Latha H.N

(9). Receiver Model:

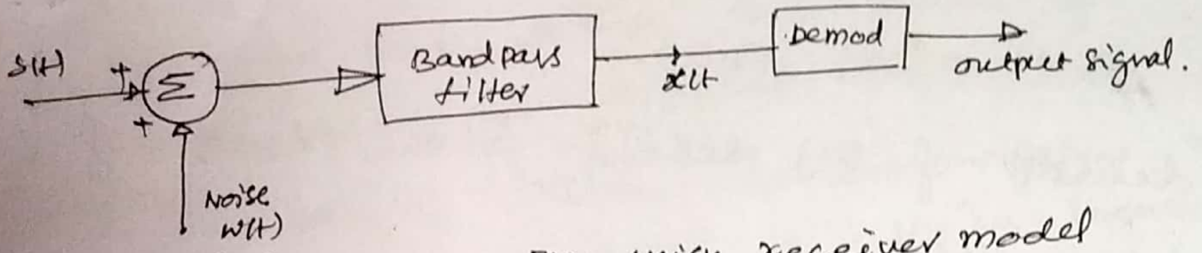

Fig: Noisy receiver model

In this figure, s(t) denotes the incoming modulated signal and w(t) denotes ~~the~~ front-end receiver noise.

→ The bandpass filter - represents the combined filtering action of the tuned amplifiers used in the actual receiver for the purpose of signal amplification prior to demodulation.

— The bandwidth of this band-pass filter is just wide enough to pass the modulated signal s(t) without distortion.

→ Let us assume noise w(t) is additive, white and Gaussian in nature.

→ P.S.D. of noise is w(t) is denoted by $N_0/2$ or $\eta/2$

where N_0 is the average noise power per unit bandwidth measured at the front-end of the receiver.

→ The filtered noise n(t) as a narrowband noise represented in the canonical form

$$n(t) = n_I(t)\cos(2\pi f_c t) - n_Q(t)\sin(2\pi f_c t).$$

where $n_I(t)$ is the in-phase noise component

$n_Q(t)$ is the quadrature noise component

.... with respect to the carrier wave

• $A_c \cdot \cos(2\pi f_c t)$. The filtered signal $x(t)$ available for demodulation is defined by

$$x(t) = s(t) + n(t).$$

The details of $s(t)$ depend on the type of modulation used.

$n(t)$ is a band-limited (narrowband noise) with the following power spectral density:

$$S_N(f) = \begin{cases} N_0/2 & f_c - B_T/2 \le |f| \le f_c + B_T/2 \\ 0, & \text{otherwise} \end{cases}$$

Figure of Merit:

$$\text{Figure of Merit} = \frac{(SNR)_O}{(SNR)_C}$$

where

$(SNR)_O$ = $\dfrac{\text{avg. power of message signal at the receiver output}}{\text{Avg. power of noise at the receiver output}}$

$(SNR)_C$ = $\dfrac{\text{Avg. power of modulated message signal at the receiver input}}{\text{avg. power of noise in message bandwidth at the receiver input}}$

For the purpose of comparing different modulation systems, we normalize the receiver performance by dividing the output ~~to~~ signal-to noise ratio by channel S-N.

Higher the value that FoM has, the better the noise performance of the receiver

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Noise in DSB-SC - Receivers:

Fig. shown below shows the Model of a DSB-SC receiver using a coherent detector:

- The use of coherent detection requires multiplication of the filtered signal $x(t)$ by a locally generated sinusoidal wave $\cos(2\pi f_c t)$ and then lowpass filtering the product.

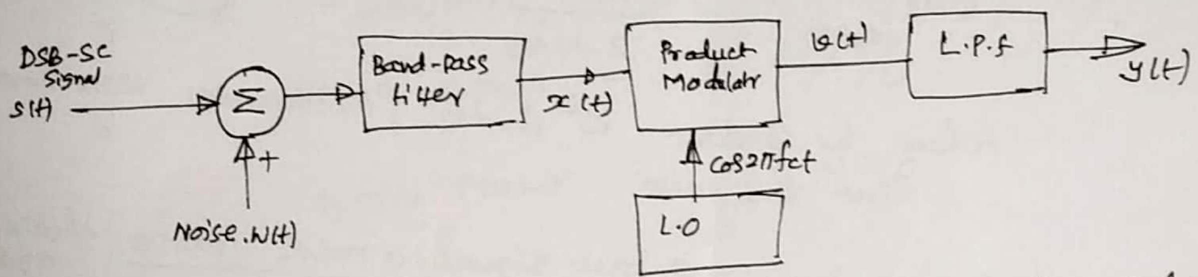

Fig:- Model of DSB-SC receiver using Coherent detection.

