Homework 5: Wed Feb 22, 2023

Due Monday March 6 2023.

Problem 1 **[20 points]**

(i) You trade options and notice that at 3:00 p.m. on March 11th 2023 the market price $C_1(K)$ of one-year European calls with strike K on a stock S is perfectly fit by the function

$$C_1(K) = \frac{100}{K+10}$$

What is the risk-neutral implied probability density function for the stock to get from S at time t to S_T in one year? Assume the one-year riskless rate is zero. [5 points]

(ii) What is the value of the stock price S at 3:00 p.m. on March 11th 2023? [10 points]

(iii) Similarly, you also notice that at the same exact time the market price of two-year European calls, $C_2(K)$, is perfectly fit by the function

$$C_2(K) = 10\exp\left(-\frac{K}{11}\right)$$

Use these prices to find the two-year riskless *annually* compounded zero-coupon interest rate? [5 points]

Solution 1

(i) $$\boxed{\rho(S, t, S_T, 1) = \frac{\partial^2}{\partial S_T^2} C_1(S_T) = \frac{200}{(S_T+10)^3}}$$

(ii) $$\boxed{S = \int_0^\infty \frac{200}{(S_T+10)^3} \times S_T dS_T = 10}$$

Another way to do this is to notice that a call with strike of zero must be equal to the stock price, and so $\boxed{C_1(0) = 10}$

(iii)

A two-year riskless bond pays \$1 for all $\boxed{S_T}$.

Its value by the usual static replication formula using options is the value of $1 times the function $\boxed{\frac{\partial^2}{\partial S_T^2} C_2(S_T) = \frac{10}{11^2}\exp\left(-\frac{S_T}{11}\right)}$ integrated over all terminal stock prices $\boxed{S_T}$

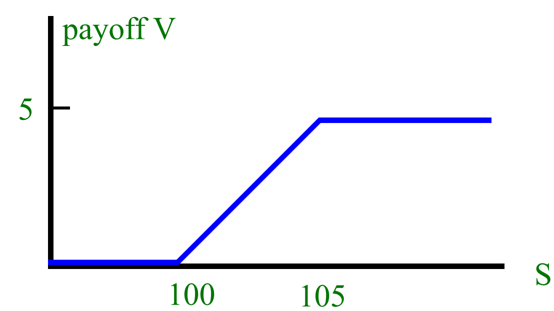
$$\boxed{B = \int_0^{\infty} \frac{10}{11^2}\exp\left(-\frac{S_T}{11}\right) dS_T = \frac{10}{11}}$$

But present value of a two-year zero-coupon bond in terms of its interest rate is given by

$$\boxed{B = \frac{1}{(1+r)^2} = \frac{10}{11}}$$

so that r = 4.88% per year for two years.

Problem 2: **Exotic option in a smile [20 points]** Consider an "exotic" option V on a stock S that one year from now pays zero if the stock price S is less than $100, (S - $100) if the stock price lies between $100 and $105, and $5 for all values of S > $105. Assume interest rates and dividend yields are zero and that the current stock price is 100.

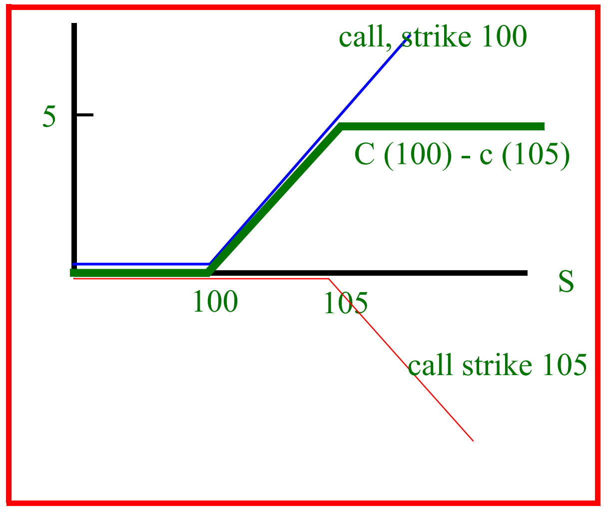


(i) What is the current value of V if the Black-Scholes implied volatility of all one-year calls, irrespective of strike, is 0.2 (that is 20% per year.)? [10]

(ii) What is the current value of V if the smile for call options is such that the current implied volatility of one-year options is given by $\Sigma(K) = 0.2 + 2(0.1)(K/100 - 1)$? Is V worth more or less than before? Why? [10]

Solution 2: Exotic option in a smile

The payoff of a call struck at 100 less a call struck at 105 is shown at right, and is the same as the payoff of V.



