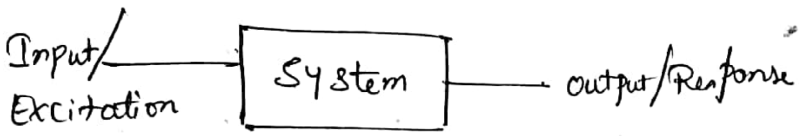
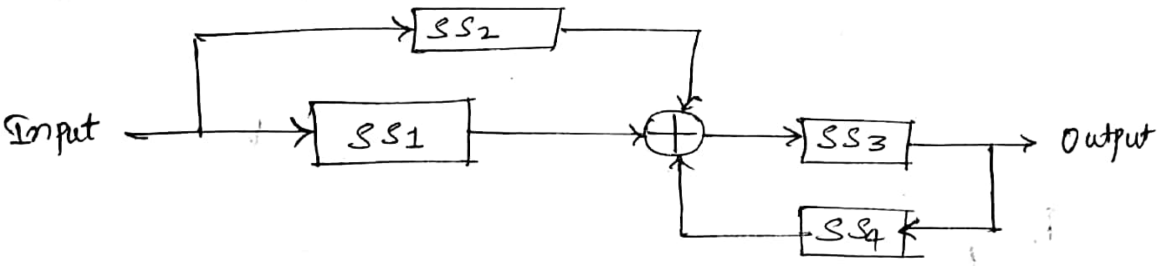
__System__: It is a collection of physical, biological (or) abstract components which together perform an intended objective.

- A system gives an output (Response) for an input (Excitation).



- A system can be a collection of multiple sub-systems:

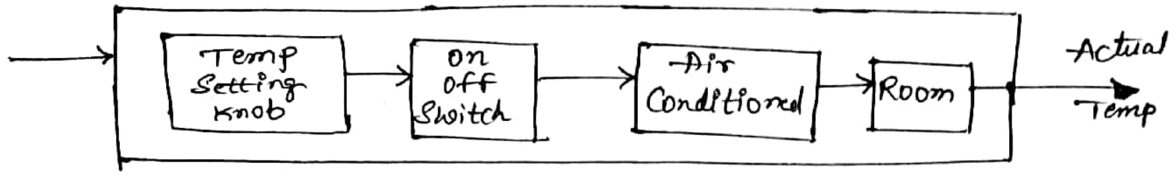


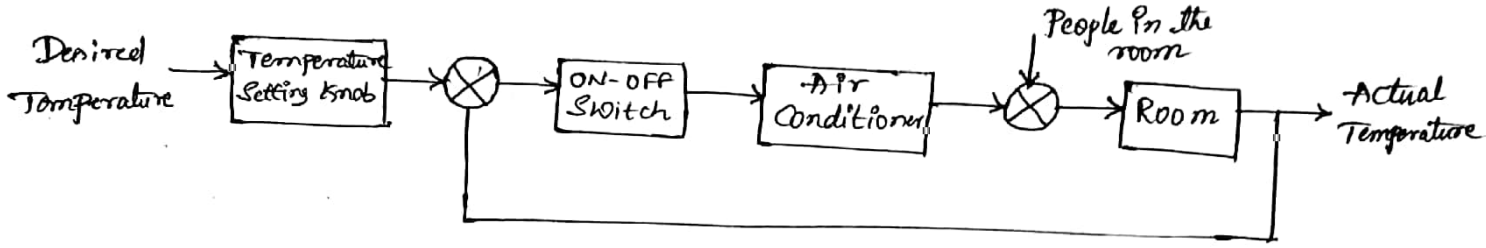
Eg: ① Motor ⟶ Electrical Energy (I/P) ⟶ Mechanical Energy (O/P)

② Air Conditioner ⟶ Electrical Energy (I/P) ⟶ Heat Energy (O/P) (changes the ambient Temperature).

③ Car (or) Bus (vehicle) ⟶ Acceleration (I/P) ⟶ vehicle Displacement (O/P)

① Examples of Control Systems: [Air Conditioner Maintaining Desired Temperature]





⇒ Plant —— Room.

Control S/m —— Air Conditioner.

Reference —— Desired Temperature.

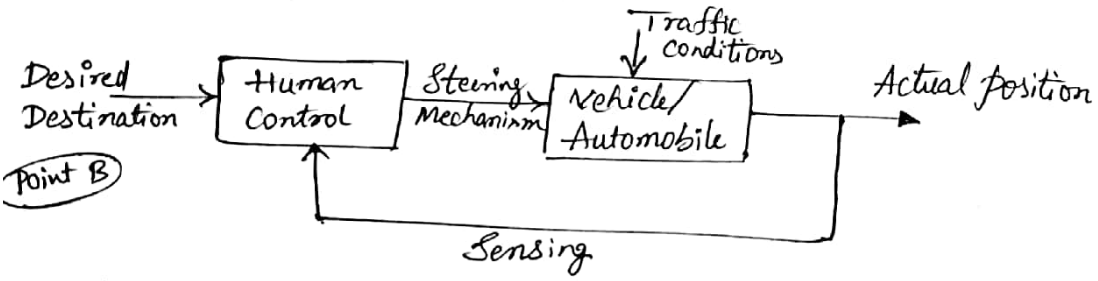
Control Input —— Compressor ON/OFF.

Output —— Output temperature.

Disturbance —— factors affecting ambient temperature.

Feedbacks —— Measured temperature.

② Human Steering an automobile. [Need to move from Point A to Point B]



⇒ Plant —— Vehicle (or) automobile.

Control S/m —— Human Control

Reference —— Desired Destination.

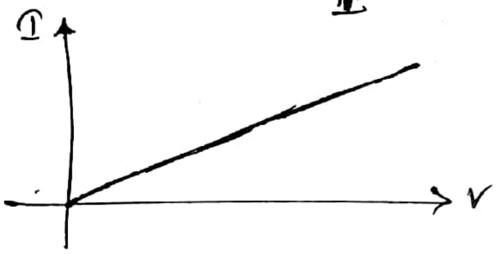
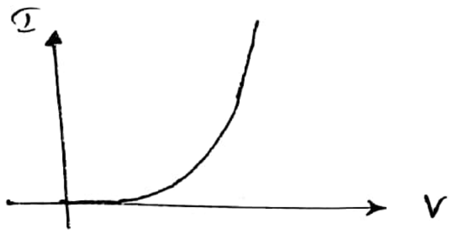
Control Input —— Steering Mechanism.

Output — Actual position

Disturbance — Traffic Conditions.

Classification of Systems:

(1) Linear & Non-Linear System :-

Linear	Non-Linear S/m
(1) Output of the System varies linearly with input.	(1) Output of the system does not vary linearly with input.
(2) Satisfy homogeneity & Superposition.	(2) Does not satisfy homogeneity & Superposition.
Eg: Resistor, $R = \frac{V}{I}$	Eg: Diode, $I = I_0 \left[e^{\frac{V}{\tau}} - 1 \right]$
	

(2) Static (M_L) & Dynamic (M) System:

Static System	Dynamic System
(1) At any time, Output of the System depends only on Present Input.	(1) Output of the System depends on Present as well as past Input.
(2) Memoryless Systems.	(2) memory Systems.
(3) $y(t) = f(u(t))$	(3) $y(t) = f(u(t), u(t-1), u(t-2), \ldots)$
Eg: Resistor. $I(t) = \frac{V(t)}{R}$	Eg: Inductor. $I(t) = \frac{1}{L} \int_0^t V(t)\, dt$

(3) Time Invariant v/s Time Variant Systems.

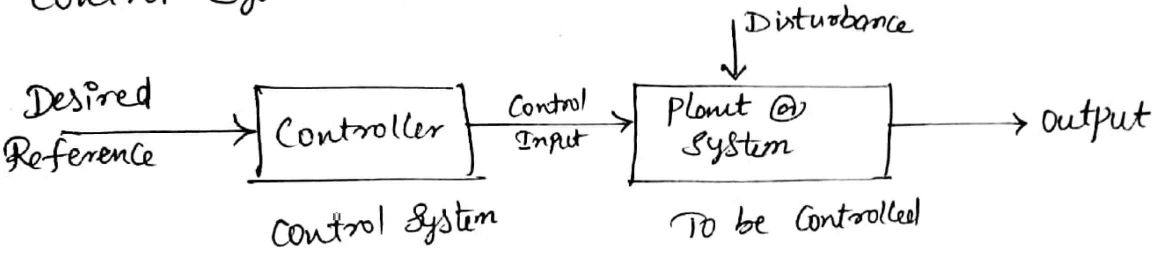
Time Invariant Systems	Time Variant Systems
(1) Output of the system is independent of the time at which the input is applied.	(1) Output of the system varies & dependent on the time at which input is applied.
(2) $y(t) = f(u(t)) \Rightarrow y(t+\delta) = f(u(t+\delta))$	(2) $y(t) = f(u(t))$ $\not\Rightarrow y(t+\delta) = f(u(t+\delta))$
Eg: An Ideal Resistor. Resistance value does not change w.r.to Time.	Eg: Aircraft: Mass (M) of the aircraft changes w.r.to time as fuel is consumed.

(4) Causal v/s Non-Causal Systems.

Causal Systems	Non-Causal Systems
(1) Output is only dependent on inputs already received (Present (or) past).	(1) Output depends on future inputs as well.
(2) Non-anticipatory s/m.	(2) System anticipates future inputs based on past.
(3) $y(t) = f(x(t), x(t-1), \dots)$	(3) $y(t) = f(x(t), x(t+1), \dots)$
Eg: ① Motor (or) generator. ② Thermostat based AC.	Eg: ① Weather forecasting system. ② Missile guidance system. ③ Anticipating questions for upcoming Exam.

What is a Control System?

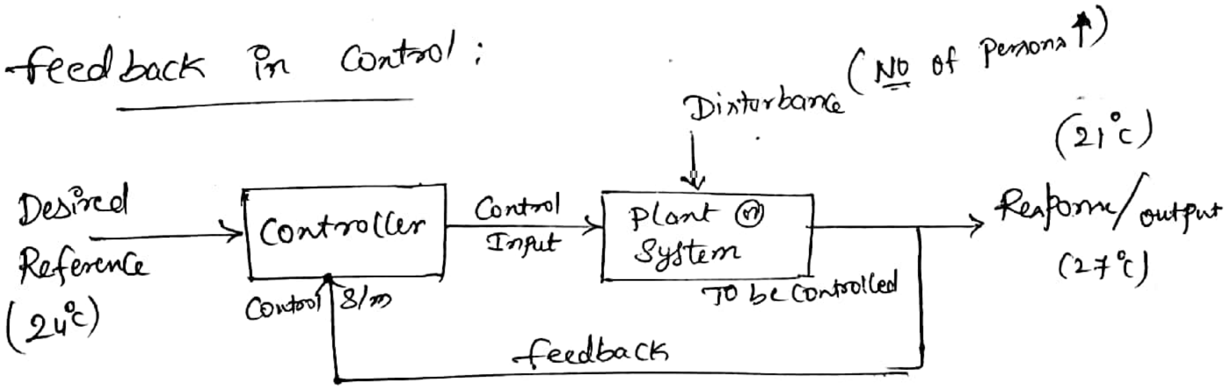
- A System (or) Mechanism which directs the input to other Systems & regulates their output.
- Control System alters the response of a plant (or) System as desired.



Disturbance:

- unwanted Signals which affect the output of the System.
  Eg: People entering & Leaving an AC room disturbs room temperature.
- Controller has to Eliminate the effects of disturbance.

feedback in Control:



Controller ⟶ It performs an action based on the feedback signal. Hence it provides a I/p to the S/m (or) plant.

→ feed back Senses the plant output & gives a signal which can be Compared to the reference.

→ Control action (Control Input) Changes based on feedback.

→ feedback Enables the Control System in Extracting the desired performance from the plant Even in Presence of disturbance.

SUPRITH KUMAR K S
Assistant Professor
ECE Dept, BMSCE

Classification of Systems:

(1) Open Loop Control System. (2) Closed-Loop Control System.

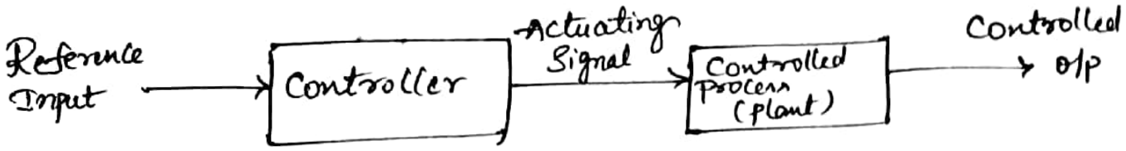
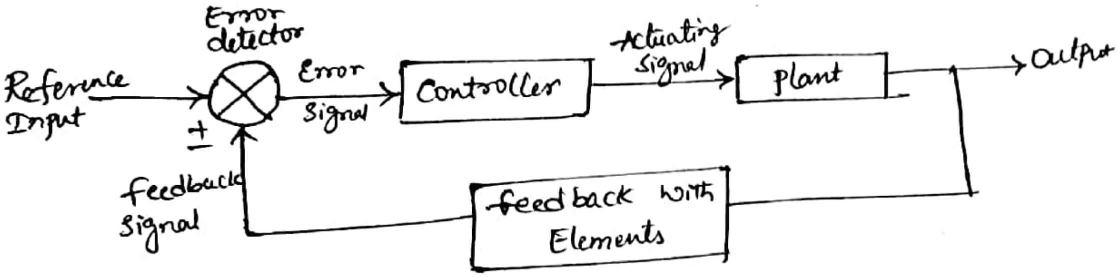


fig: open loop Control system.



IMP

Open Loop Control System	Closed Loop Control System
(1) The open loop systems are simple & Economical.	(1) The closed loop systems are complex & costlier.
(2) They consume less power.	(2) They consume more power.
(3) The open loop systems are inaccurate & unreliable.	(3) The closed loop systems are accurate & more reliable.
(4) Generally, open loop systems are stable.	(4) Stability is a major problem in closed loop control system & more care is needed to design a stable closed loop system.
(5) Open loop systems are easier to construct because of less number of components.	(5) Closed loop systems are not easy to construct because ~~of~~ more number of components.
(6) The changes in the output due to external disturbances are not corrected automatically. So they are more sensitive to noise & other disturbances.	(6) The ~~feedback~~ changes in the output due to external disturbances are corrected automatically. So they are less sensitive to noise & other disturbances.

Scanned by CamScanner

SUPRITH KUMAR KS
Assistant Professor
ECE Dept, BMSCE

Modelling of Mechanical System Elements :-

- Most of the control systems contain mechanical as well as electrical components. But from a mathematical view point the descriptions of mechanical & electrical elements are analogous.

Motion of Mechanical Elements

- Translational (motion takes place in a straight line)
- Rotational (Rotational motion)

(I) Translational:

* The variables used to describe translational motion are
(i) acceleration. (ii) velocity. (iii) Displacement.

Newton's Law of motion:

It states that, "algebraic sum of forces acting on a body in a given direction is equal to the product of mass of the body & its accelaration in the same direction."

It is given by:

$$\sum \text{forces} = M\,a(t) = M\frac{dv(t)}{dt} = M\frac{d^2x(t)}{dt^2}$$

Where,
$M \rightarrow$ Mass of the body
$a(t) \rightarrow$ acceleration of the body
$v(t) \rightarrow$ velocity of the body.
$x(t) \rightarrow$ displacement of the body

* Mechanical translational systems are modelled by three ideal elements.
(1) Mass. (2) Spring. (3) Damper.

(1) **Mass:** Mass is considered as a property of an element that stores the kinetic energy of translation motion.

$$M = \frac{W}{g}$$

$W \rightarrow$ weight of the body
$g \rightarrow$ acceleration of free fall due to gravity.

Note: One end of mass is "always connected to the ground."

(2) Spring:- In general, a spring is considered to be an element that stores Potential Energy. It is represented by "K".

(3) friction: whenever there is a motion (or) tendency of motion between two physical elements frictional force exists. Three different types of friction are commonly used in practical systems: (i) viscous friction. (ii) Static friction. (iii) Coulomb friction.

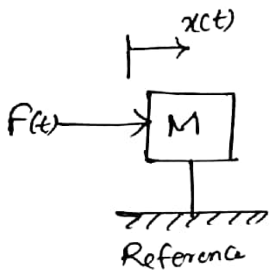
* But the predominent friction among these will be viscous friction.

- Viscous friction:- This is the force of friction between moving surfaces separated by viscous fluid (or) the force b/w a solid body & a fluid medium.

- The friction force acts in a direction opposite to that of velocity. Thus the element for viscous friction is often represented by a "dash pot".

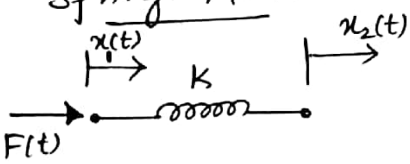
Translational Elements & Corresponding equations of motion:

(1) Mass Element:



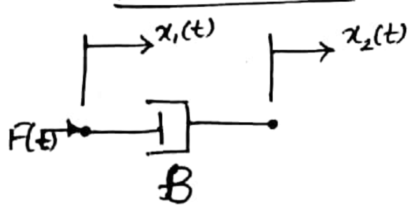
$$F = M\frac{d^2x(t)}{dt^2}$$

(2) Spring Element:



$$F = K(x_1 - x_2)$$

(3) Damper (or) Dash pot:



$$F = B\left(\frac{dx_1}{dt} - \frac{dx_2}{dt}\right)$$

(II) Rotational:

Mechanical Rotational Systems are those in which the motion is about a fixed axis.

* The variables used to describe rotational motion are
 (i) Angular acceleration (α) (ii) Angular velocity (ω) (iii) Angular displacement (θ)

Newton's Law of motion:

It states that, "the algebraic sum of moments or torques about a fixed axis is equal to the product of inertia & the angular acceleration about the axis".

It is given by:

$$\sum \text{Torques} = J\alpha(t)$$
$$= J\frac{d\omega(t)}{dt}$$
$$= J\frac{d^2\theta(t)}{dt^2}$$

where,
$J \rightarrow$ Inertia

* Mechanical Rotational Systems are modelled by three ideal Elements
 (1) Inertia (2) Torsional Spring. (3) Damper.

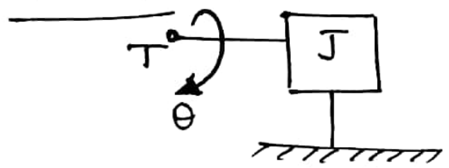
(1) Inertia: Inertia is considered to be the property of an element that stores the kinetic energy of rotational motion. One End of the inertia Element is always Connected to ground.

(2) Spring: In general, Spring can be used to represent a shaft when it is Subjected to an "applied Torque". It is represented by "K".

(3) friction: It is same as translational System. Viscous friction is the dominant one. This Element is represented by a "dash pot".

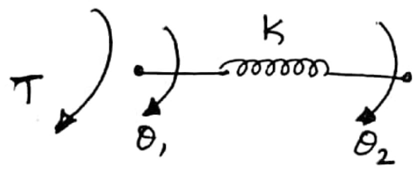
Rotational Elements & the Corresponding equations of motion:

(1) Inertia Element:



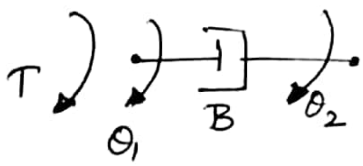
$$T = J\frac{d^2\theta}{dt^2}$$

(2) Spring Element:



$$T = K(\theta_1 - \theta_2)$$

(3) Damper (or) dash pot:



$$T = B\left(\frac{d\theta_1}{dt} - \frac{d\theta_2}{dt}\right)$$

Transfer function:

→ Transfer function is a parameter used to find System performance such as stability.

→ Transfer function is "the ratio of frequency domain representation of output to the frequency domain representation of input, with initial conditions are Zero".

$$\text{Transfer function} = \frac{\text{frequency domain representation of o/p}}{\text{frequency domain representation of i/p}}$$

$$\boxed{T.F = \frac{L.T[O/P]}{L.T[I/P]} = \frac{C(S)}{R(S)}}$$

$C(S) \longrightarrow$ Controlled O/p

$R(S) \longrightarrow$ Reference I/p

→ We are using Laplace transfer because the signals are continuous & also Laplace transform is applicable for stable as well as unstable systems. Whereas Fourier Transform is not applicable for unstable s/m. whereas Z-Transform is applicable only for discrete time s/m.

Note: (1) With initial conditions S/m becomes dynamic, in that case we will use "State Space Analysis".

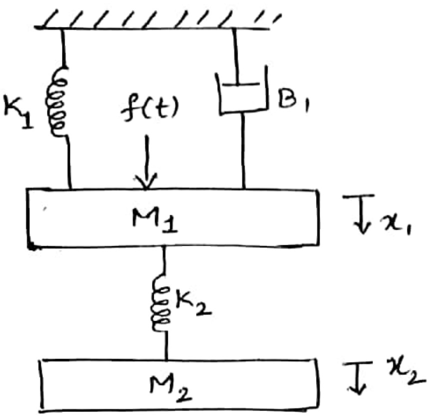
Scanned by CamScanner

Problems:-

(1) Refer the mechanical system shown below.

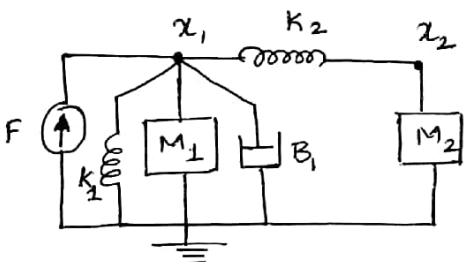
(a) Obtain the equations of motion for masses M_1 & M_2.

(b) Find the transfer function $\frac{X_2(s)}{F(s)}$.



(A) No of displacements = 2.

