IN-2023

EE23BTECH11210-Dhyana Teja Machineni*

QUESTION:

A continuous real-valued signal $x(t)$ has finite positive energy and $x(t) = 0, \forall\ t < 0$. From the list given below, select ALL the signals whose continuous-time Fourier transform is purely imaginary.

1) $x(t) + x(-t)$
2) $x(t) - x(-t)$
3) $j(x(t) + x(-t))$
4) $j(x(t) - x(-t))$

(GATE IN 2023)

Solution:

Parameter	Description
$x(t)$	Continuous real valued signal
t	time
f	frequency of the signal
$X(f)$	Fourier Transfom of $x(t)$

TABLE I
VARIABLES AND THEIR DESCRIPTIONS

Fourier transform of an real and odd signal$x(t)$ is purely imaginary.

$$\mathcal{F}\{x(t)\} = X(f) \tag{1}$$
$$X(f) = \int_{-\infty}^{\infty} x(t)\, e^{-j2\pi ft}\, dt \tag{2}$$
$$X(f)^* = \int_{-\infty}^{\infty} x(t)\, e^{j2\pi ft}\, dt \tag{3}$$
$$X(f)^* = \int_{-\infty}^{\infty} x(-t)\, e^{-j2\pi ft}\, dt \tag{4}$$
$$X(f)^* = -X(f) \tag{5}$$

Fourier transform of an imaginary even signal $jx(t)$ is purely imaginary.

$$\mathcal{F}\{x(t)\} = X(f) \tag{6}$$
$$X(f) = \int_{-\infty}^{\infty} jx(t)\, e^{-j2\pi ft}\, dt \tag{7}$$
$$X(f)^* = -\int_{-\infty}^{\infty} jx(t)\, e^{j2\pi ft}\, dt \tag{8}$$
$$X(f)^* = -\int_{-\infty}^{\infty} jx(-t)\, e^{-j2\pi ft}\, dt \tag{9}$$
$$X(f)^* = -X(f) \tag{10}$$

$$x(t) = \begin{cases} 0 & \text{for } t < 0 \\ t & \text{for } t \geq 0 \end{cases} \tag{11}$$
$$x(n) = tu(n) \tag{12}$$

1)$x(t) + x(-t)$

$$f(t) = x(t) + x(-t) \tag{13}$$
$$f(-t) = f(t) \tag{14}$$
$$f(t) = tu(t) - tu(-t) \tag{15}$$
$$\mathcal{F}\{f(t)\} = \int_{-\infty}^{\infty} f(t)\, e^{-j2\pi ft}\, dt \tag{16}$$
$$F(f) = 2\int_{0}^{\infty} tcos(2\pi ft)\ dt \tag{17}$$

$\therefore$ Fourier Transform is not Purely imaginary.

2) $x(t) - x(-t)$

$$f(t) = x(t) - x(-t) \tag{18}$$
$$f(t) = -f(-t) \tag{19}$$
$$f(t) = tu(-t) + tu(t) \tag{20}$$
$$\mathcal{F}\{f(t)\} = \int_{-\infty}^{\infty} f(t)\, e^{-j2\pi ft}\, dt \tag{21}$$
$$F(f) = 2j\int_{0}^{\infty} tsin(2\pi ft)\ dt \tag{22}$$

$\therefore$ Fourier Transform is purely imaginary.

3) $j\left(x\left(t\right)+x\left(-t\right)\right)$

$$f\left(t\right)=j\left(x\left(t\right)+x\left(-t\right)\right) \tag{23}$$

$$f\left(-t\right)=f\left(t\right) \tag{24}$$

$$f\left(t\right)=j\left(tu\left(t\right)-tu\left(-t\right)\right) \tag{25}$$

$$\mathcal{F}\{f(t)\}=\int_{-\infty}^{\infty}f\left(t\right)e^{-j2\pi ft}\ dt \tag{26}$$

$$F\left(f\right)=2j\int_{0}^{\infty}tcos\left(2\pi ft\right)\ dt \tag{27}$$

$\therefore$ Fourier Transform is Purely imaginary.

4) $j\left(x\left(t\right)-x\left(-t\right)\right)$

$$f\left(t\right)=j\left(x\left(t\right)-x\left(-t\right)\right) \tag{28}$$

$$f\left(t\right)=-f\left(-t\right) \tag{29}$$

$$f\left(t\right)=j\left(tu\left(-t\right)+tu\left(t\right)\right) \tag{30}$$

$$\mathcal{F}\{f(t)\}=\int_{-\infty}^{\infty}f\left(t\right)e^{-j2\pi ft}\ dt \tag{31}$$

$$F\left(f\right)=-2\int_{0}^{\infty}tsin\left(2\pi ft\right)\ dt \tag{32}$$

$\therefore$ Fourier Transform is not Purely imaginary.