

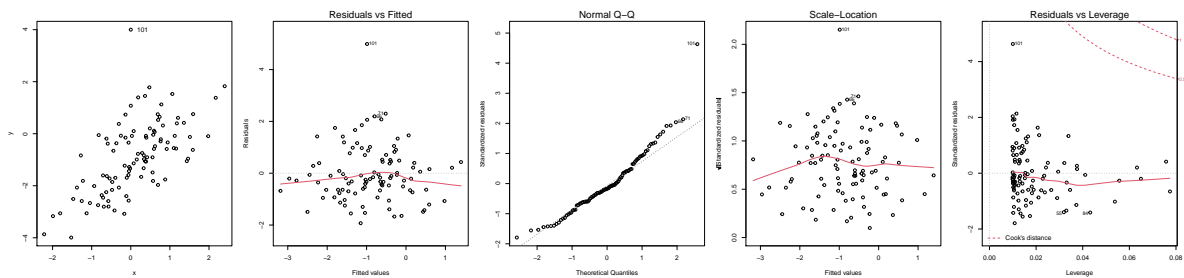
The University of British Columbia

Data Science 570 Predictive Modelling

Three types of points

1. Outlier

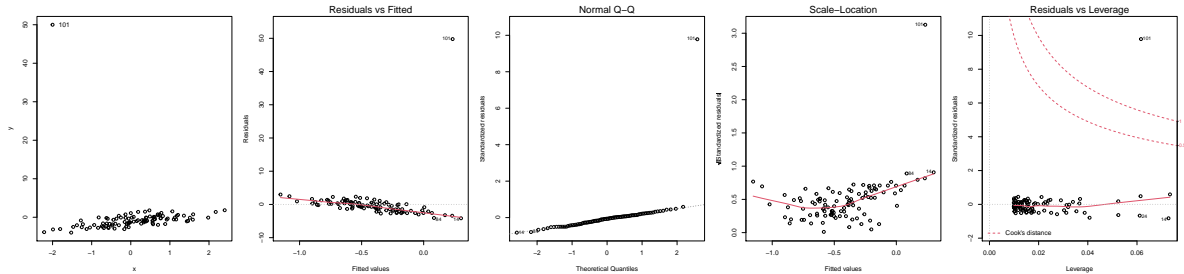
```
set.seed(1)
x=rnorm(100)
e=rnorm(100)
y=-1+x+e
fit1=lm(y~x)
#summary(fit1)
x=c(x,0)
y=c(y,4)#outlier
fit2=lm(y~x)
par(mfrow=c(1,5), mai = c(1, .5, 0.2, 0.2))
plot(x,y)
text(x[101]+0.3,y[101], "101")
plot(fit2)
```



```
#summary(fit2)
```

2. Outlier and influential point

```
set.seed(1)
x=rnorm(100)
e=rnorm(100)
y=-1+x+e
fit1=lm(y~x)
#summary(fit1)
x=c(x,-2)
y=c(y,50)#outlier and influential point
par(mfrow=c(1,5), mai = c(1, .5, 0.2, 0.2))
plot(x,y)
text(x[101]+0.3,y[101], "101")
fit2=lm(y~x)
plot(fit2)
```



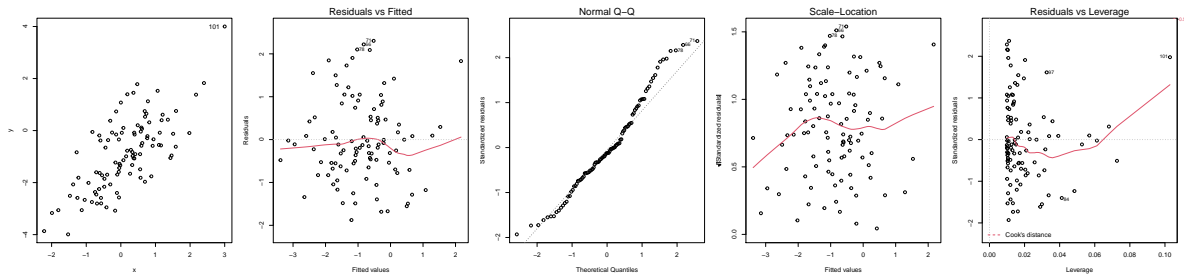
```
#summary(fit2)
cooks.distance(fit2)
```