Mechanical Network:



Mathematical Representation:

At node x_1:-

$$F = M_1 \frac{d^2x_1}{dt^2} + B_1 \frac{dx_1}{dt} + K_1 x_1 + K_2(x_1 - x_2)$$

Apply Laplace Transform on Both Sides.

$$F(s) = M_1 s^2 X_1(s) + B_1 s X_1(s) + K_1 X_1(s) + K_2(X_1(s) - X_2(s))$$

$$F(s) = X_1(s)\left[M_1 s^2 + B_1 s + K_1 + K_2\right] - X_2(s) K_2 \quad \longrightarrow (1)$$

At node x_2:-

$$M_2 \frac{d^2x_2}{dt^2} + K_2(x_2 - x_1) = 0$$

Apply L.T:

$$M_2 s^2 X_2(s) + K_2(X_2(s) - X_1(s)) = 0$$

$$K_2 X_1(s) = X_2(s)\left[M_2 s^2 + K_2\right]$$

$$X_1(s) = \frac{X_2(s)\left[M_2 s^2 + K_2\right]}{K_2} \quad \longrightarrow (2)$$

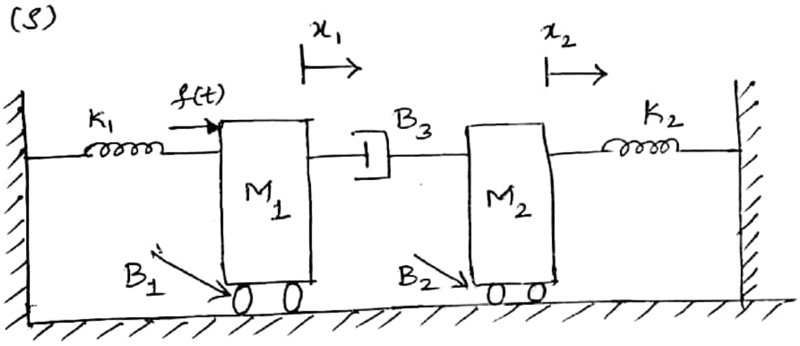
Eq ② in Eq ①

$$\Rightarrow F(s) = X_2(s)\frac{[s^2M_2+K_2]}{K_2}[s^2M_1 + sB_1+K_1+K_2] - X_2(s)K_2$$

$$F(s) = X_2(s)\left[\left(\frac{s^2M_2+K_2}{K_2}\right)(s^2M_1+sB_1+K_1+K_2) - K_2\right]$$

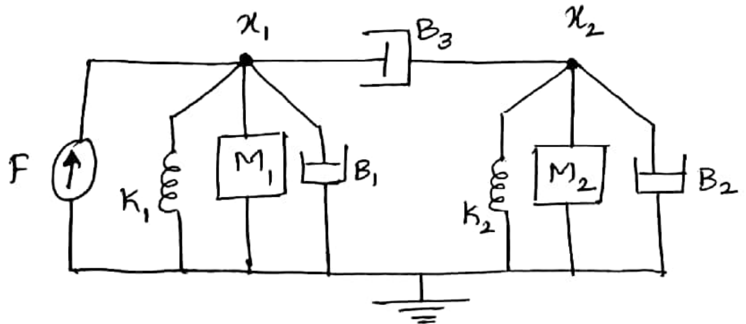
$$\boxed{\frac{X_2(s)}{F(s)} = \frac{K_2}{[s^2M_2+K_2][s^2M_1+sB_1+K_1+K_2]-K_2^2}}$$

(2) For the mechanical system shown below, find the transfer function $\frac{X_1(s)}{F(s)}$.



(A) No of displacements = 2.

Mechanical Network:



Mathematical Representation:-

At node x_1:-

$$F = M_1\frac{d^2x_1}{dt^2} + B_1\frac{dx_1}{dt} + K_1x_1 + B_3\left(\frac{dx_1}{dt} - \frac{dx_2}{dt}\right)$$

Apply L.T:

$$F(s) = M_1s^2X_1(s) + B_1sX_1(s) + K_1X_1(s) + B_3(sX_1(s) - sX_2(s))$$

$$F(s) = X_1(s)[s^2M_1 + sB_1 + K_1 + sB_3] - sB_3X_2(s) \quad \text{—— ①}$$

At node x_2:-

$$M_2 \frac{d^2x_2}{dt^2} + B_2 \frac{dx_2}{dt} + K_2 x_2 + B_3\left(\frac{dx_2}{dt} - \frac{dx_1}{dt}\right) = 0$$

Apply LT:

$$M_2 s^2 X_2(s) + B_2 s X_2(s) + K_2 X_2(s) + B_3\left(s X_2(s) - s X_1(s)\right) = 0$$

$$X_2(s)\left[M_2 s^2 + B_2 s + K_2 + B_3 s\right] - sB_3 X_1(s) = 0$$

$$X_2(s) = \frac{s B_3 X_1(s)}{\left[M_2 s^2 + B_2 s + K_2 + B_3 s\right]} \quad \text{—— (2)}$$

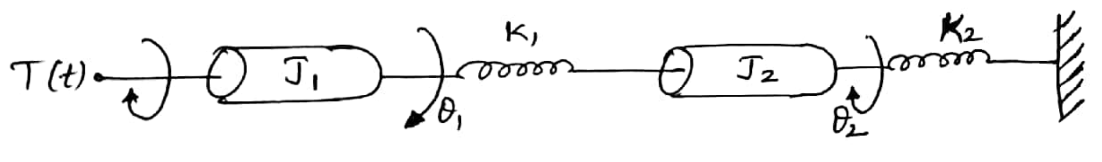
eq (2) in (1)

$$F(s) = X_1(s)\left[s^2 M_1 + s(B_1 + B_3) + K_1\right] - sB_3\left[\frac{s B_3 X_1(s)}{M_2 s^2 + B_2 s + B_3 s + K_2}\right]$$

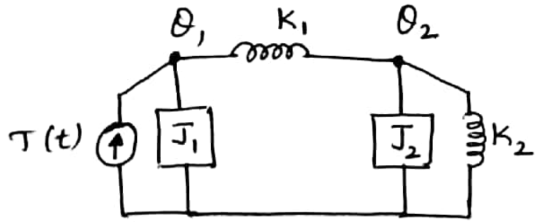
$$F(s) = X_1(s)\left[\frac{(s^2 M_1 + s(B_1 + B_3) + K_1)(M_2 s^2 + s(B_2 + B_3) + K_2) - s^2 B_3^2}{(M_2 s^2 + B_2 s + B_3 s + K_2)}\right]$$

$$\therefore \boxed{\frac{X_1(s)}{F(s)} = \frac{(M_2 s^2 + B_2 s + B_3 s + K_2)}{\left[(s^2 M_1 + s(B_1 + B_3) + K_1)(s^2 M_2 + s(B_2 + B_3) + K_2) - s^2 B_3^2\right]}}$$

(3) Refer the rotational system shown below. Obtain the equations of motion for the inertias J_1, J_2 & find the transfer function $\frac{\theta_1(s)}{T(s)}$.



(A) Mechanical Network:



At node "θ_1":-

$$T(t) = J_1 \frac{d^2\theta_1}{dt^2} + K_1(\theta_1 - \theta_2)$$

Apply LT:

$$T(s) = J_1 s^2 \theta_1(s) + K_1(\theta_1(s) - \theta_2(s))$$

$$T(s) = \theta_1(s)\left[J_1 s^2 + K_1\right] - K_1\theta_2(s) \quad \text{— (1)}$$

At node "θ_2":-

$$J_2 \frac{d^2\theta_2}{dt^2} + K_2\theta_2 + K_1(\theta_2 - \theta_1) = 0$$

Apply LT:

$$J_2 s^2 \theta_2(s) + K_2\theta_2(s) + K_1(\theta_2(s) - \theta_1(s)) = 0$$

$$\theta_2(s)\left[J_2 s^2 + K_2 + K_1\right] = K_1\theta_1(s)$$

$$\theta_2(s) = \frac{K_1\theta_1(s)}{\left[J_2 s^2 + K_2 + K_1\right]} \quad \text{— (2)}$$

Eq (2) in (1)

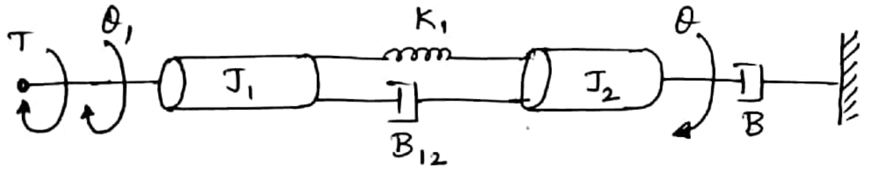
$$\Rightarrow T(s) = \theta_1(s)\left[J_1 s^2 + K_1\right] - K_1\left[\frac{K_1\theta_1(s)}{J_2 s^2 + K_2 + K_1}\right]$$

$$T(s) = \theta_1(s)\left[\frac{(J_1 s^2 + K_1)(J_2 s^2 + K_2 + K_1) - K_1^2}{(J_2 s^2 + K_2 + K_1)}\right]$$

$$\therefore \boxed{\frac{\theta_1(s)}{T(s)} = \frac{(J_2 s^2 + K_2 + K_1)}{\left[(J_1 s^2 + K_1)(J_2 s^2 + K_2 + K_1) - K_1^2\right]}}$$

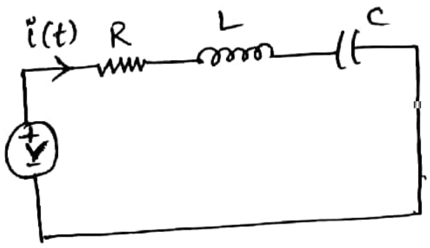
Assignment

(4) Write the differential equations governing the mechanical rotational S/m shown in figure. Determine the T.F $\theta(s)/T(s)$.



Electrical Systems:-

(a) Consider a Series RLC Circuit as shown in the below figure.



$$-V + I(t)R + L\frac{dI(t)}{dt} + \frac{1}{C}\int I(t)\,dt = 0$$

$$V = I(t)R + L\frac{dI(t)}{dt} + \frac{1}{C}\int I(t)\,dt \quad — ①$$

But WKT: $I(t) = \frac{dq(t)}{dt}$

$\therefore$ eq ① becomes

$$V = R\frac{dq(t)}{dt} + L\frac{d^2q(t)}{dt^2} + \frac{q(t)}{C}$$

This can be re-written as.

$$V = L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{q}{C} \quad — ②$$

Comparing this eq ② with the standard Mechanical s/m Equation.

$$F = M\frac{d^2x}{dt^2} + B\frac{dx}{dt} + Kx \quad — ③$$

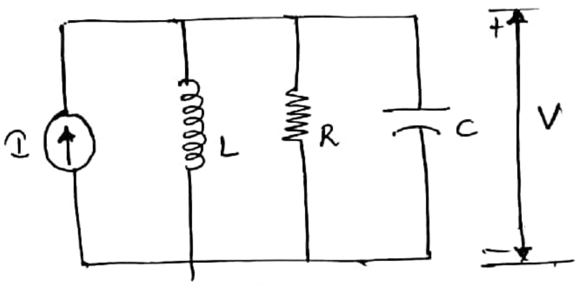
$$T = J\frac{d^2\theta}{dt^2} + B\frac{d\theta}{dt} + K\theta \quad — ④$$

By comparing eq ③ & ④ with eq ② we obtain an analogous Parameters b/w Mechanical & Electrical Systems.

Force - voltage (FV) Analogy:

Translational s/m	Rotational s/m	Electrical s/m
(1) Force, (F)	Torque (T)	Voltage (e or v)
(2) Mass, (M)	Moment of Inertia (J)	Inductance (L)
(3) Friction Co-Efficient (B)	Friction Co-Efficient (B)	Resistance (R)
(4) Spring (K)	Spring (K)	Reciprocal of Capacitance $\left(\frac{1}{C}\right)$
(5) Displacement (x)	Angular Displacement (θ)	Charge (q)

(b) Consider a parallel RLC circuit as shown in the below figure.



According to Kirchoff's Current Law:

$$\frac{V}{R} + \frac{1}{L}\int V\,dt + C\frac{dv}{dt} = I \quad \text{--- (1)}$$

But according to magnetic flux linkage.

$$\varepsilon \text{ or } V = \frac{d\phi}{dt} \quad \text{--- (2)}$$

$\varepsilon \rightarrow$ Induced emf

$\frac{d\phi}{dt} \rightarrow$ Rate of change of flux linking with the coil.

Eq (2) in Eq (1)

$$\frac{1}{R}\frac{d\phi}{dt} + \frac{1}{L}\phi + C\frac{d^2\phi}{dt^2} = I$$

Re-writing the above Equation.

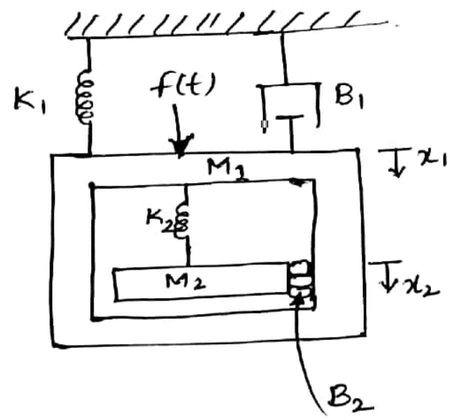
$$I = C\frac{d^2\phi}{dt^2} + \frac{1}{R}\frac{d\phi}{dt} + \frac{1}{L}\phi \quad \text{--- (3)}$$

Comparing Eq ③ with the mechanical S/m equations. We obtain the analogous parameters b/w Mechanical & Electrical System.

Force - current (FI) Analogy :-

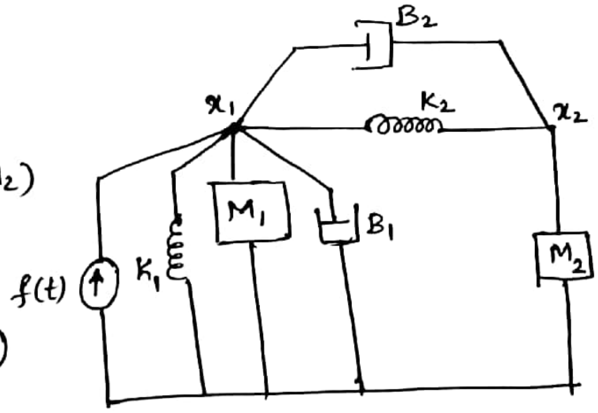
Translational S/m	Rotational S/m	Electrical S/m
(1) force, (F)	Torque (T)	Current (I)
(2) Mass, (M)	Moment of Inertia (J)	Capacitance (C)
(3) Friction Coefficient (B)	Friction Co-Efficient (B)	Conductance $(\frac{1}{R})$
(4) Spring (K)	Spring (K)	Reciprocal of Inductance $(\frac{1}{L})$
(5) Displacement (x)	Angular Displacement (θ)	Magnetic flux (ϕ)

Problem: ① For the Given Mechanical System find the Equivalent Electrical Network using F-V analogy.



Sol: At x_1:

$$F = M_1\frac{d^2x_1}{dt^2} + B_1\frac{dx_1}{dt} + K_1x_1 + K_2(x_1 - x_2) + B_2\left(\frac{dx_1}{dt} - \frac{dx_2}{dt}\right) \quad \text{--- ①}$$



FV Equation

$$V = L_1\frac{d^2q_1}{dt^2} + R_1\frac{dq_1}{dt} + \frac{1}{C_1}q_1 + \frac{1}{C_2}(q_1 - q_2) + R_2\left(\frac{dq_1}{dt} - \frac{dq_2}{dt}\right)$$

Put $\boxed{i = \frac{dq}{dt}}$

$$v(t) = L_1\frac{di_1}{dt} + R_1 i_1 + \frac{1}{C_1}\int_0^t i_1\,dt + \frac{1}{C_2}\int_0^t (i_1 - i_2)\,dt + R_2(i_1 - i_2)$$

At x_2:

$$0 = M_2 \frac{d^2 x_2}{dt^2} + B_2\left(\frac{dx_2}{dt} - \frac{dx_1}{dt}\right) + K_2(x_2 - x_1)$$

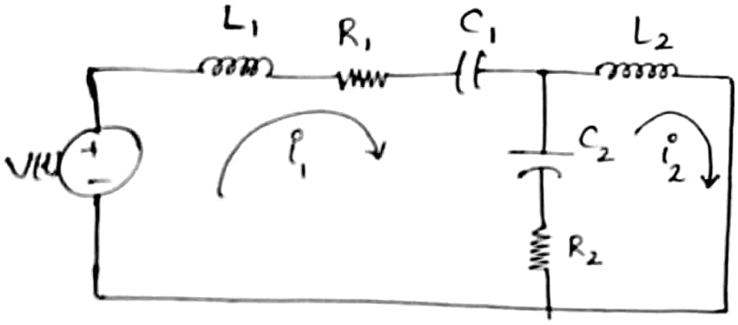
FV Equation:

$$0 = L_2 \frac{d^2 q_1}{dt^2} + R_2\left(\frac{dq_2}{dt} - \frac{dq_1}{dt}\right) + \frac{1}{C_2}(q_2 - q_1)$$

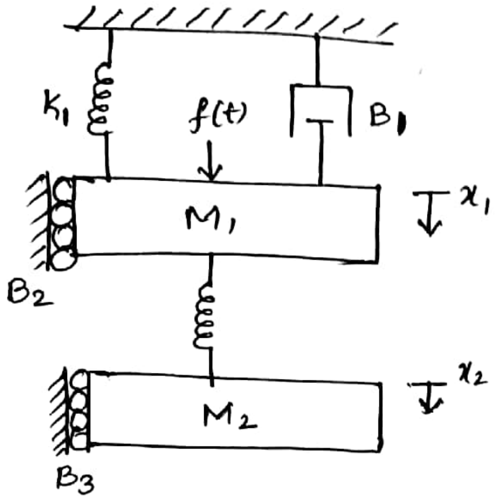
Put $i = \frac{dq}{dt}$ (or) $q = \int i\,dt$

$$0 = L_2 \frac{di_2}{dt} + R_2(i_2 - i_1) + \frac{1}{C_2}\int (i_2 - i_1)\,dt$$

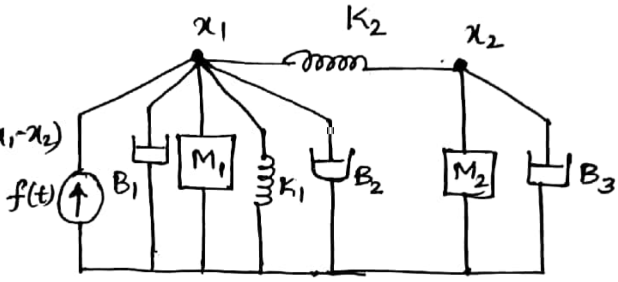
Equivalent
Electrical N/W is:



② For the given mechanical Network find the equivalent Electrical Network using F–I analogy.



Ⓐ At x_1:



$$f(t) = M_1 \frac{d^2x_1}{dt^2} + K_1x_1 + B_1\frac{dx_1}{dt} + B_2\frac{dx_1}{dt} + K_2(x_1-x_2)$$

FI

$$I(t) = C_1\frac{d^2\phi_1}{dt^2} + \frac{\phi_1}{L_1} + \frac{1}{R_1}\frac{d\phi_1}{dt} + \frac{1}{R_2}\frac{d\phi_2}{dt} + \frac{1}{L_2}(\phi_1-\phi_2)$$

Put $e = \frac{d\phi}{dt}$ (or) $\phi = \int_0^t e\,dt$

$$I(t) = C_1\frac{de_1}{dt} + \frac{1}{L_1}\int_0^t e\,dt + \frac{1}{R_1}e_1 + \frac{1}{R_2}e_2 + \frac{1}{L_2}\left(\int_0^t (e_1dt - e_2dt)\right) \quad \text{— ①}$$

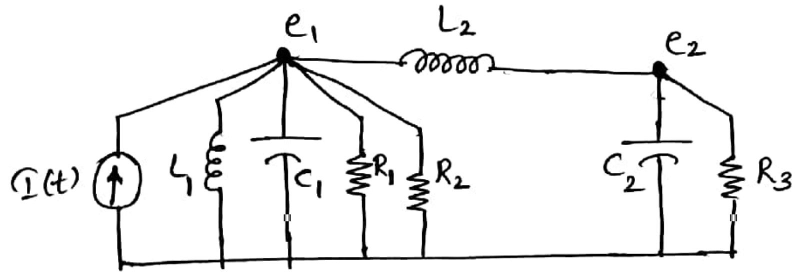
At x_2:

$$0 = M_2\frac{d^2x_2}{dt^2} + B_3\frac{dx_2}{dt} + K_2(x_2-x_1)$$

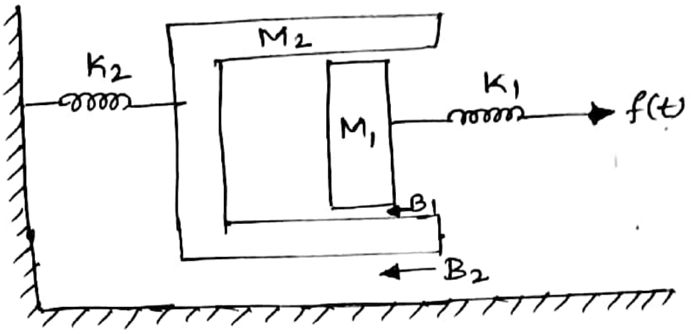
$$0 = C_2\frac{d^2\phi_2}{dt^2} + \frac{1}{R_3}\frac{d\phi_2}{dt} + \frac{1}{L_2}(\phi_2-\phi_1)$$

$$0 = C_2\frac{de_2}{dt} + \frac{1}{R_3}e_2 + \frac{1}{L_2}\int_0^t (e_2-e_1)\,dt \quad \text{— ②}$$

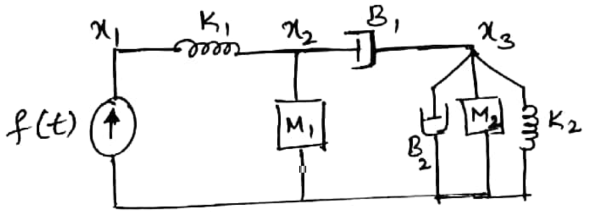
Equivalent Electrical N/w using FI analogy:



(3) Write the differential Equations for the mechanical S/m shown in figure. and Obtain F-V & F-I analogous Electrical networks.



(A)



Understand where to place the displacement.

Note: (1) Spring & dash pot has to be connected b/w 2 nodes @ displacement.

(2) It can be one displacement & one ground (or) it can be both displacement.

