AE 42

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QUESTION: Consider the equation $\frac{dy}{dx} + ay = \sin \omega x$,where a and ω are constants.Given $y = 1$ at $x = 0$, correct all the correct statement(s) from the following as $x \to \infty$.

(A) $y \to 0$ if $a \neq 0$

(B) $y \to 1$ if $a = 0$

(C) $y \to Aexp(|a|x)$ if $a < 0$; A is constant

(D) $y \to B\sin(\omega x + C)$ if $a > 0$; B and C are constants
(GATE AE 2023)

Solution: :
$y(0) = 1$

$$\frac{dy}{dx} + ay = \sin \omega x \tag{1}$$

Taking laplace transform on both sides

Function	Laplace transform
$\frac{dy}{dx}$	$xY - y(0)$
y	Y
$\sin \omega x$	$\frac{\omega}{\omega^2+x^2}$

TABLE 0
LAPLACE TRANSFORM

$$xY - y(0) + aY = \frac{\omega}{\omega^2 + x^2} \tag{2}$$

$$xY - 1 + aY = \frac{\omega}{\omega^2 + x^2} \tag{3}$$

$$\implies Y(x) = \frac{1}{x+a}\left(\frac{\omega}{\omega^2 + x^2} + \frac{1}{x+a}\right) \tag{4}$$

$$Y(x) = \frac{A}{x+a} + \frac{Bx + C}{\omega^2 + x^2} \tag{5}$$

Taking inverse laplace transform on both sides

$$y(x) = \mathcal{L}^{-1}\{Y(s)\} = Ae^{-ax} + (B\cos(\omega x) + C\sin(\omega x)) \tag{6}$$

$$y(x) = \mathcal{L}^{-1}\{Y(s)\} = Ae^{-ax} + B\sin(\omega x + C) \tag{7}$$

now as $x \to \infty$

1) $y \to 0$ if $a \neq 0$ is not true as y depend on a, ω
2) $y \to 1$ if $a = 0$ is not true as y depend on ω
3) $y \to Aexp(|a|x)$ if $a < 0$ is true as $B\sin(\omega x + C)$ is neglected compared to Ae^{-ax}
4) $y \to B\sin(\omega x + C)$ if $a > 0$; is true as $Ae^{-ax} \to 0$

$\therefore$ C,D are correct options