6.5.3.6

EE24BTECH11016 - DHWANITH M DODDAHUNDI

Question:
Find the local minimum/maximum of the given function:
$f(x) = \frac{x}{2} + \frac{2}{x}$
Solution:
For the function,

$$y(x) = \frac{x}{2} + \frac{2}{x} \tag{0.1}$$
$$y\prime(x) = \frac{1}{2} - \frac{2}{x^2} \tag{0.2}$$
$$y\prime\prime(x) = \frac{4}{x^3} \tag{0.3}$$

For critical points,

$$y\prime(x) = 0 \tag{0.4}$$
$$\implies \frac{1}{2} - \frac{2}{x^2} = 0 \tag{0.5}$$
$$\implies x^2 = 4 \tag{0.6}$$
$$\implies x = -2, 2 \tag{0.7}$$

For, critical points to be local minimum or local maximum, it should follow the following:-

$$\text{Local Minimum:}\, y\prime\prime(x) > 0 \tag{0.8}$$
$$\text{Local Maximum:}\, y\prime\prime(x) < 0 \tag{0.9}$$

For,$x = -2$ we get,

$$y\prime\prime(-2) = \frac{4}{(-2)^3} \tag{0.10}$$
$$y\prime\prime(-2) = -\frac{1}{2} < 0 \tag{0.11}$$

So, $x = -2$ is a point of local maxima.

For, $x = 2$ we get,

$$y\prime\prime(2) = \frac{4}{(2)^3} \tag{0.12}$$
$$y\prime\prime(2) = \frac{1}{2} > 0 \tag{0.13}$$

So, $x = 2$ is a point of local minima.

Computational Solution:

Finding the Local Maxima using Gradient Ascent we get,

$$x_{n+1} = x_n + \alpha f\prime(x_n) \tag{0.14}$$

$$x_{n+1} = x_n + \alpha\left(\frac{1}{2} - \frac{2}{x^2}\right) \tag{0.15}$$

Finding the Local Minima using Gradient Decent we get,

$$x_{n+1} = x_n - \alpha f\prime(x_n) \tag{0.16}$$

$$x_{n+1} = x_n - \alpha\left(\frac{1}{2} - \frac{2}{x^2}\right) \tag{0.17}$$

Taking the following conditions, we have

$$x_0 = 0.5 \tag{0.18}$$

$$h = 0.01 \tag{0.19}$$

$$\alpha = 0.1 \tag{0.20}$$

After computing we get,

$$\text{Local maxima: } x = 1.999981868996581,\ f(x) = 2.000000000082184 \tag{0.21}$$

$$\text{Local Minima: } x = -2.0000188911077013,\ f(x) = -2.0000000000892175 \tag{0.22}$$

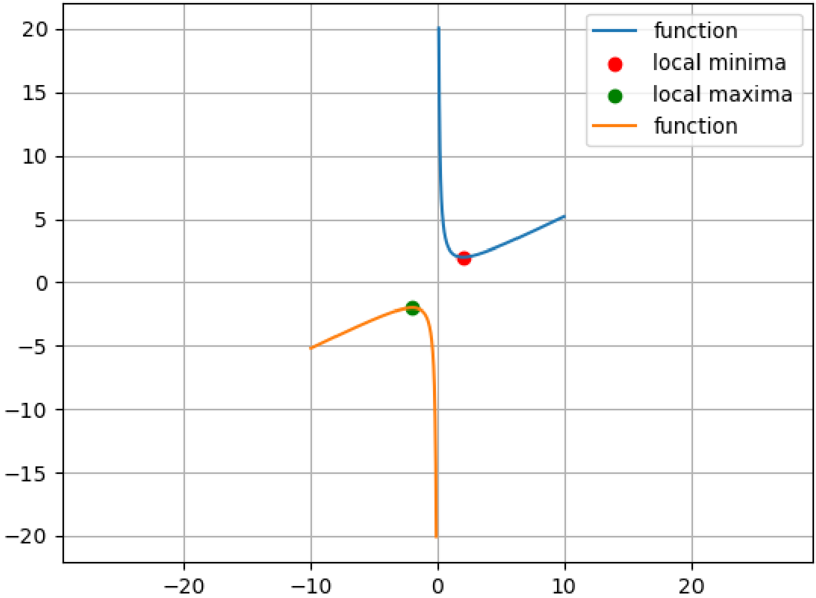

Fig. 0.1: Maxima and Minima points of the given function