Homework9

April 2, 2023

1 Homework 9

[1]:
```
\l ../columbiaHdb/
```

[2]:
```
\c 50 200
```

1.1 Exercise 1 Creating Synthetic Alphas

1.1.1 1. Compute the correlation ρ between α_t and $R_t - R_T$ for given a, b.

Because $\mathbb{E}(R_t - R_T) = 0$, $\mathbb{E}\alpha_t = a\mathbb{E}(R_t - R_T) + b\mathbb{E}(W_t - W_T) = 0$,

$$\begin{aligned} Cov(\alpha_t, R_t - R_T) =& \mathbb{E}\alpha_t(R_t - R_T) \\ =& a\mathbb{E}(R_t - R_T)^2 + b\mathbb{E}(W_t - W_T)(R_t - R_T) \\ =& a(T-t)\sigma_R^2. \end{aligned}$$

Hence,

$$\begin{aligned} \rho = Corr(\alpha_t, R_t - R_T) =& \frac{Cov(\alpha_t, R_t - R_T)}{\sqrt{Var(\alpha_t)Var(R_t - R_T)}} \\ =& \frac{a(T-t)\sigma_R^2}{\sqrt{(a^2\sigma_R^2 + b^2\sigma_W^2)(T-t)}\sqrt{(T-t)\sigma_R^2}} \\ =& \frac{a\sigma_R}{\sqrt{a^2\sigma_R^2 + b^2\sigma_W^2}}. \end{aligned}$$

1.1.2 2. Pick values of a, b such that $\alpha_t = E[R_t - R_T|\alpha_t]$ and $\rho = 0.05$.

We could find

$$\mathbb{E}[R_t - R_T|\alpha_t] = \frac{a\sigma_S^2}{a^2\sigma_R^2 + b^2\sigma_W^2}.$$

From $\frac{a}{a^2+b^2} = 1$ and $\rho = 0.05$, we get

$$\begin{aligned} a =& \rho^2 = \frac{1}{400} \\ b =& \rho\sqrt{1-\rho^2}\frac{\sigma_R}{\sigma_W} = \frac{\sqrt{399}}{400}\frac{\sigma_R}{\sigma_W} \end{aligned}$$

1.1.3 3.

[3]:
```
dt: 2019.01.03
```

Load in memory the table for the date 2019.01.03 and fill in the missing value, then show it.

[4]:
```
tbl: select from bin10 where date = dt
```

[5]:
```
tbl: update reverse fills reverse date,
 reverse fills reverse mid,
 reverse fills reverse spread,
 reverse fills reverse vol,
 reverse fills reverse adv
 by id from tbl
tbl
```

[5]:

date	time	id	trade	mid	spread	vol	adv
2019.01.03	09:30:00	0	454071.8	91.715	0.001148717	0.000446367	1.122415e+08
2019.01.03	09:30:10	0	-171833.3	91.72671	0.001072136	0.000446367	1.122415e+08
2019.01.03	09:30:20	0	-301.2193	91.58853	7.658112e-05	0.000446367	1.122415e+08
2019.01.03	09:30:30	0	-78884.65	91.65646	0.000612649	0.000446367	1.122415e+08
2019.01.03	09:30:40	0	-24705.54	91.60259	0.0008423923	0.000446367	1.122415e+08
2019.01.03	09:30:50	0	-92166.19	91.58385	0.0005871219	0.000446367	1.122415e+08
2019.01.03	09:31:00	0	-22823.01	91.56276	0.000663703	0.000446367	1.122415e+08
2019.01.03	09:31:10	0	-4940.182	91.59088	0.0005615949	0.000446367	1.122415e+08
2019.01.03	09:31:20	0	-41691.88	91.59322	0.0005360678	0.000446367	1.122415e+08
2019.01.03	09:31:30	0	-539826.4	91.60727	0.0007402842	0.000446367	1.122415e+08
2019.01.03	09:31:40	0	-53883.89	91.5487	0.0004594867	0.000446367	1.122415e+08
2019.01.03	09:31:50	0	316896.2	91.56745	0.0003063245	0.000446367	1.122415e+08
2019.01.03	09:32:00	0	8767.784	91.82033	0.0004594867	0.000446367	1.122415e+08
2019.01.03	09:32:10	0	132407.8	91.84138	0.000638176	0.000446367	1.122415e+08
2019.01.03	09:32:20	0	-52051.64	92.02614	0.0005615949	0.000446367	1.122415e+08
2019.01.03	09:32:30	0	441082.8	92.01445	0.0004339597	0.000446367	1.122415e+08
2019.01.03	09:32:40	0	-182.6154	92.23874	0.000638176	0.000446367	1.122415e+08
2019.01.03	09:32:50	0	4196.414	92.18269	0.0002807974	0.000446367	1.122415e+08
2019.01.03	09:33:00	0	112899.5	92.36479	0.0007402842	0.000446367	1.122415e+08
2019.01.03	09:33:10	0	-51038.96	92.27376	0.000612649	0.000446367	1.122415e+08
2019.01.03	09:33:20	0	-20440.02	92.19904	0.0003063245	0.000446367	1.122415e+08
2019.01.03	09:33:30	0	-121802	92.20604	0.0003318515	0.000446367	1.122415e+08
2019.01.03	09:33:40	0	24287.28	92.07989	0.0005360678	0.000446367	1.122415e+08
2019.01.03	09:33:50	0	-14046.25	92.17101	0.0004084326	0.000446367	1.122415e+08
2019.01.03	09:34:00	0	-12283.33	92.17334	0.0004850138	0.000446367	1.122415e+08
2019.01.03	09:34:10	0	-106524.7	92.16634	0.0003573786	0.000446367	1.122415e+08
2019.01.03	09:34:20	0	0	92.04016	0.000663703	0.000446367	1.122415e+08
2019.01.03	09:34:30	0	-229900.1	92.00276	0.0002552704	0.000446367	1.122415e+08
2019.01.03	09:34:40	0	18159.89	91.88817	0.0003318515	0.000446367	1.122415e+08
2019.01.03	09:34:50	0	25469.17	91.85074	0.0005360678	0.000446367	1.122415e+08
2019.01.03	09:35:00	0	-18781.33	91.94197	0.0003063245	0.000446367	1.122415e+08
2019.01.03	09:35:10	0	3026.875	91.89285	0.0005871219	0.000446367	1.122415e+08

```
2019.01.03 09:35:20 0  -21253.42 91.90689 0.0005360678 0.000446367 1.122415e+08
2019.01.03 09:35:30 0  -37882.34 91.86946 0.0007402842 0.000446367 1.122415e+08
2019.01.03 09:35:40 0  0         91.82033 0.0007147571 0.000446367 1.122415e+08
2019.01.03 09:35:50 0  38337.81  91.82267 0.000638176  0.000446367 1.122415e+08
2019.01.03 09:36:00 0  -261640.9 91.83437 0.0003063245 0.000446367 1.122415e+08
2019.01.03 09:36:10 0  -358734   91.70095 0.0004850138 0.000446367 1.122415e+08
2019.01.03 09:36:20 0  -24277.96 91.74309 0.0004339597 0.000446367 1.122415e+08
2019.01.03 09:36:30 0  -7837.575 91.63538 0.0007913382 0.000446367 1.122415e+08
2019.01.03 09:36:40 0  -68098.53 91.60962 0.0005105408 0.000446367 1.122415e+08
2019.01.03 09:36:50 0  -187543.1 91.56979 0.0003318515 0.000446367 1.122415e+08
2019.01.03 09:37:00 0  -445392.8 91.45495 0.0003063245 0.000446367 1.122415e+08
2019.01.03 09:37:10 0  24626.54  91.25089 0.0004850138 0.000446367 1.122415e+08
2019.01.03 09:37:20 0  -8152.001 91.27436 0.0004339597 0.000446367 1.122415e+08
..
```

First, calculate $R_t = \frac{S_T}{S_t} - 1$.

[6]:
```
tbl: update rtn: ((last mid) % mid) - 1 by id from tbl
`date`time`id`mid`vol`adv`rtn#/:tbl
```

[6]:
```
date       time     id mid      vol         adv          rtn
-------------------------------------------------------------------------
2019.01.03 09:30:00 0  91.715   0.000446367 1.122415e+08 -0.02958659
2019.01.03 09:30:10 0  91.72671 0.000446367 1.122415e+08 -0.02971043
2019.01.03 09:30:20 0  91.58853 0.000446367 1.122415e+08 -0.02824664
2019.01.03 09:30:30 0  91.65646 0.000446367 1.122415e+08 -0.02896681
2019.01.03 09:30:40 0  91.60259 0.000446367 1.122415e+08 -0.02839575
2019.01.03 09:30:50 0  91.58385 0.000446367 1.122415e+08 -0.02819692
2019.01.03 09:31:00 0  91.56276 0.000446367 1.122415e+08 -0.02797313
2019.01.03 09:31:10 0  91.59088 0.000446367 1.122415e+08 -0.02827149
2019.01.03 09:31:20 0  91.59322 0.000446367 1.122415e+08 -0.02829635
2019.01.03 09:31:30 0  91.60727 0.000446367 1.122415e+08 -0.02844544
2019.01.03 09:31:40 0  91.5487  0.000446367 1.122415e+08 -0.02782386
2019.01.03 09:31:50 0  91.56745 0.000446367 1.122415e+08 -0.02802287
2019.01.03 09:32:00 0  91.82033 0.000446367 1.122415e+08 -0.03069975
2019.01.03 09:32:10 0  91.84138 0.000446367 1.122415e+08 -0.030922
2019.01.03 09:32:20 0  92.02614 0.000446367 1.122415e+08 -0.03286751
2019.01.03 09:32:30 0  92.01445 0.000446367 1.122415e+08 -0.03274466
2019.01.03 09:32:40 0  92.23874 0.000446367 1.122415e+08 -0.03509664
2019.01.03 09:32:50 0  92.18269 0.000446367 1.122415e+08 -0.03450997
2019.01.03 09:33:00 0  92.36479 0.000446367 1.122415e+08 -0.03641345
2019.01.03 09:33:10 0  92.27376 0.000446367 1.122415e+08 -0.03546287
2019.01.03 09:33:20 0  92.19904 0.000446367 1.122415e+08 -0.03468117
2019.01.03 09:33:30 0  92.20604 0.000446367 1.122415e+08 -0.03475452
2019.01.03 09:33:40 0  92.07989 0.000446367 1.122415e+08 -0.03343212
2019.01.03 09:33:50 0  92.17101 0.000446367 1.122415e+08 -0.03438764
```

```
2019.01.03 09:34:00 0  92.17334 0.000446367 1.122415e+08 -0.03441211
2019.01.03 09:34:10 0  92.16634 0.000446367 1.122415e+08 -0.03433869
2019.01.03 09:34:20 0  92.04016 0.000446367 1.122415e+08 -0.03301488
2019.01.03 09:34:30 0  92.00276 0.000446367 1.122415e+08 -0.03262178
2019.01.03 09:34:40 0  91.88817 0.000446367 1.122415e+08 -0.03141545
2019.01.03 09:34:50 0  91.85074 0.000446367 1.122415e+08 -0.03102074
2019.01.03 09:35:00 0  91.94197 0.000446367 1.122415e+08 -0.03198214
2019.01.03 09:35:10 0  91.89285 0.000446367 1.122415e+08 -0.03146476
2019.01.03 09:35:20 0  91.90689 0.000446367 1.122415e+08 -0.03161265
2019.01.03 09:35:30 0  91.86946 0.000446367 1.122415e+08 -0.03121814
2019.01.03 09:35:40 0  91.82033 0.000446367 1.122415e+08 -0.03069975
2019.01.03 09:35:50 0  91.82267 0.000446367 1.122415e+08 -0.03072445
2019.01.03 09:36:00 0  91.83437 0.000446367 1.122415e+08 -0.03084793
2019.01.03 09:36:10 0  91.70095 0.000446367 1.122415e+08 -0.02943793
2019.01.03 09:36:20 0  91.74309 0.000446367 1.122415e+08 -0.02988374
2019.01.03 09:36:30 0  91.63538 0.000446367 1.122415e+08 -0.02874345
2019.01.03 09:36:40 0  91.60962 0.000446367 1.122415e+08 -0.02847028
2019.01.03 09:36:50 0  91.56979 0.000446367 1.122415e+08 -0.02804774
2019.01.03 09:37:00 0  91.45495 0.000446367 1.122415e+08 -0.02682728
2019.01.03 09:37:10 0  91.25089 0.000446367 1.122415e+08 -0.02465101
2019.01.03 09:37:20 0  91.27436 0.000446367 1.122415e+08 -0.02490177
..
```

Now, we could calculate σ_R.

[7]:
```
tbl: update rtn_diff: rtn - xprev[neg 1; rtn] by id from tbl
tbl: update rtn_sigma: sqrt(avg(rtn_diff * rtn_diff)) by id from tbl
```

From the following table we find that $\sigma_R \approx$vol.

[8]:
```
select last vol, last rtn_sigma by id from tbl
```

[8]:
```
id| vol          rtn_sigma
--| -------------------------
0 | 0.000446367  0.0004414231
1 | 0.000358252  0.0003545061
2 | 0.0004827488 0.0004584916
3 | 0.0004667023 0.0004467735
4 | 0.0003663134 0.0003470099
5 | 0.000516633  0.0005016485
6 | 0.0004753984 0.0004681693
7 | 0.0004223277 0.0004000829
8 | 0.0007492939 0.0007116007
9 | 0.0006668529 0.0006560443
10| 0.0007148106 0.0006990004
11| 0.0003864019 0.0003812525
12| 0.0003828439 0.0003630417
```

```
13| 0.0004774389 0.0004530195
14| 0.0005112646 0.0005068091
15| 0.000372391  0.0003684432
16| 0.0009286111 0.0008846601
17| 0.0006442204 0.0006265213
18| 0.0006423178 0.0006357137
19| 0.0004340049 0.0004093612
20| 0.0004842376 0.0004820598
21| 0.0003934688 0.0003871459
22| 0.0003475031 0.0003244776
23| 0.0006300022 0.0006277845
24| 0.0004235462 0.0004085966
25| 0.0004774169 0.0004748735
26| 0.0003429665 0.0003292576
27| 0.0006841965 0.0006733431
28| 0.0005421428 0.0005375367
29| 0.000553965  0.0005310981
30| 0.0004884136 0.0004804992
31| 0.0004894108 0.0004371352
32| 0.0003286248 0.0003048115
33| 0.0004340077 0.0004086145
34| 0.0004228632 0.0004147984
35| 0.0004287401 0.0004180894
36| 0.0005528349 0.0005365599
37| 0.0005698626 0.0005621711
38| 0.0004820823 0.0004798723
39| 0.00040451   0.0003907447
40| 0.0004607547 0.0004365676
41| 0.0005473872 0.0005464307
42| 0.0006445486 0.0006407746
43| 0.000435497  0.0004351681
44| 0.0004997701 0.0004871171
..
```

Then simulate a Wiener process W with volatility $\sigma_W = \sigma_R$ for each stock.

