ASSIGNMENT 2

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1. Rotation matrix R_0^1

Any rotation can be carried out by rotating the coordinate axes by γ about x axis, β about y axis and α about z axis

α, β, γ :- arbitrary angles

$$R_0^1 = R_z(\alpha)\, R_y(\beta)\, R_x(\gamma)$$

$$= \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\beta & 0 & \sin\beta \\ 0 & 1 & 0 \\ -\sin\beta & 0 & \cos\beta \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\gamma & -\sin\gamma \\ 0 & \sin\gamma & \cos\gamma \end{bmatrix}$$

$$= \begin{bmatrix} \cos\alpha\cos\beta & \cos\alpha\sin\beta\sin\gamma - \sin\alpha\cos\gamma & \cos\alpha\sin\beta\cos\gamma + \sin\alpha\sin\gamma \\ \sin\alpha\cos\beta & \sin\alpha\sin\beta\sin\gamma + \cos\alpha\cos\gamma & \sin\alpha\sin\beta\cos\gamma - \cos\alpha\sin\gamma \\ -\sin\beta & \cos\beta\sin\gamma & \cos\beta\cos\gamma \end{bmatrix}$$

$$\det(R_0^1) = \cos\alpha\cos\beta\big[\sin\alpha\sin\beta\sin\gamma\cos\beta\cos\gamma + \cos^2\gamma\cos\alpha\cos\beta$$
$$\cancel{+(\cos\alpha\sin\beta\sin\gamma - \sin\alpha\cos\gamma)[\cos\alpha\sin\gamma\sin\beta}$$
$$- \sin\alpha\sin\beta\cos\gamma\cos\beta\sin\gamma + \cos\alpha\sin^2\gamma\cos\beta\big]$$
$$+ (\cos\alpha\sin\beta\sin\gamma - \sin\alpha\cos\gamma)\big[\cos\alpha\sin\gamma\sin\beta - \sin^2\beta\sin\alpha\cos\gamma - \sin\alpha\cos^2\beta\cos\gamma\big]$$
$$+ (\cos\alpha\sin\beta\cos\gamma + \sin\alpha\sin\gamma)\big[\cos^2\beta\sin\alpha\sin\gamma + \sin\alpha\sin^2\beta\sin\gamma + \sin\beta\cos\alpha\cos\gamma\big]$$

$$= \cos^2\alpha\cos^2\beta + (\cos\alpha\sin\beta\sin\gamma - \sin\alpha\cos\gamma)^2 + (\cos\alpha\cos\gamma\sin\beta + \sin\alpha\sin\gamma)^2$$

$$= \cos^2\alpha\cos^2\beta + \cos^2\alpha\sin^2\beta\sin^2\gamma - 2\sin\alpha\cos\gamma\cos\alpha\sin\beta\sin\gamma + \sin^2\alpha\cos^2\gamma + \cos^2\alpha\cos^2\gamma\sin^2\beta + \sin^2\alpha\sin^2\gamma + 2\cos\alpha\cos\gamma\sin\beta\sin\alpha\sin\gamma$$

$$= 1$$

For orthogonality

$$AA^T = I \Rightarrow R_0' R_0'^T = I$$

$$= \cancel{-\cos^2\alpha\cos^2}$$

$$\cancel{= \gamma(\cos\alpha\cos\beta)^2 + \sin\alpha\sin\beta(\cos\alpha\sin\beta\sin\gamma - \sin\alpha\cos\gamma) - \sin\beta(\cos\alpha\sin\beta\cos\gamma + \sin\alpha\sin\gamma)} = 1$$

$$\cancel{\Rightarrow \cos^2\alpha\cos^2\beta + \cos\alpha\sin\alpha\sin^2\beta\sin\gamma - \sin^2\alpha\sin\beta\cos\gamma - \cos^2\alpha\sin^2\beta\cos\gamma - \sin\alpha\sin\beta\sin\gamma}$$