##	1	2	3	4	5	6
##	1.252317e-03	1.889337e-05	2.580950e-03	1.968290e-03	1.333061e-04	2.698618e-06
##	7	8	9	10	11	12
##	1.247070e-04	4.553409e-04	6.859262e-05	1.000112e-04	3.704282e-04	6.202731e-05
##	13	14	15	16	17	18
##	4.439617e-08	2.665886e-02	1.924767e-04	2.082011e-04	1.654674e-04	4.512468e-05
##	19	20	21	22	23	24
##	2.876573e-04	1.039379e-10	3.371474e-06	9.140262e-04	9.483638e-05	1.451897e-02
##	25	26	27	28	29	30
##	3.090508e-06	2.405553e-07	1.526460e-04	5.038627e-03	9.352508e-04	2.960635e-05
##	31	32	33	34	35	36
##	8.647710e-04	3.368814e-04	3.904218e-05	9.016607e-04	3.060965e-03	1.731254e-03
##	37	38	39	40	41	42
##	4.765330e-04	2.749169e-04	1.533236e-05	3.381889e-05	1.475379e-03	1.208705e-05
##	43	44	45	46	47	48
##	4.874045e-04	2.602137e-05	2.205599e-03	1.705981e-03	7.721658e-04	5.201953e-05
##	49	50	51	52	53	54
##	7.972182e-04	3.837782e-04	2.812559e-05	5.984786e-04	4.409145e-05	4.798151e-03
##	55	56	57	58	59	60
##	2.570567e-05	9.075049e-04	1.697771e-06	3.231850e-03	3.608455e-04	2.302249e-04
##	61	62	63	64	65	66

##	1.377212e-02	1.490203e-04	4.917170e-04	1.885602e-05	1.647142e-03	6.354078e-04
##	67	68	69	70	71	72
##	1.105536e-02	6.301551e-06	5.485625e-05	7.666813e-03	1.177524e-03	6.712528e-04
##	73	74	75	76	77	78
##	1.056523e-04	1.540645e-03	4.037810e-03	1.246113e-05	3.916109e-05	4.013288e-04
##	79	80	81	82	83	84
##	4.992687e-05	8.336974e-06	1.871343e-03	8.011014e-06	6.212583e-04	1.331531e-02
##	85	86	87	88	89	90
##	1.176624e-04	2.062779e-05	2.046775e-03	6.813147e-04	6.005081e-05	2.640341e-04
##	91	92	93	94	95	96
##	6.089614e-04	9.094228e-04	1.637773e-05	3.530008e-04	5.428422e-05	1.956834e-04
##	97	98	99	100	101	
##	4.376329e-04	1.587900e-03	1.899253e-03	6.562256e-04	3.143158e+00	

3. Outlier and high leverage point

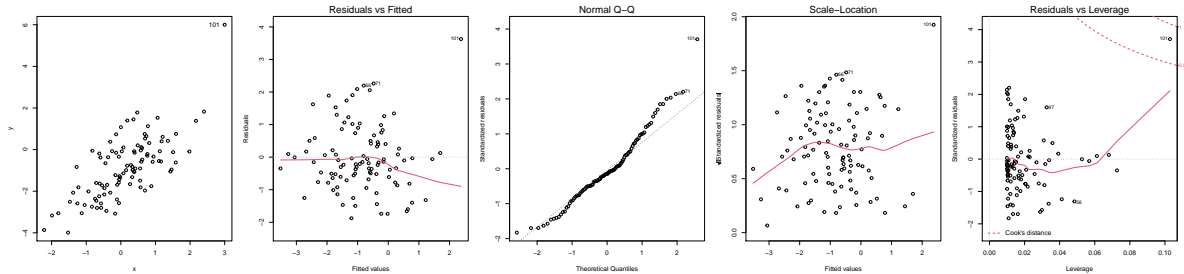
```
set.seed(1)
x=rnorm(100)
e=rnorm(100)
y=-1+x+e
fit1=lm(y~x)
#summary(fit1)
x=c(x,3)
y=c(y,4)#outlier and high leverage point
par(mfrow=c(1,5), mai = c(1, .5, 0.2, 0.2))
plot(x,y)
text(x[101]-0.3,y[101], "101")
fit2=lm(y~x)
plot(fit2)
```



```
#summary(fit2)
```