At x_1:

$$f(t) = K_1(x_1 - x_2)$$

At x_2:

$$0 = M_1 \frac{d^2x_2}{dt^2} + K_1(x_2 - x_1) + B_1\left(\frac{dx_2}{dt} - \frac{dx_3}{dt}\right)$$

At x_3:

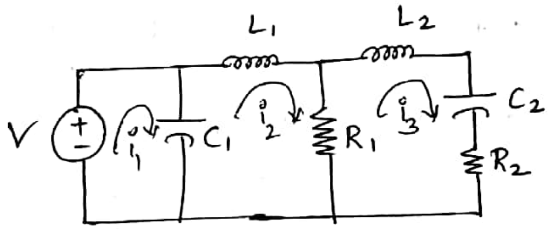
$$0 = M_2 \frac{d^2x_3}{dt^2} + K_2(x_3) + B_2 \frac{dx_3}{dt} + B_1\left(\frac{dx_3}{dt} - \frac{dx_2}{dt}\right)$$

F-V Analogy:

$$V = \frac{1}{C_1}\int(i_1 - i_2)dt$$

$$0 = L_1 \frac{di_2}{dt} + R_1(i_2 - i_3) + \frac{1}{C_1}\int(i_2 - i_1)dt$$

$$0 = L_2 \frac{di_3}{dt} + \frac{1}{C_2}\int i_3\, dt + R_2 i_3 + R_1(i_3 - i_2)$$



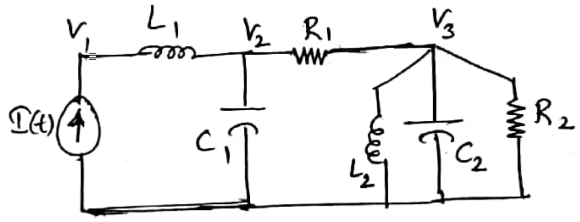
F-V Electrical N/w

F-I Analogy:

$$I = \frac{1}{L_1}\int(V_1 - V_2)dt$$

$$0 = C_1 \frac{dV_2}{dt} + \frac{1}{R_1}(V_2 - V_3) + \frac{1}{L_1}\int(V_2 - V_1)dt$$

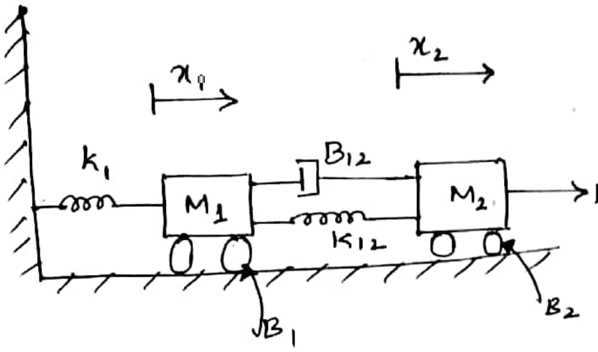
$$0 = C_2 \frac{dV_3}{dt} + \frac{1}{L_2} V_3 + \frac{V_3}{R_2} + \frac{(V_3 - V_2)}{R_1}$$



F-I Electrical N/w

Problem:

(4)

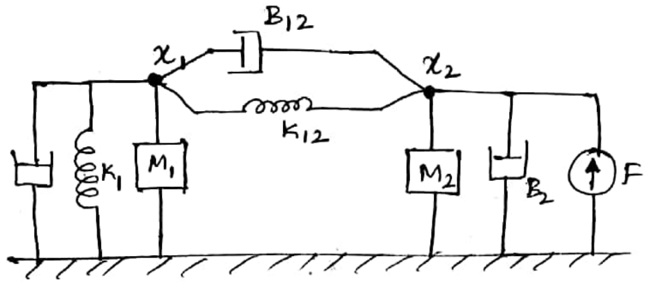


B_1 ← friction & B_2

Determine equivalent Electrical network using FV & FI analogy?

(A) ① Identify the no of displacements = 2

② what are the nodes => x_1 & x_2



At node x_1: (Sum of forces = 0)

$$M_1 \frac{d^2x_1}{dt^2} + B_1 \frac{dx_1}{dt} + K_1 x_1 + B_{12} \frac{d(x_1-x_2)}{dt} + K_{12}(x_1-x_2) = 0 \quad \text{—(1)}$$

At node x_2:

$$F = M_2 \frac{d^2x_2}{dt^2} + B_2 \frac{dx_2}{dt} + B_{12} \frac{d(x_2-x_1)}{dt} + K_{12}(x_2-x_1) \quad \text{—(2)}$$

Force to voltage Analogy:

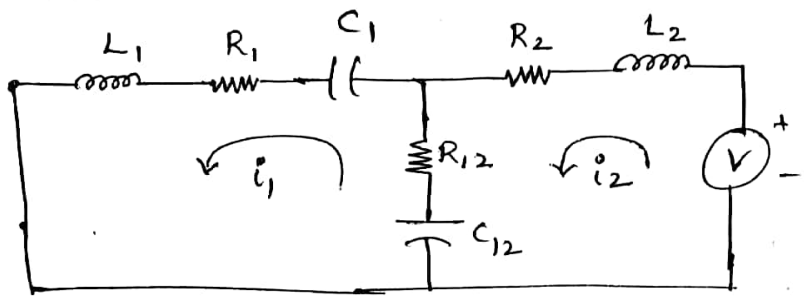
Eq ①
⇒
$$L_1 \frac{d^2q_1}{dt^2} + R_1 \frac{dq_1}{dt} + \frac{q_1}{C_1} + R_{12} \frac{d(q_1-q_2)}{dt} + \frac{1}{C_{12}}(q_1-q_2) = 0 \quad \text{—(3)}$$

$$L_1 \frac{di_1}{dt} + R_1 i_1 + \frac{1}{C_1}\int_0^t i_1 dt + R_{12}(i_1-i_2) + \frac{1}{C_{12}}\int_0^t (i_1-i_2)dt = 0 \Leftarrow$$

Eq ②
⇒
$$V = L_2 \frac{d^2q_2}{dt^2} + R_2 \frac{dq_2}{dt} + R_{12} \frac{d(q_2-q_1)}{dt} + \frac{1}{C_{12}}(q_2-q_1) \quad \text{—(4)}$$

$$V = L_2 \frac{di_2}{dt} + R_2 i_2 + R_{12}(i_2-i_1) + \frac{1}{C_{12}}\int (i_2-i_1)dt \Leftarrow$$

Electrical Model:

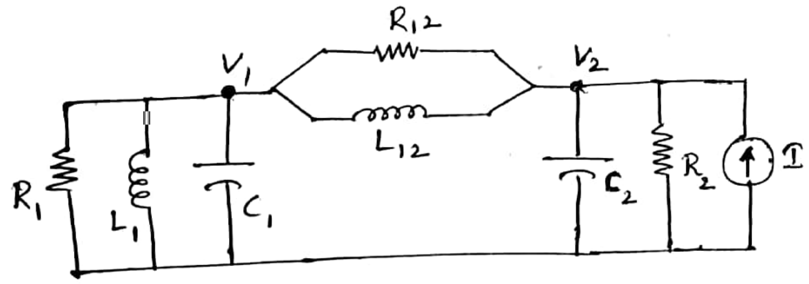


force to Current analogy:-

$$C_1 \frac{d^2\phi_1}{dt^2} + \frac{1}{L_1}\phi_1 + \frac{1}{R_1}\frac{d\phi_1}{dt} + \frac{1}{R_{12}}\left(\frac{d(\phi_1-\phi_2)}{dt}\right) + \frac{1}{L_{12}}(\phi_1-\phi_2) = 0$$

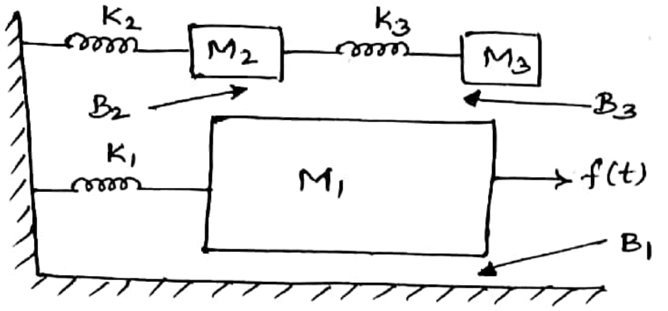
$$C_1 \frac{dV_1}{dt} + \frac{1}{L_1}\int V_1 dt + \frac{1}{R_1}V_1 + \frac{1}{R_{12}}(V_1-V_2) + \frac{1}{L_{12}}\int_0^t (V_1-V_2)dt = 0 \Leftarrow$$

$$I = C_2 \frac{d^2\phi_2}{dt^2} + \frac{1}{R_2}\frac{d\phi_2}{dt} + \frac{1}{R_{12}}\frac{d(\phi_2-\phi_1)}{dt} + \frac{1}{L_{12}}(\phi_2-\phi_1)$$

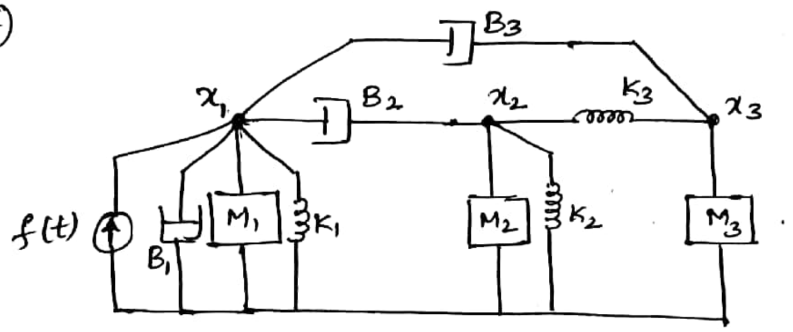


$$I = C_2\frac{dV_2}{dt} + \frac{V_2}{R_2} + \frac{(V_2-V_1)}{R_{12}} + \frac{1}{L_{12}}\int_0^t (V_2-V_1)dt \Leftarrow$$

(5) Determine equivalent Electrical network for the given mechanical system using F-V & F-I analogy.



(A)



At node x_1:

$$f(t) = M_1 \frac{d^2x_1}{dt^2} + K_1 x_1 + B_1 \frac{dx_1}{dt} + B_2\left(\frac{dx_1}{dt} - \frac{dx_2}{dt}\right) + B_3\left(\frac{dx_1}{dt} - \frac{dx_3}{dt}\right)$$

At x_2:

$$0 = M_2 \frac{d^2x_2}{dt^2} + K_2 x_2 + B_2\left(\frac{dx_2}{dt} - \frac{dx_1}{dt}\right) + K_3(x_2 - x_3)$$

At x_3:

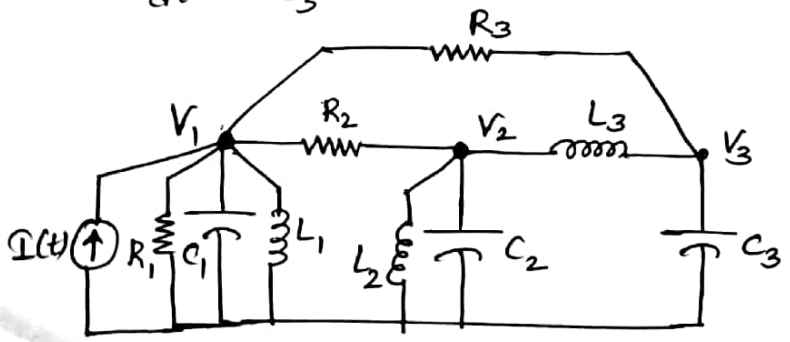
$$0 = M_3 \frac{d^2x_3}{dt^2} + K_3(x_3 - x_2) + B_3\left(\frac{dx_3}{dt} - \frac{dx_1}{dt}\right)$$

F-I:

$$I(t) = C_1 \frac{dV_1}{dt} + \frac{1}{L_1}\int V_1\,dt + \frac{1}{R_1}V_1 + \frac{1}{R_2}(V_1 - V_2) + \frac{1}{R_3}(V_1 - V_3)$$

$$0 = C_2 \frac{dV_2}{dt} + \frac{1}{L_2}\int V_2\,dt + \frac{1}{R_2}(V_2 - V_1) + \frac{1}{L_3}\left(\int (V_2 - V_3)\,dt\right)$$

$$0 = C_3 \frac{dV_3}{dt} + \frac{1}{L_3}\int (V_3 - V_2)\,dt + \frac{1}{R_3}[V_3 - V_1]$$

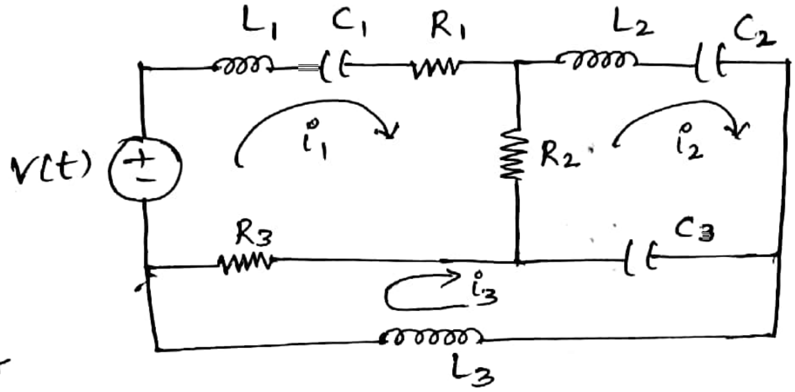


F-V :-

$$v(t) = L_1 \frac{di_1}{dt} + \frac{1}{C_1}\int i_1\,dt + R_1 i_1 + R_2(i_1 - i_2) + R_3(i_1 - i_3)$$

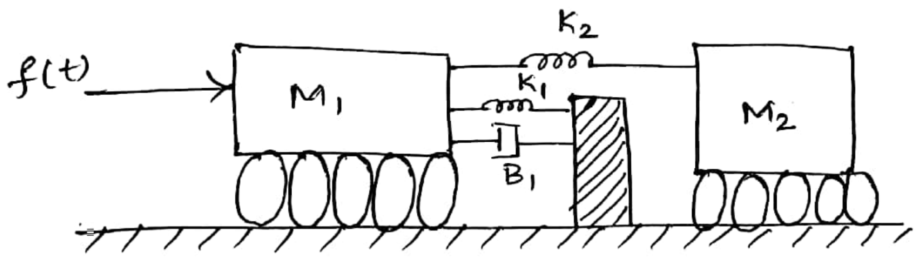
$$0 = L_2 \frac{di_2}{dt} + \frac{1}{C_2}\int i_2\,dt + R_2(i_2 - i_1) + \frac{1}{C_3}\int (i_2 - i_3)\,dt$$

$$0 = L_3 \frac{di_3}{dt} + \frac{1}{C_3}\int (i_3 - i_2)\,dt + R_3(i_3 - i_1)$$



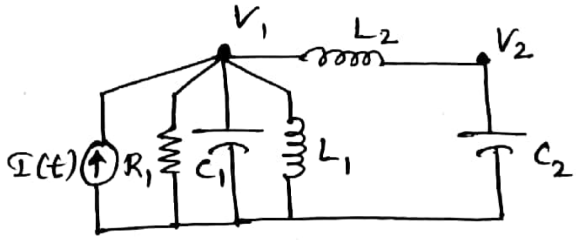
Assignment

(6) Write the equivalent Electrical network using F-I analogy for the mechanical s/m shown.



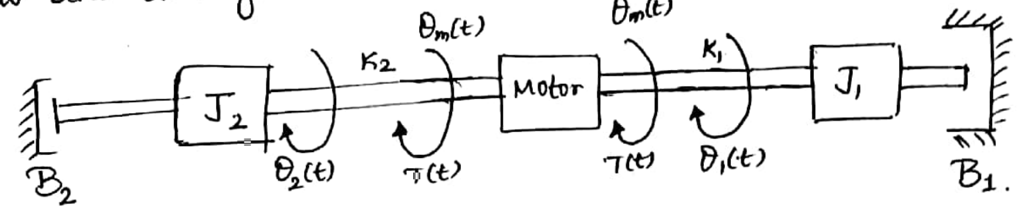
Ans:

Electrical N/W

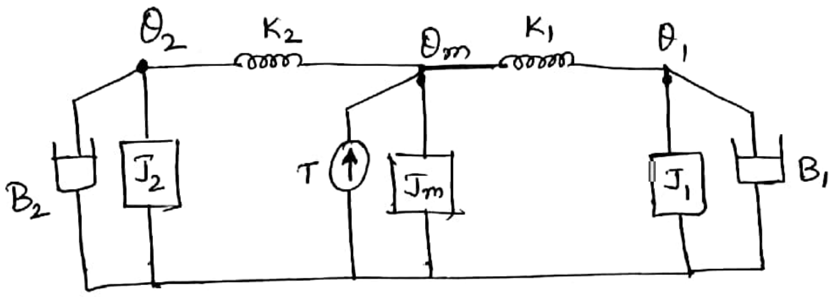


Scanned by CamScanner

① Write the Torque equations of the rotational system shown in figure. Draw the analogous electrical networks based on ⓐ T-I & ⓑ T-V.



Ⓐ M/c N/w:



At θ_2:

$$0 = J_2 \frac{d^2\theta_2}{dt^2} + B_2 \frac{d\theta_2}{dt} + K_2(\theta_2 - \theta_m)$$

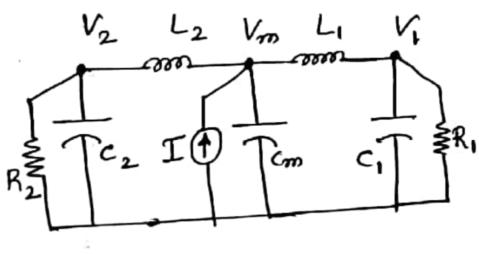
At θ_m:

$$T = J_m \frac{d^2\theta_m}{dt^2} + K_1(\theta_m - \theta_1) + K_2(\theta_m - \theta_2)$$

At θ_1:

$$0 = J_1 \frac{d^2\theta_1}{dt^2} + B_1 \frac{d\theta_1}{dt} + K_1(\theta_1 - \theta_m)$$

T-I Electrical N/w:



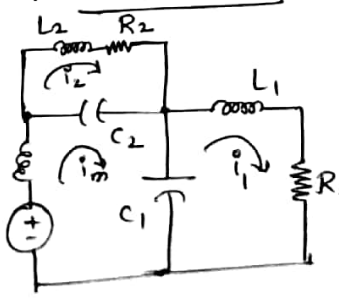
T-I:-

$$0 = C_2 \frac{dV_2}{dt} + \frac{1}{R_2} V_2 + \frac{1}{L_2}\int (V_2 - V_m)\,dt$$

$$I = C_m \frac{dV_m}{dt} + \frac{1}{L_1}\int (V_m - V_1)\,dt + \frac{1}{L_2}\int (V_m - V_2)\,dt$$

$$0 = C_1 \frac{dV_1}{dt} + \frac{1}{R_1} V_1 + \frac{1}{L_1}\int (V_1 - V_m)\,dt$$

T-V Electrical N/w:



T-V:

$$0 = L_2 \frac{di_2}{dt} + R_2 i_2 + \frac{1}{C_2}\int (i_2 - i_m)\,dt$$

$$V = L_m \frac{di_m}{dt} + \frac{1}{C_1}\int (i_m - i_1)\,dt + \frac{1}{C_2}\int (i_m - i_2)\,dt$$

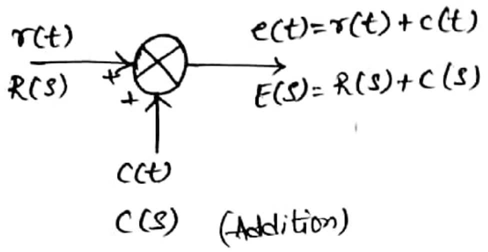
$$0 = L_1 \frac{di_1}{dt} + R_1 i_1 + \frac{1}{C_1}\int (i_1 - i_m)\,dt$$

Block Diagram:

– A Block diagram of a System is a pictorial representation of the functions performed by each component & of the flow of signals.

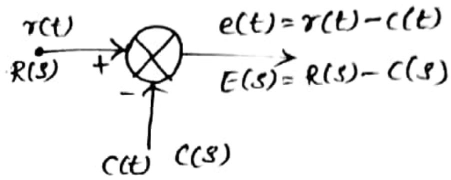
Block diagram elements used frequently in Control Systems:

(1)



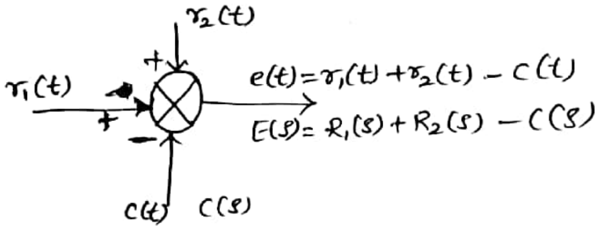
(Addition)

(2)



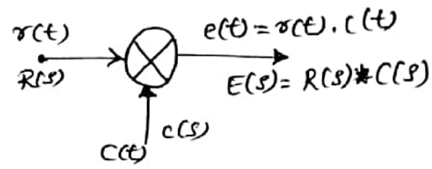
(Subtraction)

(3)



(Addition & Subtraction)

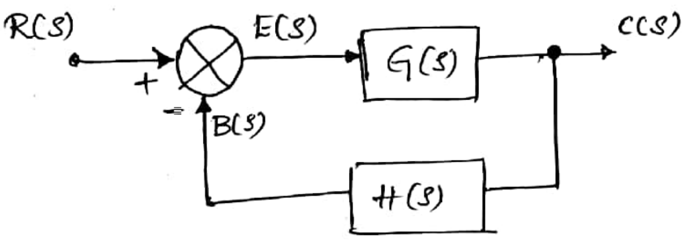
(4)



(Multiplication operation)

(5)

Block Diagram of a closed Loop System:



$R(S) \rightarrow$ Reference i/p

$C(S) \rightarrow$ Controlled o/p

$B(S) \rightarrow$ feedback Signal

$E(S) \rightarrow$ Error Signal

$G(S) \rightarrow$ Forward path T.F.

