1. Find the unit vector extending from the origin toward the point $G(2,-2,-1)$

$$\vec{G} = 2\hat{a}_x + (-2)\hat{a}_y + (-1)\hat{a}_z$$

$$|\vec{G}| = \sqrt{(2)^2 + (-2)^2 + (-1)^2} = \sqrt{9} = 3$$

$$\hat{a}_G = \frac{\vec{G}}{|\vec{G}|} = \frac{2\hat{a}_x - 2\hat{a}_y - \hat{a}_z}{3} = \underline{\underline{0.667\,\hat{a}_x - 0.667\,\hat{a}_y - 0.333\,\hat{a}_z}}$$

2. Given points $M(-1,2,1)$, $N(3,-3,0)$ and $P(-2,-3,-4)$ find a) $\vec{R}_{MN}$ b) $\vec{R}_{MN} + \vec{R}_{MP}$ c) $|\vec{R}_M|$ d) $\hat{a}_{MP}$ e) $|2\vec{R}_P - 3\vec{R}_N|$

a) $\vec{R}_M = -\hat{a}_x + 2\hat{a}_y + \hat{a}_z$

$\vec{R}_N = 3\hat{a}_x - 3\hat{a}_y$

$\vec{R}_{MN} = \vec{R}_N - \vec{R}_M = \underline{\underline{4\hat{a}_x - 5\hat{a}_y - \hat{a}_z}}$

b) $\vec{R}_P = -2\hat{a}_x - 3\hat{a}_y - 4\hat{a}_z$

$\vec{R}_{MP} = \vec{R}_P - \vec{R}_M = -\hat{a}_x - 5\hat{a}_y - 5\hat{a}_z$

$\vec{R}_{MN} + \vec{R}_{MP} = \underline{\underline{3\hat{a}_x - 10\hat{a}_y - 6\hat{a}_z}}$

c) $|\vec{R}_M| = \sqrt{(-1)^2 + (2)^2 + (1)^2} = \sqrt{6} = \underline{\underline{2.5}}$

d) $$\hat{a}_{MP} = \frac{\vec{R}_{MP}}{|\vec{R}_{MP}|} = \frac{-\hat{a}_x - 5\hat{a}_y - 5\hat{a}_z}{\sqrt{(-1)^2 + (-5)^2 + (5)^2}} = \underline{\underline{-0.14\hat{a}_x - 0.7\hat{a}_y - 0.7\hat{a}_z}}$$

e) $$2\vec{R}_P - 3\vec{R}_N = (-4\hat{a}_x - 6\hat{a}_y - 8\hat{a}_z) - (9\hat{a}_x - 9\hat{a}_y)$$
$$= -13\hat{a}_x + 3\hat{a}_y - 8\hat{a}_z$$

$$|2\vec{R}_P - 3\vec{R}_N| = \sqrt{13^2 + 3^2 + 8^2} = \underline{\underline{15.56}}$$

3) A vector field $\vec{S}$ is represented by rectangular Coordinates

as $\vec{S} = \dfrac{125}{[(x-1)^2+(y-2)^2+(z+1)^2]}[(x-1)\,\hat{a}_x+(y-2)\,\hat{a}_y+(z+1)\,\hat{a}_z]$

a) Evaluate $\vec{S}$ at $P(2,4,3)$

b) Determine a unit vector that gives the direction of $\vec{S}$ at P

c) Specify the surface $f(x,y,z)$ on which $|\vec{S}|=1$.

a) $\vec{S} = \dfrac{125}{[1+4+16]}[\hat{a}_x+2\hat{a}_y+4\hat{a}_z]$

$\vec{S} = 5.95\,\hat{a}_x + 11.904\,\hat{a}_y + 23.8\,\hat{a}_z$

b) $\hat{a}_s = \dfrac{\vec{S}}{|\vec{S}|} = 0.218\,\hat{a}_x + 0.4365\,\hat{a}_y + 0.872\,\hat{a}_z$

c) $|\vec{S}| = \dfrac{125}{[(x-1)^2+(y-1)^2+(z+1)^2]}[(x-1)^2+(y-2)^2+(z+1)^2]^{1/2} = 1$

$\sqrt{(x-1)^2+(y-1)^2+(z+1)^2} = 125$

ⓐ Give the rectangular coordinates of the point $C(\rho = 4.4, \phi = -115°, z = 2)$

ⓑ Give the cylindrical coordinates of the point $D(x = -3.1, y = 2.6, z = -3)$

ⓒ Specify the distance from C to D.

a)

$$x = \rho \cos\phi = -1.86$$
$$y = \rho \sin\phi = -3.99$$
$$z = z = 2$$

$$C(-1.86, -3.99, 2)$$

b)

$$\rho = \sqrt{x^2 + y^2} = 4.05$$
$$\phi = \tan^{-1}(y/x) = -39.98°$$
$$z = z = -3$$

$$D(\rho = 4.05, \phi = -39.98°, z = -3)$$

c) Distance from C to D is

$$= \sqrt{(x_C - x_D)^2 + (y_C - y_D)^2 + (z_C - z_D)^2}$$

$$= \sqrt{1.53 + 43.42 + 25}$$

$$= \underline{\underline{8.36 \text{ units}}}.$$

Transform to cylindrical coordinates

a) $\vec{F} = 10\,\hat{a}_x - 8\,\hat{a}_y + 6\,\hat{a}_z$ at point $P(10,-8,6)$

b) $\vec{G} = (2x+y)\,\hat{a}_x - (y-4x)\,\hat{a}_y$ at point $Q(\rho, \phi, z)$

c) Give the rectangular components of the vector

$\vec{H} = 20\,\hat{a}_\rho - 10\,\hat{a}_\phi + 3\,\hat{a}_z$ at $P(x=5, y=2, z=-1)$

a) $\vec{F} = 10\hat{a}_x - 8\hat{a}_y + 6\hat{a}_z$

$\vec{F} = A_\rho\,\hat{a}_\rho + A_\phi\,\hat{a}_\phi + A_z\,\hat{a}_z$

$A_\rho = \vec{F}\cdot\hat{a}_\rho = 10(\hat{a}_x\cdot\hat{a}_\rho) - 8(\hat{a}_y\cdot\hat{a}_\rho) + 6(\hat{a}_z\cdot\hat{a}_\rho) = 10\cos\phi - 8\sin\phi = 12.8$

$A_\phi = \vec{F}\cdot\hat{a}_\phi = 10(\hat{a}_x\cdot\hat{a}_\phi) - 8(\hat{a}_y\cdot\hat{a}_\phi) + 6(\hat{a}_z\cdot\hat{a}_\phi) = -10\sin\phi - 8\cos\phi = -2.19\times10^{-3}$

$A_z = \vec{F}\cdot\hat{a}_z = 6$

$$\boxed{\vec{F} = 12.8\,\hat{a}_\rho + 6\,\hat{a}_z}$$

b) $\vec{G} = (2x+y)\,\hat{a}_x - (y-4x)\,\hat{a}_y$

$\vec{G} = G_\rho\,\hat{a}_\rho + G_\phi\,\hat{a}_\phi + G_z\,\hat{a}_z$

$G_\rho = \vec{G}\cdot\hat{a}_\rho = [2\rho\cos\phi + \rho\sin\phi]\cos\phi - [\rho\sin\phi - 4\rho\cos\phi]\sin\phi$

