Likelihood Function: Coin example

Adopting the MLE approach, we aim to find the value of θ which maximizes $\mathcal{L}(\theta) = \theta^7(1-\theta)^3$. We can arrive at this answer using derivatives:

natural

$$\log(\mathcal{L}(\theta)) = \ell(\theta) = 7\log(\theta) + 3\log(1-\theta)$$

$$\ell'(\theta) = \frac{7}{\theta} + \frac{-3}{1-\theta}$$

Set $=0$ solve for θ

$$\frac{7}{\theta} = \frac{3}{1-\theta}$$

$$\Rightarrow \hat{\theta} = \frac{7}{10}$$ $\leftarrow$ maximum likelihood estimator (M.L.E).

Discrete

Considers three types of coins: Type A, B, and C. Each has a different probabilities of landing heads when tossed.

A coins are fair, with probability 0.5 of heads $\theta = 0.5$

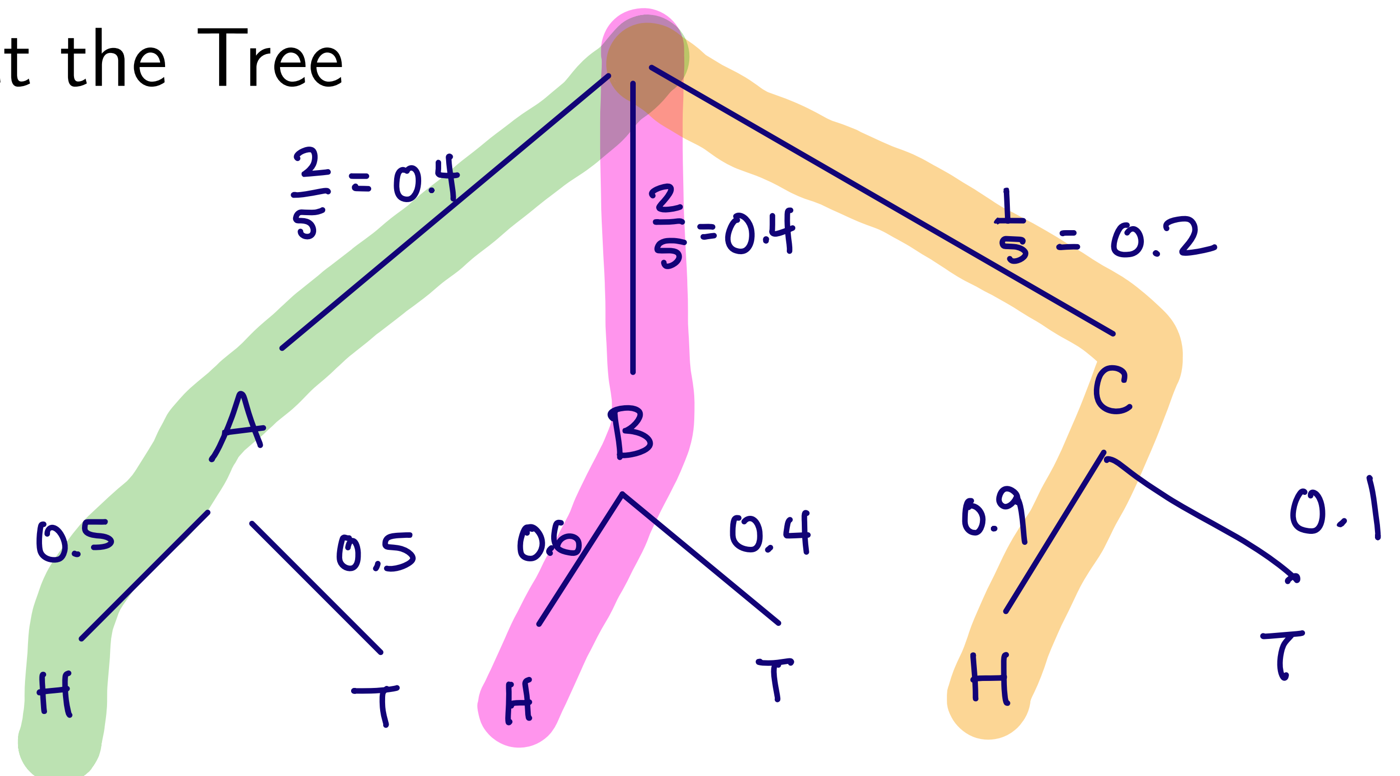
B coins are bent and have probability 0.6 of heads $\theta = 0.6$