[9]:
```
u12: {[n] -6f+sum n cut (12*n)?1f}
```

[10]:
```
tbl: update W: vol * (sums(u12[count time])) by id from tbl
`date`time`id`mid`vol`adv`rtn`W#/:tbl
```

[10]:
```
date       time     id mid      vol         adv          rtn         W
-------------------------------------------------------------------------------
--
2019.01.03 09:30:00 0  91.715   0.000446367 1.122415e+08 -0.02958659
-0.0003626282
2019.01.03 09:30:10 0  91.72671 0.000446367 1.122415e+08 -0.02971043
```

```
-0.0003141922
2019.01.03 09:30:20 0  91.58853 0.000446367 1.122415e+08 -0.02824664
-0.001206716
2019.01.03 09:30:30 0  91.65646 0.000446367 1.122415e+08 -0.02896681
-0.0005155496
2019.01.03 09:30:40 0  91.60259 0.000446367 1.122415e+08 -0.02839575
5.054547e-05
2019.01.03 09:30:50 0  91.58385 0.000446367 1.122415e+08 -0.02819692
0.0005975765
2019.01.03 09:31:00 0  91.56276 0.000446367 1.122415e+08 -0.02797313
0.0006625432
2019.01.03 09:31:10 0  91.59088 0.000446367 1.122415e+08 -0.02827149
0.0004544252
2019.01.03 09:31:20 0  91.59322 0.000446367 1.122415e+08 -0.02829635
0.0001759496
2019.01.03 09:31:30 0  91.60727 0.000446367 1.122415e+08 -0.02844544
0.0003060442
2019.01.03 09:31:40 0  91.5487  0.000446367 1.122415e+08 -0.02782386
0.0001112769
2019.01.03 09:31:50 0  91.56745 0.000446367 1.122415e+08 -0.02802287
-0.0002910298
2019.01.03 09:32:00 0  91.82033 0.000446367 1.122415e+08 -0.03069975
6.722897e-05
2019.01.03 09:32:10 0  91.84138 0.000446367 1.122415e+08 -0.030922
-0.0004550431
2019.01.03 09:32:20 0  92.02614 0.000446367 1.122415e+08 -0.03286751
-0.0002166572
2019.01.03 09:32:30 0  92.01445 0.000446367 1.122415e+08 -0.03274466
0.0006385948
2019.01.03 09:32:40 0  92.23874 0.000446367 1.122415e+08 -0.03509664
0.0008541204
2019.01.03 09:32:50 0  92.18269 0.000446367 1.122415e+08 -0.03450997
0.0008961222
2019.01.03 09:33:00 0  92.36479 0.000446367 1.122415e+08 -0.03641345 0.001131108
2019.01.03 09:33:10 0  92.27376 0.000446367 1.122415e+08 -0.03546287
0.0008855293
2019.01.03 09:33:20 0  92.19904 0.000446367 1.122415e+08 -0.03468117
0.0008984778
2019.01.03 09:33:30 0  92.20604 0.000446367 1.122415e+08 -0.03475452
0.0009198473
2019.01.03 09:33:40 0  92.07989 0.000446367 1.122415e+08 -0.03343212 0.001441632
2019.01.03 09:33:50 0  92.17101 0.000446367 1.122415e+08 -0.03438764
0.0009481515
2019.01.03 09:34:00 0  92.17334 0.000446367 1.122415e+08 -0.03441211 0.002182223
2019.01.03 09:34:10 0  92.16634 0.000446367 1.122415e+08 -0.03433869 0.002146889
2019.01.03 09:34:20 0  92.04016 0.000446367 1.122415e+08 -0.03301488 0.002340331
2019.01.03 09:34:30 0  92.00276 0.000446367 1.122415e+08 -0.03262178 0.00196064
```

```
2019.01.03 09:34:40 0  91.88817 0.000446367 1.122415e+08 -0.03141545 0.001621693
2019.01.03 09:34:50 0  91.85074 0.000446367 1.122415e+08 -0.03102074 0.00141621
2019.01.03 09:35:00 0  91.94197 0.000446367 1.122415e+08 -0.03198214 0.002134461
2019.01.03 09:35:10 0  91.89285 0.000446367 1.122415e+08 -0.03146476 0.001942889
2019.01.03 09:35:20 0  91.90689 0.000446367 1.122415e+08 -0.03161265 0.002124067
2019.01.03 09:35:30 0  91.86946 0.000446367 1.122415e+08 -0.03121814 0.002068588
2019.01.03 09:35:40 0  91.82033 0.000446367 1.122415e+08 -0.03069975 0.002092089
2019.01.03 09:35:50 0  91.82267 0.000446367 1.122415e+08 -0.03072445 0.001487925
2019.01.03 09:36:00 0  91.83437 0.000446367 1.122415e+08 -0.03084793 0.001448015
2019.01.03 09:36:10 0  91.70095 0.000446367 1.122415e+08 -0.02943793 0.001268466
2019.01.03 09:36:20 0  91.74309 0.000446367 1.122415e+08 -0.02988374
0.0008614023
2019.01.03 09:36:30 0  91.63538 0.000446367 1.122415e+08 -0.02874345
0.0007591041
2019.01.03 09:36:40 0  91.60962 0.000446367 1.122415e+08 -0.02847028
0.0006694554
2019.01.03 09:36:50 0  91.56979 0.000446367 1.122415e+08 -0.02804774 0.001380889
2019.01.03 09:37:00 0  91.45495 0.000446367 1.122415e+08 -0.02682728 0.001849998
2019.01.03 09:37:10 0  91.25089 0.000446367 1.122415e+08 -0.02465101 0.002239283
2019.01.03 09:37:20 0  91.27436 0.000446367 1.122415e+08 -0.02490177 0.002173451
..
```

Calculate the value of a and b based on $\rho = 0.05$.

[11]:
```
rho: 0.3
a: rho * rho
b: rho * sqrt (1 - rho * rho)
rho, a, b
```

[11]:
```
0.3 0.09 0.2861818
```

Calculate α_t, α'_t. We will choose 1 minute as a unit.

[12]:
```
tbl: update alpha: (a * (rtn - last rtn)) + (b * (W - last W)) by id from tbl
tbl: update dalpha: 0 ^ ((alpha - xprev[60; alpha]) % 10) by id from tbl
`date`time`id`mid`vol`adv`rtn`W`alpha`dalpha#/:tbl
```

[12]:
```
date       time     id mid      vol         adv          rtn         W
alpha        dalpha
-------------------------------------------------------------------------------
---------------------
2019.01.03 09:30:00 0  91.715   0.000446367 1.122415e+08 -0.02958659
-0.0003626282 -0.00497326  0
2019.01.03 09:30:10 0  91.72671 0.000446367 1.122415e+08 -0.02971043
-0.0003141922 -0.004970544 0
2019.01.03 09:30:20 0  91.58853 0.000446367 1.122415e+08 -0.02824664
```

```
-0.001206716  -0.005094226 0
2019.01.03 09:30:30 0  91.65646 0.000446367 1.122415e+08 -0.02896681
-0.0005155496 -0.004961243 0
2019.01.03 09:30:40 0  91.60259 0.000446367 1.122415e+08 -0.02839575
5.054547e-05  -0.004747841 0
2019.01.03 09:30:50 0  91.58385 0.000446367 1.122415e+08 -0.02819692
0.0005975765  -0.004573396 0
2019.01.03 09:31:00 0  91.56276 0.000446367 1.122415e+08 -0.02797313
0.0006625432  -0.004534662 0
2019.01.03 09:31:10 0  91.59088 0.000446367 1.122415e+08 -0.02827149
0.0004544252  -0.004621075 0
2019.01.03 09:31:20 0  91.59322 0.000446367 1.122415e+08 -0.02829635
0.0001759496  -0.004703007 0
2019.01.03 09:31:30 0  91.60727 0.000446367 1.122415e+08 -0.02844544
0.0003060442  -0.004679194 0
2019.01.03 09:31:40 0  91.5487  0.000446367 1.122415e+08 -0.02782386
0.0001112769  -0.004678991 0
2019.01.03 09:31:50 0  91.56745 0.000446367 1.122415e+08 -0.02802287
-0.0002910298 -0.004812035 0
2019.01.03 09:32:00 0  91.82033 0.000446367 1.122415e+08 -0.03069975
6.722897e-05  -0.004950426 0
2019.01.03 09:32:10 0  91.84138 0.000446367 1.122415e+08 -0.030922
-0.0004550431 -0.005119894 0
2019.01.03 09:32:20 0  92.02614 0.000446367 1.122415e+08 -0.03286751
-0.0002166572 -0.005226768 0
2019.01.03 09:32:30 0  92.01445 0.000446367 1.122415e+08 -0.03274466
0.0006385948  -0.004970954 0
2019.01.03 09:32:40 0  92.23874 0.000446367 1.122415e+08 -0.03509664
0.0008541204  -0.005120953 0
2019.01.03 09:32:50 0  92.18269 0.000446367 1.122415e+08 -0.03450997
0.0008961222  -0.005056132 0
2019.01.03 09:33:00 0  92.36479 0.000446367 1.122415e+08 -0.03641345 0.001131108
-0.005160197 0
2019.01.03 09:33:10 0  92.27376 0.000446367 1.122415e+08 -0.03546287
0.0008855293  -0.005144925 0
2019.01.03 09:33:20 0  92.19904 0.000446367 1.122415e+08 -0.03468117
0.0008984778  -0.005070867 0
2019.01.03 09:33:30 0  92.20604 0.000446367 1.122415e+08 -0.03475452
0.0009198473  -0.005071353 0
2019.01.03 09:33:40 0  92.07989 0.000446367 1.122415e+08 -0.03343212 0.001441632
-0.00480301  0
2019.01.03 09:33:50 0  92.17101 0.000446367 1.122415e+08 -0.03438764
0.0009481515  -0.005030232 0
2019.01.03 09:34:00 0  92.17334 0.000446367 1.122415e+08 -0.03441211 0.002182223
-0.004679266 0
2019.01.03 09:34:10 0  92.16634 0.000446367 1.122415e+08 -0.03433869 0.002146889
-0.004682771 0
```

```
2019.01.03 09:34:20 0  92.04016 0.000446367 1.122415e+08 -0.03301488 0.002340331
-0.004508268 0
2019.01.03 09:34:30 0  92.00276 0.000446367 1.122415e+08 -0.03262178 0.00196064
-0.004581549 0
2019.01.03 09:34:40 0  91.88817 0.000446367 1.122415e+08 -0.03141545 0.001621693
-0.00456998  0
2019.01.03 09:34:50 0  91.85074 0.000446367 1.122415e+08 -0.03102074 0.00141621
-0.004593262 0
2019.01.03 09:35:00 0  91.94197 0.000446367 1.122415e+08 -0.03198214 0.002134461
-0.004474238 0
2019.01.03 09:35:10 0  91.89285 0.000446367 1.122415e+08 -0.03146476 0.001942889
-0.004482497 0
2019.01.03 09:35:20 0  91.90689 0.000446367 1.122415e+08 -0.03161265 0.002124067
-0.004443958 0
2019.01.03 09:35:30 0  91.86946 0.000446367 1.122415e+08 -0.03121814 0.002068588
-0.004424329 0
2019.01.03 09:35:40 0  91.82033 0.000446367 1.122415e+08 -0.03069975 0.002092089
-0.004370948 0
2019.01.03 09:35:50 0  91.82267 0.000446367 1.122415e+08 -0.03072445 0.001487925
-0.004546072 0
2019.01.03 09:36:00 0  91.83437 0.000446367 1.122415e+08 -0.03084793 0.001448015
-0.004568607 0
2019.01.03 09:36:10 0  91.70095 0.000446367 1.122415e+08 -0.02943793 0.001268466
-0.004493091 0
2019.01.03 09:36:20 0  91.74309 0.000446367 1.122415e+08 -0.02988374
0.0008614023  -0.004649708 0
2019.01.03 09:36:30 0  91.63538 0.000446367 1.122415e+08 -0.02874345
0.0007591041  -0.004576358 0
2019.01.03 09:36:40 0  91.60962 0.000446367 1.122415e+08 -0.02847028
0.0006694554  -0.004577428 0
2019.01.03 09:36:50 0  91.56979 0.000446367 1.122415e+08 -0.02804774 0.001380889
-0.0043358   0
2019.01.03 09:37:00 0  91.45495 0.000446367 1.122415e+08 -0.02682728 0.001849998
-0.004091709 0
2019.01.03 09:37:10 0  91.25089 0.000446367 1.122415e+08 -0.02465101 0.002239283
-0.003784438 0
2019.01.03 09:37:20 0  91.27436 0.000446367 1.122415e+08 -0.02490177 0.002173451
-0.003825846 0
..
```

1.2 Exercise 2 Simulating Trading Strategies

1.2.1 1. Simulate the target impact state for the optimal trading strategy.

The optimal trading strategy is $I_t = \frac{1}{2}(\alpha_t - \frac{1}{\beta}\alpha'_t)$ and $I_T = \alpha_T$.

