Quantitative Financial Risk Management
Assignment 1: Portfolio VaR and ES

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DATE OF SUBMISSION	17/04/2025
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GROUP NUMBER	A 5

General Tasks

1. Data Synchronization

To prepare the data for portfolio analysis, we performed a synchronization process across all relevant time series: asset prices, exchange rates and interest rates over the common period from April 6, 2010 until April 3, 2020. This way we ensure that every data point used in return and risk calculations is properly aligned and complete across all portfolio constituents.

We first processed the combined historical price dataset by extracting individual stock data (AAPL, MSFT, ASML.AS, 6758.T, and VOW3.DE) using a prefix-matching function. Exchange rate data was retrieved using the FRED API: EUR/USD and JPY/USD series were cleaned using forward and backward filling. For USD-denominated stocks (AAPL, MSFT), we converted prices to EUR using the inverse of USD/EUR rates. For the JPY-denominated stock (6758.T), we computed a two-step conversion (JPY → USD → EUR). EUR-denominated stocks required no conversion.

For the Euribor interest rate data we used the following dataset `https://www.kaggle.com/datasets/jorijnsmit/euribor-daily-rates`, that was cleaned and filtered to the same date range. We extracted the 1-month term, applied a fixed credit spread, and converted annualized rates into daily equivalents using standard compounding.

Finally, we ensured full alignment across all datasets by retaining only dates with complete data for all assets and variables. This created a clean, synchronized dataset suitable for reliable portfolio return and risk analysis.

2. Portfolio Construction & Weight Selection

The portfolio was constructed by setting equal weights of 20% to each five of the five selcted assets: Apple (AAPL), Microsoft (MSFT), ASML Holding (ASML.AS), Sony Group (6758.T), and Volswagen (VOW3.DE). This allocation ensures balances exposure across the various sectors (such as technology), regions (North America, Europe, and Asia) and currencies (USD. EUR, and JPY), and currencies (USD, EUR,and JPY).

Equal weighting was chosen for its simplicity to avoid over concentration in any specific asset or region. Since there was no any clear reason to expect one asset to perform better than the others, equal weights were used to treat all assets the same and in a fair way. This helps spread the risk across the portfolio and lowers the impact of any single asset doing poorly.

The total initial portfolio value was set at €1,000,000., with %40 (€400,000) financed through a loan tied to the EURIBOR rate and %60 (€600,000.), allocated as equity. Each asset received an allocation of €200,000, and the number of shares purchased was based on each asset's price on the first available trading day. The number of shares for each asset was kept the same across the whole period, as the portfolio followed a buy-and-hold approach with no changes after the first purchase.

To visualize the performance of the portfolio over time, we plot the daily portfolio value in euros as seen in Figure 1 :

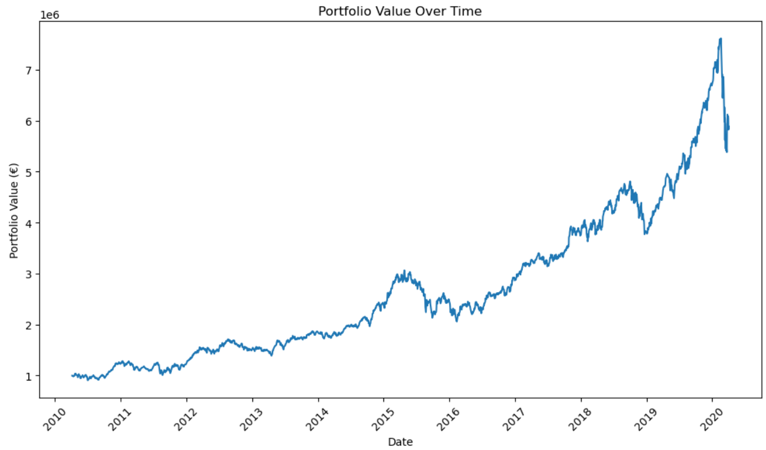

Figure 1: Portfolio Value over time.

The plot shows how the total portfolio value evolved throughout the period. The upward trend reflects the overall growth in the asset prices, although some fluctuations are visible, especially towards the end, indicating periods of increased volatility. These changes in value are influenced by stock price movements, currency fluctuations, and interest expenses on the loan. This graph helps us understand how the portfolio changes over time and is important for the risk analysis that follows, such as calculating Value-at-Risk (VaR) and Expected Shortfall (ES).

3. Computing VaR and ES

In this section, we compute and compare the Value-at-Risk (VaR) and Expected Shortfall (ES) of our portfolio at the 1-day horizon, using five different risk estimation methods:

1. Variance-Covariance method using the multivariate normal distribution for returns
2. Variance-Covariance method using the multivariate Student-t distribution for returns, using 3,4,5 or 6 degrees of freedom
3. Historical Simulation
4. GARCH(1, 1) with Constant Conditional Correlation
5. Filtered Historical Simulation with EWMA for each risk factor

Method 1:
We assume that returns follow a normal distribution and we use a rolling window of 250 days (which accounts for 1 trading year). We then compute the mean μ_t and standard deviation σ_t for each trading day.
We calculate VaR as

$$VaR_t = \mu_t + z_\alpha \sigma_t \sqrt{h}$$

where z_α is the critical value of the standard normal distribution at confidence level $\alpha = 0.05$ and h is the number of holding days (1 day in this case).
We calculate ES as

$$ES = \mu_t - \frac{\phi(\alpha)}{\alpha} \sigma_t \sqrt{h}$$

where $\phi(\alpha)$ is the probability density function of the normal distribution.

