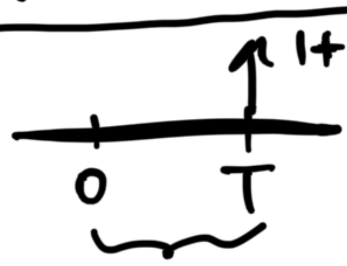
Yield curve

- fair value of derivs.
- hedge IR derivs.
- current snapshot of IR term structure.
- constructed from liquid traded IR instruments.
- mm cash instruments.
  IR futures / FRAs
  IR swaps.
- LIBOR : cash, futures / FRA, swaps.
  SOFR : futures, swaps

Valuation of vanilla IR instruments

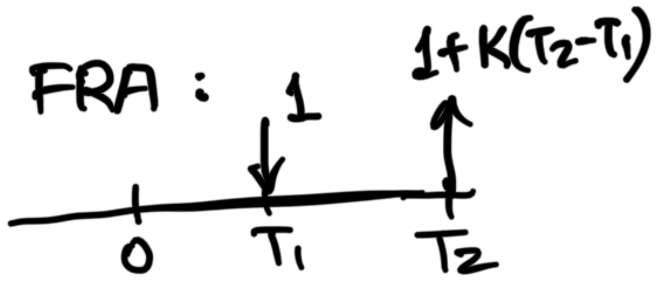
① mm cash :



$$PV(0) = Df(T)(1+rT)$$
$$\parallel$$
$$1$$

$$\boxed{DF(T) = \frac{1}{1+rT}}$$

② FRA :

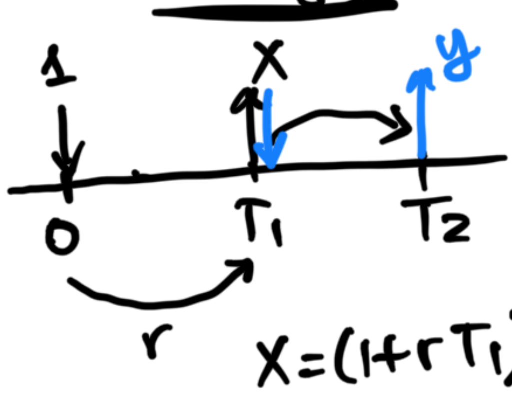


- agree upon a fixed rate K to borrow money over $[T_1, T_2)$

How to find K?
K = forward rate over $[T_1, T_2)$ observed at 0.
$= F(0, T_1, T_2)$

- no arbitrage argument.

<u>strategy</u> at time 0 :



$X = (1+rT_1)$

① borrow 1 for $[0, T_1)$
② FRA to borrow X over $[T_1, T_2)$

FRA : $y = X(1 + K\cdot(T_2 - T_1))$

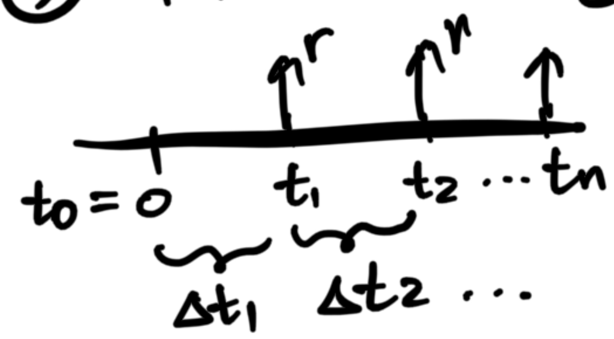
$$X = \frac{1}{DF(T_1)}$$

$$\rightarrow \quad 1 = y\,DF(T_2) \Rightarrow y = \frac{1}{DF(T_2)}$$

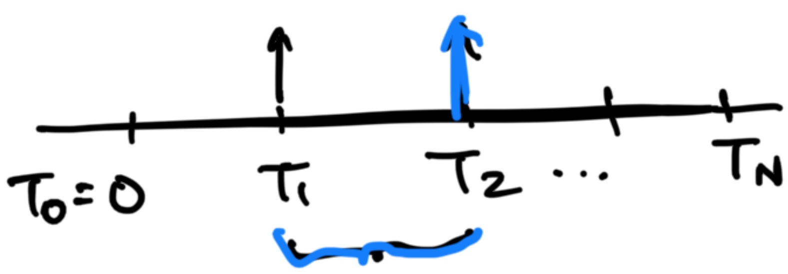
$$\frac{1}{DF(T_2)} = \frac{1}{DF(T_1)}\left(1 + K\cdot(T_2 - T_1)\right)$$

$$F(0,T_1,T_2) = \boxed{K = \left(\frac{DF(T_1)}{DF(T_2)} - 1\right)\frac{1}{T_2 - T_1}}$$

③ Fixed-Floating swaps.



