – S_2 (t)	-	10.1003 MHz to 10.1034 MHz

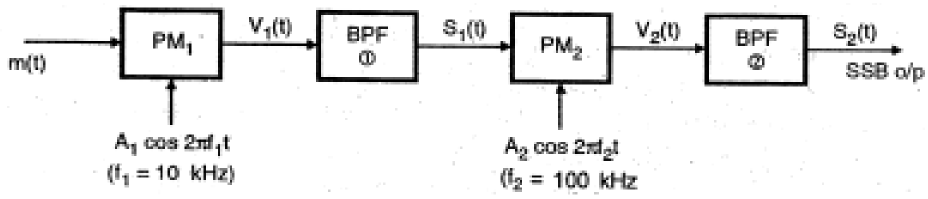
Guard Band of BPF-1 = 99.7 KHz to 100.3 KHz

Guard Band of BPF-2 = 9.8997 MHz to10.1003 MHz

- **Consider a two-stage SSB modulator where the message signal occupies a band 0.3KHz to 4 KHz and the two carrier frequencies are f1=10 KHz and f2=100 KHz. Evaluate the following:**

  i. **Sidebands of DSB-SC modulated waves at the output of the product modulators.**
  ii. **The Sidebands of SSB the modulated waves at the outputs of BPF.**
  iii. **The pass bands and the guard bands of the two BPFs.**
  iv. **The order of the two filters assuming atleast 15dB attenuation between the passband and stop band.**
  v. **Sketch the spectrum of the signal at each stage.(Assume suitable M(f))**

Sol:- **Jan -06,12M**



Given :- $m(t) = 0.3KHz$ to $4KHz$

$f_1 = 10KHz$, $f_2 = 100KHz$

o/p of PM 1 :- $\boxed{f_c = f_1 = 10KHz \,,\; f_m = 0.3KHz \text{ to } 4KHz}$

$LSB = f_c - f_m = 10KHz - (0.3KHz \text{ to } 4KHz)$

$\boxed{LSB = 6KHz \text{ to } 9.7KHz}$

$USB = f_c + f_m = 10KHz + (0.3KHz \text{ to } 4KHz)$

$\boxed{USB = 10.3KHz \text{ to } 14KHz}$

o/p of BPF 1 :- Assume that BPF1 passes only the USB

$\boxed{S_1(t) = 10.3KHz \text{ to } 14KHz}$

o/p of PM 2 :- $\boxed{f_c = f_2 = 100KHz \,,\; f_m = 10.3KHz \text{ to } 14KHz}$

$LSB = f_c - f_m = 100KHz - (10.3KHz \text{ to } 14KHz)$

$\boxed{LSB = 86KHz \text{ to } 89.7KHz}$

$USB = f_c + f_m$

$USB = 100KHz + (10.3KHz \text{ to } 14KHz)$

$\boxed{USB = 110.3KHz \text{ to } 114KHz}$

o/p of BPF 2 :- Assume that BPF2 passes only the USB

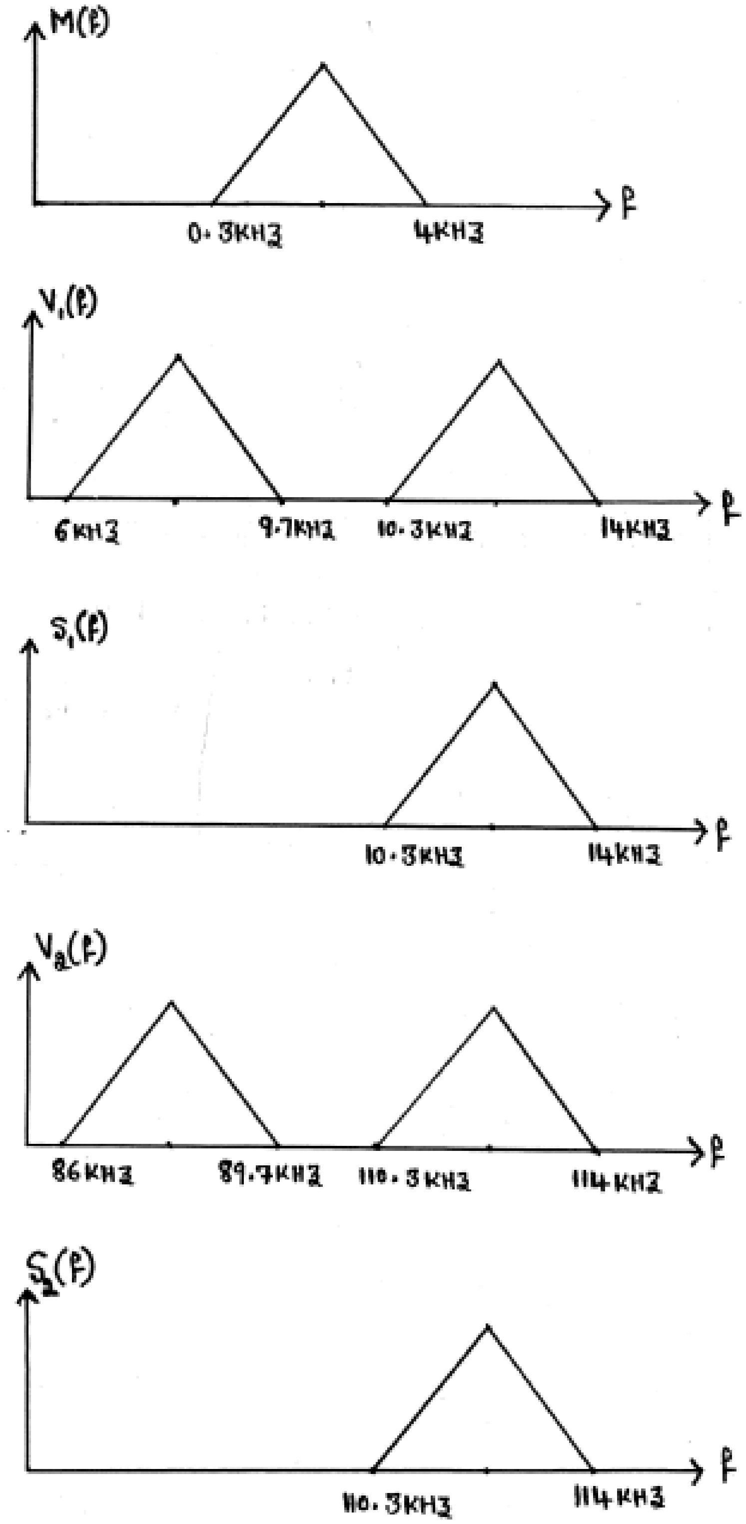
$\boxed{S_2(t) = 110.3KHz \text{ to } 114KHz}$

iii) Guard Band of BPFs :-

Guard band of BPF1	$= 9.7KHz$ to $10.3KHz$
Guardband of BPF2	$= 89.7KHz$ to $110.3KHz$

iv) The filter method gives Sideband Suppression upto 56dB. Hence both BPFs are 1st order filters.

v)



NOTE:

Parameter	LSB	USB
O/P of PM1- V_1 (t)	6 KHz to 9.7 KHz	10.3 KHz to14 KHz
O/P 0f BPF1 – S_1 (t)	-	10.3 KHz to14 KHz
O/P of PM1- V_2 (t)	86 KHz to 89.7 KHz	110.3 KHz to 114 KHz
O/P 0f BPF1 – S_2 (t)	-	110.3 KHz to 114 KHz

Guard Band of BPF-1 = 9.7 KHz to 10.3 KHz

Guard Band of BPF-2 = 89.7 KHz to110.3 KHz

---

* For the rectangular pulse shown in fig①, evaluate its hilbert - transform.

June-10, 4M

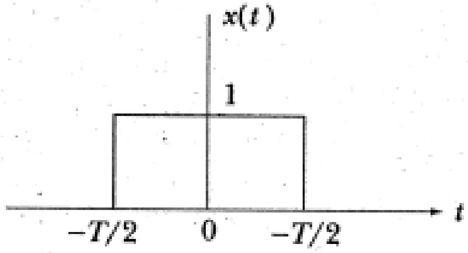


Fig ①: Rectangular pulse.

Sol :-

From fig ①, $x(t) = \begin{cases} 1, & |t| < T/2 \\ 0, & |t| > T/2 \end{cases}$

WKT

$$\hat{x}(t) = x(t) * \frac{1}{\pi t}$$

$$= \int_{-\infty}^{\infty} \frac{x(\tau)}{\pi(t-\tau)} d\tau$$

$$= \int_{-T/2}^{T/2} \frac{1}{\pi(t-\tau)} d\tau$$

$$= -\frac{1}{\pi}\int_{-T/2}^{T/2} \frac{1}{\tau - t}\, d\tau$$

$$= -\frac{1}{\pi}\left[\ln(\tau - t)\right]\Big|_{\tau=-T/2}^{T/2}$$

$$= -\frac{1}{\pi}\left[\ln\left(\frac{T}{2} - t\right) - \ln\left(-\frac{T}{2} - t\right)\right]$$

$$= \frac{1}{\pi}\ln\left[\frac{t + \frac{T}{2}}{t - \frac{T}{2}}\right]$$

VSB MODULATION

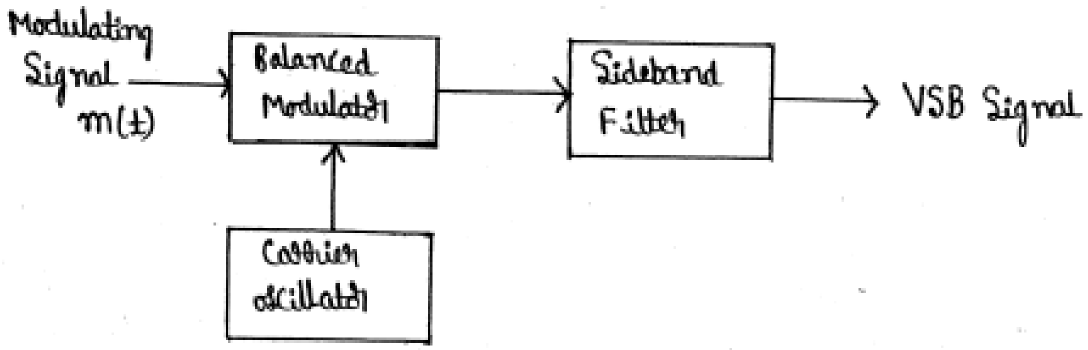
VSB Modulation:-

The Stringent (Very Strict Condition) frequency response requirements on the Sideband filter in SSB-SC modulation can be relaxed by allowing a part of the unwanted Sideband (called as vestige) to appear in the o/p of the modulator.

Due to this, the design of the Sideband filter is Simplified to a great extent. But the bandwidth of the System is increased Slightly.

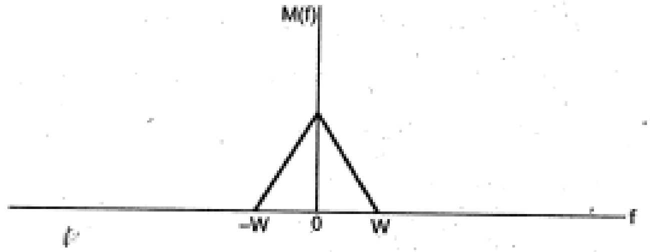
* Explain VSB modulation? Mention the advantages and - applications of VSB modulation

In VSB, one Sideband & a part of the other Sideband called as vestige is transmitted. So the bandwidth required for VSB transmission is Somewhat higher than that of SSB-modulation.

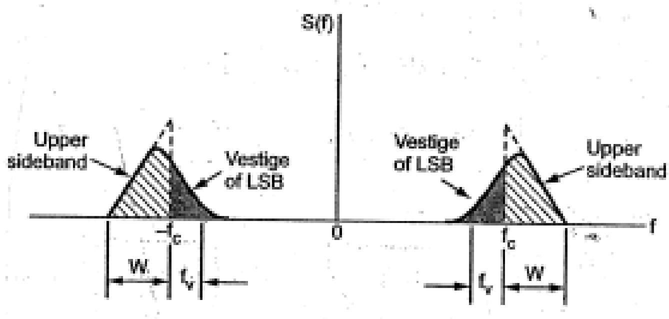


* To generate a VSB Signal, 1st we have to generate a DSB-SC Signal & then pass it through a Sideband filter as shown in figure. This filter will pass the wanted Sideband as it is along with a part of unwanted Sideband.

Frequency - domain description of VSB Wave :-



(a) Spectrum of message signal



(b) Spectrum of VSB modulated wave

* Fig shows the Spectrum of a VSB modulated Wave $S(t)$ along with the message Signal $m(t)$.

  Here lower Sideband is modified into vestigial Sideband.

* Transmission bandwidth is given by

$$\boxed{B = (W + f_v)\ Hz}$$

Where,
W is message bandwidth
f_v is the width of the vestigial Sideband.

Advantages of VSB :-

The main advantage of VSB modulation is:

1) The reduction in bandwidth. It is almost as efficient as the SSB

2) Easy to design the filter.
(Due to allowance of transmitting a part of lower sideband the constraint on the filter have been relaxed.)

Applications of VSB:-

VSB modulation has become standard for the transmission of TV signals. Because the video signals need a larger transmission bandwidth if transmitted using DSB-FC & DSB-SC techniques.

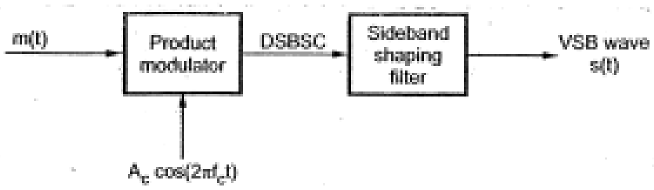
❖ **What is meant by VSB? Explain how VSB signal can be obtained from a modulating signal m(t)using a carrier $A_c\cos(2\pi f_c t)$ and later demodulated.**

July-07,8M

❖ **Explain the scheme for generation and demodulation of VSB modulated wave, with relevant spectrum of signals in the demodulation scheme. Give relevant mathematical expressions.** **June-08,10M, June-10,8M**

In VSB, one sideband & a part of the other sideband called as vestige is transmitted. So the bandwidth required for VSB transmission is some what higher than that of SSB-modulation.

Generation of VSB modulated Wave :-



* The o/p of the product modulator is the DSB-SC Wave & is given by :

$$S(t) = m(t) \cdot c(t)$$

$$\boxed{S(t) = m(t) A_c \cos(2\pi f_c t)}$$

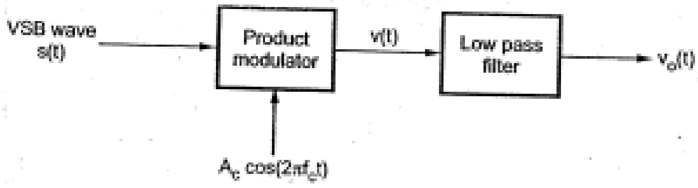
* This DSB-SC Signal is then applied to a Sideband Shaping filter. The filter will pass the wanted Sideband as it is & the vestige of the unwanted Sideband.

* Let the transfer function of the filter be H(f). Hence the Spectrum of the VSB modulated Wave is given by :

$$\boxed{S(f) = \frac{A_c}{2}\left[M(f-f_c) + M(f+f_c)\right] H(f)} \longrightarrow (1)$$

Demodulation OR Detection of VSB Modulated Wave :-

❖ **Explain the coherent detection of VSB-SC wave.** **Jan-10,7M**



* The demodulation of VSB modulated wave Can be achieved by passing VSB wave S(t) through a Coherent detector.

* Thus, multiplying $S(t)$ by a locally generated carrier wave $A_c\cos(2\pi f_c t)$, which is synchronous with the carrier wave $A_c\cos 2\pi f_c t$ in both frequency & phase as shown in fig, we get

$$\boxed{V(t) = A_c\cos 2\pi f_c t \, . \, S(t)} \longrightarrow (2)$$

Taking Fourier transform on both side of eq (2), we get

$$V(f) = \frac{A_c}{2}\left[S(f-f_c) + S(f+f_c)\right] \longrightarrow (3)$$

W.K.T

$$S(f) = \frac{A_c}{2}\left[M(f-f_c) + M(f+f_c)\right] H(f)$$

$$S(f+f_c) = \frac{A_c}{2}\left[M(f-\cancel{f_c}+\cancel{f_c}) + M(f+f_c+f_c)\right] H(f+f_c)$$

$$\boxed{S(f+f_c) = \frac{A_c}{2}\left[M(f) + M(f+2f_c)\right] H(f+f_c)} \longrightarrow (4)$$

iiily

$$S(f-f_c) = \frac{A_c}{2}\left[M(f-f_c-f_c) + M(f+\cancel{f_c}-\cancel{f_c})\right] H(f-f_c)$$

$$\boxed{S(f-f_c) = \frac{A_c}{2}\left[M(f-2f_c) + M(f)\right] H(f-f_c)} \longrightarrow (5)$$

Substituting eq (4) & (5) in eq (3), we get

$$V(f) = \frac{A_c}{2}\left\{\frac{A_c}{2}\left[M(f-2f_c)+M(f)\right]H(f-f_c) + \frac{A_c}{2}\left[M(f)+M(f+2f_c)\right]H(f+f_c)\right\}$$

$$= \frac{A_c}{4} M(f-2f_c) \, . \, H(f-f_c) + \underline{\frac{A_c}{4} M(f) H(f-f_c)} + \underline{\frac{A_c}{4} M(f) \, . \, H(f+f_c)}$$

$$+ \frac{A_c}{4} M(f+2f_c) H(f+f_c)$$

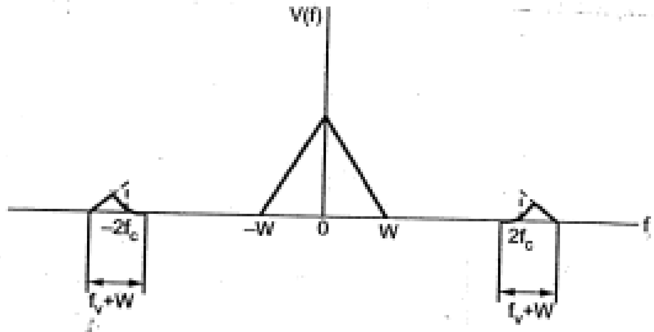
$$V(f) = \underbrace{\frac{A_c}{4} M(f)\left[H(f-f_c) + H(f+f_c)\right]}_{\text{VSB wave}} + \underbrace{\frac{A_c}{4}\left[M(f-2f_c) H(f-f_c) + M(f+2f_c) H(f+f_c)\right]}_{\text{Unwanted term}} \longrightarrow (6)$$

(5)

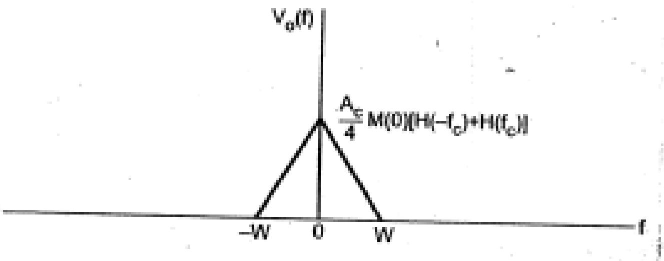
Eq ⑥ is passed through a LPF, which eliminates unwanted term & passes only wanted term i.e. VSB wave & is given by :

$$\boxed{V_o(f) = \frac{A_c}{4} M(f)\left[H(f-f_c) + H(f+f_c)\right]} \longrightarrow ⑦$$

* The Spectrum is as Shown in below figure:



(a) Spectrum of V(f)



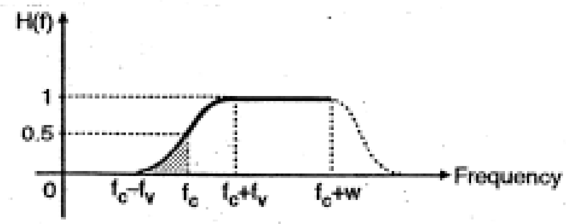
(b) Spectrum of $V_o(f)$

* To obtain the undistorted message Signal m(t) at the o/p of the demodulator, the transfer function H(f) Should Satisfy the condition as follows:

$$\boxed{H(f-f_c) + H(f+f_c) = 2H(f_c)} \longrightarrow ⑧$$

Where $H(f_c)$ is Constant.

* The Condition Stated in eq ⑧ will be Satisfied if the filter frequency response is as Shown below:



Frequency response of the sideband shaping filter for obtaining the VSB modulated wave with vestige of LSB

* Thus design of VSB filter is therefore less complicated to an SSB filter.

Envelope detection of a VSB Wave plus Carrier :-

* VSB modulation is used in the Commercial TV broadcasting in which along with VSB transmission a carrier signal of substantial size is transmitted.

∴ The modulated wave can be demodulated by using Envelope detector.

WKT. the VSB modulated wave with Full USB & a vestige of LSB is given by

$$\boxed{S(t)_{VSB} = \frac{A_c}{2}\left[m(t)\cos(2\pi f_c t) - m_Q(t)\sin(2\pi f_c t)\right]} \longrightarrow (1)$$

* Adding carrier component $A_c\cos(2\pi f_c t)$ to eq (1) scaled by a factor K_a, modifies the modulated wave applied to the envelope detector I/p as

$$S(t) = A_c\cos 2\pi f_c t + K_a S_{VSB}(t)$$

$$S(t) = \frac{A_c}{2}K_a\left[m(t)\cos(2\pi f_c t) - m_Q(t)\sin(2\pi f_c t)\right] + A_c\cos(2\pi f_c t)$$

$$S(t) = \frac{A_c}{2} K_a m(t) \cos(2\pi f_c t) - \frac{A_c}{2} K_a m_Q(t) \sin(2\pi f_c t) + A_c \cos(2\pi f_c t)$$

$$\therefore S(t) = A_c \cos(2\pi f_c t) \underbrace{\left[1 + \frac{K_a}{2} m(t)\right]}_{\text{In-phase}} - \underbrace{\frac{K_a A_c}{2} m_Q(t)}_{\text{Quadrature}} \sin(2\pi f_c t)$$

Where the Constant 'K_a' determines the percentage modulation.

* The envelope detector o/p, denoted by $a(t)$ is therefore.

$$a(t) = \sqrt{[\text{Inphase Component}]^2 + [\text{Quadrature Component}]^2}$$

$$= \sqrt{A_c^2\left[1 + \frac{K_a m(t)}{2}\right]^2 + A_c^2\left[\frac{K_a}{2} m_Q(t)\right]^2}$$

$$= \sqrt{A_c^2 \underbrace{\left[1 + \frac{K_a m(t)}{2}\right]^2}_{\text{Taking Common}} \left\{1 + \left[\frac{K_a/2\, m_Q(t)}{1 + K_a/2\, m(t)}\right]^2\right\}}$$

$$= A_c\left[1 + \frac{K_a}{2} m(t)\right] \sqrt{1 + \left[\frac{K_a/2\, m_Q(t)}{1 + K_a/2\, m(t)}\right]^2}$$

$$= A_c\left[1 + \frac{K_a}{2} m(t)\right] \left\{1 + \left[\frac{\frac{K_a}{2} m_Q(t)}{1 + \frac{K_a}{2} m(t)}\right]^2\right\}^{1/2} \longrightarrow (3)$$

Eq (3) indicates that the distortion is contributed by the quadrature component $m_Q(t)$

This distortion can be reduced using two methods:

1) Reducing the percentage modulation to reduce K_a

2) Increasing the width of the vestigial sideband to reduce $m_Q(t)$

Comparisions of Amplitude modulation Techniques :-

Sl No	parameter	DSB-FC Standard AM	DSB-SC	SSB	VSB
1)	Power	High	Medium	Less	Less than DSB-SC but greater than SSB
2)	Bandwidth	$2f_m$	$2f_m$	f_m	$f_m < BW < 2f_m$
3)	Carrier Suppression	NO	yes	yes	NO
4)	Sideband Transmission	NO	NO	one Sideband Completely	one Sideband Suppressed partly
5)	Transmission efficiency	Minimum	Moderate	Maximum	Moderate
6)	Receiver Complexity	Simple	Complex	complex	Simple
7)	Modulation type	Non-linear	Linear	Linear	Linear
8)	Applications	Radio Communication	Linear Radio Communication	Linear point to point mobile Communication	Television

Time Domain description of VSB modulated Wave:-

The procedure to be followed for obtaining the time-domain description of a VSB wave is similar to the one used for SSB:

1) Let $S(t)$ = VSB modulated wave which contains full upper-sideband (USB) & vestige of lower sideband.

2) This VSB wave can be assumed to be generated by a sideband shaping filter along with a DSB-SC signal.

3) Let the sideband filter has a transfer function $H(f)$ as shown in fig ⓐ.

4) The sideband filter can be replaced by an equivalent LPF with transfer function $\tilde{H}(f)$ as shown in fig ⓑ.

5) We can represent $\tilde{H}(f)$ of fig ⓑ as the difference between two components $\tilde{H}_u(f)$ & $\tilde{H}_v(f)$ as follows:

$$\boxed{\tilde{H}(f) = \tilde{H}_u(f) - \tilde{H}_v(f)} \longrightarrow ①$$

These two components are plotted in fig ⓒ & ⓓ respectively.

6) out of them $\tilde{H}_u(f)$ represents the LPF equivalent to a BPF which reject the LSB completely & $\tilde{H}_v(f)$ corresponds to the generation of vestige of LSB & removal of corresponding portion from USB.

7) We can represent $\tilde{H}_u(f)$ as follows:

$$\tilde{H}_u(f) = \begin{cases} \frac{1}{2}[1+\mathrm{Sgn}(f)] & , \text{ for } 0<f<W \\ 0 & , \text{ elsewhere} \end{cases} \longrightarrow ②$$

Substituting eq ② in eq ①, we get

$$\tilde{H}(f) = \frac{1}{2}\left[1 + \mathrm{Sgn}(f)\right]\tilde{H}_v(f)$$

$$\tilde{H}(f) = \begin{cases} \frac{1}{2}\left[1 + \mathrm{Sgn}(f) - 2\tilde{H}_v(f)\right], & \text{for } -f_v < f < W \\ 0, & \text{elsewhere} \end{cases} \longrightarrow ③$$

8) In eq ③, the Signum function as well as the transfer function $\tilde{H}_v(f)$ both are odd functions of frequency.

∴ Inverse Fourier transform of both of them will be purely imaginary. Hence let us introduce a new transfer function as

$$\boxed{H_Q(f) = \frac{1}{j}\left[\mathrm{Sgn}(f) - 2\tilde{H}_v(f)\right]} \longrightarrow ④$$

9) Let $h_Q(t)$ be the inverse Fourier transform of $H_Q(f)$ where $jH_Q(f)$ is as plotted as shown in fig ②.

10) Substituting eq ④ in eq ③, we get

$$\tilde{H}(f) = \begin{cases} \frac{1}{2}\left[1 + jH_Q(f)\right], & -f_v < f < W \\ 0, & \text{otherwise} \end{cases}$$

NOTE:- $jH_Q(f) = \left[\mathrm{Sgn}(f) - 2\tilde{H}_v(f)\right]$

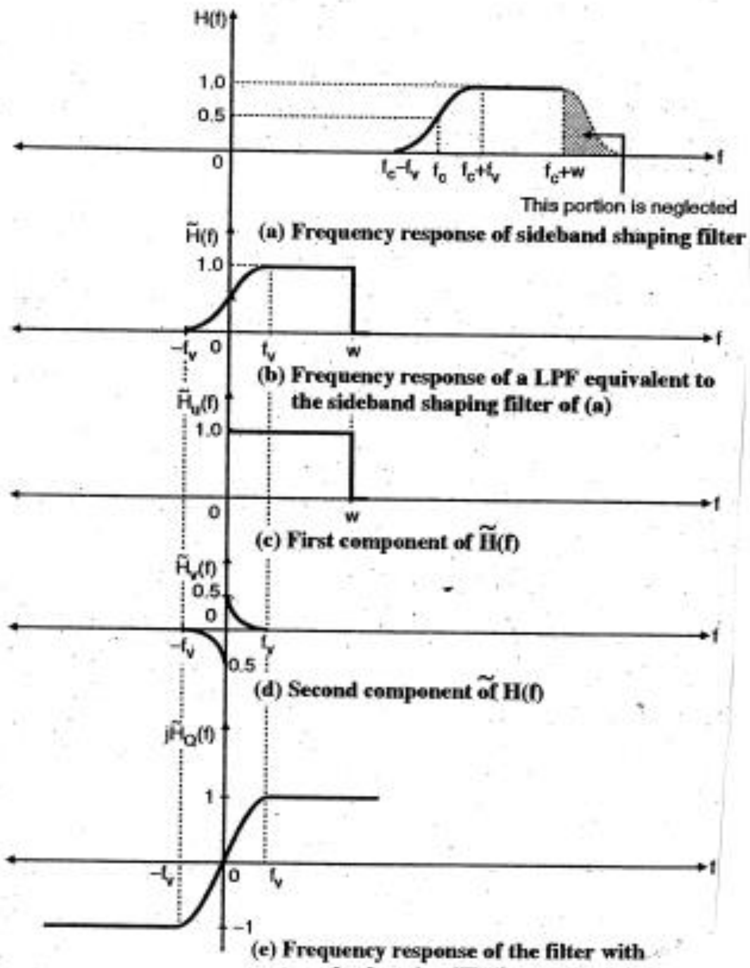
$$H_Q(f) = \frac{1}{j}\left[\mathrm{Sgn}(f) - 2\tilde{H}_v(f)\right]$$

Expression for VSB modulated Wave :-

❖ **Derive the time domain description of VSB modulated signal.** **July-09,6M**

❖ **Show that a VSB modulated wave s(t) containing a vestige of the lower sideband is defined by:**

$$S(t) = \frac{A_c}{2} m(t) \cos(2\pi f_c t) - \frac{A_c}{2} m_Q(t) \sin(2\pi f_c t)$$ **Jan-09,7M**



(a) Frequency response of sideband shaping filter
(b) Frequency response of a LPF equivalent to the sideband shaping filter of (a)
(c) First component of $\tilde{H}(f)$
(d) Second component of $\tilde{H}(f)$
(e) Frequency response of the filter with transfer function $jH_Q(f)$

* Now let us obtain the expression for VSB modulated Wave. Let this Wave be expressed as:

$$\boxed{S(t) = R_e\left[\tilde{S}(t) \cdot e^{j2\pi f_c t}\right]} \longrightarrow (1)$$

Where $\tilde{s}(t)$ is the complex envelope of $s(t)$

We get $\tilde{s}(t)$ at the o/p of the complex LPF having a transfer function $\tilde{H}(f)$ when a DSB-SC wave is applied as its I/p

$$\therefore \boxed{\tilde{s}(t) = \tilde{h}(t) * \tilde{s}_{DSBSC}(t)}$$

Where $\tilde{h}(t)$ is the impulse response of the filter & $*$ represents convolution.

* The spectrum of $\tilde{s}(t)$ can be obtained by taking the F.T of the expression for $\tilde{s}(t)$

$$\boxed{\tilde{S}(f) = \tilde{H}(f) \cdot \tilde{S}_{DSBSC}(f)} \longrightarrow (2)$$

W.K.T $\quad S_{DSBSC}(f) = A_c M(f) \longrightarrow (3)$

Substituting eq (3) in eq (2), we get

$$\tilde{S}(f) = A_c M(f) \cdot \underline{\tilde{H}(f)}$$

$$\tilde{S}(f) = A_c M(f) \left[\frac{1}{2} \left(1 + j H_Q(f) \right) \right]$$

$$\tilde{S}(f) = A_c M(f) \left[\frac{1}{2} + \frac{1}{2} j H_Q(f) \right]$$

$$\boxed{\tilde{S}(f) = \frac{A_c}{2} M(f) + j \frac{A_c}{2} H_Q(f) \cdot M(f)} \longrightarrow (4)$$

Taking IFT of eq (4), we get

$$\tilde{s}(t) = F^{-1}\left[\frac{A_c}{2} M(f) \right] + F^{-1}\left[j \frac{A_c}{2} \tilde{H}_Q(f) \cdot M(f) \right]$$

$$= \frac{A_c}{2} m(t) + j \frac{A_c}{2} \left[\tilde{h}_Q(t) * m(t) \right]$$

$$\tilde{s}(t) = \frac{A_c}{2} \left[m(t) + j \, \underline{\tilde{h}_Q(t) * m(t)} \right], \quad \boxed{\tilde{s}(t) = \frac{A_c}{2} \left[m(t) + j m_Q(t) \right]} \rightarrow (5)$$

Where $\boxed{m_Q(t) = h_Q(t) * m(t)}$

$m_Q(t)$ is the response produced by passing the message signal $m(t)$ through a LPF of impulse response $h_Q(t)$

Substituting eq ⑤ in eq ①, we get $\left(S(t) = R_e\left[\underline{\tilde{S}(t)}\, e^{j2\pi f_c t}\right] \to ①\right.$

$$S(t) = R_e\left\{\frac{A_c}{2}\left[m(t) + j m_Q(t)\right] \underline{e^{j2\pi f_c t}}\right\}$$

W.K.T $\boxed{e^{j2\pi f_c t} = \cos(2\pi f_c t) + j\sin(2\pi f_c t)}$

$$\therefore\ S(t) = R_e\left\{\frac{A_c}{2}\left[m(t) + j m_Q(t)\right]\ \cos(2\pi f_c t) + j\sin(2\pi f_c t)\right\}$$

$$S(t) = R_e\left\{\frac{A_c}{2} m(t)\cos 2\pi f_c t + j\frac{A_c}{2} m(t)\sin 2\pi f_c t + j\frac{A_c}{2} m_Q(t)\cos 2\pi f_c t + j^2\frac{A_c}{2} m_Q(t)\cdot \sin 2\pi f_c t\right\}$$

$$= R_e\left\{\frac{A_c}{2} m(t)\cos(2\pi f_c t) + \underline{j}\frac{A_c}{2} m(t)\sin(2\pi f_c t) + \underline{j}\frac{A_c}{2} m_Q(t)\cos(2\pi f_c t) - \frac{A_c}{2} m_Q(t)\sin(2\pi f_c t)\right\}$$

Selecting only the real part, we get

$$\boxed{S(t) = \frac{A_c}{2}\left[m(t)\cos(2\pi f_c t) - m_Q(t)\sin(2\pi f_c t)\right]} \longrightarrow ⑥$$

* Equation ⑥ is the expression for the VSB modulated wave in time domain.

Note that it represents the VSB wave with full <u>USB</u> & a <u>vestige</u> of <u>LSB.</u>

NOTE :-

The time-domain description for the VSB modulated wave with full LSB & vestige of USB is as follows:

$$\boxed{S(t) = \frac{A_c}{2}\left[m(t)\cos(2\pi f_c t) + m_Q(t)\sin(2\pi f_c t)\right]}$$

Frequency Translation

- **Explain how downward frequency translation is achieved with the help of a block diagram.** **Jan-05,8M**
- **Explain the frequency translation.** **Jan-09,5M**

* In the Communication Systems it is necessary to translate the modulated wave upward or downward in frequency, so that it occupies a new frequency band.

* This frequency translation is accomplished by multiplication of the signal by a locally generated carrier wave then filtering the product term as shown in fig ①.

