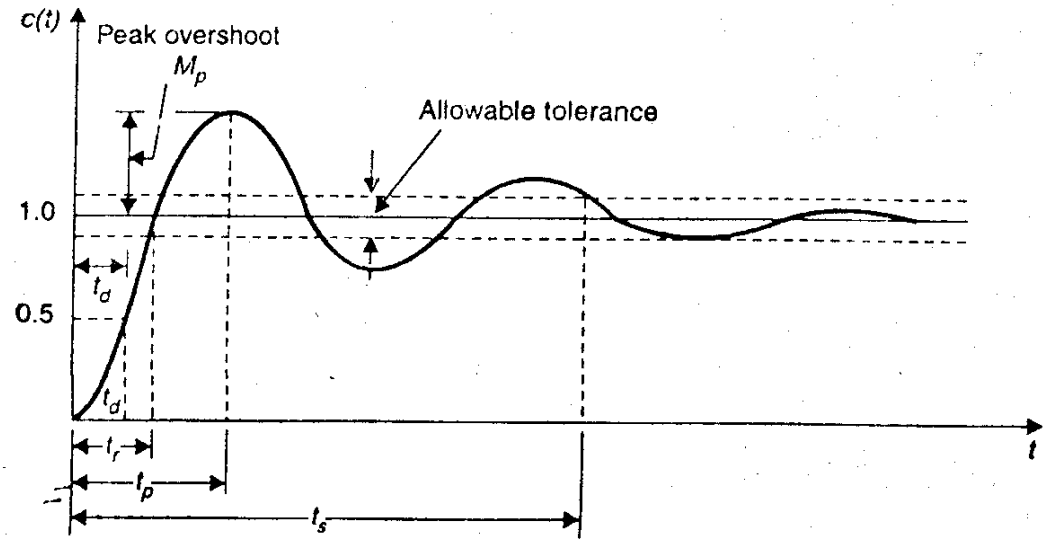
24-05-2021 CS IV Sem 'D'

Step response of first order system:

Response Specifications for step input



1. Delay time t_d : It is the time required for the response to reach 50% of its final value in first attempt.

2. Rise time t_r : It is the time required for the to rise from 10% to 90% of its final value

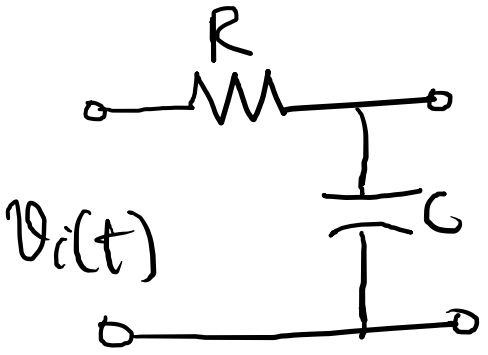
3 Peak time t_p: It is the time required for the response to reach its peak value.

4. peak overshoot M_p: It is the largest error b/n reference input and the output during the transient response

5. Settling time t_s: It is the time required for the response to settle or stay within a specified percentage (say ±2% or ±5%) of its final value

Step response of first order System:

Consider the circuit shown



Let $V_i(t)$ be step i/p

$V_i(t) = 1; t \geq 0$

$V_i(t) = 0; t < 0$

$$V_o(t) = \frac{1}{C}\int i\,dt \Rightarrow V_o(s) = \frac{I(s)}{Cs}$$

$$V_i(t) = iR + V_o(t) \Rightarrow V_i(s) = I(s)R + \frac{I(s)}{Cs}$$

$$T.F = \frac{V_o(s)}{V_i(s)} = \frac{\frac{I(s)}{Cs}}{I(s)\left(R+\frac{1}{Cs}\right)} = \frac{1}{RCs+1}$$

$\downarrow$

Pole at $s = -\frac{1}{RC}$

$v_i(t) \rightarrow$ step i/p

$\therefore V_i(s) = \frac{1}{s}$

$$V_o(s) = \frac{1}{RCs+1} \times \frac{1}{s} = \frac{1}{s(RCs+1)}$$

$$V_o(s) = \frac{A}{s} + \frac{B}{1+RCs} \Rightarrow A = 1$$

$$B = -RC$$

$$\frac{1}{s} - \frac{RC}{1+RCs} = \frac{1}{s} - \frac{1}{\frac{1}{RC}+s}$$

Taking Inverse Laplace Transform,

$$v_o(t) = 1 - e^{-t/RC}$$

→ transient term is totally dependant on RC & Rate of exponential decay is controlled by $-1/RC$, which is pole of the system