11.14-4

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QUESTION: QUESTION: Which of the following functions of time represent (a) simple harmonic, (b) periodic but not simple harmonic, and (c) non-periodic motion? Give period for each case of peiodic motion (ω is any positive constant):

1) $\sin(\omega t) - \cos(\omega t)$
2) $\sin^3(\omega t)$
3) $3\cos\left(\frac{\pi}{4} - 2\omega t\right)$
4) $\cos(\omega t) + \cos(3\omega t) + \cos(5\omega t)$
5) $exp(-\omega^2 t^2)$
6) $1 + \omega t + \omega^2 t^2$

Answer:
Definition of period:
The period is denoted by the symbol "T," and it represents the time interval required for the motion to go through one complete cycle

1) $\sin(\omega t) - \cos(\omega t)$

This function can be rewritten as

$$= \sin(\omega t) - \sin\left(\frac{\pi}{2} - \omega t\right) \tag{1}$$
$$= 2\cos\left(\frac{\pi}{4}\right)\sin\left(\omega t - \frac{\pi}{4}\right) \tag{2}$$
$$= \sqrt{2}\sin\left(\omega t - \frac{\pi}{4}\right) \tag{3}$$

$\therefore$ Simple harmonic motion with period T $=\frac{2\pi}{\omega}$
Phase angle of $\left(\frac{-\pi}{4}\right)$ or$\left(\frac{7\pi}{4}\right)$

(2) $\sin^3(wt)$

This function can be rewritten as

$$= \frac{1}{4}(3\sin(\omega t) - \sin(3\omega t)) \tag{4}$$

$\therefore$ periodc with period T $= \frac{2\pi}{\omega}$

(3) $3\cos\left(\frac{\pi}{4} - 2\omega t\right)$

This function can be rewritten as

$$= 3\cos\left(2\omega t - \frac{\pi}{4}\right) \tag{5}$$
$$\tag{6}$$

Simple harmonic motion with period T $= \frac{\pi}{\omega}$ and a phase angle of $\left(\frac{-\pi}{4}\right)$ or$\left(\frac{7\pi}{4}\right)$

(4) $\cos(\omega t) + \cos(3\omega t) + \cos(5\omega t)$

This function can be rewritten as

$$= \cos(\omega t) + \cos(5\omega t) + \cos(3\omega t) \tag{7}$$
$$= 2\cos\left(\frac{\omega t + 5\omega t}{2}\right)\cos\left(\frac{5\omega t - \omega t}{2}\right) + \cos(3\omega t) \tag{8}$$
$$= 2\cos(3\omega t)\cos(\omega t) + \cos(3\omega t) \tag{9}$$
$$= \cos(3\omega t)(1 + 2\cos(\omega t)) \tag{10}$$

Period of $\cos(3\omega t)$ is $\frac{2\pi}{3\omega}$
Period of $1 + 2\cos(\omega t)$ is $\frac{2\pi}{\omega}$
Lcm is $\frac{2\pi}{\omega}$
$\therefore$ Simple harmonic motion with period $\frac{2\pi}{\omega}$