Block diagram of frequency translator

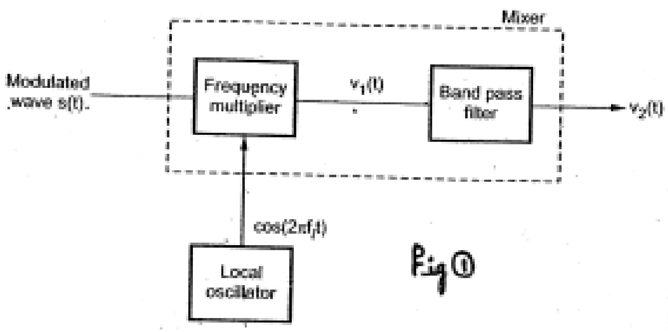


fig ①

* Consider an DSB-SC wave expressed as:

$$\boxed{S(t) = m(t)\cos(2\pi f_c t)} \longrightarrow ①$$

Where $m(t)$ is limited to the frequency band $-W \leq f \leq W$.

{ Taking F.T on both Side of eq ①, we get

$$S(f) = \frac{1}{2}\left[M(f-f_c) + M(f+f_c)\right]$$

}

* The Spectrum of $S(t)$ occupies the bands $(f_c - W)$ to $(f_c + W)$ & $(-f_c - W)$ to $(-f_c + W)$ as Shown in fig ⓑ.

* Suppose that we have to translate this modulated wave downward in frequency. To perform Such translation it is necessary to change 'f_c' to a new value 'f_o'.

Where $\boxed{f_o < f_c}$

This Can be achieved as follows:

Step 1 :- Multiply the DSB-SC wave $S(t)$ by a locally generated carrier $\cos 2\pi f_L(t)$ as Shown in fig ①.

∴ The o/p of the product modulator is given by:

$$V_1(t) = \underline{S(t)} . \cos 2\pi f_L t$$

$$V_1(t) = m(t) \cos 2\pi f_c t \, . \cos 2\pi f_L t.$$

W.K.T $\boxed{\cos A . \cos B = \frac{1}{2}\cos(A-B) + \frac{1}{2}\cos(A+B)}$

$$\boxed{V_1(t) = \frac{m(t)}{2}\cos 2\pi[f_c - f_L]t + \frac{m(t)}{2}\cos 2\pi[f_c + f_L]t} \longrightarrow ②$$

{ * Taking F.T on both Side of eq ②, we get

$$V_1(f) = \frac{1}{4}\left\{M[f-(f_c-f_L)] + M[f+(f_c-f_L)]\right\}$$

$$+\frac{1}{4}\left\{M[f-(f_c+f_L)] + M[f+(f_c+f_L)]\right\}$$

}

* The Spectrum of $V_1(f)$ is as Shown in fig ©

Step 2 :-

Pass the multiplier o/p through a BPF.

The carrier frequency is $f_0 = f_c - f_L$

* The BPF are designed to pass the Signal having $BW = 2W$ and center frequency 'f_0'.

Thus the o/p of the BPF $V_2(t)$ is given by

$$\boxed{V_2(t) = \frac{m(t)}{2}\cos 2\pi(f_c - f_L)t}$$

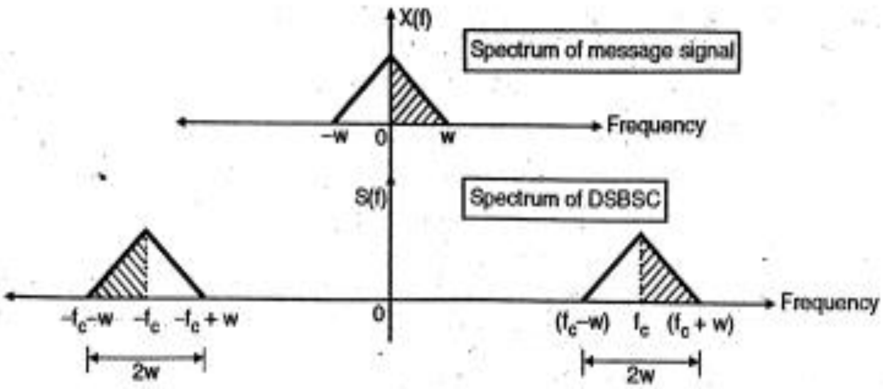
$$\boxed{V_2(t) = \frac{1}{2}m(t)\cos 2\pi f_0 t} \longrightarrow 3$$

{ Taking FT on both Side of eq 3, we get

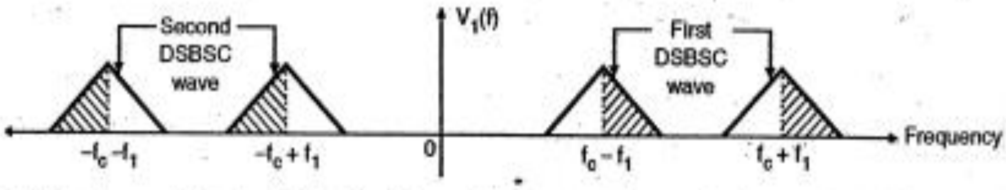
$$V_2(f) = \frac{1}{4}\left\{M(f-f_0) + M[f+f_0]\right\}$$

}

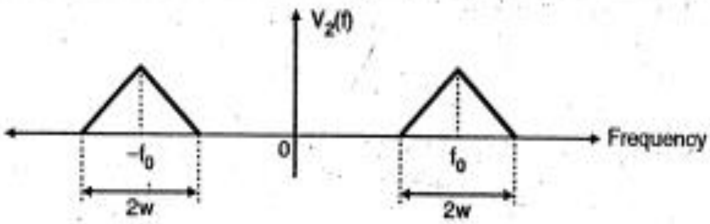
* The Spectrum of $V_2(t)$ i.e. $V_2(f)$ is as Shown in fig ④



(a) The frequency translation process illustrated using spectrums of message signal and DSBSC wave



(b) Spectrum of the signal obtained by multiplying the DSBSC wave with a local carrier



(c) Spectrum of the DSBSC wave translated downwards in frequency

Fig. : Concept of frequency translation

NOTE :-

The device which performs frequency translation is known as Mixer. The process of frequency translation is also known as Mixing or heterodyne.

Frequency translation to higher frequency:-

W.K.T the o/p of the multiplier is given by

$$V_1(t) = \frac{1}{2} m(t) \cos 2\pi(f_c - f_L)t + \frac{1}{2} m(t) \cos 2\pi(f_c + f_L)t \longrightarrow (2)$$

* Designing a BPF with center frequency $f_0 = (f_c + f_L)$ & bandwidth of $2W$.

The o/p of the BPF is given by

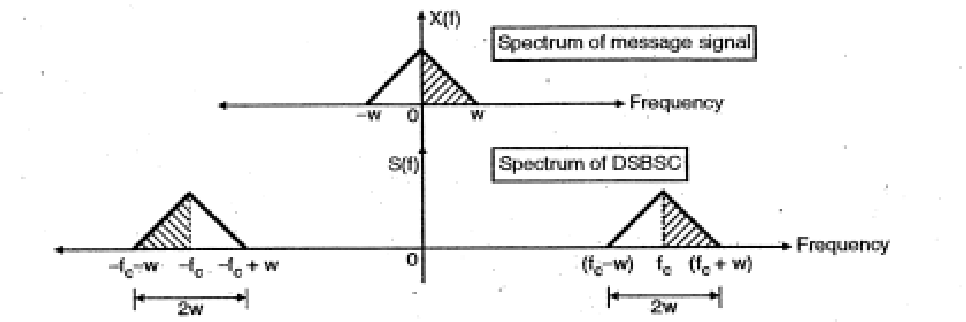
$$V_2(t) = \frac{1}{2} m(t) \cos 2\pi(f_c + f_L)t$$

$$\boxed{V_2(t) = \frac{1}{2} m(t) \cos 2\pi(f_0)t} \longrightarrow (3)$$

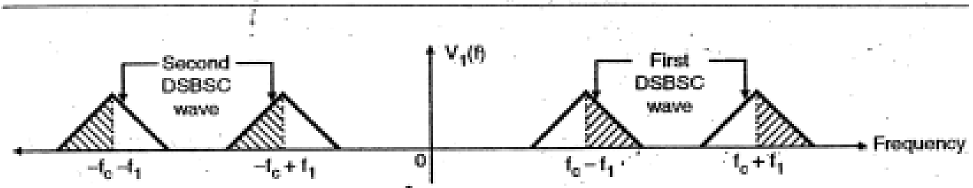
Taking F.T on both sides of eq (3), we get

$$\boxed{V_2(f) = \frac{1}{4}\left[M(f - f_c) + M(f + f_0)\right]}$$

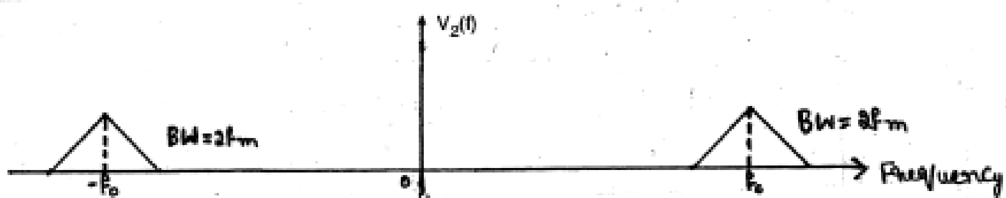
* The spectrum of $V_2(t)$ i.e. $V_2(f)$ is as shown in fig.



(a) The frequency translation process illustrated using spectrums of message signal and DSBSC wave



(b) Spectrum of the signal obtained by multiplying the DSBSC wave with a local carrier



19

(c) Spectrum of the DSBSC wave translated upwards in frequency

Concept of frequency translation

1) The incoming Signal has a midband frequency that may lie in the range 530 KHz to 1650 KHz. The allocated a bandwidth is 10 KHz. This Signal is to be translated to a fixed frequency band centered at 470 KHz. Determine the tuning range that must be provided by the local oscillator.

Sol :- Given $530\,KHz < f_c < 1650\,KHz$, $f_o = 470\,KHz$ & $BW = 10\,KHz$.

* W.K.T New Center frequency $f_o = (f_c - f_L)$

$$\boxed{f_L = f_c - f_o}$$

* When $f_c = 530\,KHz$, $f_L = 530\,KHz - 470\,KHz$, $\underline{f_L = 60\,KHz}$ &

* When $f_c = 1650\,KHz$, $f_L = 1650\,KHz - 470\,KHz$, $\underline{f_L = 1180\,KHz}$

Hence the tuning range of oscillator is $\boxed{60\,KHz \text{ to } 1180\,KHz.}$

---

2) Consider the bandwidth of a Signal 20 KHz & the midband freq-range 0.615 - 1.715 MHz. It is required to translate this Signal to a fixed frequency band centered at 0.475 MHz. Determine the range of tuning that must be provided in the local oscillator to perform necessary frequency translation.

Sol :- Given : $0.615\,MHz < f_c < 1.715\,MHz$, $f_o = 0.475\,MHz$ & $BW = 20\,KHz$

W.K.T $f_o = f_c - f_L$

$$\boxed{f_L = f_c - f_o}$$

* When

$f_c = 0.615\,MHz$, $f_L = 0.615\,MHz - 0.475\,MHz$, $\underline{f_L = 0.14\,MHz}$

* When

$f_c = 1.715\,MHz$, $f_L = 1.715\,MHz - 0.475\,MHz$, $\underline{f_L = 1.24\,MHz}$

Thus, the required tunning range of local oscillator is

$$\boxed{0.14 MHz - 1.24 MHz}$$

Multiplexing:-

Multiplexing is a technique where by a number of Independent Signals can be Combined into a Composite Signal Suitable for transmission over a Common Channel.

There are two types of multiplexing:

1) Frequency Division Multiplexing (FDM)
2) Time Division Multiplexing (TDM)

Multiplexing requires that the Signals be Kept apart So that they do not interfere with each other & thus they can be Separated at the receiving end. This is accomplished by Separating the Signals either in frequency or time.

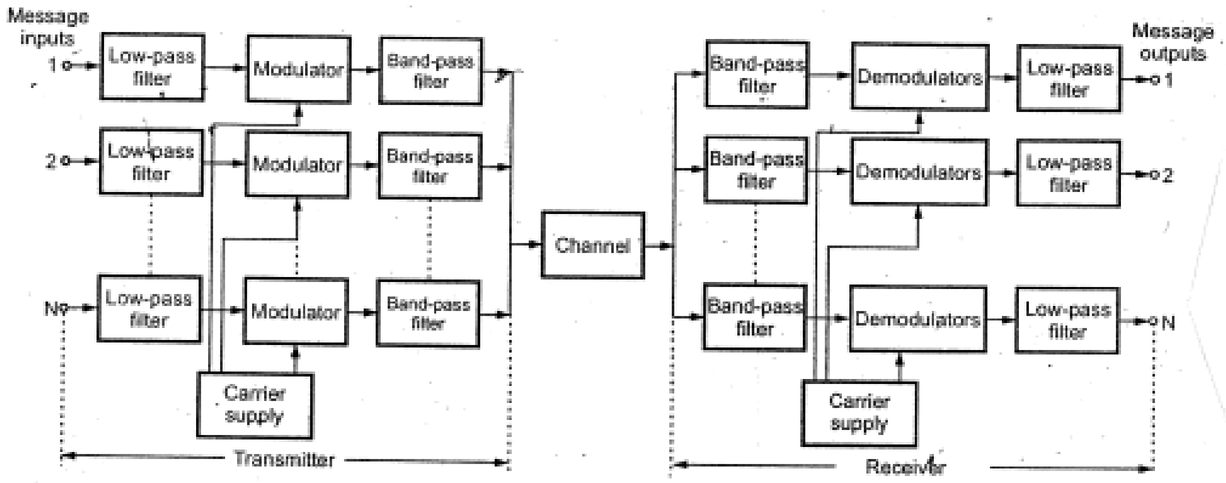
* The technique of Separating the Signals in frequency is referred to as Frequency-Division Multiplexing (FDM).

* The technique of Separating the Signals in time is Called Time-Division Multiplexing (TDM).

Frequency Division Multiplexing (FDM):-

❖ **With a neat block diagram explain the operation of FDM technique.**

July-08,5M July-10,6M June-10,6M June-10,8M July-07,5M



Block diagram of FDM system

* The I/p message signals assumed to be of the low-pass type are passed through the I/p LPF's. This LPF's are designed to remove high-frequency components that do not contribute significantly to signal representation but are capable of disturbing other message signals that share the common channel.

* The filtered message signals are then modulated with the carrier frequencies. The most widely used method of modulation in FDM is single sideband modulation, which requires a bandwidth that is approximately equal to that of original message signal.

* The BPF's following the modulators are used to restrict the band of each modulated wave to its prescribed range.

The resulting BPF o/p's are next combined in parallel to form the I/p to the common channel.

* At the receiving end, BPF's connected to the common channel in parallel to separate the message signals on the frequency occupancy basis.

* Finally, the original message signals are recovered by individual demodulators.

Transmission bandwidth:-

Consider an FDM system using SSB modulation to transmit 24 independent voice I/p's. Assume a bandwidth of 4KHz for each voice I/p. Thus in order to accomodate an FDM system using SSB modulation to transmit the 24 voice I/p's, the communication channel must provide the transmission bandwidth:

$$\boxed{BW = n \times f_m}$$

Where 'n' is the number of voice signals.

∴ For 24 voice I/p's having a bandwidth of 4KHz for each voice I/p's, the transmission bandwidth is given by:

$$BW = 24 \times 4KHz = \underline{96KHz.}$$

Advantages :-

1) A large number of Signals (Channels) can be transmitted - Simultaneously.

2) FDM does not need Synchronization between transmitter and receiver for proper operation.

3) Demodulation of FDM is easy.

Disadvantages :-

1) The Communication Channel must have a very large bandwidth

2) Large number of modulators & filters are required.

3) Cross talks occurs in FDM.

4) All the FDM Channel get affected due to Wideband fading.

Radio broadcasting :-

In radio broadcasting, a central transmitter is used to - radiate message Signals for reception at a large number of remote points.

The message Signals transmitted are usually Intended for entertainment purpose.

There are 3 types of radio broadcasting

1) AM broadcasting : use Standard amplitude modulation

2) FM broadcasting : uses Frequency modulation

3) TV broadcasting : uses VSB modulation.

AM Receiver

- **List the important characteristics of a receiver. Draw the block diagram of a super heterodyne receiver and explain the function of each section.** **July-05,10M**
- **Write the block diagram of super heterodyne receiver and specify the importance of IF value in the receiver.**

July-06,8M July-07,3M, July-09,8M June-10,6M

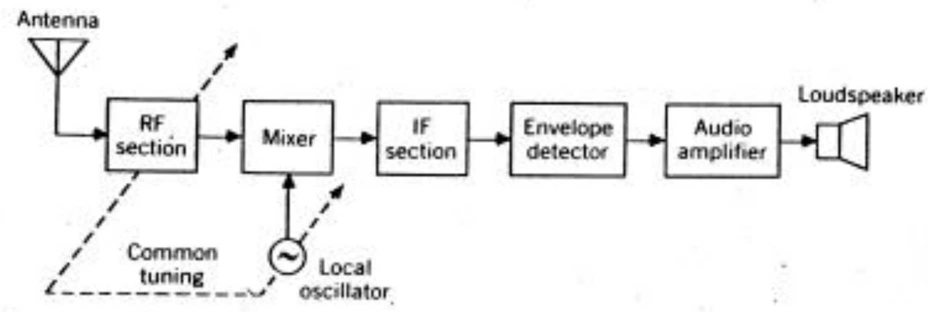


Figure
Basic elements of an AM receiver of the superheterodyne type.

* The receiver consists of a Radio frequency Section, Mixer, Local oscillator, Intermediate Section (IF) & a demodulator.

* The frequency parameters of Commercial AM radio are:

RF carrier range = 0.535 - 1.605 MHz
Mid-band frequency of IF Section = 455 KHz
IF bandwidth = 10 KHz

* The incoming amplitude modulated Wave is picked up by the receiving antenna & amplified in the RF Section. The Combination of mixer & local oscillator provides a frequency conversion or heterodyning function, whereby the incoming Signal is Converted to a pre-determined fixed Intermediate frequency i.e.

$$\boxed{f_{IF} = f_{RF} - f_{LO}}$$

Where,

f_{LO} is the frequency of the local oscillator.

f_{RF} is the carrier frequency of the incoming RF signal.

* 'f_{IF}' is called Intermediate frequency (IF), because the signal is neither at the original i/p frequency nor at the final baseband frequency.

* The IF section consists of one or more stages of tuned amplifiers provides most of the amplification & selectivity in the receiver.

* The o/p of the IF section is applied to an envelope detector to recover the base band signal.

* The recovered message signal is amplified by an audio power amplifier & finally fed to loudspeaker.

* The superheterodyne operation refers to the frequency conversion from the variable carrier frequency of the incoming RF signal to the fixed IF signal.

Advantages:-

1) No variations in bandwidth. The bandwidth remains constant over the entire operating range.

2) High sensitivity & selectivity

3) High adjacent channel rejection

Characteristics of radio receiver :-

The performance of the radio receiver can be measured in terms of following receiver characteristics:

1) Selectivity
2) Sensitivity
3) Fidelity
4) Image frequency & its rejection
5) Double spotting

FORMULAE

1) Resonant frequency : $f_r = \frac{1}{2\pi\sqrt{LC}}$

2) The Q of the tuned ckt is given by

$$Q = \frac{X_L}{R} = \frac{2\pi f_r L}{R} \quad \&$$

$$R = \frac{X_L}{Q}$$

3) Bandwidth of the tuned circuit;

$$BW = \frac{f_r}{Q}$$

4) Image rejection ratio

$$\rho = \frac{f_{si}}{f_s} - \frac{f_s}{f_{si}}$$

5) Rejection ratio

$$\alpha = \sqrt{1 + Q^2 \rho^2}$$

6) Image frequency

$$\boxed{f_{si} = f_s + 2f_i}$$

1) In a broad cast AM receiver, having a RF amplifier, loaded Q of antenna coil at the I/p of the mixer is 100. If the IF frequency is 455KHz, find the image frequency & its image rejection ratio of 1000 KHz.

July-2005, 4M

Sol:- $f_i = 455\,KHz$, $f_s = 1000\,KHz$, $Q = 100$.

* $f_{si} = f_s + 2f_i$

$= 1000\,KHz + 2 \times 455\,KHz$

$\boxed{f_{si} = 1910\,KHz}$ ← Image frequency

* Image rejection ratio:

$$p = \frac{f_{si}}{f_s} - \frac{f_s}{f_{si}} = \frac{1910\,KHz}{1000\,KHz} - \frac{1000\,KHz}{1910\,KHz} = 1.386$$

* Rejection ratio:

$$\alpha = \sqrt{1 + Q^2 p^2} = \sqrt{1 + (100)^2 \times (1.386)^2} = 100.014$$

---

2) In a broadcast Super heterodyne receiver, having no RF amplifier, the loaded Q of the antenna coupling circuit is 100. If the IF is 455KHz. Calculate:

i) The image frequency & its rejection ratio for tuning at 1000KHz

ii) The image frequency & its rejection ratio for tuning at 25MHz.

July-2006, 6M

Sol:- i) $f_{si} = f_s + 2f_i = 1000\,KHz + 2 \times 455\,KHz = 1910\,KHz$

$$p = \frac{f_{si}}{f_s} - \frac{f_s}{f_{si}} = \frac{1910\,KHz}{1000\,KHz} - \frac{1000\,KHz}{1910\,KHz} = 1.386$$

Image rejection ratio :-

$$\alpha = \sqrt{1+Q^2 p^2} = \sqrt{1+(100)^2 \times (1.386)^2} = 100.01$$

ii) $f_{si} = f_s + 2f_i = (25\times10^6) + 2\times455\times10^3 = 25.91MHz$

$$p = \frac{f_{si}}{f_s} - \frac{f_s}{f_{si}} = \frac{25.91MHz}{25MHz} - \frac{25MHz}{25.91MHz} = 0.071$$

Image rejection ratio

$$\alpha = \sqrt{1+Q^2 p^2} = \sqrt{1+(100)^2 \times (0.071)^2} = 7.17$$

---

*) The Single tone modulating Signal $m(t) = A_m \cos(2\pi f_m t)$ is used to generate the V_{SB} Signal

$$s(t) = \frac{1}{2} a A_m A_c \cos[2\pi(f_c+f_m)t] + \frac{1}{2} A_m A_c (1-a) \cos[2\pi(f_c-f_m)t]$$

Where 'a' is Constant, less than unity, representing the - attenuation of the upper Side frequency.

i) Find the Inphase & Quadrature Component of the VSB Signal $m(t)$

ii) What is the value of Constant a for which $s(t)$ reduces to a DSB-SC modulated wave?

iii) What are the values of Constant a for which it reduces to an SSB modulated wave?

iv) The VSB Singnal, plus the carrier $A_c \cos(2\pi f_c t)$, is passed through an envelope detector. Determine the distortion produced by the Quadrature Component.

v) What is the value of Constant 'a' for which this distortion reaches it worst possible Condition

Jan-2009, 8M

Sol :-

The VSB modulated wave is:

$$s(t) = \frac{1}{2} a A_m A_c \cos\left[2\pi(f_c+f_m)t\right] + \frac{1}{2} A_m A_c (1-a) \cos\left[2\pi(f_c-f_m)t\right]$$

$\cos(A+B)$ $\qquad\qquad$ $\cos(A-B)$

W.K.T

$$\cos(A+B) = \cos A \cdot \cos B - \sin A \cdot \sin B$$

$$\cos(A-B) = \cos A \cdot \cos B + \sin A \cdot \sin B$$

$$s(t) = \frac{1}{2} a A_m A_c \left\{ \cos 2\pi f_c t \cdot \cos 2\pi f_m t - \sin 2\pi f_c t \cdot \sin 2\pi f_m t \right\}$$

$$+ \frac{1}{2} A_m A_c (1-a) \left\{ \cos 2\pi f_c t \cdot \cos 2\pi f_m t + \sin 2\pi f_c t \cdot \sin 2\pi f_m t \right\}$$

$$= \frac{1}{2} a A_m A_c \underline{\cos 2\pi f_c t \cdot \cos 2\pi f_m t} - \frac{1}{2} a A_m A_c \underbrace{\sin 2\pi f_c t \cdot \sin 2\pi f_m t}$$

$$+ \frac{1}{2} A_m A_c (1-a) \underline{\cos 2\pi f_c t \cdot \cos 2\pi f_m t} + \frac{1}{2} A_m A_c (1-a) \underbrace{\sin 2\pi f_c t \cdot \sin 2\pi f_m t}.$$

$$s(t) = \cos 2\pi f_c t \cdot \cos 2\pi f_m t \left[\frac{1}{2} a A_m A_c + \frac{(1-a)}{2} A_m A_c \right]$$

$$+ \sin 2\pi f_c t \cdot \sin 2\pi f_m t \left[\frac{(1-a)}{2} A_m A_c - \frac{a}{2} A_m A_c \right]$$

$\{$ i) $\frac{a}{2} A_m A_c + \frac{(1-a)}{2} A_m A_c = \cancel{\frac{a}{2} A_m A_c} + \frac{A_m A_c}{2} - \cancel{\frac{a}{2} A_m A_c} = \frac{A_m A_c}{2}$ &

ii) $\frac{(1-a)}{2} A_m A_c - \frac{a}{2} A_m A_c = \frac{A_m A_c}{2} - \frac{a}{2} A_m A_c - \frac{a}{2} A_m A_c$

$$= \frac{A_m A_c}{2} - \frac{2a}{2} A_m A_c$$

$\frac{(1-a)}{2} A_m A_c - \frac{a}{2} A_m A_c = \frac{A_m A_c}{2} (1-2a)$

$\}$

$$\boxed{s(t) = \frac{1}{2} A_m A_c \cos 2\pi f_c t \cdot \cos 2\pi f_m t + \frac{1}{2} A_m A_c (1-2a) \cdot \sin 2\pi f_c t \cdot \sin 2\pi f_m t.} \rightarrow (1)$$

i) The In-phase Component of the VSB Wave is

$$\boxed{S_I(t) = \frac{1}{2} A_m A_c \cos 2\pi f_m t}$$

& its quadrature component is

$$\boxed{S_Q(t) = \frac{1}{2} A_m A_c (1-2a) \sin 2\pi f_m t.}$$

ii) If $a = \frac{1}{2}$, the VSB wave, $S(t)$ of eq ① reduces to DSB-SC modulated wave.

$$S(t) = \frac{1}{2} A_m A_c \cos 2\pi f_c t \cdot \cos 2\pi f_m t + \frac{1}{2} A_m A_c \Big(\underbrace{1 - \cancel{2}\big(\tfrac{1}{\cancel{2}}\big)}_{(1-1)=0}\Big) \sin 2\pi f_c t \sin 2\pi f_m t$$

$$S(t) = \frac{1}{2} A_m A_c \cos 2\pi f_c t \cdot \cos 2\pi f_m t + \frac{1}{2} A_m A_c (0) \sin 2\pi f_c t \cdot \sin 2\pi f_m t$$

$$\boxed{S(t) = \frac{1}{2} A_m A_c \cos 2\pi f_c t \cos 2\pi f_m t.}$$

iii) If $a = 0$, the $S(t)$ of eq ① reduces to an SSB wave containing only the LSB as shown by

$$S(t) = \frac{1}{2} A_m A_c \cos 2\pi f_c t \cdot \cos 2\pi f_m t + \frac{1}{2} A_m A_c (1 - \overset{0}{2(0)}) \sin 2\pi f_c t \cdot \sin 2\pi f_m t.$$

$$\boxed{S(t) = \frac{1}{2} A_m A_c \cos 2\pi f_c t \cdot \cos 2\pi f_m t + \frac{1}{2} A_m A_c \sin 2\pi f_c t \cdot \sin 2\pi f_m t.}$$

$$S(t) = A_m A_c \cos 2\pi (f_c - f_m) t \leftarrow$$

If $a = 1$, the $S(t)$ of eq ① reduces to an SSB wave containing only the USB as shown by:

$$S(t) = \frac{1}{2} A_m A_c \cos 2\pi f_c t \cdot \cos 2\pi f_m t + \frac{1}{2} A_m A_c [\overset{(-1)}{1 - 2(1)}] \sin 2\pi f_c t \cdot \sin 2\pi f_m t.$$

$$S(t) = \frac{1}{2} A_m A_c \cos 2\pi f_c t \cos 2\pi f_m t - \frac{1}{2} A_m A_c \sin 2\pi f_c t \cdot \sin 2\pi f_m t.$$

$$\boxed{S(t) = A_m A_c \cos 2\pi (f_c + f_m) t}$$

iv) Adding the carrier to the VSB modulated wave, the envelope detector I/p is

$$S'(t) = S(t) + A_c \cos 2\pi f_c t$$

$$S'(t) = \frac{1}{2} \underline{A_c} A_m \underline{\cos 2\pi f_c t} \cdot \cos 2\pi f_m t + \frac{1}{2} A_c A_m (1-2a) \sin 2\pi f_c t \cdot \sin 2\pi f_m t + \underline{A_c \cos 2\pi f_c t}$$

$$S'(t) = A_c \cos 2\pi f_c t \left[1 + \frac{1}{2} A_m \cos 2\pi f_m t\right] + \frac{1}{2} A_c A_m (1-2a) \sin 2\pi f_c t \cdot \sin 2\pi f_m t$$

The envelope detector o/p is therefore.

$$a(t) = \sqrt{(\text{In-phase Component})^2 + (\text{Quadrature Component})^2}$$

$$a(t) = \sqrt{S_I^2(t) + S_Q^2(t)}$$

$$a(t) = \sqrt{\underline{A_c^2\left[1 + \frac{1}{2} A_m \cos 2\pi f_m t\right]^2} + \left[\frac{1}{2} A_m A_c (1-2a) \sin 2\pi f_m t\right]^2}$$

$$a(t) = \sqrt{A_c^2\left[1 + \frac{1}{2} A_m \cos 2\pi f_m t\right]^2 \left\{1 + \frac{\frac{1}{2} A_m (1-2a) \sin 2\pi f_m t}{\left(1 + \frac{1}{2} A_m \cos 2\pi f_m t\right)}\right\}^2}$$

$$a(t) = A_c\left[1 + \frac{1}{2} A_m \cos 2\pi f_m t\right] \sqrt{1 + \left\{\frac{\frac{1}{2} A_m (1-2a) \sin 2\pi f_m t}{1 + \frac{1}{2} A_m \cos 2\pi f_m t}\right\}^2}$$

* The distortion, appearing as a multiplying factor, is therefore

$$d(t) = \sqrt{1 + \left\{\frac{\frac{1}{2} A_m (1-2a) \sin 2\pi f_m t}{1 + \frac{1}{2} A_m \cos 2\pi f_m t}\right\}^2}$$

v) When $a = 0$, the distortion $d(t)$ is greatest.

ANGLE MODULATION – (FM-1)

❖ Define angle modulation ?

Angle modulation is the process in which the angle of the carrier wave is varied in accordance with the instantaneous amplitude of the modulating signal, keeping constant amplitude of the carrier wave.

There are two types of angle modulation:

1. **Frequency modulation**
2. **Phase modulation**

❖ Angle Modulation : Basic Concept :-

Let the modulated wave be expressed in the general form as follows:

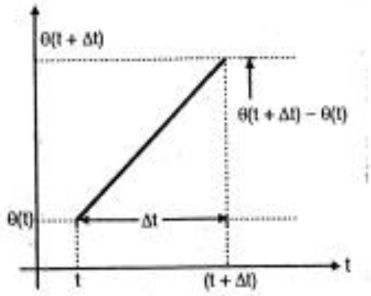
$$\boxed{S(t) = A_c \cos[\theta(t)]} \longrightarrow (1)$$

Where,

$A_c \rightarrow$ carrier amplitude (Which is maintained Constant)

$\theta(t) \rightarrow$ is the angular argument which is varied in proportion with the message Signal $m(t)$.

* The variation of $\theta(t)$ due to $m(t)$ can be expressed mathematically if we know the type of angle modulation. If $\theta(t)$ changes by 2π radians then we say that a complete oscillation has occured.
* If $\theta(t)$ is increased monotonically with time as shown in figure, then the average frequency in Hz over the interval 't' to $(t+\Delta t)$ is given by:



$$\boxed{f_{\Delta t}(t) = \frac{1}{2\pi} \frac{\theta(t+\Delta t) - \theta(t)}{\Delta t}} \longrightarrow ②$$

* The Instantaneous frequency of the angle modulated wave $s(t)$ is given by

$$f_i(t) = \lim_{\Delta t \to 0} f_{\Delta t}(t)$$

$$= \lim_{\Delta t \to 0} \left[\frac{\theta(t+\Delta t) - \theta(t)}{2\pi \, \Delta t} \right]$$

$$\boxed{f_i(t) = \frac{1}{2\pi} \frac{d\theta(t)}{dt}} \longrightarrow ③$$

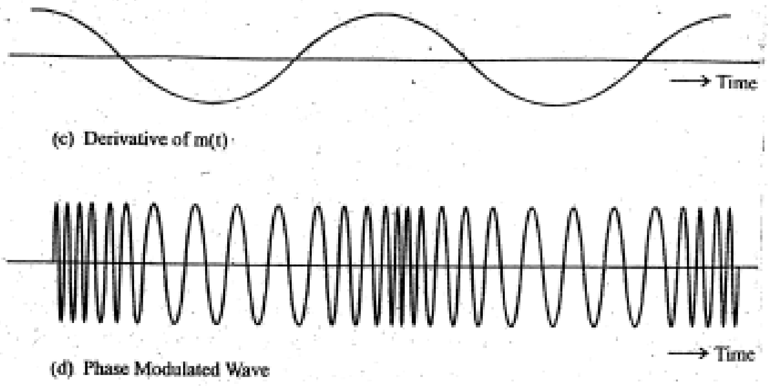
This is basic definition of derivative of a function.

* For an unmodulated carrier, the angle $\theta(t)$ is given by:

$$\boxed{\theta(t) = 2\pi f_c t + \phi_c(t)}$$

The angular frequency of the carrier is W_c, where $W_c = 2\pi f_c$, & ϕ_c is the value of $\theta(t)$ at $t=0$.

Phase Modulation (PM):-



phase modulated waves for sinusoidal signal

Phase modulation (PM) is defined as the form of angle - modulation in which the angular argument '$\theta(t)$' is varied linearly with the message signal '$m(t)$' as given below:

$$\boxed{\theta(t) = 2\pi f_c t + K_p m(t)}$$

Where,

$\omega_c t = 2\pi f_c t$ represents the angular argument of the modulated carrier &

$K_p \rightarrow$ Constant, represents the phase sensitivity of the modulator.

$\therefore$ The phase-modulated wave $S(t)$ is given by:

$$S(t) = A_c \cos[\theta(t)]$$

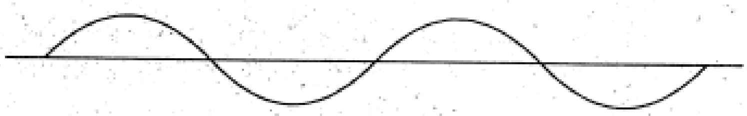
$$\boxed{S(t) = A_c \cos[2\pi f_c t + K_p m(t)]}$$

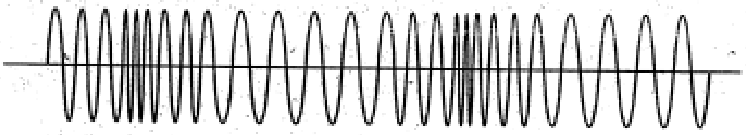
The features of phase modulation are:

i) The envelope of PM wave is a constant & equal to the amplitude of the unmodulated carrier.

2) The Zero Crossings of a PM wave no longer have a perfect - regularity in their Spacing like AM wave. This is because - instantaneous frequency of PM wave is proportional to time derivative of $m(t)$.

