R-H criteria → special cases

I. First element of a row is zero causing premature termination

Example: consider the ch. equation

$$s^4 + 10s^3 + s^2 + 10s + 20 = 0$$

s^4	1	1	20
s^3	10	10	
s^2	0		
s^1			
s^0			

$b_1 = \frac{10\times1 - 10\times1}{10} = 0$

Routh array terminates prematurely with a zero in the s^2 row

To continue with the array

1st method : Epsilon method

1) Replace zero with a small +ve number ϵ
2) Continue with the array

s^4	1	1	20
s^3	10	10	
s^2	ϵ	20	
s^1	$\frac{-200}{\epsilon}$	0	
s^0	20		

$\frac{10\epsilon - 200}{\epsilon} = \lim_{\epsilon\to0} \frac{10\epsilon - 200}{\epsilon} = -200/\epsilon$

$\frac{\frac{-200}{\epsilon}\times 20 - \epsilon\times 0}{\frac{-200}{\epsilon}}$

There are two sign changes in the first column

∴ There are 2 roots on the RHS of s-plane.

The system is unstable.

<u>2nd Method</u> is to replace s by $\frac{1}{z}$ in the ch. eqn.

(Reverse coefficients method)

$$\frac{1}{z^4} + \frac{10}{z^3} + \frac{1}{z^2} + \frac{10}{z} + 20 = 0$$

$$\frac{1 + 10z + z^2 + 10z^3 + 20z^4}{z^4} = 0$$

$$20z^4 + 10z^3 + z^2 + 10z + 1 = 0$$

z^4	20	1	1
z^3	10	10	
z^2	-19	1	
z^1	$\frac{200}{19}$		
z^0	1		

two sign changes

$$b_1 = \frac{10\times1 - 10\times20}{10}$$

$$c_1 = \frac{-190-10}{-19} = \frac{-200}{-19}$$

→ This also gives same result i.e, there are two sign changes in the first column indicating two roots on RHS of s-plane ∴ system is unstable.

II. One complete row of a Routh array is zero causing premature termination

Example: The ch. eqn of a system is given by

$$s^5 + 4s^4 + 8s^3 + 8s^2 + 7s + 4 = 0$$

s^5	1	8	7
s^4	4	8	4
s^3	6	6	0
s^2	4	4	0
s^1	0	0	
s^0			

$s^3 \rightarrow \frac{4\times8-8\times1}{4}$, $\frac{4\times7-4\times1}{4}$

$s^2 \rightarrow \frac{6\times8-6\times4}{6}$, $\frac{6\times4-4\times0}{6}$

$\frac{6\times4-6\times4}{4} \leftarrow s^1$

A row of zeros is encountered in s^1 row

To continue with the table, an auxillary eqn is formed from the row which is just above the row of zeros

Auxillary eqn is $4s^2 + 4 = 0 = A(s)$

$$\frac{dA(s)}{ds} = \underline{\underline{8s}}$$

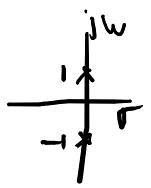
s^5	1	8	7
s^4	4	8	4
s^3	6	6	
s^2	4	4	
s^1	$\underline{\underline{8}}$		
s^0	4		

$$\frac{8\times4 - 4\times0}{8}$$

There are no sign changes in 1^{st} column.

But consider A·E $= 4s^2 + 4 = 0 \Rightarrow s^2 + 1 = 0$

$$s^2 = -1 \qquad s = \pm j1$$



∴ System is Marginally stable.

III : complex conjugate roots with symmetry about origin

Example :

The ch. eqn of a system is given by

$$s^6 + 3s^5 + 4s^4 + 6s^3 + 5s^2 + 3s + 2 = 0$$