Problem 5 Way 2

$$P\left(\sum_{i=1}^{n}\alpha_i W_i+\beta \mid W\right) \overset{\text{4th property}}{=} \sum_{i=1}^{n}\alpha_i P(W_i \mid W)+\beta \quad — ①$$

$$P(W_i \mid W) = E(W_i) + a^T(W - E(W)) \quad \text{where } \Gamma a = Cov(W_i, W) \quad ②$$

Solve for a:

$$LHS = \begin{bmatrix} Cov(W_1,W_1) & \cdots & Cov(W_1,W_n) \\ \vdots & & \vdots \\ Cov(W_n,W_1) & \cdots & Cov(W_n,W_n) \end{bmatrix}\begin{bmatrix} a_1 \\ \vdots \\ a_n \end{bmatrix} = \begin{bmatrix} \sum_{j=1}^{n} a_j Cov(W_1, W_j) \\ \vdots \\ \sum_{j=1}^{n} a_j Cov(W_n, W_j) \end{bmatrix}$$

$$RHS = \begin{bmatrix} Cov(W_i, W_1) \\ \vdots \\ Cov(W_i, W_n) \end{bmatrix}$$

$\because$ $LHS = RHS$ and $Cov(W_s, W_t) = Cov(W_t, W_s)$ $\forall s,t$

$$\therefore \quad a_j = \begin{cases} 0 & \text{if } j \neq i \\ 1 & \text{if } j = i \end{cases} \quad — ③$$

plug ③ into ② $\Rightarrow P(W_i \mid W) = \cancel{E(W_i)} + W_i - \cancel{E(W_i)} = W_i$

plug back into ① $\Rightarrow P\left(\sum_{i=1}^{n}\alpha_i W_i+\beta \mid W\right) = \sum_{i=1}^{n}\alpha_i W_i + \beta$

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