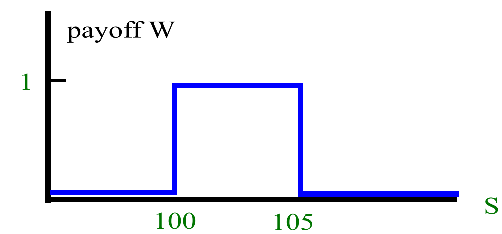
(i) $V = C(100,100,0.2,1) - C(100,105,0.2,1)$
$= 7.97 - 5.91$
$= 2.06$

(ii) $V = C(100,100,0.2,1) - C(100,105,0.21,1)$
$= 7.97 - 6.30 = 1.67$

The short position in the 105-strike call is worth more at higher volatility, so V is worth less.

Problem 3: Another exotic option in a smile **[20 points]**

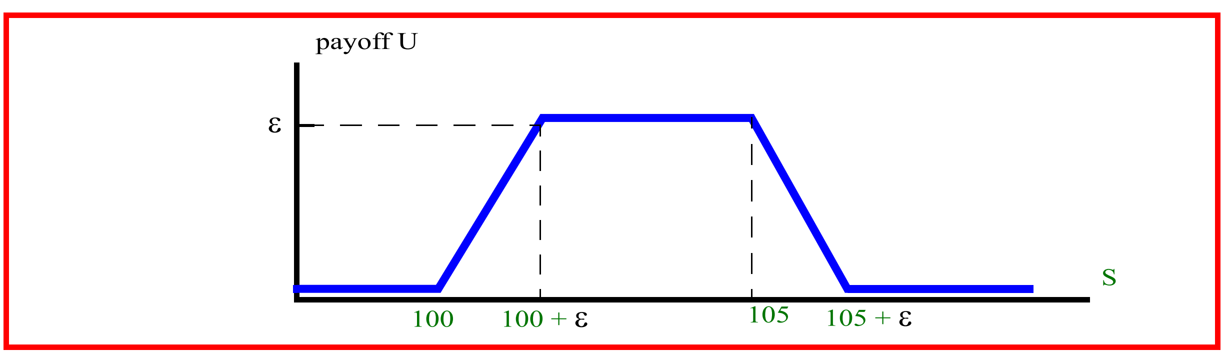
Repeat problem 2(i) and 2(ii) above for an exotic option W that has the one-year payoff of $1 between 100 and 105 and zero elsewhere, as shown in the figure.



Solution 3: Another exotic option in a smile

Consider the payoff of the portfolio $\boxed{U = C_{100} - C_{100+\varepsilon} - C_{105} + C_{105+\varepsilon}}$ where $\boxed{C_K}$ denotes the payoff of a call with strike K.

Then U looks like



and it pays off ε between 100 + ε and 105.

Therefore $\boxed{W \approx U/\varepsilon}$ has the payoff we want [$1] in the range between 100 and 105 in the limit as ε –> 0.

