28/4/2021.

Q No 1 $\quad \mu = 2040 \quad \sigma = 60$

1. In a test on 2000 electric bulbs, it was found that the life of a particular make, was normally distributed with an average life of 2040 hours and S.D of 60 hours. estimate the number of bulbs likely to burn for a) more than 2150 hours, b) less than 1950 hours c) more than 1920 hours and but less than 2160 hours. Given $A(1.5) = 0.4332$, $A(1.83) = 0.4664$ and $A(2) = 0.4772$, where $A(z)$ denotes the area under the standard normal curve from 0 to z.

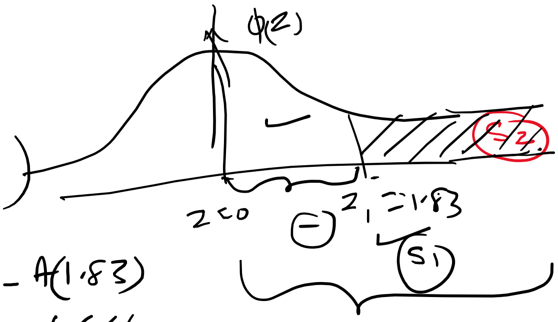
or $A(z) = \frac{1}{\sqrt{2\pi}} \int_0^z e^{-z^2/2}\, dz$.

Solution: Let x denote the life of bulb in a consignment of 2000 bulbs

(a) $P(x > 2150)$

let $z_1 = \frac{x-\mu}{\sigma} = \frac{2150 - 2040}{60} = 1.83$

$\therefore$ To find $P(z_1 > 1.83)$



$S_3 = P(0 \le z \le \infty) - P(0 < z < z_1)$

$S_4 = 0.5 - A(z_1) = 0.5 - A(1.83)$

$= 0.5 - 0.4664$

$= 0.0336$

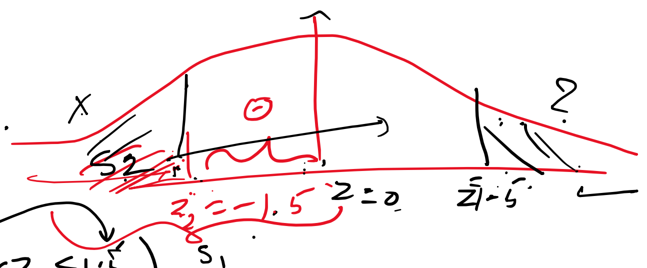
$\therefore$ No. of bulbs likely to burn more than 2150 hours

$= 2000 \times 0.0336 = 67$

(b) To find $P(x < 1950)$

$z_2 = \frac{x-\mu}{\sigma} = \frac{1950 - 2040}{60} = -1.5$

$\therefore P(z_2 < -1.5)$



$S_3 = P(0 \le z \le \infty) - P(0 \le z < 1.5)$

$S_4 = 0.5 - A(1.5) = 0.5 - 0.4332 = 0.0668$

$\therefore$ No. of bulbs which burn for less than 19 50 hours

$= 2000 \times 0.0668 = 134$

(c) $P(\overset{x_3}{\underset{z_3}{1920}} < x < \overset{x_4}{\underset{z_4}{2160}}) = ?$

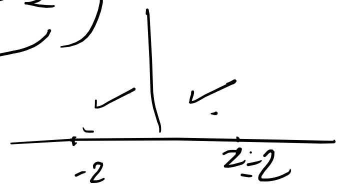
$z_3 = \dfrac{x_3 - \mu}{\sigma} = \dfrac{1920 - 2040}{60} =$

