Data 582 - Bayesian Inference

Assignment 2

Please submit a well-organized PDF or html file to **Canvas**.

Total marks: 29 marks

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1. A random man is sampled from a large population and his height is measured to be 75.59 inches. Assume the weights in the population are normally distributed with unknown mean θ and known standard deviation 2.9 inches. Suppose your prior distribution for θ is normal with mean 70 and standard deviation 4.
   (a) (2 points) What is the posterior distribution for θ given the observed data?
   (b) (2 points) Provide a symmetric 95% credibility interval for θ.
   (c) (1 point) Provide an interpretation for the CI given in part (b).

2. Apply the Metropolis algorithm to our coin flip example. Namely, flip a coin n times and observe k heads. We assume the Binomial likelihood $p(x \mid \theta) = \theta^k(1-\theta)^{n-k}$ and familiar Beta(α, β) prior on θ. Our proposal distribution will be of the form Normal(μ=$\theta^{(t)}$, σ), where $\theta^{(t)}$ is the state our chain at time t, and σ is the proposal width.

   We will generate binomal data using the following code:

```r
set.seed(582) # replace 582 with your student number
p <- runif(1) # DO NOT LOOK AT THE OUTPUT OF THIS LINE
n <- 100
x <- rbinom(1, size=n, prob=p)
```

   Hence we have some random data and we want to perform inference on θ the probability that the coin lands heads facing up. The "truth" in this case depends on our student number. You should not look at this value until the last stage of this question. Adapt the MCMC code for running the Beta-Binomial model in lab 3, this time assuming a standard uniform prior on θ, that is our hyperpareters for the beta prior are $\alpha = 1$ and $\beta = 1$.

(a) Using a starting value of 0.0001, create a MCMC with 5000 iterations using a proposal width equal to that defined below. Plot the trace plot for each.

   i. (2 points) proposal width $\sigma = 0.002$.

   ii. (2 points) proposal width $\sigma = 0.2$.

   iii. (2 points) proposal width $\sigma = 2$.

(b) (2 points) Based on the trace plots from the three different proposal widths from part (a), which would you say is optimal and why.

(c) Consider the MCMC chain referenced in part (b). Compute the ergodic mean for θ and compare it with the true value stored in variable `p` using:

   i. (1 point) the entire chain

   ii. (1 point) the entire chain with the burn-in removed.

(d) (2 points) Which estimate was more accurate in part (c)? Why do you think that is?

(e) (2 points) Calculate an estimated 90% credible interval using the most appropriate chain.

3. **Adapted from Exercise 9.20 from BayesRules! (Penguins)** Consider the Normal regression model of penguin flipper lengths (Y) by body mass in grams (X).

   (a) (2 points) Based on their study of penguins in other regions, suppose that researchers are quite certain about the relationship between flipper length and body mass, prior to seeing any data: $\beta_1 \sim N(0.01, 0.002^2)$. Describe their prior understanding.

   (b) (2 points) Plot and discuss the observed relationship between `flipper_length_mm` and `body_mass_g` among the 344 sampled penguins.

   (c) (2 points) In a simple Normal regression model of flipper length Y by one predictor X, do you think that the σ parameter is bigger when X = bill length or when X = body mass? Explain your reasoning and provide some evidence from the information you already have.

   (d) (2 points) Use `stan_glm()` to simulate the Normal regression posterior model of flipper length by body mass using the researchers' prior for β_1 and and weakly informative priors for β_{0c} and σ. Do so with 4 chains run for 10000 iterations each.

   (e) (2 points) Plot the posterior model of the β_1 `body_mass_g` coefficient. Use this to describe the researchers' posterior understanding of the relationship between flippers and mass and how, if at all, this evolved from their prior understanding.