BMS COLLEGE OF ENGINEERING, BENGALURU-19

Autonomous Institute, Affiliated to VTU

DEPARTMENT OF MATHEMATICS

Sem & Branch	FOURTH SEMESTER (BT/CH/MECH/AS/CV/EEE/ECE/EIE/ML/TCE)			
Time:	1.00-2.15 PM	Test Date:	17-5-2021	Max Marks 40

Test No.	Q. No.	(Questions in Part-A and Part-B are compulsory. Internal choice is provided in Part C)	Marks
TEST 1		**PART-A**	
	1	If θ is an angle between the two regression lines, then show that $\tan\theta = \left(\frac{1-r^2}{r}\right)\frac{\sigma_x\sigma_y}{\sigma_x{}^2+\sigma_y{}^2}$. Explain the significance when $r=0$ and $r=\pm 1$.	
		Solution:	
		Regression line of y on x: $y-\bar{y} = r\frac{\sigma_y}{\sigma_x}(x-\bar{x})$	1M
		Regression line of x on y: $x-\bar{x} = r\frac{\sigma_x}{\sigma_y}(y-\bar{y})$	
		Angle between the lines is given by	
		$\tan\theta = \frac{m_2-m_1}{1+m_1m_2}$ where $m_1 = \frac{r\sigma_y}{\sigma_x}$ and $\frac{\sigma_y}{r\sigma_x}$	1M
		$\tan\theta = \left(\frac{1-r^2}{r}\right)\frac{\sigma_x\sigma_y}{\sigma_x^2+\sigma_y^2}$	2M
		When $r=0$, the regression lines are perpendicular When $r=\pm 1$, the regression lines are parallel	1M
		PART-B	
	2	**(a)** The distributions of two stochastically independent random variables X and Y defined on the same sample space are given by the following tables:	

x	1	2
$p(x)$	0.3	0.7

y	2	3	4
$p(y)$	0.2	0.5	0.3

Find the joint distributions of X and Y. Also, evaluate $E(X)$ and $E(Y)$.

Solution:

X \ Y	2	3	4
1	0.06	0.15	0.09
2	0.14	0.35	0.21

3M

$E(X) = \sum_i x_i f(x_i) = 1.7 \qquad E(Y) = \sum_j y_j g(y_j) = 3.1$

2M

Test No.	Q. No.		Marks

(b) A source of liquid is known to contain bacteria with the mean number of bacteria per cubic centimetre equal to 2. Five 1cc test tubes are filled with the liquid. Assuming that the Poisson distribution is applicable, calculate the probability that all the five test tubes will show growth i.e. contain at least 1 bacterium each.

Solution: X- Number of bacteria

$$X \sim P(\lambda = 2);\ p(x) = \frac{e^{-\lambda}\lambda^x}{x!},\ x = 0,1,2...$$

$P(\text{Test tube will show growth}) = P(X \geq 1) = 1 - P(X < 1) = 1 - P(X = 0) = 0.8647$ **1M**

Probability that all the test tubes will show growth = $0.8647^5 = 0.4833$ **2M** **2M**

(c) In a partially destroyed laboratory record, only the lines of regression of y on x and x on y are available as $4x - 5y + 33 = 0$ and $20x - 9y = 107$ respectively. Calculate the mean of x, mean of y and the co-efficient of correlation between X and Y.

Solution: $\bar{x} = 13,\ \bar{y} = 17$ **2M**

$b_{yx} = 4/5; b_{xy} = 9/20 \dashrightarrow r = \sqrt{b_{yx}b_{xy}} = 0.6$ **1+1+1**

PART-C

TEST 1

3

(a) Estimate the chlorine residual in a swimming pool *five* hours after it has been treated with chemicals by fitting an exponential curve of the form $y = ab^x$ to the following data:

x (No.hours)	2	4	6	8	10	12
y (chlorine residual parts/million)	1.8	1.5	1.4	1.1	1.1	0.9

Solution: The linear form is $\log y = \log a + x \log b$ i.e. $Y = A + xB$

x	y	Y	xY	x^2
2	1.8	0.587787	1.175573	4
4	1.5	0.405465	1.62186	16
6	1.4	0.336472	2.018833	36
8	1.1	0.09531	0.762481	64
10	1.1	0.09531	0.953102	100
12	0.9	-0.1054	-1.2648	144
Total :42	7.8	1.415	5.2672	364

2M

Normal Equations: $\sum Y = nA + B\sum x$

$$\sum xY = A\sum x + B\sum x^2$$ **1M**

$A = 0.6996, B = -0.0663 \Rightarrow a = 2.0129, b = 0.9359$ **1+1**

Solution : At $x = 5, y = 1.445$ **1M**

OR

Test No.	Q. No.		Marks
TEST 1	3	(b)	

(b) The following table shows the recorded data of the test scores made by the salesman on an intelligence test and their weekly sales.

Salesman	1	2	3	4	5
Test scores	92	89	87	86	83
Sales (000)	86	88	91	77	68

Calculate the coefficient of correlation between the test scores and the sales. Hence find the regression line of sales on test scores and estimate the most probable weekly sales volume if a salesman makes a score of 85.