The DSB-SC Component of the filtered signal $x(t)$ is expressed as

$$s(t) = C.A_c \cos(2\pi f_c t)\, m(t). \quad \text{—— (1)}$$

Where $A_c.\cos(2\pi f_c t)$ is the sinusoidal carrier wave

$m(t)$ is the message signal.

C : a constant — a system-dependent scaling factor.

{ The purpose of which is to ensure that the signal component $s(t)$ is measured in the same units as the additive noise component $n(t)$.

- Assume that, $m(t)$ is a sample function of a stationary process of zero mean, whose power spectral density $S_m(f)$ is limited to a maximum freq W ~~by~~ where W (or f_m) is the message bandwidth

The Avg. power of the Message signal $m(t)$ $= P = \int_{-W}^{W} S_M(f)\,df$.

→ Avg. power of the DSB-SC modulated signal component

$s(t)$ — $\frac{c^2 A_c^2 P}{2}$, — ②

and Avg. Noise power in the Message band $= \frac{N_0}{2} \times 2W = \underline{\underline{W N_0}}$. — ③

from equation ② and ③ — ③.a

$$\therefore \quad (SNR)_{C,DSB} = \frac{c^2 A_c^2 P}{2 \cdot W N_0}$$

where the constant c^2 in the numerator ensures that this ratio.

→ Let us determine: output signal to noise ratio: of the system

$$x(t) = s(t) + n(t)$$

$$= C A_c \cos(2\pi f_c t)\, m(t) + n_I(t)\cos(2\pi f_c t) - n_Q(t)\sin 2\pi f_c t$$

where $n_I(t)$ and $n_Q(t)$ → are narrowband noise (In phase and quadrature components.

$$v(t) = x(t)\cos 2\pi f_c t$$

Product mod - output

$$= (C A_c \cos(2\pi f_c t)\, m(t))\cos 2\pi f_c t + n_I(t)\cos^2(2\pi f_c t) - n_Q(t)\cos 2\pi f_c t \sin 2\pi f_c t.$$

$$= \frac{1}{2} C A_c m(t) + \frac{1}{2} n_I(t) + \frac{1}{2}\left[C A_c m(t) + n_I(t)\right]\cos(4\pi f_c t) - \frac{1}{2} A_c n_Q(t) \sin$$

The Low-pass filter in the coherent detector removes the high freq. components of $v(t)$, yielding a receiver output

$$\boxed{y(t) = \frac{1}{2} C A_c m(t) + \frac{1}{2} n_I(t)} \quad — ④$$

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Equation (4) indicates the following:

1. The message signal $m(t)$ and in-phase noise component $n_I(t)$ of the filtered noise $n(t)$ appear additively at the receiver output.
2. The quadrature component $n_Q(t)$ of the noise $n(t)$ is completely rejected by the coherent detector.

— The message signal component at the receiver output is $C A_c m(t)/2$.

— Avg. power $= C^2 A_c^2 \cdot P/4$

noise power $= \frac{1}{2} W N_0$.

$$\therefore (SNR)_0 = \frac{\frac{C^2 A_c^2 P}{4}}{W N_0/2} = \frac{C^2 A_c^2 P}{2 W N_0} \quad — (5)$$

From equation (3.4) and 5

Here Figure of Merit $= \left.\frac{(SNR)_0}{(SNR)_C}\right|_{DSB} = \frac{\frac{C^2 A_c^2 P}{2WN_0}}{\frac{C^2 A_c^2 P}{2WN_0}} = \underline{\underline{1}}$

C^2 is common to both the output and channel signal to noise ratio's and therefore cancels out in evaluating the figure of merit.

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Noise in SSB Receivers:

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→ Consider the case of a receiver using coherent detection, with an incoming SSB.

→ Assume that only the Lower sideband is transmitted

$$s(t) = \frac{1}{2} C A_c \cdot \cos(2\pi f_c t) m(t) + \frac{1}{2} C A_c \cdot \sin(2\pi f_c t) \hat{m}(t) \quad \text{——} (1)$$

where $\hat{m}(t)$ is the H.T. of $m(t)$.

→ The in-phase and quadrature components of the modulated signal $s(t)$ contribute an avg. power of $C^2 A_c^2 P/8$ each where P is the Avg. power of the message signal $m(t)$.

→ Thus the Avg. power of $s(t)$ → $\frac{C^2 A_c^2 P}{4}$.

