

11.14-4

EE23BTECH11048-Ponugumati Venkata Chanakya*

QUESTION: Which of the following functions of time represent (a) simple harmonic, (b) periodic but not simple harmonic, and (c) non-periodic motion? Give period for each case of periodic motion (ω is any positive constant):

1) $\sin(\omega t) - \cos(\omega t)$

2) $\sin^3(\omega t)$

3) $3 \cos\left(\frac{\pi}{4} - 2\omega t\right)$

4) $\cos(\omega t) + \cos(3\omega t) + \cos(5\omega t)$

5) $\exp(-\omega^2 t^2)$

6) $1 + \omega t + \omega^2 t^2$

Solution: :

Periodic function:

$$x(t + T) = x(t) \quad \forall x \in \mathbb{R} \quad (1)$$

where $\min T$ s.t $T > 0$ is time period

SHM:

For a function to be in shm it must satisfy

$$\frac{d^2 x(t)}{dt^2} = -(2\pi f_0)^2 x(t) \quad (2)$$

(3)

Variable	Description	formula
$x(t)$	Displacemen wrt mean position	none
ω	Angular frequency	$2\pi f$
T	Time period	$\frac{1}{f}$
ϕ	phase angle	none

TABLE 0

INPUT PARAMETERS

1) $\sin(2\pi ft) - \cos(2\pi ft)$

The function can be rewritten as:

$$= \sin(2\pi ft) - \sin\left(\frac{\pi}{2} - 2\pi ft\right) \quad (4)$$

$$= 2 \cos\left(\frac{\pi}{4}\right) \sin\left(2\pi ft - \frac{\pi}{4}\right) \quad (5)$$

$$= \sqrt{2} \sin\left(2\pi ft - \frac{\pi}{4}\right) \quad (6)$$

$$\frac{d^2(\sin(2\pi ft) - \cos(2\pi ft))}{dt^2} = -(2\pi f)^2(\sin(2\pi ft) - \cos(2\pi ft)) \quad (7)$$

$$\frac{d^2 x(t)}{dt^2} = -(2\pi f)^2 x(t) \quad (8)$$

\therefore SHM, $T = \frac{1}{f}$ and $\phi = \left(\frac{-\pi}{4}\right)$ or $\left(\frac{7\pi}{4}\right)$

Graph of function is shown in (Fig. ??)

(2) $\sin^3(2\pi ft)$

This function can be rewritten as

$$= \frac{1}{4}(3 \sin(2\pi ft) - \sin(6\pi ft)) \quad (9)$$

$$\frac{d^2(\sin^3(2\pi ft))}{dt^2} = 9(2\pi f)^2(\sin(2\pi ft) - \sin^3(2\pi ft)) \quad (10)$$

$$\frac{d^2 x(t)}{dt^2} \neq -(2\pi f)^2 x(t) \quad (11)$$

\therefore Periodic with period $T = \frac{1}{f}$

Graph of function is shown in (Fig. ??)

(3) $3 \cos\left(\frac{\pi}{4} - 4\pi ft\right)$

This function can be rewritten as

$$= 3 \cos\left(4\pi ft - \frac{\pi}{4}\right) \quad (12)$$

$$\frac{d^2\left(3 \cos\left(\frac{\pi}{4} - 4\pi ft\right)\right)}{dt^2} = -3(4\pi f)^2 \left(\cos \frac{\pi}{4} - 4\pi ft\right) \quad (13)$$

$$\frac{d^2 x(t)}{dt^2} = -(4\pi f)^2 x(t) \quad (14)$$

\therefore SHM, $T = \frac{1}{2f}$ and $\phi = \left(\frac{-\pi}{4}\right)$ or $\left(\frac{7\pi}{4}\right)$

Graph of function is shown in (Fig. ??)

