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Laughter In The Dark

I first heard about the smile in December 1990 from Dave Rogers, our head options trader in Tokyo.

I had begun traveling to Japan regularly to bring our traders the latest releases of our risk management tools and to learn what new models and software they needed. Unlike the New York Stock Exchange, the Tokyo market closed at midday; traders grew less frenetic and went out for lunch, salespeople departed to meet clients, and there was time for leisurely conversations. While we chatted, Dave showed me the computer screen he used to watch the prices of options on the Nikkei 225 index. He pointed our a peculiar asymmetry in the Nikkei options prices: the prices of out-of-the-money puts were unexpectedly larger than those of other Nikkei options.

Everyone referred to this asymmetry as “the smile,” or “the skew.” At first it looked only mildly interesting, a peculiar anomaly you could live with. Then, when you thought about it a little more, you realized that the existence of the smile was completely at odds with Black and Scholes’s twenty-year-old foundation of options theory. And, if Black-Scholes was wrong, then so was the “delta” it predicted, the sensitivity of an index option’s price to movements in its underlying index. In that case, all traders using the Black-Scholes model’s delta were incorrectly hedging their option books. But the very essence of Black-Scholes was its prescription for replicating options by hedging. The smile, therefore, poked a small deep hole deep into the dike of theory that lay beneath all options trading. If Black-Scholes was wrong, what *was* the right delta to use for hedging an option?

During the 1990s, the smile, initially a peculiarity of equity index markets, infected other options markets, taking a slightly different form in each one, until understanding it became a dominating obsession for me and many of my quant contemporaries. It was an anomaly that sat right at the intersection of options markets and options theory, and I spent much of my intellectual energy trying to model it.

Our work started in the typical heady rush of energy and ambition that made me feel as though I were in physics again, racing to be the first to find the "right" model for something important and interesting. I had last felt that way while working on BDT, and now excited again, I fantasized about building a model that, embraced by everyone, would replace Black-Scholes. It wasn't as simple as I thought. During the next ten years I learned that "rightness" in financial modeling is a much fuzzier concept than I had imagined, and that traders, especially equity traders, are reluctant to try anything new.

One of the things you learn early on in a career in financial modeling is the importance of units. You always want the prices of securities to be quoted in a way that make it easy to compare their relative values.

When you need to compare the values of bonds, for example, their prices are insufficient, because bonds can vary by maturity and coupon. Instead, you quote their yields. A bond yield provides an estimate of the *return* the bond will generate for you irrespective of its coupon and maturity. You may not know whether a discount bond at 98 is better than a premium at 105, buy you do know that, all other things being equal, a yield of 5.3% is less attractive than 5.6%. This conversion of prices to yields is itself a model, albeit a simple one. It's a convenient way to communicate prices, and a good first step at estimating value.

In the options world as well, price alone is an insufficient measure of value; it's impossible to tell whether ¥300 for an at-the-money put is more attractive than ¥40 for a deep out-of-the-money put. A better measure of value is the stock's volatility. The Black-Scholes model views a stock option as a kind of bet on the future volatility of a stock's returns. The more volatile the stock, the more likely the bet will pay off, and therefore the more you should pay for it. You can use the model to convert an option price into the future volatility the stock must have in order for the option price to be fair. This measure is called the stock's *implied volatility*. It is, so to speak, an option's view of the stock's future volatility.

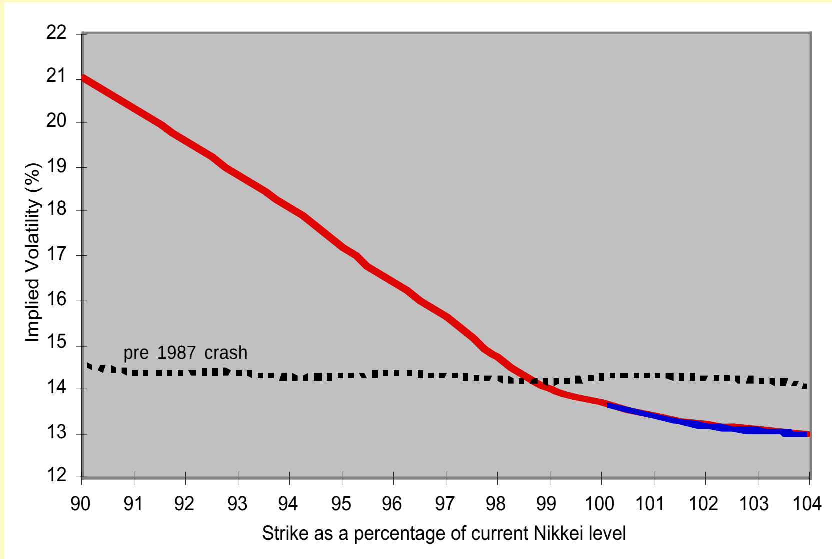
The Black-Scholes model was the market standard. On Dave's computer screen when I sat next to him in Tokyo that day, the prices were quoted in Black-Scholes implied volatilities. Even today, when no-one believes that the Black-Scholes model is absolutely the best way to estimate options value, and even though more sophisticated traders sometimes use more complex models, the Black-Scholes model's implied volatilities are still the market convention for quoting prices..

Options are generally less liquid than stocks, and implied volatility market data is consequently coarse and approximate. Nevertheless, Dave pointed out to me what I was already dimly aware of: there was a severe skew in the implied volatilities, so that three-month options of low strike had much greater implied volatilities than three-month options of higher strikes. You can see a sketch of the asymmetry in Figure 14.1, where, the lower the strike of the option, the higher its implied volatility. This lopsided shape, though it's commonly called "the smile," is more of a Bell's-palsied smirk.