$H(S) \rightarrow$ Feedback path T.F

$C(S) = E(S)\ G(S)$

$E(S) = R(S) - B(S)$

$C(S) = [R(S) - B(S)]\ G(S)$

$C(S) = G(S).R(S) - G(S).B(S)$

But $B(S) = C(S)\ H(S)$

$$\boxed{TF = \frac{C(S)}{R(S)}}$$

$C(S) = G(S)\ R(S) - G(S)\ C(S)\ H(S)$

$\Rightarrow G(S)\ R(S) = C(S) + G(S)\ C(S)\ H(S)$

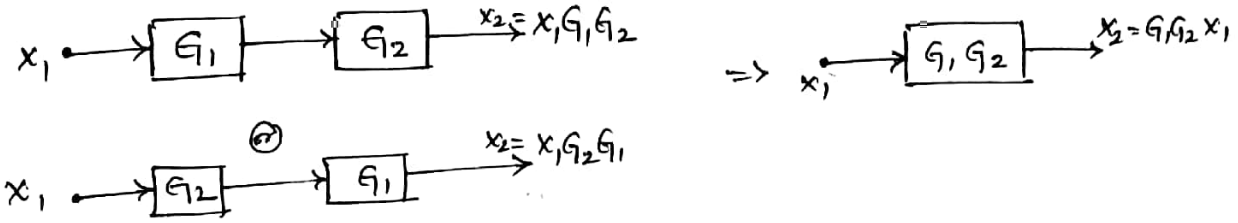
$G(S)\ R(S) = C(S)\ [1 + G(S)\ H(S)]$

$$\boxed{\frac{C(S)}{R(S)} = \frac{G(S)}{1+G(S)\ H(S)}}$$

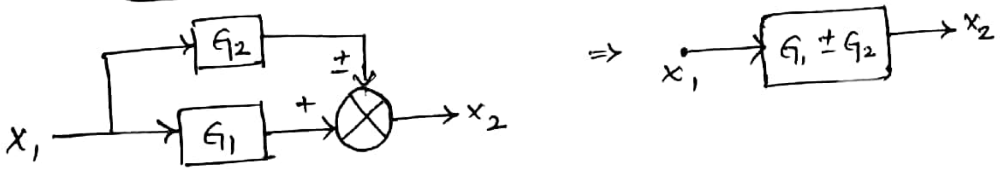
Block Diagram Reduction :-

Rules for Block Diagram Reduction:

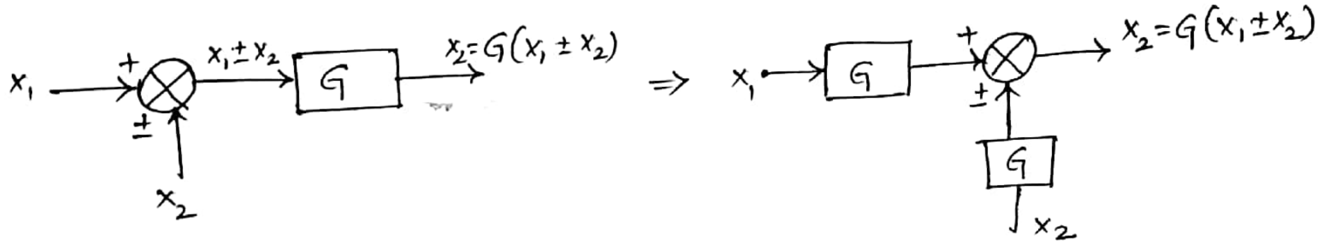
(1) Combining blocks in Cascade (Series) :-



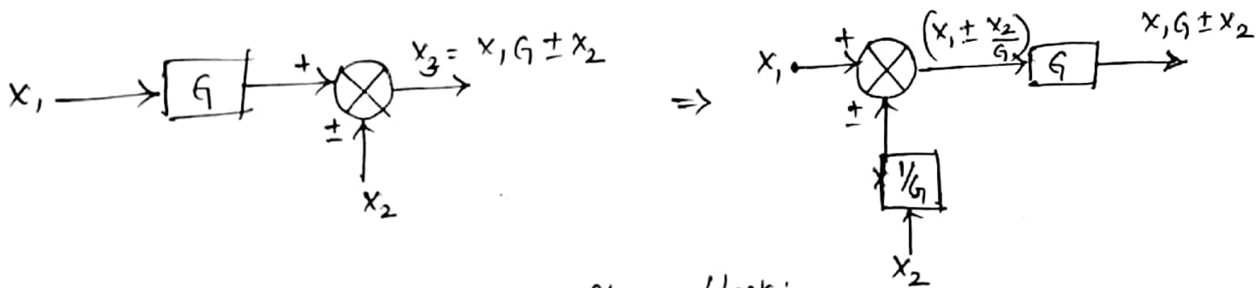
(2) Combining blocks in parallel:



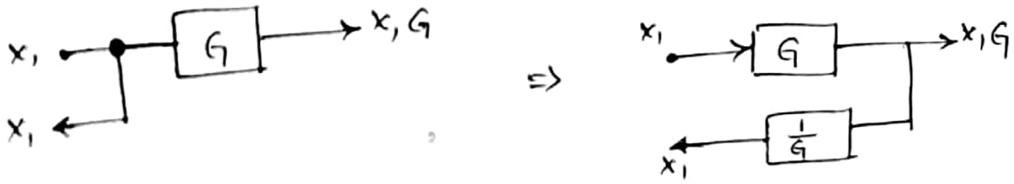
(3) Moving a Summing Point after a block:- (forward)



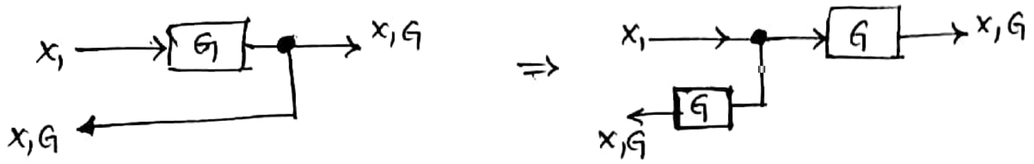
(4) Moving a Summing point ahead of a block:- (Backward)



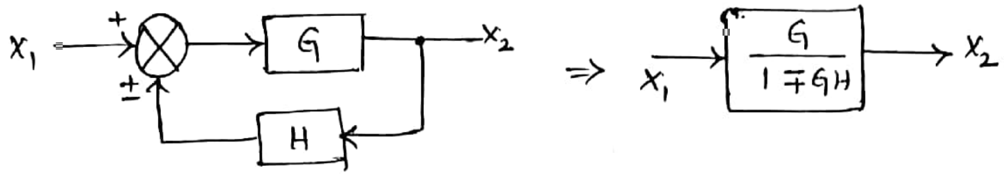
(5) Moving a take-off Point after a block: (forward)



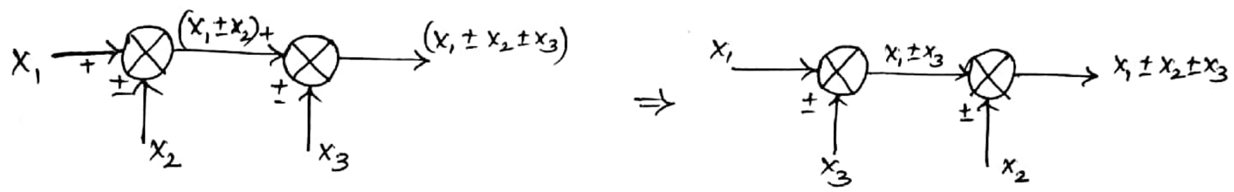
(6) Moving a take-off point ahead of a block (Backword);



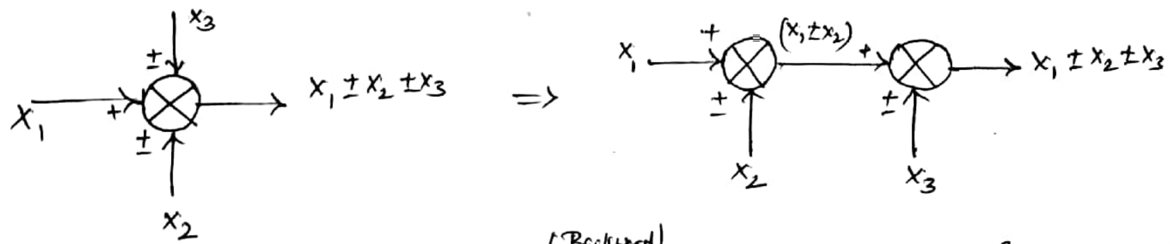
(7) Eliminating a feedback Loop:



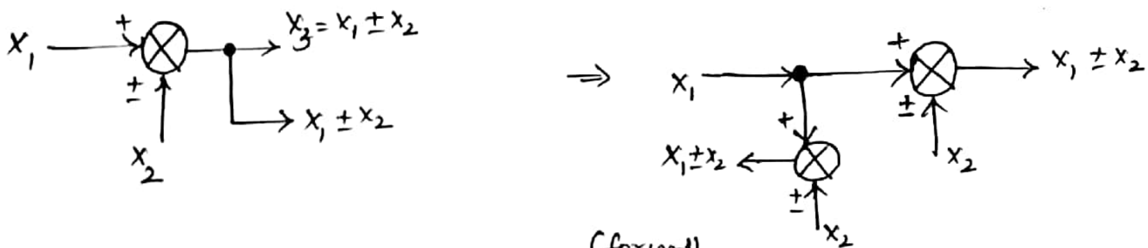
(8) Interchanging Summing Points:



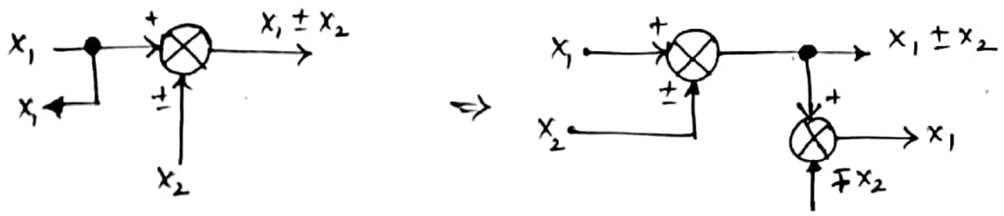
(9) Splitting a Summing Point:



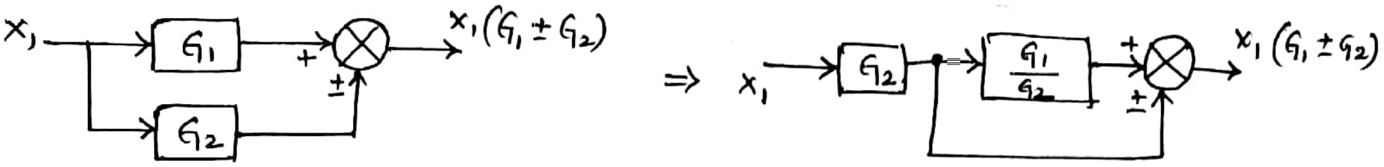
(10) Moving a take off Point (Backword) ahead Of Summing Point: {No block involved}



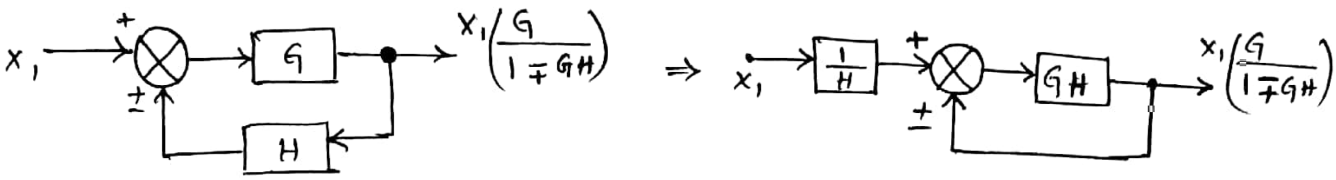
(11) Moving a take off Point (forword) after a Summing Point:– {No block involved}



(12) Removing a block from a forward path: (Same direction)



(13) Removing a block from a feedback path :- (opposite direction)



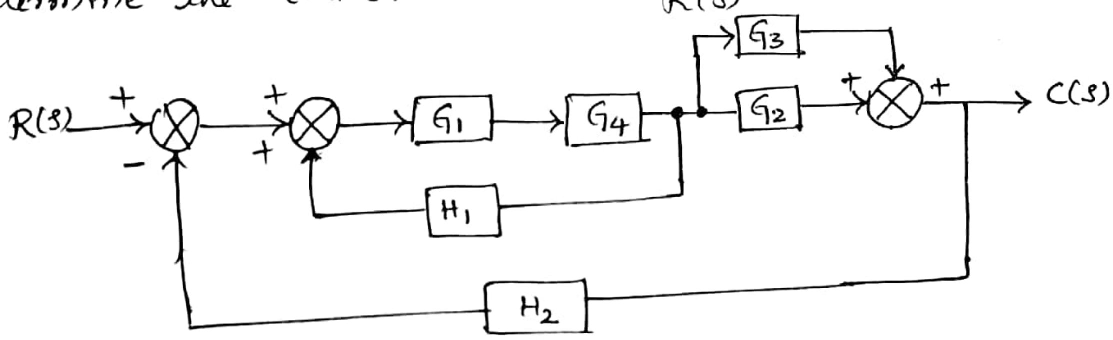
Note: While simplifying a block diagram, the following points need to be remembered.

(1) The product of the transfer functions in the feed forward direction must remain the same.

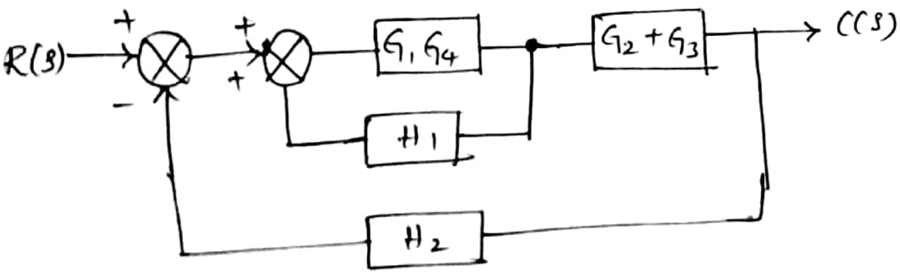
(2) The product of the transfer function around the loop must remain the same.

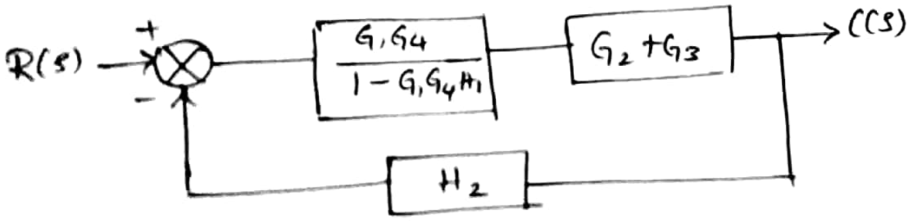
Problems:

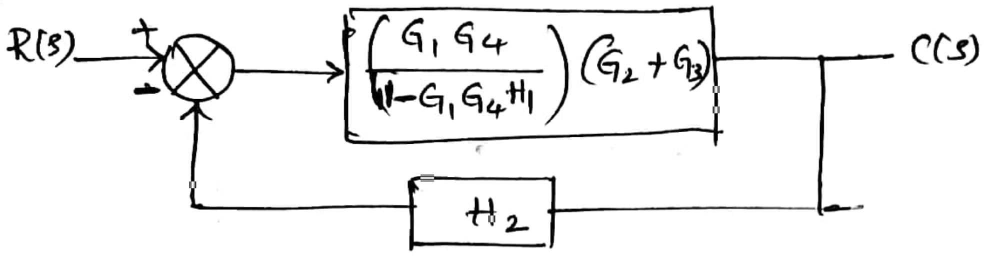
(1) Determine the transfer function $\frac{C(s)}{R(s)}$ of the system shown in the fig.



(A)





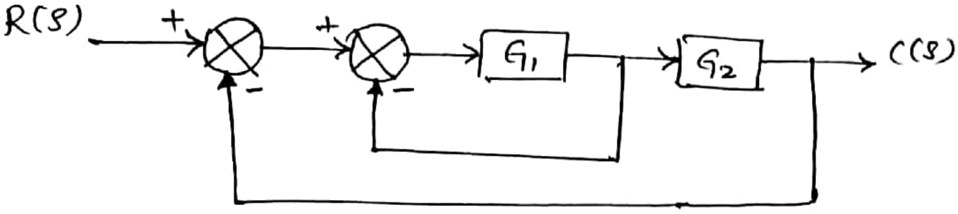


$$\frac{C(s)}{R(s)} = \frac{\dfrac{(G_1G_4)(G_2+G_3)}{1-G_1G_4H_1}}{1+\dfrac{(G_1G_4)(G_2+G_3)}{1-G_1G_4H_1}(H_2)}$$

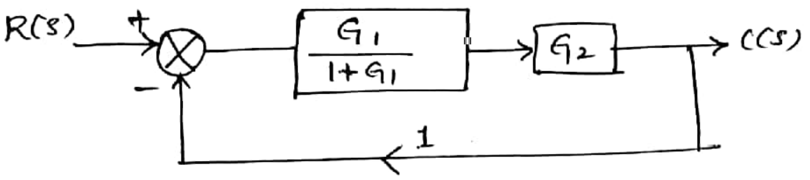
$$\boxed{\frac{C(s)}{R(s)} = \frac{(G_1G_4)(G_2+G_3)}{1-G_1G_4H_1+(G_1G_4H_2)(G_2+G_3)}}$$

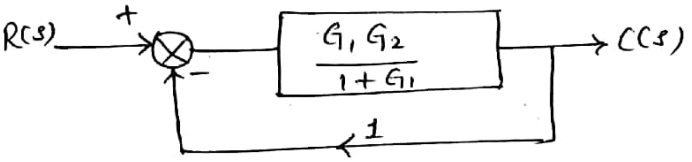
Scanned by CamScanner

(2) Refer the block diagram shown in fig. Using block diagram reduction techniques, find the overall transfer function $\frac{C(S)}{R(S)}$.



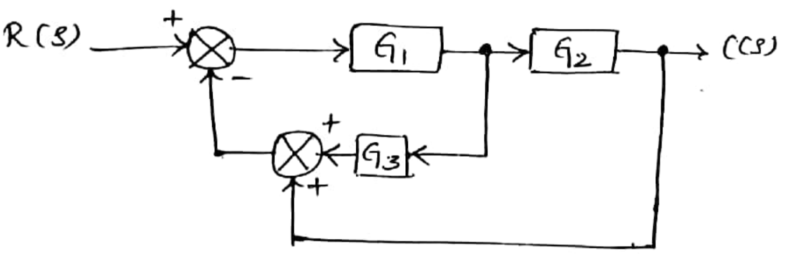
(*)



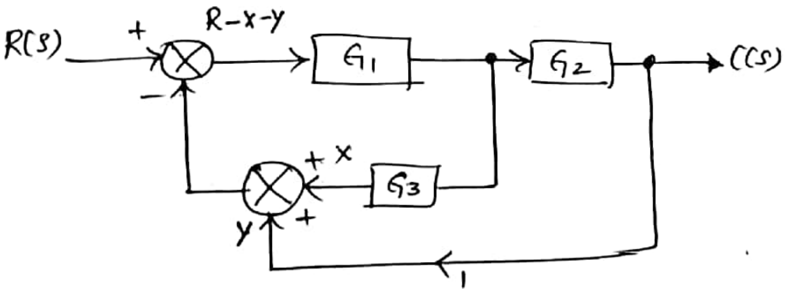


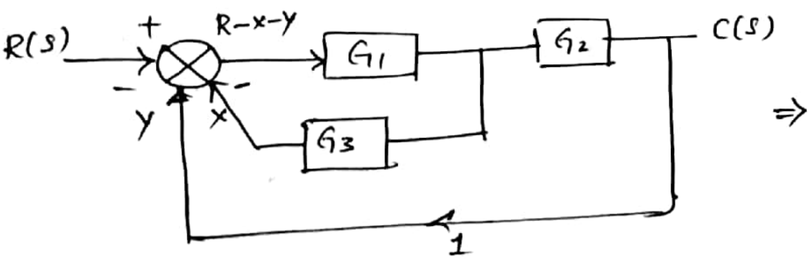
$$\frac{C(S)}{R(S)} = \frac{\frac{G_1 G_2}{1+G_1}}{1 + \frac{G_1 G_2}{1+G_1}} = \frac{G_1 G_2}{1+G_1+G_1 G_2}$$

(3) Consider the Block diagram shown in fig. find $\frac{C(S)}{R(S)}$ using BDRT.

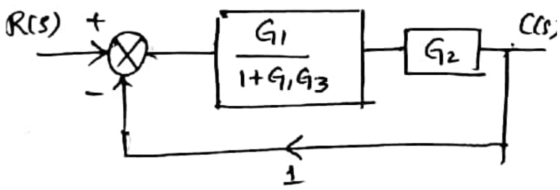


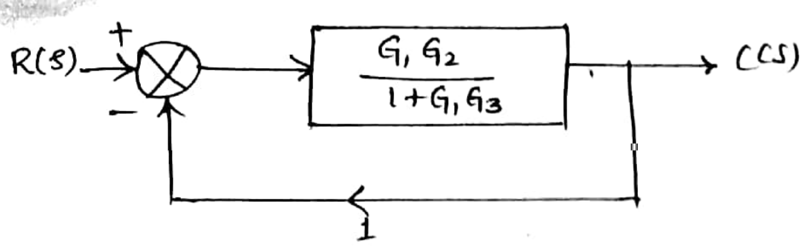
(A)





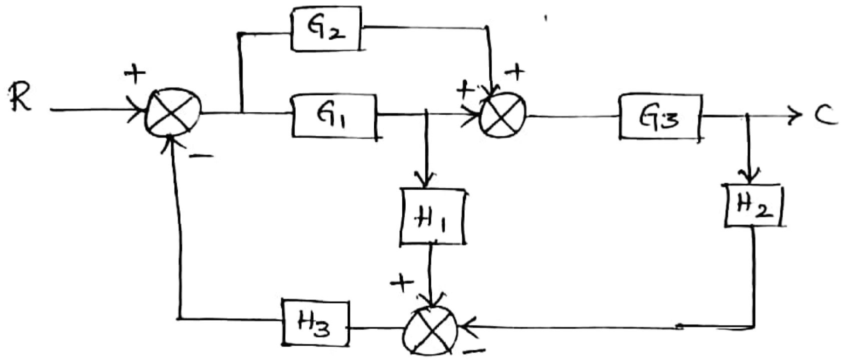
$\Rightarrow$



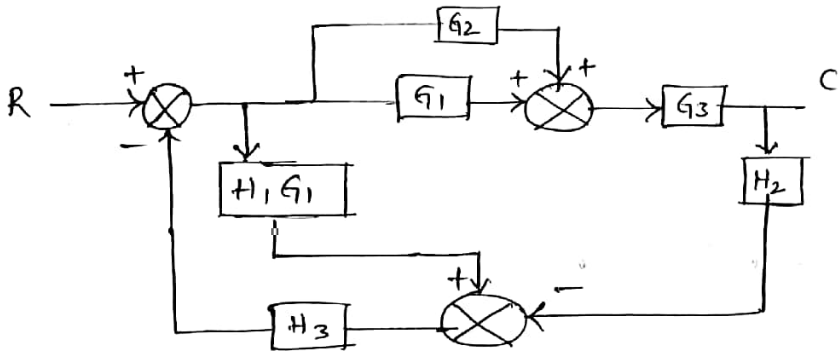


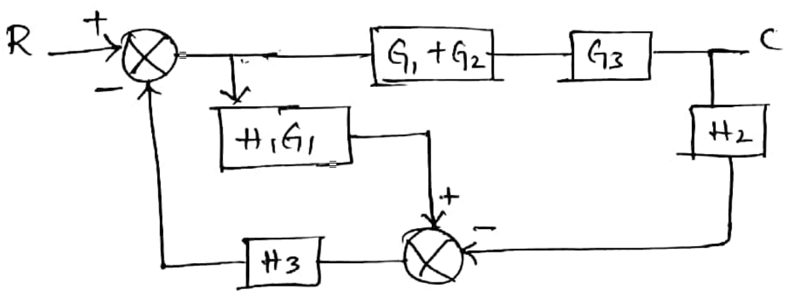
$$\Rightarrow \frac{C(s)}{R(s)} = \frac{\dfrac{G_1 G_2}{1+G_1 G_3}}{1 + \dfrac{G_1 G_2}{1+G_1 G_3}} = \frac{G_1 G_2}{1+G_1 G_3 + G_1 G_2}$$

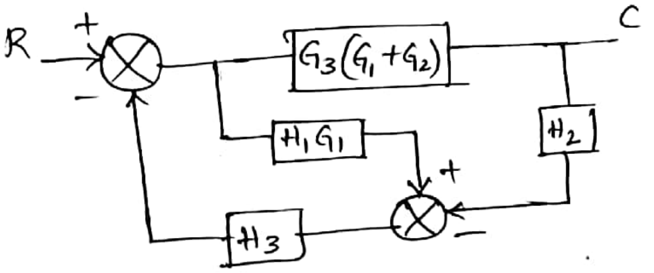
(4) Reduce the Block diagram shown in fig. find the T.F $\frac{C}{R}$.



