B.M.S. COLLEGE OF ENGINEERING, BENGALURU-19

Autonomous Institute, Affiliated to VTU

DEPARTMENT OF MATHEMATICS

Sem & Branch:	**FOURTH SEMESTER** (Common to AS/CV/EEE/ECE/EIE/ML/TCE)	**Subject:**	**Engineering Mathematics - 4**	**Sub Code:**	**19MA4BSEM4**
Time:	**1:00PM-2:15PM**	**Test Date:**	**17-05-2021**	**Max Marks:**	40

Test No.	Q. No.	**SOLUTIONS AND SCHEME OF EVALUATION**	Marks
Test -1		**PART-A**	
	1	Establish the formula $r=\frac{\sigma_x^2+\sigma_y^2-\sigma_{x-y}^2}{2\sigma_x\sigma_y}$ and hence find the coefficient of correlation between x and y when the variances of x, y and x-y respectively are 3, 2.45 and 2.45	**5**
	Solution	Let $z=x-y \Rightarrow \bar{z}=\bar{x}-\bar{y}$	**1**
		Then $(z-\bar{z})^2=(x-\bar{x})^2+(y-\bar{y})^2-2(x-\bar{x})(y-\bar{y})$	
		$\Rightarrow \sum(z-\bar{z})^2=\sum(x-\bar{x})^2+\sum(y-\bar{y})^2-2\sum(x-\bar{x})(y-\bar{y})$	
		$\Rightarrow \frac{\sum(z-\bar{z})^2}{n}=\frac{\sum(x-\bar{x})^2}{n}+\frac{\sum(y-\bar{y})^2}{n}-2\frac{\sum(x-\bar{x})(y-\bar{y})}{n}$	**1**
		$\Rightarrow \sigma_z^2=\sigma_x^2+\sigma_y^2-2r\sigma_x\sigma_y$ because $r=\frac{\sum(x-\bar{x})(y-\bar{y})}{n\sigma_x\sigma_y}$	
		Therefore we get $r=\frac{\sigma_x^2+\sigma_y^2-\sigma_{x-y}^2}{2\sigma_x\sigma_y}$ ----(1)	**2**
		Given $\sigma_x^2=3$, $\sigma_y^2=2.45$, $\sigma_{x-y}^2=2.45$	
		(1) $\Rightarrow r=0.55$	**1**
		PART-B	
	2 a)	The two lines of regression of y on x and x on y are $7x-16y+9=0$, $5y-4x-3=0$ respectively. Calculate the coefficient of correlation, $\bar{x}$ and $\bar{y}$.	**5**
	Solution	From the given lines of regression we get $b_{yx}=\frac{7}{16}$ and $b_{xy}=\frac{5}{4}$	**2**
		So that $r=\sqrt{b_{yx}\times b_{xy}}=0.7395$	**1**
		Since the regression lines pass through $(\bar{x},\bar{y})$, therefore $7\bar{x}-16\bar{y}+9=0$, $5\bar{y}-4\bar{x}-3=0$ On solving these equations we get $\bar{x}=-0.1034$ and $\bar{y}=0.5172$	**2**
	2 b)	A manufacturer of cotter pins knows that 5% of his product is defective. Pins are sold in boxes of 100. He guarantees that not more than 3 pins will be defective. Determine the probability that a box fail to meet the guarantee.	**5**
	Solution	Let x represents the number of defective cotter pins. So that p=0.05 and n =100$\Rightarrow$ Mean$(m)=np$ =5	**1**
		The probability of x number of defective blades is given by Poisson function $P(x)=\frac{m^x}{x!}e^{-m}=\frac{5^x}{x!}e^{-5}$	**1**
		The box fails to meet the guarantee if it contains more than 4 defective pins. Thus the probability that the guarantee is failed is given by $P(x>3)$=1- $P(x\leq 3)$=0.735	**3**

2 c) (5)

The table below shows the joint probability distribution of two random variables X and Y.

$X \backslash Y$	1	2	3
1	6c	4c	2c
2	3c	2c	c
3	2c	4c	0
4	4c	0	2c

(i) Find c.

(ii) Calculate the marginal distributions of X and Y.

(iii) Verify whether X and Y are independent.

Solution

(i) $\sum_{i=1}^{4}\sum_{j=1}^{3} J_{ij} = 1 \Rightarrow 30c = 1 \Rightarrow c = \frac{1}{30}$ **2**

(ii)Marginal distributions of X

x	1	2	3	4
$f(x)$	$12/30$	6/30	6/30	6/30

Marginal distributions of Y **2**

y	1	2	3
$g(y)$	15/30	10/30	5/30

(iii) Since $J_{ij} \neq f(x_i)g(y_j) \quad \forall\, i, j$, the random variable X and Y are not independent. **1**

Test -2

PART-C

3 a) (6)

The velocity V of a liquid is known to vary with temperature T according to a quadratic law $V = a + bT + cT^2$. Find the best values of a, b and c for the following table:

T	1	2	3	4	5
V	2.31	2.01	3.80	1.66	1.55

Solution

The normal equations are $\sum V = na + b\sum T + c\sum T^2$

$$\sum VT = a\sum T + b\sum T^2 + c\sum T^3$$

$$\sum VT^2 = a\sum T^2 + b\sum T^3 + c\sum T^4$$ **1**

T	V	T^2	TV	T^3	T^2V	T^4
1	2.31	1	2.31	1	2.31	1
2	2.01	4	4.02	8	8.04	16
3	3.80	9	11.40	27	34.2	81
4	1.66	16	6.64	64	26.56	256
5	1.55	25	7.75	125	38.75	625
$\sum T = 15$	$\sum V = 11.33$	$\sum T^2 = 55$	**32.12**	**225**	**109.86**	**979**

3

After solving the resulting normal equations we get
a =1.0518, b = 1.3346 and c = -0.2536 **2**

OR

	3 b)	Find the coefficient of correlation between the industrial production and export using the following data and also find the export when the production is 57 crore tons.	6

Production(crore tons)	55	56	58	59	60
Export(crore tons)	35	38	38	39	44

Solution

x (Production)	y (Export)	X=x-$\bar{x}$	Y=y-$\bar{y}$	XY	X^2	Y^2
55	35	-2.6	-3.8	9.88	6.76	14.44
56	38	-1.6	-0.8	1.28	2.56	0.64
58	38	0.4	-0.8	0.32	0.16	0.64
59	39	1.4	0.2	0.28	1.96	0.04
60	44	2.4	5.2	12.48	5.76	27.04
288	**194**			**24.24**	**17.2**	**42.8**

2

$$r = \frac{\sum XY}{\sqrt{\sum X^2}\sqrt{\sum Y^2}} = 0.89$$ 1

and $b_{yx} = \frac{\sum XY}{\sum X^2} = 1.41$ 1

The required regression line of y on x is $y - \bar{y} = b_{yx}(x - \bar{x})$

$\Rightarrow y = 1.41x - 42.412$

When x = **57** we get y = **37.95**

Thus when the production is 57 crore tons, the export is 37.95 crore tons. 2

	4 a)	Fit a Poisson distribution to the following data and hence find the theoretical frequencies	7

x	0	1	2	3	4
f	46	38	22	9	1

Solution

Mean $\Rightarrow m = \frac{\sum fx}{\sum f} = 0.9741$ 1

Therefore, the Poisson function $P(x) = \frac{m^x}{x!}e^{-m} = \frac{(0.9741)^x}{x!}e^{-0.9741}$ 2

The corresponding Poisson distribution is

x	0	1	2	3	4
$P(x)$	0.3775	0.3677	0.1791	0.0581	0.0142
f	43.79	42.65	20.77	6.74	1.64

3

Thus the required theoretical frequencies are 44, 43, 21, 7, 1 1

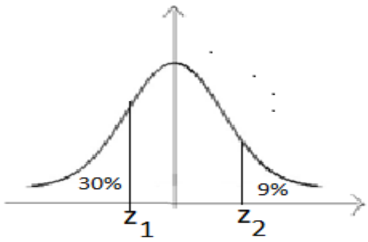
OR

	4 b)	In a normal distribution, 30% of the items are under 45 and 9% are over 64. Find the mean and S.D. of the distribution.(Given $\phi(0.53)$=0.2 and $\phi(1.35)$=0.41 where $\phi(z)$ is an area bounded by standard normal curve from 0 to z)	7

Solution

$z = \frac{x-\mu}{\sigma}$ When x=45 let $z_1 = \frac{45-\mu}{\sigma}$ ....(1)

And When x=64 let $z_2 = \frac{64-\mu}{\sigma}$......(2) 2



$\int_{-\infty}^{z_1} f(z)dz = 0.3 \Rightarrow z_1 = -0.53$ and $\int_{z_2}^{\infty} f(z)dz = 0.09 \Rightarrow z_2 = 1.35$ **3**

On solving equations (1) and (2) we get $\mu = 50.35$ and $\sigma = 10.11$ **2**

5 a) The joint probability distributions of two random variables X and Y is given below: **7**

$X \backslash Y$	1	2	3
1	0.05	0.05	0.1
2	0.05	0.1	0.35
3	0	0.2	0.1

Find $Cov(X,Y)$. Also find $P(X \leq 2, Y < 2)$

Solution

$X \backslash Y$	1	2	3	sum
1	0.05	0.05	0.1	0.2
2	0.05	0.1	0.35	0.5
3	0	0.2	0.1	0.3
sum	0.1	0.35	0.55	1

2

$E(X)$= 2.1 $E(Y)$= 2.45 $E(X, Y)$ = 5.15 $Cov(X,Y)$= 0.005 **4**

$P(X \leq 2, Y < 2) = J_{11} + J_{21}$= 0.1 **1**

OR

5 b) Two cards are selected at random from a box which contains five cards numbered 1,1,2,2 and 3. Find the joint distributions of X and Y where X denotes the product of two numbers and Y denotes the minimum of two numbers drawn. Also determine $Cov(X, Y)$. **7**

Solution

$X \backslash Y$	1	2	sum
1	0.1	0	0.1
2	0.4	0	0.4
3	0.2	0	0.2
4	0	0.1	0.1
6	0	0.2	0.2
sum	0.7	0.3	1

3

$E(X)$= 3.1 $E(Y)$= 1.3 $E(X, Y)$ = 4.7 $Cov(X,Y)$= 0.67 **4**

Suitable marks to be given for alternate methods