Mean-Variance Optimization with Technical Analysis

By

Dhun Sheth

Supervisor: Roy Kwon
April 2020

B.A.Sc. Thesis

Division of Engineering Science
UNIVERSITY OF TORONTO

Mean-Variance Optimization with Technical Analysis

By

Dhun Sheth

Supervisor: Roy Kwon

April 2020

Abstract

The following thesis addresses the gap between optimization models, specifically the mean-variance optimization and the inclusion of technical analysis strategies used in industry to direct investment decisions. A mean-variance problem using the original model and an altered version, where the risk matrix, Q, punishes stocks which do not trend, was solved for a portfolio consisting of five stocks. The portfolio was then back tested from the start of 2016 to the end of 2019. It was found that the altered MVO outperformed the portfolio found using the original model. More specifically, two methods were used to alter the original MVO, the second method punished non-trending assets more severely than the first method. Method 1 outperformed the original MVO portfolio by 2.2%, while method 2 outperformed by 8.02%. The portfolios were also compared to the SPY ETF as a benchmark and all portfolios outperformed the S&P500 index. By showing this case, it can be seen that including technical analysis can result in more optimal portfolios, however, this was not formally proved in this paper. Future work under consideration aims to formalize this finding along with testing more sophisticated optimization models with more complex technical analysis strategies.

Acknowledgments

I would like to show appreciation to Professor Kwon for his guidance and support over the last two years. I would also like to thank my peers and teaching assistants for their time, patience and help throughout the many courses and projects I had participated in.

Table of Contents

1. Introduction

Portfolio optimization is the process of determining the weights in a portfolio of assets such that, for a specific level of expected return, no less risk can be taken, or, for some level of risk, no greater expected return can be attained [1]. This idea was initially developed when Markowitz showed the effect of diversification, why investors are risk averse and want the lowest possible risk for their desired return [2]. This led Markowitz to create the Mean-Variance optimization problem which seeks to minimize variation for some desired return [2].

Although many academics argue that financial markets follow a random walk, that is, stock prices are completely random, industry professionals emphasize this behavior exists for the short term and over a longer period of time, stocks develop identifiable trends [3]. Due to the lack of a universally accepted understanding of stock price movement, many optimization models only use statistical parameters found from stock prices rather than also incorporating technical analysis strategies to identify stock trends. The aim of this research is to solve an optimization problem which combines statistical and technical analysis methodologies to reward assets which exhibit a trend over the long-term while punishing assets that do not. This will aid in the understanding of technical analysis in portfolio optimization, which can lead to the use of more sophisticated technical strategies and more robust optimization.

The following paper will first review relevant topics and literature related to the project, followed by an outline of completed and future work to be done. The optimization problem specifications will be highlighted alongside the approach to altering the known mean-variance problem.

2. Literature Review

The following section will review current literature regarding mean-variance optimization, factor models, and common technical analysis strategies.

2.1 Mean-Variance Optimization

A new perspective for asset allocation was shared when Markowitz published his Theory of Portfolio Selection in 1952 [4]. It emphasized that risk and return be considered as two sides of the same coin and that investors are risk averse. To elaborate, consider portfolio A and B [2].

Portfolio A: 100% invested in security 1 (ie. $w_1 = 1, w_i = 0 \; \forall \; i = 2...n$), with random return R_A

Portfolio B: An equally weighted portfolio (ie. $\boldsymbol{w_i = 1/n} \; \forall \; \boldsymbol{i = 1...n}$), with random return R_B

Then, the expected return of portfolio A and B is: $E[R_A] \; = \; E[R_B] \; = \mu$

The variance of portfolio A is: $var(R_A) \; = \; \sigma^2$

The variance of portfolio B is: $var(R_B) \; = \; \sigma^2/n$

The above example demonstrates that both portfolios share the same expected return, however, portfolio B offers a lower level of risk for the same return. It then becomes evident that any reasonable investor would prefer portfolio B over A, given the same expected return but lower risk. This idea of investors seeking risk averse portfolios led to the formulation of the mean-variance optimization problem shown below.

$$min \; x^T Qx$$

$$s.t \; \mu^T x \geq R$$

$$1^T x \leq 1$$

$$x_i \geq 0, i = 1...n$$

In the above problem, x represents the matrix of optimal weights for the portfolio. Q is the risk matrix calculated using the factor loadings and variance. The first constraint ensures the portfolio return is equal to or larger than the target return, while the second constraint ensures more than 100% of the budget is not used. Finally, there is no short selling allowed in this model, characterized by the last constraint.

