prior $p(\theta)$ → Observational Process → posterior $p(\theta|y)$

likelihood $p(y|\theta)$ → Observational Process

Beta-Binomial (H) (T)

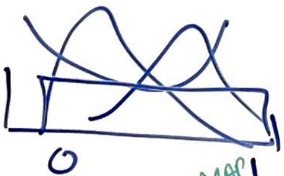
$Y \sim \text{Binomial}(n, \theta)$

$0, 1, 2, \ldots n$

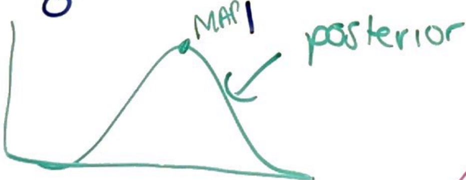
- fixed # of trials which have 2 possible outcomes

$p(\theta) \sim \text{Beta}(\underline{a}, \underline{b}) \in [0,1]$

hyperparameters



$E[\theta|y] = \frac{\alpha}{\beta+\alpha}$



Gamma-Poisson

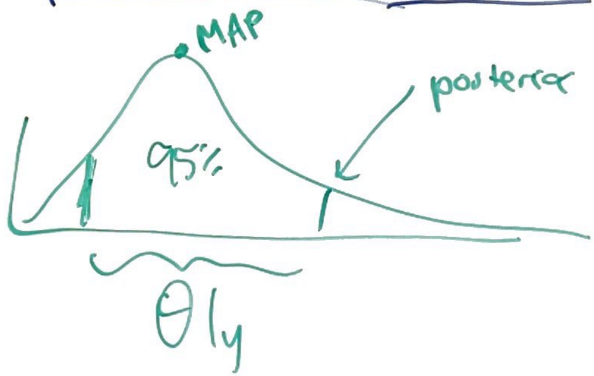
$Y \sim \text{Poisson}(\theta)$

$0, 1, 2, \ldots$

- counts the number events within a fixed tim interval, where these events happen at a fixed rate (θ)

$p(\theta) \sim \text{Gamma}(\underline{c}, \underline{d}) \in (0, \infty)$

hyperparameters



$E[\theta|y] = \frac{\alpha}{\beta}$

thin
these
θ)

$(0,\infty)$

ters

$E[\theta|y]$

$= \frac{\alpha}{\beta}$

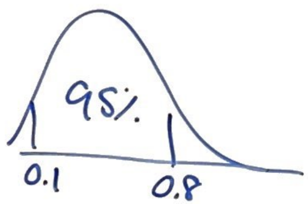
How to set prior?

① Plot various curves (different hyperparameters)

② Set prior sample size.

③ Solving for hyperparameters using mean & variance formulas

④ Prior probability intervals



qbeta(0.025, a, b) = lower

qbeta(0.975, a, b) = upper

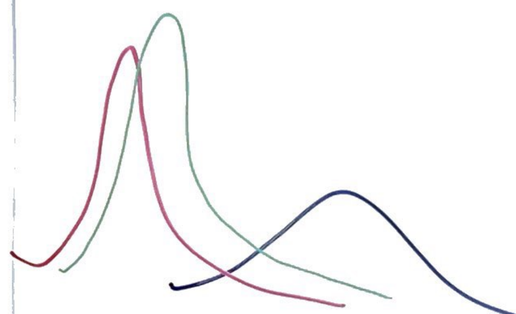
qbeta$(0.025, \tilde{\alpha}, \tilde{\beta})$ ← posterior parameters

primary concepts

- conjugate priors.
- set a reasonable prior.
- getting the posterior.
- how to summarize the posterior (MAP, $E[\theta|y]$, posterior probabilities)
- tri-plot interplay

2nd concepts

- constant of proportionality

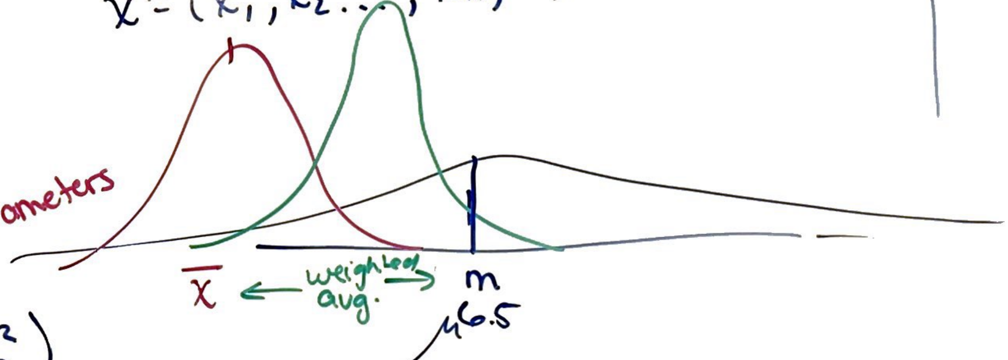


$$\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n} \Rightarrow \sum x_i = n\bar{x}$$

Normal Normal

$\vec{x} = (x_1, x_2 \ldots, x_n)$ μ

$$X_i \sim N(\underset{?}{\mu}, \underset{\text{known}}{\underline{\underline{\sigma^2}}})$$



$$\mu \sim N(\underline{m}, \underline{s^2})$$

hyperparameters

$$\mu | x \sim N(\underbrace{\mu_n, \sigma_n^2}_{\text{posterior parameter.}})$$

$$\frac{1}{\sqrt{2\pi\sigma^2}} \exp\left\{ \frac{(x-\boxed{\mu})^2}{2\boxed{\sigma^2}} \right\}$$

$$\theta | x \sim \exp\left\{ \frac{(\mu-\boxed{b/a})^2}{2\boxed{(1/a)}} \right\}$$