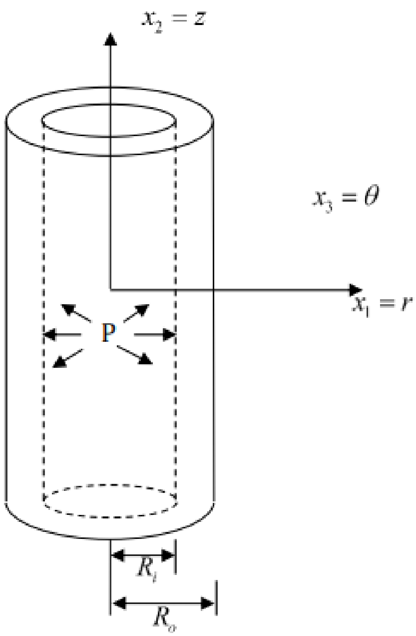
INDEX

Problem Statement:

An infinite long linear elastic tube is subjected to an internal pressure as shown below:



The tube is made of a linear elastic material with inner radius R_i = *6.0 in*, outer radius R_o = *9.0 in,*

Young's Modulus E = 1000.0 psi, Poisson's ratio ν = *0.3,* Density ρ = *0.01 lb/in*^{3}*,* and the internal pressure *P(t)* is a function of time given as,

$$P(t) = \begin{cases} \sin\left(\frac{2\pi t}{T}\right) & for\ 0 \le t \le T/2 \\ 0 & otherwise \end{cases}$$

And T = *0.05 sec*.

The objective of the project is to construct and implement radial plane strain finite element formulation to solve the above elastodynamic problem, and to study the stability and accuracy of explicit and implicit time integration methods. Linear one-dimensional finite element (2-node) along the radial direction with the following time integration methods are to be used:

Finite Element Formulation:

For the above problem, finite element formulation is carried out after converting the actual cylindrical system into radial plane strain problem by considering plane strain condition along the thickness of the tube. Finite element formulation should be done in a way to ensure the convergence to true solution by ensuring stability and consistency of finite element scheme.

After developing the weak form of the problem, the shape functions to approximate the response are taken for 2 node element (linear).

Weak Form: In general, 3-D,

$$\int_{\Omega} w_{i,j}^{h} \sigma_{i,j}^{h} \, d\Omega = \int_{\Omega} w_{i}^{h} b_{i,} \, d\Omega + \int_{\Gamma^{h}} w_{i}^{h} h_{i} d\Gamma$$

However, this can be converted to 1-D problem by considering radial plane strain condition and then employing finite element approximation.

Finite Element Approximation:

$$\Omega = \cup_{e} \Omega^{e} \text{ and } \Omega^{m} \cap \Omega^{n} = \emptyset$$

The semi discrete equation for the system, in general, can be written as:

$$M.\ddot{d}(t) + K.d(t) = f(t)$$

Note: Damping effects are not considered here.

The approximation to be used for the problem is,

At nth time step, the displacement, velocity and acceleration can be approximated as,

$$d(tn) \approx d_{n}$$

$$\dot{d}(tn) \approx v_{n}$$

$$\ddot{d}(tn) \approx a_{n}$$

Under Newmark scheme, predictor-solution-corrector flowchart is used to reach the solution, which can be explained as below:

Consider the below full discrete equation,

$$M \, a_{n+1} + K \, d_{n+1} = f_{n+1}$$

For the given initial conditions,

For example,

$$d_{o} = d(0) = u^{o}$$

$$v_{o} = \dot{d}(0) = \dot{u}^{o}$$

At time, *t = 0,*

$$a_{o} = M^{-1}(f_{o} - K.d_{o})$$

Predictor Phase,

$$\check{d}_1 = d_0 + \Delta t . v_o + \left(\frac{1}{2}\right) . (1 - 2\beta)\Delta t^2 . a_o$$

$$\check{v}_1 = v_o + (1 - \gamma)\Delta t . a_o$$

Solution Phase,

From the above two predictors,

$$a_1 = M^{*^{-1}} f_1^*$$

Where,

$$M^* = M + \beta \Delta t^2 K$$

$$f_1^* = f_1 - K . d_1$$

Corrector Phase,

$$d_1 = \check{d}_1 + \beta \Delta t^2 a_1$$

$$v_1 = \check{v}_1 + \gamma \Delta t a_1$$

Shape Functions for the approximations are,

$$N1(\xi) = \frac{1}{2}(1 - \xi)$$

$$N2(\xi) = \frac{1}{2}(1 + \xi)$$

Where, the value of ξ ranges from -1 to +1 in parametric domain. The radial direction is mapped in parametric domain as,

$$r(\xi) = N1(\xi) . r1 + N2(\xi) . r2$$

Similarly, radial displacement is approximated as,

$$u_r(\xi) = N1(\xi) . u_1 + N2(\xi) . u_2$$

Where,

r1 = coordinate of first node of an element in physical domain

r2 = coordinate of second node of an element in physical domain

u_1 = radial displacement at the first node of an element

u_2 = radial displacement at the second node of an element

Numerical Approaches

For the time integration, two methods used here are, Central Difference method and Average acceleration method. The Newmark parameters for the above two methods are shown below in table.

No.	Method	$\boldsymbol{\beta}$	$\boldsymbol{\gamma}$
1	Central Difference Method	0	0.5
2	Average Acceleration Method	0.25	0.5

In case of Central difference method, since β = 0, the modified mass matrix, $M^{*} = M$, (ignoring damping), hence, it will become explicit method, which will be simple and fast but needs an investigation for the stability. Average Acceleration (based on trapezoidal rule formulation) is an implicit method, which is always stable.

The stability of the time integration scheme employed is checked by solving generalized eigen value problem and finding the critical time step based on the maximum eigen value obtained.

Eigen value problem for the given semi-discrete equation can be formulated as,

$$(K - \lambda M)\phi = 0$$

where,

$$Eigen\ values, \lambda = \{\lambda_1, \lambda_2, \dots \dots \dots \dots., \lambda_n\}$$

$$and\ corresponding\ eigen\ vectors, \phi = \{\ \phi_1, \phi_2, \dots \dots \dots, \phi_n\}$$

Upon solving the eigen-value problem, the stability of the system can be determined and critical time step can be calculated as,

$$For\ central\ difference\ method, \Delta t_{cr} = 2/\lambda_{max}$$

$$For\ average\ acceleration\ method, \Delta t_{cr} = \sqrt{12}/\lambda_{max}$$

In the program, based on the parameters of central difference method and average acceleration method, the criteria is set in such a way that central difference method always uses lumped mass matrix and average acceleration method always uses consistent mass matrix, since this combination tends to decrease numerical period, $\bar{T}$.

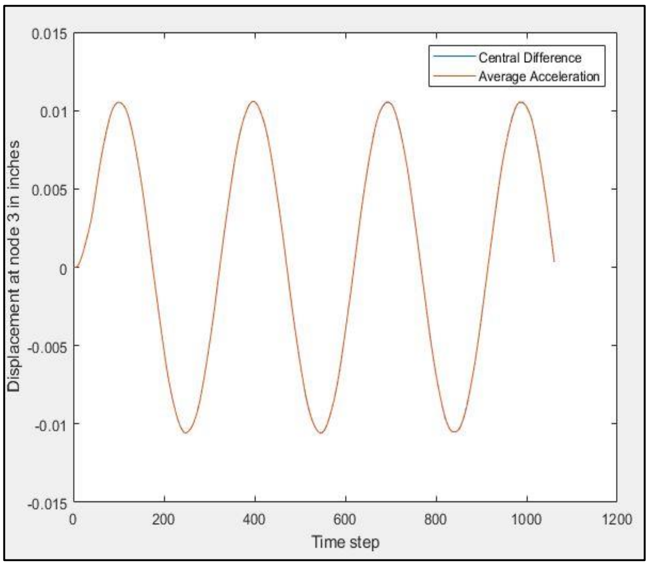
Numerical Results

1] The effect of element refinement on the accuracy

Number of elements = 10

Radial Displacement & Stresses:

Radial Displacement at node 3 and stresses at mid-point of element 3is plotted for time coefficient = 0.8



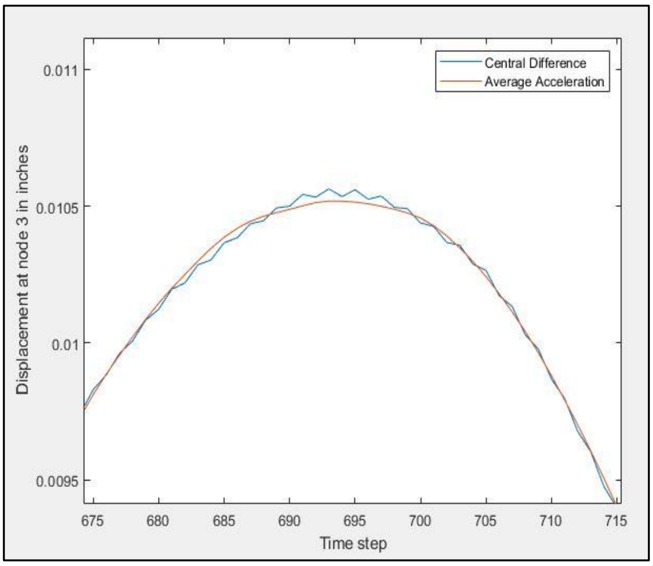
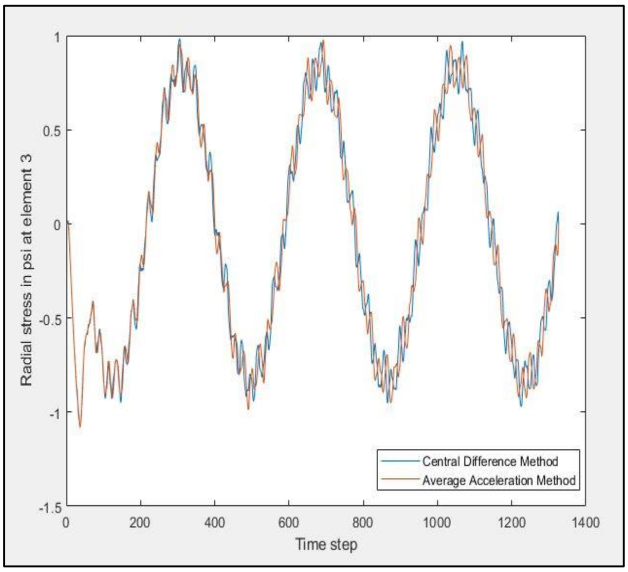


Figure 1-a- Radial Displacement Profile

Figure 1-b- Smoothening comparison of 2 methods



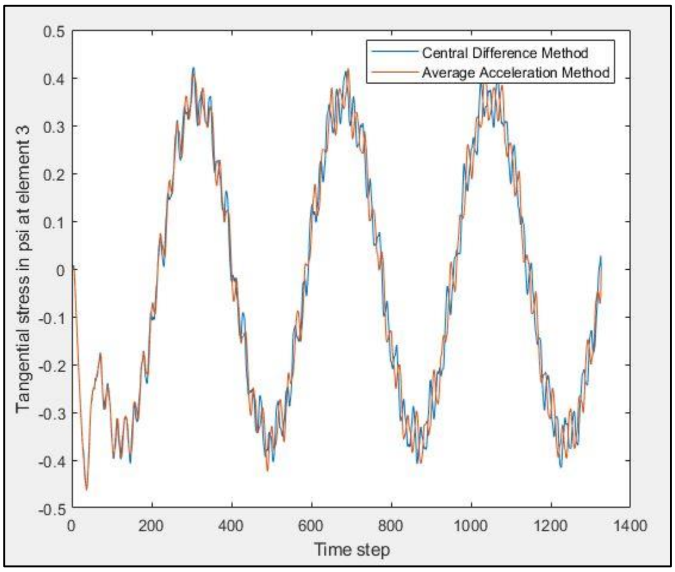


Figure 1-c- Radial Stress Profile

Figure 1-d- Tangential Stress Profile

Number of Elements = 20

Radial Displacement & Stresses:

Radial Displacement at node 3 and stresses at mid-point of element 3 with time variation coefficient = 0.8

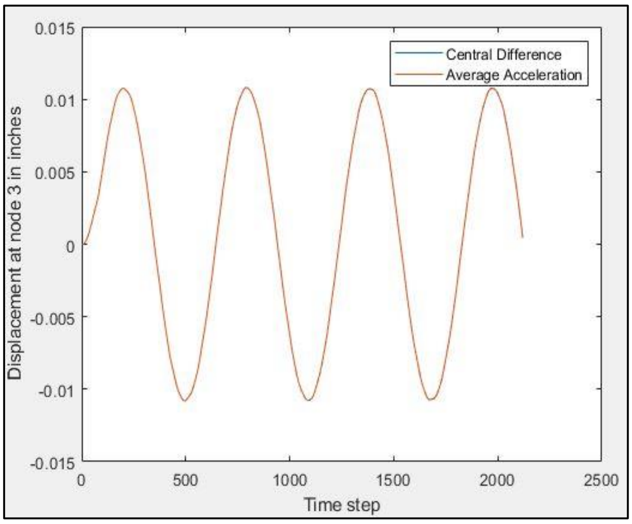


Figure 2-a- Radial Displacement Profile

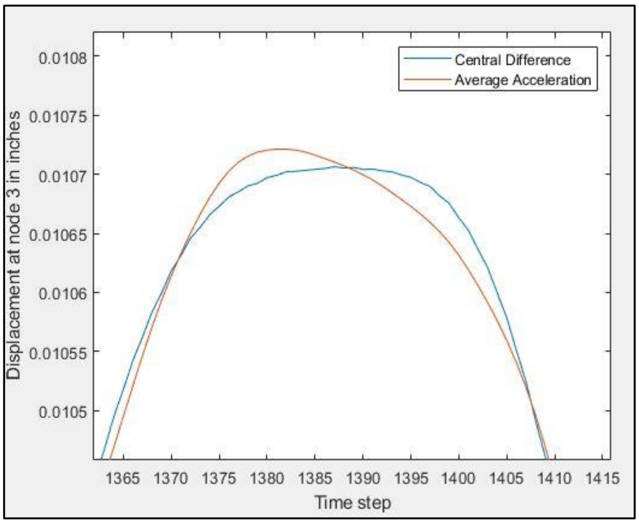


Figure 2-b- Smoothening comparison of 2 methods

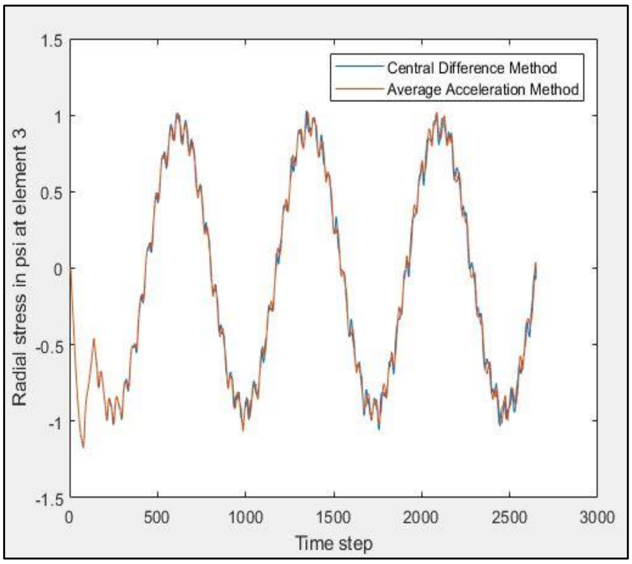


Figure 2-c- Radial Stress Profile

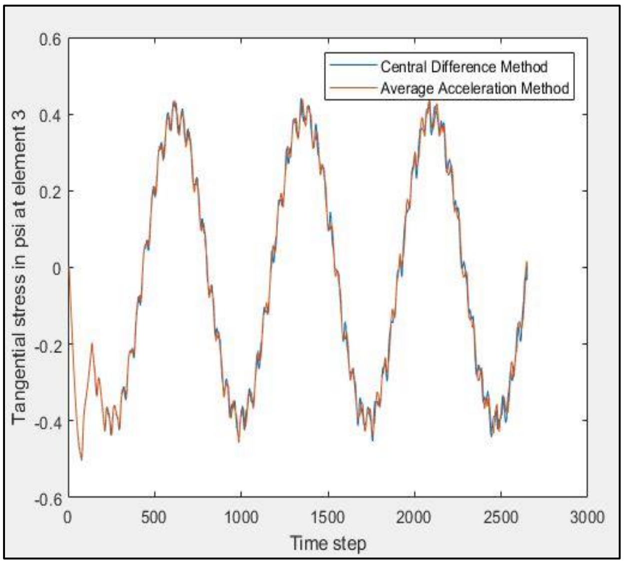
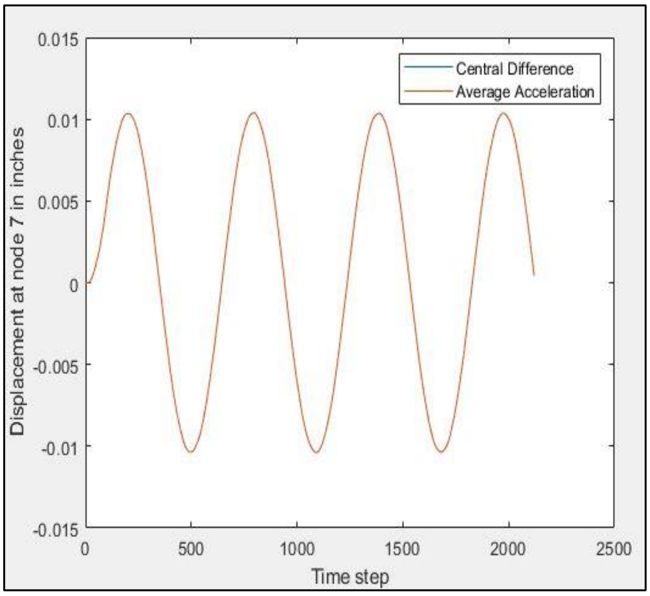


Figure 2-d- Tangential Stress Profile

Radial Displacement at node 7 and stress at the mid-point of element 6.



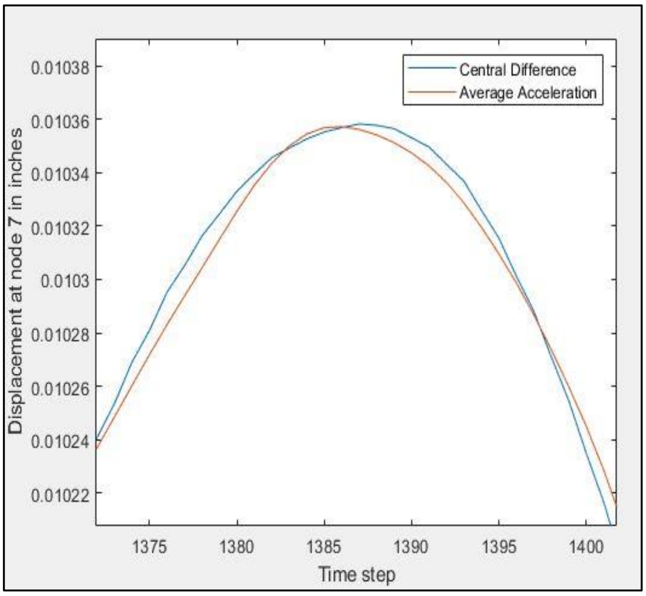
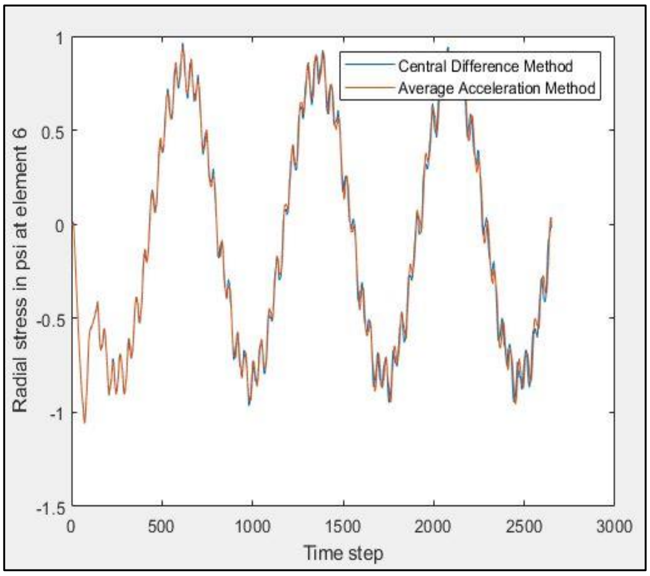


Figure 2-e- Radial Displacement Profile

Figure 2-f- Smoothening comparison of 2 methods



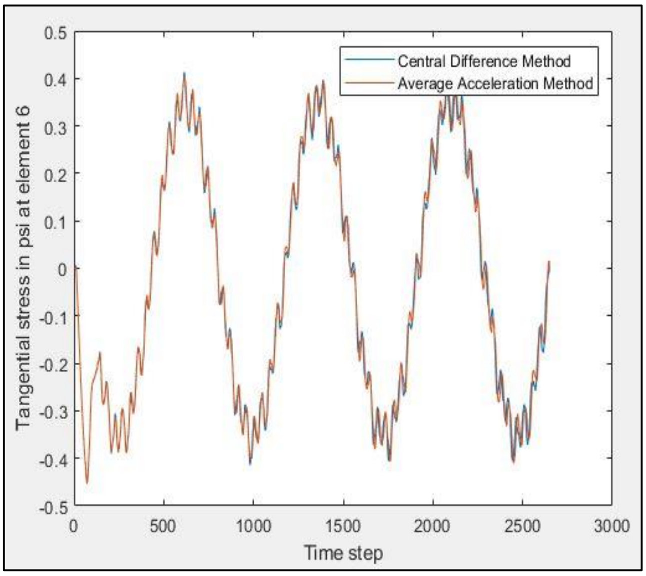


Figure 2-g- Radial Stress Profile

Figure 2-h- Tangential Stress Profile

Number of Elements = 40

Radial Displacement & Stresses:

Radial Displacement at node 3 and stresses at mid-point of element 3 with time variation coefficient = 0.8