2.2 Factor Model

Factor models are used to describe an asset's return by creating a linear function using factors which describe the asset well. Below is a very general example of an n-factor model [5].

$$r_i = \beta_0 f_0 + \beta_1 f_1 + \beta_2 f_2 + \ldots + \beta_n f_n$$

Then the expected return of asset i can be equated to the expectation of this linear function. By doing so, the asset risk can then be estimated as the variance of these factors, scaled by the factor loadings. This would come to replace the straightforward statistical variance that would have been calculated using historical asset prices, which contains significant noise.

A more specific factor model is the Fama-French three factor model, shown below. This model assumes an assets entire return can be explained by the market, small minus big, and high minus low factors [5].

$$r_i = \beta_{1i}(R_M - R_f) + \beta_{2i} SMB + \beta_{3i} HML + \epsilon_i$$

Although factor models which describe an asset's return more precisely exist, the Fama-French three factor model is a very common model used in application and study [5]. In addition, the model offers enough accuracy to be valid for this research, and also allows for readily available historical factor data to be used.

2.3 Technical Analysis

As described in the book, Technical Analysis of the Financial Markets by John Murphy, "technical analysis is the study of market action, primarily through the use of charts, for the purpose of forecasting future price trends" [6, pg.1]. He continues to describe the movement of price through trends and that without accepting price follows trends, technical analysis will provide little conformation. This clarifies that technical analysis depends on the identification of market trends to then prepare a trading strategy. But then the question of how to identify a trend arises, and although several indicators and tools can be used, a very common indicator is the moving average. The moving average is a lagging technical indicator and is also the basis for many mechanical trend-following systems used in practice [6, pg.195].

A moving average is the average closing price from the last x days. If it is a 100-day moving average, this means it is the average price over the last 100 days. The reactivity of the moving average to current price changes is determined by the number of days it is averaged over. As an example, consider a 5 and 50-day moving average, the 5-day moving average will react much more to today's price than the 50-day because it is only averaged over the last 5 days, rather than over the last 50 days. Ideally a longer moving average should be used to understand longer term trends, and an asset whose price follows this long-term moving average more closely can be said to be more predictable, thus having less volatility.

3. Methods

The following section will review the problem formulation, reasoning for specific decisions made, and where the data used in the project comes from.

3.1 Problem Formulation

Original mean-variance problem:

$$\min x^T Q x$$

$$s.t\ \mu x \geq R, 1^T x = 1, Max\ Holding\ Limit \geq x \geq Min\ Holding\ Limit$$

Altered mean-variance problem:

$$\min x^T Q^* x\, , Q^* = Q * coef.$$

$$s.t\ \mu x \geq R, 1^T x = 1, Max\ Holding\ Limit \geq x \geq Min\ Holding\ Limit$$

Markowitz's Mean-Variance optimization will be used as the base model and then altered to reward assets for showing trend-like behavior. To allow for sufficient calibration of parameters, historical data from September 1st, 2004 to December 30th, 2015 will be used in the linear regression to determine the factor loading and also calculate the moving average. This will then be used to setup the optimization to calculate the optimal weights for the original MVO as well as the altered version, and then both portfolios will be tested using data from 2016 to the end of 2019.

The Fama-French three factor model will be used to reduce the error in estimating the return and variance of assets. This is done because errors in the estimated mean and covariance matrix grow significantly to produce incorrect optimal weights.

A portfolio of five assets will be used to create a potential user portfolio for which the optimal weights must be calculated. The portfolio will cover prominent sectors driving the US economy: health care, technology, construction, retail, and nondurable manufacturing sectors [7]. It was decided that individual stock prices will be used rather than exchange traded funds (ETF's) because stocks generally show more volatility. By using assets with larger volatilities, the effectiveness of integrating technical analysis into the optimization can be better seen and measured.