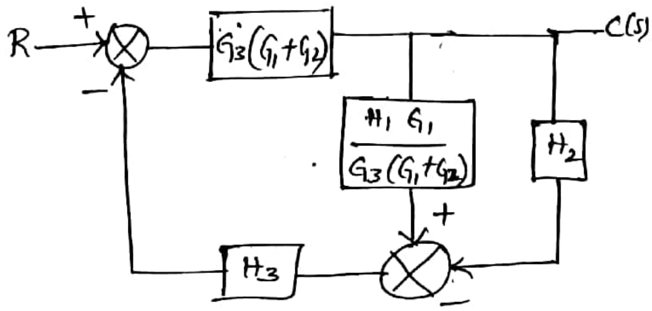
(A)

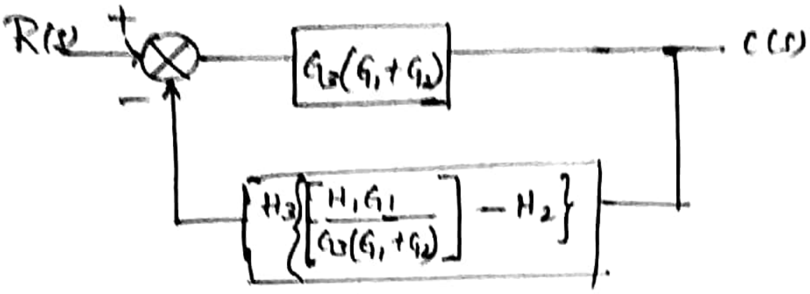






$\Rightarrow$



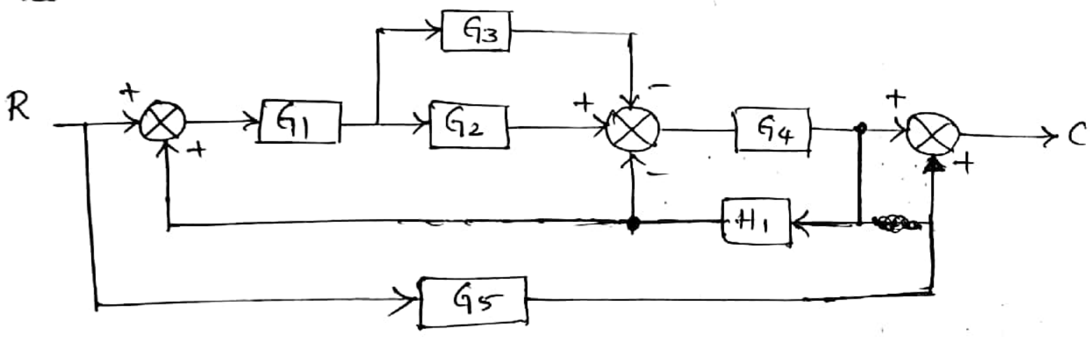


$$\frac{C(s)}{R(s)} = \frac{G_3(G_1+G_2)}{1+\left[\frac{H_3H_1G_1}{G_3(G_1+G_2)} - H_3H_2\right]\left[G_3(G_1+G_2)\right]}$$

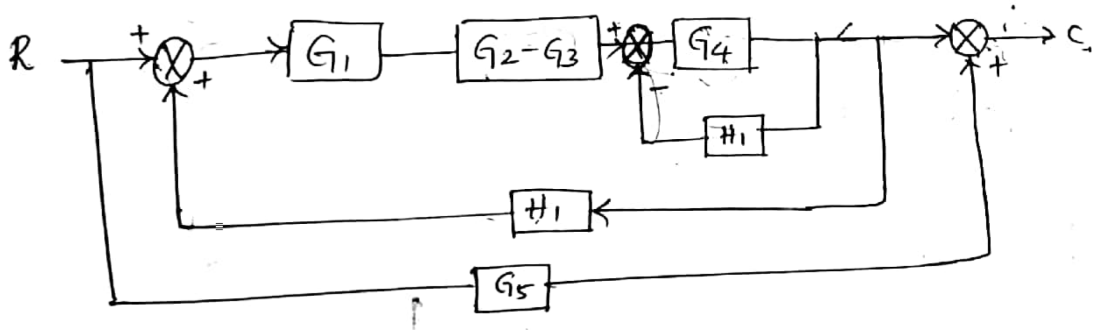
$$= \frac{G_3(G_1+G_2)}{1+\left(\frac{H_3H_1G_1 - H_3H_2G_3(G_1+G_2)}{\cancel{G_3(G_1+G_2)}}\right)\cancel{(G_3(G_1+G_2))}}$$

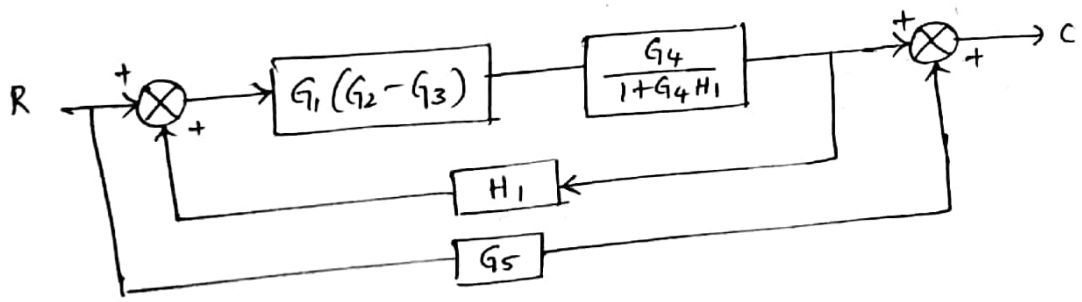
$$\boxed{\frac{C(s)}{R(s)} = \frac{G_3(G_1+G_2)}{1+H_3H_1G_1 - H_2H_3G_3(G_1+G_2)}}$$

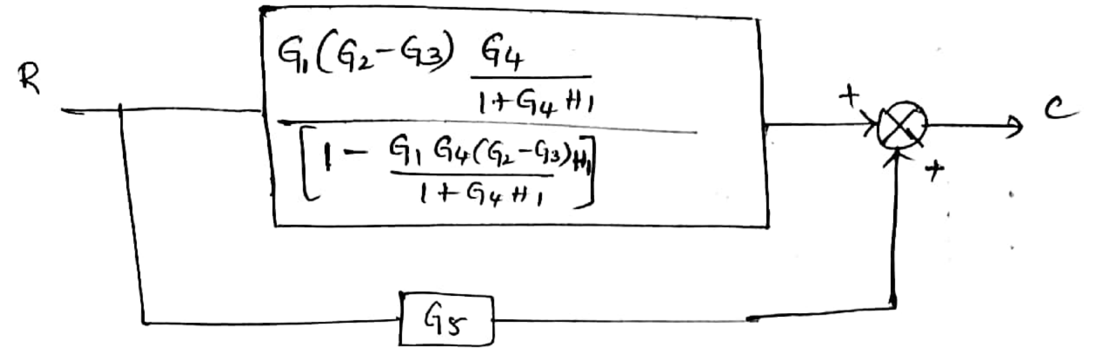
(5) Reduce the block diagram shown in fig. find $\frac{C}{R}$.



(A)





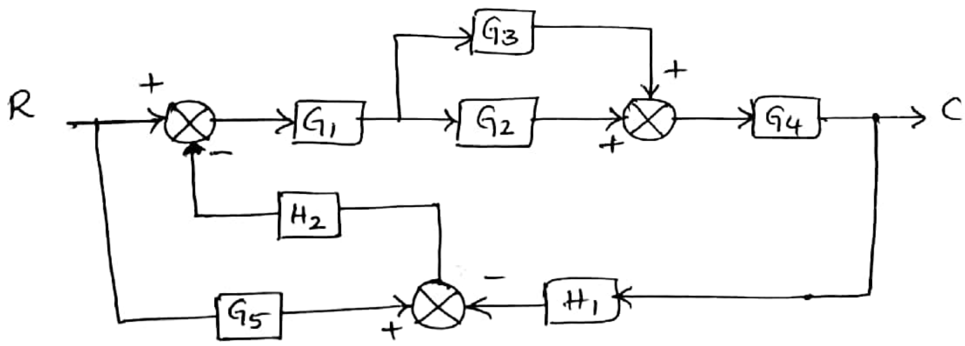


$$\frac{C(s)}{R(s)} = \frac{\dfrac{G_1(G_2-G_3)G_4}{1+G_4H_1}}{1-\dfrac{G_1G_4(G_2-G_3)H_1}{1+G_4H_1}} + G_5$$

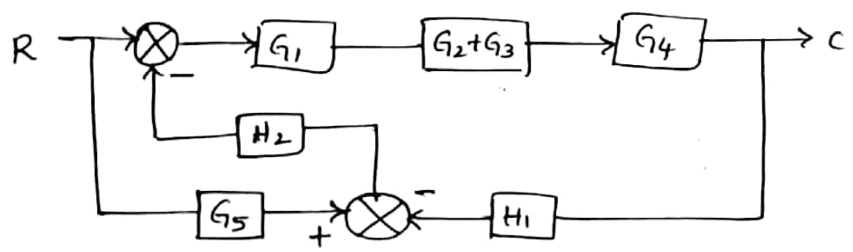
$$\boxed{\frac{C(s)}{R(s)} = \frac{G_1(G_2-G_3)G_4}{1+G_4H_1-G_1G_4H_1(G_2-G_3)} + G_5}$$

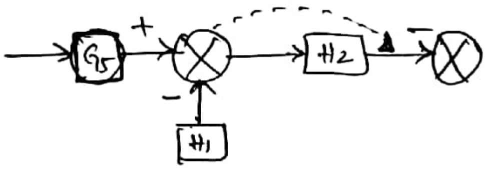
Imp

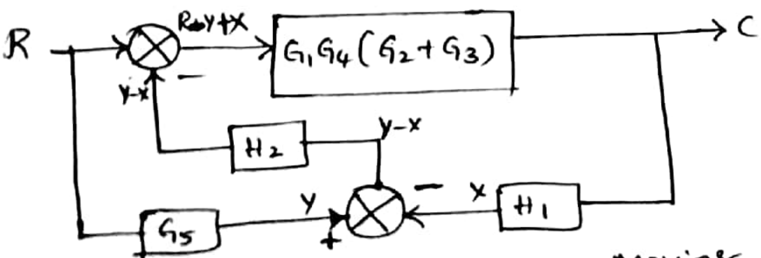
(6) find $\frac{C}{R}$ using BDRT.



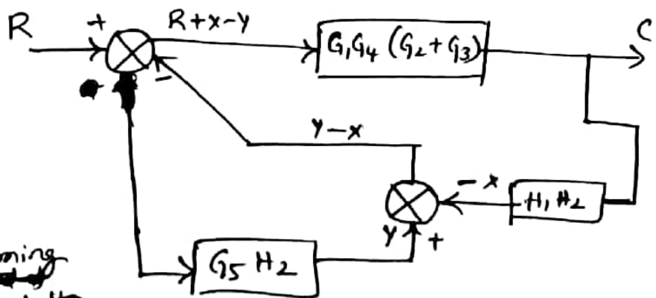
(A)



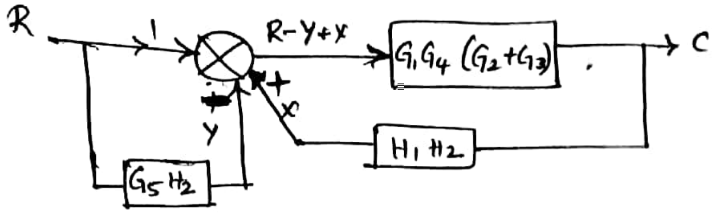


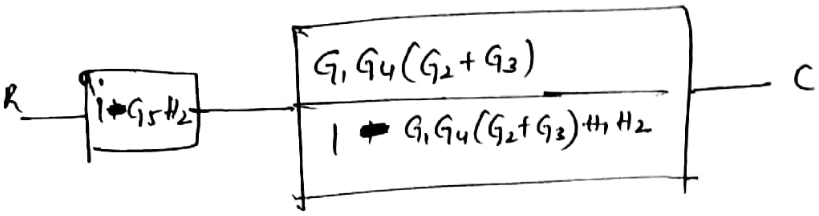


$\Rightarrow$



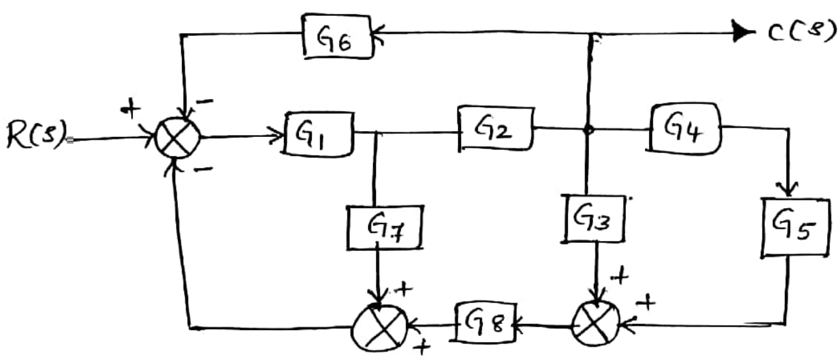
Moving Summing [redacted]
Point [redacted]
ahead of block H_2

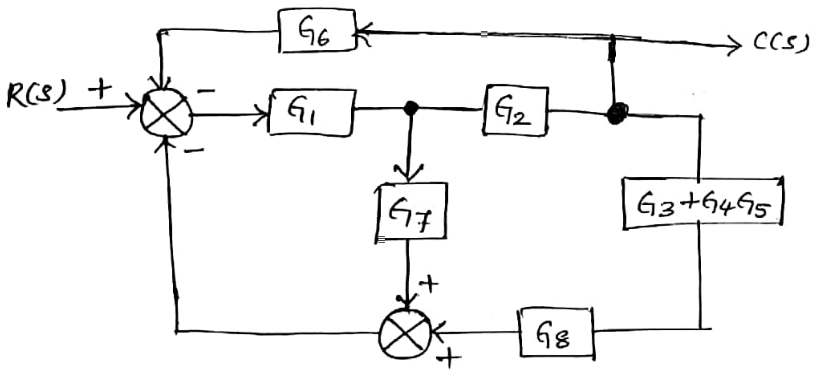




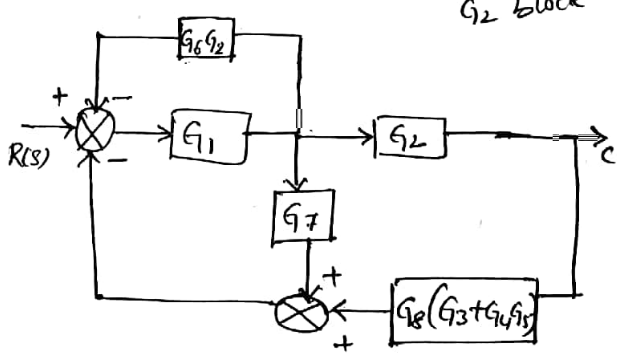
$$\boxed{\frac{C}{R} = \frac{G_1G_4(G_2+G_3)(1-G_5H_2)}{1-G_1G_4(G_2+G_3)H_1H_2}}$$

⑦ Reduce the block diagram & find $\frac{C(s)}{R(s)}$.

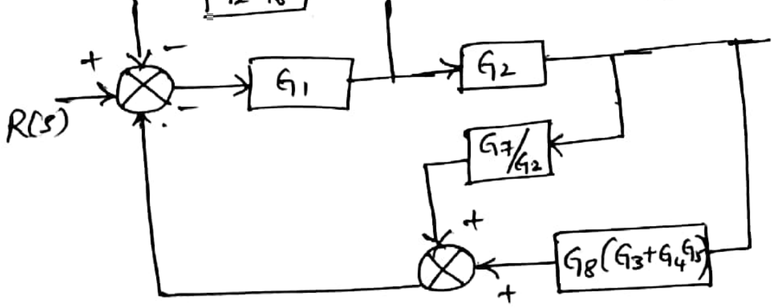


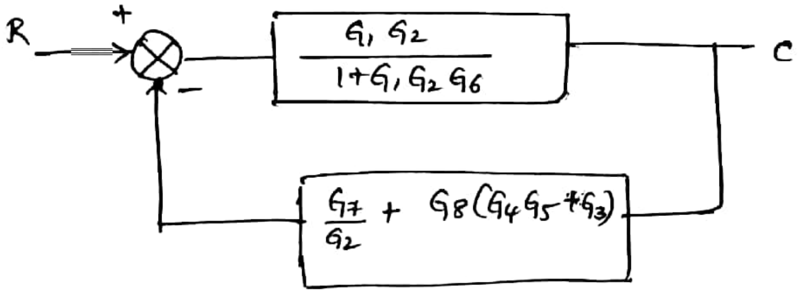


move Take off point behind G_2 block



more Take off point ahead of G_2

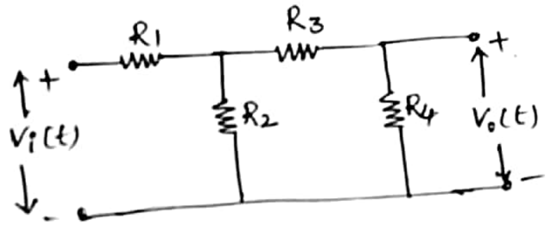




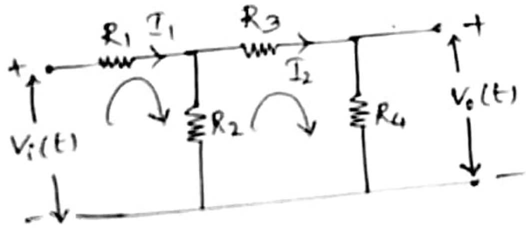
$$\frac{C(s)}{R(s)} = \frac{\frac{G_1 G_2}{1+G_1 G_2 G_6}}{1 + \left(\frac{G_1 G_2}{1+G_1 G_2 G_6}\right)\left(\frac{G_7}{G_2} + G_8(G_4 G_5 + G_3)\right)}$$

$$\boxed{\frac{C(s)}{R(s)} = \frac{G_1 G_2}{1 + G_1 G_2 G_6 + G_1 G_7 + G_1 G_2 G_8 (G_4 G_5 + G_3)}}$$

(8) obtain the block diagram for the given Electrical N/w.



(A)



KVL Loop ①:

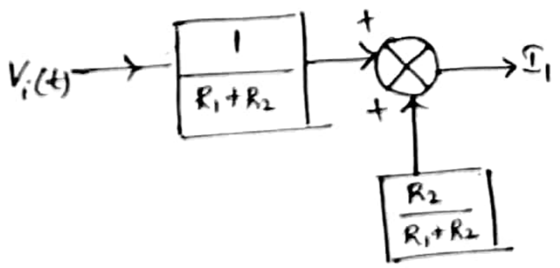
$$V_i(t) = I_1 R_1 + (I_1 - I_2) R_2$$

$$= I_1 R_1 + I_1 R_2 - I_2 R_2$$

$$V_i(t) = I_1 (R_1 + R_2) - I_2 R_2$$

(a) $$I_1 = \frac{V_i(t) + I_2 R_2}{R_1 + R_2}$$

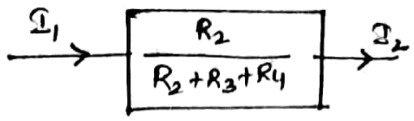
$$\boxed{I_1 = V_i(t)\left[\frac{1}{R_1+R_2}\right] + I_2\left[\frac{R_2}{R_1+R_2}\right]}$$



Scanned by CamScanner

KVL Loop ②:

$$I_2 R_3 + I_2 R_4 + R_2 (I_2 - I_1) = 0$$

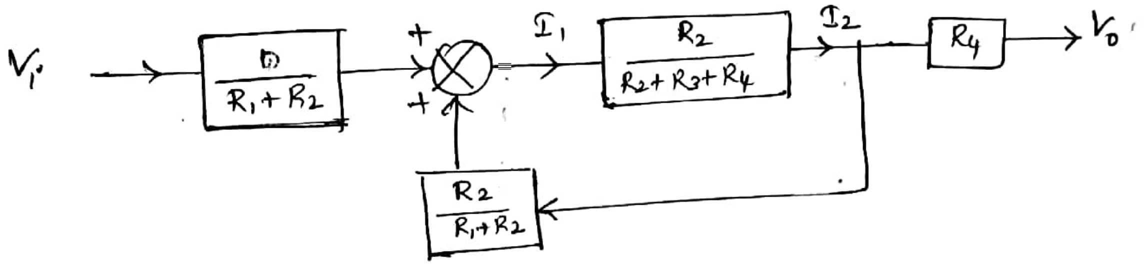


$$I_2 \left[R_3 + R_4 + R_2 \right] - I_1 R_2 = 0$$

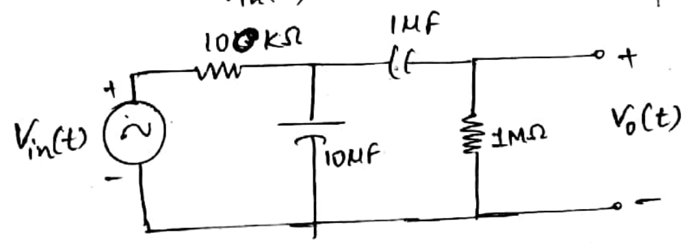
$$I_2 = I_1 \left[\frac{R_2}{R_2+R_3+R_4} \right].$$

$$\boxed{V_0 = I_2 R_4}$$

Combine all the three blocks:



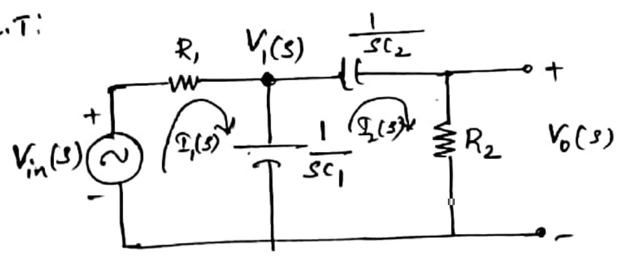
⑨ Draw Block Diagram for the Electric circuit shown & hence Evaluate the transfer function $\frac{V_0(s)}{V_{in}(s)}$



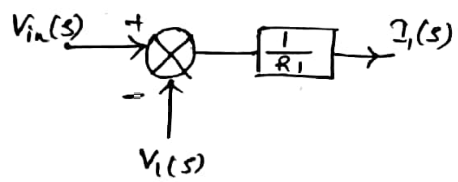
Ⓐ Let $R_1 = 100 k\Omega$ $R_2 = 1 M\Omega$

