3. ANALYSIS OF STABILITY BY ROOT LOCUS PLOT

Gain Margin (GM)

- Gain margin(GM): The gain margin indicates by how much the system gain can be increased w/o making the system unstable
- The greater the Gain Margin (GM), the greater is the stability of the system

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GAIN MARGIN FROM ROOT LOCUS

$$\mathbf{GM} = \frac{\textbf{Marginal value of K}}{\textbf{Given value of K}} = \frac{\mathbf{K_C}}{\mathbf{K}}$$

If the Root locus crosses the jω axis and

If a (pair of) closed loop pole is located on jω axis, for a particular value of K, the response is oscillatory and the system is on the verge of instability, This value of K is called marginal value of K

If K is increased further roots travel to right half of s-plane and system becomes unstable

If the root locus does not cross the $j\omega$ axis, the system has infinite Gain Margin

TIME DOMAIN RESPONSE FROM ROOT LOCUS

- IF CLOSED LOOP POLES ARE REAL AND DISTINCT,THE RESPONSE IS OVER DAMPED AND $\zeta > 1$
- IF CLOSED LOOP POLES ARE PURELY IMAGINARY,THEY LIE ON $j\omega$ AXIS AND $\zeta=0$
- WHEN $0< \zeta<1$, THE POINT OF INTERSECTION OF THE ROOT LOCUS AND THE DAMPING RATIO LINE GIVES THE 2^{ND} ORDER COMPLEX POLES ($\zeta=\cos\theta$, θ MEASURED FROM –VE REAL AXIS)

TUTORIALS

1.Sketch the closed loop poles of the feedback control system with open loop transfer function

$$G(S)H(S) = \frac{K(S+5)}{S(S^2+4S+5)}$$

Find critical value of K and GM when K=10

1.solution

- Open loop poles are at $s=0, s=-2+j1, s=-2-j1$ and

$n=3$;

- Open loop zeros are at $s=-5; m=1$
- Section on real axis:from $s=0$ to $s=-5$,since for any point in this section,total no. of poles and zeros to its right is an odd number.
- No. of branches=3, one starting from each pole.
- One branch terminates on $s=-5$ and other two branches approach infinity. Hence No. of Asymptotes$=n-m=2$

- Angle of asymtotes is calculated using

$$\theta_i = \frac{(2i+1)}{|n-m|} \times 180^\circ \quad n \neq m$$

- i=0,1 angles are 90^c and 270^o
- Centroid= $\sigma_1 = \frac{\Sigma \text{finite poles of } G(s)H(s) - \Sigma \text{finite zeros of } G(s)H(s)}{n-m}$
- =-2-2-(-5)/(3-1)=0.5

- Break away points:obtained from solution of

$$\frac{dK}{ds} = 0$$

- here is no real solution to this. Hence no break away points
- Angle of departure:

$$\theta_D = 180 + \{\text{net contribution from the zeros and poles of } G(s)H(s) \text{ evaluated at the pole in question, excluding the contribution from that pole}\}$$