Computing value of each element of $R_0' R_0'^T = A$ (let's say)

$$\Rightarrow (\cos\alpha\cos\beta)^2 + (\cos\alpha\sin\beta\sin\gamma - \sin\alpha\cos\gamma)^2 + (\cos\alpha\sin\beta\cos\gamma + \sin\alpha\sin\gamma)^2$$
$$= 1 = A_{11}$$

$$A_{12} = \cos\alpha\sin\alpha\cos^2\beta + (\cos\alpha\sin\beta\sin\gamma - \sin\alpha\cos\gamma)(\sin\alpha\sin\beta\sin\gamma + \cos\alpha\cos\gamma)$$
$$+ (\cos\alpha\sin\beta\cos\gamma + \sin\alpha\sin\gamma)(\sin\alpha\sin\beta\cos\gamma - \cos\alpha\sin\gamma)$$

$$= \cos\alpha\sin\alpha\cos^2\beta + \sin\alpha\cos\alpha\sin^2\beta\sin^2\gamma + \cos^2\alpha\sin\beta\cos\gamma\sin\gamma$$
$$- \cancel{\sin^2\alpha\sin\beta\sin\gamma\cos\gamma} - \sin\alpha\cos\alpha\cancel{\cos^2\gamma} + \sin\alpha\cos\alpha\sin^2\beta\cos^2\gamma$$
$$- \cos^2\alpha\sin\beta\cos\gamma\sin\gamma + \sin^2\alpha\sin\beta\cos\gamma\sin\gamma - \sin\alpha\cos\alpha\sin^2\gamma$$
$$= \cos\alpha\sin\alpha\cos^2\beta + \sin\alpha\cos\alpha\sin^2\beta - \sin\alpha\cos\alpha = 0$$

$$A_{13} = -\cos\alpha\cos\beta\sin\beta + \cos\alpha\sin\beta\cos\beta\sin^2\gamma - \sin\alpha\cos\beta\sin\gamma\cos\gamma$$
$$+ \cos\alpha\sin\beta\cos\beta\cos^2\gamma + \sin\alpha\cos\beta\sin\gamma\cos\gamma$$
$$= 0$$

$$A_{21} = \cos\alpha\cos^2\beta\sin\alpha + (\cos\alpha\sin\beta\sin\gamma - \sin\alpha\cos\gamma)(\sin\alpha\sin\beta\sin\gamma + \cos\alpha\cos\gamma)$$
$$+ (\cos\alpha\sin\beta\cos\gamma + \sin\alpha\sin\gamma)(\sin\alpha\sin\beta\cos\gamma - \cos\alpha\sin\gamma) = 0$$

$$A_{22} = (\sin\alpha\cos\beta)^2 + (\sin\alpha\sin\beta\sin\gamma + \cos\alpha\cos\gamma)^2 + (\sin\alpha\sin\beta\cos\gamma - \cos\alpha\sin\gamma)^2$$
$$= \sin^2\alpha\cos^2\beta + \sin^2\alpha\sin^2\beta + \cos^2\alpha$$
$$= 1$$

$A_{23} = -\sin\alpha \sin\beta \cos\beta + \cos\beta \sin\gamma (\sin\alpha \sin\beta \sin\gamma + \cos\alpha \cos\gamma)$
$+ \cos\beta \cos\gamma (\sin\alpha \sin\beta \cos\gamma - \cos\alpha \sin\gamma)$
$= 0$

$A_{31} = -\sin\beta \cos\alpha \cos\beta + \cos\beta \sin\gamma (\cos\alpha \sin\beta \sin\gamma - \sin\alpha \cos\gamma)$
$+ \cos\beta \cos\gamma (\cos\alpha \sin\beta \cos\gamma + \sin\alpha \sin\gamma)$
$= 0$

$A_{32} = -\sin\alpha \sin\beta \cos\beta + \cos\beta \sin\gamma (\sin\alpha \sin\beta \sin\gamma + \cos\alpha \cos\gamma)$
$+ \cos\gamma \cos\beta (\sin\alpha \sin\beta \cos\gamma - \cos\alpha \sin\gamma)$
$= 0$

$A_{33} = + \sin^2\beta + \cos^2\beta \sin^2\gamma + \cos^2\beta \cos^2\gamma = \underline{\underline{1}}$

Thus $A = I$ (Identity Matrix)

