IN-2023

EE23BTECH11210-Dhyana Teja Machineni*

QUESTION:
A continuous real-valued signal $x(t)$ has finite positive energy and $x(t) = 0, \forall\ t < 0$. From the list given below, select ALL the signals whose continuous-time Fourier transform is purely imaginary.

1) $x(t) + x(-t)$
2) $x(t) - x(-t)$
3) $j(x(t) + x(-t))$
4) $j(x(t) - x(-t))$

(GATE IN 2023)

Solution:

Parameter	Description
$x(t)$	Continuous real valued signal
t	time
f	frequency of the signal
$X(f)$	Fourier Transfom of $x(t)$

TABLE I
VARIABLES AND THEIR DESCRIPTIONS

Fourier transform of an real and odd signal$x(t)$ is purely imaginary.

$$\mathcal{F}\{x(t)\} = X(f) \tag{1}$$
$$X(f) = \int_{-\infty}^{\infty} x(t)\, e^{-j2\pi ft}\ dt \tag{2}$$
$$X(f)^* = \int_{-\infty}^{\infty} x(t)\, e^{j2\pi ft}\ dt \tag{3}$$
$$X(f)^* = \int_{-\infty}^{\infty} x(-t)\, e^{-j2\pi ft}\ dt \tag{4}$$
$$X(f)^* = -X(f) \tag{5}$$

Fourier transform of an imaginary even signal $jx(t)$ is purely imaginary.

$$\mathcal{F}\{x(t)\} = X(f) \tag{6}$$
$$X(f) = \int_{-\infty}^{\infty} jx(t)\, e^{-j2\pi ft}\ dt \tag{7}$$
$$X(f)^* = -\int_{-\infty}^{\infty} jx(t)\, e^{j2\pi ft}\ dt \tag{8}$$
$$X(f)^* = -\int_{-\infty}^{\infty} jx(-t)\, e^{-j2\pi ft}\ dt \tag{9}$$
$$X(f)^* = -X(f) \tag{10}$$

$$x(t) = \begin{cases} 0 & \text{for } t < 0 \\ t & \text{for } t \geq 0 \end{cases} \tag{11}$$
$$x(-t) = \begin{cases} -t & \text{for } t \leq 0 \\ 0 & \text{for } t > 0 \end{cases} \tag{12}$$

$1)x(t) + x(-t)$

$$f(t) = x(t) + x(-t) \tag{13}$$
$$f(t) = \begin{cases} -t & \text{for } t < 0 \\ t & \text{for } t \geq 0 \end{cases} \tag{14}$$
$$f(-t) = \begin{cases} t & \text{for } t > 0 \\ -t & \text{for } t \leq 0 \end{cases} \tag{15}$$
$$f(-t) = f(t) \tag{16}$$
$$\mathcal{F}\{f(t)\} = \int_{-\infty}^{\infty} f(t)\, e^{-j2\pi ft}\ dt \tag{17}$$
$$= 2\int_{0}^{\infty} t cos(2\pi ft)\ dt \tag{18}$$

$\therefore$ Fourier Transform is not Purely imaginary.

2) $x(t) - x(-t)$

$$f(t) = x(t) - x(-t) \tag{19}$$

$$f(t) = \begin{cases} t & \text{for } t < 0 \\ t & \text{for } t \geq 0 \end{cases} \tag{20}$$

$$f(-t) = \begin{cases} -t & \text{for } t > 0 \\ -t & \text{for } t \leq 0 \end{cases} \tag{21}$$

$$f(t) = -f(-t) \tag{22}$$

$$\mathcal{F}\{f(t)\} = \int_{-\infty}^{\infty} f(t)\, e^{-j2\pi ft}\, dt \tag{23}$$

$$= 2j \int_{0}^{\infty} t sin(2\pi ft)\, dt \tag{24}$$

$\therefore$ Fourier Transform is purely imaginary.
3)$j(x(t) + x(-t))$

$$f(t) = j(x(t) + x(-t)) \tag{25}$$

$$f(t) = \begin{cases} -jt & \text{for } t < 0 \\ jt & \text{for } t \geq 0 \end{cases} \tag{26}$$

$$f(-t) = \begin{cases} jt & \text{for } t > 0 \\ -jt & \text{for } t \leq 0 \end{cases} \tag{27}$$

$$f(-t) = f(t) \tag{28}$$

$$\mathcal{F}\{f(t)\} = \int_{-\infty}^{\infty} jte^{-j2\pi ft}\, dt \tag{29}$$

$$= 2j \int_{0}^{\infty} t cos(2\pi ft)\, dt \tag{30}$$

$\therefore$ Fourier Transform is Purely imaginary.
4)$j(x(t) - x(-t))$

$$f(t) = j(x(t) - x(-t)) \tag{31}$$

$$f(t) = \begin{cases} jt & \text{for } t < 0 \\ jt & \text{for } t \geq 0 \end{cases} \tag{32}$$

$$f(-t) = \begin{cases} -jt & \text{for } t > 0 \\ -jt & \text{for } t \leq 0 \end{cases} \tag{33}$$

$$f(t) = -f(-t) \tag{34}$$

$$\mathcal{F}\{f(t)\} = \int_{-\infty}^{\infty} f(t)\, e^{-j2\pi ft}\, dt \tag{35}$$

$$= -2 \int_{0}^{\infty} t sin(2\pi ft)\, dt \tag{36}$$

$\therefore$ Fourier Transform is not Purely imaginary.