

11.14-4

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QUESTION: Which of the following functions of time represent (a) simple harmonic, (b) periodic but not simple harmonic, and (c) non-periodic motion? Give period for each case of periodic motion (ω is any positive constant):

1) $\sin(\omega t) - \cos(\omega t)$

2) $\sin^3(\omega t)$

3) $3 \cos\left(\frac{\pi}{4} - 2\omega t\right)$

4) $\cos(\omega t) + \cos(3\omega t) + \cos(5\omega t)$

5) $\exp(-\omega^2 t^2)$

6) $1 + \omega t + \omega^2 t^2$

Solution: :

Periodic function:

$$x(t + T) = x(t) \quad \forall x \in \mathbb{R} \quad (1)$$

where min T s.t $T > 0$ is time period

SHM:

For a function to be in shm it must satisfy

$$\frac{d^2 x(t)}{dt^2} = -\alpha x \quad (2)$$

(3) (4) $\cos(2\pi ft) + \cos(6\pi ft) + \cos(10\pi ft)$

1) $\sin(2\pi ft) - \cos(2\pi ft)$

The function can be rewritten as:

$$= \sin(2\pi ft) - \sin\left(\frac{\pi}{2} - 2\pi ft\right) \quad (4)$$

$$= 2 \cos\left(\frac{\pi}{4}\right) \sin\left(2\pi ft - \frac{\pi}{4}\right) \quad (5)$$

$$= \sqrt{2} \sin\left(2\pi ft - \frac{\pi}{4}\right) \quad (6)$$

$$\therefore \text{SHM, } T = \frac{1}{f} \text{ and } \phi = \left(\frac{-\pi}{4}\right) \text{ or } \left(\frac{7\pi}{4}\right)$$

(2) $\sin^3(2\pi ft)$

This function can be rewritten as

$$= \frac{1}{4}(3 \sin(2\pi ft) - \sin(6\pi ft)) \quad (7)$$

$$\therefore \text{Periodic with period } T = \frac{1}{f}$$

(3) $3 \cos\left(\frac{\pi}{4} - 4\pi ft\right)$

This function can be rewritten as

$$= 3 \cos\left(4\pi ft - \frac{\pi}{4}\right) \quad (8)$$

$$(9)$$

$$\therefore \text{SHM, } T = \frac{1}{2f} \text{ and } \phi = \left(\frac{-\pi}{4}\right) \text{ or } \left(\frac{7\pi}{4}\right)$$

This function can be rewritten as

$$= \cos(2\pi ft) + \cos(10\pi ft) + \cos(6\pi ft) \quad (10)$$

$$= 2 \cos\left(\frac{2\pi ft + 10\pi ft}{2}\right) \cos\left(\frac{10\pi ft - 2\pi ft}{2}\right) + \cos(6\pi ft) \quad (11)$$

$$= 2 \cos(6\pi ft) \cos(2\pi ft) + \cos(6\pi ft) \quad (12)$$

$$= \cos(6\pi ft)(1 + 2 \cos(2\pi ft)) \quad (13)$$

Variable	Description	formula
$x(t)$	Displacement wrt mean position	none
ω	Angular frequency	$2\pi f$
T	Time period	none
ϕ	phase angle	none

TABLE 0

INPUT PARAMETERS

Period of $\cos(6\pi ft)$ is $\frac{1}{3f}$

Period of $1 + 2\cos(2\pi ft)$ is $\frac{1}{f}$

Lcm is $\frac{1}{f}$

$\therefore \text{SHM}, T = \frac{1}{f}$

(5) $\exp(-(2\pi f)^2 t^2)$

As $T \rightarrow \infty$
 $\exp(-(2\pi f)^2 t^2) \rightarrow \infty$

\therefore This never repeats and non periodic

(6) $1 + 2\pi ft + (2\pi f)^2 t^2$

As $T \rightarrow \infty$
 $1 + 2\pi ft + (2\pi f)^2 t^2 \rightarrow \infty$
 \therefore This never repeats and non periodic