$=$

$G_\phi = \vec{G}\cdot\hat{a}_\phi = [2\rho\cos\phi + \rho\sin\phi](\hat{a}_x\cdot\hat{a}_\phi) - [\rho\sin\phi - 4\rho\cos\phi](\hat{a}_y\cdot\hat{a}_\phi)$

$= [2\rho\cos\phi + \rho\sin\phi](-\sin\phi) - (\rho\sin\phi - 4\rho\cos\phi)(\cos\phi)$

$G_z = \vec{G}\cdot\hat{a}_z = 0$

$\vec{G} = [2\rho\cos^2\phi + \rho\sin\phi\cos\phi - \rho\sin^2\phi + 4\rho\cos\phi\sin\phi]\,\hat{a}_\rho +$

$[-2\rho\cos\phi\sin\phi - \rho\sin^2\phi - \rho\sin\phi\cos\phi + 4\rho\cos^2\phi]\,\hat{a}_\phi$

$\vec{G} = [2\rho\cos^2\phi + 5\rho\sin\phi\cos\phi - \rho\sin^2\phi]\,\hat{a}_\rho +$

$[4\rho^2\cos\phi - 3\rho\sin\phi\cos\phi - \rho\sin^2\phi]\,\hat{a}_\phi$

c) $\vec{H} = A_x\,\hat{a}_x + A_y\hat{a}_y + A_z\hat{a}_z$

$A_x = 20\cos\phi - 10(-\sin\phi) = 22.283$ $\quad A_y = -1.857$ $\quad A_z = 3$ $\quad \vec{H} = 22.283\hat{a}_x - 1.857\hat{a}_y + 3\hat{a}_z$

The three vertices of a triangle are located at $A(6,-1,2)$ $B(-2,3,-4)$ and $C(-3,1,5)$ Find a) R_{AB} b) R_{AC} c) the angle θ_{BAC} at vertex A; d) the vector projection of $\vec{R}_{AB}$ on $\vec{R}_{AC}$

a) $\vec{R}_{AB} = \vec{R}_B - \vec{R}_A = -8\hat{a}_x + 4\hat{a}_y - 6\hat{a}_z$

b) $\vec{R}_{AC} = \vec{R}_C - \vec{R}_A = -9\hat{a}_x + 2\hat{a}_y + 3\hat{a}_z$

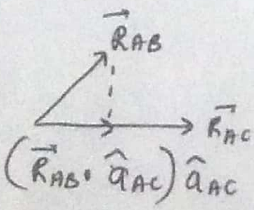
c) $\vec{R}_{BA} \cdot \vec{R}_{AC} = |\vec{R}_{BA}|\,|\vec{R}_{AC}| \cos\theta_{BAC}$

$$(-8\hat{a}_x + 4\hat{a}_y - 6\hat{a}_z)\cdot(-9\hat{a}_x + 2\hat{a}_y + 3\hat{a}_z) = [\sqrt{116}][\sqrt{94}] \cos\theta_{BAC}$$

$$62 = 104.42 \cos\theta_{BAC}$$

$$\underline{\underline{\theta_{BAC} = 53.57^\circ}}$$

d)



$$\hat{a}_{AC} = \frac{\vec{R}_{AC}}{|\vec{R}_{AC}|} = \frac{-9\hat{a}_x + 2\hat{a}_y + 3\hat{a}_z}{\sqrt{94}}$$

$$= \left[(-8\hat{a}_x + 4\hat{a}_y - 6\hat{a}_z)\cdot\frac{(-9\hat{a}_x + 2\hat{a}_y + 3\hat{a}_z)}{\sqrt{94}}\right]\left[\frac{-9\hat{a}_x + 2\hat{a}_y + 3\hat{a}_z}{\sqrt{94}}\right]$$

$$= \frac{1}{(94)}\left[\frac{(78+8-18)}{\cancel{\sqrt{94}}}\right]\left[-9\hat{a}_x + 2\hat{a}_y + 3\hat{a}_z\right]$$

Projection of $\vec{R}_{AB}$ on $\vec{R}_{AC}$ $= -5.94\hat{a}_x + 1.319\hat{a}_y + 1.97\hat{a}_z$

The three vertices of a triangle are located at $A(6,-1,2)$ $B(-2,3,-4)$ and $C(-3,1,5)$ find

a) $\vec{R}_{AB} \times \vec{R}_{AC}$ b) The area of the triangle c) a unit vector perpendicular to the plane in which the triangle is located.

a) $\vec{R}_{AB} = -8\hat{a}_x + 4\hat{a}_y - 6\hat{a}_z$

$\vec{R}_{AC} = -9\hat{a}_x + 2\hat{a}_y + 3\hat{a}_z$

$$\vec{R}_{AB} \times \vec{R}_{AC} = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ -8 & 4 & -6 \\ -9 & 2 & 3 \end{vmatrix} = \hat{a}_x(12+12) - \hat{a}_y(-24-54) + \hat{a}_z(-16+36)$$

$$\vec{R}_{AB} \times \vec{R}_{AC} = 24\hat{a}_x + 78\hat{a}_y + 20\hat{a}_z$$

b) $|\vec{R}_{AB} \times \vec{R}_{AC}|$ = Area of ▱ formed by $\vec{R}_{AB}$ & $\vec{R}_{AC}$

The area of $\Delta = \frac{1}{2}|\vec{R}_{AB} \times \vec{R}_{AC}| = \underline{42.0 \text{ units}}$

c) $\vec{R}_{AB} \times \vec{R}_{AC} = |\vec{R}_{AB}||\vec{R}_{AC}| \sin\theta \, \hat{a}_N$

$$\hat{a}_N = \left[\frac{(24\hat{a}_x + 78\hat{a}_y + 20\hat{a}_z)}{\sqrt{7060}}\right] = \frac{\vec{R}_{AB} \times \vec{R}_{AC}}{|\vec{R}_{AB} \times \vec{R}_{AC}|}$$

$$\hat{a}_N = 0.286\hat{a}_x + 0.928\hat{a}_y + 0.238\hat{a}_z$$

Given the two point $C(-3,2,1)$ and $D(r=5, \theta=20^\circ, \phi=-70^\circ)$, find

a) The spherical coordinates of C

b) The rectangular coordinates of D

c) The distance from C to D

a) $r = \sqrt{x^2+y^2+z^2} = 3.74$

$\theta = \cos^{-1} \dfrac{z}{\sqrt{x^2+y^2+z^2}} = 74.5^\circ$

$\phi = \tan^{-1}(y/x) = -33.69^\circ$

$(r, \theta, \phi) = (3.74, 74.5^\circ, -33.69^\circ)$

b) $x = r \sin\theta \cos\phi = 0.585$

$y = r \sin\theta \sin\phi = -1.607$

$z = r \cos\theta = 4.7$

$(x, y, z) = (0.585, -1.607, 4.7)$

c) Distance from C to D $= \sqrt{3.585^2 + 3.607^2 + 3.7^2} = \underline{\underline{6.9 \text{ units}}}$

