

B. M. S. COLLEGE OF ENGINEERING, BANGALORE-560 019
DEPARTMENT OF MATHEMATICS

Fourth Semester B.E. Course-(AS/ME/EEE/ECE/ET/ML/CIVIL/EIE)
Course Title: Complex Analysis, Probability and Statistical Methods
Course Code: 22MA4BSCPS

UNIT 3: STATISTICAL METHODS

Correlation and Regression

1. Find the correlation co-efficient between x and y from the given data:

| | | | | | | | | | |
|-------|-----|-----|-----|-----|-----|-----|-----|-----|------------------------|
| x : | 78 | 89 | 97 | 69 | 59 | 79 | 68 | 57 | |
| y : | 125 | 137 | 156 | 112 | 107 | 138 | 123 | 108 | Ans: $r = 0.96$ |

2. Find the co-efficient of correlation between industrial production and export using the following data and comment on the result.

| | | | | | | | | |
|-----------------------------|----|----|----|----|----|----|----|------------------------|
| Production (in crore tons): | 55 | 56 | 58 | 59 | 60 | 60 | 62 | |
| Export (in crore tons): | 35 | 38 | 38 | 39 | 44 | 43 | 45 | Ans: $r = 0.92$ |

3. Find the correlation co-efficient between x and y for the given data. Find also the two regression lines.

| | | | | | | | | | | |
|-------|----|----|----|----|----|----|----|----|----|----|
| x : | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| y : | 10 | 12 | 16 | 28 | 25 | 36 | 41 | 49 | 40 | 50 |

Ans: $r = 0.96$, $x = 0.2y - 0.64$, $y = 4.69x + 4.9$

4. Psychological tests of intelligence and of engineering ability were applied to 10 students. Here is a record of ungrouped data showing intelligence ration and engineering ratio. Calculate the co-efficient of correlation.

| | | | | | | | | | | |
|---------|-----|-----|-----|-----|-----|----|-----|----|----|----|
| Student | A | B | C | D | E | F | G | H | I | J |
| I.R. | 105 | 104 | 102 | 101 | 100 | 99 | 98 | 96 | 93 | 92 |
| E.R. | 101 | 103 | 100 | 98 | 95 | 96 | 104 | 92 | 97 | 94 |

Ans. $r = 0.59$

5. Establish the formula $r = \frac{\sigma_x^2 + \sigma_y^2 - \sigma_{x-y}^2}{2\sigma_x\sigma_y}$. Hence calculate r from the following data:

| | | | | | | | | | | |
|-------|----|----|----|----|-----|----|-----|----|-----|-----|
| x : | 21 | 23 | 30 | 54 | 57 | 58 | 72 | 78 | 87 | 90 |
| y : | 60 | 71 | 72 | 83 | 110 | 84 | 100 | 92 | 113 | 135 |

Ans: $r = 0.876$

6. Using the formula $r = \frac{\sigma_x^2 + \sigma_y^2 - \sigma_{x-y}^2}{2\sigma_x\sigma_y}$, find r from the following data:

| | | | | | | | | | | |
|-------|----|----|----|----|----|----|----|----|----|----|
| x : | 92 | 89 | 87 | 86 | 83 | 77 | 71 | 63 | 53 | 50 |
| y : | 86 | 88 | 91 | 77 | 68 | 85 | 52 | 82 | 37 | 57 |

Ans: $r = 0.7291$

7. In the following table are recorded dates showing the test scores made by salesman on an intelligence test and their weekly sales.

| | | | | | | | | | | |
|-------------|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| Salesman | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| Test scores | 40 | 70 | 50 | 60 | 80 | 50 | 90 | 40 | 60 | 60 |
| Sales (000) | 2.5 | 6.0 | 4.5 | 5.0 | 4.5 | 2.0 | 5.5 | 3.0 | 4.5 | 3.0 |

Calculate the regression line of sales on test scores and estimate the most probable weekly sales volume if a salesman makes a score of 70.