[13]:
```
beta: (log 2) % 60
beta
```

[13]:
```
0.01155245
```

[14]:
```
tbl: update I: (last alpha) ^ next prev 0.5 * (alpha - dalpha % beta) by id␣
 ↪from tbl
`date`time`id`mid`adv`rtn`W`alpha`dalpha`I#/:tbl
```

[14]:
```
date       time     id mid      adv          rtn         W             alpha
dalpha I
-------------------------------------------------------------------------------
----------------------
2019.01.03 09:30:00 0  91.715   1.122415e+08 -0.02958659 -0.0003626282
-0.00497326  0      -0.00248663
2019.01.03 09:30:10 0  91.72671 1.122415e+08 -0.02971043 -0.0003141922
-0.004970544 0      -0.002485272
2019.01.03 09:30:20 0  91.58853 1.122415e+08 -0.02824664 -0.001206716
-0.005094226 0      -0.002547113
2019.01.03 09:30:30 0  91.65646 1.122415e+08 -0.02896681 -0.0005155496
-0.004961243 0      -0.002480621
2019.01.03 09:30:40 0  91.60259 1.122415e+08 -0.02839575 5.054547e-05
-0.004747841 0      -0.00237392
2019.01.03 09:30:50 0  91.58385 1.122415e+08 -0.02819692 0.0005975765
-0.004573396 0      -0.002286698
2019.01.03 09:31:00 0  91.56276 1.122415e+08 -0.02797313 0.0006625432
-0.004534662 0      -0.002267331
2019.01.03 09:31:10 0  91.59088 1.122415e+08 -0.02827149 0.0004544252
-0.004621075 0      -0.002310538
2019.01.03 09:31:20 0  91.59322 1.122415e+08 -0.02829635 0.0001759496
-0.004703007 0      -0.002351503
2019.01.03 09:31:30 0  91.60727 1.122415e+08 -0.02844544 0.0003060442
-0.004679194 0      -0.002339597
2019.01.03 09:31:40 0  91.5487  1.122415e+08 -0.02782386 0.0001112769
-0.004678991 0      -0.002339495
2019.01.03 09:31:50 0  91.56745 1.122415e+08 -0.02802287 -0.0002910298
-0.004812035 0      -0.002406017
2019.01.03 09:32:00 0  91.82033 1.122415e+08 -0.03069975 6.722897e-05
-0.004950426 0      -0.002475213
2019.01.03 09:32:10 0  91.84138 1.122415e+08 -0.030922   -0.0004550431
-0.005119894 0      -0.002559947
2019.01.03 09:32:20 0  92.02614 1.122415e+08 -0.03286751 -0.0002166572
-0.005226768 0      -0.002613384
2019.01.03 09:32:30 0  92.01445 1.122415e+08 -0.03274466 0.0006385948
-0.004970954 0      -0.002485477
2019.01.03 09:32:40 0  92.23874 1.122415e+08 -0.03509664 0.0008541204
```

```
-0.005120953 0      -0.002560477
2019.01.03 09:32:50 0  92.18269 1.122415e+08 -0.03450997 0.0008961222
-0.005056132 0      -0.002528066
2019.01.03 09:33:00 0  92.36479 1.122415e+08 -0.03641345 0.001131108
-0.005160197 0      -0.002580099
2019.01.03 09:33:10 0  92.27376 1.122415e+08 -0.03546287 0.0008855293
-0.005144925 0      -0.002572462
2019.01.03 09:33:20 0  92.19904 1.122415e+08 -0.03468117 0.0008984778
-0.005070867 0      -0.002535433
2019.01.03 09:33:30 0  92.20604 1.122415e+08 -0.03475452 0.0009198473
-0.005071353 0      -0.002535676
2019.01.03 09:33:40 0  92.07989 1.122415e+08 -0.03343212 0.001441632
-0.00480301  0      -0.002401505
2019.01.03 09:33:50 0  92.17101 1.122415e+08 -0.03438764 0.0009481515
-0.005030232 0      -0.002515116
2019.01.03 09:34:00 0  92.17334 1.122415e+08 -0.03441211 0.002182223
-0.004679266 0      -0.002339633
2019.01.03 09:34:10 0  92.16634 1.122415e+08 -0.03433869 0.002146889
-0.004682771 0      -0.002341385
2019.01.03 09:34:20 0  92.04016 1.122415e+08 -0.03301488 0.002340331
-0.004508268 0      -0.002254134
2019.01.03 09:34:30 0  92.00276 1.122415e+08 -0.03262178 0.00196064
-0.004581549 0      -0.002290775
2019.01.03 09:34:40 0  91.88817 1.122415e+08 -0.03141545 0.001621693
-0.00456998  0      -0.00228499
2019.01.03 09:34:50 0  91.85074 1.122415e+08 -0.03102074 0.00141621
-0.004593262 0      -0.002296631
2019.01.03 09:35:00 0  91.94197 1.122415e+08 -0.03198214 0.002134461
-0.004474238 0      -0.002237119
2019.01.03 09:35:10 0  91.89285 1.122415e+08 -0.03146476 0.001942889
-0.004482497 0      -0.002241249
2019.01.03 09:35:20 0  91.90689 1.122415e+08 -0.03161265 0.002124067
-0.004443958 0      -0.002221979
2019.01.03 09:35:30 0  91.86946 1.122415e+08 -0.03121814 0.002068588
-0.004424329 0      -0.002212165
2019.01.03 09:35:40 0  91.82033 1.122415e+08 -0.03069975 0.002092089
-0.004370948 0      -0.002185474
2019.01.03 09:35:50 0  91.82267 1.122415e+08 -0.03072445 0.001487925
-0.004546072 0      -0.002273036
2019.01.03 09:36:00 0  91.83437 1.122415e+08 -0.03084793 0.001448015
-0.004568607 0      -0.002284303
2019.01.03 09:36:10 0  91.70095 1.122415e+08 -0.02943793 0.001268466
-0.004493091 0      -0.002246546
2019.01.03 09:36:20 0  91.74309 1.122415e+08 -0.02988374 0.0008614023
-0.004649708 0      -0.002324854
2019.01.03 09:36:30 0  91.63538 1.122415e+08 -0.02874345 0.0007591041
-0.004576358 0      -0.002288179
```

```
2019.01.03 09:36:40 0  91.60962 1.122415e+08 -0.02847028 0.0006694554
-0.004577428 0      -0.002288714
2019.01.03 09:36:50 0  91.56979 1.122415e+08 -0.02804774 0.001380889
-0.0043358   0      -0.0021679
2019.01.03 09:37:00 0  91.45495 1.122415e+08 -0.02682728 0.001849998
-0.004091709 0      -0.002045854
2019.01.03 09:37:10 0  91.25089 1.122415e+08 -0.02465101 0.002239283
-0.003784438 0      -0.001892219
2019.01.03 09:37:20 0  91.27436 1.122415e+08 -0.02490177 0.002173451
-0.003825846 0      -0.001912923
..
```

Confirm $I_T = \alpha_T$.

[15]:
```
select last I, last alpha by id from tbl
```

[15]:
```
id| I alpha
--| -------
0 | 0 0
1 | 0 0
2 | 0 0
3 | 0 0
4 | 0 0
5 | 0 0
6 | 0 0
7 | 0 0
8 | 0 0
9 | 0 0
10| 0 0
11| 0 0
12| 0 0
13| 0 0
14| 0 0
15| 0 0
16| 0 0
17| 0 0
18| 0 0
19| 0 0
20| 0 0
21| 0 0
22| 0 0
23| 0 0
24| 0 0
25| 0 0
26| 0 0
27| 0 0
28| 0 0
```

```
29| 0 0
30| 0 0
31| 0 0
32| 0 0
33| 0 0
34| 0 0
35| 0 0
36| 0 0
37| 0 0
38| 0 0
39| 0 0
40| 0 0
41| 0 0
42| 0 0
43| 0 0
44| 0 0
..
```

1.2.2 2. Given an impact state, simulate the corresponding trades.

Suppose $\lambda = 8 \cdot \frac{\sigma}{\text{adv}}$

[16]:
```
tbl: update lambda: 8 * (vol * sqrt 6 * 60 * 6.5) % adv by id from tbl
`date`time`id`vol`lambda#/:tbl
```

[16]:
```
date       time     id vol         lambda
-----------------------------------------------
2019.01.03 09:30:00 0  0.000446367 1.538993e-09
2019.01.03 09:30:10 0  0.000446367 1.538993e-09
2019.01.03 09:30:20 0  0.000446367 1.538993e-09
2019.01.03 09:30:30 0  0.000446367 1.538993e-09
2019.01.03 09:30:40 0  0.000446367 1.538993e-09
2019.01.03 09:30:50 0  0.000446367 1.538993e-09
2019.01.03 09:31:00 0  0.000446367 1.538993e-09
2019.01.03 09:31:10 0  0.000446367 1.538993e-09
2019.01.03 09:31:20 0  0.000446367 1.538993e-09
2019.01.03 09:31:30 0  0.000446367 1.538993e-09
2019.01.03 09:31:40 0  0.000446367 1.538993e-09
2019.01.03 09:31:50 0  0.000446367 1.538993e-09
2019.01.03 09:32:00 0  0.000446367 1.538993e-09
2019.01.03 09:32:10 0  0.000446367 1.538993e-09
2019.01.03 09:32:20 0  0.000446367 1.538993e-09
2019.01.03 09:32:30 0  0.000446367 1.538993e-09
2019.01.03 09:32:40 0  0.000446367 1.538993e-09
2019.01.03 09:32:50 0  0.000446367 1.538993e-09
2019.01.03 09:33:00 0  0.000446367 1.538993e-09
2019.01.03 09:33:10 0  0.000446367 1.538993e-09
```

```
2019.01.03 09:33:20 0  0.000446367 1.538993e-09
2019.01.03 09:33:30 0  0.000446367 1.538993e-09
2019.01.03 09:33:40 0  0.000446367 1.538993e-09
2019.01.03 09:33:50 0  0.000446367 1.538993e-09
2019.01.03 09:34:00 0  0.000446367 1.538993e-09
2019.01.03 09:34:10 0  0.000446367 1.538993e-09
2019.01.03 09:34:20 0  0.000446367 1.538993e-09
2019.01.03 09:34:30 0  0.000446367 1.538993e-09
2019.01.03 09:34:40 0  0.000446367 1.538993e-09
2019.01.03 09:34:50 0  0.000446367 1.538993e-09
2019.01.03 09:35:00 0  0.000446367 1.538993e-09
2019.01.03 09:35:10 0  0.000446367 1.538993e-09
2019.01.03 09:35:20 0  0.000446367 1.538993e-09
2019.01.03 09:35:30 0  0.000446367 1.538993e-09
2019.01.03 09:35:40 0  0.000446367 1.538993e-09
2019.01.03 09:35:50 0  0.000446367 1.538993e-09
2019.01.03 09:36:00 0  0.000446367 1.538993e-09
2019.01.03 09:36:10 0  0.000446367 1.538993e-09
2019.01.03 09:36:20 0  0.000446367 1.538993e-09
2019.01.03 09:36:30 0  0.000446367 1.538993e-09
2019.01.03 09:36:40 0  0.000446367 1.538993e-09
2019.01.03 09:36:50 0  0.000446367 1.538993e-09
2019.01.03 09:37:00 0  0.000446367 1.538993e-09
2019.01.03 09:37:10 0  0.000446367 1.538993e-09
2019.01.03 09:37:20 0  0.000446367 1.538993e-09
..
```

Then $dQ_t = \frac{1}{\lambda}(-\beta I_t dt + dI_t)$.

$$\begin{aligned}\Delta Q_n = \frac{1}{\lambda}\Delta I_n =& \frac{1}{\lambda}(I_n - I_{n-}) \\ =& \frac{1}{\lambda}(I_n - I_{n-1}e^{-\beta\Delta t})\end{aligned}$$

Calculate $I_{n-} = I_{n-1}e^{-\beta\Delta t}$

[17]:
```
tbl: update I_: 0 ^ xprev[1; I] * exp neg beta % 6 by id from tbl
`date`time`id`mid`rtn`W`alpha`dalpha`I`I_#/:tbl
```