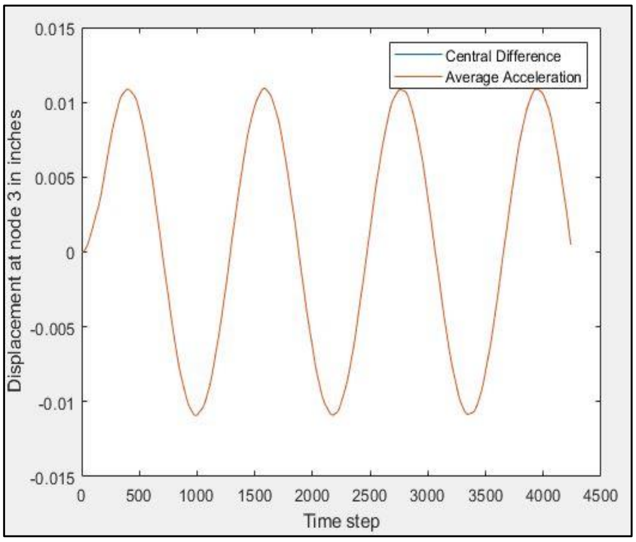


Figure 3-a- Radial Displacement Profile

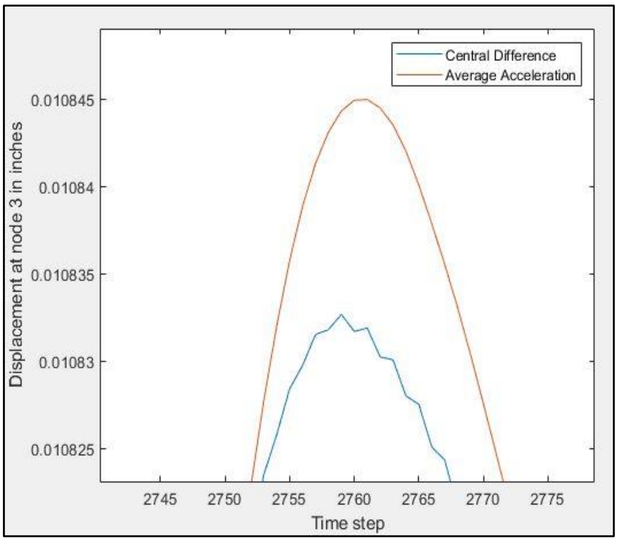


Figure 3-b- Smoothening comparison of 2 methods

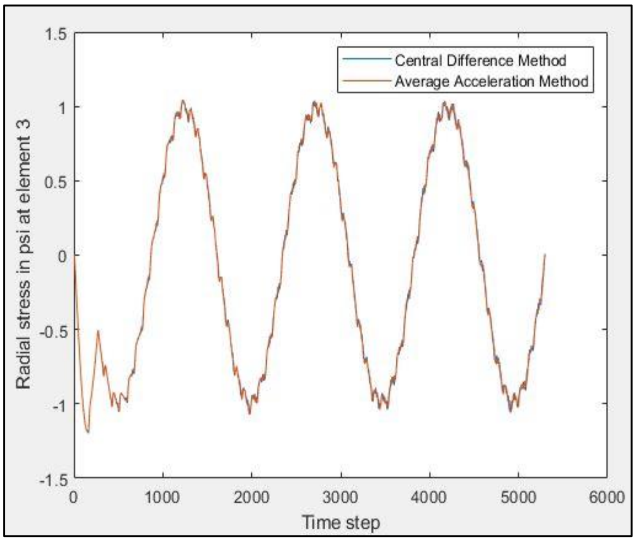


Figure 3-c- Radial Stress Profile

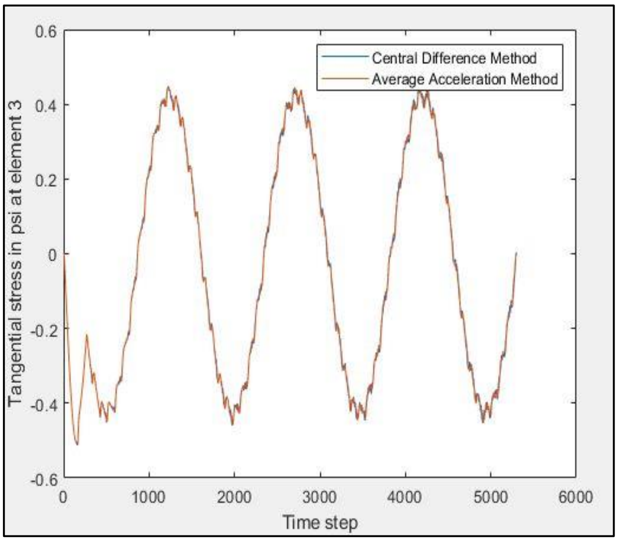


Figure 3-d- Tangential Stress Profile

Radial Displacement at node 7 and stress at the mid-point of element 6.

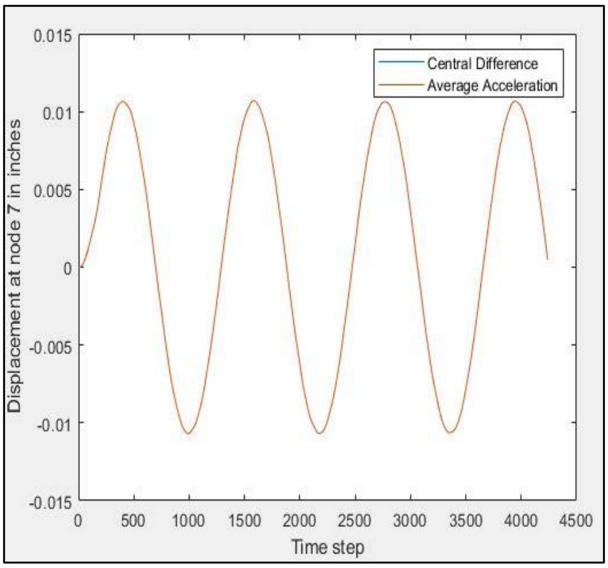


Figure 3-e- Radial Displacement Profile

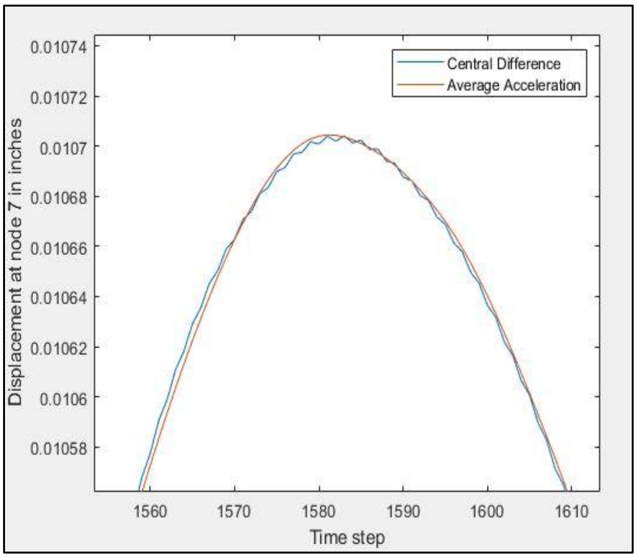


Figure 3-f- Smoothening comparison of 2 methods

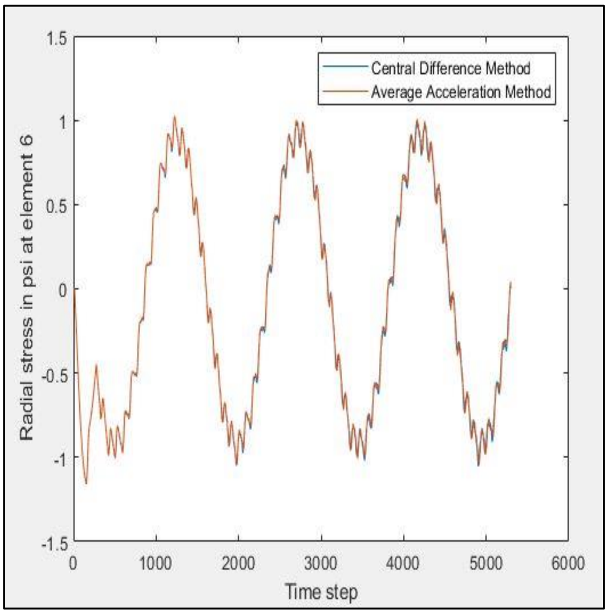


Figure 3-g- Radial Stress Profile

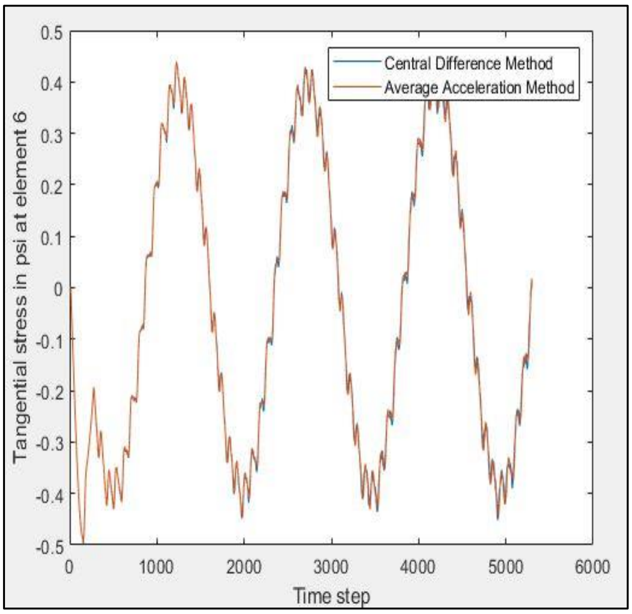
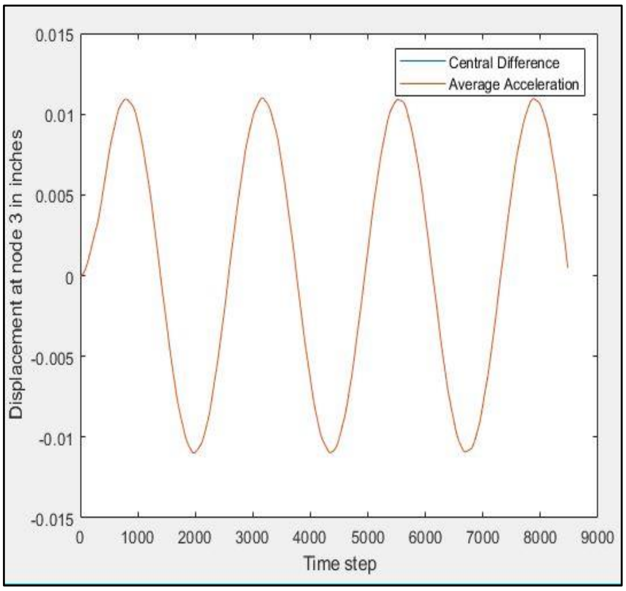


Figure 3-h- Tangential Stress Profile

Number of Elements = 80

Radial Displacement & Stresses:

Radial Displacement at node 3 and stresses at mid-point of element 3 with time variation coefficient = 0.8



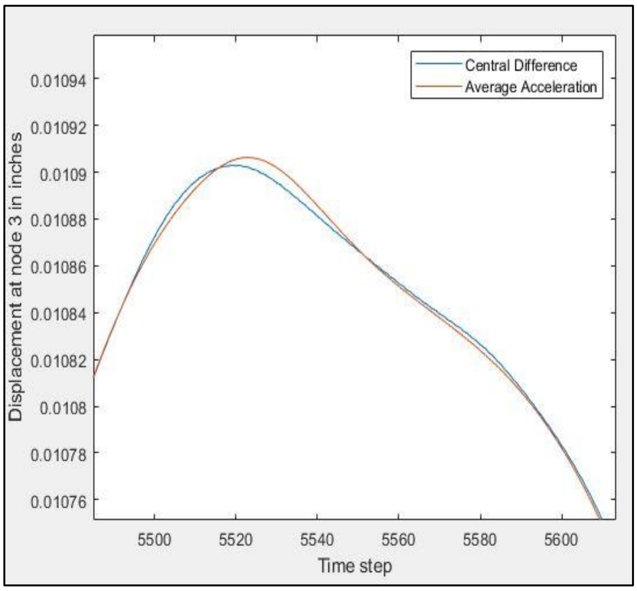
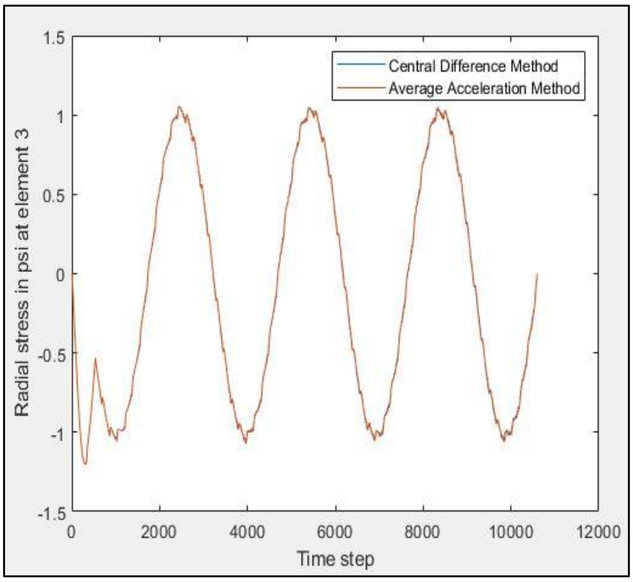