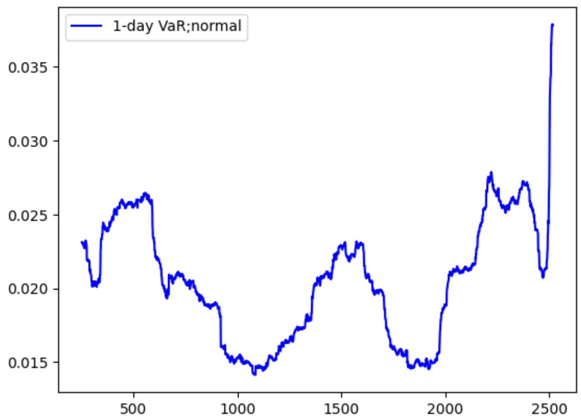

Figure 2: 1-day VaR using normal distribution

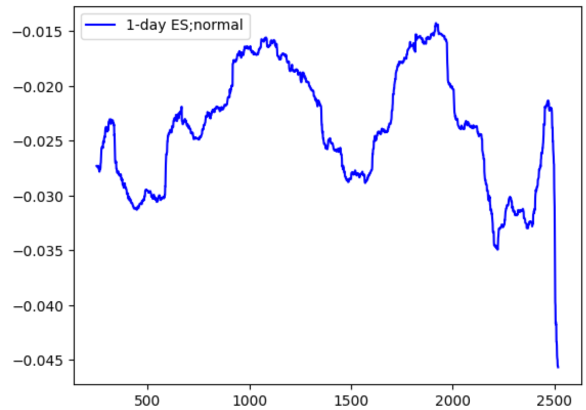

Figure 3: 1-day ES using normal distribution

The figures above show the 1-day Value-at-Risk (VaR) and Expected Shortfall (ES) under the normality assumption for the portfolio returns over time. Both are computed using a 250-day rolling window at a 95% confidence level.

The VaR plot (right) displays the estimated maximum expected daily loss that will not be exceeded with 95% confidence. The curve reflects the volatility of the portfolio: higher VaR values correspond to periods of increased return variability (e.g., approaching 2020).

The ES plot (left) represents the average loss that would be incurred if the VaR is breached — essentially quantifying the severity of worst-case losses. As expected, ES is consistently lower (more negative) than VaR and follows a similar pattern, but with larger magnitudes, especially during volatile periods.

Both series are time-varying risk measures, and their peaks coincide with known periods of market stress, such as the early 2020 COVID-19 crisis, which is clearly visible as a sharp increase in VaR and deepening ES. This confirms the sensitivity of both measures to changing portfolio risk, even under the relatively simple assumption of normally distributed returns.

Method 2:

In contrast to method 1, we assume that returns follow Student's t distribution.
We calculate VaR as

$$VaR_t = \mu_t + t_{\alpha,\nu}\sigma_t\sqrt{h}$$

where $t_{\alpha,\nu}$ is t-distribution quantile with ν degrees of freedom at confidence level $\alpha = 0.05$ and h is the number of holding days (1 day in this case).
We calculate ES as

$$ES = \mu_t - \sigma_t\sqrt{h}\frac{f(t_{\alpha,\nu})(\nu + t^2_{\alpha,\nu})}{(\nu - 1)\alpha}$$

where $f(t_{\alpha,\nu})$ is the PDF of t-distribution.

The figures above display the 1-day Value-at-Risk (VaR) and Expected Shortfall (ES) for the portfolio returns using the Student-t distribution with 3 degrees of freedom. Both measures are computed using a rolling window of 250 trading days and a confidence level of 95%.

Compared to the normal distribution, the Student-t model accounts for fat tails, allowing a greater probability of observing extreme values. As a result, both VaR and ES are more conservative (larger in absolute magnitude), especially during periods of increased market volatility. This better captures the tail risk that normal VaR may underestimate.