(4) $\cos(2\pi ft) + \cos(6\pi ft) + \cos(10\pi ft)$

This function can be rewritten as

$$= \cos(2\pi ft) + \cos(10\pi ft) + \cos(6\pi ft) \quad (15)$$

$$= 2 \cos\left(\frac{2\pi ft + 10\pi ft}{2}\right) \cos\left(\frac{10\pi ft - 2\pi ft}{2}\right) + \cos(6\pi ft) \quad (16)$$

$$= 2 \cos(6\pi ft) \cos(2\pi ft) + \cos(6\pi ft) \quad (17)$$

$$= \cos(6\pi ft)(1 + 2 \cos(2\pi ft)) \quad (18)$$

$$\frac{d^2 \cos(2\pi ft) + \cos(6\pi ft) + \cos(10\pi ft)}{dt^2} = (2\pi f)^2 \cos(2\pi ft) + (6\pi f)^2 \cos(6\pi ft) + (10\pi f)^2 \cos(10\pi ft) \quad (19)$$

$$\frac{d^2 x(t)}{dt^2} \neq -(2\pi f)^2 x(t) \quad (20)$$

Period of $\cos(6\pi ft)$ is $\frac{1}{3f}$

Period of $1 + 2 \cos(2\pi ft)$ is $\frac{1}{f}$

Lcm is $\frac{1}{f}$

\therefore SHM, $T = \frac{1}{f}$

Graph of function is shown in (Fig. ??)

$$(5) \exp(-(2\pi f)^2 t^2)$$

$$\text{As } T \rightarrow \infty \quad (21)$$

$$\exp(-(2\pi f)^2 t^2) \rightarrow \infty \quad (22)$$

$$\frac{d^2(\exp(-(2\pi f)^2 t^2))}{dt^2} = 2(2\pi f)^2 \exp(-(2\pi f)^2 t^2) + 2(2\pi f)^4 \exp(-(2\pi f)^2 t^2) \quad (23)$$

$$\frac{d^2 x(t)}{dt^2} \neq -(2\pi f)^2 x(t) \quad (24)$$

\therefore This never repeats and non periodic

Graph of function is shown in (Fig. ??)

$$(6) 1 + 2\pi ft + (2\pi f)^2 t^2$$

$$\text{As } T \rightarrow \infty \quad (25)$$

$$1 + 2\pi ft + (2\pi f)^2 t^2 \rightarrow \infty \quad (26)$$

$$\frac{d^2(1 + 2\pi ft + (2\pi f)^2 t^2)}{dt^2} = 2(2\pi f)^2 \quad (27)$$

$$\frac{d^2 x(t)}{dt^2} \neq -(2\pi f)^2 x(t) \quad (28)$$

\therefore This never repeats and non periodic

Graph of function is shown in (Fig. ??)

:

TABLE 1
SUMMARY

	Function	Periodic	Simple harmonic motion	Non Periodic	T	ϕ
(a)	$\sin(2\pi ft) - \cos(2\pi ft)$	Yes	Yes	No	$\frac{1}{f}$	$\left(\frac{-\pi}{4}\right)$
(b)	$\sin^3(2\pi ft)$	Yes	No	No	$\frac{1}{f}$	–
(c)	$3 \cos\left(\frac{\pi}{4} - 4\pi ft\right)$	Yes	Yes	No	$\frac{1}{2f}$	$\left(\frac{-\pi}{4}\right)$
(d)	$\cos(2\pi ft) + \cos(6\pi ft) + \cos(10\pi ft)$	Yes	No	No	\bar{f}	–
(e)	$\exp\left(-(2\pi ft)^2\right)$	No	No	Yes	–	–
(f)	$1 + (2\pi f)t + (2\pi f)^2 t^2$	No	No	Yes	–	–

