1\. a) $\sum_{i=1}^{n}(x_i-\theta)^2 = \sum_{i=1}^{n}\left((x_i-\bar{x})+(\bar{x}-\theta)\right)^2 = \sum(x_i-\bar{x})^2+\sum(\bar{x}-\theta)^2+2\sum(x_i-\bar{x})(\bar{x}-\theta)$

10+10

$= \sum(x_i-\bar{x})^2+n(\bar{x}-\theta)^2+2(\bar{x}-\theta)\sum(x_i-\bar{x})$

$= \sum(x_i-\bar{x})^2+n(\bar{x}-\theta)^2+2(\bar{x}-\theta)(\sum x_i-n\bar{x})$

$= \sum_{i=1}^{n}(x_i-\bar{x})^2+n(\bar{x}-\theta)^2$

b) $\frac{1}{\sigma^2}\sum(x_i-\theta)^2+\frac{1}{\tau^2}(\theta-\mu)^2 = \frac{1}{\sigma^2}\sum(x_i-\bar{x})^2+\frac{1}{\sigma^2}n(\bar{x}-\theta)^2+\frac{1}{\tau^2}(\theta-\mu)^2$

$\frac{1}{\sigma^2}n(\bar{x}-\theta)^2+\frac{1}{\tau^2}(\theta-\mu)^2 = \frac{n}{\sigma^2}\bar{x}^2-\frac{2n\bar{x}\theta}{\sigma^2}+\frac{n\theta^2}{\sigma^2}+\frac{\theta^2}{\tau^2}-2\frac{\theta\mu}{\tau^2}+\frac{\mu^2}{\tau^2}$

$= \frac{n}{\sigma^2}\bar{x}^2+\frac{1}{\tau^2}\mu^2+\left(\frac{n}{\sigma^2}+\frac{1}{\tau^2}\right)\theta^2-2\left(\frac{\mu}{\tau^2}+\frac{n\bar{x}}{\sigma^2}\right)\theta$

The last two items $= \frac{1}{\tau_n^2}\theta^2-2\frac{\mu_n}{\tau_n^2}\theta$

$= \frac{1}{\tau_n^2}(\theta-\mu_n)^2-\frac{\mu_n^2}{\tau_n^2}$

2\. Larger σ^2 $\Rightarrow$ More trust in prior than data.

10

Larger prior variance $\Rightarrow$ More trust in data than prior.

3\. Let E be the event of profits improved.

10

$p(C|E^c) = \frac{p(C)p(E^c|C)}{p(E^c)} = \frac{\frac{5}{8}\times 0.7}{\frac{1}{8}\times 0.6+\frac{2}{8}\times 0.5+\frac{5}{8}\times 0.7} = \frac{35}{51}$

4\. a) $f(x|y,z) = \frac{f(x,y,z)}{f(y,z)} = \frac{c g_1(x,z)g_2(y,z)g_3(z)}{\int_x c g_1(x,z)g_2(y,z)g_3(z)dx} = \frac{g_1(x,z)}{\int_x g_1(x,z)dx} = C_1(z)g_1(x,z)$

10+10+10

b) $f(y|x,z) = \frac{f(x,y,z)}{f(x,z)} = \frac{c g_1(x,z)g_2(y,z)g_3(z)}{\int_y c g_1(x,z)g_2(y,z)g_3(z)dy} = \frac{g_2(y,z)}{\int_y g_2(y,z)dy} = C_2(z)g_2(y,z)$

c) $f(x|z) = \int_y f(x,y|z)dy = \int_y f(x|y,z)f(y|z)dy = C_1(z)g_1(x,z)\int_y f(y|z)dy$

$= C_1(z)g_1(x,z) = f(x|y,z)$

5\. a) $f(x_1,x_2) = 2e^{-2x_1}I(x_1\geq 0)\cdot 3e^{-3x_2}I(x_2\geq 0)$

10+10+10

$= f(x_1)\cdot f(x_2)$

So $X_1 \perp\!\!\!\perp X_2$

b) $EX_2 = \int_0^{+\infty} x\,3e^{-3x}dx = \frac{1}{3}$

c) $P(X_1>X_2) = \int_0^{+\infty} P(X_1>x_2)f(x_2)dx_2 = \int_0^{+\infty} e^{-2x_2}\cdot 3e^{-3x_2}dx_2 = \frac{3}{5}$