You can see that $\boxed{W \approx -\left.\frac{\partial C}{\partial K}\right|_{K=100} + \left.\frac{\partial C}{\partial K}\right|_{K=105}}$

(i) For S = 100 and $\boxed{\Sigma=0.2}$ and T = 1 independent of strike K, $\boxed{\frac{\partial C}{\partial K} = -N(d_2)}$ and

so
$$\boxed{\begin{aligned} W &= N(d_2)\big|_{K=100} - N(d_2)\big|_{K=105} \\ &= N\left(\frac{\ln(100/100)}{0.2} - 0.1\right) - N\left(\frac{\ln(100/105)}{0.2} - 0.1\right) \\ &= N(-0.1) - N(-5\ln 1.05 - 0.1) \\ &= N(-0.1) - N(-0.34) \\ &= 0.460 - 0.367 \approx 0.093 \end{aligned}}$$

(ii) If $\boxed{\Sigma(K) = 0.2 + 0.2\left(\frac{K}{100} - 1\right)}$ and $\boxed{\frac{\partial \Sigma}{\partial K} = 0.002}$

$$\boxed{\begin{aligned} W_{ii} &\approx -\left.\frac{dC}{dK}\right|_{K=100} + \left.\frac{dC}{dK}\right|_{K=105} \\ &= -\left\{\frac{\partial C}{\partial K} + \frac{\partial C \partial \Sigma}{\partial \Sigma \partial K}\right\}_{100} + \left\{\frac{\partial C}{\partial K} + \frac{\partial C \partial \Sigma}{\partial \Sigma \partial K}\right\}_{105} \\ &= W_i - \left\{\frac{\partial C \partial \Sigma}{\partial \Sigma \partial K}\right\}_{100} + \left\{\frac{\partial C \partial \Sigma}{\partial \Sigma \partial K}\right\}_{105} \end{aligned}}$$

Now $\boxed{\frac{\partial \Sigma}{\partial K} = 0.002}$ and so $\boxed{W_{ii} = W_i - 0.002\left[\left\{\frac{\partial C}{\partial \Sigma}\right\}_{100} - \left\{\frac{\partial C}{\partial \Sigma}\right\}_{105}\right]}$

K = 100: then $\boxed{\Sigma = 0.2}$, T–t = 1 and S = 100 and $\boxed{d_1 = \frac{\ln 100/100}{0.2} + 0.1 = 0.1}$ and

$\boxed{e^{-d_1^2/2} \approx 0.995}$

$$\boxed{\frac{\partial C}{\partial \Sigma} = \frac{S\exp((-d_1^2)/2)\sqrt{T-t}}{\sqrt{2\pi}} = 0.39 \times 100 \times 0.995 = 38.8}$$

K = 105: then $\Sigma = 0.2$, T–t = 1 and S = 100 and

$d_1 = \frac{\ln 100/105}{0.21} + \frac{0.21}{2} = -\frac{0.049}{0.21} + 0.105 = -0.128$ and $e^{-d_1^2/2} \approx 0.992$

$$\frac{\partial C}{\partial \Sigma} = \frac{S\exp((-d_1^2)/2)\sqrt{T-t}}{\sqrt{2\pi}} = 0.39 \times 100 \times 0.992 = 38.7$$

So $W_{ii} = W_i - 0.002[38.8 - 38.7] = 0.093 - 0.0002) \approx 0.093$

There is a negligible change in value from the case (i) above. The W here in case (ii) is a difference of two call spreads, one at 100 and one at 105, and the smile changes the value of both of them more or less equally, so the difference cancels.

Problem 4: **[20 points]**

We showed in class that in a Black-Scholes world the value of a down-and-out call with strike K and barrier B **when the riskless rate is zero and the dividend yield is zero** is given by

$$C_{DO}(S, K, B, 0, \sigma, t, T) = C_{BS}(S, K, 0, \sigma, t, T) - \frac{S}{B}C_{BS}\left(\frac{B^2}{S}, K, 0, \sigma, t, T\right)$$

where $C_{BS}(S, K, r, \sigma, t, T)$ denotes the Black-Scholes formula with the associated spot, strike, interest rate, volatility, current time and expiration time. Note that in the above equation we have set the interest rates to be zero.

Show that $C_{DO}(S, K, B, 0, \sigma, t, T)$ satisfies the BS equation when $r = 0$ and the dividend yield is zero.

Solution to Problem 4. We know that the first term above satisfies the BS equation, by assumption.

We now prove more generally that if C(S,t) satisfies the BS equation, then so does $SC\left(\frac{X}{S}, t\right)$, where X is some constant.