The VaR plot (right) displays the estimated maximum expected daily loss that will not be exceeded with 95% confidence. It varies over time, rising in periods such as late 2018 and early 2020 — both

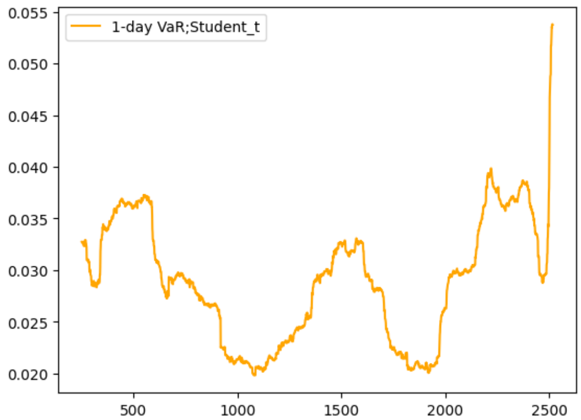

Figure 4: 1-day VaR using t-distribution

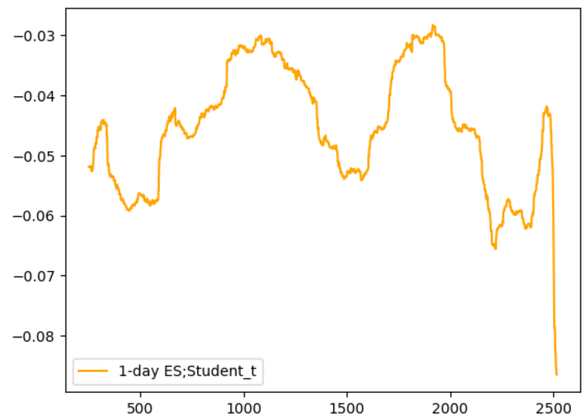

Figure 5: 1-day ES using t-distribution

corresponding to known market stress events.

The ES plot (left) illustrates the average loss on the worst 5% of days, conditional on VaR being breached. As expected, the ES curve lies consistently below the VaR curve and reflects even greater sensitivity to tail risk.

Notably, the Student-t distribution produces higher ES estimates than the normal model, capturing potential losses that are underestimated by models assuming thin-tailed distributions. The spike in both plots around early 2020 clearly illustrates the model's responsiveness to extreme downside risk during the COVID-19 market crash.

Moreover, to apply the Student-t distribution realistically, it is important to choose the appropriate degrees of freedom ν that best capture the tail behavior of the empirical return distribution. Rather than assuming an arbitrary value, we performed a quantitative goodness-of-fit test using a Q-Q plot and the Kolmogorov-Smirnov (KS) statistic.

Specifically, we used the most recent window of return data and compared it against several candidate Student-t distributions, each with different degrees of freedom. For each candidate, we computed the KS distance between the empirical cumulative distribution function (CDF) of the sample and the theoretical CDF of the t-distribution. The distribution that minimized the KS statistic was selected as the best fit. After testing candidate values $\nu = 3, 4, 5, 6$ we found that $\nu = 6$ provided the best fit according to the Kolmogorov-Smirnov statistic. This suggests that the portfolio returns exhibit moderate fat tails — more extreme than the normal distribution, but not as heavy-tailed as distributions with lower degrees of freedom.

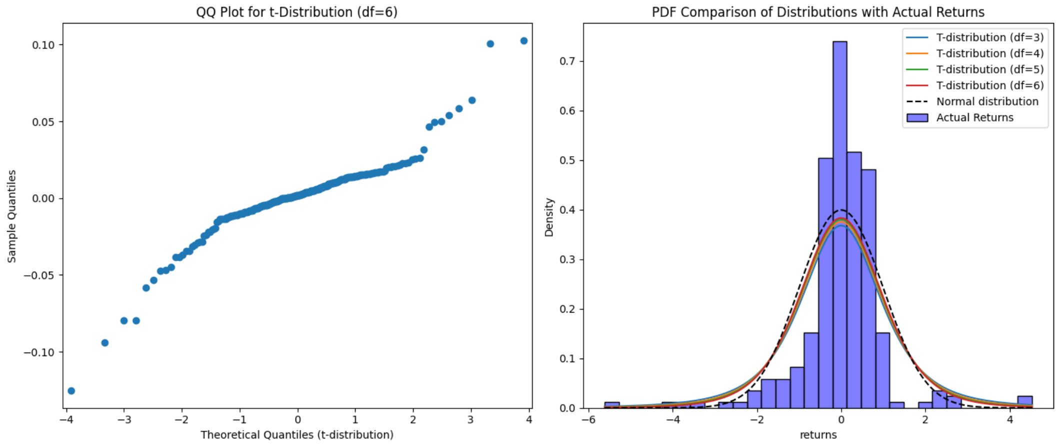

Figure 6: Enter Caption

Method 3:

For historical simulation, we use empirical quantiles of historical returns. We use a rolling window of 250 past portfolio returns and for each day, we compute the empirical α-th percentile of these returns. This gives the maximum loss under normal market conditions without assuming a specific return distribution.

$$VaR = -\text{Quantile}_{\alpha}(R_{t-\text{window}:t})$$

The historical ES is computed using the same rolling window of past returns as historical VaR. For each day, we identify the worst-performing α% of returns and calculate their average. This gives an estimate of the expected loss conditional on a VaR breach.

