

# AE 42

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**QUESTION:** Consider the equation  $\frac{dy}{dx} + ay = \sin \omega x$ , where  $a$  and  $\omega$  are constants. Given  $y = 1$  at  $x = 0$ , correct all the correct statement(s) from the following as  $x \rightarrow \infty$ .

(A)  $y \rightarrow 0$  if  $a \neq 0$

(B)  $y \rightarrow 1$  if  $a = 0$

(C)  $y \rightarrow A \exp(|a|x)$  if  $a < 0$ ;  $A$  is constant

(D)  $y \rightarrow B \sin(\omega x + C)$  if  $a > 0$ ;  $B$  and  $C$  are constants

3)  $y \rightarrow A \exp(|a|x)$  if  $a < 0$  is true as  $B \sin(\omega x + C)$  is neglected compared to  $Ae^{-ax}$

4)  $y \rightarrow B \sin(\omega x + C)$  if  $a > 0$ ; is true as  $Ae^{-ax} \rightarrow 0$   
 $\therefore$  C,D are correct options

(GATE AE 2023) **Solution:** :

$$y(0) = 1$$

$$\frac{dy}{dx} + ay = \sin \omega x \quad (1)$$

Taking laplace transform on both sides

Function	Laplace transform
$\frac{dy}{dx}$	$sY - y(0)$
$y$	$Y$
$\sin \omega x$	$\frac{\omega}{\omega^2 + s^2}$

TABLE 0

LAPLACE TRANSFORM

$$sY - y(0) + aY = \frac{\omega}{\omega^2 + s^2} \quad (2)$$

$$sY - 1 + aY = \frac{\omega}{\omega^2 + s^2} \quad (3)$$

$$\Rightarrow Y(s) = \frac{1}{s+a} \left( \frac{\omega}{\omega^2 + s^2} + \frac{1}{s+a} \right) \quad (4)$$

$$Y(s) = \frac{A}{s+a} + \frac{Bs+C}{\omega^2 + s^2} \quad (5)$$

Taking inverse laplace transform on both sides

$$y(x) = \mathcal{L}^{-1}\{Y(s)\} = Ae^{-ax} + (B \cos(\omega x) + C \sin(\omega x)) \quad (6)$$

$$y(x) = \mathcal{L}^{-1}\{Y(s)\} = Ae^{-ax} + B \sin(\omega x + C) \quad (7)$$

now as  $x \rightarrow \infty$

1)  $y \rightarrow 0$  if  $a \neq 0$  is not true as  $y$  depend on  $a, \omega$

2)  $y \rightarrow 1$  if  $a = 0$  is not true as  $y$  depend on  $\omega$