Stocks:

1. <u>Health Care:</u> Pfizer (PFE) [8]
2. <u>Technology:</u> Microsoft (MSFT) [9]
3. <u>Construction:</u> Vulcan Materials Company (VMC) [10]
4. <u>Retail:</u> Williams-Sonoma (WSM) [11]
5. <u>Nondurable Manufacturing:</u> Tyson Foods (TSN) [12]

Additionally, the MVO model has been known to create very concentrated portfolios, this will be reduced by adding a no short or long only constraint to the model [2]. To further enforce diversification, a minimum holding limit of 5% and a maximum of 30% was added. The minimum limit was chosen arbitrarily along with a daily return goal of 20%/252 = 0.079%, however, both assumptions are reasonable minimum and target return values. The return goal is being specified as a daily return target because the mu matrix was defined using daily returns. Then, portfolios using the original mean-variance problem were generated using max goals of 25%, 30%, and 35%. Each portfolio performance was determined and a max limit of 30% was chosen for superior performance under the original mean-variance problem, as seen in appendix

A. The chosen parameters for minimum holding limit = 5%, max holding limit = 30%, and a daily return goal of 20%/252 = 0.079% was used for both the original and altered optimization.

In technical analysis, moving averages are excellent ways to identify trend-like behavior because a stock with smaller deviation from its moving average shows more trend-like characteristics than one with greater deviation. As such, a long-term moving average, specifically the 100-day moving average, will be used and compared to the stock prices to help identify stocks which show better long-term trends. The 100-day moving average was chosen because it does not whipsaw as easily as the 50-day, but is also not as stagnant as the 200-day moving average, as seen in appendix B.

The risk matrix, Q, was altered by multiplying it using coefficients calculated using two methods, the difference being the first method did not punish non-trending assets as much as the second did. In the first method, the coefficient is determined by taking the highest ratio of the standard deviation of stock price minus the 100-day moving average for each stock against all the other stocks in the portfolio. In the second method, rather than choosing the highest ratio value, it is now a product of all ratios which are greater than 1, to severely punish a stock every time its standard deviation of price minus 100-day moving average is greater than another's. The results for the standard deviation of price minus the 100-day moving average and the coefficients calculated using methods one & two can be found in the below tables.

Table 1: Summary of standard deviation of price minus 100-day MA for each asset.

Stock	**Stdev(Price – 100d MA)**
MSFT	2.5066
PFE	1.5405
TSN	2.3758
VMC	7.6344
WSM	4.1665

Table 2: Summary of coefficients calculated using methods one and two.

	MSFT	PFE	TSN	VMC	WSM
MSFT	1.00	0.61	0.95	3.05	1.66
PFE	1.63	1.00	1.54	4.96	2.70
TSN	1.06	0.65	1.00	3.21	1.75
VMC	0.33	0.20	0.31	1.00	0.55
WSM	0.60	0.37	0.57	1.83	1.00
Coefficients for Alt MVO 1 – Method 1	1.6271421	1.0000000	1.5422428	4.9557831	2.7046360
Coefficients for Alt MVO 2 – Method 2	1.7167150	1.0000000	1.5422428	88.871420	7.8840291

3.2 Pricing and Factor Data

Daily closing prices for each asset were retrieved from the Yahoo Finance database and were used to calculate the 100-day simple moving average by averaging the last 100 prices. Pricing data from September 1st, 2004 to December 30th, 2015 will be used to regress over in calculating the factor loading.

The factor data for the Fama-French Three factor model will be acquired from the Kenneth R. French Data library [13]. Daily factor data will be equated to the daily returns, calculated from the historical pricing data recovered earlier from Yahoo Finance, in order to determine the factor loading values.

4. Results and Discussion

The optimal portfolios under the original mean-variance problem, and the altered problem using both methods can be found below.

Table 3: Portfolio weights for optimal portfolio under original and altered mean-variance problem.

Portfolio Problem	**MSFT**	**PFE**	**TSN**	**VMC**	**WSM**
MVO	24.89%	30.00%	30.00%	5.00%	10.11%
Alt MVO 1	26.34%	30.00%	30.00%	5.00%	8.66%
Alt MVO 2	30.00%	30.00%	30.00%	5.00%	5.00%

The portfolio performance and characteristics for each optimal portfolio can be seen in the below figure and table, each portfolio was rebalanced annually during testing.

