Control System
UNIT-1: Compensators

Need of Compensation

- In order to provide a suitable performance, it is required to adjust the control system design.
- This adjustment of a control system is called compensation.
- A device is inserted into the system for the purpose of satisfying the specification, this device is called a compensator.
- Control system are designed to perform specific tasks, given a plant with transfer function G(s), the objective is to design an overall system to meet set of

- specifications. The requirements imposed on the control system are generally related to accuracy, relative stability and speed of response.
- These specifications may have to be modified during the design since the original specifications may not be satisfied.
- If the specifications are not meet we would need to introduce a compensator to get a desired result.
- Compensate a unstable system to make it stable.
- A compensating network is used to minimize overshoot.
- These compensating networks increase the steady state accuracy of the system
- Compensating networks also introduces poles and zeros in the system thereby causes changes in the transfer function of the system. Due to this, performance specifications of the system change.

Types of Compensation

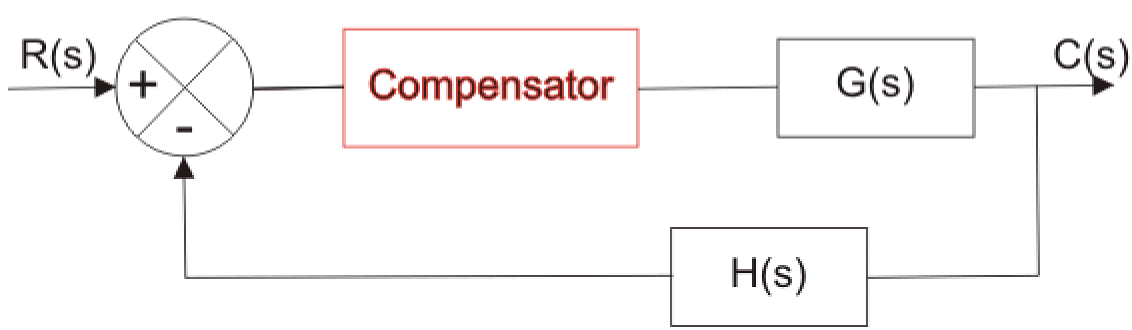
i) Series compensation

ii) parallel compensation

iii) Series- parallel compensation

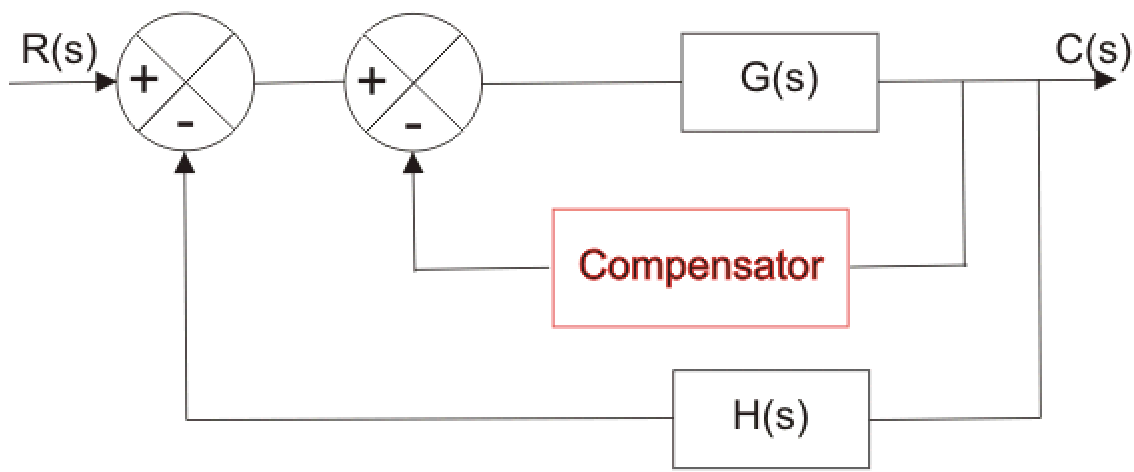
Series Compensation:

when the compensator is placed in the feed forward path, it is called a Series compensation. It is also known as a cascade compensator.