C coins are bent and have probability 0.9 of heads $\theta = 0.9$

Suppose I have a drawer containing 5 coins: 2 of type A, 2 of type B, and 1 of type C. I reach into the drawer and pick a coin at random. Without showing you the coin, I flip it once and get heads. What is the probability it is type A? Type B? Type C?

Source: Jeremy Orloff, and Jonathan Bloom. 18.05 Introduction to Probability and Statistics. Spring 2014. Massachusetts Institute of Technology: MIT OpenCourseWare, https://ocw.mit.edu.

Construct the Tree



$P(A \cap H) = P(H|A) \cdot P(A)$
$= (0.5)(0.4)$
$= 0.20$

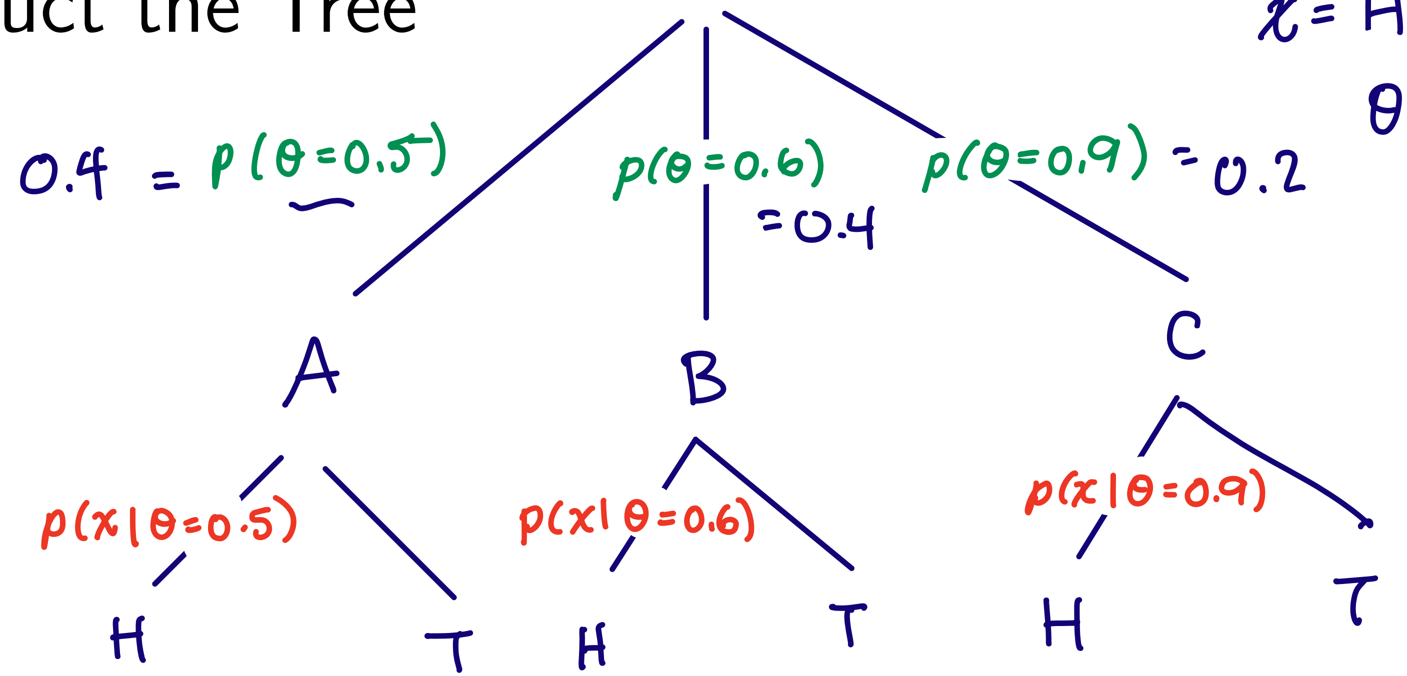
$P(B \cap H) = P(H|B) \cdot P(B)$
$= (0.6)(0.4)$
$= 0.24$