With implied volatility as your measure of value, low-strike puts are the most expensive Nikkei options. Anyone who'd been around on October 19, 1987 could easily guess why. Ever since that day when equity markets around the world had plunged, investors remained constantly aware of the possibility of an instantaneous large jump down in the market, and were willing to pay up for protection. Out-of-the-money puts were the best and cheapest insurance. Like stable-boys who shut the barn door after the horse has bolted, investors who lived through the 1987 crash were now willing to pay up for future insurance against the risks they had previously suffered. By 1990 there were similar smiles or skews in all equity markets, Before 1987, in contrast, more light-heartedly naive options markets were happy to charge about the same impied volatility for all strikes, as illustrated by the dotted line in Figure 14.1.

It was not only three-month implied volatilities that were skewed. A similar effect was visible for options of all expirations, so that implied volatility varied not only with strike but also with expiration. We began to plot this double variation of implied volatility in both the time and strike dimension as a two-dimensional *implied volatility surface*. A picture of the

FIGURE 14.1. A typical implied volatility smile for the three-month options on the Nikkei index in late 1994. The dotted line shows the lack of skew that was common prior to the 1987 crash.

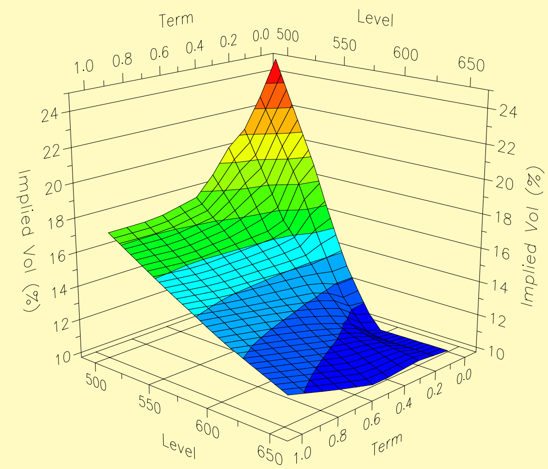


surface for S&P 500 options is illustrated in Figure 14.2. Like the yield curve, it changes continually from minute to minute and day to day.

This tent-like surface was a challenge to theorists everywhere. The Black-Scholes model couldn't account for it. Black-Scholes attributed a single volatility at all future times to an index or a stock, and therefore always produced the dull flat featureless plateau-like surface of Figure 14.3a. The best you could do, if you modified the Black-Scholes model to allow future index volatility to be different from today's, was to obtain a surface that varied in the time direction, as depicted in Figure 14.3b. But the variation in two perpendicular directions, time and strike, was a puzzle. What was wrong with the classic Black-Scholes model? And what new kind of model could possibly explain that surface?

FROM HERE Put bluntly, the Black-Scholes model over-simplifies the behavior of stock prices. It assumes that a stock price *diffuses* away into

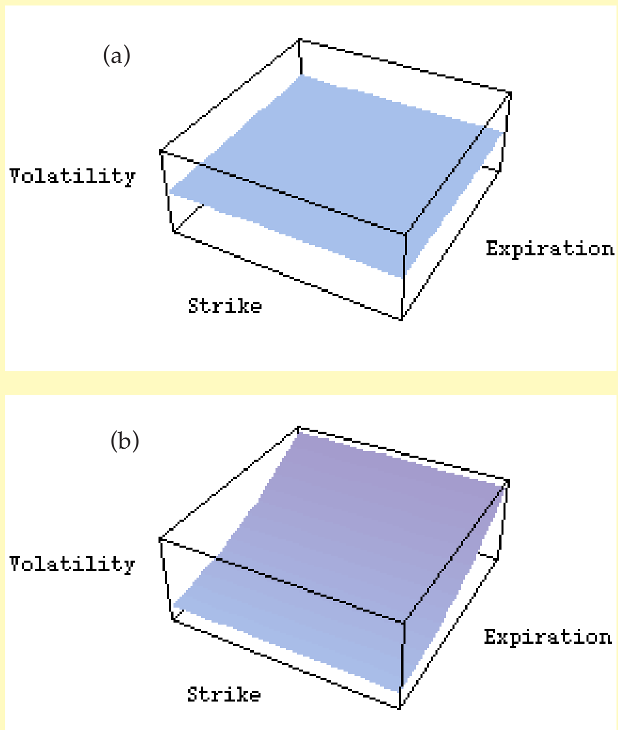
FIGURE 14.2. A typical implied volatility surface for the S&P 500 in mid 1995.



the future from its current value in a slow random continuous fashion, much like the way a cloud of smoke from the smouldering tip of a cigarette spreads through a room. Dense near the tip of the cigarette and sparse farther out, the intensity of the smoke cloud at a point represents the probability that a particle of smoke will diffuse there. In the Black-Scholes model, a similar cloud depicts the probability that the stock price will reach some particular future value at some future time. Figure 14.4 illustrates that cloud of probabilities for a stock in the Black-Scholes model. The more dispersed the cloud, the more uncertain the future stock price. A single parameter, the stock's volatility σ, determines the rate of diffusion and the width of the cloud. The greater the stock's volatility, the wider the cloud.

Though simplification is the essence of modeling, the Black-Scholes picture of smoke-cloud diffusion is too restrictive. First, stock prices don't necessarily diffuse with a constant volatility; at some times a stock diffuses more rapidly than at others. Second, and more gravely, sometimes stocks *don't diffuse at all*. Diffusion, as shown in Figure 14.4, is a slow continuous process; in diffusion, a stock price that moves from $100 to $99 passes through every possible price between them. That is not what happened during the 1987 crash; on that day the Dow-Jones index jumped its

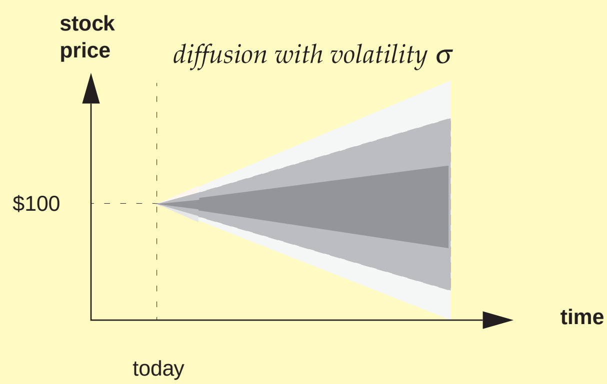
FIGURE 14.3. Implied volatility surfaces. (a) In the standard Black-Scholes model. (b) In an enhanced Black-Scholes model where volatility varies with time to expiration



way downwards through 500 points rather like an excited kid on a pogo stick.