$C_1 = 10 MF$ $C_2 = 1 MF$

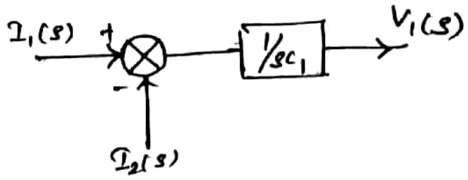
Take L.T:



$$I_1(s) = \frac{1}{R_1} \left[V_{in}(s) - V_1(s) \right]$$

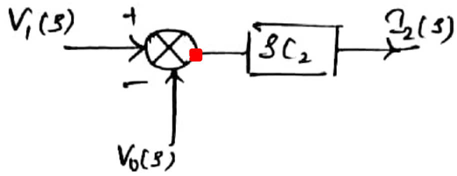


$$V_1(s) = \frac{1}{sC_1}\left[I_1(s) - I_2(s)\right]$$

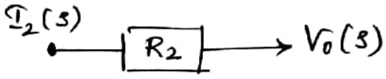


$$I_2(s) = \frac{\left[V_1(s) - V_0(s)\right]}{\frac{1}{sC_2}}$$

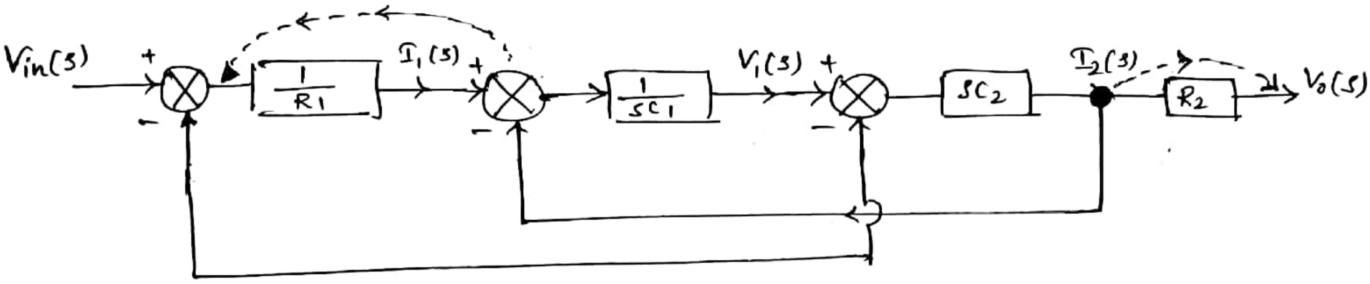
$$I_2(s) = sC_2\left[V_1(s) - V_0(s)\right]$$



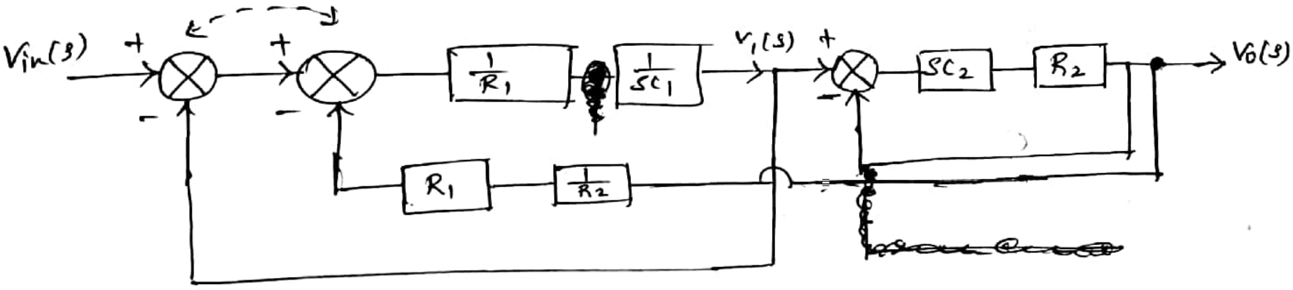
$$V_0(s) = R_2 I_2(s)$$



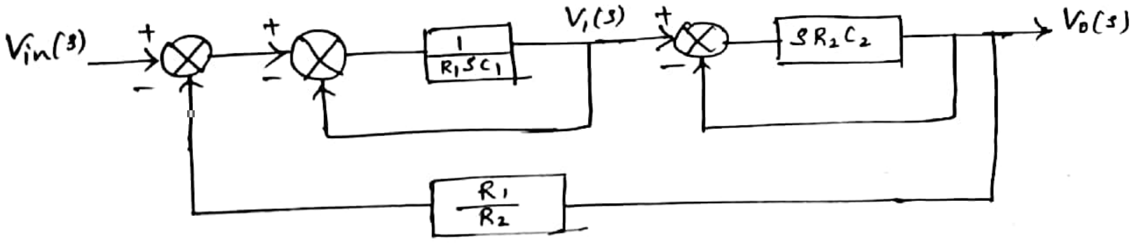
Combine all the blocks:

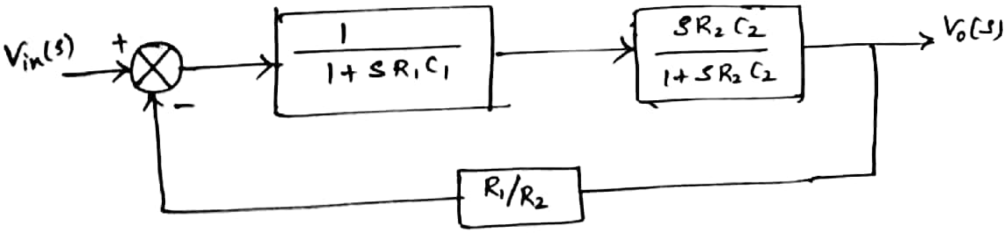


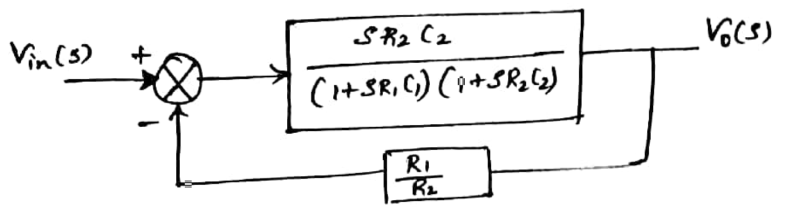
Shifting Summing Point Backwards & takeoff Point forward



Interchange The Summing Points







$$\frac{V_o(s)}{V_{in}(s)} = \frac{\dfrac{sR_2C_2}{(1+sR_1C_1)(1+sR_2C_2)}}{1+\left(\dfrac{R_1}{\cancel{R_2}}\right)\dfrac{s\cancel{R_2}C_2}{(1+sR_1C_1)(1+sR_2C_2)}}$$

$$= \frac{sR_2C_2}{(1+sR_1C_1)(1+sR_2C_2)+sR_1C_2}$$

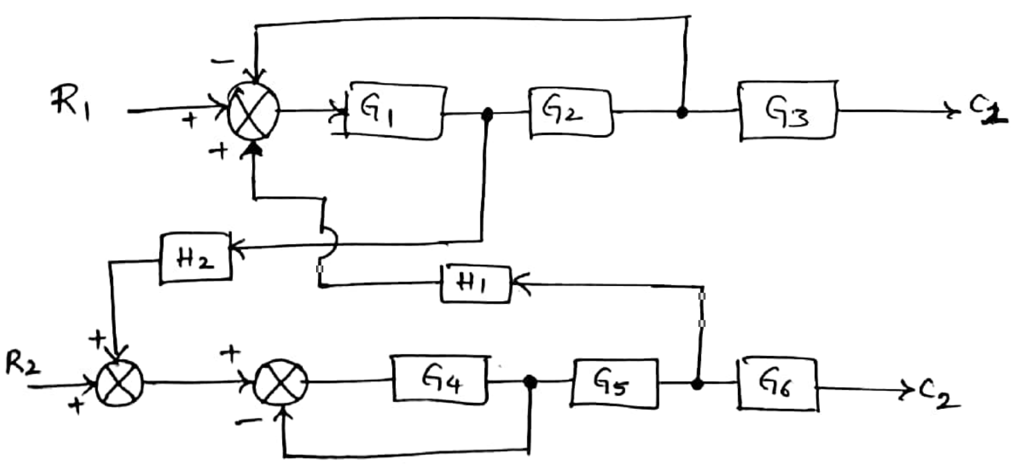
$$\boxed{\frac{V_o(s)}{V_{in}(s)} = \frac{sR_2C_2}{1+sR_2C_2+sR_1C_1+s^2R_1R_2C_1C_2+sR_1C_2}} \quad \text{—} (1)$$

Substitute the Values of R_1, R_2, C_1, C_2 in eq (1).

we get $\boxed{\dfrac{V_o(s)}{V_{in}(s)} = \dfrac{s}{s^2+2.1s+1}}$

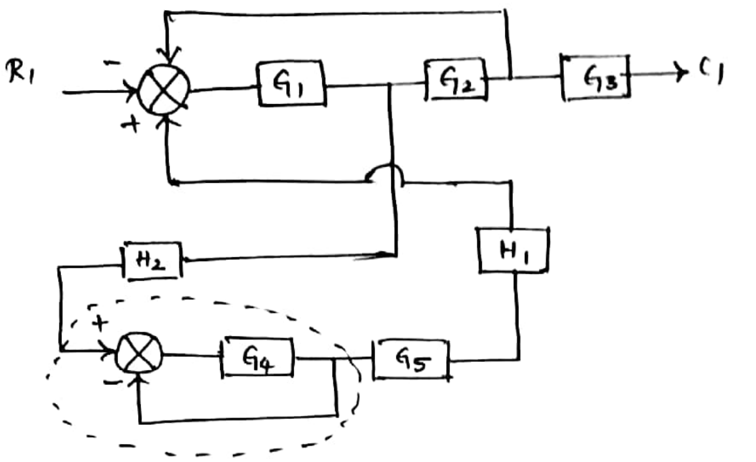
(10) For the System shown in figure determine (a) $\frac{C_1}{R_1}$ (b) $\frac{C_2}{R_2}$

(A) (a) Determination of $\frac{C_1}{R_1}$:- Since we want only $\frac{C_1}{R_1}$ hence Remove C_2 & R_2. Hence we can remove G_6 also.



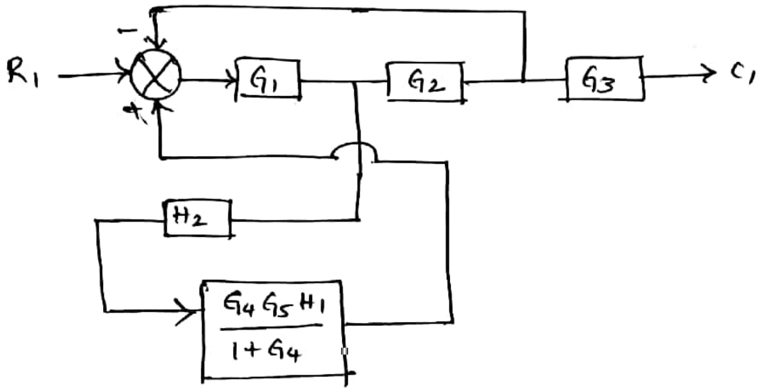
(Given Block Diagram)

Step ①: Removing C_2, R_2 & G_6

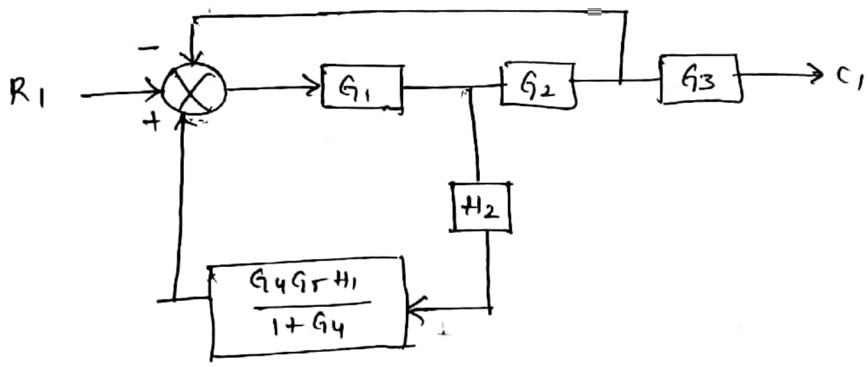


$$\boxed{\frac{G_4}{1+G_4}}$$

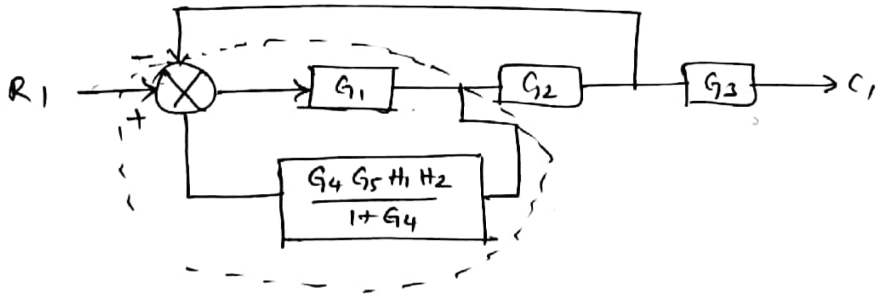
Step ②: Reducing the loop & taking G_5 & H_1 as Series



Re write the block diagram

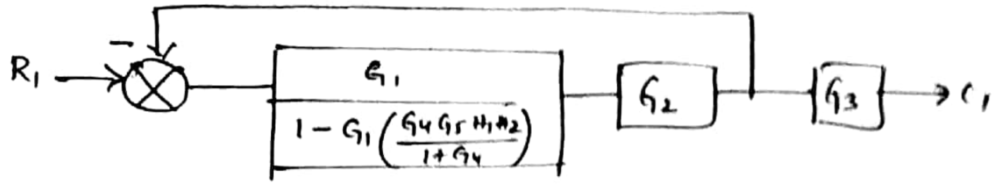


Step ③: H_2 is in Series with $\frac{G_4 G_5 H_1}{1+G_4}$

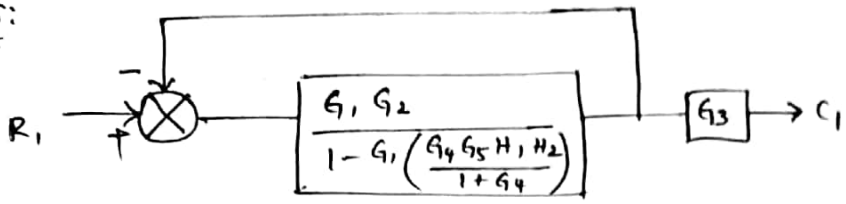


Step 4:

$$\frac{G_1}{1 - G_1\left(\frac{G_4 G_5 H_1 H_2}{1+G_4}\right)}$$



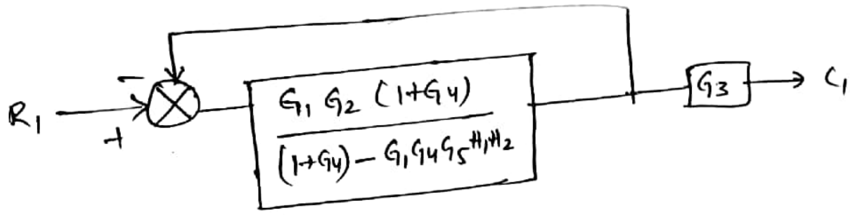
Step 5:



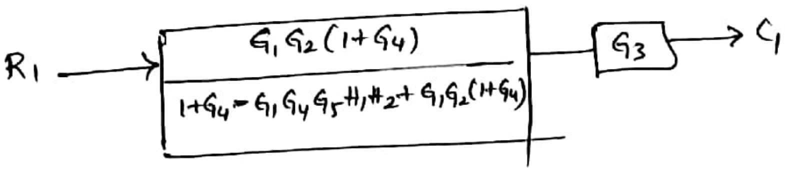
Reduce Loop:

$$\frac{\dfrac{G_1G_2}{1-G_1\left(\dfrac{G_4G_5H_1H_2}{1+G_4}\right)}}{1+\dfrac{G_1G_2}{1-G_1\left(\dfrac{G_4G_5H_1H_2}{1+G_4}\right)}} = \frac{G_1G_2}{}$$

$$\boxed{\frac{G_1G_2(1+G_4)}{(1+G_4)-G_1(G_4G_5H_1H_2)}}$$



$$\frac{\dfrac{G_1G_2(1+G_4)}{(1+G_4)-G_1G_4H_1H_2G_5}}{1+\dfrac{G_1G_2(1+G_4)}{(1+G_4)-G_1G_4G_5H_1H_2}} = \frac{G_1G_2(1+G_4)}{1+G_4-G_1G_4G_5H_1H_2+G_1G_2(1+G_4)}$$



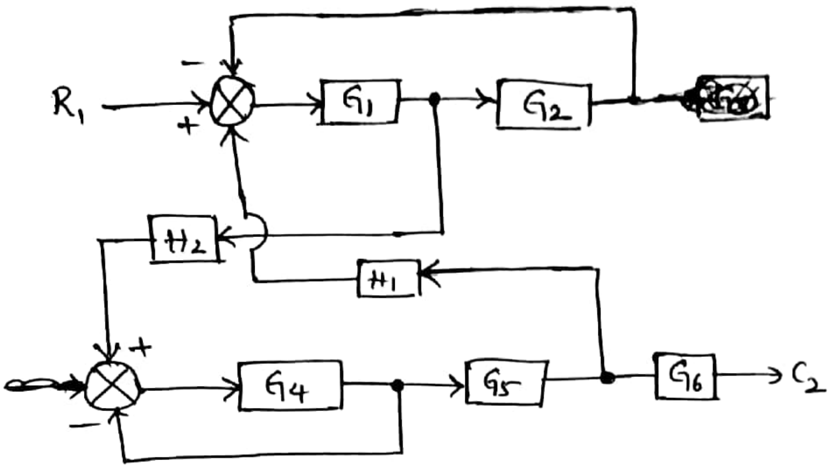
$$\frac{C_1}{R_1} = \frac{G_1G_2(1+G_4)\,G_3}{1+G_4-G_1G_4G_5H_1H_2+G_1G_2(1+G_4)} = \frac{G_1G_2G_3+G_1G_2G_3G_4}{1+G_1G_2+G_1G_2G_4+G_4-G_1G_4G_5H_1H_2}$$

Scanned by CamScanner

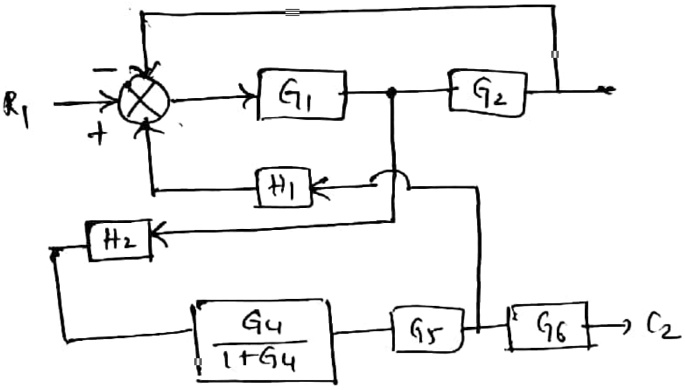
Determination of $\frac{C_2}{R_1}$:-

$R_2 = 0$, $C_1 = 0$, $G_3 = 0$

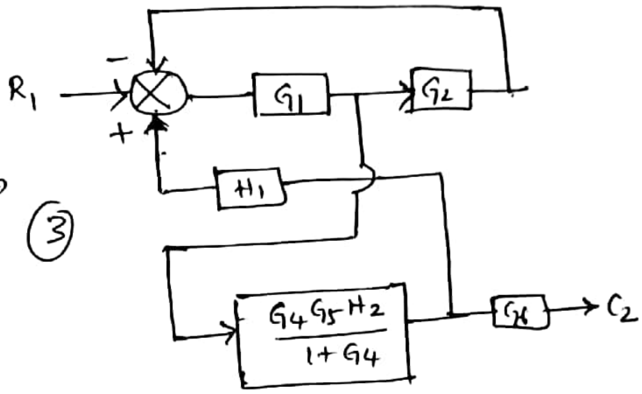
1



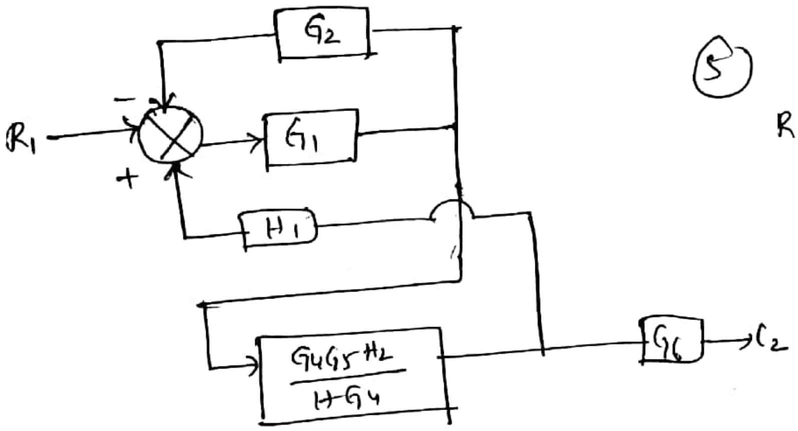
2



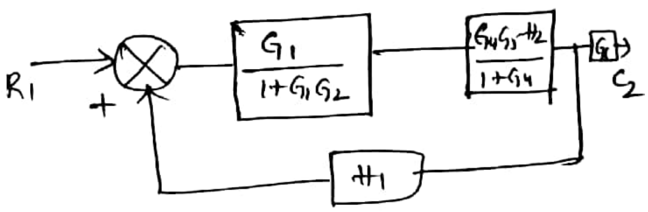
⇒ 3



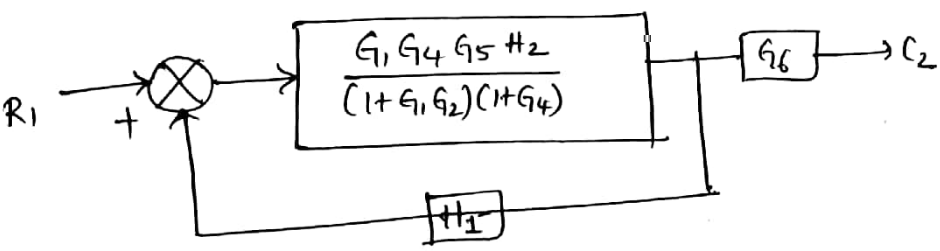
4



5



6



$$\frac{C_2}{R_1} = \left[\frac{\frac{G_1 G_4 G_5 H_2}{(1+G_1G_2)(1+G_4)}}{1 - \frac{H_1 G_1 G_4 G_5 H_2}{(1+G_1G_2)(1+G_4)}} \right] * G_6$$

$$\frac{C_2}{R_1} = \frac{G_1 G_4 G_5 H_2 G_6}{(1+G_1G_2)(1+G_4) - H_1 G_1 G_4 G_5 H_2} = \frac{G_1 G_4 G_5 G_6 H_2}{1 + G_4 + G_1G_2 + G_1G_2G_4 - H_1 G_1 G_4 G_5 H_2}$$

Signal flow Graph:

* A signal flow graph is regarded as a simplified version of block diagram.
* It is mainly used in system diagramming.
* Mason's gain formula is especially useful in reducing large & complex diagram in one step without requiring step by step reductions.

Terms used in SFG:

(1) **Node:** A node represents a system variable which is equal to the sum of all the incoming signals at the node. Outgoing signals from the node do not affect the value of the node variable.