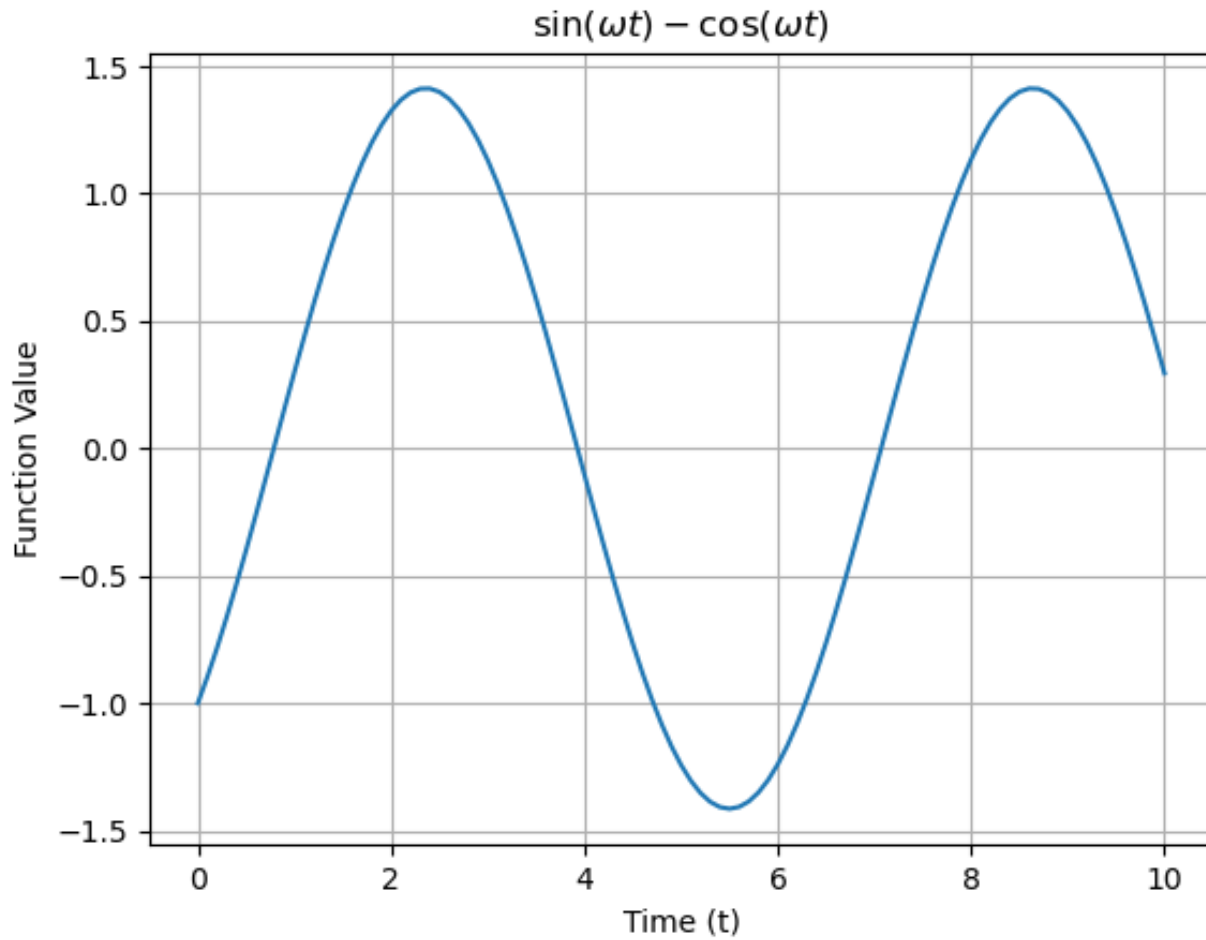


Fig. 1. $\sin(2\pi ft) - \cos(2\pi ft)$

