Econ 361: Advanced Econometrics

Linear Regression

Best Linear Predictor of Y given X under MSE

$$\text{BLP}_{\text{MSE}}(Y|X) \equiv \arg\min_{\alpha+\beta X} E[\,(\,\underbrace{(\alpha+\beta X)}_{=\hat{Y}(X)} - Y\,)^2\,]$$

$$= \alpha^* + \beta^* X$$

where

$$\alpha^* = E[Y] - b^* E[X] \;=\; \mu_Y - b^* \mu_X$$

$$\beta^* = \frac{E[XY] - E[X]\,E[Y]}{E[X^2] - (E[X])^2} \;=\; \frac{\sigma_{XY}}{\sigma_X^2}$$

OLS Predictor of Y_i given X_i

$$\hat{Y}_{ols} \equiv \arg\min_{a+bX_i} \sum_{i=1}^{N} (\,(a+bX_i) - Y_i\,)^2$$

$$= a_{ols} + b_{ols}X$$

where

$$a_{ols} = \underbrace{\frac{1}{N}\sum_{i=1}^{N} Y_i}_{\bar{Y}_N} - b_{ols}\underbrace{\frac{1}{N}\sum_{i=1}^{N} X_i}_{\bar{X}_N}$$

$$= \text{Sample Mean of } Y - b_{ols} \times \text{Sample Mean of X}$$

$$b_{ols} = \frac{\frac{1}{N}\sum_{i=1}^{N} X_i Y_i - \bar{X}_N \bar{Y}_N}{\frac{1}{N}\sum_{i=1}^{N} X_i^2 - (\bar{X}_N)^2}$$

$$= \frac{\text{Sample Covariance of } X, Y}{\text{Sample Variance of } X}$$

Analogy between BLP_{MSE} and OLS

Population		**Sample**
(Y, X)	$\Longleftrightarrow$	$\{Y_i, X_i\}_{i=1}^{N}$
Mean Squared Error (MSE) Loss	$\Longleftrightarrow$	Sum of Squared Residuals (SSR)
$E[\ (\hat{Y}(X) - Y)^2\ ]$		$\sum_{i=1}^{N} (\ (a + bX_i) - Y_i\ )^2$
$\text{BLP}_{\text{MSE}}(Y \mid X)$	$\Longleftrightarrow$	$\hat{Y}_{ols}(X)$
$=\ \alpha^* + \beta^* X$		$=\ a_{ols} + b_{ols} X$
$\alpha^* = \mu_Y - b^* \mu_X$	$\Longleftrightarrow$	$a_{ols} = \bar{Y}_N - b_{ols} \times \bar{X}_N$
$\beta^* = \dfrac{\sigma_{XY}}{\sigma_X^2}$	$\Longleftrightarrow$	$b_{ols} = \dfrac{\text{Sample Covariance of } X, Y}{\text{Sample Variance of } X}$

One Interpretation of OLS Parameters (a_{ols}, b_{ols})

(a_{ols}, b_{ols}) is the moment-based estimator (analogy principle) of the BLP$_{\text{MSE}}(Y|X)$ parameters (α^*, β^*)

Question: How *good* of an estimator is (a_{ols}, b_{ols}) of (α^*, β^*) ?

We are interested not only in deriving possible estimators of unknown parameters but also in deriving their **properties**. This will lead us to explore the **sampling distribution** of our estimators. Of particular interest will be the first and second moments associated with our estimators.

unbiasedness / consistency, “minimum” variance