Returning to New York from Tokyo, I began working with my QS colleagues Iraj Kani and Alex Bergier. I wanted to extend the Black-Scholes model just enough so that it could incorporate the smile. "Just enough" was always the aim. A model is only a model; you want to capture the essence, not the thing itself. It's far too easy, in the name of realism, to add complexity to the simple evolution of stock prices assumed by Black and Scholes, but complexity without calibration is unmanageable.

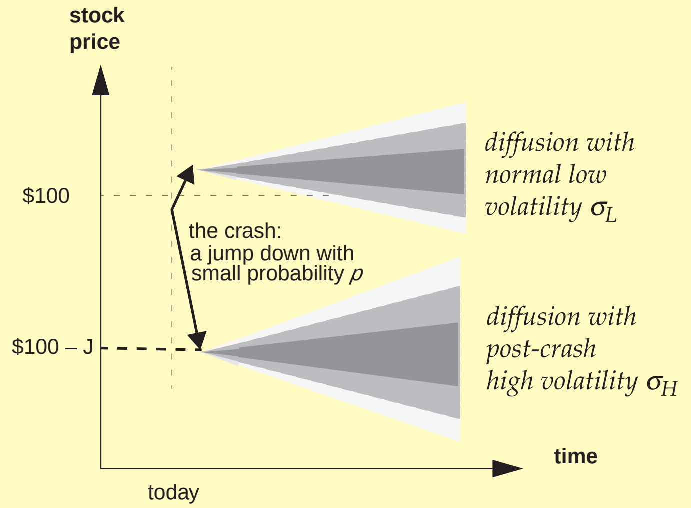
FIGURE 14.4. Simple diffusion as in the Black-Scholes model. The shaded regions illustrates the range of possible future prices for a stock whose price is known to be $100 today. The more time passes, the greater the uncertainty in the future price. The darker the shading, the more likely the price will be in that region.



The overwhelming fear of equity investors was another '87-style crash, so we added that possibility to Black-Scholes. This wasn't something especially new; Merton had done it in his so-called jump-diffusion model in the mid-1970s, and to begin with, we did it even more crudely than he had. To the constant diffusion of stock prices we added just one new feature, the small probability p that, in the next instant, the stock price might take a sizeable jump J downwards.The probability cloud for this process is depicted in Figure 14.5. which shows the two scenarios the stock can now take: a jump downwards by J and subsequent diffusion with a volatility σ_H which is likely to be high because of the after-excitement of the crash; or, more likely, continued diffusion with the normal, lower volatility σ_L.

Typically, we assumed the probability p to be of order 1%, implying that the market assigned about a one-in-a-hundred chance of another crash. We chose the σ_H to be about 40% greater than σ_L , based on a combination of intuition and experience about the after-effects of stock crashes. Our model now had only two unknown parameters, the jump size J of the

FIGURE 14.5. The range of possible future prices for a stock which can jump and then diffuse. The darker the shading, the more likely the price will be in that region.



crash scenario and the volatility σ_L associated with normal behavior. This was just one parameter richer than Black-Scholes, which contained only a single volatility. We calibrated these parameters by matching the model's option prices to the two implied volatilities that defined the shape of the three-month smile, those of an at-the-money and a 95% strike out-of-the-money put. With a normal volatility σ_L in the vicinity of 10% and a downward jump J of about 25% of the current stock price, we found we could produce smiles like those of Figure 14.1.

Our model looked at the world like this: the Nikkei on any day had a roughly one-in-a-hundred chance of dropping about 25%. That was why you paid so much more for an out-of-the-money put. We then used the model to estimate an option's delta, the hedge ratio necessary to cancel its index risk. We also used it to value the more illiquid or exotic options that were becoming increasingly fashionable, barrier options for example, whose prices were highly sensitive to the probability and size of the jump. We wanted our traders to search the market for options whose prices differed significantly from those produced by our model, to buy the apparently cheap ones and sell the rich ones in the hope these outliers would eventually revert to our model's prices and generate a profit.

Though the jump model captured one essential sentiment responsible for the smile, it was ultimately too crude. Its view of a future in which the Nikkei awoke each morning and decided either to make one large instantaneous excited jump downwards or else diffuse calmly was still too simplistic. Looking back, perhaps we should have added a distribution of possible jump sizes and jump times. But jumps occur rarely, and since there was little data about their distribution, we would have had to make many unverified assumptions, which felt unaesthetic. Rightly or wrongly, we preferred a more constrained model whose parameters were totally fixed by calibration to observed options prices. Ten years later, though, more detailed jump-diffusion models of the smile became popular again.

Our model did find users in Goldman's risk arbitrage area, where savvy traders combined worldly knowledge with quantitative methods to take educated bets. Some of them focused on mergers and acquisitions. In an acquisition, the acquiring company tenders for the stock of a target company at a public offer price that substantially exceeds its current level. If and when regulators approve the acquisition, the target's price will jump to the offer price. Until then, its price reflects an estimate of the probability of completion of the deal. For these situations, our jump model was an accurate picture, and the risk arbitrageurs made occasional use of it to see whether their estimate of the acquisition's approval matched the jump probability implied by the current price of the target.

Meanwhile, from mid-1991 through early 1993, Iraj and I and the rest of QS turned temporarily to the more pressing problem of enhancing our risk systems to handle the exotic options we increasingly traded.