$P(C \cap H) = P(H|C) \cdot P(C)$
$= (0.9)(0.2)$
$= 0.18$

Construct the Tree

$x = H$

θ = bias towards H



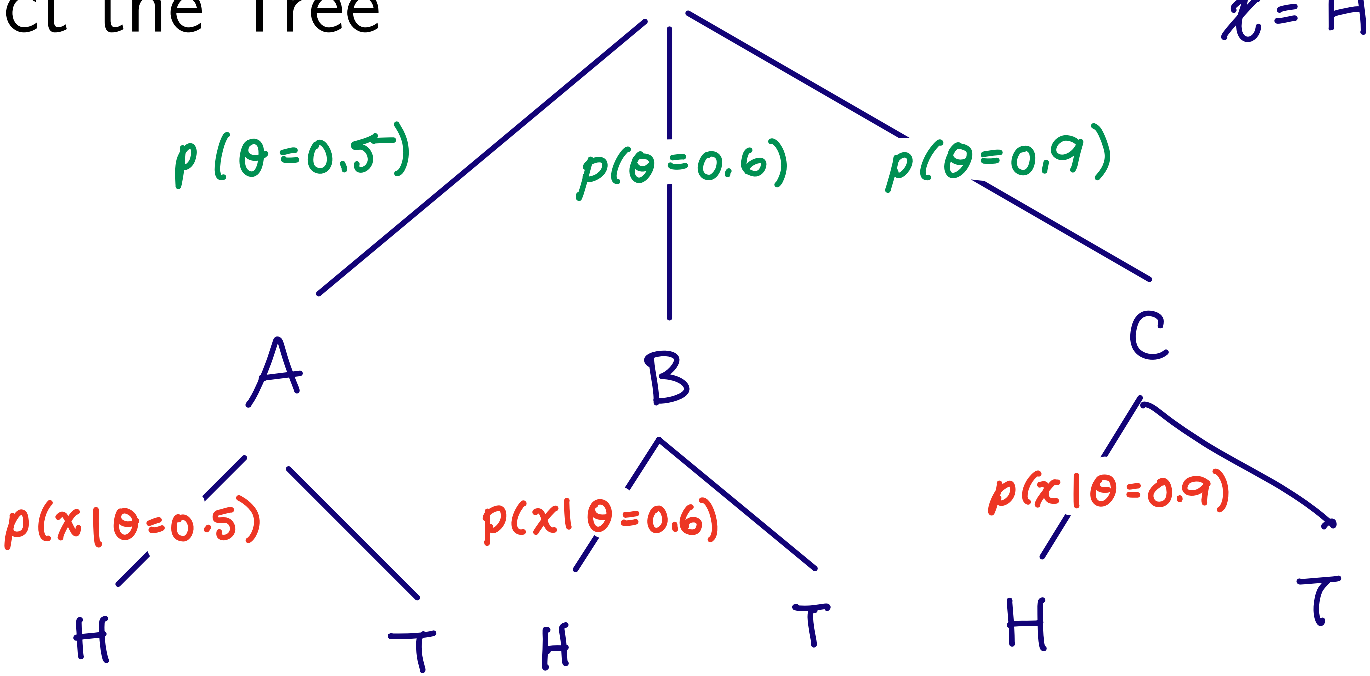
$$P(A \cap H) = P(H|A) \cdot P(A) = (0.5)(0.4) = 0.20$$