[17]:
```
date       time     id mid      rtn         W             alpha        dalpha I
I_
-------------------------------------------------------------------------------
-----------------------
2019.01.03 09:30:00 0  91.715   -0.02958659 -0.0003626282 -0.00497326  0
-0.00248663  0
2019.01.03 09:30:10 0  91.72671 -0.02971043 -0.0003141922 -0.004970544 0
-0.002485272 -0.002481847
2019.01.03 09:30:20 0  91.58853 -0.02824664 -0.001206716  -0.005094226 0
```

```
-0.002547113 -0.002480491
2019.01.03 09:30:30 0  91.65646 -0.02896681 -0.0005155496 -0.004961243 0
-0.002480621 -0.002542214
2019.01.03 09:30:40 0  91.60259 -0.02839575 5.054547e-05  -0.004747841 0
-0.00237392  -0.00247585
2019.01.03 09:30:50 0  91.58385 -0.02819692 0.0005975765  -0.004573396 0
-0.002286698 -0.002369354
2019.01.03 09:31:00 0  91.56276 -0.02797313 0.0006625432  -0.004534662 0
-0.002267331 -0.0022823
2019.01.03 09:31:10 0  91.59088 -0.02827149 0.0004544252  -0.004621075 0
-0.002310538 -0.00226297
2019.01.03 09:31:20 0  91.59322 -0.02829635 0.0001759496  -0.004703007 0
-0.002351503 -0.002306093
2019.01.03 09:31:30 0  91.60727 -0.02844544 0.0003060442  -0.004679194 0
-0.002339597 -0.00234698
2019.01.03 09:31:40 0  91.5487  -0.02782386 0.0001112769  -0.004678991 0
-0.002339495 -0.002335097
2019.01.03 09:31:50 0  91.56745 -0.02802287 -0.0002910298 -0.004812035 0
-0.002406017 -0.002334995
2019.01.03 09:32:00 0  91.82033 -0.03069975 6.722897e-05  -0.004950426 0
-0.002475213 -0.002401389
2019.01.03 09:32:10 0  91.84138 -0.030922   -0.0004550431 -0.005119894 0
-0.002559947 -0.002470452
2019.01.03 09:32:20 0  92.02614 -0.03286751 -0.0002166572 -0.005226768 0
-0.002613384 -0.002555023
2019.01.03 09:32:30 0  92.01445 -0.03274466 0.0006385948  -0.004970954 0
-0.002485477 -0.002608357
2019.01.03 09:32:40 0  92.23874 -0.03509664 0.0008541204  -0.005120953 0
-0.002560477 -0.002480696
2019.01.03 09:32:50 0  92.18269 -0.03450997 0.0008961222  -0.005056132 0
-0.002528066 -0.002555551
2019.01.03 09:33:00 0  92.36479 -0.03641345 0.001131108   -0.005160197 0
-0.002580099 -0.002523203
2019.01.03 09:33:10 0  92.27376 -0.03546287 0.0008855293  -0.005144925 0
-0.002572462 -0.002575136
2019.01.03 09:33:20 0  92.19904 -0.03468117 0.0008984778  -0.005070867 0
-0.002535433 -0.002567514
2019.01.03 09:33:30 0  92.20604 -0.03475452 0.0009198473  -0.005071353 0
-0.002535676 -0.002530556
2019.01.03 09:33:40 0  92.07989 -0.03343212 0.001441632   -0.00480301  0
-0.002401505 -0.002530799
2019.01.03 09:33:50 0  92.17101 -0.03438764 0.0009481515  -0.005030232 0
-0.002515116 -0.002396886
2019.01.03 09:34:00 0  92.17334 -0.03441211 0.002182223   -0.004679266 0
-0.002339633 -0.002510278
2019.01.03 09:34:10 0  92.16634 -0.03433869 0.002146889   -0.004682771 0
-0.002341385 -0.002335133
```

```
2019.01.03 09:34:20 0  92.04016 -0.03301488 0.002340331   -0.004508268 0
-0.002254134 -0.002336882
2019.01.03 09:34:30 0  92.00276 -0.03262178 0.00196064    -0.004581549 0
-0.002290775 -0.002249798
2019.01.03 09:34:40 0  91.88817 -0.03141545 0.001621693   -0.00456998  0
-0.00228499  -0.002286368
2019.01.03 09:34:50 0  91.85074 -0.03102074 0.00141621    -0.004593262 0
-0.002296631 -0.002280595
2019.01.03 09:35:00 0  91.94197 -0.03198214 0.002134461   -0.004474238 0
-0.002237119 -0.002292213
2019.01.03 09:35:10 0  91.89285 -0.03146476 0.001942889   -0.004482497 0
-0.002241249 -0.002232816
2019.01.03 09:35:20 0  91.90689 -0.03161265 0.002124067   -0.004443958 0
-0.002221979 -0.002236938
2019.01.03 09:35:30 0  91.86946 -0.03121814 0.002068588   -0.004424329 0
-0.002212165 -0.002217705
2019.01.03 09:35:40 0  91.82033 -0.03069975 0.002092089   -0.004370948 0
-0.002185474 -0.002207909
2019.01.03 09:35:50 0  91.82267 -0.03072445 0.001487925   -0.004546072 0
-0.002273036 -0.00218127
2019.01.03 09:36:00 0  91.83437 -0.03084793 0.001448015   -0.004568607 0
-0.002284303 -0.002268664
2019.01.03 09:36:10 0  91.70095 -0.02943793 0.001268466   -0.004493091 0
-0.002246546 -0.002279909
2019.01.03 09:36:20 0  91.74309 -0.02988374 0.0008614023  -0.004649708 0
-0.002324854 -0.002242224
2019.01.03 09:36:30 0  91.63538 -0.02874345 0.0007591041  -0.004576358 0
-0.002288179 -0.002320382
2019.01.03 09:36:40 0  91.60962 -0.02847028 0.0006694554  -0.004577428 0
-0.002288714 -0.002283777
2019.01.03 09:36:50 0  91.56979 -0.02804774 0.001380889   -0.0043358   0
-0.0021679   -0.002284312
2019.01.03 09:37:00 0  91.45495 -0.02682728 0.001849998   -0.004091709 0
-0.002045854 -0.00216373
2019.01.03 09:37:10 0  91.25089 -0.02465101 0.002239283   -0.003784438 0
-0.001892219 -0.002041919
2019.01.03 09:37:20 0  91.27436 -0.02490177 0.002173451   -0.003825846 0
-0.001912923 -0.001888579
..
```

Calculate ΔQ_n

[18]:
```
tbl: update deltaQ: (I - I_) % lambda by id from tbl
`date`time`id`mid`trade`adv`I`I_`deltaQ#/:tbl
```

[18]:
```
date       time     id mid      trade     adv          I            I_
deltaQ
-------------------------------------------------------------------------------
----------
2019.01.03 09:30:00 0  91.715   454071.8  1.122415e+08 -0.00248663  0
-1615752
2019.01.03 09:30:10 0  91.72671 -171833.3 1.122415e+08 -0.002485272 -0.002481847
-2225.612
2019.01.03 09:30:20 0  91.58853 -301.2193 1.122415e+08 -0.002547113 -0.002480491
-43289.29
2019.01.03 09:30:30 0  91.65646 -78884.65 1.122415e+08 -0.002480621 -0.002542214
40021.24
2019.01.03 09:30:40 0  91.60259 -24705.54 1.122415e+08 -0.00237392  -0.00247585
66231.14
2019.01.03 09:30:50 0  91.58385 -92166.19 1.122415e+08 -0.002286698 -0.002369354
53707.79
2019.01.03 09:31:00 0  91.56276 -22823.01 1.122415e+08 -0.002267331 -0.0022823
9726.123
2019.01.03 09:31:10 0  91.59088 -4940.182 1.122415e+08 -0.002310538 -0.00226297
-30908.34
2019.01.03 09:31:20 0  91.59322 -41691.88 1.122415e+08 -0.002351503 -0.002306093
-29506.44
2019.01.03 09:31:30 0  91.60727 -539826.4 1.122415e+08 -0.002339597 -0.00234698
4797.322
2019.01.03 09:31:40 0  91.5487  -53883.89 1.122415e+08 -0.002339495 -0.002335097
-2858.097
2019.01.03 09:31:50 0  91.56745 316896.2  1.122415e+08 -0.002406017 -0.002334995
-46148.46
2019.01.03 09:32:00 0  91.82033 8767.784  1.122415e+08 -0.002475213 -0.002401389
-47969.05
2019.01.03 09:32:10 0  91.84138 132407.8  1.122415e+08 -0.002559947 -0.002470452
-58151.66
2019.01.03 09:32:20 0  92.02614 -52051.64 1.122415e+08 -0.002613384 -0.002555023
-37921.76
2019.01.03 09:32:30 0  92.01445 441082.8  1.122415e+08 -0.002485477 -0.002608357
79844.39
2019.01.03 09:32:40 0  92.23874 -182.6154 1.122415e+08 -0.002560477 -0.002480696
-51839.37
2019.01.03 09:32:50 0  92.18269 4196.414  1.122415e+08 -0.002528066 -0.002555551
17859.17
2019.01.03 09:33:00 0  92.36479 112899.5  1.122415e+08 -0.002580099 -0.002523203
-36969.1
2019.01.03 09:33:10 0  92.27376 -51038.96 1.122415e+08 -0.002572462 -0.002575136
1736.981
2019.01.03 09:33:20 0  92.19904 -20440.02 1.122415e+08 -0.002535433 -0.002567514
20845.29
2019.01.03 09:33:30 0  92.20604 -121802   1.122415e+08 -0.002535676 -0.002530556
```

```
-3326.87
2019.01.03 09:33:40 0  92.07989 24287.28  1.122415e+08 -0.002401505 -0.002530799
84011.81
2019.01.03 09:33:50 0  92.17101 -14046.25 1.122415e+08 -0.002515116 -0.002396886
-76823.27
2019.01.03 09:34:00 0  92.17334 -12283.33 1.122415e+08 -0.002339633 -0.002510278
110881.1
2019.01.03 09:34:10 0  92.16634 -106524.7 1.122415e+08 -0.002341385 -0.002335133
-4062.876
2019.01.03 09:34:20 0  92.04016 0         1.122415e+08 -0.002254134 -0.002336882
53767.46
2019.01.03 09:34:30 0  92.00276 -229900.1 1.122415e+08 -0.002290775 -0.002249798
-26625.69
2019.01.03 09:34:40 0  91.88817 18159.89  1.122415e+08 -0.00228499  -0.002286368
895.4902
2019.01.03 09:34:50 0  91.85074 25469.17  1.122415e+08 -0.002296631 -0.002280595
-10419.87
2019.01.03 09:35:00 0  91.94197 -18781.33 1.122415e+08 -0.002237119 -0.002292213
35798.87
2019.01.03 09:35:10 0  91.89285 3026.875  1.122415e+08 -0.002241249 -0.002232816
-5479.565
2019.01.03 09:35:20 0  91.90689 -21253.42 1.122415e+08 -0.002221979 -0.002236938
9719.667
2019.01.03 09:35:30 0  91.86946 -37882.34 1.122415e+08 -0.002212165 -0.002217705
3599.957
2019.01.03 09:35:40 0  91.82033 0         1.122415e+08 -0.002185474 -0.002207909
14577.88
2019.01.03 09:35:50 0  91.82267 38337.81  1.122415e+08 -0.002273036 -0.00218127
-59627.18
2019.01.03 09:36:00 0  91.83437 -261640.9 1.122415e+08 -0.002284303 -0.002268664
-10162.29
2019.01.03 09:36:10 0  91.70095 -358734   1.122415e+08 -0.002246546 -0.002279909
21679.08
2019.01.03 09:36:20 0  91.74309 -24277.96 1.122415e+08 -0.002324854 -0.002242224
-53690.84
2019.01.03 09:36:30 0  91.63538 -7837.575 1.122415e+08 -0.002288179 -0.002320382
20924.87
2019.01.03 09:36:40 0  91.60962 -68098.53 1.122415e+08 -0.002288714 -0.002283777
-3207.8
2019.01.03 09:36:50 0  91.56979 -187543.1 1.122415e+08 -0.0021679   -0.002284312
75641.47
2019.01.03 09:37:00 0  91.45495 -445392.8 1.122415e+08 -0.002045854 -0.00216373
76592.66
2019.01.03 09:37:10 0  91.25089 24626.54  1.122415e+08 -0.001892219 -0.002041919
97271.47
2019.01.03 09:37:20 0  91.27436 -8152.001 1.122415e+08 -0.001912923 -0.001888579
-15817.99
```

..

Finally, compute the final order size as a percent of adv.

[19]:
```
tbl2: select date: last date,
             finalQ: sum deltaQ,
             finalabsQ: sum abs deltaQ,
             adv: last adv by id from tbl
tbl2: update ratio: finalQ % adv,
             absratio: finalabsQ % adv by id from tbl2
tbl2
```

[19]:

id	date	finalQ	finalabsQ	adv	ratio	absratio
0	2019.01.03	-6303077	8.997391e+08	1.122415e+08	-0.0561564	8.016101
1	2019.01.03	1232001	2.936964e+08	3.607207e+07	0.03415387	8.141935
2	2019.01.03	-124025.8	1.141529e+08	1.38682e+07	-0.00894318	8.231271
3	2019.01.03	-530740	9.684178e+07	1.192783e+07	-0.04449592	8.118973
4	2019.01.03	1186311	1.654896e+08	2.019054e+07	0.05875577	8.196394
5	2019.01.03	447382.8	6.550841e+07	7931115	0.05640857	8.259673
6	2019.01.03	-730842.2	1.462413e+08	1.790894e+07	-0.0408088	8.165826
7	2019.01.03	389890.3	8.112688e+07	9922401	0.03929394	8.176134
8	2019.01.03	-662423.7	2.789322e+08	3.452761e+07	-0.01918533	8.078525
9	2019.01.03	-218508.3	1.554682e+08	1.862319e+07	-0.01173313	8.348094
10	2019.01.03	1760990	6.864726e+08	8.626539e+07	0.02041364	7.957684
11	2019.01.03	1405119	4.741004e+08	5.807214e+07	0.02419609	8.163991
12	2019.01.03	1098764	2.523104e+08	2.996236e+07	0.03667149	8.420913
13	2019.01.03	-632599.6	1.254022e+08	1.538175e+07	-0.04112663	8.15266
14	2019.01.03	1.126941e+07	8.093299e+09	9.883369e+08	0.0114024	8.188806
15	2019.01.03	-2541685	5.945092e+08	7.305665e+07	-0.0347906	8.137647
16	2019.01.03	-1.299653e+07	1.64292e+09	1.933401e+08	-0.06722106	8.497566
17	2019.01.03	-27375.03	7.378862e+07	8571269	-0.003193813	8.608832
18	2019.01.03	-412474.1	7.111271e+07	8691514	-0.0474571	8.181855
19	2019.01.03	-1.112442e+07	9.522593e+08	1.143187e+08	-0.09731064	8.329868
20	2019.01.03	934607.8	4.535799e+08	5.476887e+07	0.01706458	8.28171
21	2019.01.03	5812481	8.519212e+08	1.036909e+08	0.05605585	8.215969
22	2019.01.03	2121918	1.099908e+08	1.357973e+07	0.1562563	8.099631
23	2019.01.03	-392944.5	1.285005e+08	1.615722e+07	-0.02432006	7.953135
24	2019.01.03	-1280892	1.513249e+08	1.804241e+07	-0.07099339	8.387176
25	2019.01.03	-96218.67	2.138514e+08	2.649014e+07	-0.003632244	8.072867
26	2019.01.03	-622905.2	1.072934e+08	1.361601e+07	-0.04574799	7.879946
27	2019.01.03	-710679.4	1.279236e+08	1.565308e+07	-0.04540188	8.172423
28	2019.01.03	34625.3	1.467594e+08	1.788349e+07	0.00193616	8.206416
29	2019.01.03	-5353687	4.667943e+08	5.844333e+07	-0.09160477	7.987127
30	2019.01.03	-1617437	3.017298e+08	3.795109e+07	-0.04261898	7.950491
31	2019.01.03	-626969.1	1.870666e+08	2.261525e+07	-0.02772329	8.271702
32	2019.01.03	-5996369	3.813882e+08	4.651718e+07	-0.1289066	8.198867

```
33| 2019.01.03 685905.2      9.900936e+07 1.219655e+07 0.05623764   8.117817
34| 2019.01.03 150018.3      2.034706e+08 2.441801e+07 0.006143756  8.332809
35| 2019.01.03 1033132       4.516881e+08 5.504771e+07 0.01876793   8.205392
36| 2019.01.03 173386.2      3.797988e+08 4.687374e+07 0.003699005  8.102593
37| 2019.01.03 2124017       1.498092e+08 1.849975e+07 0.1148133    8.097907
38| 2019.01.03 1670376       4.006783e+08 4.873656e+07 0.03427358   8.221308
39| 2019.01.03 -148789.1     1.230659e+08 1.467255e+07 -0.01014064  8.38749
40| 2019.01.03 -59526.99     1.170699e+07 1404610      -0.04237972  8.334688
41| 2019.01.03 263702.4      8.01707e+07  9919119      0.02658526   8.082441
42| 2019.01.03 -465903.6     5.833336e+07 6954907      -0.06698919  8.387368
43| 2019.01.03 1232852       1.314579e+08 1.624801e+07 0.07587709   8.090705
44| 2019.01.03 -2879542      2.116305e+08 2.639617e+07 -0.1090894   8.01747
..
```

Make a function so that we can use it to calculate trade for each date.