— Avg. noise power in the message bandwidth W is $\underline{\underline{WN_0}}$.

$$\therefore (SNR)_{C \cdot SSB} = \frac{C^2 A_c^2 P}{4 W N_0}.$$

llly.

$$(SNR)_{O, SSB} = \frac{C^2 A_c^2 P}{4 W N_0}.$$

$$F.O.M = \frac{(SNR)_O}{(SNR)_i} = \frac{C^2 A_c^2 P}{4 W N_0} \Big/ \frac{C^2 A_c^2 P}{4 W N_0} = 1.$$

∴ For the same avg. transmitted signal power and the same avg. noise power in the message bandwidth, an SSB ... will have exactly the same output signal to noise

Noise in AM. Rxs:

Let $s(t) = A_c[1 + k_a m(t)] \cos(2\pi f_c t)$.

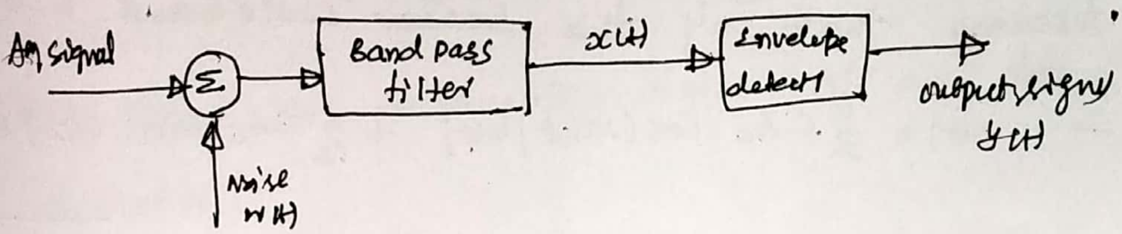


Fig: Noisy Model of AM Rxs

→ The Avg power of the carrier component in the AM signal $s(t)$ is $A_c^2/2$.

→ The Avg power of the information bearing component $A_c k_a m(t) \cos 2\pi f_c t$ is $A_c^2 k_a^2 P/2$. where P is the avg power of message signal $m(t)$.

∴ The avg power of the full AM signal $s(t)$ is therefore equal to $A_c^2(1 + k_a^2 P)/2$.

$$\therefore (SNR)_{C,AM} = \frac{A_c^2(1 + k_a^2 P)}{2WN_0}.$$

To evaluate the output signal-to-noise ratio, first represent the filtered noise $n(t)$ in terms of its inphase and quadrature components.

$\therefore x(t) = s(t) + n(t)$

$= [A_c + A_c k_a m(t)] \cos 2\pi f_c t + n_I(t) \cos 2\pi f_c t - n_Q(t) \sin(2\pi f_c t)$

Envelope of $x(t)$

$y(t) =$ envelope of $x(t)$

$$= \sqrt{\{A_c + A_c k_a m(t) + n_I(t)\}^2 + n_Q^2(t)}$$

The signal $y(t)$ defines the output of an ideal envelope detector.

- The expression defining $y(t)$ is somewhat complex and needs to be simplified in some manner in order to approximate the $y(t)$

→ we may approximate the output $y(t)$ ≈

$$y(t) \simeq \underline{A_c} + A_c k_a m(t) + n_I(t) \quad \text{— (A)}$$

The presence of the dc or constant term A_c in the envelope detector output $y(t)$ of equation (A) is due to demodulation of the transmitted carrier wave. → Hence the term may be neglected.

→ $y(t) \simeq \underline{A_c k_a m(t) + n_I(t)}$

↳ now it is similar DSB-SC Receiver Coherent detection

$$\therefore (SNR)_{O\,AM} \approx \frac{A_c^2 k_a^2 P}{2 W N_0} \quad \text{— (B)}$$

Equation - (B) is valid only if the following two conditions are satisfied.

1. the Avg noise power is small compared to the ~~average~~ avg carrier power at the

envelope detector output.

2. The amplitude sensitivity K_a is adjusted for a percentage modulation less than or equal to 100%

$$\frac{(SNR)_o}{(SNR)_c} = \frac{K_a^2 P}{1 + K_a^2 P}.$$

Ex: 1: for a single tone modulation:

let $m(t) = A_m \cdot \cos 2\pi f_m t$.

the corresponding AM wave is

$$s(t) = A_c \left[1 + \mu \cdot \cos 2\pi f_m t \right] \cos 2\pi f_c t.$$

where $\mu = K_a A_m$ is the modulation factor

$$P = \frac{1}{2} A_m^2$$

$$\frac{(SNR)_o}{(SNR)_c} = \frac{\frac{1}{2} K_a^2 A_m^2}{1 + \frac{1}{2} K_a^2 A_m^2} = \frac{\mu^2}{2 + \mu^2}$$

When $\mu = 1$, which corresponds to 100%, we get a figure of Merit equal to $\frac{1}{3}$.