Figure 4-a- Radial Displacement Profile

Figure 4-b- Smoothening comparison of 2 methods



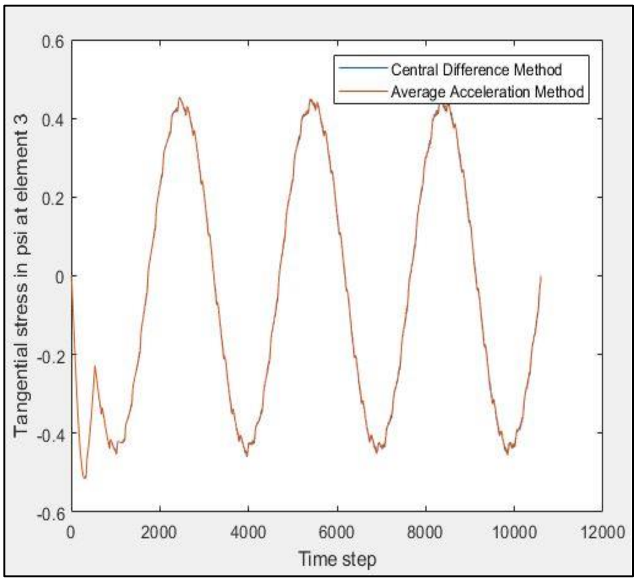
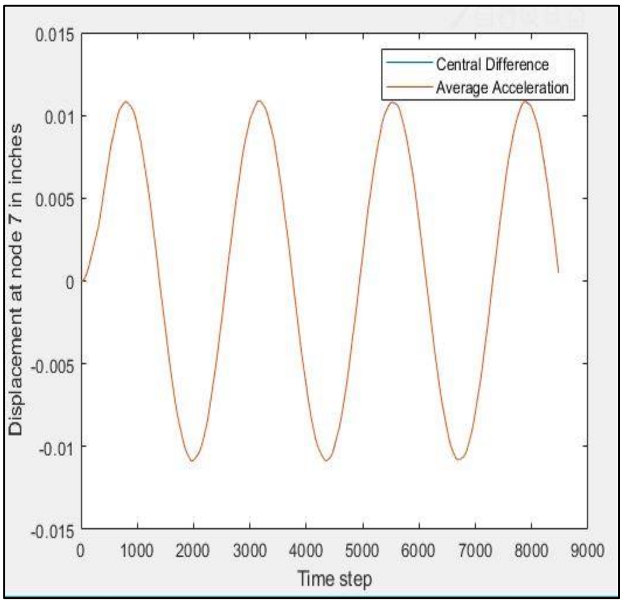


Figure 4-c- Radial Stress Profile

Figure 4-d- Tangential Stress Profile

Radial Displacement at node 7 and stresses at mid-point of element 6.



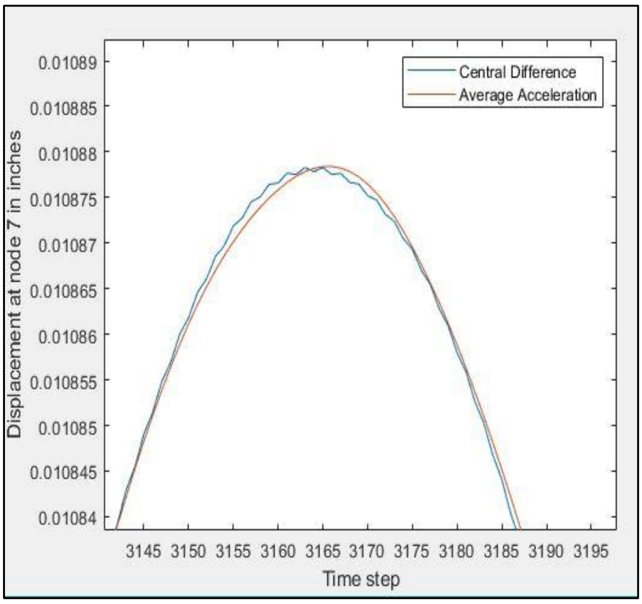
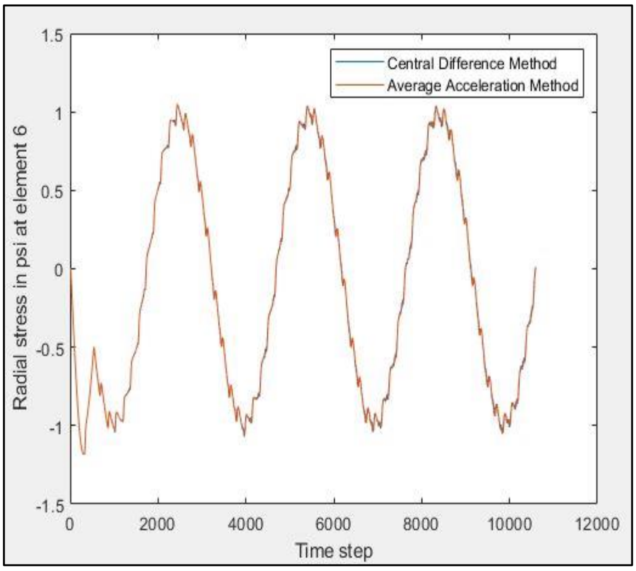


Figure 4-e- Radial Displacement Profile

Figure 4-f- Smoothening comparison of 2 methods



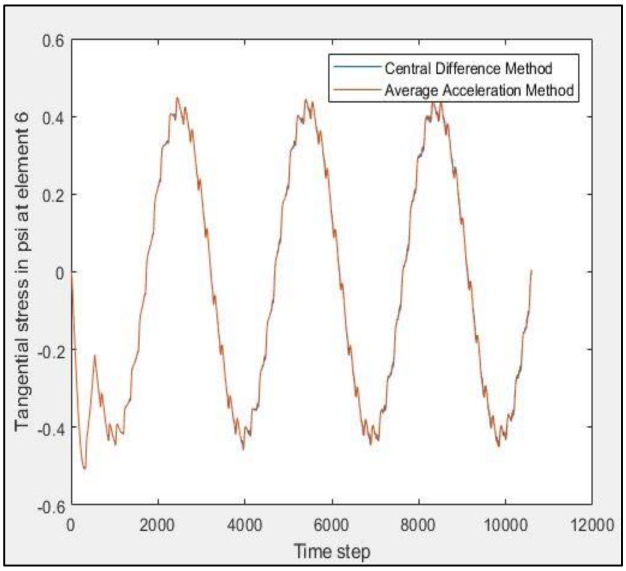


Figure 4-g- Radial Stress Profile

Figure 4-h- Tangential Stress Profile

Observations:

From the above comparative study, it is evident that, with mesh refinement, both methods seem to perform normal in terms of radial displacement. However, mesh refinement improves convergence for stress values over a time both for central difference and average acceleration method.

2] The effect of time step size on the stability

Time step size is an important criterion that indirectly affects the limit of mesh refinement. Here, in this study, time step size is controlled by time variation coefficient that are set to be 0.8, 1.0 and 1.2.

For various elements, this study is conducted and concluded with below plots.

Number of Elements = 20, Time variation coefficient = 1.0

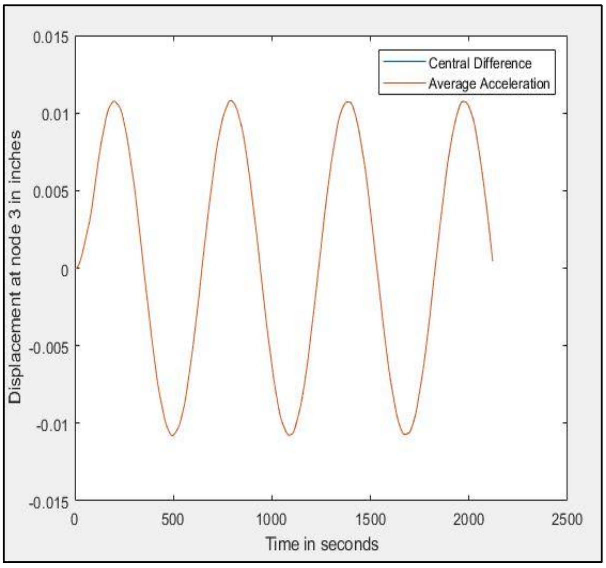


Figure 5-a- Radial Displacement Profile

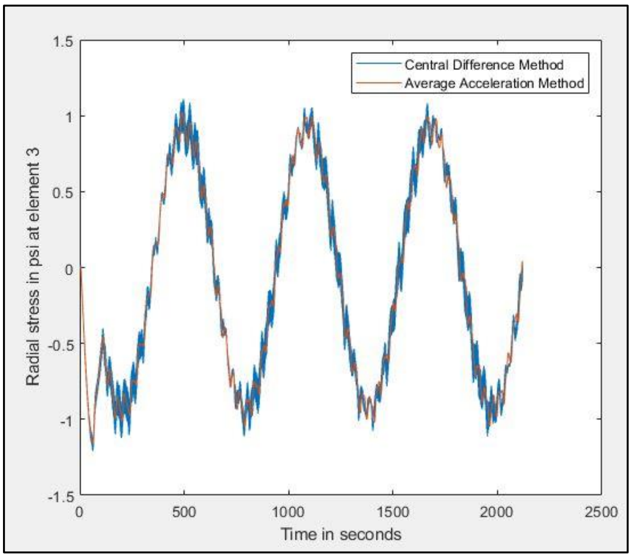


Figure 5-b- Radial Stress Profile

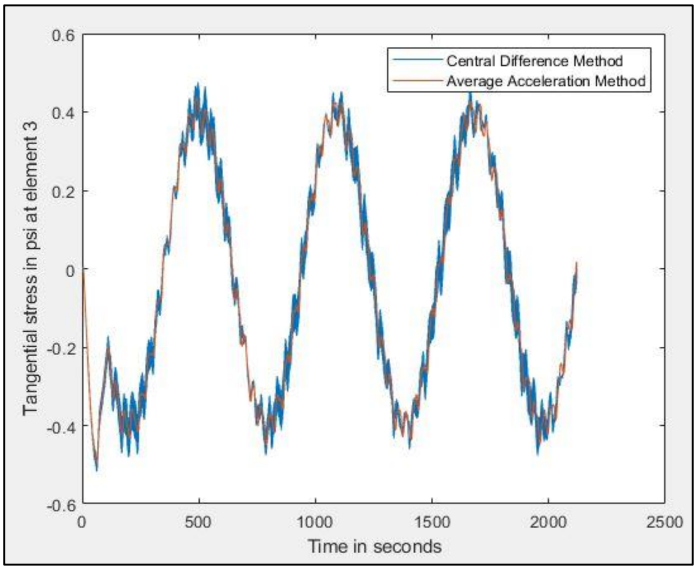
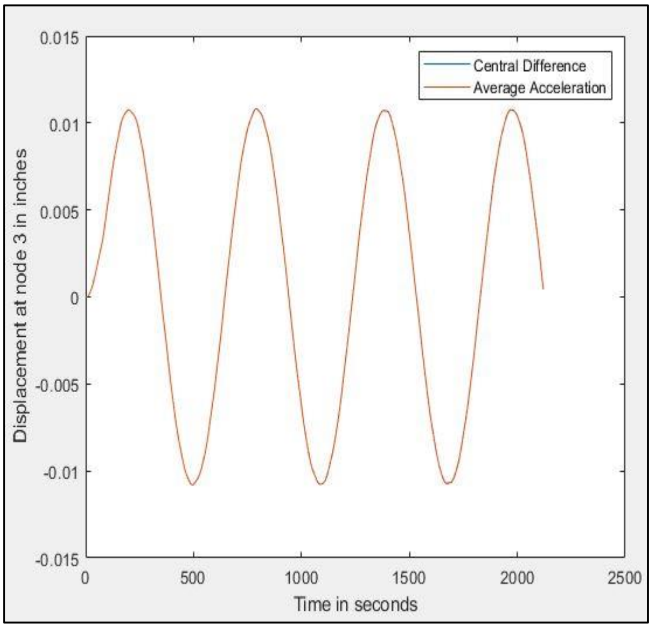


Figure 5-c- Tangential Stress Profile

Note: By mistake, on x-axis, time in seconds is written as title instead of time step. Kindly take notice of it.