Thus, R_0^1 is Orthogonal as $A = R_0^1 R_0^{1T}$

Hence proved

2. Any rotation matrix can be denoted as
$R_0^1 = R_z(\alpha) R_y(\beta) R_x(\gamma)$

$\det(R_0^1) = \det(R_z(\alpha)) \det(R_y(\beta)) \det(R_x(\gamma))$

$$R_z(\alpha) = \begin{bmatrix} C\alpha & -S\alpha & 0 \\ S\alpha & C\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad R_y(\beta) = \begin{bmatrix} C\beta & 0 & S\beta \\ 0 & 1 & 0 \\ -S\beta & 0 & C\beta \end{bmatrix} \quad R_x(\gamma) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & C\gamma & -S\gamma \\ 0 & S\gamma & C\gamma \end{bmatrix}$$

$\det(R_z(\alpha)) = 1 \quad \det(R_y(\beta)) = 1 \quad \det(R_x(\gamma)) = 1$

$\therefore \boxed{\det(R_0^1) = 1.}$ Hence proved

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5. T.P: ~~R f~~ $R\,S(\vec{a})R^T = S(R\vec{a})$

R : Rotation Matrix

We have the following properties of skew symmetric matrix $S(\vec{a})$.

(i) Linearity

$$S(\alpha\vec{a} + \beta\vec{b}) = \alpha S(\vec{a}) + \beta S(\vec{b}) \quad \text{— (i)}$$

(ii) Cross product of vectors $\vec{a}$ & $\vec{b}$ can be

$$S(\vec{a})\vec{b} = \vec{a} \times \vec{b} \quad \text{— (ii)}$$

(iii) Rotation matrix $R \in SO(3)$ (Orthogonal)

$$\therefore \quad R(\vec{a} \times \vec{b}) = R\vec{a} \times R\vec{b} \quad \text{— (iii)}$$

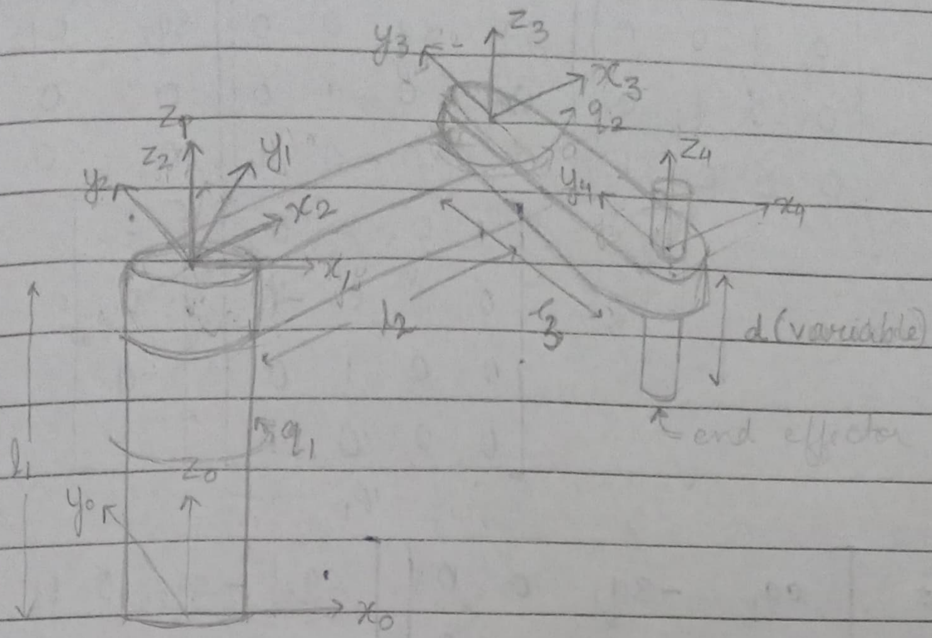
Now, let's find ($\vec{a}$ & $\vec{b}$ are vectors)

$$\begin{aligned} R\,S(\vec{a})R^T\vec{b} &= R(S(\vec{a}) \cdot R^T\vec{b}) \\ &= R(\vec{a} \times R^T\vec{b}) && \text{(from (ii))} \\ &= (R\vec{a}) \times (\vec{b}) && (RR^T = I)\text{ (from (iii))} \\ &= S(R\vec{a})\vec{b} && \text{(from (ii))} \end{aligned}$$

Thus, $\boxed{R\,S(\vec{a})R^T = S(R\vec{a})}$

Hence proved.

