

## Introduction to the Mathematics of Finance. HOMEWORK 2.

Due October 26, 2022 11:59pm

Please write a pledge that homework solutions represent your own work and that you did not copy solutions from the work of other students.

1.(5pt) What is the difference between historic and implied volatilities? Which one is higher for Microsoft stock MSFT? Go to the Bloomberg terminal and type: MSFT <Equity> HIVG <Go>. Print the output.

2.(5pt) What is the volatility smile? Using Bloomberg terminal function SKEW (CLZ2 Comdty SKEW <Go>) plot the implied volatility smiles for TYZ2 Comdty, CLZ2 Comdty, ESZ2 Index, SPY Equity. What are these securities? For description type CLZ2 Comdty DES. Compare their smiles. Submit printouts. Bloomberg terminals upgrade functions from time to time. If these functions do not work ask TAs for help.

3.(10pt) Suppose the stock price is 80, the risk-free rate is 3% continuously compounded. What is the price of a 1 year call struck at 80 if the volatility is 0. How would you hedge the call. Check your answer with the option calculator making volatility smaller and smaller.

4.(10pt) Explain why an American option on a stock paying continuous dividend yield is always worth as much as its intrinsic value. Give a numerical example of a situation when European option is worth less than intrinsic value. (Give the numerical value of stock price, strike price, time to expiration, etc.)

5.(5pt) Explain the European call-put parity argument. Why it can not be used for American options

6.(10pt) Calculate the implied volatility of Microsoft stock using November 2022 calls expiring November 18, 2022 with strike 250 and December 2022 calls expiring December 16, 2022 with strike with strike 250. Get the quotes from any data provider, for example, finance.yahoo.com and use all other necessary data. On Bloomberg type MSFT Equity CALL (page down as needed).

7.(10pt) Download Excel options model with VBA code from the courseworks. Save as a source. Open in Excel. Modify the Black model for futures to Black-Scholes model for stocks paying dividends at rate  $q$  (like in the Hull's book). Make the necessary changes in Visual Basic code. Use Excel help or consult TA's if you do not know what to do. Check that code works and submit the code printout.

8.(10pt) Download Excel Brownian Motion model from the courseworks. Save as a source. Open in Excel. Modify it to Geometric Brownian motion with growth rate  $\mu = 0.10$ , volatility  $\sigma = 0.3$  and 250 trajectories. Submit excel formulas printout. Geometric Brownian motion starts with positive  $X_0$  so you must change a starting value from 0 to a positive number that you may choose (you can choose 1, 100 or other positive number).

9.(10pt) Suppose that the value  $X$  of a variable that follows a Standard Brownian Motion is initially 70. The time is measured in years. Write the probability density function of the distribution of  $X$  after 0.5 year, 1 year, 2 years, 4 years and 5 years? What is the probability of variable after 1 year to be less than 70. (We usually defined Standard Brownian motion to start at  $X_0=0$  but you can start at any  $X_0$  (it is equivalent to adding constant  $X_0$  to all trajectories of zero start Standard Brownian Motion.)