Economics 220, January 2022 **Your Name:** Dhyey Dharmendrakumar Mavani
Homework 11

1. Consider an interaction between two peasants living in a state of anarchy. They each simultaneously decide whether to spend all their time farming or whether to spend some time arming themselves to defend their own output and, if possible, steal some of the others' output. Arming costs resources, but if one peasant arms and the other does not, the armed peasant can raid the unarmed one and steal some of their output (the rest is destroyed). The payoffs (measured in units of output) are as follows:

	farm	arm
farm	10, 10	0, 15
arm	15, 0	7, 7

(a) Suppose the peasants play this game each period and discount the future at a rate δ. For what values of δ can they sustain an equilibrium in which each agrees to spend all their time farming and not arm to raid the other?

$$\Pi_{cooperate} = 10 + 10\delta + 10\delta^2 + \cdots = \left(\frac{10}{1-\delta}\right)$$

$$\Pi_{arm} = 15 + 7\delta + 7\delta^2 + \cdots = 15 + \left(\frac{7\delta}{1-\delta}\right)$$

$$\Pi_{cooperate} > \Pi_{arm} : \left(\frac{10}{1-\delta}\right) > 15 + \left(\frac{7\delta}{1-\delta}\right)$$

$$\Rightarrow 10 - 7\delta > 15 - 15\delta$$

$$\Rightarrow 8\delta > 5$$

$$\Rightarrow \boxed{\delta > \frac{5}{8}} \text{ Ans}$$

Suppose that cooperation through repeated interaction between the peasants is not possible. However a noble warrior generously offers to become the 'lord'. In exchange for a promise that the peasants will each pay a 'tax' (or tribute) of t per period, the lord undertakes to punish any peasant who arms themselves by throwing them in a dungeon for one period – in which case the peasant gets a payoff of -D and the lord gets half their output (the rest is destroyed). However, the peasants also know that if they agree to submit to the lord's rule, and do not arm themselves, the lord will have the option of stealing half their output anyway. If the lord were

to act in this predatory manner, however, the peasants would no longer pay taxes to the lord in future periods, but would revert to the anarchic equilibrium you found in part (a).

(b) For the proposed political system to become a self-enforcing equilibrium, t must be small enough that the peasants prefer an equilibrium in which they farm and pay taxes to the lord rather than to live in a state of anarchy. What is the maximum possible value of t?

$10 - t \geq 7$

$t \leq 3$

The condition where paying taxes to the lord is more beneficial to farmers than just living in the state of anarchy

(c) Suppose t = 2 (so the lord gets a total per-period payoff of 2t = 4). By becoming a predator if both peasants farm, the lord can seize half of total output (5 + 5 = 10 total) for one period. For what values of δ would the lord prefer to act as a 'stationary bandit' (earning taxes) rather than become a predator and steal all the peasants' output for one period?

gains of the lord

$\Pi_{tax} = (2t) + (2t)\delta + \cdots = \frac{2t}{1-\delta}$

$\Pi_{pred} = 10$

$\Pi_{tax} \geq \Pi_{pred}$ (for lord to prefer "stationary bandit")

$\frac{2t}{1-\delta} \geq 10 \Rightarrow (1-\delta) \leq \frac{2}{5} \Rightarrow \boxed{\delta \geq \frac{3}{5}}$ Ans

(d) Suppose $\delta = 0.5$. For what values of t would the lord prefer to act as a 'stationary bandit' rather than become a predator and steal all the peasants' output for one period?

Condition for gains of lord: $\Pi_{tax} \geq \Pi_{pred} \Rightarrow$ (for lord to prefer "stationary bandit")

$\frac{2t}{1-\delta} \geq 10 \Rightarrow 2t \geq 5 \Rightarrow \boxed{t \geq 2.5}$ Ans