Write $V = SC\left(\frac{X}{S}, K\right)$ where $y = \frac{X}{S}$ and write $C\left(\frac{X}{S}, t\right) \equiv C(y, t)$ where $\frac{\partial y}{\partial S} = -\frac{X}{S^2}$ $V = SC(y, t)$ and assume $C(y, t)$ satisfies the BS equation with y as the stock price.

Then using the chain rule, $\boxed{V_s = C(y,t) - \frac{X}{S}C_y(y,t)}$ $\boxed{V_{ss} = \frac{X^2}{S^3}C_{yy}}$ and $\boxed{V_t = SC_t}$

Then

$$\boxed{V_t + \frac{\sigma^2 S^2}{2}V_{ss} = SC_t + \frac{\sigma^2 S^2}{2}\frac{X^2}{S^3}C_{yy} = SC_t + \frac{\sigma^2}{2}\frac{X^2}{S}C_{yy} = S\left(C_t + \frac{\sigma^2}{2}\frac{X^2}{S^2}C_{yy}\right) = S\left(C_t + \frac{\sigma^2}{2}y^2 C_{yy}\right)}$$

and the last term in brackets is zero because $\boxed{C(y,t)}$ satisfies the BS equation by assumption.

Therefore the LHS is zero and so V satisfies the BS equation too.

Problem 5: **[20 points]**

Consider an exotic option with a one-year expiration whose payoff is $V = S_T^2$ at expiration, where S_t is the price of a stock at time t and the stock pays zero dividends and interest rates are zero.

(i) Show that you can statically replicate this payoff with a portfolio of calls of all strikes between 0 and infinity, equally weighted with a strike density of 2. [10]

(ii) Let V(S,t,T) be the fair value of the exotic option above at stock price S and time t, when the expiration is later at time T. Estimate the value of V(100,0,1) by assuming that call prices C are given by the Black-Scholes formula with zero interest rates, zero dividends and 20% volatility with no smile, and then performing the integral numerically. You can use strikes $1 apart in the numerical integration. Show how you do the integral. [10]

Solution to 5(i):

The density of strikes is given by the second derivative of V w.r.t the stock price. The second derivative of V is 2. Therefore you have to sum over all strikes from zero to infinity with a density of 2 to replicate the payoff exactly.

(ii) Therefore you have shown that at the initial time t=0, $\boxed{V(S,0) = 2\int_0^{\infty} C(S, 0, X, T, \sigma)dX}$,

where C(S,t=0,T,K,σ) is the value of a Black-Scholes call with volatility σ, assuming interest rates and dividend yields are zero.

Solution to 6(ii) done by very naive numerical trapezoidal rule in Excel, using strikes one dollar apart.

Integral up to strike K is shown in last column, and it converges to about 10,408. At strikes of 200 the sum has just about converged.

strike	d1	d2	N(d1)	N(d2)	C(S,K)	0.5{C(K)+C(K+1)}dK	2. Sum
0.00	69.18	68.98	1.00	1.00	100.00	99.50	199.00
1.00	23.13	22.93	1.00	1.00	99.00	98.50	396.00
2.00	19.66	19.46	1.00	1.00	98.00	97.50	591.00
3.00	17.63	17.43	1.00	1.00	97.00	96.50	784.00
4.00	16.19	15.99	1.00	1.00	96.00	95.50	975.00
5.00	15.08	14.88	1.00	1.00	95.00	94.50	1164.00
6.00	14.17	13.97	1.00	1.00	94.00	93.50	1351.00
7.00	13.40	13.20	1.00	1.00	93.00	92.50	1536.00
8.00	12.73	12.53	1.00	1.00	92.00	91.50	1719.00
9.00	12.14	11.94	1.00	1.00	91.00	90.50	1900.00
10.00	11.61	11.41	1.00	1.00	90.00	89.50	2079.00
11.00	11.14	10.94	1.00	1.00	89.00	88.50	2256.00
12.00	10.70	10.50	1.00	1.00	88.00	87.50	2431.00
13.00	10.30	10.10	1.00	1.00	87.00	86.50	2604.00
14.00	9.93	9.73	1.00	1.00	86.00	85.50	2775.00
15.00	9.59	9.39	1.00	1.00	85.00	84.50	2944.00
16.00	9.26	9.06	1.00	1.00	84.00	83.50	3111.00