Number of Elements = 20, Time step variation coefficient = 1.2



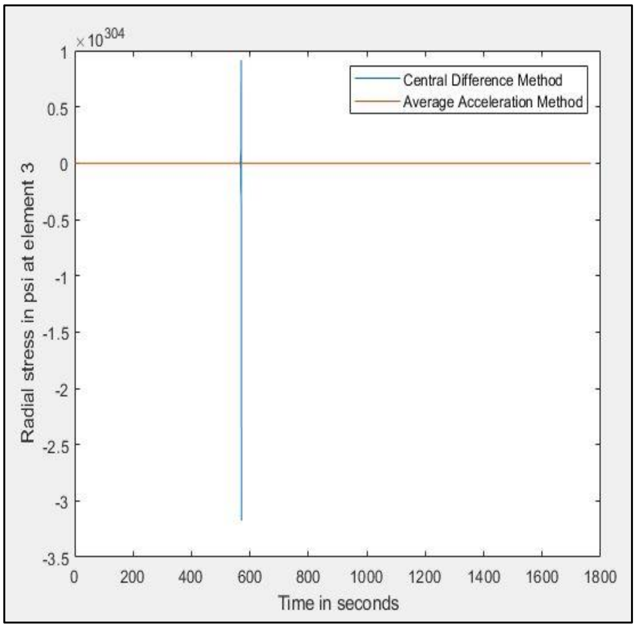


Figure 6-a- Radial Displacement Profile

Figure 6-b- Radial Stress Profile

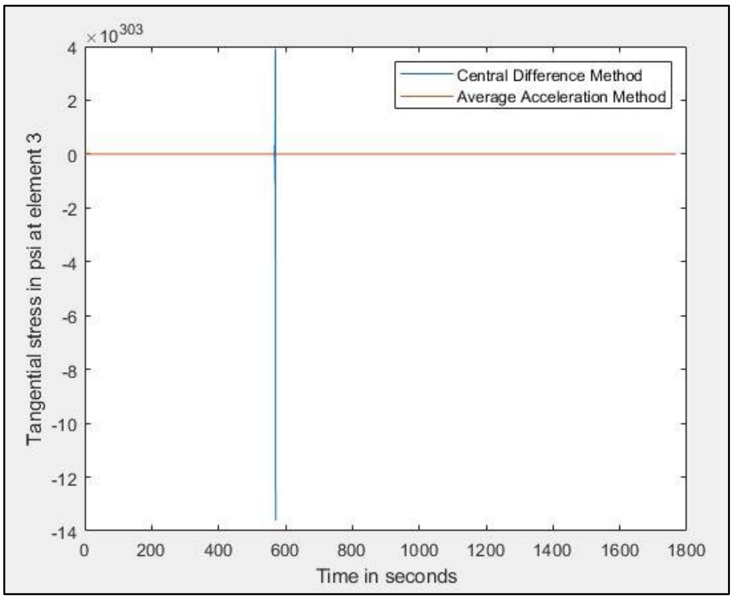
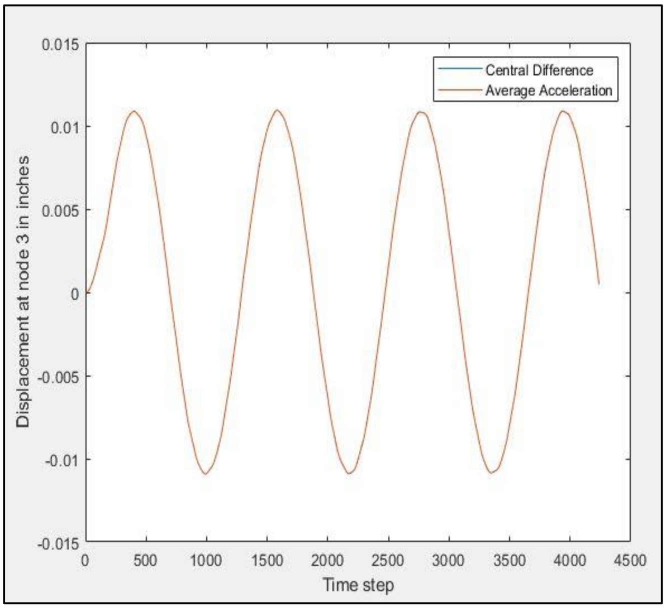


Figure 6-c- Tangential Stress Profile

The above two plots of Radial stress and Tangential Stress profile clearly shows that at coefficient 1.2, the central difference method becomes unconditionally unstable, whereas, average acceleration method behaves normally. To see, the plot of Radial Stress and Tangential Stress with average acceleration method, refer to the folder ele20coeff1-2.

Note: By mistake, on x-axis, time in seconds is written as title instead of time step. Kindly take notice of it.

Number of elements = 40, Time variation coefficient = 1.0



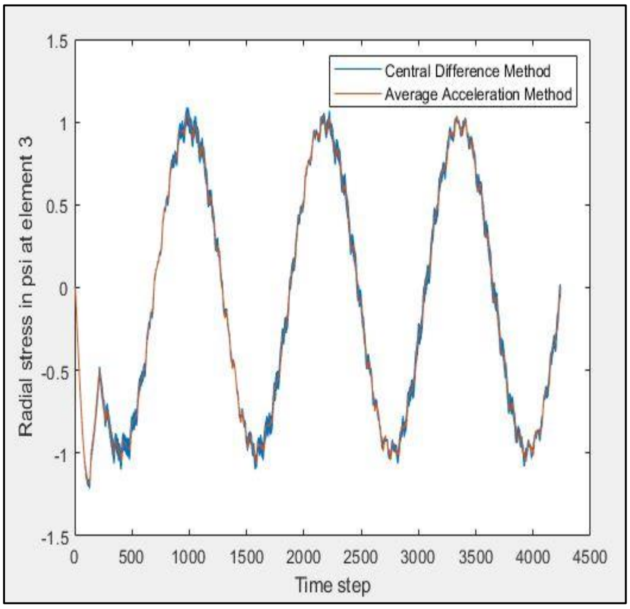


Figure 7-a- Radial Displacement Profile

Figure 7-b- Radial Stress Profile

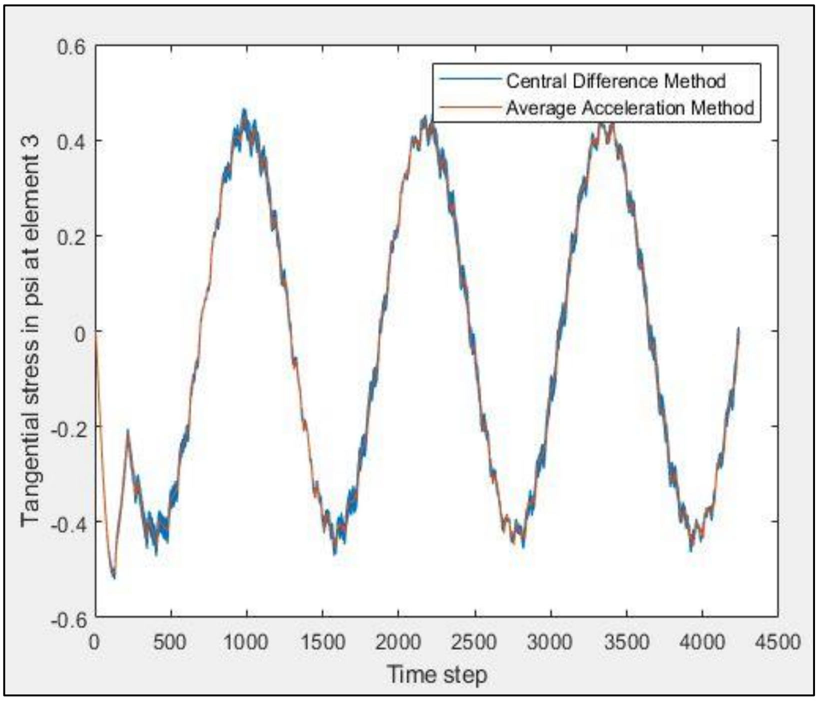
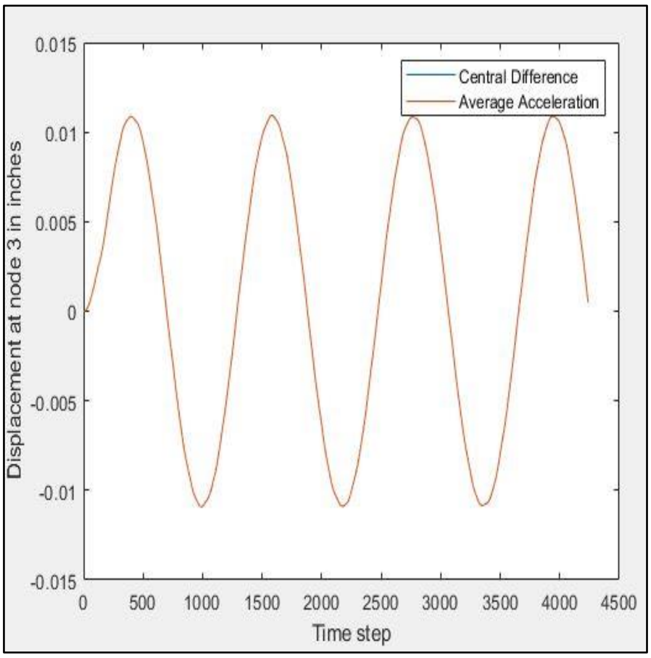


