11.14-4

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QUESTION: Which of the following functions of time represent (a) simple harmonic, (b) periodic but not simple harmonic, and (c) non-periodic motion? Give period for each case of peiodic motion ($2\pi f$ is any positive constant):

1) $\sin(2\pi ft) - \cos(2\pi ft)$

2) $\sin^3(2\pi ft)$

3) $3\cos\left(\frac{\pi}{4} - 4\pi ft\right)$

4) $\cos(2\pi ft) + \cos(6\pi ft) + \cos(10\pi ft)$

5) $\exp\left(-(2\pi ft)^2\right)$

6) $1 + 2\pi ft + 2\pi f^2 t^2$

Answer:
Definition of period:
The period is denoted by the symbol "T," and it represents the time interval required for the motion to go through one complete cycle

1) $\sin(2\pi ft) - \cos(2\pi ft)$

This function can be rewritten as

$$= \sin(2\pi ft) - \sin\left(\frac{\pi}{2} - 2\pi ft\right) \quad (1)$$

$$= 2\cos\left(\frac{\pi}{4}\right)\sin\left(2\pi ft - \frac{\pi}{4}\right) \quad (2)$$

$$= \sqrt{2}\sin\left(2\pi ft - \frac{\pi}{4}\right) \quad (3)$$

$\therefore$ Simple harmonic motion with period T $=\frac{1}{f}$
Phase angle of(ϕ) $\left(\frac{-\pi}{4}\right)$ or$\left(\frac{7\pi}{4}\right)$

(2) $\sin^3(wt)$

This function can be rewritten as

$$= \frac{1}{4}(3\sin(2\pi ft) - \sin(6\pi ft)) \quad (4)$$

$\therefore$ Periodic with period T $= \frac{1}{f}$

(3) $3\cos\left(\frac{\pi}{4} - 4\pi ft\right)$

This function can be rewritten as

$$= 3\cos\left(4\pi ft - \frac{\pi}{4}\right) \quad (5)$$

$$(6)$$

Simple harmonic motion with period T $= \frac{1}{2f}$ and a ϕ of $\left(\frac{-\pi}{4}\right)$ or$\left(\frac{7\pi}{4}\right)$

(4) $\cos(2\pi ft) + \cos(6\pi ft) + \cos(10\pi ft)$

This function can be rewritten as

$$= \cos(2\pi ft) + \cos(10\pi ft) + \cos(6\pi ft) \quad (7)$$

$$= 2\cos\left(\frac{2\pi ft + 10\pi ft}{2}\right)\cos\left(\frac{10\pi ft - 2\pi ft}{2}\right) + \cos(6\pi ft) \quad (8)$$

$$= 2\cos(6\pi ft)\cos(2\pi ft) + \cos(6\pi ft) \quad (9)$$

$$= \cos(6\pi ft)(1 + 2\cos(2\pi ft)) \quad (10)$$

Period of $\cos(6\pi ft)$ is $\frac{1}{3f}$
Period of $1 + 2\cos(2\pi ft)$ is $\frac{1}{f}$
Lcm is $\frac{1}{f}$
$\therefore$ Simple harmonic motion with period $\frac{1}{f}$