But, the more we worked on exotics, the more we ran into the problems of the smile: whenever we used the Black-Scholes framework to value the exotic options in the desk's book, we were using a model that was inconsistent with the smile, a model that produced the wrong value for much simpler standard options. That wasn't good – you can't trust a model for complex phenomena if it gets the simple stuff wrong. You wouldn't trust a NASA computer program that predicted the trajectory of an interplanetary probe from Earth to Mars if it didn't first correctly predict the orbits of Earth and Mars around the sun.

The right place to start was a model that could match the market prices of all standard options, the entire implied volatility surface. Only then, when it was correctly calibrated, could you sensibly use it calculate the value of an exotic. How could we find a model that matched any surface?

I thought back to our development of BDT. In the mid-1980s, the fixed-income options world had undergone a similar crisis: practitioners used model's like Ravi's to value a bond option but felt uncomfortable because that very model, when used to value all Treasury bonds on the yield curve, produced prices that didn't match it. BDT was one of the possible solutions to this dilemma.

We had a tremendous advantage in having come to the equity derivatives world with a background in fixed income. Iraj and I perceived an analogy between yield curves for bonds and implied volatility surfaces for stock options. We drew the following analogy between bonds and their yields and options and their volatilities:

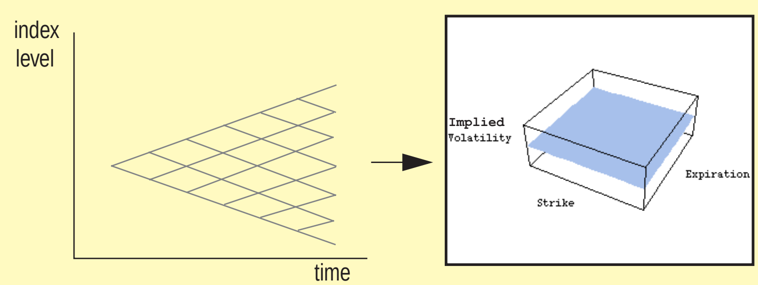
> Bond prices are quoted using current long-term yields, which reflect the market's expectation of future short-term rates.
>
> Options prices are quoted using current long-term implied volatilities, which reflect the market's expectation of future short-term volatilities.

Our ambition was to build a post-Black-Scholes model that allowed us to back out the market's expectation of future short-term volatilities from the current volatility surface. We weren't sure how to do it, but we knew that the world needed a better model, and would reward its discoverers. Throughout 1993 we felt as though we were in a race with unnamed competitors to find it.

Iraj and I were great admirers of the binomial options model, a simple, picturesque and yet accurate way of performing options-theoretic calculations on a grid-like tree of future stock prices. On a binomial tree, prices move like knights on a chess board, one discrete step forward in time and up or down a notch in price. Binomial trees are easy to draw and, in a jerky way, mimic the behavior of real prices or indexes. As the grid of the chess board on which prices make their moves becomes progressively finer, prices move more and more continuously – start to diffuse, in fact – and the binomial model became equivalent to the Black-Scholes model. Binomial trees were the Feynman diagrams of options theory, easy to picture and use, wonderful for simulating simple trading strategies or developing valuation models. Even the innumerate traders we often had to deal with could understand them. Initially invented by William Sharpe soon after Black and Scholes wrote their paper, they were then cleverly elaborated upon by John Cox, Mark Rubinstein and Steve Ross. As options theorists grew increasingly professional and better educated, binomial models fell into a low-tech disrepute, but we still found them immensely useful. Therefore, we tried to use a binomial tree of index options prices as a guide to extracting the market's view of future short-term volatilities.

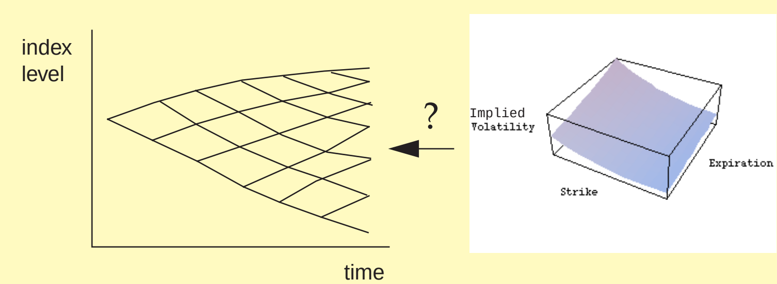
Figure 14.6 illustrates a traditional binomial tree of index levels. The left edge of the tree denotes the current price. Each step up or down from there illustrates a potential future stock price move. Traditional binomial trees make the key assumption that all moves on the tree are of equal percentage magnitude; at any future time, at any future level, the index, whether it moves up or down, grows or shrinks by an identical percentage. In technical terms, the index has a constant volatility of returns, globally the same across the entire tree, identical at each future instant of time and index level. This constancy of the index's volatility in the Black-Scholes model leads to the associated flat implied volatility surface that is inconsistent with actual options markets.

FIGURE 14.6. A binomial tree of future index moves. Each future internal move is identical and represents an equal percentage move and a constant index volatility. The inset shows the shape of the corresponding implied volatility surface.



Iraj and I developed an alternate view of the future tree of index levels. We pictured the usual constant-volatility binomial tree redrawn on a flexible sheet of rubber, which we could then stretch and distort so as to resemble the tree of Figure 14.7. On this deformed tree, the size of the moves in the index at each node of the tree could differ, representing a varying volatility whose value differed from node to node. In theorists' jargon, the index would have a varying *local volatility*, where by "local volatility" we meant the short-term volatility of the index at a particular future level and time. The constant or global volatility of Figure 14.6 was inconsistent with the market's tent-like implied volatility surface of Figure 14.2. There must be, we figured, an *implied binomial tree* whose local volatility could be chosen to match the market's implied volatility surface. We expected it to look like the tree in Figure 14.7, in which the local volatility of the index rises as the index falls and vice versa, in order to reflect the surface's variation with strike.

