11.14-4

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QUESTION:

Which of the following functions of time represent (a) simple harmonic, (b) periodic but not simple harmonic, and (c) non-periodic motion? Give period for each case of peiodic motion (ω is any positive constant):
(a)$\sin(\omega t) - \cos(\omega t)$
(b)$\sin^3(\omega t)$
(c)$3\cos\left(\frac{\pi}{4} - 2\omega t\right)$
(d)$\cos(\omega t) + \cos(3\omega t) + \cos(5\omega t)$
(e)$exp(-\omega^2 t^2)$
(f)$1 + \omega t + \omega^2 t^2$
Answer:
Definition of period:
The period is denoted by the symbol "T," and it represents the time interval required for the motion to go through one complete cycle

1) $\sin(\omega t) - \cos(\omega t)$

$$= \sin(\omega t) - \sin\left(\frac{\pi}{2} - \omega t\right) \quad (1)$$
$$= 2\cos\left(\frac{\pi}{4}\right)\sin\left(\omega t - \frac{\pi}{4}\right) \quad (2)$$
$$= \sqrt{2}\sin\left(\omega t - \frac{\pi}{4}\right) \quad (3)$$

$\therefore$ Simple harmonic motion with period T $=\frac{2\pi}{\omega}$
Phase angle of $\left(\frac{-\pi}{4}\right)$ or$\left(\frac{7\pi}{4}\right)$

2) $\sin^3(wt)$

$$= \frac{1}{4}(3\sin(\omega t) - \sin(3\omega t)) \quad (4)$$

$\therefore$ Simple harmonic motion with period T $= \frac{2\pi}{\omega}$
phase angle zero

3) $3\cos\left(\frac{\pi}{4} - 2\omega t\right)$

$$= 3\cos\left(2\omega t - \frac{\pi}{4}\right) \quad (5)$$
$$(6)$$

Simple harmonic motion with period T $= \frac{\pi}{\omega}$ and a phase angle of $\left(\frac{-\pi}{4}\right)$ or$\left(\frac{7\pi}{4}\right)$

4) $\cos(\omega t) + \cos(3\omega t) + \cos(5\omega t)$

$$= \cos(\omega t) + \cos(5\omega t) + \cos(3\omega t) \quad (7)$$
$$= 2\cos\left(\frac{\omega t + 5\omega t}{2}\right)\cos\left(\frac{5\omega t - \omega t}{2}\right) + \cos(3\omega t) \quad (8)$$
$$= 2\cos(3\omega t)\cos(\omega t) + \cos(3\omega t) \quad (9)$$
$$= \cos(3\omega t)(1 + 2\cos(\omega t)) \quad (10)$$

Period of $\cos(3\omega t)$ is $\frac{2\pi}{3\omega}$
Period of $1 + 2\cos(\omega t)$ is $\frac{2\pi}{\omega}$
Lcm is $\frac{2\pi}{\omega}$
$\therefore$ Simple harmonic motion with period $\frac{2\pi}{\omega}$