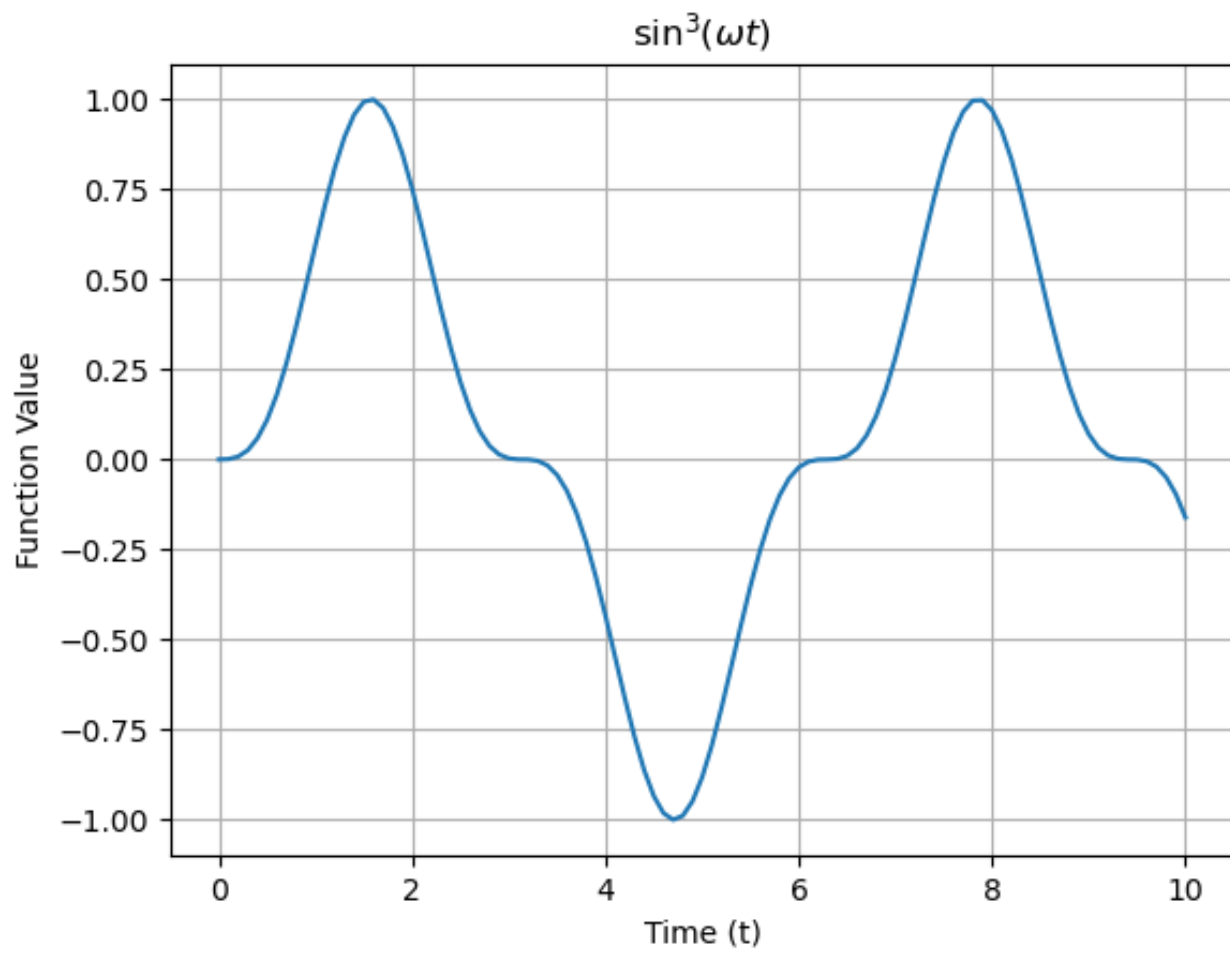


Fig. 1. $\sin^3(2\pi ft)$

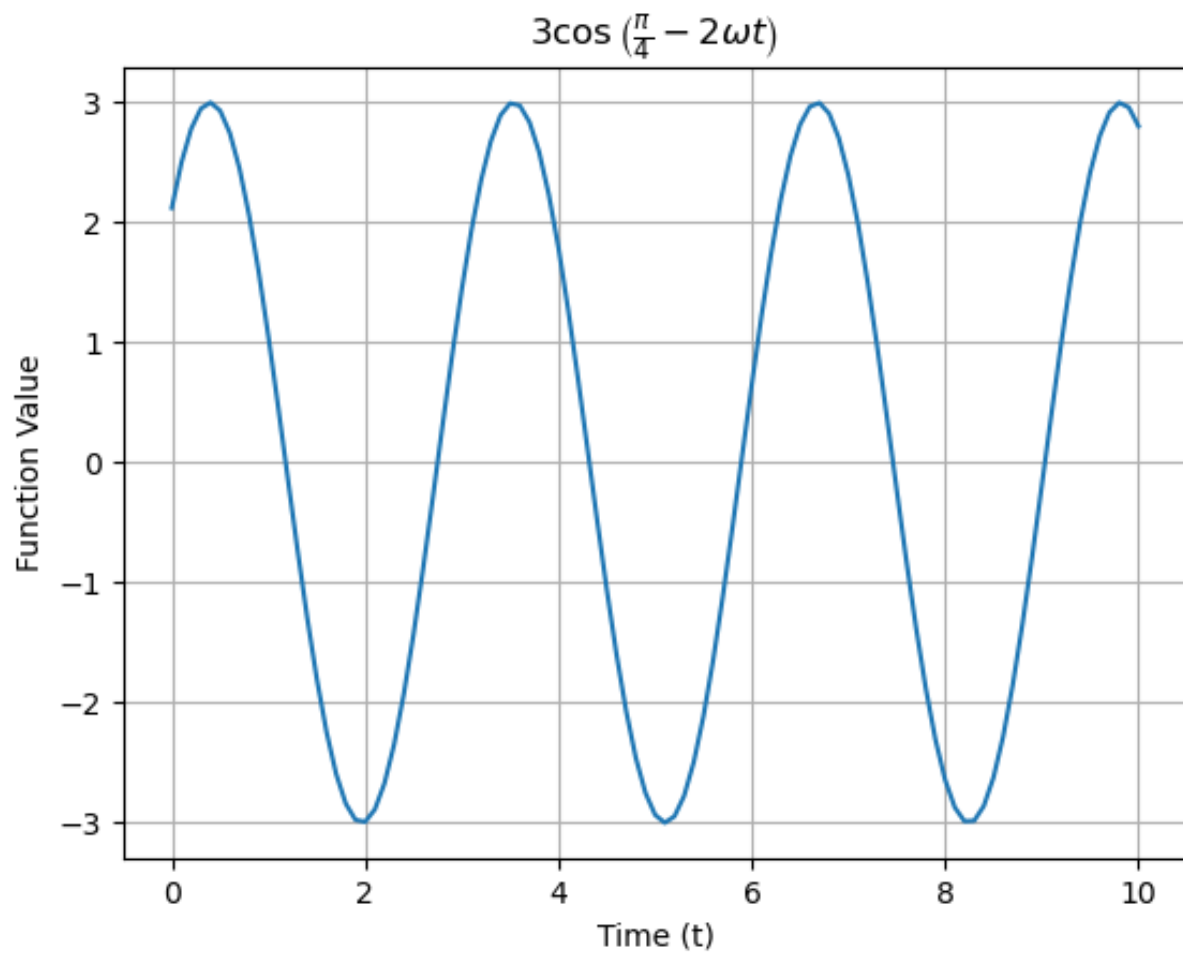


