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This exercise set has 25 questions, for a total of 46 points. These questions are representative of what might be seen on Quiz 1.

READ THE QUESTIONS CAREFULLY

Notes and electronic devices are allowed, but they MUST be kept in airplane mode. You may use the back of a page if you run out of room on the front.

SURNAME, GIVEN NAME (print) ____________________

STUDENT NUMBER. ____________________

Signature: ____________________

$\boxed{2}$ 1. Consider the multiplicative congruential generator below, with $b = 17$ and $m = 7$. If x_0 is 14, what are x_1, x_2, u_1 and u_2?

$$x_k = bx_{k-1} \pmod{2^m - 1}$$
$$u_k = \frac{x_k}{2^m}$$

This one is a straightforward calculation in R.

$\boxed{1}$ 2. A given linear congruential generator has a cycle length of 7. Suppose the first 6 elements of a generated sequence are $1, 6, 4, 2, 3, 7$. What are the next two elements of the generated sequence?

A cycle length of 7 means that the sequence will repeat after 7 entries. Thus, the 8th value will be 1, and the 7th value will be 5, assuming a maximal cycle length for the generator.

$\boxed{1}$ 3. Is a properly tuned multiplicative congruential generator capable of producing sequences of truly random numbers? If so, explain how it works.

No.

$\boxed{2}$ 4. Suppose an R function called `badrandom` takes constants n, a, b and m as input arguments and returns a vector of pseudorandom numbers `u[1], u[2], ..., u[n]` according to the iteration rule

$$x_k = a + bx_{k-1} \pmod{2^m - 1}$$
$$u_k = \frac{x_k}{2^m}$$

using $x_0 = 27$ as a starting value.

Evaluate `badrandom(2, 4, 17, 6)`.

This is a straightforward calculation in R, using $n = 2$, $a = 4$, $b = 17$ and $m = 63$.

$\boxed{2}$ 5. The ideal random number generator would achieve two simultaneous objectives. What are they?

To produce uniformly distributed numbers that are independent of each other.

$\boxed{1}$ 6. What purpose does the χ^2 goodness of fit test serve when applied to a random number generator?

To test whether the numbers are uniformly distributed.

$\boxed{1}$ 7. To compute the autocorrelation function for a sequence of symbols, the symbols need to be converted to binary values. If not, explain how it works.

Yes.

$\boxed{1}$ 8. Suppose the first 4 lag plots for a random number generator show points that scatter according to the following patterns: 1) positive trend; 2) negative trend 3) no apparent trend 4) negative trend.

Which of the following acf sequences are consistent with the above result?

(a). 0.4 -0.2 0.6 - 0.5

(b). 0.9 -0.3 0.1 -0.3

(c). 0.4 0.2 0.2 0.1

(d). 0 0.4 -0.3 -0.3

(b).

$\boxed{1}$ 9. For a Poisson process with rate 12, the time between events has the following distribution:

(a). Normal with mean 12 and variance 1.

(b). Poisson with rate 12.

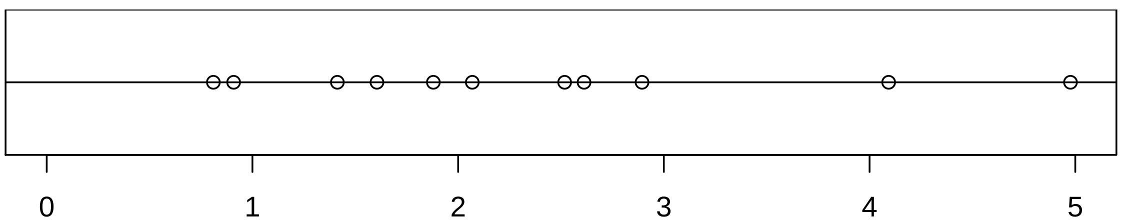
(c). Poisson with $\lambda = 12$.

(d). Exponential with rate 12.

(d).

