Week 10 Live Session

w203 Instructional Team

November2, 2016

Announcements

No class next week!

We will have class Thanksgiving week. Students in the Thursday sections are invited to attend one of the sections on Tuesday or Wednesday if possible.

Simple linear regression

Suppose we have data, represented by $(X_1, Y_1), \ldots, (X_n, Y_n)$.

Q1.1: Write a simple regression model for the i^th case.

Q1,2: The statistical errors u_i cannot be observed. Why?

Q1.3: What assumptions do we need to make?

Q1.4: Do we want the residuals to be small in magnitude? Why or why not?

Q1.5: Is it sufficient to require $\sum \hat{u_i} = 0$.

Properties of residuals

Q2.1: What are the implications of the following properties?

(1) $\sum \hat{u_i} = 0$.

(2) $\Sigma\ X_i\ \hat{u_i} = 0$

Q2.2: How many different lines through the X-Y plane would fulfill these two conditions?

Q2.2: Using the above conditions, compute $cov(\hat{Y_i}, \hat{u_i})$.

Regression in R

When a linear pattern is evident from a scatter plot, the relationship between the two variables is often modeled with a straight line. This line is expressed in a linear model between the response (or dependent) variable and the predictor (or independent) variable.

The following functions are useful for running a linear regression in R.

- Fitting a model: model <- lm(y ~ x)
- Coefficients: model$coef or coef(model)
- Fitted values: model$fitted or fitted(model)
- Residuals: model$resid or resid(model)

Install and load the BSDA package using the commands install.packages("BSDA") and library(BSDA), respectively.

```
library(BSDA)
```

```
## Loading required package: e1071
```

```
## Loading required package: lattice
```

```
##
## Attaching package: 'BSDA'
```

```
## The following object is masked from 'package:datasets':
##
##     Orange
```

We are interested in using the Gpa data frame, which is available in memory once we import the package.

Before we can find the least square regression line, we need to examine the explanatory and response variables. Briefly examine college GPA (CollGPA) and high school GPA (HSGPA).

Q3.1. Create a scatterplot of CollGPA versus HSGPA and find the correlation between the two variables. What can we infer from the correlation?

Now that we know a few things about the data, we want to find a line that best represents the relationship between the variables. In other words, we want to draw a slope that comes closest to describing the data.

Q3.2. Characterize the equation mathematically. Find the least squares estimates of β_0 and β_1 using equations,

$$\hat{\beta}_1 = \frac{\sum_{i=1}^{n}(x_i-\bar{x})(y_i-\bar{y})}{\sum_{i=1}^{n}(x_i-\bar{x})^2}$$

$$\hat{\beta}_0 = \bar{y} - \hat{\beta}_1\bar{x}$$

To perform the least square regression is R we can use the lm command. If you are interested use the help(lm) command to learn the different options for using this function. The model we are assuming is defined in the lm command using a tilde ("~") between the response variable and the explanatory variable: lm(Y ~ x).

To see the coefficients in your model, use the coef function (i.e. coef(model)).

If you would like to know what else is stored in the variable you can use the attributes command: attributes().

Q3.3. Find the least squares estimates of β_0 and β_1 using the R function lm().

Q3.4. abline() adds one or more straight lines to the current plot. Conveniently, you can pass your linear model object into abline as an argument (i.e. abline(model)).

Q3.5 Compute the correlation between your x values and your model residuals. What do you learn from this?

OLS Goodness of Fit

When building regression models, "goodness-of-fit" explains how closely our model of the data (i.e. the predictor variables) fits the outcome data. In other words, how much of the variation in an outcome can we explain with a particular model?

R-Squared

R-squared is a measure commonly used for assessing model fit. It can be understood as the proportion of variance in the outcome that can be accounted for by the model.

Looking at our simple bivariate model, we can extract R-squared as a measure of model fit in a number of ways. The easiest is simply to extract it from the lm object using model$r.squared.

But we can also calculate R-squared from our data in a number of ways. Take a couple of minutes to manually calculate R-squared.