FIGURE 14.7. An implied binomial tree of future index moves. Each future internal move has a local volatility that rises as the index level drops. Can you deduce the shape of the implied binomial tree from the implied volatility surfce at right?



It was easy to imagine such a tree. It was even easy to build such a tree by literally making up a rule for how local volatility varied within the tree and then constructing it. Given such a tree, you could use it to calculate the prices of many different options and then plot their implied volatility surface. We could see that it was possible to pick a local volatility whose variation produced a realistic looking volatility surface. But the ultimate problem we faced was the inverse of what we were doing. We needed to start with the volatility surface the market gave you and deduce from it the unique local volatilities that reflected it. The implied volatility surface was the primary object, and the whole procedure we envisaged would only constitute a true theory if you could extract from it a unique implied binomial tree.

Throughout 1993, as the QS group continued to build the more elaborate risk-management systems that occupied most of our time, we continued to ruminate over the smile. In spare moments we tinkered with implied trees, still uncertain as to whether there was a truly unique relationship between the volatility surface in Figure 14.7 and the tree we hoped it implied. We knew we could go from the tree to the surface, but what was the incontrovertible way to go from the surface to the tree? We discussed it with Dave Rogers and his traders, who, because of our sheet-of-rubber analogy, always called it the *flexible tree*. We built versions of it and used them to price and hedge varieties of options, but, too busy with supporting the software needs of the desk, we avoided an all-out attempt on the question of uniqueness.

The relationship between surface and tree reminded me of the lecture by Mark Kaç I had heard as a graduate student at Columbia thirty years earlier, when he solved the question of hearing the shape of a drum. Physicists call this an inverse-scattering problem because, whereas most models in physics proceed from the physical law to the results, inverse problems work backwards. Newton's theory of gravitation, for example, commences with the law of gravitational attraction between the sun and the planets, and deduces the planetary orbits. Inverse scattering problems go in reverse – given the observations, they ask, what law would produce them? Imagine, for example, that astronomers observed some strange perturbation in the orbit of the earth. What change in the law of gravitational attraction would account for it?

Our search for a method to extract a unique implied tree from the volatility surface was an inverse scattering problem too. This approach is more typical of financial modeling than it is of physics. For in physics, the beauty and elegance of a theory's laws, and the intuition that led to them, is often compelling, and provides a natural starting point from which to proceed to phenomena. In finance, more of a social than a natural science, there are few beautiful theories and virtually no compelling ones, and so we have no choice but to take the phenomenological approach. There, much more often, one begins with the market's data and calibrates the model's laws to fit it. This calibration is a kind of inverse-scattering approach too, and it was what we were trying to do in our attempt to construct implied trees.

Sometime in late 1993 I went to visit our trading desk in London, where I also gave a talk at a Risk magazine conference on exotic options. Between conference sessions, I met Graham Cooper, the new editor of Risk, and also ran into John Hull. During our conversations I told them what Iraj and I had been exploring. Graham and John told me that they had heard that Bruno Dupire at Paribas Capital Markets in London and Mark Rubinstein, the Berkeley finance professor who was one of the co-developers of the original constant-volatility binomial tree model, had been tackling the same problem. Worried about giving away proprietary information to our competition, I called Dave Rogers in New York and quickly got his approval to allude in public to what Iraj and I had done. I hurried back to my hotel room and quickly appended a few transparencies to my presentation to describe our implied tree approach. After my talk, Graham invited me to submit an article on our work to Risk, while John, hearing me describe our trees sometimes as "flexible" and sometimes as "implied," nudged me towards the use of "implied."

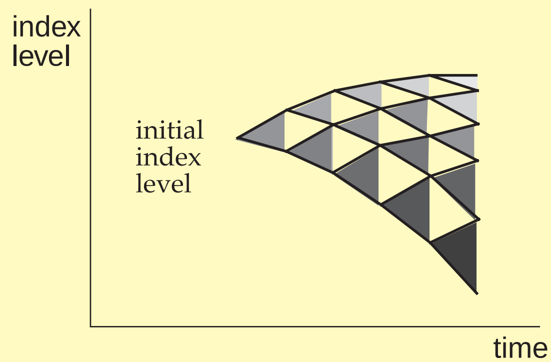
With competition at our heels, Iraj and I anxiously rededicated ourselves to proving the uniqueness our tree. Most of our day was typically spent enhancing the desk's trading models, responding to requests for pricing new structures, and building trading software. Whenever we had spare time away from desk support, we tried again to define a scheme for uniquely extracting the local volatility at each future node of the implied binomial tree.

We began with the market's implied volatility surface on a given day, as ilustrated in Equation 14.2. We then constructed a binomial tree like the one if Figure 14.8. Each shaded triangle in this tree, at each index level and future time, carries a different local volatility, whose magnitude is represented by the degree of shading. Higher index levels correspond to lower (paler) volatilities; lower index levels correspond to higher (darker) volatilities. How pale or how dark must you choose them to match the implied volatility surface of Equation 14.2? That was the question.

The local volatility in Figure 14.8 is a *local* quality of the tree, the microscopically viewed volatility within each single small internal triangle. In contrast, the implied volatility in Equation 14.2 is a *global* quality, an wide-angle overview of all the internal triangles seen from 30,000 feet. We viewed the implied volatility of an option as the average* of all the local volatilities that the index will experience during the life of that option.

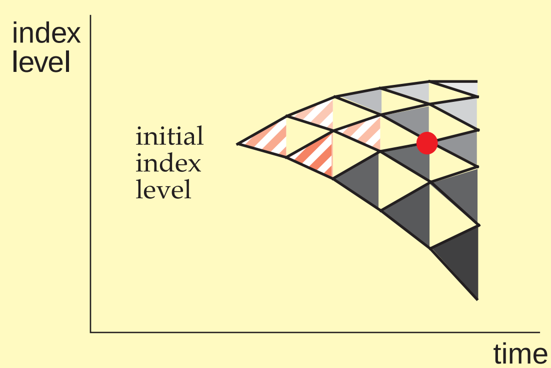
* A mathematically complex average, to be honest, but an average nonetheless.

