HW2 Solution

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Question 2

a)

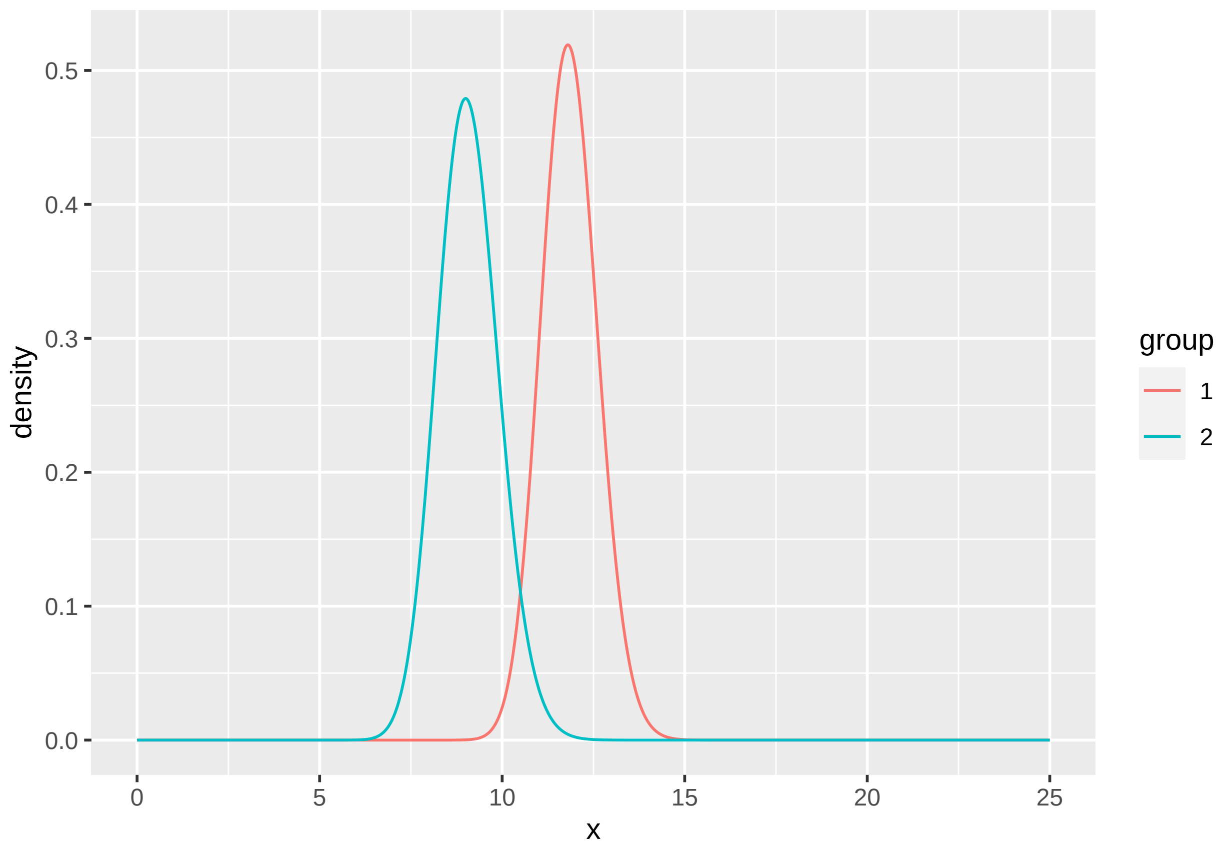
```
library(tidyverse)
```

```
## -- Attaching packages --------------------------------------- tidyverse 1.3.1 --

## v ggplot2 3.4.0     v purrr   0.3.4
## v tibble  3.1.6     v dplyr   1.0.7
## v tidyr   1.1.4     v stringr 1.4.0
## v readr   2.1.1     v forcats 0.5.1

## -- Conflicts ------------------------------------------ tidyverse_conflicts() --
## x dplyr::filter() masks stats::filter()
## x dplyr::lag()    masks stats::lag()
```

```
params <- tibble(alpha = c(237, 118), beta = c(20, 13))

library(HDInterval)
for (i in 1:2) {
  print(hdi(qgamma, 0.95, shape = params$alpha[i], rate = params$beta[i]))
}
```

```
##    lower    upper
## 10.35737 13.37075
## attr(,"credMass")
## [1] 0.95
##     lower     upper
##  7.465163 10.732044
## attr(,"credMass")
## [1] 0.95
```

```
n_grid <- 1000
plot_tb <- bind_rows(
  tibble(x = seq(from = 0, to = 25, length.out = n_grid),
                density = dgamma(x, shape = params$alpha[1], rate = params$beta[1]),
                group = '1'),
  tibble(x = seq(from = 0, to = 25, length.out = n_grid),
                density = dgamma(x, shape = params$alpha[2], rate = params$beta[2]),
                group = '2')
)
plot_tb %>%
  ggplot(aes(x = x, y = density, color = group)) +
  geom_line()
```



b)

```
B <- 50
xs <- 1:B

sum(dnbinom(xs, size = params$alpha[1], prob = params$beta[1] / (params$beta[1] + 1)) *
      pnbinom(xs - 1, size = params$alpha[2], prob = params$beta[2] / (params$beta[2] + 1)))
```

```
## [1] 0.6856978
```

```
xs <- 0:B
sum(dnbinom(xs, size = params$alpha[1], prob = params$beta[1] / (params$beta[1] + 1)) *
      pnbinom(xs, size = params$alpha[2], prob = params$beta[2] / (params$beta[2] + 1)))
```

```
## [1] 0.7575315
```

c)

