1. Within a certain region $\varepsilon = 10^{-11}$ F/m & $\mu = 10^{-5}$ H/m. If $B_x = 2\times10^{-4} \cos 10^5 t \sin 10^{-3} y$ T

a) Use $\nabla \times \vec{H} = \varepsilon \frac{\partial \vec{E}}{\partial t}$ to find $\vec{E}$

b) Find the total magnetic flux passing flux through the surface $x=0,\ 0<y<40\,m,\ 0<z<2m.$ at $t = 1\mu s$

c) Find the value of the closed line integral of E around the perimeter of the given surface.

a) $\nabla \times \vec{H} = \varepsilon \frac{\partial \vec{E}}{\partial t}$

$$\frac{\partial \vec{E}}{\partial t} = \frac{2\times10^{-4}}{\varepsilon\mu} \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \cos 10^5 t \sin 10^{-3} y & 0 & 0 \end{vmatrix}$$

$$= \frac{2\times10^{-4}}{\varepsilon}\left[\hat{a}_z\left[-\cos 10^5 t \cos 10^{-3} y\right]10^{-3}\right]$$

$$\frac{\partial \vec{E}}{\partial t} = \frac{-2\times10^{-7}}{\varepsilon\mu} \cos 10^5 t \cos 10^{-3} y\ \hat{a}_z$$

Integrate w.r.t 't' both sides

$$\vec{E} = \frac{-2\times10^{-7}}{\varepsilon\mu} \cos 10^{-3} y\ \frac{\sin 10^5 t}{10^{+5}}\ \hat{a}_z$$

$$\vec{E} = \frac{-2\times10^{-7}\times10^{-5}}{10^{-16}\times10^{5}} \sin 10^5 t \cos 10^{-3} y\ \hat{a}_z \text{ V/m}$$

$$\vec{E} = \underline{\underline{-20{,}000}} \sin(10^5 t) \cos(10^{-3} y)\ \hat{a}_z$$

b)

(2)

$$\phi = \int \vec{B}\cdot d\vec{S}$$

$$= 2\times10^{-4}\cos(10^5 t)\int_{y=0}^{40}\int_{z=0}^{2} \sin 10^{-3}y \; dy\, dz$$

$$= 2\times10^{-4}\cos(10^5 t)\left[-\frac{\cos(10^{-3}y)}{10^{-3}}\right]_{y=0}^{40}$$

$$= 3.98\times10^{-1}\left[7.99\times10^{-4}\right]$$

$$\phi = 0.318 \text{ mWb}$$

c) The line integral is $\oint \vec{E}\cdot d\vec{L}$

From Stoke's theorem

$$\oint \vec{E}\cdot d\vec{L} = \int (\nabla\times\vec{E})\cdot d\vec{S}$$

$$= \int -\frac{\partial\vec{B}}{\partial t}\cdot d\vec{S}$$

$$= \int_S \frac{\partial\left[\left(-2\times10^{-4}\cos 10^{+5}t \sin 10^{-3}y\right)\hat{a}_x\right]}{\partial t}\cdot\left(dy\,dz\;\hat{a}_x\right)$$

$$= \int_S \left[+2\times10^{-4}\sin 10^{-3}y \; \sin(10^5 t)\right]\times10^5 \; dy\, dz$$

at $t = 1\mu s$

$$= 2\times10 \sin(0.1)\int_S \sin 10^{-3}y \; dy\, dz$$

$$= -2 \;\; z\Big]_0^2 \;\; \frac{\cos 10^{-3}y}{10^{-3}}\Bigg]_0^{40} = \frac{-2\times2}{10^{-3}}\left[\cos(40\times10^{-3}) - \cos 0\right]$$

$$V = -3.19\text{V}$$

(2) Find the amplitude of displacement current density

a) Adjacent to an automobile antenna where the magnetic field intensity of an FM signal is