Transform the following vectors to spherical coordinates at the points given

a) $10\hat{a}_x$ at $P(x=-3, y=2, z=4)$

b) $10\hat{a}_y$ at $Q(\rho=5, \phi=30^\circ, z=4)$

c) $10\hat{a}_z$ at $M(r=4, \theta=110^\circ, \phi=120^\circ)$

$r = \sqrt{x^2+y^2+z^2} = 5.385$

$\theta = \cos^{-1}(z/r) = 42.029$

$\phi = \tan^{-1}(y/x) = -33.69$

a) $10\hat{a}_x$

$$\vec{A} = A_r\hat{a}_r + A_\theta\hat{a}_\theta + A_\phi\hat{a}_\phi$$

$$A_r = (\vec{A})\cdot(\hat{a}_r) = (10\hat{a}_x)\cdot(\hat{a}_r) = 10\sin\theta\cos\phi = 5.57\hat{a}_r$$

$$A_\theta = (\vec{A})\cdot(\hat{a}_\theta) = (10\hat{a}_x)\cdot(\hat{a}_\theta) = 10\cos\theta\cos\phi = 6.18\hat{a}_\theta$$

$$A_\phi = (\vec{A})\cdot(\hat{a}_\phi) = (10\hat{a}_x)\cdot(\hat{a}_\phi) = -10\sin\phi = 5.55\hat{a}_\phi$$

$$\vec{A} = 5.57\hat{a}_r + 6.18\hat{a}_\theta + 5.55\hat{a}_\phi$$

b) $10\hat{a}_y$

$x = \rho\cos\phi = 4.33$ $\quad r = 6.403$

$y = \rho\sin\phi = 2.5$ $\quad \theta = 51.33^\circ$

$z = z = 4$ $\quad \phi = 30^\circ$

$$\vec{A} = A_r\hat{a}_r + A_\theta\hat{a}_\phi + A_\phi\hat{a}_\phi$$

$$A_r = (\vec{A}\cdot\hat{a}_r) = (10\hat{a}_y)\cdot(\hat{a}_r) = 10\sin\theta\sin\phi = 3.9$$

$$A_\theta = (\vec{A}\cdot\hat{a}_\theta) = (10\hat{a}_y)\cdot(\hat{a}_\theta) = 10\cos\theta\sin\phi = 3.12$$

$$A_\phi = (\vec{A}\cdot\hat{a}_\phi) = (10\hat{a}_y)\cdot(\hat{a}_\phi) = 10\cos\phi = 8.66$$

$$\vec{A} = 3.9\hat{a}_r + 3.12\hat{a}_\theta + 8.66\hat{a}_\phi$$

c) $10\hat{a}_z$

$$\vec{A} = A_r\hat{a}_r + A_\theta\hat{a}_\theta + A_\phi\hat{a}_\phi$$

$$A_r = \vec{A}(10\hat{a}_z)\cdot\hat{a}_r = 10\cos\theta = -3.42$$

$$A_\theta = (10\hat{a}_z)\cdot\hat{a}_\theta = -10\sin\theta = -9.4$$

$$A_\phi = (10\hat{a}_z)\cdot\hat{a}_\phi = 0 =$$

$$\vec{A} = -3.42\hat{a}_r - 9.4\hat{a}_\theta$$

A charge of $Q_1 = 3\times10^{-4}$ C is located at $M(1,2,3)$ and a charge of $Q_2 = -10^{-4}$ C at $N(2,0,5)$ in a vacuum. Find the force exerted on Q_2 by Q_1.

$\vec{r}_1 = \hat{a}_x + 2\hat{a}_y + 3\hat{a}_z$

$\vec{r}_2 = 2\hat{a}_x + 5\hat{a}_z$

$\vec{R}_{12} = \vec{r}_2 - \vec{r}_1 = \hat{a}_x - 2\hat{a}_y + 2\hat{a}_z$

$$\vec{F}_{12} = \frac{(3\times10^{-4})(-10^{-4})\left[\hat{a}_x - 2\hat{a}_y + 2\hat{a}_z\right]}{4\pi\varepsilon_0 (1+4+4)^{3/2}}$$

$$\vec{F}_{12} = \frac{-3\times10^{-8}\left[\hat{a}_x - 2\hat{a}_y + 2\hat{a}_z\right]}{3\times10^{-9}}$$

$$\vec{F}_{12} = -10\hat{a}_x + 20\hat{a}_y - 20\hat{a}_z$$

A charge $Q_A = -20\mu C$ is located at $A(-6,4,7)$ and a charge $Q_B = 50\mu C$ is at $B(5,8,-2)$ in free space. If distances are given in meters, find.

a) $\vec{R}_{AB}$ b) R_{AB}. Determine the vector forces exerted on Q_A by Q_B if $\varepsilon_0 =$ c) $10^{-9}/36\pi$ F/m d) 8.854×10^{-12} F/m

a) $\vec{R}_{AB} = 11\hat{a}_x + 4\hat{a}_y - 9\hat{a}_z$

b) $R_{AB} = 14.76$ m

c) $$F_{BA} = \frac{(-20\times10^{-6})(50\times10^{-6})\left[-11\hat{a}_x - 4\hat{a}_y + 9\hat{a}_z\right]}{4\pi\varepsilon_0 (14.76)^3}$$

$$\vec{F}_{BA} = 30.76\hat{a}_x + 11.18\hat{a}_y - 25.16\hat{a}_z \text{ mN.}$$

With $\varepsilon_0 = 8.854\times10^{-12}$ F/m

$$\vec{F}_{BA} = 30.72\hat{a}_x + 11.169\hat{a}_y - 25.13\hat{a}_z \text{ mN}$$

Find $\vec{E}$ at $P(1,1,1)$ caused by four identical 3 nC charges located at $P_1(1,1,0)$, $P_2(-1,1,0)$, $P_3(-1,-1,0)$ and $P_4(1,-1,0)$

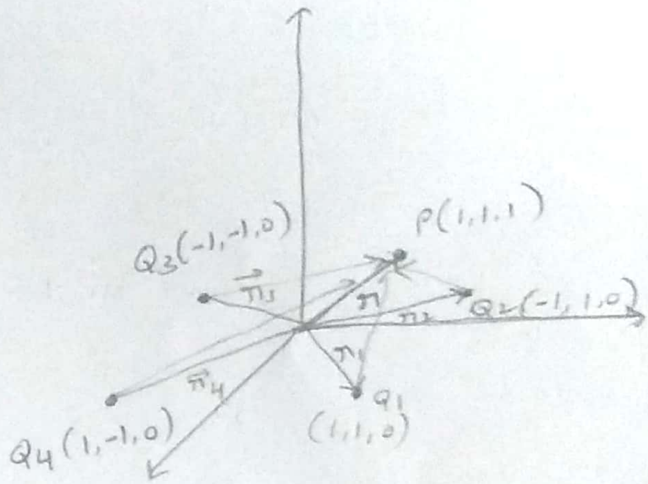
$\vec{r} = \hat{a}_x + \hat{a}_y + \hat{a}_z$

