MATH 5440: Week 7 Assignement

Due Date: March 10, 2023 at 10am

Exercise 1 Trading two consecutive orders

This exercise reconciles two points of view on consecutive trades:

(a) One can treat the second order as the first order's continuation.

(b) One can treat the second order as a separate trade from an external source.

One shows that the two points of view lead to the same optimal trading strategy. Consider a generalized OW model with parameters $\beta > 0$, $\lambda = e^{\gamma}$ satisfying the no price-manipulation condition $2\beta + \gamma' > 0$. For a given trading process Q, the price impact is the solution to the SDE

$$dI_t = -\beta_t I_t dt + \lambda_t dQ_t.$$

Consider a deterministic alpha signal α_t.

1. In the continuation approach, $Q_0 = \tilde{Q}$ and $I_0 = \tilde{I}$ are non-zero and capture the first order's effect. Derive the optimal impact state I_t^0.
2. In the separate approach, $Q_0 = 0$ and $I_0 = 0$. The external impact $\bar{I}$ captures the first order's effect. The first order's external impact satisfies

$$\forall t > 0; \quad d\bar{I}_t = -\beta_t \bar{I}_t dt.$$

   Derive the optimal impact state I_t^1.
3. Reconcile the two points of view.

Exercise 2 Absence of price manipulation strategies

This exercise finds a lower bound on the decay parameter β_t of a generalized OW model based on a liquidity profile v_t to rule out price manipulation strategies. Assume the intraday volume profile v follows the deterministic curve

$$v_t = e^{4\cdot(t-0.7)^2}.$$

Local square root model under the calendar clock I

The impact model considered is

$$dI_t = -\beta I_t dt + \frac{\lambda}{\sqrt{v_t}} dQ_t.$$

1. Establish the lower bound on β that guarantees no price manipulation.
2. For $\beta = 0.01$, find a pair of trades that lead to a price manipulation paradox.

Local log model under the volume clock II

The impact model considered is

$$dI_t = -\beta v_t I_t dt + \frac{\lambda}{v_t} dQ_t.$$

1. Establish a lower bound on β that guarantees no price manipulation.

2. For $\beta = 0.01$, find a pair of trades that lead to a price manipulation paradox.

Exercise 3 Optimal execution for large orders

This exercise solves an optimal execution problem under a globally concave AFS price impact model. The AFS model is particularly relevant when submitting sizable orders. In that regime, the instantaneous liquidity conditions are of second order, and price impact's concavity drives trading costs. The interval $[0, T]$ represents a single trading day. Consider

$$I_t = h(J_t)$$

with $h(x) = \text{sign}(x)|x|^c$ and local dynamics

$$dJ_t = -\beta J_t dt + \lambda dQ_t$$

for $c \in (0, 1]$. The corresponding self-financing equation is

$$dY_t = Q_t dS_t - h(\bar{J}_t) dQ_t - \frac{\lambda}{2} h'(\bar{J}_t) d[Q, Q]_t.$$

Assume that $I_0 = 0$.

1. Under this model, solve the optimal execution problem for the target impact state I^*. Assume given a deterministic alpha α_t and an overnight alpha $\tilde{\alpha}$.
2. What is the relationship between the order size Q_T and the implied overnight alpha $\tilde{\alpha}$ when there is no trading alpha?
3. Assume that $I_0 > 0$ from a previous order. Solve the idealized optimal execution problem for the target impact state I^* without trading alpha. How does the order size Q_T change given I_0?