$$H_x = 0.15 \cos\left[3.12(3\times10^8 t - y)\right] \text{ A/m}$$

b) In the air space at a point within a large power distribution transformer where $\vec{B} = 0.8 \cos\left[1.257\times10^{-6}(3\times10^8 t - x)\right]\hat{a}_y$ T

c) Within a large oil filled power capacitor where $\varepsilon_r = 5$ & $\vec{E} = 0.9 \cos\left[1.257\times10^{-6}(3\times10^8 t - z\sqrt{5})\right]\hat{a}_z$ MV/m

d) In a metallic conductor at 60 Hz, if $\varepsilon = \varepsilon_0$, $\mu = \mu_0$ & $\sigma = 5.8\times10^7$ S/m and $\vec{J} = \sin(377t - 117.1z)\hat{a}_x$ MA/m²

a) $\nabla\times\vec{H} = \vec{J} + \vec{J}_D$ since $J = 0$ as there is no $\sigma \Rightarrow \vec{J} = \sigma\vec{E} = 0$

$$\nabla\times\vec{H} = \vec{J}_D$$

$$\vec{J}_D = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0.15\cos[3.12(3\times10^8 t - y)] & 0 & 0 \end{vmatrix}$$

$$= 0.15\,\hat{a}_z\left[-\sin(3.12(3\times10^8 t) - y)\right](-3.12)$$

$$\vec{J}_D = 0.468\left[\sin 3.12(3\times10^8 t) - y\right] \text{ A/m}^2$$

b) $\nabla \times \vec{H} = \vec{J} + \vec{J}_D$ Since air space there is no σ

Hence $J = \sigma E = 0$

$$\nabla \times \vec{H} = \vec{J}_D$$

$$\vec{J}_D = \frac{\nabla \times \vec{B}}{\mu} = \frac{1}{\mu}\begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & 0.8\cos[1.257\times10^{-6}(3\times10^8 t - x)] & 0 \end{vmatrix}$$

$$\vec{J}_D = \frac{\hat{a}_z}{\mu}\left[-0.8\cos\left[1.257\times10^{-6}(3\times10^8 t - x)\right]\ 1.257\times10^{-6}\right]$$

$$\vec{J}_D = \frac{1.257\times10^{-6}}{\mu_0}\left[-0.8\cos\left[1.257\times10^{-6}(3\times10^8 t - x)\right]\right]\hat{a}_z$$

$$\vec{J}_D = 0.800\left[\cos 1.257\times10^{-6}(3\times10^8 t - x)\right]\hat{a}_z$$

c) $$\vec{J}_D = \frac{\partial \vec{D}}{\partial t} = \frac{\varepsilon \partial \vec{E}}{\partial t} = \varepsilon\frac{\partial}{\partial t} 0.9\cos\left[1.257\times10^{-6}(3\times10^8 t - z\sqrt{5})\right]\times10^6\hat{a}_x$$

$$\vec{J}_D = -\varepsilon_0\varepsilon_r\, 0.9\sin\left[1.257\times10^{-6}(3\times10^8 t - z\sqrt{5})\right]\times10^6\times1.257\times10^{-6}\times3\times10^8\hat{a}_x$$

$$\vec{J}_D = 0.0150\sin\left[1.257\times10^{-6}(3\times10^8 t - z\sqrt{5})\right]\hat{a}_x\ A/m^2$$

d) $\vec{J} = \sigma\vec{E}$

$\vec{E} = \vec{J}/\sigma$ & $\vec{J}_D = \frac{\varepsilon\partial\vec{E}}{\partial t} = \frac{\varepsilon}{\sigma}\frac{\partial\vec{J}}{\partial t}$

$$\vec{J}_D = \frac{\varepsilon_0}{\sigma}\frac{\partial}{\partial t}\sin(377t - 117.1z)\,\hat{a}_x\times10^6$$

$$\vec{J}_D = \frac{\varepsilon_0}{\sigma}\cos(377t - 117.1z)\,\hat{a}_x\,(377)\times10^6$$

$$\vec{J}_D = 57.551\cos(377t - 117.1z)\,10^{-12}\ A/m^2$$

③ Let $\mu = 10^{-5}$ H/m , $\varepsilon = 4\times10^{-9}$ F/m, $\sigma = 0$ & $\rho_v = 0$. Find K so that each of the following pairs of fields satisfy Maxwell's equations a) $\vec{D} = 6\hat{a}_x - 2y\hat{a}_y + 2z\hat{a}_z$ nC/m²

