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B. M. S. College of Engineering, Bengaluru - 560019

Autonomous Institute Affiliated to VTU

March - 2021 Semester End Main Examinations

Programme: B.E. **Semester: III**
Branch: CSE/ISE **Duration: 3 hrs.**
Course Code: 19MA3BSSDM **Max Marks: 100**
Course: STATISTICS AND DISCRETE MATHEMATICS **Date: 12.03.2021**

Instructions: 1. **Answer FIVE FULL questions, choosing one from each unit.**
2. **Missing data, if any, may be suitably assumed.**
3. **Use of Statistical tables is permitted.**

UNIT 1

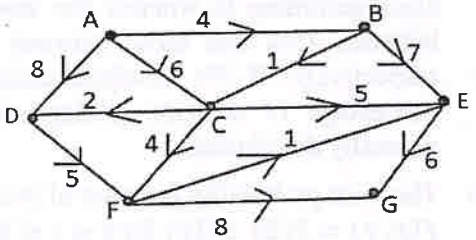
1 a) Determine the order $|V|$ of the graph $G=(V, E)$ in the following cases 6

(i) G is a cubic graph with 9 edges
(ii) G is regular with 15 edges
(iii) G has 10 edges with 2 vertices of degree 4 and all other vertices of degree 3.

b) Write a graph and its adjacency matrix whose incidence matrix is given below. 7

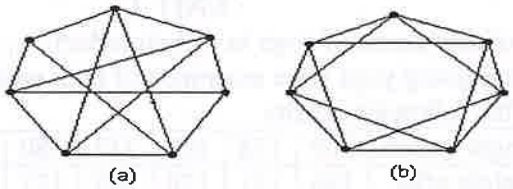
$$A(G) = \begin{array}{c} \\ a \\ b \\ c \\ d \\ e \end{array} \begin{array}{c} \begin{array}{cccccc} e_1 & e_2 & e_3 & e_4 & e_5 & e_6 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

c) Using the Dijkstra's algorithm, find the shortest path and its weight from vertex A to vertex G in the weighted directed network shown below. 7



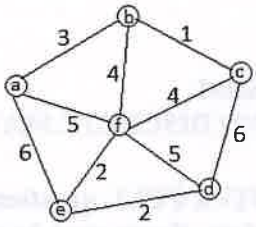
OR

2 a) Define isomorphism of a graph. Verify whether the following graphs are isomorphic. 6



b) Let G be a simple graph with n vertices and m edges where m is atleast3. If $m \geq \frac{(n-1)(n-2)}{2}+2$, prove that G is a Hamilton graph. 7

c) Construct two minimal spanning trees for the following weighted connected graph using Prim's algorithm. 7



UNIT 2

3 a) Define Catalan number. Obtain the number of paths from $(2,1)$ to $(7,6)$ and not rise above the line $y=x-1$ using the moves $R:(x,y)\to(x+1,y)$ and $U:(x,y)\to(x,y+1)$. 6

b) Find the coefficient of: 7

(i) x^9y^3 in the expansion of $(2x-3y)^{12}$

(ii) xyz^2 in the expansion of $(2x-y-z)^4$

c) Find the number of permutations of the letters A, B, C, …, X, Y, Z in which none of the patterns **FILMS, STAR** or **MOVIE** occurs? 7

UNIT 3

4 a) Suppose vehicles arrive at a signalized road intersection at an average rate of 600 per hour and the cycle of the traffic lights is set at 30 seconds. In what percentage of cycles will the number of vehicles arriving be 6

(i) Exactly 6? (ii) Less than 6?

If, after the lights change to green, there is time to clear only 6 vehicles before the signal changes to red again, what is the probability that the waiting vehicles are not cleared in one cycle?

b) A University awards distinction, first class, second class, third class or pass class according to whether the students get 80% or more; 60% or more; between 45% and 60%; between 30% and 45%; 30% or more marks respectively. If 5% obtained distinction and 10% failed, determine the percentage of students getting second class. Assuming the marks are normally distributed. 7

c) The joint probability function of two random variable X and Y is given by $f(x,y)=k(2x+3y)$ for $0\leq x\leq 2;\ 1\leq y\leq 3$. 7

(i) Find the constant k.

(ii) Calculate marginal distribution of X and Y.

(iii) Whether or not X and Y are independent.

UNIT 4

5 a) In a study of usefulness of yoga in weight reduction, a random sample of 8 persons undergoing yoga were examined of their weight before and after yoga with the following results. 6

Weight before	209	178	169	212	180	192	158	180
Weight after	196	171	170	207	177	190	159	180

Test whether yoga is useful in weight reduction at α =1%.

b) In 210 families of females with primary unipolar major depression, they found that alcoholism was present in 89. Of the 299 control families, alcoholism was present in 94. Do these data provide sufficient evidence for us to conclude that alcoholism is more likely to be present in females of subjects with unipolar depression? 7

c) An instructor has two classes, A and B, in a particular subject. Class A has 16 students while class B has 25 students. On the same examination, class A had a standard deviation of 9 while class B had a standard deviation of 12. Can we conclude at 0.01 level of significance that the variability of grades is same for both the classes? 7

OR

6 a) In an ontological examination of schoolchildren, out of 146 children examined 21 were found to have some type of ontological abnormalities. Does it confirm with statement that 20% of the schoolchildren have ontological abnormalities? 6

b) A nutritionist is interested in whether two proposed diets, A and B work equally well in providing weight-loss for customers. In order to assess a difference between the two diets, she puts 50 customers on Diet A and 60 other customers on the Diet B diet for two weeks. Those on the former had weight losses with an average of 11 pounds and a standard deviation of 3 pounds, while those on the latter lost an average of 8 pounds with a standard deviation of 2 pounds. Do the diets differ in terms of their average weight loss? 7

c) The number of cars passing a given point in 100 five second interval was observed as follows. 7

No of Cars	0	1	2	3	4	5
No of intervals	40	23	15	5	7	10

Fit a Poisson distribution and test for its goodness of fit.

UNIT 5

7 a) An auto insurance company classifies its customers in three categories: poor, satisfactory and preferred. No one moves from poor to preferred or from preferred to poor in one year. 40% of the customers in the poor category become satisfactory, 30% of those in the satisfactory category moves to preferred, while 10% become poor; 20% of those in the preferred category are downgraded to satisfactory. 6

(i) Write the transition matrix for the model.

(ii) Is the Markov chain irreducible?

b) A habitual gambler is a member of two clubs A and B. He visits either of the clubs every day for playing cards. He never visits club A on two consecutive days. But, if he visits club B on a particular day, then the next day he is as likely to visit club B or club A. (i) Find the transition matrix of this Markov chain. (ii) If the person had visited club B on Monday, find the probability that he visits club A on Thursday. (iii) In the long run, how often he visits club B. 7

c) At Bharat petrol pump, customers arrive according to a Poisson process with an average time of 5 minutes between arrivals. The service time is exponentially distributed with mean time equal to 2 minutes. On the basis of this information, find out: 7

(i) What would be the average number of customers in the queuing system?

(ii) What is the average time spent by a car in the petrol pump?

(iii) What is the average waiting time of a car before receiving petrol?

(iv) What is the probability that there are 3 customers in the system?