

## Questions:

1. For a 3 sets tennis game, would you bet on it finishing in 2 sets or 3 sets?
2. I have a square, and place three dots along the 4 edges at random. What is the probability that the dots lie on distinct edges?
3. You have 10 people in a room. How many total handshakes if they all shake hands?
4. Two decks of cards. One deck has 52 cards, the other has 104. You pick two cards separately from a same pack. If both of two cards are red, you win. Which pack will you choose?
5. What is  $39 \times 41$ ?
6. A group of people wants to determine their average salary on the condition that no individual would be able to find out anyone else's salary. Can they accomplish this, and, if so, how?
7. How many digits are in 99 to the 99th power?
8. A line of 100 passengers is waiting to board a plane. They each hold a ticket to one of the 100 seats on that flight. (For convenience, let's say that the  $n$ th passenger in line has a ticket for the seat number  $n$ .) Unfortunately, the first person in line is crazy, and will ignore the seat number on their ticket, picking a random seat to occupy. All of the other passengers are quite normal, and will go to their proper seat unless it is already occupied. If it is occupied, they will then find a free seat to sit in, at random. What is the probability that the last (100th) person to board the plane will sit in their proper seat (#100)?
9. What is the sum of the numbers one to 100?
10. You have a 3 gallon jug and 5 gallon jug, how do you measure out exactly 4 gallons? Is this possible?
11. You have 17 coins and I have 16 coins, we flip all coins at the same time. If you have more heads then you win, if we have the same number of heads or if you have less then I win. What's your probability of winning?
12. What is the probability you draw two cards of the same color from a standard 52-card deck? You are drawing without replacement.
13. You're in a room with three light switches, each of which controls one of three light bulbs in the next room. You need to determine which switch controls which bulb. All lights are off to begin, and you can't see into one room from the other. You can inspect the other room only once. How can you find out which switches are connected to which bulbs? Is this possible?
14. In world series, what are the odds it goes 7 games if each team equal chance of winning?
15. Given 100 coin flips, what is the probability that you get an even number of heads?
16. There are 5 balls, 3 red, and 2 black. What is the probability that a random ordering of the 5 balls does not have the 2 black balls next to each other?
17. What is the least multiple of 15 whose digits consist only of 1's and 0's?

18. Is 1027 a prime number?
19. Does the price of a call option increase when volatility increases?
20. 2 blue and 2 red balls, in a box, no replacing. Guess the color of the ball, you receive a dollar if you are correct. What is the dollar amount you would pay to play this game?
21. What is the singles digit for  $2^{230}$ ?

### Answers:

1. **Two sets** - Let  $p$ =prob team 1 wins and  $q$ =prob team 2 wins.  $p^2 + q^2$  = probability finish in two sets.  $2*p*q$  = probability finish in three sets.  $p^2 + q^2$  always  $\geq 2*p*q$ , so the answer is two sets.
2. **3/8** - Given the edge the first dot is on, the probability the other two dots are on distinct edges is  $(3/4)*(2/4)$
3. **45** -  $(10 \text{ choose } 2) = 45$  – this is the total number of ways two people can shake hands.
4. **104 card pack** -  $(52/104)*(51/103) > (26/52)*(25/51)$ , or  $51/103 > 25/51$
5. **1599** -  $39*41 = (40-1)*(40+1) = 40*40 - 1 = 1599$
6. **Yes, it's possible** - The first person thinks of a random number, say  $X$ . This person adds this number to her salary. The rest of the group simply adds their salary to the initial number. Then, the first person subtracts the random number  $X$  and divides the total salary sum by the size of the group to obtain the average.
7. **198** -  $99^{99} = (100)^{(99)} * (.99)^{99} = (10)^{(198)} * (.99)^{99}$ . You can convince yourself  $10^{198}$  has 199 digits, and  $0.99^{99}$  approaches  $1/e$ . Thus,  $(10)^{(198)} * (.99)^{99}$  has 198 digits.
8. **0.5** - The fate of the last passenger is determined the second either the first or last seat on the plane is taken. This statement is true because the last person will either get the first seat or the last seat. All other seats will necessarily be taken by the time the last passenger gets to pick his/her seat. Since at each choice step, the first or last seat has an equal probability of being taken, the last person will get either the first or last with equal probability: 0.5.
9. **5050** - Sum of numbers from 1,2,..., $n = n*(n+1)/2$ . You can also think about this problem by pairing off numbers - 1 and 100, 2 and 99, 3 and 98, 4 and 97, etc. We have 50 of these pairs, and each pair sums up to 101, so the final sum =  $50*101 = 5050$ .
10. **Yes, it's possible** - Fill up the 3 gallon jug. Then, pour the liquid into the 5 gallon jug. Fill the 3 gallon jug again, and then fill the 5 gallon jug until it is full. We now have 1 gallon remaining in the 3 gallon jug. We empty the five gallon jug and pour the remaining 1 gallon into our 5 gallon jug. Finally, we fill the 3 gallon jug and add this to the 5 gallon jug (which already had 1 gallon). We are left with 4 gallons in the 5 gallon jug.

11. **0.5** - Use recursion - The initial 16 flips have the same probability of everything. Thus, the game completely depends on if the last coin flip is tails or head (50/50 chance of H vs. T).
12. **25/51** - You either draw a black or a red card first. Then, there are 51 cards left in the deck and 25 of these cards have the same color. Thus, the probability is 25/51.
13. **Yes, it's possible** - Leave switch 1 off. Then, turn switch 2 on for ten minutes. After the ten minutes, turn it off and quickly turn on switch 3. Now, go into the room. The currently lit up bulb connects to switch 3. The bulb that off but still warm is from switch 2, and the remaining bulb is from switch 1.
14. **20/64** - Out of the first three games, each team needs to win three. Thus,  $(6 \text{ choose } 3) \cdot (.5^6) = 20/64$ , as each team has a  $1/2$  probability of winning each game.
15. **1/2** - Whether there is an odd or even number of heads is ultimately determined by the final flip (50/50 chance of being heads vs. tails), for any number of flips.
16. **0.6** - Because of repeats of black/red balls, there are 10 combinations of red/black balls: (5 choose 2) or (5 choose 3) spots to put the black or red balls, respectively. There are 4 places that 2 black balls can be next to each other, so the other 6 combinations do NOT have two black balls next to each other.
17. **1110** - The last digit must be zero (30, 45, 60, 75, etc.). Multiples of 15 never end in 1. Then, starting checking numbers. 10, 100, 110, 1000, 1100, 1110. You will quickly arrive at the answer if you are good with your mental math.
18. **No** -  $1027 = 1000 + 27 = 10^3 + 3^3$ . We know  $a^3 + b^3$  can be factored, so 1027 is NOT prime.
19. **Yes** - sometimes a rare finance question is included in these interviews; remember that both time and volatility increase the prices of both calls and puts
20. **17/6 dollars** - You'll always get the last ball right as your sampling w/o replacement. The first ball you have a 50% chance of getting right. The second ball you have a  $2/3$  chance of getting right.
21. **4** - Repeating patterns – 2,4,8,6,2 – follow the pattern.