

IN-2023

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QUESTION:

A continuous real-valued signal $x(t)$ has finite positive energy and $x(t) = 0, \forall t < 0$. From the list given below, select ALL the signals whose continuous-time Fourier transform is purely imaginary.

- 1) $x(t) + x(-t)$
- 2) $x(t) - x(-t)$
- 3) $j(x(t) + x(-t))$
- 4) $j(x(t) - x(-t))$

$$y(t) = x(t) + x(-t) \quad (4)$$

$$y^*(t) = y(t) \quad (5)$$

$$y(t) = y(-t) \quad (6)$$

$$Y(f) + Y^*(f) = \int_{-\infty}^{\infty} y(t) e^{-j2\pi ft} dt + \int_{-\infty}^{\infty} y^*(t) e^{j2\pi ft} dt \quad (7)$$

$$= 2 \int_{-\infty}^{\infty} y(t) \cos(2\pi ft) dt \quad (8)$$

\therefore Fourier Transform is Purely real.

(GATE IN 2023)

$$2) x(t) - x(-t)$$

Solution:

Parameter	Description
$x(t)$	Continuous real valued signal
t	time
f	frequency of the signal
$Y(f)$	Fourier Transform of $y(t)$

TABLE I
VARIABLES AND THEIR DESCRIPTIONS

$$y(t) = x(t) - x(-t) \quad (9)$$

$$y^*(t) = y(t) = -y(-t) \quad (10)$$

$$Y(f) = \int_{-\infty}^{\infty} y(t) e^{-j2\pi ft} dt \quad (11)$$

$$Y^*(f) = - \int_{-\infty}^{\infty} y(-t) e^{j2\pi ft} dt \quad (12)$$

$$= - \int_{-\infty}^{\infty} y(t) e^{-j2\pi ft} dt \quad (13)$$

$$Y(f) + Y^*(f) = 0 \quad (14)$$

\therefore Fourier Transform is purely imaginary.

Fourier transform of a signal $y(t)$

$$\mathcal{F}\{y(t)\} = Y(f) \quad (1)$$

$$Y(f) = \int_{-\infty}^{\infty} y(t) e^{-j2\pi ft} dt \quad (2)$$

$$Y^*(f) = \int_{-\infty}^{\infty} y^*(t) e^{j2\pi ft} dt \quad (3)$$

Fourier transform is purely imaginary if $Y(f) + Y^*(f) = 0$

$$1) x(t) + x(-t)$$

$$3) j(x(t) + x(-t))$$

$$y(t) = j(x(t) + x(-t)) \quad (15)$$

$$y(-t) = y(t) \quad (16)$$

$$y^*(t) = -y(t) \quad (17)$$

$$Y(f) = \int_{-\infty}^{\infty} y(t) e^{-j2\pi ft} dt \quad (18)$$

$$Y^*(f) = - \int_{-\infty}^{\infty} y(t) e^{j2\pi ft} dt \quad (19)$$

$$= - \int_{-\infty}^{\infty} y(t) e^{-j2\pi ft} dt \quad (20)$$

$$Y(f) + Y^*(f) = 0 \quad (21)$$

\therefore Fourier Transform is Purely imaginary.

$$4) \quad j(x(t) - x(-t))$$

$$y(t) = j(x(t) - x(-t)) \quad (22)$$

$$y(-t) = -y(t) \quad (23)$$

$$y^*(t) = -y(t) \quad (24)$$

$$Y(f) = \int_{-\infty}^{\infty} y(t) e^{-j2\pi ft} dt \quad (25)$$

$$Y^*(f) = - \int_{-\infty}^{\infty} y(t) e^{j2\pi ft} dt \quad (26)$$

$$= \int_{-\infty}^{\infty} y(t) e^{-j2\pi ft} dt \quad (27)$$

$$Y(f) + Y^*(f) = 2 \int_{-\infty}^{\infty} y(t) e^{-j2\pi ft} dt \quad (28)$$

\therefore Fourier Transform is not Purely imaginary.