

# 11.14-4

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**QUESTION: QUESTION:** Which of the following functions of time represent (a) simple harmonic, (b) periodic but not simple harmonic, and (c) non-periodic motion? Give period for each case of periodic motion ( $\omega$  is any positive constant):

1)  $\sin(\omega t) - \cos(\omega t)$

2)  $\sin^3(\omega t)$

3)  $3 \cos\left(\frac{\pi}{4} - 2\omega t\right)$

4)  $\cos(\omega t) + \cos(3\omega t) + \cos(5\omega t)$

5)  $\exp(-\omega^2 t^2)$

6)  $1 + \omega t + \omega^2 t^2$

Answer:

Definition of period:

The period is denoted by the symbol "T," and it represents the time interval required for the motion to go through one complete cycle

1)  $\sin(\omega t) - \cos(\omega t)$

This function can be rewritten as

$$\begin{aligned} &= \sin(\omega t) - \sin\left(\frac{\pi}{2} - \omega t\right) \\ &= 2 \cos\left(\frac{\pi}{4}\right) \sin\left(\omega t - \frac{\pi}{4}\right) \\ &= \sqrt{2} \sin\left(\omega t - \frac{\pi}{4}\right) \end{aligned}$$

$\therefore$  Simple harmonic motion with period  $T = \frac{2\pi}{\omega}$

Phase angle of  $\left(\frac{-\pi}{4}\right)$  or  $\left(\frac{7\pi}{4}\right)$

(2)  $\sin^3(\omega t)$

This function can be rewritten as

$$= \frac{1}{4}(3 \sin(\omega t) - \sin(3\omega t)) \quad (4)$$

$\therefore$  Periodic with period  $T = \frac{2\pi}{\omega}$

(3)  $3 \cos\left(\frac{\pi}{4} - 2\omega t\right)$

This function can be rewritten as

$$= 3 \cos\left(2\omega t - \frac{\pi}{4}\right) \quad (5)$$

(6)

Simple harmonic motion with period  $T = \frac{\pi}{\omega}$  and a phase angle of  $\left(\frac{-\pi}{4}\right)$  or  $\left(\frac{7\pi}{4}\right)$

(4)  $\cos(\omega t) + \cos(3\omega t) + \cos(5\omega t)$

This function can be rewritten as

$$= \cos(\omega t) + \cos(5\omega t) + \cos(3\omega t) \quad (7)$$

$$= 2 \cos\left(\frac{\omega t + 5\omega t}{2}\right) \cos\left(\frac{5\omega t - \omega t}{2}\right) + \cos(3\omega t) \quad (8)$$

$$= 2 \cos(3\omega t) \cos(\omega t) + \cos(3\omega t) \quad (9)$$

$$= \cos(3\omega t)(1 + 2 \cos(\omega t)) \quad (10)$$

(1) Period of  $\cos(3\omega t)$  is  $\frac{2\pi}{3\omega}$

(2) Period of  $1 + 2 \cos(\omega t)$  is  $\frac{2\pi}{\omega}$

(3) Lcm is  $\frac{2\pi}{\omega}$

$\therefore$  Simple harmonic motion with period  $\frac{2\pi}{\omega}$

(5)  $\exp(-\omega^2 t^2)$

This function can be rewritten as

As  $T \rightarrow \infty$

$$\exp(-\omega^2 t^2) \rightarrow \infty$$

$\therefore$  This never repeats and non periodic

$$(6) 1 + \omega t + \omega^2 t^2$$

This function can be rewritten as

$$\begin{aligned} \text{As } T &\rightarrow \infty \\ 1 + \omega t + \omega^2 t^2 &\rightarrow \infty \end{aligned}$$

