Proof-Based Exercises for MATH 5440

April 7, 2023
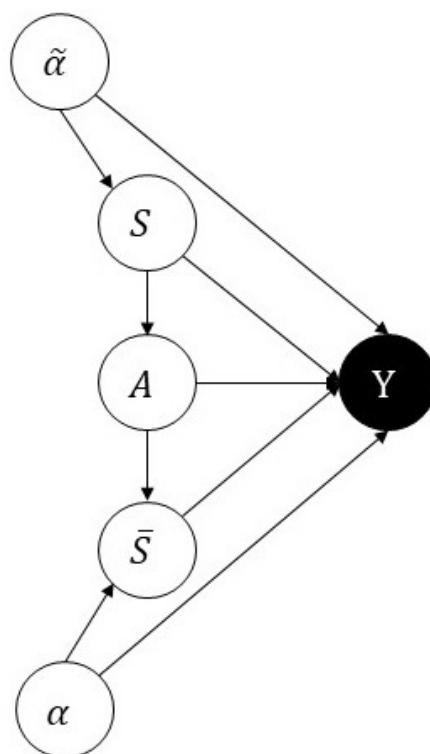

Figure 1: Causal graph for exercises.

Exercise 1   Examples of d-separation

Consider the causal structure outlined in Figure 1. From a trading perspective, this is an extension of the causal structure $\mathcal{A}$ introduced for algowheels. S represents an order's size, A the choice of algorithm, and $\bar{S}$ the

order's speed. As in the lecture, the order's size is determined before the algorithm, and determines which algorithm the order is allocated to. Conversely, the order's speed is decided by the algorithm. This exercise shows which trading variables are independent of each other and which require further conditioning to be independent. The two nodes added are:

(a) A long-term alpha $\tilde{\alpha}$ that predicts long-term returns contributing to Y, and is one of the variables determining the order size S.

(b) A short-term alpha α that predicts short-term returns contributing to Y, and is one of the variables determining the trading speed $\bar{S}$.

Assume that the two alphas are independent and that the trading infrastructure enforces the above causal graph, in particular ruling out causal paths such as $\tilde{\alpha} \to A$, $\tilde{\alpha} \to \bar{S}$, $\alpha \to A$, and $\alpha \to S$.[1]

1. Prove that α and $\tilde{\alpha}$ are independent.
2. Prove that α and $\tilde{\alpha}$ are dependent conditional on $\bar{S}$. Comment on the dependency's nature.
3. Prove that S and $\bar{S}$ are dependent. Comment on the dependency's nature.
4. Prove that S and $\bar{S}$ are independent conditional on A.
5. Prove that A and α are independent.
6. Prove that A and α are dependent conditional on $\bar{S}$.
7. Prove that A and $\tilde{\alpha}$ are dependent.
8. Prove that A and $\tilde{\alpha}$ are independent conditional on S.
9. Comment on the distinction between the dependency structure of A with α and A with $\tilde{\alpha}$

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[1]From an organizational perspective, these causal assumptions simplify the performance attribution of algorithms and alpha signals.