01/24/2023-Shift 1

EE24BTECH11016 - DHWANITH M DODDAHUNDI

1) Let the six numbers $a_1, a_2, a_3, a_4, a_5, a_6$ be in $A.P$ and $a_1 + a_2 = 10$. If the mean of these six numbers is $\frac{19}{2}$ and their variance is σ^2, then $8\sigma^2$ is equal to
   a) 220
   b) 210
   c) 200
   d) 105
2) Let $f(x)$ be a function such that $f(x+y) = f(x) \cdot f(y)$ for all x, y $\in \mathbb{N}$. If $f(1) = 3$ and $\sum_{k=1}^{n} f(k) = 3279$, then the value of n is
   a) 6
   b) 8
   c) 7
   d) 9
3) The number of real solutions of the equations $3\left(x^2 + \frac{1}{x^2}\right) - 2\left(x + \frac{1}{x}\right) + 5 = 0$, is
   a) 4
   b) 0
   c) 3
   d) 2
4) If $f(x) = \frac{2^{2x}}{2^{2x}+2}, x \in \mathbb{R}$,
   then $f\left(\frac{1}{2023}\right) + f\left(\frac{2}{2023}\right) + \cdots\cdots + f\left(\frac{2022}{2023}\right)$ is equal to
   a) 2011
   b) 1010
   c) 2010
   d) 1011
5) If $f(x) = x^3 - x^2 f'(1) + x f''(2) - f'''(3), x \in \mathbb{R}$, then
   a) $3f(1) + f(2) = f(3)$
   b) $f(3) - f(2) = f(1)$
   c) $2f(0) - f(1) + f(3) = f(2)$
   d) $f(1) + f(2) + f(3) = f(0)$
6) The number of integers, greater than 7000 that can be formed, using the digits $3, 5, 6, 7, 8$ without repetition, is
   a) 120
   b) 168
   c) 220
   d) 48
7) If the system of equations
   $x + 2y + 3z = 3$

$4x + 3y - 4z = 4$
$8x + 4y - \lambda z = 9 + \mu$
has infinitely many solutions, then the ordered pair (λ, μ) is equal to

a) $\left(\frac{72}{5}, \frac{21}{5}\right)$
b) $\left(\frac{-72}{5}, \frac{-21}{5}\right)$
c) $\left(\frac{72}{5}, \frac{-21}{5}\right)$
d) $\left(\frac{-72}{5}, \frac{21}{5}\right)$

8) The value of $\left(\frac{1+\sin\frac{2\pi}{9}+i\cos\frac{2\pi}{9}}{1+\sin\frac{2\pi}{9}-i\cos\frac{2\pi}{9}}\right)^3$ is

a) $\frac{-1}{2}(1 - i\sqrt{3})$
b) $\frac{1}{2}(1 - i\sqrt{3})$
c) $\frac{-1}{2}(\sqrt{3} - i)$
d) $\frac{1}{2}(\sqrt{3} + i)$

9) The equations of the sides AB and AC of a triangle ABC are $(\lambda + 1)x + \lambda y = 4$ and $\lambda x + (1 - \lambda)y + \lambda = 0$ respectively. Its vertex A is on the y-axis and its orthocentre is $(1, 2)$. The length of the tangent from the point C to the part of the parabola $y^2 = 6x$ in the first quadrant is

a) $\sqrt{6}$
b) $2\sqrt{2}$
c) 2
d) 4

10) The set of all values of a for which $\lim_{x\to a}([x - 5] - [2x + 2]) = 0$, where $[x]$ denotes the greatest integer less than or equal to ∞ is equal to

a) $(-7.5, -6.5)$
b) $(-7.5, -6.5]$
c) $[-7.5, -6.5]$
d) $[-7.5, -6.5)$

11) If $({}^{30}C_1)^2 + 2({}^{30}C_2)^2 + 3({}^{30}C_3)^2 + \ldots\ldots + 30({}^{30}C_{30})^2 = \frac{\alpha 60!}{(30!)^2}$, then α is equal to

a) 30
b) 60
c) 15
d) 10

12) Let the plane containing the line of intersection of the planes
$P1 : x + (\lambda + 4)y + z = 1$ and
$P2 : 2x + y + z = 2$ pass through the points $(0, 1, 0)$ and $(1, 0, 1)$. Then the distance of the point $(2\lambda, \lambda, -\lambda)$ from the plane $P2$ is

a) $5\sqrt{6}$
b) $4\sqrt{6}$
c) $2\sqrt{6}$
d) $3\sqrt{6}$

13) $\overrightarrow{\alpha} = 4\hat{i}+3\hat{j}+5\hat{k}$ and $\overrightarrow{\beta} = \hat{i}+2\hat{j}-4\hat{k}$. Let $\overrightarrow{\beta}_1$ be parallel to $\overrightarrow{\alpha}$ and $\overrightarrow{\beta}_2$ be perpendicular

to $\overrightarrow{\alpha}$. If $\overrightarrow{\beta} = \overrightarrow{\beta}_1 + \overrightarrow{\beta}_2$, then the value of $5\overrightarrow{\beta}_2 \cdot (\hat{i} + \hat{j} + \hat{k})$ is

a) 6
b) 11
c) 7
d) 9

14) The locus of the mid points of the chords of the circle $C_1 : (x-4)^2 + (y-5)^2 = 4$ which subtend an angle θ_i at the center of the circle C_1, is a circle of radius r_i. If $\theta_1 = \frac{\pi}{3}, \theta_3 = \frac{2\pi}{3}$ and $r_1^2 = r_2^2 + r_3^2$ then θ_2 is equal to

a) $\frac{\pi}{4}$
b) $\frac{3\pi}{4}$
c) $\frac{\pi}{6}$
d) $\frac{\pi}{2}$

15) If the foot of the perpendicular drawn from $(1, 9, 7)$ to the line passing through the point $(3, 2, 1)$ and parallel to the planes $x + 2y + z = 0$ and $3y - z = 3$ is (α, β, γ), then $\alpha + \beta + \gamma$ is equal to

a) -1
b) 3
c) 1
d) 5