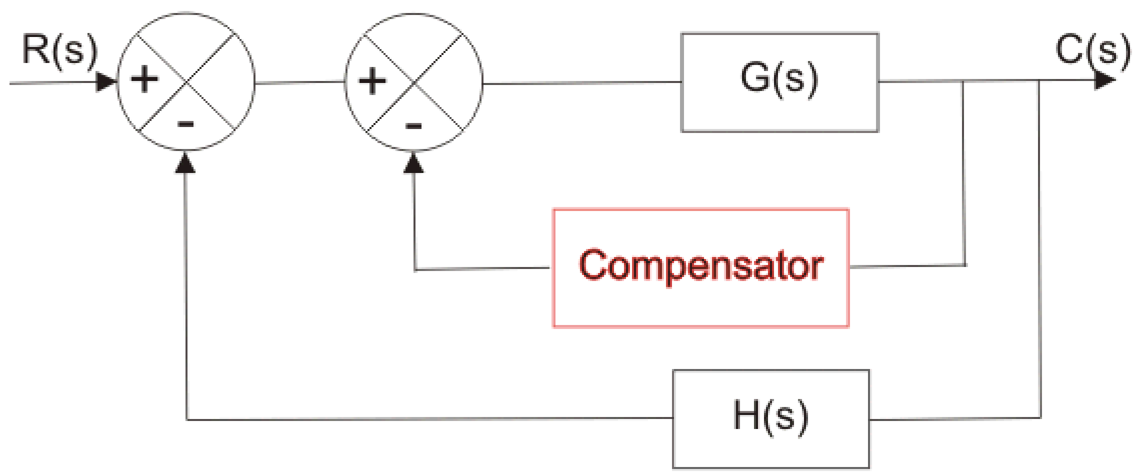
***parallel compensation*:**

When the compensator is introduced in the feedback path to produce an additional feedback, it is called a parallel compensation.



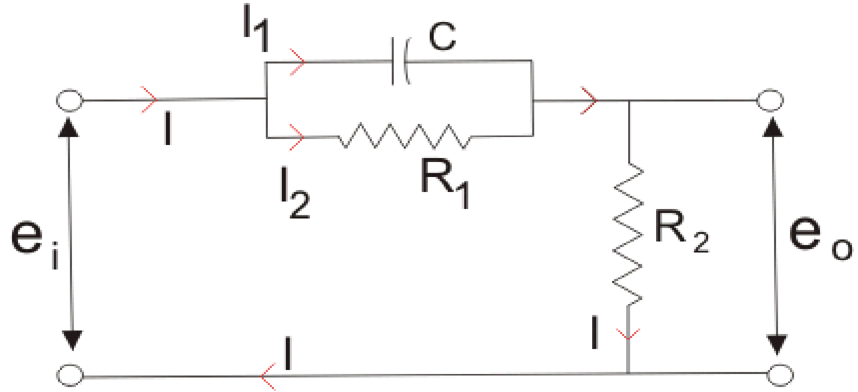
Series-parallel compensation

- In certain application, we need to introduce compensation in the forward path as well as in the feedback path. This type of compensation is called Series-parallel compensation



- Compensating devices are may be in the form of electrical, mechanical, hydraulic etc.
- Most electrical compensator are RC filter.
- The simplest network used for compensator are known as lead, lag network.

- Phase Lead Compensation Network(high pass filter)



From above circuit we get,

$$
\begin{aligned}
I_1 &= C\frac{d}{dt}(e_i - e_o) \\
I_2 &= \frac{e_i - e_o}{R_1} \\
I &= I_1 + I_2 = C\frac{d}{dt}(e_i - e_o) + \frac{e_i - e_o}{R_1} \\
Again, I &= \frac{e_o}{R_2}
\end{aligned}
$$

Equating above expression of I we get,

$$\frac{e_o}{R_2} = C\frac{d}{dt}(e_i - e_o) + \frac{e_i - e_o}{R_1}$$

- Transfer function of the given network is the ratio of the Laplace transform output voltage to the Laplace transform input voltage.
So taking Laplace transform of both side of above equations,

$$\frac{1}{R_2}E_o(s) = \frac{1}{R_1}[E_i(s)-E_o(s)]+Cs[E_i(s)-E_o(s)] \quad (\textit{neglecting initial condition})$$

$$\Rightarrow \frac{1}{R_2}E_o(s) + \frac{1}{R_1}E_o(s) + CsE_o(s) = \frac{E_i(s)}{R_1} + CsE_i(s)$$

$$\Rightarrow \frac{E_o(s)}{E_i(s)} = \frac{\frac{1+sCR_1}{R_1}}{\frac{R_1+R_2+sR_1R_2C}{R_2R_1}}$$

$$\Rightarrow \frac{E_o(s)}{E_i(s)} = \frac{R_2}{R_1+R_2}\left[\frac{1+sCR_1}{1+\frac{sR_1R_2C}{R_1+R_2}}\right]$$

