# CHAPTER 4

**D4.1.** Given the electric field $\mathbf{E} = (1/z^2)(8xyz\mathbf{a}_x + 4x^2z\mathbf{a}_y - 4x^2y\mathbf{a}_z)$ V/m, find the differential amount of work done in moving a 6-nC charge a distance of 2 μm, starting at $P(2, -2, 3)$ and proceeding in the direction $\mathbf{a}_L =$ (*a*) $-6/7\mathbf{a}_x + 3/7\mathbf{a}_y + 2/7\mathbf{a}_z$; (*b*) $6/7\mathbf{a}_x - 3/7\mathbf{a}_y - 2/7\mathbf{a}_z$; (c) $3/7\mathbf{a}_x + 6/7\mathbf{a}_y$.

(*a*) $dW = -Q\mathbf{E} \cdot d\mathbf{L}$

Finding first the differential length $d\mathbf{L}$,

$$d\mathbf{L} = \mathbf{a}_L \cdot dL = (-6/7\mathbf{a}_x + 3/7\mathbf{a}_y + 2/7\mathbf{a}_z)(2 \times 10^{-6}) = (-12/7\mathbf{a}_x + 6/7\mathbf{a}_y + 4/7\mathbf{a}_z)(1 \times 10^{-6})$$

$$dW = -(6 \times 10^{-9})(1/z^2)(8xyz\mathbf{a}_x + 4x^2z\mathbf{a}_y - 4x^2y\mathbf{a}_z) \cdot (-12/7\mathbf{a}_x + 6/7\mathbf{a}_y + 4/7\mathbf{a}_z)(1 \times 10^{-6})$$

$$= -6 \times 10^{-15}[(1/z^2)((-96/7)xyz + (24/7)x^2z - (16/7)x^2y)]_{x=2,\,y=-2,\,z=3} = -6 \times 10^{-15}(224/9)$$

$$= -149.3 \text{ fJ}$$

$\vec{dL} \to 2$

$dW = -Q\vec{E} \cdot dL$

Put $x = 2$
$y = -2$
$z = 3$

(*b*) Same procedure in (*a*),

$$d\mathbf{L} = (12/7\mathbf{a}_x - 6/7\mathbf{a}_y + 4/7\mathbf{a}_z)(1 \times 10^{-6})$$

$$dW = -(6 \times 10^{-9})(1/z^2)(8xyz\mathbf{a}_x + 4x^2z\mathbf{a}_y - 4x^2y\mathbf{a}_z) \cdot (12/7\mathbf{a}_x - 6/7\mathbf{a}_y + 4/7\mathbf{a}_z)(1 \times 10^{-6})$$

$$= -6 \times 10^{-15}[(1/z^2)((96/7)xyz - (24/7)x^2z - (16/7)x^2y)]_{x=2,\,y=-2,\,z=3} = -6 \times 10^{-15}(-224/9)$$

$$= 149.3 \text{ fJ}$$

(*c*) Same procedure in (*a*) and (*b*),

$$d\mathbf{L} = (6/7\mathbf{a}_x + 12/7\mathbf{a}_y + 0\mathbf{a}_z)(1 \times 10^{-6})$$

$$dW = -(6 \times 10^{-9})(1/z^2)(8xyz\mathbf{a}_x + 4x^2z\mathbf{a}_y - 4x^2y\mathbf{a}_z) \cdot (6/7\mathbf{a}_x + 12/7\mathbf{a}_y + 0\mathbf{a}_z)(1 \times 10^{-6})$$

$$= -6 \times 10^{-15}[(1/z^2)((48/7)xyz + (48/7)x^2z]_{x=2,\,y=-2,\,z=3} = -6 \times 10^{-15}(0)$$

$$= 0$$

D4.2 Calculate the work done in moving a 4-C charge from B(1, 0, 0) to A(0, 2, 0) along the path

$= 0$

D4.2. Calculate the work done in moving a 4-C charge from $B(1, 0, 0)$ to $A(0, 2, 0)$ along the pa
$y = 2 - 2x$, $z = 0$ in the field $\mathbf{E} =$ (*a*) $5\mathbf{a}_x$ V/m; (*b*) $5x\mathbf{a}_x$ V/m; (*c*) $5x\mathbf{a}_x + 5y\mathbf{a}_y$ V/m. Path

