Math Methods – Financial Price Analysis

Spring 2023, Mathematics, GR5360

Homework #1

Distributed: 2/11/2023,
Due Date: 2/25/2023.

All problems can be solved or answered using the class handouts.

Problem 1. Using the class handouts related to BlackJack, please, answer the following questions (all settings for standard BlackJack game are considered in the below, same as the one described in handouts).

Following the Basic Blackjack Strategy described in the class handouts, please, choose your most rational action in the following situations:

a. Dealer Upcard is: 6; Your Hand has: A,5 (aka "soft 16"); choose(circle) your action:
   - Hit;
   - Stand;
   - Split;
   - Double.

b. Dealer Upcard is: 10; Your Hand: A,9 (aka "soft 20"); choose(circle) your action:
   - Hit;
   - Stand;
   - Split;
   - Double.

c. Following the Basic BlackJack Strategy, described in the class handouts, irrespectively of what the Dealer Upcard is, which one of these Your Hands is statistically better for you (choose or circle one of the following choices):
   - 9;
   - 10;
   - 11.

d. Against the Dealers Upcard being a "small card", which of the following choices of Your Hand cards is statistically better to have (choose or circle one of the following choices):
   - 9, 10, or 11;
   - 13, 14, 15, 16, or 17.

e. If your Bankroll (the amount of money you came to play BlackJack with) is equal to $5,000 and the True Count is equal to 7, what would be your optimal bet size, if rounded up to a penny?

f. Please, in 4-6 sentences, explain in your own words, how a Player can have an edge against the Dealer in the game of BlackJack if both the Player and the Dealer follow nearly the same rules and share nearly the same outcomes (such as they can both have blackjacks, for example)?

Problem 2. Using the class handouts, your notes, and/or Kelly 1956 article ("KellyBettingFormalism.pdf"), please, derive and write detailed accompanying notes for the optimal fractional bet size for "slightly tampered with" or "slightly unfair" coin toss betting problem. In other words, derive the Kelly-optimal fixed-fraction bet size. Try to use your own words and thoughts.

Problem 3. Following the class notes (handouts), please, spell out explicitly the following Bloomberg tickers for futures (Name/Contract Description; Expiry Month; Expiry Year):

- ESH3;
- CFG3;
- NGJ23.

Problem 4. Following the class notes, within the framework of our course, what does it mean when we say that we would like *to study a financial time series* $p = p(t)$? In other words, what is the basic set of steps that one should be taking to produce inference about a high-frequency financial price series? Give both brief and detailed answers. Try to use your own words and thoughts.

Problem 5. Please, explain, what is the relationship between the "energy spectrum" and the auto-correlation function? Be detailed, try using your own thoughts rather than copying the handouts.

Problem 6. Please, explain the meaning of skewness and kurtosis for any real-life financial price (or NAV) time series as compared to the Gaussian random variable. Please, be detailed.

Problem 7. Please, following the class notes, derive the formula for the probability $p_1(m,n)$ to find the large particle (or a frog) at the position $a \cdot m$, at the time $n \cdot \tau$, for the discrete 1-dimensional Random Walk problem described in the class. Try to write your own explanations.

Problem 8. Using the above solution, please, derive the formulas for $\langle m \rangle$ and $\langle m^2 \rangle$. Justify the intermediate steps with your own explanations.

Problem 9. Please, explain what it means that a financial time series has a:

- Short(-term) memory;
- Long(-term) memory.

Be detailed and offer original explanations and examples.

Problem 10. Following the class notes, derive the formula for the variance ratio $VR(3)$, or more explicitly, $VR(3\tau) = \frac{Var(3\tau)}{3 \cdot Var(\tau)}$ as a function of correlation coefficients $\rho_1, \rho_2, \ldots$.

By analogy, please, derive formulas for $VR(4\tau)$ and $VR(3\tau)$. Accompany your derivations with your own detailed explanations.

Problem 11. Using Matlab, Python, or similar language and the real-life 1-minute data for the ES (approximately, 2.4 million minutes) and FT (approximately, 3.2 million minutes) markets, which are already back-adjusted since inception data for the S&P 500 E-Mini and FTSE-100 futures in the ASCII "Date,Time,Open,High,Low,Close,Volume" (header) comma-separated format, please, measure and plot in lin-lin (linear-linear) and log-log (logarithm with base 10) scales the function $\sigma = \sigma(\tau)$ for $1[\text{min}] \le \tau \le 10{,}000[\text{mins}]$. The data files "ES" and "FT" are uploaded into "Lecture #4" folder on CourseWorks. Using these results and standard regular least squares linear regression output, estimate the algebraic slope ν: $\sigma(\tau) \propto \tau^{\nu}$. Provide it with some of the additional standard outputs of the least squares procedure: the slope ν and R^2 coefficients in regression between $y = \log_{10}(\sigma(\tau))$ and $x = \log_{10}(\tau)$. Please, accompany your findings with detailed notes and code that was used. You can use the handouts charts for comparisons against your results. In a few sentences write down possible inferences from your results and comparisons. Write detailed explanations.