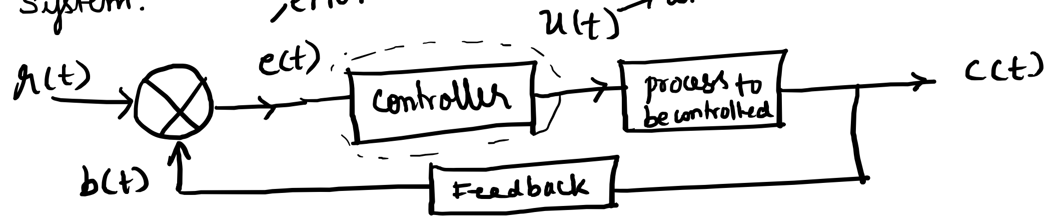
UNIT-2:

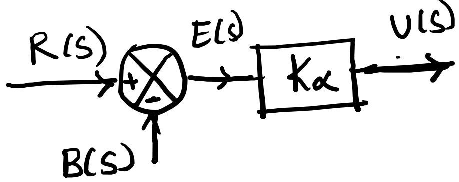
Effect of PI, PD and PID controllers on the time response of the system.



- → Two position on-off switch
- → proportional controllers
- → Integral controller
- → P-I controller
- → P-D controller
- → PID controller

Derivative controller mode is not used alone since it operates on rate of change of e(t) & not on the e(t) itself

1) proportional controller: controller o/p is proportional to error signal



$$u(t) \propto e(t)$$

$$u(t) = K_\alpha e(t)$$

$$\frac{U(s)}{E(s)} = K_\alpha$$

Controller error signal $e_{ss} = \frac{K_\alpha}{1+K_p}$ → position control error constant

$$e_{ss} = \frac{K_\alpha}{K_v}.$$

Good response → Settling time t_s → small
Rise time t_r → small
peak overshoot M_p → small
Steady state error (e_{ss}) → small

If $K_v \uparrow$, e_{ss} reduces but this requires increase in system gain → instability

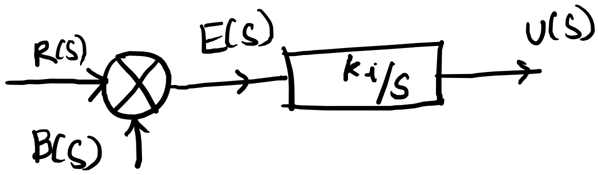
→ Main drawback of proportional controller is it produces steady state error.

2) Integral controller

controller o/p is proportional to integral of error signal

$$u(t) = K_i \int e(t)\,dt$$

$$\frac{U(S)}{E(S)} = \frac{K_i}{s}$$



when actuating error is zero, o/p remains stationary (reset control).

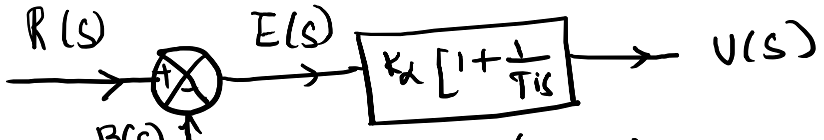
3) P-I controller:

Proportional + Integral

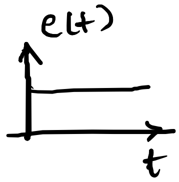
$$u(t) \propto e(t) + \frac{1}{T_i}\int e(t)\,dt$$

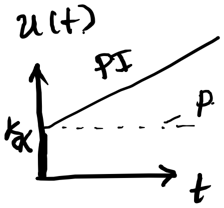
$$u(t) = K_\alpha e(t) + \frac{K_\alpha}{T_i}\int e(t)$$

$$\frac{U(S)}{E(S)} = \left[K_\alpha + \frac{K_\alpha}{T_i S}\right] = K_\alpha\left[1 + \frac{1}{T_i S}\right]$$



$$\hookrightarrow K_\alpha \frac{(T_i s + 1)}{T_i s}$$





→ Both K_α and T_i are adjustable.

→ Increases loop gain & reduces steady state error

Controller T.F, $G_c(S) = K_\alpha\left(\frac{T_i s + 1}{T_i s}\right)$

$$G_p(S) = \frac{\omega_n^2}{S^2 + 2\xi\omega_n s + \omega_n^2}$$

$$G(S) = G_c(S)\,G_p(S)$$

$$C.L.T.F = \frac{G(S)}{1+G(S)H(S)}$$ let $H(S) = 1$.

$$G(S) = \frac{(T_i S + 1)}{T_i S} \cdot \frac{\omega_n^2}{S^2 + 2\xi\omega_n S + \omega_n^2}$$

