B. M. S. COLLEGE OF ENGINEERING, BANGALORE-560 019
DEPARTMENT OF MATHEMATICS

Fourth Semester B.E. Course-(AS/ME/EEE/ECE/ET/ML/CIVIL/EIE)
Course Title: Complex Analysis, Probability and Statistical Methods
Course Code: 22MA4BSCPS

UNIT 5: STATISTICAL INFERENCE

I. Test of significance for single mean

Test statistic	Null Hypothesis	Alternative Hypothesis	Accept H_o
$Z = \frac{\overline{x} - \mu_O}{\sigma / \sqrt{n}}$	$H_0 : \mu = \mu_O$	$H_1 : \mu \neq \mu_O$	$\lvert z \rvert < z_{\alpha/2}$
		$H_1 : \mu < \mu_O$	$z > z_\alpha$
		$H_1 : \mu > \mu_O$	$z < z_\alpha$

1. The length of life X of certain computers is approximately normally distributed with mean 800 hours and S.D 40 hours. If a random sample of 30 computers has an average life of 788 hours, test the null hypothesis that μ=800 hours at (a) 5% (b) 1% (c) 10% (d) 15% level of significance
2. Mice with an average lifespan of 32 months will live upto 40 months when fed by a certain nutrious food. If 64 mice fed on this diet have an average lifespan of 38 months and standard deviation of 5.8 months, is there any reason to believe that average lifespan is less than 40 months.
3. A machine runs on an average of 125 hours/year. A random sample of 49 machines has an annual average use of 126.9 hours with standard deviation 8.4 hours. Does this suggest to believe that machines are used on the average more than 125 hours annually at 0.05 level of significance?
4. A company claims that the mean thermal efficiency of diesel engines produced by them is 32.3%. To test this claim, a random sample of 40 engines were examined which showed the mean thermal efficiency of 31.4% and S.D of 1.6%. Can the claim be accepted or not, at 0.01 L.O.S.?
5. It has previously been recorded that the average depth of ocean at a particular region is 67.4 fathoms. Is there reason to believe this at 0.01 L.O.S. if the readings at 40 random locations in that particular region showed a mean of 69.3 with S.D of 5.4 fathoms?
6. A company producing computers states that the mean lifetime of its computers is 1600 hours. Test this claim at 0.01 L.O.S. against the A.H.: μ < 1600 hours if 100 computers produced by this company has mean lifetime of 1570 hours with S.D. of 120 hours.
7. A manufacturer of tyres guarantees that the average lifetime of its tyres is more than 28000 miles. If 40 tyres of this company tested, yields a mean lifetime of 27463 miles with S.D. of 1348 miles, can the guarantee be accepted at 0.01 L.O.S.?

II. Test of significance for difference between two means

Test statistic	Null Hypothesis	Alternative Hypothesis	Accept H_o
$Z=\dfrac{\bar{x}-\mu_O}{\sigma/\sqrt{n}}$	$H_0:\mu=\mu_O$	$H_1:\mu\neq\mu_O$	$\lvert z\rvert<z_{\alpha/2}$
		$H_1:\mu<\mu_O$	$z>z_\alpha$
		$H_1:\mu>\mu_O$	$z<z_\alpha$
$Z=\dfrac{(\bar{x}_1-\bar{x}_2)-\delta}{\sqrt{\dfrac{\sigma_1^{\,2}}{n_1}+\dfrac{\sigma_2^{\,2}}{n_2}}}$	$H_0:\mu_1-\mu_2=\delta$	$H_1:\mu_1-\mu_2\neq\delta$	$\lvert z\rvert<z_{\alpha/2}$
		$H_1:\mu_1-\mu_2>\delta$	$z<z_\alpha$
		$H_1:\mu_1-\mu_2<\delta$	$z>z_\alpha$