The transfer function of this lead compensator is -

$$\frac{V_o(s)}{V_i(s)} = \beta\left(\frac{s\tau + 1}{\beta s\tau + 1}\right)$$

Where,

$$\tau = R_1 C$$

$$\beta = \frac{R_2}{R_1 + R_2}$$

From the transfer function, we can conclude that the lead compensator has pole at $s=-\frac{1}{\beta}$ and zero at $s=-\frac{1}{\beta\tau}$.

Substitute, $s=j\omega$ in the transfer function.

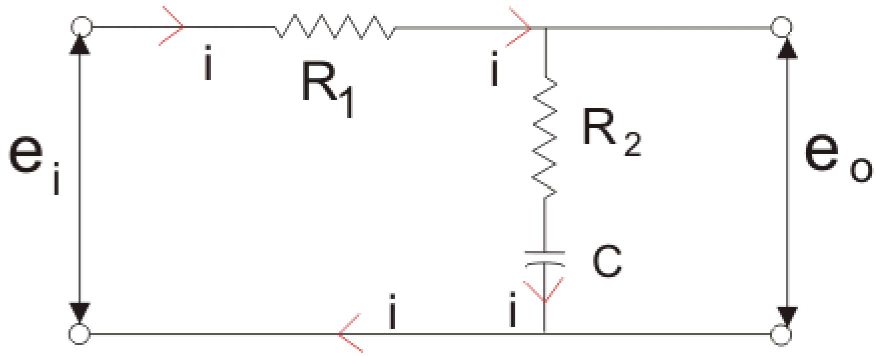
$$\frac{V_o(j\omega)}{V_i(j\omega)}=\beta\left(\frac{j\omega\tau+1}{\beta j\omega\tau+1}\right)$$

Phase angle $\phi=tan^{-1}\omega\tau-tan^{-1}\beta\omega\tau$

We know that, the phase of the output sinusoidal signal is equal to the sum of the phase angles of input sinusoidal signal and the transfer functio
So, in order to produce the phase lead at the output of this compensator, the phase angle of the transfer function should be positive.
This will happen when $0<\beta<1$. Therefore, zero will be nearer to origin in pole-zero configuration of the lead compensator.

- **Phase Lag Compensation(low pass circuit)**
- (one zero and one dominating pole, response will be slower)

Circuit diagram for the phase **lag compensation** network



We will have the output at the series combination of the resistor R_2 and the capacitor C.
From the above circuit diagram, we get

$$e_i = iR_1 + iR_2 + \frac{1}{C}\int i\,dt$$
$$e_o = iR_2 + \frac{1}{C}\int i\,dt$$

- Taking Laplace transform of two equations we get

$$E_i(s) = R_1 I(s) + R_2 I(s) + \frac{1}{Cs} I(s)$$

$$E_o(s) = R_2 I(s) + \frac{1}{Cs} I(s)$$

$$Transfer\ function,\ G_{lag}(s) = \frac{E_o(s)}{E_i(s)} = \frac{R_2 + \frac{1}{Cs}}{R_1 + R_2 + \frac{1}{Cs}}$$

$$\Rightarrow G_{lag}(s) = \frac{R_2 Cs + 1}{(R_1 + R_2)Cs + 1}$$

- The Lag Compensator is an electrical network which produces a sinusoidal output having the phase lag when a sinusoidal input is applied.

The transfer function of this lag compensator is -

$$\frac{V_o(s)}{V_i(s)} = \frac{1}{\alpha}\left(\frac{s+\frac{1}{\tau}}{s+\frac{1}{\alpha\tau}}\right)$$

Where,

$$\tau = R_2 C$$

$$\alpha = \frac{R_1 + R_2}{R_2}$$

From the above equation, α is always greater than one.

From the transfer function, we can conclude that the lag compensator has one pole at $s = -\frac{1}{\alpha\tau}$ and one zero at $s = -\frac{1}{\tau}$. This means, the pole will be nearer to origin in the pole-zero configuration of the lag compensator.

Substitute, $s = j\omega$ in the transfer function.

