Experimental Design Formulas

Completely Randomized Design

Balanced:

$$Y_{ij} = \mu + \tau_i + R_{ij}$$

where

- $i = 1, 2, \ldots, t$
- $j = 1, 2, \ldots, r$
- $R_{ij} \sim N(0, \sigma^2)$
- The balanced design constraint is $\sum_{i=1}^{t} \tau_i = 0$

Unbalanced:

$$Y_{ij} = \mu + \tau_i + R_{ij}$$

where

- $i = 1, 2, \ldots, t$
- $j = 1, 2, \ldots, n_i$
- $R_{ij} \sim N(0, \sigma^2)$
- the unbalanced constraint is $\sum_{i=1}^{t} n_i \tau_i = 0$

The pooled overall sample variance is:

$$\hat{\sigma}^2 = \frac{(n_1 - 1)\hat{\sigma}_1^2 + (n_2 - 1)\hat{\sigma}_2^2}{n_1 + n_2 - 2}$$

The test statistic for the difference between two treatments is:

$$\frac{(\hat{\tau}_1 - \hat{\tau}_2) - (\tau_1 - \tau_2)}{\hat{\sigma}\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} \sim t_{n_1 + n_2 - 2}$$

where $\hat{\sigma}$ is the square root of the pooled variance estimate

The $(1 - \alpha) \times 100\%$ confidence interval for the difference between two treatments:

$$\hat{\tau}_1 - \hat{\tau}_2 \pm c \times \text{s.e.}(\hat{\tau}_1 - \hat{\tau}_2)$$

where c is chosen such that

$$P(|T_{n_1+n_2-2}| \leq c) = 1 - \alpha$$

and

$$\text{s.e.}(\hat{\tau}_1 - \hat{\tau}_2) = \hat{\sigma}\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$$

Randomized Block Design

$$Y_{ij} = \mu + \tau_i + \beta_j + R_{ij} \quad i = 1, 2, \ldots, t \quad j = 1, ..., b$$

- μ: overall average across treatments and blocks.
- τ_i: treatment effect
- β_j: block effect
- $R_{ij} \sim N(0, \sigma^2)$
- Constraints: $\sum_{i=1}^{t} \tau_i = 0$ and $\sum_{j=1}^{b} \beta_j = 0$

For a contrast $\theta = \sum_i a_i \tau_i$ where $\sum_i a_i = 0$

$$\hat{\theta} = \sum_i a_i \hat{\tau}_i$$

and

$$s.e(\hat{\theta}) = \hat{\sigma}_R^2 \frac{\sum_i a_i^2}{b}$$

Factorial Design

$$Y_{ijk} = \mu + \tau_i + \beta_j + (\tau\beta)_{ij} + R_{ijk}$$

where

- μ: overall mean effect
- τ_i: treatment effect of the ith level of the row factor A ($i = 1, \ldots, a$)
- β_j: is the effect of the jth level of column factor B ($j = 1, \ldots, b$)
- $(\tau\beta)_{ij}$: is the effect of the interaction between i and j
- R_{ijk}: is a random error component ($k = 1, \ldots, r$)

Constraints: $\sum_{i=1}^{a} \tau_i = 0$, $\sum_{j=1}^{b} \beta_j = 0$ and $\sum_{i=1}^{a}\sum_{j=1}^{b} (\tau\beta)_{ij} = 0$.