6. RRP SCARA Robot



The coordinate frames are as shown above

$$R_0^1 = I \qquad d_0^1 = \begin{bmatrix} 0 \\ 0 \\ l_1 \end{bmatrix}$$

$$R_1^2 = \begin{bmatrix} cq_1 & -sq_1 & 0 \\ sq_1 & cq_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad d_1^2 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\cancel{R_2^3 = I} \qquad R_2^3 = \begin{bmatrix} cq_2 & -sq_2 & 0 \\ sq_2 & cq_2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad d_2^3 = \begin{bmatrix} l_2 \\ 0 \\ 0 \end{bmatrix}$$

$$\cancel{R_3^4 =} \qquad R_3^4 = I \qquad d_3^4 = \begin{bmatrix} 0 \\ -l_3 \\ 0 \end{bmatrix} \qquad P_4 = \begin{bmatrix} 0 \\ 0 \\ -d \end{bmatrix}$$

$$\begin{bmatrix} P_0 \\ 1 \end{bmatrix} = H_0^1 H_1^2 H_2^3 H_3^4 \begin{bmatrix} P_4 \\ 1 \end{bmatrix}$$

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$$\begin{bmatrix} p_0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} cq_1 & -sq_1 & 0 & 0 \\ sq_1 & cq_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} cq_2 & -sq_2 & 0 & l_2 \\ sq_2 & cq_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -l_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ -d \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} cq_1 & -sq_1 & 0 & 0 \\ sq_1 & cq_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} cq_2 & -sq_2 & 0 & l_2 \\ sq_2 & cq_2 & 0 & -l_3 cq_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ -d \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} cq_1 cq_2 - sq_1 sq_2 & -cq_1 sq_2 - sq_1 cq_2 & 0 & l_2 cq_1 + l_3 sq_1 cq_2 \\ sq_1 cq_2 + sq_2 cq_1 & -sq_1 sq_2 + cq_1 cq_2 & 0 & l_2 sq_1 - l_3 cq_1 cq_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ -d \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} l_2 cq_1 + l_3 sq_1 cq_2 \\ l_2 sq_1 - l_3 cq_1 cq_2 \\ -d \\ 1 \end{bmatrix}$$

$$\therefore \quad p_0 = \begin{pmatrix} l_2 \cos q_1 + l_3 \sin q_1 \cos q_2 \\ l_2 \sin q_1 - l_3 \cos q_1 \cos q_2 \\ -d \end{pmatrix}$$

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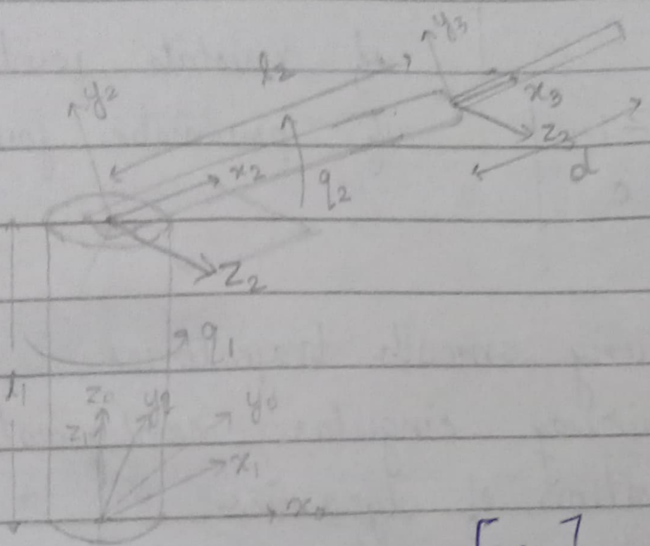
8. RRP Stanford Manipulator



$$R_2^3 = I \qquad d_2^3 = \begin{bmatrix} l_2 \\ 0 \\ 0 \end{bmatrix} \qquad p_3 = \begin{bmatrix} d \\ 0 \\ 0 \end{bmatrix}$$