[20]:
```
opt_trade: {[dt]
    tbl1: select from bin10 where date = dt;
    tbl1: update reverse fills reverse date,
                 reverse fills reverse mid,
                 reverse fills reverse spread,
                 reverse fills reverse vol,
                 reverse fills reverse adv
                 by id from tbl1;

    tbl1: update rtn: ((last mid) % mid) - 1 by id from tbl1;

    tbl1: update W: vol * (sums(u12[count time])) by id from tbl1;

    tbl1: update alpha: (a * (rtn - last rtn)) + (b * (W - last W)) by id from␣
↪tbl1;
    tbl1: update dalpha: 0 ^ ((alpha - xprev[60; alpha]) % 10) by id from tbl1;

    tbl1: update I: (last alpha) ^ next prev 0.5 * (alpha - dalpha % beta) by␣
↪id from tbl1;

    tbl1: update lambda: 8 * (vol * sqrt 6 * 60 * 6.5) % adv by id from tbl1;
    tbl1: update I_: 0 ^ xprev[1; I] * exp neg beta % 6 by id from tbl1;
    tbl1: update deltaQ: (I - I_) % lambda by id from tbl1;

    tbl2: select alpha: first alpha,
             finalQ: sum deltaQ,
             finalabsQ: sum abs deltaQ,
             adv: last adv by date, id from tbl1;
    tbl2: update ratio: finalQ % adv,
             absratio: finalabsQ % adv by id from tbl2;
```

```
    return: `id xasc tbl2;
    return};
```

Check whether this function works.

[21]:
```
opt_trade[dt]
```

[21]:
```
date       id| alpha         finalQ        finalabsQ    adv          ratio
absratio
-------------|
-------------------------------------------------------------------------------
2019.01.03 0 | -0.01334278   -1.4564e+07   9.183493e+08 1.122415e+08 -0.1297559
8.181906
2019.01.03 1 | -0.009694622  -841911.3     2.934796e+08 3.607207e+07 -0.0233397
8.135924
2019.01.03 2 | -0.01133848   -1216939      1.160932e+08 1.38682e+07  -0.08775031
8.371177
2019.01.03 3 | 0.003326134   274512.7      1.000215e+08 1.192783e+07 0.02301447
8.385551
2019.01.03 4 | -0.005097134  -1416231      1.622616e+08 2.019054e+07 -0.0701433
8.036514
2019.01.03 5 | 0.005436319   438257.7      6.641755e+07 7931115      0.05525802
8.374301
2019.01.03 6 | 0.0009837467  -59384.62     1.433129e+08 1.790894e+07
-0.003315921  8.002311
2019.01.03 7 | -0.005622434  -618284.6     7.815469e+07 9922401      -0.062312
7.876591
2019.01.03 8 | 0.008692787   668744.8      2.823999e+08 3.452761e+07 0.01936841
8.178959
2019.01.03 9 | 0.002005186   840360.2      1.50175e+08  1.862319e+07 0.04512439
8.063873
2019.01.03 10| -0.002328725  3680620       7.211892e+08 8.626539e+07 0.04266624
8.360123
2019.01.03 11| 0.007026243   1377702       4.812012e+08 5.807214e+07 0.02372398
8.286266
2019.01.03 12| -0.001560131  -407350.3     2.429468e+08 2.996236e+07 -0.0135954
8.1084
2019.01.03 13| 0.008305634   1008002       1.279337e+08 1.538175e+07 0.06553233
8.317237
2019.01.03 14| 0.004032849   -2.323279e+07 8.210277e+09 9.883369e+08 -0.02350695
8.307164
2019.01.03 15| 0.002279387   2416604       6.056537e+08 7.305665e+07 0.03307849
8.290192
2019.01.03 16| 0.00675711    4760081       1.558342e+09 1.933401e+08 0.02462024
8.060107
2019.01.03 17| -0.01783212   -856160.5     7.006373e+07 8571269      -0.09988725
8.174254
2019.01.03 18| 0.007375103   94398.44      7.072624e+07 8691514      0.01086099
```

```
8.13739
2019.01.03 19| -0.008568304  -1.365327e+07 9.06175e+08  1.143187e+08 -0.1194317
7.926747
2019.01.03 20| -0.003813939  -4465493      4.578314e+08 5.476887e+07 -0.08153342
8.359336
2019.01.03 21| -0.002062755  3177874       8.738667e+08 1.036909e+08 0.03064757
8.427612
2019.01.03 22| 0.001220702   115859        1.095489e+08 1.357973e+07 0.008531761
8.067089
2019.01.03 23| -0.003268964  -355591.6     1.330617e+08 1.615722e+07 -0.02200822
8.235436
2019.01.03 24| 0.008271999   1615024       1.496543e+08 1.804241e+07 0.08951265
8.294587
2019.01.03 25| -0.01376878   -3052130      2.227409e+08 2.649014e+07 -0.1152176
8.408443
2019.01.03 26| 0.001993387   381138.2      1.116686e+08 1.361601e+07 0.02799191
8.201268
2019.01.03 27| 0.005081485   1230816       1.268956e+08 1.565308e+07 0.07863089
8.106747
2019.01.03 28| -0.001318901  -635585.4     1.522761e+08 1.788349e+07 -0.03554034
8.514897
2019.01.03 29| -0.0007024328 -36655.67     4.764771e+08 5.844333e+07
-0.0006272002 8.152805
2019.01.03 30| -0.002709989  -1003161      3.124522e+08 3.795109e+07 -0.026433
8.233022
2019.01.03 31| -0.00797438   -1199163      1.855951e+08 2.261525e+07 -0.05302454
8.206638
2019.01.03 32| -0.002855737  -1867717      3.774103e+08 4.651718e+07 -0.04015112
8.113353
2019.01.03 33| 0.002712697   685460.1      9.939394e+07 1.219655e+07 0.05620115
8.149349
2019.01.03 34| 0.0003113098  -776693.4     1.992495e+08 2.441801e+07 -0.03180822
8.159938
2019.01.03 35| -0.008192905  -2992038      4.541077e+08 5.504771e+07 -0.05435353
8.249346
2019.01.03 36| 0.01623438    5320946       3.943669e+08 4.687374e+07 0.1135166
8.413387
2019.01.03 37| -0.004667279  -360132.7     1.56982e+08  1.849975e+07 -0.0194669
8.485628
2019.01.03 38| -0.0005399176 -146646.8     3.928908e+08 4.873656e+07
-0.003008968  8.061521
2019.01.03 39| 0.005910632   1154920       1.205342e+08 1.467255e+07 0.07871295
8.214944
2019.01.03 40| 0.007516086   101168.4      1.143066e+07 1404610      0.07202596
8.137955
2019.01.03 41| -0.003187025  -377774.6     7.938249e+07 9919119      -0.0380855
8.002978
```

```
2019.01.03 42| -0.002446193  8386.373      5.875223e+07 6954907      0.001205821
8.447594
2019.01.03 43| -0.002054127  -568642.6     1.298686e+08 1.624801e+07 -0.03499767
7.992892
2019.01.03 44| -0.006009744  -133080.7     2.129719e+08 2.639617e+07
-0.005041666  8.068287
..
```

Get all the dates. I delete the file on 2019.01.09, so there are only 249 days and I don't need to consider that day.

[22]:
```
dt_list: "D"$ system "ls ../columbiaHdb/"
count dt_list
```

[22]:
```
249
```

Calculate the final order size for each stock and each date.

[23]:
```
answer: `date`id xasc raze opt_trade peach dt_list
```

[24]:
```
answer
```

[24]:
```
date       id| alpha         finalQ    finalabsQ    adv          ratio
absratio
-------------|
-------------------------------------------------------------------------
2019.01.02 0 | -0.002737095  -437132.6 9.43226e+07  1.161895e+07 -0.03762239
8.117999
2019.01.02 1 | 0.002996551   2967552   3.211401e+08 4.081003e+07 0.07271625
7.869147
2019.01.02 2 | -0.006517938  -1027846  8.674047e+07 1.052341e+07 -0.09767233
8.242623
2019.01.02 3 | 0.002751787   102602    5.589034e+07 6987232      0.01468421
7.998924
2019.01.02 4 | -0.001811368  -173372.7 1.509553e+08 1.907262e+07 -0.009090137
7.914766
2019.01.02 5 | 0.009282064   2127314   1.781874e+08 2.222419e+07 0.09572065
8.017722
2019.01.02 6 | -0.0001223209 -164387.2 8.397125e+07 1.009195e+07 -0.01628894
8.320614
2019.01.02 7 | -0.004412123  307620.7  1.691004e+08 2.039854e+07 0.01508053
8.289833
2019.01.02 8 | -0.002554365  -638971.7 2.312347e+08 2.821476e+07 -0.02264672
8.195523
2019.01.02 9 | -0.004865517  -495580.7 1.568493e+08 1.895952e+07 -0.02613888
8.272852
```

```
2019.01.02 10| 0.0001853087  -2583186  4.446597e+08 5.362121e+07 -0.04817471
8.292608
2019.01.02 11| 0.006378425   945182.3  1.08749e+08  1.371147e+07 0.06893371
7.931248
2019.01.02 12| 0.00490453    -235420.2 4.060761e+08 4.953557e+07 -0.004752549
8.197667
2019.01.02 13| 0.005780924   1052624   9.031441e+07 1.092683e+07 0.09633382
8.265377
2019.01.02 14| 0.0028533     329873.1  9.34229e+07  1.145875e+07 0.02878788
8.152973
2019.01.02 15| -0.001890349  582155.4  1.866878e+08 2.327105e+07 0.02501629
8.022319
2019.01.02 16| 0.004605275   405279.1  1.000653e+08 1.236677e+07 0.03277163
8.091467
2019.01.02 17| -0.007973917  -511752.1 1.317607e+08 1.599064e+07 -0.03200323
8.239861
2019.01.02 18| -0.003838137  -261828.9 5.594626e+07 7045968      -0.0371601
7.94018
2019.01.02 19| 0.008195705   369138.6  7.754366e+07 9231475      0.03998696
8.39992
2019.01.02 20| -0.01003056   -2219961  1.326619e+08 1.589806e+07 -0.1396373
8.344536
2019.01.02 21| 0.01239179    611387.9  5.085471e+07 6176131      0.09899206
8.234072
2019.01.02 22| 3.263918e-05  -392281.5 1.899349e+08 2.387497e+07 -0.01643066
7.955398
2019.01.02 23| 0.003544378   588196.6  1.230927e+08 1.505667e+07 0.03906553
8.175296
2019.01.02 24| 0.002340105   -590228.3 2.19943e+08  2.74124e+07  -0.02153144
8.023487
2019.01.02 25| 0.009785605   301776.3  7.93345e+07  9743904      0.03097078
8.141962
2019.01.02 26| 0.005175695   819408.2  9.556822e+07 1.197435e+07 0.06843028
7.981078
2019.01.02 27| -0.0007278826 -9359.661 7.238911e+07 8665796      -0.001080069
8.353429
2019.01.02 28| -0.004396734  -532787.6 1.156651e+08 1.431196e+07 -0.03722674
8.081711
2019.01.02 29| -0.00745491   -2224060  4.943757e+08 6.449573e+07 -0.03448384
7.665247
2019.01.02 30| 0.009636759   642929.1  6.953278e+07 8555770      0.07514568
8.127005
2019.01.02 31| 0.003446523   889305.7  3.069632e+08 3.708067e+07 0.023983
8.278253
2019.01.02 32| 0.002576564   178772.9  6.908176e+07 8523391      0.02097439
8.104961
2019.01.02 33| -0.005131416  -375672.3 1.215054e+08 1.481507e+07 -0.02535745
```

```
8.201478
2019.01.02 34| -0.005249198  -2017478  2.118811e+08 2.629457e+07 -0.07672604
8.057979
2019.01.02 35| -0.001171809  86186.75  1.181808e+08 1.425214e+07 0.006047284
8.292143
2019.01.02 36| 0.002134748   2899508   4.734221e+08 5.577244e+07 0.05198819
8.488459
2019.01.02 37| -0.007601686  -2625526  2.339146e+08 2.805252e+07 -0.09359324
8.338455
2019.01.02 38| 0.003231055   88977.04  1.016214e+08 1.255333e+07 0.007087925
8.095175
2019.01.02 39| -0.001222969  -541158.4 3.398109e+08 4.104227e+07 -0.01318539
8.279536
2019.01.02 40| 0.00334371    604807.8  6.126645e+07 7441732      0.08127245
8.23282
2019.01.02 41| 0.0001963716  71414.09  1.010495e+08 1.215736e+07 0.005874143
8.311792
2019.01.02 42| 0.001539668   908773    1.086615e+09 1.34563e+08  0.006753515
8.075139
2019.01.02 43| 0.0002870119  1105770   4.193537e+08 5.173965e+07 0.0213718
8.105075
2019.01.02 44| 0.000508148   1390228   1.805556e+08 2.218375e+07 0.06266876
8.139093
..
```

