Math 280, Spring 2022: Graph Theory

MWF 11:00 – 11:50am, SMUD 205 and Tu 9:00 – 9:50am, SMUD 205

Webpage: `https://rlbenedetto.people.amherst.edu/math280/`
(Also linked from the Math 280 moodle page.)

Instructor: Rob Benedetto

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Office Hours: Mon, 2:30–3:30pm; Tue, 2–3:30pm; Thu, 1–2:30pm; or by appointment.

Our Math 280 Fellow (or "TA"): Anna Dietrich
Email: `adietrich22@amherst.edu` Office hours: TBA

Text: J.M. Harris, J.L. Hirst, and M.J. Mossinghoff, *Combinatorics and Graph Theory*, 2nd ed., Springer, 2008.
Available as an e-book through the Amherst College library.

Exams:

- **Midterm 1**: **Wednesday, March 9**, in class.
- **Midterm 2**: **Wednesday, April 20**, in class.
- **Final**: **Take-home; details TBA**

The only excuses for missing an exam are incapacitating illness, religious conflict, or the like.

Using calculators, cell phones, etc. during an in-class exam is CHEATING.
Consulting outside sources during any exam is CHEATING.
All cheating will be prosecuted, leading to an F in the course, and worse.

Homework:

- Reading from the textbook will be assigned each week.
- Problem sets will be due (usually) once a week, **on gradescope by 11:59pm ET**.
See page 3 of this syllabus for important homework information.

Grading:

- **Effort**: 5%
- **Problem Sets**: 20%
- **Midterm Exams**: Better one: 25%. Worse one: 15%. (Total 40%.)
- **Final Exam**: 35%

"Effort" is a combination of class attendance, class participation, and handing in problem sets. It is not computed linearly; a student deficient in any one of those areas will get a very low Effort grade. (See pages 2 and 4 of this handout for more on attendance and participation.)

"Problem Sets" means actual grades on the problem sets. Late problem sets will be marked down substantially in the Problem Sets portion of your grade (see the webpage for details); but all problem sets handed in by the last day of classes count towards Effort.

If an individual student's Midterm, Final, or Problem Set grade is substantially higher or lower than their other grades, and if the student's Effort grade is strong, I will tweak the above percentages a little for that student to favor the better grades. Overall course grades will be curved.

About Cell Phones and Mobile Devices

Cell phones and similar devices have no place in my classroom. Don't use them. Not for talking, not for texting, not for anything. So at every class:

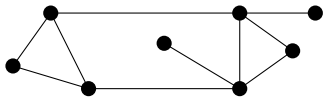
Silence your cell phone, put it away, and pay attention.

Necessary Background

The prerequisite for this course is linear algebra (Math 271 or Math 272), with other experience with proofs (as in Math 220 or other theory-based math courses) recommended. We will make some use of matrices, but the more important requisite is exposure to abstract concepts and proofs. I don't expect you to feel completely comfortable with proofs yet, and this course is a good place to get more experience with them, but if you feel completely bewildered by mathematical proofs and haven't taken Math 220 yet (Reasoning and Proof), then Math 220 may be a better course for you first.

Course Content

In our context, a *graph* is a set of nodes, or *vertices*, any two of which may or may not be connected by an *edge*. Drawing dots for vertices and lines for edges, we think of a graph as a picture like this:



Graphs can be used to model airline networks (vertices as airports, edges as flight routes), computer networks (vertices as computers, edges as wired or wireless connections), social networks (vertices as people, edges as friendships), maps (vertices as countries, edges as sharing a border), and much more. But for our purposes, we will focus on the mathematical theory of graphs, and not on their applications. In particular, that means that the intuitive ideas and pictures will have to be backed up by formal definitions and rigorous proofs. Here's a more detailed list of some of what we'll see, almost all of which is from Part 1 (pages 1–127) of our textbook:

- Section 1.1 is mostly terminology and definitions, but already with some basic proofs appearing.
- Section 1.2 introduces a notion of *distance* on graphs, as well as the use of matrices (from linear algebra) to encode a graph and facilitate certain computations.
- Section 1.3 is about particular class of graphs called *trees*.
- Section 1.4 is about *walks, paths, trails, cycles, and circuits*, which are different ways of tracing routes from vertex to vertex along the edges of a particular graph.
- Section 1.5 is about *planar graphs*, which are graphs that can be drawn on a piece of paper without any of the edges crossing each other.
- Section 1.6 is about *graph coloring*, which means assigning a value (called a *color*) to each vertex. Different vertices may be assigned the same color, but we usually impose the restriction that no two vertices that share an edge have the same color. This idea is inspired by geographical maps, where we often want to color in each country with a solid color, and while it's OK to color two different countries the same color, we don't want to do that for countries that share a border.
- Sections 1.7 and 1.8 are about slightly more advanced topics: matchings, factoring, edge colorings, and Ramsey theory.
- If time permits, we'll learn some topics not in the textbook, like spectral graph theory.

Class Participation and Classroom Dynamics

Class participation is part of the Effort portion of your grade. If you are quiet by nature, don't worry; as long as you attend class devotedly, pay close attention, and do the homework, you will get full Effort credit. That said, finding your voice in class helps you learn better. So for your own benefit, **speak up, ask questions, and even try to answer questions I may ask in class**.

On the flip side, **respect your fellow students** both in and out of the classroom. Treat every person's ideas the same way you would if it had been me or Biddy sharing them.