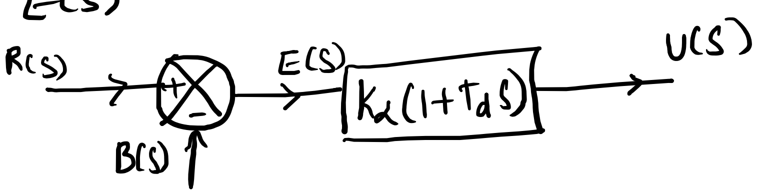
$$C.L.T.F = \frac{G(S)}{1+G(S)H(S)} = \frac{(K_d/T_i)\,\omega_n^2 + \omega_n^2 K_d S}{S^3 + 2\xi\omega_n S^2 + \omega_n^2 K_d S + \omega_n^2 \frac{K_d}{T_i}}$$

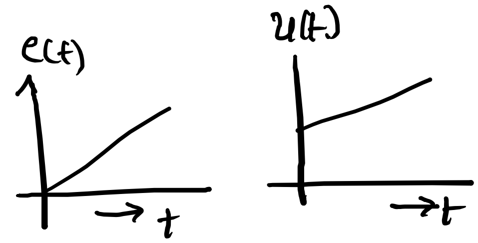
→ Introduced a zero & increased the order — increases loop gain, reduces e_{ss} & hence system is relatively less stable.

4) P-D controller. (proportional + Derivative)

$$u(t) = K_d\, e(t) + K_d T_d \frac{d\,e(t)}{dt}.$$

$$\frac{U(S)}{E(S)} = K_d (1 + T_d S)$$





K_d and T_d are adjustable.

$$G_C = K_d (1 + T_d S)$$

$$G_P = \frac{\omega_n^2}{S^2 + 2\xi\omega_n S + \omega_n^2}$$ → T.F of 2^{nd} order system.

$$G(S) = K_d (1 + T_d S)\, \frac{\omega_n^2}{S^2 + 2\xi\omega_n S + \omega_n^2}$$

$$C.L.T.F = \frac{G(S)}{1+G(S)H(S)}$$; $H(S) = 1$.

$$\frac{C(S)}{R(S)} = \frac{K_d \omega_n^2 + K_d T_d S \omega_n^2}{S^2 + 2\xi\omega_n S + K_d T_d S + K_d \omega_n^2}$$

→ Transient response improves ($\because \xi \uparrow$)
→ e_{ss} unchanged.

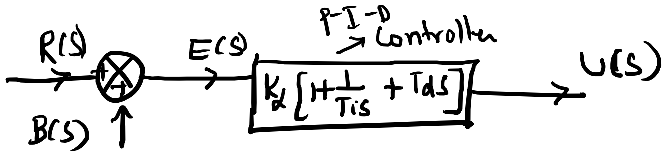
5) P-I-D controller: o/p of controller is given by

$$u(t) \propto \left\{ e(t) + \frac{1}{T_i}\int e(t)\,dt + T_d \frac{d\,e(t)}{dt} \right\}$$

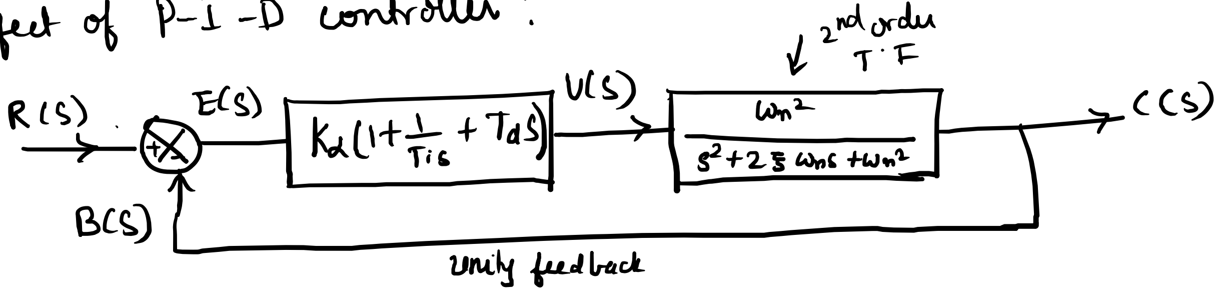
$$u(t) = K_\alpha \left[e(t) + \frac{1}{T_i}\int e(t)\,dt + T_d \frac{d\,e(t)}{dt} \right]$$

Taking L.T

$$\frac{U(s)}{E(s)} = K_\alpha \left[1 + \frac{1}{T_i s} + T_d s \right]$$



Effect of P-I-D controller:



$$C.L.T.F = \frac{G(s)}{1+G(s)H(s)}$$ where $H(s) = 1$.

$$G(s) = K_\alpha\left[1+\frac{1}{T_i s}+T_d s\right]\left[\frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}\right]$$

$$C.L.T.F = \frac{C(s)}{R(s)} = \frac{K_\alpha \omega_n^2 \left(1+\frac{1}{T_i s} + T_d s\right)}{s^2 + 2\xi\omega_n s + K_\alpha \omega_n^2\left(1+\frac{1}{T_i s}+T_d s\right)}$$

→ Two zeros → improves stability
Type of system increases (pole at origin) → reduces steady state error.