FIGURE 14.8. An implied binomial tree with varying local volatilities represented by the shaded triangles in the tree.



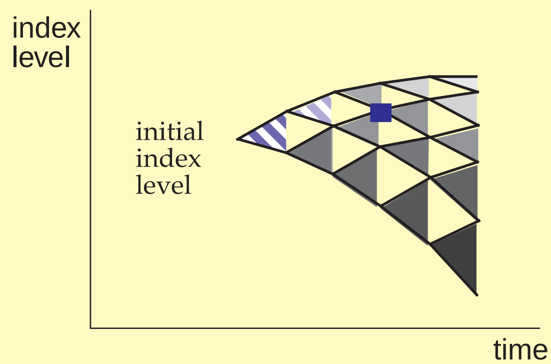
Consider the option whose expiration and strike correspond to the time and index level of the small circle in the next-to-last-row of the tree in Figure 14.9. The value of its implied volatility depends upon the values of the local volatilities in the shaded right-striped triangles; those are the local volatility regions that the index can traverse in moving towards the strike during the life of the option. It's useful to think of the option expiring at the small circle as an X-ray source that illuminates the local volatilities in internal right-striped nodes of the tree.

FIGURE 14.9. The implied volatility of the option whose strike lies at the circle illuminates the local volatilities in the right-striped region of the tree.



Similarly, the option whose strike lies at the little square in the tree of Figure 14.10 illuminates the local volatilities in the shaded left-striped triangles.

FIGURE 14.10. The implied volatility of the option whose strike lies at the square illuminates the local volatilities in the left-striped region of the tree.

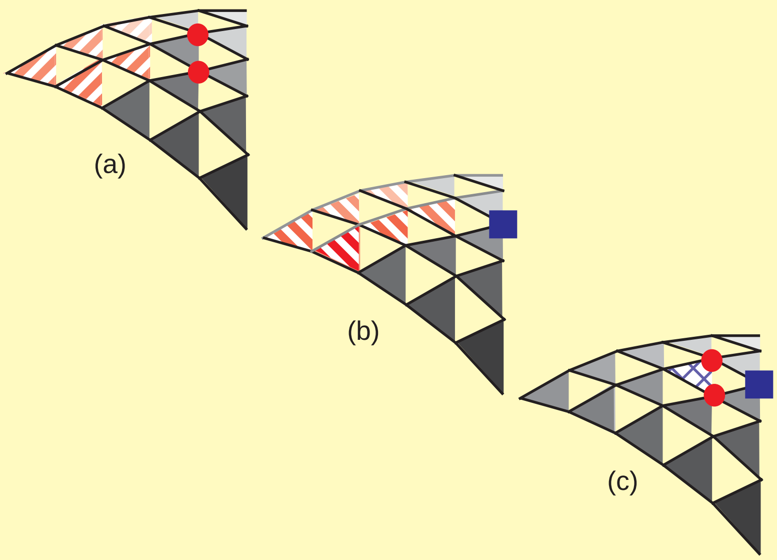


The option with a strike at the circle illuminates one part of the tree, while the option whose strike lies at the square illuminates another part. But no single option, neither the one struck at the circle nor the one struck at the square, sheds light on just one triangle in the tree, the single node whose volatility was the obscure object of our desire.

We kept struggling. We wanted a set of options that illuminated the volatilities at just one internal node. But each scheme we tried failed – there seemed to be no recipe for the local volatility at a single node.

Then, one day, as we played around with a five-row toy version of our tree on a spreadsheet, we found something miraculous happened, something so strange that for a few minutes we thought it was due to a programming error in the spreadsheet. We noticed, almost by accident, that if you used *three* distinct option strikes to illuminate the interior of the tree, two of them with adjacent earlier strikes and one with a strike one period later – if, so to speak, you directed X-rays into the tree from three different angles – the illumination cancelled everywhere except at the single node where they intersected, as illustrated in Figure 14.11. It was astonishing: you could determine the local volatility at a single node in terms of the market implied volatilities of options with strikes at the three surrounding nodes.

FIGURE 14.11. How three options can illuminate one node. (a) Two options with adjacent strikes illuminate the triangles below them. (b) A single option expiring one period later illuminates all the previous triangles plus one more. (c) Subtracting the earlier expirations from the later leaves just one internal triangle illuminated.



Now we knew how to find every local volatility, step by step. We could select a node on the implied tree, read off the implied volatilities of the three options adjacent to it off from the market's implied volatility surface, and then extract the local volatility at that node via our algorithm. One step at a time, we could find all the local volatilities. With those local volatilities and the implied binomial tree on which they sat, we could value and hedge any option on the index in a manner consistent with the smile. We were immensely elated, believing we had made the next big breakthrough in options pricing, an extension of the Black-Scholes model to make it consistent with reality.

We weren't the only excited ones. Mark Rubinstein and Bruno Dupire had also spent the past year developing similar extensions of Black-Scholes. A few weeks later Mark delivered his presidential address at the January 1994 meeting of the American Finance Association on the topic of *Implied Binomial Trees*. Speaking to him over subsequent years, I learned that he too imagined he had achieved a great breakthrough.

In the interim John Hull mailed me a copy of a talk Bruno had given on his version of implied trees at a meeting of the International Association of Financial Engineers in New York a few weeks earlier. In it, Bruno also claimed to have found a unique way to extract local volatilities from implieds.

Each of us – Iraj and I together, Mark and Bruno separately – had tackled the inverse scattering problem in characteristically different styles. Iraj and I were used to dealing with the denizens of the investment banking world, the less numerate traders, salespeople and clients, and so we wrote our paper as simply and clearly as possible. We wanted anyone reading it to know exactly how to build their own implied tree. We explained, step by step, exactly how you could build the tree and calibrate it to a given volatility surface, and we illustrated it with a fully worked-out numerical example involving a five-period tree that anyone could check for themselves.

