

11.9.3.7

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EXERCISE 9.3

7. Find the sum to indicated number of terms in each of the geometric progressions in 0.15, 0.015, 0.0015 ... 20 terms.

Solution:

TABLE I
VARIABLES AND THEIR DESCRIPTIONS

Parameter	Description	Value
n	No. of terms in the G.P	20
$x(0)$	first term in the G.P	0.15
r	common ratio in the G.P	0.1

$$x(n) = x(0)r^n$$

$$X(z) = \frac{x(0)}{1 - rz^{-1}} \quad |z| > |r|$$

$$U(z) = \frac{1}{1 - z^{-1}}, \quad |z| > 1$$

$$y(n) = x(n) * u(n)$$

$$Y(z) = X(z)U(z)$$

$$= \left(\frac{0.15}{1 - 0.1z^{-1}} \right) \left(\frac{1}{1 - z^{-1}} \right) \quad |z| > 1$$

Using Contour integration

$$y(20) = \frac{1}{2\pi j} \oint_C \frac{0.15z^2}{(z-1)(z-0.1)} z^{19} dz$$

$$= \frac{1}{2\pi j} \oint_C \frac{0.15}{0.9} \left(\frac{1}{z-1} - \frac{1}{z-0.1} \right) z^{21} dz$$

$$= \frac{1}{6} \left(\left(\lim_{z \rightarrow 1} \frac{z^{21}}{z-1} (z-1) \right) - \left(\lim_{z \rightarrow 0.1} \frac{z^{21}}{z-0.1} (z-0.1) \right) \right)$$

$$= \frac{1}{6} (1 - 0.1^{21})$$

$$= 0.16667$$

∴ Sum of 20 terms of the given GP is 0.16667

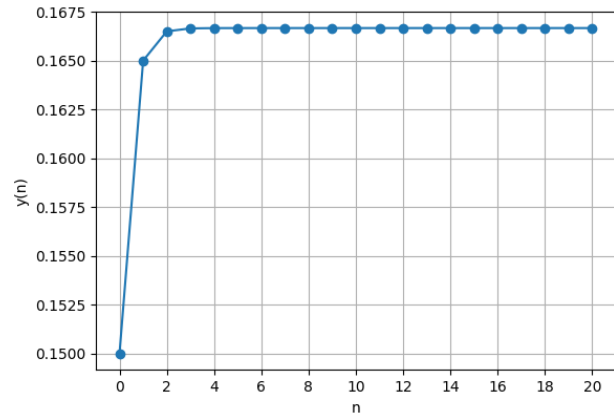


Fig. 0. Stem plot of y(n)