# AE 42

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**QUESTION:** Consider the equation $\frac{dy}{dx} + ay = \sin\omega x$,where $a$ and $\omega$ are constants.Given $y = 1$ at $x = 0$, correct all the correct statement(s) from the following as $x \to \infty$.

(A) $y \to 0$ if $a \neq 0$

(B) $y \to 1$ if $a = 0$

(C) $y \to Aexp(|a|x)$ if $a < 0$; A is constant

(D) $y \to B\sin(\omega x + C)$ if $a > 0$; B and C are constants

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**Solution:** :

$$y(0) = 1 \tag{1}$$

$$\frac{dy}{dx} + ay = \sin\omega_0 x \tag{2}$$

Taking Fourier transform on both sides

| Function | Fourier transform |
|---|---|
| $\frac{dy}{dx}$ | $\jmath\omega Y$ |
| $y$ | $Y$ |
| $\sin\omega x$ | $\frac{\omega}{2\jmath}(\delta(\omega-\omega_0) - \delta(\omega+\omega_0)$ |

TABLE 0
FOURIER TRANSFORM

$$\jmath\omega Y + aY = \frac{\omega}{2\jmath}(\delta(\omega-\omega_0) - \delta(\omega+\omega_0) \tag{3}$$

$$\implies Y = \frac{\omega}{2\jmath(\jmath\omega + a)}(\delta(\omega-\omega_0) - \delta(\omega+\omega_0) \tag{4}$$

Taking inverse fourier transform on both sides

$$y(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty} Y\, e^{\jmath sx}\, ds \tag{5}$$

$$y(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{\omega}{2\jmath(\jmath\omega + a)}\,(\delta(\omega-\omega_0) - \delta(\omega+\omega_0))\, e^{\jmath sx}\, ds \tag{6}$$

$$y(x) = \mathcal{F}^{-1}\{Y\} = Ae^{-ax} + (B\cos(\omega x) + C\sin(\omega x)) \tag{7}$$

$$y(x) = \mathcal{F}^{-1}\{Y\} = Ae^{-ax} + B\sin(\omega x + C) \tag{8}$$

Now as $x \to \infty$

1) $y \to 0$ if $a \neq 0$ is not true as $y$ depend on $a, \omega$
2) $y \to 1$ if $a = 0$ is not true as y depend on $\omega$
3) $y \to Aexp(|a|x)$ if $a < 0$ is true as $B\sin(\omega x + C)$ is neglected compared to $Ae^{-ax}$
4) $y \to B\sin(\omega x + C)$ if $a > 0$; is true as $Ae^{-ax} \to 0$

$\therefore$ C,D are correct options