→ This means that, other factors being equal, an AM system (using envelope detection) must transmit three times as much avg power of a ~~SC system~~ Suppressed-Carrier (DSB) system ~~SC~~ in order to achieve the same quality of noise performance.

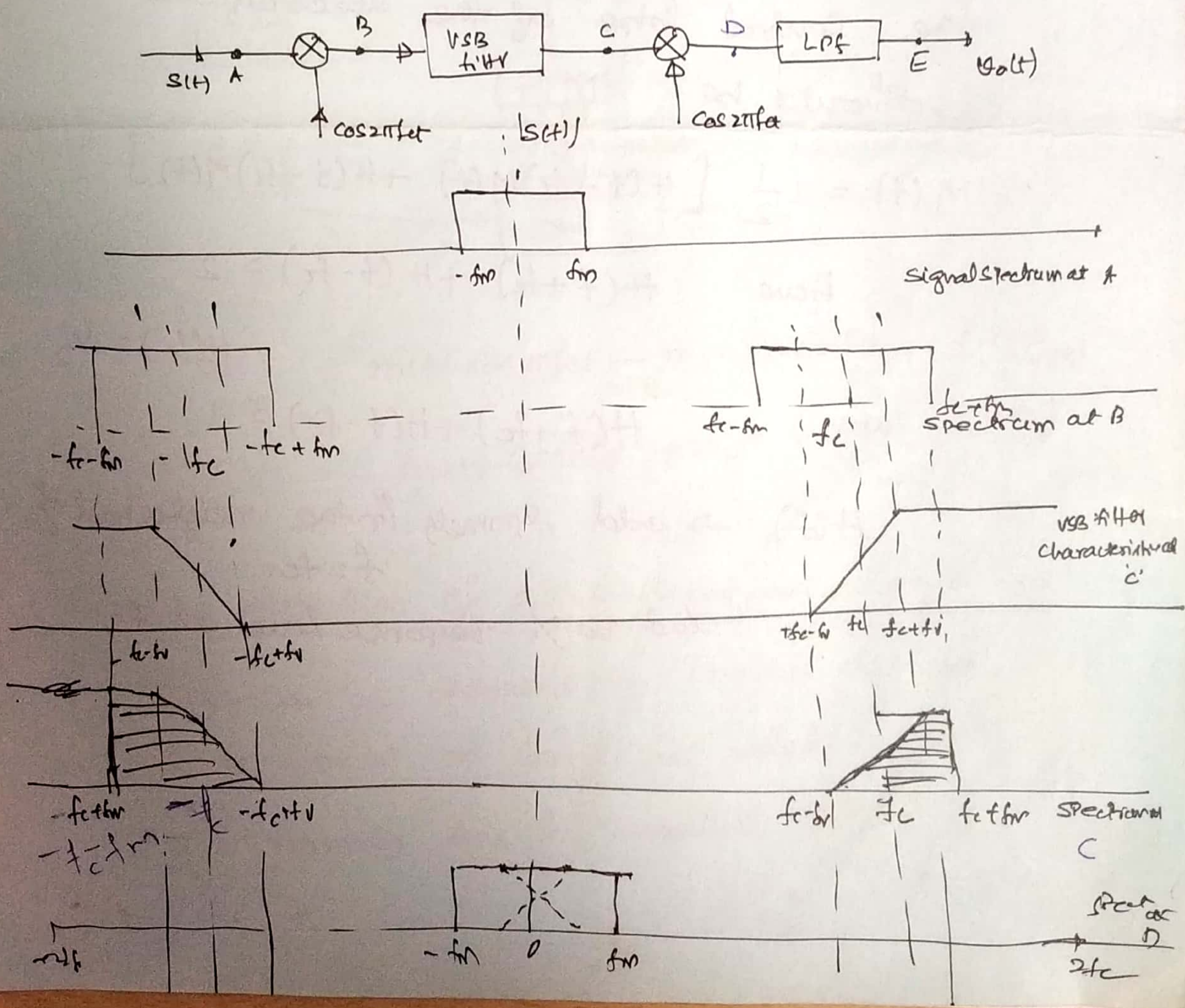
VSB: [Vestigial sideband transmission]

* SSB - suitable for voice signals.

* When the baseband signal contains significant component at extremely low freq. components {ex. T.V. and telegraph}

* The use of SSB modulation is inappropriate for the transmission of such baseband signals due to the difficulty of isolating one sideband

* VSB is used for T.V. Transmission.



