

11.9.3.7

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EXERCISE 9.3

7. Find the sum to indicated number of terms in each of the geometric progressions in

0.15, 0.015, 0.0015 ... 20 terms

Solution:

TABLE I
VARIABLES AND THEIR DESCRIPTIONS

Parameter	Description	Value
n	No. of terms in the G.P	20
$x(0)$	first term in the G.P	0.15
r	common ratio in the G.P	0.1

∴ Sum of 20 terms of the given GP is 0.16667

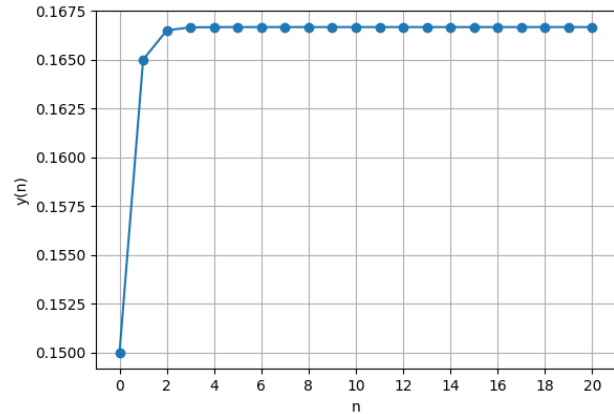


Fig. 0. Stem plot of y(n)

$$x(n) = x(0)r^n \quad (1)$$

$$X(z) = \frac{x(0)}{1 - rz^{-1}} \quad |z| > |r| \quad (2)$$

$$U(z) = \frac{1}{1 - z^{-1}}, \quad |z| > 1 \quad (3)$$

$$y(n) = x(n) * u(n) \quad (4)$$

$$Y(z) = X(z)U(z) \quad (5)$$

$$= \left\{ \frac{0.15}{1 - 0.1z^{-1}} \right\} \left\{ \frac{1}{1 - z^{-1}} \right\} \quad |z| > 1 \quad |z| > |0.1| \quad (6)$$

Use Counter integration to find the inverse of the z transform which gives sum of n terms

$$y(20) = \frac{1}{2\pi j} \oint_C \frac{0.15z^2}{(z-1)(z-0.1)} z^{19} dz \quad (7)$$

$$= \frac{1}{2\pi j} \oint_C \frac{0.15}{0.9} \left\{ \frac{1}{z-1} - \frac{1}{z-0.1} \right\} z^{21} dz \quad (8)$$

$$= \frac{1}{6} \left\{ \left\{ \lim_{z \rightarrow 1} \frac{z^{21}}{z-1} (z-1) \right\} - \left\{ \lim_{z \rightarrow 0.1} \frac{z^{21}}{z-0.1} (z-0.1) \right\} \right\} \quad (9)$$

$$= \frac{1}{6} (1 - 0.1^{21}) \quad (10)$$

$$= 0.16667 \quad (11)$$