BMS COLLEGE OF ENGINEERING, BENGALURU-19

Autonomous Institute, Affiliated to VTU

DEPARTMENT OF MATHEMATICS

Sem & Branch:	**Fourth Semester (Common to AS/CV/EEE/ECE/EIE/ ML/TCE)**	**Subject:**	**ENGINEERING MATHEMATICS - 4**	**Sub Code:**	**19MA4BSEM4**
Duration	**75 MINUTES**	**Test Date:**	**17.05.2 021 (Monday)**	**Max Marks:**	40

Test No.	Q. No.	**ANSWER ALL QUESTIONS** (Questions in Part-A and Part-B are compulsory. Internal choice is provided in Part C)	Marks	CO
		PART-A		
Test -1	1	Obtain mean and standard deviation expression for the Poisson Probability distribution. obtaining, mean = μ — 2 variance = μ — 2 standard deviation = $\sqrt{\mu}$ — 1	5	1
		PART-B		
	2	(a) The regression equations of two variables x and y are $x = 0.7y + 5.2$, $y = 0.3x + 2.8$. Find the means of the variables and the coefficient of correlation between them. obtaining, $\bar{x} = 8.86$, $\bar{y} = 5.518$ — 2 $r = \sqrt{(\text{coefficient of } x)(\text{co-efficient of } y)}$ $= \sqrt{(0.3)(0.7)}$ — 3	5	1
	2	(b) A car hire firm has two cars, which it hires out day by day. The demand for a car on each day is distributed as a Poisson distribution with mean 1.5. Calculate the probability that on a certain randomly chosen day (i) neither car is used (ii) some demand is refused. $P(\mu, x) = \frac{e^{-\mu}\mu^{x}}{x!}$, $\mu = 1.5$ — 1 (1) $P(x=0) = 0.2231$ — 2 (2) P(of demand refused) = $P(x>2) = 1 - P(x \le 2)$ $= 1 - [P(0) + P(1) + P(2)]$ $= 0.1913$ — 2	5	1

2.	**(c)** The table below shows the joint probability distribution of two random variables X and Y.		

X \ Y	0	1	2	3
0	0	k	2k	3k
1	2k	3k	4k	5k
2	4k	5k	6k	7k

(i) Find k.
(ii) Calculate the marginal distributions of X and Y.
(iii) Verify whether X and Y are independent.

5 1

① $42K = 1 \Rightarrow K = \frac{1}{42}$ — ②

② x_i: 0 1 2 y_j: 0 1 2 3
$f(x_i)$: $\frac{1}{7}$ $\frac{1}{3}$ $\frac{11}{21}$ $g(y_j)$: $\frac{1}{7}$ $\frac{3}{14}$ $\frac{2}{7}$ $\frac{5}{14}$ — ②

③ $f(x_i)\cdot g(y_j) \neq J_{ij}$ — ①
$\therefore$ X and Y are dependent.

PART-C

3 (a) Fit a least squares curve of the form $y = a + bx^2$ for the following data

x	1	2.5	3.5	4.0
y	3.8	15.0	26.0	33.0

Put $x^2 = X \Rightarrow y = a + bX$, $\Sigma y = na + b\Sigma X$, $\Sigma Xy = a\Sigma X + b\Sigma X^2$ — ①

$n = 4$, $\Sigma x = 11$, $\Sigma y = 77.8$, $\Sigma X = 35.5$, $\Sigma X^2 = 446.725$
$\Sigma Xy = 944.05$ — ③

On solving normal equations, we get
$a = 2.278$, $b = 1.93$
$\therefore y = 2.278 + (1.93)x^2$ — ②

3 6 1

OR

3 (b) Obtain the lines of regression and hence find the coefficient of correlation for the data.

x	1	3	4	2	5
y	8	6	10	8	12

$\bar{x} = \frac{\Sigma x}{n} = 3$, $\bar{y} = \frac{\Sigma y}{n} = 8.8$, $X = x - \bar{x} = x - 3$, $Y = y - \bar{y} = y - 8.8$ — ①

$\Sigma X^2 = 10$, $\Sigma Y^2 = 20.8$, $\Sigma XY = 10$ — ②

$Y = \frac{\Sigma XY}{\Sigma X^2} X \Rightarrow y = x + 5.8$ — ①

$X = \frac{\Sigma XY}{\Sigma Y^2} Y \Rightarrow x = 0.48y - 1.23$ — ①

$r = \sqrt{(1)(0.48)} = 0.69$ — ①

4 | (a) The number of accidents per day (x) as recorded in a textile industry over a period of 400 days is given below. Test whether x can be Poisson variable by obtaining the theoretical frequencies or otherwise.