Solution: $\bar{x}=\frac{\sum x}{n}=87.4;\ \bar{y}=\frac{\sum y}{n}=82;$

$$\sigma_x^{\ 2}=\frac{\sum\left(x_i-\bar{x}\right)^2}{n}=3.0067^2;\ \sigma_y^{\ 2}=\frac{\sum\left(y_i-\bar{y}\right)^2}{n}=8.4143^2$$

$$r=\frac{\sum\left(x-\bar{x}\right)\left(y-\bar{y}\right)}{\sqrt{\sum\left(x-\bar{x}\right)^2\sum\left(y-\bar{y}\right)^2}}=0.7352$$ **3M**

2M

$$b_{yx}=r\frac{\sigma_y}{\sigma_x}=2.0575$$

$$\left(y-\bar{y}\right)=b_{yx}\left(x-\bar{x}\right)\Rightarrow y-82=2.0575\left(x-87.4\right)\Rightarrow y=2.0575x-97.8255$$ **1M**

At $x=85\Rightarrow y=77.0620$ **1M**

Test No.	Q. No.		Marks
TEST 1	4	(a)	

(a) A radioactive material was observed for α particles in 100 emissions, each time the duration being ten seconds. The data consisting of 100 observations are arranged in a frequency table as follows : Fit a Poisson distribution.

No. of particle(x)	0	1	2	3	4	5
No. of emissions(f)	11	20	28	24	9	8

Solution: $X-$ Alpha particle emission by a radioactive material.

$X\sim P(\lambda);\ p(x)=\frac{e^{-\lambda}\lambda^x}{x!},x=0,1,2...$; Mean $\bar{x}=\lambda=\frac{\sum fx}{\sum f}=\frac{221}{100}=2.21$ **2M**

x	f	fx	$p(x)$	$E=Np(x)$
0	11	0	0.1097	10.97~11
1	20	20	0.2424	24.24~24
2	28	56	0.2679	26.79~27
3	24	72	0.1973	19.73~20
4	12	48	0.1090	10.90~11
5	5	25	0.0482	4.82~5
>5	0	0	0.0255	2.55~2
15	100	221	0.9745	

5M

Test No.	Q. No.		Marks
		OR	
	4	**(b)** Business schools require applicants for admission to take the Graduate Management Admission Council's GMAT examination. Scores on the GMAT are normally distributed with a mean of 527 and a standard deviation of 112. (i) What is the probability of an individual scoring above 500 on the GMAT? (ii) How high must an individual score on the GMAT in order to score in the highest 5%? (Given $\phi(0.24)$=0.0948 and $\phi(1.65)$=0.45 where $\phi(z)$ is an area bounded by standard normal curve from 0 to z) **Solution:** X - Scores in GMAT $X \sim N(\mu = 527, \sigma^2 = 112^2); f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\left(\frac{x-\mu}{\sqrt{2}\sigma}\right)^2}, -\infty < x, \mu < \infty$ i) $P(x > 500) = P(z > -0.241) = 0.5948$ ii) $P(x > a) = 0.05; P(z > A) = 0.05$ where $A = \frac{a-\mu}{\sigma}$, $A = 1.65$ from normal table $a = 711.8 \sim 712$	3M 4M
TEST 1	5.	**(a)** The joint probability distribution of two random variables X and Y is given below: (table below) Find the coefficient of correlation between the variables X and Y. **Solution:** $E(X) = \sum_i x_i f(x_i) = 24/36 = 2/3$ $\quad E(Y) = \sum_j y_j g(y_j) = 16/36 = 4/9$ $E(XY) = \sum_i \sum_j x_i y_j p_{ij} = 6/36 = 1/6$ ${\sigma_x}^2 = E(X^2) - (E(X))^2 = 5/6 - (2/3)^2 = 0.3889$ ${\sigma_y}^2 = E(Y^2) - (E(Y))^2 = (1/2) - (4/9)^2 = 0.3024$ $\text{cov}(X,Y) = E(XY) - E(X)E(Y) = -0.1296$ $\rho(X,Y) = \frac{\text{cov}(X,Y)}{\sigma_x \sigma_y} = -0.3779$	2M 1M 1M 1M 1M 1M
		OR	

X \ Y	0	1	2
0	1/6	2/9	1/36
1	1/3	1/6	0
2	1/12	0	0

Test No.	Q. No.		Marks
TEST 1	5	**(b)** Two flashcards are selected at random from a box which contains five cards numbered 1,2,2,3,3. Find the joint distributions of X and Y where X denotes the sum of two numbers and Y denote the minimum of two numbers drawn. Also determine $Cov(X, Y)$.	
		Solution:	
		$X=\{3,4,5,6\}$, $Y=\{1,2,3\}$	
		(i) Joint Probability distribution of X and Y	4M

X \ Y	1	2	3	
3	4/20	0	0	4/20
4	4/20	2/20	0	6/20
5	0	8/20	0	8/20
6	0	0	2/20	2/20
	8/20	10/20	2/20	1

$E(X)=\sum_i x_i f(x_i)=88/20=4.4$ $\quad$ $E(Y)=\sum_j y_j g(y_j)=17/10=1.7$

$E(XY)=\sum_i\sum_j x_i y_j p_{ij}=8$

$\text{cov}(X,Y)=E(XY)-E(X)E(Y)=0.52$

3M

Suitable marks to be allotted for alternate methods
Scheme and Solutions prepared by
(Mrs. Shazia P.A.)