

DAA LABORATORY 4

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SY Btech Comp eng.

TASK 1:

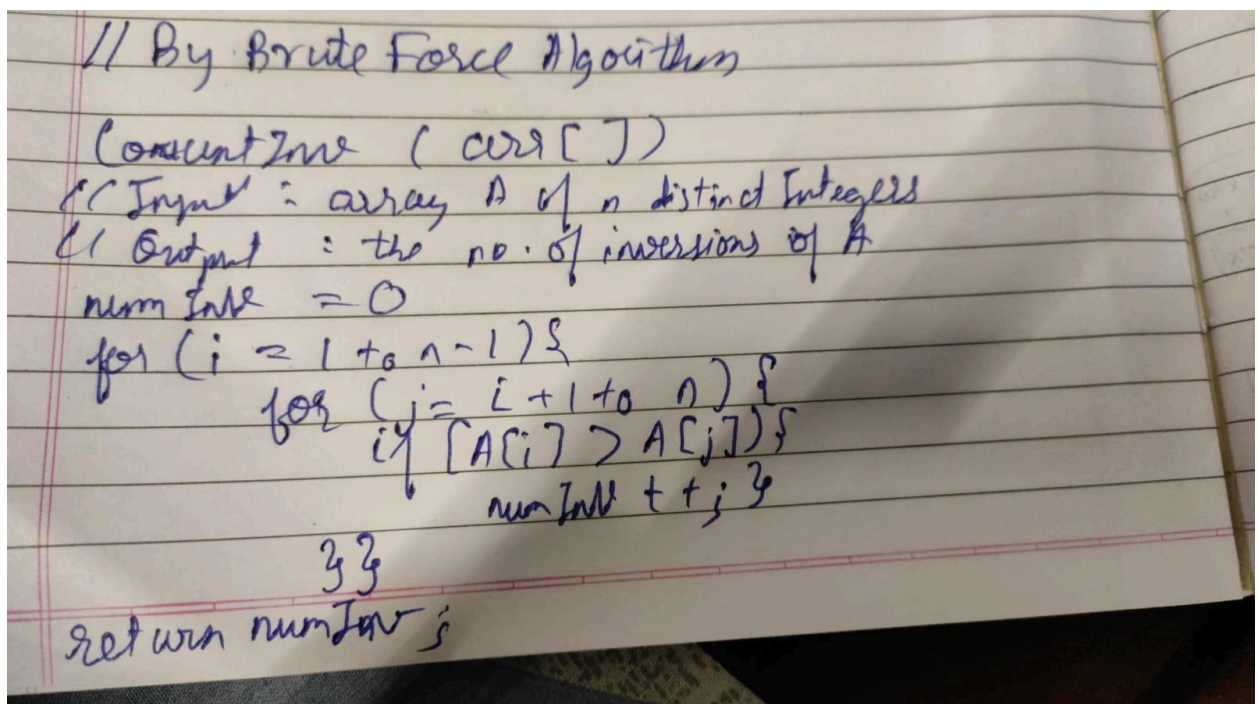
Aim:

Consider first/second year course-code choices of 100 students.

Find the inversion count of these choices.

Find students with zero, one, two, three inversion counts and comment on your result.

ALGO by Brute Force:



```
// By Brute Force Algorithm
CountInve ( arr[] )
/* Input : array A of n distinct Integers
   Output : the no. of inversions of A
   numInve = 0
   for (i = 1 to n-1) {
       for (j = i+1 to n) {
           if (A[i] > A[j]) {
               numInve ++;
           }
       }
   }
   return numInve;
```

ALGO by Divide And Conquer:

1) To count the no. of inversions in a course card
Count-Inversion (vector) (7) divided conquer technique

// Input : array A of n distinct integers
// Output : the number of inversions of A

if $(n = 0 \text{ or } n = 1)$ {
 return 0; }

else {

 left Inv = Count Inversion (first half of A)

 right Inv = Count Inversion (second half of A)

 split Inv = Count ~~split~~ ^{inversion} Inv (A)

 return left Inv + right Inv + split Inv

To break down all the in half

Sort and Count

if $n = 0 \text{ or } n = 1$ {
 return (A, 0) }

else {

 (C, left Inv) = sort and Count (first half of A)

 (D, right Inv) = sort and Count (second half of A)

 (B, split Inv) = sort and Count (C, D)

 return (B, left Inv + right Inv + split Inv);

}

(11)

Merge and Count Split Inverse

// Input sorted array (and D)

// Output: sorted array B and the no. of split Inverse

// Assumption: n is even.

i = 1, j = 1
 splitInv = 0;

~~do {~~
 for (k = 1 to n) {

~~do {~~
 if (C[i] < D[j]) {
 B[k] = C[i];
 i++;

 } else {
 B[k] = D[j];
 j++;

 splitInv = splitInv + $\left(\frac{n}{2} - i + 1\right)$;
 }
 }

return (B, splitInv).

