Math Methods – Financial Price Analysis

Spring 2023, Mathematics, GR5360

Homework #2

Distributed: 3/4/2023,
Due Date: 3/18/2023.

All problems can be solved or answered using the class handouts.

Although you should use the class handouts as a guide for the following problems, try to not copy the handouts directly. Offer your own insights for questions involving written explanation and make sure to derive what is needed from the first principles, trying to explain the non-trivial steps involved.

Problem 1. Please define the symmetric bi-parametric (parameters α –scaling and γ-amplitude) Lévy probability distribution function (PDF) of a random function x as function of time τ.

Problem 2. Following the class handouts, introduce a set of new self-similar variables (instead of x and τ) that will express the Lévy PDF in simpler and more universal, scale-independent form.

Problem 3. Following the class handouts, derive a "near 0" asymptotic series expansion for the Lévy PDF for $x \to 0^+$. Please provide your own detailed explanations of the steps in derivation.

Problem 4. Using the 1-min data for the S&P500 E-Mini and FTSE-100 index futures used for homework 1, please develop a histogram of 1-min price changes $\Delta p = p(t+1) - p(t)$, where t represents trading minutes. Please make use of all possible realizations of Δp (i.e., $p(1+1) - p(1)$, $p(2+1) - p(2)$, etc.), and not only of non-overlapping intervals like in the previous homework. For this problem, please do the following:

- Normalize the results of your measurements such that $\int_{-\infty}^{\infty} P(x,\tau)dx = 1$.
- Graph the results for $P(x,\tau=1)$ on a log-lin scale (i.e., $\log_{10}(P(x,\tau=1))$ as a function of x). Use a symmetric range of positive and negative values of x.
- On this graph, superimpose a true Gaussian PDF which has the empirically correct σ (standard deviation of the 1-min price changes Δp) and μ (=sample average of Δp's) which you will need to measure from the data, with the purpose of a clear visual illustration of the existence of non-Gaussian "fat" tails in the PDF.

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- So far, we've only used $\tau = 1$. Using Matlab, Python, C or any other high-level programming language, write a function to do this for any given time shift τ . Use your code to redo what you've done for this problem with other values of $\tau = 5, 30, 60, 120, 180, 360$ mins. Submit all resulting graphs.

- Using the leading-order term from the formula for the Taylor-series expansion of the Lévy PDF, please, infer the Lévy exponent α for both of the ES and FT markets. Please provide additional insight into how you were able to determine this from using your code.

- Finally, submit your code.

<u>Problem 5.</u> This is a question related to the "analogies with fluid turbulence" lecture. Please explain in detail what structures or price formations are responsible for the fat tails in the S&P 500 E-Mini futures. As a reminder, coherent vortices and shock waves were responsible for the fat tails in fluid turbulence.

<u>Problem 6.</u> Please explain the basic key basic similarities between the velocity differences in fluid turbulence and fluctuations in S&P 500 E-Mini futures prices. Provide as much details and as many of your own words and thought as possible.