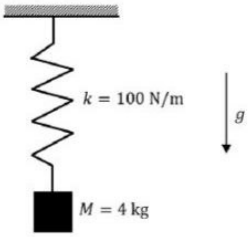
XE 71

EE23BTECH11048-Ponugumati Venkata Chanakya*

QUESTION: A spring mass system is shown in the figure . Take the value of acceleration due to gravity as $g = 9.81 m/s^2$.The static deflection due to weight and the time period of the oscillations,respectively,are

(GATE 2023 XE)



Solution:

1) Static deflection due to weight(sdw)
let x be sdw.
0 At mean position in equilibrium
??

$$Mg = kx \tag{1}$$
$$x = 39.24cm \tag{2}$$

2) Time period of oscillation

$$F = -kx \tag{3}$$
$$m\left(\frac{d^2x}{dt^2}\right) = -kx \tag{4}$$

Initial Conditions be at extreme point of SHM

$$x(0) = 0.3924 \tag{5}$$
$$\frac{dx}{dt} = 0 \text{ at } t = 0 \text{ (released from rest)} \tag{6}$$

Taking Laplace transform:

$$m(s^2X(s) - sx(0) - mx'(0)) + kX(s) = 0 \tag{7}$$

$$X(s) = \frac{x(0)ms + mx'(0)}{ms^2 + k} \tag{8}$$

$$X(s) = x(0)\frac{s}{s^2 + \frac{k}{m}} + \left(x'(0)\sqrt{\frac{k}{m}}\right)\frac{\sqrt{\frac{k}{m}}}{s^2 + \left(\sqrt{\frac{k}{m}}\right)^2} \tag{9}$$

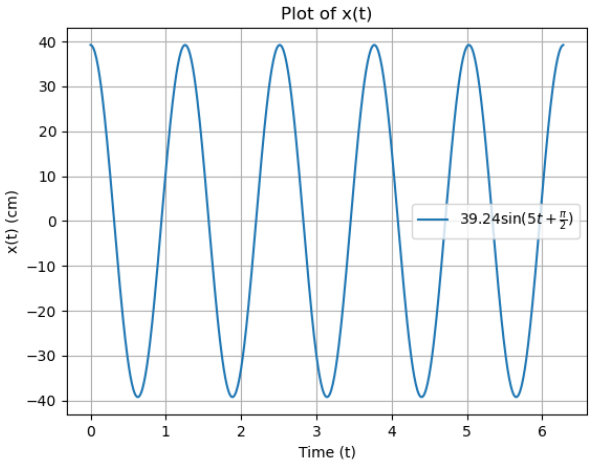
Taking Inverse Laplace Transform:

$$x(t) = x(0)\cos\left(\sqrt{\frac{k}{m}}t\right) + \left(x'(0)\sqrt{\frac{k}{m}}\right)\sin\left(\sqrt{\frac{k}{m}}t\right) \tag{10}$$
$$\tag{11}$$

Using 5 and 6

$$x(t) = 0.3924\cos\left(\sqrt{\frac{k}{m}}t\right) \tag{12}$$
$$x(t) = 39.24\sin\left(5t + \frac{\pi}{2}\right) \text{ cm} \tag{13}$$

The static deflection due to weight and the time period of the oscillations,respectively are 39.24 cm and $\frac{2\pi}{5}$ s



Variable	Description	Value
M	weight of block	4 kg
K	spring constant	$100\frac{N}{m}$
x	Static deflection due to weight	39.24 cm
$x(t)$	Displacement of particle from mean position at time t	none
$x(0)$	Initial Displacement of particle from mean position	39.24cm
$x'(t)$	velocity of particle	none
$x'(0)$	initial velocity of particle	0

TABLE 2
INPUT PARAMETERS