Frequency Modulation (FM):-


(a) Message signal m(t)



(b) Frequency modulated wave

Frequency modulation is the form of angle modulation in which the instantaneous frequency $f_i(t)$ is varied linearly with the message Signal $m(t)$ as given below:

$$\boxed{f_i(t) = f_c + K_f m(t)} \longrightarrow (1)$$

Where,

$f_c \rightarrow$ frequency of the unmodulated carrier

$K_f \rightarrow$ frequency Sensitivity of the modulator expressed in hertz's per volt.

W.K.T, (From eq ③)

$$f_i(t) = \frac{1}{2\pi} \frac{d\theta}{dt}$$

$$\frac{d\theta}{dt} = 2\pi f_i(t) \longrightarrow ②$$

Integrating on both sides of eq ② w.r.to 't'

$$\theta(t) = \int_0^t 2\pi f_i(t) \cdot dt \longrightarrow ③$$

$$\theta(t) = \int_0^t 2\pi \left[f_c + K_f m(t) \right] dt$$

$$= \int_0^t 2\pi f_c \, dt + \int_0^t 2\pi K_f \, m(t) \cdot dt$$

$$= 2\pi f_c \int_0^t (1) \, dt + 2\pi K_f \int_0^t m(t) \cdot dt$$

$$\boxed{\theta(t) = 2\pi f_c t + 2\pi K_f \int_0^t m(t) \cdot dt} \longrightarrow ④$$

* The FM wave in time domain can be written as

$$S(t) = A_c \cos\left[\theta(t)\right] \longrightarrow ⑤$$

Substituting eq ④ in eq ⑤, we get

$$\boxed{S(t) = A_c \cos\left[2\pi f_c t + 2\pi K_f \int_0^t m(t) \, dt\right]}$$

Relationship between FM & PM :-

In both FM & PM, the instantaneous angle $\theta(t)$ changes but in a different manner.

* The expressions for the FM & PM waves in the time domain are as follows:

PM Wave : $S(t) = A_c \cos[2\pi f_c t + K_p m(t)]$

FM Wave : $S(t) = A_c \cos\left[2\pi f_c t + 2\pi K_f \int_0^t m(t)\, dt\right]$

Comparing these expressions we can conclude that an FM Wave is actually a PM Wave having a modulating Signal $\int_0^t m(t)\, dt$ instead of $m(t)$.

Note:-

1) In FM Wave, $\theta(t)$ is directly proportional to the Integral of $m(t)$ i.e. $\int_0^t m(t).dt$

2) PM can be generated by 1st differentiating modulating Signal $m(t)$ w.r. to 't' & then $\frac{d\,m(t)}{dt}$ is modulated by using - a Sinusoidal carrier.

❖ **Define angle modulation. Describe with the help of block diagrams schemes for generating. i. FM wave using PM ii. PM wave using FM**

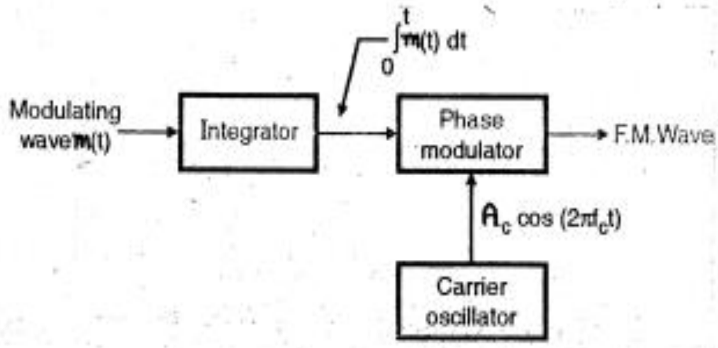
June-08,8m Jan-05,8M June-09,8M

Angle modulation is the process in which the angle of the carrier wave is varied in accordance with the instantaneous amplitude of the modulating signal, keeping constant amplitude of the carrier wave.

There are two types of angle modulation:

1. **Frequency modulation**
2. **Phase modulation**

i. Generation of FM using PM (Phase Modulator):-

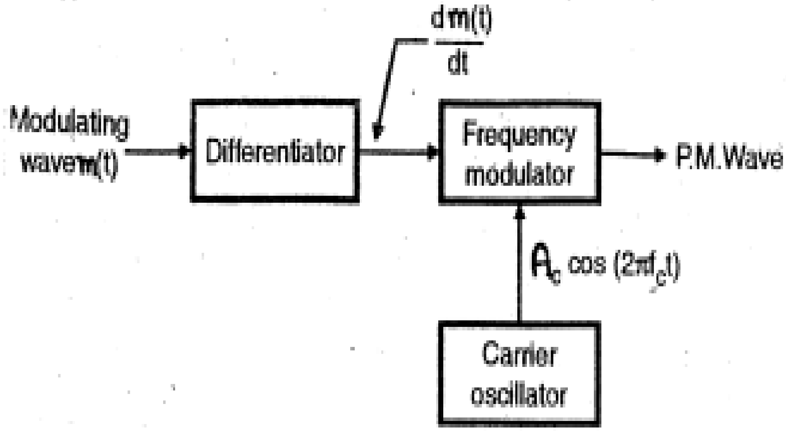


Generation of FM from phase modulator

* FM can be generated by 1st integrating m(t) & then using the result as the I/p to a phase modulator as shown in above figure.

$$\boxed{s(t) = A_c \cos\left[2\pi f_c t + \underset{K_p}{\underset{\uparrow}{2\pi K_f}} \int_0^t m(t)\cdot dt\right]}$$

ii. Generation of PM using FM (Frequency Modulator):-



Generation of P.M. wave using frequency modulator

* The PM signal can be generated by 1st differentiating m(t) & then using the result as the I/p to a frequency-modulator as shown in fig above.

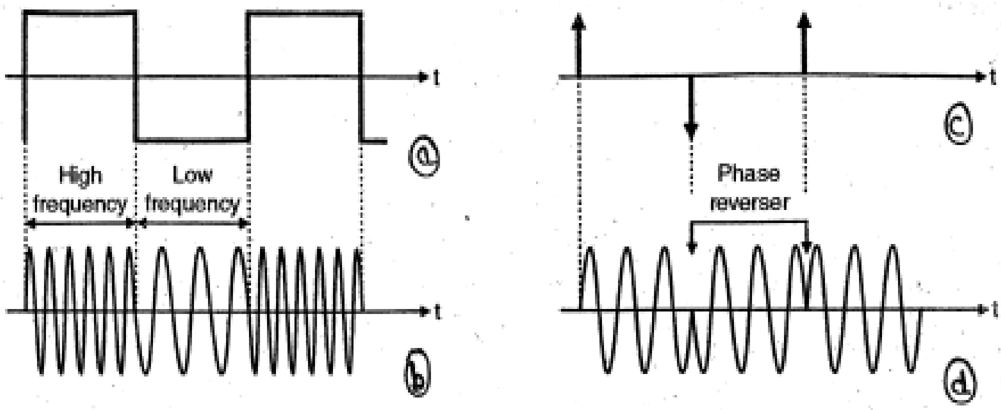
$$\therefore \quad s(t) = A_c \cos\left[2\pi f_c t + 2\pi K_f \int_0^t \frac{d}{dt} m(t) \cdot dt\right]$$

Substituting $2\pi K_f = K_p$

$$s(t) = A_c \cos\left[2\pi f_c t + \underline{2\pi K_f}\, m(t)\right]$$

$$\boxed{s(t) = A_c \cos\left[2\pi f_c t + K_p m(t)\right]}$$

Square Modulation:-



Frequency modulated wave   Phase modulated wave

Square modulation

* Consider two full cycles of square modulating wave m(t) as shown in fig ⓐ. The FM wave produced by this modulating wave is plotted in fig ⓑ.

* The PM wave has been plotted by using the differentiated version of m(t) i.e. $\frac{d}{dt}m(t)$ as the modulating signal as shown in fig ⓒ.

* Note that $\frac{d}{dt}m(t)$ is a train of alternate (+ve & -ve) delta function.

The desired PM wave is plotted in fig ⓓ.

❖ **An FM wave is defined by**

$$S(t) = A_c \cos[10\pi t + \sin(4\pi t)]$$

find the instantaneous frequency of S(t).

Sol:- W.K.T $f_i(t) = \frac{1}{2\pi} \frac{d}{dt} \theta(t)$ Hz &

$$S(t) = A_c \cos[\theta(t)]$$

$$\therefore \theta(t) = 10\pi t + \sin 4\pi t$$

$$f_i(t) = \frac{1}{2\pi} \frac{d}{dt} [10\pi t + \sin 4\pi t] \text{ Hz}$$

$$= \frac{1}{\cancel{2\pi}} [\overset{5}{\cancel{10\pi}} + \cos(4\pi t) \cdot \overset{2}{\cancel{4\pi}}] \text{ Hz}$$

$$\boxed{f_i(t) = 5 + 2\cos(4\pi t) \text{ Hz}}$$

❖ **An FM wave is defined by**

$$S(t) = 10\cos[2 + \sin(6\pi t)]$$

find the instantaneous frequency of S(t).

Sol:- Given : $\theta(t) = 2\pi t + \sin(6\pi t)$

W.K.T $f_i(t) = \frac{1}{2\pi} \frac{d}{dt} \theta(t)$ Hz

$$= \frac{1}{2\pi} \frac{d}{dt} [2\pi t + \sin(6\pi t)] \text{ Hz}$$

$$= \frac{1}{\cancel{2\pi}} [\cancel{2\pi} + \cos(6\pi t) \cdot \overset{3}{\cancel{6\pi}}] \text{ Hz}$$

$$= [1 + \cos(6\pi t) \cdot 3] \text{ Hz}$$

$$\boxed{f_i(t) = [1 + 3\cos(6\pi t)] \text{ Hz}}$$

❖ **Derive an expression for single tone sinusoidal FM wave, find its spectrum**

June-10,10M

❖ **Explain single tone frequency modulation.**

* The frequency modulated wave in time domain is given by:

$$\boxed{S(t) = A_c \cos[\theta(t)]} \longrightarrow (1)$$

* The sinusoidal modulating signal is defined by

$$\boxed{m(t) = A_m \cos(2\pi f_m t)} \longrightarrow (2)$$

* The instantaneous frequency of the FM signal is given by:

$$f_i(t) = f_c + K_f \underline{m(t)}$$

$$f_i(t) = f_c + \underline{K_f A_m} \cos(2\pi f_m t)$$

$$\boxed{f_i(t) = f_c + \Delta f \cos(2\pi f_m t)} \longrightarrow (3)$$

Where, $\Delta f = K_f A_m$ & it is called as frequency deviation

{ The quantity Δf is called the frequency deviation. The frequency deviation Δf is proportional to the amplitude of the modulating signal & is independent of the modulating frequency.
}

W.K.T the angular velocity $\omega_i(t)$ is the rate of change of $\theta(t)$.

$$\omega_i(t) = \frac{d}{dt}\theta(t)$$

$$2\pi f_i(t) = \frac{d}{dt}\theta(t) \longrightarrow (4)$$

Integrating eq ④ w.r.to dt

$$\int_0^t \frac{d}{dt}\theta(t)\,dt = \int_0^t 2\pi f_i(t)\cdot dt$$

$$\theta(t) = \int_0^t 2\pi \underline{f_i(t)}\cdot dt \longrightarrow ⑤$$

Substituting eq ③ in eq ⑤

$$\theta(t) = \int_0^t 2\pi\left[f_c + \Delta f \cos(2\pi f_m t)\right] dt$$

$$= \int_0^t 2\pi f_c\, dt + \int_0^t 2\pi\Delta f \cos(2\pi f_m t)\, dt$$

$$= 2\pi f_c t + \cancel{2\pi}\Delta f \cdot \frac{\sin 2\pi f_m t}{\cancel{2\pi} f_m}$$

$$\theta(t) = 2\pi f_c t + \frac{\Delta f}{f_m}\sin 2\pi f_m t$$

W.K.T

$$\boxed{\int_0^t \cos at\, dt = \frac{\sin at}{a}}$$

$$\boxed{\theta(t) = 2\pi f_c t + \underline{\beta}\sin(2\pi f_m t)} \rightarrow ⑥$$

Where $\beta = \frac{\Delta f}{f_m}$

Substitution eq ⑥ in eq ①, we get

$$\therefore \boxed{s(t) = A_c \cos[2\pi f_c t + \beta \sin(2\pi f_m t)]}$$

Modulation Index (β or m_f):-

Modulation index is defined as the ratio of frequency deviation 'Δf' to the modulating frequency 'f_m'.

$$\beta = \frac{\text{Frequency deviation}}{\text{Modulating frequency}}$$

$$\boxed{\beta \text{ or } m_f = \frac{\Delta f}{f_m}}$$

NOTE:- 1) In FM the modulation index can be greater than 1.

2) The modulation index is very important in FM because it decides the bandwidth of the FM wave.

3) The modulation index also decides the number of sidebands having significant amplitude.

Frequency Deviation (Δf):-

* The instantaneous frequency of the FM signal varies w.r.to time. The maximum change in the instantaneous frequency from the average value carrier frequency 'f_c' is known as frequency deviation.

$$\boxed{\Delta f = |K_f m(t)|_{max}}$$

❖ **Derive the equation for FM waves. Define modulation index, maximum deviation and bandwidth of a FM signal** **June-05,6M June-09,M(old)**

* Let the modulated wave be expressed in the general form as follows:

$$\boxed{S(t) = A_c \cos[\theta(t)]} \longrightarrow (1)$$

* The Instantaneous frequency of the FM Signal is given by:

$$\boxed{f_i(t) = f_c + \Delta f \cos(2\pi f_m t)} \longrightarrow (2)$$

WKT the angular velocity $\omega_i(t)$ is the rate of change of $\theta(t)$

$$\omega_i(t) = \frac{d}{dt}\theta(t)$$

$$2\pi f_i(t) = \frac{d}{dt}\theta(t) \longrightarrow (3)$$

Integrating eq (3) w.r.to dt

$$\int_0^t \frac{d}{\cancel{dt}}\theta(t).\cancel{dt} = \int_0^t 2\pi f_i(t)\, dt$$

$$\theta(t) = \int_0^t 2\pi \underline{f_i(t)}.\, dt \longrightarrow (4)$$

Substituting equation (2) in eq (4), we get

$$\theta(t) = \int_0^t 2\pi[f_c + \Delta f \cos(2\pi f_m t)]\, dt$$

$$= \int_0^t 2\pi f_c\, dt + \int_0^t 2\pi \Delta f \cos(2\pi f_m t)\, dt$$

$$= 2\pi f_c t + \cancel{2\pi}\Delta f \frac{\sin 2\pi f_m t}{\cancel{2\pi} f_m}$$

$$\theta(t) = 2\pi f_c t + \frac{\Delta f}{f_m}\sin 2\pi f_m t$$

$$\boxed{\theta(t) = 2\pi f_c t + \underline{\beta}\sin(2\pi f_m t)} \longrightarrow (5)$$

Where, $\beta = \frac{\Delta f}{f_m}$.

Substituting eq ⑤ in eq ①, we get

$$\therefore \boxed{S(t) = A_c \cos[2\pi f_c t + \beta \sin(2\pi f_m t)]}$$

Modulation Index :-

$$\beta = \frac{\text{Frequency deviation}}{\text{Modulating frequency}} = \frac{\Delta f}{f_m}.$$

Maximum deviation :-

The maximum change in the instantaneous frequency from the average value carrier frequency 'f_c' is known as frequency deviation.

$$\boxed{\Delta f = |K_f m(t)|_{max}}$$

$$\Delta f = K_f A_m$$

Bandwidth :-

The FM wave consists infinite number of sidebands. Thus bandwidth of a FM signal is infinite.

By carson's rule :

$$BW = 2[\Delta f + f_m]$$

❖ Types of FM or Classification of FM :-

Depending on the value of the modulation index 'β' FM wave is classified as follows:

1. **Narrow band FM (NBFM)**
2. **Wide band FM (WBFM)**

i) Narrow band FM :- property 1

* A NBFM is the FM wave with a small bandwidth. The modulation index 'β' of NBFM is small as compared to one radian.
* The NBFM has a narrow bandwidth which is equal to twice the message bandwidth.

2) Wide band FM :- property 2

* The WBFM has a much larger value of 'β' which is theoretically infinite.
* For larger value of modulation index 'β', the FM wave ideally contains the carrier & an infinite number of sidebands located symmetrically around the carrier.

  Such a FM wave has infinite bandwidth & hence called Wideband FM.

❖ Mention the properties of FM?

There are three properties of FM:

1. **Narrow band FM (NBFM)**
2. **Wide band FM (WBFM)**
3. **Constant average power.**

Narrow band Frequency Modulation:-

- **Describe with necessary equations and block diagram, the generation of narrow band FM.** **Jan-05,6M**
- **Narrow band FM** **June-09,5M(old)**

The time-domain expression for an FM wave is

$$S(t) = A_c \cos[2\pi f_c t + \beta \sin(2\pi f_m t)] \longrightarrow ①$$

Using the trignometric identity

$$\boxed{\cos(A+B) = \cos A . \cos B - \sin A . \sin B}$$

$A = 2\pi f_c t$ & $B = \beta \sin(2\pi f_m t)$

$$S(t) = A_c\left[\cos(2\pi f_c t) . \cos(\beta \sin 2\pi f_m t) - \sin(2\pi f_c t) . \sin(\beta \sin 2\pi f_m t)\right] \rightarrow ②$$

In NBFM, β is Small, hence it possible to approximate

$$\boxed{\begin{aligned} &\cos(\beta . \sin 2\pi f_m t) \approx 1 \\ &\sin(\beta \sin 2\pi f_m t) \approx \beta \sin 2\pi f_m t \end{aligned}} \longrightarrow ③$$

Substituting eq ③ in eq ②, we get

$$S(t) = A_c \cos 2\pi f_c t (1) - A_c \sin 2\pi f_c t . (\beta \sin 2\pi f_m t)$$

$$S(t) = A_c \cos 2\pi f_c t - \beta A_c . \underline{\sin 2\pi f_c t . \sin 2\pi f_m t} . \rightarrow ④$$

W.K.T

$$\boxed{\sin A . \sin B = \frac{1}{2} \cos(A-B) - \frac{1}{2} \cos(A+B)}$$

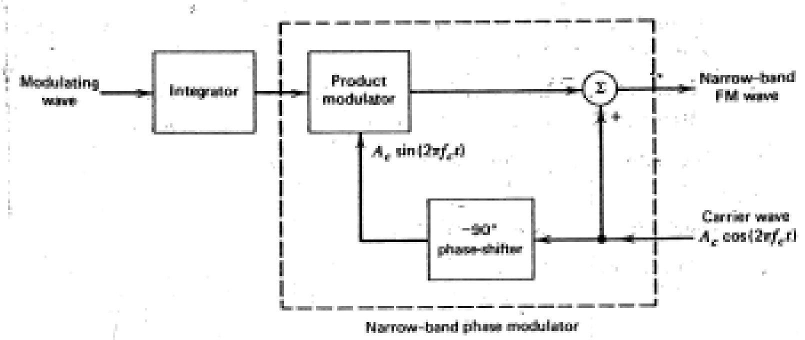
$$S(t) = A_c \cos 2\pi f_c t - \left[\frac{\beta A_c}{2} \cos 2\pi(f_c - f_m)t - \frac{\beta A_c}{2} \cos 2\pi(f_c + f_m)t\right]$$

$$S(t) = A_c \cos 2\pi f_c t - \frac{\beta A_c}{2} \cos 2\pi(f_c - f_m)t + \frac{\beta A_c}{2} \cos 2\pi(f_c + f_m)t \rightarrow ⑤$$

W.K.T the amplitude modulated wave is given by

$$S(t) = A_c \cos 2\pi f_c t + \frac{\mu A_c}{2} \cos 2\pi (f_c - f_m)t + \frac{\mu A_c}{2} \cos 2\pi (f_c + f_m)t \rightarrow ⑥$$

* Comparing equation ⑤ & equation ⑥. The only difference observed between NBFM wave & AM wave is the Sign reversal of the lower Sideband.
* Thus NBFM requires the Same bandwidth as that of AM.



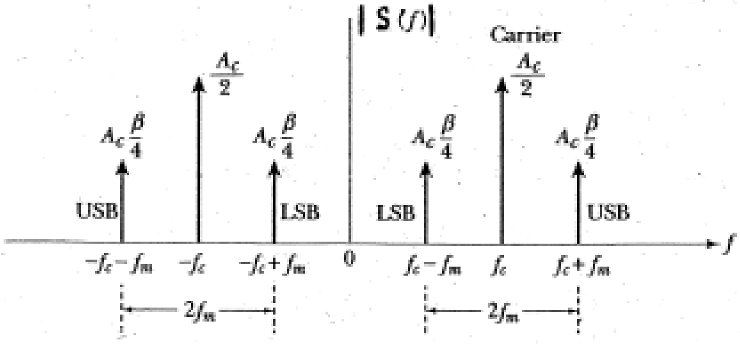
Block diagram of a method for generating a narrow-band FM signal.

Taking Fourier Transform on both Sides of eq ⑤, we get

$$S(f) = \frac{A_c}{2}\left[\delta(f-f_c)+\delta(f+f_c)\right] - \frac{\beta A_c}{4}\left\{\delta[f-(f_c-f_m)]+\delta[f+(f_c-f_m)]\right\}$$

$$+ \frac{\beta A_c}{4}\left\{\delta[f-(f_c+f_m)]+\delta[f+(f_c+f_m)]\right\}$$

* The transmission bandwidth of a NBFM wave is $2f_m$.
* The NBFM wave & Conventional AM wave are Identical but there is No amplitude variation in FM.



18

Spectral content of a NBFM wave for single-tone modulation.

Complex envelope of FM wave:-

The FM wave for sinusoidal modulation is given by:

$$s(t) = A_c \cos[2\pi f_c t + \beta \sin 2\pi f_m t] \longrightarrow ①$$

by using trignometric function

$$\boxed{\cos(A+B) = \cos A \cdot \cos B - \sin A \cdot \sin B}$$

Where $A = 2\pi f_c t$, $B = \beta \sin 2\pi f_m t$

$$s(t) = A_c \cos(2\pi f_c t) \cdot \cos(\beta \sin 2\pi f_m t) - A_c \sin(2\pi f_c t) \cdot \sin(\beta \sin 2\pi f_m t)$$

The inphase & Quadrature components of FM wave $s(t)$ are given by

$$s_I(t) = A_c \cos[\beta \sin(2\pi f_m t)]$$

$$s_Q(t) = A_c \sin[\beta \sin(2\pi f_m t)].$$

* The Complex envelope of the FM wave is

$$\hat{s}(t) = s_I(t) + j s_Q(t)$$

$$\hat{s}(t) = A_c \cos[\beta \sin(2\pi f_m t)] + j A_c \sin[\beta \sin(2\pi f_m t)] \rightarrow ②$$

expressing $\hat{s}(t)$ in terms of $e^{j\theta}$

i.e. $e^{j\theta} = \cos\theta + j\sin\theta$

From eq ②, $\theta = \beta \sin(2\pi f_m t)$

$$\boxed{\hat{s}(t) = A_c e^{j\beta \sin(2\pi f_m t)}}$$

Formulae & Basic Concepts :-

1) Complex number expressed as a trignometric function & exponential form

$$\boxed{e^{j\theta} = \cos\theta + j\sin\theta}$$

Where,

Real part $\rightarrow$ $R_e[e^{j\theta}] = \cos\theta$

Imaginary part $\rightarrow$ $Img[e^{j\theta}] = \sin\theta$

2) Complex Fourier Series

$$\hat{s}(t) = \sum_{n=-\infty}^{\infty} C_n e^{j2\pi n f_m t}$$

3) Complex Fourier Co-efficient

$$C_n = f_m \int_{-1/2f_m}^{1/2f_m} \hat{s}(t) \cdot e^{-j2\pi n f_m t} \cdot dt$$

4) Bessel function

$$J_n(\beta) = \frac{1}{2\pi} \int_{-\pi}^{\pi} e^{j(\beta \sin x - nx)} \cdot dx$$

- **Derive an expression for the spectrum of FM wave with sinusoidal modulation** **Jan-05,7M**
- **Derive time-domain expression for a wideband FM wave.** **Jan-10,8M**
- **Derive the equation of a FM signal s(t), from basic principles and further analyse the sinusoidal FM wave interms of S(f) and Bessel function J_n (β).** **July-07,10M**
- **Derive an expression for spectrum of FM wave with sinusoidal modulation.** **Jan-07,10M**
- **Derive expression for the spectrum of FM wave with sinusoidal modulation** **June-09,7M**

* The FM Wave for Sinusoidal modulation is given by:

$$S(t) = A_c \cos[2\pi f_c t + \beta \sin 2\pi f_m t] \longrightarrow (1)$$

Taking real part of eq (1)

{ NOTE:- Equation (1) is not having imaginary part. It has only real part. $\boxed{\theta = 2\pi f_c t + \beta \sin 2\pi f_m t}$
}

$$S(t) = R_e[A_c e^{j\theta}]$$

$$S(t) = R_e[A_c e^{j(2\pi f_c t + \beta \sin 2\pi f_m t)}]$$

$$= R_e[A_c e^{j2\pi f_c t} \cdot e^{j\beta \sin 2\pi f_m t}]$$

$$= R_e[e^{j2\pi f_c t} \cdot A_c e^{j\beta \sin 2\pi f_m t}]$$

$$S(t) = R_e[e^{j2\pi f_c t} \cdot \hat{S}(t)] \longrightarrow (2)$$

Where, $\hat{S}(t) = A_c e^{j\beta \sin 2\pi f_m t} \longrightarrow (3)$

* $\hat{S}(t)$ is a periodic time function with a fundamental frequency 'f_m'. This can be expressed using Complex Fourier Series as:

$$\boxed{\hat{s}(t) = \sum_{n=-\infty}^{\infty} C_n e^{j2\pi n f_m t}} \longrightarrow (4)$$

Where C_n is a complex Fourier co-efficient given by

$$\boxed{C_n = f_m \int_{-1/2f_m}^{1/2f_m} \hat{s}(t) \cdot e^{-j2\pi n f_m t} \cdot dt} \longrightarrow (5)$$

Substituting eq (3) in eq (5), we get

$$C_n = f_m \int_{-1/2f_m}^{1/2f_m} A_c e^{j\beta \sin(2\pi f_m t)} \cdot e^{-j2\pi n f_m t} \cdot dt \longrightarrow (5)$$

$$C_n = A_c f_m \int_{-1/2f_m}^{1/2f_m} e^{j[\beta \sin(2\pi f_m t) - 2\pi n f_m t]} \cdot dt$$

Let $x = 2\pi f_m t \rightarrow (a)$

differentiating eq (a) w.r.to t

$$\frac{dx}{dt} = 2\pi f_m \quad (1)$$

$$dt = \frac{dx}{2\pi f_m}$$

Giving the limits

WKT $x = 2\pi f_m t$	
When $t = -\frac{1}{2f_m}$ $x = \cancel{2}\pi \cancel{f_m} \cdot -\frac{1}{\cancel{2}\cancel{f_m}}$ $\boxed{x = -\pi}$	When $t = \frac{1}{2f_m}$ $x = \cancel{2}\pi \cancel{f_m} \cdot \frac{1}{\cancel{2}\cancel{f_m}}$ $\boxed{x = \pi}$

$$C_n = A_c f_m \int_{-\pi}^{\pi} e^{j(\beta \sin x - nx)} \frac{dx}{2\pi f_m}$$

$$C_n = A_c \frac{\cancel{f_m}}{2\pi \cancel{f_m}} \int_{-\pi}^{\pi} e^{j(\beta \sin x - nx)} . dx$$

$$C_n = \frac{A_c}{2\pi} \int_{-\pi}^{\pi} e^{j(\beta \sin x - nx)} . dx$$

$$\boxed{C_n = A_c J_n(\beta)} \longrightarrow (6)$$

Where, $J_n(\beta) = \frac{1}{2\pi} \int_{-\pi}^{\pi} e^{j(\beta \sin x - nx)} . dx$

$J_n(\beta)$ is a bessel function of the 1st kind, n^{th} order with an argument β.

Substituting eq (6) in eq (4), we get $\left\{ \hat{S}(t) = \sum_{n=-\infty}^{\infty} C_n e^{j2\pi n f_m t} \rightarrow (4) \right\}$

$$\boxed{\hat{S}(t) = \sum_{n=-\infty}^{\infty} A_c J_n(\beta) e^{j2\pi n f_m t}} \longrightarrow (7)$$

Substituting eq (7) in eq (2), we get $\left\{ S(t) = R_e[\hat{S}(t) . e^{j2\pi f_c t}] \rightarrow (2) \right\}$

$$S(t) = R_e\left[A_c \sum_{n=-\infty}^{\infty} J_n(\beta)\, e^{j2\pi n f_m t} . e^{j2\pi f_c t} \right]$$

$$S(t) = R_e\left[A_c \sum_{n=-\infty}^{\infty} J_n(\beta)\, e^{j2\pi [f_c + n f_m] t} \right]$$

W.K.T $\quad e^{j\theta} = \cos\theta + j\sin\theta$

$$R_e[e^{j\theta}] = \cos\theta \quad \& \quad \theta = 2\pi(f_c + n f_m) t$$

$$\text{III}^{ly} \quad \boxed{s(t) = A_c \sum_{n=-\infty}^{\infty} J_n(\beta) \cdot \cos[2\pi(f_c + nf_m)t]} \longrightarrow (8)$$

Giving the values for n b/w $-\infty$ to $+\infty$

i.e. $n = 0, 1, -1, 2, -2, \cdots +\infty, -\infty$.

$$s(t) = A_c \Big[J_0(\beta)\cos 2\pi f_c t + J_1(\beta)\cos 2\pi(f_c+f_m)t + J_{-1}(\beta)\cos 2\pi(f_c-f_m)t$$
$$+ J_2(\beta)\cos 2\pi[f_c+2f_m]t + J_{-2}(\beta)\cos 2\pi[f_c-2f_m]t$$
$$+ J_3(\beta)\cos 2\pi[f_c+3f_m]t + J_{-3}(\beta)\cos 2\pi[f_c-3f_m]t + \cdots \Big] \longrightarrow (9)$$

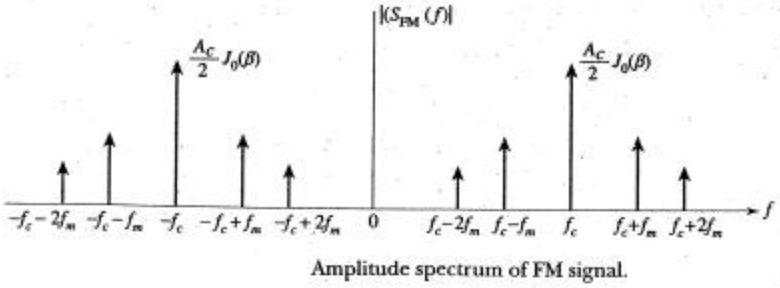
$$s(t) = A_c \Big\{ J_0(\beta)\cos 2\pi f_c t + J_1(\beta)\left[\cos 2\pi(f_c+f_m)t - \cos 2\pi(f_c-f_m)t\right]$$
$$+ J_2(\beta)\left[\cos 2\pi(f_c+2f_m)t - \cos 2\pi(f_c-2f_m)t\right]$$
$$+ J_3(\beta)\left[\cos 2\pi(f_c+3f_m)t - \cos 2\pi(f_c-3f_m)t\right] + \cdots \Big\}$$

* Thus the modulated signal has a carrier component & an infinite number of side frequencies $f_c \pm f_m$, $f_c \pm 2f_m$, $f_c \pm 3f_m$, $\cdots$, $f_c \pm nf_m$.

* Taking Fourier Transform on both sides of eq (9), we get

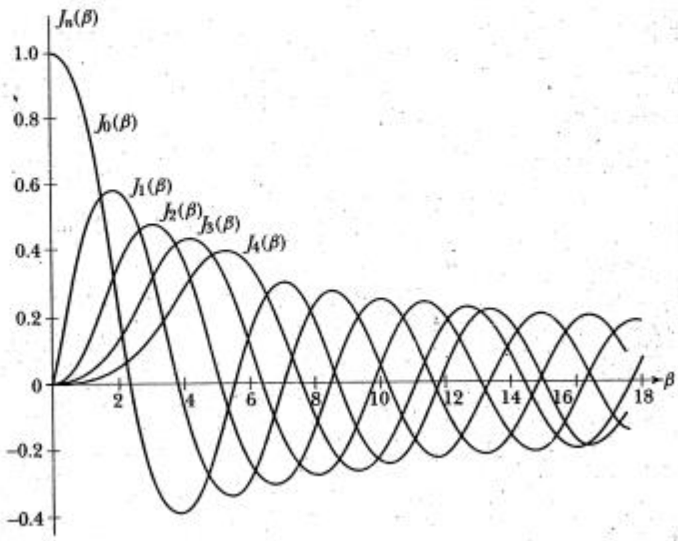
$$S(f) = \frac{A_c}{2} J_0(\beta)\left[\delta(f-f_c) + \delta(f+f_c)\right] + \frac{A_c}{2} J_1(\beta)\left\{\delta(f-[f_c+f_m]) + \delta(f+[f_c+f_m])\right\}$$
$$+ \frac{A_c}{2} J_{-1}(\beta)\left\{\delta[f-(f_c-f_m)] + \delta[f+(f_c-f_m)]\right\} + \cdots\cdots$$
$$+ \frac{A_c}{2} J_n(\beta)\left\{\delta[f-(f_c+nf_m)] + \delta[f+(f_c+nf_m)]\right]$$
$$+ \frac{A_c}{2} J_{-n}(\beta)\left\{\delta[f-(f_c-nf_m)] + \delta[f+(f_c-nf_m)]\right\} \longrightarrow (10)$$

Now plotting spectrum for above equation



Amplitude spectrum of FM signal.

* The amplitude of Side frequency component depends upon the bessel function. The Bessel variations as a function of 'β' fixing the values of 'n' as shown in figure below.



Plots of Bessel function of the first kind.

Constant average power :- (3rd Property)

* The envelope of an FM wave is constant, so that the average power of such a wave dissipated in 1-ohm resistor is also constant.

* The FM wave $s(t)$ has a constant envelope equal to A_c.

$\therefore$ power dissipation $= \frac{A_c^2}{2R}$

* The average power dissipated by $s(t)$ in a 1-ohm resistor is given by:

$$P = \frac{A_c^2}{2(1)}$$

$$\boxed{P = \frac{A_c^2}{2}}$$

* The average power of a single tone FM wave $s(t)$ may be expressed in the form of a corresponding series as:

$$\boxed{P = \frac{A_c^2}{2} \sum_{n=-\infty}^{\infty} J_n^2(\beta)}$$

but

$$\sum_{n=-\infty}^{\infty} J_n^2(\beta) = 1$$

Thus

$$P = \frac{A_c^2}{2}(1)$$

$$\boxed{P = \frac{A_c^2}{2}}$$

- **Explain difference between wideband FM and Narrow band FM** Jan-09,4M
- **Compare Narrowband and Wideband FM** June-09,6M

Sl No	parameters	NBFM	WBFM
1)	Modulation Index	$\beta < 1$	$\beta > 1$
2)	Spectrum	The Spectrum of NBFM is same as that of AM	The Spectrum of WBFM differs from AM
3)	Maximum deviation $[\Delta f]_{max}$	$\Delta f_{max} = 5KHz$	$\Delta f_{max} = 75KHz$
4)	Modulating freq Range.	30Hz - 3KHz	30Hz - 15KHz
5)	Maximum modulation Index	β may be slightly greater than 1.	β may be between 5 to 2500
6)	Bandwidth	Small i.e. approximately same as that of AM	Large i.e. about 15 times higher than Bw of NBFM.
7)	Pre-emphasis & De-emphasis	Needed	Needed.
8)	Applications	used in Speech transmission ex:- FM mobile Comm	used for high quality music transmission ex:- Entertainment broadcasting

❖ Bandwidth of Angle modulated wave:-

Case i : Sinusoidal modulation :-

Theoretically FM has infinite number of Sidebands. So the bandwidth required for transmission is also infinite.