$$ES_t = -\mathbb{E}(R | R < \text{VaR}_t)$$

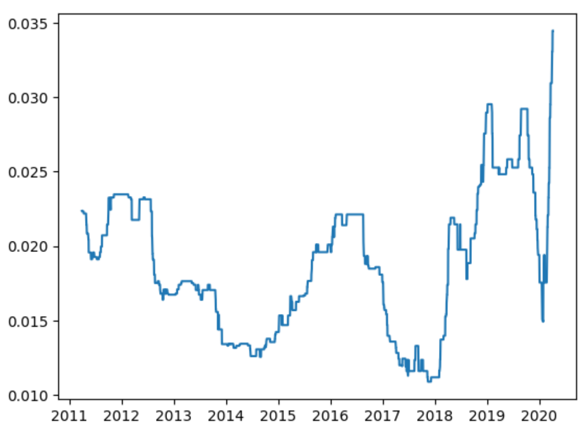

Figure 7: 1-day VaR using historical returns

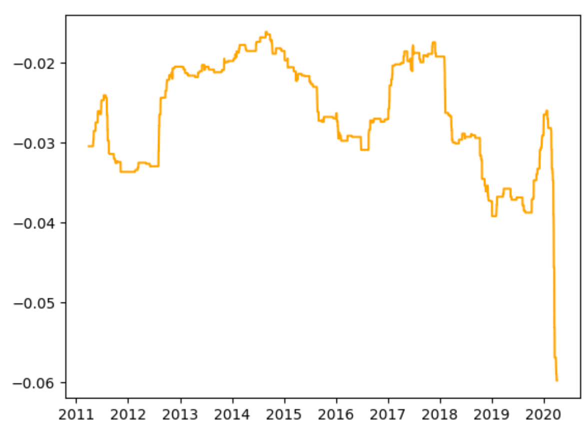

Figure 8: 1-day ES using historical returns

The figures above show the 1-day Value-at-Risk (VaR) and Expected Shortfall (ES) estimated using the historical simulation method over a rolling 250-day window.

Method 4:

In contrast to previous methods where we assumed constant or historical volatility for the portfolio's

returns, the GARCH(1,1) model relies on time-varying volatility.
For each day, we fit a GARCH(1,1) model using a rolling window of 1000 previous returns. The GARCH model used is

$$\sigma_t^2 = \alpha_0 + \alpha_1 \epsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2$$

where σ_t^2 is the conditional variance (volatility forecast), ϵ_{t-1}^2 is the previous day's return shock and $\alpha_0, \alpha_1, \beta_1$ are the model's parameters. Once we fit the model, we use the 1-step ahead forecasted volatility values to compute 1-day VaR assuming normality as

$$VaR_t = -z_\alpha \hat{\sigma_t}$$

where z_α is the critical value of the standard normal distribution and $\hat{\sigma_t}$ is the conditional standard deviation forecasted by the GARCH model.
In addition, using the GARCH-forecasted volatility, we compute 1-day ES as

$$ES_t = \frac{\phi(\alpha)}{\alpha} \hat{\sigma_t}$$

This approach, allows VaR and ES to adapt to changes in market volatility.

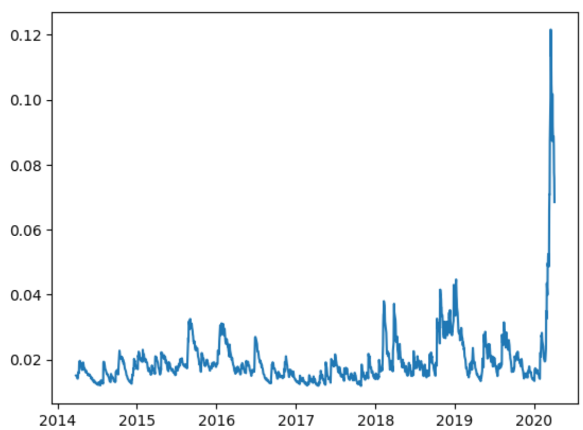

Figure 9: 1-day VaR using GARCH(1,1) model

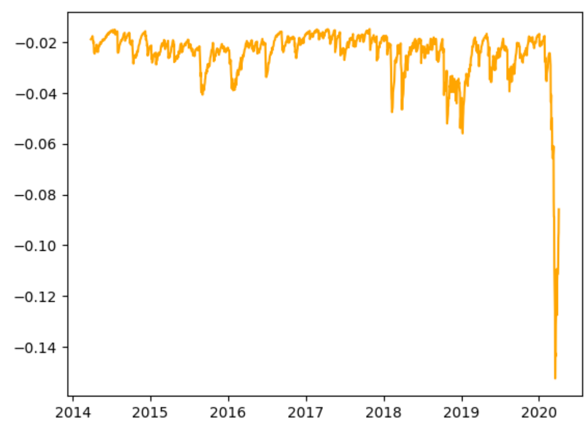

Figure 10: 1-day ES using GARCH(1,1) model

Method 5:
Filtered Historical simulation with Exponentially Weighted Moving Average (EWMA) of variance combines historical simulation with time-varying volatility. The EWMA factor rescales past returns to reflect their relative volatility at each point in time. This is done by applying exponentially decaying weights to past squared returns as

$$\sigma_t^2 = (1-\lambda) \sum_{i=1}^{n} \lambda^{i-1} r_{t-i}^2$$

where $\lambda = 0.94$ is the decaying factor
Once the current volatility estimate is obtained, historical returns are standardized (divided by volatility), and historical simulation is applied to this filtered series. The final VaR is then rescaled back using the current volatility as

$$VaR_t = -q_\alpha \sigma_t$$

where q_α is the α-th quantile of the filtered returns.