$\therefore$  This never repeats and non periodic

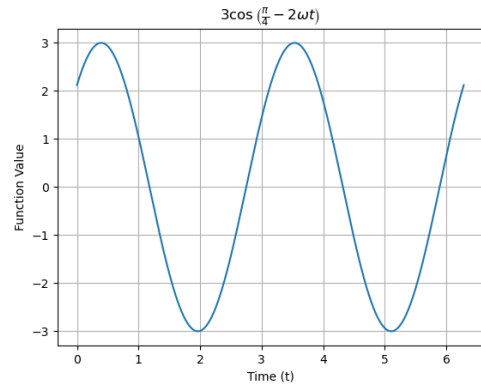


Fig. 0.  $3 \cos\left(\frac{\pi}{4} - 2\omega t\right)$

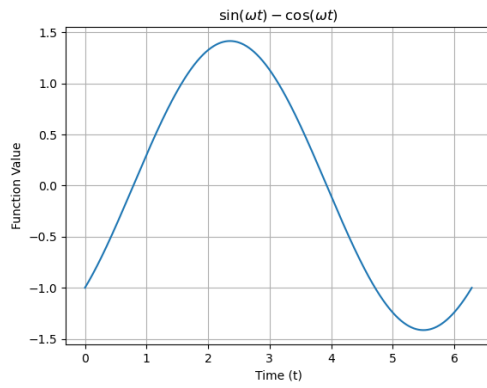


Fig. 0.  $\sin(\omega t) - \cos(\omega t)$

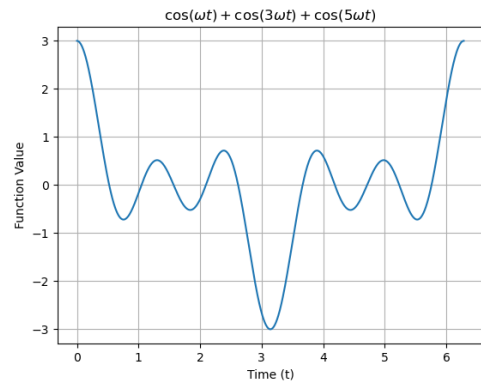


Fig. 0.  $\cos(\omega t) + \cos(3\omega t) + \cos(5\omega t)$

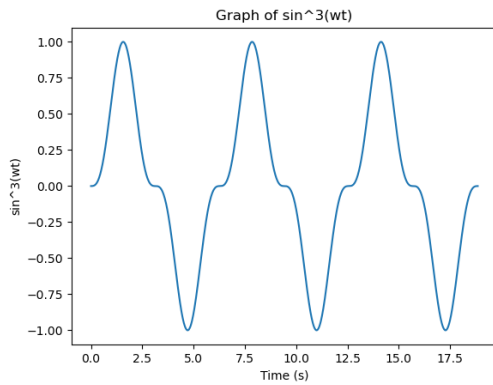


Fig. 0.  $\sin^3(\omega t)$

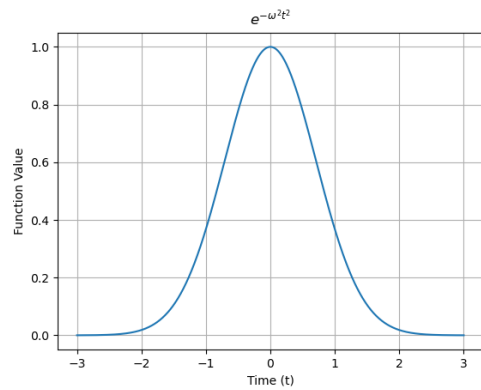


Fig. 0.  $\exp(-\omega^2 t^2)$

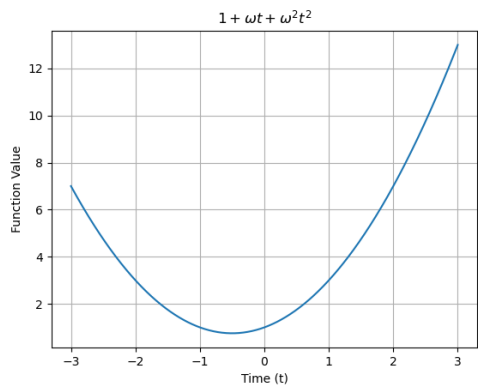


Fig. 0.  $1 + \omega t + \omega^2 t^2$

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TABLE 0  
SUMMARY

|     | Function   | Periodic | Simple harmonic motion | Non Periodic | Period                |
|-----|--|----------|------------------------|--------------|-----------------------|
| (a) | $\sin(\omega t) - \cos(\omega t)$                    | Yes      | Yes                    | No           | $\frac{2\pi}{\omega}$ |
| (b) | $\sin^3(\omega t)$                                   | Yes      | Yes                    | No           | $\frac{2\pi}{\omega}$ |
| (c) | $3\cos\left(\frac{\pi}{4} - 2\omega t\right)$        | Yes      | Yes                    | No           | $\frac{\pi}{\omega}$  |
| (d) | $\cos(\omega t) + \cos(3\omega t) + \cos(5\omega t)$ | Yes      | Yes                    | No           | $\frac{2\pi}{\omega}$ |
| (e) | $\exp(-\omega^2 t^2)$                                | No       | No                     | Yes          | —                     |
| (f) | $1 + \omega t + \omega^2 t^2$                        | No       | No                     | Yes          | —                     |