TIME COMPLEXITY:

Time Complexity :

// Brute Force Algo.

We are using a nested for-loop in which first we iterate over all element of the array and in the next for loop we compare them with all other elements in the array to compare if there is any inversion.

and so as we know for 2 (nested for loop) the time complexity would be $O(n^2)$.

// Divide & Conquer Technique.

Divide: The divide step; just finds the middle of the subarray, which takes constant time.

$\therefore O(1)$.

Conquer: We recursively solve two subproblems, each of size $n/2$, which contribute $2T(n/2)$.

Combine: We have already noted so Merge on a n -element subarray takes $O(n)$ time.

\therefore Total Time

$$T(n) = \begin{cases} O(1) & n = 1 \\ 2T(n/2) + O(n) & \text{if } n > 1 \end{cases}$$

By master's Thm.

$$T(n) = 2T(n/2) + O(n)$$

$$f(n) = O(n) \quad a=2, b=2, d=1$$

$$a=b$$

$$\therefore T(n) = O(n^d \log n) \\ = O(n \log n)$$

POSITIVE TESTCASES:

1) INput:

Output

Clear

Total inversion count (Brute Force) across all students: 244
Total inversion count (Divide and Conquer) across all students: 244

Categorized Inversion Counts (Brute Force):
Inversion Count 0: Students [12, 18, 27, 29, 38, 41, 45, 49, 55, 61, 90, 94, 97, 98, 99]
Inversion Count 1: Students [2, 5, 13, 21, 22, 33, 37, 46, 58, 78, 85, 87, 88, 95]
Inversion Count 2: Students [1, 4, 15, 19, 23, 25, 35, 40, 50, 54, 56, 60, 66, 69, 73, 77, 79, 81, 83, 84, 86, 92, 100]
Inversion Count 3: Students [6, 8, 11, 14, 34, 36, 39, 43, 44, 48, 57, 62, 63, 64, 65, 70, 71, 72, 76, 80, 93]
Inversion Count 4: Students [7, 9, 10, 17, 24, 30, 32, 47, 51, 59, 67, 74, 75, 89, 91, 96]
Inversion Count 5: Students [16, 20, 26, 28, 31, 52, 53, 68, 82]
Inversion Count 6: Students [3, 42]

Categorized Inversion Counts (Divide and Conquer):
Inversion Count 0: Students [12, 18, 27, 29, 38, 41, 45, 49, 55, 61, 90, 94, 97, 98, 99]
Inversion Count 1: Students [2, 5, 13, 21, 22, 33, 37, 46, 58, 78, 85, 87, 88, 95]
Inversion Count 2: Students [1, 4, 15, 19, 23, 25, 35, 40, 50, 54, 56, 60, 66, 69, 73, 77, 79, 81, 83, 84, 86, 92, 100]
Inversion Count 3: Students [6, 8, 11, 14, 34, 36, 39, 43, 44, 48, 57, 62, 63, 64, 65, 70, 71, 72, 76, 80, 93]
Inversion Count 4: Students [7, 9, 10, 17, 24, 30, 32, 47, 51, 59, 67, 74, 75, 89, 91, 96]
Inversion Count 5: Students [16, 20, 26, 28, 31, 52, 53, 68, 82]
Inversion Count 6: Students [3, 42]

=== Code Execution Successful ===

Activate Windows
Go to Settings to activate Windows.

2)

NEGATIVE TESTCASES:

If no input is given(empty array)

INPUT:[]

```
Output Clear
Categorized Inversion Counts (Valid Entries):
Negative Integer Entries:
Student 1: Inversion count can be found since course code cant be negative.
=== Code Execution Successful ===
```

IF negative no is inputted

INPUT:[5, 2, 3, 6], [-3, -1, -5, -2], [-7, -6, -4, -1], [-6, -2, -5, -7], [2, 3, 8, 4], [5, 5, 5, 4]

```
Output Clear
Categorized Inversion Counts (Valid Entries):
Student 1: Brute Force Inversion Count = 2, Divide and Conquer Inversion Count = 2
Student 5: Brute Force Inversion Count = 1, Divide and Conquer Inversion Count = 1
Student 6: Brute Force Inversion Count = 3, Divide and Conquer Inversion Count = 3
Negative Integer Entries:
Student 2: Inversion count can be found since course code cant be negative.
Student 3: Inversion count can be found since course code cant be negative.
Student 4: Inversion count can be found since course code cant be negative.
=== Code Execution Successful ===
```