4. Outlier, high leverage, and influential point

```
set.seed(1)
x=rnorm(100)
e=rnorm(100)
y=-1+x+e
fit1=lm(y~x)
#summary(fit1)
x=c(x,3)#outlier, high leverage point, and influential point
y=c(y,6)
par(mfrow=c(1,5), mai = c(1, .5, 0.2, 0.2))
plot(x,y)
text(x[101]-0.3,y[101], "101")
fit2=lm(y~x)
plot(fit2)
```



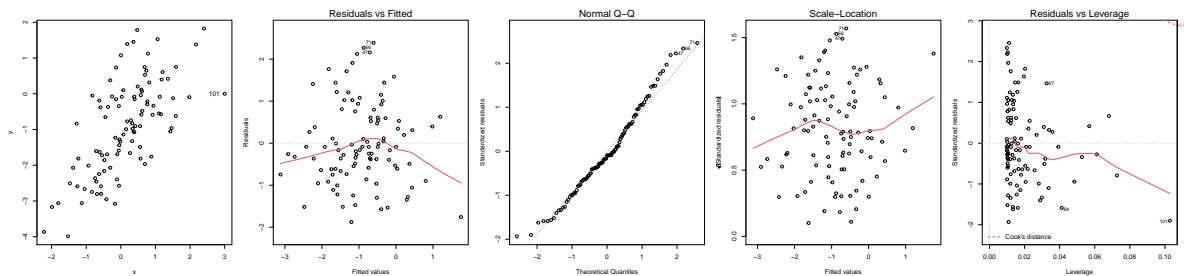
```
#summary(fit2)
cooks.distance(fit2)
```

##	1	2	3	4	5	6
##	2.193411e-03	5.473491e-06	6.246755e-03	1.964689e-05	2.297821e-03	3.546845e-02
##	7	8	9	10	11	12
##	2.425573e-03	4.651737e-03	6.133712e-04	1.760651e-02	1.045208e-02	1.256236e-03
##	13	14	15	16	17	18
##	1.863161e-02	4.766415e-03	1.172980e-03	6.785951e-04	4.446676e-04	1.252374e-03
##	19	20	21	22	23	24
##	1.194944e-03	3.366857e-04	3.038531e-03	1.115973e-02	2.071044e-04	2.949230e-04
##	25	26	27	28	29	30
##	1.645855e-04	2.677833e-03	7.358389e-06	5.734538e-04	2.499164e-03	6.818148e-04
##	31	32	33	34	35	36
##	1.354579e-04	1.585552e-03	1.255266e-03	1.105632e-02	4.562417e-03	1.385889e-02
##	37	38	39	40	41	42
##	3.456310e-04	1.262630e-03	6.065793e-03	1.367630e-04	1.848634e-02	8.390719e-03
##	43	44	45	46	47	48
##	1.963539e-02	1.547328e-03	8.762767e-03	3.621820e-03	2.123189e-02	3.118554e-05
##	49	50	51	52	53	54
##	8.022460e-03	2.372892e-02	8.773232e-04	4.658954e-05	5.974694e-04	8.207304e-03
##	55	56	57	58	59	60
##	3.960551e-02	4.355022e-02	6.966749e-03	2.806804e-03	1.203037e-02	1.864885e-02
##	61	62	63	64	65	66

##	6.045470e-04	2.330663e-04	6.241637e-03	3.879952e-03	2.340561e-03	2.295259e-02
##	67	68	69	70	71	72
##	5.225578e-07	3.815863e-02	1.047153e-04	1.063668e-04	2.749244e-02	4.020461e-04
##	73	74	75	76	77	78
##	9.256788e-04	4.079579e-05	3.836305e-04	1.578840e-05	4.910287e-03	2.117005e-02
##	79	80	81	82	83	84
##	5.085489e-03	1.302579e-02	9.817796e-03	5.331129e-03	7.376399e-05	3.303263e-02
##	85	86	87	88	89	90
##	1.245210e-03	1.719671e-04	1.725034e-02	2.942700e-03	1.078636e-03	4.316984e-03
##	91	92	93	94	95	96
##	6.036342e-05	7.638659e-04	8.026379e-03	3.708305e-03	3.317645e-02	6.990006e-03
##	97	98	99	100	101	
##	4.304348e-02	6.484570e-03	5.331140e-03	6.295700e-04	7.881521e-01	

