U.S.N.										

B.M.S. College of Engineering, Bengaluru-560019

Autonomous Institute Affiliated to VTU

April 2023 Semester End Main Examinations

Programme: B.E. — Semester: III

Branch: CSE/ISE — Duration: 3 hrs.

Course Code: 22MA3BSSDM — Max Marks: 100

Course: Statistics and Discrete Mathematics — Date: 10.04.2023

Instructions: 1. Answer any FIVE full questions, choosing one full question from each unit.
2. Missing data, if any, may be suitably assumed.
3. Use of Statistical tables is permitted.

Important Note: Completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages. Revealing of identification, appeal to evaluator will be treated as malpractice.

UNIT - I

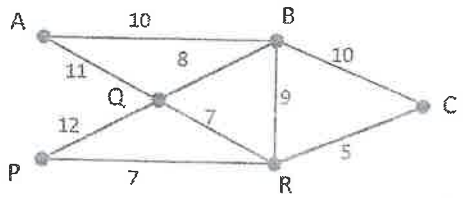
1 a) Determine |V| for the following graphs. — 6

i) G has nine edges and all vertices have degree 3.

ii) G is regular with 15 edges.

b) Define isomorphism of two graphs. Determine whether the two graphs given below are isomorphic or not. — 7

c) Apply Kruskal's algorithm to find a minimal spanning tree for the weighted graph shown below. — 7



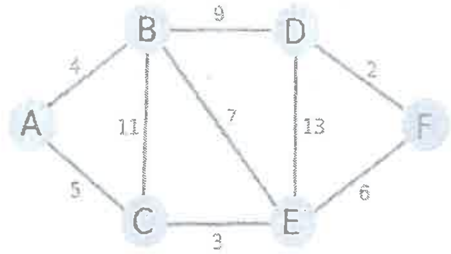
OR

2 a) Prove that a connected graph G remains connected after removing an edge e from G if and only if e is a part of some cycle in G. — 6

b) Obtain the incidence matrix for the graph whose adjacency matrix is given below. — 7

$$X(G) = \begin{array}{c} \\ a \\ b \\ c \\ d \\ e \end{array} \begin{array}{c} \begin{array}{ccccc} a & b & c & d & e \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 \end{bmatrix} \end{array}$$

c) Apply Dijkstra's algorithm to obtain the shortest path from vertex A to each of the other vertices in the weighted, directed network shown below. 7



UNIT - II

3 a) Find the coefficient of 6

i) x^9y^3 in the expansion of $(2x - 3y)^{12}$.

ii) $a^2b^3c^2d^5$ in the expansion of $(a + 2b - 3c + 2d + 5)^{16}$.

b) In how many ways can the 26 letters of the English alphabet be permuted so that none of the patterns CAR, DOG, PUN or BYTE occurs? 7

c) By using the expansion formula, find the rook polynomial for the labelled board shown below. 7

1	2	3
4		5
6	7	8

UNIT - III

4 a) If P is the pull required to lift a load W by means of a pulley block, find a linear law of the form $P = mW + c$ connecting P and W for the following data: 6

P	12	15	21	25
W	50	70	100	120

where P and W are taken in kg. Compute P when W=150 kg.

b) Obtain the regression lines and hence find the coefficient of correlation for the following data: 7

x	1	2	3	4	5	6	7
y	10	12	16	28	25	36	41

c) In a normal distribution, 31% of the items are under 45 and 8% are over 64. Find the mean and standard deviation of the distribution. 7

UNIT - IV

5 a) In a random sample of 100 tube lights produced by company A, the mean lifetime (mlt) of tube light is 1190 hours with standard deviation of 90 hours. Also, in a random sample of 75 tube lights from company B the mean lifetime is 1230 hours with standard deviation of 120 hours. Is there a difference between the mean lifetimes of the two brands of tube lights at a significance level of 0.05? 6

b) A certain stimulus administered to each of the 12 patients resulted in the following change in blood pressure: 5, 2, 8, -1, 3, 0, 6, -2, 1, 5, 0 and 4. Can it be concluded that the stimulus will increase the blood pressure?