$$PV(\text{fixed}) = \sum_{i=1}^{n} (r\,\Delta t_i) DF(t_i)$$



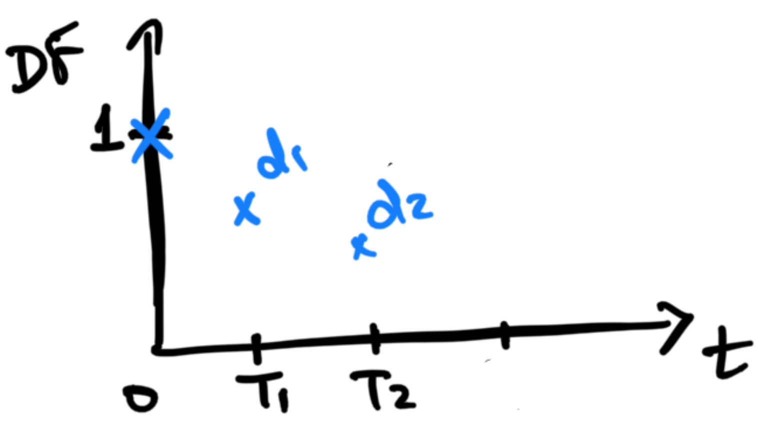
$$PV(\text{float}) = \sum_{i=1}^{N} F(0, T_{i-1}, T_i)\,\Delta T_i\, DF(T_i)$$

④ Futures. (later)

<u>Bootstrapping</u>

- use cash, FRA, swaps. inputs

1m cash	3%	1x3	FRA	3.15% ← K
2m cash	3.1%	2x3	FRA	3.25% ↙
⋮		2Y	swap	5%
		3Y	"	5.01%