Let us derive the transfer function for VSB filter

* Spectrum at A :- is Proportional to $M(f)$

* Spectrum at B : is proportional to

$$\frac{1}{2}\left[M(f-f_c)+M(f+f_c)\right]$$

* Spectrum at C :- $\frac{1}{2}H(f)\left[M(f-f_c)+M(f+f_c)\right]$

Spectrum at D : $\frac{1}{4}\cdot H(f-f_c)\left[M(f)+M(f-2f_c)\right]$

$$+\frac{1}{2}H(f+f_c)\left[M(f)+M(f+2f_c)\right]$$

the central lobe of the spectrum at D' should be $M(f)$

$$M(f)=\frac{1}{2}\left[H(f+f_c)M(f)+H(f-f_c)M(f)\right]$$

Here $H(f+f_c)+H(f-f_c)=2$

$$H(f_c)=\frac{1}{2}$$

$$H(f+f_c)+H(f-f_c)=1$$

$H(f)\rightarrow$ odd symmetry in the neighborhood of $f=f_c$.

and 50% response level at f_c.

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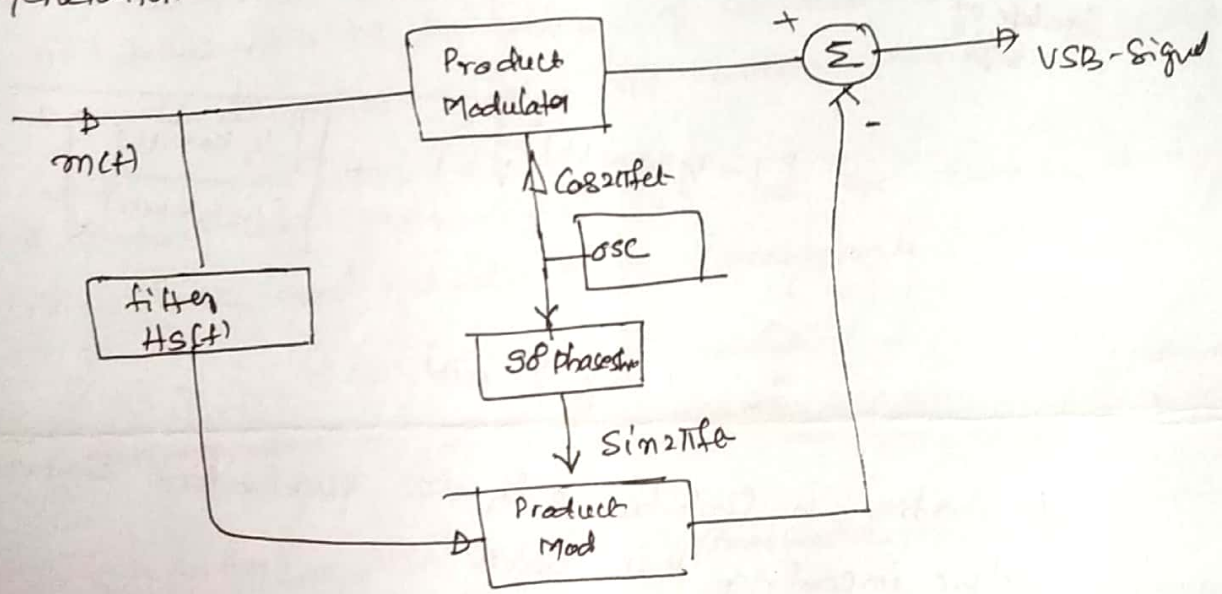
VSB signal can be represented in the time domain as

$$s(t) = m(t)\cos 2\pi f_c t - m_s(t)\sin 2\pi f_c t$$

where $m_s(t)$: output of the filter of transfer function $H_s(f)$. where

$$H_s(f) = j\left[H(f-f_c) - H(f+f_c)\right]$$

Generation:



$$s(t)_{VSB} = m(t)\cos 2\pi f_c t - m_s(t)\sin 2\pi f_c t \quad \left(\begin{matrix}V.S.B \\ +USB\end{matrix}\right)$$

$$= m(t)\cos 2\pi f_c t + m_s(t)\sin 2\pi f_c t \quad \left(\begin{matrix}VSB \\ -LSB\end{matrix}\right)$$

Envelope detection of a VSB signal wave + Carrier signal:

VSB can be detected by envelope detector, if a large amount of carrier is added.