Figure 7-c- Tangential Stress Profile

Number of elements = 40, Time variation coefficient = 1.2



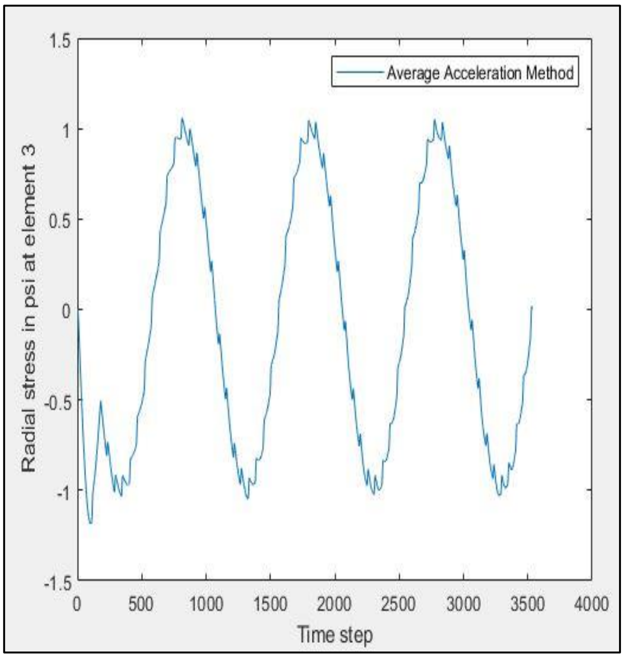


Figure 8-a- Radial Displacement Profile

Figure 8-b- Radial Stress Profile

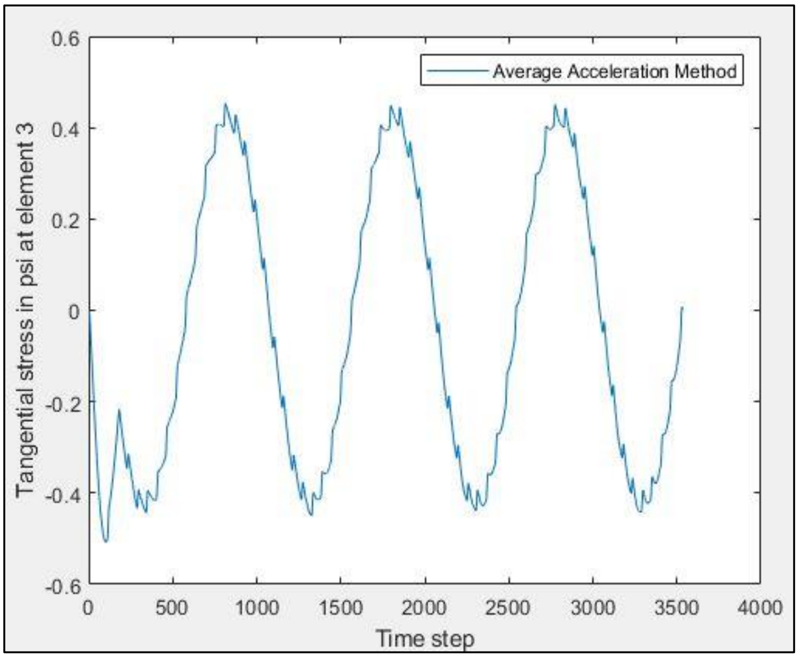
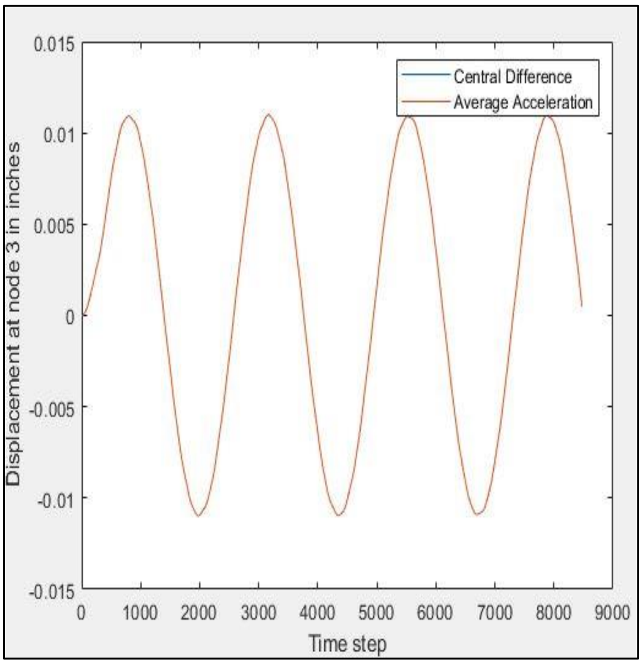


Figure 8-c- Tangential Stress Profile

Note: As shown in Figure 7-b & 7-c, the radial and tangential stress values for central difference method phased out, for all other elements, radial stress and tangential stress are plotted for average acceleration method only.

Number of Elements = 80, Time variation coefficient = 1.0



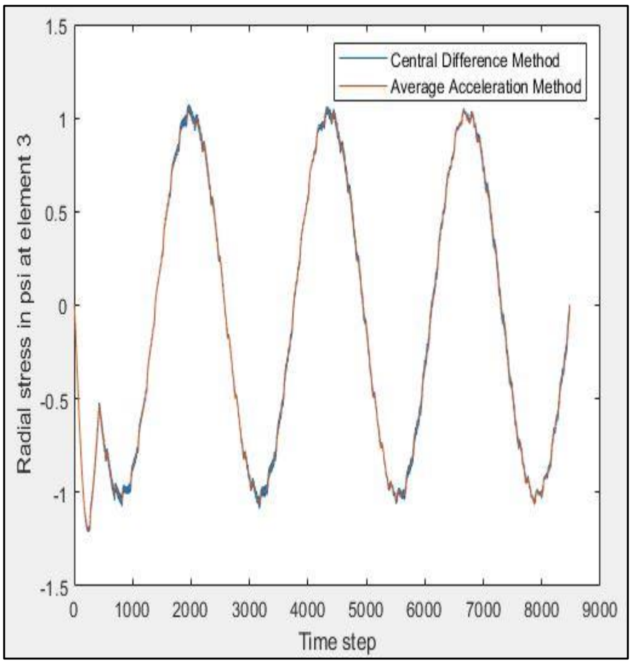


Figure 9-a- Radial Displacement Profile

Figure 9-b- Radial Stress Profile

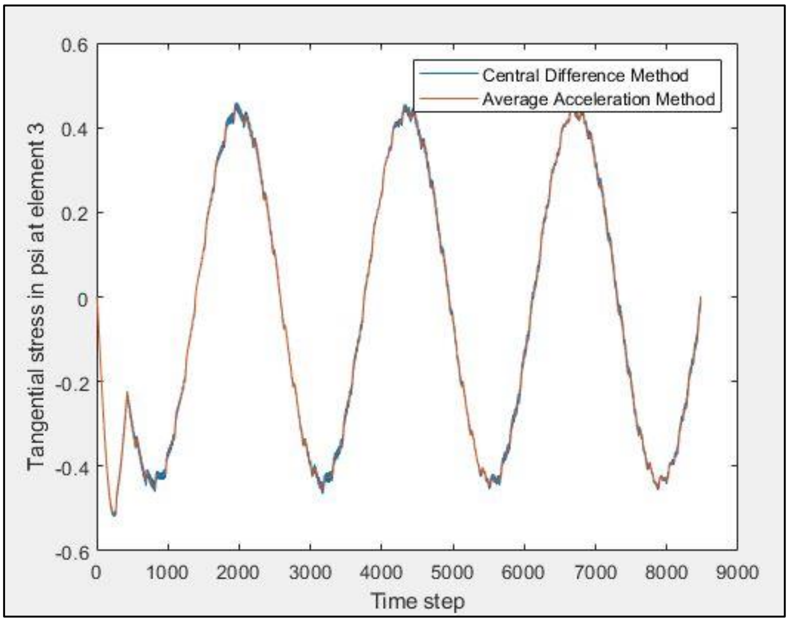
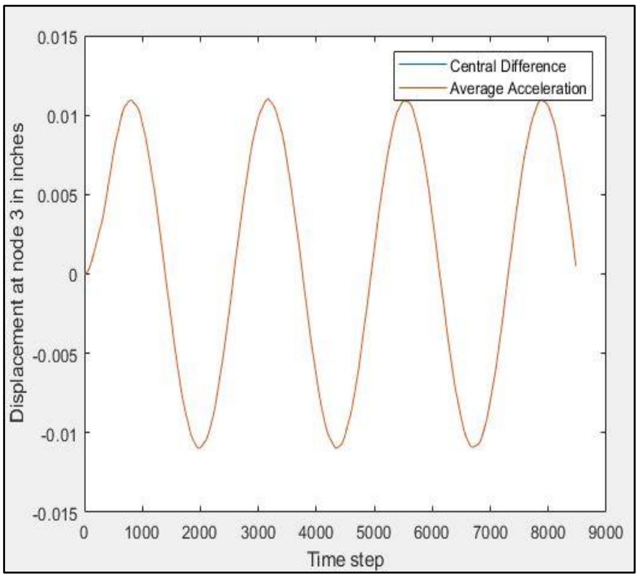


Figure 9-c- Tangential Stress Profile

Number of Elements = 80, Time variation coefficient = 1.2



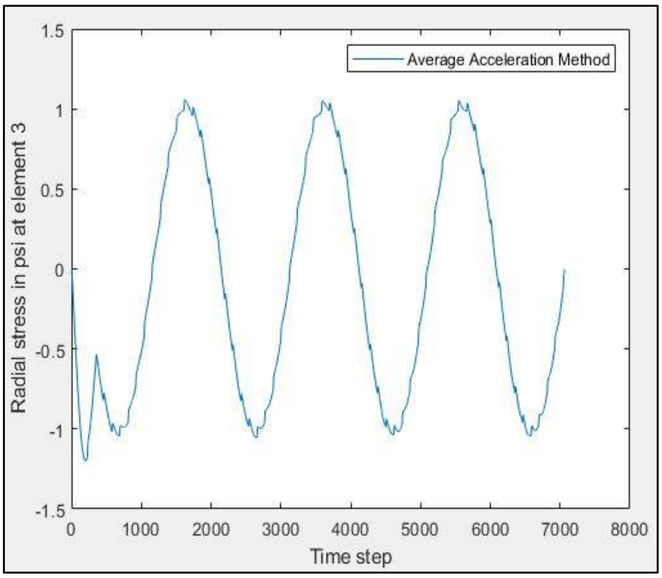


Figure 10-a- Radial Displacement Profile

Figure 10-b- Radial Stress Profile

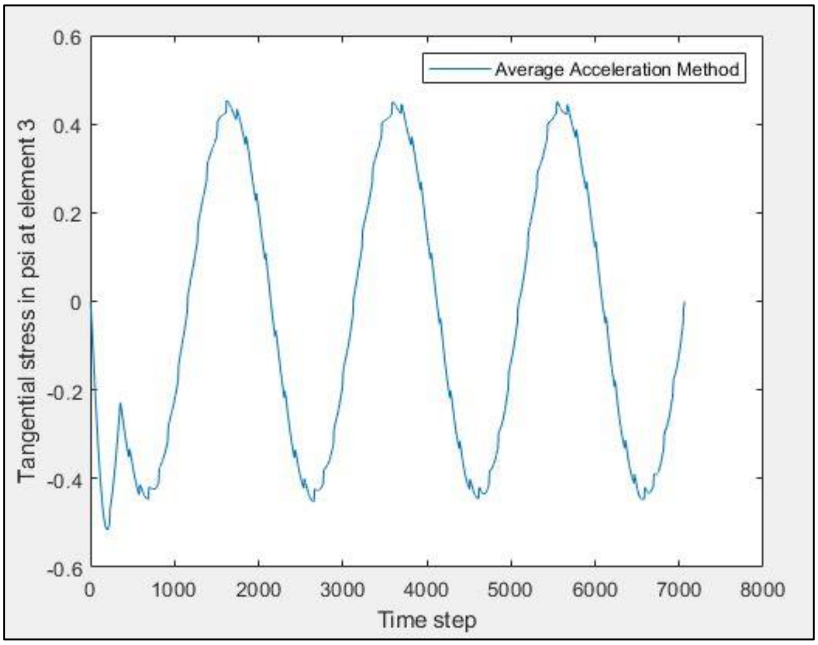


Figure 10-c- Tangential Stress Profile

Observations:

With increase in time step coefficient, that governs the size of time step, the central difference method seems to become unstable as compared to average acceleration method, which is particularly evident from the plots of radial and tangential stresses.

3] Comparison of Central Difference Method and Average Acceleration Method

Difference between the above two methods is less evident for radial displacement, however at very small time steps, the performance of Average Acceleration Method seems to be very smooth compared to central difference method for a given time step variation coefficient. As seen, in the previous plot, with increase in time step variation coefficient, the average acceleration method performs normally, however, central difference method gives phased out values of stresses which shows unstable performance.

With mesh refinement, for a given time step, it is observed that, central difference method approaches to average acceleration method (slight smoothening in the plot). Some of the plots to see the behavior of both methods for radial displacement are presented below.

