11.14-4

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QUESTION: Which of the following functions of time represent (a) simple harmonic, (b) periodic but not simple harmonic, and (c) non-periodic motion? Give period for each case of periodic motion (ω is any positive constant):

1) $\sin(\omega t) - \cos(\omega t)$
2) $\sin^3(\omega t)$
3) $3\cos\left(\frac{\pi}{4} - 2\omega t\right)$
4) $\cos(\omega t) + \cos(3\omega t) + \cos(5\omega t)$
5) $\exp\left(-\omega^2 t^2\right)$
6) $1 + \omega t + \omega^2 t^2$

Solution: :

1) Periodic function:
$$x(t+T) = x(t) \quad \forall x \in \mathbb{R} \tag{1}$$
where min T s.t $T > 0$ is time period

2) SHM:
For a function to be in shm it must satisfy
$$\frac{d^2 x(t)}{dt^2} = -(2\pi f_0)^2 x(t) \tag{2}$$
$$\tag{3}$$

Variable	**Description**	formula
$x(t)$	Displacemen wrt mean position	none
ω	Angular frequncy	$2\pi f$
T	Time period	$\frac{1}{f}$
ϕ	phase angle	none

TABLE 0
INPUT PARAMETERS

1) $\sin(2\pi ft) - \cos(2\pi ft)$
The function can be rewritten as:

$$= \sin(2\pi ft) - \sin\left(\frac{\pi}{2} - 2\pi ft\right) \tag{4}$$

$$= 2\cos\left(\frac{\pi}{4}\right)\sin\left(2\pi ft - \frac{\pi}{4}\right) \tag{5}$$

$$= \sqrt{2}\sin\left(2\pi ft - \frac{\pi}{4}\right) \tag{6}$$

$$\frac{d^2(\sin(2\pi ft) - \cos(2\pi ft))}{dt^2} = -(2\pi f)^2(\sin(2\pi ft) - \cos(2\pi ft)) \tag{7}$$

$$\frac{d^2x(t)}{dt^2} = -(2\pi f)^2 x(t) \tag{8}$$

$\therefore$ SHM, $T = \frac{1}{f}$ and $\phi = \left(\frac{-\pi}{4}\right)$ or $\left(\frac{7\pi}{4}\right)$
$\sin\left(2\pi f\left(t + \frac{1}{f}\right)\right) - \cos\left(2\pi f\left(t + \frac{1}{f}\right)\right) = \sin(2\pi ft) - \cos(2\pi ft)$
Graph of function is shown in (Fig. **??**)

(2) $\sin^3(2\pi ft)$

This function can be rewritten as

$$= \frac{1}{4}(3\sin(2\pi ft) - \sin(6\pi ft)) \tag{9}$$

$$\frac{d^2(\sin^3(2\pi ft))}{dt^2} = 9(2\pi f)^2(\sin(2\pi ft) - \sin^3(2\pi ft)) \tag{10}$$

$$\frac{d^2x(t)}{dt^2} \neq -(2\pi f)^2 x(t) \tag{11}$$

$\therefore$ Periodic with period T = $\frac{1}{f}$
$\sin^3\left((2\pi f\left(t + \frac{1}{f}\right)\right) = \sin^3(2\pi ft)$
Graph of function is shown in (Fig. **??**)

(3) $3\cos\left(\frac{\pi}{4} - 4\pi ft\right)$

This function can be rewritten as

$$= 3\cos\left(4\pi ft - \frac{\pi}{4}\right) \tag{12}$$

$$\frac{d^2\left(3\cos\left(\frac{\pi}{4} - 4\pi ft\right)\right)}{dt^2} = -3(4\pi f)^2\left(\cos\frac{\pi}{4} - 4\pi ft\right) \tag{13}$$

$$\frac{d^2x(t)}{dt^2} = -(4\pi f)^2 x(t) \tag{14}$$

$\therefore$ SHM, $T = \frac{1}{2f}$ and $\phi = \left(\frac{-\pi}{4}\right)$ or $\left(\frac{7\pi}{4}\right)$
$3\cos\left(\frac{\pi}{4} - 4\pi f\left(t + \frac{1}{2f}\right)\right) = 3\cos\left(\frac{\pi}{4} - 4\pi ft\right)$
Graph of function is shown in (Fig. **??**)