$$P(B \cap H) = P(H|B) \cdot P(B) = (0.6)(0.4) = 0.24$$

$$P(C \cap H) = P(H|C) \cdot P(C) = (0.9)(0.2) = 0.18$$

Construct the Tree

$x = H$



$= P(H|A)\cdot P(A)$
$= (0.5)(0.4)$
$= 0.20$

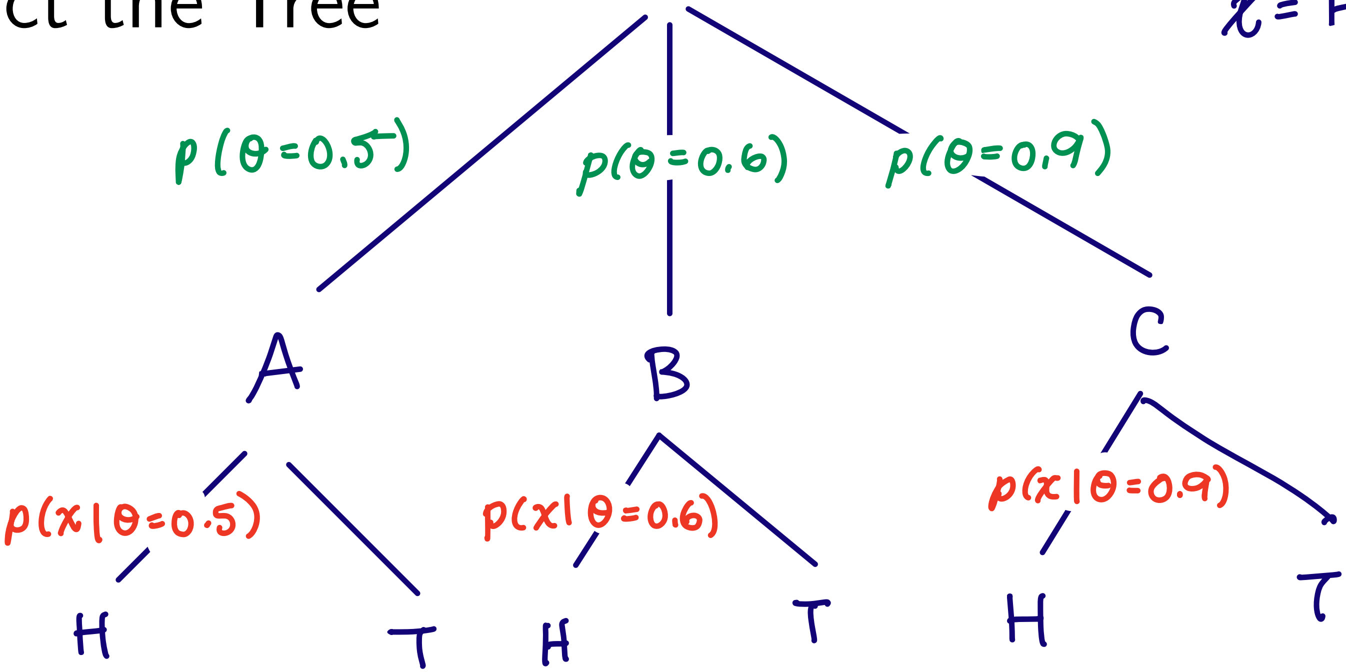
$= P(H|B)\cdot P(B)$
$= (0.6)(0.4)$
$= 0.24$

$= P(H|C)\cdot P(C)$
$= (0.9)(0.2)$
$= 0.18$

Bayes numerator
(unnormalized posterior)

Construct the Tree

$x = H$



$$P(A \cap H) = P(H|A)\cdot P(A) = (0.5)(0.4) = 0.20$$

$$P(B \cap H) = P(H|B)\cdot P(B) = (0.6)(0.4) = 0.24$$

$$P(C \cap H) = P(H|C)\cdot P(C) = (0.9)(0.2) = 0.18$$

$$p(x) = P(H \cap A) + P(H \cap B) + P(H \cap C)$$
$$= P(H|A)\cdot P(A) + P(H|B)\cdot P(B) + P(H|C)\cdot P(C)$$
$$= 0.20 + 0.24 + 0.18 = 0.62$$