17.00	8.96	8.76	1.00	1.00	83.00	82.50	3276.00
18.00	8.67	8.47	1.00	1.00	82.00	81.50	3439.00
19.00	8.40	8.20	1.00	1.00	81.00	80.50	3600.00
20.00	8.15	7.95	1.00	1.00	80.00	79.50	3759.00
21.00	7.90	7.70	1.00	1.00	79.00	78.50	3916.00
22.00	7.67	7.47	1.00	1.00	78.00	77.50	4071.00
23.00	7.45	7.25	1.00	1.00	77.00	76.50	4224.00
24.00	7.24	7.04	1.00	1.00	76.00	75.50	4375.00
25.00	7.03	6.83	1.00	1.00	75.00	74.50	4524.00
26.00	6.84	6.64	1.00	1.00	74.00	73.50	4671.00
27.00	6.65	6.45	1.00	1.00	73.00	72.50	4815.99
28.00	6.47	6.27	1.00	1.00	72.00	71.50	4958.99
29.00	6.29	6.09	1.00	1.00	71.00	70.50	5099.99
30.00	6.12	5.92	1.00	1.00	70.00	69.50	5238.99
31.00	5.96	5.76	1.00	1.00	69.00	68.50	5375.99
32.00	5.80	5.60	1.00	1.00	68.00	67.50	5510.99
33.00	5.64	5.44	1.00	1.00	67.00	66.50	5643.99
34.00	5.49	5.29	1.00	1.00	66.00	65.50	5774.99
35.00	5.35	5.15	1.00	1.00	65.00	64.50	5903.99
36.00	5.21	5.01	1.00	1.00	64.00	63.50	6030.99
37.00	5.07	4.87	1.00	1.00	63.00	62.50	6155.99
38.00	4.94	4.74	1.00	1.00	62.00	61.50	6278.99
39.00	4.81	4.61	1.00	1.00	61.00	60.50	6399.99
40.00	4.68	4.48	1.00	1.00	60.00	59.50	6518.99
41.00	4.56	4.36	1.00	1.00	59.00	58.50	6635.99
42.00	4.44	4.24	1.00	1.00	58.00	57.50	6750.99
43.00	4.32	4.12	1.00	1.00	57.00	56.50	6863.99
44.00	4.21	4.01	1.00	1.00	56.00	55.50	6974.99
45.00	4.09	3.89	1.00	1.00	55.00	54.50	7083.99
46.00	3.98	3.78	1.00	1.00	54.00	53.50	7190.99

