

11.9.3.7

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QUESTION:

Find the sum to indicated number of terms in each of the geometric progressions in 0.15, 0.015, 0.0015, ... 20 terms.

SOLUTION

TABLE I
VARIABLES AND THEIR DESCRIPTIONS

Parameter	Description	Value
n	No. of terms in the G.P	20
$x(0)$	first term in the G.P	0.15
r	common ratio in the G.P	0.1
$x(n)$	nth term in the G.P	none
$X(z)$	Z transform of $x(n)$	none
$S(z)$	Z transform of $s(n)$	none
$y(n)$	Sum of n terms of GP	none

$$x(n) = x(0)r^n$$

$$X(z) = \frac{x(0)}{1 - rz^{-1}} \quad |z| > |r|$$

$$U(z) = \frac{1}{1 - z^{-1}}, \quad |z| > 1$$

$$S(z) = \sum_{n=-\infty}^{\infty} s(n)z^{-n}$$

$$y(n) = x(n) * u(n)$$

$$S(z) = X(z)U(z)$$

$$= \left(\frac{0.15}{1 - 0.1z^{-1}} \right) \left(\frac{1}{1 - z^{-1}} \right), \quad |z| > 1 \quad |z| > |r|$$

transform which gives sum of n terms

$$y(20) = \frac{1}{2\pi j} \oint_C \frac{0.15z^2}{(z-1)(z-0.1)} z^{19} dz \quad (8)$$

$$= \frac{1}{2\pi j} \oint_C \frac{0.15}{0.9} \left(\frac{1}{z-1} - \frac{1}{z-0.1} \right) \quad (9)$$

$$= \frac{1}{6} \left(\left(\lim_{z \rightarrow 1} \frac{z^{n+1}}{z-1} (z-1) \right) - \left(\lim_{z \rightarrow 0.1} \frac{z^{n+1}}{z-0.1} (z-0.1) \right) \right) \quad (10)$$

$$= \frac{1}{6} (1 - 0.1^{21}) \quad (11)$$

$$= 0.16667 \quad (12)$$

\therefore Sum of 20 terms of the given GP is 0.16667

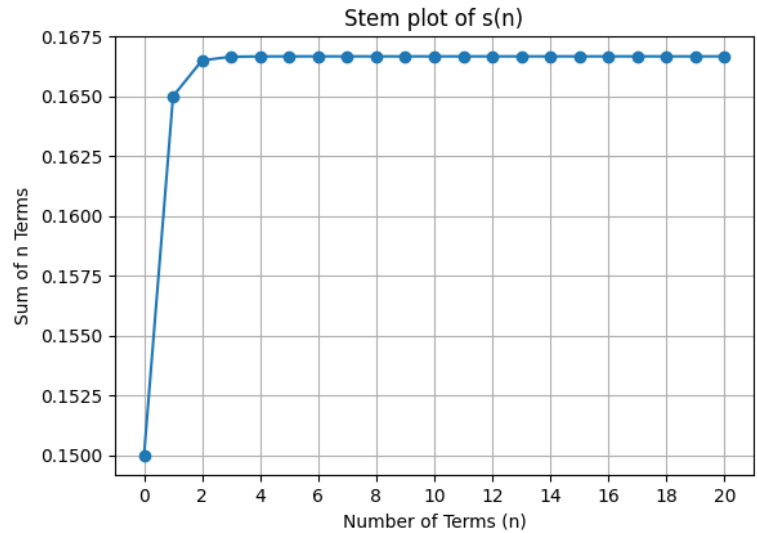


Fig. 0. SUM OF n TERMS OF GP

Use Counter integration to find the inverse of the z