

AE 42

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QUESTION: Consider the equation $\frac{dy}{dx} + ay = \sin \omega x$, where a and ω are constants. Given $y = 1$ at $x = 0$, correct all the correct statement(s) from the following as $x \rightarrow \infty$.

(A) $y \rightarrow 0$ if $a \neq 0$

(B) $y \rightarrow 1$ if $a = 0$

(C) $y \rightarrow A \exp(|a|x)$ if $a < 0$; A is constant

(D) $y \rightarrow B \sin(\omega x + C)$ if $a > 0$; B and C are constants

3) $y \rightarrow A \exp(|a|x)$ if $a < 0$ is true as $B \sin(\omega x + C)$ is neglected compared to Ae^{-ax}

4) $y \rightarrow B \sin(\omega x + C)$ if $a > 0$; is true as $Ae^{-ax} \rightarrow 0$
 \therefore C,D are correct options

(GATE AE 2023) **Solution:** :

$$y(0) = 1$$

$$\frac{dy}{dx} + ay = \sin \omega x \quad (1)$$

Taking laplace transform on both sides

Function	Laplace transform
$\frac{dy}{dx}$	$sY - y(0)$
y	Y
$\sin \omega x$	$\frac{\omega}{\omega^2 + s^2}$

TABLE 0

LAPLACE TRANSFORM

$$sY - y(0) + aY = \frac{\omega}{\omega^2 + s^2} \quad (2)$$

$$sY - 1 + aY = \frac{\omega}{\omega^2 + s^2} \quad (3)$$

$$\Rightarrow Y(s) = \frac{1}{s+a} \left(\frac{\omega}{\omega^2 + s^2} + \frac{1}{s+a} \right) \quad (4)$$

$$Y(s) = \frac{A}{s+a} + \frac{Bx+C}{\omega^2 + s^2} \quad (5)$$

Taking inverse laplace transform on both sides

$$y(x) = \mathcal{L}^{-1}\{Y(s)\} = Ae^{-ax} + (B \cos(\omega x) + C \sin(\omega x)) \quad (6)$$

$$y(x) = \mathcal{L}^{-1}\{Y(s)\} = Ae^{-ax} + B \sin(\omega x + C) \quad (7)$$

now as $x \rightarrow \infty$

1) $y \rightarrow 0$ if $a \neq 0$ is not true as y depend on a, ω

2) $y \rightarrow 1$ if $a = 0$ is not true as y depend on ω