Mark's presentation was more discursive and academic. His solution to the inverse scattering problem was not as complete: whereas Iraj and I could match the entire implied volatility surface – all strikes and all expirations – Mark's method could match all strikes at only a single expiration.

Bruno's IAFE talk was the most tantalizing. Being French, he had a taste for formal mathematics, and his very brief report proposed an elegant formula for the local volatilities in terms of the slope and curvature of the implied volatility surface at the same strike and expiration. He wasn't easy to understand and people I spoke to weren't sure it was correct.

We studied Bruno's report and soon realized that his concise formula was exactly equivalent to what Iraj and I had developed on a discrete tree. Where he had used calculus, we had used algebra. Though it wasn't anywhere near as important or grand, we were playing Feynman to his Schwinger or Black & Scholes to his Merton. In an appendix to our paper we rederived Bruno's results and included a more transparent proof and a reference to his work.

In late December 1993 both Bruno and Iraj & I submitted our respective papers to Graham Cooper at Risk Magazine. Bruno's appeared in Risk's January 1994 issue, together with an editorial explaining that ours would follow in February. The January issue also contained a page-long news report by Graham Cooper about the work that Mark, Bruno and Iraj & I had done, referring to it as the new "supermodel," and rather accurately describing the relative merits of our individual approaches. We were all suitably flattered.

For the next several years we toured the seminar and conference circuit. I spoke at dozens of university finance departments and business schools, at the Vienna options exchange, and at countless industry conferences. Salespeople in Tokyo, France, Switzerland, Spain and Italy, countries where the more sophisticated traders loved learning about quantitative theories, took me to see client after client. We gave day-long seminars to large groups in Zurich and London, Bilbao and Paris, Milan and Munich. It was exhilarating.

Iraj and I, together with Mike Kamal and Joe Zou, two more ex-physicists who joined our group in 1994, continued to enhance the model and to use it to value exotic options. We also worked with two software engineers in QS, Deniz Ergener and Alan Buckwalter, to embed the model in the desk's trading software. Most of all, we struggled to abstract the model's mathematical features into a visceral understanding that would make it comprehensible to traders. Since all their intuition was based on the Black-Scholes model, we developed a series of simple approximate correctionss, rules of thumb as we called them, that traders 'could use to nudge the Black-Scholes model in the direction of implied trees.

It turned out to be much harder to explain the model to our traders than to our clients. The traders were busy, their life dominated by watching screens, servicing salespeople and entering the terms of their transactions into their computer systems. Each day they had to book hundreds or thousands of new trades, often staying late into the night to make sure all the details were correctly reconciled between the front-office risk management system on their desks and the back-office mainframe that held the firm's books and records. They were more interested in automating the the book-keeping aspects of their life than in improving their pricing.

And the traders were in charge, and used whatever model they wanted without reason or justification. If you wanted them to hedge differently, the burden of proof was on you. They weren't stupid, just sensibly philistine, averse to using new models they didn't understand, but also, unfortunately, averse to spending time on gaining understanding. When traders have no model at all, it's easy to get them to use the very first model you invent. Once they have something they rely on, it's much harder to get them to accept an improvement.

So, they simply stuck to using the Black-Scholes single-volatility framework for valuing exotics, even though it produced a flat volatility surface. To compensate, they put all their inventive energy and intuition into picking the "right" single volatility to use in the wrong model. One senior trader insisted that even if the implied tree model was correct and the Black-Scholes model was wrong, he would always be able to think his way through to the correspondingly appropriate single volatility that, when inserted into the wrong model, would nevertheless produce the correct value for an option. I was therefore very pleased one day to discover that there were certain exotic options whose correct value in the implied tree model lay completely outside the range of values you could obtain from Black-Scholes model, no matter what single volatility you entered into it. There was simply no appropriate single volatility that gave the correct answer from the wrong model. I showed this example to him with great glee and some vindictiveness. It was a minor victory, and it failed to alter anyone's behavior.

By 2000, the tide had begun to turn a little. Increasingly, at Goldman and its competitors, trading desks were becoming obliged to provide some justification that the models they used were appropriate for their market. Nevertheless, in the tug of war between traders and risk managers, traders had greater pull.

One day in late 1994 the tension between traders and quants led me to do something particularly stupid. I had spent all day dealing with impatient demands for systems support I couldn't provide. That evening I took a limo out to Kennedy airport to catch a flight to Vienna, where I was scheduled to give a talk on implied trees at a conference at the OTÖB, the Austrian options exchange. Boarding my plane, I sat down in my business-class aisle seat and finally relaxed. I was frustrated by the constant battles at work, and swore that I would be be pushed around no longer.

As I unwound before takeoff, a family of three boarded the plane at the last minute and began taking their seats, none of which were contiguous. The father, a gentleman of about fifty, took the window seat on my right, his son sat across the aisle from me, and his wife sat further towards the front of the plane. As I flipped through the pages of the OTÖB conference program, the father conversed with the flight attendant, trying to see if he could find a set of three contiguous seats for his family. Finally, after about ten minutes of unsuccessful agitation, he turned to me and asked if I would switch seats with his son.

Still smarting from being pushed around all day, I remembered my "No more Mr. Nice Guy" promise to myself. I was seated in the aisle seat I had requested and now I wasn't going to give it up for anyone. Turning to him with misplaced firmness, I said, "I'm sorry, but I'd rather stay where I am."

As we took off for Vienna, I was appalled at my pointless recalcitrance. Both my seat and his son's seat were on the aisle. I was gaining nothing by being stubborn. And worse, I had now condemned myself to sit for ten hours next to someone to whom I had been unnecessarily objectionable. Guilt began to overwhelm me as I debated with myself how to undo what I had done.