1. By squaring the correlation between X and Y.
2. By taking the ratio of the variance of the fitted values to the variance of Y.
3. By weighting the slope coefficient: $R^2 = \beta_1^2 \frac{var(X)}{var(Y)}$

Adjusted R Square

The "Adjusted R-squared" is commonly used in place of the "regular" R-squared, which is sensitive to the number of independent variables in the model. In other words, as we put more variables into the model, R-squared increases even if those variables are unrelated to the outcome.

Adjusted R-squared attempts to correct for this by deflating R-squared by the expected amount of increase from including irrelevant additional predictors.

We can see this property of R-squared and Adjusted R-squared by adding a completely random variables unrelated to our other covariates or the outcome into our model and examine the impact on R-squared and Adjusted R-squared.

```
tmp1 <- rnorm(10, 0, 10)
```

Add this variable to your simple regression model, creating a new lm object, then observe what happens to R-squared.

Now extract the adjusted R-squared from both models using lm$adj.r.squared. It may also go down, but by less than regular r-squared.

OLS: Issues to be Aware of

Unfortunately, the pitfalls of applying least squares are not often well understood by many of the people who attempt to apply it. What follows is a list of some of the biggest problems with using least squares regression in practice, along with some brief comments about how these problems may be mitigated or avoided

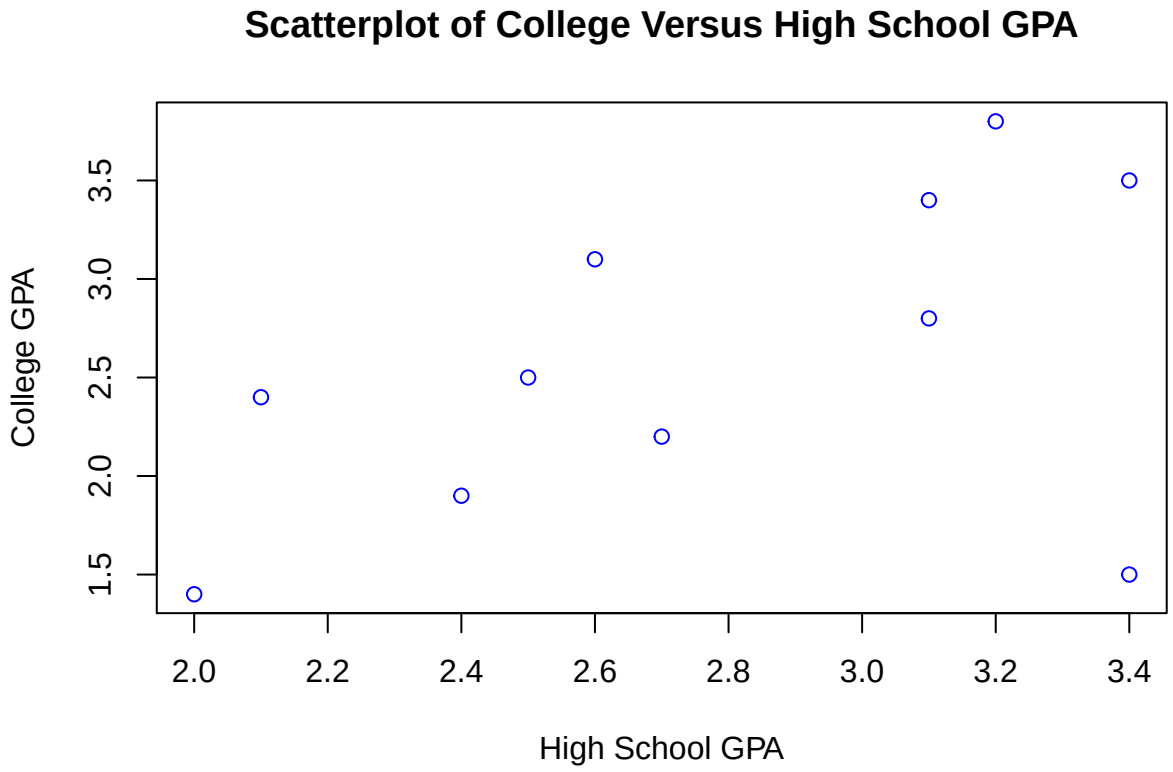
Outliers

Least squares regression can perform very badly when some points in the training data have excessively large or small values for the dependent variable compared to the rest of the training data. The reason for this is that since the least squares method is concerned with minimizing the sum of the squared error, any training point that has a dependent value that differs a lot from the rest of the data can have a disproportionately large effect on the resulting constants that are being solved for.