This is normally done in commercial TV.

$$s(t) = \cos 2\pi f_c t + \frac{1}{2} k_a m(t) \cos 2\pi f_c t - \frac{1}{2} k_a m_s(t) \sin 2\pi f_c t$$

$$= \left[1 + \frac{1}{2} k_a m(t)\right] \cos 2\pi f_c t - \frac{1}{2} k_a m_s(t) \sin 2\pi f_c t$$

k_a → determines the % Modulation

$$a(t) = \sqrt{\left(1 + \frac{1}{2} k_a m(t)\right)^2 + \left[\frac{1}{2} k_a m(t)\right]^2}$$

↓
Envelope of sign

$$= \left[1 + \frac{1}{2} k_a m(t)\right] \sqrt{1 + \left[\frac{\left[\frac{1}{2} k_a m(t)\right]}{\left[1 + \frac{1}{2} k_a m(t)\right]}\right]^2}$$

$$\simeq 1 + \frac{1}{2} k_a m(t)$$

Distortion is contributed by the quadrature component of the incoming VSB wave

The distortion can be reduced by Reducing % Mod (ka)

Typical Q's: (1) suggest a suitable AM technique to transmit a message signal which contains significant components at extremely low frequency such as TV signal and provide a specifications of filter transfer function H(f) of a sideband shaping filter to extract the desigend modulated wave considering coherent detector output

(2) Justify the VSB signal + carrier can be demodulated using Envelope detector.

(3) Formulate the expression for VSB signal.

then, the envelope will be

(P) The single tone modulating signal $m(t) = A_m \cdot \cos(2\pi f_m t)$ is used to generate the following VSB signal:

$$s(t) = \frac{1}{2} \alpha A_m A_c [\cos[2\pi(f_c + f_m)t] + \frac{1}{2} A_m A_c (1-\alpha) \cos(2\pi(f_c - f_m)t)]$$

where α is constant, less than unity, representing the attenuation of the upper side freq.

(i) Determine the quadrature component of the VSB signal $s(t)$,

(ii) The VSB signal, plus the carrier $A_c \cdot \cos(2\pi f_c t)$, is passed through an envelope detector. Determine the distortion produced by the quadrature component.

(iii) What will be the value of constant 'α' for which this distortion reaches its worst possible condition.

→ We know that any signal $s(t)$ can be expressed in terms of inphase and quadrature components, as

(i)

$$s(t) = s_I(t) \cos 2\pi f_c t - s_Q(t) \sin 2\pi f_c t.$$

$\cos(A+B) = \cos A \cos B - \sin A \sin B$

$\cos(A-B) = \cos A \cos B + \sin A \sin B$

→ expanding the given signal $s(t)$

$$s(t) = \frac{1}{2} \alpha A_m A_m \{\cos(2\pi f_c t) \cos(2\pi f_m t) - \frac{1}{2} \alpha A_m A_c \sin(2\pi f_c t) \sin(2\pi f_m t)$$

$$+ \frac{1}{2}(1-a) A_c A_m \cdot \cos(2\pi f_c t) \cos(2\pi f_m t) + \frac{1}{2}(1-a) A_m A_c \sin(2\pi f_c t) \sin 2\pi f_m t.$$

$$s(t) = \frac{1}{2} A_m A_c \cos(2\pi f_c t) \cos(2\pi f_m t) + \frac{1}{2} A_m A_c (1-2\alpha) \sin(2\pi f_c t) \sin(2\pi f_m t)$$

Comparing:

$$s_Q(t) = -\frac{1}{2} A_c A_m (1-2\alpha) \sin(2\pi f_m t)$$

(ii) The VSB signal after adding carrier signal $A_c \cos(2\pi f_c t)$ will be

$$s(t) = A_c \left[1 + \frac{A_m}{2} \cdot \cos(2\pi f_m t)\right] \cos 2\pi f_c t + \frac{1}{2} A_c A_m (1-2\alpha) \sin(2\pi f_m t) \sin 2\pi f_c t$$

then, the envelope will be equal

$$a(t) = A_c \sqrt{\left[1 + \tfrac{1}{2} A_m \cdot \cos(2\pi f_m t)\right]^2 + \left[\tfrac{1}{2} A_m (1-2\alpha) \sin(2\pi f_m t)\right]^2}$$

$$a(t) = A_c \left[1 + \tfrac{1}{2} A_m \cos(2\pi f_m t)\right] \sqrt{1 + \left[\frac{\tfrac{1}{2} A_m (1-2\alpha) \sin(2\pi f_m t)}{1 + \tfrac{1}{2} A_m \cos(2\pi f_m t)}\right]^2}$$

or $a(t) = A_c \left[1 + \tfrac{1}{2} A_m \cdot \cos(2\pi f_m t)\right] d(t)$

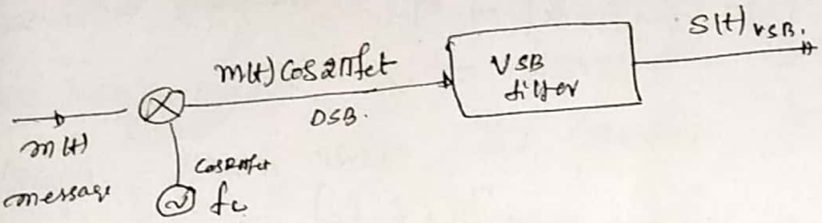
$\boxed{\text{where } d(t) = \text{distortion term}}$

$$d(t) = \sqrt{1 + \left[\frac{\tfrac{1}{2} A_m (1-2\alpha) \sin(2\pi f_m t)}{1 + \tfrac{1}{2} A_m \cos(2\pi f_m t)}\right]^2}$$

(iii) → The distortion will be maximum when $\alpha = 0$.