While I writhed, I continued looking through the conference schedule. Then, I noticed, the man on my right extracted a similar schedule from his briefcase and began to look at it too. I glanced at his face again, and suddenly realized I was sitting next to Bob Merton himself, the developer of continuous-time financial modeling, a Harvard professor and also a partner at Long-Term Capital Management. I had seen his picture just that morning on the inside flap of the cover of his famous book on options theory when I had been reading about jump-diffusion models.

I turned to him shamefacedly and apologized for my rudeness. By now his son had fallen asleep, and there was no point in exchanging seats, so we talked for a couple of hours about the history of options pricing models. Though Bob was very gracious, I felt mortified at having displayed tis unpleasant part of my character, and for the duration of the conference I found myself instinctively avoiding him and his family. Next time, I promised myself, I'd be tough on the people who deserve it.

Though Mark, Bruno and Iraj & I were among the first theorists to use local volatility to tackle the smile, various other people had similar ideas. In particular, I came across a closely connected paper by Avi Bick, an Israeli finance professor at Simon Fraser University in Canada. The time seemed ripe for exploiting exploiting our inverse-scattering approach to valuation. In November 1994, speaking at NYU conference on *Derivatives: The State of the Art*, I was buoyed to hear Gary Gastineau, himself the author of an options text, comment that our model would make markets more liquid by making options pricing more accurate. But though we did live to see local volatility become a household world and a textbook topic, I discovered that it was much more difficult than I had imagined to create a truly successful financial model.

Local volatility was an improvement on Black-Scholes in that it could account for the smile, but it had some genuine failings. First, our model excluded the possibility that an index or stock could jump, and most market participants nowadays regard that possibility as the main factor determining the shape of the very short-term volatility smile. Our very first attempt to model the smile had indeed involved such jumps. We were never fond of jump models – since jumps are too violent and discontinuous to be hedged, when you include them you lose much of the consistency of the Black-Scholes model. But jumps are real, and omitting them made our model less realistic.

Second, implied trees were difficult to calibrate. Often, as you tried to build progressively finer-meshed trees for better computational accuracy, the local volatility surface grew wild, displaying unrealistic waves and troughs as it varied from point to point. Over time we developed methods for smoothing these fluctuations, but the need to smooth them made it difficult to automate the production of implied trees for the desk. These waves and troughs were themselves a consequence having excluded jumps; we were trying to model a violent phenomenon with only a tranquil diffusion, which was bound to make calibration unstable.

Finally, our model ignored the random nature of volatility itself. Iraj tried to enhance our implied tree model a few years later by adding a random component to the local volatilities, but it made calibration and calculation even more complex and unwieldy.

Over time we discovered that you can model the smile with local volatility, jumps, random volatility, or some mixture of all three of these effects. So, local volatility and implied trees didn't become "the" single model of the smile. Years later, at a dinner in Manhattan discussing the past, Mark Rubinstein and I both laughed ruefully at the mismatch between what we had expected and the way things turned out .

And yet, and yet ..... I was ultimately satisfied with the result. Iraj & I had been among the first to propose a consistent model of a new and strange phenomenon; we had created a new framework and vocabulary, and we had done it from the front lines, on Wall St., not from a leisurely research position in academia. The model was somewhat simplistic, as all models are, and it wasn't the whole story, but it was a plausible, self-consistent little world that captured one true and essential feature of equity volatility markets, that volatility tended to increase when the market falls. Local volatility models have become part of the standard arsenal of tools used by academics and practitioners.

During the 1990's, volatility smiles spread to almost all other options markets. Wherever there was fear of large market moves that could hurt investors – downwards for equities, upwards for gold, in either direction for many currencies and interest rates – there smiles and skews appeared, and our model became a key tool in explaining at least some of these features. By 2000 smile models were ubiquitous, with Risk magazine conferences devoting entire sessions, year after year, to these issues. In the Firmwide Risk area at Goldman, Sachs, I ran a team of about twelve Ph.Ds in the Derivatives Analysis group that had to approve the prices set and the models used throughout firm's derivatives businesses, and we soon found that every desk had its own smile model (all of them different), and that most of our work involved the verification of these models. The models differed from desk to desk because their markets differed; each market had its own characteristic smile for its own idiosyncratic reasons. Equity markets feared a crash; gold markets, after years of low prices, feared a sudden upward move; in interest-rate markets, bond investors feared the high rates that would devalue their assets while insurance companies, who often guaranteed their clients a minimum rate of interest, feared the low rates that would diminish their incoming cash flow; in currency markets, investors feared a move outside some stable band. Each fear, based on bitter experience, corresponded to a different pattern and required a different model. There was no single model for the smile.

Years ago, when I first became aware of the smile and hoped to find the "right" model, I used to ask colleagues at other firms which model they thought was correct. But now there is such a profusion of models that I ask more practical questions – not "What do you believe?" but "When you hedge a standard S&P 500 option, do you use the Black-Scholes hedge ratio, something larger or something smaller?" Local volatility models produce smaller hedge ratios, while stochastic volatility models tend to produce larger ratios. The differences between the models are even more dramatic for exotic options.

In 2003, at a derivatives meeting in Barcelona, I led a small round-table discussion group on the smile. There were fifteen of us, traders and quants from derivatives desks all over the world. I asked my simple question: Would you use the Black-Scholes hedge ratio, something larger or something smaller? I was surprised that ten years after the first smile models appeared, with the smile a fact of life in almost every derivatives market, after thousands of published papers, there was still no consensus on how to respond to it.

There still isn't. Though we know much more about the *theories* of the smile, we are still on a darkling plain as regards what's correct. A decade of speaking with traders and theorists has made me wonder what "correct" means. If you are a theorist you must never forget that you are travelling through lawless roads where the local inhabitants don't respect your principles. The more I look at the conflict between markets and theories, the more limitations of models of the financial and human world become apparent to me.