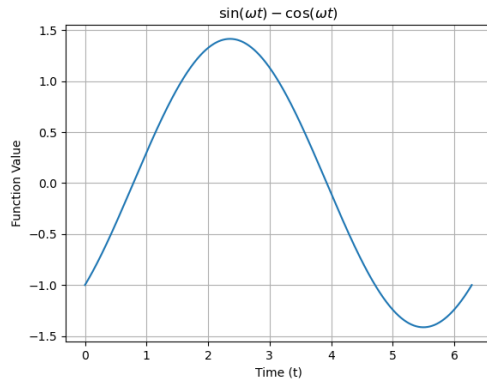


Fig. 1. $\sin(2\pi ft) - \cos(2\pi ft)$

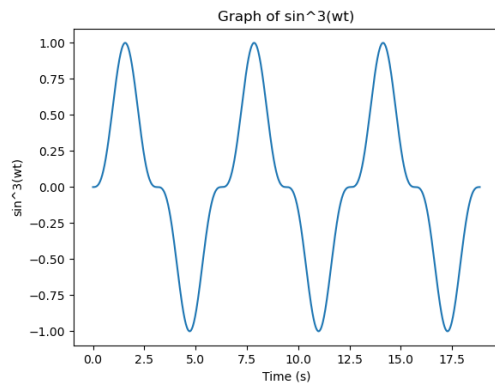


Fig. 1. $\sin^3(2\pi ft)$

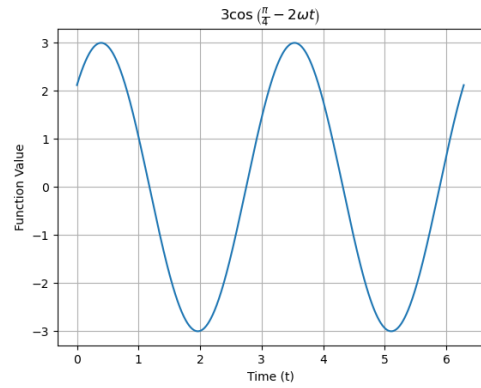


Fig. 1. $3\cos\left(\frac{\pi}{4} - 4\pi ft\right)$

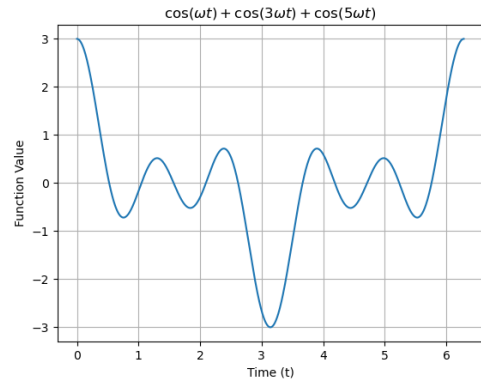


Fig. 1. $\cos(2\pi ft) + \cos(6\pi ft) + \cos(10\pi ft)$

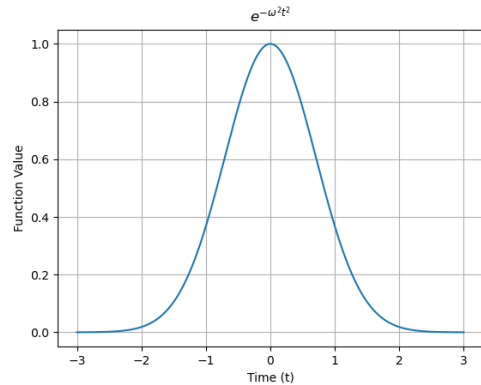


Fig. 1. $\exp(-(2\pi ft)^2)$

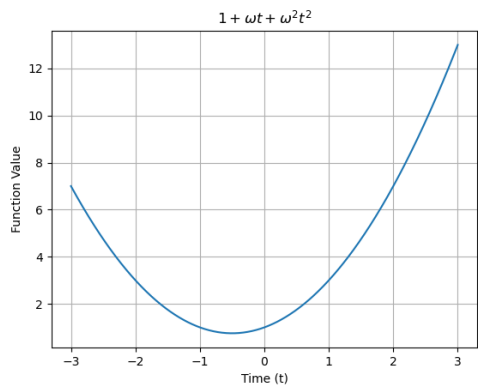


Fig. 1. $1 + 2\pi ft + (2\pi ft)^2$

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TABLE 1
SUMMARY

	Function	Periodic	Simple harmonic motion	Non Periodic	T	ϕ
(a)	$\sin(2\pi ft) - \cos(2\pi ft)$	Yes	Yes	No	$\frac{1}{f}$	$\left(\frac{-\pi}{4}\right)$
(b)	$\sin^3(2\pi ft)$	Yes	No	No	$\frac{1}{f}$	—
(c)	$3 \cos\left(\frac{\pi}{4} - 4\pi ft\right)$	Yes	Yes	No	$\frac{1}{2f}$	$\left(\frac{-\pi}{4}\right)$
(d)	$\cos(2\pi ft) + \cos(6\pi ft) + \cos(10\pi ft)$	Yes	No	No	$\frac{1}{f}$	—
(e)	$\exp\left(-(2\pi ft)^2\right)$	No	No	Yes	—	—
(f)	$1 + (2\pi f)t + (2\pi f)^2 t^2$	No	No	Yes	—	—