* Carson generalized the bandwidth formula for an FM wave. According to him, the approximate formula for computing the bandwidth of an FM signal generated by a single tone modulating signal frequency 'f_m' is

$$\boxed{B_T \simeq 2(1+\beta) f_m} \longrightarrow (1)$$

The above formula holds good for all values of β.

* The transmission bandwidth 'B_T' can also be expressed interms of frequency deviation 'Δf'

W.K.T $\quad \beta = \frac{\Delta f}{f_m}$

$$\boxed{\Delta f = \beta f_m}$$

From equation (1)

$$B_T = 2(1+\beta) f_m$$
$$= 2f_m + 2\beta f_m$$
$$= 2f_m + 2\Delta f$$
$$= 2\Delta f \left[1 + \frac{f_m}{\Delta f}\right] \qquad \because \frac{1}{\beta} = \frac{f_m}{\Delta f}$$

$$\boxed{B_T = 2\Delta f \left[1 + 1/\beta\right]}$$

Case ii : Non Sinusoidal or Arbitrary modulation :-

* For an angle modulated Signal with an arbitrary modulating - Signal $m(t)$, bandlimited to 'W' Hz, we define the deviation ratio as $\boxed{D = \frac{\Delta f}{W}} \longrightarrow (1)$

* The deviation ratio 'D' plays the Same role for non-Sinusoidal modulation that the modulation Index 'β' plays for the case of Sinusoidal modulation.

Then, replacing 'β' by 'D' & replacing 'f_m' with 'W' we may use carson's rule

W.K.T $\boxed{B_T = 2(1+\beta)f_m} \longrightarrow (2)$

replacing $f_m = W$ & $\beta = D$ in eq (2)

$$\boxed{B_T = 2(1+D)W}$$

The above relation is also Known as Carson's Formula.

Universal curve for evaluating FM bandwidth:-

Suppose, n_{max} is the largest value of Integer 'n' Such that $|J_n(\beta)| > 0.01$. Then, we define the transmission bandwidth as

$$\boxed{B_T = 2n_{max}f_m}$$

Generation of FM waves:-

❖ **Explain the methods of FM generation.** **June-10,5M**

There are two basic methods of generating FM waves:

1. **Indirect method or Armstrong method**
2. **Direct method or Direct FM**

1) Indirect Method or ARMSTRONG Method OR Stereo FM :-

In this method, a Narrow-band FM (NBFM) Wave is generated. Frequency multipliers are then used to increase the frequency deviation which results in Wideband-FM (WBFM).

2) Direct FM or Direct Method :-

In direct FM, the carrier frequency 'f_c' is directly varied in accordance with the amplitude of the modulating signal.

{ * Direct FM is not feasible, practically as it involves - maintaining high frequency stability of the carrier with adequate frequency deviation.
}

❖ **Indirect Method** or **ARMSTRONG method :-**

❖ **Explain with a neat circuit diagram, the direct method of generating FM waves. Derive an expression for instantaneous frequency of FM for sinusoidal modulation.** **June-10,10M(old)**

❖ **Explain the FM generation using indirect method.** **Jan-09,8M**

❖ **Briefly explain indirect method of generating FM wave.** **Jan-08,10M**

❖ **With neat block diagram, explain ARMSTRONG method of FM generation.** **July-09,7M**

❖ **Explain how FM wave can be generated using indirect method. Write the spectrum of FM with sinusoidal modulation along with relevant equations**

❖ **Explain how FM wave can be generated using indirect method.** **July-05,8M**

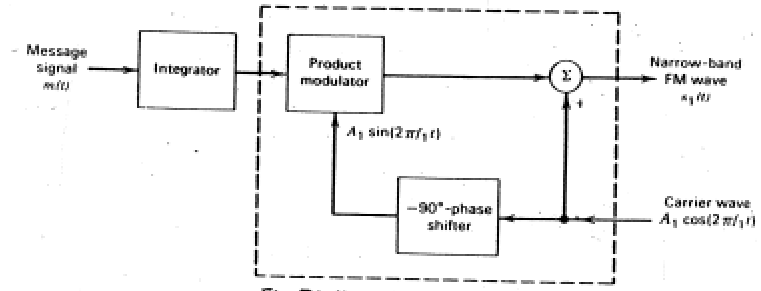


Fig ①: Narrow-band phase modulator

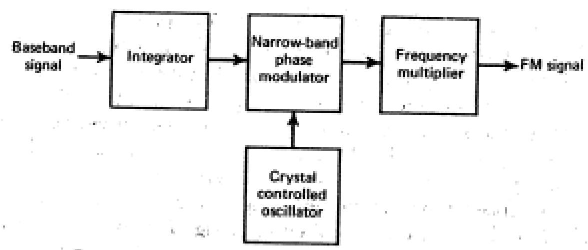


Figure ②: Block diagram of the indirect method of generating a wide-band FM signal.

Fig Shows the block diagram of a Indirect FM System.

* In Indirect method, the message Signal m(t) is 1st passed through an Integrator before applying it to the phase modulator, as Shown in fig ①.

(31)

* The carrier signal is generated by using crystal oscillator because it provides very high frequency stability.

The operation of Indirect method is divided into two parts as follows:

i) Generate a NBFM wave using a phase modulator.

ii) Using the frequency multipliers & mixer to obtain the required values of frequency deviation & modulation index. (ie. WBFM)

* In order to minimize the distortion in the phase modulator, the maximum phase deviation or modulation index 'β' is kept small there by resulting in a NBFM signal.

* Let $S_1(t)$ be the NBFM wave, then we have

$$\boxed{S_1(t) = A_c \cos\left[2\pi f_1 t + 2\pi K_p \int_0^t m(t) \cdot dt\right]} \longrightarrow ①$$

Where, f_c is the frequency of the crystal oscillator & K_p is the frequency sensitivity constant in Hz/volt.

* For a single-tone modulation signal defined by

$m(t) = A_m \cos 2\pi f_m t$, then eq ① becomes

$$\boxed{S_1(t) = A_c \cos\left[2\pi f_1 t + \beta_1 \sin 2\pi f_m t\right]} \longrightarrow ②$$

Where, β_1 is the modulation index for single-tone modulation & is kept below 0.3 radians to minimize the distortion.

The instantaneous frequency of eq ② is

$$\boxed{f_i(t) = f_1 + K_p m(t)}$$

ii) Generation of WBFM :-

* The o/p of the Narrow band phase modulator is then multiplied by a frequency multiplier, producing the desired WBFM wave as shown in fig ③.

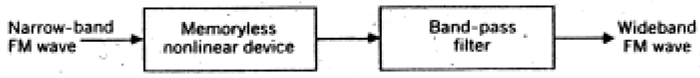


Fig ③: Frequency Multiplier.

* A frequency multiplier consists of a memoryless non-linear device followed by a BPF as shown in fig ③.
The I/p - o/p relation of such a non-linear device may be expressed in the general form.

$$\boxed{V(t) = a_1 S_1(t) + a_2 S_1^2(t) + \cdots + a_n S_1^n(t)} \longrightarrow ③$$

Where $a_1, a_2, \cdots, a_n$ are co-efficients & n is the highest order of non-linearity.

Substituting eq ① in eq ③ & simplifying, we find the frequency modulated wave having carrier frequencies $f_1, 2f_1, \cdots, nf_1$ with frequency deviation $\Delta f_1, 2\Delta f_1, \cdots, n\Delta f_1$.

The BPF has two functions to perform:

1) To pass the FM wave centered at carrier frequency nf_1 & having the frequency deviation $n\Delta f_1$.

2) To suppress all other FM spectra.

* The o/p of the frequency multiplier produces the desired WBFM wave having the following time-domain description.

$$\boxed{s(t) = A_c \cos\left[2\pi n f_1 t + 2\pi n K_f \int_0^t m(t)\, dt\right]} \longrightarrow (4)$$

Whose instantaneous frequency is

$$\boxed{f_i'(t) = n f_1 + n K_f m(t)}$$

Direct Method:-

- **Explain the direct method of generating FM waves.** **Jan-10,8M**
- **Explain FM generation using direct method.** **Jan-07,7M July-06,5M June-06,3M**

* In direct FM system, the instantaneous frequency of the carrier wave is varied directly in accordance with the message signal by means of a device called a "voltage controlled oscillator" (VCO).

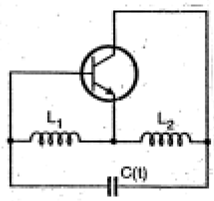
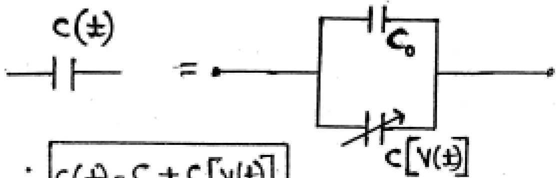


Fig (1) Hartley oscillator

* Fig (1) shows a hartley oscillator in which the capacitive component of the frequency determining N/w in the oscillator consists of a fixed capacitor shunted by a voltage-variable capacitor.

{



$$\therefore \boxed{C(t) = C_0 + C[V(t)]}$$

}

* The frequency of oscillation of the hartley oscillator is given by:

$$\boxed{f_i(t) = \frac{1}{2\pi\sqrt{(L_1+L_2)\,C(t)}}} \longrightarrow ①$$

Where,

$C(t) = C_0 + C[V(t)]$

L_1 & L_2 → are the two inductances in the frequency - determining the oscillator.

* Assume that a Sinusoidal modulating wave of frequency 'f_m', the Capacitance $C(t)$ is expressed as:

$$\boxed{C(t) = C_0 + \Delta C \cos(2\pi f_m t)} \longrightarrow ②$$

Where,

C_0 is the total capacitance in the absence of modulation i.e. $f_m = 0$ &

ΔC is the maximum change in capacitance.

Substituting eq ② in eq ①, we get

$$f_i(t) = \frac{1}{2\pi\sqrt{(L_1+L_2)\quad C_0 + \Delta C \cos(2\pi f_m t)}}$$

$$f_i(t) = \frac{1}{2\pi\sqrt{(L_1+L_2)\, C_0\left[1+\frac{\Delta C}{C_0}\cos(2\pi f_m t)\right]}}$$

$$f_i(t) = f_0 \frac{1}{\sqrt{1+\frac{\Delta C}{C_0}\cos(2\pi f_m t)}}$$

Where, $f_0 = \frac{1}{2\pi\sqrt{(L_1+L_2)\,C_0}}$, unmodulated frequency of oscillation.

$$f_i(t) = f_0 \frac{1}{\left[1+\frac{\Delta C}{C_0}\cos(2\pi f_m t)\right]^{1/2}}$$

$$f_i(t) = f_0\left[1+\frac{\Delta C}{C_0}\cos(2\pi f_m t)\right]^{-1/2}$$

Recall the binomial theorem

$$[1+x]^{-1/2} \approx 1-\frac{x}{2}$$

if $|x| \ll 1$

$$f_i(t) = f_0\left[1-\frac{\Delta C}{2C_0}\cos(2\pi f_m t)\right]$$

Let $\boxed{-\frac{\Delta C}{2C_0} = \frac{\Delta f}{f_0}}$

$$f_i(t) = f_0\left[1+\frac{\Delta f}{f_0}\cos(2\pi f_m t)\right]$$

$$f_i(t) = f_0 + \frac{\cancel{f_0}\Delta f}{\cancel{f_0}}\cos(2\pi f_m t)$$

$$\boxed{f_i(t) = f_0 + \Delta f\cos(2\pi f_m t)} \longrightarrow (3)$$

Equation (3) is the Instantaneous frequency of an FM wave, assuming Sinusoidal modulation.

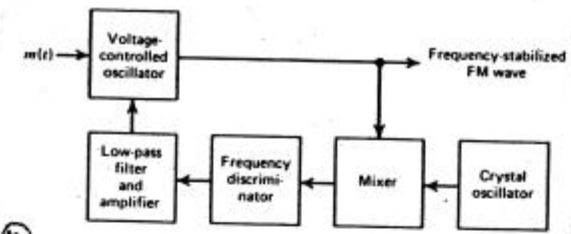


Figure (4)
A feedback scheme for the frequency stabilization of a frequency modulator.

* In order to generate a WBFM with the required frequency deviation, fig (4) is used. It Consists of VCO, frequency - multiplier & mixers.

* This Configuration provides good oscillator Stability, Constant proportionality b/w o/p frequency change to I/p voltage change, & the necessary frequency deviation to achieve WBFM.

ANGLE MODULATION – (FM-2)

Demodulation:-

Frequency demodulation is the process of recovering the original modulating wave from the frequency modulated wave.

Demodulation of FM waves:-

The FM demodulators are classified into:

1. Direct method

i. **Frequency discriminator**
ii. **Zero crossing detector.**

2. Indirect method

i. **Phase-Locked Loop.**

Requirements of FM detectors (Demodulators):-

The FM demodulator must satisfy the following requirements:

1) It must convert frequency variation into amplitude variations
2) The conversion must be linear & efficient.
3) The demodulator ckt should be insensitive to amplitude changes. It should respond only to the frequency changes.
4) It should not be too critical in its adjustment & operation.

Frequency discriminator or Simple Slope detector :- (Not in syllabus)

Principle of Slope detection:-

* Let us consider a Tuned Ckt shown in fig. A frequency modulated (FM) signal is applied to this tuned ckt. The centre frequency of the FM signal is 'f_c' & the frequency deviation is Δf. The resonant frequency of the tuned ckt depends on frequency of the tuned ckt depends on the frequency deviation of the I/p FM signal.

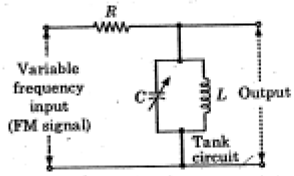


Fig ① *Tuned circuit.*

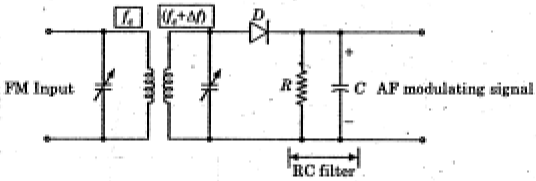


Fig ② *Simple slope detector.*

Fig ② shows the circuit diagram of a Simple Slope detector.

* When frequency of the I/p signal increases, it becomes more close to the resonant frequency, increasing the o/p voltage.

* When frequency of the I/p signal decreases, it moves away from the resonant frequency, decreasing the o/p voltage.

∴ Frequency variations in the I/p signal about the carrier center frequency produce proportional o/p voltage variations as shown in fig ③.

* The o/p voltage is applied to a diode detector with a RC load of suitable time constant to get the original modulating signal.

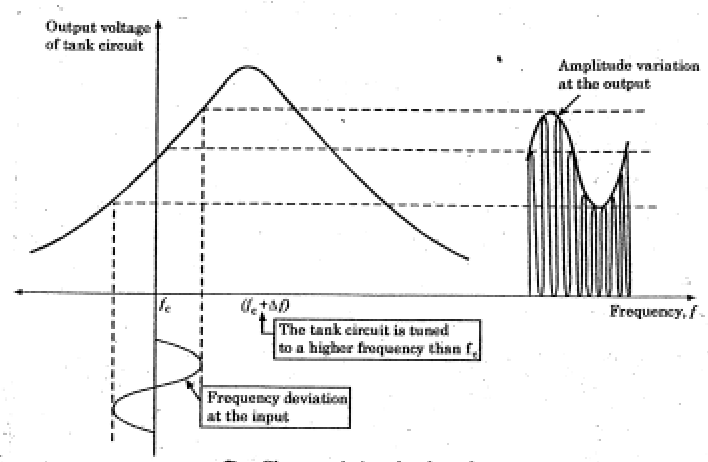


Fig ③ *Characteristics of a slope detector.*

Advantages:-

1) The circuit is Simple.

Disadvantages:-

1) The ckt Sensitivity is poor i.e. Change in amplitude for a given Change of frequency is Small. (Inefficient)

2) It is linear only over a limited frequency range.

3) It is difficult to adjust as the primary & Secondary windings of the transformer must be tuned Slightly different frequencies

4) The circuit non-linearity causes harmonic distortion.

Balanced Frequency discriminator or Balanced slope detector or Round – Travis Detector

- Explain the detection process of FM signals using balanced frequency discriminator with relevant diagrams. **July-09,6M(old)**
- Draw the block diagram of balance frequency discriminator and explain it for demodulation of FM signal. **Jan-09,8M**
- Explain clearly how a balanced slope detector is used for FM demodulation. **June-08,7M**
- Explain the detection process of FM signals using balanced frequency discriminator with relevant diagrams. **Jan-06,6M**
- With associated diagrams and equations, explain how FM wave can be detected using ratio detector. **July-05,7M June-09,6M**

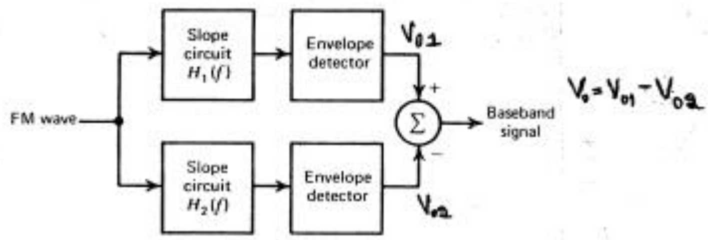


Fig: (a) Block Diagram

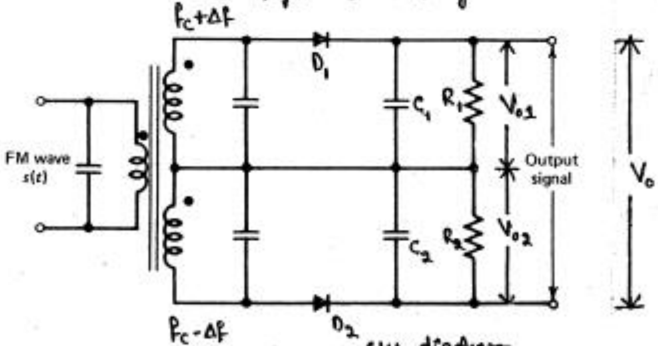


Fig (b): Ckt diagram

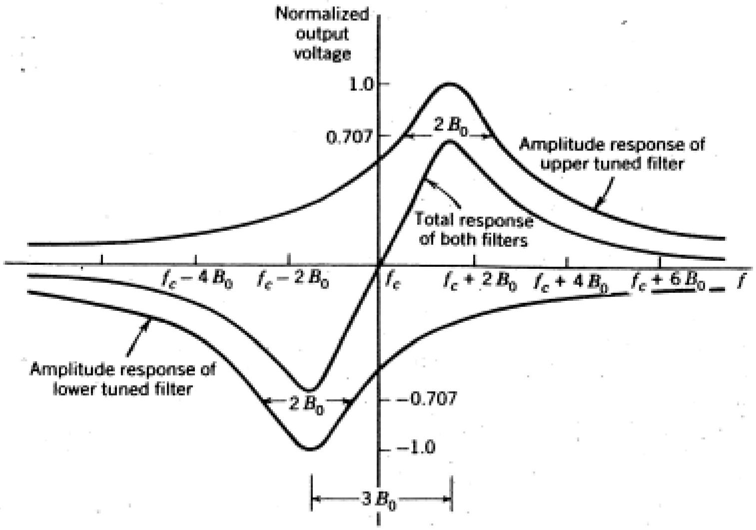


Fig ©: Frequency response

* The balanced Slope detector Consists of Two Slope detector Ckts.
* The I/p transformer has a center tapped Secondary. Hence the I/p voltages to the two Slope detectors are 180° out of phase.
* There are 3 tuned Ckts
  i) The primary is tuned to IF i.e. f_c.
  ii) The upper tuned Ckt of the Secondary (T_1) is tuned above f_c by Δf i.e. its resonant frequency is $f_c + \Delta f$.
  iii) The lower tuned Ckt of the Secondary (T_2) is tuned below f_c by Δf i.e. its resonant frequency is $f_c - \Delta f$
* R_1C_1 & R_2C_2 are the filter Ckts.
* V_{01} & V_{02} are the o/p voltages of the two Slope detectors.
* The final o/p voltage V_0 is obtained by taking the difference

of the Individual o/p voltages V_{01} & V_{02}.

i.e. $\boxed{V_0 = V_{01} - V_{02}}$

operation of the Ckt:-

We can understand the operation by dividing the I/p frequency into three ranges as follows:

i) $f_{in} = f_c$:-

When I/p frequency is equal to carrier freq 'f_c', the Induced voltage in the T_1 winding of Secondary is exactly equal to that Induced in the winding T_2.

Thus the I/p voltages to both the diodes D_1 & D_2 will be Same.

∴ The dc o/p voltages V_{01} & V_{02} will also be Identical but they have opposite polarities hence $V_0 = 0V$.

ii) $f_{in} > f_c$:-

$\boxed{f_{in} > f_c \; (f_c + \Delta f)}$ i.e. $f_{in} \simeq f_c + \Delta f$

When I/p frequency is greater than 'f_c', the Induced voltage in 'T_1' winding is higher than that Induced in 'T_2'.

∴ The I/p to D_1 is higher than D_2. So +ve o/p V_{01} (of D_1) is higher than the -ve o/p V_{02} (of D_2).

Thus o/p voltage V_0 is positive. (The +ve o/p voltage increases as the I/p frequency increases towards $f_c + \Delta f$.)

iii) $f_{in} < f_c$:-

$\boxed{\text{i.e. } f_{in} \simeq f_c - \Delta f}$

When I/p frequency is less than 'f_c', the Induced voltage in 'T2' winding is higher than in 'T1', So I/p voltage to diode D_2 is higher than that of D_1.

Hence the -ve o/p 'V_{o2}' is greater than V_{o1}.

∴ The o/p voltage of the balanced Slope detector is -ve in this frequency range. { The -ve o/p voltage increases as f_{in} goes closer to '$f_c - \Delta f$' }

$$\therefore \quad \boxed{V_0 = \begin{cases} 0, & f_{in} = f_c \\ +ve, & f_{in} > f_c \\ -ve, & f_{in} < f_c \end{cases}}$$

Advantages :-

1) This ckt is more efficient than Simple Slope detector.
2) It has better linearity than the Simple Slope detector.

Disadvantages :-

1) This ckt is difficult to tune Since the three tuned ckts are to be tuned at different frequencies i.e. f_c, $(f_c+\Delta f)$, $(f_c-\Delta f)$.
2) Amplitude limiting is not provided.

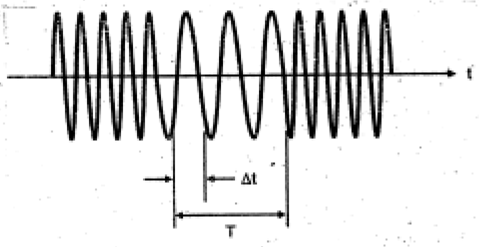
Zero – Crossing Detector:-

❖ **Explain FM demodulation using Zero crossing detector** **July-06,8M Jan-05,7M**

* The Zero Crossing detector operates on the principle that the instantaneous frequency of an FM wave is approximately given by

$$\boxed{f_i \simeq \frac{1}{2\Delta t}}$$

Where,
Δt is the time difference b/w adjacent Zero Crossing of the FM wave as shown in fig ①.



Definitions of T and Δt for an FM wave

* The time Interval 'T' is chosen in accordance with the following two conditions:

1) The Interval 'T' is small compared to the reciprocal of the message bandwidth 'W' i.e. $\left(\frac{1}{W}\right)$

2) The Interval 'T' is large compared to the reciprocal of the carrier frequency 'f_c' of the FM wave i.e. $(1/f_c)$.

* Let 'n_0' denote the number of Zero Crossings inside the Interval 'T'. Hence Δt is the time between the adjacent Zero Crossing points given by

$$\boxed{\Delta t = \frac{T}{n_0}}$$

∴ Instantaneous frequency is given by

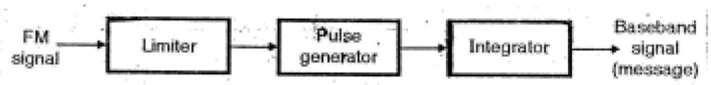
$$f_i \approx \frac{1}{2\Delta t}$$

$$f_i \simeq \frac{1}{2\frac{T}{n_0}}$$

$$\boxed{f_i \simeq \frac{n_0}{2T}}$$

* By the definition of Instantaneous frequency, W.K.T there is a linear relation b/w f_i & message signal $m(t)$. Hence we can recover $m(t)$ if n_0 is known.

* The Simplified block diagram of the zero crossing detector based on this principle is shown below.



Block diagram of zero crossing detector

Phase Locked Loop:-

- **Starting from block diagram of PLL obtain its non-linear and linear model. Show that o/p of PLL is scaled version modulating signal** **June-10,12M**
- **With relevant analysis, explain the FM demodulation, using PLL** **Jan-10,10M**
- **Explain how first order PLL can be used for FM detection** **June-10,8M**
- **Explain with relevant mathematical expression the demodulation of a FM signal using PLL.** **June-09,10M**

Jan-09,6M July-08,5M Jan-08,10M June-07,5M Jan-07,7M June-05,5M

* PLL is a -ve Feedback System that Consists of three major - Components
  i) A multiplier
  ii) A loop filter
  iii) A voltage Controlled oscillator (VCO)

Connected in the form of a feedback loop as shown in fig ①.

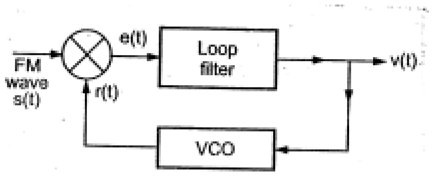


Fig ①: Block diagram of PLL

* The VCO is a Sine-wave generator whose frequency depends on the I/p Control voltage.

{ Any frequency modulator may serve as a VCO.}

* Initially assume that VCO is adjusted so that when the control voltage is Zero, 2 Conditions are Satisfied:

1) The frequency of the VCO is precisely set at the unmodulated carrier frequency 'f_c' &

2) The VCO o/p has a 90° phase-shift w.r.to the unmodulated - carrier wave.

* Suppose that the I/p Signal applied to the PLL is an FM wave defined by $s(t) = A_c \sin\left[2\pi f_c t + 2\pi K_f \int_0^t m(t)\cdot dt\right]$

$$\boxed{s(t) = A_c \sin\left[2\pi f_c t + \phi_1(t)\right]} \longrightarrow ①$$

Where A_c is the carrier amplitude with a modulating wave $m(t)$

We have $\phi_1(t) = 2\pi K_f \int_0^t m(t)\, dt$.

Where K_f is the frequency Sensitivity of the frequency modulator.

Let the VCO o/p be defined as

$$\boxed{r(t) = A_v \cos[2\pi f_c t + \phi_2(t)]} \longrightarrow ②$$

Where 'A_v' is the amplitude of VCO o/p. If the control voltage applied to VCO is $v(t)$ then

$$\boxed{\phi_2(t) = 2\pi K_v \int_0^t v(t) \cdot dt}$$

Where 'K_v' is the frequency sensitivity constant of the VCO having the unit Hz/V.

* When VCO I/p $v(t)$ equal to zero i.e. $v(t)=0$, then $\phi_2(t)=0$.

* The incoming FM wave $S(t)$ & the VCO o/p $r(t)$ are applied to the multiplier.

The o/p of the multiplier is

$$\boxed{e(t) = S(t)\, r(t)} \longrightarrow ③$$

Substituting eq ① & ② in eq ③, we get

$$e(t) = \underbrace{A_c \sin[2\pi f_c t + \phi_1(t)]}_{S(t)} \cdot \underbrace{A_v \cos[2\pi f_c t + \phi_2(t)]}_{r(t)}$$

$$e(t) = A_c A_v \sin[2\pi f_c t + \phi_1(t)] \cdot \cos[2\pi f_c t + \phi_2(t)]$$

W.K.T $\boxed{\sin A \cdot \cos B = \frac{1}{2}\sin[A+B] + \frac{1}{2}\sin[A-B]}$

Put $A = [2\pi f_c t + \phi_1(t)]$ & $B = [2\pi f_c t + \phi_2(t)]$

$$e(t) = \frac{A_c A_v}{2}\sin[\underline{2\pi f_c t} + \phi_1(t) + \underline{2\pi f_c t} + \phi_2(t)] + \frac{A_c A_v}{2}\sin[\cancel{2\pi f_c t} + \phi_1(t) - \cancel{2\pi f_c t} - \phi_2(t)]$$

$$\boxed{e(t) = \frac{A_c A_v}{2}\sin[4\pi f_c t + \phi_1(t) + \phi_2(t)] + \frac{A_c A_v}{2}\sin[\phi_1(t) - \phi_2(t)]} \longrightarrow ④$$

$$\boxed{e(t) = K_m A_c A_v \sin\left[4\pi f_c t + \phi_1(t) + \phi_2(t)\right] + K_m A_c A_v \sin\left[\phi_1(t) - \phi_2(t)\right]} \longrightarrow (5)$$

Where $K_m = \frac{1}{2}$ is the multiplier gain measured in volts.

* Equation (5) is the o/p of the product modulator & it has two components

i) A high-frequency component represented by

$$K_m A_c A_v \sin\left[4\pi f_c t + \phi_1(t) + \phi_2(t)\right]$$

2) A low frequency component represented by

$$K_m A_c A_v \sin\left[\phi_1(t) - \phi_2(t)\right]$$

* The high frequency component is eliminated by the LPF.

Thus the I/p to the loop filter is given by:

$$e(t) = K_m A_c A_v \sin\left[\phi_1(t) - \phi_2(t)\right]$$

$$\boxed{e(t) = K_m A_c A_v \sin\left[\phi_e(t)\right]} \longrightarrow (6)$$

Where, $\phi_e(t)$ is the phase ~~error~~ defined by

$$\boxed{\phi_e(t) = \phi_1(t) - \phi_2(t)} \longrightarrow (7)$$

$$\phi_e(t) = \phi_1(t) - 2\pi K_v \int_0^t v(t) \cdot dt$$

* The loop filter operates on its I/p $e(t)$ to produce the o/p

$$v(t) = e(t) * h(t).$$

$$\boxed{v(t) = \int_{-\infty}^{\infty} e(\tau) \cdot h(t-\tau)\, d\tau}$$

* Differentiating eq (7) w.r.to t, we get

$$\frac{d\phi_e(t)}{dt} = \frac{d\phi_1(t)}{dt} - \frac{d\phi_2(t)}{dt}$$

$$= \frac{d\phi_1(t)}{dt} - \left[\frac{d}{\cancel{dt}}\left(2\pi K_v \int_0^t v(t)\cdot \cancel{dt}\right)\right]$$

$$= \frac{d\phi_1(t)}{dt} - \left[2\pi K_v v(t)\right] \qquad \because v(t) = e(t) * h(t)$$

$$\frac{d\phi_e(t)}{dt} = \frac{d\phi_1(t)}{dt} - \left[2\pi K_v \left(\underline{e(t)} * h(t)\right)\right] \longrightarrow ⑧$$

Substituting eq ⑥ in eq ⑧, we get

$$= \frac{d\phi_1(t)}{dt} - \left[2\pi K_v\left(\underline{K_m A_c A_v \sin\phi_e(t)} * h(t)\right)\right]$$

$$= \frac{d\phi_1(t)}{dt} - 2\pi K_v K_m A_c A_v\left[\sin \underline{\phi_e(t) * h(t)}\right]$$

$$\boxed{\frac{d\phi_e(t)}{dt} = \frac{d\phi_1(t)}{dt} - 2\pi K_0 \int_{-\infty}^{\infty} \sin\left[\phi_e(\tau)\cdot h(t-\tau)\, d\tau\right]}$$

Where 'K_0' is a loop parameter defined by

$$K_0 = K_m K_v A_c A_v$$

Equation necessary for developing the block diagram of PLL:-

W.K.T $\quad \phi_e(t) = \phi_1(t) - \phi_2(t) \rightarrow ①$

Where, $\phi_1(t) = 2\pi K_f \int_0^t m(t)\cdot dt$

$\phi_2(t) = 2\pi K_v \int_0^t V(t)\cdot dt$

Differentiating $\phi_2(t)$ w.r.to 't' we get

$$\frac{d\phi_2(t)}{dt} = 2\pi K_v \cancel{\frac{d}{dt}} \cancel{\int_0^t} V(t)\cdot \cancel{dt}$$

$$\frac{d\phi_2(t)}{dt} = 2\pi K_v \cdot \underline{V(t)} \longrightarrow ②$$

W.K.T

$$\boxed{V(t) = e(t) * h(t)}$$

$$\frac{d\phi_2(t)}{dt} = 2\pi K_v \left[\underline{e(t)} * h(t) \right] \longrightarrow ③$$

W.K.T

$$\boxed{e(t) = K_m A_c A_v \sin\phi_e(t)}$$

Substituting $\underline{e(t)}$ in eq ③, we get

$$\frac{d\phi_2(t)}{dt} = 2\pi K_v \left[K_m A_c A_v \sin\phi_e(t) * h(t) \right]$$

$$\boxed{\frac{d\phi_2(t)}{dt} = 2\pi \underline{K_o} \sin\phi_e(t) * h(t)} \longrightarrow ④$$

Where, $K_o = K_v K_m A_c A_v$

From equation ②, We can write

$$V(t) = \frac{1}{2\pi K_v} \frac{d\phi_2(t)}{dt} \longrightarrow ⑤$$

Substituting eq ⑤ in eq ④ we get

$$\boxed{V(t) = \frac{1}{2\pi K_v} 2\pi K_o \sin\phi_e(t) * h(t)}$$

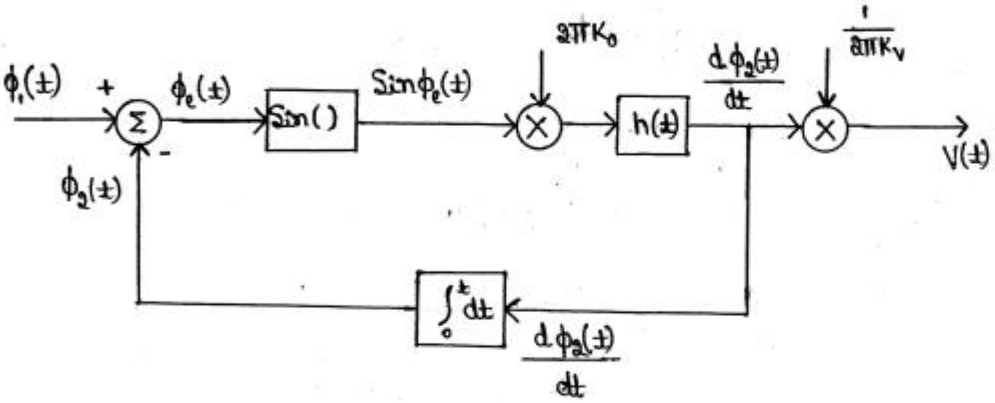


Fig ②: Non-linear model of PLL

* We can observe that the multiplier of Fig ① is replaced by a Sinusoidal non linearity & the VCO by an Integrator because of the Sinusoidal Non linearity, the above representation is known as the non-linearity representation of PLL.

