ECON 111: Fall 2021
Tyler Porter
Discussion: October 27, 2021

1. Suppose that the market demand for ink pens is given as:

$$Q_d = 20 - 5P$$

where quantity is in thousands of pens. Suppose that the marginal cost of producing a pen is constant at \$1 for each firm in the market, regardless of how firms are present. Assume that fixed costs are 0, so that entry into the market is free.

(a) Sketch the market demand and marginal cost curve for this market.

(b) Suppose that the market is populated by a very large number of competitive firms. This is the case of perfect competition. Find the market equilibrium in this instance and compute total surplus. How much profit does each firm earn?

(c) Suppose now that, rather than a large number of perfectly competitive firms, there is a single monopolist in the market. Sketch the marginal revenue curve for this monopolist. What quantity should the monopolist produce?

(d) Compute the deadweight loss associated with the inefficiently low production of the monopolist. Briefly provide an economic interpretation of this deadweight loss.

(e) Finally, let's consider an instance in which the market is populated by two firms that sell differentiated ink pens. For example, firm A sells ink pens that have a rubber grip and a clicky-top, while firm B sells ink pens that have a foam grip and a twisty-bottom. These firms face the same market demand curve as before. In this case, their joint production determines the market price:

$$P = 4 - \frac{Q_A + Q_B}{5}$$

Suppose that firm B decides to produce 5 thousand ink-pens. Sketch the market price as a function of firm A's quantity, then sketch firm A's marginal revenue curve. What quantity should firm A produce?

(f) Let's now suppose keep firm B's production as a variable, just Q_B. Write down firm A's marginal revenue curve.

(g) Using your answer from part (f), write down firm A's profit-maximizing quantity Q_A as a function of Q_B. Sketch this curve.

(h) Also sketch firm B's profit-maximizing quantity as a function of Q_A and sketch the curve in your plot from part (g). Find the equilibrium quantity for each firm.

(i) How does the total quantity (for both firms) from part (h) compare to the competitive equilibrium and monopolistic quantity? Compute the deadweight loss associated with the oligopoly equilibrium from part (h).

1. (a) Demand and marginal cost are sketched in the following figure:

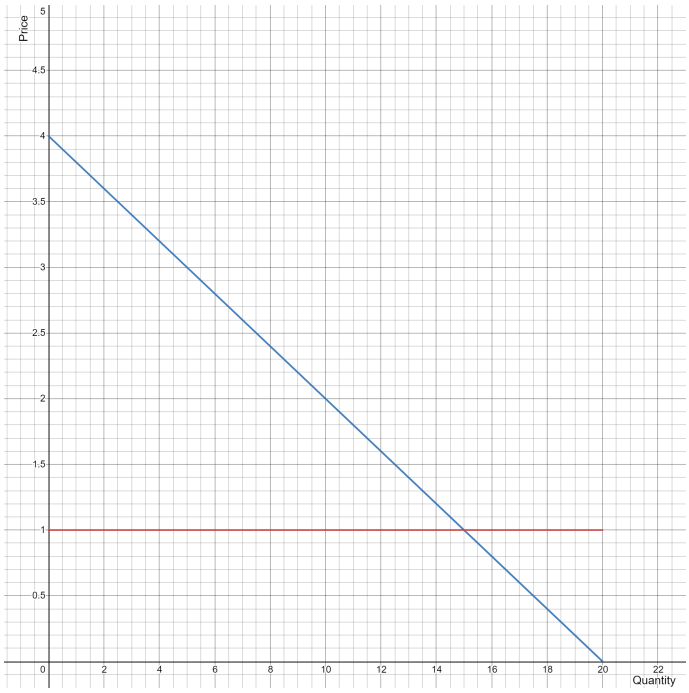

Figure 1: Demand (blue) and marginal cost (red)

(b) The equilibrium would be where marginal cost intersects the demand curve. This can be found by rewriting the demand equation in terms of price:

$$P = 4 - \frac{Q_d}{5}$$

Equating this equal to 1 yields our efficient outcome of:

$$4 - \frac{Q_d}{5} = 1 \Rightarrow Q_d = 15$$

Total surplus is then:

$$(4-1) \cdot \frac{15}{2} = \frac{45}{2}$$

in dollars. Profit in this market is 0, as each firm is charging a price equal to its constant marginal cost.

(c) The following figure sketches the monopolist's marginal revenue curve:

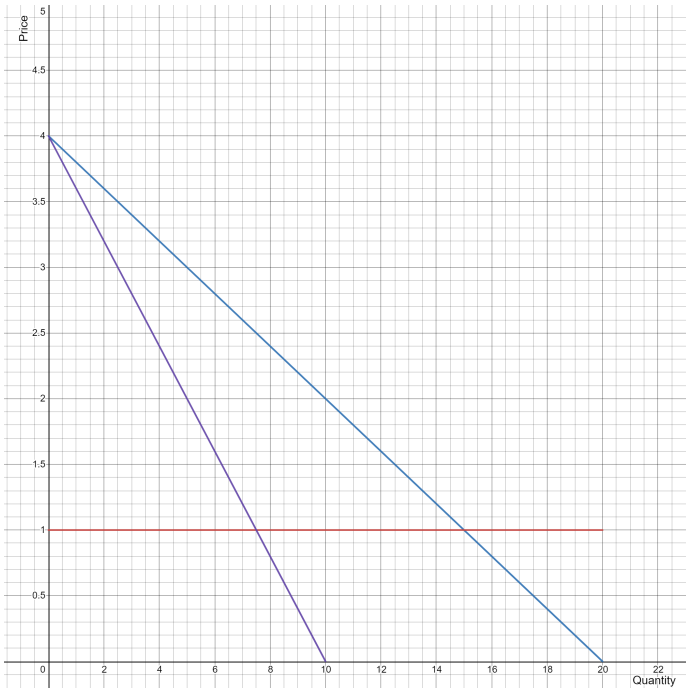

Figure 2: Marginal revenue (purple)

The monopolist's profit maximizing quantity occurs when MR = MC. The equation for the marginal revenue curve is given by:

$$MR = 4 - \frac{2Q}{5}$$

Setting this equal to the marginal cost of 1 yields:

$$4 - \frac{2Q}{5} = 1 \Rightarrow Q = \frac{15}{2}$$

(d) The following figure sketches the deadweight loss associated with the monopolist's profit-maximizing price policy:

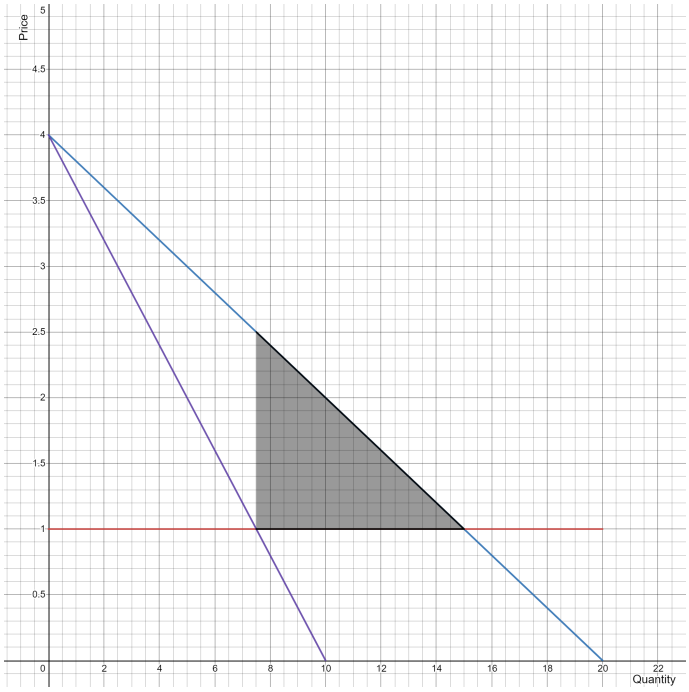

Figure 3: Deadweight loss (gray)

This can be compute as:

$$(2.5 - 1) \cdot \frac{(15 - 7.5)}{2} = \frac{45}{8}$$

Here, deadweight represents transactions which would generate positive surplus, but which are precluded due to the monopolist choosing to set a price above their marginal cost.

(e) If firm B chooses to produce 5 thousand ink-pens, firm A faces the following relationship between the market price P and thier own quantity Q_A:

$$P = 3 - \frac{Q_A}{5}$$

This implies that the marginal revenue curve for firm A, given firm B's production, is given by:

$$MR = 3 - \frac{2Q_A}{5}$$

Each of these curves are sketched in the following figure:

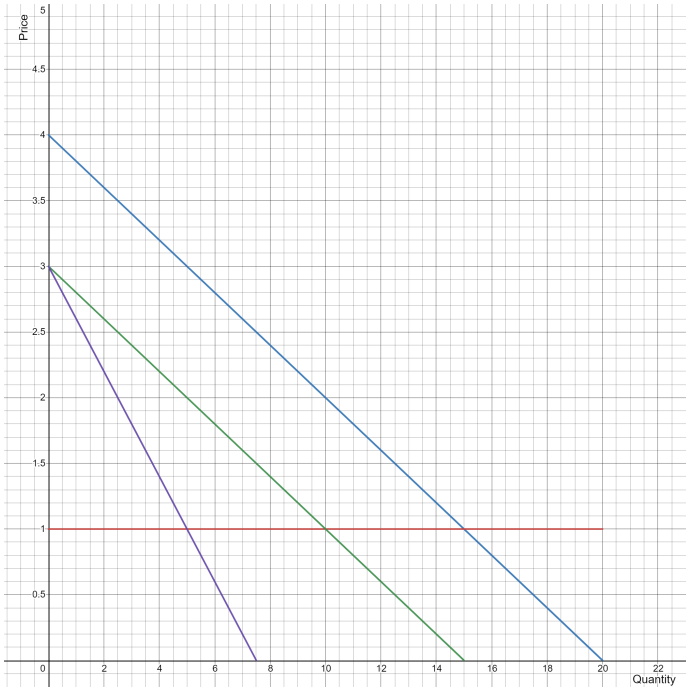

Figure 4: Firm A's marginal revenue (purple) and demand (green) given firm B's production of 5 thousand ink-pens

Firm A's profit-maximizing quantity is 5 thousand ink-pens in response to firm B's quantity.