With 95% probability, the range of asymptotic confidence interval is $2 * z_{0.975}\sqrt{\frac{\sigma^2}{n}}$ which should equals 0.05 * 2.

```
variance <- sum(params$alpha / params$beta^2)
variance / 0.05^2 * qnorm(0.975)^2
```

```
## [1] 1983.307
```

d)

```r
set.seed(0)
n <- 10000
theta_as <- rgamma(n, params$alpha[1], params$beta[1])
theta_bs <- rgamma(n, params$alpha[2], params$beta[2])
mean(theta_as - theta_bs)
```

```
## [1] 2.768717
```

```r
mean(theta_as > theta_bs)
```

```
## [1] 0.9911
```

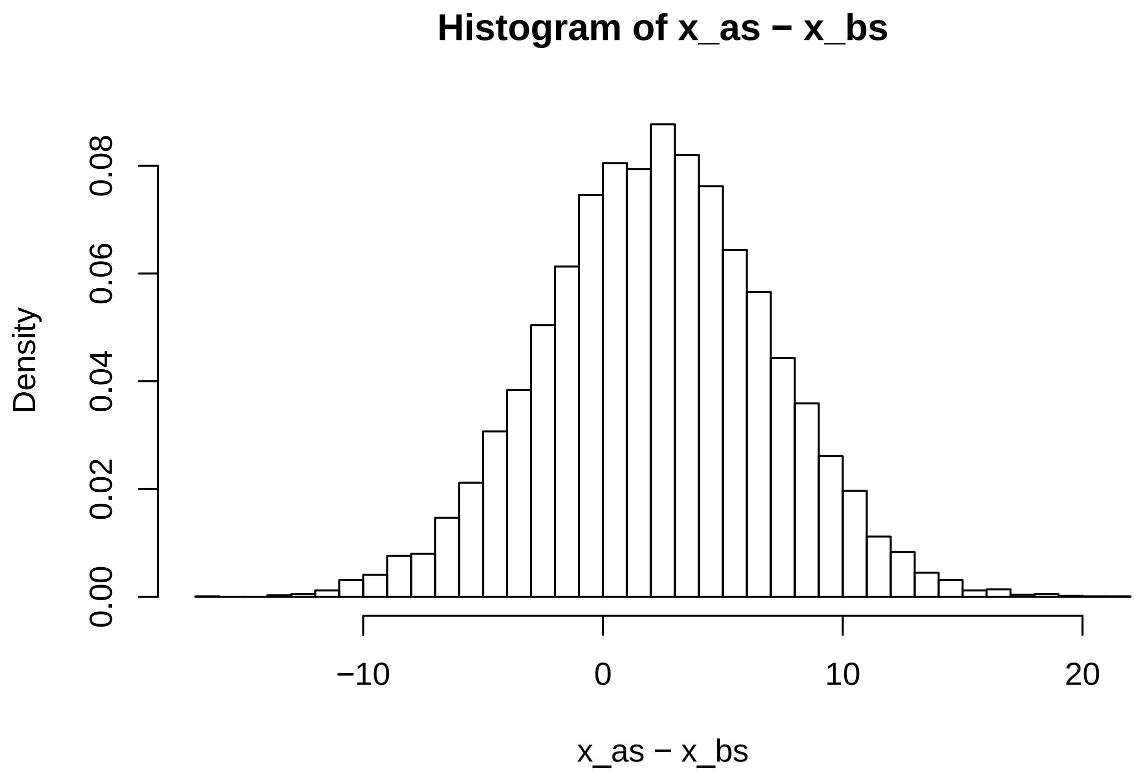
```r
quantile(theta_as - theta_bs, probs = c(0.025, 0.975))
```

```
##      2.5%     97.5%
## 0.4943001 4.9350317
```

From the confidence interval, we know θ_a is significantly larger than θ_b, which is consistent with the $P(\theta_a > \theta_b)$ estimation.

e)

```r
set.seed(0)
n <- 10000
x_as <- rnbinom(n, size = params$alpha[1], prob = params$beta[1] / (params$beta[1] + 1))
x_bs <- rnbinom(n, size = params$alpha[2], prob = params$beta[2] / (params$beta[2] + 1))

hist(x_as - x_bs, freq = FALSE, breaks = 50)
```



The result is supposed to be the difference between two negative binomial random variables.

Question 4

```
set.seed(0)
n <- 10000
xs <- runif(n)
mean(4 * sqrt(1 - xs^2))
```

```
## [1] 3.134347
```

```
set.seed(0)
n <- 10000
xs <- rbeta(n, 2, 2)
mean(4 * sqrt(1 - xs^2) / dbeta(xs, 2, 2))
```

```
## [1] 3.125043
```