$\vec{r}_1 = \hat{a}_x + \hat{a}_y$

$\vec{r}_2 = -\hat{a}_x + \hat{a}_y$

$\vec{r}_3 = -\hat{a}_x - \hat{a}_y$

$\vec{r}_4 = \hat{a}_x - \hat{a}_y$



$$\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + \vec{E}_4$$

$$= \frac{Q_1}{4\pi\varepsilon_0}\frac{(\vec{r}-\vec{r}_1)}{|\vec{r}-\vec{r}_1|^3} + \frac{Q_2}{4\pi\varepsilon_0}\frac{(\vec{r}-\vec{r}_2)}{|\vec{r}-\vec{r}_2|^3} + \frac{Q_3}{4\pi\varepsilon_0}\frac{(\vec{r}-\vec{r}_3)}{|\vec{r}-\vec{r}_3|^3} + \frac{Q_4}{4\pi\varepsilon_0}\frac{(\vec{r}-\vec{r}_4)}{|\vec{r}-\vec{r}_4|^3}$$

$$= \frac{Q}{4\pi\varepsilon_0}\left[\frac{\hat{a}_z}{1} + \frac{+2\hat{a}_x+\hat{a}_z}{(5)^{3/2}} + \frac{2\hat{a}_x+2\hat{a}_y+\hat{a}_z}{(9)^{3/2}} + \frac{2\hat{a}_y+\hat{a}_z}{(5)^{3/2}}\right]$$

$$= 26.96\left[\hat{a}_z + \frac{2\hat{a}_x+\hat{a}_z}{5^{3/2}} + \frac{2\hat{a}_x+2\hat{a}_y+\hat{a}_z}{3^3} + \frac{2\hat{a}_y+\hat{a}_z}{5^{3/2}}\right]$$

$$\vec{E} = \underline{6.82\,\hat{a}_x + 6.82\,\hat{a}_y + 32.8\,\hat{a}_z} \text{ V/m}$$

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A charge of $-0.3\,\mu C$ is located at $A(25,-30,15)$ in cm and a second charge of $0.5\,\mu C$ is at $B(-10,8,12)$ cm find $\vec{E}$ at a) The origin b) $P(15,20,50)$ cm

a) $$\vec{E} = \frac{-0.3\times10^{-6}\left(-25\hat{a}_x+30\hat{a}_y-15\hat{a}_z\right)\times10^{-2}}{4\pi\varepsilon_0\left[0.25^2+0.3^2+0.15^2\right]^{3/2}} +$$

$$\frac{0.5\times10^{-6}\left(10\hat{a}_x-8\hat{a}_y-12\hat{a}_z\right)\times10^{-2}}{4\pi\varepsilon_0\left[0.1^2+0.08^2+0.12^2\right]^{3/2}}$$

$$\vec{E} = \left(9207.77\hat{a}_x - 11049.33\hat{a}_y + 5524.66\hat{a}_z\right) +$$
$$\left(83137\hat{a}_x - 66509.6\hat{a}_y - 99764.4\hat{a}_z\right)$$

$$\vec{E} = \underline{92.3}\,\hat{a}_x - 77.55\hat{a}_y - 94.23\hat{a}_z \text{ kV/m}$$

b) $$\vec{E} = \frac{-0.3\times10^{-6}}{4\pi\varepsilon_0}\frac{\left(-10\hat{a}_x+50\hat{a}_y+35\hat{a}_z\right)\times10^{-2}}{\left[0.1^2+0.5^2+0.35^2\right]^{3/2}} + \frac{0.5\times10^{-8}}{4\pi\varepsilon_0}$$

$$\frac{\left(25\hat{a}_x+12\hat{a}_y+38\hat{a}_z\right)}{\left[0.25^2+0.12^2+0.38^2\right]^{3/2}}$$

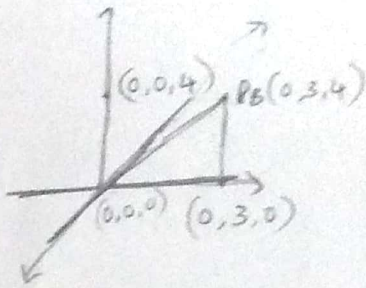
$$\vec{E} = \left(-1138.78\hat{a}_x - 5693.94\hat{a}_y - 3985.76\hat{a}_z +\right.$$
$$\left.10791.66\hat{a}_x + 5179.999\hat{a}_y + 16403.33\hat{a}_z\right)$$

$$\vec{E} = \underline{11.9\hat{a}_x} - 0.519\hat{a}_y + 12.4\hat{a}_z \text{ kV/m}$$

Infinite uniform line charges of 5nC/m lie along the x and y axes in free space find $\vec{E}$ at

a) $P_A(0,0,4)$ b) $P_B(0,3,4)$

a) $$\vec{E}_A = \frac{5\times10^{-9}}{2\pi\varepsilon_0(4)}\hat{a}_z + \frac{5\times10^{-9}}{2\pi\varepsilon_0(4)}\hat{a}_z$$



$$\vec{E}_A = 45\,\hat{a}_z \text{ V/m}$$

b) $$\vec{E}_B = \frac{5\times10^{-9}\,(4\hat{a}_z)}{2\pi\varepsilon_0\,(4)^2} + \frac{5\times10^{-9}\,(3\hat{a}_y+4\hat{a}_z)}{2\pi\varepsilon_0\,(\sqrt{9+16})^2}$$

$$\vec{E}_B = 10.78\,\hat{a}_y + 14.38\,a_z + 22.46\,\hat{a}_z$$

$$\vec{E}_B = 10.8\,\hat{a}_y + 36.9\,\hat{a}_z \text{ V/m}$$

Three infinite uniform sheets of charge are located in free space as follows: $3\,nC/m^2$ at $z=-4$, $6\,nC/m^2$ at $z=1$ and $-8\,nC/m^2$ at $z=4$. Find $\vec{E}$ at the point

a) $P_A(2,5,-5)$ b) $P_B(4,2,-3)$ c) $P_C(-1,-5,2)$ d) $P_D(-2,4,5)$

a) $P_A(2,5,-5)$

$$\vec{E}_A = \vec{E}_1 + \vec{E}_2 + \vec{E}_3$$

$$= \frac{3\times10^{-9}}{2\varepsilon_0}(-\hat{a}_z) + \frac{6\times10^{-9}}{2\varepsilon_0}(\hat{a}_z) + \frac{-8\times10^{-9}}{2\varepsilon_0}(\hat{a}_z)$$

$$\vec{E}_A = -56.46\,\hat{a}_z$$

b) $\vec{E}_B = \frac{3\times10^{-9}}{2\varepsilon_0}(\hat{a}_z) - \frac{6\times10^{-9}}{2\varepsilon_0}(\hat{a}_z) - \frac{(-8\times10^{-9})}{2\varepsilon_0}\hat{a}_z$

$\vec{E}_B = 283\,\hat{a}_z$

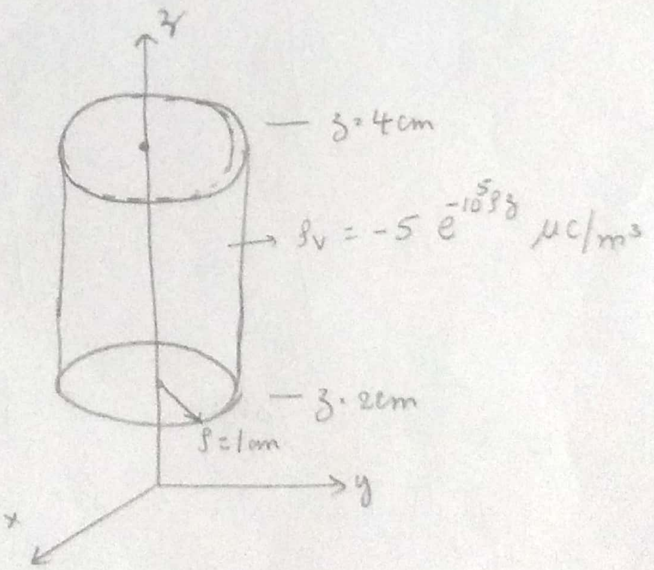
c) $\vec{E} = \frac{3\times10^{-9}}{2\varepsilon_0}\hat{a}_z + \frac{6\times10^{-9}}{2\varepsilon_0}\hat{a}_z - \frac{(-8\times10^{-9})}{2\varepsilon_0}\hat{a}_z$

