11.9.3.7

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QUESTION:

Find the sum to indicated number of terms in each of the geometric progressions in 0.15, 0.015, 0.0015, ... 20 terms.

SOLUTION:

TABLE I
VARIABLES AND THEIR DESCRIPTIONS

Parameter	Description	Value
n	No. of terms in the G.P	20
$x(0)$	first term in the G.P	0.15
r	common ratio in the G.P	0.1
$x(n)$	nth term in the G.P	none
$X(z)$	Z transform of x(n)	none
$Y(z)$	Z transform of y(n)	none
$y(n)$	Sum of n terms of GP	none

$$x(n) = x(0)r^n \tag{1}$$

$$X(z) = \frac{x(0)}{1 - rz^{-1}} \qquad |z| > |r| \tag{2}$$

$$U(z) = \frac{1}{1 - z^{-1}}, \qquad |z| > 1 \tag{3}$$

$$y(n) = x(n) * u(n) \tag{4}$$

$$= \sum_{k=-\infty}^{\infty} x(k)u(n-k) \tag{5}$$

$$= \sum_{k=0}^{\infty} x(0)r^k u(n-k) \tag{6}$$

$$y(n) = 0.15\left(\frac{1 - (0.1)^{n+1}}{0.9}\right) \tag{7}$$

$$Y(z) = \sum_{n=-\infty}^{\infty} y(n)z^{-n} \tag{8}$$

$$Y(z) = X(z)U(z) \tag{9}$$

$$= \left(\frac{0.15}{1 - 0.1z^{-1}}\right)\left(\frac{1}{1 - z^{-1}}\right), \qquad |z| > 1 \qquad |z| > |r| \tag{10}$$

Use Counter integration to find the inverse of the z transform which gives sum of n terms

$$y(20) = \frac{1}{2\pi j} \oint_C \frac{0.15z^2}{(z-1)(z-0.1)} z^{19} dz \tag{11}$$

$$= \frac{1}{2\pi j} \oint_C \frac{0.15}{0.9}\left(\frac{1}{z-1} - \frac{1}{z-0.1}\right) z^{21} dz \tag{12}$$

$$= \frac{1}{6}\left(\left(\lim_{z \to 1} \frac{z^{21}}{z-1}(z-1)\right) - \left(\lim_{z \to 0.1} \frac{z^{21}}{z-0.1}(z-0.1)\right)\right) \tag{13}$$

$$= \frac{1}{6}(1 - 0.1^{21}) \tag{14}$$
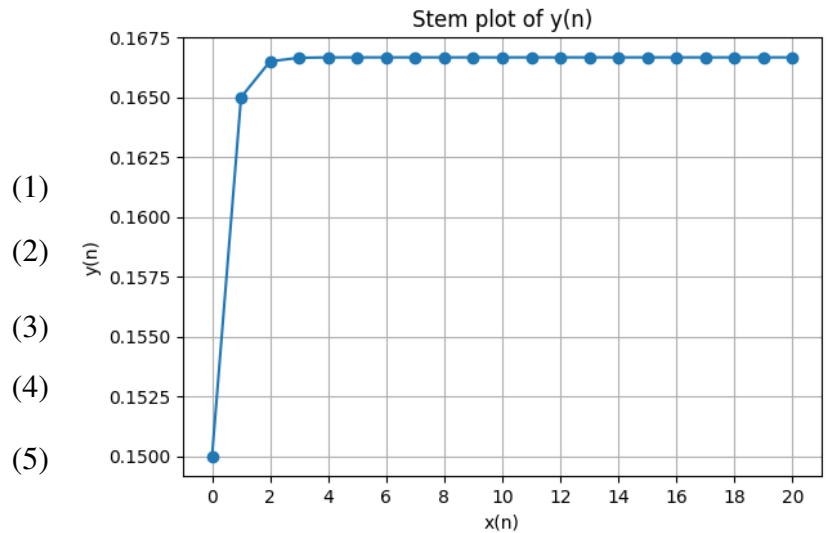
$$= 0.16667 \tag{15}$$

∴ Sum of 20 terms of the given GP is 0.16667

Fig. 0. SUM OF n TERMS OF GP