5) $exp^{(-\omega^2 t^2)}$

$$\text{As } T \to \infty$$
$$exp\left(-\omega^2 t^2\right) \to \infty$$

$\therefore$ This never repeats and non periodic

6) $1 + \omega t + \omega^2 t^2$

$$\text{As } T \to \infty$$
$$1 + \omega t + \omega^2 t^2 \to \infty$$

$\therefore$ This never repeats and non periodic

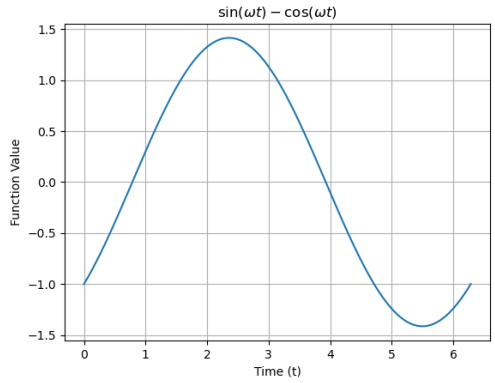

Fig. 6. $\sin(\omega t) - \cos(\omega t)$

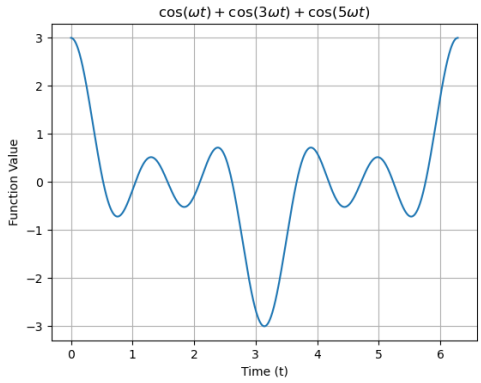

Fig. 6. $\cos(\omega t) + \cos(3\omega t) + \cos(5\omega t)$

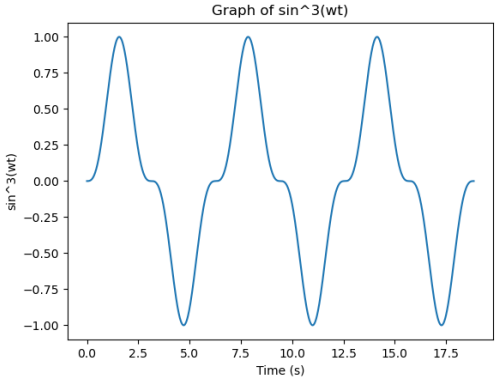

Fig. 6. $\sin^3(\omega t)$

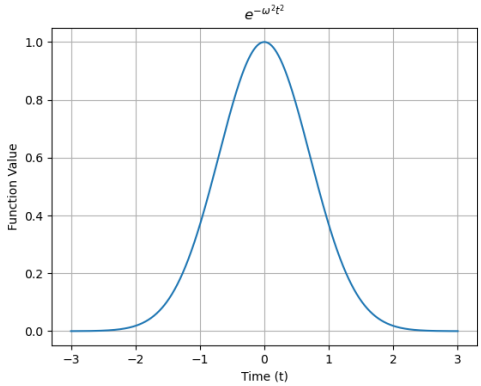

Fig. 6. $exp^{(-\omega^2 t^2)}$

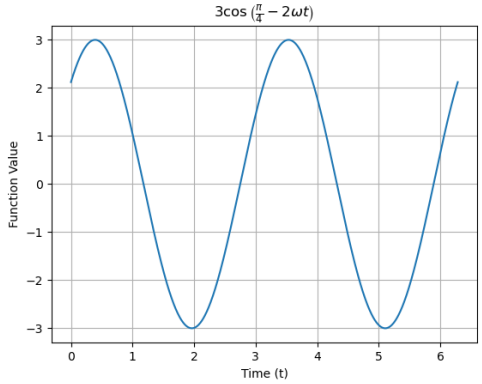

Fig. 6. $3\cos\left(\frac{\pi}{4} - 2\omega t\right)$

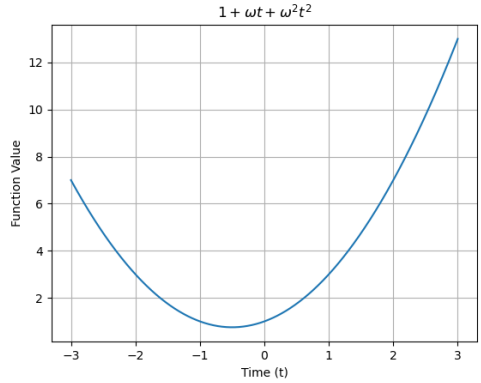

Fig. 6. $1 + \omega t + \omega^2 t^2$

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TABLE 6
SUMMARY

	Function	Periodic	Simple harmonic motion	Non Periodic	Period
(a)	$sin(\omega t) - cos(wt)$	Yes	Yes	No	$\frac{2\pi}{\omega}$
(b)	$sin^3(\omega t)$	Yes	Yes	No	$\frac{2\pi}{\omega}$
(c)	$3cos\left(\frac{\pi}{4} - 2\omega t\right)$	Yes	Yes	No	$\frac{\pi}{\omega}$
(d)	$cos(\omega t) + cos(3\omega t) + cos(5\omega t)$	Yes	Yes	No	$\frac{2\pi}{\omega}$
(e)	$exp^{(-\omega^2 t^2)}$	No	No	Yes	–
(f)	$1 + \omega t + \omega^2 t^2$	No	No	Yes	–