WARNING: Do not ever remove an observation just because it's an outlier.

Returning to our example, let's add an outlier.

```
y_out <- c(Gpa$CollGPA,1.5)
x_out <- c(Gpa$HSGPA,3.4)
plot(x_out, y_out, col="blue", main="Scatterplot of College Versus High School GPA",
     xlab="High School GPA",ylab="College GPA")
```




```
cor(x_out,y_out)
```

```
## [1] 0.4986491
```

We can see the outlier pulls the correlation off a lot. Let's see what it does to the linear model.

```
model_out <- lm(y_out ~ x_out)
summary(model_out)
```

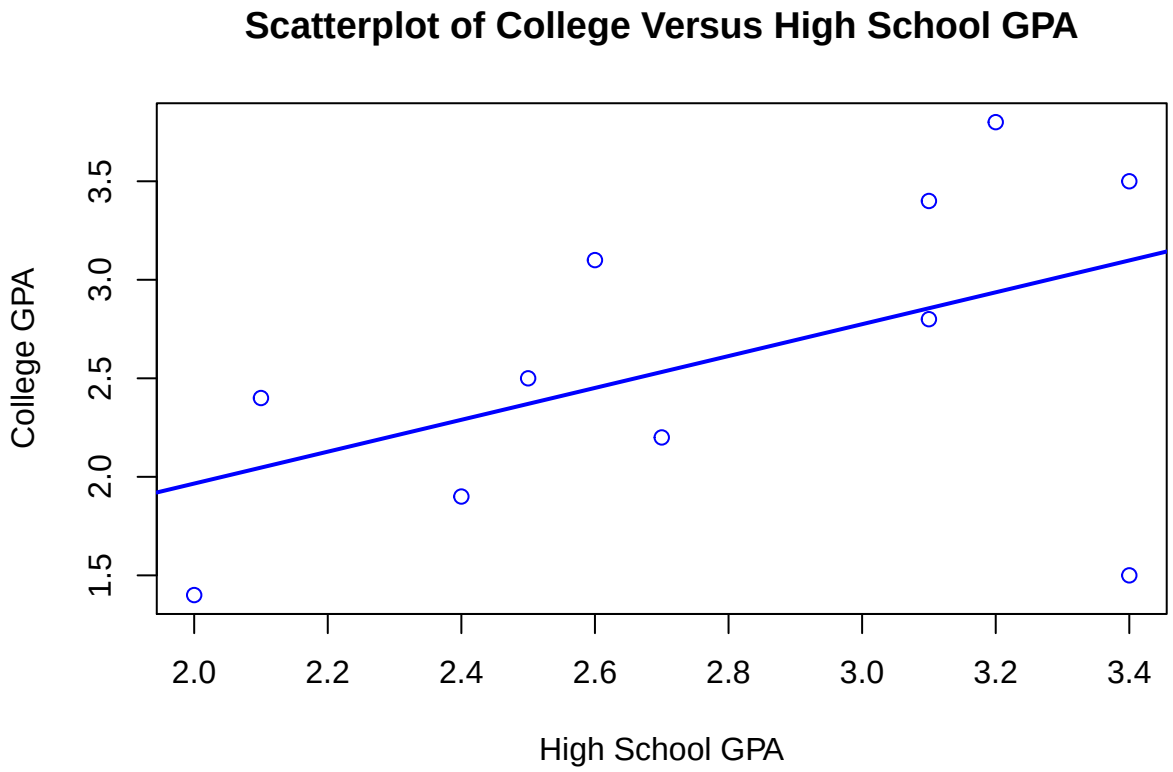
```
##
## Call:
## lm(formula = y_out ~ x_out)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -1.5982 -0.3608  0.1297  0.4731  0.8635
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)   0.3484     1.3183   0.264    0.798
## x_out         0.8088     0.4686   1.726    0.118
##
## Residual standard error: 0.7383 on 9 degrees of freedom
## Multiple R-squared:  0.2487, Adjusted R-squared:  0.1652
## F-statistic: 2.978 on 1 and 9 DF,  p-value: 0.1185
```

```
model_out$coef
```

```
## (Intercept)       x_out
##   0.3483516   0.8087912
```