c) According to a theory in genetics, the proportion of beans of four types A, B, C and D in a generation should be 9:3:3:1. In an experiment, among 1600 beans, the frequency of beans of each of the above four types were 882, 313, 287 and 118 respectively. Does the result support the theory? 7

OR

6 a) Two samples of sizes 9 and 8 give the sum of squares of deviations from their respective means is equal to 160 inches^2 and 91 inches^2 respectively. Can these be regarded as drawn from the normal population with equal variances? 6

b) A company claims that the mean thermal efficiency of diesel engines produced by them is 32.3. To test this claim, a random sample of 40 engines were examined which showed the mean thermal efficiency of 31.4 and standard deviation of 1.6. Can the claim be accepted or not at 0.01 level of significance? 7

c) A random sample of specimens of coal from two mines A and B were drawn and their heat producing capacity (in millions of calories/ton) were measured yielding the following results: 7

Mine A	8350	8070	8340	8130	8260	-
Mine B	7900	8140	7920	7840	7890	7950

Is there a significant difference between the means of these two samples at 1% level of significance?

UNIT - V

7 a) Solve the linear congruence equation $9x \equiv 21(mod 30)$. 6

b) Solve the system of linear congruences $x \equiv 2(mod 3)$, $x \equiv 3(mod 5)$ and $x \equiv 2(mod 7)$. 7

c) Apply Fermat's Little theorem to find the remainder when 24^{1947} is divided by 17. 7

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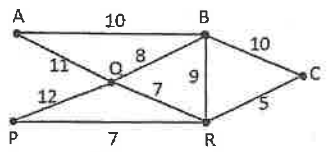
BMS College of Engineering, Bangalore – 560019
(Autonomous Institute, Affiliated to VTU, Belgaum)
April 2023 – SEE Examinations

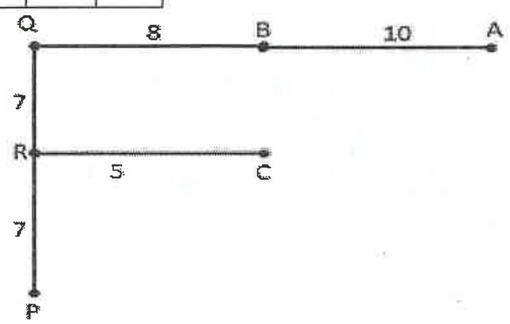
Exam: **BE** Branch : **CSE/ISE** Sem : **III**
Course: **STATISTICS AND DISCRETE MATHEMATICS** Course Code : ~~19MA3BSSDM~~ 22MA3BSSDM
Total Number of Pages submitted: 9
Signature of the Scheme Setter : Signature of the H.O.D.
Date: 08-05-2023 Date: 08-05-2023

Q. No	UNIT-1	Marks
1.a	Determine the order $\lvert V \rvert$ of the graph G = (V, E) in the following cases (i) G has nine edges and all vertices have degree 3. (ii) G is regular with 15 edges	**6M**
Soln	According to handshaking property, $\sum \deg(V) = 2m$ Let there be 'n' vertices (i) Degree of each vertex is 3 and given $m = 9$ Hence $n = 6$ (ii) G is a regular graph with 15 edges $nk = 2X15$ n should be a divisor of 30 i.e. n = 2,3,5,6,15 and 30	**1M** **2M** **3M**
1.b	Define isomorphism of two graphs. Determine whether the two graphs given below are isomorphic or not.	**7M**
Soln	Definition- one to one correspondence of both vertex and edge set. The given graphs are not isomorphic as there is no vertex correspondence between the two graphs. Graph 1 has 2 vertices of degree 3 each and graph 2 has all the vertices of degree 3.	**3M** **4M**
1.c	Apply Kruskal's algorithm to find a minimal spanning for the weighted graph shown below.	