(9)

(P) Evaluate the condition for distortionless demodulation of a VSB signal, initialy generated by passing a DSB signal through a VSB filter, using synchronus detector



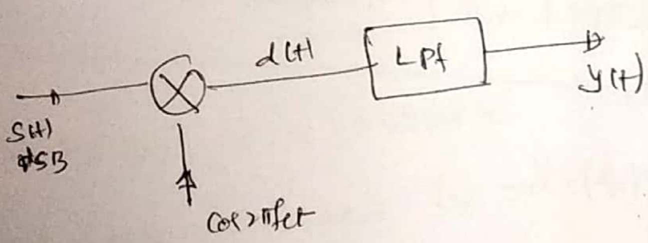
$$s(t)_{DSB} = m(t)\cos 2\pi f_c t.$$

$$\hookrightarrow \underset{\substack{DSB \\ \text{Spectrum}}}{S(f)} = \frac{1}{2}\left[M(f-f_c)+M(f+f_c)\right]$$

$H(f)_{VSB} \rightarrow$ Transfer function of VSB-filter

then $$S(f)_{VSB} = S(f)_{DSB} \cdot H(f)$$

$$S(f)_{VSB} = \frac{1}{2}\left[M(f-f_c)+M(f+f_c)\right] H(f) \quad \text{— (1)}$$



For demodulation using synchronus demodulator, we have

$$d(t) = s(t)_{VSB} \cdot \cos 2\pi f_c t$$

$\downarrow$ Taking fourier

$$D(f) = \frac{1}{2}\left[S_{VSB}(f-f_c) + S_{VSB}(f+f_c)\right] \quad \text{— (2)}$$

From equation (1) and (2).

$$S_{vsb}(f-f_c) = \frac{1}{2}\left[M(f-f_c-f_c) + M(f-f_c+f_c)\right] H(f-f_c)$$

$$S(f+f_c) = \frac{1}{2}\left[M(f+f_c-f_c) + M(f+f_c+f_c)\right] H(f+f_c)$$

$$D(f) = \frac{1}{2}\cdot\frac{1}{2}\left[\left[M(f-2f_c) + M(f)\right]H(f-f_c) + \left(M(f) + M(f+2f_c)\right) H(f+f_c)\right]$$

$$D(f) = \frac{1}{4}\left[M(f)\left[H(f-f_c) + H(f+f_c)\right] + M(f-2f_c)H(f-f_c) + M(f+2f_c)H(f+f_c)\right]$$

(eliminate the spectr at $2f_c$)

LPF $\to$ output

$$Y(f) = \frac{1}{4}\left[M(f)\right]\left[\underline{H(f-f_c) + H(f+f_c)}\right]$$

$$\boxed{\text{If } H(f-f_c) + H(f+f_c) = \text{constant} = K \text{ then}}$$

then $Y(f) = \frac{1}{4} M(f) \cdot K$

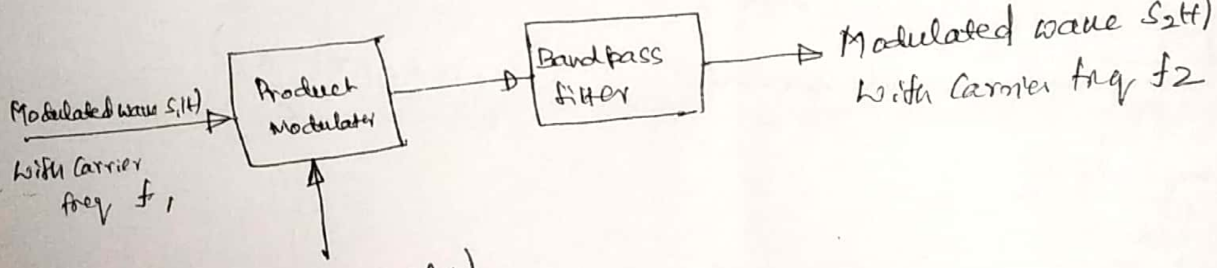
$= M(f) K'$

$$\boxed{y(t) = K' m(t)}$$

Frequency Translation: (Also called as freq mixing, freq changing)

* Suppose that we have a modulated wave $s_1(t)$ whose spectrum is centered on a carrier freq f_1, and the requirement is to translate it upward in frequency such that its carrier freq is changed from f_1 to a new value f_2.