(*a*) $W = -Q\int_B^A \mathbf{E} \bullet d\mathbf{L}$ where $d\mathbf{L} = dx\, \mathbf{a}_x + dy\, \mathbf{a}_y + dz\, \mathbf{a}_z$

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$$= -4\int_1^0 (5\mathbf{a}_x + 0\mathbf{a}_y + 0\mathbf{a}_z) \bullet (dx\,\mathbf{a}_x + dy\,\mathbf{a}_y + dz\,\mathbf{a}_z) = -4\int_1^0 5\, dx = -20x\Big|_1^0 = -20(-1) = 20$$

(*b*) Same procedure in (*a*),

$$W = -4\int_1^0 5x\, dx = -20(x^2/2)\Big|_1^0 = -20(-0.5) = 10\text{ J}$$

(*c*) Same procedure in (*a*) and (*b*),

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$$W = -4\int_{(1,0)}^{(0,2)} (5x\, dx + 5y\, dy) = -20\left(\frac{x^2}{2} + \frac{y^2}{2}\Bigg|_{(1,0)}^{(0,2)}\right) = -20(1.5) = -30\text{ J}$$

5. If the path selected is such that it is forming a **closed contour** i.e. starting point is same as the terminating point then the **work done** is **zero.**

**Example 4.3.1** *An electrostatic field is given by* $\bar{E} = -8xy\,\bar{a}_x - 4x^2\,\bar{a}_y + \bar{a}_z$ *V/m. The charge of 6 C is to be moved from B (1, 8, 5) to A (2, 18, 6). Find the work done in each of the following cases, 1. The path selected is* $y = 3x^2 + z,\ z = x + 4$ *2. The straight line from B to A. Show that work done remains same and is independent of the path selected.*

**Jan.-12, Marks 8**

**Solution :** The work done is given by,

$$W = -Q\int_B^A \bar{E} \bullet d\bar{L}$$

Let us differential length $d\bar{L}$ in cartesian co-ordinate system is,

$$d\bar{L} = dx\,\bar{a}_x + dy\,\bar{a}_y + dz\,\bar{a}_z$$

$$\therefore \quad \bar{E} \bullet d\bar{L} = \left(-8xy\,\bar{a}_x - 4x^2\,\bar{a}_y + \bar{a}_z\right) \bullet \left(dx\,\bar{a}_x + dy\,\bar{a}_y + dz\,\bar{a}_z\right)$$

$$= -8\,xy\,dx - 4x^2\,dy + dz$$

As $\bar{a}_x \bullet \bar{a}_x = \bar{a}_y \bullet \bar{a}_y = \bar{a}_z \bullet \bar{a}_z = 1$, other dot products are zero.

$$\therefore \quad W = -Q\int_B^A -8\,xy\,dx - 4x^2 dy + dz = -Q\left[\int_B^A -8\,xy\,dx - \int_B^A 4\,x^2 dy + \int_B^A dz\right]$$

**Case 1 :** The path is $y = 3x^2 + z,\ z = x+4\ \ y = 3x^2+x+4$ differentiate i.e. $dy = (6x+1)\,dx$

$W = -Q\int_B^A \vec{E}\cdot d\vec{L}$

given

For $\int_B^A -8\,xy\,dx \rightarrow$ The limits are $x = 1$ to $x = 2$.

For $\int_B^A -4x^2 y \rightarrow$ The limits are $y = 8$ to $y = 18$

For $\int_B^A dz \rightarrow$ The limits are $z = 5$ to $z = 6$.

$$\therefore \quad W = -Q\left[\int_{x=1}^{2} -8\,xy\,dx - \int_{y=8}^{18} 4\,x^2 dy + \int_{z=5}^{6} dz\right]$$

