```
# Performing Necessary library imports
import numpy as np
import scipy as sp
from scipy import optimize
import matplotlib.pyplot as plt
```

QUESTION 1

In this question, we will use swap rate quotes to construct a discount factor curve using log-linear interpolation.

Suppose that a discount factor curve is represented by the discount factors for a set of dates D_i's where D_0 is today and D_i $(0 < i \leq n)$ is the maturity date of the ith swap. Discount factor for any other date is interpolated from the discount factor curve.

Let t_i be the time in years from today to the date D_i and let DF_i be its associated discount factor for $0 \leq i \leq n$. Assume that $t_0 = 0$ and $DF_0 = 1.0$.

For a given time t where $t_i \leq t < t_{i+1}$, the log-linearly interpolated discount factor for time t is:

$$DF(t) = \exp\{\ln(DF_i) + (t - t_i)(\ln(DF_{i+1}) - \ln(DF_i))/(t_{i+1} - t_i)\}$$

a) Define a function to return the log-linearly interpolated discount factor for a given time t. For $t > t_n$, this function shall return the discount factor computed from the zero rate of the last discount factor DF_n in the curve. That is: $DF(t) = \exp(-zt)$ and $z = -\ln(DF_n)/t_n$

```
# Question 1 (a)

'''
A function to return the log-linearly interpolated
discount factor for a given time t
'''
def get_df(t, df_times, df_values):

    # if t is less than the smallest element in df_times
    if t < df_times[0]:
        return df_values[0]
    # if t is greater than the largest element in df_times
    elif t > df_times[-1]:
        # calculating df by zero rate of last df
        z = -np.log(df_values[-1]) / df_times[-1]
        df = np.exp(-z * t)
        return df
    # if t is between two elements of df_times
    else:
        # searching for index i where df_times[i] < t < df_times[i+1]
        i = np.searchsorted(df_times, t) - 1
        # calculating df by log-linear interpolation
        df = np.exp(np.log(df_values[i]) + (t - df_times[i]) *
                    (np.log(df_values[i+1]) - np.log(df_values[i])) /
                    (df_times[i+1] - df_times[i]))
        return df
```

b) Define a function to return the simple forward rate of a period. The forward rate output is implied from a given discount factor curve as follows.

For a given forward rate period from t_S to t_E, the simple forward rate is computed as:

$$F(t_S, t_E) = \left(\frac{DF(t_S)}{DF(t_E)} - 1\right)/(t_E - t_S)$$

```
# Question 1 (b)

'''
A function to return the simple forward rate of a period
'''
def get_forward_rate(t1, t2, df_times, df_values):

  # using get_df to calculate df_s and df_e respectively
  df_t1 = get_df(t1, df_times, df_values)
  df_t2 = get_df(t2, df_times, df_values)
```

```
  # calculating forward rate as per given formula
  F = ((df_t1/df_t2) - 1)/(t2 - t1)

  # returning the forward rate
  return F
```

Whenever you have defined some functions, it's always a good practice to test your functions for some boundary cases.

c) Create a discount factor with only two points. $t_0 = 0$ and $DF_0 = 1.0$ and $t_1 = 1.0$ and $DF_1 = 1.0/(1 + 0.05)$. Then call your get_df function as follows for sanity checks.

```
print('DF(0.0):', get_df(0.0, df_times, df_values))
print('DF(0.5):', get_df(0.5, df_times, df_values))
print('DF(1.0):', get_df(1.0, df_times, df_values))
print('DF(1.5):', get_df(1.5, df_times, df_values))
```

```
# Question 1 (c)
df_times = [0.0, 1.0]
df_values = [1.0, 1.0/1.05]
print('DF(0.0):', get_df(0.0, df_times, df_values))
print('DF(0.5):', get_df(0.5, df_times, df_values))
print('DF(1.0):', get_df(1.0, df_times, df_values))
print('DF(1.5):', get_df(1.5, df_times, df_values))
```

```
DF(0.0): 1.0
DF(0.5): 0.9759000729485332
DF(1.0): 0.9523809523809523
DF(1.5): 0.9294286409033649
```

Now we define a function for pricing swaps.

For simplicity, we only consider swaps receiving fixed rate annually and paying floating rate quarterly. We also assume that the swaps start from today (ie. zero spot days) and the swap periods are evenly distributed. For example, for a 1-year swap with annual fixed periods and quarterly floating periods, the fixed period starts at time 0.0 and ends at time 1.0. The floating periods start at time 0.0, 0.25, 0.5,0.75 and end at time 0.25, 0.5,0.75,1.0.

