11.14-4

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QUESTION: Which of the following functions of time represent (a) simple harmonic, (b) periodic but not simple harmonic, and (c) non-periodic motion? Give period for each case of peiodic motion (ω is any positive constant):

1) $\sin(\omega t) - \cos(\omega t)$
2) $\sin^3(\omega t)$
3) $3\cos\left(\frac{\pi}{4} - 2\omega t\right)$
4) $\cos(\omega t) + \cos(3\omega t) + \cos(5\omega t)$
5) $\exp\left(-\omega^2 t^2\right)$
6) $1 + \omega t + \omega^2 t^2$

Answer:
Periodic function:

$$x(t+T) = x(t) \forall x \in \mathbb{R} \tag{1}$$

where min T is time period

SHM:
For a function to be in shm it must satisfy $d^2x(t)/dt^2 = -\alpha x$
$x(t)$ is displacement
α is constant
t is time

ω is angular frequency $\omega = 2\pi f$

1) $\sin(2\pi ft) - \cos(2\pi ft)$
The function can be rewritten as:

$$= \sin(2\pi ft) - \sin\left(\frac{\pi}{2} - 2\pi ft\right) \tag{2}$$

$$= 2\cos\left(\frac{\pi}{4}\right)\sin\left(2\pi ft - \frac{\pi}{4}\right) \tag{3}$$

$$= \sqrt{2}\sin\left(2\pi ft - \frac{\pi}{4}\right) \tag{4}$$

$\therefore$ SHM, $T = \frac{1}{f}$ and $\phi = \left(\frac{-\pi}{4}\right)$ or $\left(\frac{7\pi}{4}\right)$

(2) $\sin^3(wt)$

This function can be rewritten as

$$= \frac{1}{4}(3\sin(2\pi ft) - \sin(6\pi ft)) \tag{5}$$

$\therefore$ Periodic with period T = $\frac{1}{f}$

(3) $3\cos\left(\frac{\pi}{4} - 4\pi ft\right)$

This function can be rewritten as

$$= 3\cos\left(4\pi ft - \frac{\pi}{4}\right) \tag{6}$$

$$\tag{7}$$

SHM, T = $\frac{1}{2f}$ and $\phi = \left(\frac{-\pi}{4}\right)$ or$\left(\frac{7\pi}{4}\right)$

(4) $\cos(2\pi ft) + \cos(6\pi ft) + \cos(10\pi ft)$

This function can be rewritten as

$$= \cos(2\pi ft) + \cos(10\pi ft) + \cos(6\pi ft) \tag{8}$$

$$= 2\cos\left(\frac{2\pi ft + 10\pi ft}{2}\right)\cos\left(\frac{10\pi ft - 2\pi ft}{2}\right) + \cos(6\pi ft) \tag{9}$$

$$= 2\cos(6\pi ft)\cos(2\pi ft) + \cos(6\pi ft) \tag{10}$$

$$= \cos(6\pi ft)(1 + 2\cos(2\pi ft)) \tag{11}$$

Period of $\cos(6\pi ft)$ is $\frac{1}{3f}$
Period of $1 + 2\cos(2\pi ft)$ is $\frac{1}{f}$
Lcm is $\frac{1}{f}$
$\therefore$ SHM,$T = \frac{1}{f}$

(5) $\exp\left(-(2\pi f)^2 t^2\right)$

This function can be rewritten as

$$\text{As } T \rightarrow \infty$$
$$\exp\left(-(2\pi f)^2 t^2\right) \rightarrow \infty$$
$$\therefore \text{ This never repeats and non periodic}$$

(6) $1 + 2\pi f t + (2\pi f)^2 t^2$

This function can be rewritten as

$$\text{As } T \rightarrow \infty$$
$$1 + 2\pi f t + (2\pi f)^2 t^2 \rightarrow \infty$$
$$\therefore \text{ This never repeats and non periodic}$$