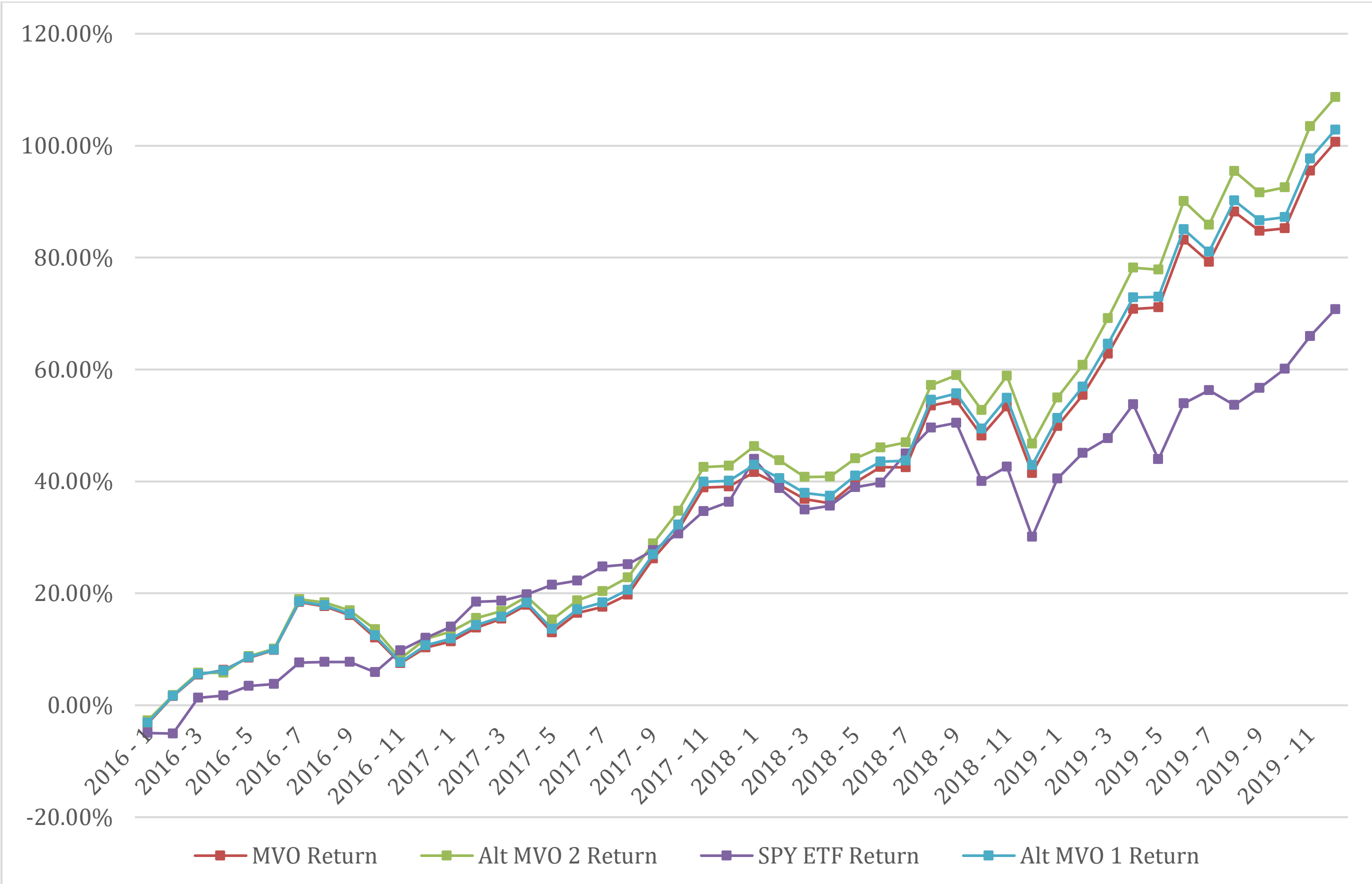

Figure 1: Portfolio performance of MVO, Altered MVO, and SPY for the period of 2016 to 2019

Table 4: A comparison of performance metrics for each portfolio for the period of 2016 to 2019

Comparison Metric	**MVO**	**Alt MVO 1**	**Alt MVO 2**	**SPY ETF**
Average Annual Return	19.83%	20.15%	20.98%	15.05%
Annual Volatility	11.89%	11.83%	11.74%	11.51%
Sharpe Ratio	1.43	1.46	1.53	1.11
Best Year	41.84%	41.95%	42.22%	31.22%
Worst Year	1.72%	2.02%	2.76%	-4.56%
Max. Drawdown	-9.24%	-9.16%	-8.95%	-13.52%
US Market Correlation	0.64	0.64	0.65	1.00
Annual Alpha	8.99%	9.26%	9.94%	0.00%

