8.1.1.Ex 1

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Question: Find the area enclosed by the circle $x^2 + y^2 = a^2$.
Solution:
Theoretical Solution:
Finding Area

$$A = 4\int_0^a y dx \tag{1}$$

Since $x^2 + y^2 = a^2$,we get $y = \pm\sqrt{a^2 - x^2}$

$$A = 4\int_0^a \sqrt{a^2 - x^2} dx \tag{2}$$

$$A = 4\left[\frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right)\right]_0^a \tag{3}$$

$$A = 4\left[\frac{a}{2} \times 0 + \frac{a^2}{2}\sin^{-1}(1) - 0\right] \tag{4}$$

$$A = 4\left(\frac{a^2}{2}\right)\left(\frac{\pi}{2}\right) \tag{5}$$

$$A = \pi a^2 \tag{6}$$

Computational Solution:
Using the trapezoidal rule to get the area
The trapezoidal rule is as follows.

$$A = \int_a^b f(x)\, dx \approx h\left(\frac{1}{2}f(a) + f(x_1) + f(x_2) \cdots + f(x_{n-1}) + \frac{1}{2}f(b)\right) \tag{7}$$

$$h = \frac{b-a}{n} \tag{8}$$

$$A = j_n, \text{ where, } j_{i+1} = j_i + h\frac{f(x_{i+1}) + f(x_i)}{2} \tag{9}$$

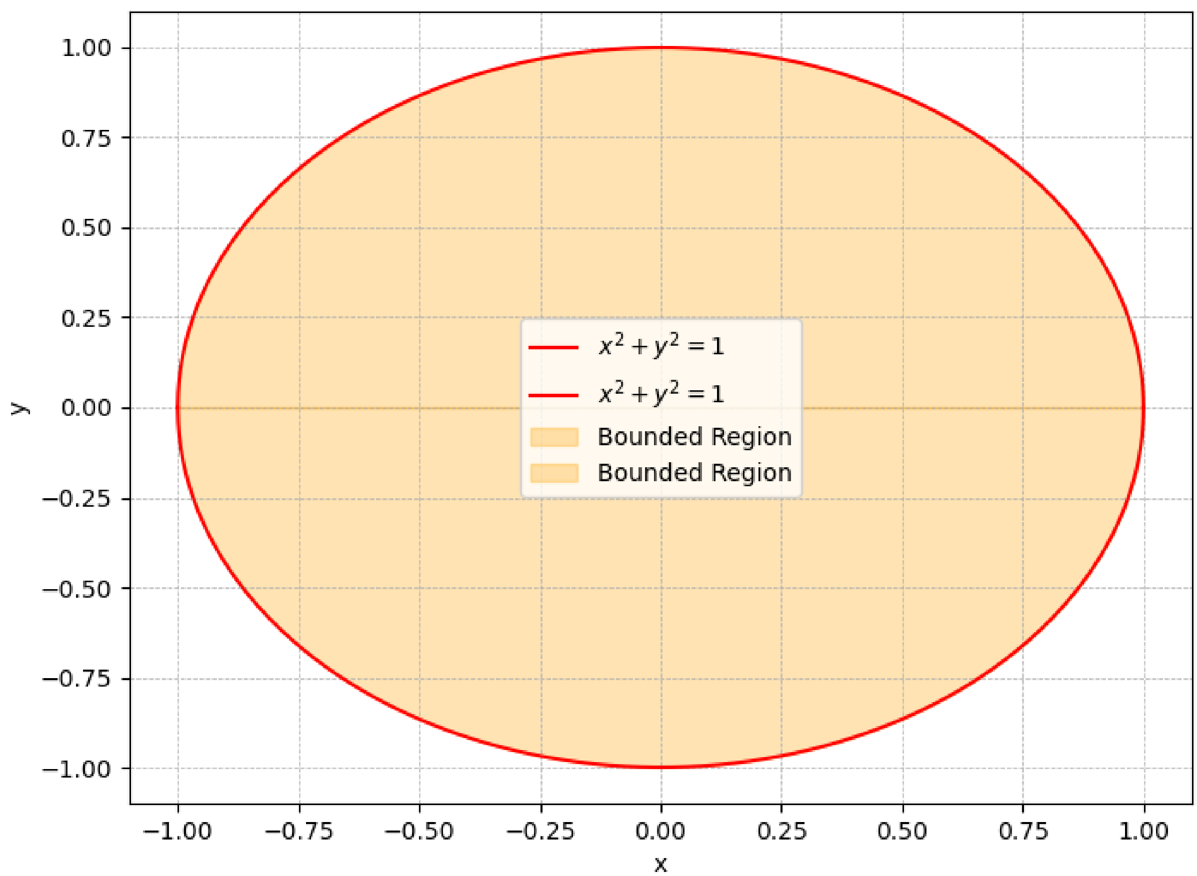
$$\rightarrow j_{i+1} = j_i + h\left(\sqrt{x_{i+1}} + \sqrt{x_i}\right) \tag{10}$$

$$x_{i+1} = x_i + h \tag{11}$$

$$h = \frac{1}{30000} \tag{12}$$

$$n = 30000 \tag{13}$$

Using the code answer obtained is $A = 3.141592427302344$ sq. units for a radius of 1 unit.


$x^2 + y^2 = 1$
$x^2 + y^2 = 1$
Bounded Region
Bounded Region
y
x
1.00
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