1m cash : T_1

$$d_1 = \frac{1}{1 + r_1 T_1}$$

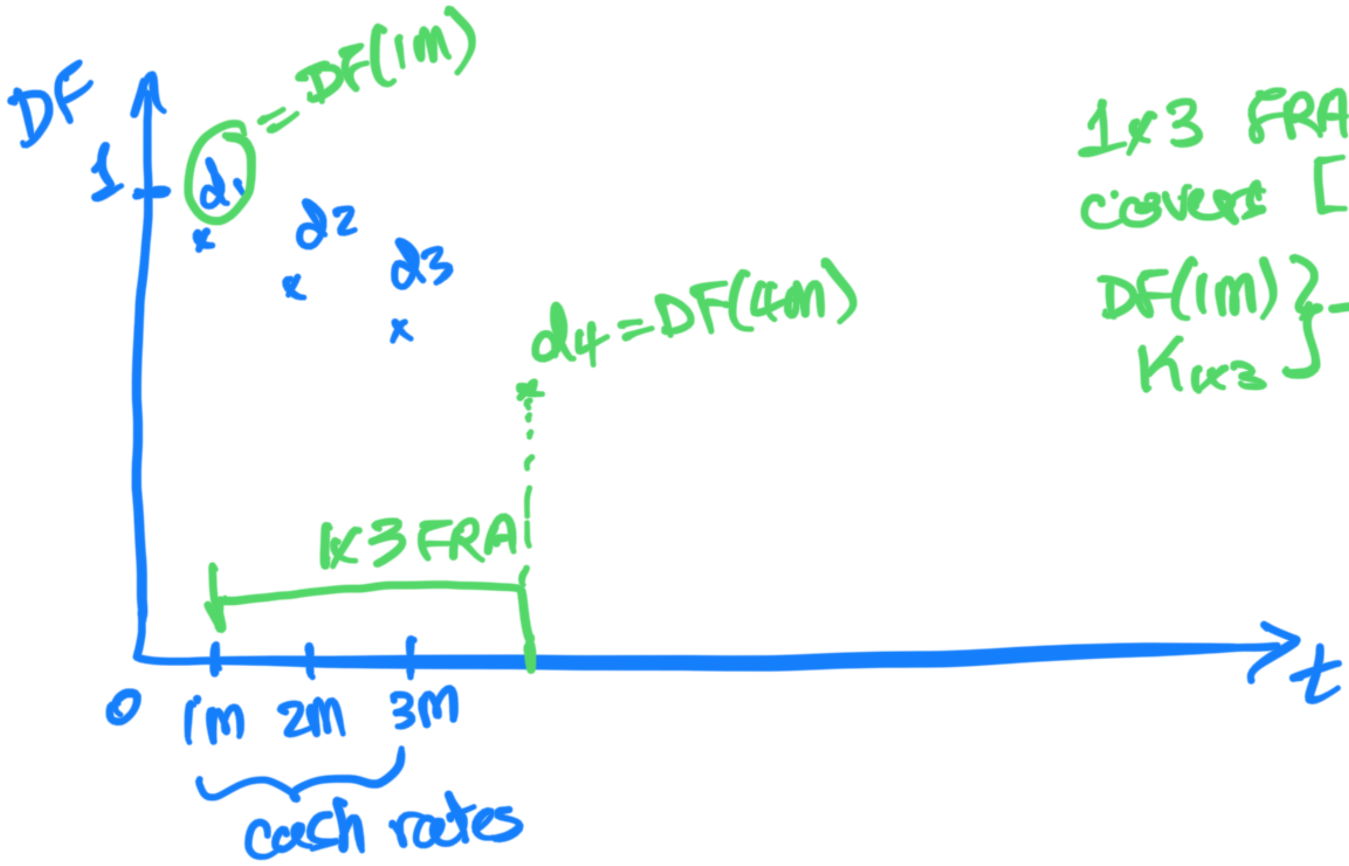
1m×3m FRA : $[T_1, T_2)$

- this rate depends on $DF(T_1)$, $DF(T_2)$

1x3 FRA quote K ✓

$$= \left(\frac{DF(T_1)}{DF(T_2)} - 1\right)\frac{1}{T_2 - T_1}$$

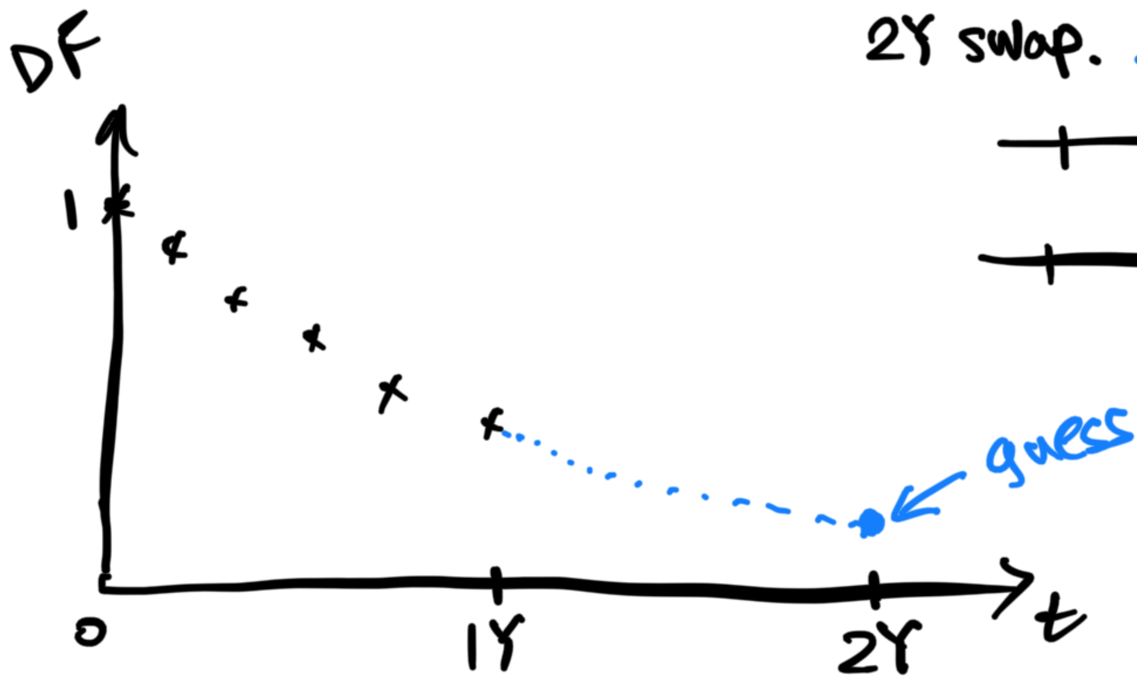
$$d_2 = DF(T_2) = \frac{DF(T_1)}{1 + K\cdot(T_2 - T_1)}$$



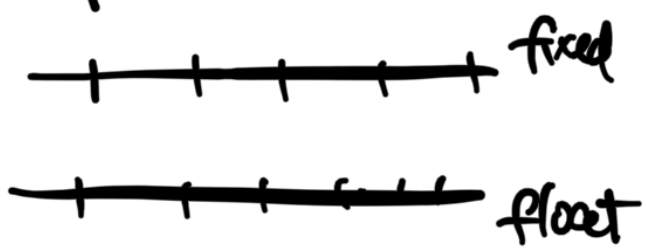
1x3 FRA K_{1x3}

covers [1m, 4m]

$$\left.\begin{matrix} DF(1m) \\ K_{1x3} \end{matrix}\right\} \rightarrow DF(4m)$$



2Y swap. 5%



$$PV(\text{2Y swap } 5\%) = f(\underbrace{DF(2Y)}_{x})$$

Solve $f(x) = 0$ ← Newton's method