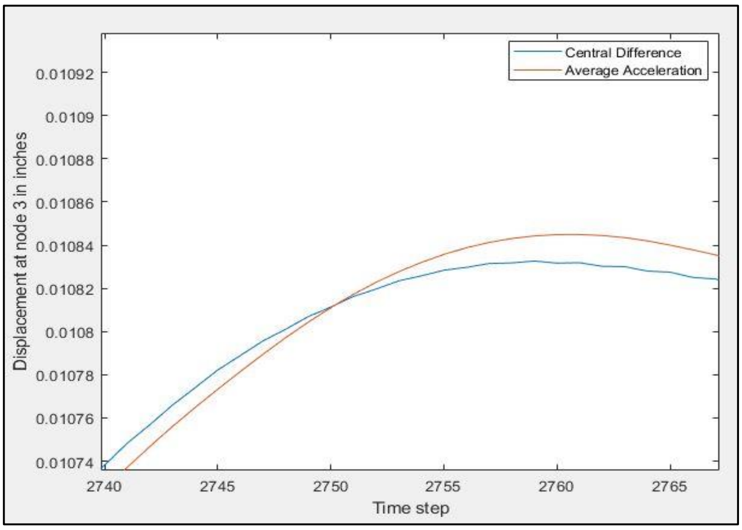


Figure 11 Radial Displacement Comparison for 40 element mesh and time variation coefficient of 1.0

From Figure 11, it is evident that Average Acceleration method has smooth response compared to central difference method for radial displacement calculation.

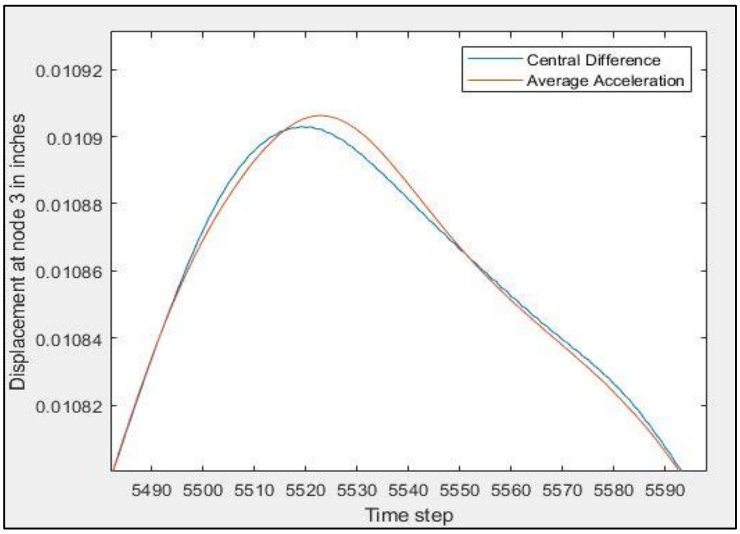


Figure 12 Radial Displacement Comparison for 80 element mesh and time variation coefficient of 1.0

From Figure 12, it is evident that Average acceleration method has smooth response, however, compared to Figure 11 with mesh size of 40, central difference method performs better and it approaches the smooth behavior.

Conclusion

1] Observations on Accuracy

Radial displacement, radial stresses and tangential stresses are plotted at locations 6.30 inches and 6.90 inches (node-3 and node -7 respectively) from 0s to 0.05s for a critical time step, that is, ct*$\Delta t_{critical}$, where ct = 1.0.

By keeping ct = 0.8, it is found that, there's very less effect of mesh refinement from 10 elements to 80 elements on radial displacement response. However, at very discrete time step, smoothness of average acceleration method is evident. Besides, looking at the plots of radial stress and tangential stress, it is clear that average acceleration performs better. The initial kinks in the plots of stresses is however less predicted. However, the roughness of the stress plot can be explained by considering the fact that, the consideration for stress plot is based on the middle point of each element, which is less accurate than the nodal solution. While framing the problem, through weak form, the nodal exactness for displacement was made essential and hence, the displacement plots have better accuracy compared to stress plots.

With mesh refinement, it is observed that, the solution from central difference method reduces its roughness and attain smooth behavior as that of average acceleration method.

2] Observations on Stability

To understand the stability of two-time integration schemes, radial displacement and stresses are plotted at 6.30 inches and 6.90 inches for critical time step with coefficient 0.8, 1.0 and 1.2 for time 0 s to 0.05s. From the plots of radial displacement, both methods, that are, central difference and average acceleration, seem to give more or less same behavior for different time step. However, a comparison of stability become crucial for radial and tangential stresses at critical time step 1.2*$\Delta t_{critical}$, when the central difference method gives phased out results for stresses, which simply means that it become unstable for that critical time step condition as it is higher than the theoretical critical time step for the method.

Effect of mesh refinement along with different critical time step is also evident from the plots of stresses since the behavior of plot starts getting changed from very rough to slightly smooth. However, the overall behavior for both displacement and stresses continue to be periodic in nature given that, the external pressure is a function of sine.

From the above two broad outlines on accuracy and stability, it seems that most of the results are similar to predicted results. The initial kinks in the plots of stress needs further investigation.

Contents


```
%Matrix of Elastic Constants
E = 1000; %psi
v = 0.3;  %poisson's ratio
rho=0.01; %density of the material
D = (1/((1+v)*(1-2*v)))*[E*(1-v), v*E; v*E, E*(1-v)]; %Matrix of Elastic Constants

% 4 pt gaussian integration data
% weight = [0.652145154862546,0.652145154862546,0.347854845137453,0.347854845137453];
% intpt = [-0.339981043584856,0.339981043584856,-0.861136311594052,0.861136311594052];
```

Time based Parameters

```
t = 0.05;
t_end = 0.5; %end of time loop
% parameters for variation in delta-t
ct1 = 0.8;
ct2 = 1;
ct3 = 1.2;
```

Newmark Scheme Parameters

```
%parameters for central difference method
a1_beta = 0; a1_gamma = 0.5;
%parameters for average acceleration method
a2_beta = 0.25; a2_gamma = 0.5;
```

Mesh Generation

```
Ri = 6; %inch
Ro = 9; %inch
n = 20;
nn = n+1;
np = linspace(Ri, Ro, n+1);
le = (Ro - Ri)/n; %length of each element
x1 = zeros(1,n); %coordinates of first node for each element
x2 = zeros(1,n); %coordinates of seconde node for each element
for i = 1:n
    x1(1,i) = x1(1,i) + 6 + (i-1)*le;
    x2(1,i) = x2(1,i) + (6+le) + (i-1)*le;
end
```

```
element_coord = [x1' x2'];
```

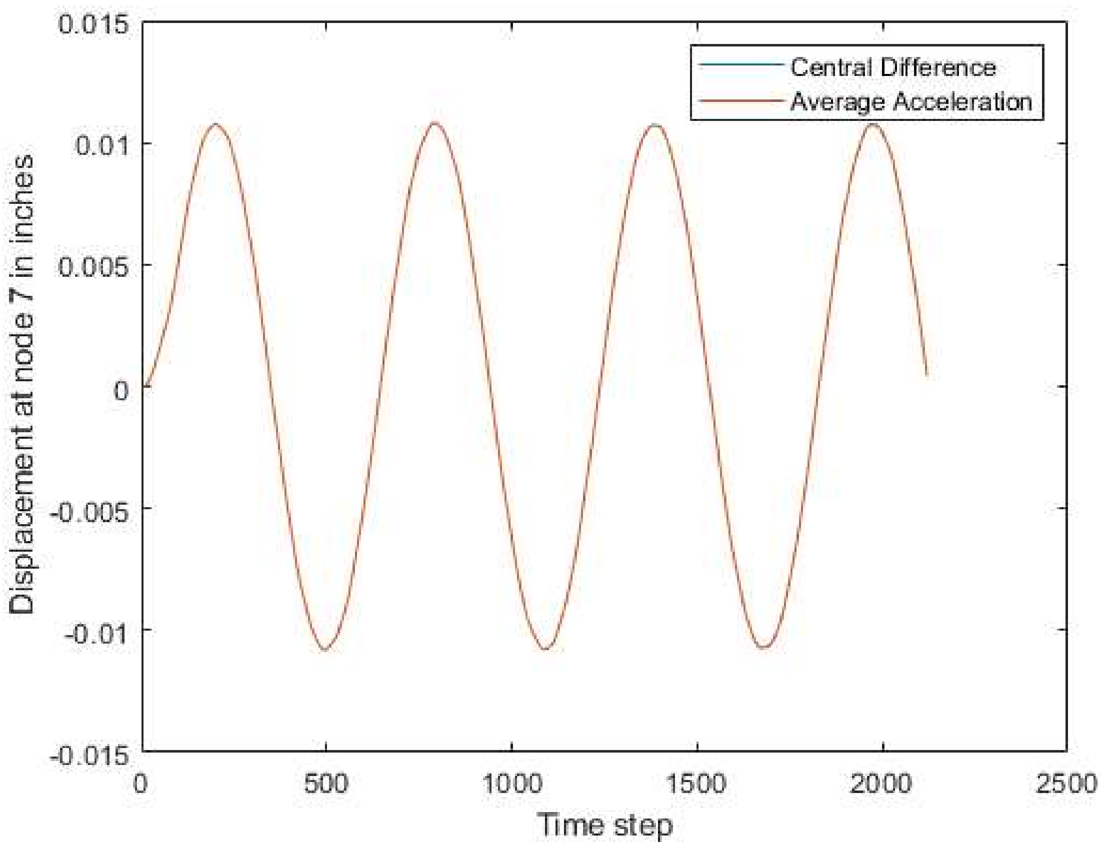
Stiffness Matrix, Force Vector, Mass Matrix generation

```
K_global = zeros(n+1,n+1);
F_global = zeros(n+1,1);
M_global = zeros(n+1, n+1);
eigen_global = zeros(n,1);
beta = [0 0.25];
gamma = [0.5 0.5];
for i = 1:n
    syms eps;
    N1 = 0.5*(1-eps);
    N2 = 0.5*(1+eps);
    r = N1*x1(1,i) + N2*x2(1,i);
    B = [-1/le  1/le; N1/r N2/r];
    L = @(eps) B'*D*B*r*(le/2);
    K_local = 2*pi*vpaintegral(L,eps,[-1 1]);
    M_local_1 = @(eps)[N1, N2]'*[N1 N2].*r.*((x2(1,i)-x1(1,i))/2);
    cons_mass_mat = 2*pi*rho*vpaintegral(M_local_1,eps,[-1 1]);
    for j = 1:2
        if abs(beta(1,j) - 0.25) <= 10^-10
            Mass_Matrix = cons_mass_mat;
        else
            M = diag(cons_mass_mat);
            M(1) = M(1) + cons_mass_mat(1,2);
            M(2) = M(2) + cons_mass_mat(2,1);
            Mass_Matrix = diag(M);
        end
    end
    M_local = Mass_Matrix;
    lamda = eig(M_local\K_local);
    max_lamda = max(lamda);
    eigen_global(i,1) = max_lamda;
    id1 = i; id2 = i+1;
    K_global(id1:id2, id1:id2) = K_global(id1:id2,id1:id2) + K_local;
    M_global(id1:id2, id1:id2) = M_global(id1:id2,id1:id2) + M_local;
end
```

critical time step & evaluation of displacement, velocity and acceleration

```
big_lamda = max(eigen_global(:,1));

% for the given elements
% central difference calculation
[disp_t_n, vel_t_n, ac_t_n, dt_1,nt_1] = Eqsolver(nn,a1_beta,a1_gamma,K_global, F_global, M_g
lobal,big_lamda,Ri,t,t_end,ct1);
[disp_t_n_1, vel_t_n_1, ac_t_n_1, dt_2,nt_2] = Eqsolver(nn,a1_beta,a1_gamma,K_global, F_globa
l, M_global,big_lamda,Ri,t,t_end,ct2);
[disp_t_n_2, vel_t_n_2, ac_t_n_2, dt_3, nt_3] = Eqsolver(nn,a1_beta,a1_gamma,K_global, F_glob
al, M_global,big_lamda,Ri,t,t_end,ct3);

% average acceleration calculation
[disp_t_n_aa, vel_t_n_aa, ac_t_n_aa, dt_aa_1,nt_aa_1] = Eqsolver(nn,a2_beta,a2_gamma,K_global
```

