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QUESTION:
A continuous real-valued signal $x(t)$ has finite positive energy and $x(t) = 0, \ \forall \ t < 0$. From the list given below, select ALL the signals whose continuous-time Fourier transform is purely imaginary.

1) $x(t) + x(-t)$
2) $x(t) - x(-t)$
3) $j(x(t) + x(-t))$
4) $j(x(t) - x(-t))$

(GATE IN 2023)

Solution:

Parameter	Description
$x(t)$	Continuous real valued signal
t	time
f	frequency of the signal
$Y(f)$	Fourier Transfom of $y(t)$

TABLE I
VARIABLES AND THEIR DESCRIPTIONS

Fourier transform of an real siganl $y(t)$

$$\mathcal{F}\{y(t)\} = Y(f) \tag{1}$$
$$Y(f) = \int_{-\infty}^{\infty} y(t)\, e^{-j2\pi ft}\ dt \tag{2}$$
$$Y^*(f) = \int_{-\infty}^{\infty} y^*(t)\, e^{j2\pi ft}\ dt \tag{3}$$

Fourier transform is purely imaginary if $Y(f) + Y^*(f) = 0$
1)$x(t) + x(-t)$

$$y(t) = x(t) + x(-t) \tag{4}$$
$$y^*(t) = y(t) \tag{5}$$
$$Y(f) + Y^*(f) = \int_{-\infty}^{\infty} y(t)\, e^{-j2\pi ft}\ dt + \int_{-\infty}^{\infty} y^*(t)\, e^{j2\pi ft}\ dt \tag{6}$$
$$= 2\int_{-\infty}^{\infty} y(t)\, cos(2\pi ft)\ dt \tag{7}$$

$\therefore$ Fourier Transform is Purely real.
2) $x(t) - x(-t)$

$$y(t) = x(t) - x(-t) \tag{8}$$
$$y^*(t) = -y(-t) \tag{9}$$
$$Y(f) = \int_{-\infty}^{\infty} y(t)\, e^{-j2\pi ft}\ dt \tag{10}$$
$$Y^*(f) = -\int_{-\infty}^{\infty} y(-t)\, e^{j2\pi ft}\ dt \tag{11}$$
$$= -\int_{-\infty}^{\infty} y(t)\, e^{-j2\pi ft}\ dt \tag{12}$$
$$Y(f) + Y^*(f) = 0 \tag{13}$$

$\therefore$ Fourier Transform is purely imaginary.
3)$j(x(t) + x(-t))$

$$y(t) = j(x(t) + x(-t)) \tag{14}$$
$$y^*(t) = -y(t) \tag{15}$$
$$Y(f) = \int_{-\infty}^{\infty} y(t)\, e^{-j2\pi ft}\ dt \tag{16}$$
$$Y^*(f) = -\int_{-\infty}^{\infty} y(t)\, e^{j2\pi ft}\ dt \tag{17}$$
$$= -\int_{-\infty}^{\infty} y(t)\, e^{-j2\pi ft}\ dt \tag{18}$$
$$Y(f) + Y^*(f) = 0 \tag{19}$$

$\therefore$ Fourier Transform is Purely imaginary.
4) $j\left(x\left(t\right)-x\left(-t\right)\right)$

$$y\left(t\right)=j\left(x\left(t\right)-x\left(-t\right)\right) \tag{20}$$

$$y^{*}\left(t\right)=-y\left(t\right) \tag{21}$$

$$Y\left(f\right)=\int_{-\infty}^{\infty}y\left(t\right)e^{-j2\pi ft}\,dt \tag{22}$$

$$Y^{*}\left(f\right)=-\int_{-\infty}^{\infty}y\left(t\right)e^{j2\pi ft}\,dt \tag{23}$$

$$=\int_{-\infty}^{\infty}y\left(t\right)e^{-j2\pi ft}\,dt \tag{24}$$

$$Y\left(f\right)+Y^{*}\left(f\right)=2\int_{-\infty}^{\infty}y\left(t\right)e^{-j2\pi ft}\,dt \tag{25}$$

$\therefore$ Fourier Transform is not Purely imaginary.