The present value of the swap is:

$$PV = \sum_{i=1}^{k-1} R(\mathcal{T}_i - \mathcal{T}_{i-1})DF(\mathcal{T}_i) - \sum_{j=1}^{m-1} F(T_{j-1}, T_j)(T_j - T_{j-1})DF(T_j)$$

Where:

R is the swap fixed rate

$\mathcal{T}_1, \dots, \mathcal{T}_{k-1}$ are the time (in years) to the fixed period end dates.

$T_1, \dots, T_{m-1}$ are the time (in years) to the floating rate period end dates.

$\mathcal{T}_0 = T_0 = 0.0$

d) Define a function to return the present value of a swap with given fixed rate and maturity.

```
# Question 1 (d)

'''
A function to return the present value of a swap with given fixed rate and maturity.
'''
def swap_price(rate, maturity, df_times, df_values):

  # Making the Tau array with annual increments till maturity
  Tau = [0.0]
  i = 1.0
  mat = maturity
  while mat > 1.0:
    Tau.append(i)
    i += 1
    mat -= 1
  Tau.append(maturity)
  #print(Tau)
```

```
  # calculating the part of PV stemming from fixed periods
  fixed_part_of_PV = 0
  for i in range(1, len(Tau)):
    fixed_part_of_PV += (rate*(Tau[i] - Tau[i-1])*
                         get_df(Tau[i], df_times, df_values))

  # Making the T array with quarterly increments till maturity
  T = [0.0]
  t = maturity
  cur = 0.0
  while t > 0.25:
    t -= 0.25
    cur += 0.25
    T.append(cur)
  T.append(cur + t)
  #print(T)

  # calculating the part of PV stemming from floating periods
  floating_part_of_PV = 0
  for j in range(1, len(T)):
    floating_part_of_PV += (get_forward_rate(T[j-1], T[j], df_times, df_values)*
                            (T[j] - T[j-1])*get_df(T[j], df_times, df_values))

  # calculating the PV to be returned
  PV = fixed_part_of_PV - floating_part_of_PV

  # returning the PV
  return PV
```

e) Now test your function. Create a discount factor with three points.

$t_0 = 0$ and $DF_0 = 1.0$, $t_1 = 0.5$ and $DF_1 = 1.0/(1 + 0.04 * 0.5)$, $t_2 = 1.0$ and $DF_2 = 1.0/(1 + 0.05)$.

Then call your swap_price function as follows for sanity checks. Both results should be zero (up to a machine error).

```
# Question 1 (e)
df_times = [0.0, 0.5, 1.0]
df_values = [1.0, 1.0/1.02, 1.0/1.05]
print('Swap PV:', swap_price(0.05, 1.0, df_times, df_values))
print('Swap PV:', swap_price(0.04, 0.5, df_times, df_values))
print("These results are zero upto a machine error which is a good sanity check!")
```

```
Swap PV: -2.0816681711721685e-17
Swap PV: -6.938893903907228e-17
These results are zero upto a machine error which is a good sanity check!
```

We will use optimize.newton method in scipy to solve for the discount factor for the swap maturity t_i given that the discount factors $DF_1, \ldots, DF_{i-1}$ are solved.

f) The method optimize.newton requires an input for the function to solve. Define such function to return the present value of a swap if the DF_i in the discount factor curve is replaced with the input value df_input. The function signature shall look something like this:

```
def function_solve(df_input, i, rate, maturity, df_times, df_values)
```

```
# Question 1 (f)

'''
A function to return the present value of a swap if the DF_i in the
discount factor curve is replaced with the input value df_input.
'''
def function_solve(df_input, i, rate, maturity, df_times, df_values):

  # replacing the discount factor at index i with the input value df_input
  df_values[i] = df_input

  # calling the swap_price function to calculate the present value of the swap
  PV = swap_price(rate, maturity, df_times, df_values)

  # returning the difference between the present value and the target PV
```

```
  return PV
```

