12.10.16

EE23BTECH11210-Dhyana Teja Machineni*

QUESTION: In double-slit experiment using light of wavelength 600 nm, the angular width of a fringe formed on a distant screen is 0.1°. What is the spacing between the two slits?

SOLUTION: Let the equation of the light waves coming from the source be

$$y_1 = A\sin(2\pi ft - kx_1) \tag{1}$$
$$y_2 = A\sin(2\pi ft - kx_2) \tag{2}$$
$$\tag{3}$$

where k is wave number of the wave equation

$$k = \frac{2\pi}{\lambda} \tag{4}$$

Since both the light waves are from the same source so the frequency of both the waves is same frequency f and amplitude A.

Using principle of superposition, we get

$$y = y_1 + y_2 \tag{5}$$

where y is resultant wave equation

$$y = y_1 + y_2 \tag{6}$$
$$y = Asin(2\pi ft - kx_1) + Asin(2\pi ft - kx_2) \tag{7}$$

using

$$sin(c) + sin(d) = 2sin(\frac{c+d}{2})cos(\frac{c-d}{2}) \tag{8}$$

we get

$$y = 2Asin(2\pi ft + \frac{kx_2 - kx_1}{2})cos(\frac{kx_1 - kx_2}{2}) \tag{9}$$

For constructive interference to happen

$$cos(\frac{k(x_1 - x_2)}{2}) = +/-1 \tag{10}$$
$$k(x_1 - x_2) = 2n\pi \tag{11}$$
$$k(\Delta x) = 2n\pi \tag{12}$$

From equation(4)

$$\frac{2\pi}{\lambda}(\Delta x) = 2n\pi \tag{13}$$
$$\therefore \Delta x = n\lambda \tag{14}$$

By approximations the light rays from the slits are parallel and the line drawn from the centre of the slits to interference is also parallel so the perpendicular drawn from the slit 1 to the light ray from the second slit makes the same angle as angular fringe width with the slit 1

$$\Delta x = dsin(\theta) \tag{15}$$

from the equations (14) and (15)

$$dsin(\theta) = n\lambda \tag{16}$$

Now, for small values of θ, we can approximate

$$sin(\theta) \approx tan(\theta) \tag{17}$$
$$\frac{n\lambda}{d} = \frac{y}{D} \tag{18}$$
$$y = n\frac{D\lambda}{d} \tag{19}$$

Now let us find the fringe width for this interference Let fringe width be β

$$y_{n+1} = (n+1)\frac{D\lambda}{d} \tag{20}$$
$$y_n = n\frac{D\lambda}{d} \tag{21}$$
$$\beta = y_{n+1} - y_n \tag{22}$$
$$\beta = \frac{D\lambda}{d} \tag{23}$$

Angluar Fringe width for light rays in YDSE is given by θ

$$Tan(\theta) = \frac{\beta}{D} \tag{24}$$

For small angles we can assume

$$Tan(\theta) \approx \theta \tag{25}$$

From equation (21)

$$\theta = \frac{\frac{\lambda D}{d}}{D} \tag{26}$$
$$\therefore \theta = \frac{\lambda}{d} \tag{27}$$

Given

$$\theta = 0.1^\circ \tag{28}$$
$$= \frac{\pi}{1800} \tag{29}$$
$$\lambda = 600nm \tag{30}$$

$$d = \frac{600}{\frac{\pi}{1800}} \tag{31}$$
$$\therefore d = 3.44 * 10^{-4} m. \tag{32}$$

TABLE 0
VARIABLES AND THEIR DESCRIPTIONS

VARIABLE	**Description**	**Value**
y_1	Equation of first wave	none
y_2	Equation of the second wave	none
k	Wave number of the light ray	none
f	Frequency of the light ray	none
y	Equation of the resultant light wave	none
A	Amplitude of the light wave	none
f	Frequency of both the wave equations	none
Δx	Path difference between the light rays	none
ϕ	Phase difference between the light rays	none
β	Fringe width of the interface formed by the light rays	none
D	Distance between the centre of the slits and the screen	none
d	Spacing between the slits used in the YDSE	NEED TO BE FOUND
λ	Wavelength of the light used	600nm
θ	Angular fringe width	0.1°