ME 36

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QUESTION: The value of Integral

$$\oint\left(\frac{6z}{2z^4-3z^3+7z^2-3z+5}\right)dz$$

evaluated over a counter-clockwise circular contour in the complex plane enclosing only the pole $z = \jmath$, where $\jmath$ is the imaginary unit,is

1) $(-1+\jmath)\pi$
2) $(1+\jmath)\pi$
3) $2(1-\jmath)\pi$
4) $(2+\jmath)\pi$

(GATE 2022 ME)

Solution:

Given $z = \jmath$ is only enclosing pole

$$\oint\left(\frac{6z}{2z^4-3z^3+7z^2-3z+5}\right)dz = \oint\left(\frac{\frac{6z}{2z^3+(2\jmath-3)z^2+(5-3\jmath)z+5\jmath}}{z-\jmath}\right)dz \tag{1}$$

$$= 2\pi\jmath\left(\frac{6z}{2z^3+(2\jmath-3)z^2+(5-3\jmath)z+5\jmath}\right) \text{ At } z=\jmath \tag{2}$$

(By Cauchy's integral formula)

$$= 2\pi\jmath\left(\frac{6\jmath}{2\jmath^3+(2\jmath-3)\jmath^2+(5-3\jmath)\jmath+5\jmath}\right) \tag{3}$$

$$= 2\pi\jmath\left(\frac{\jmath}{\jmath+1}\right) \tag{4}$$

$$= -2\pi\frac{\jmath-1}{\jmath^2-1} \tag{5}$$

$$= (-1+\jmath)\pi \tag{6}$$