g) Define a function to generate a discount factor curve from a list of swap rate quotes. The function signature shall look something like this:

```
def generate_discount_curve(maturs, swap_rates)
```

```
# Question 1 (g)

'''
A function to generate a discount factor curve from a list of swap rate quotes
'''
def generate_discount_curve(maturs, swap_rates):

  # initializing a DF array to store our df values
  DF = [0 for i in range(len(maturs))]

  # for maturity in maturs array we calculate DF
  for i in range(len(maturs)):
      # keeping our df_times updated
      updated_times = [0] + maturs[:i+1]
      updated_values = [1] + DF[:i+1]
      # solving for the unknown df using numerical newton's method solver
      DF[i] = optimize.newton(function_solve, 0.01, args=(i+1, swap_rates[i], maturs[i], updated_times, updated_values))
  print("The upated times list is: ", updated_times)
  print("The updated values list is: ", updated_values)
  plt.plot(DF, maturs)
```

h) Use the following list of swap rate quotes to generate a discount factor curve.

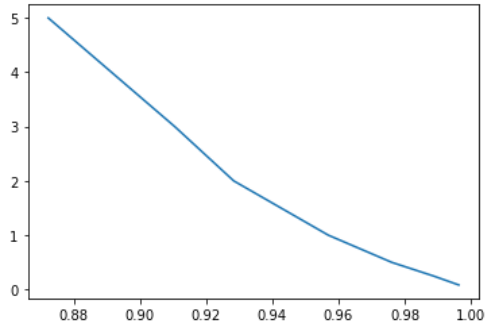
Maturity	1M	3M	6M	1Y	2Y	3Y	5Y
Swap rate %	4.4	4.6	4.9	4.5	3.8	3.2	2.8

```
# Question 1 (h)

# defining the inputs as per the question
maturs = [1/12,3/12,6/12,1,2,3,5]
swap_rates = [4.4/100,4.6/100,4.9/100,4.5/100,3.8/100,3.2/100,2.8/100]
df_times = [0.0, 0.5, 1.0]
df_values = [1.0, 1.0/1.02, 1.0/1.05]

# generating the discount factor curve
generate_discount_curve(maturs,swap_rates)
```

```
The upated times list is:  [0, 0.08333333333333333, 0.25, 0.5, 1, 2,
The updated values list is:  [1, 0.9963467286615743, 0.988630746416
```



i) Use the curve generated above to compute the daily forward rates for the next 2000 calendar days. Plot the computed daily forward rates. Comment on the shape of the curve and explain why this is the case.

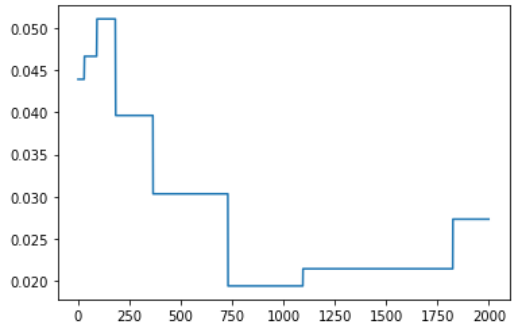
```
# Question 1 (i)
```

```
# initializing the arrays using the information from part (h)
daily_forward_rates = []
times = []
final_df_times = [0, 0.08333333333333333, 0.25, 0.5, 1, 2, 3, 5]
final_df_values = [1, 0.9963467286615743, 0.9886307464162135, 0.9760858955588092,
                   0.9569377990430623, 0.9283587318269398, 0.9105334409032555,
                   0.872336872927112]

# for each calendar day calculating the daily forward rate and storing it
for i in range(2000):
    daily_forward_rates.append(get_forward_rate(i/365, (i+1)/365, final_df_times, final_df_values))
    times.append(i)

# plotting the daily forward rates vs time (in calendar days)
print(daily_forward_rates)
plt.plot(times, daily_forward_rates)
```

```
[0.04392217244698404, 0.04392217244698404, 0.043922172446984056, 0.(
[<matplotlib.lines.Line2D at 0x7f66e27acc70>]
```



We can see that initially, the curve of computed daily forward rates is upward-sloping, then it becomes downward-sloping while being flat for some periods. Finally, it is upward-sloping while flat for some time.

- A downward-sloping forward rate vs. time curve is usually indicative of an environment of declining interest rates. This indicated that the market anticipates that the central bank's monetary policy will ease and interest rates will reduce.
- A flat forward rate vs. time curve indicates that the market perceives the interest rates as stable and not likely to change soon.
- An upward-sloping forward rate curve implies that the market expectations for future short-term interest rates increase over time. This may mean a potential future increase in the overall level of interest rates, reflecting higher demand for credit or inflationary pressures in the economy.