→ This requirement may be accomplished using the mixer as shown below.



Block diagram of Mixer:

The Mixer is a device that consists of a product modulator followed by a bandpass filter.

— The BPF is designed to have a bandwidth equal to that of the modulated signal $s_1(t)$ used as input.

$$f_2 = f_1 + f_L$$

$$f_L = f_2 - f_1.$$

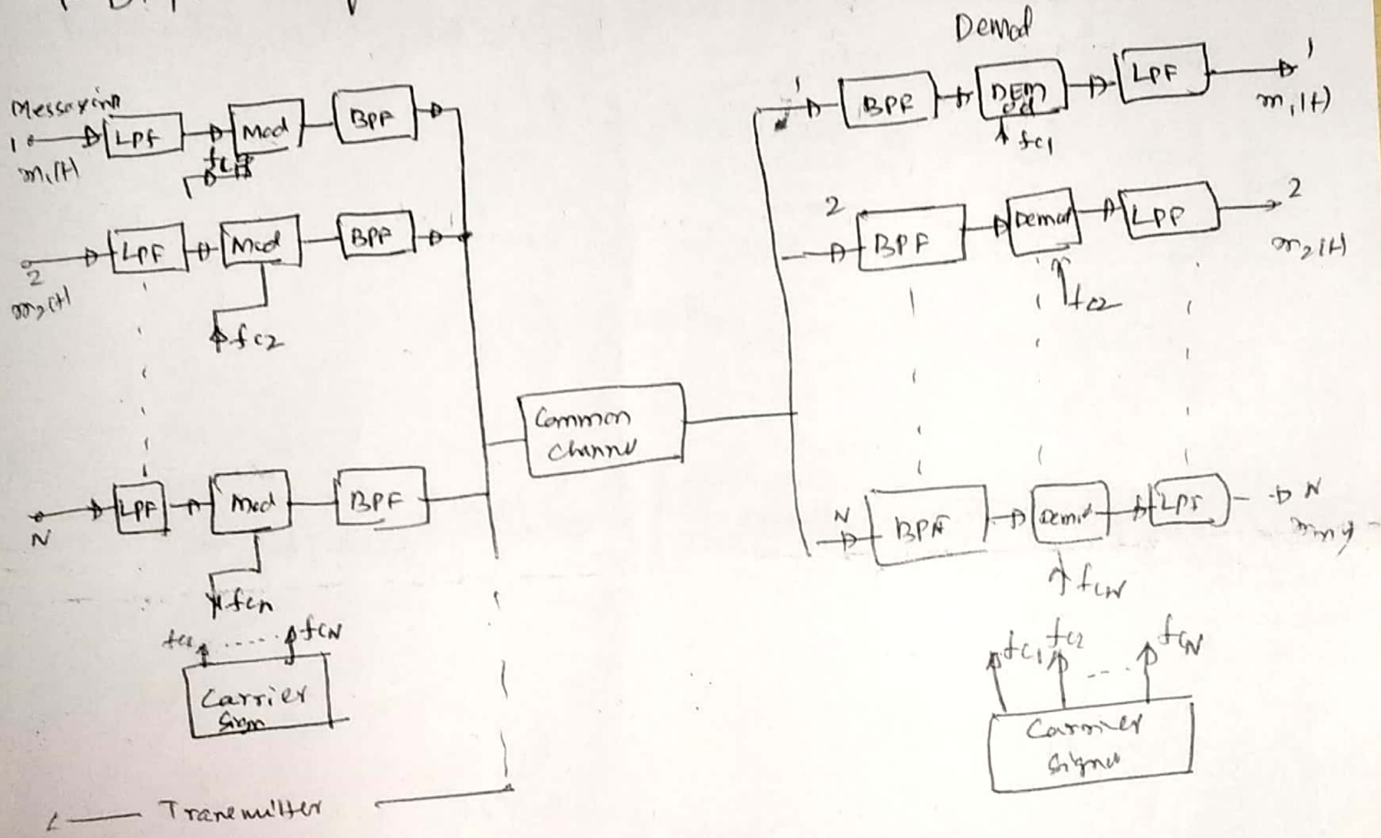
... translated

upward

if $f_1 > f_2$, $(f_c = f_1 - f_2)$. the carrier freq is

translated downward.

· FDM :- Freq. division Multiplex.



Block diagram of FDM System.