

IN-2023

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QUESTION:

A continuous real-valued signal $x(t)$ has finite positive energy and $x(t) = 0, \forall t < 0$. From the list given below, select ALL the signals whose continuous-time Fourier transform is purely imaginary.

- 1) $x(t) + x(-t)$
- 2) $x(t) - x(-t)$
- 3) $j(x(t) + x(-t))$
- 4) $j(x(t) - x(-t))$

(GATE IN 2023)

Solution:

Parameter	Description
$x(t)$	Continuous real valued signal
t	time
f	frequency of the signal
$X(f)$	Fourier Transform of $x(t)$

TABLE I

VARIABLES AND THEIR DESCRIPTIONS

Fourier transform of an real and odd signal $x(t)$ is purely imaginary.

$$\mathcal{F}\{x(t)\} = X(f) \quad (1)$$

$$X(f) = \int_{-\infty}^{\infty} x(t) e^{-j2\pi ft} dt \quad (2)$$

$$X(f)^* = \int_{-\infty}^{\infty} x(t) e^{j2\pi ft} dt \quad (3)$$

$$X(f)^* = \int_{-\infty}^{\infty} x(-t) e^{-j2\pi ft} dt \quad (4)$$

$$X(f)^* = -X(f) \quad (5)$$

Fourier transform of an imaginary even signal $jx(t)$ is purely imaginary.

$$\mathcal{F}\{x(t)\} = X(f) \quad (6)$$

$$X(f) = \int_{-\infty}^{\infty} jx(t) e^{-j2\pi ft} dt \quad (7)$$

$$X(f)^* = - \int_{-\infty}^{\infty} jx(t) e^{j2\pi ft} dt \quad (8)$$

$$X(f)^* = - \int_{-\infty}^{\infty} jx(-t) e^{-j2\pi ft} dt \quad (9)$$

$$X(f)^* = -X(f) \quad (10)$$

$$x(t) = \begin{cases} 0 & \text{for } t < 0 \\ t & \text{for } t \geq 0 \end{cases} \quad (11)$$

$$x(n) = tu(n) \quad (12)$$

$$1) x(t) + x(-t)$$

$$f(t) = x(t) + x(-t) \quad (13)$$

$$f(-t) = f(t) \quad (14)$$

$$f(t) = tu(t) - tu(-t) \quad (15)$$

$$\mathcal{F}\{f(t)\} = \int_{-\infty}^{\infty} f(t) e^{-j2\pi ft} dt \quad (16)$$

$$F(f) = 2 \int_0^{\infty} t \cos(2\pi ft) dt \quad (17)$$

\therefore Fourier Transform is not Purely imaginary.

$$2) x(t) - x(-t)$$

$$f(t) = x(t) - x(-t) \quad (18)$$

$$f(t) = -f(-t) \quad (19)$$

$$f(t) = tu(-t) + tu(t) \quad (20)$$

$$\mathcal{F}\{f(t)\} = \int_{-\infty}^{\infty} f(t) e^{-j2\pi ft} dt \quad (21)$$

$$F(f) = 2j \int_0^{\infty} t \sin(2\pi ft) dt \quad (22)$$

∴ Fourier Transform is purely imaginary.

3) $j(x(t) + x(-t))$

$$f(t) = j(x(t) + x(-t)) \quad (23)$$

$$f(-t) = f(t) \quad (24)$$

$$f(t) = j(tu(t) - tu(-t)) \quad (25)$$

$$\mathcal{F}\{f(t)\} = \int_{-\infty}^{\infty} jte^{-j2\pi ft} dt \quad (26)$$

$$F(f) = 2j \int_0^{\infty} t \cos(2\pi ft) dt \quad (27)$$

∴ Fourier Transform is Purely imaginary.

4) $j(x(t) - x(-t))$

$$f(t) = j(x(t) - x(-t)) \quad (28)$$

$$f(t) = -f(-t) \quad (29)$$

$$f(t) = j(tu(-t) + tu(t)) \quad (30)$$

$$\mathcal{F}\{f(t)\} = \int_{-\infty}^{\infty} f(t) e^{-j2\pi ft} dt \quad (31)$$

$$F(f) = -2 \int_0^{\infty} t \sin(2\pi ft) dt \quad (32)$$

∴ Fourier Transform is not Purely imaginary.