(4) $\cos(2\pi ft) + \cos(6\pi ft) + \cos(10\pi ft)$

This function can be rewritten as

$$
\begin{aligned}
&= \cos(2\pi f t) + \cos(10\pi f t) + \cos(6\pi f t) && (15)\\
&= 2\cos\left(\frac{2\pi f t + 10\pi f t}{2}\right)\cos\left(\frac{10\pi f t - 2\pi f t}{2}\right) + \cos(6\pi f t) && (16)\\
&= 2\cos(6\pi f t)\cos(2\pi f t) + \cos(6\pi f t) && (17)\\
&= \cos(6\pi f t)(1 + 2\cos(2\pi f t)) && (18)\\
\frac{d^2\cos(2\pi f t) + \cos(6\pi f t) + \cos(10\pi f t)}{dt^2} &= (2\pi f)^2\cos(2\pi f t) + (6\pi f)^2\cos(6\pi f t) + (10\pi f)^2\cos(10\pi f t) && (19)\\
\frac{d^2 x(t)}{dt^2} &\neq -(2\pi f)^2 x(t) && (20)
\end{aligned}
$$

Period of $\cos(6\pi f t)$ is $\frac{1}{3f}$
Period of $1 + 2\cos(2\pi f t)$ is $\frac{1}{f}$
Lcm is $\frac{1}{f}$
$\therefore$ SHM,$T = \frac{1}{f}$
$\cos\left(2\pi f\left(t + \frac{1}{f}\right)\right) + \cos\left(6\pi f\left(t + \frac{1}{f}\right)\right) + \cos\left(10\pi f\left(t + \frac{1}{f}\right)\right) = \cos(2\pi f t) + \cos(6\pi f t) + \cos(10\pi f t)$
Graph of function is shown in (Fig. **??**)

(5) $\exp\left(-(2\pi f)^2 t^2\right)$

$$
\begin{aligned}
\text{As } T &\to \infty && (21)\\
\exp\left(-(2\pi f)^2 t^2\right) &\to \infty && (22)\\
\frac{d^2(\exp\left(-(2\pi f)^2 t^2\right))}{dt^2} &= 2(2\pi f t)^2\exp\left(-(2\pi f)^2 t^2\right) + 2(2\pi f t)^4\exp\left(-(2\pi f)^2 t^2\right) && (23)\\
\frac{d^2 x(t)}{dt^2} &\neq -(2\pi f)^2 x(t) && (24)
\end{aligned}
$$

$\therefore$ This never repeats and non periodic
Graph of function is shown in (Fig. **??**)

(6) $1 + 2\pi f t + (2\pi f)^2 t^2$

$$
\begin{aligned}
\text{As } T &\to \infty && (25)\\
1 + 2\pi f t + (2\pi f)^2 t^2 &\to \infty && (26)\\
\frac{d^2(1 + 2\pi f t + (2\pi f)^2 t^2)}{dt^2} &= 2(2\pi f)^2 && (27)\\
\frac{d^2 x(t)}{dt^2} &\neq -(2\pi f)^2 x(t) && (28)
\end{aligned}
$$

$\therefore$ This never repeats and non periodic
Graph of function is shown in (Fig. **??**)

:

TABLE 1
SUMMARY

	Function	Periodic	Simple harmonic motion	Non Periodic	T	ϕ
(a)	$\sin(2\pi ft) - \cos(2\pi ft)$	Yes	Yes	No	$\frac{1}{f}$	$\left(\frac{-\pi}{4}\right)$
(b)	$\sin^3(2\pi ft)$	Yes	No	No	$\frac{1}{f}$	–
(c)	$3\cos\left(\frac{\pi}{4} - 4\pi ft\right)$	Yes	Yes	No	$\frac{1}{2f}$	$\left(\frac{-\pi}{4}\right)$
(d)	$\cos(2\pi ft) + \cos(6\pi ft) + \cos(10\pi ft)$	Yes	No	No	$\frac{}{f}$	–
(e)	$\exp\left(-(2\pi ft)^2\right)$	No	No	Yes	–	–
(f)	$1 + (2\pi f)t + (2\pi f)^2 t^2$	No	No	Yes	–	–

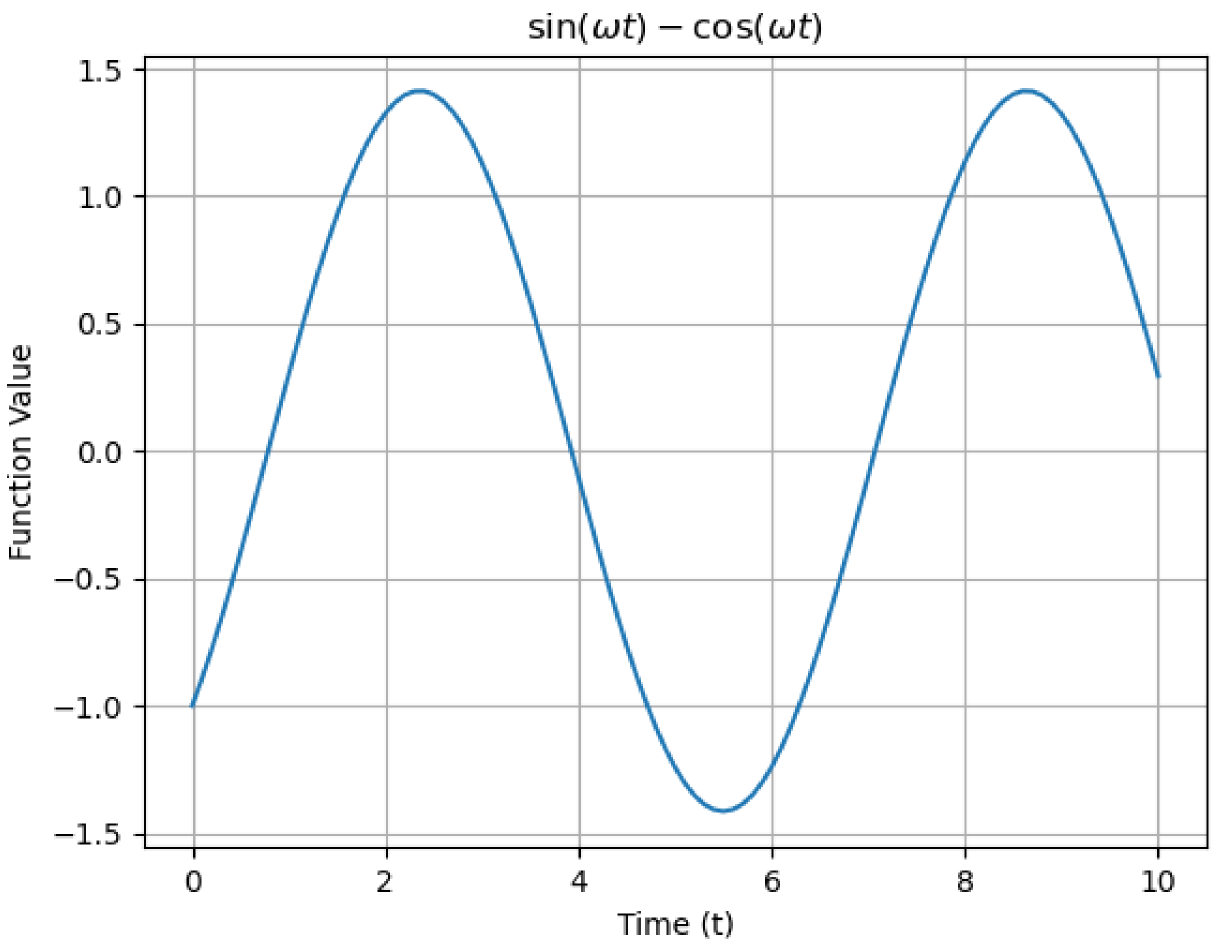

Fig. 1. $\sin(2\pi ft) - \cos(2\pi ft)$

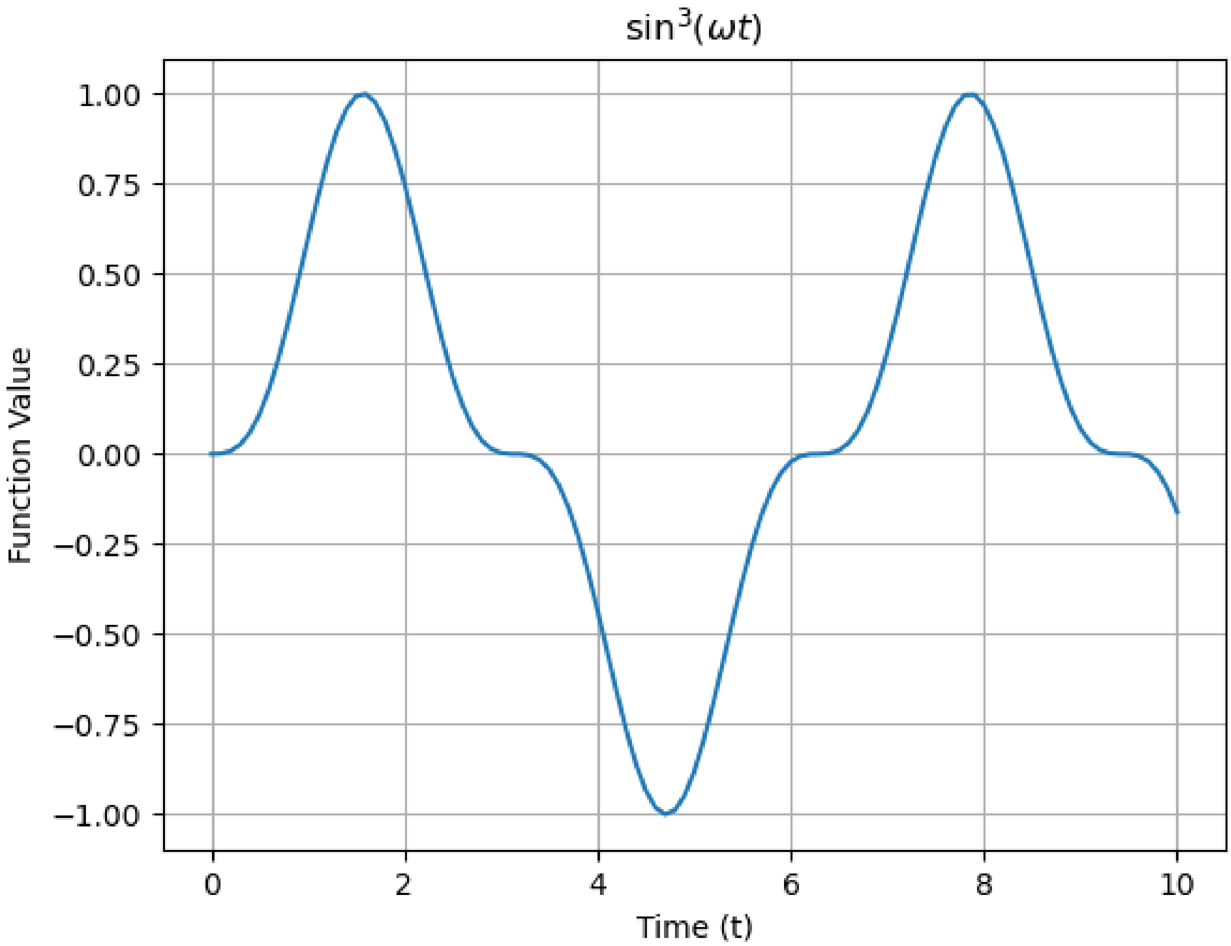

Fig. 1. $\sin^3(2\pi f t)$

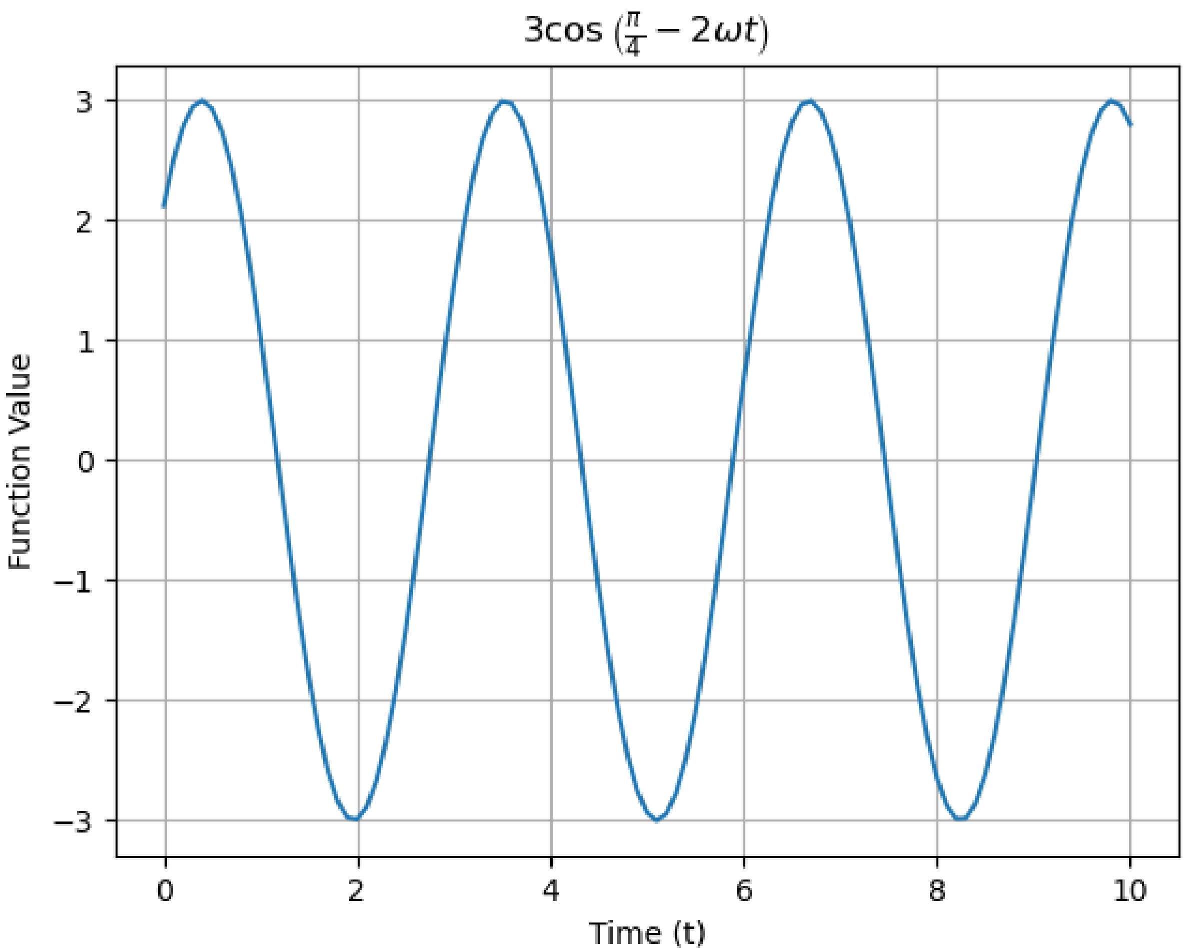

