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EE23BTECH11210-Dhyana Teja Machineni*

QUESTION:44

A continuous real-valued signal $x(t)$ has finite positive energy and $x(t) = 0,\ \forall\ t < 0$. From the list given below, select ALL the signals whose continuous-time Fourier transform is purely imaginary.

1) $x(t) + x(-t)$
2) $x(t) - x(-t)$
3) $j(x(t) + x(-t))$
4) $j(x(t) - x(-t))$

(GATE IN 2023)

Solution: $X(\omega)=\int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\ dt$

Parameter	Description
$x(t)$	Continuous real valued signal
t	time
ω	angular velocity of the signal
$X(\omega)$	Fourier Transfom of x(t)
$X(-\omega)$	Fourier Transform of x(-t)

TABLE I
VARIABLES AND THEIR DESCRIPTIONS

Fourier transform of a continuous signal

$$\mathcal{F}\{x(t)\} = X(\omega) \tag{1}$$
$$X(\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\ dt \tag{2}$$
$$\mathcal{F}\{x(-t)\} = X(-\omega) \tag{3}$$
$$X(-\omega) = \int_{-\infty}^{\infty} x(-t)\, e^{j\omega t}\ dt \tag{4}$$
$$\tag{5}$$

1)$x(t) + x(-t)$

$$X(\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\ dt \tag{6}$$
$$X(\omega) = \int_{0}^{\infty} x(t)\, e^{-j\omega t}\ dt \tag{7}$$
$$X(-\omega) = \int_{-\infty}^{0} x(-t)\, e^{j\omega t}\ dt \tag{8}$$
$$= \int_{0}^{\infty} x(t)\, e^{j\omega t}\ dt \tag{9}$$
$$X(\omega) + X(-\omega) = \int_{0}^{\infty} x(t)\, e^{-j\omega t}\ dt + \int_{0}^{\infty} x(t)\, e^{j\omega t}\ dt \tag{10}$$
$$= 2\int_{0}^{\infty} x(t)\, cos(\omega t)\ \ dt \tag{11}$$

$\because$ Integral of a real number is real, Continuous time Fourier transform of $(x(t) + x(-t))$ is not Purely imaginary.

2) $x(t) - x(-t)$

$$X(\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\ dt \tag{12}$$
$$X(\omega) = \int_{0}^{\infty} x(t)\, e^{-j\omega t}\ dt \tag{13}$$
$$X(-\omega) = \int_{-\infty}^{0} x(-t)\, e^{j2\pi f t}\ dt \tag{14}$$
$$= \int_{0}^{\infty} x(t)\, e^{j\omega t}\ dt \tag{15}$$
$$X(\omega) - X(-\omega) = \int_{0}^{\infty} x(t)\, e^{-j\omega t}\ dt - \int_{0}^{\infty} x(t)\, e^{j\omega t}\ dt \tag{16}$$
$$= -2\int_{0}^{\infty} jx(t)\, sin(\omega t)\ \ dt \tag{17}$$

$\because$ Integral of a purely imaginary number is imaginary, Continuous time Fourier transform of $(x(t) - x(-t))$ is Purely imaginary.

3) $j\left(x(t)+x(-t)\right)$

$$X(\omega)=\int_{-\infty}^{\infty} jx(t)\,e^{-j\omega t}\ dt \tag{18}$$

$$X(\omega)=\int_{0}^{\infty} jx(t)\,e^{-j\omega t}\ dt \tag{19}$$

$$X(-\omega)=\int_{-\infty}^{0} jx(-t)\,e^{j\omega t}\ dt \tag{20}$$

$$=\int_{0}^{\infty} jx(t)\,e^{j\omega t}\ dt \tag{21}$$

$$X(\omega)+X(-\omega)=\int_{0}^{\infty} jx(t)\,e^{-j\omega t}\ dt+\int_{0}^{\infty} jx(t)\,e^{j\omega t}\ dt \tag{22}$$

$$=2\int_{0}^{\infty} jx(t)\,cos(\omega t)\ \ dt \tag{23}$$

$\because$ Integral of a purely imaginary number is imaginary, Continuous time Fourier transform of $j\left(x(t)+x(-t)\right)$ is Purely imaginary.

4) $j\left(x(t)-x(-t)\right)$

$$X(\omega)=\int_{-\infty}^{\infty} jx(t)\,e^{-j\omega t}\ dt \tag{24}$$

$$X(\omega)=\int_{0}^{\infty} jx(t)\,e^{-j\omega t}\ dt \tag{25}$$

$$X(-\omega)=\int_{-\infty}^{0} jx(-t)\,e^{j\omega t}\ dt \tag{26}$$

$$=\int_{0}^{\infty} jx(t)\,e^{j\omega t}\ dt \tag{27}$$

$$X(\omega)-X(-\omega)=\int_{0}^{\infty} jx(t)\,e^{-j\omega t}\ dt-\int_{0}^{\infty} jx(t)\,e^{j\omega t}\ dt \tag{28}$$

$$=2\int_{0}^{\infty} x(t)\,sin(2\pi ft)\ \ dt \tag{29}$$

$\because$ Integral of a real number is real, Continuous time Fourier transform of $j\left(x(t)+x(-t)\right)$ is not Purely imaginary.