9.2.5

EE24BTECH11016 - DHWANITH M DODDAHUNDI

QUESTION:
Consider the differential equation $xy' = y(x \neq 0)$. Verify that $y = Ax$ is a solution for it, given the initial conditions $y(1) = A$.
SOLUTION:
Consider the differential equation,

$$xy' = y(x \neq 0) \tag{0.1}$$

$$y' = \frac{y}{x} \tag{0.2}$$

$$\frac{dy}{dx} = \frac{y}{x} \tag{0.3}$$

Since the Laplace transform works best with initial conditions defined at $x = 0$, we change the variable by letting:

$$t = \log(x), x = e^t. \tag{0.4}$$

$$\frac{d}{dx} = \frac{d}{dt} \cdot \frac{dt}{dx} = \frac{d}{dx} \cdot \frac{1}{x} \tag{0.5}$$

Thus, the equation becomes:

$$\frac{dy}{dt} \cdot \frac{1}{x} = \frac{y}{x} \tag{0.6}$$

$$\frac{dy}{dt} = y \tag{0.7}$$

Taking the Laplace transform of both sides:

$$\mathcal{L}\left(\frac{dy}{dt}\right) = \mathcal{L}(y) \tag{0.8}$$

$$sY(s) - y(0) = Y(s) \tag{0.9}$$

Rearranging,

$$(s-1)Y(s) = y(0) \tag{0.10}$$

$$Y(s) = \frac{y(0)}{s-1} \tag{0.11}$$

From the original problem, $y(1) = A$. Substituting $x = 1$ (or $t = \log(1) = 0$), we find $y(0) = A$. So:

$$Y(s) = \frac{A}{s-1} \tag{0.12}$$

$$\mathcal{L}(y(t)) = \frac{A}{s-1} \tag{0.13}$$

Now, take the inverse laplace transform

$$y(t) = \mathcal{L}^{-1}\left(\frac{A}{s-1}\right) \tag{0.14}$$

$$y(t) = A \cdot \mathcal{L}^{-1}\left(\frac{1}{s-1}\right) \tag{0.15}$$

$$y(t) = A \cdot e^{t} \tag{0.16}$$

Returning to the original variable $x = e^t$, we get:

$$y(x) = A \cdot e^{\ln x} = Ax \tag{0.17}$$

Hence, verified.

ALGORITHM :

$$x_0 = 1 \tag{0.18}$$

$$y_0 = 1(\text{when } A = 1) \tag{0.19}$$

$$h = 0.01 \tag{0.20}$$

$$x_{n+1} = x_n + h \tag{0.21}$$

$$y_{n+1} = y_n + h(y') \tag{0.22}$$

From (0.2),

$$y_{n+1} = y_n + h\left(\frac{y_n}{x_n}\right) \tag{0.23}$$

$$y_{n+1} = y_n + h(1) \tag{0.24}$$

which is the required difference equation.

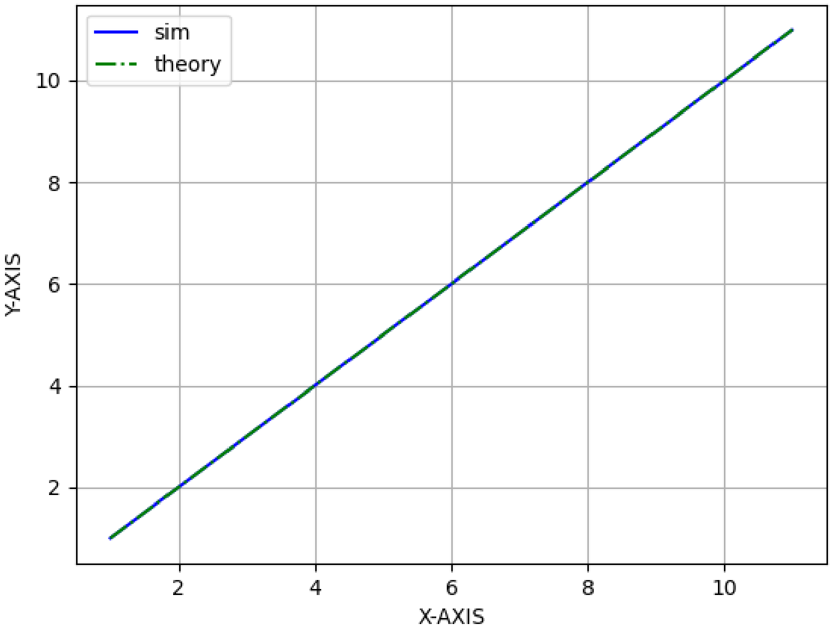

Fig. 0.1: A plot of the given question.