Fig. 1. $3\cos\left(\frac{\pi}{4} - 4\pi ft\right)$

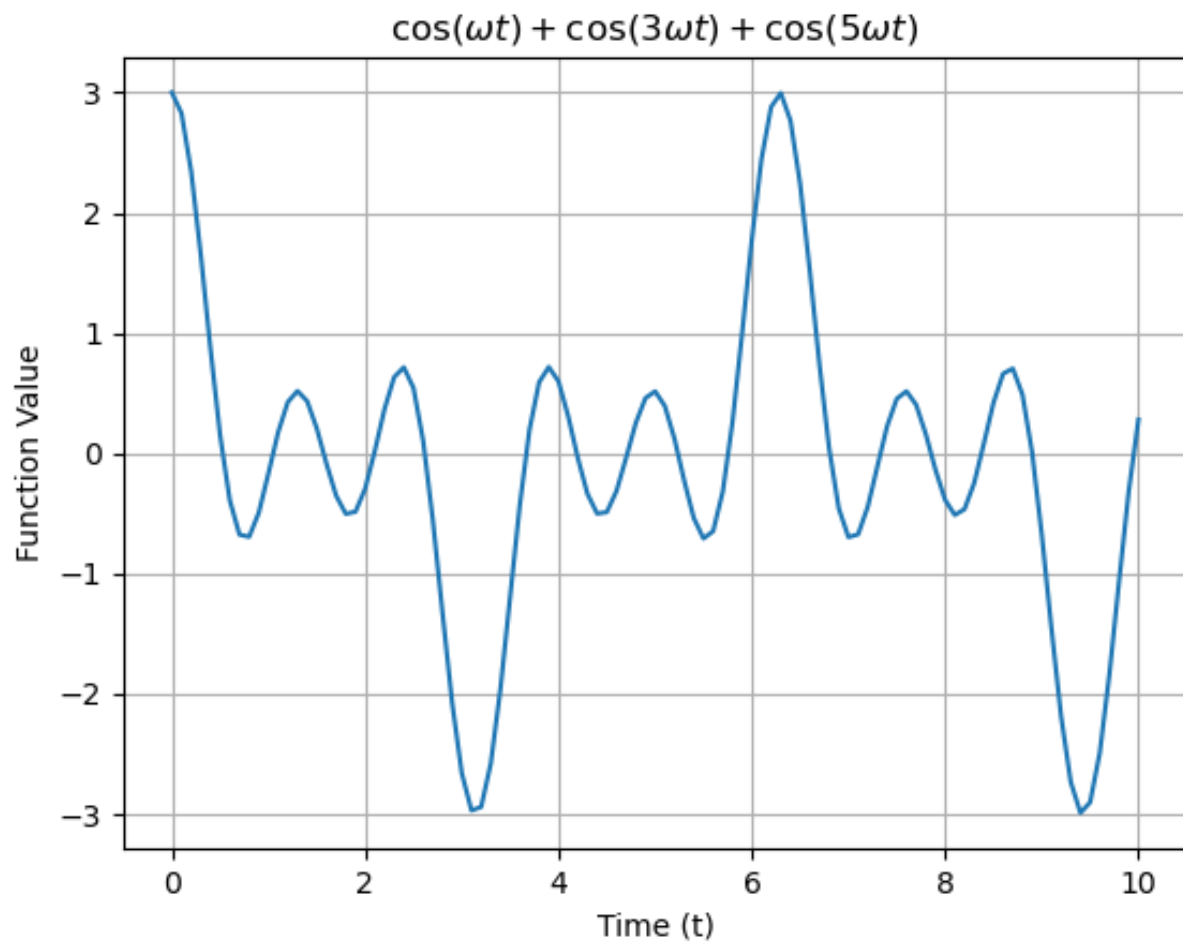


