$$\therefore \lim_{\substack{n\to\infty \\ p\to 0}} p(x) = \frac{m\,(m-0)\,(m-0)\,\ldots\,(m-0)}{x!} \cdot \frac{\lim\limits_{n\to m}\left(1-\frac{m}{n}\right)^n}{\lim\limits_{p\to 0}(1-p)^x}$$

$$= \frac{m^x}{x!}\cdot\frac{e^{-m}}{1} = e^{-m}\cdot\frac{m^x}{x!}, \text{ which is poisson distribution.}$$

Thus Poisson distribution can be treated as binomial distribution with large n and small p.

Observations :

Poisson distribution is used in all cases where Bernoulli trials are involved i.e., where binomial distribution type of situation arises but with the difference that

(i) there is no fixedness for the number of trials or the number of trials is large.

(ii) number of failures is of no concern.

A great variety of natural and man - made phenomena are found to follow Poisson's distribution. Some common features of such phenomena are :

(i) the probability of success during a small interval (of time or space) is proportional to the interval.

(ii) the probability of more than one success in the interval is negligible.

(iii) the probability of success in a given interval does not depend on what happened prior to that interval.

If we take into consideration a time interval T and divide it into n sub-intervals of duration Δt, then $T = n\cdot\Delta t$.

Suppose that the probability of a success in the sınall interval Δt is $\alpha\cdot\Delta t$ where α is constant. The interval T has n sub-intervals of duration Δt and the probability of success in each sub-interval is $p = \alpha\Delta t$. We have now a Bernoulli trial in each sub-interval and there are n such trials over the period T. Thus we have a binomial distribution with $n = \frac{T}{\Delta t}$ and $p = \alpha\Delta t$ since n is large and Δt small, we can approximate by Poisson distribution with m = mean = np i.e., $= \frac{T}{\Delta t}\cdot\alpha\Delta t$ i.e., with $m = \alpha T$. Then $p(x) = \frac{e^{-m}\cdot m^x}{x!}$ gives the probability for x successes in the interval T.

Some phenomena which fit to this model are :

- arrival of telephone calls at a switchboard,
- passing by of trains at an unmanned gate.
- number of deaths due to accidents in a month on a National Highway.

- number of typographical mistakes in a page of this book.
- number of α-particles emitted by a radioactive source in a unit of time.
- number of electrons released from the cathode of a vacuum tube in a unit of time.

These are some examples in respect of time intervals. We may also have space intervals. Some examples are

- number of blood cells visible under the microscope. Here the surface area is T,
- number of stars appearing a portion of the Milky way.
- number of particles of impurities found in a quantity of a molten substance. Here the volume is T,
- number of germinating seeds on a plot of land. Here the area of the plot is T,
- number of deaths due to an epidemic in a given locality.
- number of germs in a bacteria culture experiment.

(e) Recurrence relation $\quad p(x+1) = \frac{m}{x+1}\cdot p(x).$

Proof : By definition of Poisson distribution,

$$p(x) = \frac{e^{-m}\cdot m^x}{x!}.$$

Changing x to $x + 1$, we have

$$p(x+1) = \frac{e^{-m}\cdot m^{x+1}}{(x+1)!}$$

We can write $p(x+1) = \frac{e^{-m}\cdot m\cdot \alpha^x}{(x+1)\cdot x!}$

$$\text{i.e.,} \quad = \frac{m}{x+1}\cdot\frac{e^{-m}\,m^x}{x!}$$

$$= \frac{m}{x+1}\cdot p(x) \quad \text{by above.}$$

$$\text{Thus } p(x+1) = \frac{m}{x+1}\cdot p(x).$$

WORKED EXAMPLES

Example 5.67 *With the usual notation prove that for a Poisson distribution.*

$$\mu_{r+1} = m\left(r\cdot\mu_{r-1} + \frac{d}{dm}\mu_r\right)$$