EC-2022

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QUESTION:

Consider the signals $x(n)=2^{n-1}u(-n+2)$ and $y(n)=2^{-n+2}u(n+1)$, where $u(n)$ is the unit step sequence. Let $X\left(e^{j\omega}\right)$ and $Y\left(e^{j\omega}\right)$ be the discrete-time Fourier of $x(n)$ and $y(n)$,respectively. The value of the integral $\frac{1}{2\pi}\int_0^{2\pi} X\left(e^{j\omega}\right)Y\left(e^{-j\omega}\right)d\omega$ (rounded off to one decimal place) is _____

(GATE EC 2022)

Solution:

Parameter	Description
$u(n)$	unit step function
$z(n)$	convolution of $x(n)$,$y(-n)$
$Z\left(e^{j\omega}\right)$	DFT of $z(n)$

TABLE I

Variables and their descriptions

$$V = \frac{1}{2\pi}\int_0^{2\pi} X\left(e^{j\omega}\right)Y\left(e^{-j\omega}\right)d\omega \tag{1}$$

$$z(n) = x(n) * y(-n) \tag{2}$$

$$= \sum_{k=-\infty}^{\infty} 2^{k-1}u(-k+2)\,2^{n-k+2}u(-n+k+1) \tag{3}$$

$$= \sum_{k=-\infty}^{2} 2^{n+1}u(k-n+1) \tag{4}$$

$$\mathcal{F}\{z[n]\} = Z\left(e^{j\omega}\right) \tag{5}$$

$$Z\left(e^{j\omega}\right) = X\left(e^{j\omega}\right)Y\left(e^{-j\omega}\right) \tag{6}$$

$$z(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} Z\left(e^{j\omega}\right)e^{j\omega n}d\omega \tag{7}$$

$$z(0) = \frac{1}{2\pi}\int_0^{2\pi} Z\left(e^{j\omega}\right)d\omega \tag{8}$$

$$z(0) = \sum_{k=-\infty}^{2} 2u(k+1) \tag{9}$$

$$= 2\sum_{k=-1}^{2} u(k+1) \tag{10}$$

$$\therefore z(0) = 8 \tag{11}$$

$$\therefore \frac{1}{2\pi}\int_0^{2\pi} X\left(e^{j\omega}\right)Y\left(e^{-j\omega}\right)d\omega = 8$$