Fig. 1. $\cos(2\pi ft) + \cos(6\pi ft) + \cos(10\pi ft)$

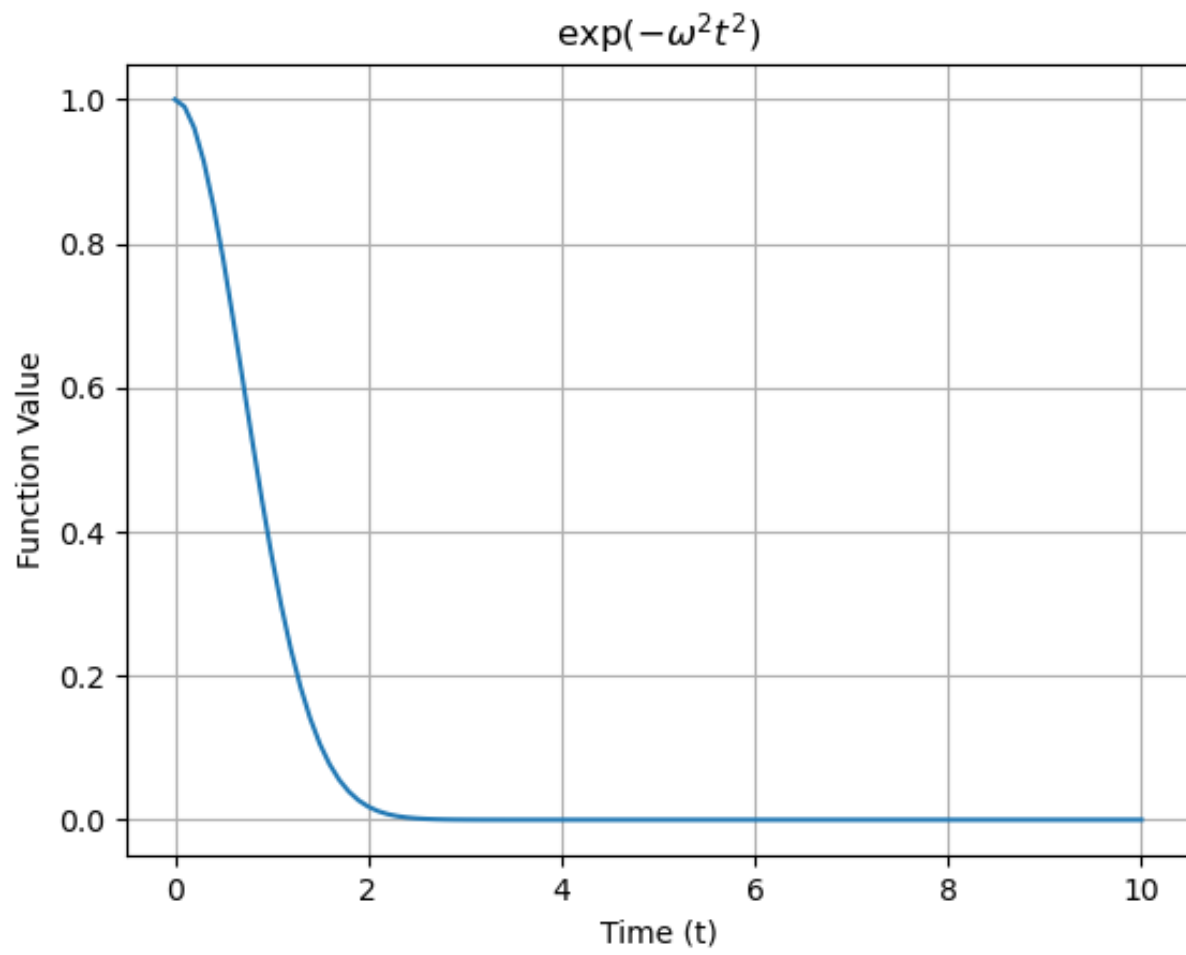