$\vec{E} = 961\,\hat{a}_z$

d) $\vec{E}_D = \frac{3\times10^{-9}}{2\varepsilon_0}\hat{a}_z + \frac{(6\times10^{-9})}{2\varepsilon_0}\hat{a}_z + \frac{(-8\times10^{-9})}{2\varepsilon_0}\hat{a}_z$

$\vec{E} = 56.5\,\hat{a}_z$

Find the total charge contained in a electron beam shown in figure.



$$Q = \int_v \rho_v\,dv$$

$$Q = \int_{0.02}^{0.04}\int_{0}^{2\pi}\int_{0}^{0.01} -5\times10^{-6}\,e^{-10^5\rho z}\,\rho\,d\rho\,d\phi\,dz$$

$$= \int_{0.02}^{0.04}\int_{0}^{0.01} (-5\times10^{-6})\,e^{-10^5\rho z}(2\pi)\,\rho\,d\rho\,d\phi\,dz.$$

$$= \int_{z=0.02}^{0.04}\int_{0}^{0.01} (-10^{-5}\pi)\, e^{-10^5 \rho z}\, \rho\, d\rho\, dz$$

$$= \int_{\rho=0}^{0.01} \left. \frac{-10^{-5}\pi\, e^{-10^5 \rho z}\, \rho}{-10^5 \rho} \right]_{z=0.02}^{0.04}$$

$$= \int_{\rho=0}^{0.01} \left[-10^{-10}\pi\, e^{-10^5 \rho z}\, d\rho \right]_{z=0.02}^{0.04}$$

$$= -10^{-10}\pi \left[\int_{\rho=0}^{0.01} \left(e^{-2000\rho} - e^{-4000\rho} \right) d\rho \right]$$

$$= -10^{-10}\pi \left[\frac{e^{-2000\rho}}{-2000} - \frac{e^{-4000\rho}}{4000} \right]_0^{0.01}$$

$$Q = -10^{-10}\pi \left[\frac{1}{2000} - \frac{1}{4000} \right] = -\frac{\pi}{40} = \underline{\underline{0.0785\,pC}}$$

Calculate the total charge within each of indicated volumes.

a) $0.1 \le |x|, |y|, |z| \le 0.2$ & $\rho_v = 1/x^3y^3z^3$

b) $0 \le \rho \le 0.1$, $0 \le \phi \le \pi$, $2 \le z \le 4$ & $\rho_v = \rho^2 z^2 \sin(0.6\phi)$

c) Universe $\rho_v = e^{-2r}/r^2$

a) $$Q = \int_{x=0.1}^{0.2}\int_{y=0.1}^{0.2}\int_{z=0.1}^{0.2} \frac{1}{x^3y^3z^3}\, dx\, dy\, dz = \int_{x=0.1}^{0.2}\int_{y=0.1}^{0.2} \left. \frac{1}{x^3y^3}\, \frac{z^{-2}}{-2} \right]_{0.1}^{0.2} dx\, dy$$

$$= 37.5 \int_{x=0.1}^{0.2} \frac{1}{x^3} \frac{y^{-2}}{-2}\Big]_{0.1}^{0.2} = 1406.25 \frac{\cancel{x}^{-2}}{-2}\Big]_{0.1}^{0.2}$$

$$\underline{\underline{Q}} = 52.73 \times 10^{3}\ C$$

b) $$Q = \int_{\rho=0}^{0.1} \int_{\phi=0}^{\pi} \int_{z=2}^{4} \rho^2 z^2 \sin(0.6\phi)\ \rho\, d\rho\, d\phi\, dz$$

$$= \int_{\rho=0}^{0.1} \int_{\phi=0}^{\pi} \rho^3 \sin(0.6\phi) \frac{z^3}{3}\Big]_2^4 d\rho\, d\phi$$

$$= 18.66 \int_{\phi=0}^{\pi} \sin(0.6\phi) \frac{\rho^4}{4}\Big]_0^{0.1} d\phi$$

$$= 4.665 \times 10^{-4} \quad -\frac{\cos(0.6\phi)}{0.6}\Big]_0^{\pi}$$

$$\underline{\underline{Q}} = 1.077\ mC$$

c) $$Q = \int_{r=0}^{\infty} \int_{\theta=0}^{\pi} \int_{\phi=0}^{2\pi} \frac{e^{-2r}}{\cancel{r^2}} \cancel{r^2} \sin\theta\, dr\, d\theta\, d\phi$$

$$= 2\pi \int_{r=0}^{\infty} \int_{\theta=0}^{2\pi} e^{-2r} \sin\theta\, dr\, d\theta$$

$$= 4\pi \int_{r=0}^{\infty} e^{-2r}\, dr = 2\pi(2) \frac{e^{-2r}}{-2}\Big]_{r=0}^{\infty}$$

$$Q = -2\pi[0-1] = 2\pi = 6.28C$$

Given a $60\mu C$ point charge located at the origin, find the total electric flux passing through a) That portion of sphere $r = 26cm$ bouded by $0 < \theta < \pi/2$ & $0 < \phi < \pi/2$

b) The closed surface defined by $\rho = 26cm$ and $z = \pm 26cm$

c) The plane $z = 26cm$

a)
$$A' = \int_0^{\pi/2} \int_0^{\pi/2} r^2 \sin\theta \, d\theta \, d\phi = r^2 (\pi/2)(1) = (26)^2 \frac{\pi}{2}$$

Portion of area $= A' = 1061.858 \, cm^2$

Total area $= A = 4\pi r^2 = 8494.86 \, cm^2$.

The flux passing through portion of area A is

$$\psi' = \frac{Q}{A} \times A' = \frac{60 \times 10^{-6}}{8494.86} \times 1061.858 = \underline{\underline{7.5 \mu C}}$$

b) The closed surface defined by $\rho = 26cm$ & $z = \pm 26cm$ is closed cylinder hence total flux is equal to total charges enclosed $= 60 \mu C$

c) Approximately half of the flux pass through plane $= 26cm$ hence Flux crossing this plane is $\underline{\underline{30 \mu C}}$

Calculate $\vec{D}$ in rectangular coordinates at point $P(2,-3,6)$ produced by ⓐ a point charge $Q_A = 55 \, mC$ at $Q(-2,3,-6)$

ⓑ a uniform line charge $\rho_{LB} = 20 mC/m$ on the x-axis.

ⓒ a uniform surface charge density $\rho_{SC} = 120 \mu C/m^2$ on the plane $z = -5m$.