Soln **4M** **3M**

Edge	RC	RQ	PR	QB	BR	BC	AB	AQ	PQ
Weight	5	7	7	8	9	10	10	11	12
Select	Y	Y	Y	Y	N	N	Y	N	N



OR

2.a Prove that a connected graph G remains connected after removing an edge *e* from G if and only if *e* is a part of some cycle in G. **6M**

Soln Suppose *e* is a part of some cycle *C* in *G*. Then the end vertices of *e* are joined by at least two paths, one of which is *e* and the other *C-e*. Hence the removal of *e* from G will not affect the connectivity of G; because even after the removal of *e* the end vertices of *e* remain connected through the path *C-e*. **3M**

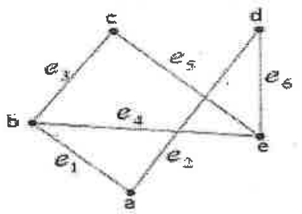
Conversely, suppose *e* is not a part of any cycle in *G*. Then the end vertices of *e* are connected by at most one path. Hence the removal of *e* from G disconnects these end points. This means that *G-e* is a disconnected graph. Thus, if *e* is not a part any cycle in *G* then *G-e* is disconnected. This is equivalent to saying that if *G-e* is connected then *e* belongs to some cycle in *G*. Hence proved. **4M**

2.b Obtain the incidence matrix for the graph whose adjacency matrix is given below. **7M**

$$\begin{array}{c} \\ a \\ b \\ c \\ d \\ e \end{array}\begin{array}{c} \begin{array}{ccccc} a & b & c & d & e \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 \end{bmatrix} \end{array}$$

Soln

GRAPH

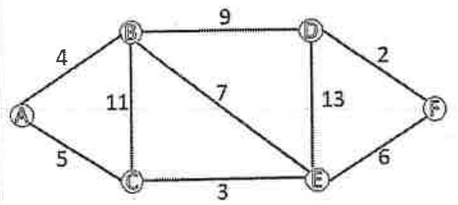


ADJACENCY MATRIX

$$\begin{array}{c} \\ a \\ b \\ c \\ d \\ e \end{array}\begin{array}{c} \begin{array}{cccccc} e_1 & e_2 & e_3 & e_4 & e_5 & e_6 \end{array} \\ \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 \end{bmatrix} \end{array}$$

3M+4M

2.c Apply Dijkstra's algorithm to obtain the shortest path from vertex A to each of the other vertices in the weighted network shown below. **7M**



Soln

	A	B	C	D	E	F
A	**0**	∞	∞	∞	∞	∞
B	0	**4**	5	∞	∞	∞
C	0	4	**5**	13	11	∞
E	0	4	5	13	**8**	∞
D	0	4	5	**13**	8	14
F	0	4	5	13	8	**14**

4M

Shortest paths from vertex A to other vertices

$A -> C -> E -> F = 14$

$A -> C -> E = 8$

$A -> C = 5$

$A -> B -> D = 13$

$A -> B = 4$

3M

<table>
<tr><td></td><td>UNIT-2</td><td></td></tr>
<tr><td>3.a</td><td>Find the coefficient of
(i) x^9y^3 in the expansion of $(2x-3y)^{12}$
(ii) $a^2b^3c^2d^5$ in the expansion of $(a+2b-3c+2d+5)^{16}$</td><td>6M</td></tr>
<tr><td>Soln</td><td>The general term in the expansion of $(2x-3y)^{12} = \sum_{r=0}^{12}\binom{12}{r}(2x)^{12-r}(-3y)^r$

For $r=3,$ the coefficient of $x^9y^3 \, is \; -2^9X3^3X\binom{12}{3}$ =1946

The general term in the expansion of $(a+2b-3c+2d+5)^{16} =$
$\binom{16}{n_1,n_2,n_3,n_4,n_5}(a)^{n_1}(2b)^{n_2}(-3c)^{n_3}(2d)^{n_4}(5)^{n_5}$
Comparing the expression with $a^2b^3c^2d^5$,we get
$n_1=2, n_2=3, n_3=2, n_4=5 \to n_5=4$
Therefore, the required coefficient is $\binom{16}{2,3,2,5,4}(1)^2(2)^3(-3)^2(2)^5 5^4$</td><td>1M

