For tutorials (Tuesday)

1. Suppose we have 600 voters: 350 blue, 250 green, to be divided into five constituencies of 120.

   - What is the maximum number of representatives of each kind that can be achieved by gerrymandering?

     We can get 5 blue representatives by making each district 70 blue + 50 green. The resulting efficiency gap is (250-50)/500 = 0.33

     We can get 4 green and 1 blue representative by packing 120 blue voters into one district and putting 57 or 58 in each of the other four. The resulting efficiency gap is (290-10)/600 = 0.467

   - What is the efficiency gap in each case? *(Efficiency gap = the difference between the two parties' numbers of wasted votes, divided by the total number of votes.)*

   - What electoral outcome would minimize the efficiency gap?

     3 blue + 2 green => efficiency gap is (170-130)/600 = 0.067

2. 1,000 households live uniformly distributed along a road 100 miles long (10 households per mile). The government plans to divide these households into school 'districts' in order to maximize total welfare. It need to decide how many schools, s, to build, to be built at uniform intervals along the road.

   Suppose the cost of building a school is C = 1000 + 5n, where n is the number of households that use the school.

   Each household's utility is $U = 200 - t - d$

   where t is their tax bill (which will be C*s/1000) and d is their distance (in miles) from the nearest school.

   - What is a household's average distance from school, as a function of s? (assuming schools are distributed so as to minimize total commute distance)

     100/4s

   - If the government builds s schools, what is the value of n for each school?

     1000/s

- What is each household's tax bill, as a function of s?

  $t = sC/1000 = s+5$

- What is an average household's utility, as a function of s?

  $U = 200 - t - d = 200 - (s + 5) - 100/4s = 195 - s - 25/s$

- What is average household utility, for s =…
  - 4? $195 - s - 25/s = 184.75$
  - 5? $195 - s - 25/s = 185$
  - 6? $195 - s - 25/s = 184.83$

1. **Ans: avg distance is 100/4 if 1, 100/8 if 2, 100/12 if 3, …, ie., 100/4s.**
2. **Tax bill is Cs/1000.**
3. **Note, n = 1000/s. So, C = 1000 + 5000/s.**
4. **So, tax bill is s + 5**
5. **Util = 5000 – s – 5 – 100/(4s)**
6. **max: $1 = 25/s^2$. $s^2 = 25$. s = 5.**