Using $y = 3x^2 + x + 4$ and $dy = (6x + 1)\,dx$ and changing limits of y from 8 to 18 interms of x from 1 to 2 we get

$$\therefore \quad W = -Q\left[\int_{x=1}^{2} -8x\left[3x^2+x+4\right]dx - \int_{x=1}^{2} 4\,x^2[6x+1]dx + \int_{z=5}^{6} dz\right]$$

$$= -Q\left[\int_{x=1}^{2}\left[-24x^3 - 8x^2 - 32x\right]dx - \int_{x=1}^{2}\left(24\,x^3 + 4x^2\right)dx + \int_{z=5}^{6} dz\right]$$

$$= -Q\left[\left(-6x^4 - \frac{8}{3}x^3 - 16x^2 - 6x^4 - \frac{4}{3}x^3\right)_{x=1}^{2} + (z)_5^6\right]$$

$$= -Q\{-256+1\} = -6\times -255 = \mathbf{1530\ J}$$

**Case 2 :** Straight line path from B to A.

To obtain the equations of the straight line, any two of the following three equations of planes passing through the line are sufficient,

B (1, 8, 5) and A (2, 18, 6)

$$(y - y_B) = \frac{y_A - y_B}{x_A - x_B}(x - x_B),\quad (z - z_B) = \frac{z_A - z_B}{y_A - y_B}(y - y_B),\quad (x - x_B) = \frac{x_A - x_B}{z_A - z_B}(z - z_B)$$

Using the co-ordinates of A and B,

$$y - 8 = \frac{18-8}{2-1}(x-1) \quad \text{i.e.} \quad y - 8 = 10\,(x - 1)$$

$$\therefore \quad y = 10x - 2 \quad \ldots ($$

$$\therefore \quad dy = 10\,dx$$

And $$z - 5 = \frac{6-5}{18-8}(y-8) \quad \text{i.e.} \quad z - 5 = \frac{1}{10}(y-8)$$

$$\therefore \quad 10\,z = y + 42 \qquad \dots (2)$$

Now $$W = -Q\left[\int_{x=1}^{2} -8\,xy\,dx - \int_{y=8}^{18} 4\,x^2 dy + \int_{z=5}^{6} dz\right]$$

$$= -Q\left[\int_{x=1}^{2} -8\,x(10x-2)\,dx - \int_{x=1}^{2} 4\,x^2(10dx) + \int_{z=5}^{6} dz\right]$$

$$= -Q\left\{\left[\frac{-80}{3}x^3 + \frac{16x^2}{2} - \frac{40x^3}{3}\right]_{x=1}^{2} + [z]_5^6\right\}$$

$$= -Q\{-213.33 + 32 - 106.667 + 26.667 - 8 + 13.33 + 1\} = -Q[-255] = -6\times -255 = \mathbf{1530\ J}$$

This shows that **irrespective of path selected, the work done in moving a charge from B to A remains same.**

**Example 4.3.2** *Consider an infinite line charge along z-axis. Show that the work done is zero*

**Example 4.5.1** *A point charge Q = 0.4 nC is located at the origin. Obtain the absolute potential of A (2, 2, 3).*

**Solution :** The potential of A due to point charge Q at the origin is given by,

$$V_A = \frac{Q}{4\pi\varepsilon_0 r_A} \quad \text{and} \quad A\ (2, 2, 3),\ Q \text{ at } (0, 0, 0)$$

where $r_A = \sqrt{(2-0)^2+(2-0)^2+(3-0)^2} = \sqrt{17}$

$$\therefore \quad V_A = \frac{0.4\times10^{-9}}{4\pi\times8.854\times10^{-12}\times\sqrt{17}} = 0.8719 \text{ V}$$

... The reference is at infinity.

**Example 4.5.2** *If same charge Q = 0.4 nC in above example is located at (2, 3, 3) then obtain the absolute potential of point A (2, 2, 3).*

**Solution :** Now the Q is located at (2, 3, 3).

The potential at A is given by

$$V_A = \frac{Q}{4\pi\varepsilon_0 R_A} \quad \text{where}$$

$$R_A = |r - r'|$$

$$= \sqrt{(2-2)^2 + (2-3)^2 + (3-3)^2} = 1$$

... by distance formula

$$\therefore \quad V_A = \frac{0.4 \times 10^{-9}}{4\pi \times 8.854 \times 10^{-12} \times 1} = 3.595 \text{ V}$$