$$R_1^2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & C\pi/2 & -S\pi/2 \\ 0 & S\pi/2 & C\pi/2 \end{bmatrix} \begin{bmatrix} cq_2 & -sq_2 & 0 \\ sq_2 & cq_2 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} cq_2 & -sq_2 & 0 \\ 0 & 0 & -1 \\ sq_2 & cq_2 & 0 \end{bmatrix}$$

$$d_1^2 = \begin{bmatrix} 0 \\ 0 \\ l_1 \end{bmatrix}$$

$$R_0^1 = \begin{bmatrix} cq_1 & -sq_1 & 0 \\ sq_1 & cq_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad ; \quad d_0^1 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} p_0 \\ 1 \end{bmatrix} = H_0^1 H_1^2 H_2^3 \begin{bmatrix} p_3 \\ 1 \end{bmatrix}$$

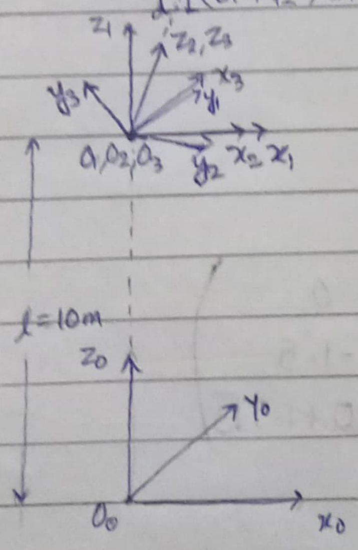
$$= \begin{bmatrix} cq_1 & -sq_1 & 0 & 0 \\ sq_1 & cq_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} cq_2 & -sq_2 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ sq_2 & cq_2 & 0 & l_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & l_2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} d \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} cq_1 cq_2 & -cq_1 sq_2 & sq_1 & 0 \\ sq_1 cq_2 & -sq_1 sq_2 & -cq_1 & 0 \\ sq_2 & cq_2 & 0 & l_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} d + l_2 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} (d+l_2)\, cq_1 cq_2 \\ (d+l_2)\, sq_1 cq_2 \\ (d+l_2)\, sq_2 + l_1 \\ 1 \end{bmatrix}$$

$$\therefore \quad p_0 = \begin{bmatrix} (d+l_2)\cos q_1 \cos q_2 \\ (d+l_2)\sin q_1 \cos q_2 \\ (d+l_2)\sin q_2 + l_1 \end{bmatrix}$$

9.



$O_0(x_0, y_0, z_0)$ is base/reference frame

$$R_2^3 = \begin{bmatrix} C_{60} & -S_{60} & 0 \\ S_{60} & C_{60} & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1/2 & -\sqrt{3}/2 & 0 \\ \sqrt{3}/2 & 1/2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$d_2^3 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \quad p_3 = \begin{bmatrix} 0 \\ 0 \\ 3 \end{bmatrix}$$

$$R_1^2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & C_{30} & -S_{30} \\ 0 & S_{30} & C_{30} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \sqrt{3}/2 & -1/2 \\ 0 & 1/2 & \sqrt{3}/2 \end{bmatrix}, \quad d_1^2 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$R_0^1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad d_0^1 = \begin{bmatrix} 0 \\ 0 \\ l \end{bmatrix}$$

~~p_0 : obstacle w.r.t.~~

p_0 : obstacle w.r.t. reference frame.

$$\begin{bmatrix} p_0 \\ 1 \end{bmatrix} = H_0^1 H_1^2 H_2^3 \begin{bmatrix} p_3 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & l \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \sqrt{3}/2 & -1/2 & 0 \\ 0 & 1/2 & \sqrt{3}/2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1/2 & -\sqrt{3}/2 & 0 & 0 \\ \sqrt{3}/2 & 1/2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 3 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \sqrt{3}/2 & -1/2 & 0 \\ 0 & 1/2 & \sqrt{3}/2 & l \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 3 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ -3/2 \\ 3\sqrt{3}/2 + l \\ 1 \end{bmatrix}$$

$$\therefore \quad p_0 = \begin{pmatrix} 0 \\ -3/2 \\ 3\sqrt{3}/2 + l \end{pmatrix}$$

Here, $l = 10$

$$\therefore \quad p_0 = \begin{pmatrix} 0 \\ -1.5 \\ 10 + 1.5\sqrt{3} \end{pmatrix}$$

11. RRP SCARA.

$\rho_1 = 1$, $Z_0 = \hat{k}_0 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$

$\rho_2 = 1$, $Z_1 = \hat{k}_1 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$

$\rho_3 = 0$.