$\vec{H} = Kx\,\hat{a}_x + 10y\,\hat{a}_y - 25z\,\hat{a}_z$ A/m

b) $\vec{E} = (20y - kt)\,\hat{a}_x$ V/m $\vec{H} = (y + 2\times10^{6}t)\,\hat{a}_z$ A/m

a) $\nabla\cdot\vec{B} = 0$

$\mu\,\nabla\cdot\vec{H} = 0$

$$\mu\left[\left(\frac{\partial}{\partial x}\hat{a}_x + \frac{\partial}{\partial y}\hat{a}_y + \frac{\partial}{\partial z}\hat{a}_z\right)\cdot\left(H_x\hat{a}_x + H_y\hat{a}_y + H_z\hat{a}_z\right)\right] = 0$$

$$\frac{\partial}{\partial x}(kx) + \frac{\partial}{\partial y}(10y) + \frac{\partial}{\partial z}(-25z) = 0$$

$$k + 10 - 25 = 0$$

$$\underline{\underline{k = 15\ \text{A/m}^2}}$$

b) $\nabla\times\vec{H} = \vec{J}_c + \vec{J}_D$ Since $\sigma = 0$, $\vec{J} = \sigma\vec{E} = 0$

$$\nabla\times\vec{H} = \varepsilon\frac{\partial\vec{E}}{\partial t} = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & 0 & (y + 2\times10^{6}t) \end{vmatrix} = \varepsilon(-k)$$

$$-k\varepsilon\,\hat{a}_x = [1 + 0]\,\hat{a}_x$$

$$\underline{\underline{k = -1/\varepsilon = -2.5\times10^{8}\ \text{V/ms}}}$$

4) The unit vector $0.64\hat{a}_x + 0.6\hat{a}_y - 0.48\hat{a}_z$ is dedicated from region 2 ($\varepsilon_r = 2$, $\mu_r = 3$, $\sigma_2 = 0$) towards region 1 $\varepsilon_r = 4$, $\mu_r = 2$, $\sigma = 0$). If $B_1 = (\hat{a}_x - 2\hat{a}_y + 3\hat{a}_z)\sin 300t$ T at point P in region 1 adjacent to the boundary find the amplitude at P of a) B_{N1} b) B_{t1} c) B_{N2} d) B_2

a) Normal component is given by

$$H_{N1} = \vec{H}_1 \cdot \hat{a}$$

$$\vec{B}_{N1} = \vec{B}_1 \cdot \hat{a} = \left[(\hat{a}_x - 2\hat{a}_y + 3\hat{a}_z)\sin(300t)\right] \cdot \left[0.64\hat{a}_x + 0.6\hat{a}_y - 0.48\hat{a}_z\right]$$

$$\vec{B}_{N1} = (0.64 - 1.2 - 1.44)\sin(300t) = 0.88\sin(300t)$$

$$\vec{B}_{N1} = B_{N1}\,\hat{a} = -2\sin(300t)\left[0.64\hat{a}_x + 0.6\hat{a}_y - 0.48\hat{a}_z\right]$$

$$\vec{B}_{N1} = B_{N1}\,\hat{a} = (-1.28\hat{a}_x - 1.2\hat{a}_y + 0.96\hat{a}_z)\sin(300t)$$