47.00	3.88	3.68	1.00	1.00	53.00	52.50	7295.99
48.00	3.77	3.57	1.00	1.00	52.00	51.50	7398.99
49.00	3.67	3.47	1.00	1.00	51.00	50.50	7499.99
50.00	3.57	3.37	1.00	1.00	50.00	49.50	7599.00
51.00	3.47	3.27	1.00	1.00	49.00	48.50	7696.00
52.00	3.37	3.17	1.00	1.00	48.00	47.50	7791.00
53.00	3.27	3.07	1.00	1.00	47.00	46.50	7884.01
54.00	3.18	2.98	1.00	1.00	46.00	45.51	7975.02
55.00	3.09	2.89	1.00	1.00	45.01	44.51	8064.04
56.00	3.00	2.80	1.00	1.00	44.01	43.51	8151.05
57.00	2.91	2.71	1.00	1.00	43.01	42.51	8236.08
58.00	2.82	2.62	1.00	1.00	42.02	41.52	8319.11
59.00	2.74	2.54	1.00	0.99	41.02	40.52	8400.16
60.00	2.65	2.45	1.00	0.99	40.03	39.53	8479.22
61.00	2.57	2.37	1.00	0.99	39.03	38.54	8556.30
62.00	2.49	2.29	0.99	0.99	38.04	37.55	8631.40
63.00	2.41	2.21	0.99	0.99	37.06	36.56	8704.53
64.00	2.33	2.13	0.99	0.98	36.07	35.58	8775.69
65.00	2.25	2.05	0.99	0.98	35.09	34.60	8844.89
66.00	2.18	1.98	0.99	0.98	34.11	33.62	8912.13
67.00	2.10	1.90	0.98	0.97	33.14	32.65	8977.44
68.00	2.03	1.83	0.98	0.97	32.17	31.69	9040.81
69.00	1.96	1.76	0.98	0.96	31.21	30.73	9102.27
70.00	1.88	1.68	0.97	0.95	30.25	29.77	9161.81
71.00	1.81	1.61	0.97	0.95	29.30	28.83	9219.47
72.00	1.74	1.54	0.96	0.94	28.36	27.89	9275.24
73.00	1.67	1.47	0.95	0.93	27.42	26.96	9329.16
74.00	1.61	1.41	0.95	0.92	26.50	26.04	9381.24
75.00	1.54	1.34	0.94	0.91	25.58	25.13	9431.49
76.00	1.47	1.27	0.93	0.90	24.68	24.23	9479.96

77.00	1.41	1.21	0.92	0.89	23.79	23.35	9526.65
78.00	1.34	1.14	0.91	0.87	22.91	22.47	9571.59
79.00	1.28	1.08	0.90	0.86	22.04	21.61	9614.81
80.00	1.22	1.02	0.89	0.85	21.19	20.77	9656.35
81.00	1.15	0.95	0.88	0.83	20.35	19.94	9696.22
82.00	1.09	0.89	0.86	0.81	19.53	19.12	9734.47
83.00	1.03	0.83	0.85	0.80	18.72	18.33	9771.12
84.00	0.97	0.77	0.83	0.78	17.93	17.55	9806.22
85.00	0.91	0.71	0.82	0.76	17.16	16.79	9839.79
86.00	0.85	0.65	0.80	0.74	16.41	16.04	9871.87
87.00	0.80	0.60	0.79	0.73	15.67	15.32	9902.50
88.00	0.74	0.54	0.77	0.71	14.96	14.61	9931.73
89.00	0.68	0.48	0.75	0.69	14.26	13.93	9959.58
90.00	0.63	0.43	0.74	0.67	13.59	13.26	9986.10
91.00	0.57	0.37	0.72	0.65	12.93	12.62	10011.34
92.00	0.52	0.32	0.70	0.62	12.30	11.99	10035.32
93.00	0.46	0.26	0.68	0.60	11.69	11.39	10058.10
94.00	0.41	0.21	0.66	0.58	11.09	10.81	10079.71
95.00	0.36	0.16	0.64	0.56	10.52	10.24	10100.20
96.00	0.30	0.10	0.62	0.54	9.97	9.70	10119.60
97.00	0.25	0.05	0.60	0.52	9.44	9.18	10137.96
98.00	0.20	0.00	0.58	0.50	8.93	8.68	10155.33
99.00	0.15	-0.05	0.56	0.48	8.44	8.20	10171.73
100.00	0.10	-0.10	0.54	0.46	7.97	7.74	10187.21
101.00	0.05	-0.15	0.52	0.44	7.52	7.30	10201.81
102.00	0.00	-0.20	0.50	0.42	7.08	6.88	10215.56
103.00	-0.05	-0.25	0.48	0.40	6.67	6.48	10228.52
104.00	-0.10	-0.30	0.46	0.38	6.28	6.09	10240.70
105.00	-0.14	-0.34	0.44	0.37	5.91	5.73	10252.16
106.00	-0.19	-0.39	0.42	0.35	5.55	5.38	10262.92