Thus, $J\omega = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 0 \end{bmatrix}$

For velocities

$$\cancel{\frac{\partial q_0}{\partial q_i}} \quad \frac{\partial d_0^3}{\partial q_1} = Z_0 \times (R_0^0 \cdot d_0^n)$$

$$= \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} (R_0^1 R_1^2 R_2^3 R_3^4 d_4)$$

$$= \cancel{\begin{bmatrix} l_2 cq_1 + l_3 sq_1 cq_2 \\ l_2 sq_1 - l_3 cq_1 cq_2 \\ -d \end{bmatrix}}$$

$$= \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} l_2 cq_1 + (l_3 cq_2) cq_1 \\ l_2 sq_1 - (l_3 sq_2) cq_1 \\ l_1 \end{bmatrix}$$

$$= \begin{bmatrix} -l_2 sq_1 + l_3 sq_2 cq_1 \\ l_2 cq_1 + l_3 cq_2 cq_1 \\ 0 \end{bmatrix}$$

$$\frac{\partial d_0^2}{\partial q_1} = Z_1 \times R_0^1 d_1^2$$

$$= (0,0,1) \times \begin{bmatrix} cq_1 & -sq_1 & 0 \\ sq_1 & cq_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} l_2 cq_1 \\ l_2 sq_1 \\ l_1 \end{bmatrix}$$

$$= (0,0,1) \times (l_2 (cq_1)^2 - l_2 (sq_1)^2, 2l_2 cq_1 sq_1, l_1)$$

$$= \begin{bmatrix} -2l_2 cq_1 sq_1 \\ l_2 (cq_1)^2 - l_2 (sq_1)^2 \\ 0 \end{bmatrix}$$

$$\frac{\partial d_0^1}{\partial q_1} = z_2 \times R_0^2 d_2^3$$

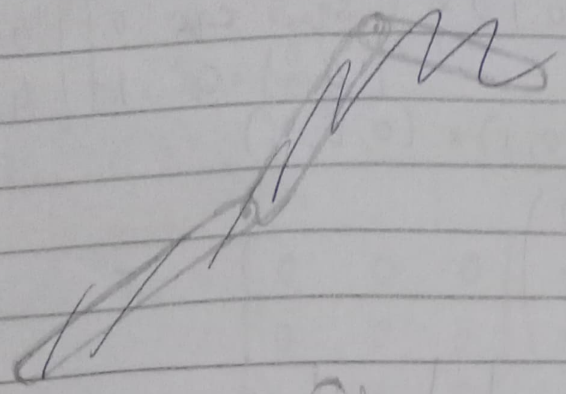
$$= (0,0,1) \times \begin{bmatrix} cq_2 & -sq_2 & 0 \\ sq_2 & cq_2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ l_1 \end{bmatrix}$$

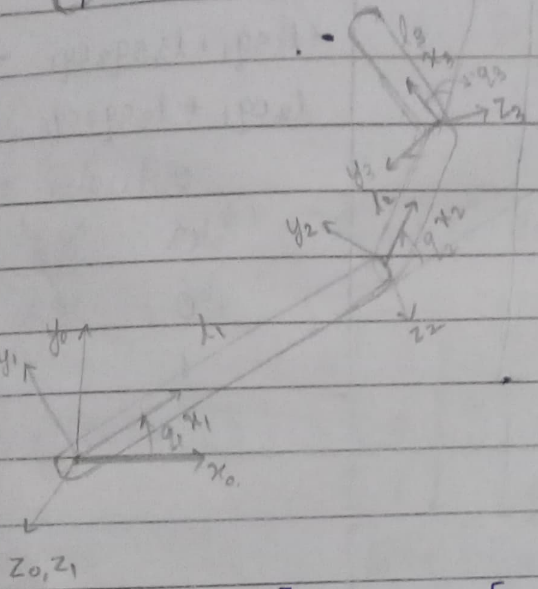
$$= (0,0,1) \times (0,0,l_1)$$

$$= \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$J = \begin{bmatrix} J_v \\ \\ J_\omega \end{bmatrix} = \begin{bmatrix} -l_2 sq_1 + l_3 sq_2 cq_1 & -2 l_2 cq_1 sq_1 & 0 \\ l_2 cq_1 + l_3 cq_2 cq_1 & l_2\, c(2q_1) & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 0 \end{bmatrix}$$