It can be seen in figure 1, the portfolio found from the altered mean-variance problem which punishes non-trending assets more severely, outperformed the other portfolios along with the SPY ETF. The Alt MVO 2 portfolio yielded a total return which is 5.82% greater than the Alt MVO 1 portfolio, and 8.02% greater than the MVO portfolio. In addition, the Alt MVO 2 portfolio averages a higher annual return over the 2016 to 2019 test period. It also then follows that the Alt MVO 2 portfolio has the lowest Sharpe ratio because it has the highest annual return and lowest annual volatility, resulting in a Sharpe ratio of 1.53, as seen in Table 4. Furthermore, this portfolio has the highest US market correlation among the optimal portfolios, then it is not unreasonable to also assume it would have the largest max drawdown or worst year, however, it not only performs better in its worst year than the other optimal portfolios, but it also has the lowest max drawdown, as seen in Table 4. This can also be partially attributed to it having the lowest annual volatility.

The benefit of including technical analysis into portfolio optimization can be seen in the above case. During times of increased volatility, assets which show better trend-like behavior may resist movement against its trend a lot more efficiently than assets which do not have this behavior. Although the case observed in this paper does show better performance from a portfolio which punishes stocks more harshly for not having trend-like behavior, the difference between how the coefficients are calculated in methods one and two for the altered mean-variance problem, it does not always guarantee it. Also, many combinations of parameters were tested to better understand if including technical analysis, i.e. the Alt MVO, always leads to superior performance. In all tested cases the Alt MVO did outperform, however, this has yet to be more concretely proven for it to be a guaranteed. The first example below will demonstrate a case where the portfolio from Alt MVO 1 outperforms Alt MVO 2. The second example will show a case where the performance of MVO, Alt MVO 1, and Alt MVO 2 are very close, however, the MVO is still beaten by the Alt MVO.

Example 1: Alt MVO 1 outperforming Alt MVO 2

<u>Parameters:</u> min holding limit = 5%, max holding limit = 45%, daily return target is 20%/252 = 0.079%.

Table 5: Optimal weights under different test parameters to show superior Alt MVO 1 performance.

Portfolio Problem	**MSFT**	**PFE**	**TSN**	**VMC**	**WSM**
MVO	5.00%	45.00%	40.00%	5.00%	5.00%
Alt MVO 1	8.31%	45.00%	36.69%	5.00%	5.00%
Alt MVO 2	7.69%	45.00%	37.31%	5.00%	5.00%

Portfolio Returns

Portfolio	Initial Balance	Final Balance	CAGR	Stdev	Best Year	Worst Year	Max. Drawdown	Sharpe Ratio	Sortino Ratio	US Mkt Correlation
Portfolio 1	$10,000	$17,887	15.65%	13.52%	34.18%	-2.00%	-14.23%	1.05	1.89	0.44
Portfolio 2	$10,000	$18,163	16.09%	12.84%	33.64%	-0.23%	-13.32%	1.13	2.03	0.48
Portfolio 3	$10,000	$18,112	16.01%	12.96%	33.74%	-0.56%	-13.49%	1.12	2.01	0.47
SPDR S&P 500 ETF Trust	$10,000	$17,071	14.31%	11.51%	31.22%	-4.56%	-13.52%	1.11	1.70	1.00

Figure 2: To show Portfolio 2 → Alt MVO 1 outperformance [14]

It can be seen in Table 5 and Figure 2, the Alt MVO 1 outperforms Alt MVO 2 slightly, for the test parameters in Example 1. This case is important because it questions which method of altering the risk matrix, Q, based on the calculated coefficients is better. Although this is not the topic for this report, it can be seen as an important idea to explore later on, that is, what is the best method for judging trend-like behavior and how should assets not exemplifying this behavior be punished.

Example 2: Performance of MVO and Alt MVO becoming very similar

Parameters: min holding limit = 5%, max holding limit = 28%, daily return target is 20%/252 = 0.079%.