Homework

Your homework consists **BOTH** of reading the book **AND** doing the assigned problems. Skim textbook sections **before** the corresponding lecture, and then go back and read more carefully to solidify your understanding and to help with the problem sets.

Please note the following **Important Problem Set Rules:**

1. Problem sets are due **on Gradescope**, each by its specified deadline.
2. You must "tag" your problems in Gradescope, marking where each problem's solution appears.
3. **Write legibly**, and organize your work clearly. Make it a pleasure to read!
4. If you worked with other students or got help from a source other than me, Anna (our Math Fellow), the book, or other course resources, then say so explicitly on the first page of your problem set. (See the discussion below on the Statement of Intellectual Responsibility.)
5. The Problem Sets grade for any late assignment will be substantially reduced. The later it is, the greater the reduction; see the course webpage under "Problem Set Rules" for details.

See Homework 0 and the handout on Gradescope Instructions for more information on Gradescope.

I am often willing to grant penalty-free extensions on problem sets; but see "Attendance, Extensions, and Extra Office Hours" on page 4.

I strongly encourage you to work on problem sets together, in pairs or small groups, provided you follow the common-sense guidelines below.

About the Statement of Intellectual Responsibilty

Exams: Your work must be entirely your own, so no looking at other people's papers, no communication, and no outside help. For the in-class midterms, no books, notes, calculators, or other resources are allowed, either. For the take-home exam, you may use only course materials, **your own** notes, and the textbook; you may also consult Prof. Benedetto, but not Anna (our Math Fellow) or anyone else. No other books, notes, online resources, or communications with other people are allowed.

Problem sets: I urge you to collaborate with each other, under the following ground rules:

1. If you collaborate with, say, Jane and Joe, write a note on the front of your problem set saying, "I worked with Jane and Joe." Use similar notation if you got help from a tutor, fellow student, another professor, another book, the web, etc. However, you do **not** need to write about help you got from me, Anna, the textbook, or course materials.
2. Working together does not mean that Joe does the first half of the problem set and Jane does the second half; everyone should work on every problem.
3. Each student must hand in their own problem set; you can't hand in a single packet as the work of multiple people.
4. Each student must write up each problem **in their own words**. Working together means discussing the problems. Copying someone else's solution (even when the source doesn't mind) is plagiarism and a violation of intellectual responsibility.

A common question: What if Joe asks Jane about a homework problem she has already solved? If Joe copies Jane's solution, both Joe and Jane would be guilty of academic dishonesty, leading to an F in the course for both of them and potentially to dismissal from the college. Instead, Jane can explain her solution to Joe (even showing him what she wrote), before Joe writes up his own solution himself, **in his own words**. Joe would then have to write that he got help from Jane (see rule 1 above), but Jane doesn't need to write anything unless she also got help in return.

If at any time you aren't sure about what's OK and what's not as far as intellectual responsibility is concerned for this course, talk to me about it.

Class Attendance, Extensions, and Extra Office Hours

Attendance: You should be at every class meeting, and you should be on time. Of course, if you're sick, are in mandated isolation, have a religious conflict, or the like, just let me know (in advance, when possible). One or two accidental misses are OK, too; oversleeping can happen. Otherwise, however, **I expect you to be in class, and on time, for every class meeting**. I don't plan to take formal attendance, but I will easily be able to tell who misses class too much; those students' Effort grades will take a hit. (And after more than just a very few unexcused absences, your Effort grade will truly plummet, since I do not compute it linearly.)

Extensions: You may request **up to two** homework extensions over the course of the semester, each for up to 48 hours. To claim an extension, you must:

1. Not have used both your extensions yet,
2. Request the extension (by email, or in person) **no later than 4pm ET on the due date**,
3. Have been attending class devotedly and meeting homework deadlines, and
4. Attend class **on time** on the original due date **and** the following day that the class meets.

Note: you do **not** need to provide an excuse or reason for your extension request; just ask.

Office Hours: you are always welcome to attend my regularly scheduled office hours. In addition, **IF you have been attending class and doing the homework**, you are also welcome to make appointments to see me **outside** of my regularly scheduled office hours.

Getting Help

If you're ever stuck or confused, seek help immediately:

Office Hours: Stop by (unannounced) to see me during my scheduled office hours, or make an appointment to see me another time.

Math 280 Fellow: Visit Anna's office hours, too. See the course website for details.

Tutoring: If you feel you need regular one-on-one help from a tutor, we can probably set that up. To do so, talk to me first.

Advice, and What to Expect

The amount of proof and rigor in Math 280 will be somewhere between the amount in Math 271/272 the amount in Math 350/355. So Math 280 is a good place to hone your existing proof-writing skills. That said, the weekly problem sets will be challenging, so you should **start working on each problem set the same day it is assigned**; do *not* put it off until the due date. You'll need time to ponder and mess around with some of the trickier problems, and you may want to seek help (e.g., office hours or asking questions in class). So every week, **read and think about ALL of the problems several days in advance**.

The homework will be a mix of working with explicit examples and crafting more general proofs, and you will have to explain your solutions with **words, complete sentences, and well-written paragraphs**. Get in the habit of working things out on scratch paper in advance, figuring out what needs to be said, what **doesn't** need to be said, and what **order** everything should come in. You don't need to explain the intuition; you need to **solve the problem** or **prove the theorem**.

And if you're ever feeling lost, there's always office hours: mine or Anna's.