Columbia University
MATH GR5260 Spring 2023
Programming for Quant and Computational Finance
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Homework Assignment 3
Release date: March 25th 2023 (Sat)
Due date: April 16th 2023 (Sun) 11:00pm

HOMEWORK GUIDELINE

Submit your solution file(s) onto CourseWorks by the specified due date and time.

A. Theory parts

   Your solution to theory questions can be (i) hand-written and scanned as a pdf file or (ii) prepared using Word and be converted into a pdf file or (iii) be included in the solution for practice parts.

B. Practice parts

   Your Python source code and outputs shall be prepared and submitted in a format (eg. pdf, html, .ipynb, .py, etc.) that the graders can understand. Before submitting your solution, make sure that you have run your program successfully to generate all required outputs. Points may be deducted if some outputs are missing.

   You are free to use standard python packages distributed by Anaconda.

   Do not submit additional packages that can be downloaded from the web. Just provide instructions in the solution file.

C. Other files

   Include files that may be requested by the homework assignment. Make proper references in the solution file.

Note: If you suspect there are typos in this homework, or some questions are wrong, please first discuss with your TAs.

BACKGROUND

Suppose you are working in the risk department and have been asked to investigate into which VaR methodology shall be used to evaluate an equity portfolio. The risk manager tells you to look at linear parametric VaR and historical VaR which are commonly used in the market.

QUESTION 1

Suppose you have a portfolio of stock positions as follows.

Symbol	Number of shares (in million)
JNJ	1.2

MRK	2.5
PFE	3.5

a) Write a Python program to compute the parametric 1-day VaR (with 99% confidence) for the above portfolio. The attached files of historical market prices for the above stocks shall be used to estimate the variance-covariance matrix for the portfolio using the exponentially weighted averaging method. The most recent date in the files is considered as the current date. The most recent market price in the files is used as the current market price. The Pandas' built-in EWM classes shall be used to compute the exponentially weighted volatilities and correlations.
b) Write a Python program to compute the historical 1-day VaR (with 99% confidence) for the same portfolio as in part (a). The attached files of historical market prices shall be used to generate the portfolio loss distribution.
c) Comment on the results in (a) and (b). Which method would you recommend to the risk manager?

QUESTION 2

The risk manager would also like to know the VaR on a swap portfolio. He explains that the cash flow profile of the portfolio has already been generated. He wonders how linear parametric VaR can be applied to those cash flows as they fall on dates which are different from the dates of the underlying risk factors. The underlying risk factors are benchmark rates for 1M, 2M, 3M, 6M, 1Y, 2Y, …, 10Y.

You've learnt in class about the general approach of applying parametric VaR to a portfolio of trades. Below is a further analysis of the approach being applied to a cash flow on a date between two benchmark rate dates.

Suppose a cash flow N falls on date T where $T_1 \leq T \leq T_2$ and T_1, T_2 are the two benchmark rate dates of the underlying risk factors.

Let $S = Ne^{-zT}$ be the present value of the cash flow N at T and $B_i = e^{-y_i T_i}$ be the zero coupon bond price at T_i, $i = 1,2$.

Suppose the zero rate at T is linearly interpolated from y_1, y_2. That is, $z = ay_1 + (1-a)y_2$ for some a, $0 \leq a \leq 1$.

a) Use the 1st order Taylor approximation to derive an expression for ΔS in terms of $\frac{\partial S}{\partial B_1}, \frac{\partial S}{\partial B_2}$ and B_1, B_2

The terms $\frac{\partial S}{\partial B_1}$ and $\frac{\partial S}{\partial B_2}$ are considered as the cash flow equivalents of the cash flow N. That is the cash flow on date T is decomposed and represented by two synthetic cash flows on the neighboring benchmark rate dates.

b) Compute these cash flow equivalents in terms of the zero rates z, y_1, y_2.

c) Is the sum of the present value of the two cash flow equivalents in (b) equal to the present value of the actual cash flow on date T?

d) Suppose that we use log-linear interpolation instead of linear interpolation on the zero rates. That is, the assumption of $z = ay_1 + (1-a)y_2$ is replaced by:

$-zT = a\log(B_1) + (1-a)\log(B_2)$.

Compute the cash flow equivalents in terms of the zero rates z, y_1, y_2.

In this case, is the sum of the present value of the two cash flow equivalents equal to the present value of the actual cash flow on date T? Which interpolation method would you recommend in computing parametric VaR of this portfolio and why?

--end of Assignment--