Ans: $\bar{x} = 60$ $\bar{y} = 4.05$, $y = 0.06x + 0.45$, At $x = 70$, $y = 4.65$



8. In a partially destroyed laboratory record, only the lines of regression of y on x and x on y are available as $4x - 5y + 33 = 0$ and $20x - 9y = 107$ respectively. Calculate \bar{x} , \bar{y} and the co-efficient of correlation between x and y .
Ans: $\bar{x} = 13$, $\bar{y} = 17$, $r = 0.6$
9. The two regression equations of the variables x and y are $x = 19.13 - 0.87y$ and $y = 11.64 - 0.50x$. Find (i) Mean of x 's (ii) mean of y 's (iii) the correlation co-efficient between x and y .
Ans. Mean of x 's = 15.79, Mean of y 's = 3.74 and $r = -0.66$
10. If the co-efficient of correlation between two variables x and y is 0.5 and the acute angle between their lines of regression is $\tan^{-1}\left(\frac{3}{8}\right)$, show that $\sigma_x = \frac{1}{2}\sigma_y$.
11. While calculating correlation co-efficient between two variables x and y from 25 pairs of observations, the following results were obtained: $n = 25$, $\sum x = 125$, $\sum x^2 = 650$, $\sum y = 100$, $\sum y^2 = 460$, $\sum xy = 508$. Later it was discovered at the time of checking that the pairs of values
 $x: \begin{matrix} 8 & 6 \\ y: 12 & 8 \end{matrix}$ were copied down as $x: \begin{matrix} 6 & 8 \\ y: 14 & 6 \end{matrix}$ obtain the correct value of correlation co-efficient.
Ans: $r = 0.67$
12. The following results were obtained from records of age (x) and blood pressure (y) of a group of 10 men.
 Mean: $x : 53$ 142
 Variance: 130 165 and $\sum (x - \bar{x})(y - \bar{y}) = 122$
 Find the appropriate regression equation and use it to estimate the blood pressure of a man whose age is 45.
Ans: $y = 0.006x + 141.682$ At $x = 45$ $y = 141.952$

Rank Correlation

1. Find the rank correlation for the following data:

i.

| | | | | | | | | | | | | |
|----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| x: | 56 | 42 | 72 | 36 | 63 | 47 | 55 | 49 | 38 | 42 | 68 | 60 |
| y: | 147 | 125 | 160 | 118 | 149 | 128 | 150 | 145 | 115 | 140 | 152 | 155 |

ii.

| | | | | | | |
|----|----|----|----|---|---|----|
| x: | 2 | 4 | 5 | 6 | 8 | 11 |
| y: | 18 | 12 | 10 | 8 | 7 | 5 |

iii.

| | | | | | | | | | | |
|----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| x: | 14 | 17 | 28 | 17 | 16 | 13 | 24 | 25 | 18 | 31 |
| y: | 0.9 | 1.1 | 1.6 | 1.3 | 1.0 | 0.8 | 1.5 | 1.4 | 1.2 | 2.0 |

iv.

| | | | | | | | | |
|----|---|---|---|---|---|---|---|---|
| x: | 2 | 5 | 7 | 6 | 8 | 1 | 3 | 4 |
| y: | 4 | 6 | 8 | 5 | 9 | 2 | 1 | 3 |



v.

| | | | | | | |
|----|---|---|---|---|---|---|
| x: | 1 | 2 | 3 | 4 | 5 | 6 |
| y: | 2 | 1 | 5 | 3 | 4 | 6 |

2. Ten competitors in a beauty contest were given ranking x and y as follows:

| | | | | | | | | | | |
|----|---|---|---|----|---|---|---|----|---|---|
| x: | 1 | 6 | 5 | 10 | 3 | 2 | 4 | 9 | 7 | 8 |
| y: | 6 | 4 | 9 | 8 | 1 | 2 | 3 | 10 | 5 | 7 |

Compute the coefficient of rank correlation.