107.00	-0.24	-0.44	0.41	0.33	5.21	5.05	10273.01
108.00	-0.29	-0.49	0.39	0.31	4.89	4.74	10282.48
109.00	-0.33	-0.53	0.37	0.30	4.58	4.44	10291.36
110.00	-0.38	-0.58	0.35	0.28	4.29	4.16	10299.67
111.00	-0.42	-0.62	0.34	0.27	4.02	3.89	10307.44
112.00	-0.47	-0.67	0.32	0.25	3.76	3.64	10314.71
113.00	-0.51	-0.71	0.31	0.24	3.51	3.40	10321.50
114.00	-0.56	-0.76	0.29	0.23	3.28	3.17	10327.85
115.00	-0.60	-0.80	0.28	0.21	3.06	2.96	10333.76
116.00	-0.64	-0.84	0.26	0.20	2.86	2.76	10339.28
117.00	-0.69	-0.89	0.25	0.19	2.66	2.57	10344.42
118.00	-0.73	-0.93	0.23	0.18	2.48	2.39	10349.21
119.00	-0.77	-0.97	0.22	0.17	2.31	2.23	10353.67
120.00	-0.81	-1.01	0.21	0.16	2.15	2.07	10357.81
121.00	-0.85	-1.05	0.20	0.15	2.00	1.93	10361.66
122.00	-0.89	-1.09	0.19	0.14	1.86	1.79	10365.24
123.00	-0.94	-1.14	0.18	0.13	1.72	1.66	10368.56
124.00	-0.98	-1.18	0.17	0.12	1.60	1.54	10371.64
125.00	-1.02	-1.22	0.16	0.11	1.48	1.43	10374.50
126.00	-1.06	-1.26	0.15	0.11	1.37	1.32	10377.14
127.00	-1.10	-1.30	0.14	0.10	1.27	1.23	10379.60
128.00	-1.13	-1.33	0.13	0.09	1.18	1.14	10381.87
129.00	-1.17	-1.37	0.12	0.09	1.09	1.05	10383.96
130.00	-1.21	-1.41	0.11	0.08	1.01	0.97	10385.91
131.00	-1.25	-1.45	0.11	0.07	0.93	0.90	10387.70
132.00	-1.29	-1.49	0.10	0.07	0.86	0.83	10389.36
133.00	-1.33	-1.53	0.09	0.06	0.80	0.77	10390.89
134.00	-1.36	-1.56	0.09	0.06	0.74	0.71	10392.30
135.00	-1.40	-1.60	0.08	0.06	0.68	0.65	10393.60
136.00	-1.44	-1.64	0.08	0.05	0.63	0.60	10394.80

137.00	-1.47	-1.67	0.07	0.05	0.58	0.55	10395.91
138.00	-1.51	-1.71	0.07	0.04	0.53	0.51	10396.93
139.00	-1.55	-1.75	0.06	0.04	0.49	0.47	10397.87
140.00	-1.58	-1.78	0.06	0.04	0.45	0.43	10398.73
141.00	-1.62	-1.82	0.05	0.04	0.41	0.40	10399.53
142.00	-1.65	-1.85	0.05	0.03	0.38	0.37	10400.26
143.00	-1.69	-1.89	0.05	0.03	0.35	0.34	10400.93
144.00	-1.72	-1.92	0.04	0.03	0.32	0.31	10401.55
145.00	-1.76	-1.96	0.04	0.03	0.30	0.28	10402.12
146.00	-1.79	-1.99	0.04	0.02	0.27	0.26	10402.64
147.00	-1.83	-2.03	0.03	0.02	0.25	0.24	10403.12
148.00	-1.86	-2.06	0.03	0.02	0.23	0.22	10403.56
149.00	-1.89	-2.09	0.03	0.02	0.21	0.20	10403.96
150.00	-1.93	-2.13	0.03	0.02	0.19	0.18	10404.33
151.00	-1.96	-2.16	0.03	0.02	0.18	0.17	10404.66
152.00	-1.99	-2.19	0.02	0.01	0.16	0.16	10404.97
153.00	-2.03	-2.23	0.02	0.01	0.15	0.14	10405.26
154.00	-2.06	-2.26	0.02	0.01	0.14	0.13	10405.52
155.00	-2.09	-2.29	0.02	0.01	0.12	0.12	10405.76
156.00	-2.12	-2.32	0.02	0.01	0.11	0.11	10405.97
157.00	-2.16	-2.36	0.02	0.01	0.10	0.10	10406.17
158.00	-2.19	-2.39	0.01	0.01	0.10	0.09	10406.36
159.00	-2.22	-2.42	0.01	0.01	0.09	0.08	10406.52
160.00	-2.25	-2.45	0.01	0.01	0.08	0.08	10406.67
161.00	-2.28	-2.48	0.01	0.01	0.07	0.07	10406.81
162.00	-2.31	-2.51	0.01	0.01	0.07	0.06	10406.94
163.00	-2.34	-2.54	0.01	0.01	0.06	0.06	10407.06
164.00	-2.37	-2.57	0.01	0.01	0.06	0.05	10407.16
165.00	-2.40	-2.60	0.01	0.01	0.05	0.05	10407.26
166.00	-2.43	-2.63	0.01	0.00	0.05	0.04	10407.35