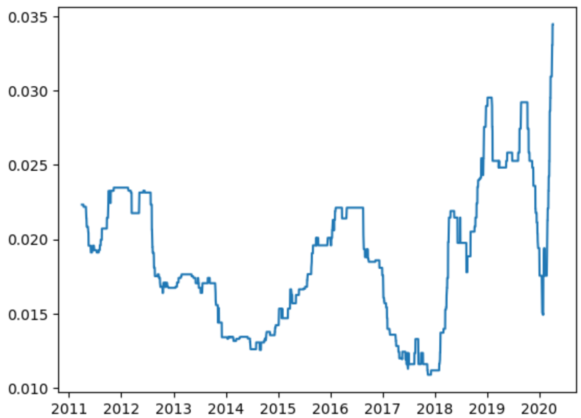

Figure 11: 1-day VaR using filtered historical simulation with EWMA

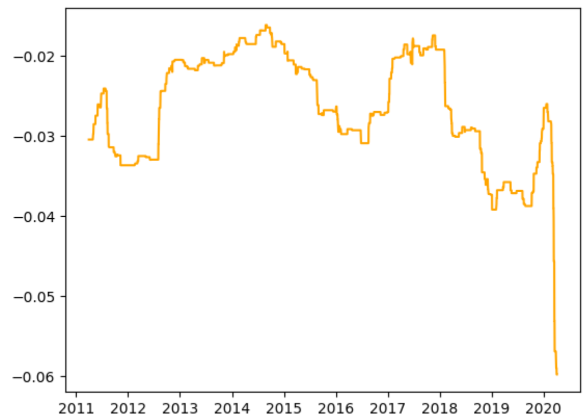

Figure 12: 1-day ES using filtered historical simulation with EWMA

4. VaR and ES Backtesting

VaR Backtesting:
To assess the reliability of our VaR models, we conducted a backtest by comparing the actual number of VaR violations per year against the expected number, under a 95% confidence level and under the Student's-t distribution for the returns.

A violation is recorded whenever the actual portfolio return falls below the predicted VaR threshold (when losses are worse than what the model predicted). Since a 95% VaR should only be violated in approximately 5% of trading days, we compared the actual number of violations per year against the expected number (which is 5% of trading days in each year)

We therefore implemented a function that identifies and counts all violations, aggregates violations per year and finally visualizes the results.

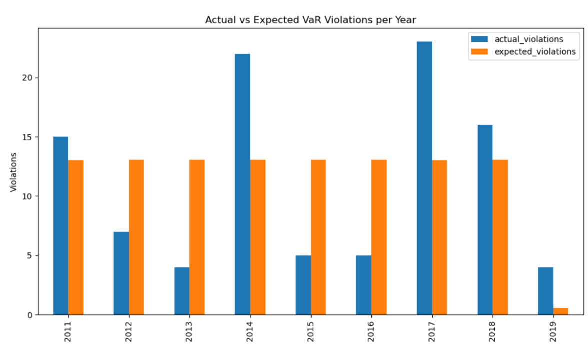

Figure 13: Actual vs Expected VaR violations

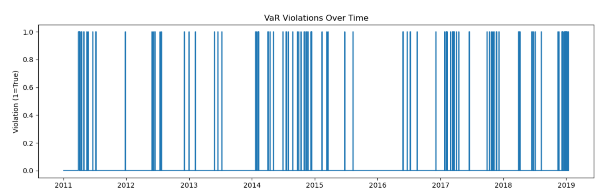

Figure 14: Time Series of violations

The bar chart demonstrates the actual number of VaR violations per year (blue) versus the expected number (orange) assuming a 95% confidence level. We note that in most years, the number of actual violations slightly exceeds the expected value, suggesting that the model may slightly underestimate downside risk.

The second plot displays a binary time series of violations; a value of 1 indicates a day when the actual return fell below the VaR estimate. We notice that the plot shows clustering of violations, especially during volatile periods like late 2018, early 2020, and 2022–2023.

We conclude that these visuals validate the importance of backtesting and show that while the Student-t VaR model captures much of the tail behavior, some deviation still exists which is expected in real-life applications. Meanwhile, by comparing the second frequency chart with the VAR distribution plot, we find that violations are less likely to occur when the economy is stabilizing and VaR starts to fall, whereas they will occur in large numbers at the very beginning of a crisis which means VaR will increase in the future.

5. 5- and 10-day VaR

To understand how risk changes over longer periods, we calculated the portfolio's Value-at-Risk (VaR) for 1-day, 5-day, and 10-day horizons using the historical simulation method. For the 5-day and 10-day VaRs, we used non-overlapping return windows, meaning that each multi-day return is calculated from independent blocks of 5 or 10 days without overlapping periods. This avoids reusing the same days in multiple calculations and gives a more accurate view of risk over time. Historical Simulation method computes VaR directly from historical portfolio returns without making any distributional assumptions. Our results show that the 1-day historical VaR was approximately 0.0224, while the 5-day and 10-day historical VaRs were 0.0563 and 0.0749, respectively.

