

- 1) Let the six numbers $a_1, a_2, a_3, a_4, a_5, a_6$ be in $A.P$ and $a_1 + a_2 = 10$. If the mean of these six numbers is $\frac{19}{2}$ and their variance is σ^2 , then $8\sigma^2$ is equal to
- 220
 - 210
 - 200
 - 105
- 2) Let $f(x)$ be a function such that $f(x+y) = f(x).f(y)$ for all $x, y \in \mathbb{N}$. If $f(1) = 3$ and $\sum_{k=1}^n f(k) = 3279$, then the value of n is
- 6
 - 8
 - 7
 - 9
- 3) The number of real solutions of the equations $3\left(x^2 + \frac{1}{x^2}\right) - 2\left(x + \frac{1}{x}\right) + 5 = 0$, is
- 4
 - 0
 - 3
 - 2
- 4) If $f(x) = \frac{2^{2x}}{2^{2x}+2}$, $x \in \mathbb{R}$, then $f\left(\frac{1}{2023}\right) + f\left(\frac{2}{2023}\right) + \dots + f\left(\frac{2022}{2023}\right)$ is equal to
- 2011
 - 1010
 - 2010
 - 1011
- 5) If $f(x) = x^3 - x^2 f'(1) + x f''(2) - f'''(3)$, $x \in \mathbb{R}$, then
- $3f(1) + f(2) = f(3)$
 - $f(3) - f(2) = f(1)$
 - $2f(0) - f(1) + f(3) = f(2)$
 - $f(1) + f(2) + f(3) = f(0)$
- 6) The number of integers, greater than 7000 that can be formed, using the digits 3, 5, 6, 7, 8 without repetition, is
- 120
 - 168
 - 220
 - 48
- 7) If the system of equations
- $$x + 2y + 3z = 3$$

$$4x + 3y - 4z = 4$$

$$8x + 4y - \lambda z = 9 + \mu$$

has infinitely many solutions, then the ordered pair (λ, μ) is equal to

- a) $\left(\frac{72}{5}, \frac{21}{5}\right)$
- b) $\left(\frac{-72}{5}, \frac{-21}{5}\right)$
- c) $\left(\frac{72}{5}, \frac{-21}{5}\right)$
- d) $\left(\frac{-72}{5}, \frac{21}{5}\right)$

8) The value of $\left(\frac{1+\sin \frac{2\pi}{9}+i \cos \frac{2\pi}{9}}{1+\sin \frac{2\pi}{9}-i \cos \frac{2\pi}{9}}\right)^3$ is

- a) $\frac{-1}{2}(1-i\sqrt{3})$
- b) $\frac{1}{2}(1-i\sqrt{3})$
- c) $\frac{-1}{2}(\sqrt{3}-i)$
- d) $\frac{1}{2}(\sqrt{3}+i)$

9) The equations of the sides AB and AC of a triangle ABC are

$(\lambda + 1)x + \lambda y = 4$ and $\lambda x + (1 - \lambda)y + \lambda = 0$ respectively. Its vertex A is on the y-axis and its orthocentre is $(1, 2)$. The length of the tangent from the point C to the part of the parabola $y^2 = 6x$ in the first quadrant is

- a) $\sqrt{6}$
- b) $2\sqrt{2}$
- c) 2
- d) 4

10) The set of all values of a for which

$\lim_{x \rightarrow a} ([x - 5] - [2x + 2]) = 0$, where $[x]$ denotes the greatest integer less than or equal to x is equal to

- a) $(-7.5, -6.5)$
- b) $(-7.5, -6.5]$
- c) $[-7.5, -6.5]$
- d) $[-7.5, -6.5)$

11) If $({}^{30}C_1)^2 + 2({}^{30}C_2)^2 + 3({}^{30}C_3)^2 + \dots + 30({}^{30}C_{30})^2 = \frac{\alpha 60!}{(30!)^2}$, then α is equal to

- a) 30
- b) 60
- c) 15
- d) 10

12) Let the plane containing the line of intersection of the planes

$$P_1 : x + (\lambda + 4)y + z = 1 \text{ and}$$

$P_2 : 2x + y + z = 2$ pass through the points $(0, 1, 0)$ and $(1, 0, 1)$. Then the distance of the point $(2\lambda, \lambda, -\lambda)$ from the plane P_2 is

- a) $5\sqrt{6}$
- b) $4\sqrt{6}$
- c) $2\sqrt{6}$
- d) $3\sqrt{6}$

13) $\vec{\alpha} = 4\hat{i} + 3\hat{j} + 5\hat{k}$ and $\vec{\beta} = \hat{i} + 2\hat{j} - 4\hat{k}$. Let $\vec{\beta}_1$ be parallel to $\vec{\alpha}$ and $\vec{\beta}_2$ be perpendicular

to $\vec{\alpha}$. If $\vec{\beta} = \vec{\beta}_1 + \vec{\beta}_2$, then the value of $5\vec{\beta}_2 \cdot (\hat{i} + \hat{j} + \hat{k})$ is

- a) 6
- b) 11
- c) 7
- d) 9

14) The locus of the mid points of the chords of the circle $C_1 : (x - 4)^2 + (y - 5)^2 = 4$ which subtend an angle θ_i at the center of the circle C_1 , is a circle of radius r_i . If $\theta_1 = \frac{\pi}{3}$, $\theta_3 = \frac{2\pi}{3}$ and $r_1^2 = r_2^2 + r_3^2$ then θ_2 is equal to

- a) $\frac{\pi}{4}$
- b) $\frac{3\pi}{4}$
- c) $\frac{\pi}{6}$
- d) $\frac{\pi}{2}$

15) If the foot of the perpendicular drawn from $(1, 9, 7)$ to the line passing through the point $(3, 2, 1)$ and parallel to the planes $x + 2y + z = 0$ and $3y - z = 3$ is (α, β, γ) , then $\alpha + \beta + \gamma$ is equal to

- a) -1
- b) 3
- c) 1
- d) 5