(5) $\exp\left(-\omega^2 t^2\right)$

This function can be rewritten as

$$\text{As } T \to \infty$$

$$\exp\left(-\omega^2 t^2\right) \to \infty$$

$\therefore$ This never repeats and non periodic

(6) $1 + \omega t + \omega^2 t^2$

This function can be rewritten as

$$\text{As } T \to \infty$$
$$1 + \omega t + \omega^2 t^2 \to \infty$$

$\therefore$ This never repeats and non periodic

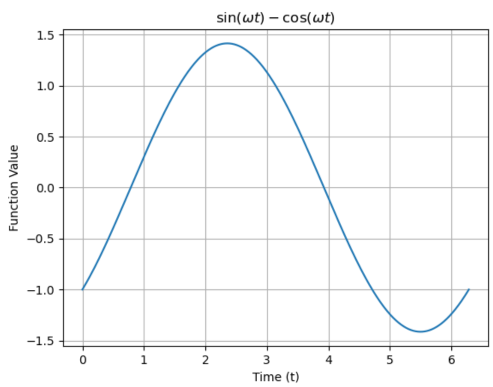

Fig. 0. $\sin(\omega t) - \cos(\omega t)$

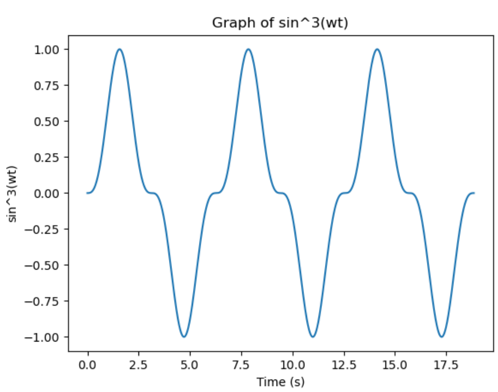

Fig. 0. $\sin^3(\omega t)$

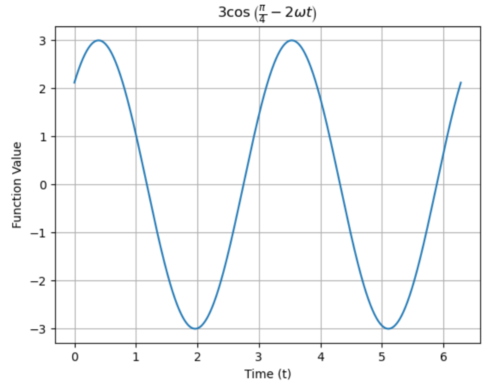

Fig. 0. $3\cos\left(\frac{\pi}{4} - 2\omega t\right)$

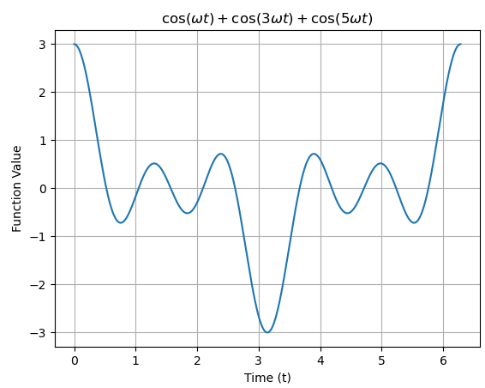

Fig. 0. $\cos(\omega t) + \cos(3\omega t) + \cos(5\omega t)$

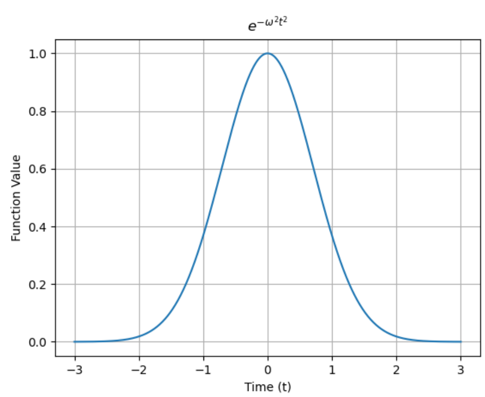

Fig. 0. $exp^{\left(-\omega^2 t^2\right)}$

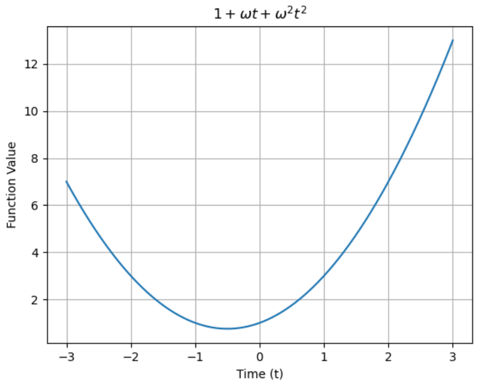

Fig. 0. $1 + \omega t + \omega^2 t^2$

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TABLE 0
SUMMARY

	Function	**Periodic**	**Simple harmonic motion**	**Non Periodic**	**Period**
(a)	$sin(\omega t) - cos(wt)$	Yes	Yes	No	$\frac{2\pi}{\omega}$
(b)	$sin^3(\omega t)$	Yes	Yes	No	$\frac{2\pi}{\omega}$
(c)	$3cos\left(\frac{\pi}{4} - 2\omega t\right)$	Yes	Yes	No	$\frac{\pi}{\omega}$
(d)	$cos(\omega t) + cos(3\omega t) + cos(5\omega t)$	Yes	Yes	No	$\frac{2\pi}{\omega}$
(e)	$exp^{(-\omega^2 t^2)}$	No	No	Yes	$-$
(f)	$1 + \omega t + \omega^2 t^2$	No	No	Yes	$-$