5. High leverage point

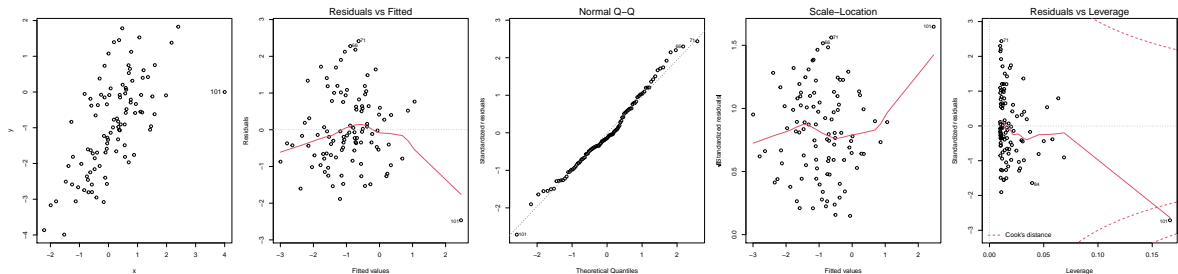
```
set.seed(1)
x=rnorm(100)
e=rnorm(100)
y=-1+x+e
fit1=lm(y~x)
#summary(fit1)
x=c(x,3)
y=c(y,0)#high leverage point
par(mfrow=c(1,5), mai = c(1, .5, 0.2, 0.2))
plot(x,y)
text(x[101]-0.3,y[101], "101")
fit2=lm(y~x)
plot(fit2)
```



```
#summary(fit2)
```

6. High leverage and influential point

```
set.seed(1)
x=rnorm(100)
e=rnorm(100)
y=-1+x+e
fit1=lm(y~x)
#summary(fit1)
x=c(x,4)
y=c(y,0)#high leverage point and influential point
par(mfrow=c(1,5), mai = c(1, .5, 0.2, 0.2))
plot(x,y)
text(x[101]-0.3,y[101], "101")
fit2=lm(y~x)
plot(fit2)
```



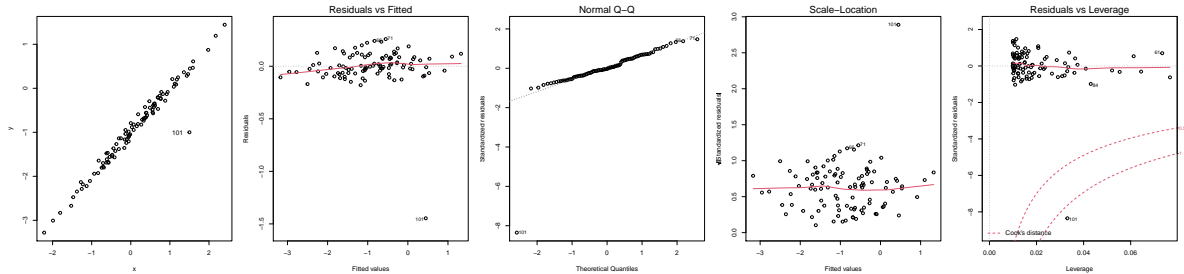
```
#summary(fit2)
cooks.distance(fit2)
```

##	1	2	3	4	5	6
##	3.537797e-03	6.589205e-05	9.784962e-03	2.765143e-03	1.687245e-03	3.061285e-02
##	7	8	9	10	11	12
##	3.904579e-03	7.747013e-03	1.548376e-03	1.789891e-02	2.571285e-03	7.226275e-04
##	13	14	15	16	17	18
##	1.662176e-02	3.024160e-02	4.972574e-06	6.454387e-04	3.898491e-04	1.153242e-04
##	19	20	21	22	23	24
##	3.155000e-03	1.950702e-05	1.005251e-03	1.623891e-02	1.251358e-04	4.550527e-03
##	25	26	27	28	29	30
##	3.710602e-06	3.036008e-03	1.044748e-05	5.669077e-04	3.468507e-03	2.768389e-04
##	31	32	33	34	35	36
##	1.012083e-03	1.658567e-03	2.133722e-03	1.158224e-02	6.643350e-04	1.620800e-02
##	37	38	39	40	41	42
##	6.035201e-04	1.260291e-03	2.251467e-03	5.192873e-05	2.002044e-02	8.551006e-03
##	43	44	45	46	47	48
##	1.588870e-02	7.245743e-04	1.193325e-02	5.703729e-03	2.546833e-02	1.850811e-04
##	49	50	51	52	53	54
##	8.553591e-03	1.786797e-02	1.632797e-03	1.572820e-05	2.743314e-04	1.492829e-02
##	55	56	57	58	59	60
##	2.329508e-02	1.549221e-02	6.631562e-03	6.437315e-03	9.754886e-03	2.003469e-02
##	61	62	63	64	65	66