Amplitude of $\vec{B}_{N1} = 2\ T$

b) $\vec{B}_{t1} = \vec{B}_1 - \vec{B}_{N1} =$

$$= (\hat{a}_x - 2\hat{a}_y + 3\hat{a}_z)\sin 300t - (-1.28\hat{a}_x - 1.2\hat{a}_y + 0.96\hat{a}_z)\sin 300t$$

$$= \sin(300t)\left[+2.28\hat{a}_x - 0.8\hat{a}_y + 2.04\hat{a}_z\right]$$

Amplitude of $\vec{B}_{t1} = \sqrt{(2.28)^2 + (0.8)^2 + (2.04)^2} = 3.18\ T$

c) $\vec{B}_{N2} = \vec{B}_{N1} \Rightarrow$ Amplitude of $\vec{B}_{N2} = 2T$.

d) $\vec{B}_2 = \vec{B}_{tan2} + \vec{B}_{N2}$

$$\frac{\vec{B}_{t2}}{\mu_2} = \frac{\vec{B}_{t1}}{\mu_1}$$

$$\vec{B}_{t2} = \frac{\mu_2}{\mu_1}\vec{B}_{t1} = \frac{3}{2}(3.18\ T)$$

$$= 3/2\left[\sin(300t)\,(2.28\hat{a}_x - 0.8\hat{a}_y + 2.04\hat{a}_z)\right]$$

$$\vec{B}_2 = \vec{B}_{N2} + \vec{B}_{t2} = \left[(-1.28\hat{a}_x - 1.2\hat{a}_y + 0.96\hat{a}_z) + \frac{3}{2}(2.28\hat{a}_x - 0.8\hat{a}_y + 2.04\hat{a}_z)\right]\sin(300t)$$

$$\vec{B}_2 = \left[(2.14\hat{a}_x - 2.4\hat{a}_y + 4.02\hat{a}_z)\sin(300t)\right] \Rightarrow \text{Amplitude of } \vec{B}_2 = 5.147\ T$$

5) Find the frequency at which conduction current density & displacement current density are equal in medium with $\sigma = 2\times10^{-4}\ \mho/m$ & $\varepsilon_r = 81$

The ratio of two current densities is given as

$$\frac{|\vec{J}_c|}{|\vec{J}_D|} = \frac{\sigma}{\omega\varepsilon} = 1$$

$$\omega = \frac{\sigma}{\varepsilon} = \frac{\sigma}{\varepsilon_0 \varepsilon_r}$$

$$\omega = \frac{2\times10^{-4}}{(8.854\times10^{-12})\,81} = 0.2788\times10^{6}\ \text{rad/sec}$$

$$\omega = 2\pi f$$

$$f = \omega/2\pi = \underline{\underline{44.372\ \text{kHz}}}$$

6) A circular cross-section conductor of radius 1.5 mm carries a current $i = 5.5\sin(4\times10^{10}t)\ \mu A$. Find the amplitude of displacement current density if $\sigma = 35\ \mho/m$ & $\varepsilon_r = 10$.

W.K.T $\quad \dfrac{|\vec{J}_c|}{|\vec{J}_D|} = \dfrac{\sigma}{\omega\varepsilon}$

$$|\vec{J}_c| = \frac{\text{Conduction Current}}{\text{Area of cross section}} = \frac{5.5\times10^{-6}}{\pi(1.5\times10^{-3})^2}$$

$$\vec{J}_c = 0.77809\ A/m^2$$

$$|\vec{J}_D| = |\vec{J}_c|\,\frac{\omega\varepsilon}{\sigma} = \frac{0.77809\times 4\times10^{10}\times10\times8.854\times10^{-12}}{35}$$

$$\underline{\underline{|\vec{J}_D| = 0.07873\ A/m^2}}$$

7) Do the fields $\vec{E} = E_m \sin x \sin t \ \hat{a}_y$ & $\vec{H} = \frac{E_m}{\mu_0} \cos x \cos t \ \hat{a}_z$ satisfy Maxwell's equations?