Table 6: Optimal weights which show similar performance of MVO and Alt MVO

Portfolio Problem	MSFT	PFE	TSN	VMC	WSM
MVO	27.76%	28.00%	28.00%	5.00%	11.24%
Alt MVO 1	28.00%	28.00%	28.00%	5.00%	11.00%
Alt MVO 2	28.00%	28.00%	28.00%	5.00%	11.00%

Portfolio Returns

Portfolio	Initial Balance	Final Balance	CAGR	Stdev	Best Year	Worst Year	Max. Drawdown	Sharpe Ratio	Sortino Ratio	US Mkt Correlation
Portfolio 1	$10,000	$20,362	19.45%	11.75%	42.72%	2.49%	-8.84%	1.47	2.81	0.67
Portfolio 2	$10,000	$20,399	19.51%	11.74%	42.74%	2.53%	-8.81%	1.48	2.82	0.67
Portfolio 3	$10,000	$20,399	19.51%	11.74%	42.74%	2.53%	-8.81%	1.48	2.82	0.67
SPDR S&P 500 ETF Trust	$10,000	$17,071	14.31%	11.51%	31.22%	-4.56%	-13.52%	1.11	1.70	1.00

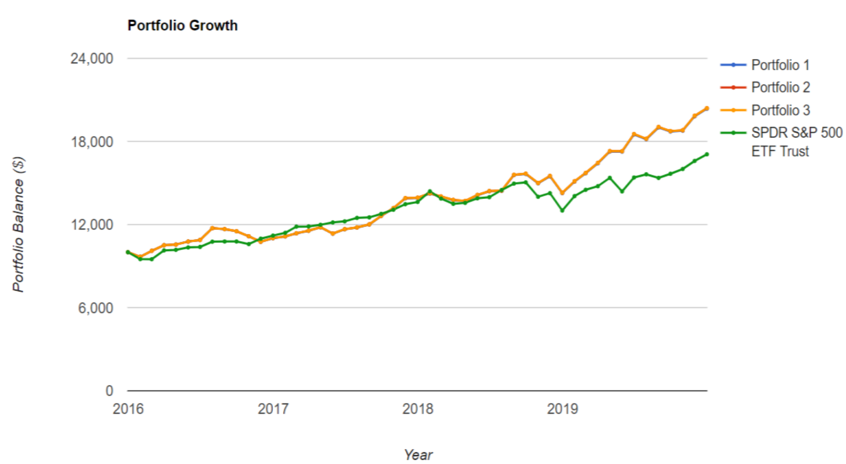

Figure 3: To show similar performance of MVO and Alt MVO [15]

As seen in Table 6 and Figure 3, the MVO, Alt MVO 1, and Alt MVO 2 have an almost identical value, however, the Alt MVO's just beat the MVO by a small margin. This is true for their Sharpe ratios as well. From this example, it can be seen that a more concrete justification is necessary to conclude that including technical analysis into the mean-variance problem always yields a better performing portfolio. However, with this example and the preceding cases it is also clear that including technical analysis can also lead to more optimal portfolios, widening the possible combinations and problem complexity.

5. Conclusion

This paper has shown the possibility of improving portfolio performance by including technical analysis in the standard mean-variance problem. As mentioned before however, although better performance is likely to be observed, it is not guaranteed. Additional work still needs to be done to formally prove including technical analysis always leads to better performance.

Future work under consideration to build on this paper includes the proof mentioned above, as well as pursuing more complex optimization models like Robust MVO or the Black-Litterman model and utilizing more complex technical analysis indicators such as Bollinger Bands and Kelter Channels to identify trend like characteristics.

6. References

1. Macedo, Luís Lobato, et al. "Mean-Semivariance Portfolio Optimization with Multiobjective Evolutionary Algorithms and Technical Analysis Rules." *Expert Systems with Applications*, Pergamon, 22 Feb. 2017, www.sciencedirect.com/science/article/pii/S0957417417301252.

2. Haugh, Martin. *Mean-Variance Optimization and the CAPM*. Columbia University, 2016, www.columbia.edu/~mh2078/FoundationsFE/MeanVariance-CAPM.pdf.

3. Staff, Motley Fool. "What Is the Random Walk Hypothesis?" *The Motley Fool*, The Motley Fool, 28 June 2016, www.fool.com/knowledge-center/what-is-the-random-walk-hypothesis.aspx.

4. Kolm, Petter N., et al. "60 Years of Portfolio Optimization: Practical Challenges and Current Trends." *European Journal of Operational Research*, North-Holland, 1 Nov. 2013, www.sciencedirect.com/science/article/abs/pii/S0377221713008898.

5. Li, Jialiang, et al. "Factor Models for Asset Returns Based on Transformed Factors." *Journal of Econometrics*, North-Holland, 3 Oct. 2018, www.sciencedirect.com/science/article/abs/pii/S0304407618301660.

6. Murphy, John J. *Technical Analysis of the Financial Markets: a Comprehensive Guide to Trading Methods and Applications*. New York Institute of Finance, 1999.