##	2.138651e-02	1.992785e-04	9.555248e-03	4.562597e-03	4.164580e-03	2.649329e-02
##	67	68	69	70	71	72
##	5.045573e-03	2.161759e-02	2.985994e-05	8.088038e-03	3.319222e-02	4.396263e-05
##	73	74	75	76	77	78
##	2.117350e-03	2.335092e-04	3.131314e-03	1.295485e-05	4.318043e-03	2.353338e-02
##	79	80	81	82	83	84
##	6.039961e-03	1.143673e-02	1.246164e-02	5.724803e-03	1.885130e-03	5.522207e-02
##	85	86	87	88	89	90
##	2.558737e-03	2.525957e-05	2.649265e-02	3.537728e-03	6.065115e-04	3.689746e-03
##	91	92	93	94	95	96
##	2.906315e-04	4.096503e-03	3.199541e-03	6.322705e-03	1.595385e-02	5.229560e-03
##	97	98	99	100	101	
##	2.999589e-02	8.587693e-03	1.559762e-03	1.097302e-03	7.343219e-01	

7. Influential point

```
set.seed(1)
x=rnorm(100)
e=rnorm(100,0,0.1)
y=-1+x+e
fit1=lm(y~x)
#summary(fit1)
x=c(x,1.5)
y=c(y,-1)#influential point
par(mfrow=c(1,5), mai = c(1, .5, 0.2, 0.2))
plot(x,y)
text(x[101]-0.3,y[101], "101")
fit2=lm(y~x)
plot(fit2)
```



```
#summary(fit2)
cooks.distance(fit2)
```

##	1	2	3	4	5	6
##	1.087522e-03	9.692572e-05	3.324978e-03	3.244519e-03	2.972141e-04	1.026271e-02
##	7	8	9	10	11	12
##	1.881622e-03	3.779175e-03	9.613529e-04	6.207635e-03	5.584729e-05	7.681003e-05
##	13	14	15	16	17	18
##	5.678826e-03	1.616814e-02	2.014578e-04	1.046787e-04	4.770160e-05	3.982810e-05
##	19	20	21	22	23	24
##	1.938023e-03	3.407651e-05	4.140980e-05	7.270761e-03	2.716499e-06	3.306404e-03
##	25	26	27	28	29	30
##	9.562957e-05	1.232749e-03	3.140927e-06	4.657547e-04	9.958561e-04	7.309053e-06
##	31	32	33	34	35	36
##	1.523855e-03	3.663605e-04	1.090160e-03	3.206879e-03	8.312386e-05	4.914344e-03
##	37	38	39	40	41	42
##	1.316455e-04	2.556780e-04	1.739675e-04	2.098794e-04	5.827787e-03	3.080837e-03
##	43	44	45	46	47	48
##	4.106739e-03	5.671937e-05	3.864910e-03	1.839849e-03	9.521169e-03	3.350696e-04
##	49	50	51	52	53	54
##	2.341288e-03	4.501556e-03	8.882594e-04	9.401963e-07	1.018755e-05	5.583593e-03
##	55	56	57	58	59	60
##	4.986403e-03	1.641675e-03	2.393392e-03	2.408670e-03	2.401794e-03	6.983980e-03
##	61	62	63	64	65	66

##	1.942815e-02	1.474588e-05	4.414164e-03	1.808759e-03	1.358882e-03	9.493252e-03
##	67	68	69	70	71	72
##	3.096881e-03	4.437163e-03	3.957765e-06	9.322105e-03	1.256607e-02	2.020470e-05
##	73	74	75	76	77	78
##	1.244325e-03	1.000895e-04	1.403052e-03	6.339187e-05	1.578310e-03	8.245665e-03
##	79	80	81	82	83	84
##	2.354391e-03	3.947367e-03	3.899049e-03	2.151941e-03	1.799517e-03	2.182469e-02
##	85	86	87	88	89	90
##	1.423218e-03	1.110478e-05	1.241086e-02	9.481298e-04	5.785390e-05	8.267542e-04
##	91	92	93	94	95	96
##	6.468307e-05	3.059462e-03	3.123971e-04	3.130270e-03	2.659499e-03	1.140656e-03
##	97	98	99	100	101	
##	9.465909e-03	2.663317e-03	3.646772e-04	2.826429e-04	1.193439e+00	