

- 40) In a parallel plate capacitor, the plate at $x = 0$ is grounded and the plate at $x = d$ is maintained at a potential V_0 . The space between the two plates is filled with a linear dielectric of permittivity $\epsilon = \epsilon_0 \left(1 + \frac{x}{d}\right)$, where ϵ_0 is the permittivity of free space.

Neglecting the edge effects, the electric field (\vec{E}) inside the capacitor is

- a) $-\frac{V_0}{(d+x)\ln 2} \hat{x}$
- b) $-\frac{V_0}{d} \hat{x}$
- c) $-\frac{V_0}{(d+x)} \hat{x}$
- d) $-\frac{V_0 d}{(d+x)x} \hat{x}$

- 41) The equation of motion for the forced simple harmonic oscillator is

$$\ddot{x}(t) + \omega^2 x(t) = F \cos(\omega t)$$

where $x(t=0) = 0$ and $\dot{x}(t=0) = 0$. Which one of the following options is correct?

- a) $x(t) \propto t \sin(\omega t)$
- b) $x(t) \propto t \cos(\omega t)$
- c) $x(t) = \infty$
- d) $x(t) \propto e^{\omega t}$

- 42) An atom is subjected to a weak uniform magnetic field \vec{B} . The number of lines in its Zeeman spectrum for transition from $n = 2, l = 1$ to $n = 1, l = 0$ is

- a) 8
- b) 10
- c) 12
- d) 5

- 43) Consider two matrices: $P = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$ and $Q = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

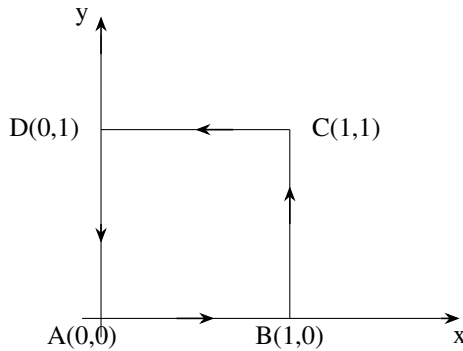
Which of the following statement is/are true?

- a) P and Q have same set of eigenvalues
- b) P and Q commute with each other
- c) P and Q have different sets of linearly independent eigenvectors
- d) P is diagonalizable

- 44) An infinite one dimensional lattice extends along x -axis. At each lattice site there exists an ion with spin $\frac{1}{2}$. The spin can point either in $+z$ or $-z$ direction only. Let S_P, S_F , and S_A denote the entropies of paramagnetic, ferromagnetic and antiferromagnetic configurations, respectively. Which of the following relation is/are true?

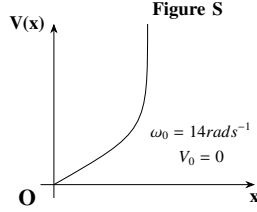
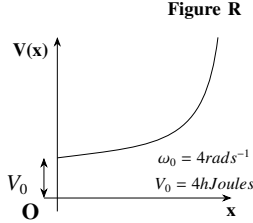
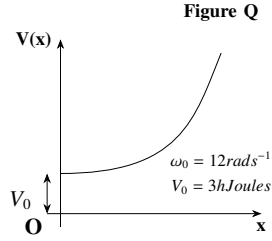
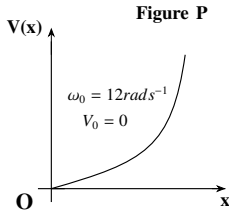
- a) $S_P > S_F$
- b) $S_A > S_F$
- c) $S_A = 4S_F$
- d) $S_P > S_A$

- 45) Consider a vector field $\vec{F} = (2xz + 3y^2)\hat{y} + 4yz^2\hat{z}$. The closed path $(\Gamma : A \rightarrow B \rightarrow C \rightarrow D \rightarrow A)$ in $z = 0$ plane is shown in figure.



$\oint_{\Gamma} \vec{F} \cdot d\vec{l}$ denotes the line integral of \vec{F} along the closed path Γ . Which of the following option is/are true?

- a) $\oint_{\Gamma} \vec{F} \cdot d\vec{l} = 0$
 - b) \vec{F} is non-conservative
 - c) $\vec{\nabla} \cdot \vec{F} = 0$
 - d) \vec{F} can be written as the gradient of a scalar field
- 46) Two point charges of charge $+q$ each are placed a distance $2d$ apart. A grounded solid conducting sphere of radius a is placed midway between them. Assume $a^2 \ll d^2$. Which of the following statement is/are true?
- a) If $a > \frac{d}{8}$ the net force acting on the charges is directed towards each other
 - b) The potential at the surface of the sphere is zero
 - c) Total induced charge on the sphere is $\left(-\frac{2aq}{d}\right)$
 - d) The potential at the center of the sphere is non-zero
- 47) A particle of mass m is moving in the potential
- $$V(x) = \begin{cases} V_0 + \frac{1}{2}m\omega_0^2 x^2 & x > 0 \\ \infty & x \leq 0 \end{cases}$$
- Figures P, Q, R and S show different combinations of the values of ω_0 and V_0 .