7. Deutsch, Alison L. "The 5 Industries Driving the U.S Economy." *Investopedia*, Investopedia, 18 Nov. 2019, www.investopedia.com/articles/investing/042915/5-industries-driving-us-economy.asp.

8. Farley, Alan. "Top Health Care Stocks for January 2020." *Investopedia*, Investopedia, 13 Jan. 2020, www.investopedia.com/investing/top-healthcare-stocks/.

9. Green, Timothy. "The 10 Biggest Tech Stocks." *The Motley Fool*, The Motley Fool, 28 July 2019, www.fool.com/investing/10-biggest-tech-stocks.aspx.

10. Olson, Sheila. "Top 4 Long-Term Infrastructure Stocks of 2019." *Investopedia*, Investopedia, 6 Jan. 2020, www.investopedia.com/investing/infrastructure-stocks/.

11. Goldman, M. Corey, et al. "Best-Retail-Stocks-for-This-Year: Top Rated Stocks:" *TheStreet*, TheStreet, www.thestreet.com/topic/25866/top-rated-equity-multiline-retail.html.

12. Carnevale. "11 Investable Consumer Non-Durables: Finding Value In Food, Footwear, Apparel, Alcohol And Tobacco: Part 4." *Seeking Alpha*, Seeking Alpha, 9 Feb. 2019, https://seekingalpha.com/article/4239637-11-investable-consumer-non-durables-finding-value-in-food-footwear-apparel-alcohol-and.

13. French, Kenneth R. "Current Research Returns Data." *Kenneth R. French - Data Library*, https://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html.
14. https://www.portfoliovisualizer.com/backtest-portfolio?s=y&timePeriod=4&startYear=2016&firstMonth=1&endYear=2019&lastMonth=12&calendarAligned=true&initialAmount=10000&annualOperation=0&annualAdjustment=0&inflationAdjusted=true&annualPercentage=0.0&frequency=4&rebalanceType=1&absoluteDeviation=5.

0&relativeDeviation=25.0&showYield=false&reinvestDividends=true&benchmark=-1&benchmarkSymbol=SPY&symbol1=MSFT&allocation1_1=5&allocation1_2=8.31&allocation1_3=7.69&symbol2=PFE&allocation2_1=45&allocation2_2=45&allocation2_3=45&symbol3=TSN&allocation3_1=40&allocation3_2=36.69&allocation3_3=37.31&symbol4=VMC&allocation4_1=5&allocation4_2=5&allocation4_3=5&symbol5=WSM&allocation5_1=5&allocation5_2=5&allocation5_3=5

15. https://www.portfoliovisualizer.com/backtest-portfolio?s=y&timePeriod=4&startYear=2016&firstMonth=1&endYear=2019&lastMonth=12&calendarAligned=true&initialAmount=10000&annualOperation=0&annualAdjustment=0&inflationAdjusted=true&annualPercentage=0.0&frequency=4&rebalanceType=1&absoluteDeviation=5.0&relativeDeviation=25.0&showYield=false&reinvestDividends=true&benchmark=-1&benchmarkSymbol=SPY&symbol1=MSFT&allocation1_1=27.76&allocation1_2=28&allocation1_3=28&symbol2=PFE&allocation2_1=28&allocation2_2=28&allocation2_3=28&symbol3=TSN&allocation3_1=28&allocation3_2=28&allocation3_3=28&symbol4=VMC&allocation4_1=5&allocation4_2=5&allocation4_3=5&symbol5=WSM&allocation5_1=11.24&allocation5_2=11&allocation5_3=11

7. Appendix

[A] Portfolios generated under the original MVO and then compared to determine the best upper holding limit to enforce. Parameters: Min = 5% and daily return goal = 20%/252.

Max Holding percent	MSFT	PFE	TSN	VMC	WSM	Final Balance	Sharpe Ratio
25	25.00%	25.00%	25.00%	5.00%	20.00%	$ 19,711.00	1.37
30	24.89%	30.00%	30.00%	5.00%	10.11%	$ 20,067.00	1.43
35	17.71%	35.00%	35.00%	5.00%	7.29%	$ 19,341.00	1.29

[B] A comparison between the 200, 100 and 50-day moving averages for MSFT from 2014 to 2017.