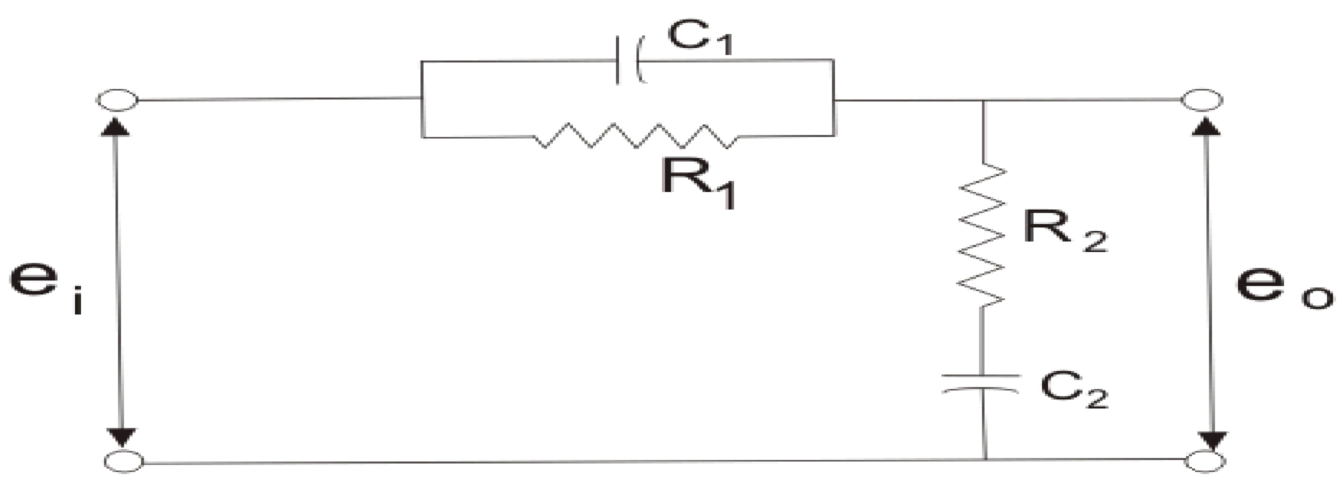
$$\frac{V_o(j\omega)}{V_i(j\omega)} = \frac{1}{\alpha}\left(\frac{j\omega + \frac{1}{\tau}}{j\omega + \frac{1}{\alpha\tau}}\right)$$

We know that, the phase of the output sinusoidal signal is equal to the sum of the phase angles of input sinusoidal signal and the transfer function.
So, in order to produce the phase lag at the output of this compensator,
the phase angle of the transfer function should be negative. This will happen when $\alpha>1$.

Phase Lag Lead Compensation:

- Provides phase lead at one frequency region and phase lag at another frequency region

Circuit diagram for the phase **lag- lead compensation** network.



$$Transfer\ function\ , G_{lag-lead}(s) = \frac{E_0(s)}{E_i(s)} = \frac{\left(s + \frac{1}{R_1C_1}\right)\left(s + \frac{1}{R_2C_2}\right)}{s^2 + \left(\frac{1}{R_1C_1} + \frac{1}{R_2C_2} + \frac{1}{R_2C_1}\right)s + \frac{1}{R_1R_2C_1C_2}}$$

$$\Rightarrow G_{lag-lead}(s) = \frac{(R_1C_1s + 1)(R_2C_2s + 1)}{R_1R_2C_1C_2s^2 + (R_1C_1 + R_2C_2 + R_1C_2)s + 1}$$