$E_j^{(P)}$, $E_j^{(Q)}$, $E_j^{(R)}$ and $E_j^{(S)}$ with $j = 0, 1, 2, \dots$ are the eigen-energies of the j -th level for the potentials shown in figures P, Q, R and S, respectively. Which of the statement is/are true?

- a) $E_0^{(P)} = E_0^{(Q)}$ c) $E_0^{(P)} = E_0^{(R)}$
 b) $E_0^{(Q)} = E_0^{(S)}$ d) $E_0^{(R)} \neq E_0^{(Q)}$

48) The non-relativistic Hamiltonian for a single electron atom is

$$H_0 = \frac{p^2}{2m} - V(r)$$

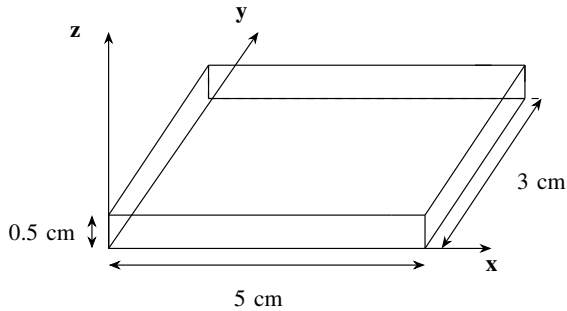
where $V(r)$ is the Coulomb potential and m is the mass of the electron. Considering the spin-orbit interaction term

$$H' = \frac{1}{2m^2c^2} \frac{1}{r} \frac{dV}{dr} \vec{L} \cdot \vec{S}$$

added to H_0 , which of the following statement is/are true?

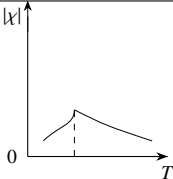
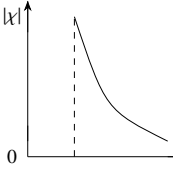
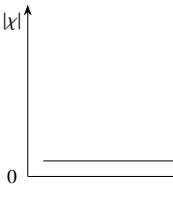
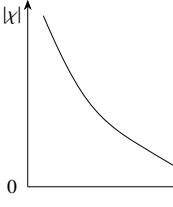
- a) H' commutes with L^2
 b) H' commutes with L_z and S_z
 c) For a given value of principal quantum number n and orbital angular momentum quantum number l , there are $2(2l+1)$ degenerate eigenstates of H_0
 d) H_0, L^2, S^2, L_z and S_z have a set of simultaneous eigenstates
- 49) Decays of mesons and baryons can be categorized as weak, strong and electromagnetic decays depending upon the interactions involved in the processes. Which of the following option is/are true?
- a) $\pi^0 \rightarrow \gamma\gamma$ is a weak decay
 b) $\Lambda^0 \rightarrow \pi^0 + p$ is an electromagnetic decay
 c) $K^0 \rightarrow \pi^+ + \pi^-$ is a weak decay
 d) $\nabla^{++} \rightarrow p + \pi^+$ is a strong decay

- 50) An extrinsic semiconductor shown in figure carries a current of $2mA$ along its length parallel to $+x$ axis



When the majority charge carrier concentration is $12.5 \times 10^{13} \text{ cm}^{-3}$ and the sample is exposed to a constant magnetic field applied along the $+z$ direction, a Hall voltage of 20 mV is measured with the negative polarity at $y = 0$ plane. Take the electric charge as $1.6 \times 10^{-19} \text{ C}$. The concentration of minority charge carrier is negligible. Which of the following statement is/are true?

- a) The majority charge carrier is electron
 - b) The magnitude of the applied magnetic field is 1 Tesla
 - c) The electric field corresponding to the Hall voltage is in the $+y$ direction
 - d) The magnitude of Hall coefficient is $50,000 \text{ m}^3 \text{ C}^{-1}$
- 51) A^α and B_β ($\alpha, \beta = 1, 2, 3, \dots, n$) are contravariant and covariant vectors, respectively. By convention, any repeated indices are summed over. Which of the following expression is/are tensors?
- a) $A^\alpha B_\beta$
 - b) $\frac{A^\alpha B_\beta}{A^\alpha B_\alpha}$
 - c) $\frac{A_\alpha}{B_\beta}$
 - d) $A^\alpha + B_\beta$
- 52) The temperature T dependence of magnetic susceptibility γ (Column I) of certain magnetic materials (Column II) are given below. Which of the following option is/are correct?

Column 1	Column 2
<p>(1)</p> 	(P) Diamagnetic
<p>(2)</p> 	(Q) Paramagnetic
<p>(3)</p> 	(R) Ferromagnetic
<p>(4)</p> 	(S) Antiferromagnetic

- a) 2 - P, 4 - Q, 3 - S
 b) 4 - P, 1 - Q, 2 - R
 c) 4 - Q, 2 - R, 1 - S
 d) 3 - P, 4 - Q, 2 - R