1. In a random sample of 100 tube lights produced by company A, the mean lifetime (mlt) of tube light is 1190 hours with standard deviation of 90 hours. Also in a random sample of 75 tube lights from company B the mean lifetime is 1230 hours with standard deviation of 120 hours. Is there a difference between the mean lifetimes of the two brands of tube lights at a significance level of (a) 0.05 (b) 0.01?
2. To test the effects a new pesticide on rice production, a farm land was divided into 60 units of equal areas, all portions having identical qualities as to soil, exposure to sunlight etc. The new pesticide is applied to 30 units while old pesticide to the remaining 30. Is there reason to belive that the new pesticide is better than the old pesticide if the mean number of kgs of rice harvested/units using new pesticide (N.P.) is 496.31 with S.D. of 17.18 kgs. Test at a level of significance (a) 0.05 (b) 0.01?
3. A random sample of 40 'geyers' produced by company A have a mean lifetime (mlt) of 647 hours of continuous use with a S.D. of 27 hours, while a sample 40 produced by another company B have mlt of 638 hours with S.D. 31 hours. Does this substantiate the claim of company A that their 'geyers' are superior to those produced by company B at (a) 0.05 (b) 0.01 L.O.S.
4. Test at 0.05 L.O.S. a manufacturer's claim that the mean tensile strength (mts) of a tread A exceeds the mts of thread B by at least 12 kgs. If 50 pieces of each type of thread are tested under similar conditions yielding the following data:

	Sample Size	Mts (kgs)	S.D. (kgs)
Type A	50	86.7	6.28
Type B	50	77.8	5.61

5. Test the N.H.: $\mu_A-\mu_B=0$ against the A.H.: $\mu_A-\mu_B\neq 0$ at 0.01 L.O.S. for the following data

	Sample Size	Mts (kgs)	S.D. (kgs)
Type A	40	247.3	15.2

Type B	30	254.1	18.7

6. If a random sample data show that 42 men earn on the average $\bar{x}_1 = 744.85$ with S.D. $s_1 = 397.7$ while 32 women earn on the average $\bar{x}_2 = 516.78$ with S.D. $s_2 = 162.523$, test at 0.05 level of significance whether the average income for men and women is same or not.
7. A company claims that alloying reduces resistance of electric wire by more than 0.05 ohm. To test this claim samples of standard wire and alloyed wire are tested yielding the following results:

Type of wire	Sample Size	Mean resistance (ohms)	S.D. (ohms)
Standard	32	0.136	0.004
Alloyed	32	0.083	0.005

III. Small Sample Test Concerning Single Mean: t-Distribution

Test statistic	Null Hypothesis	Alternative Hypothesis	Accept H_o
$t = \dfrac{\bar{x} - \mu_O}{S/\sqrt{n}}$ here $S^2 = \dfrac{\sum (x_i - \bar{x})^2}{n-1}$ OR $t = \dfrac{\bar{x} - \mu_O}{s/\sqrt{n-1}}$ here $s^2 = \dfrac{\sum (x_i - \bar{x})^2}{n}$	$H_0 : \mu = \mu_O$	$H_1 : \mu \neq \mu_O$	$\lvert t \rvert < t_{\alpha/2, n-1}$
		$H_1 : \mu < \mu_O$	$t > t_\alpha$
		$H_1 : \mu > \mu_O$	$t < t_\alpha$

1. Find the student's *t* for the following variable values in a sample of eight: $-4, -2, -2, 0, 2, 2, 3, 3,$ taking the mean of the universe to be zero.
2. A machine is expected to produce nails of length 3 inches. A random sample of 25 nails gave an average length of 3.1 inch with standard deviation 0.3. Can it be said that the machine is producing nails as per specification? ($t_{0.05}$ for 24 $d.f$ is 2.064)

3. Ten individuals are chosen at random from a population and their heights in inches are found to be $63, 63, 66, 67, 68, 69, 70, 70, 71, 71.$ Test the hypothesis that the mean height of the universe is 66 inches. ($t_{.05} = 2.262$ for 9 $d.f$)

4. A sample of 10 measurements of the diameter of a sphere gave a mean of 12cm and a standard deviation 0.15cm. Find the 95% confidence limits for the actual diameter.

5. A certain stimulus administrated to each of the 12 patients resulted in the following change in blood pressure $5, 2, 8, -1, 3, 0, 6, -2, 1, 5, 0, 4.$ Can it be concluded that the stimulus will increase the blood pressure? ($t_{.05}$ for 11 $d.f$ = 2.201)

6. A group of boys and girls were given an intelligence test. The mean score, S.D score and numbers in each group are as follows.

	Boys	Girls
Mean	74	70
SD	8	10
n	12	10

Is the difference between the mean of the two groups significant at 5% level of significance ($t_{.05}$ = 2.086 for 20 $d.f$)

7. A sample of 11 rats from a central population had an average blood viscosity of 3.92 with a standard deviation of 0.61. On the basis of this sample, establish 95% fiducial limits for μ the mean blood viscosity of the central population ($t_{.05}$ = 2.228 for 10 d . f)

8. Two types of batteries are tested for their length of life and the following results were obtained.

   Battery A: $n_1 = 10, \bar{x}_1 = 500hrs, \sigma_1^2 = 100$

   Battery B: $n_2 = 10, \bar{x}_2 = 500hrs, \sigma_2^2 = 121$

9. A group of 10 boys fed on a diet *A* and another group of 8 boys fed on a different diet B for a period of 6 months recorded the following increase in weights (lbs).