1M

1M

1M

2M</td></tr>
<tr><td>3.b</td><td>In how many ways can the 26 letters of the English alphabets be permuted so that none of the pattern CAR, DOG, PUN or BYTE occurs?</td><td>7M</td></tr>
<tr><td></td><td>S – set of all permutations of the letters $\Rightarrow |S| = 26!$
$|A_1|$-number of permutations of the word CAR $\Rightarrow |A_1| = 24!$
$|A_2|$-number of permutations of the word DOG $\Rightarrow |A_2| = 24!$
$|A_3|$ -number of permutations of the word PUN $\Rightarrow |A_3| = 24!$
$|A_4|$ -number of permutations of the word BYTE $\Rightarrow |A_4| = 23!$
$|A_1 \cap A_2| = (22)! = |A_1 \cap A_3| = |A_3 \cap A_2|; |A_1 \cap A_4| = |A_{12} \cap A_4| = |A_3 \cap A_4| = (21)!$
$|A_1 \cap A_2 \cap A_3| = (20)!\,; |A_1 \cap A_4 \cap A_3| = (19)!\,; |A_1 \cap A_2 \cap A_4| = (19)!;$
$|A_2 \cap A_4 \cap A_3| = (19)!$ And $|A_1 \cap A_2 \cap A_4 \cap A_3| = (17)!$
$|\overline{A_1 \cup A_2 \cup A_3}| =$
$= |S| - \sum |A_i| + \sum |A_i \cap A_j| - \sum |A_1 \cap A_2 \cap A_3| + \sum |A_1 \cap A_2 \cap A_3 \cap A_4|$
= 26! -{3X24! + (23!)} + 3{22! + 21!}-{20! + 3X19!}+17!</td><td>1M

2M

2M

2M</td></tr>
<tr><td>3.c</td><td>By using the expansion formula, find the rook polynomial for the labelled board shown below:
<table><tr><td>1</td><td>2</td><td>3</td></tr><tr><td>4</td><td></td><td>5</td></tr><tr><td>6</td><td>7</td><td>8</td></tr></table></td><td>7M</td></tr>
<tr><td>Soln</td><td>Expansion formula: $r(C,x) = xr(D,x) + r(E,x)$ where
D is the board obtained from C by deleting the row and column containing the * cell and
E is the board obtained from C by deleting only the * cell.
Thus, for the given board, the rook polynomial is
Obtaining $r(D,x)$
Obtaining $r(E,x)$
$\boxed{r(C,x) = 1+8x+14x^2+4x^3}$</td><td>1M

2M
2M
2M</td></tr>
</table>

UNIT-3

4.a If P is the pull required to lift a load W by means of a pulley block, find a linear law of the form $P = mW + c$ connecting P and W using the following data. Also, compute P when W =150kg **6M**

P	12	15	21	25
W	50	70	100	120

Soln

$P = mW + c$

Normal Equations: $\sum P = m\sum W + nc$

$$\sum PW = m\sum W^2 + c\sum W$$ **1M**

P	W	W^2	PW
12	50	2500	600
15	70	4900	1050
21	100	10000	2100
25	120	14400	3000
73	**340**	**31800**	**6750**

3M

$P = 0.1879W + 2.2759$ **1M**

$\mathbf{P(150) = 30.4635}$ **1M**

4.b Obtain the regression lines and hence find the correlation co-efficient between x and y for the following data. **7M**

x	1	2	3	4	5	6	7
y	10	12	16	28	25	36	41

Soln

$$\overline{x} = \frac{\sum x}{n} = 5.5, \overline{y} = \frac{\sum y}{n} = 30.7$$

$$\sigma_x{}^2 = \frac{\sum(x_i - \overline{x})^2}{n} = 8.25 \Rightarrow \sigma_x = 2.8723$$

$$\sigma_y{}^2 = \frac{\sum(y_i - \overline{y})^2}{n} = 196.21 \Rightarrow \sigma_y = 14.007$$

$$b_{yx} = \frac{\sum(x-\overline{x})(y-\overline{y})}{\sqrt{\sum(x-\overline{x})^2}} = 4.686$$

$$b_{xy} = \frac{\sum(x-\overline{x})(y-\overline{y})}{\sqrt{\sum(y-\overline{y})^2}} = 0.197$$ **2M**

Regression equation of y on x is $(y - \overline{y}) = b_{yx}(x - \overline{x}) \rightarrow y = 4.686x + 4.927$ **2M**

Regression equation of x on y is $(x - \overline{x}) = b_{xy}(y - \overline{y}) \rightarrow x = 0.197y - 0.548$ **2M**