(f) Similar to the previous part, firm A's marginal revenue curve can be written as:

$$MR = (4 - \frac{Q_B}{5}) - \frac{2Q_A}{5}$$

(g) Firm A should maximize profit by choosing a quantity such that MR = MC. Given the marginal revenue curve from part (f), we see that they should solve:

$$(4 - \frac{Q_B}{5}) - \frac{2Q_A}{5} = 1$$

This gives us firm A's profit-maximizing quantity in response to firm B's production:

$$Q_A = \frac{15}{2} - \frac{Q_B}{2}$$

This curve, as well as firm B's best-response function in part (h), are sketched in the following figure:

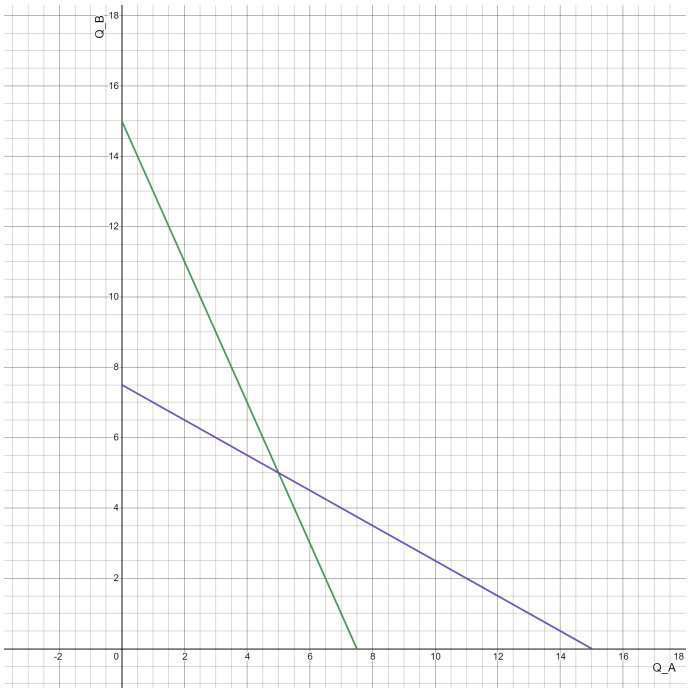

Figure 5: Firm A's best response function (green) and firm B's best-response function (purple)

Notice that when either firm chooses to produce nothing, the other firm will produce $\frac{15}{2}$, the monopoly quantity.

(h) To find the equilibrium quantity, we solve the following system of equations for each firm's best response to the other firm's quantity decision:

$$Q_A = \frac{15}{2} - \frac{Q_B}{2}$$
$$Q_B = \frac{15}{2} - \frac{Q_A}{2}$$

Solving this system by, for example, substitution, yields $Q_A = Q_B = 5$.

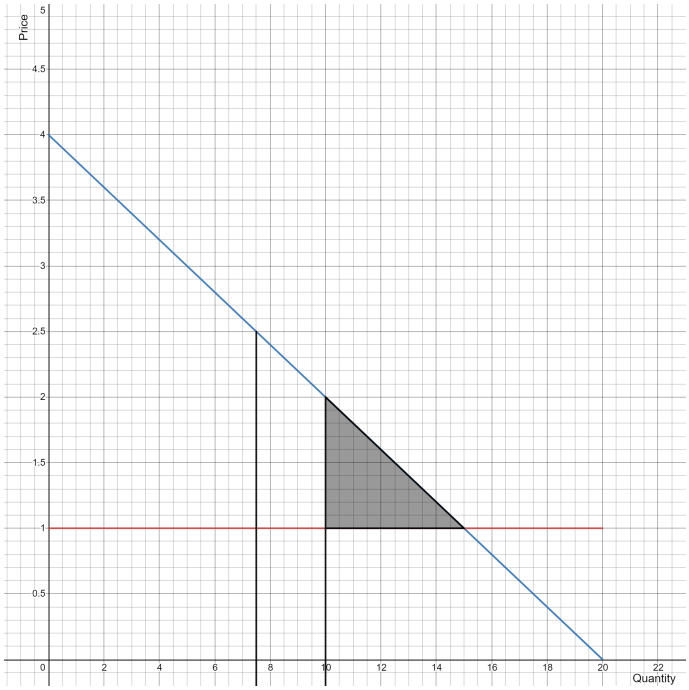

Figure 6: Deadweight loss (gray)

(i) The total quantity produced in the duopoly outcome is $Q_A + Q_B = 5 + 5 = 10$. We see that this is larger than the monopoly quantity of $\frac{15}{2}$. Sketching this on the graph with the monopoly quantity, we obtain the following figure: The deadweight loss is calculated as:

$$(2-1) \cdot \frac{15-10}{2} = \frac{5}{2}$$