

First, let me remind you the Gaussian probability density function:

$$G(x; \mu, \sigma) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}},$$

Here  $x$  is a random variable, which is a price change  $p(t+\tau) - p(t)$  in our case, and  $\mu$  and  $\sigma$  are parameters, for which, as you can check the following equalities are true:

$$\langle x \rangle = \mu \text{ and } \langle x^2 \rangle = \sigma^2,$$

where by  $\langle \dots \rangle$ , I have denoted averaging over a PDF.

Let us, say, you chose to histogram an empirically given to you random variable  $y$  (for example, using an Histogram tool in Excel). For that you need to select an equidistant range of bins, separated with step  $\Delta$ . Then the PDF value obtained from the numerical histogram measurements can be calculated as follows:

$$P(x_i) = \frac{1}{\Delta} \frac{N_i}{N},$$

where  $i$  is the bin number,  $N_i$  is the count of values in bin  $i$ , and  $N = \sum_{i=1}^M N_i$ , where  $M$  is the number

of bins. This empirical PDF needs to be calculated as an function of  $\hat{x}_i = x_i - \frac{\Delta}{2}$ , as the center of bin having  $x_i$  as its larger endpoint.