Consider $\nabla \times \vec{E} = \frac{-\partial \vec{B}}{\partial t}$

Let us assume that the EM fields in free space

$$\nabla \times \vec{E} = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & E_m \sin x \sin t & 0 \end{vmatrix}$$

$$\nabla \times \vec{E} = \underline{\underline{E_m \sin t \cos x \ \hat{a}_z}}$$

$$\frac{-\partial \vec{B}}{\partial t} = \mu_0 \frac{\partial \vec{H}}{\partial t} = -\mu_0 \frac{\partial}{\partial t}\left[\frac{E_m}{\mu_0} \cos x \cos t\right] = \underline{\underline{E_m \cos x \sin t \ \hat{a}_z}}$$

Hence $\underline{\underline{\nabla \times \vec{E} = \frac{-\partial \vec{B}}{\partial t}}}$.

8) A uniform plane wave $\vec{E}_y = 10 \sin(2\pi \times 10^8 t - \beta x) \ \hat{a}_y$ is travelling in x-direction in free space. Find i) Phase Constant ii) Phase velocity iii) Expression for H_z. Assume $E_z = 0 = H_y$

The wave travelling in x-direction hence E_y component can be written as

$$\vec{E}_y = E_{y0} \sin(\omega t - \beta x)$$

By comparing the given expression $\omega = 2\pi \times 10^8$ rad/s
$E_{y0} = 10$ V/m

In free space $\varepsilon_r = \mu_r = 1$ & $c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = 3\times10^8$ m/s

1. $\beta = \frac{\omega}{v} = \frac{2\pi\times10^8}{3\times10^8} = 2.09435$ rad/m

2. Phase velocity $v = 1/\sqrt{\mu_0\varepsilon_0} = 3\times10^8$ m/s

3. For free space $\eta_0 = 377\,\Omega$

$$H_{z0} = \frac{E_{y0}}{\eta_0} = \frac{10}{377} = 0.026525 \text{ A/m}$$

$$H_z = H_{z0} \sin(\omega t - \beta x)$$

$$H_z = 26.52 \sin(2\pi\times10^8 t - \beta z) \text{ mA/m}$$

9) An $\vec{E}$ field in free space is given by $\vec{E} = 800 \cos(10^8 t - \beta y)\hat{a}_z$ V/m Find i) β ii) λ iii) $\vec{H}$ at $P(0.1, 1.5, 0.4)$ at $t = 8$ ns

$$\vec{E}_z = E_{z0} \cos(10^8 t - \beta y)\ \hat{a}_z \text{ V/m}$$

i) $\beta = \omega/c = 10^8/3\times10^8 = 0.333$ rad/m

ii) $\lambda = c/f = 3\times10^8/(\omega/2\pi) = \frac{3\times10^8\times2\pi}{10^8} = 6\pi = 18.85$ m

iii) $\vec{H}_x = H_{x0} \cos(\omega t - \beta y)\ \hat{a}_x$ A/m

$$H_{x0} = E_{z0}/\eta_0 = \frac{800}{120\pi} = 2.122$$

$$\vec{H}_x = 2.122 \cos(10^8 t - \beta y)\ \hat{a}_x \text{ A/m}$$

at $P(0.1, 1.5, 0.4)$ & $t = 8$ ns

$$\underline{\vec{H}_x = 2.027}\ \hat{a}_x \text{ A/m}$$

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10) A 300 MHz uniform plane wave propagates through fresh water for which $\sigma = 0$, $\mu_r = 1$ & $\varepsilon_r = 78$ Calculate i) Attenuation Constant ii) Phase Constant iii) Wavelength iv) Intrensic Impedance.