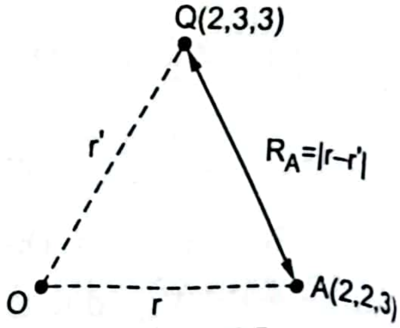


Fig. 4.5.5

**Example 4.5.3** *If the point B is at (– 2, 3, 3) in the above example, obtain the potential difference between the points A and B.*

**Solution :** $V_{AB} = V_A - V_B$

where $V_A$ and $V_B$ are the absolute potentials of A and B.

Now $V_A = 3.595$ V ... As calculated earlier.

$$V_B = \frac{Q}{4\pi\varepsilon_0 R_B}$$ where $R_B$ is distance between point B and Q (2, 3, 3)

$$\therefore \quad R_B = \sqrt{(-2-2)^2 + (3-3)^2 + (3-3)^2} = 4$$

$$\therefore \quad V_B = \frac{0.4 \times 10^{-9}}{4\pi \times 8.854 \times 10^{-12} \times 4} = 0.8987 \text{ V}$$

$$\therefore \quad V_{AB} = V_A - V_B = 3.595 - 0.8987 = \mathbf{2.6962 \text{ V}}$$

**Example 4.5.4** *If three charges, 3 μC, 4 μC and 5 μC are located at (0, 0, 0), (2 – 1, 3) and (0, 4, – 2) respectively. Find the potential at (1, 0, 1) assuming zero potential at infinity.*

**Solution :** Let $Q_1 = 3\ \mu C$, $Q_2 = -4\ \mu C$

and $Q_3 = 5\ \mu C$

The potential of A due to $Q_1$ is,

$$V_{A1} = \frac{Q_1}{4\pi\varepsilon_0 R_1}$$

and $$R_1 = \sqrt{(1-0)^2 + (0-0)^2 + (1-0)^2}$$

$$= \sqrt{2}$$

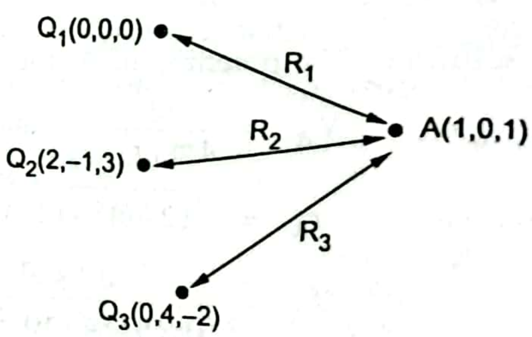


Fig. 4.5.6

$$\therefore \quad V_{A1} = \frac{3 \times 10^{-6}}{4\pi\varepsilon_0 \times \sqrt{2}} = 19.0658 \text{ kV}$$

The potential of A due to $Q_2$ is,

$$V_{A2} = \frac{Q_2}{4\pi\varepsilon_0 R_2}$$

and $$R_2 = \sqrt{(1-2)^2 + [0-(-1)]^2 + (1-3)^2} = \sqrt{6}$$

$\therefore$ $$V_{A2} = \frac{-4\times10^{-6}}{4\pi\varepsilon_0 \times \sqrt{6}} = -14.6769 \text{ kV}$$

The potential of A due to $Q_3$ is,

$$V_{A3} = \frac{Q_3}{4\pi\varepsilon_0 R_3}$$

and $$R_3 = \sqrt{(1-0)^2 + (0-4)^2 + [1-(-2)]^2} = \sqrt{26}$$

$\therefore$ $$V_{A3} = \frac{5\times10^{-6}}{4\pi\varepsilon_0 \times \sqrt{26}} = 8.8132 \text{ kV}$$

$\therefore$ $$V_A = V_{A1} + V_{A2} + V_{A3} = +\mathbf{13.2021\ kV}$$



### 4.5.4 Potential Calculation When Reference is Other Than Infinity