$z_4 = \dfrac{x_4 - \mu}{\sigma} = \dfrac{2160 - 2040}{\sigma} =$

To find $P(z_3 < Z < z_4) = P(-2 < Z < 2)$

$P(|Z| < 2)$

$= 2P(0 < Z < 2)$



$= 2 \times A(2)$

$= 2 \times 0.4772 = 0.9544$

Required number of bulbs $= 2000 \times 0.9544 = 1909$

E2 The mean height of 500 students is 151 cm and the S.D. is 15 cm. assuming that the heights are normally distributed , find how many student's heights lie between 120 and 155 cm. Given $\phi(0.27) = 0.1064$, $\phi(2.07) = 0.4808$, where $\phi(z)$ denotes the area under the Standard normal curve from $z=0$ to $z=Z$

i.e., $\phi(z) = \dfrac{1}{\sqrt{2\pi}} \displaystyle\int_0^z e^{-z^2/2}\, dz.$

ϕ or A or $P(0 < z < 2.07)$

Solution

$\mu = 151\text{ cm}$ $\sigma = 15\text{ cm}$

Let x denote the height of students

To find $\boxed{P(\underset{x_1}{120} < x < \underset{x_2}{155})}$

It is enough to find $P(z_1 < Z < z_2)$

$$Z_1 = \frac{x_1 - \mu}{\sigma} = \frac{120 - 151}{15} = -2.066 = -2.07$$

$$Z_2 = \frac{x_2 - \mu}{\sigma} = \frac{155 - 151}{15} =$$

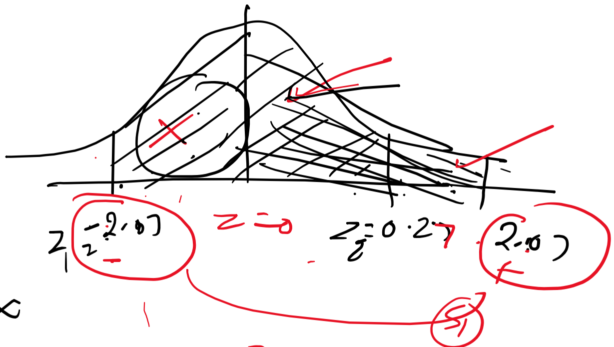
$$Z_2 = 0.266 = 0.27$$

$\therefore P(-2.07 < Z < 0.27)$

(S3) $P(0 < Z < 0.27) + P(0 < Z < 2.07)$

(S4) $= \phi(0.27) + \phi(2.07)$

$= 0.1064 + 0.4808 = 0.5872$

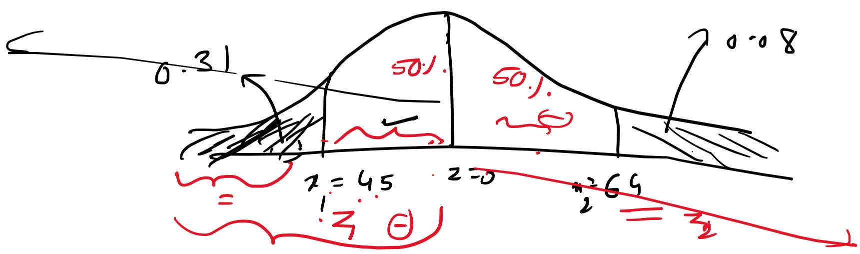


$\therefore$ No. of students whose height lie between 120 cm and 155 cm are $500 \times 0.5872 = 294$

E3 In a normal distribution, 31% of the items are under 45 and 8% are over 64. Find the mean and S.D. of the distribution. Given $A(0.5) = 0.19$ $A(1.4) = 0.42$ where $A(z) = \frac{1}{\sqrt{2\pi}} \int_0^z e^{-z^2/2} \, dz$.

