Economics 361
Distribution Tables
for Standard Normal and t

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Overview

To calculate the critical region of a hypothesis test for some desired significance level, one must know the pdf and/or cdf of the sampling distribution of the test statistic when the null hypothesis is true ("under H_o").

For the Z-statistic and the t-statistic, the sampling distribution under H_o is the Standard Normal and t-distribution, respectively. The required information about the pdf/cdf for such distribution is usually provided in a table format. Such tables are attached to this handout. This handout provides some pointer on how to read such tables.

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Standard Normal

Let Z be a random variable distributed Standard Normal, i.e. $N(0, 1)$.

Standard Normal is unimodal and symmetric around 0. This implies

- $f_Z(c) = f_Z(-c)$ for any real valued $c \geq 0$
- $\text{Prob}(Z \leq -c) = \text{Prob}(Z \geq c)$ for any real valued $c \geq 0$
  - Note: $\text{Prob}(Z \leq -c) = \int_{-\infty}^{-c} f_Z(z)\, dz$ and $\text{Prob}(Z \geq c) = \int_{c}^{\infty} f_Z(z)\, dz$

A special case of the second implication is $\text{Prob}(Z \leq 0) = 0.5 = \text{Prob}(Z \geq 0)$. Half the probability mass is less than zero and the other half greater than zero.[1]

The attached Table D.1 provides the area underneath the Standard Normal pdf between 0 and some positive value z. That value + 0.5 provides the value of the CDF at z: $F_Z(z) = \text{Prob}(Z \leq z)$.

$$F_Z(z) \quad = \quad \text{Prob}(Z \leq z) \quad = \quad \text{Prob}(Z \leq 0) \ + \ \text{Prob}(0 \leq Z \leq z) \quad = \quad 0.5 + \text{Prob}(0 \leq Z \leq z)$$

Example: To find the value of $\text{Prob}(0 \leq Z \leq 1.64)$, see the element in the row labeled "1.6" and the column labeled ".04" to get "0.4495." This implies that

- $\text{Prob}(0 \leq Z \leq 1.64) = 0.4495$
- $F_Z(1.64) \quad = \quad \text{Prob}(Z \leq 1.64) = 0.9495$
- $\text{Prob}(Z \geq 1.64) = 0.0505 = \text{Prob}(Z \leq -1.64)$

Example: To find the value of z such that $\text{Prob}(Z \geq z) = 0.025$, look for the element in the table valued at (or nearest to) 0.5 - 0.025 = 0.475. This is the element in row "1.9" and column ".06" Thus, $\text{Prob}(Z \geq 1.96) = 0.025$. Similarly, $\text{Prob}(Z \leq -1.96) = 0.025$.

Example: To find that value z such that $F_Z(z) = 0.99$, look for the element in the table valued at (or nearest to) 0.99 - 0.5 = 0.49. This is the element in row "2.3" and column ".03" Thus, $F_Z(2.33) = \text{Prob}(Z \leq 2.33) \approx 0.99$

[1] We do not have to worry about the exact case of zero as Normal random variables are continuous

t-Distribution

Let T be a random variable distributed by the t-distribution.

The t-distribution is also unimodal and symmetric around 0. This implies

- $f_Z(c) = f_Z(-c)$ for any real valued $c \geq 0$
- $\text{Prob}(Z \leq -c) = \text{Prob}(Z \geq c)$ for any real valued $c \geq 0$
  - Note: $\text{Prob}(Z \leq -c) = \int_{-\infty}^{-c} f_Z(z)\ dz$ and $\text{Prob}(Z \geq c) = \int_{c}^{\infty} f_Z(z)\ dz$

A special case of the second implication is $\text{Prob}(Z \leq 0) = 0.5 = \text{Prob}(Z \geq 0)$. Half the probability mass is less than zero and the other half greater than zero.[2]

Sometimes distribution tables show the area underneath the pdf not for values between 0 and some value t but between t and $+\infty$. The attached Table D.2 is such a table for the t-distribution.

While the Standard Normal is a specific version of the Normal distribution, the t-distribution is a class of distribution. In the same way the parameters (mean, variance) precisely characterize the Normal distribution, the parameter "degrees of freedom" (denoted v in Table D.2) precisely characterizes the t-distribution. A random variable T distributed t-distribution with v degrees of freedom is denoted $T \sim t_v$.

