11.16.2.1

EE24BTECH11016 - DHWANITH M DODDAHUNDI

Question:
A die is rolled. Let E be the event 'die shows 4' and F be the event 'die shows even number'. Are E and F mutually exclusive?

Theoretical Solution:
The sample space S of a fair six-sided die is

$$S = \{1, 2, 3, 4, 5, 6\} \tag{0.1}$$

The event E is defined when the die shows 4

$$E = \{4\} \tag{0.2}$$

The event F is defined when the die shows an even number in the sample space

$$F = \{2, 4, 6\} \tag{0.3}$$

The event $E \cap F$ is defined when the die shows an even number and 4. (i,e intersection of E and F)

$$E \cap F = \{4\} \tag{0.4}$$

$$|E \cap F| = 1 \tag{0.5}$$

$$|S| = 6 \tag{0.6}$$

$$P(E \cap F) = \frac{|E \cap F|}{|S|} \tag{0.7}$$

$$\tag{0.8}$$

On substituting,

$$P(E \cap F) = \frac{1}{6} \tag{0.9}$$

Since $E \cap F \neq \phi$ and $P(E \cap F) \neq 0$ the events E and F are not mutually exclusive

Computational Solution:
The goal of this task was to compute the probability distribution of outcomes when rolling a six-sided die. The outcomes $1, 2, 3, 4, 5, 6$ represent the faces of the die, and each face is expected to have an approximately equal probability if the die is fair. The computed probabilities (PMF) were plotted as a stem plot to visualize the distribution. The probability of the die showing 4 and an even number was to be calculated in order to find if both the events were mutually exclusive.

Process Overview

The process involved two main steps: 1. Computation of the probabilities using a C program. 2. Visualization of the results using Python.

Step 1: Probability Computation in C

- A simulation was performed by rolling a virtual six-sided die N times, where $N = 1,000,000$, to ensure accurate probabilities. - Each roll was simulated using a random number generator that produced values between 1 and 6. - A count was maintained for how many times each outcome occurred during the simulation. - The probability of each outcome (PMF) was calculated by dividing the count of each outcome by the total number of rolls.

Step 2: Data Export via Shared Library

- The C program was compiled into a shared library (`.so` file) that could be accssed from Python. - This ensured that the computationally heavy task of rolling the die and calculating probabilities was handled efficiently in C.

Step 3: Visualization in Python

- The computed probabilities were imported from the shared library into Python. - A stem plot was used to visualize the probability distribution. Each outcome $1, 2, 3, 4, 5, 6$ was plotted on the x-axis, and its corresponding probability was plotted on the y-axis. - The stem plot highlighted the uniform distribution of probabilities for a fair die, with each outcome having a probability close to $1/6$.

Results and Insights

Probability Mass Function (PMF)

The PMF represents the probability of each individual outcome $x \in \{1, 2, 3, 4, 5, 6\}$. The table below shows the PMF for a six-sided die based on simulation:

Outcome (x)	$P_X(x)$
1	0.1667
2	0.1665
3	0.1666
4	0.1668
5	0.1669
6	0.1665

As expected, the probabilities are close to $1/6 \approx 0.1667$, with minor variations due to random sampling.

Cumulative Distribution Function (CDF)

The CDF represents the cumulative probability up to each outcome $x \in \{1, 2, 3, 4, 5, 6\}$. The table below shows the CDF for a six-sided die:

Outcome (x)	$F_X(x)$
1	0.1667
2	0.3332
3	0.4998
4	0.6666
5	0.8335
6	1.0000

The CDF starts with the PMF of $x = 1$ and accumulates to 1.0 at $x = 6$, confirming the correctness of the cumulative probabilities.

CONCLUSION

This task demonstrates the integration of C and Python for simulating and visualizing a probabilistic experiment. By combining the computational efficiency of C with the graphical capabilities of Python, we achieve an effective solution for analyzing and representing data. From the code we get the probabilities of the events defined,

$$P(E) = 0.1667 \text{ (Die shows 4)} \tag{0.10}$$

$$P(F) = 0.4998 \text{ (Die shows an even number)} \tag{0.11}$$

$$P(E \cap F) = 0.1667 \text{ (Die shows 4 and is even)} \tag{0.12}$$

Since $P(E \cap F) \neq 0$ the events E and F are not mutually exclusive