11.14-4

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QUESTION: QUESTION: Which of the following functions of time represent (a) simple harmonic, (b) periodic but not simple harmonic, and (c) non-periodic motion? Give period for each case of peiodic motion (ω is any positive constant):

1) $\sin(\omega t) - \cos(\omega t)$
2) $\sin^3(\omega t)$
3) $3\cos\left(\frac{\pi}{4} - 2\omega t\right)$
4) $\cos(\omega t) + \cos(3\omega t) + \cos(5\omega t)$
5) $\exp\left(-\omega^2 t^2\right)$
6) $1 + \omega t + \omega^2 t^2$

Answer:
Periodic function:
If for a function $f(x)$ is periodic if there exists a positive number such that for all x in the domain of f , the following equation holds:

$$f(x+T) = f(x) \tag{1}$$

Period: The smallest positive value of T which holds is above equation

Simple Harmonic Motion:
Mathematically, simple harmonic motion (SHM) is described by an equation that represents the displacement of an object undergoing such motion as a function of time. The general form of this equation is often given by:

$$x(t) = A\cos(\omega t + \phi) \tag{2}$$

$x(t)$ is displacement
A is amplitude
ω is angular frequency $\omega = 2\pi f$
t is time
ϕ is phase angle

1) $\sin(2\pi f t) - \cos(2\pi f t)$

This function can be rewritten as

$$= \sin(2\pi f t) - \sin\left(\frac{\pi}{2} - 2\pi f t\right) \tag{3}$$
$$= 2\cos\left(\frac{\pi}{4}\right)\sin\left(2\pi f t - \frac{\pi}{4}\right) \tag{4}$$
$$= \sqrt{2}\sin\left(2\pi f t - \frac{\pi}{4}\right) \tag{5}$$

$\therefore$ Simple harmonic motion with period T $=\frac{1}{f}$
Phase angle of(ϕ) $\left(\frac{-\pi}{4}\right)$ or$\left(\frac{7\pi}{4}\right)$

(2) $\sin^3(wt)$

This function can be rewritten as

$$= \frac{1}{4}(3\sin(2\pi f t) - \sin(6\pi f t)) \tag{6}$$

$\therefore$ Periodic with period T $= \frac{1}{f}$

(3) $3\cos\left(\frac{\pi}{4} - 4\pi f t\right)$

This function can be rewritten as

$$= 3\cos\left(4\pi f t - \frac{\pi}{4}\right) \tag{7}$$
$$\tag{8}$$

Simple harmonic motion with period T $= \frac{1}{2f}$ and a ϕ of $\left(\frac{-\pi}{4}\right)$ or$\left(\frac{7\pi}{4}\right)$

(4) $\cos(2\pi f t) + \cos(6\pi f t) + \cos(10\pi f t)$

This function can be rewritten as

$$
\begin{aligned}
&= \cos(2\pi f t) + \cos(10\pi f t) + \cos(6\pi f t) && (9)\\
&= 2\cos\left(\frac{2\pi f t + 10\pi f t}{2}\right)\cos\left(\frac{10\pi f t - 2\pi f t}{2}\right) + \cos(6\pi f t) && (10)\\
&= 2\cos(6\pi f t)\cos(2\pi f t) + \cos(6\pi f t) && (11)\\
&= \cos(6\pi f t)(1 + 2\cos(2\pi f t)) && (12)
\end{aligned}
$$

Period of $\cos(6\pi f t)$ is $\frac{1}{3f}$
Period of $1 + 2\cos(2\pi f t)$ is $\frac{1}{f}$
Lcm is $\frac{1}{f}$
$\therefore$ Simple harmonic motion with period $\frac{1}{f}$

(5) $\exp\left(-(2\pi f)^2 t^2\right)$

This function can be rewritten as

$$
\begin{gathered}
\text{As } T \to \infty\\
\exp\left(-(2\pi f)^2 t^2\right) \to \infty
\end{gathered}
$$

$\therefore$ This never repeats and non periodic

(6) $1 + 2\pi f t + (2\pi f)^2 t^2$

This function can be rewritten as

$$
\begin{gathered}
\text{As } T \to \infty\\
1 + 2\pi f t + (2\pi f)^2 t^2 \to \infty
\end{gathered}
$$