13. RRR Planar Robot.





$$R_0^1 = \begin{bmatrix} cq_1 & -sq_1 & 0 \\ sq_1 & cq_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad R_1^2 = \begin{bmatrix} cq_2 & -sq_2 & 0 \\ sq_2 & cq_2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad R_2^3 = \begin{bmatrix} cq_3 & -sq_3 & 0 \\ sq_3 & cq_3 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$d_0^1 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad d_1^2 = \begin{bmatrix} l_1 \\ 0 \\ 0 \end{bmatrix} \quad d_2^3 = \begin{bmatrix} l_2 \\ 0 \\ 0 \end{bmatrix}$$

$$p_3 = \begin{bmatrix} l_3 \\ 0 \\ 0 \end{bmatrix}$$

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$$J_\omega = \begin{bmatrix} \rho_1 R_0 \hat{k}_0 & \rho_2 R_0^1 \cdot \hat{k}_1 & \rho_3 R_0^2 \cdot \hat{k}_2 \end{bmatrix}$$

$$\rho_1 = \rho_2 = \rho_3 = 1$$

$$J_{\omega,1} = 1 \cdot R_0 \cdot \hat{k}_0 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$J_{\omega,2} = 1 \cdot R_0^1 \cdot \hat{k}_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$J_{\omega,3} = 1 \cdot R_0^2 \cdot \hat{k}_2 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

We got J_ω.

Now find J_v.

$$J_v = \begin{bmatrix} J_{v,1} & J_{v,2} & J_{v,3} \end{bmatrix} = \begin{bmatrix} Z_1 \times (O_n - O_0) & Z_2 \times (O_n - O_1) & Z_3 \times (O_n - O_2) \end{bmatrix}$$

$$= \begin{bmatrix} Z_1 \times (R_0 \cdot d_0^n) & Z_2 \times (R_0^1 d_1^n) & Z_3 \times (R_0^2 d_2^n) \end{bmatrix}$$

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$$O_n - O_0 = R_0 d_0^n \qquad (O_0 = O_1)$$
$$= R_0^1 d_1^n$$
$$= R_0^1 (d_1^2 + R_1^2 d_2^n)$$
$$= R_0^1 d_1^2 + R_0^1 R_1^2 d_2^3 + R_0^1 R_1^2 R_2^3 d_3^n$$

$\therefore$ ~~$J_{v,1}$~~ $J_{v,1} = Z_1 \times (O_n - O_0)$. here $Z_1 = (0,0,1)$

On calculating and simplifying, we get

$$J_{v,1} = \begin{bmatrix} -l_1 s q_1 - l_2 s(q_1+q_2) - l_3 s(q_1+q_2+q_3) \\ l_1 c q_1 + l_2 c(q_1+q_2) + l_3 c(q_1+q_2+q_3) \\ 0 \end{bmatrix}$$

$$O_n - O_2 = R_0^2 d_2^n = R_0^2 (d_2^3 + R_2^3 d_3^n)$$
$$= \begin{bmatrix} c(q_1+q_2) & -s(q_1+q_2) & 0 \\ s(q_1+q_2) & c(q_1+q_2) & 0 \\ 0 & 0 & 1 \end{bmatrix} \left(\begin{bmatrix} l_2 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} cq_3 & -sq_3 & 0 \\ sq_3 & cq_3 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} l_3 \\ 0 \\ 0 \end{bmatrix} \right)$$

and $J_{v,2} = Z_2 \times (O_n - O_2)$, here $Z_2 = (0,0,1)$

Thus,

$$J_{v,2} = \begin{bmatrix} -(l_2 s(q_1+q_2) + l_3 (s(q_1+q_2+q_3))) \\ l_2 c(q_1+q_2) + l_3 c(q_1+q_2+q_3) \\ 0 \end{bmatrix}$$

$$O_n - O_3 = R_0^3 d_3^n = \begin{bmatrix} c(q_1+q_2+q_3) & -s(q_1+q_2+q_3) & 0 \\ s(q_1+q_2+q_3) & c(q_1+q_2+q_3) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} l_3 \\ 0 \\ 0 \end{bmatrix}$$

