AE 42

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QUESTION: Consider the equation $\frac{dy}{dx}+ay = \sin\omega x$,where a and ω are constants.Given $y = 1$ at $x = 0$, correct all the correct statement(s) from the following as $x \to \infty$.

(A) $y \to 0$ if $a \neq 0$

(B) $y \to 1$ if $a = 0$

(C) $y \to Aexp(|a|x)$ if $a < 0$; A is constant

(D) $y \to B\sin(\omega x + C)$ if $a > 0$; B and C are constants

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Solution: :

$$\frac{dy}{dx} + ay = \sin(\omega_0 x) \quad \text{with} \quad y(0) = 1 \tag{1}$$

Taking Fourier transform on both sides

Function	Fourier transform
$\frac{dy}{dx}$	$i\omega Y$
y	Y
$\sin\omega_0 x$	$\frac{\omega}{2J}(\delta(\omega+\omega_0) - \delta(\omega-\omega_0)$

TABLE 0
FOURIER TRANSFORM

$$i\omega Y + aY = \frac{\omega_0}{2i}\left(\delta(\omega+\omega_0) - \delta(\omega-\omega_0)\right) \tag{2}$$

$$\implies Y = \frac{\omega_0}{2i(i\omega + a)}\left(\delta(\omega+\omega_0) - \delta(\omega-\omega_0)\right) \tag{3}$$

Taking inverse fourier transform on both sides

$$y(x) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} Y(\omega)\, e^{-i\omega x}\, d\omega \tag{4}$$

$$y(x) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} \frac{\omega_0}{2i(i\omega + a)}\left(\delta(\omega-\omega_0) - \delta(\omega+\omega_0)\right) e^{-i\omega x}\, d\omega \tag{5}$$

$$y(x) = \mathcal{F}^{-1}\{Y\} = \frac{w_0}{2i}e^{-ax} + \frac{\pi}{2i}\sin(\omega_0 x + \phi) \tag{6}$$

$$y(x) = \mathcal{F}^{-1}\{Y\} = Ae^{-ax} + B\sin(\omega_0 x + C) \tag{7}$$

Now as $x \to \infty$

1) $y \to 0$ if $a \neq 0$ is not true as y depend on a, ω_0
2) $y \to 1$ if $a = 0$ is not true as y depend on ω_0
3) $y \to Aexp(|a|x)$ if $a < 0$ is true as $B\sin(\omega_0 x + C)$ is neglected compared to Ae^{-ax}
4) $y \to B\sin(\omega_0 x + C)$ if $a > 0$; is true as $Ae^{-ax} \to 0$

$\therefore$ C,D are correct options