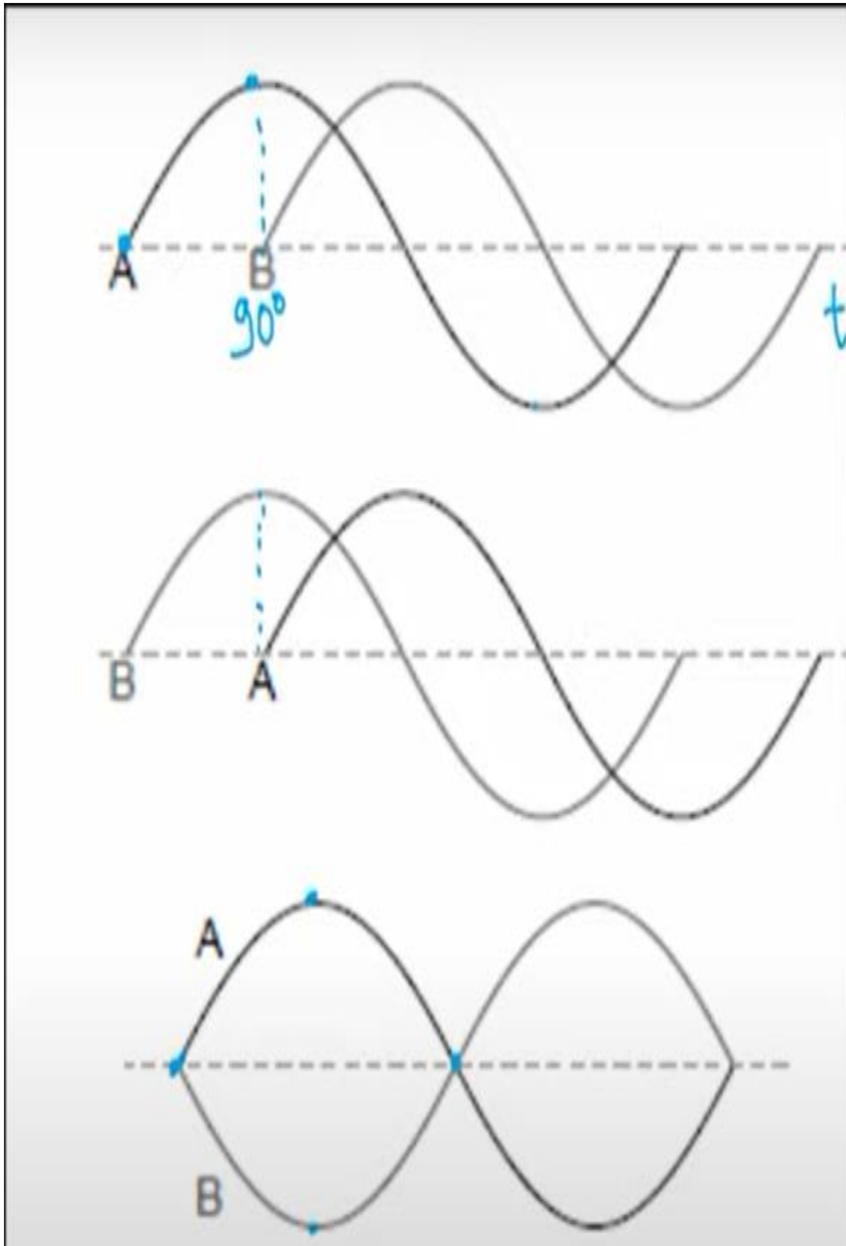
- Transfer function of the circuit will be the product of transfer functions of the lead and the lag compensators.

$$\frac{V_o(s)}{V_i(s)} = \beta\left(\frac{s\tau_1 + 1}{\beta s\tau_1 + 1}\right)\frac{1}{\alpha}\left(\frac{s + \frac{1}{\tau_2}}{s + \frac{1}{\alpha\tau_2}}\right)$$

We know $\alpha\beta = 1$

$$\Rightarrow \frac{V_o(s)}{V_i(s)} = \left(\frac{s + \frac{1}{\tau_1}}{s + \frac{1}{\beta\tau_1}}\right)\left(\frac{s + \frac{1}{\tau_2}}{s + \frac{1}{\alpha\tau_2}}\right)$$