   Diet A: 5 6 8 1 12 4 3 9 6 10

   Diet B: 2 3 6 8 10 1 2 8

   Test whether diets *A* and *B* differ significantly regarding their effect on increase in weight.

10. Two horses *A* and *B* were tested according to the time (in seconds) to run a particular race with the following results.

    Horse A: 28 30 32 33 33 29 34

    Horse B: 29 30 30 24 27 29

    Test whether you can discriminate between the two horses.

11. A die is thrown 264 times and the number appearing on the face (x) follows the following frequency distribution.

x	1	2	3	4	5	6
f	40	32	28	58	54	60

Calculate the value of x^2

12. Five dice were thrown 96 times and the numbers 1, 2 or 3 appearing on the face of the dice follows the frequency distribution as below.

No. of dice showing 1,2 or 3	5	4	3	2	1	0
Frequency	7	19	35	24	8	3

Test the hypothesis that the data follows a binomial distribution ($x^2_{0.05} = 11.07$ for 5 $d.f$)

13. A sample analysis of examination results of 500 students was made. It was found that 220 students had failed, 170 had secured third class 90 had secured second class and 20 had secured first class. Do these figures support the general examination result which is in the ratio 4 : 3 : 2 : 1 for the respective categories ($x^2_{0.05} = 7.81$ for 3 $d.f$)

14. 4 coins are tossed 100 times and the following results were obtained. Fit a binomial distribution for the data and test the goodness of fit ($x^2_{0.05} = 9.49$ for 4 d . f)

No. of heads	0	1	2	3	4
Frequency	5	29	36	25	5

15. Fit a Poisson distribution for the following data and test the goodness of fit given that $x^2_{0.05} = 7.815$ for 3 d . f

x	0	1	2	3	4
f	122	60	15	2	1

16. The number of accidents per day (x) as recorded in a textile industry over a period of 400 days id given below. Test the goodness of fit in respect of Poisson distribution of fit to yhr given data ($x^2_{0.05} = 9.49$ for 4 d . f)

x	0	1	2	3	4	5

f	173	168	37	18	3	1

IV. Small Sample Test Concerning Difference Between Two Means: t-Distribution

Test statistic	Null Hypothesis	Alternative Hypothesis	Accept H_o
$t=\dfrac{(\bar{x}_1-\bar{x}_2)-\delta}{S\sqrt{\dfrac{1}{n_1}+\dfrac{1}{n_2}}}$ (i) $S^2=\dfrac{\sum(x_{1i}-\bar{x}_1)^2+\sum(x_{2i}-\bar{x}_2)^2}{n_1+n_2-2}$ OR (ii) $S^2=\dfrac{n_1s_1^{\ 2}+n_2s_2^{\ 2}}{n_1+n_2-2}$ if $s_1^{\ 2}=\dfrac{\sum(x_{1i}-\bar{x}_1)^2}{n_1}$ and $s_2^{\ 2}=\dfrac{\sum(x_{2i}-\bar{x}_2)^2}{n_2}$ OR (iii) $S^2=\dfrac{(n_1-1)S_1^{\ 2}+(n_2-1)S_2^{\ 2}}{n_1+n_2-2}$ if $S_1^{\ 2}=\dfrac{\sum(x_{1i}-\bar{x}_1)^2}{n_1-1}$ and $S_2^{\ 2}=\dfrac{\sum(x_{2i}-\bar{x}_2)^2}{n_2-1}$	$H_0:\mu_1-\mu_2=\delta$	$H_1:\mu_1-\mu_2\neq\delta$	$\lvert t\rvert<t_{\alpha/2,n_1+n_2-2}$
		$H_1:\mu_1-\mu_2>\delta$	$t<t_\alpha$
		$H_1:\mu_1-\mu_2<\delta$	$t>t_\alpha$

1. In a mathematics examination 9 students of class A and 6 students of class B obtained the following marks. Test at 1% L.O.S. whether the performance in mathematics is same or not for the two classes A and B. assume that the samples are drawn from normal populations having same variance.

