IN-2023

EE23BTECH11210-Dhyana Teja Machineni*

QUESTION:44

A continuous real-valued signal $x(t)$ has finite positive energy and $x(t) = 0, \forall\ t < 0$. From the list given below, select ALL the signals whose continuous-time Fourier transform is purely imaginary.

1) $x(t) + x(-t)$
2) $x(t) - x(-t)$
3) $j(x(t) + x(-t))$
4) $j(x(t) - x(-t))$

(GATE IN 2023)

Solution: $X(f)=\int_{-\infty}^{\infty} x(t)\, e^{-j2\pi ft}\ dt$

Parameter	Description
$x(t)$	Continuous real valued signal
t	time
f	frequency of the signal
$X(f)$	Fourier Transfom of x(t)
$X(-f)$	Fourier Transform of x(-t)

TABLE I
VARIABLES AND THEIR DESCRIPTIONS

Fourier transform of a continuous signal

$$\mathcal{F}\{x(t)\} = X(f) \tag{1}$$

$$X(f) = \int_{-\infty}^{\infty} x(t)\, e^{-j2\pi ft}\ dt \tag{2}$$

$$\mathcal{F}\{x(-t)\} = X(-f) \tag{3}$$

$$X(-f) = \int_{-\infty}^{\infty} x(-t)\, e^{j2\pi ft}\ dt \tag{4}$$

$$\tag{5}$$

1)$x(t) + x(-t)$

$$X(f) = \int_{-\infty}^{0} x(t)\, e^{-j2\pi ft}\ dt + \int_{0}^{\infty} x(t)\, e^{-j2\pi ft}\ dt \tag{6}$$

$$X(f) = \int_{0}^{\infty} x(t)\, e^{-j2\pi ft}\ dt \tag{7}$$

$$X(-f) = \int_{-\infty}^{0} x(-t)\, e^{j2\pi ft}\ dt \tag{8}$$

$$= \int_{0}^{\infty} x(t)\, e^{j2\pi ft}\ dt \tag{9}$$

$$X(f) + X(-f) = \int_{0}^{\infty} x(t)\, e^{-j2\pi ft}\ dt + \int_{0}^{\infty} x(t)\, e^{j2\pi ft}\ dt \tag{10}$$

$$= 2\int_{0}^{\infty} jx(t)\, cos(2\pi ft)\ dt \tag{11}$$

$\because$ Integral of a real number is real, Continuous time Fourier transform of $(x(t) + x(-t))$ is not Purely imaginary.

2) $x(t) - x(-t)$

$$X(f) = \int_{-\infty}^{0} x(t)\, e^{-j2\pi ft}\ dt - \int_{0}^{\infty} x(t)\, e^{-j2\pi ft}\ dt \tag{12}$$

$$X(f) = \int_{0}^{\infty} x(t)\, e^{-j2\pi ft}\ dt \tag{13}$$

$$X(-f) = \int_{-\infty}^{0} x(-t)\, e^{j2\pi ft}\ dt \tag{14}$$

$$= \int_{0}^{\infty} x(t)\, e^{j2\pi ft}\ dt \tag{15}$$

$$X(f) - X(-f) = \int_{0}^{\infty} x(t)\, e^{-j2\pi ft}\ dt - \int_{0}^{\infty} x(t)\, e^{j2\pi ft}\ dt \tag{16}$$

$$= -2\int_{0}^{\infty} jx(t)\, sin(2\pi ft)\ dt \tag{17}$$

$\because$ Integral of a purely imaginary number is imaginary, Continuous time Fourier transform of

$(x(t) - x(-t))$ is Purely imaginary.

3) $j(x(t) + x(-t))$

$$X(f) = \int_{-\infty}^{0} jx(t)\, e^{-j2\pi ft}\ dt + \int_{0}^{\infty} jx(t)\, e^{-j2\pi ft}\ dt \tag{18}$$

$$X(f) = \int_{0}^{\infty} jx(t)\, e^{-j2\pi ft}\ dt \tag{19}$$

$$X(-f) = \int_{-\infty}^{0} jx(-t)\, e^{j2\pi ft}\ dt \tag{20}$$

$$= \int_{0}^{\infty} jx(t)\, e^{j2\pi ft}\ dt \tag{21}$$

$$X(f) + X(-f) = \int_{0}^{\infty} jx(t)\, e^{-j2\pi ft}\ dt + \int_{0}^{\infty} jx(t)\, e^{j2\pi ft}\ dt \tag{22}$$

$$= 2\int_{0}^{\infty} jx(t)\, cos(2\pi ft)\ \ dt \tag{23}$$

$\because$ Integral of a purely imaginary number is imaginary, Continuous time Fourier transform of $j(x(t) + x(-t))$ is Purely imaginary.

4) $j(x(t) - x(-t))$

$$X(f) = \int_{-\infty}^{0} jx(t)\, e^{-j2\pi ft}\ dt - \int_{0}^{\infty} jx(t)\, e^{-j2\pi ft}\ dt \tag{24}$$

$$X(f) = \int_{0}^{\infty} jx(t)\, e^{-j2\pi ft}\ dt \tag{25}$$

$$X(-f) = \int_{-\infty}^{0} jx(-t)\, e^{j2\pi ft}\ dt \tag{26}$$

$$= \int_{0}^{\infty} jx(t)\, e^{j2\pi ft}\ dt \tag{27}$$

$$X(f) - X(-f) = \int_{0}^{\infty} jx(t)\, e^{-j2\pi ft}\ dt - \int_{0}^{\infty} jx(t)\, e^{j2\pi ft}\ dt \tag{28}$$

$$= 2\int_{0}^{\infty} x(t)\, sin(2\pi ft)\ \ dt \tag{29}$$

$\because$ Integral of a real number is real, Continuous time Fourier transform of $j(x(t) + x(-t))$ is not Purely imaginary.