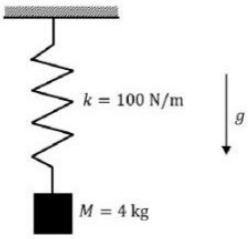
XE 71

EE23BTECH11048-Ponugumati Venkata Chanakya*

QUESTION: A spring mass system is shown in the figure . Take the value of acceleration due to gravity as $g = 9.81 m/s^2$.The static deflection due to weight and the time period of the oscillations,respectively,are

(GATE 2023 XE)



Solution:

1) Static deflection due to weight(sdw)
let x be sdw.
At mean position in equilibrium

$$Mg = kx \tag{1}$$
$$x = 39.24cm \tag{2}$$

2) Time period of oscilattion

$$F = -kx \tag{3}$$
$$m\left(\frac{d^2x}{dt^2}\right) = -kx \tag{4}$$

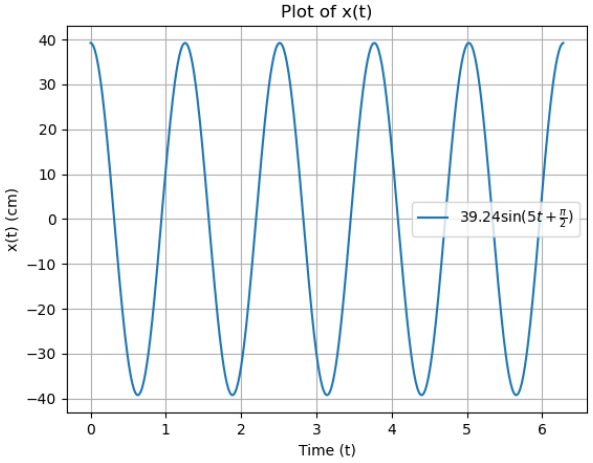
Taking Laplace transform:

$$X(s) = \frac{msx(0) + x'(0)}{ms^2 + k} \tag{5}$$
$$X(s) = \frac{1}{\sqrt{\frac{k}{m}}}\left(A\frac{s - i\sqrt{\frac{k}{m}}}{s^2 + \frac{k}{m}} + B\frac{s + i\sqrt{\frac{k}{m}}}{s^2 + \frac{k}{m}}\right) \tag{6}$$

Taking Inverse Laplace Transform:
Initial Conditions be at extreme point of SHM

$$x(t) = A\left(B\sin(\sqrt{\frac{k}{m}}t) + C\cos(\sqrt{\frac{k}{m}}t)\right) \tag{7}$$
$$x(t) = P\left(\sin(\sqrt{\frac{k}{m}}t + Q)\right) \tag{8}$$
$$x(t) = 0.3924\left(\sin(\sqrt{\frac{100}{5}}t + \frac{\pi}{2})\right)\ \text{m} \tag{9}$$
$$x(t) = 39.24\sin(5t + \frac{\pi}{2})\ \text{cm} \tag{10}$$

The static deflection due to weight and the time period of the oscillations,respectively are 39.24 cm and $\frac{2\pi}{5}$ s



Variable	Description	Value
M	weight of block	4 kg
K	spring constant	$100\frac{N}{m}$
x	Static deflection due to weight	39.24 cm
P	Amplitude	39.24 cm
Q	phase angle	$\frac{\pi}{2}$
A, B, C	integrating constants	none

TABLE 2
INPUT PARAMETERS