R-H Criteria

1) Examine the stability of a system with characteristic equation

$$s^6 + 3s^5 + 4s^4 + 6s^3 + 5s^2 + 3s + 2 = 0$$ using R-H criteria.

Soln:

s^6	1	4	5	2
s^5	3	6	3	0
s^4	2	4	2	0
s^3	0	0	0	0
s^2				
s^1				
s^0				

s^3 row → a row of zeros

A row of zeros is encountered in s^3 row.

$\therefore$ Form the Auxillary eqn using coefficients in s^4 row

$$A(s) = 2s^4 + 4s^2 + 2 = 0 \Rightarrow s^4 + 2s^2 + 1 = 0$$

$$\frac{dA(s)}{ds} = 4s^3 + 4s + 0$$

s^6	1	4	5	2
s^5	3	6	3	0
s^4	2	4	2	0
s^3	4	4	0	0
s^2	2	2	0	0
s^1	0	0	0	0
s^0				

s^3 row → row of zeros replaced by coefficients of $\frac{dA(s)}{ds}$

s^1 row → A row of zeros for the 2nd time

$$\therefore A \cdot E = 2s^2 + 2 = 0$$

$$A(s) = s^2 + 1 = 0.$$

$$\frac{dA(s)}{ds} = 2s + 0$$

s^6	1	4	5	2
s^5	3	6	3	0
s^4	2	4	2	0
s^3	4	4	0	0
s^2	2	2	0	0
s^1	2	0	0	0
s^0	2	0	0	0

s^1 row → row of zeros replaced by coefficients of $\frac{dA(s)}{ds}$.

Interpretations from routh array

→ No sign changes in the first column but row of zeros occured while forming the array.

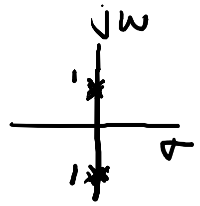
∴ The system may be marginally stable or unstable.

To examine, find the roots of A·E

i.e., $2s^4+4s^2+2=0 \Rightarrow s^4+2s^2+1=0$

$(s^2+1)^2=0 \quad s^2+1=0$

or $s^2=-1$. ; $s=\pm j1$



$2s^2+2=0 \Rightarrow s^2+1=0 \Rightarrow s=\pm j1.$

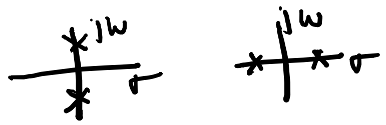
As there are repeated roots on the Imaginary axis system is unstable.

→ All the coefficients of the polynomial should have same sign. & None of the coefficient is zero

→ If there are any sign changes in the first column of routh array, system is unstable (No. of sign changes = No. of roots on RH side of s-plane)

→ If a zero occurs in the first column only, there may be roots on Imaginary axis or in RHP

→ If a row of zeros occurs, there are roots of equal magnitude & opposite sign



The system stability can be predicted from the roots of the Auxillary eqn $A(s)=0$

Applications of RH criteria

→ To determine the limits of the stable gain

→ To determine relative stability

Advantages & limitations of R-H criteria

→ stability is predicted w/o solving ch. eqn.

→ can be used to determine range of values of gain (K) for system stability

→ Relative stability can be determined

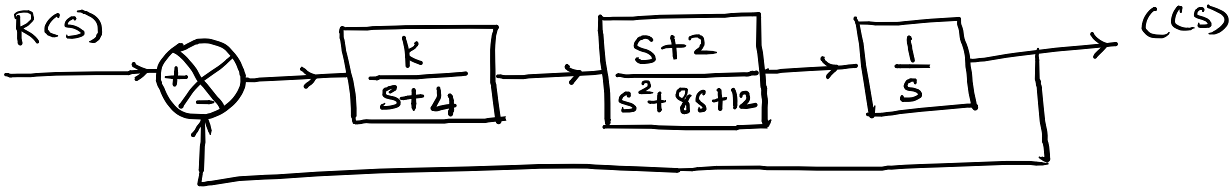
Limitations are → valid only for real coefficients of ch. eqn