If letters are inputted in array

INPUT: [5, 2, 3, 6], ['a', 1, 5, 2], [7, 6, 4, 1], [6, 2, 'b', 7], [2, 3, 8, 4], [5, 5, 5, 4]

```
Output
Clear

Categorized Inversion Counts (Valid Entries):
Student 1: Brute Force Inversion Count = 2, Divide and Conquer Inversion Count = 2
ERROR!
Student 3: Brute Force Inversion Count = 6, Divide and Conquer Inversion Count = 6
Student 5: Brute Force Inversion Count = 1, Divide and Conquer Inversion Count = 1
Student 6: Brute Force Inversion Count = 3, Divide and Conquer Inversion Count = 3

Error Messages for Invalid Entries:
Student 2: Error: Array contains non-integer values, inversion count can't be performed.
Student 4: Error: Array contains non-integer values, inversion count can't be performed.

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```

Both negative nd letters

input: [5, 2, 3, 6], ['a', 1, 5, 2], [-7, -6, -4, -1], [-6, -2, -5, -7], [2, 3, 8, 4], [5, 5, 5, 4]

```
Output Clear

ERROR!

Categorized Inversion Counts (Valid Entries):
Student 1: Brute Force Inversion Count = 2, Divide and Conquer Inversion
          Count = 2
Student 5: Brute Force Inversion Count = 1, Divide and Conquer Inversion
          Count = 1
Student 6: Brute Force Inversion Count = 3, Divide and Conquer Inversion
          Count = 3

Negative Integer Entries:
Student 3: Inversion count can be found since course code cant be negative.
Student 4: Inversion count can be found since course code cant be negative.

Error Messages for Invalid Entries:
Student 2: Error: Array contains letters instead of integer values,
          inversion count can't be performed.

=== Code Execution Successful ===
```

CONCLUSION:

IN this task, we understand by divide and conquer we can do task with less time like in this case we could count the total no of inversions in just $O(n \log n)$ time while it took $O(n^2)$ time while we did it in brute force algorithm.

TASK 2:

AIM:

Consider large integers of size 10, 50, 100, 500 and 1000 digits.

Write integer multiplication program

Write integer multiplication program using divide and conquer technique.

ALGO by Brute Force:

1/ Algo for Integer Multiplication (Brute Force)

1/ Input: two n -digit positive integers x and y

2/ Output: The product $x \cdot y$

Assumption: n is a power of 2.

$n(n=2)$ {

return $x \cdot y$; }

else {

$a =$ first half of x ;

$b =$ first half of y ;

$c =$ first half of x ;

$d =$ second half of y ;

recursively {

$ac = a \times b$;

$bd = b \times d$;

$ad = a \times d$;

$bc = b \times c$; }

return: $10^n ac + 10^{n/2} (ad + bc) + bd$;

Time complexity:

So, here we multiply ac , bd , ad , bc , recursively for $n/2$ elements, which take $T(n/2)$ and $O(n)$ for other work;

$$\therefore T(n) = 4T(n/2) + O(n)$$

By master's theorem,

$$a = 4, b = 2, d = 1;$$

$$T(n) = T(n^{\log_2 4})$$

$$= T(n^2)$$

ALGO by Divide And Conquer:

Algo for Integer Multiplication (Karatsuba)

// Input: Two n-digit positive integers x and y

// Output: The product x.y

// Assumption: n is a power of 2.

if $n = 1$ &

return x.y

else

a, b = first and

a = first half of x;

b = second half of x;

c = first half of y;

d = second half of y;

p = a + b;

q = c + d;

recursively

ac = a.c;

bd = b.d;

pq = p.q;

abd = pq - ac - bd;

return $(10^n \cdot ac + 10^{n/2} \cdot abd + bd)$;

y.

Time complexity:

so, we multiply a, b, p, q recursively for $n/2$

element which takes $T(n/2)$ time.

and $O(n)$ to add and other time.

Hence, $T(n) = 3T(n/2) + cn$

by Master Theorem

a = 3, b = 2, d = 1

$\therefore T(n) = T(n^{log_2 3}) = T(n^{1.585})$

$\approx T(n^{1.566})$

POSITIVE TESTCASES:

10 digits:

```
Output Clear  
/tmp/idTn4Dijt0.o  
Enter the first large number: 1234567890  
Enter the second large number: 9876543210  
Multiplication result: 12193263111263526900  
  
=== Code Execution Successful ===
```

50 digits:

```
Output Clear  
/tmp/LIt0cyHG4n.o  
Enter the first large number:  
1234567890123456789012345678901234567890  
  