Solution: Given $P(x < 45) = 31\% = 0.31$

$P(x > 64) = 8\% = \frac{8}{100} = 0.08$



$P(z_1 < z < 0)$

Let $x = x_1 = 45$; $z = z_1 = \frac{x_1 - \mu}{\sigma} \Rightarrow z_1 = \frac{45 - \mu}{\sigma} \Rightarrow \mu + z_1\sigma = 45$ —(1)

$x = x_2 = 64$; $z = z_2 = \frac{x_2 - \mu}{\sigma} \Rightarrow z_2 = \frac{64 - \mu}{\sigma} \Rightarrow \mu + z_2\sigma = 64$ —(2)

$\therefore P(z < z_1) = 0.31$

$P(z_1 < z < 0)$

$P(-\infty < z < 0) - P(0 \le z \le z_1) = 0.31$

$\Rightarrow 0.5 - A(z_1) = 0.31$

$\Rightarrow A(z_1) = 0.5 - 0.31 = 0.19$

$\therefore A(z_1) = 0.19$

But $A(0.5) = 0.19$

$z_1 = -0.5$ ($\because z_1$ is negative)

$P(z > z_2) = 0.08$

$P(0 < z < \infty) - P(0 < z < z_2) = 0.08$

$\therefore 0.5 - A(z_2) = 0.08$

$\therefore A(z_2) = 0.5 - 0.08 = 0.42$

But $A(1.4) = 0.42$

$z_2 = 1.4$

(1) and (2) become $\mu - 0.5\sigma = 45$

$\mu + 1.4\sigma = 64$

Solving:- $\mu = 50$ $\sigma = 10$

E4

E4 In a normal distribution, 7% of the items are under 35 and 89% are over 63. Find the mean and S.D. of the distribution. Given A(1.23)=0.39 and A(1.48)=0.43 in the usual notation.

under

under

Solution: Given $P(x < 35) = 7\% = 0.07$

$P(x < 63) = 89\% = 0.89$

Let $x_1 = 35$ $\quad z_1 = \frac{x_1 - \mu}{\sigma} \Rightarrow z_1 = \frac{35-\mu}{\sigma} \Rightarrow \mu + \sigma z_1 = 35$ —(1)

Shading &

$P(Z < z_1)$ $\quad x_2 = 63$, $\quad z_2 = \frac{x_2 - \mu}{\sigma}$ & $z_2 = \frac{63-\mu}{\sigma} \Rightarrow \mu + \sigma z_2 = 63$ —(2)

$\therefore P(-\infty < Z < 0) - P(0 < Z < z_1) = 0.07$

$P(-\infty < Z < 0) + P(0 < Z < z_2) = 0.89$

$\Rightarrow 0.5 + A(z_2) = 0.89$

$\therefore A(z_1) = 0.5 - 0.07 = 0.43$

$\boxed{A(z_2) = 0.89 - 0.5 = 0.39}$

But $A(1.48) = 0.43$

$\therefore z_1 = -1.48$

But $A(1.23) = 0.39$

$z_2 = 1.23$

(1) and (2) becomes

$\mu - 1.48\sigma = 35$

$\mu + 1.23\sigma = 63$

Solving $\mu = 50.29$ $\quad \sigma = 10.33$

E5 In a examination taken by 500 candidates, the average and S.D. of marks obtained (normally distributed) are 40% and 10%. Find approximately (i) how many will pass, if 50% is fixed as a minimum? (ii)What should be the minimum if 350 candidates are to pass? (iii) How many have scored marks above 60%?

Given $A(1) = 0.3413$, $A(2) = 0.4772$ and $A(0.55) = 0.2$

where $\quad A(z) = \frac{1}{\sqrt{2\pi}} \int_0^z e^{-z^2/2}\, dz$.

A sample of 100 dry battery cells tested had a mean of 12 hours and standard deviation of 3 hours. Assuming the data to be normally distributed, find percentage of battery cells have life

(i) more than 15 hours $P(x>15) \rightarrow$ z_1

(ii) less than 6 hours $P(x<6) \rightarrow$ z_2

(iii) between 10 and 14 hours. $(10<x<14) \rightarrow$ z_3 z_4

$\times 100$

(given A(1)=0.3413, A(2)=0.4772 and A(0.67)=0.2487).

$P(0<Z<1) = 0.3413$

Given $\phi(0.67) = 0.2487$, $P(0<z<2) = 0.4772$ and