$r^2 = b_{yx}. b_{xy} \rightarrow r = 0.96$ **1M**

4.c	In a normal distribution, 31% of the items are under 45 and 8% are over 64. Find the mean and S.D. of the distribution.	**7M**
Soln	$X \sim N(\mu,\sigma^2); f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\left(\frac{x-\mu}{\sqrt{2}\sigma}\right)^2}, -\infty < x,\mu < \infty$ i)$P(x < 45) = 0.31 \rightarrow P(z < a) = 0.31$ where a= $\frac{45-\mu}{\sigma}$ From Normal table, we get a= -0.5 hence $\mu - 0.5\sigma = 45$ ii) $P(x > 64) = 0.08 \rightarrow P(z > b) = 0.08$ where b= $\frac{64-\mu}{\sigma}$ From Normal table, we get $b = 1.4$ hence $\mu + 1.4\sigma = 64$ Therefore $\mu = 50$ *and* $\sigma = 10$	**2M** **2M** **2M**
	UNIT-4	
5.a	In a random sample of 100 tube lights produced by company A, the mean lifetime (mlt) of tube light is 1190 hours with standard deviation of 90 hours. Also in a random sample of 75 tube lights from company B the mean lifetime is 1230 hours with standard deviation of 120 hours. Is there a difference between the mean lifetimes of the two brands of tube lights at a significance level of 0.05?	**6M**
Soln	X_1 –Lifetime of electric bulbs produced by manufacturer A. X_2 – Lifetime of electric bulbs produced by manufacturer B. H0 :$\mu_1 = \mu_2$, there is no difference between the mean lifetime of two brands H1: $\mu_1 \neq \mu_2$, there is a significant difference between the mean lifetime of two brands. Test statistic under H0 :$z = \frac{(\overline{x_1}-\overline{x_2})-(\mu_1-\mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1}+\frac{\sigma_2^2}{n_2}}} \sim N(0,1)$ Decision Rule: At $\alpha = 5\%$, H0 is accepted when $\|z\| < z_{\alpha/2}$=1.96 Numerical Computation:$n_1 = 100, \overline{x_1} = 1190, \widehat{\sigma_1} = s_1 = 90$ $n_2 = 75, \overline{x_2} = 1230, \widehat{\sigma_2} = s_2 = 120$ $z = -2.4209$ Conclusion: Reject H0 there is a significant difference between the mean lifetimes of two brands.	**1M** **2M** **2M** **1M**
5.b	A certain stimulus administered to each of the 12 patients resulted in the following change in blood pressure: 5, 2, 8, -1, 3, 0, 6, -2, 1, 5, 0 and 4. Can it be concluded that the stimulus will increase the blood pressure?	**7M**
Soln	X_1 – BP before administering stimulus. X_2 – BP after administering stimulus. **Null Hypothesis**: H0: $\mu_d \leq 0$ *or* $\mu_1 \leq \mu_2$ **Alternate Hypothesis** H1: $\mu_d > 0 or \mu_1 > \mu_2$ **Test statistic under H0:** $d = 5,2,8,-1,3,0,6,-2,1,5,0,4$ $\bar{d} = \frac{\sum d_i}{n} = 2.5833, S_d = \sqrt{\frac{\sum(d_i-\bar{d})^2}{n-1}} = 3.088$ Test statistic: $t = \frac{\bar{d}-\mu_d}{\frac{S_d}{\sqrt{n}}} = 2.8979$ **Critical value:** At $\alpha = 5\%, v = n-1 = 11 \Rightarrow t_{\alpha,v} = t_{0.05,11} = 2.201$ At $\alpha = 1\%, v = n-1 = 11 \Rightarrow t_{\alpha,v} = t_{0.01,11} = 2.718$	**1M** **3M** **1M**

	Decision: At 5% or 1% L.O.S, H_o is accepted if $t < t_{\alpha,v}$. $\Rightarrow 2.8979 > 2.201$ or $2.8979 > 2.718$ **Conclusion:** H_o is rejected; there is a significant increase in the BP after administering the stimulus.	1M 1M
5.c	According to a theory in genetics, the proportion of beans of four types A, B, C and D in a generation should be 9:3:3:1. In an experiment, among 1600 beans, the frequency of beans of each of the above four types were 882, 313, 287, and 118 respectively. Does the result support the theory?	7M
Soln	**Null Hypothesis**: H_0: Result support theory **Alternate Hypothesis** H_1: Result does not support theory Test statistic under H_0: $\chi^2 = \sum_i \frac{(O_i-E_i)^2}{N} \sim \chi^2_{(3,0.05)}$	1M

