ECON 111: Fall 2021
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Today we'll go over some example problems on surplus and elasticity.

1. Consider the following market supply and demand curves:

$$P = 20 + \frac{Q_s}{3}$$
$$P = 50 - \frac{5Q_d}{3}$$

   (a) Sketch the supply and demand curves, labeling the axes. Compute the equilibrium price and quantity.
   (b) Compute consumer and producer surplus at the market equilibrium.
   (c) Compute the price elasticity of demand and price elasticity of supply at the equilibrium point.
   (d) Suppose now that the price of an input for suppliers increases. After the market adjusts to the new equilibrium, do you expect demand to be elastic or inelastic at the equilibrium point? Why?
   (e) Suppose now that instead of the change from part (d), a price floor of 30 is imposed.
      i. Would this result in excess supply or excess demand? How much?
      ii. Would the equilibrium after the price floor be efficient? If it is efficient, explain. If it is not efficient, compute the deadweight loss at the new outcome.
      iii. Which side of the market, if any, has a higher surplus after this change?
   (f) Suppose now that the market is populated by a single firm, instead of a large collection of similar price-taking firms. Suppose also that this single firm is able to perfectly price discriminate in the sense that they are able to charge every consumer their exact willingness to pay for the good. Reason about consumer and producer surplus when this lone firm is able to engage in this behavior. Is the outcome efficient? Why or why not?

1. (a) A sketch of the supply and demand curves is provided below:

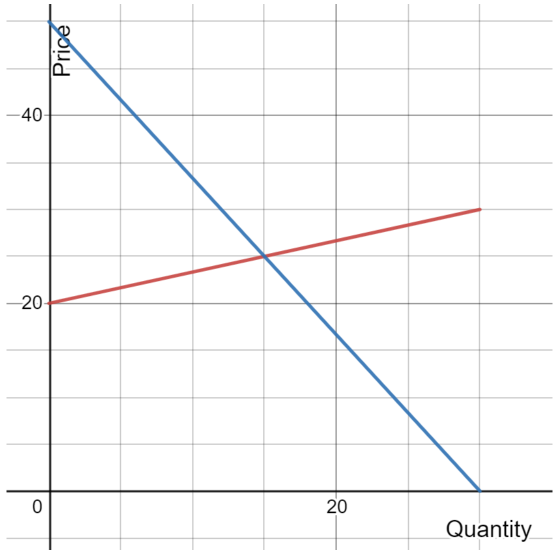

Figure 1: Demand (blue) and Supply (red)

The equilibrium quantity is 15 and the price is 25.

(b) The following figure provides the shaded regions for consumer and producer surplus in this case.

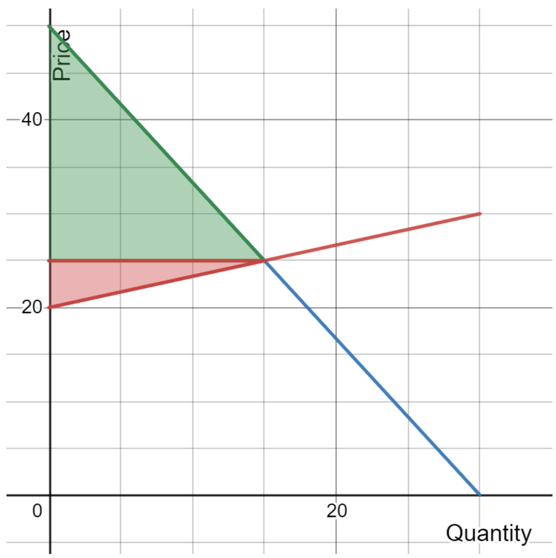

Figure 2: Consumer surplus (green) and producer surplus (red)

Consumer surplus in this case is: $(50-25)\cdot\frac{15}{2}=\frac{375}{2}$ Producer surplus in this case is: $(25-20)\cdot\frac{15}{2}=\frac{75}{2}$

(c) Computing price elasticity of demand at equilibrium, it helps to rewrite the demand curve as:

$$Q_d = 30 - \frac{3}{5}P$$

So that we have $\frac{\Delta Q_d}{\Delta P}=-\frac{3}{5}$. Price elasticity of demand is then:

$$\epsilon_{p,d} = \frac{\Delta Q_d}{\Delta P}\frac{P}{Q} = -\frac{3}{5}\cdot\frac{25}{15} = -1$$

Similarly, to compute the price elasticity of supply we can rewrite supply as:

$$Q_s = 3P - 60$$

so that $\frac{\Delta Q_s}{\Delta P} = 3$. Price elasticity of supply at the equilibrium point is then:

$$\epsilon_{p,s} = \frac{\Delta Q_s}{\Delta P}\frac{P}{Q} = 3 \cdot \frac{25}{15} = 5$$