3. Ten students get the following percentage of marks in two subjects A and B . Find the rank correlation coefficient

| | | | | | | | | | | |
|--------------|----|----|----|----|----|----|----|----|----|----|
| Marks in A | 78 | 36 | 98 | 25 | 75 | 82 | 90 | 62 | 65 | 39 |
| Marks in B | 84 | 51 | 91 | 60 | 68 | 62 | 86 | 58 | 53 | 47 |

Curve Fitting

I. Least Squares Straight line fitting:

1. If P is the pull required to lift a load W by means of a pulley block, find a linear law of the form $P = a + bW$ connecting P and W using the following data

| | | | | |
|-----|----|----|-----|-----|
| P | 12 | 15 | 21 | 25 |
| W | 50 | 70 | 100 | 120 |

Ans: $P = 2.2759 + 0.1879W$ $P(150) = 30.4635$

2. Find a least squares straight line for the following data and estimate y at $x=4$ and x at $y=4$.

| | | | | | | |
|-----|---|---|---|---|---|---|
| X | 1 | 2 | 3 | 4 | 5 | 6 |
| Y | 6 | 4 | 3 | 5 | 4 | 2 |

Ans: $y = 5.7999 - 0.514x$, $y(4) = 3.743$, $x = 7.1 - 0.94y$, $x(4) = 3.5$.

3. Fit a least squares straight line to the following data

| | | | | | | |
|-----|----|----|----|----|----|----|
| x | 2 | 7 | 9 | 1 | 5 | 12 |
| y | 13 | 21 | 23 | 14 | 15 | 21 |

Ans: $y = 12.45 + 0.8977x$

4. In some determinations of the volume V of carbon dioxide dissolved in a given volume of water at different temperature the following pairs of values were obtained:

| | | | | |
|----------|---|---|----|----|
| θ | 0 | 5 | 10 | 15 |
|----------|---|---|----|----|



| | | | | |
|---|------|------|------|------|
| V | 1.80 | 1.45 | 1.18 | 1.00 |
|---|------|------|------|------|

Obtain a relation of the form $V = a + b\theta$ by the method of least squares.

Ans: $V = 1.758 - 0.053\theta$.

5. Fit a straight line to the following data

| | | | | | |
|----------------------------|------|------|------|------|------|
| x(year) | 1961 | 1971 | 1981 | 1991 | 2001 |
| Y(production in 1000 tons) | 8 | 10 | 12 | 14 | 16 |

Find the expected production in 2006.

6. A simply supported beam carries a concentrated load P at its midpoint. Corresponding to various values of P , the maximum deflection y is measured. The data are given below:

| | | | | | | |
|-----|------|------|------|------|------|------|
| P | 100 | 120 | 140 | 160 | 180 | 200 |
| y | 0.45 | 0.55 | 0.60 | 0.70 | 0.80 | 0.85 |

Find a law of

the form $y = a + bP$.

Ans: $y = 0.004P + 0.048$

7. The results of measurement of electric resistance R of a copper bar at various temperatures $t^\circ\text{C}$ are listed below:

| | | | | | | | |
|-----|----|----|----|----|----|----|----|
| t | 19 | 25 | 30 | 36 | 40 | 45 | 50 |
| R | 76 | 77 | 79 | 80 | 82 | 83 | 85 |

Find a relation $R = a + bt$ where a & b are constants.

Ans: $R = 70.052 + 0.292t$.

II. Least squares quadratic curve (or) parabolic curve fitting:

1. Fit a least squares quadratic curve to the following data:

| | | | | |
|-----|-----|-----|-----|-----|
| x | 1 | 2 | 3 | 4 |
| y | 1.7 | 1.8 | 2.3 | 3.2 |

Estimate y (2.4)

Ans: $y = 2 - 0.5x + 0.2x^2$ & $y(2.4) = 1.952$

2. Fit a least squares parabola to the following data:

| | | | | | | |
|-----|-------|-------|-------|-------|-------|-------|
| x | 0.0 | 0.2 | 0.4 | 0.7 | 0.9 | 7.0 |
| y | 1.016 | 0.768 | 0.648 | 0.401 | 0.272 | 0.193 |

Ans: $y = 0.999 - 1.0006x + 0.210x^2$

3. Find the quadratic equation that fits the following data by least squares method

| | | | | | | |
|-----|-------|-------|-------|-------|-------|-------|
| x | 1 | 2 | 3 | 4 | 5 | 6 |
| y | 13235 | 11528 | 11600 | 12747 | 14940 | 18400 |

Ans: $y = 11953 + 531.5x + 153.3x^2$.