$Z_3 = (0,0,1)$

$$J_{v,3} = Z_3 \times (O_n - O_3) = \begin{bmatrix} -l_3 s(q_1+q_2+q_3) \\ l_3 c(q_1+q_2+q_3) \\ 0 \end{bmatrix}$$

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Thus, final Jacobian turns out to be,

$$J = \begin{bmatrix} -l_1 s q_1 - l_2 s(q_1+q_2) - l_3 s(q_1+q_2+q_3) & -l_2 s(q_1+q_2) - l_3 s(q_1+q_2+q_3) & -l_3 s(q_1+q_2+q_3) \\ l_1 c q_1 + l_2 c(q_1+q_2) + l_3 c(q_1+q_2+q_3) & +l_2 c(q_1+q_2) + l_3 s(q_1+q_2+q_3) & l_3 c(q_1+q_2+q_3) \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

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7. Code link: [19110046_ME 639 Assignment 2 Q7.ipynb - Colaboratory (google.com)]
8. Code link: [19110046_ME 639 Assignment 2 Q8.ipynb - Colaboratory (google.com)]

10. Read about a few different types of gearboxes typically used with motors in a robotic application and explain in 2-3 sentences in your own words some key pros and cons of each gearbox type and where it is typically used. Further, explain if you would typically see a gearbox used along with a motor in a drone application. Explain the reasons.

i) Types of gearboxes:

- Planetary Gearbox: It is named so because it has a central sun gear surrounded by 3-4 planet gears. The gears are held together by outer ring gear with internal teeth. It is capable of providing high system rigidity and high torque in compact space. It has good endurance and accuracy.
- Spiral bevel gearbox: It has versatile applications. However, it is available in relatively larger sizes. On the positive side, it can provide good efficiency.
- Hybrid Gearbox: This gearbox has features of both spiral bevel gearbox and planetary gearbox.
- Helical Gearbox: Compact in size, thus gives space efficiency. Also, it can only be used in applications with low power consumption. It is typically used for heavy-duty operations. A unique feature of this gearbox is that it is fixed at an angle which allows more teeth to interact in the same direction when in motion. This provides constant contact.
- Bevel Helical Gearbox: Its application is to provide rotary motion between non-parallel shafts. It has a curved set of teeth located on a cone-shaped surface very close to the rim of the unit.
- Worm Reduction Gearbox: Applications in heavy-duty operations. It has a large diameter and can give increased speed reduction between non-intersecting crossed-axis shafts. The worm meshes with teeth on the peripheral area and causes the wheel to move similar to the worm.
- Skew bevel helical gearbox: It has a rigid monolithic structure. It can be used in heavy load applications. It is highly customisable. One can change the number of gears and teeth according to the need.
- Coaxial helical inline gearbox: It has high manufacturing specifications, thus giving the best load and transmission ratios. It is also suitable for heavy-duty applications. It has appreciable quality and efficiency.

References: 6 Types of Gearbox: The Ultimate Guide in 2021 | Linquip, How to Choose the Best Gearbox for Your Servo Motor (automate.org)

ii) Generally gearboxes are not attached to motors in drones. Instead, high torque motors are used. Drones need to be flown, thus, weight needs to be minimised. In drones, ESCs are given signals by flight controllers which send signals and power to the motors. Also, the propellers are directly attached to the motors. Attaching the gearbox will add to the weight and also consume space on the drone, both of which are undesirable.

12. Code Link: 19110046_ME 639 Assignment 2 Q12.ipynb - Colaboratory (google.com)
14. Code Link: 19110046_ME 639 Assignment 2 Q14.ipynb - Colaboratory (google.com)