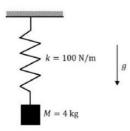
1

XE 71

EE23BTECH11048-Ponugumati Venkata Chanakya*

QUESTION: A spring mass system is shown in the figure. Take the value of acceleration due to gravity as $g = 9.81m/s^2$. The static deflection due to weight and the time period of the oscillations, respectively, are

(GATE 2023 XE)



Solution:

1) Static deflection due to weight(sdw) let x be sdw.

At mean position in equilibrium ??

$$Mg = kx \tag{1}$$

$$x = 39.24cm \tag{2}$$

2) Time period of oscillation

$$F = -kx \tag{3}$$

$$m\left(\frac{d^2x}{dt^2}\right) = -kx\tag{4}$$

Taking Laplace transform:

$$X(s) = \frac{msx(0) + x'(0)}{ms^2 + k}$$
 (5)

$$X(s) = \frac{1}{\sqrt{\frac{k}{m}}} \left(A \frac{s - i\sqrt{\frac{k}{m}}}{s^2 + \frac{k}{m}} + B \frac{s + i\sqrt{\frac{k}{m}}}{s^2 + \frac{k}{m}} \right)$$
(6)

Taking Inverse Laplace Transform: Initial Conditions be at extreme point of SHM

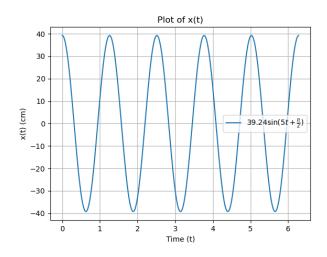
$$x(t) = A\left(B\sin(\sqrt{\frac{k}{m}}t) + C\cos(\sqrt{\frac{k}{m}}t)\right)$$
 (7)

$$x(t) = P\left(\sin(\sqrt{\frac{k}{m}}t + Q)\right) \tag{8}$$

$$x(t) = 0.3924 \left(\sin\left(\sqrt{\frac{100}{5}}t + \frac{\pi}{2}\right) \right)$$
 (9)

$$x(t) = 39.24 \sin(5t + \frac{\pi}{2}) \text{ cm}$$
 (10)

The static deflection due to weight and the time period of the oscillations, respectively are 39.24 cm and $\frac{2\pi}{5}$ s



Variable	Description	Value
M	weight of block	4 kg
K	spring constant	$100\frac{N}{m}$
х	Static deflection due to weight	39.24 cm
P	Amplitude	39.24 cm
Q	phase angle	$\frac{\pi}{2}$
A, B, C	integrating constants	none

TABLE 2 INPUT PARAMETERS