4. Using least squares method, fit a second degree polynomial. Estimate y at $x = 6.5$



| | | | | | | | | | |
|-----|------|------|------|-----|-----|-----|-----|-----|-----|
| x | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| y | 12.0 | 10.5 | 10.0 | 8.0 | 7.0 | 8.0 | 7.5 | 8.5 | 9.0 |

Ans: $y = 12.2 - 1.85x + 0.183x^2$ & $y(6.5) = 7.9$

5. Fit a second degree parabola to the following data:

| | | | | | | | |
|-----|-----|-----|-----|-----|-----|-----|-----|
| x | 1.0 | 1.5 | 2.0 | 2.5 | 3.0 | 3.5 | 4.0 |
| y | 1.1 | 1.3 | 1.6 | 2.0 | 2.7 | 3.4 | 4.1 |

Ans: $y = 1.04 - 198x + 0.244x^2$

6. If V (km/hr) and R (kg/ton) are related by a relation of the type $R = a + bV^2$, find a & b by the method of least squares with the help of the following table.

| | | | | | |
|-----|----|----|----|----|----|
| V | 10 | 20 | 30 | 40 | 50 |
| R | 8 | 10 | 15 | 21 | 30 |

Ans: $a = 6.68$ & $b = 0.0092$

7. The following table gives the results of the measurements of train resistances, V is the velocity in miles per hour. R is the resistance in pounds per ton.

| | | | | | | |
|-----|-----|-----|------|------|------|------|
| V | 20 | 40 | 60 | 80 | 100 | 120 |
| R | 5.5 | 9.1 | 14.9 | 22.8 | 33.3 | 46.0 |

If R is related to V by the relation $R = a + bV + cV^2$, find. a , b & c .

Ans: $a = 3.48$, $b = -0.002$ & $c = 0.0029$

8. Fit a second-degree polynomial (or parabola) to the following data:

| | | | | | | | | | |
|-----|------|------|------|------|------|------|------|------|------|
| x | 1929 | 1930 | 1931 | 1932 | 1933 | 1934 | 1935 | 1936 | 1937 |
| y | 352 | 356 | 357 | 358 | 360 | 361 | 361 | 360 | 359 |

Ans: $y - 358 = 2.0043(x - 1933) - 0.2673(x - 1933)^2$

III. Exponential growth curve fitting: $y = ax^b$.

1. Fit a power function of the form $y = ax^b$ to the following data and estimate y at $x = 12$;

| | | | | | |
|--------------|----|----|-----|----|----|
| x (Price) | 20 | 16 | 10 | 11 | 14 |
| y (Demand) | 22 | 41 | 120 | 89 | 56 |

2. Fit a power function $y = ax^b$ to the following data pertaining to demand for a product and its price charged at five different cities. Predict the demand when price of the product is Rs. 12

| | | | | | | |
|------------------------|-----|----|----|-----|----|----|
| Price (Rs.) | x | 20 | 16 | 10 | 11 | 14 |
| Demand (1000 units) | y | 22 | 41 | 120 | 89 | 56 |



3. Fit a geometric curve $y = ax^b$ to the following data and estimate $y(2.5)$.

| | | | | |
|-----|---|---|---|---|
| x | 1 | 2 | 4 | 6 |
| y | 6 | 4 | 2 | 2 |

4. Fit a power function of the form $y = ax^b$ to the following data.

| | | | | | |
|-----|------|---|-----|---|-----|
| x | 1 | 2 | 3 | 4 | 5 |
| y | 12.5 | 8 | 4.5 | 2 | 0.5 |

5. Fit a least square geometric curve $y = ax^b$ to the following data.

| | | | | | |
|-----|-----|---|-----|---|------|
| x | 1 | 2 | 3 | 4 | 5 |
| y | 0.5 | 2 | 4.5 | 8 | 12.5 |