7.7.3.1

EE24BTECH11016 - Dhwanith M Doddahundi

Question:
Find the equation of the circle passing through $(0, 0)$ and making intercepts a and b on the coordinate axes.
Solution:

Variable	**Description**
$\mathbf{c}$	Center of the circle
$\mathbf{u}$	$-\mathbf{c}$
r	Radius of the circle
f	$\|\|\mathbf{u}\|\|^2 - r^2$
$\mathbf{x_1}$	First point $(a, 0)$
$\mathbf{x_2}$	Second point$(0, b)$

TABLE 0: Variables Used

Let the equation of circle be $||\mathbf{x}||^2 + 2\mathbf{u}^\top \mathbf{x} + f = 0$
Since the circle is passing through $(0, 0)$, we get

$$f = 0 \tag{0.1}$$

Then, the equation of circle is given by $||\mathbf{x}||^2 + 2\mathbf{u}^\top \mathbf{x} = 0$
So,

$$||\mathbf{x_1}||^2 + 2\mathbf{u}^\top \mathbf{x_1} = 0 \tag{0.2}$$

$$||\mathbf{x_2}||^2 + 2\mathbf{u}^\top \mathbf{x_2} = 0 \tag{0.3}$$

Turning them into matrix form gives

$$\begin{pmatrix} 2\mathbf{x_1}^\top \\ 2\mathbf{x_2}^\top \end{pmatrix} \begin{pmatrix} \mathbf{u} \end{pmatrix} = -\begin{pmatrix} ||\mathbf{x_1}||^2 \\ ||\mathbf{x_2}||^2 \end{pmatrix} \tag{0.4}$$

Given $\mathbf{x_1} = \begin{pmatrix} a \\ 0 \end{pmatrix}, \mathbf{x_2} = \begin{pmatrix} 0 \\ b \end{pmatrix}$. Substituting into the matrix equation gives

$$\begin{pmatrix} 2a & 0 \\ 0 & 2b \end{pmatrix} \begin{pmatrix} \mathbf{u} \end{pmatrix} = \begin{pmatrix} -a^2 \\ -b^2 \end{pmatrix} \tag{0.5}$$

The Augmented matrix is

$$\begin{pmatrix} 2a & 0 & -a^2 \\ 0 & 2b & -b^2 \end{pmatrix} \tag{0.6}$$

Solving the matrix equation

$$\begin{pmatrix} 2a & 0 & -a^2 \\ 0 & 2b & -b^2 \end{pmatrix} \xleftrightarrow[R_1 \leftarrow \frac{R_1}{2a}]{R_2 \leftarrow \frac{R_2}{2b}} \begin{pmatrix} 1 & 0 & -\frac{a}{2} \\ 0 & 1 & -\frac{b}{2} \end{pmatrix} \tag{0.7}$$

The value of $\mathbf{u}$ is

$$\mathbf{u} = \begin{pmatrix} -\frac{a}{2} \\ -\frac{b}{2} \end{pmatrix} \tag{0.8}$$

Center of the circle is

$$\mathbf{c} = \begin{pmatrix} \frac{a}{2} \\ \frac{b}{2} \end{pmatrix} \tag{0.9}$$

Therefore, the equation of circle is

$$\|\mathbf{x}\|^2 - 2\begin{pmatrix} \frac{a}{2} & \frac{b}{2} \end{pmatrix}\mathbf{x} = 0 \tag{0.10}$$

$$\|\mathbf{x}\|^2 - \begin{pmatrix} a & b \end{pmatrix}\mathbf{x} = 0 \tag{0.11}$$

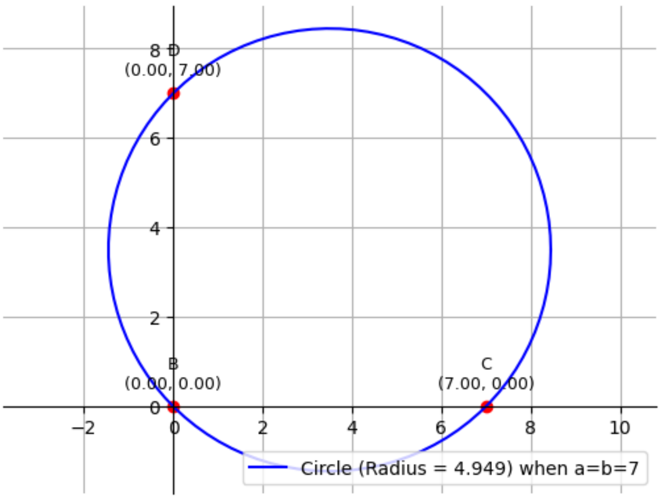

Fig. 0.1: Stem Plot of y(n)