x	0	1	2	3	4	5
f	173	168	37	18	3	1

$\mu = \bar{x} = \frac{\Sigma fx}{\Sigma f} = \frac{313}{400} = 0.7825$ —— 1

The poisson distribution of fit $= N \cdot \frac{e^{-\mu} \cdot \mu^x}{x!} = \frac{400 \cdot e^{-0.7825} (0.7825)^x}{x!}$ → 2

For $x = 0, 1, 2, 3, 4, 5$, we get
183, 143, 56, 15, 3, 0 as the required theoretical frequencies. } → 3

As the sum of these $= 400 \Rightarrow x$ is Poisson variate — 1

OR

(b) In a normal distribution, 7% of the items are under 35 and 89% are under 63. Find the mean and standard deviation. Given that $A(1.23) = 0.39$ and $A(1.48) = 0.43$.
4 Where, $A(z)$ is an area bounded by standard normal curve from 0 to z. 7 1

$P(x < 35) = 0.07$.

$t = \frac{x-\mu}{\sigma} = \frac{35-\mu}{\sigma} = t_1$ — 1 Equation —— 1 Mark

i.e. $P(t < t_1) = 0.07 \Rightarrow 0.5 - \phi(t_1) = 0.07 \Rightarrow t_1 = -1.48$ —— 1 Mark

$P(x < 63) = 0.89$. —— 1 Mark

$t = \frac{63-\mu}{\sigma} = t_2$ — 2 Equation.

i.e. $P(t < t_2) = 0.89 \Rightarrow P(0 < t < \infty) - P(0 < t < t_2)$

$\therefore P(t < t_2) = 0.89 = 0.5 - \phi(t_2)$

$\therefore \phi(t_2) = 0.39 \Rightarrow t_2 = 1.23$ —— 1 Mark

on solving equations 1 and 2, we get

$35 - \mu = -1.48\sigma$ and $63 - \mu = 1.23\sigma$ — 1 Mark

$\mu = 50.29, \quad \sigma = 10.33$ —— 2 Marks

5	**(a)** The joint distribution of two random variables X and Y are given below:		

Y \ X	-2	-1	4	5
1	0.1	0.2	0	0.3
2	0.2	0.1	0.1	0

Find the marginal distributions of X and Y. Also determine μ_x, μ_y and the correlation of X and Y.

x_i : 1 2 y_j : -2 -1 4 5 — (2)

$f(x_i)$: 0.6 0.4 $g(y_j)$: 0.3 0.3 0.1 0.3

$E(X) = 1.4$, $E(Y) = 1.0$, $E(XY) = 0.9$ — (1)

$Cov(X,Y) = E(XY) - E(X).E(Y) = -0.5$ — (2)

$\sigma_X^2 = E(X^2) - [E(X)]^2 = 2.2 - 1.96 = 0.24$

$\sigma_Y^2 = E(Y^2) - [E(Y)]^2 = 10.6 - 1 = 9.6$

$\sigma_X = 0.48$, $\sigma_Y = 3.09$. — (1)

$\rho(X,Y) = \dfrac{Cov(X,Y)}{\sigma_X \sigma_Y} = -0.337$ — (1)

5 7 1

OR

5 **(b)** A coin is tossed three times. Let X be equal to 0 or 1 according as a head or tail occurs on the first toss. Let Y be equal to the total number of heads which occur. Determine (i) the marginal distributions of X and Y. (ii) the joint distributions of X and Y, (iii) $Cov(X,Y)$.

$S = \{HHH, HHT, HTH, HTT, TTT, TTH, THT, THH\}$, $o(S) = 8$ and probability is $1/8$.

x_i : 0 1 y_j : 0 1 2 3 — (2)

$f(x_i)$: $4/8$ $4/8$ $g(y_j)$: $1/8$ $3/8$ $3/8$ $1/8$

X \ Y	0	1	2	3
0	0	$1/8$	$2/8$	$1/8$
1	$1/8$	$2/8$	$1/8$	0

— (2)

$E(X) = 1/2$, $E(Y) = 3/2$, $E(XY) = \frac{1}{2}$ — (2)

$Cov(X,Y) = E(XY) - E(X).E(Y) = \frac{1}{2} - 3/4$

$Cov(X,Y) = -1/4$ — (1)

Course Outcome:

CO 1	Demonstrate an understanding of concepts of statistical analysis and probability distributions.

Note: Suitable marks to be allotted for alternate methods

Shivalli Hy