$\therefore$ This never repeats and non periodic

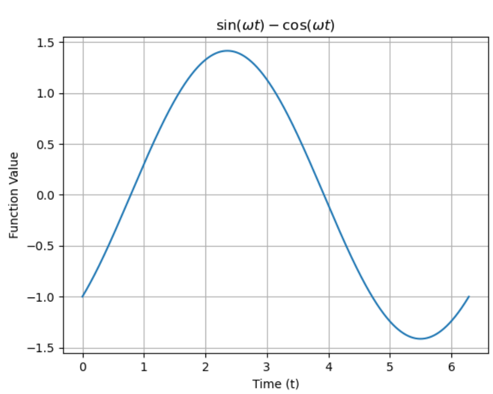

Fig. 0. $\sin(2\pi f t) - \cos(2\pi f t)$

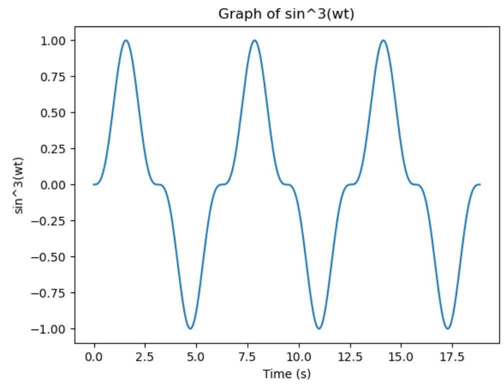

Fig. 0. $\sin^3(2\pi f t)$

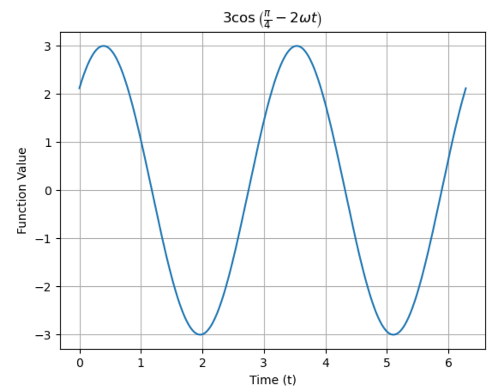

Fig. 0. $3\cos\left(\frac{\pi}{4} - 4\pi f t\right)$

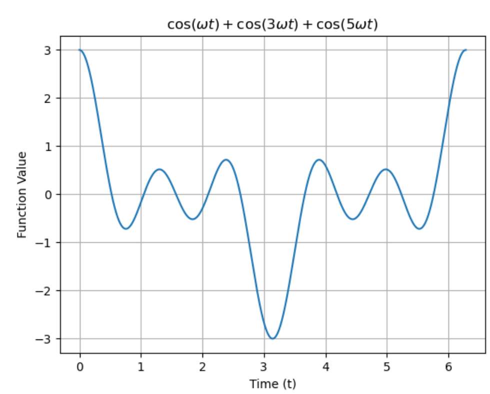

Fig. 0. $\cos(2\pi f t) + \cos(6\pi f t) + \cos(10\pi f t)$

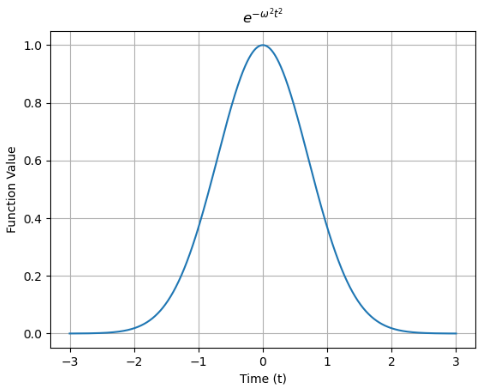

Fig. 0. $exp^{\left(-(2\pi f t)^2\right)}$

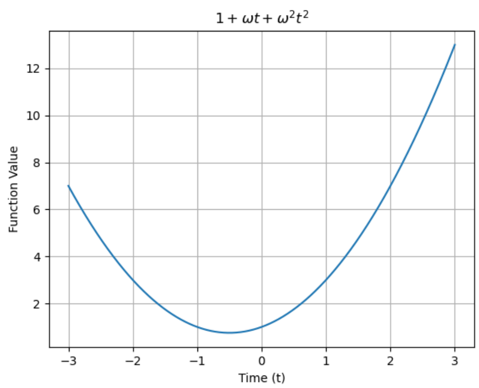

Fig. 0. $1 + 2\pi f t + (2\pi f t)^2$

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TABLE 0
SUMMARY

	Function	**Periodic**	**Simple harmonic motion**	**Non Periodic**	**Period**
(a)	$sin(2\pi f t) - cos(2\pi f t)$	Yes	Yes	No	$\frac{1}{f}$
(b)	$sin^3(2\pi f t)$	Yes	No	No	$\frac{1}{f}$
(c)	$3cos\left(\frac{\pi}{4} - 4\pi f t\right)$	Yes	Yes	No	$\frac{1}{2f}$
(d)	$cos(2\pi f t) + cos(6\pi f t) + cos(10\pi f t)$	Yes	No	No	$\frac{}{f}$
(e)	$\exp\left(-(2\pi f t)^2\right)$	No	No	Yes	–
(f)	$1 + (2\pi f)t + (2\pi f)^2 t^2$	No	No	Yes	–