Enter the second large number:  
9876543210987654321098765432109876543210  
Multiplication result:  
121932631137021795226185032733866788594511507391561194939744871208653362  
2923332237463801111263526900  
  
=== Code Execution Successful ===
```

100 digits:

Output

Clear

```
/tmp/F49oSBrmjz.o
Enter the first large number:
12345678901234567890123456789012345678901234567890123456789012
3456789012345678901234567890

Enter the second large number:
98765432109876543210987654321098765432109876543210987654321098
7654321098765432109876543210

Multiplication result:
121932631137021795226185032733866788594511507391563633592367611644557885
992987901082152001356500521236092058011126352589864349937861606461673677
79295611949397448712086533622923332237463801111263526900

=== Code Execution Successful ===
```

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500 digits:

Output

Clear

```
/tmp/m9UE5Jv40j.o
Enter the first large number:
123456789012345678901234567890123456789012345678901234567890
123456789012345678901234567890123456789012345678901234567890
123456789012345678901234567890123456789012345678901234567890
123456789012345678901234567890123456789012345678901234567890

Enter the second large number:
987654321098765432109876543210987654321098765432109876543210
987654321098765432109876543210987654321098765432109876543210
987654321098765432109876543210987654321098765432109876543210
987654321098765432109876543210987654321098765432109876543210
```


Multiplication result:

```
1219326311370217952261850327338667885945115073915636335923676116445578
8599298790108215200135650052126047858423853071163510135648463496418057
5979271268846212448009449776913427830902591068411383935373250876390536
3359243747584209695883249501700807803381329065659257735098038256363014
8300563603581771039233348571810852003969836915075858862975473403444336
0920591124843773791359548847023472031491098917827985063250686023471857
3540618646105776543485749122236092059012360920580111263525898643499378
6160646167367779295611949397448712086533622923332237463801111263526900
```

Activate Windows

Go to Settings to activate Windows.

=== Code Execution Successful ===

1000 digits:

Output

Clear

/tmp/04Iak0VrFH.o

Enter the first large number:

```
1234567890123456789012345678901234567890123456789012345678901234567890
1234567890123456789012345678901234567890123456789012345678901234567890
1234567890123456789012345678901234567890123456789012345678901234567890
1234567890123456789012345678901234567890123456789012345678901234567890
1234567890123456789012345678901234567890123456789012345678901234567890
1234567890123456789012345678901234567890123456789012345678901234567890
1234567890123456789012345678901234567890123456789012345678901234567890
```

Enter the second large number:

```
9876543210987654321098765432109876543210987654321098765432109876543210
9876543210987654321098765432109876543210987654321098765432109876543210
9876543210987654321098765432109876543210987654321098765432109876543210
9876543210987654321098765432109876543210987654321098765432109876543210
9876543210987654321098765432109876543210987654321098765432109876543210
9876543210987654321098765432109876543210987654321098765432109876543210
98765432109876543210
```

Activate Windows

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Output

Clear

98765432109876543210

Multiplication result:

1219326311370217952261850327338667885945115073915636335923676116445578
8599298790108215200135650052126047858423853071163510135648463496418057
5979271268846212448009449776913427830902591068411383935373250876390536
3359243747584209695883249501700807803381329065659257735098038256363015
0739216226322206943757049226488187775860067062907131534822825636335493
8119188705060204191630848951275567748338805060155005334547613016307972
5499161183798201439492455412949245541294924554129480262149835543361772
3060508561057765534980948021385611948927312909571606462425048163389293
5680536082319768287107148296598231976448574912323732662700260783416814
8300563603581771039233348571810852003969836915075858862975473403444336
0920591124843773791359548847023472031491098917827985063250686023471857
3540618646105776543485749122236092059012360920580111263525898643499378
6160646167367779295611949397448712086533622923332237463801111263526900

Activate Windows

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=== Code Execution Successful ===

NEGATIVE TESTCASES:

negative Testcase

1) $a = \text{"abcde"}$
 $b = 1375$

output: multiplication not possible as one of the input is a string text

2) $a = \text{"12345"}$
 $b = 12345$

output: multiplication not possible as one of the input is a string

3) $a = 123.45$
 $b = 123$

output = multiplication not possible of float and integer

4) $a = \text{True}$

$b = 231$

output: multiplication not possible of boolean and integer

5) $a = 1+2j$

$b = 1234$

output: Multiplication not possible as one of the input is complex.

CONCLUSION:

By this task we learned that our normal multiplying method take n^2 time while by using karatsuba's algo we can do it in approx $n^{1.566}$ which make our work easy.