11.9.3.7

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QUESTION:
Find the sum to indicated number of terms in each of the geometric progressions in 0.15, 0.015, 0.0015, ... 20 terms.

Solution

Let $x(0)$ denote the first term and r the common ratio. The sum of a geometric progression with n terms:

$$x(n) = x(0)r^n \tag{1}$$

$$X(z) = \sum_{n=0}^{\infty} x(n)z^{-n} \tag{2}$$

$$X(z) = \frac{x(0)}{1 - rz^{-1}} \qquad |z| > |r| \tag{3}$$

$$y(n) = \frac{x(0)(r^n - 1)}{r - 1} \tag{4}$$

$$Y(z) = \sum_{n=0}^{\infty} y(n)z^{-n} \tag{5}$$

$$Y(z) = \frac{x(0)}{r - 1} \sum_{n=0}^{\infty} (r^n - 1)z^{-n} \tag{6}$$

$$Y(z) = \frac{x(0)}{r - 1} \left(\frac{1}{1 - rz^{-}1} - \frac{1}{1 - z^{-}1} \right) \tag{7}$$

$$= \frac{x(0)z}{(z - 1)(z - r)} \qquad |z| > 1 \qquad |z| > |r| \tag{8}$$

Use Counter integration to find the inverse of the z transform which gives sum of n terms

$$= \frac{1}{2\pi j} \oint_C X(z)\, z^{n-1} dz \tag{9}$$

$$= \frac{1}{2\pi j} \oint_C \frac{x(0)z}{(z - 1)(z - r)} z^{n-1} dz \tag{10}$$

$$= \frac{1}{(m - 1)!} \lim_{z \to a} \frac{d^{m-1}}{dz^{m-1}} \left((z - a)^m f(z) \right) \tag{11}$$

$$= \lim_{z \to 1} \frac{d}{dz} \left((z - 1)^2 \frac{x(0)z^n}{(z - 1)(z - r)} \right) \tag{12}$$

$$= \lim_{z \to 1} \frac{d}{dz} \left((z - 1) \frac{x(0)z^n}{(z - r)} \right) \tag{13}$$

solving equation(13) we get sum of n terms of the given GP

$$s(n) = \frac{x(0)}{1 - r} \tag{14}$$

$$= \frac{0.15}{0.9} \tag{15}$$

$$= 0.16667 \tag{16}$$

$\therefore$ Sum of 20 terms of the given GP is 0.16667

TABLE 0
Variables and their descriptions

Parameter	Description	Value
n	Number of terms in the G.P (positive even integer)	20
$x(0)$	first term in the G.P	0.15
r	common ratio in the G.P	0.1
$x(n)$	nth term in the G.P	none
$X(z)$	Z transform of x(n)	none
$Y(z)$	Z transform of s(n)	none
$s(n)$	Sum of n terms of GP	none

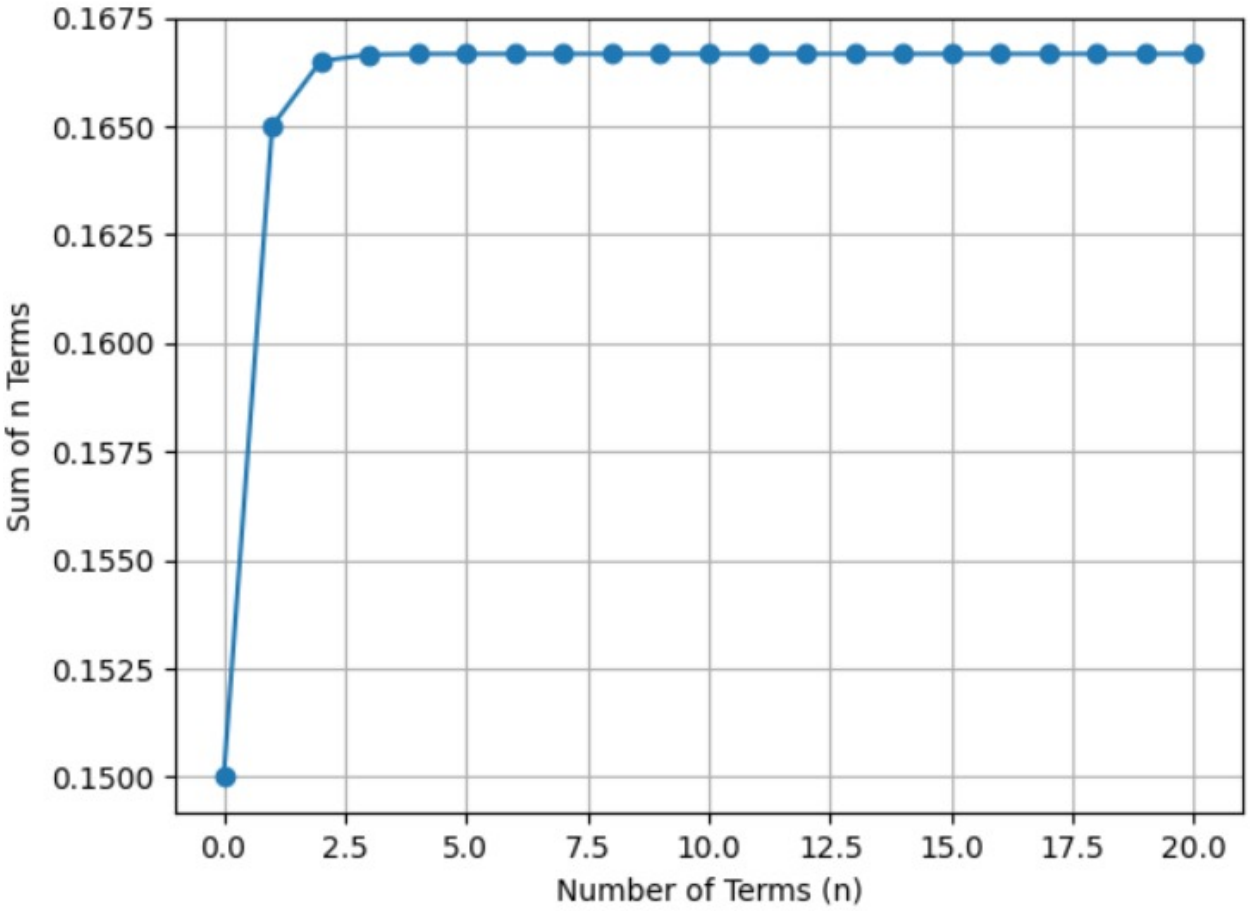

Fig. 0. SUM OF n TERMS OF GP