Fig. 1. $3\cos\left(\frac{\pi}{4} - 4\pi f t\right)$

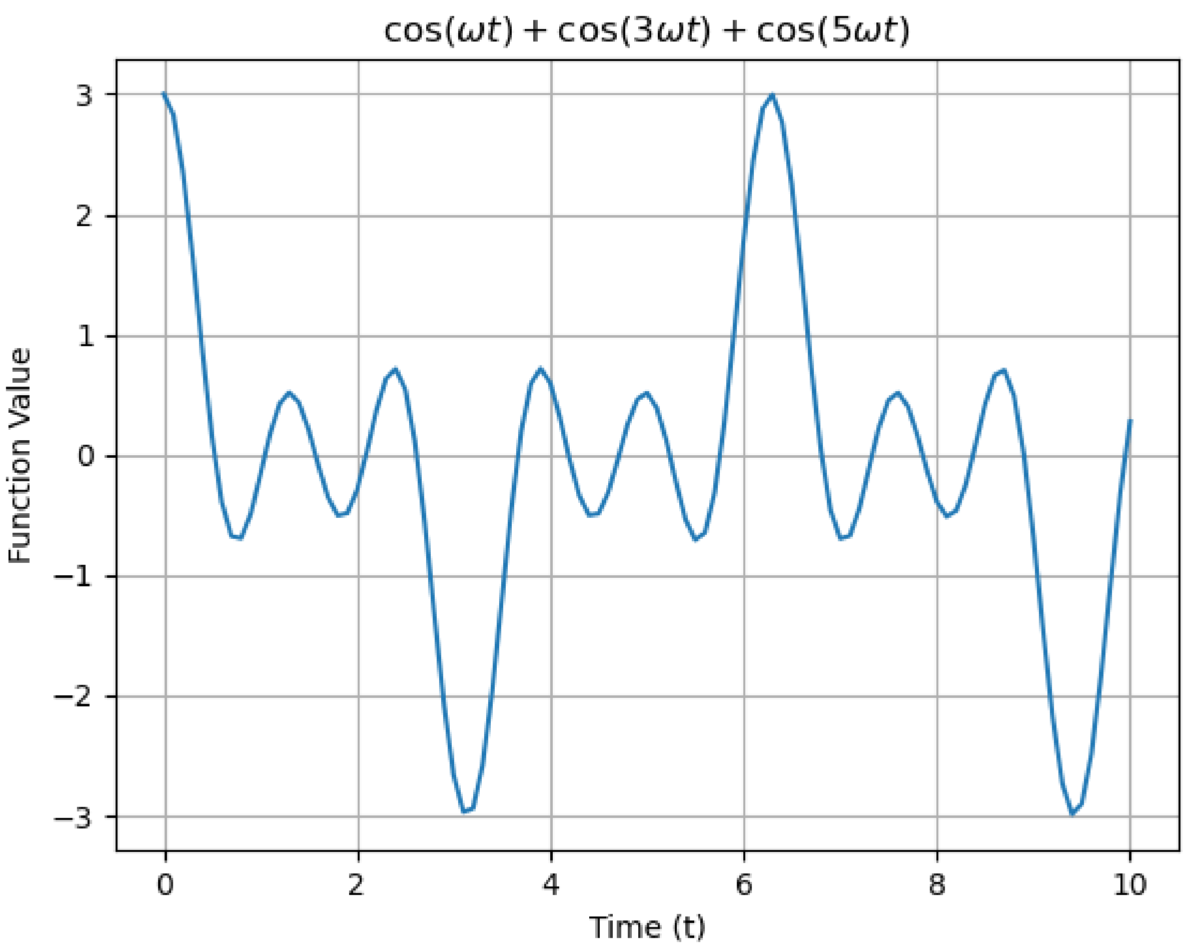

Fig. 1. $\cos(2\pi f t) + \cos(6\pi f t) + \cos(10\pi f t)$