The row in Table D.2 indicates the specific type of t-distribution (degrees of freedom). The column indicates the area underneath the pdf of that t-distribution for values between t and $+\infty$. The table element indicates the value of t ... I know, confusing notation ... but unfortunately standard.

Note that as the degrees of freedome get larger, the t-distribution approaches the Standard Normal. In fact, the t-distribution converges to the Standard Normal as $v \to +\infty$. So, for very large v, one may use the Standard Normal table instead.

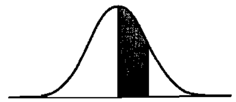
Example: To find the value of t such that $\text{Prob}(T \geq t) = .01$ for $T \sim t_{18}$, find the element in row "18" and column ".01" The element value is 2.552. Thus, $\text{Prob}(T \geq 2.552) = .01$ and $F_T(2.552) = \text{Prob}(T \leq 2.552) = 1 - .01 = 0.99$.

Example: To find the value of t such that $\text{Prob}(T \leq t) = .05$ for $T \sim t_{10}$, find the element in row "10" and column ".05" and multiply that value by -1. Thus, $\text{Prob}(T \leq -1.812) = .05$.

[2] We do not have to worry about the exact case of zero as T is also a continuous random variable

TABLE D.1
Areas of a standard normal distribution

An entry in the table is the proportion under the entire curve that is between $z = 0$ and a positive value of z. Areas for negative values of z are obtained by symmetry.



z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359
0.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0753
0.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
0.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517
0.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
0.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
0.6	.2257	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2517	.2549
0.7	.2580	.2611	.2642	.2673	.2703	.2734	.2764	.2794	.2823	.2852
0.8	.2881	.2910	.2939	.2967	.2995	.3023	.3051	.3078	.3106	.3133
0.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.4319
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441
1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545
1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857
2.2	.4861	.4864	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4986
3.0	.4987	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4990	.4990

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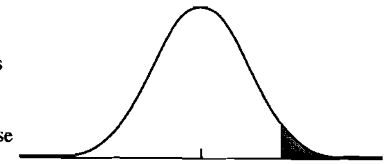
TABLE D.2
Student's t distribution

The first column lists the number of degrees of freedom (ν). The headings of the other columns give probabilities (P) for t to exceed the entry value. Use symmetry for negative t values.



ν \ P	.10	.05	.025	.01	.005
1	3.078	6.314	12.706	31.821	63.657
2	1.886	2.920	4.303	6.965	9.925
3	1.638	2.353	3.182	4.541	5.841
4	1.533	2.132	2.776	3.747	4.604
5	1.476	2.015	2.571	3.365	4.032
6	1.440	1.943	2.447	3.143	3.707
7	1.415	1.895	2.365	2.998	3.499
8	1.397	1.860	2.306	2.896	3.355
9	1.383	1.833	2.262	2.821	3.250
10	1.372	1.812	2.228	2.764	3.169
11	1.363	1.796	2.201	2.718	3.106
12	1.356	1.782	2.179	2.681	3.055
13	1.350	1.771	2.160	2.650	3.012
14	1.345	1.761	2.145	2.624	2.977
15	1.341	1.753	2.131	2.602	2.947
16	1.337	1.746	2.120	2.583	2.921
17	1.333	1.740	2.110	2.567	2.898
18	1.330	1.734	2.101	2.552	2.878
19	1.328	1.729	2.093	2.539	2.861
20	1.325	1.725	2.086	2.528	2.845
21	1.323	1.721	2.080	2.518	2.831
22	1.321	1.717	2.074	2.508	2.819
23	1.319	1.714	2.069	2.500	2.807
24	1.318	1.711	2.064	2.492	2.797
25	1.316	1.708	2.060	2.485	2.787
26	1.315	1.706	2.056	2.479	2.779
27	1.314	1.703	2.052	2.473	2.771
28	1.313	1.701	2.048	2.467	2.763
29	1.311	1.699	2.045	2.462	2.756
30	1.310	1.697	2.042	2.457	2.750
40	1.303	1.684	2.021	2.423	2.704
60	1.296	1.671	2.000	2.390	2.660
120	1.289	1.658	1.980	2.358	2.617
∞	1.282	1.645	1.960	2.326	2.576

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