

# IN-2023

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## QUESTION:

A continuous real-valued signal  $x(t)$  has finite positive energy and  $x(t) = 0, \forall t < 0$ . From the list given below, select ALL the signals whose continuous-time Fourier transform is purely imaginary.

- 1)  $x(t) + x(-t)$
- 2)  $x(t) - x(-t)$
- 3)  $j(x(t) + x(-t))$
- 4)  $j(x(t) - x(-t))$

(GATE IN 2023)

## Solution:

Parameter	Description
$x(t)$	Continuous real valued signal
$t$	time
$\omega$	angular velocity of the signal
$X(\omega)$	Fourier Transform of $x(t)$

TABLE I

VARIABLES AND THEIR DESCRIPTIONS

Fourier transform of an real and odd signal  $x(t)$  is purely imaginary.

$$\mathcal{F}\{x(t)\} = X(\omega) \quad (1)$$

$$X(\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt \quad (2)$$

$$X(\omega)^* = \int_{-\infty}^{\infty} x(t) e^{j\omega t} dt \quad (3)$$

$$X(\omega)^* = \int_{-\infty}^{\infty} x(-t) e^{-j\omega t} dt \quad (4)$$

$$X(\omega)^* = -X(\omega) \quad (5)$$

$$\mathcal{F}\{x(t)\} = X(\omega) \quad (6)$$

$$X(\omega) = \int_{-\infty}^{\infty} jx(t) e^{-j\omega t} dt \quad (7)$$

$$X(\omega)^* = - \int_{-\infty}^{\infty} jx(t) e^{j\omega t} dt \quad (8)$$

$$X(\omega)^* = - \int_{-\infty}^{\infty} jx(-t) e^{-j\omega t} dt \quad (9)$$

$$X(\omega)^* = -X(\omega) \quad (10)$$

$$1) x(t) + x(-t)$$

$$f(t) = x(t) + x(-t) \quad (11)$$

$$f(-t) = x(-t) + x(t) \quad (12)$$

$$f(t) = f(-t) \quad (13)$$

$\therefore$  Fourier Transform is not Purely imaginary.

$$2) x(t) - x(-t)$$

$$f(t) = x(t) - x(-t) \quad (14)$$

$$f(-t) = x(-t) - x(t) \quad (15)$$

$$f(-t) = -f(t) \quad (16)$$

$\therefore$  Fourier Transform is purely imaginary.

$$3) j(x(t) + x(-t))$$

$$f(t) = j(x(t) + x(-t)) \quad (17)$$

$$f(-t) = j(x(-t) + x(t)) \quad (18)$$

$$f(t) = f(-t) \quad (19)$$

$\therefore$  Fourier Transform is Purely imaginary.

$$4) j(x(t) - x(-t))$$

$$f(t) = j(x(t) - x(-t)) \quad (20)$$

$$f(-t) = j(x(-t) - x(t)) \quad (21)$$

$$f(t) = -f(-t) \quad (22)$$

$\therefore$  Fourier Transform is not Purely imaginary.

Fourier transform of an imaginary even signal  $jx(t)$  is purely imaginary.