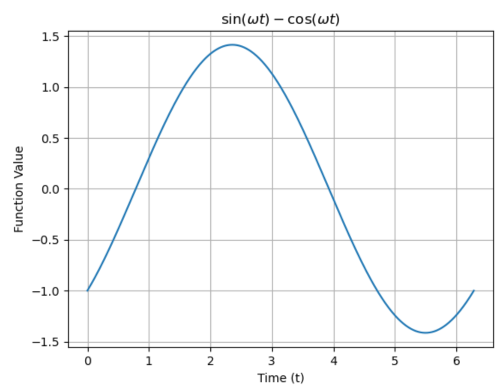

Fig. 1. $\sin(2\pi f t) - \cos(2\pi f t)$

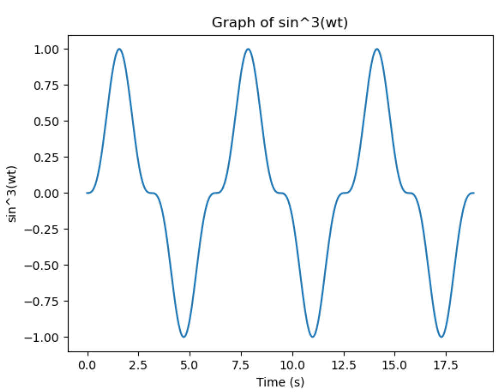

Fig. 1. $\sin^3(2\pi f t)$

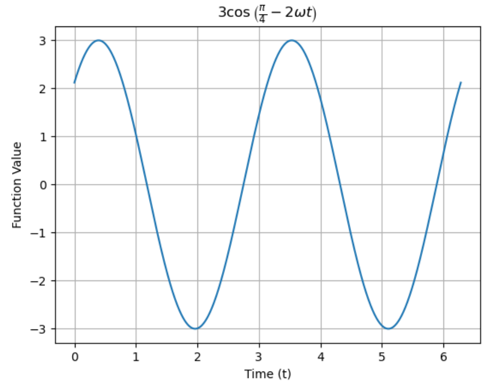

Fig. 1. $3\cos\left(\frac{\pi}{4} - 4\pi f t\right)$

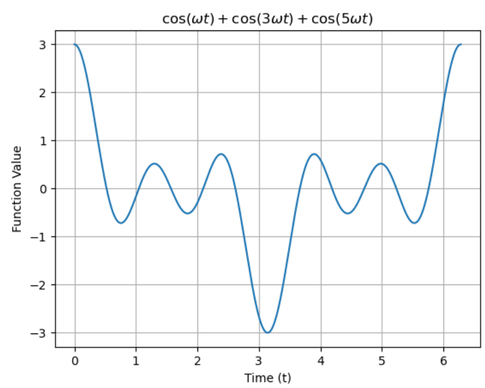

Fig. 1. $\cos(2\pi f t) + \cos(6\pi f t) + \cos(10\pi f t)$

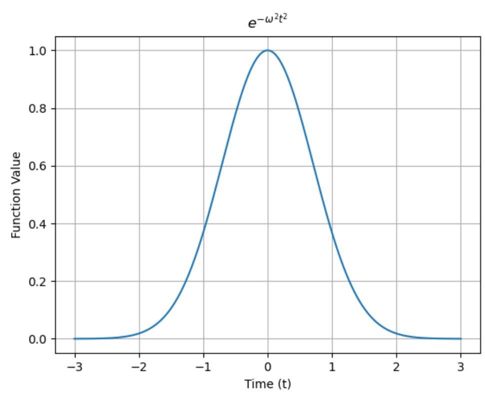

Fig. 1. $exp^{\left(-(2\pi f t)^2\right)}$

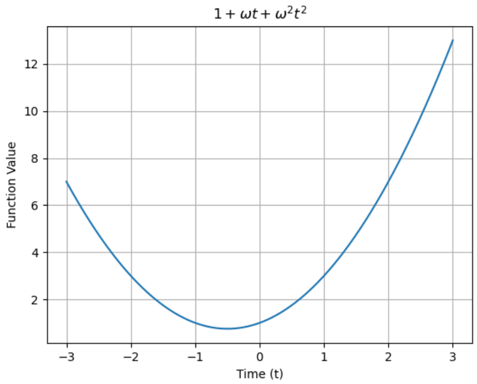

Fig. 1. $1 + 2\pi f t + (2\pi f t)^2$

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TABLE 1
SUMMARY

	Function	**Periodic**	**Simple harmonic motion**	**Non Periodic**	**T**	ϕ
(a)	$sin(2\pi f t) - cos(2\pi f t)$	Yes	Yes	No	$\frac{1}{f}$	$\left(\frac{-\pi}{4}\right)$
(b)	$sin^3(2\pi f t)$	Yes	No	No	$\frac{1}{f}$	–
(c)	$3cos\left(\frac{\pi}{4} - 4\pi f t\right)$	Yes	Yes	No	$\frac{1}{2f}$	$\left(\frac{-\pi}{4}\right)$
(d)	$cos(2\pi f t) + cos(6\pi f t) + cos(10\pi f t)$	Yes	No	No	$\frac{}{f}$	–
(e)	$\exp\left(-(2\pi f t)^2\right)$	No	No	Yes	–	–
(f)	$1 + (2\pi f)t + (2\pi f)^2 t^2$	No	No	Yes	–	–