

11.9.3.7

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QUESTION:

Find the sum to indicated number of terms in each of the geometric progressions in 0.15, 0.015, 0.0015, ... 20 terms.

SOLUTION

TABLE I
VARIABLES AND THEIR DESCRIPTIONS

Parameter	Description	Value
n	No. of terms in the G.P	20
$x(0)$	first term in the G.P	0.15
r	common ratio in the G.P	0.1
$x(n)$	nth term in the G.P	none
$X(z)$	Z transform of $x(n)$	none
$Y(z)$	Z transform of $s(n)$	none
$s(n)$	Sum of n terms of GP	none

Use Counter integration to find the inverse of the z transform which gives sum of n terms

$$= \frac{1}{2\pi j} \oint_C X(z) z^{n-1} dz \quad (9)$$

$$= \frac{1}{2\pi j} \oint_C \frac{x(0)z}{(z-1)(z-r)} z^{n-1} dz \quad (10)$$

$$= \frac{1}{(m-1)!} \lim_{z \rightarrow a} \frac{d^{m-1}}{dz^{m-1}} ((z-a)^m f(z)) \quad (11)$$

$$= \lim_{z \rightarrow 1} \frac{d}{dz} \left((z-1)^2 \frac{x(0)z^n}{(z-1)(z-r)} \right) \quad (12)$$

$$= \lim_{z \rightarrow 1} \frac{d}{dz} \left((z-1) \frac{x(0)z^n}{(z-r)} \right) \quad (13)$$

solving equation(13) we get sum of n terms of the given GP

$$s(n) = \frac{x(0)}{1-r} \quad (14)$$

$$= \frac{0.15}{0.9} \quad (15)$$

$$= 0.16667 \quad (16)$$

∴ Sum of 20 terms of the given GP is 0.16667

$$x(n) = x(0)r^n \quad (1)$$

$$X(z) = \sum_{n=0}^{\infty} x(n)z^{-n} \quad (2)$$

$$X(z) = \frac{x(0)}{1-rz^{-1}} \quad |z| > |r| \quad (3)$$

$$y(n) = \frac{x(0)(r^n - 1)}{r - 1} \quad (4)$$

$$Y(z) = \sum_{n=0}^{\infty} y(n)z^{-n} \quad (5)$$

$$Y(z) = \frac{x(0)}{r-1} \sum_{n=0}^{\infty} (r^n - 1)z^{-n} \quad (6)$$

$$Y(z) = \frac{x(0)}{r-1} \left(\frac{1}{1-rz^{-1}} - \frac{1}{1-z^{-1}} \right) \quad (7)$$

$$= \frac{x(0)z}{(z-1)(z-r)} \quad |z| > 1 \quad |z| > |r| \quad (8)$$

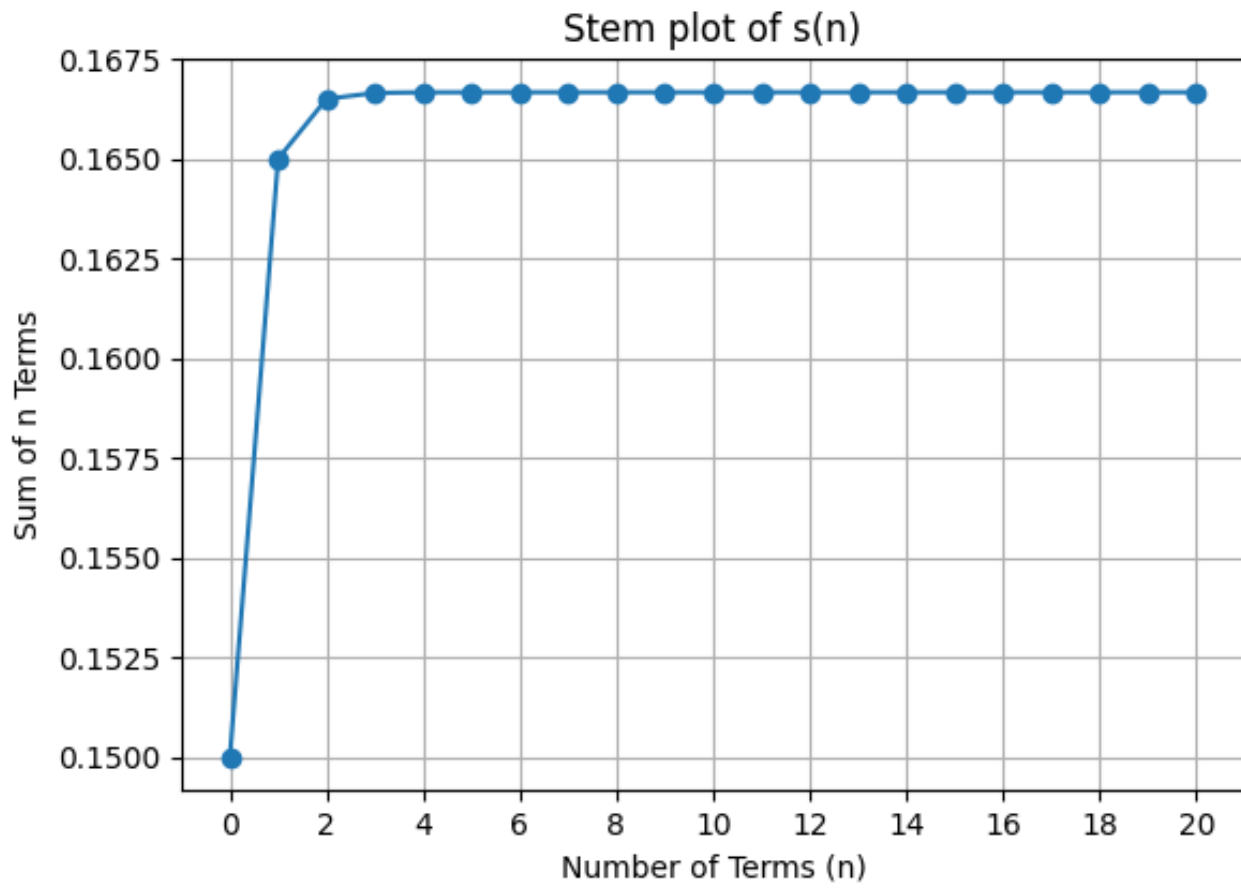


Fig. 0. SUM OF n TERMS OF GP