IN-2023

EE23BTECH11210-Dhyana Teja Machineni*

QUESTION:44

A continuous real-valued signal $x(t)$ has finite positive energy and $x(t) = 0, \ \forall \ t < 0$. From the list given below, select ALL the signals whose continuous-time Fourier transform is purely imaginary.

1) $x(t) + x(-t)$
2) $x(t) - x(-t)$
3) $j(x(t) + x(-t))$
4) $j(x(t) - x(-t))$

(GATE IN 2023)

Solution:

Parameter	Description
$x(t)$	Continuous real valued signal
t	time
ω	angular velocity of the signal
$X(\omega)$	Fourier Transfom of x(t)
$X(\omega)^*$	Conjugate of $X(\omega)$

TABLE I
VARIABLES AND THEIR DESCRIPTIONS

Fourier transform of an real and odd signal$x(t)$ is purely imaginary.

$$\mathcal{F}\{x(t)\} = X(\omega) \tag{1}$$
$$X(\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\ dt \tag{2}$$
$$= \int_{-\infty}^{\infty} x(-t)\, e^{j\omega t}\ dt \tag{3}$$
$$= -\int_{-\infty}^{\infty} x(t)\, e^{j\omega t}\ dt \tag{4}$$
$$X(\omega)^* = \int_{-\infty}^{\infty} x(t)\, e^{j\omega t}\ dt \tag{5}$$
$$X(\omega) = -X(\omega)^* \tag{6}$$

Fourier transform of an imaginary even signal $jx(t)$ is purely imaginary.

$$\mathcal{F}\{x(t)\} = X(\omega) \tag{7}$$
$$X(\omega) = \int_{-\infty}^{\infty} jx(t)\, e^{-j\omega t}\ dt \tag{8}$$
$$= \int_{-\infty}^{\infty} jx(-t)\, e^{j\omega t}\ dt \tag{9}$$
$$= \int_{-\infty}^{\infty} jx(t)\, e^{j\omega t}\ dt \tag{10}$$
$$X(\omega)^* = -\int_{-\infty}^{\infty} jx(t)\, e^{j\omega t}\ dt \tag{11}$$
$$X(\omega) = -X(\omega)^* \tag{12}$$

1)$x(t) + x(-t)$

$$f(t) = x(t) + x(-t) \tag{13}$$
$$f(-t) = x(-t) + x(t) \tag{14}$$
$$f(t) = f(-t) \tag{15}$$

$\therefore\ x(t) + x(-t)$ is not Purely imaginary.

2) $x(t) - x(-t)$

$$f(t) = x(t) - x(-t) \tag{16}$$
$$f(-t) = x(-t) - x(t) \tag{17}$$
$$f(-t) = -f(t) \tag{18}$$

$\therefore\ x(t) - x(-t)$ is purely imaginary.

3)$j(x(t) + x(-t))$

$$f(t) = j(x(t) + x(-t)) \tag{19}$$
$$f(-t) = j(x(-t) + x(t)) \tag{20}$$
$$f(t) = f(-t) \tag{21}$$

$\therefore\ j(x(t) + x(-t))$ is Purely imaginary.

4)$j(x(t) - x(-t))$

$$f(t) = j(x(t) - x(-t)) \tag{22}$$
$$f(-t) = j(x(-t) - x(t)) \tag{23}$$
$$f(t) = -f(-t) \tag{24}$$

$\therefore\ j(x(t) + x(-t))$ is not Purely imaginary.