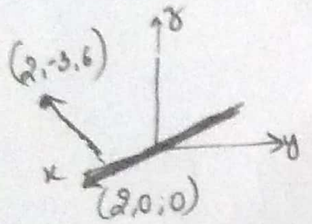
a) $\vec{D} = \varepsilon_0 \vec{E}$

$$\vec{D} = \frac{Q}{4\pi R^2}\hat{a}_n$$

$$= \frac{55\times10^{-3}\left[4\hat{a}_x - 6\hat{a}_y + 12\hat{a}_z\right]}{4\pi\left[\sqrt{4^2+6^2+12^2}\right]^{3/2}}$$

$$\vec{D} = 6.38\,\hat{a}_x - 9.57\,\hat{a}_y + 19.14\,\hat{a}_z \ \mu C/m^2$$

b) $\rho_{LB} = 20\,mC/m$

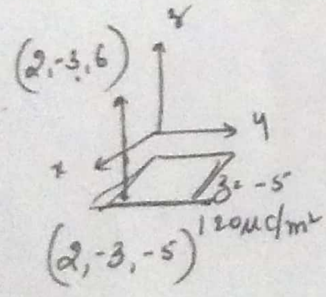


$$\vec{D} = \frac{\rho_L}{2\pi\rho}\hat{a}_\rho$$

$$= \frac{20\times10^{-3}\left[-3\hat{a}_y + 6\hat{a}_z\right]}{2\pi\left[\sqrt{+3^2+6^2}\right]^2}$$

$$\vec{D} = -212.2\,\hat{a}_y + 424\,\hat{a}_z \ \mu C/m^2$$

c)



$$\vec{D} = \frac{\rho_s}{2}\hat{a}_z$$

$$= \frac{120\times10^{-6}(11\hat{a}_z)}{2\,(11)}$$

$$\vec{D} = 60\,\hat{a}_z \ \mu c/m^2$$

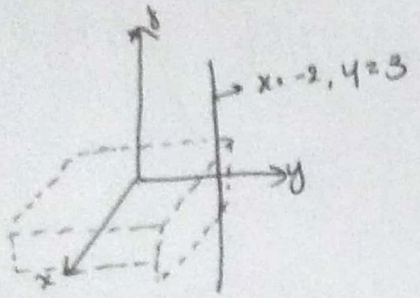
Calculate the total electric flux leaving the cubical surface formed by the six planes $x, y, z = \pm 5$ if the charge distribution is a) Two point charges $0.1\,\mu c$ at $(1,-2,3)$ and $\frac{1}{7}\,\mu c$ at $(-1,2,-2)$ b) A Uniform line charge of $\pi\,\mu c/m$ at $x = -2,\ y = 3$ c) A uniform surface charge of $0.1\,\mu c/m^3$ on the plane $y = 3x$.

a) Two point charges closed inside the cube of $x, y, z = \pm 5$

Total flux $= \psi = Q = \left(0.1 + \frac{1}{7}\right) 10^{-6} C$

$\psi = 0.242 \mu C$

b)

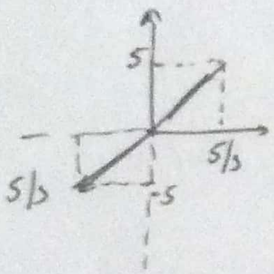


The total charge inside the cube of ± 5

$Q = \psi = 10 \times \pi \ \mu C$

$\psi = 31.4 \mu C$

c)



$y = 5 \Rightarrow x = 5/3$ Length of sheet $= 2\sqrt{5^2 + (5/3)^2} = 10.5409$ m

$y = -5 \Rightarrow x = -5/3$

Area of plane surface inside cube

$A = 10 \times 10.5409 \text{ m} = 105.401 \text{ m}^2$.

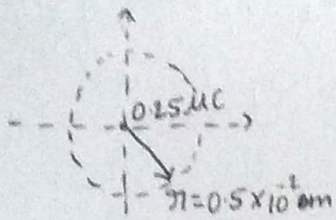
$Q = \rho_s A = 0.1 \times 10^{-6} \times 105.401 \text{ m}^2$

$Q = 10.54 \mu C$

A point charge of $0.25 \mu C$ is located at $r = 0$, and uniform surface charge densities are located as follows. 2 mC/m^2 at $r = 1 \text{ cm}$, and -0.6 mC/m^2 at $r = 1.8 \text{ cm}$ Calculate $\vec{D}$ at

a) $r = 0.5 \text{ cm}$, b) $r = 1.5 \text{ cm}$ c) $r = 2.5 \text{ cm}$ d) what uniform surface charge densities should establish at $r = 3 \text{ cm}$ to cause $\vec{D} = 0$ at $r = 3.5 \text{ cm}$?

a)

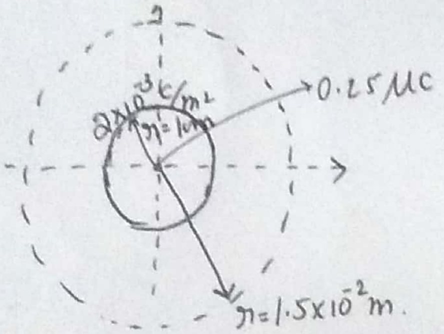


$$\vec{D} = \frac{\text{charge enclosed}}{\text{Total surface area}}$$

$$\vec{D} = \frac{0.25\times10^{-6}}{4\pi\left(0.5\times10^{-2}\right)^2}$$

$$\underline{\underline{\vec{D} = 796\ \hat{a}_r\ \mu C/m^2}}$$

b)

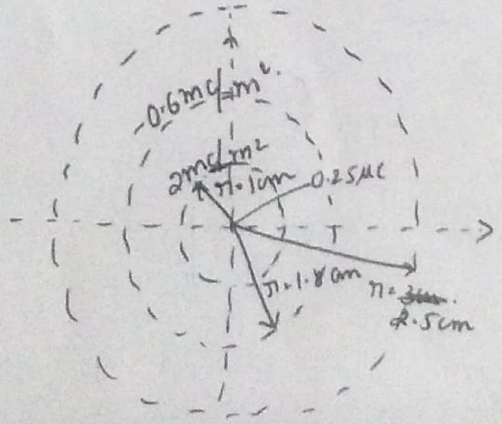


$$\vec{D} = \frac{Q_{point} + Q_{spherical\ surface}}{4\pi r^2}$$

$$= \frac{\left(0.25\times10^{-6}\right) + 2\times10^{-3}\times 4\pi\left(1\times10^{-2}\right)^2}{4\pi\left(1.5\times10^{-2}\right)^2}$$

$$\underline{\underline{\vec{D} = 977.30\ \hat{a}_r\ \mu C/m^2}}$$

c)



$$\vec{D} = \frac{Q_{point} + Q_{sphere\ of\ r=1.8cm} + Q_{sphere\ of\ r=1cm}}{4\pi r^2}$$

$$= \frac{\left(0.25\times10^{-6}\right) + \left(2\times10^{-3}\right)\left[4\pi\left(1\times10^{-2}\right)^2\right]\left[4\pi\left(1.8\times10^{-2}\right)^2 - 0.6\times10^{-3}\right]}{4\pi\left(2.5\times10^{-2}\right)^2}$$

$$\underline{\underline{\vec{D} = 3.2037\times10^{-7}\ C/m^2}}$$

d) $\vec{D} = \left.\frac{Q}{4\pi r^2}\right|_{r=3.5cm} = 0 \Rightarrow Q = 0$

$Q_{at\ 3cm} = -Q_{at\ cm} = -0.32037\ \mu C$

$$\rho_s = \frac{-Q_{at\ 3cm}}{4\pi\left(3\times10^{-2}\right)^2} = -28.32\ \mu C/m^2$$