Let's see that scatterplot again with our new regression line.

```
plot(x_out, y_out, col="blue", main="Scatterplot of College Versus High School GPA",
     xlab="High School GPA",ylab="College GPA")
abline(model_out,col="blue",lwd=2)
```



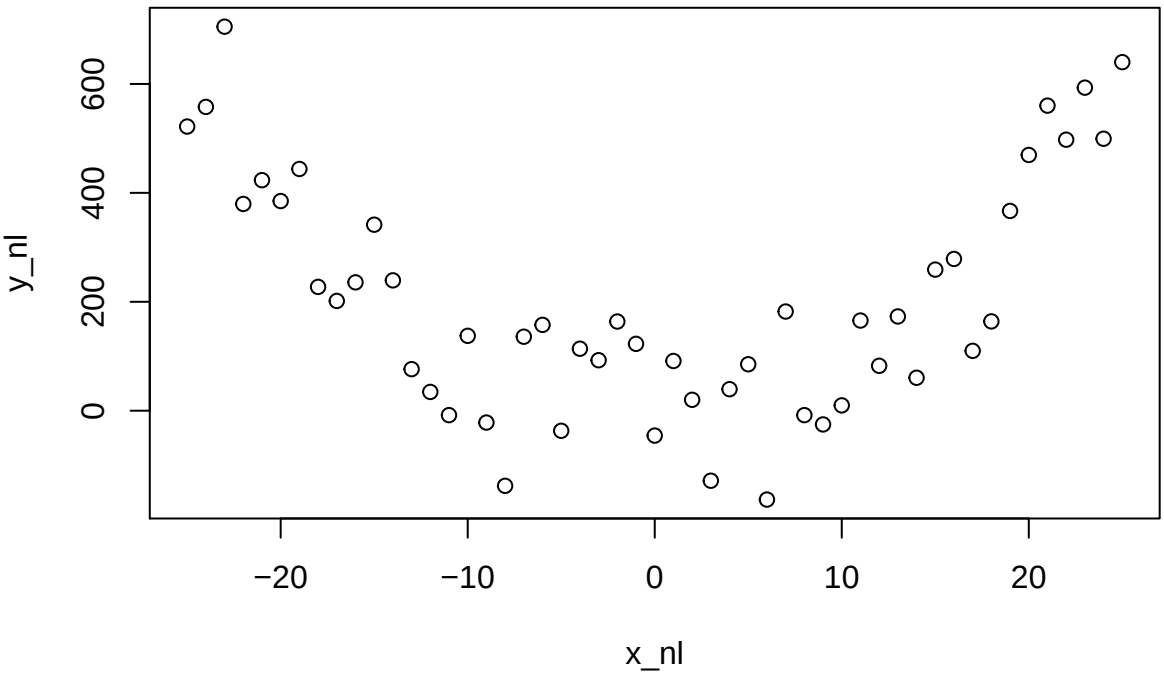
WARNING: Do not ever remove an observation just because it's an outlier.

Non-Linearities

All linear regression methods (including, of course, least squares regression), suffer from the major drawback that in reality most systems are not linear.

Let's take another dataset that is clearly non-linear.

```
x_nl<-seq(-25,25,1)
y_nl<-x_nl^2+rnorm(51,0,100)
plot(x_nl,y_nl)
```



There's definitely a relationship here, but we will need to do a transformation prior to OLS.