i) For the medium of fresh air $\sigma = 0$, hence the medium can be assumed as lossless medium $\alpha = 0$

ii) Phase Constant $\beta = \omega\sqrt{\mu\varepsilon} = \omega\sqrt{(\mu_0\mu_r)(\varepsilon_0\varepsilon_r)}$

$$\beta = (2\pi \times 300 \times 10^{6})\sqrt{(4\pi \times 10^{-7})(8.854 \times 10^{-12})(78)}$$

$$\beta = \underline{\underline{55.529 \text{ rad/m}}}$$

iii) $$\lambda = 2\pi/\beta = \frac{2\pi}{55529} = 0.1131 \text{ m}$$

iv) $$\eta = \sqrt{\frac{\mu}{\varepsilon}} = \sqrt{\frac{\mu_0\mu_r}{\varepsilon_0\varepsilon_r}} = \underline{\underline{42.656\ \Omega}}$$

11) A lossy dielectric is characterized by $\varepsilon_r = 2.5$, $\mu_r = 4$ & $\sigma = 10^{-3}$ ℧/m at a frequency 10 MHz. Find i) Attenuation constant ii) Phase constant iii) Velocity of constant iv) Wavelength v) Intrensic Impedance

For lossy dielectric

i) The propagation constant

$$\gamma = \sqrt{j\omega\mu(\sigma + j\omega\varepsilon)}$$

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$$= \sqrt{j(2\pi f)(\mu_0\mu_r)[\sigma + j(2\pi f)\varepsilon_0\varepsilon_r]}$$

$$= \sqrt{j(2\pi 10\times10^6)(4\pi\times10^{-7}\times4)[10^{-3} + j(2\pi\times10\times10^6)(8.854\times10^{-12}\times2.5)}$$

$$= \sqrt{j(315.82)[1\times10^{-3} + j\,1.3907\times10^{-3}}$$

$$\gamma = 0.2255 + j\,0.7$$

Attenuation Constant $\alpha = 0.2255$ Np/m

ii) $\beta = 0.7$ rad/m

iii) $v = \omega/\beta = \dfrac{2\pi\times10\times10^6}{0.7} = 8.9759\times10^7$ m/s

iv) $\lambda = 2\pi/\beta = 2\pi/0.7 = 8.975$ m

v) $\eta = \sqrt{\dfrac{j\omega\mu}{\sigma + j\omega\varepsilon}} = \sqrt{\dfrac{j(2\pi10\times10^6)(4\pi\times10^{-7}\times4)}{10^{-3} + j(2\pi10\times10^6)(8.854\times10^{-12}\times2.5)}}$

$$\eta = \sqrt{\frac{315.82\angle 90^\circ}{1.7129\times10^{-3}\angle 54.28^\circ}}$$

$$\eta = \underline{\underline{429.39}}\angle 17.86^\circ$$

12) A 160 MHz plane wave penetrates through aluminium of conductivity 10^5 ℧/m, $\varepsilon_r = \mu_r = 1$. Calculat skin depth and also depth at which the wave amplitude decreases to 13.5% of its initial value.

The skin depth is given by ($\sigma = 10^5$ for aluminium)

$$\delta = 1/\sqrt{\pi f \mu \sigma} = \frac{1}{\sqrt{\pi \times 160 \times 10^6 \times 4\pi \times 10^{-7} \times 10^5}}$$

$$\underline{\underline{\delta = 125.82 \mu m}}$$

For good conductor ($\sigma >> 1$)

$$\alpha = \beta = 1/\delta = \frac{1}{125.82 \times 10^{-6}} = \underline{\underline{7947.6725}}$$

At $z = z'$ the amplitude decreases to 13.5% of its initial value. Let

$$|\vec{E}| = E_0 \, e^{-\beta z'} = \frac{13.5}{100} E_0$$

$$e^{-\beta z'} = 0.135$$

Taking ln on both sides

$$-\beta z' \ln(e) = \ln(0.135) = -2.0025$$

$$z' = 0.25196 \times 10^{-3} \, m \approx \underline{\underline{252 \mu m}}$$

Thus at $z = z' = 252 \mu m$ the amplitude decreases to 13.5% of its initial value.