A:	44	71	63	59	68	46	69	54	48
B:	52	70	41	62	36	50			

2. Out of random sample of 9 mice, suffering with a disease, 5 mice were treated with a new serum while the remaining were not treated. From the time commencement of experiment, the following are the survival times:

Treatment	2.1	5.3	1.4	4.6	0.9
NoTreatment	1.9	0.5	2.8	3.1	

Test whether the serum treatment is effective in curing the disease at 5% L.O.S., assuming that the two distributions are normally distributed with equal variances.

3. Random samples of specimens of coal from two mines A and B are drawn and their heat producing capacity (in millions of calories/ton) were measured yielding the following results:

Mine A:	8350	8070	8340	8130	8260	
Mine B:	7900	8140	7920	7840	7890	7950

Is there significant difference between the means of these two samples at 1% L.O.S.?

4. A study is conducted to determine whether the wear of material A exceeds that of B by more than 2 units. If test of 12 pieces of material A yielded a mean wear of 85 units and S.D. of 4 while test of 10 pieces of material B yielded a mean of 81 and S.D. 5, what conclusion can be drawn at 5% L.O.S. Assume that populations are approximately normally distributed with equal variances.

5. To determine whether vegetarian and non-vegetarian diets effects significantly on increase in weight a study was conducted yielding the following data of gain in weight.

Veg	34	24	14	32	25	32	30	24	30	31	35	25			
Non-Veg	22	10	47	31	44	34	22	40	30	32	35	18	21	35	29

Can we claim that the two diets differ pertaining to weight gain, assuming that samples are drawn from normal populations with same variance?

V. Paired Sample t-Test

Test statistic	Null Hypothesis	Alternative Hypothesis	Accept H_o
$t=\frac{\bar{d}-\mu_d}{S_d/\sqrt{n}}$ here $S_d^2=\frac{\sum\left(d_i-\bar{d}\right)^2}{n-1}$ OR $t=\frac{\bar{d}-\mu_d}{S_d/\sqrt{n-1}}$ here $S_d^2=\frac{\sum\left(d_i-\bar{d}\right)^2}{n}$	$H_0:\mu=\mu_d$	$H_1:\mu\neq\mu_d$	$\lvert t\rvert<t_{\alpha/2,n-1}$
		$H_1:\mu<\mu_d$	$t>t_\alpha$
		$H_1:\mu>\mu_d$	$t<t_\alpha$

1. Use paired sample test at 0.05 L.O.S. to test from the following data whether the differences of means of the weights obtained by two different scales (weighting machines) is significant.

Weight (gms)	Scale: I	11.23	14.36	8.33	10.50	23.42	9.15	13.47	6.47	12.40	19.38

	Scale: II	11.27	14.41	8.35	10.52	23.41	9.17	13.52	6.46	12.45	19.35

2. The average weekly losses of man-hours due to strikes in an institute before and after a disciplinary program was implemented are as follows: Is there reason to believe that the disciplinary program is effective at 5% L.O.S.?

Before	45	73	46	124	33	57	83	34	26	17
After	36	60	44	119	35	51	77	29	24	11

3. The pulsality index (P.I.) of 11 patients before and after contracting a disease are given below. Test at 0.05 L.O.S. whether there is a significant increase of the mean of P.I. values.

Before	0.4	0.45	0.44	0.54	0.48	0.62	0.48	0.60	0.45	0.46	0.35
After	0.5	0.60	0.57	0.65	0.63	0.78	0.63	0.80	0.69	0.62	0.68

4. The blood pressure (B.P.) of 5 women before and after intake of a certain drug are given below:

Before	110	120	125	132	125
After	120	118	125	136	121

Test at 1% L.O.S. whether there is significant change in B.P.

5. Marks obtained in mathematics by 11 students before and after intensive coaching are given below:

Before	24	17	18	20	19	23	16	18	21	20	19
After	24	20	22	20	17	24	20	20	18	19	22

Test at 5% L.O.S. whether the intensive coaching is useful?

VI. Chi-Square Distribution: Goodness of Fit

Test statistic	Null Hypothesis	Alternative Hypothesis	Accept H_o
$\chi^2 = \dfrac{\sum_{i=1}^{n}(o_i - e_i)^2}{e_i}$	H_0 : There is no significant difference between experimental and theoretical values	H_1 : There is significant difference between experimental and theoretical values	$\chi^2 < \chi^2_{n-k,\alpha}$

1. Test for goodness of fit of a Poisson distribution at 5% L.O.S. to the following frequency distribution:

No. of patients arriving/hour: (x)	0	1	2	3	4	5	6	7	8
Frequency	52	151	130	102	45	12	5	1	2