Linearized PLL:- June-07,5M

Linearized Model :-

* When the phase error $\phi_e(t)$ is Zero, the PLL is said to be in phase-locked.
* When $\phi_e(t)$ is very small compared to 0.5 radian, at all times, we may use the following approximation.

$$\boxed{\sin\phi_e(t) \approx \phi_e(t)}$$

Thus Fig ② reduces to Fig ③.

Fig ③ is known as the Linearized model of PLL.

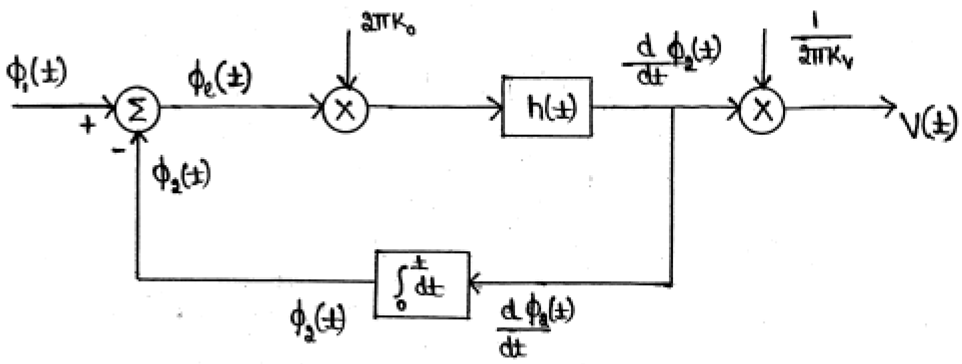


Fig ③: Linear model of PLL.

From Fig ③, we have

$$\phi_e(t) = \phi_1(t) - \phi_2(t) \longrightarrow ①$$

$$\phi_1(t) = \phi_e(t) + \phi_2(t) \longrightarrow ②$$

differentiating both Sides of eq ②, we get

$$\frac{d}{dt}\phi_1(t) = \frac{d}{dt}\phi_e(t) + \underline{\frac{d}{dt}\phi_2(t)}$$

$$\frac{d}{dt}\phi_1(t) = \frac{d}{dt}\phi_e(t) + 2\pi K_0 \underline{\sin\phi_e(t)} * h(t)$$

$\because \frac{d}{dt}\phi_2(t) = 2\pi K_0 \sin\phi_e(t) * h(t)$ & W.K.T, $\sin\phi_e(t) \approx \phi_e(t)$

$$\boxed{\frac{d}{dt}\phi_1(t) = \frac{d}{dt}\phi_e(t) + 2\pi K_0 \phi_e(t) * h(t)} \longrightarrow ③$$

Taking Fourier transform on both Side of the eq ③, we get

$$j2\pi f\,\phi_1(f) = j2\pi f\,\phi_e(f) + 2\pi K_0 \phi_e(f)\cdot H(f)$$

{ NOTE :-

$$\frac{d}{dt}\phi_1(t) \xrightarrow{FT} j2\pi f\,\phi_1(f)$$

$$\frac{d}{dt}\phi_e(t) \xrightarrow{FT} j2\pi f\,\phi_2(f)$$

$$\phi_e(t) * h(t) \xrightarrow{FT} \phi_e(f)\cdot H(f)$$

}

$$\cancel{j2\pi f}\,\phi_1(f) = \cancel{j2\pi f}\left[\phi_e(f) + \frac{1}{jf} K_0 \phi_e(f)\cdot H(f)\right]$$

$$\phi_1(f) = \phi_e(f) + \frac{K_0 H(f)}{jf}\cdot\phi_e(f)$$

Let $L(f) = \frac{K_0 H(f)}{jf}$

Then,

$$\phi_1(f) = \phi_e(f) + L(f)\cdot\phi_e(f)$$

$$\phi_1(f) = \phi_e(f)\left[1 + L(f)\right]$$

$$\boxed{\phi_e(f) = \frac{\phi_1(f)}{1 + L(f)}} \longrightarrow ④$$

Where $L(f)$ is called the open loop transfer function of the PLL.

W.K.T

$$V(t) = \frac{1}{2\pi K_v} \cdot 2\pi K_0 \underline{\sin\phi_e(t)} * h(t)$$

$\because$ $\boxed{\sin\phi_e(t) \approx \phi_e(t)}$

$$\boxed{V(t) = \frac{1}{2\pi K_v} 2\pi K_0 \, \phi_e(t) * h(t)} \longrightarrow 5$$

Eq 5 is the o/p of the PLL.

* In frequency domain the o/p of the PLL is given

$$V(f) = \frac{1}{\cancel{2\pi} K_v} \cancel{2\pi} K_0 \, \phi_e(f) \cdot H(f)$$

$$V(f) = \frac{K_0}{K_v} \cdot \underline{\phi_e(f)} \cdot H(f) \longrightarrow 6$$

Substituting eq 4 in eq 6, we get

$$V(f) = \frac{K_0}{K_v} \cdot \frac{\phi_1(f)}{[1+L(f)]} \cdot H(f)$$

$\because$ $\boxed{\phi_e(f) = \frac{\phi_1(f)}{1+L(f)}}$

Since $L(f) \gg 1$, we can write $1+L(f) \approx L(f)$

hence, $V(f) = \frac{K_0}{K_v} \frac{\phi_1(f)}{L(f)} H(f)$

$\because$ $\boxed{L(f) = \frac{K_0 H(f)}{jf}}$

$$V(f) = \frac{\cancel{K_0}}{K_v} \frac{\phi_1(f)}{\frac{\cancel{K_0 H(f)}}{jf}} \cdot \cancel{H(f)}$$

$$\boxed{V(f) = \frac{1}{K_v} \cdot jf \, \phi_1(f)} \longrightarrow 7$$

X^lying & ÷ing RHS of eq 7 by 2π

$$V(f) = \frac{1}{2\pi K_v} \cdot j2\pi f \phi_1(f)$$

W.K.T $\frac{d}{dt}\phi_1(t) \xrightarrow{FT} j2\pi f \phi_1(f)$

hence, $V(f) = \frac{1}{2\pi K_v} \frac{d}{dt}\phi_1(t) \longrightarrow (8)$

Substituting $\phi_1(t) = 2\pi K_f \int_0^t m(t).\,dt$ in eq (8), we get

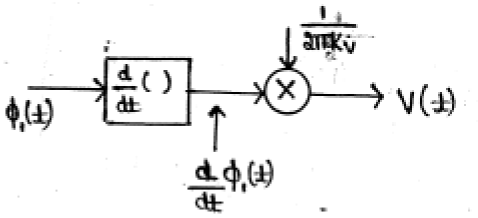
$$V(f) = \frac{1}{\cancel{2\pi} K_v} \cancel{\frac{d}{dt}}\left[\cancel{2\pi} K_f \cancel{\int_0^t} m(t). \cancel{dt}\right]$$

$$\boxed{V(f) = \frac{K_f}{K_v}\, m(t)}$$

The corresponding time-domain relation of eq (8) is

$$\boxed{V(t) = \frac{1}{2\pi K_v} \frac{d}{dt}\phi_1(t)} \longrightarrow (9)$$

From eq (9), we can write



Linearized Model :-

From Godse - Book

* When the phase error $\phi_e(t)$ is zero, the PLL is said to be in phase - locked.

* When $\phi_e(t)$ is very small compared to 0.5 radians, at all times, we may use the following approximation.

$$\boxed{\sin\phi_e(t) \approx \phi_e(t)}$$

Thus fig (2) reduces to fig (3).

Fig ③ is known as the Linearized model of PLL.

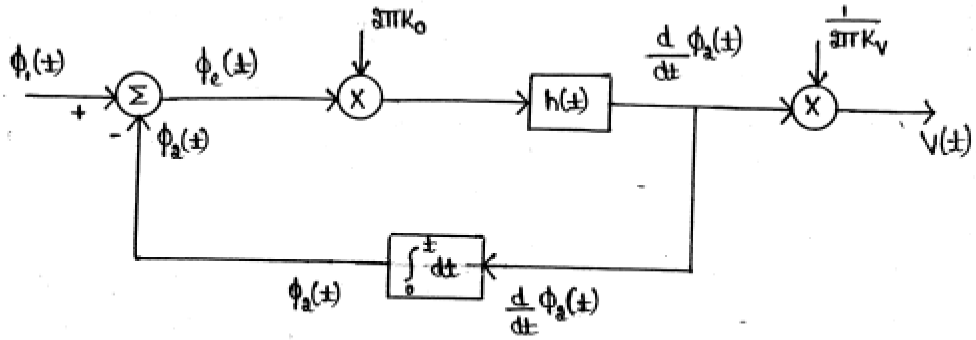


Fig ③: Linear model of PLL

W.K.T

$$\phi_1(t) = 2\pi K_f \int_0^t m(t)\cdot dt \quad \&$$

$$\phi_2(t) = 2\pi K_v \int_0^t V(t)\cdot dt.$$

From fig ③, We have

$$\phi_e(t) = \phi_1(t) - \phi_2(t). \longrightarrow ①$$

Assuming Small error. So $\phi_e(t) \approx 0$

$$0 = \phi_1(t) - \phi_2(t)$$

$$\phi_2(t) = \phi_1(t) \longrightarrow ②$$

$$\cancel{2\pi} K_v \int_0^t V(t)\cdot dt = \cancel{2\pi} K_f \int_0^t m(t)\, dt \longrightarrow ③$$

Differentiating both Sides of eq ③ w.r.to time 't'

$$K_v V(t) = K_f m(t)$$

$$\boxed{V(t) = \frac{K_f}{K_v}\, m(t)}$$

Non- Linear effects in FM:-

❖ **Explain the non linearity and its effect in FM systems** Jan-09,6M July-06,6M

* Non-linearities are present in all electrical networks. Consider a communication channel having a non-linear transfer characteristic given by

$$\boxed{V_o(t) = a_1 V_i(t) + a_2 V_i^2(t) + a_3 V_i^3(t)} \longrightarrow (1)$$

Where,

$V_i(t) \longrightarrow$ I/p Signal

$V_o(t) \longrightarrow$ O/p Signal

$a_1, a_2 \& a_3 \longrightarrow$ Constants

* Let the I/p to the channel be an FM wave given by

$$V_i(t) = A_c \cos\left[2\pi f_c t + \phi(t)\right] \longrightarrow (2)$$

Where, $\phi(t) = 2\pi K_f \int_0^t m(t).dt$

Substituting eq (2) in eq (1), we get

$$V_o(t) = a_1 A_c \cos[2\pi f_c t + \phi(t)] + a_2 A_c^2 \cos^2[2\pi f_c t + \phi(t)] + a_3 A_c^3 \cos^3[2\pi f_c t + \phi(t)]$$

W.K.T

$$\boxed{\begin{aligned} \cos^2\theta &= \frac{1}{2} + \frac{\cos 2\theta}{2} \\ \cos^3\theta &= \frac{3\cos\theta}{4} + \frac{\cos(3\theta)}{4} \end{aligned}}$$

$$\boxed{\theta = \left[2\pi f_c t + \phi(t)\right]}$$

$$\Big\{$$

$$a_2 A_c^2 \cos^2[2\pi f_c t + \phi(t)] = \frac{a_2 A_c^2}{2} + \frac{a_2 A_c^2}{2} \cos\left[4\pi f_c t + 2\phi(t)\right]$$

$$a_3 A_c^3 \cos^3[2\pi f_c t + \phi(t)] = \frac{3a_3 A_c^3}{4} \cos[2\pi f_c t + \phi(t)] + \frac{a_3 A_c^3}{4} \cos[6\pi f_c t + 3\phi(t)]$$

$\}$

$$V_0(t) = a_1 A_c \cos[2\pi f_c t + \phi(t)] + \frac{a_2 A_c^2}{2} + \frac{a_2 A_c^2}{2} \cos[4\pi f_c t + 2\phi(t)] + \frac{a_3 A_c^3}{4} 3\cos[2\pi f_c t + \phi(t)] + \frac{a_3 A_c^3}{4} \cos[6\pi f_c t + 3\phi(t)] \longrightarrow (3)$$

* The equation (3) indicates that the channel o/p consists of a DC component & three frequency modulated signals with carrier frequencies f_c, $2f_c$ & $3f_c$.

* The required FM wave centered at f_c is obtained by passing '$V_0(t)$' through a BPF.

* The o/p of the BPF is

$$V_0(t) = a_1 A_c \cos[2\pi f_c t + \phi(t)] + \frac{a_3 A_c^3}{4} 3\cos[2\pi f_c t + \phi(t)]$$

$$\boxed{V_0(t) = \cos[2\pi f_c t + \phi(t)] \left(a_1 A_c + \frac{3}{4} a_3 A_c^3 \right)} \longrightarrow (4)$$

* Equation (4) reveals that $V_0(t)$ is the original FM signal except for the change in amplitude. Thus, amplitude non-linearities of the channel does not affect an FM signal (unlike in amplitude modulation).

* For this reason, FM is widely used in <u>Microwave</u> & <u>Satellite Communication</u>.

Compare FM with AM and PM **June-06,6M**

Sl No	FM	AM
1)	The equation for FM wave is: $S(t)_{FM} = A_c \sin[\omega_c t + \beta \sin\omega_m t]$	The equation for AM wave is $S(t)_{AM} = A_c[1 + \mu \sin\omega_m t]\sin\omega_c t$.
2)	The modulation Index can have any value i.e. less than 1 or more than 1.	The modulation Index is always in between 0 and 1.
3)	All the transmitted power is useful	Carrier power and one Sideband power are useless.
4)	$P = \frac{A_c^2}{2R}$	$P_T = P_c[1+\mu^2/2]$
5)	The modulation Index determines the number of Sidebands in an FM Signal	In AM, only two Sidebands are produced, irrespective of the modulation Index.
6)	$BW = 2[\Delta f + f_m]$. The BW depends on modulation Index	$BW = 2f_m$. The BW does not depends on modulation Index
7)	For FM, $\%\text{ Modulation} = \frac{\text{Actual freq deviation}}{\text{Max allowed freq deviation}} \times 100$	For AM, $\%\text{ modulation} = \frac{A_m}{A_c} \times 100$
8)	The main advantage of FM over AM is its noise Immunity.	The AM System is more Susceptible to Noise & more affected by Noise than FM.

Sl No	FM	AM
9)	The BW required to transmit FM Signal is much larger than the BW of AM (i.e ≈200KHz)	The BW required to transmit AM Signal is much less than that of FM (i.e. ≈10KHz)
10)	FM transmission & reception equipment are more Complex	AM equipments are less Complex.
11)	FM transmission is expensive than AM transmission	AM transmission is Cheaper than FM transmission.
12)	Used for Short distance Comm	Used for long distance Comm

Sl No	FM	PM
1)	The equation for FM Wave is $S(t)_{FM} = A_c \cos[W_c t + 2\pi K_f m(t)]$	The equation for PM Wave is $S(t)_{PM} = A_c \cos[W_c t + K_p m(t)]$
2)	Amplitude of FM Wave is Constant	Amplitude of PM Wave is Constant
3)	Frequency deviation is proportional to modulating voltage.	phase deviation is proportional to the modulating voltage.
4)	The modulation Index of an FM Signal is the ratio of the frequency deviation to the modulating frequency.	The modulation Index is proportional to the maximum amplitude of the modulating Signal.
5)	Noise Immunity is better than AM & PM	Noise Immunity is better than AM but worse than FM.

6)	Signal to Noise ratio is better than that of PM	Signal to Noise ratio is Inferior to that of FM.
7)	FM is widely used	PM is used in Some mobile - Systems
8)	It is possible to receive FM on a PM receiver.	It is possible to receive PM on a FM receiver.
9)	FM is direct method of producing FM Signal	PM is Indirect method of producing FM.
10)	Noise is better Suppressed in FM Systems as Compared to PM System	Noise Immunity is Inferior to that of FM.
11)	To have better quality of - transmission & reception of higher audio frequencies, pre-emphasis & de-emphasis Circuits are used.	The amount of frequency Shift produced by a phase modulator increases with the modulating frequency. Hence an audio equalizer is required to - Compensate this.
12)	FM is mainly used for FM broadcasting. i.e. Entertainment purpose	PM is used in mobile Comm System

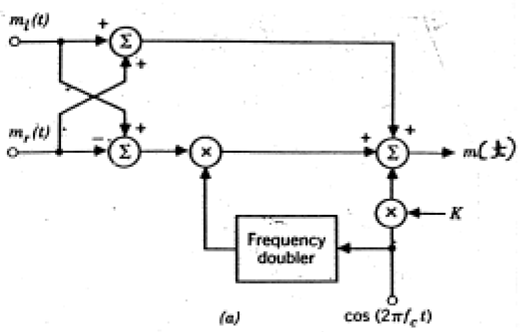
FM Stereo multiplexing:-

- **Explain with relevant block diagram FM stereo multiplexing** **June-10,8M**
- **FM stereo multiplexing** **Jan-10,5M**
- **With a neat block diagram, explain the operation of FM stereo multiplexing and demultiplexing** **June-10,10M(OLD)**
- **Stereo FM** **June-06,3M**

* Stereo multiplexing is a form of frequency-division multiplexing (FDM) designed to transmit two separate signals via the same carrier.

* It is widely used in FM broadcasting to send two different elements of a program (eg: two different sections of an orchestra, a vocalist & an accompanist) so as to give a spatial dimension to its perception by a listener at the receiving end.

FM Stereo Transmitter:-



* Let $m_l(t)$ & $m_r(t)$ denotes the signals picked up by left-hand & right-hand microphones at the transmitting end of the system.

* These signals are then applied to a simple matrixer that generates the sum signal i.e. $m_l(t)+m_r(t)$, & the difference

Signal i.e. $m_l(t) - m_r(t)$.

* The Sum Signal is left unprocessed in its baseband form: it is available for monophonic reception.

* The difference Signal & a 38KHz Subcarrier are applied to a product modulator, producing a DSB-SC modulated wave.

* The Sum Signal, DSB-SC modulated wave & a 19KHz pilot Signal are Combined to form a multiplexed Signal $S(t)$.

{ Here pilot Signal is included to provide a reference for the Coherent detection of the difference Signal at the Stereo receiver. }

Thus multiplexed Signal $m(t)$ is

$$\boxed{m(t) = [m_l(t) + m_r(t)] + [m_l(t) - m_r(t)] \cos 4\pi f_c t + K \cos 2\pi f_c t}$$

Where $f_c = 19KHz$.

FM Stereo Receiver:-

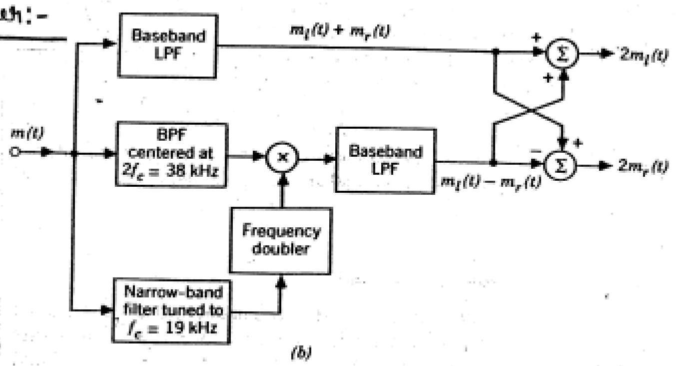


Figure 7.58
(a) Multiplexer in transmitter of FM stereo. (b) Demultiplexer in receiver of FM stereo.

* At a Stereo receiver, the multiplexed Signal $m(t)$ is recovered from the incoming FM wave.

* The $m(t)$ is then applied to the demultiplexing as shown in fig ⓑ.
* The individual components of the multiplexed signal $m(t)$ are - separated by the use of three appropriate filters.
* The recovered pilot signal is frequency - doubled to produce the desired 38KHz subcarrier which enables the coherent detection of the DSB-SC modulated wave, thereby recovering the difference signal $m_l(t) - m_r(t)$.
* The baseband LPF in the top path of fig ⓑ is designed to pass the sum signal $m_l(t) + m_r(t)$.
* Finally, the simple matrixer reconstruct the left-hand signal $m_l(t)$ & right hand signal $m_r(t)$ & applies them to their respective speaker.

FM FORMULAE

1. Carrier Frequency	$W_c = 2\pi f_c$,	$f_c = W_c/2\pi$
2. Modulating Frequency	$W_m = 2\pi f_m$,	$f_m = W_m/2\pi$
3. Modulation Index (β or m_f)	$\beta = \Delta f / f_m$	
4. Power dissipation	$P = A^2{}_c/2R$	
5. Frequency deviation	$\Delta f = K_f A_m$ $\Delta f = \beta f_m$	
6. Frequency sensitivity	$K_f = \Delta f / A_m$	
7. Deviation ratio	$D = \Delta f_{max} / f_{max}$	
8. Highest frequency reached	$(f_i)_{max} = f_c + \Delta f$	
9. Lowest frequency reached	$(f_i)_{min} = f_c - \Delta f$	
10. Carrier Swing	$(f_i)_{max} - (f_i)_{min}$	
11. Carrier Swing	$2 x \Delta f$	
12. Frequency deviation	Δf = Carrier Swing / 2 $\Delta f = (f_i)_{max} - f_C$	
13. Bandwidth (Carson rule)	$BW = 2(\Delta f + f_m)$ or $BW = 2\Delta f\,(1 + 1/\beta)$	
14. Message signal	$S(t) = A_c\cos[W_{ct} + \beta \sin W_m t]$ $S(t) = A_c\sin[W_{ct} + \beta \sin W_m t]$ $S(t) = A_c\cos[2\pi f_{ct} + 2\pi k_f \int m(t)\,dt]$ $S(t) = A_c\cos[W_{ct} + 2\pi k_f \int m(t)\,dt]$ $S(t) = A_c\cos[W_{ct} + 2\pi k_f\, m(t)]$	

PM FORMULAE

1. **Phase deviation** — $\Delta P = K_p A_m f_m$
2. **Bandwidth** — $BW = 2(\Delta f + f_m)$ or
   (Carson rule) — $BW = 2\Delta f\,(1+1/\beta)$
3. **Message signal** — $S(t) = A_c \cos[W_c t + K_p m(t)]$
   $S(t) = A_c \cos[2\pi f_c t + K_p m(t)]$

1. **The equation for a FM wave is $S(t) = 10\cos[5.7\times10^{8}t + 5\sin(12\times10^{3})t]$.**
   Calculate. i. Carrier frequency ii. Modulating frequency
   iii. Modulation index iv. Frequency deviation
   v. Power dissipated in a 100Ω resistor load.

June-10,6M July-09,5M (old) June-08,10M June-06,6M

Sol :-

$$S(t) = 10\cos[5.7\times10^{8}t + 5\sin(12\times10^{3})t] \rightarrow (1)$$

Compare eq (1) with Standard equation for FM

$$S(t) = A_c\cos[\omega_c t + \beta\sin\omega_m t] \longrightarrow (2)$$

$A_c = 10V$, $\omega_c = 5.7\times10^{8}$, $\beta = 5$ & $\omega_m = 12\times10^{3}$

i) Carrier frequency $f_c = \dfrac{\omega_c}{2\pi} = \dfrac{5.7\times10^{8}}{2\pi}$

$$\boxed{f_c = 90.7183\,MHz}$$

ii) Modulating frequency $f_m = \dfrac{\omega_m}{2\pi} = \dfrac{12\times10^{3}}{2\pi}$

$$\boxed{f_m = 1.909\,KHz}$$

iii) Modulation Index

$$\boxed{\beta = 5}$$

iv) Frequency deviation $\Delta f = \beta f_m = 5\times1.909\,KHz$

$$\boxed{\Delta f = 9.545\,KHz}$$

v) power dissipated in a 100Ω resistor load

$$P = \frac{A_c^2}{2R} = \frac{10^2}{2\times100}$$

$$\boxed{P = 0.5\,W}$$

2. **A FM signal has sinusoidal modulation with f_m=15KHz and modulation index β=2. Using carson's rule, find the transmission bandwidth and deviation ratio. Assume Δf =75 KHz.** June-10,6M

Given :-

$f_m = 15\,KHz$, $\beta = 2$, $\Delta f = 75\,KHz$

BW = ? & Deviation ratio 'D' = ?

* $BW = 2(\Delta f + f_m) = 2(75KHz + 15KHz) = \underline{180\,KHz.}$
* $D = \frac{\Delta f}{f_m} = \frac{75KHz}{15KHz} = \underline{5}$

---

3. **A sinusoidal modulating voltage of amplitude 5V and frequency 1 KHz is applied to frequency modulator. The frequency sensitivity of modulator is 40 Hz/V. The carrier frequency is 100KHz. Calculate i. Frequency deviator ii. Modulation index** June-10,5M

Given :- $A_m = 5V$, $f_m = 1KHz$, $K_f = 40\,Hz/V$ & $f_c = 100KHz$.

i) Frequency deviation $\Delta f = K_f A_m = 40 \times 5 = \underline{200\,Hz}$

ii) Modulation Index 'β' $= \frac{\Delta f}{f_m} = \frac{200}{1000} = \underline{0.2}$

---

4. **A sinusoidal modulating waveform of amplitude 10V and a frequency of 1 KHz is applied to FM generator that has a frequency sensitivity constant of is 40 Hz/V. Determine the i. Frequency deviation and ii. Modulation index** Jan-10,4M

Given : $A_m = 10V$, $f_m = 1KHz$, $K_f = 40\,Hz/V$.

i) Frequency deviation $\Delta f = K_f A_m = 40 \times 10 = 400\,Hz$

ii) Modulation Index $\beta = \frac{\Delta f}{f_m} = \frac{400\,Hz}{1\,KHz} = 0.4$

---

5. A carrier wave of 100 MHz is frequency modulated by a 100 KHz sinewave of amplitude 20V, the sensitivity of the modulator is 25 KHz/V.

i. Determine the frequency deviation and bandwidth of the modulated signal using Carson's rule.

ii. Repeat your calculation for PM wave, assume k_p=k_f **June-10,6M(IT)**

Given : $f_c = 100\,MHz$, $f_m = 100\,KHz$, $A_m = 20V$, $K_f = 25\,KHz/V$.

i) $BW = 2[\Delta f + f_m]$

$\Delta f = K_f A_m = 25\,KHz \times 20 = \underline{500\,KHz}$

$BW = 2[500\,KHz + 100\,KHz]$

$\boxed{BW = 1200\,KHz}$

OR

$BW = 2f_m(1+\beta)$

$\beta = \frac{500\,KHz}{100\,KHz} = \underline{5}$

$BW = 2 \times 100\,KHz(1+5)$

$\boxed{BW = 1200\,KHz}$

ii) Assuming that $K_p = K_f$ for PM wave

$\Delta f = K_p A_m f_m = 25\,KHz \times 20 \times 100\,KHz$

$\boxed{\Delta f = 50000\,KHz}$

---

6. A single tone FM signal is given by: $S(t) = 10 \sin[16\pi \times 10^6 t + 20 \sin 2\pi \times 10^3)t]$.

Calculate. i. Modulation index **ii. Modulation Frequency**

iii. Frequency deviation **iv. Carrier frequency**

v. Power of the FM signal. **Jan-09,8M**

Sol :- $S(t) = 10 \sin[16\pi \times 10^6 t + 20 \sin 2\pi \times 10^3 t] \longrightarrow (1)$

Comparing eq (1) with Standard equation for FM

$$S(t) = A_c \sin[\omega_c t + \beta \sin \omega_m t] \longrightarrow (2)$$

we get, $A_c = 10V$, $\omega_c = 16\pi \times 10^6$, $\omega_m = 2\pi \times 10^3$, $\beta = 20$

i) Modulation Index $\beta = 20$.

ii) Modulating Frequency $f_m = \frac{\omega_m}{2\pi} = \frac{2\pi \times 10^3}{2\pi} = 1KHz$

iii) Frequency deviation $\Delta f = \beta f_m = 20 \times 1 \times 10^3 = 20KHz$

iv) Carrier Frequency $f_c = \frac{\omega_c}{2\pi} = \frac{16\pi \times 10^6}{2\pi} = 8MHz$

v) Power 'p' $= \frac{A_c^2}{2R} = \frac{10^2}{2R} = \frac{50}{R} W$

---

7. An angle modulated signal is defined by S(t) = 10 sin[$2\pi x10^6t$ + 0.2 sin(2000π)t] volts. Find the following:

i. Power in the modulated signal **ii. Frequency deviation**

iii. Phase deviation **iv. Approximate transmission bandwidth.**

Jan-08,10M

Given :- $S(t) = 10 \cos[2\pi \times 10^6 t + 0.2 \sin(2000\pi t)] \longrightarrow (1)$

Comparing eq (1) with Standard equation for FM

$$S(t) = A_c \cos[\omega_c t + \beta \sin \omega_m t] \longrightarrow (2)$$

We get, $A_c = 10V$, $\beta = 0.2$, $\omega_m = 2000\pi$, $\omega_c = 2\pi \times 10^6$

* $f_m = \frac{\omega_m}{2\pi} = \frac{2000\pi}{2\pi} = 1KHz$

* $f_c = \frac{\omega_c}{2\pi} = \frac{2\pi \times 10^6}{2\pi} = 1MHz$

i) $P = \frac{A_c^2}{2R} = \frac{10^2}{2 \cdot R} = \frac{50}{R} W$

ii) $\Delta f = \beta f_m = 0.2 \times 1000 = \underline{200 Hz}$

iii) Phase deviation $\Delta\theta = \beta = \frac{\Delta f}{f_m} = \frac{200 Hz}{1 KHz} = \underline{0.2}$

iv) $BW = 2(\Delta f + f_m) = 2(200 + 1000) = \underline{2400 Hz}$

OR

$BW = 2\Delta f\left(1 + \frac{1}{\beta}\right) = 2 \times 200\left(1 + \frac{1}{0.2}\right) = \underline{2400 Hz}$

---

8. A given angle modulated signal is S(t) given by the equation:

$S(t) = 12 \cos(12\pi 10^8 t + 200 \cos 2\pi 10^3 t)$. Find its bandwidth.

June-07,5M

Given :-

$$S(t) = 12 \cos(12\pi 10^8 t + 200 \cos 2\pi 10^3 t) \longrightarrow ①$$

Compare eq ① with Standard equation for FM

$$S(t) = A_c \cos(\omega_c t + \beta \sin \omega_m t) \longrightarrow ②$$

We get, $A_c = 12V$, $\omega_c = 12\pi 10^8$, $\beta = 200$, $\omega_m = 2\pi 10^3$

* $f_m = \frac{\omega_m}{2\pi} = \frac{2\pi 10^3}{2\pi} = 1 KHz$

* $\Delta f = \beta f_m = 200 \times 1 KHz = 200 KHz$

* $BW = 2(\Delta f + f_m) = 2(200 KHz + 1 KHz) = 402 KHz.$

---

9. A modulated signal $5 \cos 2\pi\, 15 \times 10^3 t$, angle modulates a carrier $A \cos w_c t$. Find the modulation index and the bandwidth for the FM system. Determine the change in the bandwidth and modulation index if f_m is reduced to 5 KHz. What is the conclusion of the two results?
Assume $k_p = k_f = 15 KHz/Volt$.

Jan-07,13M

Given :- $A_m = 5V$, $f_m = 15 KHz$, $K_p = K_f = 15 KHz/V$.

For FM System:

i) Frequency deviation $\Delta f = K_f A_m = 15KHz \times 5 = 75KHz$

ii) Modulation Index $\beta = \frac{\Delta f}{f_m} = \frac{75KHz}{15KHz} = 5$

iii) $BW = 2(\Delta f + f_m) = 2(\Delta f + f_m) = 2(75KHz + 15KHz) = \underline{180KHz}$.

When f_m is reduced to 5KHz i.e. now $\boxed{f_m = 5KHz}$

i) $\Delta f = K_f A_m = 15KHz \times 5 = 75KHz$

ii) $\beta = \frac{\Delta f}{f_m} = \frac{75KHz}{5KHz} = 15$

iii) $BW = 2(\Delta f + f_m) = 2(75KHz + 5KHz) = \underline{160KHz}$

Conclusion:

<u>Bandwidth</u> changes only <u>Slightly</u> with modulating frequency 'f_m'.

---

10. Find the carrier and modulating frequencies along with modulation index and maximum deviation of the FM wave represented by deviation of the FM wave represented by the voltage equation: V = 12 sin ($6x10^8$t + 5 sin 1250t). What power will the FM wave dissipated in a 10Ω resistor? **July-05,5M**

Given :- $S(t) = 12 \sin(6\times10^8 t + 5 \sin 1250t) \longrightarrow ①$

Comparing eq① with standard equation for FM

$S(t) = A_c \sin(\omega_c t + \beta \sin \omega_m t) \longrightarrow ②$

We get, $A_c = 12V$, $\omega_c = 6\times10^8$, $\beta = 5$, $\omega_m = 1250$

i) $f_c = \frac{\omega_c}{2\pi} = \frac{6\times10^8}{2\pi} = \underline{95.5MHz}$

ii) $f_m = \frac{\omega_m}{2\pi} = \frac{1250}{2\pi} = \underline{199Hz}$

iii) $\beta = \underline{5}$

iv) $\Delta f = \beta f_m = 5 \times 199 = \underline{995\ Hz}$

v) $P = \frac{A_c^2}{2R} = \frac{12^2}{2 \times 100} = \underline{7.2W}$

11. An angle modulated signal is described by $S(t) = 10\cos[2\pi(10^6)t + 0.1\sin(10^3)t$. Find the message signal m(t).

i. Considering S(t) is PM with k_p=10.

ii. Considering S(t) is FM with k_f=5. **Jan-05,5M**

Sol:- The equation for PM wave is given by:

$$S(t) = A_c \cos[\omega_c t + K_p m(t)]$$

Comparing this equation with given equation, we have

$$K_p m(t) = 0.1 \sin(10^3)\pi t$$

$$m(t) = \frac{0.1}{K_p} \sin(10^3)\pi t$$

$$m(t) = \frac{0.1}{10} \sin(10^3)\pi t$$

$$\boxed{m(t) = 0.01 \sin(10^3)\pi t}$$

ii) The equation for FM wave is given by

$$S(t) = A_c \cos[\omega_c t + 2\pi K_f m(t)]$$

Comparing this equation with given equation, we have

$$2\pi K_f m(t) = 0.1 \sin(10^3)\pi t$$

$$m(t) = \frac{0.1}{2\pi K_f} \sin(10^3)\pi t$$

$$m(t) = \frac{0.1}{\cancel{2\pi}\times 5} \sin(10^3)\cancel{\pi} t$$

$$\boxed{m(t) = 0.01 \sin(10^3)\, t}$$

An angle modulated Signal is described by **Aug-2000**

$x_c(t) = 10\cos[2\pi(10^6)t + 0.1\sin(10^3)\pi t]$ Considering $x_c(t)$ as a PM Signal With $K_p = 10$. Find $m(t)$.

Sol:- The equation for PM wave is given by

$$s(t) = A_c \cos[\omega_c t + K_p m(t)]$$

Comparing this equation with the given equation, we have

$$K_p m(t) = 0.1 \sin(10^3)\pi t$$

$$m(t) = \frac{0.1}{K_p} \sin(10^3)\pi t$$

$$= \frac{0.1}{10} \sin(10^3)\pi t$$

$$\boxed{m(t) = 0.01 \sin(10^3)\, t}$$

---

* In the block diagram shown in Fig. find out the carrier frequency, frequency deviation and modulation index at the points A and B. Assume that at the output of the mixer, the additive frequency component is being selected.

