Time Series Analysis

GU4221/GR5221

Lecture I

01/17/2022

Objectives of Time series Analysis

- Drawing inferences for TS $\underbrace{\{Y_t : t=1,\dots,n\}}_{\text{data set}}$

- Forecast ($\hat{Y}_{n+1}$ or $\hat{Y}_{n+d}$)

$$\hat{Y}_{n+d} \pm \underbrace{ME}_{\text{Margin of error of the forecast (Inference)}}$$

Other Considerations

- Recognize and remove seasonal component so that its not confused with the long term trend.

  (Seasonal adjustment)

- Remove both seasonality & trend. study the noise

$$Z_t = Y_t - m_t - s_t$$

- Is $Z_t \sim WN$ (white noise) ?
- How can you test this?

<u>Def</u> $Z_1, \ldots, Z_n \sim WN$ (white noise)

if $\operatorname{cov}(Z_i, Z_j) = 0$ for $i \neq j$

and $E Z_i = 0$, $\operatorname{var}(Z_i) = \sigma^2$.

- center WN if $EZ = \mu \neq 0$
- WN is <u>not</u> iid !
- $\{Z_t\}$ iid $\Longrightarrow$ $\{Z_t\}$ WN
- $\{Z_t\}$ WN $\not\Longrightarrow$ $\{Z_t\}$ iid

Example AR(1)

First-order Autoregressive Model

$$\text{(EQ1)} \quad \begin{cases} Y_t = \phi Y_{t-1} + Z_t \\ t \in \mathbb{Z}, \quad \mathbb{Z} = \{\dots, -2, -1, 0, 1, 2, \dots\} \\ Z_t \sim WN(0, \sigma^2) \end{cases}$$

- Find a solution to (EQ1)

$$Y_t = ?$$

- Answer

$$Y_t = \sum_{j=0}^{\infty} \phi^j Z_{t-j} \qquad |\phi| < 1$$

- why?

$$\phi Y_{t-1} + Z_t = \phi \left(\sum_{j=0}^{\infty} \phi^j Z_{(t-1)-j} \right) + Z_t$$

$$= \underbrace{\sum_{j=0}^{\infty} \phi^{j+1} Z_{t-(j+1)}}_{K = j+1} + Z_t$$

$$= \sum_{k=1}^{\infty} \phi^k Z_{t-k} + \underbrace{Z_t}_{\phi^0 Z_{t-0}}$$

$$= \sum_{k=0}^{\infty} \phi^k Z_{t-k}$$

$$= Y_t$$ ✓

- why do we require $|\phi| < 1$
  (see later section)

- How does an infinite sum solution help practitioners?
  (see later section)

Application Aspect of AR(1) Example

$$Y_t = \phi Y_{t-1} + Z_t, \quad Z_t \sim wn(0, \sigma^2)$$
$$t \in \mathbb{Z}$$

- can we estimate ϕ?

- $Y_t \sim Y_{t-1}$ (regress Y_t on Y_{t-1})

In R: `lm(Y[2:n] ~ Y[1:(n-1)])`

- The Least Squares estimator of ϕ can be solved regardless of if $|\phi| < 1$.

$$\min \sum (Y_t - \phi Y_{t-1})^2$$