XE 71

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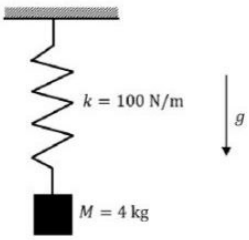
QUESTION: A spring mass system is shown in the figure . Take the value of acceleration due to gravity as $g = 9.81 m/s^2$.The static deflection due to weight and the time period of the oscillations,respectively,are

(GATE 2023 XE)



Solution:

1) Static deflection due to weight(sdw)
let x be sdw.
At mean position in equilibrium

$$Mg = kx \tag{1}$$
$$4 \cdot 9.81 = 100x \tag{2}$$
$$39.24 = 100x \tag{3}$$
$$x = 0.3924m \tag{4}$$
$$x = 39.24cm \tag{5}$$

2) Time period of oscilattion

$$F = -kx \tag{6}$$
$$ma = -kx \tag{7}$$
$$m\left(\frac{d^2x}{dt^2}\right) = -kx \tag{8}$$

Taking Laplace transform:

$$X(s) = \frac{msx(0) + x'(0)}{ms^2 + k} \tag{9}$$
$$X(s) = \frac{1}{\sqrt{\frac{k}{m}}}\left(A\frac{s - i\sqrt{\frac{k}{m}}}{s^2 + \frac{k}{m}} + B\frac{s + i\sqrt{\frac{k}{m}}}{s^2 + \frac{k}{m}}\right) \tag{10}$$

Taking Inverse Laplace Transform:
Initial Conditions be at extreme point of SHM

$$x(t) = A\left(B\sin(\sqrt{\frac{k}{m}}t) + C\cos(\sqrt{\frac{k}{m}}t)\right) \tag{11}$$
$$x(t) = 0.3924\left(\sin(\sqrt{\frac{100}{5}}t + \frac{\pi}{2}\right) \text{ m} \tag{12}$$
$$x(t) = 39.24\sin(5t + \frac{\pi}{2}) \text{ cm} \tag{13}$$

The static deflection due to weight and the time period of the oscillations,respectively are 39.24 cm and $\frac{2\pi}{5}$ s