11.9.3.7

EE23BTECH11210-Dhyana Teja Machineni*

QUESTION:
Find the sum to indicated number of terms in each of the geometric progressions in 0.15, 0.015, 0.0015...20 terms.
Solution:

TABLE I
VARIABLES AND THEIR DESCRIPTIONS

Parameter	Description	Value
n	No. of terms in the G.P	20
$x(0)$	first term in the G.P	0.15
r	common ratio in the G.P	0.1

$$x(n) = x(0)r^n \tag{1}$$

$$X(z) = \frac{x(0)}{1 - rz^{-1}} \qquad |z| > |r| \tag{2}$$

$$U(z) = \frac{1}{1 - z^{-1}}, \qquad |z| > 1 \tag{3}$$

$$y(n) = x(n) * u(n) \tag{4}$$

$$Y(z) = X(z)U(z) \tag{5}$$

$$= \left(\frac{0.15}{1 - 0.1z^{-1}}\right)\left(\frac{1}{1 - z^{-1}}\right) \quad |z| > 1 \quad |z| > |0.1| \tag{6}$$

Use Counter integration to find the inverse of the z transform which gives sum of n terms

$$y(20) = \frac{1}{2\pi j}\oint_C \frac{0.15z^2}{(z-1)(z-0.1)} z^{19} dz \tag{7}$$

$$= \frac{1}{2\pi j}\oint_C \frac{0.15}{0.9}\left(\frac{1}{z-1} - \frac{1}{z-0.1}\right) z^{21} dz \tag{8}$$

$$= \frac{1}{6}\left(\left(\lim_{z\to 1} \frac{z^{21}}{z-1}(z-1)\right) - \left(\lim_{z\to 0.1} \frac{z^{21}}{z-0.1}(z-0.1)\right)\right) \tag{9}$$

$$= \frac{1}{6}(1 - 0.1^{21}) \tag{10}$$

$$= 0.16667 \tag{11}$$

$\therefore$ Sum of 20 terms of the given GP is 0.16667

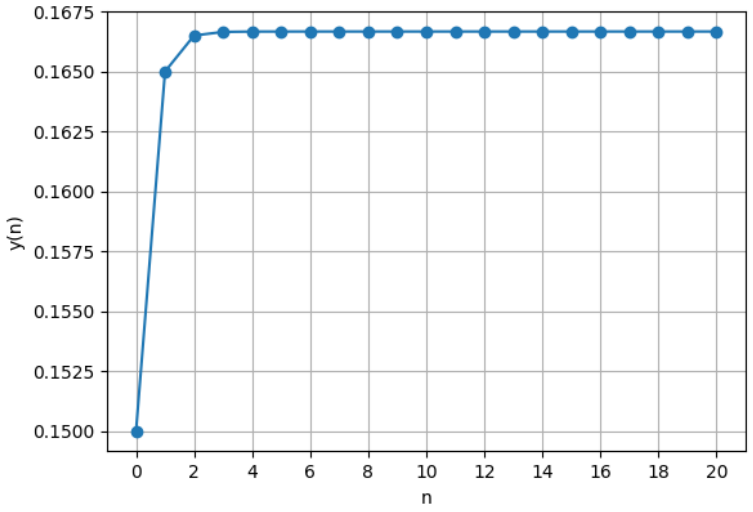

Fig. 0. SUM OF n TERMS OF GP