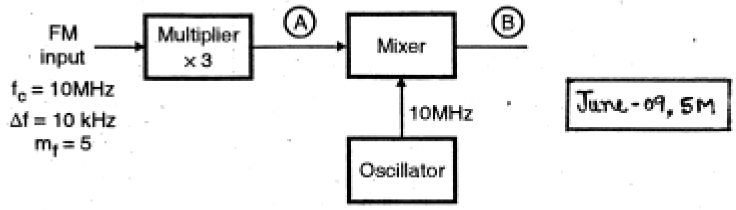


Fig.

Soln. :

(i) At point (A) :

The carrier f_c = 3×10 MHz = 30 MHz.

The frequency deviation δ = 3×10 kHz = 30 kHz and modulation index $m_f = 3 \times 5 = 15$.

The minimum frequency f_{min} = 30 MHz – 30 kHz = 29.970 MHz

The maximum frequency f_{max} = 30 MHz + 30 kHz = 30.030 MHz.

(ii) At point (B) :

Carrier frequency f_c = 30 MHz + 10 MHz = 40 MHz.

Maximum frequency f_{max} = 30.03 + 10 = 40.03 MHz

Minimum frequency f_{min} = 29.970 + 10 = 39.970 MHz.

As there is no change in deviation due to mixing, the modulation index will remain same i.e. $m_f = 15$.

Determine the bandwidth of FM Signal, if the maximum value of frequency deviation Δf is fixed at 75KHZ for Commercial FM broadcasting by radio & modulation frequency is $W = 15KHZ$.

$\boxed{\text{Aug-2001}}$

Sol:- Given : $\Delta f = 75KHZ$

$W = 15KHZ$ or f_m

i) Deviation ratio : $D = \frac{\Delta f}{W} = \frac{75KHZ}{15KHZ} = 5$

ii) Using Carson rule

$$B_T = 2[1+D]W$$

$$= 2[1+5]\, 15\times10^3$$

$$B_T = 180KHZ$$

$\boxed{\text{OR}}$

* $\beta = D = \frac{\Delta f}{W} = \frac{75KHZ}{15KHZ} = 5$

* $B_T = 2[1+\beta] f_m$

$= 2[1+5]\, 15KHZ$

$B_T = 180KHZ.$

$\boxed{\text{OR}}$

* $B_T = 2\Delta f + 2f_m$

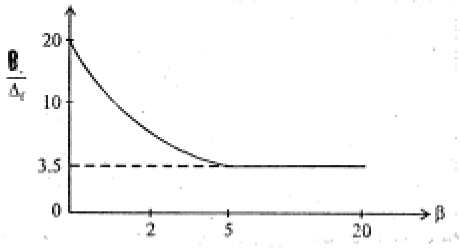
$= 2(75KHZ) + 2(15KHZ)$

$B_T = 180KHZ$

A carrier wave of 100MHz is frequency modulated by a sine wave of amplitude 20 volts & frequency 100KHz. The frequency sensitivity of modulator is 25 KHz/v. Determine

i) Transmission bandwidth using Carson's rule.

ii) Transmission bandwidth using universal rule (The universal graph is as shown below).



<u>Sol:-</u> Given: $f_c = 100MHz$, $A_m = 20V$, $f_m = 100KHz$, $K_f = 25KHz/volt$.

ii) Modulation Index

$$\beta = \frac{\Delta f}{f_m} = \frac{K_f A_m}{f_m} = \frac{25\times10^3 \times 20V}{100\times10^3} = 5$$

From the given universal graph the value of $\frac{B}{\Delta_f}$ for $\beta = 5$ is

$$\frac{B}{\Delta_f} = 3.5 \;\&$$

$$\Delta f = K_f A_m = (25\times10^3)(20) = 500KHz$$

$$B = \Delta_f \times 3.5$$

$$= 500KHz \times 3.5$$

$$\therefore \boxed{B = 1750 KHz}$$

i) The transmission BW using Carson's rule is

$$B_T = 2[\Delta f + f_m] = 2[500KHz + 100KHz]$$

$$\boxed{B_T = 1200KHz}$$

Sol:- Given: $f_c = 100MHz$, $f_m = 100KHz$, $A_m = 20V$, $K_f = 25KHz/V$

i) $B_T = 2[\Delta f + f_m]$

$\Delta f = K_f A_m = (25KHz/\cancel{V}) \times 20\cancel{V} = 500KHz$

$B_T = 2[500KHz + 100KHz]$

$\boxed{B_T = 1.2MHz}$

ii) $\beta = \frac{\Delta f}{f_m} = \frac{500KHz}{100KHz} = \underline{5}$

From universal Curve, for $\underline{\beta = 5}$, We have $\boxed{\frac{B}{\Delta f} = 3.2}$

$B = \Delta f \times 3.2$

$B = 500KHz \times 3.2$

$\boxed{B = 1.6MHz}$

iii) Modulating voltage is doubled $= 2 \times 20V = 40V$.

* $\Delta f = K_f A_m = (25KHz/\cancel{V})(40\cancel{V}) = \underline{1MHz.}$

* BW using Carson's rule

$B_T = 2[\Delta f + f_m]$

$= 2[1MHz + 100KHz]$

$\boxed{B_T = 2.2 MHz}$

* $\beta = \frac{\Delta f}{f_m} = \frac{1 MHz}{100 KHz} = \underline{10}$

From universal Curve, for $\beta = 10$, We have $\frac{B}{\Delta f} = 3$

A carrier wave frequency 100MHZ is frequency modulated by a sinusoidal wave of amplitude 20V & frequency 100KHZ. The frequency sensitivity of the modulator is 25KHZ per volt.

i) Determine the approximate bandwidth of the FM signal, using Carson's rule.

ii) Determine the bandwidth by transmitting only those side frequencies whose amplitude exceed 1 percent of the unmodulated carrier amplitude. Use the universal curve of fig ① for this calculation.

iii) Repeat the calculation, assuming that the amplitude of the modulating signal is doubled.

iv) Repeat the calculations, assuming the modulation freq is doubled

July - 2008, 8M

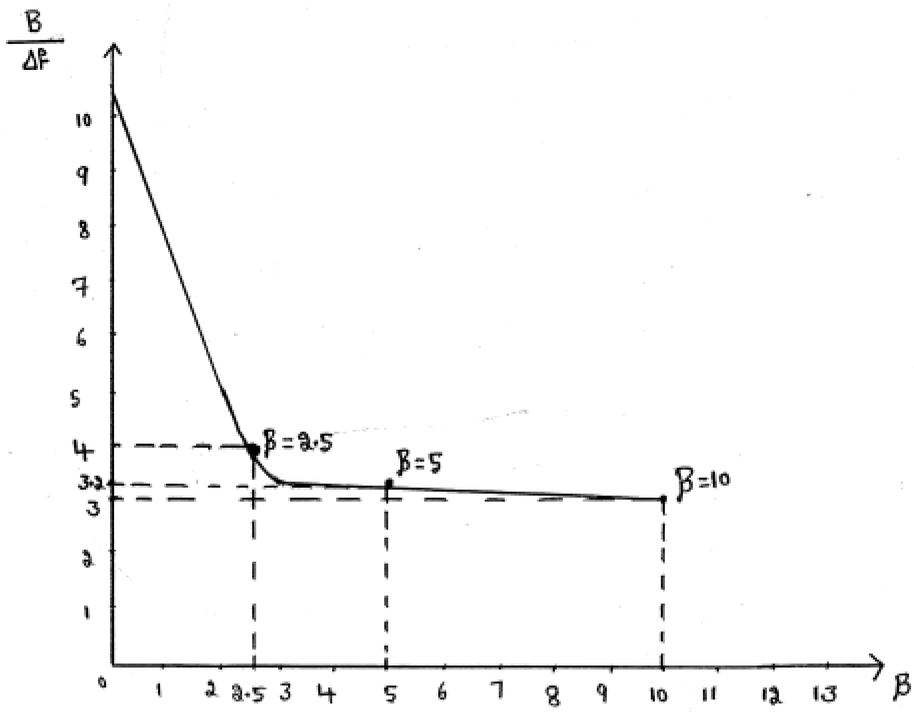


Figure ①: universal curve

$B = \Delta f \times 3 = 1MHz \times 3$

$\boxed{B = 3MHz}$

iv) Given : $f_m = 200KHz$, $A_m = 20V$

* Frequency deviation $\Delta f = K_f A_m$

$= (25 KHz/V) \times 20V$

$\boxed{\Delta f = 500KHz}$

* Bandwidth using Carson's rule

$B_T = 2[\Delta f + f_m]$

$= 2[500KHz + 200KHz]$

$\boxed{B_T = 1.4 MHz}$

$\beta = \frac{\Delta f}{f_m} = \frac{500KHz}{200KHz} = 2.5$

* From universal Curve, for $\beta = 2.5$,

We have, $\frac{B}{\Delta f} = 4.0$

$B = \Delta f \times 4.0$

$= 500KHz \times 4$

$\boxed{B = 2MHz}$

Sketch the variations of the frequency of the resulting FM & PM Signal as a function of time where a carrier Signal is modulated by a modulating Signal $m(t) = \frac{A}{T_0} t \quad 0 \le t \le T_0$, which is periodic with period T_0. Assume the following:

Carrier frequency $f_c = 100\text{ KHz}$, $A = 5\text{ volts}$, $T_0 = 1\text{ msec}$, $K_p = 0.2\pi\text{ rad/sec}$ & $K_f = 2\text{KHz/V}$.

Derive the equation for the PM & FM Signals & draw the relevant block diagram.

Jan-2006, 11M

Sol:-

i) FM Wave :-

W.K.T the FM Wave is given by:

$$S(t) = A_c \cos\left[\omega_c t + 2\pi K_f \int_0^t m(t) \cdot dt\right] \longrightarrow ①$$

$$= A_c \cos\left[2\pi f_c + 2\pi K_f \int_0^t \frac{A}{T_0} t \cdot dt\right]$$

$$= 5 \cos\left[2\pi \times 10^5 t + (2\pi \times 2\times10^3)\left\{\frac{A}{T_0} \frac{t^2}{2}\right\}\right]$$

$$= 5 \cos\left[2\pi \times 10^5 t + \frac{4\pi \times 10^3}{2}\left(\frac{5}{1\times10^{-3}} \cdot t^2\right)\right]$$

$$\boxed{S(t) = 5 \cos\left[2\pi \times 10^5 t + 10\pi \times 10^6 t^2\right]} \longrightarrow ②$$

Eq ② is the FM modulated wave for the given values.

* The Instantaneous frequency is given by

$$f_i = f_c + K_f m(t)$$

$$= 100\times10^3 + 2\times10^3 \left(\frac{A}{T_0} t\right)$$

$$= 10^5 + 2\times10^3 \times \frac{5}{1\times10^{-3}} t$$

$$f_i = 10^5 + 2\times10^3 (5000 t)$$

$$\boxed{f_i = 10^5 + 10\times10^6 t}$$

at $\underline{t = 1 msec}$

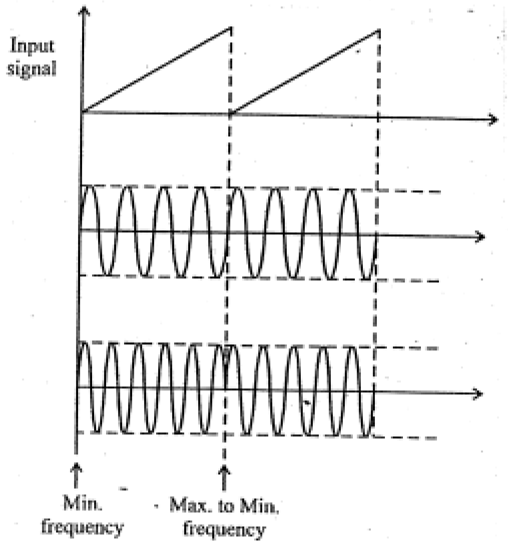
$$f_i = 10^5 + 10\times10^6 (1\times10^{-3})$$

$$\boxed{f_i = 110 KHz}$$

t in msec	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
$m(t) = \left(\frac{A}{T_0} t\right)$ in V	0	0.5	1	1.5	2	2.5	3	3.5	4	4.5	5

Where, $T_0 = 1 msec$

$A = 5V$



WKT the PM Wave is given by

$$S(t) = A_c \cos[\theta(t)]$$

$$S(t) = A_c \cos[2\pi f_c t + K_p m(t)]$$

$$S(t) = 5 \cos\left[2\pi \times 10^5 t + 0.2\pi \left(\frac{A}{T_0} t\right)\right]$$

$$S(t) = 5 \cos\left[2\pi \times 10^5 t + 0.2\pi \frac{5}{1\times 10^{-3}} t\right]$$

$$\boxed{S(t) = 5 \cos[2\pi \times 10^5 t + \pi(1\times 10^3) t]} \longrightarrow (3)$$

Equation (3) is the modulated equation of PM Wave.

* The Instantaneous frequency of the phase modulated wave is given by

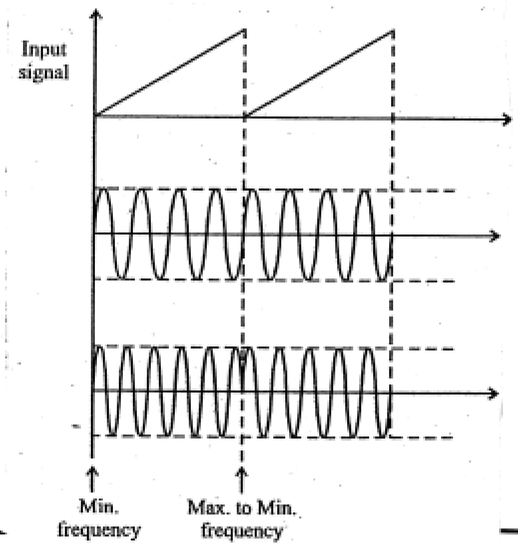
$$\omega_i(t) = \frac{d}{dt}\theta(t) \rightarrow (4)$$

From eq (3), $\theta = [2\pi \times 10^5 t + \pi(1\times 10^3) t]$

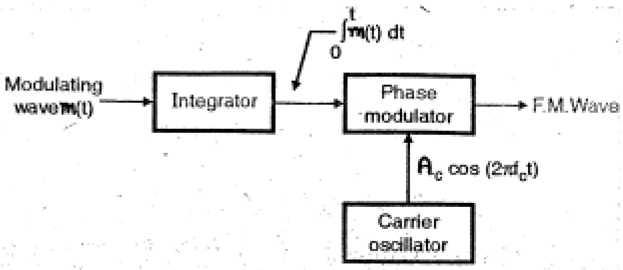
$$2\pi f_i(t) = \frac{d}{dt}[2\pi \times 10^5 t + \pi(1\times 10^3) t]$$

$$f_i(t) = \frac{1}{2\pi}\left[2\pi \times 10^5 + \frac{1000\pi}{500}\right]$$

$$\boxed{f_i(t) = [1\times 10^5 + 500]\ Hz}$$



i) Generation of FM using PM (phase Modulator) :-

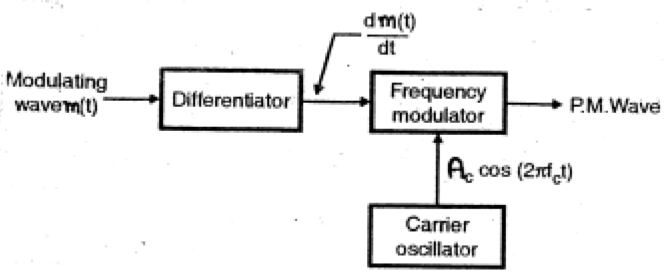


Generation of FM from phase modulator

* FM can be generated by 1st Integrating m(t) & then using the result as the I/p to a phase modulator as shown in above figure.

$$\boxed{S(t) = A_c \cos\left[2\pi f_c t + \underset{\substack{\uparrow \\ K_p}}{2\pi K_p} \int_0^t m(t) \cdot dt\right]}$$

ii) Generation of PM using a FM (Frequency Modulator) :-



Generation of P.M. wave using frequency modulator

* The PM Signal can be generated by 1st differentiating m(t) & then using the result as the I/p to a Frequency - modulator as shown in fig above.

$$\therefore \quad S(t) = A_c \cos\left[2\pi f_c t + 2\pi K_f \int_0^t \frac{\cancel{d}}{\cancel{dt}} m(t) \cdot \cancel{dt}\right.$$

Substituting $2\pi K_f = K_p$

$$s(t) = A_c \cos\left[2\pi f_c t + \underline{2\pi K_f}\, m(t)\right]$$

$$\boxed{s(t) = A_c \cos\left[2\pi f_c t + K_p\, m(t)\right]}$$

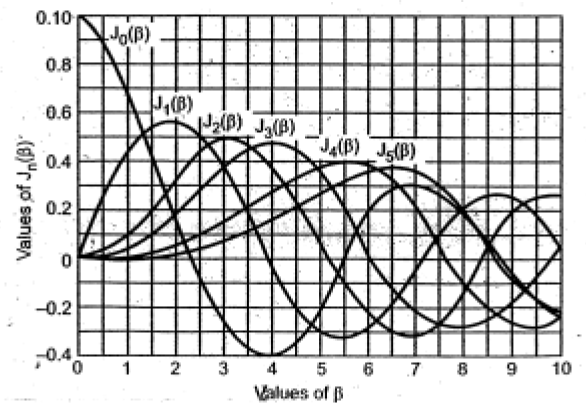
Bessels Functions Table:-

Bessel functions

x (β)	J_0	J_1	J_2	J_3	J_4	J_5	J_6	J_7	J_8	J_9	J_{10}	J_{11}	J_{12}	J_{13}	J_{14}	J_{15}	J_{16}
	n or Order																
0.00	1.00	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
0.25	0.98	0.12	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
0.5	0.94	0.24	0.03	—	—	—	—	—	—	—	—	—	—	—	—	—	—
1.0	0.77	0.44	0.11	0.02	—	—	—	—	—	—	—	—	—	—	—	—	—
1.5	0.51	0.56	0.23	0.06	0.01	—	—	—	—	—	—	—	—	—	—	—	—
2.0	0.22	0.58	0.35	0.13	0.03	—	—	—	—	—	—	—	—	—	—	—	—
2.5	−0.05	0.50	0.45	0.22	0.07	0.02	—	—	—	—	—	—	—	—	—	—	—
3.0	−0.26	0.34	0.49	0.31	0.13	0.04	0.01	—	—	—	—	—	—	—	—	—	—
4.0	−0.40	−0.07	0.36	0.43	0.28	0.13	0.05	0.02	—	—	—	—	—	—	—	—	—
5.0	−0.18	−0.33	0.05	0.36	0.39	0.26	0.13	0.05	0.02	—	—	—	—	—	—	—	—
6.0	0.15	−0.28	−0.24	0.11	0.36	0.36	0.25	0.13	0.06	0.02	—	—	—	—	—	—	—
7.0	0.30	0.00	−0.30	−0.17	0.16	0.35	0.34	0.23	0.13	0.06	0.02	—	—	—	—	—	—
8.0	0.17	0.23	−0.11	−0.29	−0.10	0.19	0.34	0.32	0.22	0.13	0.06	0.03	0.01	—	—	—	—
9.0	−0.09	0.24	0.14	−0.18	−0.27	−0.06	0.20	0.33	0.30	0.21	0.12	0.06	0.03		—	—	—
10.0	−0.25	0.04	0.25	0.06	−0.22	−0.23	−0.01	0.22	0.31	0.29	0.20	0.12	0.06	0.03	0.01	—	—
12.0	0.05	−0.22	−0.08	0.20	0.18	−0.07	−0.24	−0.17	0.05	0.23	0.30	0.27	0.20	0.12	0.07	0.03	0.01
15.0	−0.01	0.21	0.04	−0.19	−0.12	0.13	0.21	0.03	−0.17	−0.22	−0.09	0.10	0.24	0.28	0.25	0.18	0.12

▶ **Figure**

Bessel functions



18

A Carrier wave of amplitude 5V & frequency 90MHz is frequency modulated by a sinusoidal voltage of amplitude 5V & frequency - 15KHz. The frequency deviation constant is 1KHz/V. Sketch the spectrum of the modulated FM wave.

Sol:- Given : $A_c = 5V$, $f_c = 90MHz$, $A_m = 5V$, $f_m = 15KHz$. $K_f = 1KHz/V$.

* Frequency deviation $\Delta f = K_f A_m = 1KHz/V \times 5V = 5KHz$.

* $\beta = \frac{\Delta f}{f_m} = \frac{5KHz}{15KHz} = 0.333$

From the table of Bessel functions, for $\beta = 0.333$

use approximate values for J_0, J_1 & J_2.

i) For carrier : $J_0 = 0.96$

ii) 1^{st} Side frequency : $J_1 = 0.18$

iii) 2^{nd} Side frequency : $J_2 = 0.02$

Higher order side frequencies are negligible since β is small.

i) Amplitude Spectrum of the carrier : $A_c J_0(\beta) = 5V \times 0.96 = 4.8V$

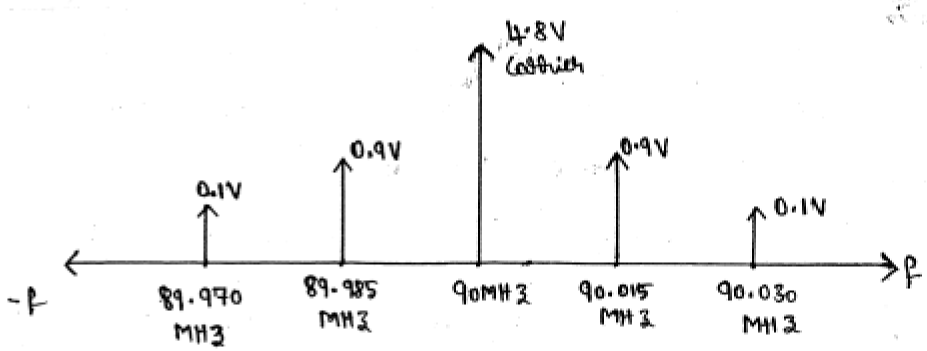
Carrier frequency $f_c = 90MHz$

ii) Amplitude Spectrum of the 1^{st} Side frequency : $A_c J_1(\beta) = 5 \times 0.18V = 0.9V$

1^{st} Side frequency : $f_c + f_m = 90MHz + 15KHz = 90.015 MHz$

iii) Amplitude Spectrum of the 2^{nd} Side frequency : $A_c J_2(\beta) = 5 \times 0.02V = 0.1V$

2^{nd} Side frequency : $f_c + 2f_m = 90MHz + 2(15KHz) = 90.030 MHz$



$f_c - f_m = 90MHz - 15KHz = 89.985\ MHz$

$f_c - 2f_m = 90MHz - 2(15\ KHz) = 89.970MHz.$

NOTE :-

1) Carrier Signal $\rightarrow A_c J_0(\beta) \cos 2\pi f_c t$

2) 1^{st} pair of Side frequencies $\rightarrow A_c J_1(\beta) \cos 2\pi(f_c \pm f_m) t$

3) 2^{nd} pair of Side frequencies $\rightarrow A_c J_2(\beta) \cos 2\pi(f_c \pm 2f_m) t$

$\vdots$

n) n^{th} pair of Side frequencies $\rightarrow A_c J_n(\beta) \cos 2\pi(f_c \pm nf_m) t$

A carrier wave is frequency modulated using a sinusoidal signal of frequency f_m & amplitude A_m.

i) Determine the values of modulation index β for which the carrier component of the FM wave is reduced to zero.

ii) In a certain experiment conducted with $f_m = 1KHz$ and increasing A_m from zero, it is found that the carrier component of FM wave is reduced to zero for the 1^{st} time when $A_m = 2.0V$. Find the frequency sensitivity of the modulator.

iii) What is the value of A_m for which the carrier component becomes zero for the second time?

$\boxed{\text{Jan-2006, 6M}}$

Sol:- Given: $f_m = 1KHz$, $A_m = 2V$, $K_f = ?$

i) From Bessel function table & plot of Bessel function of the 1^{st} kind, the carrier disappears for the modulation index

$$\boxed{\beta = 2.408,\ 5.52,\ 8.6,\ 11.8 \text{ and so on.}}$$

ii)
$$\beta = \frac{\Delta f}{f_m} = \frac{K_f A_m}{f_m}$$

$J_0(\beta) = 0$ i.e. ~~First time~~ carrier is 0 at $\beta = 2.405$

$$K_f = \frac{\beta f_m}{A_m} = \frac{(2.405) \times 1 \times 10^3}{2V} = \underline{1.2025\ KHz/V}$$

iii) Now $J_0(\beta) = 0$ i.e. Second time carrier is 0 at $\beta = 5.52$, $A_m = ?$

$$\beta = \frac{\Delta f}{f_m} = \frac{K_f A_m}{f_m}$$

$$A_m = \frac{\beta f_m}{K_f} = \frac{5.52 \times 1 \times 10^3}{1.2025 \times 10^3} = \underline{4.9V}$$

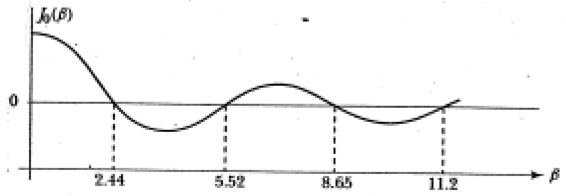
A Carrier wave is frequency modulated using $m(t) = A_m \cos 2\pi f_m t$.

a) List the values of Index β for which the carrier component of FM wave is reduced to zero.

b) In a certain experiment conducted using spectrum analyser with $f_m = 1KHz$ & increasing A_m starting from zero volts, it is found that carrier component of FM wave is reduced to zero for the 1st time with $A_m = 2V$. What is the frequency sensitivity of the modulator? What is the value of A_m for which the carrier component is reduced to zero for the second time?

Sol :- Given : $A_m = 2V$, $f_m = 1KHz$

$\beta = 2.44$, Since the 1st time $J_0(\beta)$ is zero



■ Plot of $J_0(\beta)$ v/s β.

a) The amplitude of the carrier in FM wave is $A_c J_0(\beta)$. This means that if we can make $J_0(\beta) = 0$, the carrier gets suppressed in the FM waveform. The typical values of β for which $J_0(\beta) = 0$ are 2.44, 5.52, 8.65, 11.2 etc.

b) W.K.T. $\beta = \dfrac{\Delta f}{f_m} = \dfrac{K_f A_m}{f_m}$

$$K_f = \frac{\beta f_m}{A_m} = \frac{(2.44) \times 1 \times 10^3}{2} = 1.22 \times 10^3 \text{ Hz/V}$$

Also, $\beta = 5.52$, Since for the Second time $J_0(\beta)$ is Zero

Hence, $A_m = \frac{\beta f_m}{K_f} = \frac{(5.52) \times 1 \times 10^3}{1.22 \times 10^3}$

$$\boxed{A_m = 4.52V}$$

A Carrier wave of amplitude 10V & frequency 100MHz is frequency modulated by a Sinusoidal voltage. The modulating - voltage has an amplitude of 5V & frequency $f_m = 20KHz$. The frequency deviation Constant is 2KHz/V. Draw the frequency Spectrum of FM wave.

Sol :- Given : $A_c = 10V$, $f_c = 100MHz$, $K_f = 2KHz/V$.
$A_m = 5V$, $f_m = 20KHz$.

* $\Delta f = K_f A_m = 2KHz/\cancel{V} \times 5\cancel{V} = 10KHz$

* $\beta = \frac{\Delta f}{f_m} = \frac{10KHz}{20KHz} = \underline{0.5}$

* From the table of Bessel functions, for $\beta = 0.5$ we approximate values of J- Coefficients are :

$$\boxed{J_0 = 0.94, \; J_1 = 0.24, \; J_2 = 0.03}$$

* The amplitude, frequencies of the Carrier & Sidebands are as follows :

i) Carrier amplitude → $A_c J_0(\beta) = 10V \times 0.94 = 9.4V$

Carrier frequency → 100MHz

ii) Frequency of 1st Sideband → ; $f_c + f_m = 100.2\,MHz$

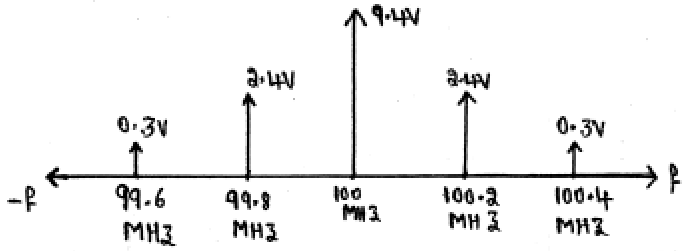
$f_c - f_m = 99.8\,MHz$

Amplitude of 1st Sideband → $A_c J_1(\beta) = 10V \times 0.24 = 2.4V$

iii) Frequency of 2nd Sideband → $f_c + 2f_m = 100.4\,MHz$

$f_c - 2f_m = 99.6\,MHz$

Amplitude of 2nd Sideband → $A_c J_2(\beta) = 10V \times 0.03 = 0.3V$.



Spectrum of FM wave

An unmodulated carrier has amplitude 10V & frequency 100MHz. A Sinusoidal Waveform of frequency 1KHz, frequency-modulates this carrier such that the frequency deviation is 75KHz. The modulated waveform passes through Zero & is increasing at time $t=0$. Write the time-domain expression for the modulated carrier waveform.

Sol :- $A_c = 10V$, $f_c = 100MHz$, $f_m = 1KHz$, $\Delta f = 75KHz$.

W.K.T the time-domain expression of FM wave is

$$S(t) = A_c \cos\left[2\pi f_c t + \beta \sin 2\pi f_m t\right]$$

* $$\beta = \frac{\Delta f}{f_m} = \frac{75KHz}{1KHz} = 75$$

24 $$\boxed{S(t) = 10\cos\left[2\pi \times 100\times10^6 t + 75 \sin 2\pi \times 1\times10^3 t\right]}$$

An angle modulated Signal is represented by

$$S(t) = 10\cos\left[2\pi \times 10^6 t + 5\sin 2000\pi t + 10\sin 3000\pi t\right] \text{ volts.}$$

Find the following:

a) The power in the modulated Signal

b) The frequency deviation Δf

c) The deviation ratio

d) The phase deviation $\Delta\theta$

e) The approximate transmission bandwidth B_T.

Sol:- Given: $A_c = 10V$.

a) $$P = \frac{A_c^2}{2R} = \frac{(10)^2}{2R} = \frac{50}{R}$$

assume $R = 1\Omega$

$$P = \frac{50}{1} = 50 \text{ Watts}$$

b) $$\Delta f = \frac{|\omega_i(t) - \omega_c|_{max}}{2\pi} \quad \text{(OR)} \quad \Delta W = |W_i(t) - W_c|_{max}$$

$$\because 2\pi\Delta f = \Delta W$$

W.K.T $S(t) = A_c\cos[\theta_i(t)] \rightarrow ①$

Comparing eq ① with given equation, we get

$$\theta_i(t) = 2\pi \times 10^6 t + 5\sin 2000\pi t + 10\sin 3000\pi t$$

* The instantaneous angular frequency is

$$W_i(t) = \frac{d}{dt}\theta_i(t)$$

$$W_i(t) = \frac{d}{dt}\left[2\pi \times 10^6 t + 5\sin 2000\pi t + 10\sin 3000\pi t\right]$$

$$W_i(t) = 2\pi \times 10^6 + 5\cos 2000\pi t\,(2000\pi) + 10\cos 3000\pi t\,(3000\pi)$$

$$\boxed{W_i(t) = 2\pi \times 10^6 + 5 \times 2000\pi \cdot \cos 2000\pi t + 10 \times 3000\pi \cdot \cos 3000\pi t}$$

Hence,

$$\Delta W = \left| W_i(t) - W_c \right|_{max}$$

Where, $\boxed{W_c = 2\pi \times 10^6}$

$$\Delta W = \left| \cancel{2\pi \times 10^6} + 5 \times 2000\pi \cdot \cos 2000\pi t + 10 \times 3000\pi \cdot \cos 3000\pi t - \cancel{2\pi \times 10^6} \right|_{max}$$

$$\Delta W = \left| \underline{5 \times 2000\pi} \cdot \cos 2000\pi t + \underline{10 \times 3000\pi} \cdot \cos 3000\pi t \right|_{max}$$

$$\Delta W = \left| \underline{10{,}000\pi} \cos 2000\pi t + \underline{30{,}000\pi} \cos 3000\pi t \right|_{max}$$

$$\Delta W = 10000\pi + 30000\pi \text{ rad/Sec}$$

$$\boxed{\Delta W = 40{,}000\pi \text{ rad/Sec}}$$

example

$$\boxed{\begin{array}{l} \Delta f = \left| \underline{5} \cos\theta \right|_{max} \\ \Delta f = 5 \end{array}}$$

$$2\pi \Delta f = 40{,}000\pi \text{ rad/Sec}$$

$$\Delta f = \frac{40{,}000\cancel{\pi}}{\cancel{2\pi}} = 20\,KHz$$

c) Deviation ratio, $D = \dfrac{\Delta f}{f_m} = \dfrac{20\,KHz}{1500\,Hz} = 13.33$

* In given equation, f_m is the highest significant frequency present in the modulating signal i.e. $\underline{3000\pi}$

$$W_m = 3000\pi$$

$$2\pi f_m = 3000\pi$$

$$f_m = \frac{3000\cancel{\pi}}{\cancel{2\pi}} = 1500\,Hz$$

d) $\Delta\theta = \left| \theta_i(t) - \theta_c \right|_{max}$ Where $\theta_c = 2\pi \times 10^6 t$

$$\Delta\theta = \left| \cancel{2\pi \times 10^6 t} + 5 \sin 2000\pi t + 10 \sin 3000\pi t - \cancel{2\pi \times 10^6 t} \right|_{max}$$

$$\Delta\theta = \left| \underline{5} \sin 2000\pi t + \underline{10} \sin 3000\pi t \right|_{max}$$

$$\Delta\theta = 5 + 10 \text{ radians}$$

$$\boxed{\Delta\theta = 15 \text{ radians}}$$

e) Approximate bandwidth using Carson's formula is

$$B_T = 2[D+1] f_m$$

$$= 2[13.33+1]\, 1500$$

$$\boxed{B_T = 43 \text{ KHz}}$$

---

26) An angle modulated Signal is defined by

$S(t) = 10 \cos[2\pi \times 10^6 t + 0.2 \sin 2000\pi t]$ volts.

Find the following:

a) The power in the modulated Signal

b) The frequency deviation

c) phase deviation $\Delta\theta$

d) The approximate transmission bandwidth.