Fig. 1. $\exp(-2\pi f t^2)$

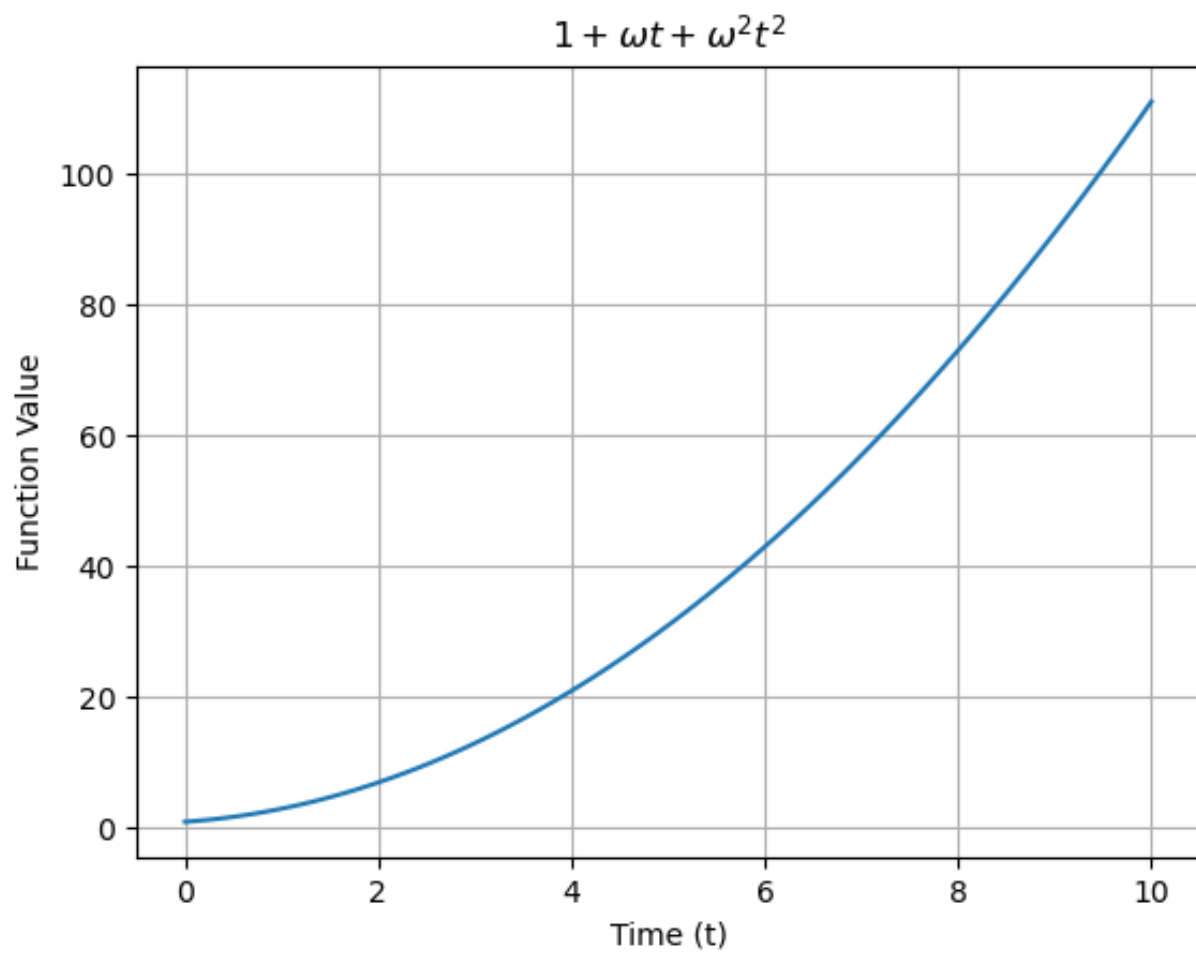


Fig. 1. $1 + 2\pi f t + (2\pi f t)^2$