167.00	-2.46	-2.66	0.01	0.00	0.04	0.04	10407.43
168.00	-2.49	-2.69	0.01	0.00	0.04	0.04	10407.50
169.00	-2.52	-2.72	0.01	0.00	0.04	0.03	10407.57
170.00	-2.55	-2.75	0.01	0.00	0.03	0.03	10407.63
171.00	-2.58	-2.78	0.01	0.00	0.03	0.03	10407.69
172.00	-2.61	-2.81	0.01	0.00	0.03	0.03	10407.74
173.00	-2.64	-2.84	0.00	0.00	0.02	0.02	10407.78
174.00	-2.67	-2.87	0.00	0.00	0.02	0.02	10407.83
175.00	-2.70	-2.90	0.00	0.00	0.02	0.02	10407.87
176.00	-2.73	-2.93	0.00	0.00	0.02	0.02	10407.90
177.00	-2.76	-2.96	0.00	0.00	0.02	0.02	10407.93
178.00	-2.78	-2.98	0.00	0.00	0.02	0.02	10407.96
179.00	-2.81	-3.01	0.00	0.00	0.01	0.01	10407.99
180.00	-2.84	-3.04	0.00	0.00	0.01	0.01	10408.01
181.00	-2.87	-3.07	0.00	0.00	0.01	0.01	10408.03
182.00	-2.89	-3.09	0.00	0.00	0.01	0.01	10408.05
183.00	-2.92	-3.12	0.00	0.00	0.01	0.01	10408.07
184.00	-2.95	-3.15	0.00	0.00	0.01	0.01	10408.09
185.00	-2.98	-3.18	0.00	0.00	0.01	0.01	10408.10
186.00	-3.00	-3.20	0.00	0.00	0.01	0.01	10408.12
187.00	-3.03	-3.23	0.00	0.00	0.01	0.01	10408.13
188.00	-3.06	-3.26	0.00	0.00	0.01	0.01	10408.14
189.00	-3.08	-3.28	0.00	0.00	0.01	0.01	10408.15
190.00	-3.11	-3.31	0.00	0.00	0.01	0.01	10408.16
191.00	-3.14	-3.34	0.00	0.00	0.00	0.00	10408.17
192.00	-3.16	-3.36	0.00	0.00	0.00	0.00	10408.18
193.00	-3.19	-3.39	0.00	0.00	0.00	0.00	10408.18
194.00	-3.21	-3.41	0.00	0.00	0.00	0.00	10408.19
195.00	-3.24	-3.44	0.00	0.00	0.00	0.00	10408.20
196.00	-3.27	-3.47	0.00	0.00	0.00	0.00	10408.20

197.00	-3.29	-3.49	0.00	0.00	0.00	0.00	10408.21
198.00	-3.32	-3.52	0.00	0.00	0.00	0.00	10408.21
199.00	-3.34	-3.54	0.00	0.00	0.00	0.00	10408.22
200.00	-3.37	-3.57	0.00	0.00	0.00	0.00	10408.22