

EC-2022

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QUESTION:

Consider the signals $x(n)=2^{n-1}u(-n+2)$ and $y(n)=2^{-n+2}u(n+1)$, where $u(n)$ is the unit step sequence. Let $X(e^{j\omega})$ and $Y(e^{j\omega})$ be the discrete-time Fourier of $x(n)$ and $y(n)$, respectively. The value of the integral $\frac{1}{2\pi} \int_0^{2\pi} X(e^{j\omega}) Y(e^{-j\omega}) d\omega$ (rounded off to one decimal place) is _____
(GATE EC 2022)

Solution:

Parameter	Description
$u(n)$	unit step function
$z(n)$	$x(n) * y(-n)$
$Z(e^{j\omega})$	$X(e^{j\omega}) Y(e^{-j\omega})$

TABLE I
VARIABLES AND THEIR DESCRIPTIONS

$$V = \frac{1}{2\pi} \int_0^{2\pi} X(e^{j\omega}) Y(e^{-j\omega}) d\omega \quad (1)$$

$$Z(e^{j\omega}) = X(e^{j\omega}) Y(e^{-j\omega}) \quad (2)$$

$$z(n) \xrightarrow{\mathcal{F}} Z(e^{j\omega}) \quad (3)$$

$$z(n) = \frac{1}{2\pi} \int_{-\pi}^{\pi} Z(e^{j\omega}) e^{j\omega n} d\omega \quad (4)$$

$$z(0) = \frac{1}{2\pi} \int_0^{2\pi} Z(e^{j\omega}) d\omega \quad (5)$$

$$z(n) = x(n) * y(-n) \quad (6)$$

$$= \sum_{k=-\infty}^{\infty} 2^{k-1} u(-k+2) 2^{n-k+2} u(-n+k+1) \quad (7)$$

$$= \sum_{k=-\infty}^2 2^{n+1} u(k-n+1) \quad (8)$$

$$z(0) = \sum_{k=-\infty}^2 2u(k+1) \quad (9)$$

$$= 2 \sum_{k=-1}^2 u(k+1) \quad (10)$$

$$\therefore z(0) = 8 \quad (11)$$

$$\therefore \frac{1}{2\pi} \int_0^{2\pi} X(e^{j\omega}) Y(e^{-j\omega}) d\omega = 8$$

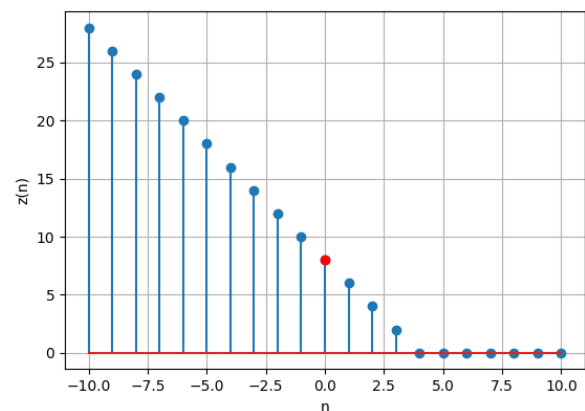


Fig. 0. Stem Plot of $z(n)$