(2) **Forward Path:-** A f/w path is a path that starts at an input node & ends at an o/p node & along which no node is traversed more than once.

(3) **Loop:** A Loop is a path which originates & terminates at the same node & along which no node is traversed more than once.

(4) **Non-touching Loops:** Non-touching loops are loops which do not posses any common node.

(5) **Self-Loop:** A Self Loop is a loop consisting of a single branch.

Comparison of Block Diagram & Signal flow Graph Method:

Block Diagram	Signal flow Graph
(1) It is a pictorial representation of the functions performed by each Component & of the flow of Signals.	(1) It is a graphical representation of a relationship b/w variables of a set of linear algebraic equations.
(2) It can be used to represent linear as well as non linear systems.	(2) It can be used to represent only Linear Systems.
(3) No direct formula is available to find Transfer function.	(3) Mason's gain formula is available to find the overall transfer function.
(4) Step by step procedure is to be followed to find transfer function.	(4) Transfer function can be obtained in one step.
(5) It is not a systematic method.	(5) It is a Systematic method.

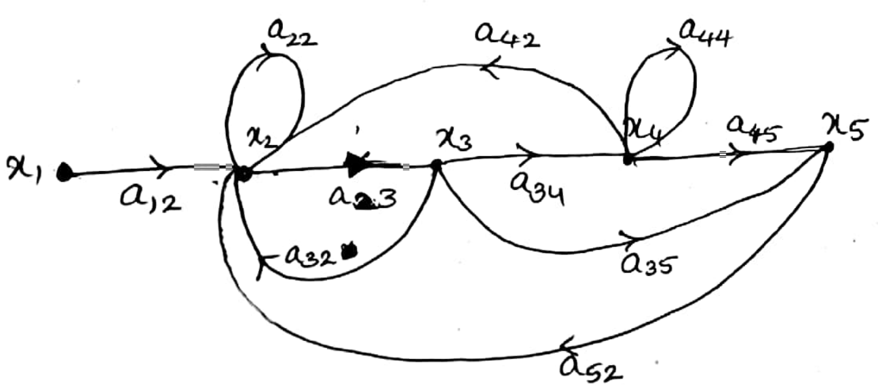
Example ①: The System equations is as follows. write the SFG, where x_1 is the input variable & x_5 is the output variable.

$$x_2 = a_{12}x_1 + a_{22}x_2 + a_{32}x_3 + a_{42}x_4 + a_{52}x_5$$

$$x_3 = a_{23}x_2$$

$$x_4 = a_{34}x_3 + a_{44}x_4$$

$$x_5 = a_{35}x_3 + a_{45}x_4$$



Mason's Gain formula:

"Mason's Gain formula is used to find transfer function in a single step."

$$\boxed{T.F = \frac{\sum_K M_K \Delta_K}{\Delta}}$$

where,

$M_K \rightarrow$ "Path gain" of the K^{th} forward path.

$\Delta_K \rightarrow$ Value of Δ for that part of the graph not touching the K^{th} forward path.

$T \rightarrow$ overall gain of the system.

$\Delta \rightarrow$ determinant of the SFG.

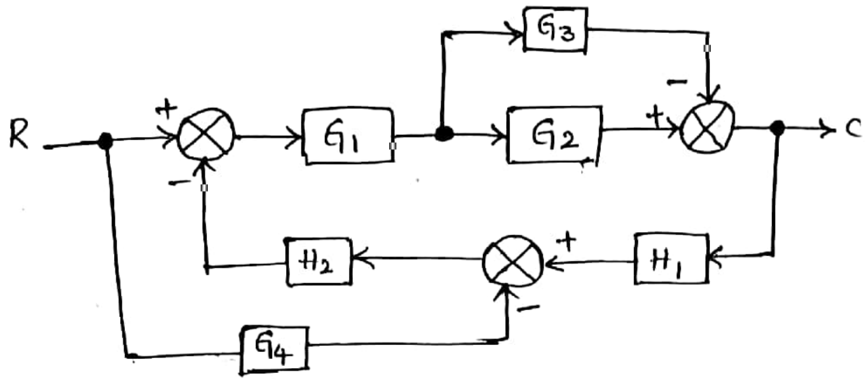
$\Delta = 1 -$ (Sum of the loop gains of all individual loops)
+ (Sum of the gain products of all possible combinations of two non-touching loops)
− (Sum of the gain products of all possible combinations of three non-touching loops) + ... etc

$$\therefore \Delta = 1 - \sum_P M_{P_1} + \sum_P M_{P_2} - \sum_P M_{P_3} + \cdots\cdots$$

where

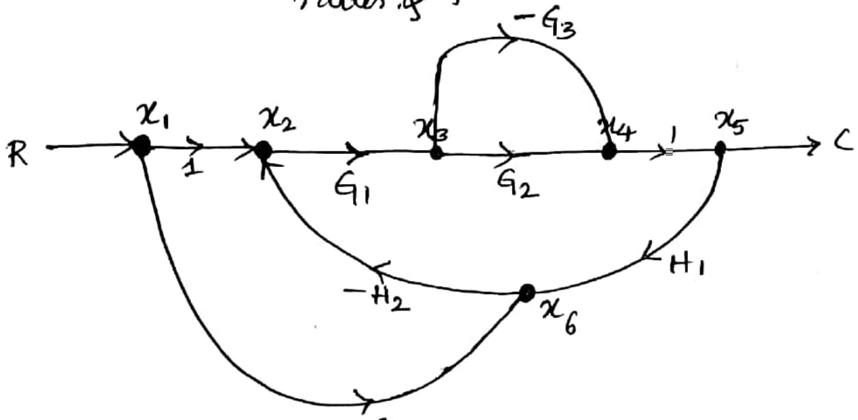
$M_{pr} \longrightarrow$ Is the gain product of p^{th} possible combination of r non-touching loops.

Problems ①: obtain the transfer function of the control system whose block diagram is shown in fig. using Signal flow Graph Method



Sol: step 1: consider all the take off points & summing points as nodes & plot the SFG.

[Note: Consider the ⊖ve sign w.r.to the gains]



Step 2: find the no of forward paths & its gain.

$M_1 \longrightarrow G_1 G_2$

$M_2 \rightarrow -G_1 G_3$

$M_3 \rightarrow + G_4 H_2 G_1 G_2$

$M_4 \rightarrow - G_4 H_2 G_1 G_3$

Step 3: find the no of Loops:

$L_1 \rightarrow \{-G_1 G_2 H_1 H_2\}$

$L_2 \rightarrow \{G_1 G_3 H_1 H_2\}$

Step 4: find the no of 2 non-touching Loops:

$\boxed{\text{Zero}}$

Step 5: find the no of 3 non-touching Loops: $\boxed{\text{Zero}}$

Step 6: Find the determinant value:

$$\Delta = 1 - (L_1 + L_2)$$

$$\Delta = 1 - (-G_1 G_2 H_1 H_2 + G_1 G_3 H_1 H_2)$$

$$\boxed{\Delta = 1 + G_1 G_2 H_1 H_2 - G_1 G_3 H_1 H_2}$$

Step 7: Find the forward paths touching the Loops & based on that find "Δ".

Note: * If the forward path touches all the loops then

$$\boxed{\Delta = 1}$$

* If the forward path does not touch some of the loops then gain of such Loops need to be subtracted from the "Δ".

Here,

$M_1 \longrightarrow$ Touches all Loops $\therefore \Delta_1 = 1$

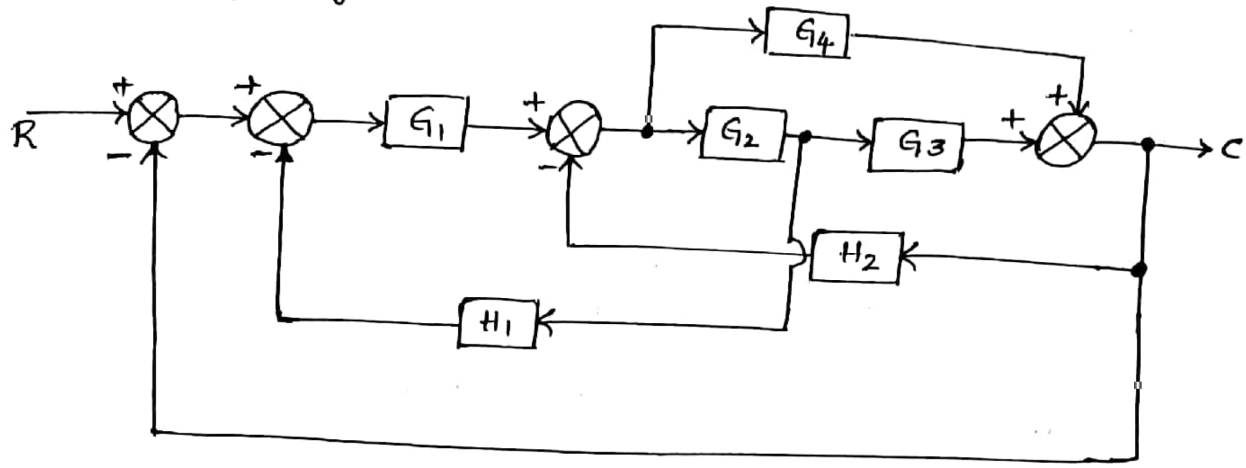
$M_2, M_3, M_4 \longrightarrow$ Touches all Loops $\therefore \Delta_2 = \Delta_3 = \Delta_4 = 1$

Step 8: Find the T.F using Mason's gain formula. $\boxed{T.F = \frac{\sum_K M_K \Delta_K}{\Delta}}$

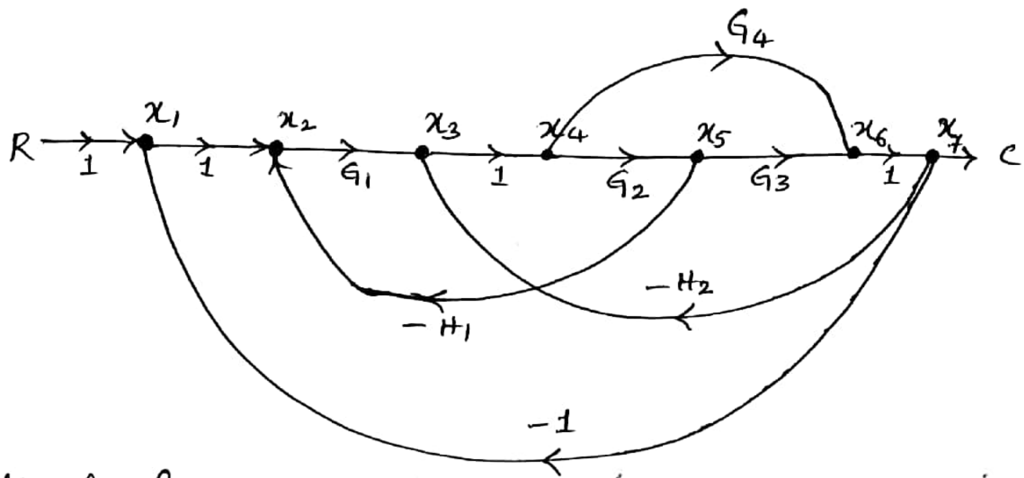
$$\frac{C}{R} = \frac{M_1 \Delta_1 + M_2 \Delta_2 + M_3 \Delta_3 + M_4 \Delta_4}{\Delta}$$

$$\therefore \boxed{\frac{C}{R} = \frac{G_1 G_2 - G_1 G_3 + G_4 H_2 G_1 G_2 - G_4 H_2 G_1 G_3}{1 + G_1 G_2 H_1 H_2 - G_1 G_3 H_1 H_2}}$$

② Obtain the transfer function of the feedback control s/m shown in fig. using signal flow graph method.



Sol:



① No of forward paths:

$M_1 \longrightarrow G_1 G_2 G_3$ $\qquad \Delta_1 = 1$

$M_2 \longrightarrow G_1 G_4$ $\qquad \Delta_2 = 1$

② No of Loops:

$L_1 = -G_1 G_2 H_1$

$L_2 = -G_2 G_3 H_2$

$L_3 = -G_4 H_2$

$L_4 = -G_1 G_2 G_3$

$L_5 = -G_1 G_4$

③ Non-touching Loops = 0

④ $\Delta = 1 - [L_1 + L_2 + L_3 + L_4 + L_5]$

$\Delta = 1 - [-G_1 G_2 H_1 - G_2 G_3 H_2 - G_4 H_2 - G_1 G_2 G_3 - G_1 G_4]$

$\Delta = 1 + G_1 G_2 H_1 + G_2 G_3 H_2 + G_4 H_2 + G_1 G_2 G_3 + G_1 G_4$

$$T.F = \frac{C}{R} = \frac{M_1\Delta_1 + M_2\Delta_2}{\Delta}$$

$$\boxed{\frac{C}{R} = \frac{G_1G_2G_3 + G_1G_4}{1 + G_1G_2H_1 + G_2G_3H_2 + G_4H_2 + G_1G_2G_3 + G_1G_4}}$$

(3) The s/m. equations is as follows. find the T.F for $\frac{x_5}{x_1}$ using SFG.

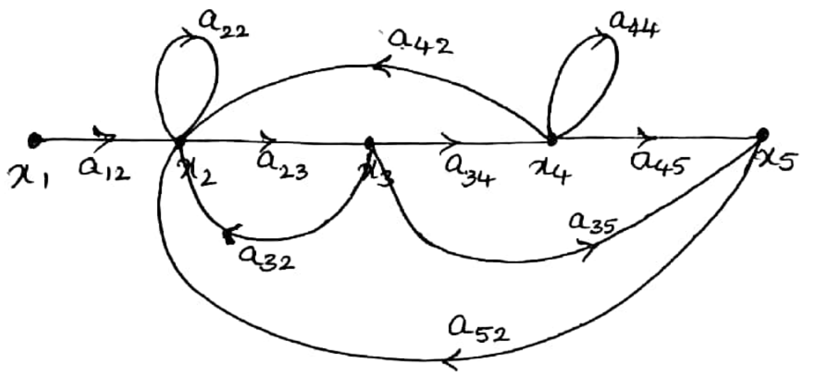
$$x_2 = a_{12}x_1 + a_{22}x_2 + a_{32}x_3 + a_{42}x_4 + a_{52}x_5$$

$$x_3 = a_{23}x_2$$

$$x_4 = a_{34}x_3 + a_{44}x_4$$

$$x_5 = a_{35}x_3 + a_{45}x_4$$

(A) From the given question we understood "x_1" in the i/p & "x_5" in o/p & there are 5 nodes.



(1) NO of f/w paths:

$M_1 = a_{12}\,a_{23}\,a_{34}\,a_{45}$ $\qquad \Delta_1 = 1$

$M_2 = a_{12}\,a_{23}\,a_{35}$ $\qquad \Delta_2 = 1 - a_{44}$

(2) NO of Loops:

$L_1 = a_{22}$

$L_2 = a_{44}$

$L_3 = a_{23}\,a_{32}$

$L_4 = a_{23}\,a_{34}\,a_{42}$

$L_5 = a_{23}\,a_{34}\,a_{45}\,a_{52}$

$L_6 = a_{23}\,a_{35}\,a_{52}$

(3) Two non-touching loops:

$L_{12} = a_{22}\,a_{44}$

$L_{23} = a_{44}\,a_{23}\,a_{32}$

$L_{26} = a_{44}\,a_{23}\,a_{35}\,a_{52}$

(4) 3 non-touching loops:

Zero

(5) $T.F = \dfrac{\sum_K M_K \Delta_K}{\Delta}$

find Δ :-

$\Delta = 1 - (\text{Sum of all Loops}) + (\text{Sum of two non-touching Loops})$

$= 1 - [L_1 + L_2 + L_3 + L_4 + L_5 + L_6) + [L_{12} + L_{23} + L_{26}]$

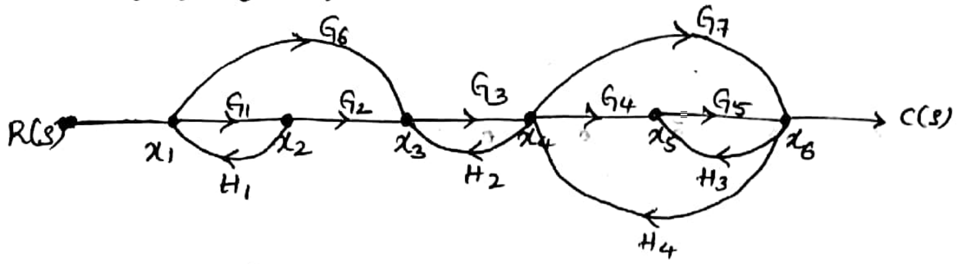
$= 1 - [a_{22} + a_{44} + a_{23}a_{32} + a_{23}a_{34}a_{42} + a_{23}a_{34}a_{45}a_{52} + a_{23}a_{35}a_{52}]$

$+ [a_{22}a_{44} + a_{44}a_{23}a_{32} + a_{44}a_{23}a_{35}a_{52}]$

$\therefore \dfrac{x_5}{x_1} = \dfrac{M_1\Delta_1 + M_2\Delta_2}{\Delta}$

$$\boxed{\frac{x_5}{x_1} = \frac{[a_{12}a_{23}a_{34}a_{45}] + [a_{12}a_{23}a_{35}]}{1 - a_{22} - a_{44} - a_{23}a_{32} - a_{23}a_{34}a_{42} - a_{23}a_{34}a_{45}a_{52} - a_{23}a_{35}a_{52} + a_{22}a_{44} + a_{44}a_{23}a_{32} + a_{44}a_{23}a_{35}a_{52}}}$$

(4) find the transfer function of the s/m whose SFG is as shown in fig.



(A)

(1) NO of f/w paths:

$M_1 = G_1 G_2 G_3 G_4 G_5$

$M_2 = G_6 G_3 G_4 G_5$

$M_3 = G_1 G_2 G_3 G_7$

$M_4 = G_6 G_3 G_7$

(2) NO of Loops:

$L_1 = G_1 H_1$

$L_2 = G_3 H_2$

$L_3 = G_5 H_3$

$L_4 = G_4 G_5 H_4$

$L_5 = G_7 H_4$

(3) Pairs of 2 non-touching Loops:

$L_{12} = (G_1 H_1)(G_2 H_3)$

$L_{13} = G_1 G_5 H_1 H_3$

$L_{23} = G_3 G_5 H_2 H_3$

$L_{15} = G_1 G_7 H_1 H_4$

$L_{14} = G_1 G_4 G_5 H_1 H_4$

(4) Combination ~~Pairs~~ of 3 non-touching Loops:

$L_{123} = G_1 G_3 G_5 H_1 H_2 H_3$

(5) $\Delta = 1 - (L_1 + L_2 + L_3 + L_4 + L_5) + (L_{12} + L_{13} + L_{23} + L_{14} + L_{15}) - (L_{123})$

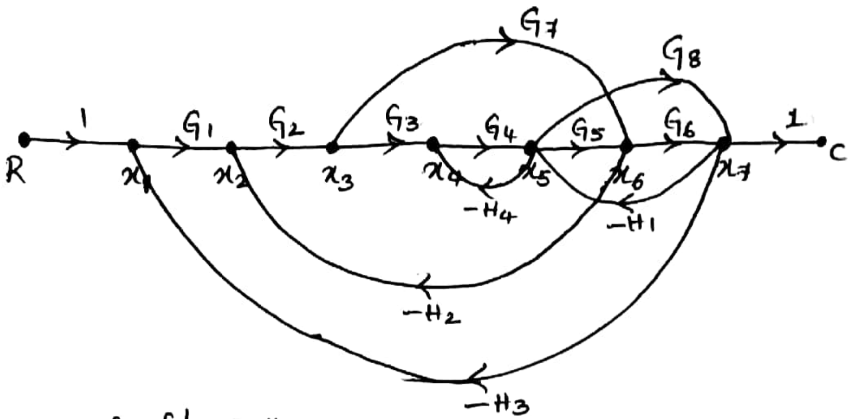
$= 1 - [G_1 H_1 + G_3 H_2 + G_5 H_3 + G_4 G_5 H_4 + G_7 H_4] + [G_1 G_2 H_1 H_3 + G_1 G_5 H_1 H_3$
$+ G_3 G_5 H_2 H_3 + G_1 G_7 H_1 H_4 + G_1 G_4 G_5 H_1 H_4] - G_1 G_3 G_5 H_1 H_2 H_3$

(6) $\Delta_1 = \Delta_2 = \Delta_3 = \Delta_4 = 1$

$$\therefore \frac{C}{R} = \frac{M_1 \Delta_1 + M_2 \Delta_2 + M_3 \Delta_3 + M_4 \Delta_4}{\Delta}$$

$$\boxed{\frac{C}{R} = \frac{(G_1 G_2 G_3 G_4 G_5) + (G_3 G_4 G_5 G_6) + (G_1 G_2 G_3 G_7) + (G_3 G_6 G_7)}{1 - [G_1 H_1 + G_3 H_2 + G_5 H_3 + G_4 G_5 H_4 + G_7 H_4] + [G_1 G_2 H_1 H_3 + G_1 G_5 H_1 H_3 + G_3 G_5 H_2 H_3 + G_1 G_7 H_1 H_4 + G_1 G_4 G_5 H_1 H_4] - G_1 G_3 G_5 H_1 H_2 H_3}}$$

(5) find the transfer function $\frac{C}{R}$ of the signal flow graph shown in figure.



