B. M. S. COLLEGE OF ENGINEERING, BANGALORE-560 019
DEPARTMENT OF MATHEMATICS

Fourth Semester B.E. Course-(AS/ME/EEE/ECE/ET/ML/CIVIL/EIE)
Course Title: Complex Analysis, Probability and Statistical Methods
Course Code: 22MA4BSCPS

UNIT 4: PROBABILITY DISTRIBUTIONS

POISSON DISTRIBUTION

1. Derive an expression fore expectation and variance of Poisson distribution.
2. The number of accidents per day (x) as recorded in a textile industry over a period of 400 days is given. Fit a Poisson distribution for the data and calculate the theoretical frequencies.

x	0	1	2	3	4	5
f	173	168	37	18	3	1

3. Fit a Poisson distribution for the following frequency distribution.

x	0	1	2	3	4
f	122	60	15	2	1

4. The frequency of accidents per shift in a factory is as shown in the following table:

Accidents per shifts	0	1	2	3	4
Frequency	180	92	24	3	1

   Calculate the mean number of accidents per shifts and the corresponding Poisson distribution and compare with actual observations.
5. Fit a Poisson distribution for the following data and calculate the theoretical frequencies

x	0	1	2	3	4
f	111	63	22	3	1

6. In a certain factory turning out razors blades, there is a small chance of 0.002 for any blade to be defective. The blades are supplied in packets of 10. Use Poisson distribution to calculate the approximate number of packets containing (i) no defective (ii) one defective and (iii) two defective blades respectively in a consignment of 10,000 packets.
7. The number of accidents in a year to taxi in a city follows a Poisson distribution with mean 3. Out of 1000 taxi drivers find approximately the number of the drivers with
   (i) No accident in a year
   (ii) More than 3 accidents in a year.
8. 2% of the fuses manufactured by a firm are found to be defective. Find the probability that a box containing 200 fuses contains
   (i) No defective fuses
   (ii) 3 or more defective fuses.
9. If the probability of a bad reaction from a certain injection is 0.001, determine the chance that out of 2000 individuals,
   (i) Exactly 3
   (ii) More than 2 will suffer a bad reaction
10. A communication channel receives independent pulses at the rate of 12 pulses per micro second. The probability of transmission error is 0.001 for each micro second. Compute the probabilities of

(i) no error during a micro second
(ii) one error per micro second
(iii) atleast one error per micro second
(iv) two errors
(v) atmost two errors.

11. If X follows a Poisson law such that P (X=2) = (2/3) P (X=1), find P (X=0) and P (X=3).
12. If x is a Poisson variate such that P(x = 2) = 9 P (x = 4) + 90 P (x = 6), compute the mean and variance of the Poisson distribution.
13. A switch board can handle only 4 telephone calls per minute. If the incoming calls per minute follow a Poisson distribution with parameter 3, find the probability that the switch board is over taxed in any one minute.
14. If a random variable has a Poisson distribution such that p (1) = p (2), find
i) Mean of the distribution ii) p (4).
15. X is a Poisson variable and it is found that the probability that X=1. Find the probability that X=0 and the probability that X=3. What is the probability that X exceeds 3?
16. A car hire firm has two cars which it hires out day by day. The number of demands for a car on each day is distributed as a Poisson distribution with mean 1.5.calculate the proportion of days on which
(i) there is no demand
(ii) demand is refused.
17. A shop has 4 diesel generator sets which it hires every day. The demand for a gen set on an average is a Poisson variate with value 5/2. Obtain the probability that on a particular day
(i) There was no demand
(ii) A demand is refused.
18. A source of liquid is known to contain bacteria with the mean number of bacteria per cubic centimeter equal to 3.ten 1cc test tubes are filled with the liquid. Assuming that Poisson distribution is applicable, calculate the probability that all the test tubes will show growth i.e. contain at least 1 bacterium each.

NORMAL DISTRIBUTION

1. In a test on 2000 electric bulbs, it was found that the life of a particular make, was normally distributed with an average life of 2040 hours and S.D of 60 hours. estimate the number of bulbs likely to burn for a) more than 2150 hours, b) less than 1950 hours c) more than 1920 hours and but less than 2160 hours.
2. In a normal distribution, 31% of the items are under 45 and 8% are over 64. Find the mean and S.D. of the distribution.
3. In a certain examination, the percentage of candidates passing and getting distinctions were 45 and 9 respectively. Estimate the average marks obtained by the candidates, the minimum pass and distinction marks being 40 and 75 respectively.
4. A manufacturer of air-mail envelopes knows from experience that weight of the envelopes is normally distributed with mean 1.95 gm and S.D. 0.05 gm. About how many envelopes weighting (i) 2 gm or more (ii) 2.05 gm or more can be expected in a given packet of 100 envelopes.