In free space let $\vec{D} = 8xyz^4\,\hat{a}_x + 4x^2z^4\,\hat{a}_y + 16x^2yz^3\,\hat{a}_z$ pC/m²

a) Find the total electric flux passing through the rectangular surface $z=2, 0<x<2, 1<y<3$ in $\hat{a}_z$ direction

b) Find $\vec{E}$ at $P(2,-1,3)$

c) Find an approximate value for the total charge contained in an incremental sphere located at $P(2,-1,3)$ and having a volume of $10^{-12}\,m^3$.

a) $z=2, \; 0<x<2, \; 1<y<3$

$$\psi = \int \vec{D}\cdot d\vec{s} = \int_{x=0}^{2}\int_{y=1}^{3} 16\,x^2yz^3\times10^{-12}\,dx\,dy = 16\times10^{-12}(2)^3\int_{x=0}^{2}\int_{y=1}^{3} x^2y\,dx\,dy$$

$$= 128\times10^{-12}\left.\frac{x^3}{3}\right]_0^2 \left.\frac{y^2}{2}\right]_1^3 = \frac{128\times10^{-12}}{6}(8-0)(9-1)$$

$$\underline{\underline{\psi = 1365.33\ pC}}$$

b) $\vec{E}$ at $P(2,-1,3)$

$$\vec{E} = \frac{\vec{D}}{\varepsilon_0} = \left[\frac{8xyz^4\,\hat{a}_x + 4x^2z^4\hat{a}_y + 16x^2yz^3\,\hat{a}_z}{\varepsilon_0}\right]\times10^{-12}$$

at $P(2,-1,3)$

$$\vec{E} = \left[-146.37\,\hat{a}_x + 146.37\,\hat{a}_y - 195.16\,\hat{a}_z\right]\ V/m$$

c) Total charge enclosed in volume is

$$Q_{\Delta v} = \left[\frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z}\right]\Delta V$$

$$= \left[\frac{\partial(8xyz^4)}{\partial x} + \frac{\partial(4x^2z^4)}{\partial y} + \frac{\partial(16x^2yz^3)}{\partial z}\right]10^{-12}\,m^3$$

$$= \left[(-64\times10^{-12}) + (0) + (-1728)\times10^{-12}\right]10^{-12} = \underline{\underline{-2.376\times10^{-21}\ C}}$$

In each of the following parts, find a numerical value for div $\vec{D}$ at the point specified

a) $\vec{D} = (2xyz - y^2)\,\hat{a}_x + (x^2 z - 2xy)\,\hat{a}_y + x^2 y\,\hat{a}_z$ C/m² at $P_A(2,3,-1)$

b) $\vec{D} = 2\rho z^2 \sin^2\phi\,\hat{a}_\rho + \rho z^2 \sin 2\phi\,\hat{a}_\phi + 2\rho^2 z \sin^2\phi\,\hat{a}_z$ C/m² at $P_B(\rho = 2,\ \phi = 110^\circ,\ z = -1)$

c) $\vec{D} = 2r\sin\theta\cos\phi\,\hat{a}_r + r\cos\theta\cos\phi\,\hat{a}_\theta - r\sin\phi\,\hat{a}_\phi$ C/m². at $P_C(r = 1.5,\ \theta = 30^\circ,\ \phi = 50^\circ)$

a) $$\text{div}\ \vec{D} = \left[\frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z}\right] = \left[2yz + (-2x) + 0\right]$$

at $P_A(2,3,-1)$ $\quad$ div $\vec{D} = -6 - 4 = \underline{\underline{-10}}$

b) $$\text{div}\ \vec{D} = \frac{1}{\rho}\frac{\partial}{\partial \rho}(\rho D_\rho) + \frac{1}{\rho}\frac{\partial D_\phi}{\partial \phi} + \frac{\partial D_z}{\partial z}$$

$$= \frac{1}{2}\frac{\partial}{\partial \rho}\left(2\rho^2 z^2 \sin^2\phi\right) + \frac{1}{2}\frac{\partial}{\partial \phi}\left(\rho z^2 \sin 2\phi\right) + \frac{\partial}{\partial z}\left(2\rho^2 z \sin^2\phi\right)$$

$$\text{div}\ \vec{D} = \frac{1}{2}\,\frac{2\rho^3 z^2}{3}\sin^2\phi + \frac{1}{2}\,\frac{\rho z^2 \cos(2\phi)}{2} + 2\rho^2 \sin^2\phi$$

div $\vec{D}$ at $P_B = \underline{\underline{9.0358}}$

c) $$\text{div}\ \vec{D} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 D_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(\sin\theta\, D_\theta) + \frac{1}{r\sin\theta}\frac{\partial D_\phi}{\partial \phi}$$

$$\vec{D} = 2r\sin\theta\cos\phi\,\hat{a}_r + r\cos\theta\cos\phi\,\hat{a}_\theta - r\sin\phi\,\hat{a}_\phi \ \text{C/m}^2$$

$$\text{div}\ \vec{D} = \frac{1}{r^2}\left[6r^2 \sin\theta\cos\phi\right] + \frac{1}{r\sin\theta}\left[r\cos 2\theta\cos\phi\right] + \frac{1}{r\sin\theta}\left[-r\cos\phi\right]$$

at $(P_C = r = 1.5,\ \theta = 30^\circ,\ \phi = 50^\circ)$

div $\vec{D} = 1.92836 + 0.64278 - 1.2855$

div $\vec{D} = 1.285$ C/m³

Determine an expression for the volume charge density associated with each $\vec{D}$ field following

a) $\vec{D} = \frac{4xy}{z}\hat{a}_x + \frac{2x^2}{z}\hat{a}_y - \frac{2x^2y}{z^2}\hat{a}_z$

b) $\vec{D} = z\sin\phi\,\hat{a}_\rho + z\cos\phi\,\hat{a}_\phi + \rho\sin\phi\,\hat{a}_z$

c) $\vec{D} = \sin\theta\sin\phi\,\hat{a}_r + \cos\theta\sin\phi\,\hat{a}_\theta + \cos\phi\,\hat{a}_\phi$

a) $\operatorname{div}\vec{D} = \rho_v = \frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z}$

$= \frac{4y}{z} - 2x^2y\left[-2z^{-3}\right] = \left[\frac{4y}{z}\right] + \left[\frac{4x^2y}{z^3}\right] = \frac{4yz^2+4x^2y}{z^3}$

$\rho_v = \frac{4y}{z^3}\left[x^2+z^2\right]$

b) $\rho_v = \operatorname{div}\vec{D} = \frac{1}{\rho}\frac{\partial(\rho D_\rho)}{\partial\rho} + \frac{1}{\rho}\frac{\partial D_\phi}{\partial\phi} + \frac{\partial D_z}{\partial z}$

$\rho_v = \frac{1}{\rho}\frac{\partial(\rho z\sin\phi)}{\partial\rho} + \frac{1}{\rho}\frac{\partial(z\cos\phi)}{\partial\phi} + \frac{\partial(\rho\sin\phi)}{\partial z}$