(5) $\exp\left(-(2\pi f)^2 t^2\right)$

This function can be rewritten as

$$\text{As } T \to \infty$$
$$\exp\left(-(2\pi f)^2 t^2\right) \to \infty$$

$\therefore$ This never repeats and non periodic

(6) $1 + 2\pi f t + (2\pi f)^2 t^2$

This function can be rewritten as

$$\text{As } T \to \infty$$
$$1 + 2\pi f t + (2\pi f)^2 t^2 \to \infty$$

$\therefore$ This never repeats and non periodic

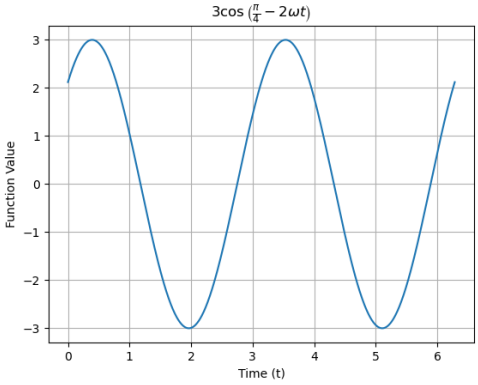

Fig. 0. $3\cos\left(\frac{\pi}{4} - 4\pi f t\right)$

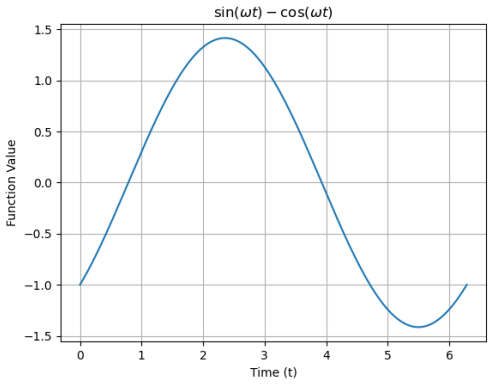

Fig. 0. $\sin(2\pi f t) - \cos(2\pi f t)$

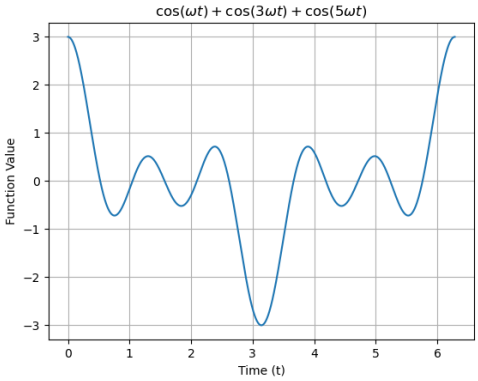

Fig. 0. $\cos(2\pi f t) + \cos(6\pi f t) + \cos(10\pi f t)$

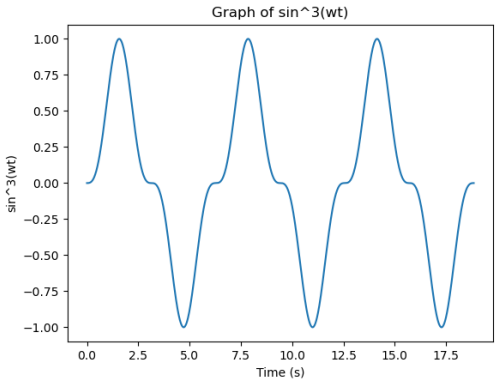

Fig. 0. $\sin^3(2\pi f t)$

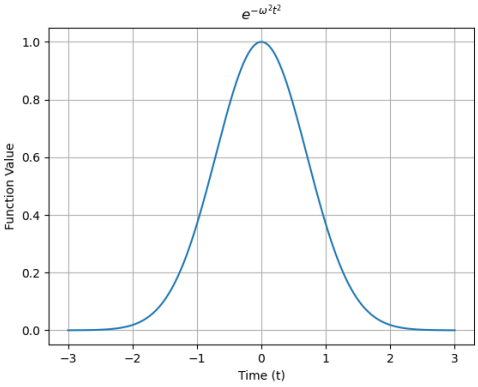

Fig. 0. $exp^{(-(2\pi f t)^2)}$

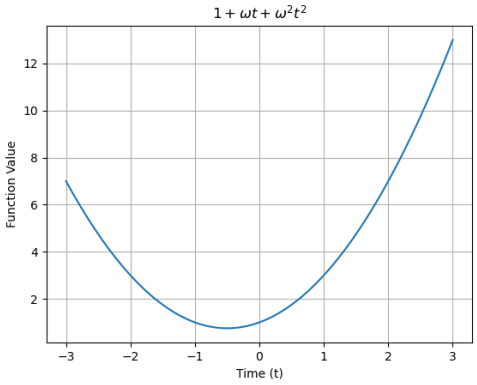

Fig. 0. $1 + 2\pi ft + (2\pi ft)^2$

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TABLE 0
SUMMARY

	Function	Periodic	Simple harmonic motion	Non Periodic	Period
(a)	$sin(2\pi ft) - cos(2\pi ft)$	Yes	Yes	No	$\frac{1}{f}$
(b)	$sin^3(2\pi ft)$	Yes	No	No	$\frac{1}{f}$
(c)	$3cos\left(\frac{\pi}{4} - 4\pi ft\right)$	Yes	Yes	No	$\frac{1}{2f}$
(d)	$cos(2\pi ft) + cos(6\pi ft) + cos(10\pi ft)$	Yes	No	No	$\frac{}{f}$
(e)	$\exp\left(-(2\pi ft)^2\right)$	No	No	Yes	–
(f)	$1 + (2\pi f)t + (2\pi f)^2 t^2$	No	No	Yes	–