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13) Wet Marshy soil is characterized by $\sigma = 10^{-2}$ S/m, $\varepsilon_r = 15$ and $\mu_r = 1$. At frequencies 60 Hz, 1 MHz, 100 MHz and 10 GHz indicate whether soil be considered a conductor or dielectric

i) At $f = 60$ Hz

$$\frac{\sigma}{\omega\varepsilon} = \frac{\sigma}{(2\pi f)(\varepsilon_0 \varepsilon_r)} = \frac{10^{-2}}{(2\pi \times 60)(8.854 \times 10^{-12} \times 15)}$$

$$\frac{\sigma}{\omega\varepsilon} = 1.99472 \times 10^{5} \gg 1$$

The wet marshy soil acts as conductor at 60 Hz.

ii) At $f = 1$ MHz

$$\frac{\sigma}{\omega\varepsilon} = \frac{10^{-2}}{(2\pi \times 1 \times 10^{6})(\varepsilon_0 \varepsilon_r)} = 11.9836 \gg 1$$

At 1 MHz also soil acts as conductor

iii) At $f = 100$ MHz

$$\frac{\sigma}{\omega\varepsilon} = \frac{10^{-2}}{(2\pi \times 100 \times 10^{6})(\varepsilon_0 \varepsilon_r)} = 0.1198 < 1$$

At $f = 100$ MHz soil acts as lossy dielectric

iv) At $f = 10$ GHz

$$\frac{\sigma}{\omega\varepsilon} = \frac{10^{-2}}{(2\pi \times 10 \times 10^{9})(\varepsilon_0 \varepsilon_r)}$$

$$\frac{\sigma}{\omega\varepsilon} = 0.0012 \ll 1$$

At $f = 10$ GHz the wet marshy soil acts as perfect dielectric

19) In free space $\vec{E} = 150 \sin(\omega t - \beta z)\, \hat{a}_x$ V/m. Calculate the total power passing through a rectangular area of sides 30 mm and 15 mm in $z=0$ plane. Assume $\frac{E_m}{H_m} = \eta_0$ & $\eta_0 = 120\pi\ \Omega$

$$\vec{E} = 150 \sin(\omega t - \beta z)\, \hat{a}_x \text{ V/m}$$

$$\vec{H} = \frac{150}{\eta_0} \sin(\omega t - \beta z)\, \hat{a}_y \text{ A/m}$$

converting both the sinusoidal function to cosinusoidal function

$$\vec{E} = 150 \cos(\omega t - \beta z - \pi/2)\, \hat{a}_x \text{ and}$$

$$\vec{H} = \frac{150}{\eta_0} \cos(\omega t - \beta z - \pi/2)\, \hat{a}_y$$

writing in phasor form

$$\vec{E} = 150\, e^{j(-\beta z - \pi/2)}\, \hat{a}_x$$

$$\vec{H} = \frac{150}{\eta_0}\, e^{j(-\beta z - \pi/2)}\, \hat{a}_y$$

Complex Conjugate of $\vec{H}$ is given by

$$\vec{H}^* = \frac{150}{\eta_0}\, e^{j(\beta z + \pi/2)}\, \hat{a}_y$$

Hence average power density is given by

$$\vec{P}_{avg} = \frac{1}{2} \mathrm{Re}\left[\vec{E} \times \vec{H}^*\right]$$

$$= \frac{1}{2}\left(\frac{150}{\eta_0}\right)\left(e^{j(-\beta z - \omega t)}\hat{a}_x \times e^{j(\omega t + \beta z)}\hat{a}_y\right)$$

$$\vec{P}_{avg} = \frac{1}{2} \frac{150}{120\pi}\, \hat{a}_z = 29.841\, \hat{a}_z \text{ W/m}^2$$

Now the total power crossing area is given by

$$P = (P_{avg})(\text{area}) = (29.841)(15 \times 30 \times 10^{-6}) = \underline{13.428 \text{ mW}}$$