Sol:- a) $P = \frac{A_c^2}{2R} = \frac{(10)^2}{2R} = \frac{50}{R}$

Assume $R = 1\,\Omega$, $P = \frac{50}{1} = \underline{50 \text{ Watts.}}$

b) The instantaneous phase of the angle modulated Signal is

$$\theta_i(t) = 2\pi \times 10^6 t + 0.2 \sin 2000\pi t$$

$$\Delta W = \left| W_i(t) - W_c \right|_{max}$$

* WKT

$$\omega_i(t) = \frac{d}{dt}\theta_i(t)$$

$$\omega_i(t) = \frac{d}{dt}\left[2\pi \times 10^6 t + 0.2 \sin 2000\pi t\right]$$

$$\omega_i(t) = 2\pi \times 10^6 + 0.2 \cos(2000\pi t) \cdot 2000\pi$$

$$\boxed{\omega_i(t) = 2\pi \times 10^6 + 0.2 \times 2000\pi \cos(2000\pi t)}$$

$$\Delta W = \left| 2\pi \cancel{\times 10^6} + \underline{0.2 \times 2000\pi} \cos(2000\pi t) - 2\pi \cancel{\times 10^6} \right|_{max}$$

Where $W_c = 2\pi \times 10^6$

$$\Delta W = 0.2 \times 2000\pi$$

$$2\pi \Delta f = 0.2 \times 2000\pi$$

$$\Delta f = \frac{0.2 \times \cancel{2000\pi}^{1000}}{\cancel{2\pi}}$$

$$\boxed{\Delta f = 200 Hz}$$

c) $\Delta\theta = \left| \theta_i(t) - \theta_c \right|_{max}$

Where $\theta_c = 2\pi \times 10^6 t$

$$\Delta\theta = \left| 2\pi \cancel{\times 10^6} t + 0.2 \sin 2000\pi t - 2\pi \cancel{\times 10^6} t \right|$$

$$\Delta\theta = \left| \underline{0.2} \sin 2000\pi t \right|$$

$$\boxed{\Delta\theta = 0.2 \text{ radians}}$$

d) Transmission bandwidth

$$B_T = 2[\Delta f + f_m]$$

W.K.T $\quad W_m = 2000\pi$

$$2\pi f_m = 2000\pi$$

$$f_m = \frac{\cancel{2000\pi}^{1000}}{\cancel{2\pi}}$$

$$\boxed{f_m = 1KHz}$$

$$B_T = 2[200 + 1000]$$

$$\boxed{B_T = 2400Hz}$$

A Carrier is frequency modulated by a Sinusoidal modulating Signal of frequency 2KHz, resulting in a frequency deviation of 5KHz.

a) What is the bandwidth occupied by the modulated waveform?

b) If the amplitude of the modulating Signal is increased by a factor of 2 & its frequency lowered to 1KHz, what is the new bandwidth?

Sol:- Given: $f_m = 2KHz$, $\Delta f = 5KHz$.

a) $B_T = 2[\Delta f + f_m] = 2[5KHz + 2KHz]$

$$\boxed{B_T = 14KHz}$$

b) Now A_m is increased by a factor 2 & f_m is decreased to 1KHz, BW=?

W.K.T $\boxed{\Delta f = K_f A_m}$

Since the amplitude of the modulating signal is increased by a factor of 2, the frequency deviation also increases by the same factor. Hence, the new frequency deviation is

$$\Delta f = 2 \times 5\text{KHz} = 10\text{KHz} \quad \& \quad \text{Now } f_m = 1\text{KHz}$$

$$\therefore \quad B_T = 2[\Delta f + f_m] = 2[10\text{KHz} + 1\text{KHz}]$$

$$\boxed{B_T = 22\text{KHz}}$$

A sinusoidal modulating wave $m(t) = A_m \cos 2\pi f_m t$ is applied to a phase modulator with phase sensitivity 'K_p'. The unmodulated carrier wave has frequency 'f_c' & amplitude 'A_c'. Determine the spectrum of the resulting phase modulated wave, assuming that maximum phase deviation $\beta_p = K_p A_m$ does not exceed 0.3 radian.

Sol :-

The time-domain expression for the PM wave is

$$S(t) = A_c \cos[2\pi f_c t + K_p m(t)] \longrightarrow (1)$$

Where K_p is the phase sensitivity constant in rad/volt.

Substituting $m(t) = A_m \cos 2\pi f_m t$ in eq (1), we get

$$s(t) = A_c \cos\left[2\pi f_c t + \underline{k_p A_m} \cos 2\pi f_m t\right]$$

$$s(t) = A_c \cos\left[2\pi f_c t + \beta_p \cos 2\pi f_m t\right]$$

W.K.T

$$\boxed{\cos(A+B) = \cos A \cdot \cos B - \sin A \cdot \sin B}$$

$$s(t) = A_c \cos(2\pi f_c t) \cdot \cos(\beta_p \cos 2\pi f_m t) - A_c \sin(2\pi f_c t) \cdot \sin(\beta_p \cos 2\pi f_m t) \rightarrow (2)$$

Since, $\beta_p \leq 0.3$ rad, we can make the following approximations

$$\cos(\beta_p \cos 2\pi f_m t) \approx 1 \quad \text{and}$$

$$\sin(\beta_p \cos 2\pi f_m t) \approx \beta_p \cos 2\pi f_m t$$

* Making use of above approximations in eq (2), we get expressions for NBFM Signal

$$s(t) = A_c \cos(2\pi f_c t) - A_c \sin(2\pi f_c t) \cdot \beta_p \cos(2\pi f_m t).$$

W.K.T

$$\boxed{\sin A \cdot \cos B = \frac{1}{2}\sin(A-B) + \frac{1}{2}\sin(A+B)}$$

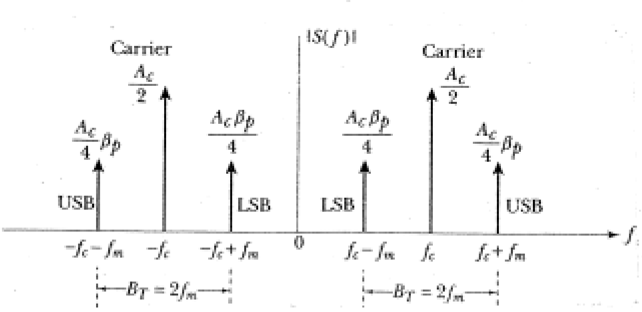
$$s(t) = A_c \cos(2\pi f_c t) - \left[\frac{A_c \beta_p}{2} \sin 2\pi(f_c - f_m)t + \frac{A_c \beta_p}{2} \sin 2\pi(f_c + f_m)t\right]$$

$$\boxed{s(t) = A_c \cos(2\pi f_c t) - \frac{\beta_p A_c}{2} \sin 2\pi(f_c - f_m)t - \frac{\beta_p A_c}{2} \sin 2\pi(f_c + f_m)t} \rightarrow (3)$$

Taking FT on both Sides of eq (3), we get

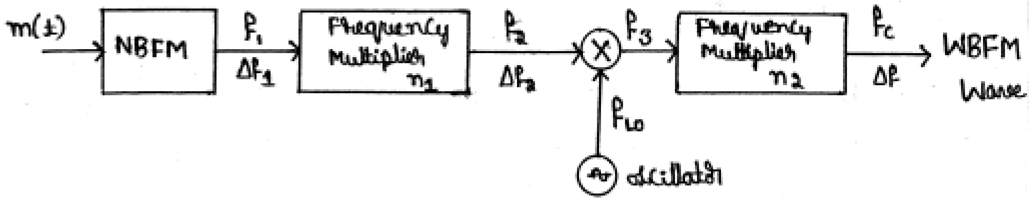
$$S(f) = \frac{A_c}{2}\left[\delta(f - f_c) + \delta(f + f_c)\right] - \frac{\beta_p A_c}{4j}\left[\delta(f - (f_c - f_m)) - \delta(f + (f_c - f_m))\right]$$

$$- \frac{\beta_p A_c}{4j}\left[\delta(f - (f_c + f_m)) - \delta(f + (f_c + f_m))\right]$$



31

The block diagram of an Armstrong AM transmitter is shown in Figure ①. Compute the maximum frequency deviation & o/p frequency of the transmitter. Take $f_1 = 200\,KHz$, $f_{LO} = 10.8\,MHz$, $\Delta f_1 = 25\,Hz$, $n_1 = 64$ & $n_2 = 48$.



Sol:-

1) The o/p of the 1st frequency multiplier

i) $f_2 = n_1 \times f_1 = 64 \times 200\,KHz = 12.8\,MHz$

ii) $\Delta f_2 = n_1 \times \Delta f_1 = 64 \times 25\,Hz = 1600\,Hz$

2) The o/p of the product modulator

$$\boxed{f_3 = f_2 \pm f_{LO}}$$

i) $f_3 = f_2 + f_{LO} = 23.6\,MHz$

ii) $f_3 = f_2 - f_{LO} = 2\,MHz$

* The o/p of the 2nd frequency multiplier i.e. Final o/p

i) $f_c = n_2 f_3 = 1.1328\,GHz$, When $f_3 = 23.6\,MHz$

$f_c = n_2 f_3 = 96\,MHz$, When $f_3 = 2\,MHz$

ii) $\Delta f = n_2 \times \Delta f_2 = 76.8\,MHz$

Calculate the carrier swing, carrier frequency, frequency deviation & modulation index for an FM signal which reaches a maximum frequency of 99.047 MHz & a minimum freq of 99.023 MHz. The frequency of the modulating signal is 7KHz.

Sol:- Given : $(f_i)_{max} = 99.047\,MHz$, $(f_i)_{min} = 99.023\,MHz$ & $f_m = 7KHz$.

* Carrier Swing $= (f_i)_{max} - (f_i)_{min} = 99.047MHz - 99.023MHz$
$= 24KHz.$

* $\Delta f = \frac{\text{Carrier Swing}}{2} = \frac{24KHz}{2} = 12KHz.$

* W.K.T $(f_i)_{max} = f_c + \Delta f$

Carrier freq $f_c = (f_i)_{max} - \Delta f$

$= 99.047MHz - 12KHz$

$f_c = 99.035\,MHz.$

* Modulation Index 'β' $= \frac{\Delta f}{f_m}$

$= \frac{12KHz}{7KHz}$

$$\boxed{\beta = 1.714}$$

A 93.2 MHz carrier is frequency modulated by a 5 KHz sine wave. The resultant FM signal has a frequency deviation of 40 KHz.

a) Find the carrier swing of the FM signal.

b) What are the highest & lowest frequencies attained by the frequency modulated signal.

c) Calculate the modulation index for the wave.

Sol:- Given: $f_c = 93.2\,MHz$, $f_m = 5\,KHz$, $\Delta f = 40\,KHz$.

a) Carrier swing $= 2 \times \Delta f$

$= 2 \times 40\,KHz$

$= 80\,KHz$

b) $[f_i]_{max} = f_c + \Delta f$

$= 93.2\,MHz + 40\,KHz$

$= 93.24\,MHz$

$(f_i)_{min} = f_c - \Delta f$

$= 93.2\,MHz - 40\,KHz$

$= 93.16\,MHz$

c) Modulation Index $= \dfrac{\Delta f}{f_m} = \dfrac{40\,KHz}{5\,KHz} = 8$

NOTE:-

$\Delta f = (f_i)_{max} - (f_i)_{min}$

$= 93.24\,MHz - 93.16\,MHz$

$= 80\,KHz$

When a 50.4 MHz carrier is frequency modulated by a sinusoidal AF modulating signal, the highest frequency reached is 50.405 MHz. Calculate

a) The frequency deviation produced

b) Carrier swing of the wave

c) Lowest frequency reached.

Sol :-

a) $\Delta f = (f_i)_{max} - f_c = 50.405\ MHz - 50.4 MHz = 5KHz$

b) Carrier swing $= 2 \times \Delta f = 2 \times 5\ KHz = 10\ KHz$

c) $(f_i)_{min} = f_c - \Delta f = 50.4MHz - 5\ KHz = 50.395MHz$

What is the bandwidth required for a FM signal if the modulating frequency is 1KHz & the maximum deviation is 10KHz? What is bandwidth required for a DSBSC (AM) transmission? Comment on the result.

Sol :- Given : $\Delta f = 10KHz$, $f_m = 1KHz$.

* The carson's rule to find out the bandwidth of FM :

$$BW = 2[\Delta f + f_m]$$

$$= 2[10+1]$$

$$\boxed{BW = 22KHz}$$

* The BW of DSBSC (AM) system $= 2f_m = 2(1KHz) = 2KHz$.

* This result shows that for the transmission of same signal the bandwidth required for FM is very much higher than that of an AM system.

35

In an FM System, When the audio frequency is 500Hz & modulating voltage 2.5 V, the deviation produced is 5KHz. If the modulating voltage is now increased to 7.5V, Calculate the new value of frequency deviation produced. If the AF voltage is raised to 10V while the modulating frequency dropped to 250Hz, what is the frequency deviation? Calculate the modulation index in each case.

Sol :- Given : $f_m = 500Hz$, $A_m = 2.5V$, $\Delta f = 5KHz$.

i) W.K.T $\Delta f = K_f A_m$

$$K_f = \frac{\Delta f}{A_m} = \frac{5KHz}{2.5V} = 2KHz/V.$$

$$\beta = \frac{\Delta f}{f_m} = \frac{5KHz}{500Hz} = 10$$

ii) When modulating voltage is raised to 7.5V i.e. $A_m = 7.5V$

$\Delta f = ?$, $\beta = ?$

$$\Delta f = K_f A_m = 2KHz/v \times 7.5V = 15KHz.$$

$$\beta = \frac{\Delta f}{f_m} = \frac{15KHz}{500Hz} = 30$$

iii) When modulating voltage is raised to 10V i.e. $A_m = 10V$ &

$f_m = 250Hz$.

$$\Delta f = K_f A_m = 2KHz/v \times 10V = 20KHz$$

$$\beta = \frac{\Delta f}{f_m} = \frac{20KHz}{250Hz} = 80$$

A 2KHz Sinusoidal Signal phase modulates a carrier at 100MHz With a peak phase deviation of 45°. Using Carson's rule evaluate the approximate bandwidth of the PM Signal.

Sol:- The time-domain expression for a PM Wave is

$$S(t) = A_c \cos[2\pi f_c t + K_p m(t)]$$

W.K.T $m(t) = A_m \cos 2\pi f_m t$, We get

$$S(t) = A_c \cos[2\pi f_c t + K_p A_m \cos 2\pi f_m t]$$

$$S(t) = A_c \cos[2\pi f_c t + \beta_p \cos 2\pi f_m t]$$

$$S(t) = A_c \cos[\theta_c + \beta_p \cos 2\pi f_m t]$$

$$\boxed{S(t) = A_c \cos[\theta_i(t)]}$$

* The Instantaneous phase,

$$\theta_i(t) = \theta_c + \beta_p \cos 2\pi f_m t.$$

* Peak-phase deviation:-

$$\Delta\theta = |\theta_i(t) - \theta_c|_{max}$$

$$\boxed{\Delta\theta = \beta_p}$$

Given $\beta_p = \frac{\pi}{4}$ radians

* Using Carson's rule, the transmission bandwidth of the PM Wave is given by:

$$B_T = 2[\beta_p + 1] f_m$$

$$= 2\left[\frac{\pi}{4} + 1\right] 2\times10^3$$

$$= 2[0.785 + 1]\, 2\times10^3$$

$$\boxed{B_T = 7.14\,\text{KHz}}$$

In a FM System, the modulating frequency $f_m = 1\,\text{KHz}$, the modulating voltage $A_m = 2V$ & the deviation is $6\,\text{KHz}$. If the modulating voltage is raised to $4V$ then what is the new deviation? If the modulating voltage is further increased to $8V$ & modulating frequency is reduced to $500\,\text{Hz}$. what will be deviation? Calculate the modulation Index in each Case. Comment on the result.

Sol:- Given : $f_m = 1\,\text{KHz}$, $A_m = 2V$, $\Delta f = 6\,\text{KHz}$.

WKT $\quad \Delta f = K_f A_m$

$$K_f = \frac{\Delta f}{A_m} = \frac{6\,\text{KHz}}{2V} = \underline{3\,\text{KHz}/V}$$

$$\beta = \frac{\Delta f}{f_m} = \frac{6\,\text{KHz}}{1\,\text{KHz}} = \underline{6}$$

* $\therefore$ When modulating voltage is raised to 4V i.e. $A_m = 4V$.

$\Delta f = ?$

$$\Delta f = K_f A_m = (3\,\text{KHz}/V) \times 4V = \underline{12\,\text{KHz}}$$

$$\beta = \frac{\Delta f}{f_m} = \frac{12\,\text{KHz}}{1\,\text{KHz}} = \underline{12}$$

* For $A_m = 8V$ & $f_m = 500\,\text{Hz}$

$$\Delta f = K_f A_m = (3\,\text{KHz}/V) \times 8V = \underline{24\,\text{KHz}}$$

$$\beta = \frac{\Delta f}{f_m} = \frac{24\,\text{KHz}}{500\,\text{Hz}} = \underline{48}$$

Comment:-

The modulation index is dependent on the value of deviation as well as the modulating frequency.

The Carrier Swing of a FM Signal is 70KHz & the modulating Signal is a 7KHz Sine wave. Determine the modulation-Index of the FM Signal.

Sol:- Given $\text{Carrier Swing} = 70\,KHz$, $f_m = 7\,KHz$. $\beta = ?$

$$\beta = \frac{\Delta f}{f_m}$$

$$\Delta f = \frac{\text{Carrier Swing}}{2} = \frac{70\,KHz}{2} = 35\,KHz$$

$$\beta = \frac{35\,KHz}{7\,KHz} = 5$$

NOISE

1. Define: Shot noise, Thermal noise, Noise figure. Jan 09 (6)
2. Derive the relation between noise figure and equivalent noise temperature.
   Jan 09 (6)
3. Explain the following terms: July 09 (10)
   a. Shot noise
   b. Thermal noise
   c. White noise
   d. Noise figure
   e. Transit time noise
4. Derive an expression for overall Equivalent noise temperature of the cascade connection of any number of noises for two port network. July 09 (5)
5. Define and explain the following and obtain the relation between them: July 09 (7)
   a. Noise figure
   b. Equivalent noise temperature.
6. What is noise equivalent band width? Derive an expression for noise equivalent bandwidth
   Jan 05 (8)
7. List and explain the types of noise which occur in an electronic circuit. July 05 (8)
8. Define noise figure and noise temperature along with related equations. July 05 (6)
9. Define and explain the following and obtain the relation between them: Jan 06 (6)
   a. Noise figure
   b. Equivalent noise temperature

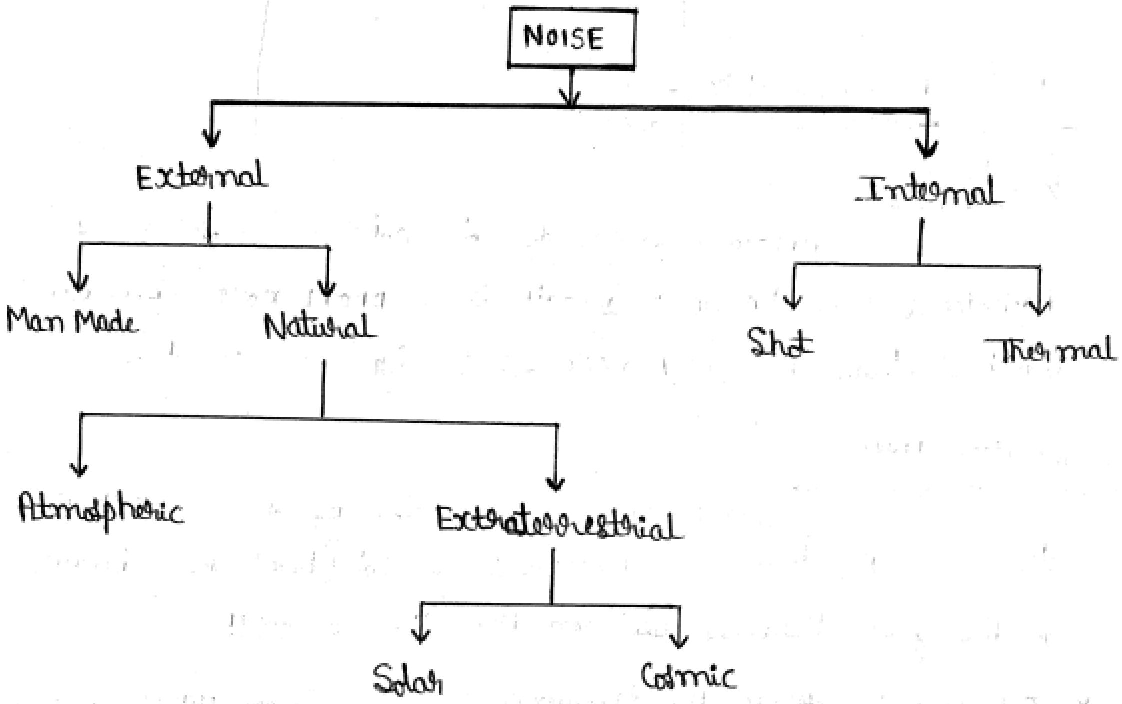
NOISE

NOISE:-

Noise is an unwanted Signal. Noise is random in nature & interferes with the desired Signal.

* Noise disturb the proper reception & reproduction of transmitte-d Signals.

Classification of Noise:-



I) External Noise:-

1) Atmospheric Noise:-

Atmospheric noise is also called as Static Noise. The atmospheric noise is mainly due to electrical disturbances such as lightning.

* Lightning refer to electric discharges that occur between clouds or between the earth & clouds.
  This results in a transient electrical signals that generates harmonic energy that can travel extremely long distances.

* Atmospheric noise effects the reception at frequencies less than 30MHz & less effected for the frequencies above 30MHz.

2) Extra terrestrial Noise:-

i) Solar Noise:-

Primary source of solar noise is Sun. The Sun radiates a wide range of signals in a broad noise spectrum which includes the frequencies we use for communication.

ii) Cosmic Noise:-

Cosmic Noise is generated by stars, which are also suns. The level of cosmic noise is not (high) great because of the great distances between the stars & earth.

* Cosmic noise effects the frequency ranging from 15KHz to 150MH

II) Internal Noise :-

Internal noise is generated internally in the circuit. Electronic components such as resistors, diodes and transistors etc produce this noise.

1) Shot Noise :-

* Shot Noise arises in electronic devices because of the discrete (pulse) nature of current flow in the device.
* Shot noise appears in the active devices due to random behaviour of charge carriers (electrons or holes).
* In vaccum tubes shot noise is generated due to random emission of electrons from the cathode.
* In semiconductor devices due to random diffusion of electrons or the random recombination of electrons with holes.
* In a photo diode, it is the random emission of photons.
* The nature of current variation wrt. to time in a vaccum diode is as shown in fig below.

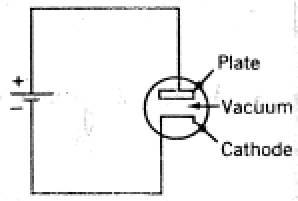


Fig ①: vaccum diode

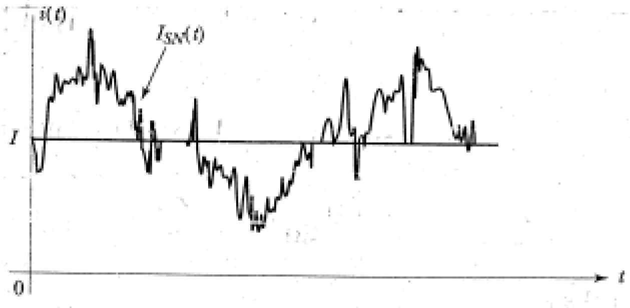


Figure 2 ■ Shot noise current.

* Fig ② shows the current fluctuation over the mean value 'I'.

* The fluctuating current is called Shot Noise & denoted as 'I_{SN}'. The fluctuating current $I_{SN}(t)$ is not observed by normal measuring instrument i.e. it look like constant current 'I'.

* The fluctuating nature of $I_{SN}(t)$ can be seen only in fast acting oscilloscopes.

∴ The total current flowing in the vaccum diode

$$\boxed{i(t) = I + I_{SN}(t)}$$

* For a vaccum diode, the mean square value of randomly - fluctuating component of current is given by :

$$\boxed{I_{SN}^2 = 2qIB_N \text{ amps}^2}$$

Where,

$q \rightarrow$ electron charge equal to 1.6×10^{-19} Coulombs.

$I \rightarrow$ The mean value of the current in amperes &

$B_N \rightarrow$ Noise bandwidth in Hz.

* Shot Noise has a uniform spectral density (like Thermal Noise)

* The mean square shot noise current for a diode is given as:

$$\boxed{I_{SN}^2 = 2q(I + 2I_S)B_N}$$

Where, $I \rightarrow$ dc current across the junction

$I_S \rightarrow$ Reverse Saturation current

$q \rightarrow$ electron charge $= 1.6\times10^{-19}$ C

$B_N \rightarrow$ Noise Bandwidth.

---

1) A noise generator using diode is required to produce 15 µV noise-voltage in a receiver which has an I/p impedance of 75 Ω (purely resistive). The receiver has a noise power bandwidth of 200 KHz. Calculate the current through the diode.

Sol :- Given: $V_{SN} = 15\,\mu V$, $R = 75\,\Omega$, $B_N = 200\,KHz$, $I = ?$

W.K.T. $$I_{SN}^2 = 2q(I + 2I_S)B_N$$

$I \gg I_S$, neglecting I_S.

$$I_{SN}^2 = 2q(I)B_N. \rightarrow (1)$$

W.K.T. $$I_{SN} = \frac{V_{SN}}{R} = \frac{15\,\mu V}{75\,\Omega} = 0.2\,\mu A$$

* From eq (1),

$$I = \frac{I_{SN}^2}{2qB_N} = \frac{(0.2\,\mu A)^2}{2\times1.6\times10^{-9}\times 200\,KHz}$$

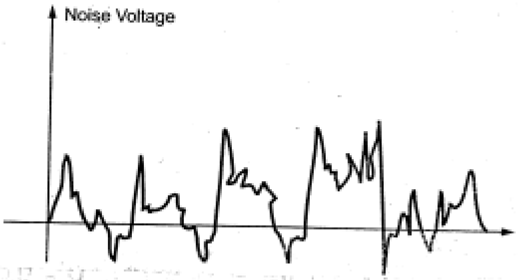
$$\therefore \boxed{I = 625\,mA}$$

2) Thermal Noise & Johnson's Noise :-

* The random movement of electrons inside the conductor resulting in a randomly varying voltage across the conductor as shown in fig.

Figure

Thermal noise



* This randomly varying noise voltage produced across the conductor is called as thermal Noise. It is also known as Johnson Noise.

* The power spectral density of thermal noise produced by a resistor is given by:

$$\boxed{S_{TN}(f) = \frac{2h|f|}{\exp(h|f|/KT)-1}} \longrightarrow ①$$

Where, $T \rightarrow$ absolute temperature in degrees Kelvin.

$K \rightarrow$ Boltzmann's constant i.e. 1.38×10^{-23} Joules/°K

$h \rightarrow$ Planck's constant i.e. 6.63×10^{-34} Joules/Sec.

* The power spectral density for low frequency is defined by $f \ll \frac{KT}{h}$ we may use the approximation $\boxed{\exp\left(\frac{h|f|}{KT}\right) = 1 + \frac{h|f|}{KT}} \longrightarrow ②$

Substituting eq ② in eq ①, we get

$$S_{TN}(f) = \frac{2h|f|}{\cancel{1} + \frac{h|f|}{KT} - \cancel{1}} = \frac{2h|f|}{\frac{h|f|}{KT}} = \frac{\cancel{2h|f|}\,KT}{\cancel{h|f|}}$$

$$\boxed{S_{TN}(f) = 2KT} \longrightarrow ③$$

* The mean Square value of the thermal noise voltage measured across the terminals of the resistor equals

$$\boxed{V_{TN}^2 = 2RB_N S_{TN}(f)} \longrightarrow ④$$

Substituting eq ③ in eq ④, we get

$$V_{TN}^2 = 2RB_N(2KT)$$

$$\boxed{V_{TN}^2 = 4KTB_N R \text{ volts}^2}$$

Where,

$V_{TN} \rightarrow$ root-mean Square noise voltage

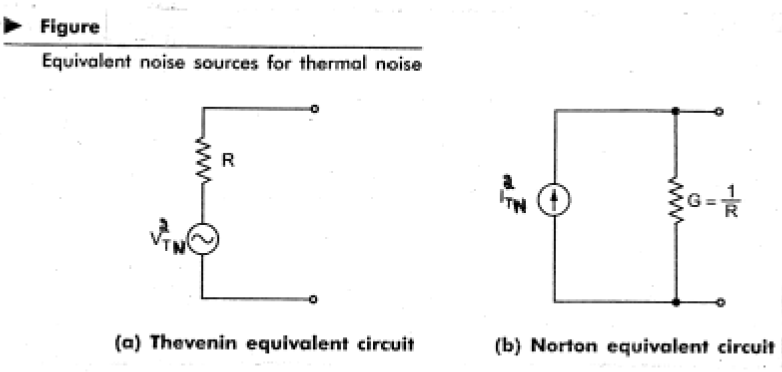
$K \rightarrow$ Boltzmann's Constant

$T \rightarrow$ Temperature of the Conductor in Kelvins

$B_N \rightarrow$ Noise bandwidth in Hz.

$R \rightarrow$ Resistance of the Conductor in ohms.

Equivalent Noise Sources for the thermal Noise :-



* Fig ⓐ Shows a model of a Noisy resistor.
  The Thevenin equivalent ckt Consisting of a Noise voltage generator with a mean-Square values of V^2_{TN} in Series with a noiseless resistor.

* Similarly Fig ⓑ Shows Norton equivalent ckt Consisting of a Noise current generator in parallel with a Noiseless Conductance.

  The mean-Square value of the Noise current generator is:

$$I^2_{TN} = \frac{V^2_{TN}}{R^2} = \frac{4KTB_N \cancel{R}}{R^{\cancel{2}}} = 4KTB_N \frac{1}{R}$$

$$\boxed{I^2_{TN} = 4KTB_N G} \quad \text{amps}^2$$

Where, $G = \frac{1}{R}$ is the Conductance.

Available Noise power :-

* The Root-mean Square value of the voltage V_{RMS} across the matched load R_L is

$$\boxed{V_{RMS} = \frac{\sqrt{V^2_{TN}}}{2}}$$

* The maximum average Noise power delivered to the load is :

$$P_n = \frac{V_{RMS}^2}{R} = \frac{V_{TN}^2}{4R} = \frac{\cancel{4}KTB_N\cancel{R}}{\cancel{4R}}$$

$$\boxed{P_n = KTB_N}$$

Thus, the available Noise power 'P_a' is equal to 'KTB_N' & is independent of 'R'.

FORMULAE :

1) RMS Noise voltage : $V_{TN}^2 = 4KTB_NR$

$$\boxed{V_{TN} = \sqrt{4KTB_NR}}$$

2) Thermal Noise power

$$\boxed{P_n = KTB_N}$$

---

1) Calculate the rms noise voltage and thermal noise power appearing across a 20 kΩ resistor at 25°C temperature with an effective noise bandwidth of 10 KHz.

Sol :- Given : $R = 20k\Omega$, $T = 273 + 25 = 298K$, $B_N = 10\,KHz$, $K = 1.38 \times 10^{-23}$

* $V_{TN} = \sqrt{4KTB_NR} = \sqrt{4 \times 1.38 \times 10^{-23} \times 298 \times 10 \times 10^3 \times 20 \times 10^3}$

$$\boxed{V_{TN} = 1.81\,\mu V}$$

* $P_n = KTB_N = 1.38 \times 10^{-23} \times 298 \times 10 \times 10^3$

$$\boxed{P_n = 4.11 \times 10^{-17}}$$

2) A receiver has a noise power bandwidth of 12 KHz. A resistor which matches with the receiver I/p Impedance is connected across the antenna terminals. What is the noise power contributed by this resistor in the receiver bandwidth? Assume temperature to be 30°c.

Sol :- Given : $B_N = 12KHz$, $T = 30°c + 273 = 303°K$, $K = 1.38 \times 10^{-23}$

$$P_n = KTB_N = 1.38 \times 10^{-23} \times 303 \times 12 \times 10^{3}$$

$$\boxed{P_n = 5.01768 \times 10^{-17} W}$$

---

3) A 600Ω resistor is connected across the 600Ω antenna input of a radio receiver. The bandwidth of the radio receiver is 20KHz & the resistor is at room temperature of 27°c. Calculate the noise power & the noise voltage applied at the I/p of the receiver.

Sol:- Given : $R_1 = 600\Omega$, $R_2 = 600\Omega$, $B_N = 20 \times 10^{3} Hz$, $T = 27 + 273 = 300°K$.

$P_n = ?$, $V_{TN} = ?$, $K = 1.38 \times 10^{-23}$

* Noise power : $P_n = KTB_N = 1.38 \times 10^{-23} \times 300 \times 20 \times 10^{3}$

$$\boxed{P_n = 8.28 \times 10^{-17} W}$$

* Noise voltage at the receiver I/p :

Since the two resistors are in parallel

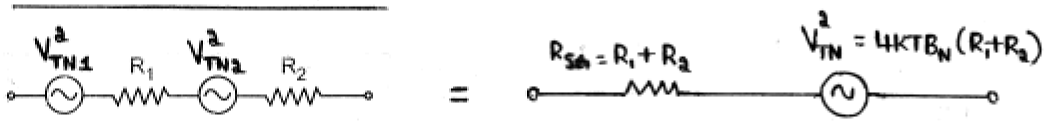
* $R = R_1 \| R_2 = 600\Omega \| 600\Omega = 300\Omega$.

∴ Noise voltage $V_{TN} = \sqrt{4KTB_N R} = \sqrt{4 \times 1.38 \times 10^{-23} \times 300 \times 20 \times 10^{3} \times 300}$

$$\boxed{V_{TN} = 0.3152 \mu V}$$

Thermal Noise Calculation :-

1) In Series resistors :-



(a) Series resistors   (b) Equivalent Ckt

* Fig (a) Shows two resistors R_1 & R_2 connected in Series.

∴ The total Series resistance is given as

$$R_{Ser} = R_1 + R_2$$

W.K.T
$$V^2_{TN} = 4KTB_N R_{Ser}$$
$$= 4KTB_N[R_1+R_2]$$
$$V^2_{TN} = 4KTB_N R_1 + 4KTB_N R_2$$
$$V^2_{TN} = V^2_{TN1} + V^2_{TN2}$$

$$\boxed{V_{TN} = \sqrt{V^2_{TN1} + V^2_{TN2}}}$$

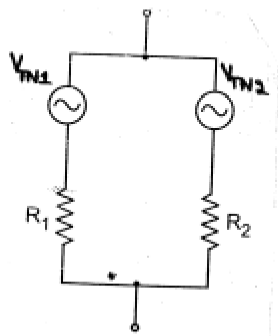
2) In parallel resistor :-

* When two resistors R_1 & R_2 are connected in parallel, then total Noise voltage V_{TN} is obtained.

$$V^2_{TN} = 4KTB_N R_p$$

Where R_p = Equivalent resistance of parallel resistors R_1 & R_2 i.e.

$$R_p = R_1 \| R_2$$



Parallel resistors

1) Three 5 kΩ resistors are Connected in Series. For room temperature $KT = 4\times10^{-21}$ & an effective noise bandwidth of 1MHz, determine

a) The noise voltage appearing across each resistor.

b) The noise voltage appearing across the Series Combination.

c) What is the rms noise voltage which appears across Same three resistors Connected in parallel under the Same Conditions?

Sol :- Given : $R_1 = R_2 = R_3 = 5k\Omega$, $KT = 4\times10^{-21}$, $B_N = 1MHz$.

a) $V_{TN} = \sqrt{4KTB_NR} = \sqrt{4\times4\times10^{-21}\times1\times10^{6}\times5\times10^{3}}$

$\boxed{V_{TN} = 8.94\ \mu V}$

b) $R_{ser} = R_1+R_2+R_3 = 5k\Omega + 5k\Omega + 5k\Omega = 15k\Omega.$

$V_{TN} = \sqrt{4KTB_NR_{ser}} = \sqrt{4\times4\times10^{-21}\times1\times10^{6}\times5\times10^{3}}$

$\boxed{V_{TN} = 15.5\ \mu V}$

c) $R_p = R_1 \| R_2 \| R_3$

$\frac{1}{R_p} = \frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3} = \frac{1}{5k\Omega}+\frac{1}{5k\Omega}+\frac{1}{5k\Omega} = \frac{1}{6\times10^{-4}}$

$\boxed{R_p = 1.66\ k\Omega}$

$V_{TN} = \sqrt{4KTB_NR_p} = \sqrt{4\times4\times10^{-21}\times1\times10^{6}\times1.66\times10^{3}}$

$\boxed{V_{TN} = 5.15\ \mu V}$

2) Two resistors 20kΩ & 50kΩ are at room temperature 290°K. Calculate for the bandwidth of 100KHz, the thermal noise for the following Conditions:

i) For each resistor ii) For two resistors in Series

iii) For two resistors in parallel.

