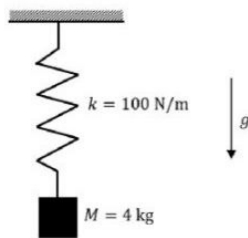# XE 71

EE23BTECH11048-Ponugumati Venkata Chanakya*

QUESTION: A spring mass system is shown in the figure . Take the value of acceleration due to gravity as $g = 9.81 m/s^2$.The static deflection due to weight and the time period of the oscillations,respectively,are

(GATE 2023 XE)

Solution:

1) Static deflection due to weight(sdw)
   let x be sdw.
   At mean position in equilibrium
   ??

$$Mg = kx \qquad (1)$$
$$x = 39.24cm \qquad (2)$$

2) Time period of oscillation

$$F = -kx \qquad (3)$$
$$m\left(\frac{d^2x}{dt^2}\right) = -kx \qquad (4)$$

Taking Laplace transform:

$$X(s) = \frac{msx(0) + x'(0)}{ms^2 + k} \qquad (5)$$

$$X(s) = \frac{1}{\sqrt{\frac{k}{m}}}\left(A\frac{s - i\sqrt{\frac{k}{m}}}{s^2 + \frac{k}{m}} + B\frac{s + i\sqrt{\frac{k}{m}}}{s^2 + \frac{k}{m}}\right) \qquad (6)$$

Taking Inverse Laplace Transform:

$$x(t) = A\left[B\sin(\sqrt{\frac{k}{m}}t) + C\cos(\sqrt{\frac{k}{m}}t)\right] \qquad (7)$$

$$x(t) = P\left[\sin(\sqrt{\frac{k}{m}}t + Q)\right] \qquad (8)$$

$$x(t) = P\sin(5t + Q) \qquad (9)$$
$$(10)$$

Initial Conditions be at extreme point of SHM

$$x(0) = 0.3924 = PsinQ \qquad (11)$$

$$\frac{dx}{dt} = 0 \text{ at } t = 0 \text{ (released from rest)} \qquad (12)$$
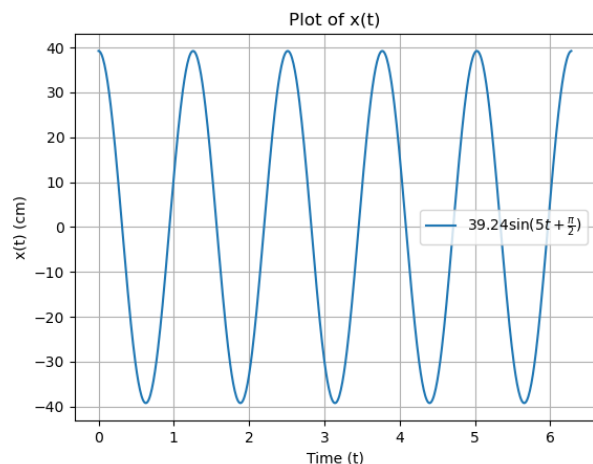
$$\frac{dx}{dt} = 5P\cos(Q) = 0 \qquad (13)$$

$$(14)$$

solving **??** using **??** and **??**

$$x(t) = 0.3924\sin\left(5t + \frac{\pi}{2}\right) \text{ m} \qquad (15)$$

$$x(t) = 39.24\sin\left(5t + \frac{\pi}{2}\right) \text{ cm} \qquad (16)$$

The static deflection due to weight and the time period of the oscillations,respectively are 39.24 cm and $\frac{2\pi}{5}$ s

Variable	Description	Value
M	weight of block	4 kg
K	spring constant	$100\frac{N}{m}$
x	Static deflection due to weight	39.24 cm
$\frac{dx}{dt}$	velocity of particle	none
P	Amplitude	39.24 cm
Q	phase angle	$\frac{\pi}{2}$
A, B, C	integrating constants	none

TABLE 2

INPUT PARAMETERS