EC 32

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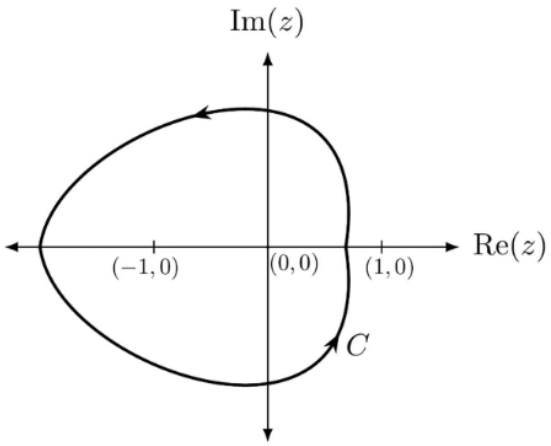
QUESTION: A Simple closed path C in the Complex Plane is shown in the figure.

$$\oint_C \frac{2^z}{z^2-1}dz = -i\pi A$$

Where $1 = \sqrt{-1}$, Then find the value of A is _(Rounded of to two decimals) (GATE 2022 EC)



Solution:

Let

$$f(z) = \frac{2^z}{z^2-1}$$

For poles

$$z^2 - 1 = 0 \tag{1}$$
$$\implies z = \pm 1 \tag{2}$$

As $Z = -1$ lies inside the C and $z = 1$ lies outside C

$$\oint_C f(z)dz = \oint_C \frac{\frac{2^z}{z-1}}{z+1}dz \tag{3}$$
$$= 2\pi i\left(\frac{2^z}{z-1}\right) \text{ At } z = -1 \tag{4}$$

(By Cauchy's integral formula)

$$= 2\pi i\left(\frac{-1}{4}\right) \tag{5}$$
$$= -\pi i\left(\frac{1}{2}\right) \tag{6}$$

By comparing

$$A = \frac{1}{2} = 0.50 \tag{7}$$