5. The mean height of 500 students is 151 cm and the S.D. is 15 cm. assuming that the heights are normally distributed , find how many student's heights lie between 120 and 155 cm.
6. The mean and S.D. of the marks obtained by 1000 students in an examination are respectively 34.4 and 16.5. Assuming the normality of the distribution, find the approximate number of students expected to obtain marks between 30 and 60.
7. In a examination taken by 500 candidates, the average and S.D. of marks obtained (normally distributed) are 40% and 10%. Find approximately (i) how many will pass, if 50% is fixed as a minimum? (ii)What should be the minimum if 350 candidates are to pass? (iii) How many have scored marks above 60%?
8. The mean inside diameter of a sample of 200 washers produced by a machine is 5.02 mm and the S.D. is 0.05 mm. the purpose for which these washers are intended allows a maximum tolerance in the diameter of 4.96 to 5.08 mm, otherwise the washers are considered defective. Determine the percentage of defective washers produced by the machine, assuming the diameters are normally distributed.
9. It is given that the age of thermostats of a particular makes follow the normal law with mean 5 years and S.D. 2 years. 1000 units are sold out every month. How many of them will have to be replaced at the end of the second year.
10. The income of a group of 10,000 persons was found to be normally distributed with mean Rs. 750 p.m. and S.D. of Rs. 50. Show that, of this group, about 95%had income exceeding Rs. 668 and only 5% had income exceeding Rs. 832. Also find the lowest income among the richest 100.
11. The diameter of an electric cable is normally distributed with a mean 0.8 and standard deviation 0.02. What is the probability that the diameter will exceed 0.81? If a cable is considered defective, if the diameter differs from the mean by more than 0.025, what is the probability that a cable is defective?
12. The average life of a certain instrument is 10 years with standard deviation 2 years. The manufacturer wants to give a guarantee of replacement. If he is willing to replace at most 3% of the instruments that he sells, what should be the number of years of guarantee that may be given?

Joint Probability Distribution:

1. Find (a) marginal distributions $f(x)$ and $g(y)$, (b) $E(x)$ and $E(Y)$, (c) $Cov(x,y)$ (d) $\sigma x, \sigma y$ and (e) $\int(x,y)$ for the following distribution. (f) are x and y independent random variables?

x \ y	-4	2	7
1	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{8}$
5	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{8}$

2. Find the joint distribution of x and y, which are independent random variables with the following respective distributions.

x_i :	1	2
$f(x_i)$:	0.7	0.3

and

y_j :	-2	5	8
$g(y_j)$:	0.3	0.5	0.2

3. Determine (a) marginal distributions of x and y (b) $cov(x,y)$ (c) $\int(x, y)$ for the following joint distribution (d) determine where x and y are independent.

x \ y	-3	2	4
1	0.1	0.2	0.2
3	0.3	0.1	0.1

4. If x and y are independent random variables, find the joint distribution of x and y with the following marginal distribution of x and y.

x_i :	1	2
$f(x_i)$:	0.6	0.4

and

y_j :	5	10	15
$g(y_j)$:	0.2	0.5	0.3

5. Given the joint distribution:

y \ x	0	1	2
0	0.1	0.4	0.1
1	0.2	0.2	0

(a) determine the marginal distributions of x and y.

6. Find the marginal distributions of x and y and find $p(y = 3, x = 2)$, if the joint distribution is:

y \ x	1	2	3
1	0.05	0.05	0.1
2	0.05	0.1	0.35
3	0	0.2	0.1

Markov Chains:

1. Show that $p = \begin{bmatrix} 0 & 3/4 & 1/4 \\ 1/2 & 1/2 & 0 \\ 0 & 1 & 0 \end{bmatrix}$ is a regular stochastic matrix. Also find associated unique fixed probability vector.

2. Find the unique fixed probability vector of each matrix:

   $(1)\ A = \begin{bmatrix} 2/3 & 1/3 \\ 2/5 & 3/5 \end{bmatrix}$ $(2)\ B = \begin{bmatrix} 1/4 & 3/4 \\ 5/6 & 1/6 \end{bmatrix}$ $(3)\ C = \begin{bmatrix} 0.2 & 0.8 \\ 0.5 & 0.5 \end{bmatrix}$

   $(4)\ A = \begin{bmatrix} 0 & 1/2 & 1/2 \\ 1/3 & 2/3 & 0 \\ 0 & 1 & 0 \end{bmatrix}$ $(5)\ B = \begin{bmatrix} 0 & 1 & 0 \\ 1/2 & 0 & 1/2 \\ 1/2 & 1/4 & 1/4 \end{bmatrix}$

3. Show that $p = \begin{bmatrix} 0 & 1 \\ 1/2 & 1/2 \end{bmatrix}$ is a regular stochastic matrix. Also find associated unique fixed probability vector.

4. For a Markov chain, the transition matrix $p\begin{bmatrix} 1/2 & 1/2 \\ 3/4 & 1/4 \end{bmatrix}$ with initial distribution $p^{(0)} = \left(1/4, 3/4\right)$.

   Find $(1)\ p_{21}^{(2)}$ $(2)\ p_{12}^{(2)}$ $(3)\ p^{(2)}$ $(4)\ p_1^{(2)}$

5. A salesman's territory consists of 3 cities A,B and C. He never sells in the same city on successive days. If he sells in city A, then the next day he sells in city B. However if he sells in either B or C, then the next day he is twice as likely to sell in city A as in other city. In the long run, how often does he sell in each of the cities.

6. A man's smoking habit are as follows. If he smokes filter cigarettes one week, he switches to nonfilter cigarettes the next week with probability 0.2, on the other hand if he smokes nonfilter cigarettes one week, then is a probability of 0.7 that he will smoke nonfilter cigarettes the next week as well. In the long run how often does he smoke filter cigarettes.