$\rho_v = \frac{1}{\rho}z\sin\phi - \frac{1}{\rho}z\sin\phi$

$\underline{\underline{\rho_v = 0}}$

c) $\operatorname{div}\vec{D} = \frac{1}{r^2}\frac{\partial(r^2D_r)}{\partial r} + \frac{1}{r\sin\theta}\frac{\partial(\sin\theta\, D_\theta)}{\partial\theta} + \frac{1}{r\sin\theta}\frac{\partial D_\phi}{\partial\phi}$

$= \frac{1}{r^2}(2r\sin\theta\sin\phi) + \frac{1}{r\sin\theta}(\cos 2\theta\sin\phi) - \frac{\sin\phi}{r\sin\theta}$

$\operatorname{div}\vec{D} = \frac{2\sin\theta\sin\phi}{r} + \frac{\cos 2\theta\sin\phi}{r\sin\theta} - \frac{\sin\phi}{r\sin\theta}$

$= \frac{2\sin\theta\sin\phi}{r} - \frac{2\sin\theta\sin\phi}{r}$

$\rho_v = \operatorname{div}\vec{D} = 0$

Given the field $\vec{D} = 6\rho \sin(\frac{1}{2}\phi)\,\hat{a}_\rho + 1.5\rho \cos(\frac{1}{2}\phi)\,\hat{a}_\phi$ C/m², evaluate both sides of the divergence for region bounded by $\rho = 2$, $\phi = 0$ & $\phi = \pi$, $z = 0$ & $z = 5$

Divergence theorem can be written as $\oint \vec{D}\cdot d\vec{S} = \int_{vol} \nabla\cdot\vec{D}\, dv$

Consider
$$\oint \vec{D}\cdot d\vec{S} = \int_{\phi=0}^{\pi}\int_{z=0}^{5} (\vec{D})_{\rho=2}\cdot(\rho\, d\phi\, dz\, \hat{a}_\rho) + \int_{\rho=0}^{2}\int_{z=0}^{5} (\vec{D})_{\phi=0}\cdot(-d\rho\, dz\, \hat{a}_\phi) +$$
$$\int_{\rho=0}^{2}\int_{z=0}^{5} (\vec{D})_{\phi=\pi}\cdot(d\rho\, dz\, \hat{a}_\phi) + \int_{\phi=0}^{\pi}\int_{\rho=0}^{2} (\vec{D})_{z=0}\cdot(-\rho\, d\phi\, d\rho\, \hat{a}_z) + \int_{\phi=0}^{\pi}\int_{\rho=0}^{2} (\vec{D})_{z=2}\cdot(\rho\, d\phi\, d\rho\, \hat{a}_z)$$

$$= \int_{\phi=0}^{\pi}\int_{z=0}^{5} 6\rho^2 \sin(\phi/2)\, d\phi\, dz + \int_{\rho=0}^{2}\int_{z=0}^{5} -1.5\rho \cos(\frac{1}{2}\phi)\, d\rho\, dz +$$
$$\int_{\rho=0}^{2}\int_{z=0}^{5} 1.5\rho \cos(\frac{1}{2}\phi)\, d\rho\, dz + 0 + 0$$

$$\oint \vec{D}\cdot d\vec{S} = 240 - 15 = \underline{\underline{225\ C}}$$

$$\nabla\cdot\vec{D} = \frac{1}{\rho}\frac{\partial(\rho D_\rho)}{\partial\rho} + \frac{1}{\rho}\frac{\partial(D_\phi)}{\partial\phi} + \frac{\partial D_z}{\partial z}$$
$$= \frac{1}{\rho}\frac{\partial}{\partial\rho}\left[6\rho^2 \sin(\phi/2)\right] + \frac{1}{\rho}\frac{\partial}{\partial\phi}\left[1.5\rho \cos(\phi/2)\right] + 0$$
$$= \frac{12\cancel{\rho} \sin(\phi/2)}{\cancel{\rho}} + \frac{1.5\cancel{\rho}(-\sin\phi/2)}{2\cancel{\rho}}$$

$$\nabla\cdot\vec{D} = 12\sin(\phi/2) - \frac{3}{4}\sin(\phi/2) = 11.25\sin(\phi/2)\ \text{C/m}^3$$

$$\int_{vol} \nabla\cdot\vec{D} = \int_{z=0}^{5}\int_{\phi=0}^{\pi}\int_{\rho=0}^{2} 11.25 \sin(\phi/2)\, \rho\, d\rho\, d\phi\, dz = \underline{\underline{225\ C}}$$

Vefify the divergence theorem for the field $\vec{D} = 2xy\,\hat{a}_x + x^2\,\hat{a}_y$ C/m^2 in the region bounded by plane $x = 0$ & 1; $y = 0$ & 2; $z = 0$ & 3.

From divergence theorem $\int \vec{D}\cdot d\vec{s} = \int_{Vol} \nabla\cdot\vec{D}\, dv$

$$\oint \vec{D}\cdot d\vec{s} = \int_{y=0}^{2}\int_{z=0}^{3} (\vec{D})_{x=0}\cdot(-dy\,dz\,\hat{a}_x) + \int_{y=0}^{2}\int_{z=0}^{3}(\vec{D})_{x=1}\cdot(dy\,dz\,\hat{a}_x) + \int_{x=0}^{1}\int_{z=0}^{3}(\vec{D})_{y=0}\cdot(-dz\,dx\,\hat{a}_y) +$$

$$\int_{x=0}^{1}\int_{z=0}^{3}(\vec{D})_{y=2}\cdot(dz\,dx\,\hat{a}_y) + \int_{x=0}^{1}\int_{y=0}^{2}(\vec{D})_{z=0}\cdot(-dx\,dy\,\hat{a}_z) + \int_{x=0}^{1}\int_{y=0}^{2}(\vec{D})_{z=3}\cdot(dx\,dy\,\hat{a}_z)$$

$$= -\int_{y=0}^{2}\int_{z=0}^{3}(2xy\,dy\,dz)_{x=0} + \int_{y=0}^{2}\int_{z=0}^{3}(2xy\,dy\,dz)_{x=1} + \int_{x=0}^{1}\int_{z=0}^{3}(-x^2\,dx\,dz)_{y=0} +$$

$$\int_{x=0}^{1}\int_{z=0}^{3}(x^2\,dx\,dz)_{y=2} + 0 + 0$$

$$= \left\{ 2\left.\frac{y^2}{2}\right]_0^2 \left. z\right]_0^3 \right\} - \left\{ \left. z\right]_0^3 \left.\frac{x^3}{3}\right]_0^1 \right\} + \left\{ \left. z\right]_0^3 \left.\frac{x^3}{3}\right]_0^1 \right\}$$

$$= (4)(3) - (3)(1/3) + (3)(1/3) = \underline{\underline{12\,C}}$$

$$\nabla\cdot\vec{D} = \frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z} = \frac{\partial(2xy)}{\partial x} + \frac{\partial(x^2)}{\partial y} + \frac{\partial(0)}{\partial z} = 2y \text{ C/m}^3$$

$$\int_{Vol} \nabla\cdot\vec{D}\, dv = \int_{x=0}^{1}\int_{y=0}^{2}\int_{z=0}^{3} 2y\,dx\,dy\,dz = 6\left.\frac{y^2}{2}\right]_0^2 = 3[4] = \underline{\underline{12\,C}}$$

Hence $\left\langle \oint \vec{D}\cdot d\vec{s} = \int_{Vol} \nabla\cdot\vec{D}\, dv \right\rangle$