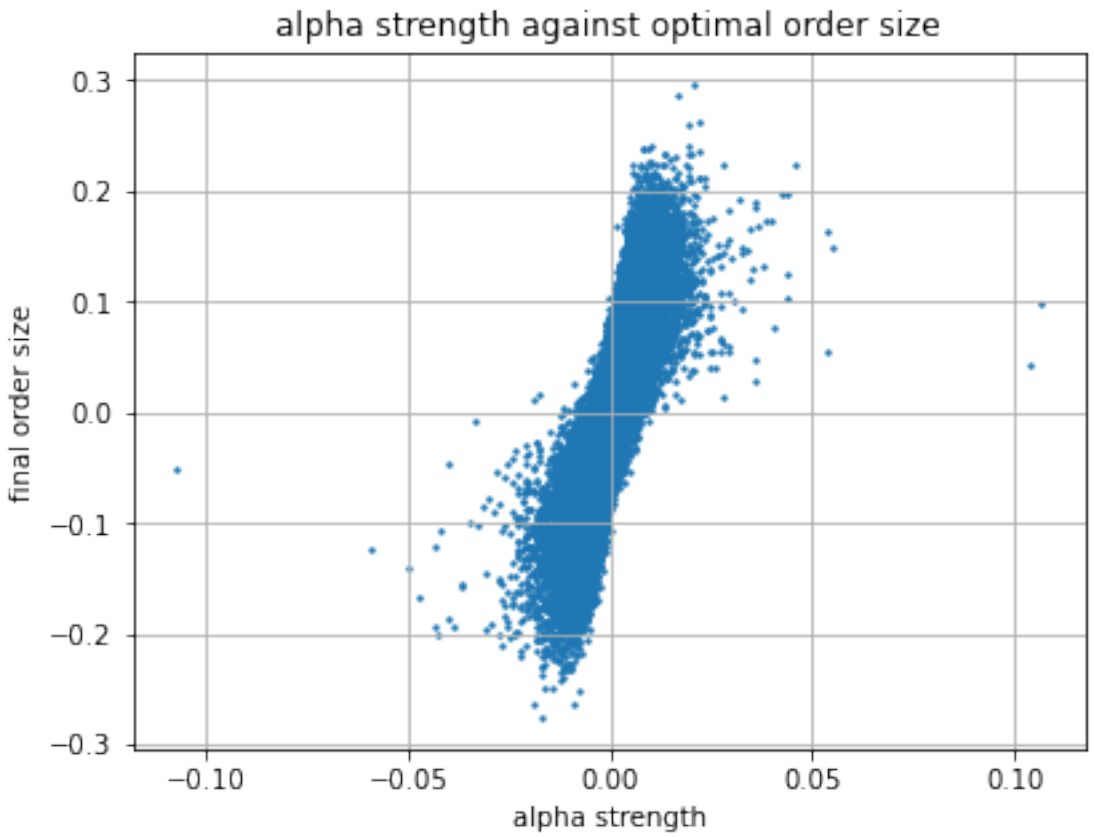
Draw a scatter plot.

[25]:
```
alpha: select alpha from answer
ratio: select ratio from answer
```

[26]:
```
\l ../importmatplotlib.q

plt:.matplotlib.pyplot[]
plt.

plt.scatter[alpha `alpha; ratio `ratio; s:2];
plt.xlabel"alpha strength";
plt.ylabel"final order size";
plt.title"alpha strength against optimal order size";
plt.grid 1b;
plt.show[];
```



1.3 Exercise 3 Backtesting Trading Strategies

1.3.1 1. Simulate a VWAP strategy for an order of the same size as the optimal trading strategy.

Write a function to implement VMAP strategy.

[27]:
```
VWAPsimschedule: {[trade; adv; size]
    mytrade: size * (abs(trade) % adv);
    mytrade};
```

Implement VMAP strategy.

[28]:
```
tbl: update VMAPtrade: VWAPsimschedule[trade; adv; sum deltaQ] by id from tbl
`date`time`id`mid`trade`adv`deltaQ`VMAPtrade#/:tbl
```

[28]:
```
date       time     id mid      trade     adv          deltaQ    VMAPtrade
-------------------------------------------------------------------------
2019.01.03 09:30:00 0  91.715   454071.8  1.122415e+08 -1615752  -25499.04
2019.01.03 09:30:10 0  91.72671 -171833.3 1.122415e+08 -2225.612 -9649.539
2019.01.03 09:30:20 0  91.58853 -301.2193 1.122415e+08 -43289.29 -16.91539
```

```
2019.01.03 09:30:30 0  91.65646 -78884.65 1.122415e+08 40021.24  -4429.878
2019.01.03 09:30:40 0  91.60259 -24705.54 1.122415e+08 66231.14  -1387.374
2019.01.03 09:30:50 0  91.58385 -92166.19 1.122415e+08 53707.79  -5175.721
2019.01.03 09:31:00 0  91.56276 -22823.01 1.122415e+08 9726.123  -1281.658
2019.01.03 09:31:10 0  91.59088 -4940.182 1.122415e+08 -30908.34 -277.4228
2019.01.03 09:31:20 0  91.59322 -41691.88 1.122415e+08 -29506.44 -2341.266
2019.01.03 09:31:30 0  91.60727 -539826.4 1.122415e+08 4797.322  -30314.7
2019.01.03 09:31:40 0  91.5487  -53883.89 1.122415e+08 -2858.097 -3025.925
2019.01.03 09:31:50 0  91.56745 316896.2  1.122415e+08 -46148.46 -17795.75
2019.01.03 09:32:00 0  91.82033 8767.784  1.122415e+08 -47969.05 -492.3672
2019.01.03 09:32:10 0  91.84138 132407.8  1.122415e+08 -58151.66 -7435.545
2019.01.03 09:32:20 0  92.02614 -52051.64 1.122415e+08 -37921.76 -2923.033
2019.01.03 09:32:30 0  92.01445 441082.8  1.122415e+08 79844.39  -24769.62
2019.01.03 09:32:40 0  92.23874 -182.6154 1.122415e+08 -51839.37 -10.25502
2019.01.03 09:32:50 0  92.18269 4196.414  1.122415e+08 17859.17  -235.6555
2019.01.03 09:33:00 0  92.36479 112899.5  1.122415e+08 -36969.1  -6340.028
2019.01.03 09:33:10 0  92.27376 -51038.96 1.122415e+08 1736.981  -2866.164
2019.01.03 09:33:20 0  92.19904 -20440.02 1.122415e+08 20845.29  -1147.838
2019.01.03 09:33:30 0  92.20604 -121802   1.122415e+08 -3326.87  -6839.963
2019.01.03 09:33:40 0  92.07989 24287.28  1.122415e+08 84011.81  -1363.886
2019.01.03 09:33:50 0  92.17101 -14046.25 1.122415e+08 -76823.27 -788.787
2019.01.03 09:34:00 0  92.17334 -12283.33 1.122415e+08 110881.1  -689.7874
2019.01.03 09:34:10 0  92.16634 -106524.7 1.122415e+08 -4062.876 -5982.044
2019.01.03 09:34:20 0  92.04016 0          1.122415e+08 53767.46  -0
2019.01.03 09:34:30 0  92.00276 -229900.1 1.122415e+08 -26625.69 -12910.36
2019.01.03 09:34:40 0  91.88817 18159.89  1.122415e+08 895.4902  -1019.794
2019.01.03 09:34:50 0  91.85074 25469.17  1.122415e+08 -10419.87 -1430.257
2019.01.03 09:35:00 0  91.94197 -18781.33 1.122415e+08 35798.87  -1054.692
2019.01.03 09:35:10 0  91.89285 3026.875  1.122415e+08 -5479.565 -169.9784
2019.01.03 09:35:20 0  91.90689 -21253.42 1.122415e+08 9719.667  -1193.515
2019.01.03 09:35:30 0  91.86946 -37882.34 1.122415e+08 3599.957  -2127.335
2019.01.03 09:35:40 0  91.82033 0          1.122415e+08 14577.88  -0
2019.01.03 09:35:50 0  91.82267 38337.81  1.122415e+08 -59627.18 -2152.913
2019.01.03 09:36:00 0  91.83437 -261640.9 1.122415e+08 -10162.29 -14692.81
2019.01.03 09:36:10 0  91.70095 -358734   1.122415e+08 21679.08  -20145.21
2019.01.03 09:36:20 0  91.74309 -24277.96 1.122415e+08 -53690.84 -1363.363
2019.01.03 09:36:30 0  91.63538 -7837.575 1.122415e+08 20924.87  -440.1299
2019.01.03 09:36:40 0  91.60962 -68098.53 1.122415e+08 -3207.8   -3824.168
2019.01.03 09:36:50 0  91.56979 -187543.1 1.122415e+08 75641.47  -10531.74
2019.01.03 09:37:00 0  91.45495 -445392.8 1.122415e+08 76592.66  -25011.66
2019.01.03 09:37:10 0  91.25089 24626.54  1.122415e+08 97271.47  -1382.938
2019.01.03 09:37:20 0  91.27436 -8152.001 1.122415e+08 -15817.99 -457.787
..
```

Calculate the impact I_t and I_{t-}.

[29]:
```
computeImpact : {[trade; adv; vol]
    / exponential decay constant per 10 seconds,
    / i.e. \beta * \delta t in the formula above
    h: 60;
    beta: (log 2) % (6 * h);
    lambda: 8;
    dailyVol: vol * sqrt 6 * 60 * 6.5;
    / initialize impact to be zero
    / drop the first element from ema in the output
    (neg count trade)#ema[beta; 0, lambda * dailyVol * trade % beta * adv]};
```

[30]:
```
tbl: update VMAP_I: computeImpact[VMAPtrade; adv; vol] by id from tbl
tbl: update VMAP_I_: 0 ^ xprev[1; VMAP_I] * exp neg beta % 6 by id from tbl
`date`time`id`mid`deltaQ`I`I_`VMAPtrade`VMAP_I`VMAP_I_#/:tbl
```

