Time domain response specifications of 2nd order system: (under damped)

$$c(t) = 1 - \frac{e^{-\xi \omega_n t}}{\sqrt{1-\xi^2}} \sin(\omega_d t + \theta) \; ; \qquad \text{—①}$$

$\cos\theta = \xi$

$\omega_d = \omega_n \sqrt{1-\xi^2}$

1) Rise time (t_r): The time required for the response to reach from 0 to 100%.

$c(t) = 1$ at $t = t_r$

$$1 = 1 - \frac{e^{-\xi \omega_n t_r}}{\sqrt{1-\xi^2}} \sin(\omega_d t_r + \theta)$$

$$\Rightarrow \quad \frac{e^{-\xi \omega_n t_r}}{\sqrt{1-\xi^2}} \sin(\omega_d t_r + \theta) = 0$$

→ not zero when $0 < \xi < 1$.

$$\therefore \sin(\omega_d t_r + \theta) = 0$$

$$\Rightarrow \omega_d t_r + \theta = n\pi \text{ where } n = 0, 1, 2 \ldots$$

$$\omega_d t_r = n\pi - \theta$$

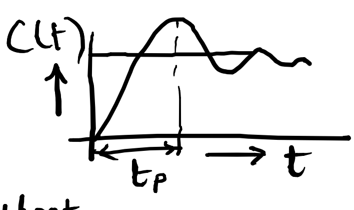
$$t_r = \frac{n\pi - \theta}{\omega_d}$$

if $n = 1$

$$\boxed{t_r = \frac{\pi - \theta}{\omega_n \sqrt{1-\xi^2}}}$$

2) Peak time (t_p):

It is the time taken by the response to reach peak of the overshoot



$$c(t) = 1 - \frac{e^{-\xi \omega_n t}}{\sqrt{1-\xi^2}} \sin(\omega_d t + \theta)$$

Maximum value is obtained when $\frac{d\,c(t)}{dt} = 0$ | $\xi = \cos\theta$, $\sqrt{1-\xi^2} = \sin\theta$

$$\frac{d\,c(t)}{dt} = 0 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}\left[\cos(\omega_d t_p + \theta)\right]\omega_d - \frac{\xi\omega_n}{\sqrt{1-\xi^2}} e^{-\xi\omega_n t_p} \sin(\omega_d t + \theta)$$

$$= \frac{-e^{-\xi\omega_n t_p}}{\sqrt{1-\xi^2}}\left[\cos(\omega_d t_p + \theta)\,\omega_n\sqrt{1-\xi^2} - \xi\omega_n \sin(\omega_d t_p + \theta)\right]$$

$$\frac{d\,c(t)}{dt} = \underbrace{\frac{-e^{-\xi\omega_n t_p}}{\sqrt{1-\xi^2}}}_{\text{not zero}}\,\omega_n\Big[\underbrace{\cos(\omega_d t_p + \theta)\sin\theta - \sin(\omega_d t_p + \theta)\cos\theta}_{\sin(A-B) = \sin A\cos B - \cos A\sin B}\Big] = 0$$

For $0 < \xi < 1$

$$\Rightarrow \left[\sin(\omega_d t_p + \theta - \theta)\right] = 0$$

i.e., $\sin(\omega_d t_p) = 0 \Rightarrow \omega_d t_p = n\pi$ where $n = 0, 1, 2 \ldots$

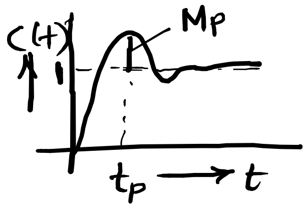
$$t_p = \frac{n\pi}{\omega_d}$$

For $n = 1$, $\boxed{t_p = \frac{\pi}{\omega_d} = \frac{\pi}{\omega_n\sqrt{1-\xi^2}}}$

3) The Maximum over shoot M_p:

Maximum over-shoot occurs at $t = t_p$

$$c(t) = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}} \sin(\omega_d t + \theta)$$



overshoot:

$$M_p = c(t_p) - 1$$

$$M_p = -\frac{e^{-\xi\omega_n t_p}}{\sqrt{1-\xi^2}} \sin(\omega_d t_p + \theta)$$

$\hookrightarrow t_p = \frac{\pi}{\omega_d}$.

$$M_p = -\frac{e^{-\xi\omega_n t_p}}{\sqrt{1-\xi^2}} \sin(\pi+\theta)$$

$\sin(\pi+\theta) = -\sin\theta$

$\sqrt{1-\xi^2} = \sin\theta$

$$= \frac{-e^{-\xi\omega_n t_p}}{\sin\theta} \times (-\sin\theta)$$

$$M_p = e^{-\xi\omega_n t_p}$$ — $\omega_n = \frac{\omega_d}{\sqrt{1-\xi^2}}$, $t_p = \frac{\pi}{\omega_d}$.

$$M_p = e^{-\pi\xi/\sqrt{1-\xi^2}}$$

$\frac{\xi\omega_d}{\sqrt{1-\xi^2}} \cdot \frac{\pi}{\omega_d}$

$$\boxed{M_p = e^{-\frac{\pi\xi}{\sqrt{1-\xi^2}}}}$$

4) Settling time (t_s): Time required by the response to reach and stay within a specified tolerance band of its final value

$\downarrow$ $\pm 2\%$ or $\pm 5\%$

$$c(t) = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}} \sin(\omega_d t + \theta)$$

Let the tolerance band be $\pm 2\%$

$$\frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}} = 0.02.$$

For underdamped case $\xi < 1$.

& $\xi^2 << 1$

$$e^{-\xi\omega_n t_s} \approx 0.02$$

$$-\xi\omega_n t_s \approx \ln(0.02)$$

$$\boxed{t_s = \frac{3.912}{\xi \omega_n}} \rightarrow \pm 2\%$$

$\approx 4T$

$$\frac{1}{\xi \omega_n} = T$$

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$$\left.\begin{aligned} t_s &\approx 3T \\ &\approx \frac{3}{\xi \omega_n} \end{aligned}\right\} \rightarrow \pm 5\%$$