10. The following code simulates a 1 dimensional Poisson process with intensity λ on the interval $[0, 5]$.

```
N <- rpois(1, 9)
U1 <- runif(N, min=0, max=5)
```



$\boxed{1}$ (a) What is the value of λ?

We have that $\lambda T = 9$ with $T = 5$, so $\lambda = 9/5 = 1.8$.

$\boxed{2}$ (b) If you didn't know the value of λ, what value would be a reasonable estimate, based on the figure above?

We observe 7 points in 5 time units, so the observed rate is 7/5 or 1.4. This would be a reasonable guess for λ, based on the available data.

$\boxed{1}$ 11. If you are measuring length with a ruler with a millimeter scale, which probablity model would be most appropriate for modelling the measurement error?

Uniform on [-.5, .5].

12. Refer to the previous question. You have measured the length of a pencil and found it to be 153 mm, and suppose you are using a uniform distribution on the interval $[a, b]$ to model the measurement error.

$\boxed{1}$ (a) What are the values of a and b?

$a = -.5$ *and* $b = .5$.

$\boxed{1}$ (b) What is the expected value of the measurement error?

$\frac{a+b}{2} = 0$.

$\boxed{2}$ (c) What is the standard deviation of the measurement error?

$\sqrt{\frac{(b-a)^2}{12}} = \sqrt{1/12} = 0.289$.

$\boxed{1}$ 13. The number of surface abrasions on manufactured refrigerators is monitored with a control chart. What kind of probability distribution might be appropriate for modelling the numbers of abrasions from unit to unit?

The Poisson distribution is a good model for counts where there is no clear upper limit on the number.

$\boxed{1}$ 14. Refer to the previous question. A random sample of 10 units results in the following counts of surface abrasions: 2, 0, 0, 1, 2, 0, 0, 1, 3, 1. The sample mean is 1 and the sample variance is 1.11. Do you think a Poisson model is reasonable for these data?

We see that it is 1.11 and the sample mean is 1. These are pretty close to each other, so we don't have a reason to complain about the use of a Poisson model here.

15. Manufactured lots of piston rings are being inspected before use in diesel engine assembly. Random samples of 50 rings are taken from each lot. When the manufacturing process is under control, no more than 3 percent of the rings should be defective.

$\boxed{1}$ (a) What is an appropriate probability model for counts of defective piston rings in a sample.

Binomial with $n = 50$ *and unknown* p.

$\boxed{1}$ (b) For a marginally in-control lot, what is the expected number of defective piston rings.

A lot is marginally in-control if it is borderline, i.e. 3 percent defective. This mean that $p = .03$, *and the expected value is* $np = 50(.03) = 1.5$.

$\boxed{1}$ (c) For a marginally in-control lot, what is the variance of the number of defective piston rings.

The variance is $np(1-p) = 1.455$.

$\boxed{1}$ (d) If we approximate the binomial model using the Poisson model, then what's the rate λ?

$\lambda = np = 1.5$.

$\boxed{1}$ 16. To simulate 100000 Poisson random numbers with mean 40 in R, what command would you use?

```
rpois(100000, lambda = 40)
```

$\boxed{1}$ 17. To simulate 100000 binomial random numbers with $n = 10$ and $p = .3$ in R, what command would you use?

```
rbinom(100000, size = 10, prob = 0.3)
```

$\boxed{1}$ 18. To simulate 100 standard normal random variables in R, what command would you use?

```
rnorm(100)
```

$\boxed{1}$ 19. I have simulated 1000 standard normal random variables in R and have stored them in an object called `Z`. I would like to convert them to normal variables having mean 3 and standard deviation 4 and to store the result in an object called `X`. What command should I use?

```
X <- 4*Z + 3
```

$\boxed{1}$ 20. If you ran the following code, what would you expect the output to be, at least approximately?

```
x <- rpois(1000000, 9)
mean(x)
```

The sample mean should be pretty close to the expected value which is λ, especially if the sample size is large. Therefore, the result would be expected to be very close to 9.