[30]:
```
date       time     id mid      deltaQ    I            I_           VMAPtrade
VMAP_I        VMAP_I_
-------------------------------------------------------------------------------
-------------------------
2019.01.03 09:30:00 0  91.715   -1615752  -0.00248663  0            -25499.04
-3.924283e-05 0
2019.01.03 09:30:10 0  91.72671 -2225.612 -0.002485272 -0.002481847 -9649.539
-5.401784e-05 -3.916734e-05
2019.01.03 09:30:20 0  91.58853 -43289.29 -0.002547113 -0.002480491 -16.91539
-5.393987e-05 -5.391393e-05
2019.01.03 09:30:30 0  91.65646 40021.24  -0.002480621 -0.002542214 -4429.878
-6.065356e-05 -5.383611e-05
2019.01.03 09:30:40 0  91.60259 66231.14  -0.00237392  -0.00247585  -1387.374
-6.267193e-05 -6.053689e-05
2019.01.03 09:30:50 0  91.58385 53707.79  -0.002286698 -0.002369354 -5175.721
-7.051666e-05 -6.255138e-05
2019.01.03 09:31:00 0  91.56276 9726.123  -0.002267331 -0.0022823   -1281.658
-7.235335e-05 -7.038102e-05
2019.01.03 09:31:10 0  91.59088 -30908.34 -0.002310538 -0.00226297  -277.4228
-7.264099e-05 -7.221417e-05
2019.01.03 09:31:20 0  91.59322 -29506.44 -0.002351503 -0.002306093 -2341.266
-7.610432e-05 -7.250126e-05
2019.01.03 09:31:30 0  91.60727 4797.322  -0.002339597 -0.00234698  -30314.7
-0.0001226119 -7.595793e-05
2019.01.03 09:31:40 0  91.5487  -2858.097 -0.002339495 -0.002335097 -3025.925
-0.0001270327 -0.000122376
2019.01.03 09:31:50 0  91.56745 -46148.46 -0.002406017 -0.002334995 -17795.75
-0.0001541756 -0.0001267883
2019.01.03 09:32:00 0  91.82033 -47969.05 -0.002475213 -0.002401389 -492.3672
-0.0001546365 -0.0001538791
2019.01.03 09:32:10 0  91.84138 -58151.66 -0.002559947 -0.002470452 -7435.545
-0.000165782  -0.0001543391
```

```
2019.01.03 09:32:20 0  92.02614 -37921.76 -0.002613384 -0.002555023 -2923.033
-0.0001699614 -0.0001654631
2019.01.03 09:32:30 0  92.01445 79844.39  -0.002485477 -0.002608357 -24769.62
-0.0002077544 -0.0001696344
2019.01.03 09:32:40 0  92.23874 -51839.37 -0.002560477 -0.002480696 -10.25502
-0.0002073702 -0.0002073548
2019.01.03 09:32:50 0  92.18269 17859.17  -0.002528066 -0.002555551 -235.6555
-0.0002073336 -0.0002069713
2019.01.03 09:33:00 0  92.36479 -36969.1  -0.002580099 -0.002523203 -6340.028
-0.0002166916 -0.0002069347
2019.01.03 09:33:10 0  92.27376 1736.981  -0.002572462 -0.002575136 -2866.164
-0.0002206854 -0.0002162748
2019.01.03 09:33:20 0  92.19904 20845.29  -0.002535433 -0.002567514 -1147.838
-0.000222027  -0.0002202609
2019.01.03 09:33:30 0  92.20604 -3326.87  -0.002535676 -0.002530556 -6839.963
-0.0002321262 -0.0002215999
2019.01.03 09:33:40 0  92.07989 84011.81  -0.002401505 -0.002530799 -1363.886
-0.0002337782 -0.0002316797
2019.01.03 09:33:50 0  92.17101 -76823.27 -0.002515116 -0.002396886 -788.787
-0.0002345421 -0.0002333286
2019.01.03 09:34:00 0  92.17334 110881.1  -0.002339633 -0.002510278 -689.7874
-0.000235152  -0.0002340909
2019.01.03 09:34:10 0  92.16634 -4062.876 -0.002341385 -0.002335133 -5982.044
-0.0002439056 -0.0002346997
2019.01.03 09:34:20 0  92.04016 53767.46  -0.002254134 -0.002336882 -0
-0.000243436  -0.0002434364
2019.01.03 09:34:30 0  92.00276 -26625.69 -0.002290775 -0.002249798 -12910.36
-0.0002628362 -0.0002429677
2019.01.03 09:34:40 0  91.88817 895.4902  -0.00228499  -0.002286368 -1019.794
-0.0002638996 -0.0002623306
2019.01.03 09:34:50 0  91.85074 -10419.87 -0.002296631 -0.002280595 -1430.257
-0.0002655927 -0.000263392
2019.01.03 09:35:00 0  91.94197 35798.87  -0.002237119 -0.002292213 -1054.692
-0.0002667044 -0.0002650818
2019.01.03 09:35:10 0  91.89285 -5479.565 -0.002241249 -0.002232816 -169.9784
-0.0002664525 -0.0002661914
2019.01.03 09:35:20 0  91.90689 9719.667  -0.002221979 -0.002236938 -1193.515
-0.0002677763 -0.00026594
2019.01.03 09:35:30 0  91.86946 3599.957  -0.002212165 -0.002217705 -2127.335
-0.0002705347 -0.0002672612
2019.01.03 09:35:40 0  91.82033 14577.88  -0.002185474 -0.002207909 -0
-0.0002700138 -0.0002700143
2019.01.03 09:35:50 0  91.82267 -59627.18 -0.002273036 -0.00218127  -2152.913
-0.0002728072 -0.0002694944
2019.01.03 09:36:00 0  91.83437 -10162.29 -0.002284303 -0.002268664 -14692.81
-0.0002948941 -0.0002722825
2019.01.03 09:36:10 0  91.70095 21679.08  -0.002246546 -0.002279909 -20145.21
```

```
-0.0003253296 -0.0002943268
2019.01.03 09:36:20 0  91.74309 -53690.84 -0.002324854 -0.002242224 -1363.363
-0.0003268014 -0.0003247038
2019.01.03 09:36:30 0  91.63538 20924.87  -0.002288179 -0.002320382 -440.1299
-0.0003268496 -0.0003261728
2019.01.03 09:36:40 0  91.60962 -3207.8   -0.002288714 -0.002283777 -3824.168
-0.0003321056 -0.0003262208
2019.01.03 09:36:50 0  91.56979 75641.47  -0.0021679   -0.002284312 -10531.74
-0.0003476744 -0.0003314668
2019.01.03 09:37:00 0  91.45495 76592.66  -0.002045854 -0.00216373  -25011.66
-0.0003854978 -0.0003470057
2019.01.03 09:37:10 0  91.25089 97271.47  -0.001892219 -0.002041919 -1382.938
-0.0003868839 -0.0003847563
2019.01.03 09:37:20 0  91.27436 -15817.99 -0.001912923 -0.001888579 -457.787
-0.0003868435 -0.0003861397
..
```

1.3.2 2. Backtest the trading costs and P&L of the optimal and the VWAP strategy across all stocks and dates.

In order to derive the trading cost and P&L, we first start from the unit of Q is share and S is the price per share.

$$\begin{aligned}
\Delta_n Y =& Q_{n-1}\Delta_n S + \Delta_n Q \Delta_n S - I_{n-}\Delta_n Q - \frac{1}{2}\Delta_n I \Delta_n Q \\
=& Q_{n-1}(S_n - S_{n-1}) + \Delta_n Q \Delta_n S - I_{n-}\Delta_n Q - \frac{1}{2}\Delta_n I \Delta_n Q \\
=& Q_{n-1}S_{n-1}\frac{S_n}{S_T}(R_{n-1} - R_n) + \Delta_n Q \Delta_n S - I_{n-}\Delta_n Q - \frac{1}{2}\Delta_n I \Delta_n Q \\
=& -\tilde{Q}_{n-1}\frac{S_n}{S_T}\Delta_n R + \Delta_n Q \Delta_n S - \tilde{I}_{n-}\Delta_n \tilde{Q} - \frac{1}{2}\Delta_n \tilde{I} \Delta_n \tilde{Q},
\end{aligned}$$

where $\tilde{Q}_n = Q_n S_n$ is the dollar trade, $\Delta_n \tilde{Q} = (Q_n - Q_{n-1})S_n = \Delta_n Q S_n$ is the instant trade and $\tilde{I}$ is the impact on return.

Taking the sum of $\Delta_n Y$ we get the final P&L:

$$
\begin{aligned}
Y_T &= \sum_{n=0}^{T} \Delta_n Y \\
&= \sum_{n=1}^{T} Q_{n-1}(S_n - S_{n-1}) + \sum_{n=0}^{T} \Delta_n Q \Delta_n S - \sum_{n=0}^{T} (\tilde{I}_{n-} \Delta_n \tilde{Q} + \frac{1}{2} \Delta_n \tilde{I} \Delta_n \tilde{Q}) \\
&= \sum_{n=1}^{T} (S_n - S_{n-1})(\sum_{m=0}^{n-1} \Delta_m Q) + \sum_{n=0}^{T} \Delta_n \tilde{Q} \frac{\Delta_n S}{S_n} - \sum_{n=0}^{T} \frac{1}{2} (\tilde{I}_{n-} + \tilde{I}_n) \Delta_n \tilde{Q} \\
&= \sum_{m=0}^{T-1} \Delta_m Q (\sum_{n=m+1}^{T} (S_n - S_{n-1})) + \sum_{n=0}^{T} \Delta_n \tilde{Q} (1 - \frac{S_{n-1}}{S_n}) - \sum_{n=0}^{T} \frac{1}{2} (\tilde{I}_{n-} + \tilde{I}_n) \Delta_n \tilde{Q} \\
&= \sum_{m=0}^{T-1} \frac{\Delta_m \tilde{Q}}{S_m} (S_T - S_m) + \sum_{n=0}^{T} \Delta_n \tilde{Q} (1 - \frac{S_{n-1}}{S_n}) - \sum_{n=0}^{T} \frac{1}{2} (\tilde{I}_{n-} + \tilde{I}_n) \Delta_n \tilde{Q} \\
&= \sum_{n=0}^{T} \Delta_n \tilde{Q} (R_n + 1 - \frac{S_{n-1}}{S_n}) - \sum_{n=0}^{T} \frac{1}{2} (\tilde{I}_{n-} + \tilde{I}_n) \Delta_n \tilde{Q}
\end{aligned}
$$

Now, we could calculate the trading costs and P&L of the optimal.

[31]:
```
tbl3: select opt_alpha: sum deltaQ * (rtn + 1 - ((first mid) ^ (prev mid)) %␣
 ↪mid),
             opt_TC: sum deltaQ * 0.5 * (I + I_),
             VMAP_alpha: sum VMAPtrade * (rtn + 1 - ((first mid) ^ (prev mid))␣
 ↪% mid),
             VMAP_TC: sum VMAPtrade * 0.5 * (VMAP_I + VMAP_I_) by id from tbl
tbl3: update opt_PaL: opt_alpha - opt_TC,
             VMAP_PaL: VMAP_alpha - VMAP_TC by id from tbl3
```

[32]:
```
tbl3
```

[32]:
```
id| opt_alpha opt_TC   VMAP_alpha VMAP_TC  opt_PaL   VMAP_PaL
--| ---------------------------------------------------------
0 | 212396    50295.15 58036.93   10548.43 162100.9  47488.5
1 | -15761.73 12528.86 -11776.27  951.4372 -28290.59 -12727.71
2 | 10161.49  7924.268 503.3789   35.62276 2237.226  467.7561
3 | 2734.194  6280.439 1443.221   734.9044 -3546.245 708.3166
4 | 11340.86  10646.66 -1253.319  1708.651 694.1916  -2961.97
5 | 3419.564  4547.707 -600.5958  863.3032 -1128.142 -1463.899
6 | 11967.02  9000.153 3383.902   911.4586 2966.869  2472.444
7 | 3850.185  3166.525 -1411.188  422.0441 683.6595  -1833.232
8 | 102939.3  25721.2  11485.16   635.1244 77218.07  10850.03
9 | 13997.28  10504.73 3212.016   110.2298 3492.546  3101.786
10| 18603.16  62516.36 -29353.43  1678.39  -43913.2  -31031.82
11| -6375.663 29822.15 -10978.17  860.9616 -36197.81 -11839.13
12| -8256.828 12790.87 -4207.038  1006.274 -21047.7  -5213.312
13| 27471.11  9151.506 7611.594   855.2844 18319.61  6756.31
```

```
14| 989929.2  483656.9 -61995.68  4237.37  506272.4  -66233.05
15| 40468.99  31242.42 12938.28   2175.961 9226.575  10762.32
16| 422423.7  362398.6 148769.3   51428.62 60025.08  97340.67
17| 19144.87  5547.17  624.1669   3.505957 13597.7   620.661
18| 15812.72  5953.212 3989.655   864.6277 9859.507  3125.027
19| 240943.9  94380.21 76885.11   31200.79 146563.7  45684.32
20| 24119.66  40526.79 1727.374   528.6094 -16407.12 1198.765
21| 21992.12  49097.47 -38920.57  8453.735 -27105.35 -47374.31
22| -7430.63  15310.66 -13421.23  7942.458 -22741.3  -21363.69
23| 10245.22  7987.033 373.3393   396.1309 2258.184  -22.79161
24| 16388.03  11338.5  915.3499   2501.321 5049.53   -1585.972
25| 22000.98  13361.36 345.9876   10.6808  8639.623  335.3068
26| 6864.523  4346.923 2420.855   683.3291 2517.599  1737.526
27| 25612.16  12767.22 9767.499   1401.947 12844.94  8365.552
28| 24867.04  12145.61 -252.4714  2.34368  12721.43  -254.8151
29| 141814.9  60140.74 52806.94   18388.22 81674.18  34418.72
30| 11072.86  17581.34 13944.96   2247.995 -6508.477 11696.96
31| 12926.11  10333.79 256.467    714.5431 2592.314  -458.0761
32| 18162.77  41627.87 27666.17   16901.61 -23465.1  10764.56
33| 3641.61   7550.529 -1312.52   1150.469 -3908.919 -2462.989
34| 15968.98  11036.38 -807.9798  25.51122 4932.607  -833.491
35| 49209.85  34375.1  -15834.54  531.4332 14834.75  -16365.97
36| 29985.06  28668.47 -707.1835  22.48733 1316.595  -729.6708
37| 17735.91  22140.09 -5917.796  9166.478 -4404.179 -15084.27
38| 39394.51  35270.94 -10155.26  1800.761 4123.565  -11956.02
39| 11004     6802.889 402.4244   39.98421 4201.113  362.4402
40| 1274.298  814.8427 420.147    80.69695 459.4552  339.4501
41| 9282.951  5741.851 -533.3202  246.762  3541.101  -780.0822
42| 4831.233  6546.027 -2955.833  1377.241 -1714.794 -4333.075
43| 21828.96  12382.27 -1556.788  2676.156 9446.684  -4232.944
44| 80278.15  26758.55 23032.1    10837.94 53519.6   12194.16
..
```

Calculate the average of three interested values for two strategies.

[33]:
```
select avg opt_PaL, avg opt_alpha, avg opt_TC, avg VMAP_PaL, avg VMAP_alpha,␣
↪avg VMAP_TC from tbl3
```

[33]:
```
opt_PaL  opt_alpha opt_TC  VMAP_PaL  VMAP_alpha VMAP_TC
-----------------------------------------------------------
1766.611 28173.51  26406.9 -4088.655 346.4304   4435.086
```

Wrap up the previous code into a function so that we can use it to calculate trade for each date.

[34]:
```
PaLcalc:{[dt]
    tbl1: select from bin10 where date = dt;
```

```
   tbl1: update reverse fills reverse date,
                reverse fills reverse mid,
                reverse fills reverse spread,
                reverse fills reverse vol,
                reverse fills reverse adv
                by id from tbl1;

   tbl1: update rtn: ((last mid) % mid) - 1 by id from tbl1;

   tbl1: update W: vol * (sums(u12[count time])) by id from tbl1;

   tbl1: update alpha: (a * (rtn - last rtn)) + (b * (W - last W)) by id from␣
↪tbl1;
   tbl1: update dalpha: 0 ^ ((alpha - xprev[60; alpha]) % 10) by id from tbl1;

   tbl1: update I: (last alpha) ^ next prev 0.5 * (alpha - dalpha % beta) by␣
↪id from tbl1;

   tbl1: update lambda: 8 * (vol * sqrt 6 * 60 * 6.5) % adv by id from tbl1;
   tbl1: update I_: 0 ^ xprev[1; I] * exp neg beta % 6 by id from tbl1;
   tbl1: update deltaQ: (I - I_) % lambda by id from tbl1;

   tbl1: update VMAPtrade: VWAPsimschedule[trade; adv; sum deltaQ] by id from␣
↪tbl1;

   tbl1: update VMAP_I: computeImpact[VMAPtrade; adv; vol] by id from tbl1;
   tbl1: update VMAP_I_: 0 ^ xprev[1; VMAP_I] * exp neg beta % 6 by id from␣
↪tbl1;

   tbl3: select opt_alpha: sum deltaQ * (rtn + 1 - ((first mid) ^ (prev mid))␣
↪% mid),
            opt_TC: sum deltaQ * 0.5 * (I + I_),
            VMAP_alpha: sum VMAPtrade * (rtn + 1 - ((first mid) ^ (prev mid))␣
↪% mid),
            VMAP_TC: sum VMAPtrade * 0.5 * (VMAP_I + VMAP_I_),
            vol: last vol by date, id from tbl1;

   tbl3: update opt_PaL: opt_alpha - opt_TC,
            VMAP_PaL: VMAP_alpha - VMAP_TC by id from tbl3;

   tbl3: `id xasc tbl3}
```

Check whether the function works.

