

# HW4 solutions for Intro

December 2022

Q1

a)

```
M=8000; %number of trajectories
N=250; %Number of steps in one trajectory
X0=100; %initial point
T=1; %Final Time in years in trajectory
dt=T/N; %time step
Sqrtdt=sqrt(dt);
mu=0.02 ;
Sigma=0.3;
alpha=0.75; %elasticity
%dx_t=mu*x_t*dt+Sigma*x_t^alpha*dW_t
X(1:M,1)=X0; % Initial value X(j,1)=X0 for all j=1:M
for j=1:M %generate M traject.of constant elasticity of variance process
    for i = 2:N+1 %generate j-th trajectory
        X(j,i)=X(j,i-1) + mu*X(j,i-1)*dt +Sigma*(X(j,i-1)^alpha)*randn*Sqrtdt;
    end
end
t=0:dt:T; %creating time array for plotting
plot(t,X(:,,:)); %Plotting graph of trajectories
```

Figure 1: MC Code

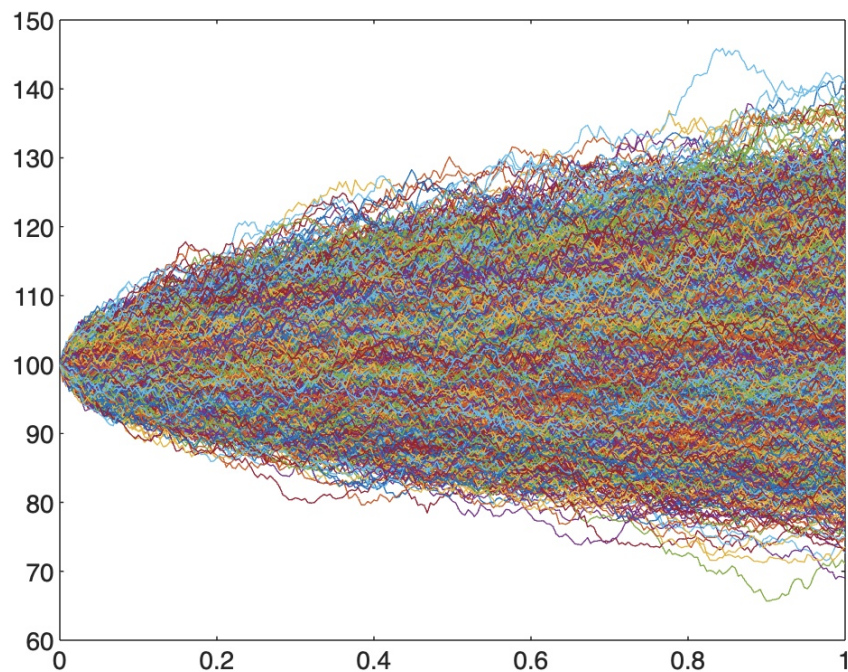


Figure 2: MC trajectories

b)

```
tic
M=100000;    %number of trajectories
N=250;      %Number of steps in one trajectory
X0=100;      %initial point
T=1;        %Final Time in years in trajectory
dt=T/N;      %time step
Sqrtdt=sqrt(dt);
mu=0.04 ;
Sigma=0.3;
alpha=0.75; %elasticity
K=99;
r=4/100;

%dx_t=mu*x_t*dt+Sigma*x_t^alpha*dt
X(1:M,1)=X0;    % Initial value  X(j,1)=X0 for all j=1:M

for j=1:M %generate M traject.of constant elasticity of variance process
    for i = 2:N+1 %generate j-th trajectory
        X(j,i)=X(j,i-1) + mu*X(j,i-1)*dt +Sigma*(X(j,i-1)^alpha)*randn*Sqrtdt;
    end
end
payoff=max(X(:,N+1)-K,0);
price=exp(-r*T)*mean(payoff)
toc
```

Figure 3: Pricing Code

```
price =

    6.6218

Elapsed time is 3.428186 seconds.
```

Figure 4: Price and time it took to run the code

The run time depends on how strong your computer's processor is.

## Q2

VaR measures the max potential loss on a group of equities over some time period  $T$ , given a specific probability. Once we set the confidence interval, VaR is the maximum loss a group of securities is allowed to have. Whereas, CVaR (equivalent to Expected Shortfall) represents the average of the most severe losses given 100 periods. The same time period and probability is used to calculate VaR and CVaR.

### Q3

Stock	Closing Price (USD)	Number of Stocks
MSFT	241.68	500 (long)
AAPL	150.72	400 (long)
MSFT	394.24	100 (short)

a)

$$\text{NAV} = 241.68(500) + 150.72(400) - 394.24(100) + 100000 = \$241,704$$

b)

$$\begin{aligned} \text{Gross Leverage} &= \frac{241.68(500) + 150.72(400) + 394.24(100)}{\text{NAV}} \\ &= \frac{220,552}{241,704} \\ &= 91.25\% \end{aligned}$$

$$\begin{aligned} \text{Net Leverage} &= \frac{241.68(500) + 150.72(400) - 394.24(100)}{\text{NAV}} \\ &= \frac{141,704}{241,704} \\ &= 58.63\% \end{aligned}$$

c

Earn 1% annually with daily compounding  $\rightarrow \frac{1\%}{365} = 0.00274\%$  daily compounding so  $S_1 = S_0(1 + 0.00274\%$   
Please see the excel file "HW4 Excel Solutions" to see how the calculations are done.