Sol: Given: $T = 290°K$, $B_N = 100KHz$, $R_1 = 20k\Omega$, $R_2 = 50k\Omega$,

$K = 1.38 \times 10^{-23}$

* $KT = 1.38 \times 10^{-23} \times 290 = 4 \times 10^{-21}$

i) For 20kΩ resistor:

$$V_{TN} = \sqrt{4KTB_N R_1} = \sqrt{4 \times 4 \times 10^{-21} \times 100 \times 10^3 \times 20 \times 10^3}$$

$$\boxed{V_{TN} = 5.65\ \mu V}$$

For 50kΩ resistor

$$V_{TN} = \sqrt{4KTB_N R_2} = \sqrt{4 \times 4 \times 10^{-21} \times 100 \times 10^3 \times 50 \times 10^3}$$

$$\boxed{V_{TN} = 8.94\ \mu V}$$

ii) $R_{Ser} = R_1 + R_2 = 20k\Omega + 50k\Omega = 70k\Omega$

$$V_{TN} = \sqrt{4KTB_N R_{Ser}} = \sqrt{4 \times 4 \times 10^{-21} \times 100 \times 10^3 \times 70 \times 10^3}$$

$$\boxed{V_{TN} = 10.58\ \mu V}$$

iii) $R_p = R_1 \parallel R_2 = 20k\Omega \parallel 50k\Omega = 14.28\ k\Omega$

$$V_{TN} = \sqrt{4KTB_N R_p} = \sqrt{4 \times 4 \times 10^{-21} \times 100 \times 10^3 \times 14.28 \times 10^3}$$

$$\boxed{V_{TN} = 4.78\ \mu V}$$

3) An amplifier operating over a frequency range from 17 to 19 MHz has a I/p resistance of 5 kΩ. What is the rms thermal noise voltage at the I/p of this amplifier? Assume the operating - temperature to be 27°c.

Sol:- W.K.T. Bandwidth is given by: $f_2 - f_1$

$$B_N = 19MHz - 17MHz = \underline{2MHz.}$$

Given: $R = 5k\Omega$, $T = 27°c + 273 = 300°K$, $B_N = 2MHz$,

$K = 1.38 \times 10^{-23}$

$$V_{TN} = \sqrt{4KTB_N R} = \sqrt{4 \times 1.38 \times 10^{-23} \times 300 \times 2 \times 10^6 \times 5 \times 10^3}$$

$$\boxed{V_{TN} = 12.86 \mu V}$$

Man made Noise & Industrial Noise :-

* Industrial noise is produced by aircraft ignition, electrical motors, welding machines, ignition system of the automobile etc.
* This noise effect the signal having frequency range <u>1MHz</u> to <u>600MHz.</u>

Transit time Noise & High frequency Noise :-

* Transit time is the time taken by an electron to travel from cathode to anode & the time taken by the charge carriers to cross a p-n junction.

* The transit time occupies a small portion of the I/p period at lower frequencies. As the frequency is increased, the transit time occupies a considerable portion of the I/p period.

  In such a situation, the charge carriers may start diffuse back to the source i.e. emitter in the case of a transistor without reaching the collector.

* The diffusion of the carrier back to the source give rise to an I/p admittance in which the conductance increases with frequency.

* The noise current generator associated with this conductance increases with frequency.

* At very high frequencies it becomes a predominant noise component.

Flicker Noise or Low frequency Noise :-

* The flicker noise will appear at low frequency's. It is sometimes called as "1/f" noise.

* In the semiconductor devices, the flicker noise is generated - to the fluctuations in the carrier density.

Partition Noise :-

* partition Noise is generated when the current gets divided between two or more paths.

* It is generated due to the random fluctuations in the current divisions.

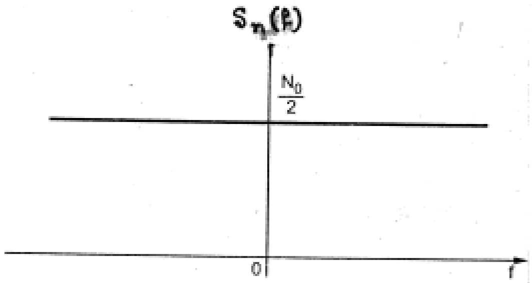
∴ The partition noise in a transistor will be higher than that in a diode.

White Noise :-

White Noise is the noise whose power spectral density is uniform over the entire frequency range as shown in fig.

▶ Figure

(a) Power spectral density of white noise



* The spectral density of white Noise is given by

$$\boxed{S_n(f) = \frac{N_0}{2}}$$

Where, $N_0 = KT_e$

$K \rightarrow$ Boltzmann's Constant

$T_e \rightarrow$ Equivalent noise temperature of the System.

Formulae :-

1) The Fourier transform of auto correlation function

$$R(\tau) \xrightarrow{FT} S(f)$$

2) The Inverse Fourier transform of $S(f)$ is $R(\tau)$ i.e.

$$R(\tau) = IFT|S(f)|$$

3) The IFT of

$$\frac{N_0}{2} \xrightarrow{IFT} \frac{N_0}{2} \delta(t)$$

1) Derive and plot the auto correlation function of a white gaussian noise which has a power spectral density of $N_0/2$.

Sol:- The auto correlation function is denoted by $R(\tau)$

* Taking FT of $R(\tau)$

$$FT[R(\tau)] \longrightarrow S(f)$$

or

$$IFT[S(f)] = R(\tau)$$

* Thus

$$R(\tau) = IFT[S(f)] \longrightarrow ①$$

WKT $\boxed{S(f) = \frac{N_0}{2}} \longrightarrow ②$

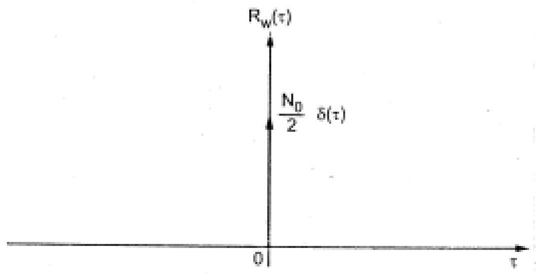
Substituting eq ② in eq ①, we get

$$R(\tau) = IFT\left[\frac{N_0}{2}\right]$$

$$\boxed{R(\tau) = \frac{N_0}{2} \delta(t)}$$

▶ **Figure**

Autocorrelation function of white noise



Noise Equivalent Bandwidth:-

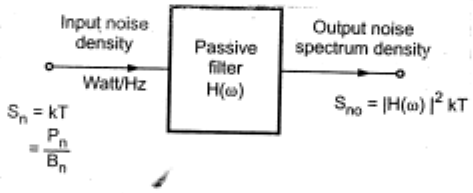
*) What is noise equivalent bandwidth? Derive an expression for noise equivalent bandwidth.

$\boxed{\text{Jan - 2005, 8M}}$

* Consider a passive filter having voltage-ratio transfer function $H(\omega)$. Let the I/p noise spectrum density be

$$S_n = KT = \frac{P_n}{B_n}$$

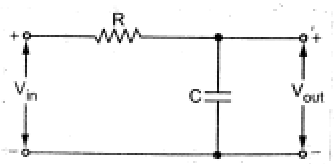
Where, P_n is noise power.



* The o/p Noise Spectrum density S_{no}, for an I/p density of $S_n = KT$ is

$$\boxed{S_{no} = |H(\omega)|^2 \cdot KT} \longrightarrow (1)$$

* Consider the passive R-C LPF shown below.



* The transfer function is given by:

$$H(s) = \frac{V_{out}}{V_{in}} = \frac{1/sC}{R+\frac{1}{sC}} = \frac{1/\cancel{sC}}{\frac{RsC+1}{\cancel{sC}}}$$

$$H(s) = \frac{1}{1+sCR}$$

$$|H(\omega)| = \frac{1}{\sqrt{1+(\omega CR)^2}}$$

$$\boxed{|H(\omega)|^2 = \frac{1}{1+(\omega CR)^2}} \longrightarrow (2)$$

Substituting eq (2) in eq (1), we get

$$S_{no} = \frac{1}{1+(\omega CR)^2} \cdot KT$$

$$\boxed{S_{no} = \frac{KT}{1+(\omega CR)^2}} \longrightarrow (3)$$

* The o/p Spectrum density, S_{no} decreases as the frequency increases. The total noise power is obtained by integrating S_{no} over the frequency range from 0 to ∞.

i.e. $P_{no} = \int_0^\infty \underline{S_{no}}\, df \longrightarrow (4)$

Substituting eq (3) in eq (4), we get

$$P_{no} = \int_0^\infty \frac{KT}{1+(\omega CR)^2} \cdot df$$

WKT $\boxed{\omega = 2\pi f}$

$$P_{no} = \int_0^{\pm} \frac{KT}{1+(\underline{2\pi f}RC)}\, df$$

Let $2\pi f RC = x \rightarrow$ ⓐ

differentiating eq ⓐ w.r.to 'f'

$$\frac{df}{dx} 2\pi RC = (1)$$

$$df = \frac{dx}{2\pi RC}$$

The limits remain unchanged

$$P_{no} = \int_0^{\infty} \frac{KT}{1+x^2} \cdot \frac{dx}{2\pi RC}$$

$$P_{no} = \frac{KT}{2\pi RC} \int_0^{\infty} \frac{dx}{1+x^2}$$

$$= \frac{KT}{2\pi RC} \int_0^{\infty} \frac{1}{1+x^2} dx$$

$$= \frac{KT}{2\pi RC} \left[\tan^{-1} x \right]_0^{\infty}$$

$$= \frac{KT}{2\pi RC} \left[\tan^{-1}(\infty) - \tan^{-1}(0) \right]$$

$$P_{no} = \frac{KT}{2\cancel{\pi} RC} \left[\frac{\cancel{\pi}}{2} - 0 \right] = \frac{KT}{2RC} \cdot \frac{1}{2}$$

∴ The total noise power at the o/p is

$$\boxed{P_{no} = \frac{KT}{4RC}} \longrightarrow (5)$$

Comparing eq (5) with $P_n = KTB_N$, we get

Effective Noise BW as $\boxed{B_N = \frac{1}{4RC}} \longrightarrow (6)$

* The rms noise voltage, V_N will be given by

$$V_N^2 = 4KTB_N \cdot R \longrightarrow (7)$$

Substituting eq ⑥ in eq ⑦, we get

$$V_N^2 = \cancel{4}KT\left(\frac{1}{\cancel{4R}C}\right)\cdot\cancel{R}$$

$$\boxed{V_N^2 = \frac{KT}{C}}$$

* Although the capacitance 'C' does not contribute to the noise, it acts as a limiting factor to the rms noise voltage.

---

1) A signal circuit is equivalent to a parallel combination of R = 1KΩ & C = 0.47 μF. Calculate the effective noise bandwidth.

Sol :- Effective bandwidth

$$B_N = \frac{1}{4RC} = \frac{1}{4\times1\times10^{3}\times0.47\times10^{-6}} = \underline{531.915\ Hz}$$

Signal to Noise Ratio :- (SNR)

* Signal to Noise ratio is defined as the ratio of signal power to Noise power.

$$(SNR) = \frac{S}{N} = \frac{P_s}{P_n}$$

$$= \frac{V_s^2/\cancel{R}}{V_N^2/\cancel{R}}$$

$$\boxed{\frac{S}{N} = \left(\frac{V_s}{V_N}\right)^2}$$

* The Signal to Noise ratio interms of decibels :

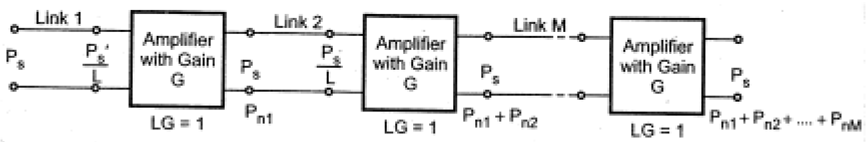
$$\left(\frac{S}{N}\right)_{dB} = 10\log\left(\frac{V_s}{V_N}\right)^2$$

$$\boxed{\left(\frac{S}{N}\right)_{dB} = 20 \log\left(\frac{V_S}{V_N}\right)}$$

* Signal to Noise ratio of a Tandem Connection :-

▶ **Figure**

Tandem connection



* In telephone systems, telephone cables are used as media to transmit signals. The signals gets attenuated as it travels through telephone cables due to power loss in the telephone cables. To make up this power loss the signal is amplified such that, if the power loss of a line section is 'L', then the amplified power gain 'G' is chosen so that $LG = 1$.

* A long telephone line is divided into equal sections called links.

* As signals travel through these links, each amplifier adds its own noise to the system.
Therefore at the receiving end we get the accumulated Noise power as shown in fig above.

* The total Noise power at the o/p of the M^{th} link is

$$P_n = P_{n1} + P_{n2} + P_{n3} + \dots + P_{nM}$$

Where,

P_{n1} = Noise power added at the end of 1^{st} link

P_{n2} = Noise power added at the end of 2^{nd} link.

P_{n3} = Noise power added at the end of 3^{rd} link.

⋮

P_{nM} = Noise power added at the end of M^{th} link.

* If links are identical such that each link adds Noise power 'P_n' then the total Noise power is given as:

$$\boxed{P_{n\,total} = M \times P_n}$$

∴ The o/p Signal to Noise ratio is:

$$\left(\frac{S}{N}\right)_{M\,dB} = 10\log\left(\frac{P_s}{P_{n\text{-}total}}\right)$$

$$= 10\log\left(\frac{P_s}{MP_n}\right)$$

$$= 10\log\left(\frac{P_s}{P_n}\right) - 10\log(M)$$

$$\boxed{\left(\frac{S}{N}\right)_{M\,dB} = \left(\frac{S}{N}\right)_{1\,dB} - (M)_{dB}}$$

Where,

$(M)_{dB} \rightarrow$ Signal to Noise ratio at the end of M- links

$\left(\frac{S}{N}\right)_{1\,dB} \rightarrow$ Signal to Noise ratio at the end of 1^{st} link.

1) Calculate the o/p Signal to Noise ratio in decibels for Four identical links. Assume that Signal to Noise of each link is 80 dB.

Sol:- Given : $\left(\frac{S}{N}\right)_{dB} = 80\,dB$, $M = 4$.

$$(M)_{dB} = 10\log(M)$$
$$(M)_{dB} = 10\log(4) = 6.02\,dB$$

$$\left(\frac{S}{N}\right)_{M\,dB} = \left(\frac{S}{N}\right)_{1\,dB} - (M)_{dB}$$

$$= 80\,dB - 6.02\,dB$$

$$\boxed{\left(\frac{S}{N}\right)_{M\,dB} = 73.98\,dB}$$

Noise Factor :-

* The Noise factor 'F' of an amplifier or any Network is defined in terms of Signal to Noise ratio is defined as:

$$\text{Noise factor, } F = \frac{\text{available S/N power ratio at the I/p}}{\text{available S/N power ratio at the o/p}} = \frac{(SNR)_i}{(SNR)_o}$$

$$F = \frac{P_{si}/P_{ni}}{P_{so}/P_{no}}$$

$$\boxed{F = \frac{P_{si}}{P_{ni}} \times \frac{P_{no}}{P_{so}}} \longrightarrow (1)$$

{* In general any amplifier will add Noise to the I/p Signal, therefore the SNR at the o/p of the amplifier is less than the SNR at the I/p. Hence the Noise factor is a measure of degradation of the Signal to Noise ratio or the amount of noise added by the S/M}

* The available power gain 'G' is given by

$$G = \frac{\text{Signal power at the o/p}}{\text{Signal power at the I/p}}$$

$$\boxed{G = \frac{P_{so}}{P_{si}}} \longrightarrow (2)$$

From eq (1), we can re-arrange

$$F = \left(\frac{P_{si}}{P_{so}}\right) \times \frac{P_{no}}{P_{ni}} \longrightarrow (3)$$

Substituting eq (2) in eq (3), we get

$$F = \frac{1}{G} \cdot \frac{P_{no}}{P_{ni}}$$

$$F = \frac{P_{no}}{G P_{ni}}$$

$$P_{no} = F G P_{ni}$$

W.K.T the Noise power at I/p, $P_{ni} = KTB_N$

$$\boxed{P_{no} = FGKTB_N}$$

- Thus with increase in the Noise factor 'F', the noise power at the o/p will increase.

NOISE Figure :-

* When noise factor is expressed in decibels, it is called Noise figure.

$$\text{Noise Figure} = 10 \log_{10}(F)$$

$$= 10 \log_{10}\left[\frac{S/N \text{ at the I/p } ``(S/N)_i"}{S/N \text{ at the o/p } ``(S/N)_o"}\right]$$

$$= 10 \log_{10}\left[\frac{(S/N)_i}{(S/N)_o}\right]$$

$$\boxed{\text{Noise Figure } (F)_{dB} = 10 \log_{10}(S/N)_i - 10 \log_{10}(S/N)_o}$$

* The Ideal value of Noise figure is 0 dB.

---

1) The Signal power & Noise power measured at the I/p of an amplifier are 150 μw & 1.5 μw respectively. If the Signal power at the o/p 1.5W & Noise power is 40mW, calculate the amplifier noise factor & Noise figure.

Sol :- Given ; $P_{Si} = 150\mu w$, $P_{ni} = 1.5\mu w$, $P_{So} = 1.5W$, $P_{no} = 40mw$.

* Noise Factor 'F' $= \frac{P_{Si}}{P_{ni}} \times \frac{P_{no}}{P_{So}}$

$$= \frac{150 \times 10^{-6}}{1.5 \times 10^{-6}} \times \frac{40 \times 10^{-3}}{1.5}$$

$$\boxed{F = 2.666}$$

* Noise Figure $(F)_{dB} = 10 \log_{10}(F) = 10 \log_{10}(2.666)$

$$\boxed{(F)_{dB} = 4.26 \text{ dB}}$$

3) The Signal to Noise ratio at the I/p of an amplifier is 40 dB. If the Noise Figure of an amplifier is 20 dB, Calculate the Signal to Noise ratio in dB at the amplifier o/p.

Sol :- Given : $(S/N)_i = 40\,dB$, $(S/N)_o = ?$, $(S/N)_o = 20\,dB$

W.K.T Noise Figure $(F)_{dB} = (S/N)_{i\,dB} - (S/N)_{o\,dB}$

$$(S/N)_{o\,dB} = (S/N)_{i\,dB} - (F)_{dB}$$

$$= 40\,dB - 20\,dB$$

$$\boxed{(S/N)_{o\,dB} = 20\,dB}$$

Amplifier I/p Noise interms of 'F' (P_{na}) :-

The total noise at the I/p of the amplifier is given by:

$$\boxed{\text{Total } P_{ni} = \frac{P_{no}}{G}} \longrightarrow (1)$$

W.K.T $P_{no} = FGKTB_N$.

Substituting 'P_{no}' value in eq (1), we get

$$\text{Total } P_{ni} = \frac{F\cancel{G}KTB_N}{\cancel{G}}$$

$$\therefore \boxed{\text{Total } P_{ni} = FKTB_N}$$

* out of this total I/p noise power, the I/p Source Contribution is only KTB_N & the remaining is contributed by the amplifier :

$$P_{ni(total)} = P_{ni} + P_{na}$$

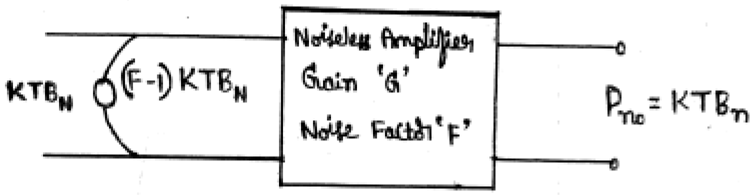
$$P_{na} = P_{ni(total)} - P_{ni}$$

$$P_{na} = FKTB_N - KTB_N$$

$$P_{na} = KTB_N(F-1)$$

$$\boxed{P_{na} = (F-1)\,KTB_N}$$

This can be Shown in below figure:



$\therefore$ The fraction of total available noise Contributed by the amplifier

$$\frac{(F-1)\,\cancel{KTB_N}}{(F)\,\cancel{KTB_N}} = \frac{(F-1)}{F}$$

Equivalent Noise Temperature at Amplifier I/P:-

Jan-06, 4M
July-09, 4M

* W.K.T, the noise power due to amplifier, having a noise factor 'F' is

$$\boxed{P_{na} = (F-1)\,KTB_N} \longrightarrow (1)$$

* If 'T_e' represents the equivalent noise temperature representing noise Power, then

$$\boxed{P_{na} = KT_eB_N} \longrightarrow (2)$$

Equating eq (1) & (2), we get

$$\cancel{K}T_e\cancel{B_N} = (F-1)\cancel{K}T\cancel{B_N}$$

$$\boxed{T_e = (F-1)\,T} \longrightarrow (3)$$

$$(F-1) = \frac{T_e}{T}$$

$$\boxed{F = \frac{T_e}{T} + 1}$$

- Noise Temperature of Cascaded N/W :-

1) Derive an expression for overall Equivalent Noise temperature of the cascade connection of any number of noises for two port N/w

July-09, 5M

* It is possible to develop an expression for the overall Noise temperature using Friis formula i.e.

$$F = F_1 + \frac{F_2-1}{G_1} + \frac{F_3-1}{G_1G_2} + \dots \longrightarrow (1)$$

Subtract 1 from both Sides of eq ①, we get

$$F - 1 = F_1 - 1 + \frac{F_2 - 1}{G_1} + \frac{F_3 - 1}{G_1 G_2} + \cdots$$

* If 'T_e' is overall equivalent Noise temperature of the Cascade, While T_{e1}, T_{e2}, ... are corresponding values for each amplifier in Cascade, then from eq ③ "$\frac{T_e}{T} = (F-1)$", we have

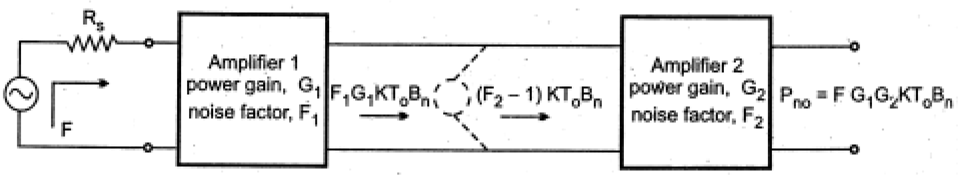
$$\frac{T_e}{T} = \frac{T_{e1}}{T} + \frac{T_{e2}/T}{G_1} + \frac{T_{e3}/T}{G_1 G_2} + \cdots$$

$$\boxed{T_e = T_{e1} + \frac{T_{e2}}{G_1} + \frac{T_{e3}}{G_1 G_2} + \cdots}$$

Noise Factor of amplifier in Cascade :-

▶ **Figure**

Noise factor of two amplifiers in cascade



* Consider two amplifiers connected in Cascade as Shown above. The available Noise power at the o/p of 1st amplifier is

$$\boxed{P_{no1} = F_1 G_1 K T_o B_N} \longrightarrow ①$$

* This is available to the 2nd amplifier & 2nd amplifier has noise $(F_2-1)KTB_N$ of its own at its I/p of the 2nd amplifier is

$$\boxed{P_{ni2} = F_1 G_1 K T B_N + (F_2 - 1) K T B_N} \longrightarrow ②$$

* Consider 2^{nd} amplifier as a Noiseless amplifier with amplifier gain 'G_2'

We have

$$\boxed{P_{no2} = G_2 P_{ni2}} \longrightarrow (3)$$

Substituting eq (2) in eq (3), we get

$$P_{no2} = G_2 \left[F_1 G_1 KTB_N + (F_2 - 1) KTB_N \right] \longrightarrow (4)$$

* WKT, the overall voltage gain of the two amplifiers in cascade is

$$G = G_1 G_2 \;\&$$

* From figure, the overall Noise power is

$$\boxed{P_{no} = F G_1 G_2 KTB_N} \longrightarrow (5)$$

* Equating eq (4) & (5), we get

$$P_{no} = P_{no2}$$

$$F \underbrace{G_1 G_2 KTB_N} = G_2 \left[F_1 G_1 KTB_N + (F_2 - 1) KTB_N \right]$$

$$F = \frac{F_1 G_1 G_2 KTB_N + (F_2 - 1) G_2 KTB_N}{G_1 G_2 KTB_N}$$

$$F = \frac{\cancel{F_1 G_1 G_2 KTB_N}}{\cancel{G_1 G_2 KTB_N}} + \frac{(F_2 - 1) \cancel{G_2 KTB_N}}{G_1 \cancel{G_2 KTB_N}}$$

$$\boxed{F = F_1 + \frac{(F_2 - 1)}{G_1}}$$

By having G_1 large, the noise contribution of the 2^{nd} stage can be made negligible.

* For Multistage amplifier

$$\boxed{F = F_1 + \frac{(F_2-1)}{G_1} + \frac{(F_3-1)}{G_1 G_2} + \cdots} \longrightarrow (7)$$

Equation (7) is Known as "Friis" Formula.

NOTE :-

For 4- Stage amplifier

$$\boxed{F = F_1 + \frac{(F_2-1)}{G_1} + \frac{(F_3-1)}{G_1 G_2} + \frac{(F_4-1)}{G_1 G_2 G_3}}$$

Important Formulae

Sr. No.			Expression
1.	Speed of light	:	$C = \lambda \times f$
2.	Thermal noise power	:	$P_n = kTB$
3.	Shot noise	:	$I_n^2 = 2(I + 2I_o)\,qB$
4.	Signal to noise ratio	:	$\frac{S}{N} = \left[\frac{V_s}{V_n}\right]^2$
5.	Noise factor	:	$F = \frac{S/N \text{ ratio at the input}}{S/N \text{ ratio at the output}}$
6.	Noise figure	:	$F_{dB} = 10 \log_{10}$ (Noise factor)
7.	Noise temperature	:	$T_e = (F - 1)\,T_o$
8.	Thermal noise voltage	:	$V_n = \sqrt{4\,k\,TBR}$
9.	Noise voltage for resistors in series	:	$V_n = \left[V_{n1}^2 + V_{n2}^2\right]^{1/2}$ and $R = R_1 + R_2$
10.	Total noise voltage for resistors in series	:	$V_n = 4\,k\,TB\,R_P$ where $R_P = R_1 \parallel R_2$
11.	Noise power contributed by an amplifier	:	$P_{na} = (F - 1)\,k\,T_o\,B$
12.	Friiss formula	:	$F = F_1 + \frac{(F_2 - 1)}{G_1} + \frac{(F_3 - 1)}{G_1 G_2} + \ldots$
13.	Friiss formula for noise temperature	:	$T_{eq.} = T_{eq.1} + \frac{T_{eq2}}{G_1} + \frac{T_{eq3}}{G_1 G_2} \ldots$

UNIT – 1

RANDOM PROCESS: Random variables: Several random variables. Statistical averages: Function of Random variables, moments, Mean, Correlation and Covariance function: Principles of autocorrelation function, cross – correlation functions. Central limit theorem, Properties of Gaussian
process. **7 Hrs**

TEXT BOOKS:

1. **Communication Systems**, Simon Haykins, 5th Edition, John Willey, India Pvt. Ltd, 2009.

2. **An Introduction to Analog and Digital Communication**, Simon Haykins, John Wiley India Pvt. Ltd., 2008

REFERENCE BOOKS:

1. **Modern digital and analog Communication systems** B. P. Lathi, Oxford University Press., 4th ed, 2010

2. **Communication Systems**, Harold P.E, Stern Samy and A Mahmond, Pearson Edn, 2004.

3. **Communication Systems**: Singh and Sapre: Analog and digital TMH 2nd , Ed 2007.

1.1. Random Process

A random process is a signal that takes on values, which are determined (at least in part) by chance. A sinusoid with amplitude that is given by a random variable is an example of a random process. A random process cannot be predicted precisely. However, a deterministic signal is completely predictable.

An ergodic process is one in which time averages may be used to replace ensemble averages. As signals are often functions of time in signal processing applications, egodicity is a useful property. An example of an ensemble average is the mean win at the blackjack tables across a whole casino, in one day. A similar time average could be the mean win at a particular blackjack table over every day for a month, for example. If these averages were approximately the same, then the process of blackjack winning would appear to be ergodic. Ergodicity can be difficult to prove or demonstrate, hence it is often simply assumed.

1.2. Probabilistic Description of a Random Process

Although random processes are governed by chance, more typical values and trends in the signal value can be described. A probability density function can be used to describe the typical intensities of the random signal (over all time). An autocorrelation function describes how similar the signal values are expected to be at two successive time instances. It can distinguish between 'erratic' signals versus a 'lazy random walk'.

1.3. Autocorrelation

$$R_x(t_1, t_2) = E[X(t_1)X(t_2)]$$

If the autocorrelation for a process depends only on the time difference t2 – t1, then the random process is deemed 'wide sense stationary'. For a WSS process:

$$R_x(\tau) = E[X(t)\bar{X}(t+\tau)]$$

For a WSS, ergodic, process autocorrelation may be evaluated via a time average

$$R_x(\tau) = \lim_{T\to\infty} \frac{1}{T}\int_0^T X(t)X(t+\tau)dt$$

(Autocorrelation function, for continuous-time)

$$R_x(m) = \lim_{N\to\infty} \frac{1}{N}\sum_{k=1}^{N} X(k)X(k+m)$$

(Autocorrelation sequence, for discrete-time)

Autocorrelation properties of a stationary process:

- $R_x(0)$ is the mean-square value of the process. (Compare to definition of E[X²]).
- $R_x(\tau)$ is an even function.
- $|R_x(\tau)| \le R_x(0)$ for all τ.
- If $X(t)$ is periodic then $R_x(\tau)$ is also periodic.

Crosscorrelation Function for a Stationary Process

$$R_{xy}(\tau) = E[X(t)Y(t+\tau)]$$

- Properties: $R_{xy}(\tau) = R_{yx}(-\tau)$. Also peak doesn't necessarily occur at $\tau = 0$.

1.4. Power Spectral Density

For a stationary process, the Wiener-Khinchine relation states:

$$S_x(j\omega) = \Im[R_x(\tau)]$$

Where denotes Fourier transform. can be thought of as the 'expected power' of the random process. This is because

$$S_x(j\omega) = \lim_{T\to\infty} E\left[\frac{1}{T}|\Im[X_T(t)]|^2\right]$$

Where is a truncated version of the random process. The quantity

$$\frac{1}{T}|\Im[X_T(t)]|^2$$

is known as the periodogram of X(t). The PSD may be estimated computationally, by taking the average of many examples of the power spectrum, over a relatively long intervals of the process X(t). This reveals the expected power density at various frequencies. (One such interval of X(t) would not typically have the power density of the time averaged version).

- Properties: $S_x(j\omega)$ is real, symmetric and nonnegative.

Cross Spectral Density for a Stationary Process

$$S_{xy}(j\omega) = \Im[R_{xy}(\tau)]$$

- Properties: $S_{xy}(j\omega) = S^*_{yx}(j\omega)$. Cross spectral densities are not typically real.

Common Random Processes and Sequences

Gauss-Markov Random Process (Continuous)

- Probability Density Function: Gaussian, with zero mean and variance σ^2.
- $R_x(\tau) = \sigma^2 e^{-\beta|\tau|}$ $S_x(j\omega) = 2\sigma^2\beta/(\omega^2 + \beta^2)$
- Stationary and physically realizable.
- Note: Particularly useful because it is simple and provides a reasonably accurate model for a wide range of physical processes.

Gaussian-White Sequence (Discrete)

- Probability Density Function: Gaussian, with zero mean and variance σ^2.
- $R_x(m) = \sigma^2\delta(m)$ Samples of (discrete) spectrum have magnitude σ^2. Spectrum is band-limited due to the Fs sample rate.
- Stationary and physically realizable.

Gaussian-White Process (Continuous)

- Probability Density Function: Gaussian, with zero mean and infinite variance.
- $R_x(\tau) = A\delta(\tau)$ $S_x(j\omega) = A$
- Stationary and **not** physically realizable. (However still useful in analyses).
- Signal changes "infinitely far, infinitely fast"

Band-limited White Process (Continuous)

- Probability Density Function: Unspecified – can vary. $\sigma^2 = 2WA$
- $R_x(\tau) = 2WA\sin(2\pi W\tau)/(2\pi W\tau)$ $S_x(j\omega)$ is a rectangular pulse with amplitude A and bandwidth 2 pi W (W is in Hertz)
- Stationary and physically realizable.

Weiner Process ("Random Walk" or "Brownian Motion") (Continuous)

Defined as $X(t) = \int_0^t n(u)du$ where n() is a Gaussian-White Process.

- *Non-stationary*, with $E[X(t=t_1)] = 0$ and $E[X^2(t=t_1)] = t_1$.
- Probability Density Function: Gaussian, with zero mean and $\sigma^2 = t$.
- Autocorrelation Function: $R_x(t_1,t_2) = \min(t_1,t_2)$

Estimating Autocorrelation from Real Data

An autocorrelation sequence may be estimated from N samples of a discrete-time process X(k) via

$$R_x(m) \approx V_x(m) = \frac{1}{N}\sum_{k=1}^{N} X(k)X(k+m)$$

This is an unbiased estimate. The variance of an estimate $V_x(\tau)$ for a continuous-time process $X_T(t)$ of duration T is, in general,

$$Variance\{V_x(\tau)\} = \frac{4}{T}\int_0^{\infty} R_x^2(\tau)d\tau$$

For a Gauss-Markov Process this is

$$Variance\{V_x(\tau)\} = \frac{2\sigma^4}{\beta T}$$

Description of a Random Process Using a Multivariable Probability Density Function

Another way to describe changes in a random process is to use a multivariable PDF with values of the process, X(t), at successive time instances. For example the PDF for $f_r(r)$ could be defined for a Gauss-Markov process with

$$r = \begin{bmatrix} x_0 = X(t) \\ x_1 = X(t+\Delta T) \\ x_2 = X(t+2\Delta T) \end{bmatrix}$$

A multivariable Gaussian PDF then describes how likely a given triplet of values are. The PDF would have a covariance matrix

$$C = \begin{bmatrix} E[x_0^2] & E[x_0 x_1] & E[x_0 x_2] \\ E[x_1 x_0] & E[x_1^2] & E[x_1 x_2] \\ E[x_2 x_0] & E[x_2 x_1] & E[x_2^2] \end{bmatrix}$$

The above E[] may be computed directly from the autocorrelation function of the Gauss-Markov process. The PDF in the above example is $f_{x_1,x_2,x_3}(x_1,x_2,x_3)$. As with any Gaussian process, higher order PDFs such as this may be found directly from the process model.

Recommended questions:

1. Define mean, Auto correlation and auto co variance of a random process
2. Define power spectral density and explain its properties.
3. Explain cross correlation and Auto correlation.
4. State the properties of Gaussian process?
5. Define conditional probability, Ramdom variable amd mean?
6. Explain Joint probability of the events A & B?
7. Explain Conditional probability of the events A & B?
8. Explain the properties Gaussian process?
9. Compare auto correlation and cross correlation functions?