11.9.3.7

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QUESTION:
Find the sum to indicated number of terms in each of the geometric progressions in 0.15, 0.015, 0.0015, ... 20 terms.

Solution

TABLE I
Variables and their descriptions

Parameter	Description	Value
n	No. of terms in the G.P	20
$x(0)$	first term in the G.P	0.15
r	common ratio in the G.P	0.1
$x(n)$	nth term in the G.P	none
$X(z)$	Z transform of x(n)	none
$S(z)$	Z transform of s(n)	none
$s(n)$	Sum of n terms of GP	none

$$x(n) = x(0)r^n \tag{1}$$

$$X(z) = \sum_{n=-\infty}^{\infty} x(0)r^n u(n) z^{-n} \tag{2}$$

$$X(z) = \sum_{n=0}^{\infty} x(0)r^n z^{-n} \tag{3}$$

$$X(z) = \frac{x(0)}{1 - rz^{-1}} \qquad |z| > |r| \tag{4}$$

$$U(z) = \frac{1}{1 - z^{-1}}, \qquad |z| > 1 \tag{5}$$

$$S(z) = \sum_{n=-\infty}^{\infty} s(n) z^{-n} \tag{6}$$

$$s(n) = x(n) * u(n) \tag{7}$$

$$S(z) = X(z)U(z) \tag{8}$$

$$= \left(\frac{x(0)}{1 - rz^{-1}}\right)\left(\frac{1}{1 - z^{-1}}\right), \qquad |z| > 1 \qquad |z| > |r| \tag{9}$$

Use Counter integration to find the inverse of the z transform which gives sum of n terms

$$s(n) = \frac{1}{2\pi j} \oint_C S(z) z^{n-1} dz \tag{10}$$

$$= \frac{1}{2\pi j} \oint_C \frac{x(0)z^2}{(z-1)(z-r)} z^{n-1} dz \tag{11}$$

$$= \frac{1}{(m-1)!} \lim_{z \to a} \frac{d^{m-1}}{dz^{m-1}} \left((z-a)^m f(z)\right) \tag{12}$$

$$= \lim_{z \to 1} \frac{d}{dz} \left((z-1)^2 \frac{x(0)z^{n+1}}{(z-1)(z-r)}\right) \tag{13}$$

$$= \lim_{z \to 1} \frac{d}{dz} \left((z-1) \frac{x(0)z^{n+1}}{(z-r)}\right) \tag{14}$$

solving equation(13) we get sum of n terms of the given GP

$$s(n) = \frac{x(0)}{1-r} \tag{15}$$

$$= \frac{0.15}{0.9} \tag{16}$$

$$= 0.16667 \tag{17}$$

$\therefore$ Sum of 20 terms of the given GP is 0.16667

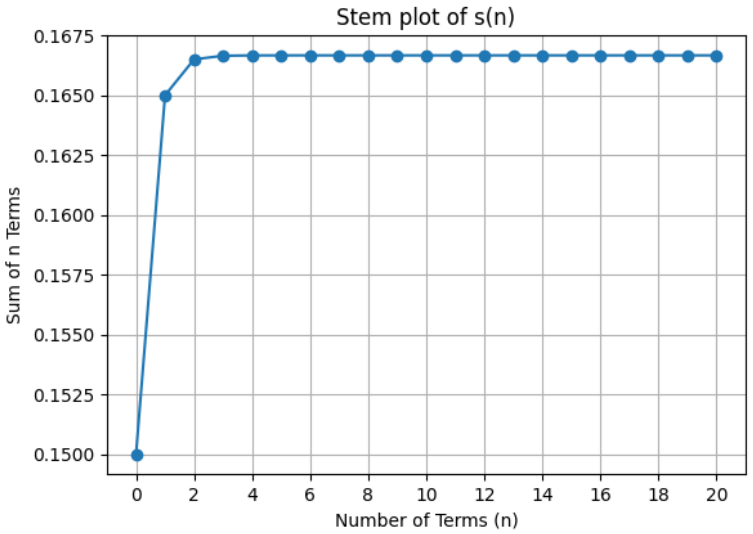

Fig. 0. SUM OF n TERMS OF GP