To asses how well the square root of time rule approximated multi-day VaR, we scaled the 1-day historical VaR by $\sqrt{5}$ and $\sqrt{10}$ to obtain expected 5-day and 10-day VaRs. The ratio of 5-day to 1-day historical VaR was approximately 2.52, while $\sqrt{5}$ is approximately 2.236. Similarly, the 10-day to 1-day ratio was about 3.35, compared to the expected $\sqrt{10} \approx 3.162$. This indicated that the actual risk, as observed from historical returns, grows slightly faster than the square root of time rule predicts. This difference may suggest that the square root of time rules assume daily returns are similar and not linked to each other, but in our case, this assumption might not fully hold , which is why the actual risk increases a bit faster than expected.

We repeated this analysis using the Variance-Covariance method assuming normally distributed returns, which showed smaller deviations between observed and expected ratios. For example, the 5-day/1-day VaR ratio from the normal method was approximately 2.20 which is closer to $\sqrt{5}$, and the 10-day/1-day ratio was about 3.09, close to $\sqrt{10}$.

All the results discussed can be clearly shown in the following table:

Metric	**Historical VaR**	**Normal VaR**	**Expected Value**
1-day VaR	0.0224	0.0231	–
5-day VaR	0.0563	0.0508	–
10-day VaR	0.0785	0.0715	–
5-day / 1-day Ratio	2.516613	2.195645	$\sqrt{5} \approx 2.236$
10-day / 1-day Ratio	3.351552	3.091565	$\sqrt{10} \approx 3.162$

Table 1: Comparison of Historical and Normal VaR Estimates and Ratios

Therefore, we conclude that while the square-root-of-time rule provides quick approximation, especially under the normality assumption, our findings suggest it may slightly underestimate multi-day risk when using historical data. For accurate risk management, especially longer horizons, directly calculating multi-day VaR is preferable.

6. Stress testing

The stress test scenarios are used to quantify the consequences of extreme market moves. Specifically, since our portfolio consists of a leveraged long position, we will only consider shocks that negatively affect our portfolio for obvious reasons. In table 2 the scenarios we consider are shown, and as a bonus we are also considering what could be called a "black swan" event, with all of the previously mentioned shocks occurring altogether. As these shocks are more likely than not to propagate through markets and affect each other.

Table 2: Stress-testing scenarios

Risk factor	**Levels applied**
Stocks	−20%, −40%
Foreign exchange	−10% on USD & JPY crosses; −20%
Interest rates (Euribor 1 m)	+200 bp, +300 bp
Combined scenario	EQ −40%, FX −10/ − 20%, IR +300 bp

The date the shocks are introduced is 2014-04-02, which is around a third of the way through the entire sample range. For stocks positions, the shocks were introduced in a multiplicative way, so every value from the date of the crisis is multiplied by $(1 - shock)$. This ensures that the movement after the crisis date is relative to the lower values. The forex changes are handled in the same way. we add the equivalent of the +bps increase (annualized to a 1/360 daily increment) to the EURIBOR daily rate. All VaR calculations in the stress tests (Figures 15–20) are based on net equity returns, that is, the portfolio value minus the fixed loan and cumulative interest.

A summary of the initial effects of the crisis on the VaR:

- **Figure 15 (–20 % Equity)**: Parametric VaR spikes from 2.3 % to 3.5 %; Historical VaR is slightly elevated.
- **Figure 16 (–40 % Equity)**: Parametric VaR jumps from 2.3 % to 5.6 %; Historical VaR is elevated even more.
- **Figure 17 (FX –10/–20 %)**: Parametric VaR edges from 2.3 % to 3 %; Histroical VaR remains relatively similair.
- **Figure 18 (IR +200 bp)**: Parametric VaR barely changes; Historical VaR also remains similair
- **Figure 19 (IR +300 bp)**: Parametric VaR rises a tad bit; Historical VaR remains is slightly elevated.
- **Figure 20 (Combined)**: Parametric VaR roughly triples from 2.3 % to 6.8 %; Historical VaR approximately ticks up from 1.9 % to 2.2 %.

In all stock-shock scenarios, VaR jumps immediately after the initial drop and then gradually decays as the extreme "return" rolls out of the 250-day window. However, the VaR on net equity remains elevated until the end of the sample. This comes from the fact that our financing remains unchanged: the loan size is fixed, so after the shock each subsequent absolute price move represents a larger percentage change in net equity.