Date	MSFT Adj Close	AAPL Adj Close	SPY Adj Close	Cash	NAV	Gross Leverage	Net Leverage	Return	Daily 90-Day	Annualized vol	95% var
11/1/22	227.528793	150.400497	384.519989	99967.1259	235439.72	0.902042324	0.57540246	-0.012071812	0.013475	0.213909428	0.0221664
11/2/22	219.481476	144.78981	374.869995	99969.865	230139.53	0.89138821	0.56561193	-0.022511897	0.0136644	0.216916303	0.022478
11/3/22	213.647903	138.650009	371.01001	99972.6041	225155.56	0.885543123	0.55598429	-0.021656294	0.0136861	0.217260199	0.0225136
11/4/22	220.767838	138.380005	376.350006	99975.3434	228076.26	0.891679468	0.56165827	0.012971945	0.0137091	0.217624873	0.0225514
11/7/22	227.22963	138.919998	379.950012	99978.0827	231165.9	0.89623002	0.56750505	0.013546486	0.0137581	0.218402638	0.022632
11/8/22	228.226822	139.5	382	99980.8221	231694.23	0.898224389	0.56847945	0.002285534	0.0137343	0.2180247	0.0225929
11/9/22	223.879074	134.869995	374.130005	99983.5616	228458.1	0.889881072	0.56235492	-0.013967275	0.0137537	0.218333519	0.0226249
11/10/22	242.297165	146.869995	394.690002	99986.3011	240413.88	0.912449728	0.58410762	0.052332509	0.0148241	0.235324919	0.0243856
11/11/22	246.415558	149.699997	398.51001	99989.0408	243225.82	0.916591754	0.58890449	0.01169623	0.0148555	0.235824327	0.0244374
11/14/22	240.871185	148.279999	395.119995	99991.7805	240227.37	0.912716935	0.58376192	-0.012327822	0.0149093	0.236677907	0.0245258
11/15/22	241.290009	150.039993	398.48999	99994.5202	240806.52	0.9157144	0.58475161	0.00241084	0.0148913	0.236392236	0.0244962
11/16/22	241.729996	148.789993	395.450012	99997.2601	240833.25	0.913187829	0.58478633	0.000111007	0.0147641	0.2343735	0.024287
11/17/22	241.679993	150.720001	394.23999	100000	241704	0.912487993	0.58627081	0.003615546	0.0147685	0.234441942	0.0242941

Figure 5: Excel Spreadsheet showing NAV, Gross Leverage, and Net Leverage

### Q4

Look at the plots on the next page



Figure 6: Daily 90-Day Rolling Vol

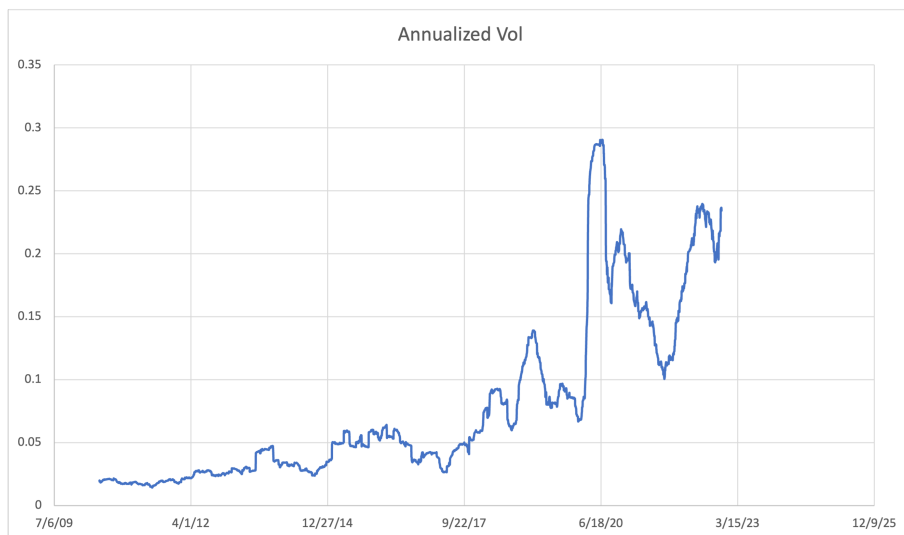


Figure 7: Annual Vol

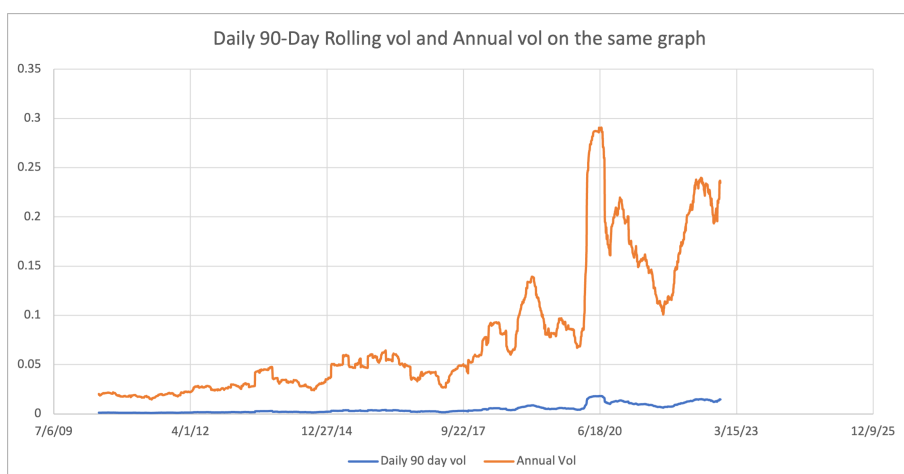


Figure 8: Daily 90-Day Rolling Vol and Annual Vol on the same plot



Figure 9: 95% VaR