1.2.23

EE24BTECH11016 - Dhwanith M Doddahundi

Question:

A motorboat is racing towards north at $25km/h$ and the water current in that region is $10km/h$ in the direction of $60°$ east of south. Find the resultant velocity of the boat.

Solution:

Variable	Description
$\mathbf{v_b}(0, 25)$	velocity vector of boat in north direction
$\mathbf{v_c}$	velocity vector of water current at $60°$ east of south direction
$\mathbf{v_r}$	resultant velocity vector of boat

TABLE 0: Input parameters

$$\mathbf{v}_c = 10 \begin{pmatrix} \sin 60° \\ -\cos 60° \end{pmatrix} \tag{1}$$

$$\implies v_c = 10 \begin{pmatrix} \frac{\sqrt{3}}{2} \\ \frac{-1}{2} \end{pmatrix} \tag{2}$$

$$\implies v_c = \begin{pmatrix} 5\sqrt{3} \\ -5 \end{pmatrix} \tag{3}$$

$$\implies \therefore v_r = v_b + v_c \tag{4}$$

$$\implies v_r = \begin{pmatrix} 0 \\ 25 \end{pmatrix} + \begin{pmatrix} 5\sqrt{3} \\ -5 \end{pmatrix} \tag{5}$$

$$\implies v_r = \begin{pmatrix} 5\sqrt{3} \\ 20 \end{pmatrix} \tag{6}$$

$$\implies \|v_r\| = \sqrt{v_r^\top v_r} \tag{7}$$

$$\implies \|v_r\| = \sqrt{\left(5\sqrt{3}\right)^2 + (20)^2} \tag{8}$$

$$\implies \|v_r\| = \sqrt{475} \approx 21.8km/h \tag{9}$$

Let the angle of the resultant vector be θ from north direction

$$\tan\theta = \frac{5\sqrt{3}}{20} \tag{10}$$

$$\implies \theta = \tan^{-1}\frac{\sqrt{3}}{4} \approx 23.4° \tag{11}$$

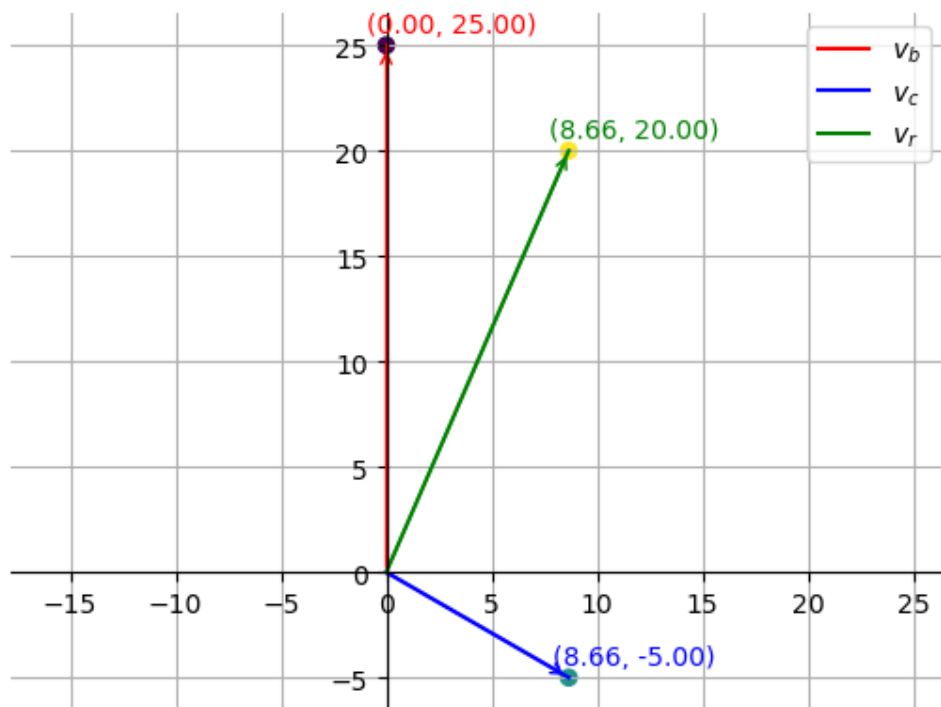So, the resultant velocity vector v_r is $21.8km/h$ at an angle of $23.4°$ from north direction.

Fig. 0: Resultant velocity vector of boat