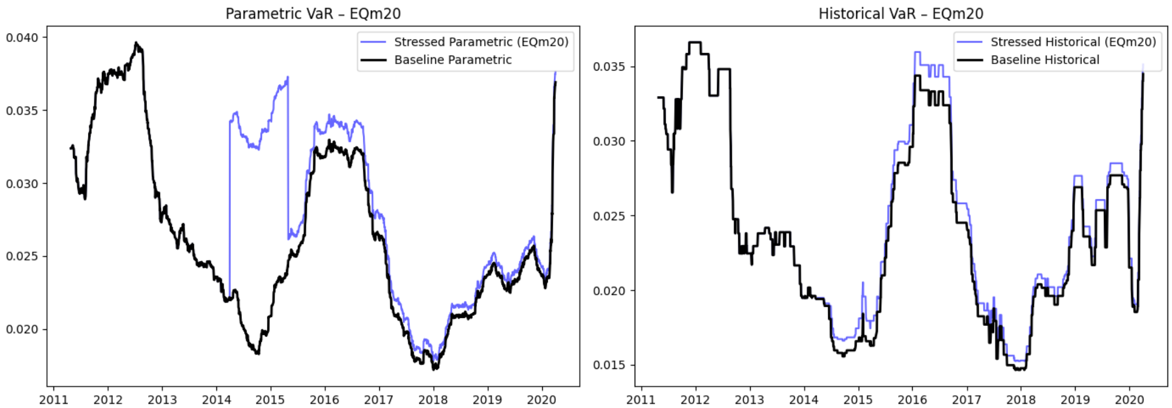

Figure 15: VaR, -20% stock shocks

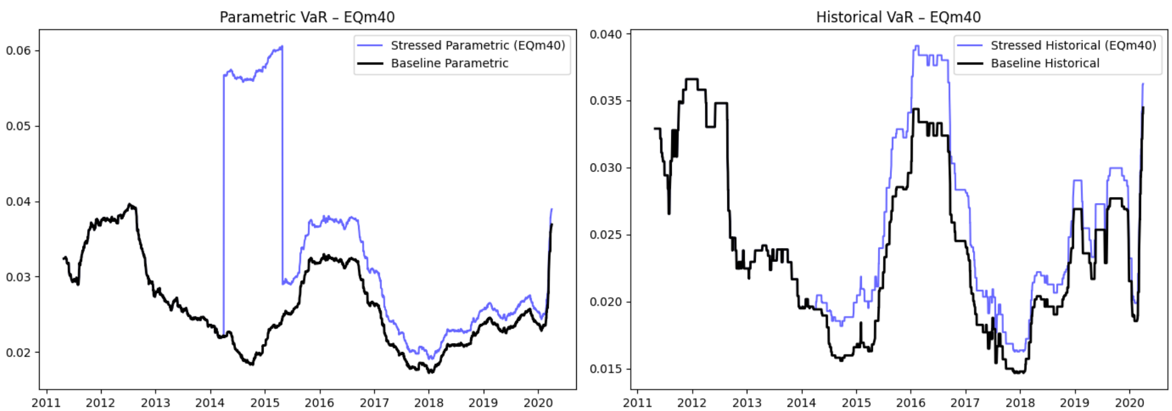

Figure 16: VaR, -40% stock shocks

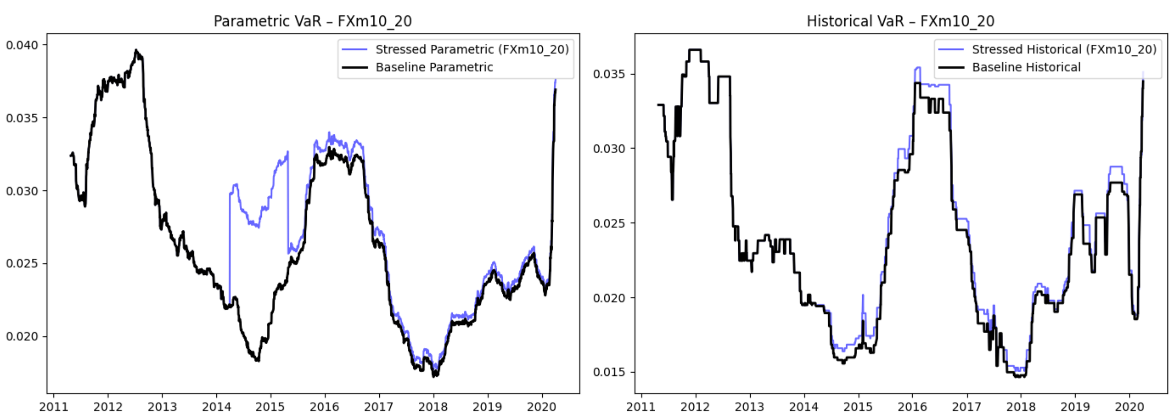

Figure 17: VaR, FX shock

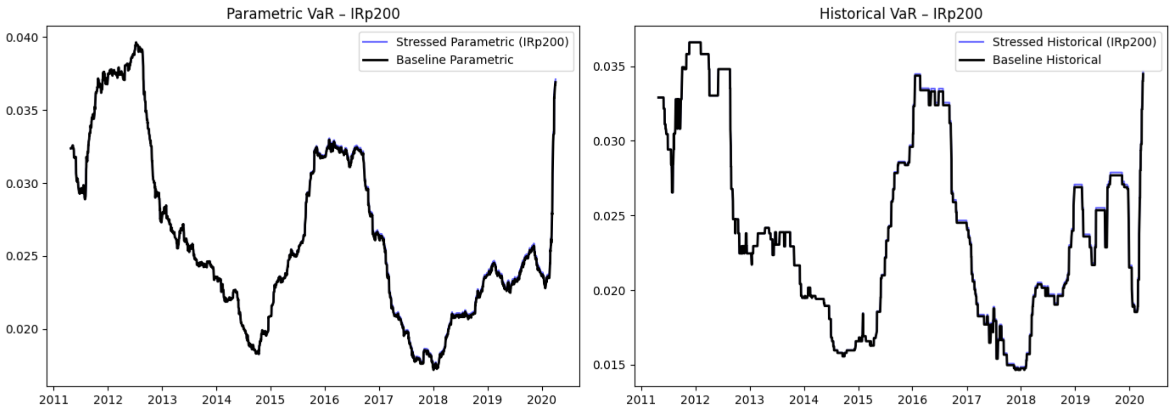

Figure 18: VaR, Interest +200 bp. shock

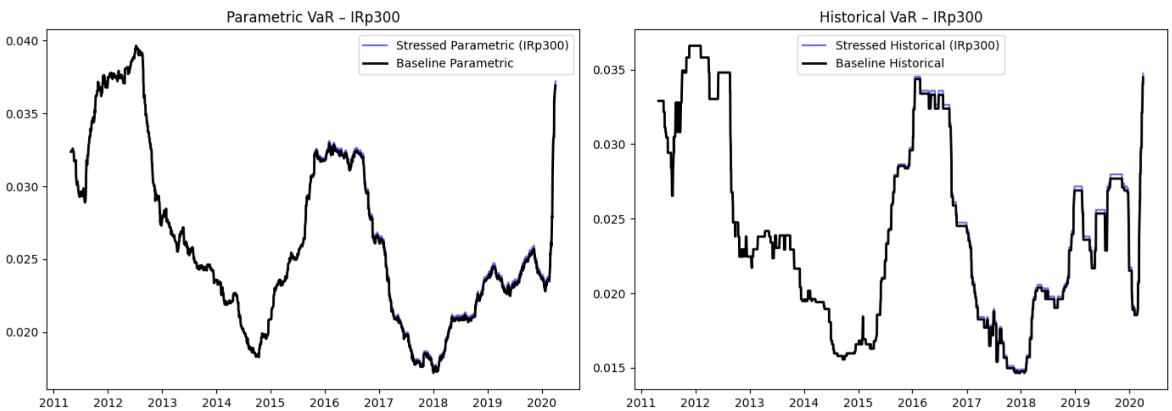

Figure 19: VaR, Interest +300 bp. shock

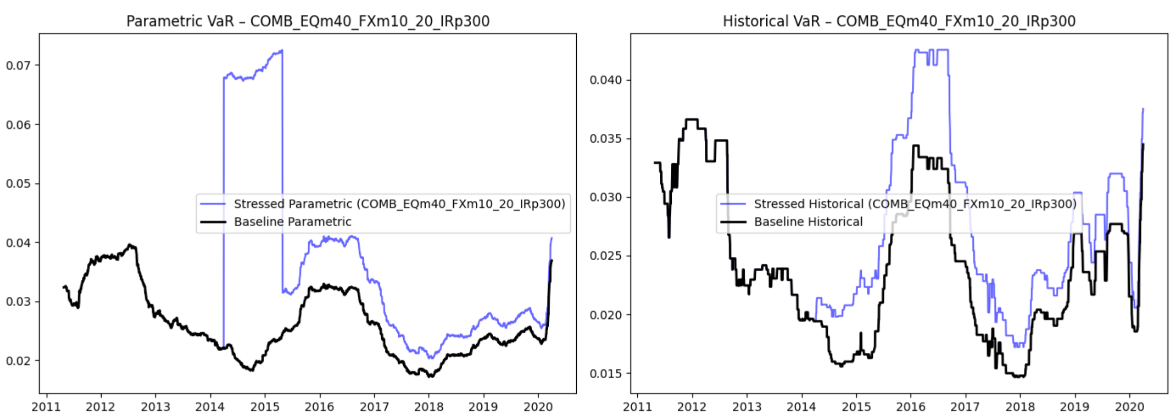

Figure 20: VaR, all combined

7. Conclusion and Recommendations for further actions.

To summarize, the variance-covariance method is fast to calculate which means it can be used in scenarios where portfolios need to be quickly adjusted based on stock prices. Historical simulation calculates VaR based on the previous data set, which is also fast to calculate, but does not allow for a quick response to new crises and does not take into account relationships between portfolios. Both garch-CCC and ewma filtered history simulations take into account the correlation between different stocks, but their parameters are complex, the run-time increases exponentially with the number of stocks, and the effect of the parameters on the results cannot be ignored, so that they are suitable for scenarios that do not require a fast response.