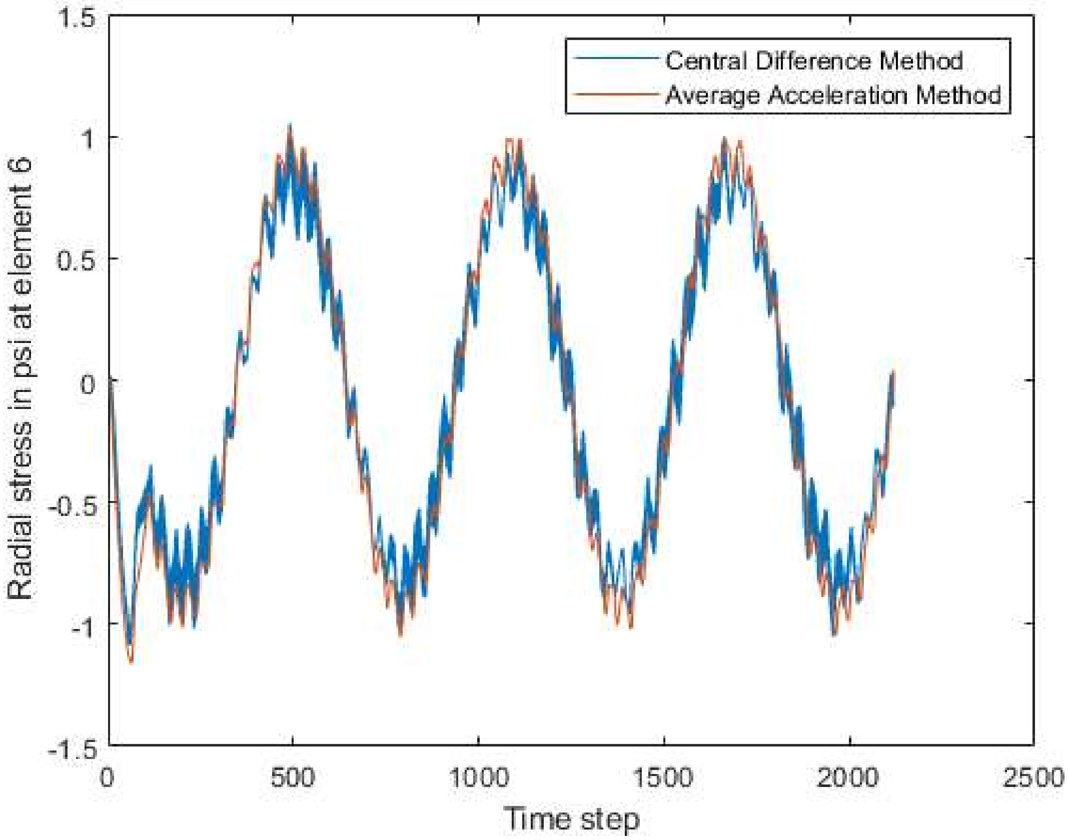
```
, F_global, M_global,big_lamda,Ri,t,t_end,ct1);
[disp_t_n_aa_1, vel_t_n_aa_1, ac_t_n_aa_1, dt_aa_2,nt_aa_2] = Eqsolver(nn,a2_beta,a2_gamma,K_
global, F_global, M_global,big_lamda,Ri,t,t_end,ct2);
[disp_t_n_aa_2, vel_t_n_aa_2, ac_t_n_aa_2, dt_aa_3, nt_aa_3] = Eqsolver(nn,a2_beta,a2_gamma,K
_global, F_global, M_global,big_lamda,Ri,t,t_end,ct3);

figure(1)
plot(disp_t_n_1(3,:))
hold on
plot(disp_t_n_aa_1(3,:))
xlabel('Time step')
ylabel('Displacement at node 7 in inches')
legend('Central Difference', 'Average Acceleration')
hold off
```



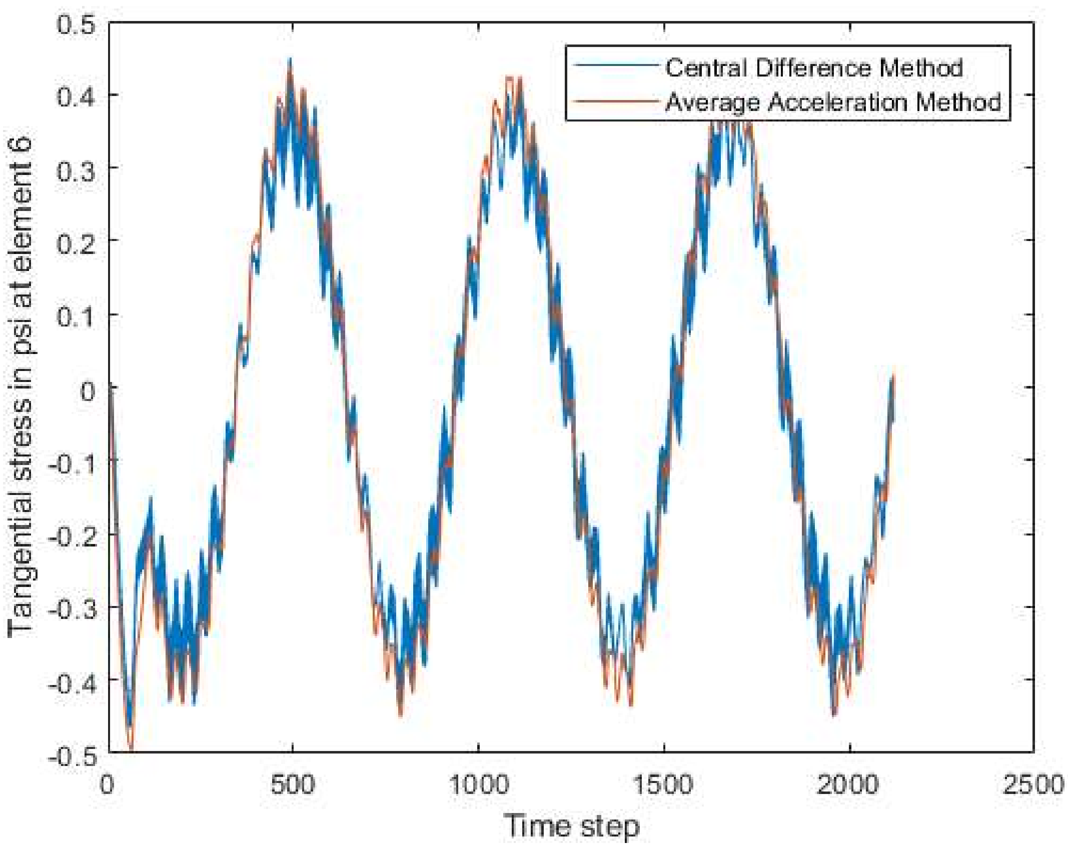
Radial and Tangential Stress Calculation

```
Be = [-1/le 1/le];
du_dr_cd = zeros(n,nt_2+1);
du_dt_cd = zeros(n,nt_2+1);
sigma_rr_cd = zeros(n,nt_2+1);
sigma_tt_cd = zeros(n,nt_2+1);
r1_cd = zeros(n,nt_2+1);
u_middle_cd = zeros(n,nt_2+1);

for i = 1: nt_2
    for j = 1:n
        u_e_cd = disp_t_n_1(j:j+1,i);
```

```
        du_dr_cd(j,i) = Be*u_e_cd;
        u_middle_cd(j,i) = 0.5*(disp_t_n_1(j) + disp_t_n_1(j+1));
        r1_cd(j,i) = x1(1,j)+ u_middle_cd(j,i) + le/2;
        du_dt_cd(j,i) = u_middle_cd(j,i)/r1_cd(j,i);
        sigma_rr_cd(j,i) = (E/((1+v)*(1-2*v)))*((1-v)*du_dr_cd(j,i) + v*du_dt_cd(j,i));
        sigma_tt_cd(j,i) = (E/((1+v)*(1-2*v)))*(v*du_dr_cd(j,i) + (1-v)*du_dt_cd(j,i));
    end
end


du_dr_aa = zeros(n,nt_aa_2+1);
du_dt_aa = zeros(n,nt_aa_2+1);
sigma_rr_aa = zeros(n,nt_aa_2+1);
sigma_tt_aa = zeros(n,nt_aa_2+1);
r1_aa = zeros(n,nt_aa_2+1);
u_middle_aa = zeros(n,nt_aa_2+1);

for i = 1: nt_aa_2
    for j = 1:n
        u_e_aa = disp_t_n_aa_1(j:j+1,i);
        du_dr_aa(j,i) = Be*u_e_aa;
        u_middle_aa(j,i) = 0.5*(disp_t_n_aa_1(j) + disp_t_n_aa_1(j+1));
        r1_aa(j,i) = x1(1,j)+ u_middle_aa(j,i) + le/2;
        du_dt_aa(j,i) = u_middle_aa(j,i)/r1_aa(j,i);
        sigma_rr_aa(j,i) = (E/((1+v)*(1-2*v)))*((1-v)*du_dr_aa(j,i) + v*du_dt_aa(j,i));
        sigma_tt_aa(j,i) = (E/((1+v)*(1-2*v)))*(v*du_dr_aa(j,i) + (1-v)*du_dt_aa(j,i));
    end
end


figure(2)
plot(sigma_rr_cd(6,:))
hold on
plot(sigma_rr_aa(3,:))
xlabel('Time step')
ylabel('Radial stress in psi at element 6')
legend('Central Difference Method','Average Acceleration Method')


figure(3)
plot(sigma_tt_cd(6,:))
hold on
plot(sigma_tt_aa(3,:))
xlabel('Time step')
ylabel('Tangential stress in psi at element 6')
legend('Central Difference Method','Average Acceleration Method')
```



Central Difference Method
Average Acceleration Method
Radial stress in psi at element 6
Time step
1.5
1
0.5
0
-0.5
-1
-1.5
0
500
1000
1500
2000
2500



Central Difference Method
Average Acceleration Method
Tangential stress in psi at element 6
Time step
0.5
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4
-0.5
0
500
1000
1500
2000
2500

```
function[disp_t, vel_t, ac_t, dt, nt] = Eqsolver(nn,beta,gamma,K_global, F_global, M_global,b
ig_lamda,Ri,t,t_end,ct)
dt = ct*2/sqrt(big_lamda);

nt = ceil(t_end/dt);

def = zeros(nn,1);
vel = zeros(nn,1);
ac = zeros(nn,1);
disp_t = zeros(nn,nt+1);
vel_t = zeros(nn,nt+1);
ac_t = zeros(nn,nt+1);

%Start time loop
for j = 1:nt
    tn = j*dt;
    % predictor phase

        def1 = def + dt*vel+0.5*(dt^2)*(1-2*beta)*ac;
        def2(:,j)= def1;
        v1= vel+(1-gamma)*dt*ac;
        % solution phase
        A = (M_global + beta*(dt^2)*K_global);
        P = [2*pi*pressure(tn,t)*Ri];
        F_global(1) = P;
        F_star = F_global - K_global*def1;
%         F_new(:,j) = F_global;
        ac = A\F_star;
        % correction phase
        def = def1 + beta*dt*dt*ac;
        vel = v1 + gamma*dt*ac;
        % final values of displacement, acceleration and velocity
        disp_t(:,j+1) = def;
        vel_t(:,j+1) = vel;
        ac_t(:,j+1) = ac;

end
end
```

```
Not enough input arguments.

Error in Eqsolver (line 2)
dt = ct*2/sqrt(big_lamda);
```

```
function P= pressure(tn,t)
if (tn-(t/2))<= (10^(-10))
    P = sin(2*pi*tn/t);
else
    P = 0;
end
end
```

```
Not enough input arguments.

Error in pressure (line 2)
if (tn-(t/2))<= (10^(-10))
```