- Pole at s=-2+j1,=-45^o, pole at s=-2-j1= 45^o

- Imaginary axis crossing point.
- The Ch. Eqn is

$$s^3 + 4s^2 + (5+K)s + 5K = 0$$

forming routh array,

$s^3 : 1 \quad \mathrm{K}+5$

$s^2 : 4 \quad 5\mathrm{K}$

$\mathrm{s} : \dfrac{20 - \mathrm{K}}{4} \quad 0$

$s^0 : 5\mathrm{K}$

when $\mathrm{K} = 20$, there is a vanishing row. Hence $\mathrm{K_C} = 20$

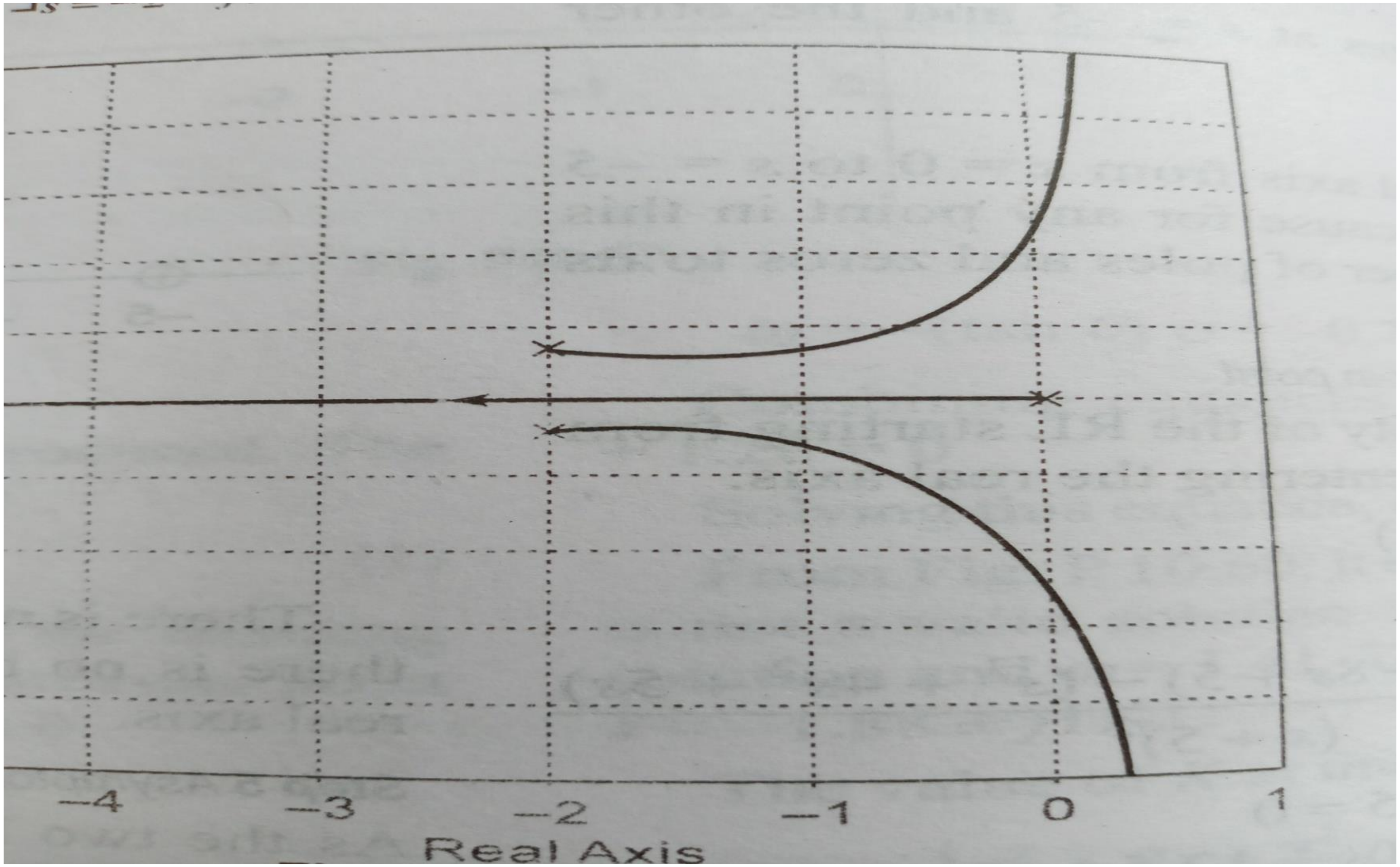
solving 2nd row $4s^2 + 5\mathrm{K_C} = 0$, imaginary axis crossing point is

$\mathrm{s} = \pm \mathrm{j}5$

Gain Margin

$$\mathbf{GM} = \frac{\textbf{Marginal value of K}}{\textbf{Given value of K}} = \frac{\mathbf{K_C}}{\mathbf{K}} = \frac{20}{10} = 2 = 6.02dB$$

Root locus of $G(S)H(S) = \dfrac{K(S+5)}{S(S^2+4S+5)}$



2. Construct the root locus for a feedback control system with open loop TF

$$G(s)H(s) = \frac{K}{s(s^2+6s+10)}$$

Show all the salient points on the sketch. Determine the value of K for which the closed loop poles are all real.

Solution

- Open loop poles are at $s=0, s= -3\pm j1; n=3$
- Open loop zeros nil; $m=0$
- There are 3 branches all which approach infinity.
- Sections on real axis: Entire –ve real axis is part of root locus
- There are $(3-0)=3$ Asymptotes
- Centroid$=(-3-3)-0/3=-2$, Angle of Asymptotes

$=(2i+1)180/(n-m)=60^{\circ}, 180^{\circ}, 300^{\circ}$

- Break away points are obtained using

$$\frac{dK}{ds} = 0$$

$$K = -(s^3 + 6s^2 + 10s)$$

$$\frac{dK}{ds} = -(3s^2 + 12s + 10) = 0,\ \text{solving, s} = \text{-}2 \pm 0.82 = \text{-}1.18, \text{-}2.82$$

- Angles of departure: s

$$\theta_D = 180 + \{\text{net contribution from the zeros and poles of G(s)H(s) evaluated at the pole in question, excluding the contribution from that pole}\}$$

- At s=-3+j1, 180+{0-90-161.57)}=-71.57°
- At s=-3-j1, 180+{0-(-90)-(-161.57)}=+71.57°

