Chapter 6 Sequences and Series

EE24BTECH11016- DHWANITH M

6) If N is a natural number such that $n = p_1^{a_1}.p_2^{a_2}.p_3^{a_3}......p_k^{a_k}$ and $p_1, p_2, ..., p_k$ are distinct primes, then show that $lnn \geq kln2$
(1985 – 5Marks)
7) Find the sum of the series : $\sum_{r=0}^{n} (-1)^r \binom{n}{r} \left[\frac{1}{2^r} + \frac{3^r}{2^{2r}} + \frac{7^r}{2^{3r}} + \frac{15^r}{2^{4r}}.....\right.$ up to m terms,]
(1985 – 5Marks)
8) Solve for x the following equation:
(1987 – 3Marks)
$\log_{(2x+3)} \left(6x^2 + 23x + 21\right) = 4 - \log_{(3x+7)} \left(4x^2 + 12x + 9\right)$
9) If $\log_3 2, \log_3 2^x - 3$ and $\log_3 \left(2^x - \frac{7}{2}\right)$ are in arithmetic progression. Determine the value of x.
(1990 – 4Marks)
10) Let p be the first of n arithmetic means between two numbers and q the first of n harmonic means between the same numbers , Show that q does not lie between p and $\left(\frac{n+1}{n-1}\right)^2 p$
(1991 – 4Marks)
11) If $S_1, S_2, S_3......, S_n$ are the sums of infinite geometric series whose first terms are $1, 2, 3, ..., n$ and whose common ratios are $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, ..., \frac{1}{n+1}$ respectively, then find the values of $S_1^2 + S_2^2 + S_3^2 + ... + S_{2n-1}^2$
(1991 – 4Marks)
12) The real numbers x_1, x_2, x_3 satisfying the equation $x^3 - x^2 + \beta x + \gamma = 0$ are in AP. Find the intervals in which β and γ lie.
(1996 – 3Marks)
13) Let a, b, c, d be real numbers in GP. If u, v, w satisfy the system of equations
(1999 – 10Marks)
$u + 2v + 3w = 6$
$4u + 5v + 6w = 12$
$6u + 9v = 4$
then show that the roots of the equation
$\left(\frac{1}{u} + \frac{1}{v} + \frac{1}{w}\right) x^2 + \left[(b-c)^2 + (c-a)^2 + (d-b)^2\right] x + u + v + w = 0$
and $20x^2 + 10 (a-d)^2 x - 9 = 0$ are reciprocals of each other.
14) The fourth power of the common difference of an arithmetic progression with integer entries is added to the product of any four consecutive terms of it.Prove that the resulting sum is the square of an integer.
(2000 – 4Marks)
15) Let $a_1, a_2, ..., a_n$ be positive real numbers in geometric progression. For each n,let A_n, G_n, H_n be respectively, the arithmetic mean, geometric mean ,and harmonic mean of $a_1, a_2, ..., a_n$. Find an expression for the geometric mean of $G_1, G_2, ..., G_n$ in terms of $A_1, A_2, ..., A_n, H_1, H_2, ..., H_n$.
(2001 – 5Marks)
16) Let a, b be positive real numbers.If a, A_1, A_2, b are in arithmetic progression, a, G_1, G_2, b are in geometric progression and a, H_1, H_2, b are in harmonic progression, show that $\frac{G_1 G_2}{H_1 H_2} = \frac{A_1 + A_2}{H_1 + H_2} = \frac{(2a+b)(a+2b)}{9ab}$
(2002 – 4Marks)
17) If a, b, c are in A.P.,a^2, b^2, c^2 are in H.P., then prove that either $a = b = c$ or $a, b, -\frac{c}{2}$ form a G.P.
(2003 – 4Marks)
18) If $a_n = \frac{3}{4} - \left(\frac{3}{4}\right)^2 + \left(\frac{3}{4}\right)^3 + ... (-1)^{n-1} \left(\frac{3}{4}\right)^n$ and $b_n = 1 - a_n$, then find the least natural number n_0 such that $b_n > a_n \forall n \geq n_0$.
(2006 – 6Marks)

PASSAGE-1

Let V_r denote the sum of first r terms of an arithmetic progression $(A.P)$ whose first term is r and the common difference is $(2r - 1)$.Let $T_r = V_{r+1} - V_r - 2$ and $Q_r = T_{r+1} - T_r$ for $r = 1, 2, ...$

1) The sum $V_1 + V_2 + ... + V_n$ is
(2007 – 4Marks)
   a) $\frac{1}{12} n (n+1) \left(3n^2 - n + 1\right)$
   b) $\frac{1}{12} n (n+1) \left(3n^2 + n + 2\right)$
   c) $\frac{1}{2} n \left(2n^2 - n + 1\right)$
   d) $\frac{1}{3} \left(2n^3 - 2n + 3\right)$

2) T_r is always

$(2007 - 4Marks)$

a) an odd number
b) an even number
c) a prime number
d) composite number

3) Which one of the following is a correct statement?

$(2007 - 4Marks)$

a) $Q_1, Q_2, Q_3, ...$ are in A.P. with common difference 5
b) $Q_1, Q_2, Q_3, ...$ are in A.P. with common difference 6
c) $Q_1, Q_2, Q_3, ...$ are in A.P. with common difference 11
d) $Q_1 = Q_2 = Q_3 = ...$