Probability

Econ 361 Week 1

Two Dice Roll Experiment

	Second Die Roll							
First Die Roll		“Roll 1”	“Roll 2”	“Roll 3”	“Roll 4”	“Roll 5”	“Roll 6”	
	“Roll 1”							
	“Roll 2”							
	“Roll 3”							
	“Roll 4”							
	“Roll 5”							
	“Roll 6”							

36 Atomistic, mutually exclusive events (”simple” events) that span the Sample Space

For simplicity, denote (X, Y) where X indicates first die roll and Y second die roll

e.g. (2, 3) denotes the simple event where first die roll results in 2 and second die roll in 3

Two (*Fair*) Dice Roll Experiment

		Second Die Roll						
First Die Roll		“Roll 1”	“Roll 2”	“Roll 3”	“Roll 4”	“Roll 5”	“Roll 6”	
	“Roll 1”	1/36	1/36	1/36	1/36	1/36	1/36	
	“Roll 2”	1/36	1/36	1/36	1/36	1/36	1/36	
	“Roll 3”	1/36	1/36	1/36	1/36	1/36	1/36	
	“Roll 4”	1/36	1/36	1/36	1/36	1/36	1/36	
	“Roll 5”	1/36	1/36	1/36	1/36	1/36	1/36	
	“Roll 6”	1/36	1/36	1/36	1/36	1/36	1/36	

36 Atomistic, mutually exclusive events (”simple” events) that span the Sample Space
For simplicity, denote (X, Y) where X indicates first die roll and Y second die roll
e.g. (2, 3) denotes the simple event where first die roll results in 2 and second die roll in 3

Two (*Fair*) Dice Roll Experiment: Why 1/36?

	Second Die Roll							
First Die Roll		"Roll 1"	"Roll 2"	"Roll 3"	"Roll 4"	"Roll 5"	"Roll 6"	
	"Roll 1"	1/36	1/36	1/36	1/36	1/36	1/36	
	"Roll 2"	1/36	1/36	1/36	1/36	1/36	1/36	
	"Roll 3"	1/36	1/36	1/36	1/36	1/36	1/36	
	"Roll 4"	1/36	1/36	1/36	1/36	1/36	1/36	
	"Roll 5"	1/36	1/36	1/36	1/36	1/36	1/36	
	"Roll 6"	1/36	1/36	1/36	1/36	1/36	1/36	

Kolmogorov's Axioms and understanding that each of these 36 simple events are mutually exclusive, equally likely, and span the Sample Space …

Two (*Fair*) Dice Roll Experiment: Probability Function Domain is a Borel Field

	Second Die Roll							
First Die Roll		"Roll 1"	"Roll 2"	"Roll 3"	"Roll 4"	"Roll 5"	"Roll 6"	
	"Roll 1"	1/36	1/36	1/36	1/36	1/36	1/36	
	"Roll 2"	1/36	1/36	1/36	1/36	1/36	1/36	
	"Roll 3"	1/36	1/36	1/36	1/36	1/36	1/36	
	"Roll 4"	1/36	1/36	1/36	1/36	1/36	1/36	
	"Roll 5"	1/36	1/36	1/36	1/36	1/36	1/36	
	"Roll 6"	1/36	1/36	1/36	1/36	1/36	1/36	

Interested in making probability statements that involve not just simple events but also ***composite*** events
e.g. "First Die roll is 1," "Sum of the two dice rolls is 7," "Second Die Roll is ***not*** 1"

Borel Field: collection of subsets of Sample Space that is closed un complementary, under countable union, and allows for null set

Two (*Fair*) Dice Roll Experiment: Conditional Probability

	Second Die Roll							
First Die Roll		"Roll 1"	"Roll 2"	"Roll 3"	"Roll 4"	"Roll 5"	"Roll 6"	
	"Roll 1"	1/36	1/36	1/36	1/36	1/36	1/36	**1/6**
	"Roll 2"	1/36	1/36	1/36	1/36	1/36	1/36	**1/6**
	"Roll 3"	1/36	1/36	1/36	1/36	1/36	1/36	**1/6**
	"Roll 4"	1/36	1/36	1/36	1/36	1/36	1/36	**1/6**
	"Roll 5"	1/36	1/36	1/36	1/36	1/36	1/36	**1/6**
	"Roll 6"	1/36	1/36	1/36	1/36	1/36	1/36	**1/6**
		1/6	**1/6**	**1/6**	**1/6**	**1/6**	**1/6**	

Conditional Probability: restricts Sample Space, redefines probability function over this restricted Sample Space
Probability mass for the restricted Sample Space falls
  e.g. if we condition on first die roll being 3, the probability mass for the remaining simple events is now 1/6
Conditional probability must account for this reduced probability mass

Two (*Fair*) Dice Roll Experiment: Conditioning on First Die Roll being 3

		Second Die Roll						
First Die Roll		"Roll 1"	"Roll 2"	"Roll 3"	"Roll 4"	"Roll 5"	"Roll 6"	
	"Roll 1"							**1/6**
	"Roll 2"							**1/6**
	"Roll 3"	1/36	1/36	1/36	1/36	1/36	1/36	**1/6**
	"Roll 4"							**1/6**
	"Roll 5"							**1/6**
	"Roll 6"							**1/6**
		1/6	**1/6**	**1/6**	**1/6**	**1/6**	**1/6**	

Conditional Probability: restricts Sample Space, redefines probability function over this restricted Sample Space

Probability mass for the restricted Sample Space falls

e.g. if we condition on first die roll being 3, the probability mass for the remaining simple events is now 1/6

Conditional probability must account for this reduced probability mass

Two (*Fair*) Dice Roll Experiment: Conditioning on First Die Roll being 3

	Second Die Roll							
First Die Roll		"Roll 1"	"Roll 2"	"Roll 3"	"Roll 4"	"Roll 5"	"Roll 6"	
	"Roll 1"							
	"Roll 2"							
	"Roll 3"	1/36 / 1/6 = 1/6	1/36 / 1/6 = 1/6	1/36 / 1/6 = 1/6	1/36 / 1/6 = 1/6	1/36 / 1/6 = 1/6	1/36 / 1/6 = 1/6	**1/6 / 1/6 = 1**
	"Roll 4"							
	"Roll 5"							
	"Roll 6"							

Normalizing all of the conditional probability for the reduced probability mass

Conditioning is often associated with the concept of "**control**" as in **controlled experiment**

There is some truth to this association … but this association also causes some confusion/problem

Conditional Probability "Definition"

$$P(A|B) = \frac{P(A \text{ and } B)}{P(B)}$$ for $P(B) > 0$

$P(A \text{ and } B) = P(A|B)\ P(B)$ (Gateway to Bayes' Rule)

Statistical Independence

- Events A and B are considered (pair-wise) statistically independent if $P(A|B) = P(A)$

  Note: $P(A|B) = P(A)$ implies $P(B|A) = P(B)$ assuming $P(A) > 0$

- Conditional probability is sometimes viewed as a statement concerning how informative the conditioning event is about the unconditioned event

  In this view, what does statistical independence mean?

- Later, we link conditional probability with ***prediction***