$\boxed{2}$ 21. Refer to the previous question, identify the approximate output be of the additional line commands

```
var(x)
```

Since the variance and mean are identical for a Poisson distribution, we expect the sample variance to be close to the sample mean: 9.

```
sd(x)
```

Taking the square root gives us a standard deviation close to 3.

22. Consider a Poisson process on the interval $[0, 10]$ where the intensity is 40.0.

$\boxed{1}$ (a) How many points do you expect in the interval?

The expected value is $\lambda T = 40 \times 10 = 400$.

$\boxed{1}$ (b) What is the standard deviation of the number of points?

The variance is 400 so the standard deviation must be the square root of 400: 20.

$\boxed{1}$ (c) How many points do you expect in the subinterval $[3, 7]$?

The expected value is $\lambda \times 4 = 160$.

$\boxed{2}$ (d) Fill in the blanks in the code below in order to simulate the process.

```
N <- ________(1, 400)

U <- ________(___, max = 10)

sort(__)
```

```
N <- rpois(1, 400)
U <- runif(N, max = 10)
sort(U)
```

(e) Consider a Poisson process on a 3×7 rectangle where the intensity is 21.0.

$\boxed{1}$ (a) How many points do you expect in the rectangle?

$21 \times 21 = 441$.

$\boxed{1}$ (b) What is the standard deviation of the number of points?

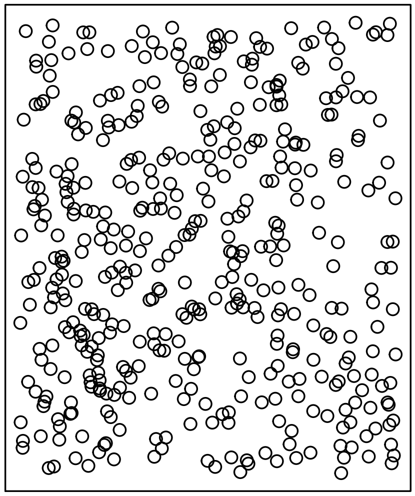
$\sqrt{441} = 21$.

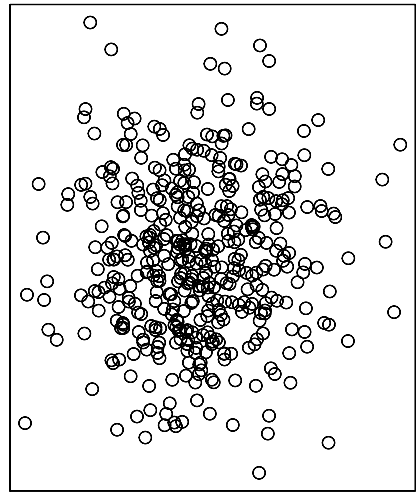
$\boxed{2}$ (c) Fill in the blanks in the code below in order to simulate the process.

```
N <- rpois(1, _______)

U1 <- runif(___, max = _____)

U2 <- runif(___, max = _____)
```

```
N <- rpois(1, 441)
U1 <- runif(N, max = 7)
U2 <- runif(N, max = 3)
```

1 (d) Refer to the previous question. Which of the following plots is the result of the given code?





The plot on the left matches a Poisson process realization. The one on the right is too sparse near the corners.

3 (e) Suppose X is a nonnegative random variable for which the following is true

$$P(X \leq x) = 1 - \frac{1}{1+x}$$

when $x \geq 0$. Show that if U is a uniform random number on the interval $[0, 1]$, then values of X can be simulated using the formula

$$X = \frac{1}{1-U} - 1.$$

Show that the expression for X is the inverse of the cdf $P(X \leq x)$.
$Moreprecisely, weneedtoshowthatif Xisdefinedintermsof Uasgivenabove, that P(X \leq x) = 1 - \frac{1}{1+x}$, *as follows:*

$$P(X \leq x) = P\left(\frac{1}{1-U} - 1 \leq x\right) = P\left(U \leq 1 - \frac{1}{1+x}\right) = 1 - \frac{1}{1+x}.$$