Beans	O	E	$\frac{(O-E)^2}{E}$
A	882	900	0.36
B	313	300	0.5633
C	287	300	0.5633
D	118	100	3.24
	1600	1600	4.7266

3M

Test statistic: $\chi^2 = \sum \frac{(O-E)^2}{E} = 4.7266$ — 1M

Critical value: Given, $\alpha = 5\%$, $k = 1, v = n - k = 3 \Rightarrow \chi^2_{\alpha,v} = \chi^2_{0.05,3} = 7.815$ — 1M

Decision: At 5% L.O.S, H_o is accepted if $\chi^2 < \chi^2_{\alpha,v}$. $\Rightarrow 4.7266 < 7.815$

Conclusion: H_o is accepted. — 1M

Result support theory.

OR

6.a	Two samples of sizes 9 and 8 give the sum of squares of deviations from their respective means is equal to 160 inches² and 91 inches² respectively. Can these be regarded as drawn from the normal population with equal variances?	6M

Soln

H_0: $\sigma_1^2 = \sigma_2^2$, Population variances are equal. — 1M

H_1: $\sigma_1^2 \neq \sigma_2^2$, Population variances are not equal.

Test statistic under H_0: $F = \frac{S_1^2}{S_2^2} \sim F_{(n_1-1,n_2-1)}$

Decision Rule: At $\alpha = 5\%$, H_0 is accepted when $F < F_{n_1-1,n_2-1,\alpha} = F_{8,7,0.05} = 3.73$

(OR)

$F_{n_1-1,n_2-1,1-\alpha} < F < F_{n_1-1,n_2-1,\alpha}$ *where* $F_{8,7,0.95} = \frac{1}{F_{7,8,0.05}} = \frac{1}{3.5} = 0.2857$ — 2M

$0.2857 < F < 3.73$

Numerical Computation:

$S_1^2 = \frac{\sum(x_i-\bar{x})^2}{n-1} = \frac{160}{8} = 20;\ S_2^2 = \frac{\sum(x_i-\bar{x})^2}{n-1} = \frac{91}{7} = 13$ — 2M