Imaginary axis crossing point

The characterstic equation is

$$s(s^2 + 6s + 10) + K = 0$$

$$= s^3 + 6s^2 + 10s + K = 0$$

forming routh array

$$s^3 : 1 \qquad 10$$

$$s^2 : 6 \qquad K$$

$$s \; : \frac{60 - K}{6} \quad 0$$

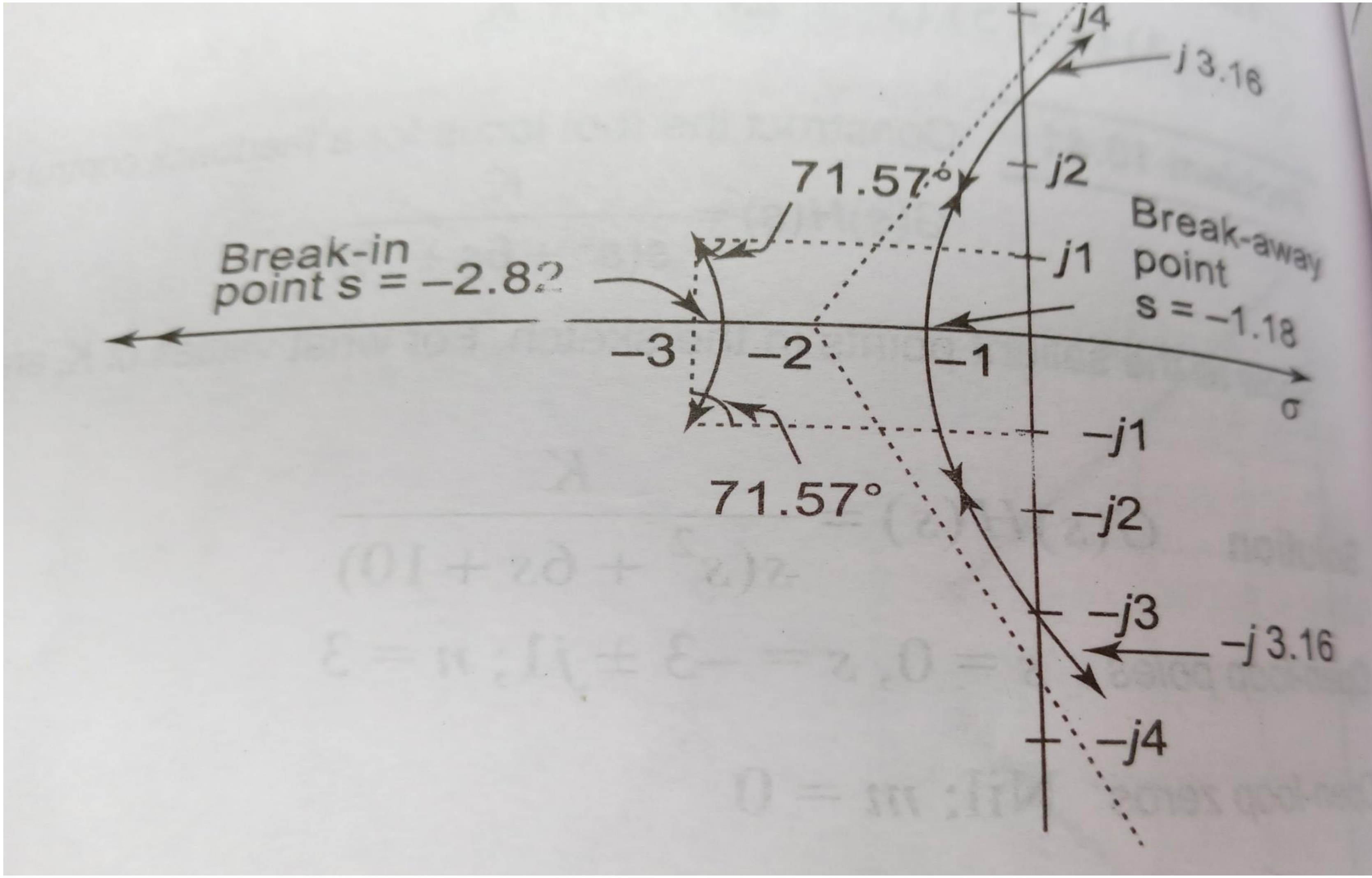
$$s^0 : K$$

when $K = 60$, there is a vanishing row,

$\therefore$ At $K = 60$, root locus crosses Imaginary axis,

Solving $6s^2 + 60 = 0$, we get $s = \sqrt{10} = \pm j3.16$

Construction



K for which all closed loop poles are real

- Lie between breakin and break away points
- At $s=-2.82$, $K=2.91$
- At $s=-1.18$, $K=5.09$

Closed loop poles are real for $2.91 \leq K \leq 5.09$