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QUESTION: In double-slit experiment using light of wavelength 600 nm, the angular width of a fringe formed on a distant screen is 0.1Â°. What is the spacing between the two slits?

SOLUTION: Let the equation of the light waves coming from the source be

$$y_1 = A\sin(2\pi ft) \tag{1}$$

$$y_2 = A\sin(2\pi ft + \phi) \tag{2}$$

Since both the light waves are from the same source so the frequency of both the waves is same frequency f and amplitude A.

USing principle of superposition, we get

$$y = y_1 + y_2 \tag{3}$$

where y is resultant wave equation

$$y = y_1 + y_2 \tag{4}$$

$$y = Asin(2\pi ft) + Asin(2\pi ft + \phi) \tag{5}$$

using

$$sin(c) + sin(d) = 2sin(\frac{c+d}{2})cos(\frac{c-d}{2}) \tag{6}$$

we get

$$y = 2Asin(2\pi ft + \frac{\phi}{2})cos(\frac{\phi}{2}) \tag{7}$$

For constructive interference to happen

$$cos(\phi/2) = +/-1 \tag{8}$$

$$\phi = 2n\pi \tag{9}$$

Equation(8) is the condition for constructive interference In YDSE setup, the path difference between the light rays is given by

$$\Delta x = \frac{\lambda}{2\pi}\phi \tag{10}$$

$$\Delta x = dsin(\theta) \tag{11}$$

from the equations (9) and (10)

$$\frac{\lambda}{2\pi}2n\pi = dsin(\theta) \tag{12}$$

$$dsin(\theta) = n\lambda \tag{13}$$

Now, for small values of θ, we can approximate

$$sin(\theta) \approx tan(\theta)\frac{n}{d} \qquad = \frac{y}{D}y = n\frac{D\lambda}{d} \tag{14}$$

Now let us find the fringe width for this interference Let fringe width be β

$$y_{n+1} = (n+1)\frac{D\lambda}{d} \tag{15}$$

$$y_n = n\frac{D\lambda}{d} \tag{16}$$

$$\beta = y_{n+1} - y_n \tag{17}$$

$$\beta = \frac{D\lambda}{d} \tag{18}$$

Angluar Fringe width for light rays in YDSE is given by

$$Tan(\theta) = \frac{\beta}{D} \tag{19}$$

For small angles we can assume

$$Tan(\theta) \approx \theta \tag{20}$$

From equation (21)

$$\theta = \frac{\frac{\lambda D}{d}}{D} \tag{21}$$

$$\therefore \theta = \frac{\lambda}{d} \tag{22}$$

Given

$$\theta = 0.1^\circ \tag{23}$$

$$= \frac{\pi}{1800} \tag{24}$$

$$\lambda = 600nm \tag{25}$$

$$d = \frac{600}{\frac{\pi}{1800}} \tag{26}$$

$$\therefore d = 3.44 * 10^{-4}m. \tag{27}$$

TABLE 0
VARIABLES AND THEIR DESCRIPTIONS

VARIABLE	**Description**	**Value**
y_1	Equation of first wave	none
y_2	Equation of the second wave	none
y	Equation of the resultant light wave	none
A	Amplitude of the light wave	none
f	Frequency of both the wave equations	none
Δx	Path difference between the light rays	none
ϕ	Phase difference between the light rays	none
β	Fringe width of the interface formed by the light rays	none
D	Distance between the centre of the slits and the screen	none
d	Spacing between the slits used in the YDSE	NEED TO BE FOUND
λ	Wavelength of the light used	600nm
θ	Angular fringe width	0.1°