(d) Note that since the price of an input rose, supply should decrease. That is to say that at any price, the market quantity supplied is lower. The following figure demonstrates a possible shift in supply:

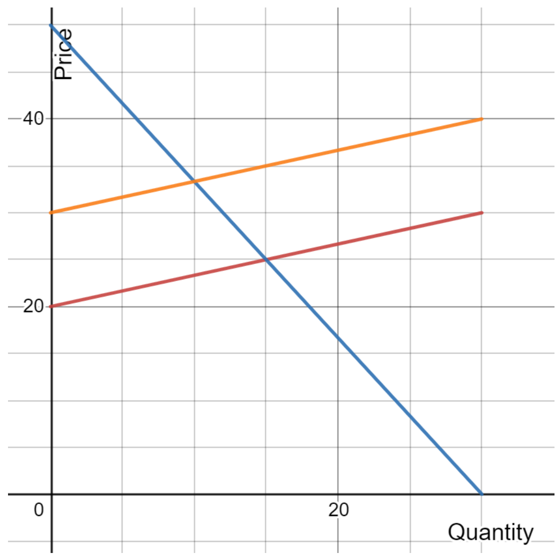

Figure 3: New supply (yellow)

Now, we see that we have moved to a point on the demand curve corresponding to a higher price and a lower quantity. Let us once again review the formula for the price elasticity of demand:

$$\epsilon_{p,d} = \frac{\Delta Q_d}{\Delta P}\frac{P}{Q}$$

Notice that at *any* point along the demand curve, the expression $\frac{\Delta Q_d}{\Delta P}$ is the same. Hence the only thing that changes at the new equilibrium is the ratio $\frac{P}{Q}$. Since the price is higher at the new equilibrium, and the quantity is lower, it must be that the ratio $\frac{P}{Q}$ is higher at the new equilibrium. Therefore, the price elasticity of demand must be higher as a result of this change. This is a general feature of linear demand curves: demand is more elastic at points with higher P and lower Q, and more inelastic at points with lower P and higher Q.

(e) The following figure summarizes the results in part (e).

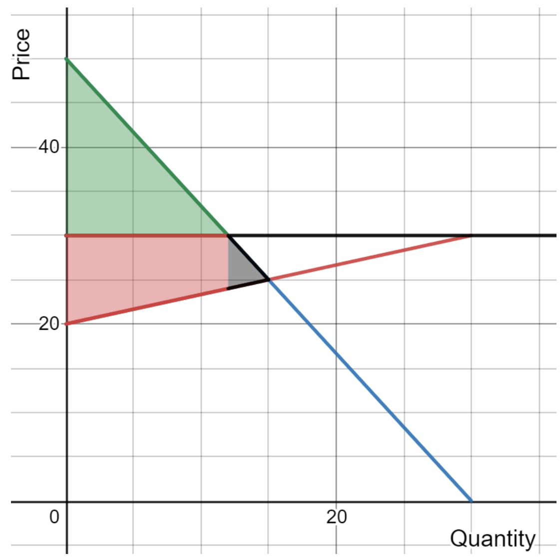

Figure 4: Price floor (black line), consumer surplus (green), producer surplus (red) and deadweight loss (gray)

We see that the price floor results in excess supply (a surplus). To compute this, all we need to do is find the quantities corresponding to a price of \$30 on the demand and supply curves. These are:

$$30 = 20 + \frac{Q_s}{3} \Rightarrow Q_s = 30$$
$$30 = 50 - \frac{5Q_d}{3} \Rightarrow Q_d = 12$$

and hence we have a surplus of 18 units.

Since the original equilibrium was efficient (there were no externalities, and we had perfect competition), then the new outcome after the price floor features some deadweight loss. To compute this deadweight loss, notice that the new equilibrium quantity is $Q = 12$. Now, we need to determine the prices on the supply and demand curves that correspond to a quantity of 12. We have:

$$P = 20 + \frac{12}{3} \Rightarrow P = 24$$
$$P = 50 - \frac{5}{3}(12) \Rightarrow P = 30$$

And hence deadweight loss is calculated as the area of the gray triangle:

$$DWL = (30 - 24) \cdot \frac{12}{2} = 36$$

2. The short answer is that since the single firm can charge every consumer their willingness to pay, then this new outcome will be efficient because the price-discriminating firm will continue to sell until their marginal cost curve meets the demand curve. They receive *all* of the surplus in the market.

   Do not worry too much about this problem for now. We will return to this later when we discuss the competitive firm.