$F = \frac{S_1^2}{S_2^2} = 1.54$ — 1M

Conclusion: Accept H_0, Population variances are equal. — 1M

6.b	A company claims that the mean thermal efficiency of diesel engines produced by them is 32.3. to test this claim, a random sample of 40 engines were examined which showed the mean thermal efficiency of 31.4 and standard deviation of 1.6. can the claim be accepted or not at 0.01 level of significance?	**7M**
Soln	$X -$ Thermal efficiency of diesel engines. **Null Hypothesis**: H0: $\mu = 32.3$, mean thermal efficiency is 32.3 **Alternate Hypothesis**: $\mu \neq 32.3$, mean thermal efficiency is not equal to 32.3 Test statistic under H0 : $z = \frac{\bar{x}-\mu}{\frac{\sigma}{\sqrt{n}}}$ Given: $\overline{x} = 31.4; \hat{\sigma} = s = 1.6; n = 40$ $z = -3.558$ **Critical value:** $\alpha = 0.005; z_{\alpha/2} = 2.58$ **Decision:** At $\alpha = 0.05$, H_o is accepted if $\lvert z \rvert < z_{\alpha/2}$. **Conclusion:** H0 is rejected; $\mu \neq 32.3$; claim is false.	**1M** **2M** **2M** **1M**
6.c	Random samples of specimens of coal from two mines A and B are drawn and their heat producing capacity (in millions of calories/ton) were measured yielding the following results: *Mine A*: 8350, 8070, 8340, 8130, 8260, – *Mine B*: 7900, 8140, 7920, 7840, 7890, 7950 Is there significant difference between the means of these two samples at 1% L.O.S.?	**7M**
Soln	X_1 – Heat producing capacity of coal from mine A. X_2 – Heat producing capacity of coal from mine B. H0: $\mu_1 = \mu_2$, Heat producing capacity of coal from mine A and B is same. H1: $\mu_1 \neq \mu_2$, Heat producing capacity of coal from mine A and B is not same. Test statistic under H0 : $t = \frac{(\overline{x_1} - \overline{x_2})}{S\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}}$ Decision Rule: At $\alpha = 1\%$, H0 is accepted when $\lvert t \rvert < t_{\frac{\alpha}{2}, n_1+n_2-2} = t_{9,0.005} = 3.25$ Numerical Computation: $\overline{x}_1 = 8230; \hat{\sigma}_1{}^2 = s_1^2 = 12600; n_1 = 5$ $\overline{x}_2 = 7940; \hat{\sigma}_2{}^2 = s_2^2 = 9100; n_2 = 6$ $S^2 = \frac{n_1 s_1{}^2 + n_2 s_2{}^2}{n_1+n_2-2} = 114.3095^2; t = 4.1897$ Conclusion: Reject H0, Heat producing capacity of coal from mine A and B is not same.	**1M** **1M** **2M** **2M** **1M**
	UNIT-5	
7.a	Solve the linear congruence $9x \equiv 21(\text{mod}\,30)$.	**6M**
Soln	$a = 9,\ b = 21,\ m = 30$ *and* $\gcd(a,m) = \gcd(9,30) = 3$ Therefore, the given congruence has 3 solutions $3x \equiv 7(\text{mod}\,10)$ $3x - 7 = 10k \Rightarrow k = 3t - 7, t = 0,1,2$ Solutions: $x \equiv 9(\text{mod}\,30),\ x \equiv 19(\text{mod}\,30),\ x \equiv 29(\text{mod}\,30)$	**1M** **3M** **2M**

Table in 6.c:

Mine A	8350	8070	8340	8130	8260	–
Mine B	7900	8140	7920	7840	7890	7950

7.b	Solve the system of linear congruences $x \equiv 2(\bmod 3), x \equiv 3(\bmod 5), x \equiv 2(\bmod 7)$.	**7M**
Soln	Given, $x \equiv 2(\bmod 3)$, $x \equiv 3(\bmod 5)$, $x \equiv 2(\bmod 7)$ $a_1 = 2, a_2 = 3, a_3 = 2 \text{ and } m_1 = 3, m_2 = 5, m_3 = 7$ $M = 105,\ M_1 = 35,\ M_2 = 21,\ M_3 = 15$ $M_1 x_1 \equiv 1(\bmod m_1) \Rightarrow 35x_1 \equiv 1(\bmod 3) \Rightarrow x_1 \equiv 2(\bmod 3)$ $M_2 x_2 \equiv 1(\bmod m_2) \Rightarrow 21x_2 \equiv 1(\bmod 5) \Rightarrow x_2 \equiv 1(\bmod 5)$ $M_3 x_3 \equiv 1(\bmod m_3) \Rightarrow 15x_3 \equiv 1(\bmod 7) \Rightarrow x_3 \equiv 1(\bmod 7)$ $\boxed{x \equiv (M_1 a_1 x_1 + M_2 a_2 x_2 + M_3 a_3 x_3)(\bmod M) \equiv 233(\bmod 105) \equiv 23(\bmod 105)}$	**1M** **1M** **3M** **2M**
7.c	Apply Fermat's theorem to find the remainder when 24^{1947} is divided by 17.	**7M**
Soln	By Fermat's little theorem, $a^{p-1} \equiv 1(\bmod p) \Rightarrow 24^{16} \equiv 1(\bmod 17)$ Express 1947 in terms of $16 \Rightarrow 1947 = 121 \times 16 + 11$ $24^{1947} \equiv 24^{11}(\bmod 17)$ **<u>To find the remainder for</u>** $24^{11}(\bmod 17)$ $24 \equiv 7(\bmod 17) \Rightarrow 24^{11} \equiv 14(\bmod 17)$ Therefore, 14 is the remainder when 24^{1947} is divided by 17.	**2M** **2M** **3M**

NOTE: Full marks to be rewarded for alternate methods.

COE