[35]:
```
answer2: PaLcalc[dt]
answer2
```

[35]:
```
date       id| opt_alpha opt_TC   VMAP_alpha VMAP_TC  vol          opt_PaL
VMAP_PaL
-------------|
-------------------------------------------------------------------------
2019.01.03 0 | 224497.5  54640.82 47974.28   7207.686 0.000446367  169856.7
40766.6
2019.01.03 1 | 47440.71  14863.08 7033.335   339.3807 0.000358252  32577.63
6693.955
2019.01.03 2 | 13778.54  5995.11  342.6303   16.504   0.0004827488 7783.435
326.1263
2019.01.03 3 | 9712.052  7095.319 -1103.381  429.552  0.0004667023 2616.732
-1532.933
2019.01.03 4 | 4787.639  7903.326 1125.269   1377.346 0.0003663134 -3115.687
-252.0767
2019.01.03 5 | 6717.977  4054.927 98.32608   23.13856 0.000516633  2663.05
75.18753
2019.01.03 6 | 6376.834  8642.13  3859.141   1185.449 0.0004753984 -2265.296
2673.693
2019.01.03 7 | 8125.474  5071.521 639.203    86.58963 0.0004223277 3053.953
552.6133
2019.01.03 8 | 55245.33  28768.64 21505.65   2226.845 0.0007492939 26476.69
19278.81
2019.01.03 9 | 65950.68  24145.66 31081.73   10321.76 0.0006668529 41805.02
20759.96
2019.01.03 10| 240009.8  187181.4 166341.3   53898.36 0.0007148106 52828.43
112443
2019.01.03 11| -51774.7  29861.54 -33259.52  7902.339 0.0003864019 -81636.24
-41161.86
2019.01.03 12| 15265.68  10243.63 2621.62    390.7536 0.0003828439 5022.044
2230.867
2019.01.03 13| 44720.81  8637.115 10612.27   1662.555 0.0004774389 36083.7
8949.717
2019.01.03 14| 2319436   572863.8 116278.6   14906.42 0.0005112646 1746572
101372.2
2019.01.03 15| 43021.21  27030.04 18029.29   4225.278 0.000372391  15991.18
13804.01
2019.01.03 16| -22077.98 295493.7 -162420.1  61299.66 0.0009286111 -317571.7
-223719.8
2019.01.03 17| -2037.422 5067.26  -8354.557  628.1335 0.0006442204 -7104.682
-8982.69
2019.01.03 18| 23444.03  11585.2  9982.751   5413.26  0.0006423178 11858.84
4569.491
2019.01.03 19| 286016.5  156698.9 114030.6   68631.54 0.0004340049 129317.7
45399.03
2019.01.03 20| 8925.946  43130.29 3456.062   2116.046 0.0004842376 -34204.35
1340.016
2019.01.03 21| 67166.79  43007.23 17487.61   1706.675 0.0003934688 24159.56
```

```
15780.93
2019.01.03 22| 3993.258  6286.525 2022.908   180.4357 0.0003475031 -2293.268
1842.473
2019.01.03 23| 19598.45  17559.98 777.0021   1715.836 0.0006300022 2038.47
-938.8335
2019.01.03 24| 7128.85   19280.11 -1657.109  8197.8   0.0004235462 -12151.26
-9854.909
2019.01.03 25| -2675.881 16664.03 777.6853   53.96227 0.0004774169 -19339.91
723.723
2019.01.03 26| 13105.66  6902.975 2830.17    933.9368 0.0003429665 6202.688
1896.233
2019.01.03 27| 9877.229  12315.27 926.3686   12.61051 0.0006841965 -2438.039
913.7581
2019.01.03 28| 18585.14  11755.12 8307.454   2537.522 0.0005421428 6830.023
5769.932
2019.01.03 29| 196013.6  123451.6 98735.27   64283.91 0.000553965  72562.04
34451.35
2019.01.03 30| 43148     17584.02 11997.92   1664.076 0.0004884136 25563.98
10333.85
2019.01.03 31| 29038.05  12114.54 84.21393   77.04302 0.0004894108 16923.51
7.170908
2019.01.03 32| 22437.63  19337.74 -7221.843  1151.665 0.0003286248 3099.891
-8373.509
2019.01.03 33| 13078.91  11226.91 1995.279   2658.706 0.0004340077 1852.003
-663.4272
2019.01.03 34| 22312.05  14641.7  9927.04    3850.966 0.0004228632 7670.355
6076.074
2019.01.03 35| 73894.39  28383.93 16471.71   575.063  0.0004287401 45510.46
15896.65
2019.01.03 36| 6814.15   41176.86 -8825.683  3502.438 0.0005528349 -34362.71
-12328.12
2019.01.03 37| 40749.59  19335.04 4822.294   6086.813 0.0005698626 21414.55
-1264.519
2019.01.03 38| 57803.45  45598.68 29867.79   15576.86 0.0004820823 12204.77
14290.92
2019.01.03 39| 11400.47  4528.483 326.874    26.38036 0.00040451   6871.99
300.4937
2019.01.03 40| 846.6486  634.6849 -152.7897  10.67193 0.0004607547 211.9637
-163.4617
2019.01.03 41| 4154.18   6027.642 -765.0254  507.7547 0.0005473872 -1873.462
-1272.78
2019.01.03 42| 25365.91  7205.331 -1991.08   624.9242 0.0006445486 18160.58
-2616.004
2019.01.03 43| 15821.59  13835.37 -2103.294  4884.867 0.000435497  1986.216
-6988.162
2019.01.03 44| 36973.01  17816.03 13775.5    3876.986 0.0004997701 19156.99
9898.51
```

..

[36]:
```
select avg opt_PaL, avg opt_alpha, avg opt_TC, avg VMAP_PaL, avg VMAP_alpha,␣
↪avg VMAP_TC from answer2
```

[36]:
```
opt_PaL opt_alpha opt_TC   VMAP_PaL  VMAP_alpha VMAP_TC
-----------------------------------------------------
3597.87 33991.8   30393.93 -5894.246 1987.044   7881.29
```

Backtest the trading costs and P&L of the optimal and the VWAP strategy across all stocks and dates

[37]:
```
answer3: `date`id xasc raze PaLcalc peach dt_list
```

[38]:
```
answer3
```

[38]:
```
date       id| opt_alpha opt_TC   VMAP_alpha VMAP_TC   vol          opt_PaL
VMAP_PaL
-------------|
-------------------------------------------------------------------------
2019.01.02 0 | 3598.625  5380.254 47.8237    9.348172  0.0004776026 -1781.629
38.47553
2019.01.02 1 | 23937.21  13690.93 1418.254   1167.333  0.0004289543 10246.28
250.9208
2019.01.02 2 | 1302.187  8104.044 -4879.916  2249.991  0.0004655141 -6801.856
-7129.907
2019.01.02 3 | 7503.663  3259.041 -223.996   6.918396  0.0005068115 4244.621
-230.9144
2019.01.02 4 | 8475.087  9905.228 -1793.14   104.8129  0.0003931664 -1430.141
-1897.953
2019.01.02 5 | 54522.53  17187.61 24024.16   5329.39   0.0004659031 37334.92
18694.77
2019.01.02 6 | 2288.168  5253.597 -1713.662  60.05754  0.0003448442 -2965.429
-1773.719
2019.01.02 7 | 11714.61  14019.69 3959.85    585.4173  0.0007937001 -2305.071
3374.433
2019.01.02 8 | 52048.22  13483.55 12855.48   2788.78   0.0004189908 38564.67
10066.7
2019.01.02 9 | 30261.42  15841.6  -2876.918  788.587   0.0005368464 14419.82
-3665.505
2019.01.02 10| 36555.02  15708.79 -1355.872  271.3171  0.0002936445 20846.23
-1627.189
2019.01.02 11| 4670.474  8408.324 -1335.799  156.4231  0.0005445913 -3737.851
-1492.222
2019.01.02 12| 177394.9  86200.81 17624.34   32404.15  0.0004928323 91194.07
-14779.81
```

```
2019.01.02 13| 15177.94  5246.05  806.3422   36.623    0.0004186735 9931.893
769.7192
2019.01.02 14| 7804.285  4749.264 21.25228   0.4407691 0.000342972  3055.02
20.81151
2019.01.02 15| -5652.11  7582.074 32.44179   8.310318  0.0003834546 -13234.18
24.13147
2019.01.02 16| 7479.045  5526.539 -148.1462  139.428   0.0003747228 1952.506
-287.5743
2019.01.02 17| 20535.77  9778.798 5776.925   416.6719  0.0004778426 10756.97
5360.253
2019.01.02 18| 3206.031  5071.046 -2574.777  287.8645  0.0006521663 -1865.014
-2862.641
2019.01.02 19| 27216.43  10506.2  4479.557   398.4434  0.0009456223 16710.22
4081.113
2019.01.02 20| 21105.7   11430.76 -15.92993  1.412086  0.0006752427 9674.944
-17.34201
2019.01.02 21| 6075.881  3267.802 1064.719   781.8336  0.0004560371 2808.079
282.885
2019.01.02 22| 30520.61  11831.59 683.516    88.39     0.0003638657 18689.02
595.126
2019.01.02 23| 9474.103  10865.77 -520.2888  175.6337  0.000587086  -1391.662
-695.9225
2019.01.02 24| 41497.13  15019.69 -1349.524  173.4227  0.0003823996 26477.44
-1522.947
2019.01.02 25| 31085.29  12913.52 2543.613   3237.563  0.0004814114 18171.77
-693.9501
2019.01.02 26| 8619.168  6180.48  260.429    10.02351  0.0004513634 2438.688
250.4055
2019.01.02 27| 33320.68  8195.65  6962.698   2214.305  0.0004495159 25125.03
4748.393
2019.01.02 28| 6857.837  6689.58  -1994.057  357.1088  0.0005898566 168.2577
-2351.166
2019.01.02 29| 68442.68  47701.28 11875.38   14419.96  0.0004995882 20741.4
-2544.58
2019.01.02 30| 7278.181  4473.6   2071.797   197.3269  0.000452326  2804.58
1874.47
2019.01.02 31| 70770.71  32164.41 13215.81   4753.718  0.000633535  38606.3
8462.093
2019.01.02 32| 1931.576  6289.771 10.78187   2167.923  0.0005513465 -4358.195
-2157.141
2019.01.02 33| 27851.3   14250.52 12152.1    5842.722  0.0005069254 13600.78
6309.376
2019.01.02 34| -8385.021 24111.12 2538.144   9344.506  0.0005094877 -32496.14
-6806.362
2019.01.02 35| 12344.61  9526.578 -1655.225  648.6814  0.0005509134 2818.028
-2303.907
2019.01.02 36| 40260.57  26330.04 1048.77    1921.875  0.0003135381 13930.52
```

```
-873.1048
2019.01.02 37| 29448.26  12274.36 507.3197   696.5547  0.0003975732 17173.91
-189.2351
2019.01.02 38| 2257.512  8204.697 -195.128   27.64009  0.0005353687 -5947.185
-222.768
2019.01.02 39| -108.9444 33507.69 -13549.82  7061.768  0.0004394434 -33616.63
-20611.59
2019.01.02 40| 5587.142  5519.293 414.6429   141.5732  0.0006714575 67.84929
273.0697
2019.01.02 41| 15446.49  9858.052 4101.173   2072.271  0.0003883097 5588.442
2028.902
2019.01.02 42| 42542.1   94193.05 -5199.046  34456.17  0.00041801   -51650.95
-39655.21
2019.01.02 43| 4071.46   22310.73 -1955.927  261.0494  0.0003074411 -18239.27
-2216.976
2019.01.02 44| 37567.55  30981.84 549.5154   8153.793  0.0005207574 6585.707
-7604.278
..
```

1.3.3 3. Provide summary statistics: average P&L, sharpe ratio, and alpha-impact ratios. Then, repeat the analysis bucketing by vol.

[39]:
```
select opt_aPaL: avg opt_PaL,
       opt_SP: (avg opt_PaL) % (sdev opt_PaL),
       opt_ratio: avg (opt_alpha % opt_TC),
       VMAP_aPaL: avg VMAP_PaL,
       VMAP_SP: (avg VMAP_PaL) % (sdev VMAP_PaL),
       VMAP_ratio: avg (VMAP_alpha % VMAP_TC) from answer3
```

[39]:
```
opt_aPaL opt_SP   opt_ratio VMAP_aPaL VMAP_SP     VMAP_ratio
-----------------------------------------------------------
4028.306 0.097773 1.334197  -431.7276 -0.02195973 4.198539
```

Bucket all stocks into three equal-size groups: low, medium, and high volatility. Then calculate the summary statistics.

[40]:
```
select opt_aPaL: avg opt_PaL,
       opt_SP: (avg opt_PaL) % (sdev opt_PaL),
       opt_ratio: avg (opt_alpha % opt_TC),
       VMAP_aPaL: avg VMAP_PaL,
       VMAP_SP: (avg VMAP_PaL) % (sdev VMAP_PaL),
       VMAP_ratio: avg (VMAP_alpha % VMAP_TC) by bucket:3 xrank vol from answer3
```

[40]:
```
bucket| opt_aPaL opt_SP     opt_ratio VMAP_aPaL VMAP_SP     VMAP_ratio
------| -----------------------------------------------------------
0     | 2563.787 0.08839132 1.293439  -320.3752 -0.02482088 1.879916
```

```
1      | 3617.321 0.1019815  1.349199  -320.1558 -0.02160988 -0.4389018
2      | 5903.809 0.1080175  1.359952  -654.6518 -0.02354021 11.1546
```