

Data-580 Lab 2

Dhun Sheth

2023-09-21

Question 1

knitr::include_graphics("dhun-data580-Lab2-question1.jpg")

Dhun Sheth
 32576881
 Campus: UBC Okanagan
 Program: MSc

Sept 21 / 2023
 Data 580
 $f(x) = \begin{cases} \frac{1}{b-a} & x \in [a, b] \\ 0 & \text{otherwise} \end{cases}$

1) a) $E[X] = \int_a^b x f(x) dx = \int_a^b x \frac{1}{b-a} dx = \frac{1}{b-a} \int_a^b x dx = \frac{1}{b-a} \left[\frac{x^2}{2} \right]_a^b = \frac{1}{b-a} \left(\frac{b^2}{2} - \frac{a^2}{2} \right) = \frac{b^2 - a^2}{2(b-a)} = \frac{(b-a)(b+a)}{2(b-a)} = \frac{a+b}{2}$

b) $E[(X-M)^3] = \int_a^b (x-M)^3 \frac{1}{b-a} dx$
 $y = x-M$
 $dy = dx$
 $x = a \rightarrow y = a-M$
 $x = b \rightarrow y = b-M$
 $y = b-M$
 $y = a-M$
 $\frac{b^2 - a^2}{2(b-a)} = \frac{(b-a)(b+a)}{2(b-a)} = \frac{a+b}{2}$

c) $E[(X-M)^4] = \int_a^b (x-M)^4 \frac{1}{b-a} dx$
 $y = x-M$
 $dy = dx$
 $x = a \rightarrow y = a-M$
 $x = b \rightarrow y = b-M$
 $y = b-M$
 $y = a-M$
 $\frac{b^2 - a^2}{2(b-a)} = \frac{(b-a)(b+a)}{2(b-a)} = \frac{a+b}{2}$

Change variable
 $y = x-M$
 $dy = dx$
 $x = a \rightarrow y = a-M$
 $x = b \rightarrow y = b-M$
 $y = b-M$
 $y = a-M$
 $\frac{b^2 - a^2}{2(b-a)} = \frac{(b-a)(b+a)}{2(b-a)} = \frac{a+b}{2}$

Change variable
 $y = x-M$
 $dy = dx$
 $x = a \rightarrow y = a-M$
 $x = b \rightarrow y = b-M$
 $y = b-M$
 $y = a-M$
 $\frac{b^2 - a^2}{2(b-a)} = \frac{(b-a)(b+a)}{2(b-a)} = \frac{a+b}{2}$

Question 2

Part A

```
x <- runif(100000, min=0, max=1)

m <- mean(x)
print(m)
```

```
## [1] 0.50149
```

Computed mean of x is 0.50149

Theoretical mean is $(a+b)/2 = (0+1)/2 = 0.5$

Part B

```
k <- mean( (x-mean(x))**3 )

print(k)
```

```
## [1] -0.0001526693
```

Computed value of skewness is $-1.5266931 \times 10^{-4}$

Theoretical skewness of uniform distribution is 0.

Question 3

Part A

```
binsim<- rbinom(10000,20,0.3)

prob_less_than_or_equal_to_5 <- sum(binsim <= 5)/10000
print(prob_less_than_or_equal_to_5)
```

```
## [1] 0.4153
```

Simulated $P(X \leq 5) = 0.4153$

Part B

```
prob_equal_5 <- sum(binsim == 5)/10000
print(prob_equal_5)
```

```
## [1] 0.1811
```

Simulated $P(X = 5) = 0.1811$

Part C

```
mean_binsim <- mean(binsim)
print(mean_binsim)
```

```
## [1] 6.0113
```

Simulated mean of binsim is 6.0113

Part D

```
var_binsim <- var(binsim)
print(var_binsim)
```

```
## [1] 4.152788
```

Simulated var of binsim is 4.1527876

Question 4

```
pois<- rpois(10000,7.2)
m_pois<-mean(pois)
var_pois<-var(pois)
print(m_pois)
```

```
## [1] 7.2207
```

```
print(var_pois)
```

```
## [1] 7.261318
```

Simulated mean of Poisson is 7.2207 and variance is 7.2613176
Theoretical mean and variance is 7.2.

Question 5

Part A

```
p1<- rpois(10000,5)
p2<- rpois(10000,25)
p3<- rpois(10000,125)
p4<- rpois(10000,625)

p1_mean<- mean(p1)
p2_mean<- mean(p2)
```

```
p3_mean<- mean(p3)
p4_mean<- mean(p4)

print(p1_mean)
```

```
## [1] 4.9907
```

```
print(p2_mean)
```

```
## [1] 24.931
```

```
print(p3_mean)
```

```
## [1] 124.9927
```

```
print(p4_mean)
```

```
## [1] 625.0981
```

Part B

```
p1_log_mean<- mean(log(1+p1))
p2_log_mean<- mean(log(1+p2))
p3_log_mean<- mean(log(1+p3))
p4_log_mean<- mean(log(1+p4))

print(p1_log_mean)
```

```
## [1] 1.710925
```

```
print(p2_log_mean)
```

```
## [1] 3.23638
```

```
print(p3_log_mean)
```

```
## [1] 4.832203
```

```
print(p4_log_mean)
```

```
## [1] 6.43871
```

Part C

```
p1_root_mean<- mean(sqrt(p1))  
p2_root_mean<- mean(sqrt(p2))  
p3_root_mean<- mean(sqrt(p3))  
p4_root_mean<- mean(sqrt(p4))
```

```
print(p1_root_mean)
```

```
## [1] 2.167722
```

```
print(p2_root_mean)
```

```
## [1] 4.967661
```

```
print(p3_root_mean)
```

```
## [1] 11.16863
```

```
print(p4_root_mean)
```

```
## [1] 24.99697
```