Where, $\tau_1 = R_1C_1$ $\tau_2 = R_2C_2$



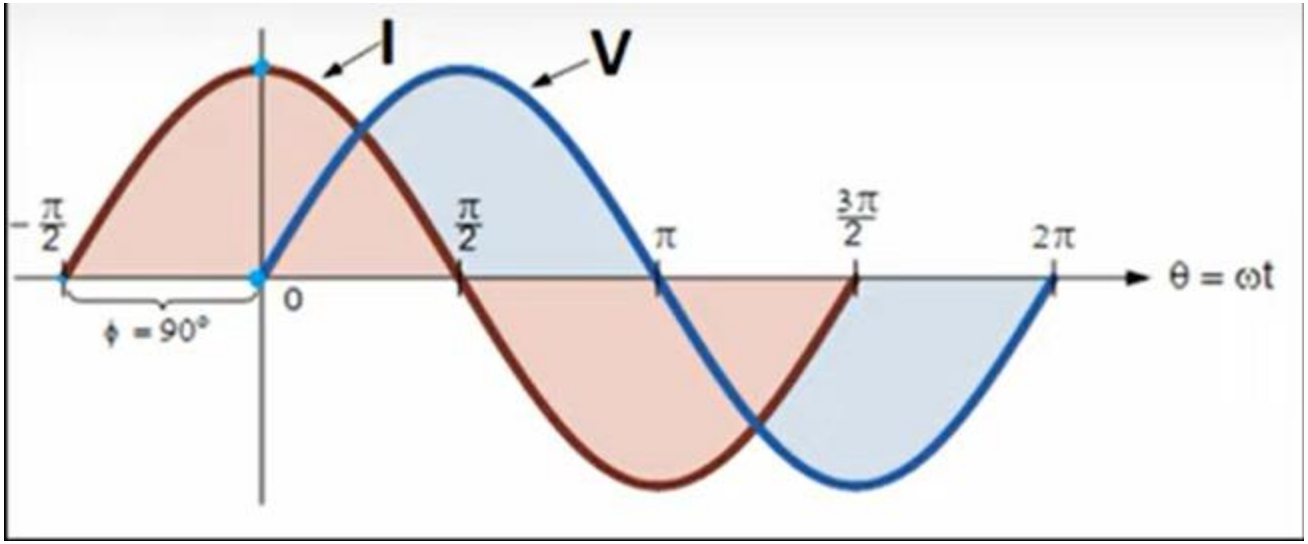
A leads B by $90^\circ / \frac{\pi}{2}$ rad

B lags A by $90^\circ / \frac{\pi}{2}$ rad

B Leads A

A Lags B

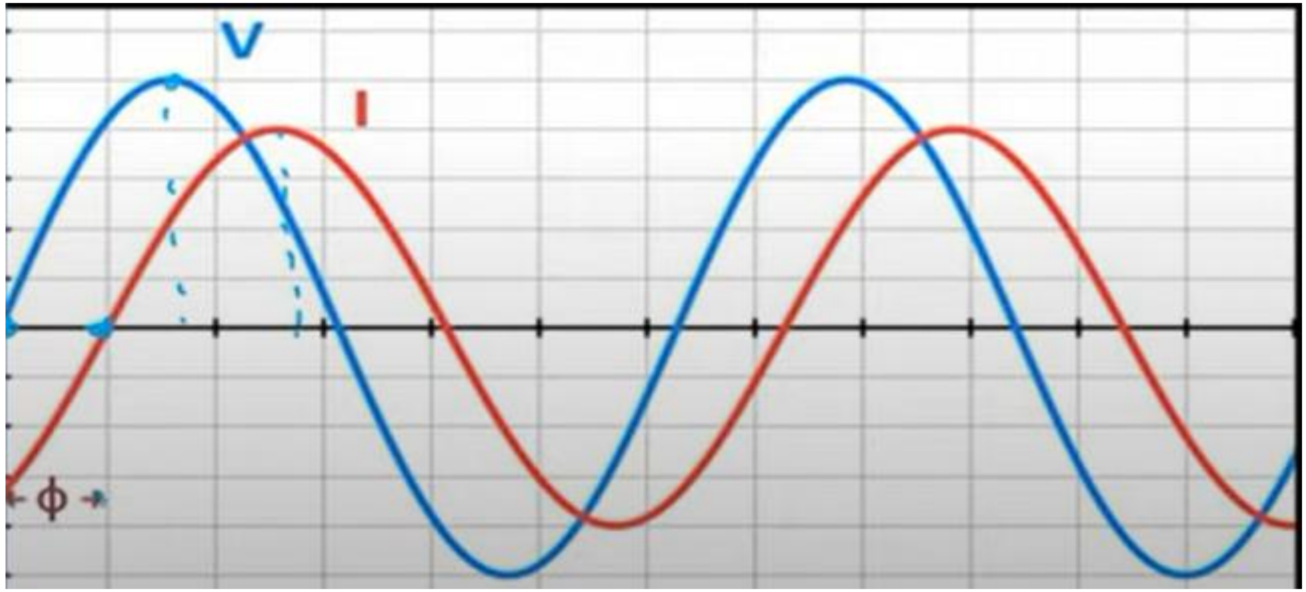
A and B are 180° out of phase



$$V = V_m \sin \omega t$$

$$I = I_m \sin(\omega t + 90^\circ)$$

lead



$V = V_m \sin \omega t$

$I = I_m \sin(\omega t - \phi)$

lag