Math 280, Spring 2022 Professor Rob Benedetto

Homework #7

Due **Wednesday, April 6** in Gradescope by **11:59 pm ET**

READ Textbook Sections 1.4.3 and 1.5.1, lightly read 1.4.4, and start 1.5.2

WRITE AND SUBMIT solutions to the following problems.

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1. (10 points) For the graph $G = C_4$, determine:

(a) is it Eulerian? (b) is it Hamiltonian?

(c) is it traceable? (d) what is its independence number $\alpha(G)$?

As always, be sure to (briefly) justify your answers.

2. (10 points) For the graph $G = K_{3,3}$, determine:

(a) is it Eulerian? (b) is it Hamiltonian?

(c) is it traceable? (d) what is its independence number $\alpha(G)$?

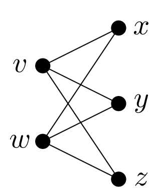
As always, be sure to (briefly) justify your answers.

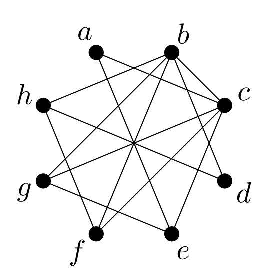
3. (18 points) Textbook Section 1.4.3, problem 4:
Find the connectivity and the independence number of the Petersen graph.
Make sure to prove your answers!

4. (12 points) Textbook Section 1.4.3, problem 8:
Let G be a graph. Prove that the line graph $L(G)$ is claw-free.

5. (10 points) Textbook Section 1.4.3, problem 9:
Let G be a K_3-free graph. Prove that its complement, $\overline{G}$ is claw-free.
(**Note**: don't misread that: it says K_3, not $K_{3,3}$. And recall $K_3 = C_3$, just three vertices and three edges. So we're saying G doesn't contain induced subgraphs isomorphic to C_3.)

6. (12 points) Textbook Section 1.5.1, problem 1:
Find planar representations of each of the following graphs:

(a): 

(b): 

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7. (14 points) Textbook Section 1.5.1, converse of problem 6:
Let G be a planar graph, and let $e \in E(G)$. Suppose that in some planar representation of G, the edge e does *not* bound a region. Prove that e is a bridge.

(**Suggestion**: If the same region R is on both sides of e, what happens if you draw a curve through R from one side of e to the other side?)

8. (12 points) Textbook Section 1.5.1, problem 8, slightly modified:
Prove that there exist planar graphs G_1 and G_2 that have the same number n of vertices, the same number q of edges, and the same number r of regions, **but** which are not isomorphic.
That is, write down the two graphs, compute the numbers n, q, r for each and verify they match, and then prove that G_1 and G_2 are *not* isomorphic.

Optional Challenges (do NOT hand in): Textbook Section 1.4.3, Problem 11; Section 1.5.1, Problem 5

Questions? You can ask in:

Class: MWF 11:00–11:50am, SMUD 205
Tu 9:00–9:50am, SMUD 205

My office hours: Mon 2:30–3:30pm, Tue 2–3:30pm, and Thu, 1–2:30pm, SMUD 406

Anna's Math Fellow office hours:
Sundays, 7:30–9:00pm, and Tuesdays, 6:00–7:30pm,
SMUD 007

Also, you may email me any time at `rlbenedetto@amherst.edu`