[C] Python Code for Original Mean-Variance Optimization

```
import pandas as pd
import numpy as np
from sklearn import linear_model
import scipy
import cvxpy as cp

data = pd.read_csv('Data/data.csv', header=0, index_col=0)
data_val = data.values

factor_data = pd.read_csv('Data/factor_data.csv', header=0, index_col=0)
factor_data_val = factor_data.values

i = 2
y = 1
beta = ()
while i < 15:
    y_val = data_val[:,i]
    x_val = factor_data_val

    linreg = linear_model.LinearRegression()
    model = linreg.fit(x_val, y_val)
    beta = np.append(beta, np.ndarray.tolist(model.coef_[0:3]))
    beta = np.reshape(beta, (y,3))
    y = y + 1
    i = i + 3

factor_var = []
i = 0
while i <= 2:
    factor_var = np.append(factor_var, np.var(factor_data_val[:,i]))
    i = i+1

F = np.diag(factor_var)
Q = np.matmul(np.matmul(beta, F), np.transpose(beta))

i = 0
mu = ()
while i <= 4:
    mu_temp = beta[i,0]*np.average(factor_data_val[:,0]) + beta[i,1]*np.average(factor_data_val[:,1]) +
beta[i,2]*np.average(factor_data_val[:,2]) + np.average(factor_data_val[:,3])
    mu = np.append(mu, mu_temp)
    i = i + 1

R = 0.2/252
# w_weights = []
# j = 0.01
# i = 0
# while i < 10:
w = cp.Variable(5)
ret = mu@w
risk = cp.quad_form(w, Q)
prob = cp.Problem(cp.Minimize(risk),
    [np.ones(5)@w == 1,
```

```
    w >= 0.05,
    w <= 0.45,
    ret >= R])
prob.solve()
w_weights = w.value
```

[D] Python Code for Alternate Mean-Variance Optimization

```
import pandas as pd
import numpy as np
from sklearn import linear_model
import scipy
import cvxpy as cp

data = pd.read_csv('Data/data.csv', header=0, index_col=0)
data_val = data.values

factor_data = pd.read_csv('Data/factor_data.csv', header=0, index_col=0)
factor_data_val = factor_data.values

i = 2
y = 1
beta = ()
while i < 15:
    y_val = data_val[:,i]
    x_val = factor_data_val

    linreg = linear_model.LinearRegression()
    model = linreg.fit(x_val, y_val)
    beta = np.append(beta, np.ndarray.tolist(model.coef_[0:3]))
    beta = np.reshape(beta, (y,3))
    y = y + 1
    i = i + 3

factor_var = []
i = 0
while i <= 2:
    factor_var = np.append(factor_var, np.var(factor_data_val[:,i]))
    i = i+1

F = np.diag(factor_var)
Q = np.matmul(np.matmul(beta, F), np.transpose(beta))

# Risk matrix, Q, altered to account for variation in the (Price "minus" 100_day_moving_average)
alternating_coef = np.array([[ 1.71671497,  1,  1,  1,  1],
                             [1,  1,  1,  1,  1],
                             [1,  1,  1.54224283,  1,  1],
                             [1,  1,  1, 88.87141951,  1],
                             [1,  1,  1,  1,  7.88402912]])
# alternating_coef = np.array([[ 0.5,  1,  1,  1,  1],
#                              [1,  1,  1,  1,  1],
#                              [1,  1,  1,  1,  1],
#                              [1,  1,  1, 88.87141951,  1],
#                              [1,  1,  1,  1,  4.30273645]])

# alternating_coef = np.array([[1.6271421,  1,  1,  1,  1],
#                              [1,  1,  1,  1,  1],
#                              [1,  1,  1.54224283,  1,  1],
#                              [1,  1,  1, 4.9557831,  1],
#                              [1,  1,  1,  1,  2.7046360]])
```

```
Q = Q*alternating_coef

i = 0
mu = ()
while i <= 4:
    mu_temp = beta[i,0]*np.average(factor_data_val[:,0]) + beta[i,1]*np.average(factor_data_val[:,1]) +
beta[i,2]*np.average(factor_data_val[:,2]) + np.average(factor_data_val[:,3])
    mu = np.append(mu, mu_temp)
    i = i + 1

R = 0.2/252
w = cp.Variable(5)
ret = mu@w
risk = cp.quad_form(w, Q)
prob = cp.Problem(cp.Minimize(risk),
               [np.ones(5)@w == 1,
                w >= 0.05,
                w <= 0.45,
                ret >= R])
prob.solve()
w_weights = w.value
```