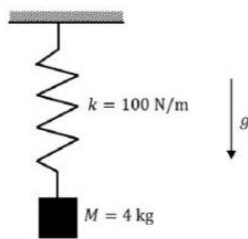


XE 71

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QUESTION: A spring mass system is shown in the figure . Take the value of acceleration due to gravity as $g = 9.81 \text{ m/s}^2$. The static deflection due to weight and the time period of the oscillations, respectively, are

(GATE 2023 XE)



Solution:

- 1) Static deflection due to weight (sdw)
let x be sdw.
0 At mean position in equilibrium
??

$$Mg = kx \quad (1)$$

$$x = 39.24 \text{ cm} \quad (2)$$

- 2) Time period of oscillation

$$F = -kx \quad (3)$$

$$m \left(\frac{d^2x}{dt^2} \right) = -kx \quad (4)$$

Initial Conditions be at extreme point of SHM

$$x(0) = 0.3924 \quad (5)$$

$$\frac{dx}{dt} = 0 \text{ at } t = 0 \text{ (released from rest)} \quad (6)$$

Taking Laplace transform:

$$m(s^2X(s) - sx(0) - mx'(0)) + kX(s) = 0 \quad (7)$$

$$X(s) = \frac{x(0)ms + mx'(0)}{ms^2 + k} \quad (8)$$

$$X(s) = x(0) \frac{s}{s^2 + \frac{k}{m}} + \left(x'(0) \sqrt{\frac{k}{m}} \right) \frac{\sqrt{\frac{k}{m}}}{s^2 + \left(\sqrt{\frac{k}{m}} \right)^2} \quad (9)$$

Taking Inverse Laplace Transform:

$$x(t) = x(0) \cos \left(\sqrt{\frac{k}{m}} t \right) + \left(x'(0) \sqrt{\frac{k}{m}} \right) \sin \left(\sqrt{\frac{k}{m}} t \right) \quad (10)$$

$$(11)$$

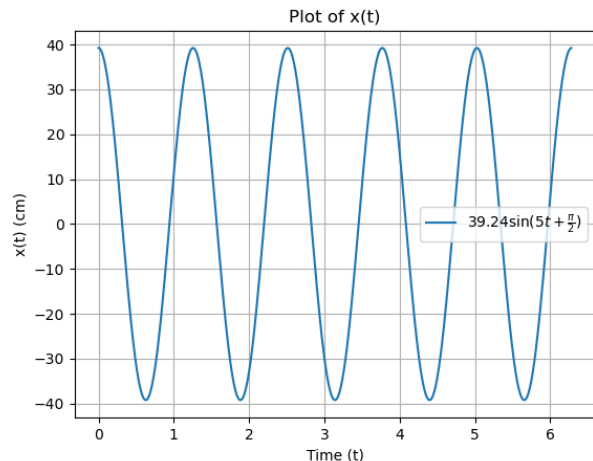
Using ?? and ??

$$x(t) = 0.3924 \cos \left(\sqrt{\frac{k}{m}} t \right) \quad (12)$$

$$x(t) = 0.3924 \sin \left(5t + \frac{\pi}{2} \right) \text{ m} \quad (13)$$

$$x(t) = 39.24 \sin \left(5t + \frac{\pi}{2} \right) \text{ cm} \quad (14)$$

The static deflection due to weight and the time period of the oscillations, respectively are 39.24 cm and $\frac{2\pi}{5}$ s



Variable	Description	Value
M	weight of block	4 kg
K	spring constant	$100 \frac{N}{m}$
x	Static deflection due to weight	39.24 cm
$x(t)$	Displacement of particle from mean position at time t	none
$x(0)$	Initial Displacement of particle from mean position	39.24cm
$x'(t)$	velocity of particle	none
$x'(0)$	initial velocity of particle	0

TABLE 2
INPUT PARAMETERS