→ does not provide actual location of closed loop poles

→ cannot be used to stabilize an unstable system

→ applicable only to linear systems

Example: For the system shown, determine the range of stable gain (K) and the oscillation frequency of the system at the maximum value of gain



$$\frac{C(S)}{R(S)} = \frac{\dfrac{K(S+2)}{(S+4)(S^2+8S+12)S}}{1 + \dfrac{K(S+2)}{S(S+4)(S^2+8S+12)}} = \frac{K(S+2)}{S(S+4)(S^2+8S+12)+K(S+2)}$$

$$C.L.T.F = \frac{K(S+2)}{S^4+12S^3+44S^2+(48+K)S+2K} = \frac{G(S)}{1+G(S)H(S)}$$

∴ ch. eqn $1+G(S)H(S)=0$ is

$$S^4+12S^3+44S^2+(48+K)S+2K=0$$

Forming Routh array,

$$\begin{array}{c|ccc}
s^4 & 1 & 44 & 2K \\
s^3 & 12 & (48+K) & 0 \\
s^2 & \frac{480-K}{12} & 2K & 0 \\
s^1 & \frac{-K^2+144K+23040}{480-K} & 0 & 0 \\
s^0 & 2K & &
\end{array}$$

s^2 row

$$\frac{12 \times 44 - (48+K) \times 1}{12}$$

s^1 row $\rightarrow$ all zeros

$$\frac{\frac{480-K}{12} \times (48+K) - 24K}{\frac{480-K}{12}}$$

$$-K^2 + 144K + 23040$$

$$K^2 - 144K - 23040$$

$$\frac{+144 \pm \sqrt{(144)^2 - 4 \times 23040}}{2}$$

$K_1 = 240$

$K_2 = -96 \rightarrow$ ignore -ve gain

$-96 \le K \le 240$

A.E $\quad \frac{480-K}{12} s^2 + 2K = 0$

$$\frac{480-240}{12} s^2 + 2 \times 240 = 0$$

$$20 s^2 + 480 = 0$$

$$s^2 = \frac{-48\cancel{0}}{2\cancel{0}} = -24$$

$$s = \pm j 4.9 \text{ rad/sec.}$$

$K = 240, \quad \omega = 4.9$ rad/sec.

Example: open loop Transfer function of a unity feedback system is $G(S) = \frac{K}{S(S^2+S+1)(S+2)}$. Find the range of K for stability. For what value of K system will oscillate & what is the freq of oscillations?

$1 + G(S)H(S) = 0 \rightarrow$ ch. eqn. here $H(S) = 1$.

$$1 + \frac{K}{s(s^2+s+1)(s+2)} = 0$$

$$s(s^2+s+1)(s+2) + K = 0$$

$$(s^3+s^2+s)(s+2)+k = 0$$

$$s^4+s^3+s^2+2s^3+2s^2+2s+k=0$$

$$s^4+3s^3+3s^2+2s+k=0$$

Forming the Routh array

s^4	1	3	K
s^3	3	2	0
s^2	$\frac{7}{3}$	K	
s^1	$\frac{14-9K}{7}$	0	
s^0			

$$\frac{\frac{14}{3}-3K}{7/3} = \frac{14-9K}{7.}$$

$$14-9K=0$$

$$K = \frac{14}{9}$$

$$= 1.55$$

when $k = 1.55 \rightarrow s^1$ row $\rightarrow 0\ 0.$

$\therefore K = 1.55 \rightarrow$ is critical value.

$K > 0$

$K < 1.55$

$\underline{\underline{0 < K < 1.55}}$

considering A.E $\rightarrow \frac{7}{3}s^2 + K = 0$

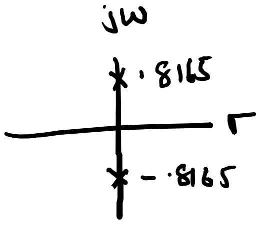
$$7s^2+3k=0$$

$$7s^2+3\times\frac{14}{9_3}=0$$

$$s^2+\frac{2}{3}=0 \qquad s^2 = -\frac{2}{3}$$

$$s = \pm j\sqrt{\frac{2}{3}} = \pm j\omega$$

$$= \pm j\,0.8165 \text{ rad/se.}$$



Range of $k \rightarrow 0 < K < 1.55$

At $K = 1.55$, $\omega = 0.8165$ rad/sec.