Sol: (1) No of flw paths:

$M_1 = G_1 G_2 G_3 G_4 G_5 G_6$

$M_2 = G_1 G_2 G_7 G_6$

$M_3 = G_1 G_2 G_3 G_4 G_8$

(2) No of Loops:

$L_1 = -G_4 H_4$

$L_2 = -G_2 G_3 G_4 G_5 H_2$

$L_3 = -G_2 G_7 H_2$

$L_4 = -G_5 G_6 H_1$

$L_5 = -G_8 H_1$

$L_6 = -G_1 G_2 G_3 G_4 G_5 G_6 H_3$

$L_7 = -G_1 G_2 G_7 G_6 H_3$

$L_8 = -G_1 G_2 G_3 G_4 G_8 H_3$

(3) Pair of 2 non-touching loop:

$L_{13} = G_2 G_4 G_7 H_2 H_4$

$L_{17} = G_1 G_2 G_4 G_6 G_7 H_3 H_4$

$L_{35} = G_2 G_7 G_8 H_1 H_2$

(4) $\Delta = 1 - (L_1 + L_2 + L_3 + L_4 + L_5 + L_6 + L_7 + L_8) + (L_{13} + L_{17} + L_{35})$

$$\Delta = 1 - [-G_4 H_4 - G_2 G_3 G_4 G_5 H_2 - G_2 G_7 H_2 - G_5 G_6 H_1 - G_8 H_1 - G_1 G_2 G_3 G_4 G_5 G_6 H_3 - G_1 G_2 G_7 G_6 H_3 - G_1 G_2 G_3 G_4 G_8 H_3] + [G_2 G_4 G_7 H_2 H_4 + G_1 G_2 G_4 G_6 G_7 H_3 H_4 + G_2 G_7 G_8 H_1 H_2]$$

(5) $\Delta_1 = 1$

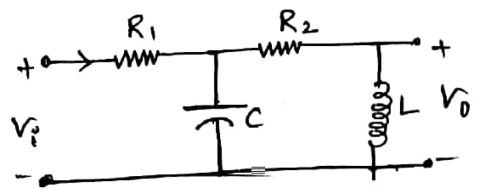
$\Delta_2 = 1 + G_4 H_4$

$\Delta_3 = 1$

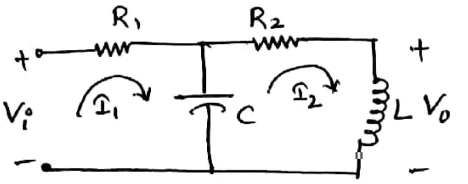
(6) $\frac{C}{R} = \frac{M_1 \Delta_1 + M_2 \Delta_2 + M_3 \Delta_3}{\Delta}$

$$\boxed{\frac{C}{R} = \frac{G_1 G_2 G_3 G_4 G_5 G_6 + G_1 G_2 G_7 G_6 (1 + G_4 H_4) + G_1 G_2 G_3 G_4 G_8}{\Delta}}$$

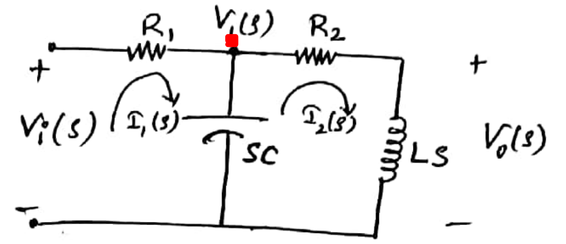
1) find T.F for the given network. using SFG method.



(A)



ALT $\Rightarrow$



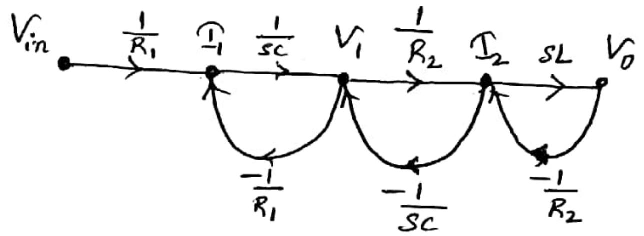
$$I_1(s) = \frac{V_i - V_1}{R_1} = \frac{V_i}{R_1} - \frac{V_1}{R_1} \rightarrow (1)$$

$$V_1(s) = \frac{(I_1 - I_2)}{sC} \rightarrow (2)$$

$$I_2(s) = \frac{V_1 - V_0}{R_2} = \frac{V_1}{R_2} - \frac{V_0}{R_2} \rightarrow (3)$$

$$V_0 = I_2 Ls \rightarrow (4)$$

SFG:



1) No of forward paths:

$$M_1 \rightarrow \frac{\cancel{s}L}{\cancel{s}R_1R_2C} = \frac{L}{R_1R_2C}$$

2) No of loops:

$$L_1 \rightarrow \frac{-1}{SCR_1}$$

$$L_2 \rightarrow \frac{-1}{SCR_2}$$

$$L_3 \rightarrow -\frac{SL}{R_2}$$

3) No of 2 non-touching loops:

$$L_{13} = \frac{+\cancel{s}L}{+\cancel{s}CR_1R_2} = \frac{L}{R_1R_2C}$$

4) $\Delta = 1 - (L_1 + L_2 + L_3) + L_{13}$

$$\Delta = 1 - \left[\frac{-1}{SCR_1} \frac{-1}{SCR_2} - \frac{SL}{R_2}\right] + \frac{L}{R_1R_2C}$$

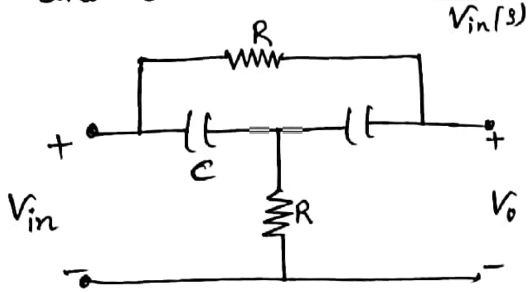
$$\Delta = 1 + \frac{1}{SCR_1} + \frac{1}{SCR_2} + \frac{SL}{R_2} + \frac{L}{R_1R_2C}$$

$$\Delta = \frac{SCR_1R_2 + R_2 + R_1 + s^2LCR_1 + SL}{SCR_1R_2}$$

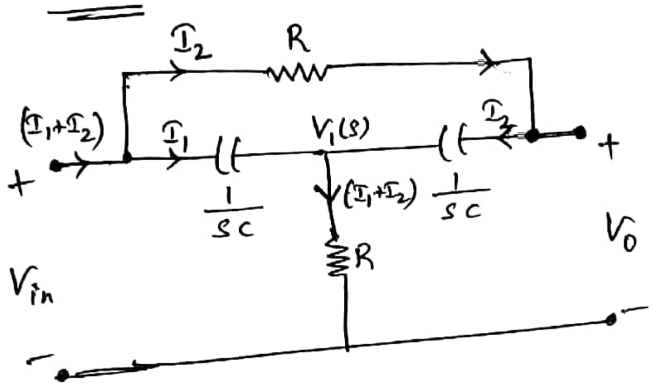
5) $\Delta_1 \rightarrow 1$

6) $$\frac{V_0(s)}{V_i(s)} = \frac{M_1\Delta_1}{\Delta} = \frac{\frac{L}{\cancel{R_1R_2C}}}{\frac{SCR_1R_2 + s^2LCR_1 + sL + R_1 + R_2}{s\,\cancel{CR_1R_2}}} = \frac{SL}{s^2CLR_1 + s[L + CR_1R_2] + R_1 + R_2}$$

(2) Obtain the transfer function $\frac{V_o(s)}{V_{in}(s)}$ for the n/w shown using SFG Method.



(A) Apply LT:



$$I_1(s) = \frac{V_{in} - V_1}{\frac{1}{sC}} = sC\left[V_{in} - V_1\right] = sC\,V_{in} - sC\,V_1 \quad — (1)$$

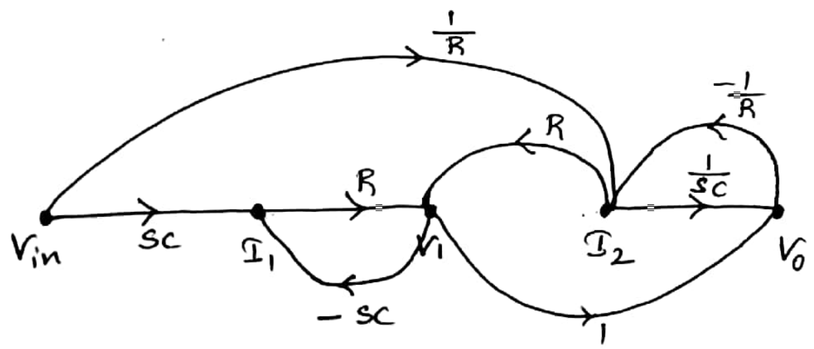
$$I_2(s) = \frac{V_{in} - V_o}{R} = \frac{V_{in}}{R} - \frac{V_o}{R} \quad — (2)$$

$$V_1(s) = (I_1 + I_2)R = I_1 R + I_2 R \quad — (3)$$

$$I_2(s) = \frac{V_o - V_1}{\frac{1}{sC}} = sC\,V_o - sC\,V_1 \quad — (4)$$

$$V_o = V_1 + \frac{I_2}{sC} \quad — (5)$$

→ one term has to be represented.
(Don't represent Both)



① No of forward paths:

$M_1 \longrightarrow SCR$

$M_2 \longrightarrow \frac{1}{SCR}$

$M_3 \longrightarrow \frac{1}{R} \times R = 1$

② No of Loops:

$L_1 \longrightarrow -SCR$

$L_2 \longrightarrow -\frac{1}{SCR}$

$L_3 \longrightarrow -\frac{1}{R} \times R = -1$

③ No of 2 non-touching loops:

$L_{12} \longrightarrow \cancel{to1} \, (-SCR) \times \left(\frac{-1}{SCR}\right) = +1$

④ $\Delta_1 \longrightarrow 1$

$\Delta_2 \longrightarrow 1 - (-SCR)$

$\boxed{\Delta_2 \Rightarrow 1 + SCR}$

$\Delta_3 \longrightarrow 1$

⑤ find Δ:

$$\Delta = 1 - [L_1 + L_2 + L_3] + L_{12}$$

$$= 1 - \left[(-SCR) - \frac{1}{SCR} - 1\right] + 1$$

$$= 1 + SCR + \frac{1}{SCR} + 2$$

$$\boxed{\Delta = \frac{3SCR + (SCR)^2 + 1}{SCR}}$$

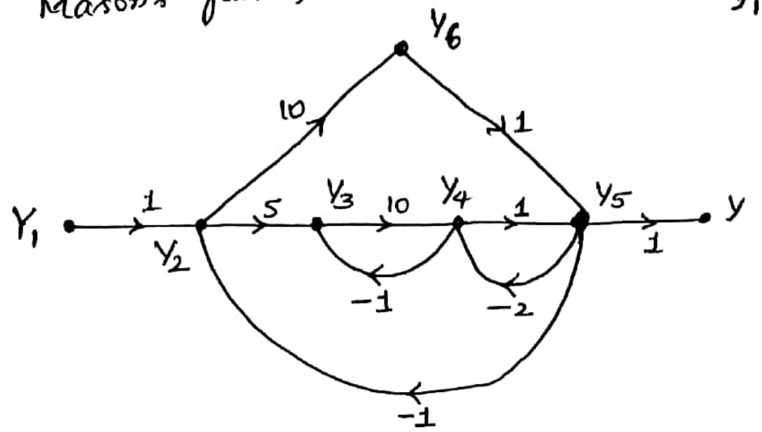
⑥

$$\frac{V_o(s)}{V_{in}(s)} = \frac{M_1 \Delta_1 + M_2 \Delta_2 + M_3 \Delta_3}{\Delta}$$

$$= \frac{SCR + \frac{1}{SCR}(1 + SCR) + 1}{\frac{3SCR + (SCR)^2 + 1}{SCR}}$$

$$\boxed{\frac{V_o(s)}{V_{in}(s)} = \frac{\frac{(SCR)^2 + (1 + SCR) + SCR}{\cancel{SCR}}}{\frac{(SCR)^2 + 3SCR + 1}{\cancel{SCR}}} = \frac{(SCR)^2 + 2SCR + 1}{(SCR)^2 + 3SCR + 1}}$$

(3) Use Mason's gain formula to find out $\frac{Y_5}{Y_1}$, for signal flow graph shown.



(A) ① No. of flw paths:

$M_1 \rightarrow 1\times10\times1 = 10$

$M_2 \rightarrow 1\times5\times10\times1 = 50$

② Loops:

$L_1 = -10$ $\quad L_4 = -50$

$L_2 = -2$

$L_3 = -10$

③ 2 non-touching Loops:

$L_{13} = -10\times-10 = 100$

④ find Δ:

$\Delta = 1-(L_1+L_2+L_3+L_4)+L_{13}$

$\Delta = 1-[-10-2-10-50]+100$

$= 1-[-72]+100$

$\boxed{\Delta = 173}$

⑤ $\Delta_1 \rightarrow 1+10 = 11$

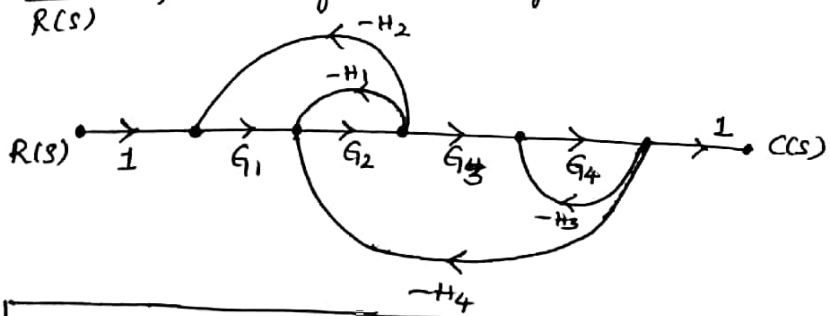
$\Delta_2 \rightarrow 1$

⑥ $\frac{Y_5}{Y_1} = \frac{M_1\Delta_1 + M_2\Delta_2}{\Delta}$

$$\boxed{\frac{Y_5}{Y_1} = \frac{(10\times11)+(50\times1)}{173} = \frac{160}{173}}$$

Assignment

(4) $\frac{C(S)}{R(S)}$ find using Mason's gain formula.

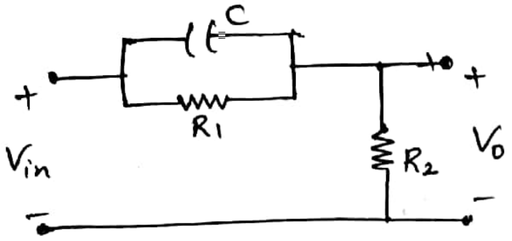


Sol:

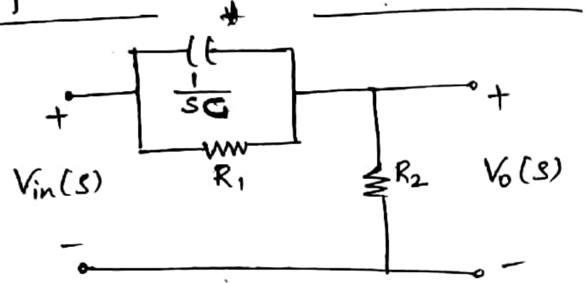
$$\boxed{\frac{C(s)}{R(s)} = \frac{G_1G_2G_3G_4}{1+G_2H_1+G_1G_2H_2+G_4H_3+G_2G_3G_4H_4+G_2G_4H_1H_3+G_1G_2G_4H_2H_3}}$$

Phase-Lead Compensator:-

SUPRITH KUMAR . K . S
Assistant Professor
ECE Dept, BMSCE



Take Laplace Transform to find transfer function:



- Apply Voltage division rule:

$$V_o(s) = V_{in}(s)\left[\frac{R_2}{R_2 + \dfrac{R_1\left[\frac{1}{sC}\right]}{\left(R_1+\frac{1}{sC}\right)}}\right]$$

$$= V_{in}(s)\left[\frac{R_2}{\dfrac{R_2\left(R_1+\frac{1}{sC}\right)+\frac{R_1}{sC}}{\left(R_1+\frac{1}{sC}\right)}}\right]$$

$$= V_{in}(s)\left[\frac{R_2\left(R_1+\frac{1}{sC}\right)}{R_1R_2+\frac{R_2}{sC}+\frac{R_1}{sC}}\right]$$

$$= V_{in}(s)\left[\frac{\dfrac{R_2(R_1sC+1)}{\cancel{sC}}}{\dfrac{R_1R_2sC+R_2+R_1}{\cancel{sC}}}\right]$$

$$= V_{in}(s)\left[\frac{R_2(1+sR_1C)}{R_1+R_2+sR_1R_2C}\right]$$

$$\boxed{\frac{V_o(s)}{V_{in}(s)} = \frac{R_2}{(R_1+R_2)}\left[\frac{(1+sCR_1)}{1+\left(\frac{R_1R_2}{R_1+R_2}\right)sC}\right]} \quad \text{— (A)}$$

Compare equation (A) with standard ~~lead~~ Transfer Function of Compensator

$$\frac{V_o}{V_{in}} = \frac{\alpha(1+s\tau)}{(1+s\alpha\tau)}$$

$$\therefore \quad \boxed{\alpha = \frac{R_2}{R_1+R_2}}, \quad \boxed{\tau = R_1 C} \quad \& \quad \boxed{\alpha < 1}$$

WKT: from the transfer function

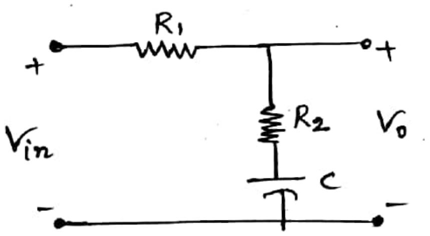
(1) Zero is at $-\frac{1}{\tau}$

(2) Pole is at $-\frac{1}{\alpha\tau}$ $\qquad \boxed{0 < \alpha < 1}$

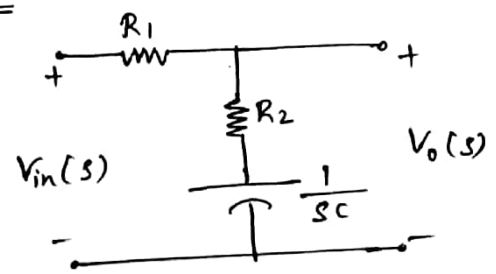
(3) $$\phi = \tan^{-1}(\omega\tau) - \tan^{-1}(\omega\alpha\tau)$$

(4) for phase Lead if we plot magnitude response it will be High pass filter response (HPF)

Phase-Lag compensator:



Apply L.T:



Find the o/p voltage by using Voltage Divider Rule:

$$V_o(s) = \frac{V_{in}(s)\left[R_2 + \frac{1}{sC}\right]}{R_1 + R_2 + \frac{1}{sC}}$$

$$V_o(s) = \frac{V_{in}(s)\left[\frac{R_2 sC + 1}{\cancel{sC}}\right]}{\frac{R_1 sC + R_2 sC + 1}{\cancel{sC}}}$$

$$\frac{V_o(s)}{V_{in}(s)} = \frac{[1 + sR_2C]}{[1 + (R_1+R_2)sC]}$$

Compare it with standard Transfer function of compensator

$$\frac{V_o}{V_{in}} = \frac{\beta(1+s\tau)}{(1+s\beta\tau)}$$

$$\boxed{\tau = R_2 C}$$

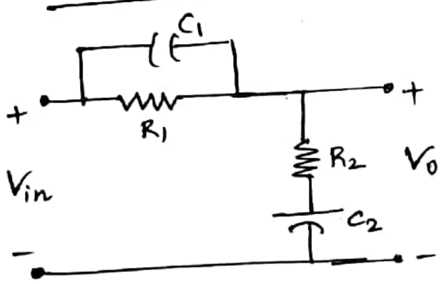
$$\boxed{\beta = \frac{(R_1+R_2)}{R_2}} \quad \& \quad \boxed{\beta > 1}$$

Note: (1) Zero is at $-\frac{1}{\tau}$

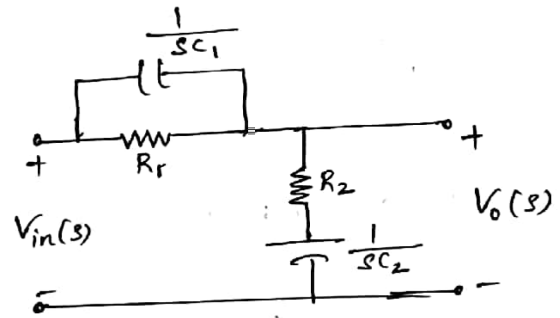
(2) pole is at $\frac{-1}{\beta\tau}$

(3) for phase lag compensator if we plot magnitude response it will be Low Pass filter response (LPF).

Phase Lag-Lead compensator:



L.T ⇒



* Lag-Lead compensator is an electrical n/w which produces phase Lag at one frequency region & phase Lead at other freq region.

General Lag-Lead Transfer function:

$$\frac{V_0}{V_{in}} = \alpha\frac{(1+s\tau_1)}{(1+s\tau_1\alpha)}\;\beta\frac{(1+s\tau_2)}{(1+s\tau_2\beta)}$$

where
$\tau_1, \tau_2 > 0$
$\alpha < 1$
$\beta > 1$

$$\boxed{\frac{V_0}{V_{in}} = \alpha\beta\frac{(1+s\tau_1)(1+s\tau_2)}{(1+s\tau_1\alpha)(1+s\tau_2\beta)} = \frac{\left(s+\frac{1}{\tau_1}\right)\left(s+\frac{1}{\tau_2}\right)}{\left(s+\frac{1}{\alpha\tau_1}\right)\left(s+\frac{1}{\beta\tau_2}\right)}} \quad \text{—(1)}$$

By Applying Voltage divider rule:

$$V_0(s) = V_{in}(s)\left[\frac{R_2 + \frac{1}{sC_2}}{\left(R_2 + \frac{1}{sC_2}\right) + \frac{R_1\left(\frac{1}{sC_1}\right)}{R_1 + \frac{1}{sC_1}}}\right]$$

$$= V_{in}(s)\left[\frac{\left(R_2 + \frac{1}{sC_2}\right)}{\frac{\left(R_1 + \frac{1}{sC_1}\right)\left(R_2 + \frac{1}{sC_2}\right) + \frac{R_1}{sC_1}}{\left(R_1 + \frac{1}{sC_1}\right)}}\right]$$

$$= V_{in}(s)\left[\frac{\left(R_2 + \frac{1}{sC_2}\right)\left(R_1 + \frac{1}{sC_1}\right)}{R_1R_2 + \frac{R_1}{sC_2} + \frac{R_2}{sC_1} + \frac{1}{s^2C_1C_2} + \frac{R_1}{sC_1}}\right]$$

$$\frac{V_0}{V_{in}} = \frac{\left(\frac{R_2sC_2+1}{\cancel{sC_2}}\right)\left(\frac{sC_1R_1+1}{\cancel{sC_1}}\right)}{\frac{s^2C_1C_2R_1R_2 + R_1sC_1 + R_2sC_2 + 1 + R_1sC_2}{\cancel{s^2C_1C_2}}}$$

$$= \frac{(1+sR_1C_1)(1+sR_2C_2)}{\left[1 + s^2(C_1C_2R_1R_2) + s\left[R_1C_1 + R_2C_2 + R_1C_2\right]\right]}$$

$$= \frac{\cancel{R_1C_1}\left(s + \frac{1}{R_1C_1}\right)\cancel{R_2C_2}\left(s + \frac{1}{R_2C_2}\right)}{\cancel{R_1R_2C_1C_2}\left[s^2 + s\left[\frac{1}{R_2C_2} + \frac{1}{R_1C_1} + \frac{1}{R_2C_1}\right] + \frac{1}{R_1R_2C_1C_2}\right]}$$

$$\therefore \boxed{\frac{V_0(s)}{V_{in}(s)} = \frac{\left(s + \frac{1}{R_1C_1}\right)\left(s + \frac{1}{R_2C_2}\right)}{s^2 + s\left[\frac{1}{R_1C_1} + \frac{1}{R_2C_2} + \frac{1}{R_2C_1}\right] + \frac{1}{R_1R_2C_1C_2}}}$$

By comparing with Equation (1)

$\tau_1 \longrightarrow R_1C_1$

$\tau_2 \longrightarrow R_2C_2$

$\alpha\beta \longrightarrow 1$

$\alpha \longrightarrow \frac{R_2}{R_1+R_2}$

$\beta \longrightarrow \frac{R_1+R_2}{R_2}$