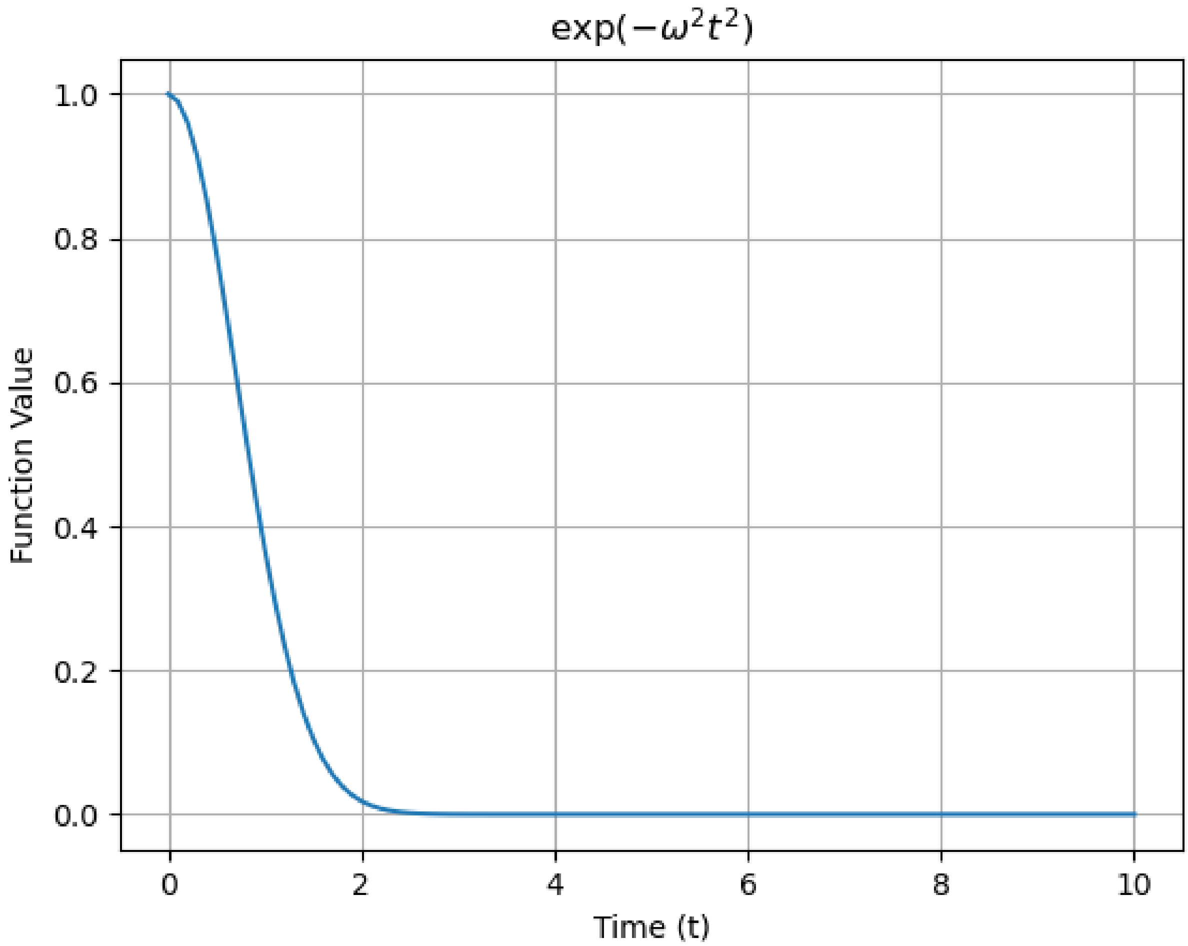

Fig. 1. $exp^{\left(-(2\pi f t)^2\right)}$

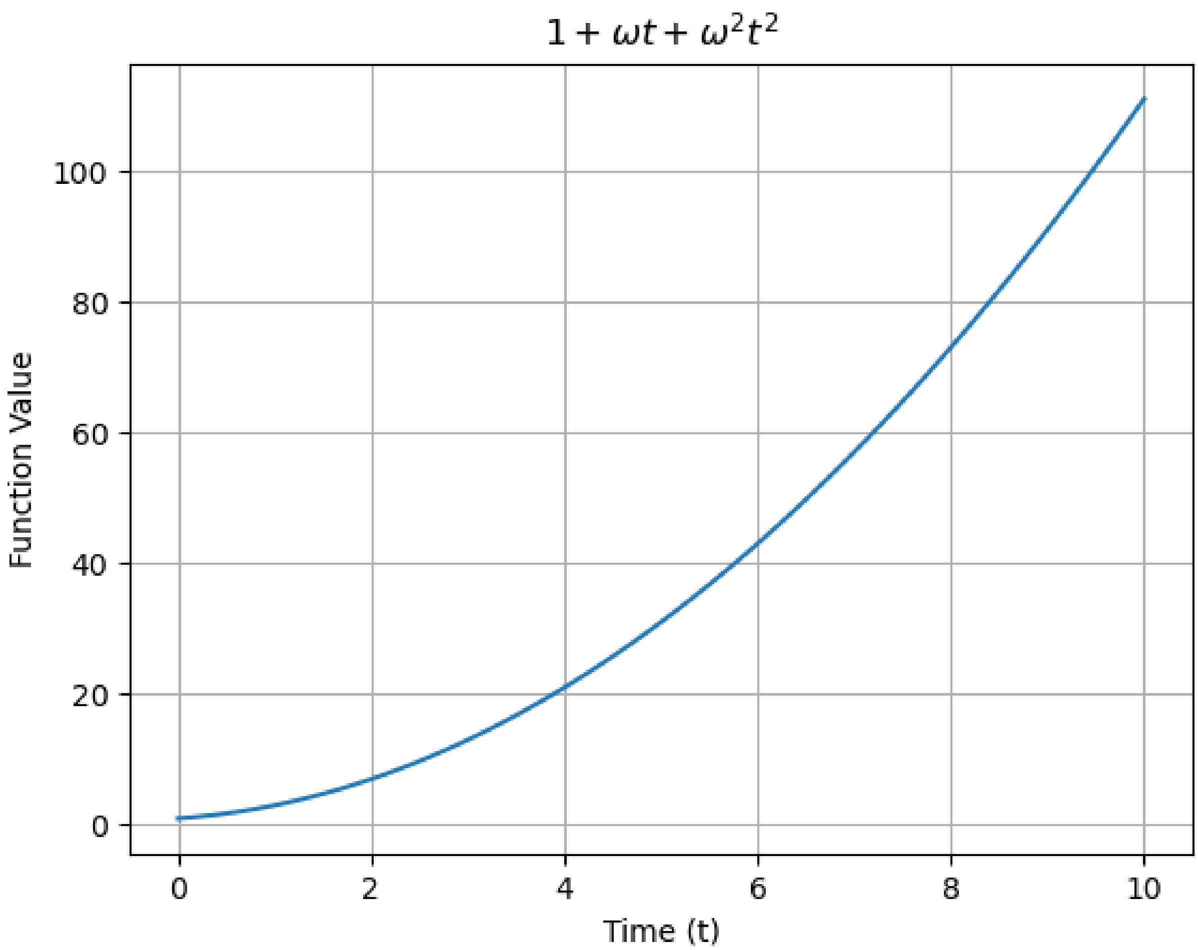

Fig. 1. $1 + 2\pi f t + (2\pi f t)^2$