Price Impact Models and Applications

Introduction to Algorithmic Trading

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Plan

Last Week
Applying price impact to risk management.

For this Week
Introduction to Module 3: Measuring Price Impact

(a) Bouchaud's list of four trading biases.
(b) An informal primer on causal graphs.
(c) Introduction to live trading experiments.

Next Week
The Mathematics of Causal Inference (1/2)

Summary of Module 2 (1/6)

Given a trading strategy Q, there are three prices

(a) the unobserved, unperturbed price S,

(b) the observed price $P = S + I(Q)$, and

(c) the transaction price $\tilde{P} = P \pm s$

The OW price impact model is an exponential kernel of past trades.

$$dI_t = -\beta I_t dt + \lambda dQ_t$$

The optimal strategy is best expressed in impact space

$$\forall t \in (0, T) \quad I_t = \frac{1}{2}\left(\alpha_t - \beta^{-1}\alpha_t'\right); \quad I_T = \alpha_T$$

where

$$\alpha_t = \mathbb{E}\left[S_T - S_t \middle| \mathcal{F}_t \right].$$

Summary of Module 2 (2/6)

Liquidity is dynamic
it exhibits time-of-day patterns, as well as stochastic behavior. Trading strategies react strongly to these liquidity fluctations.

Beware of price manipulation
Trading algorithms, especially complicated black-box algorithms, can easily be tricked into thinking there is a price manipulation opportunity.

Round-trip trade manipulation
happens when liquidity increases too fast. For the generalized OW model

$$dI_t = -\beta_t I_t dt + e^{\gamma_t} dQ_t$$

the no-price manipulation condition is

$$2\beta_t + \gamma_t' > 0.$$

Summary of Module 2 (3/6)

Communicate a pre-trade cost model
E.g., under the OW model,

$$TC(Q) = -\frac{\Lambda_T}{2} Q^2; \quad \mathbb{E}\left[Y_T\right] = \frac{\alpha}{2} Q$$

where

$$\Lambda_T = \frac{\sigma}{(2 + \beta T)\text{adv}}.$$

Consider intraday signals for the trading schedule
Under the OW model, an intraday alpha signal α_t changes the order by

$$Q_T(\alpha) = \int_0^T \frac{\text{adv}}{2\sigma} \left(\beta \alpha_t - \alpha_t'\right) dt.$$

Summary of Module 2 (4/6)

Micro-alphas are de-centralized signals.
They lead to tactical deviations form the trading schedule. For the OW model,

$$\mathbb{E}\left[Y_T(Q^r)\right] - \mathbb{E}\left[Y_T(Q^*)\right] = -\mathbb{E}\left[\beta \int_0^T \frac{\mathsf{adv}}{\sigma} \left(\delta I_t\right)^2 dt + \frac{\mathsf{adv}}{2\sigma} \left(\delta I_T\right)^2\right].$$

For multiple orders, update the implied alpha.
The myopic relationship updates the target impact state.

Summary of Module 2 (5/6)

Waelbroeck's backtest algorithm

> *"To make a good assessment of alternative strategies, one may wish to first subtract out the impact of those strategies to then be able to simulate accurately alternative strategies."*

Statistical arbitrage theory
Consider alpha decay in alpha research and trading strategies.

Statistical arbitrage implementation

(a) Given alpha level and decay, implement target impact state.

(b) Invert the map to translate impact into trades.

Summary of Module 2 (6/6)

Accounting P&L over-estimates a position's closing P&L

Price impact matters for liquidity risk management
Price impact causes both position inflation and liquidation costs.

Managing liquidity risk starts before leveraging the position up
Portfolio managers, risk managers and regulators can measure price impact in real time and predict mechanical P&L movements ahead of time.

Module 3: Measuring Price Impact with Causal Inference

What is Causal Inference?

Causal inference extends Bayesian statistics
Causal inference distinguishes between observations and interventions.

(*) If I observe X, what is the expected Y?

$$\mathbb{E}\left[Y|X\right]$$

(*) If I change X, what is the expected Y?

$$\mathbb{E}\left[Y|\,\mathrm{do}(X)\right]$$

A Brief History of Causal Inference

Development of the theory
mostly done in the 90s in Statistics and Computer Science departments (Pearl, Rubin...).

Rubin (2004)

> *"Fortunately, the days of 'statistics can only tell us about associations, and association is not causation' seem to be permanently over." (p. 1)*

Recent focus in machine learning
Yoshua Bengio (2018 Turing award for contributions to deep learning) identified causal machine learning as a key next step to improving model robustness.

Recent Use-Cases in the Technology Industry

The Microsoft Research Summit of 2021
had over a dozen talks within its causal machine learning track, one of its seven science tracks.

> *"This track focuses on emerging causal machine learning technologies and the opportunities for practical impact at the intersection of academia and industry, with contributions from researchers at Microsoft and the broader academic and industrial research communities."*

Netflix Causal Inference and Experimentation Summit 2022

> *"The weeklong conference brought speakers from across the content, product, and member experience teams to learn about methodological developments and applications in estimating causal effects."*

Recommended Reading

References on causal inference

(a) *Causality* by Pearl (2008, link): *not* a finance book, but core to causal inference.

(b) Lopez de Prado (2022, ADIA, link) outlines applications of causal inference to finance.

(c) Gitlin et al. (2022, Uber, link) summarizes the causal machine learning **infrastructure** at Uber.

(d) Netflix (link) has an expansive list of research articles on **applications** of causal inference and causal machine learning.

(e) Microsoft Research Summit 2021, Causal Machine Learning (link) leans heavier on the **statistics** side.

Why Causal Inference in Trading?

Causal Inference has applications to trading
AB testing and live trading experiments have emerged as a crucial tool to complement backtests. For example,

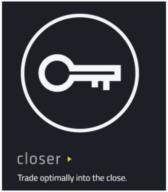
(a) to estimate algorithms' performance in "algo-wheels" and Transaction Cost Analysis (TCA).

(b) to disentangle price impact from alpha.

(c) to remove statistical artifacts and biases due to market microstructure.

Example: `quantitativebrokers.com/whitepapers/2022-03a`

A/B testing of New Closer in Production
Recall QB's *closer* trading algorithm optimizing for the close.



QB upgraded it in late 2020 and reported its live trading performance ("production") in 2022. Their report uses randomize A/B testing to convince clients of the upgrade's merits.

Introduction to Trading Biases

Motivation (1/3)

Transaction Cost Analysis (TCA)

(a) Quarterly, *quantitative* review of trading algorithms.
(b) Mandated by MiFID II for all investment and trading firms since 2018.

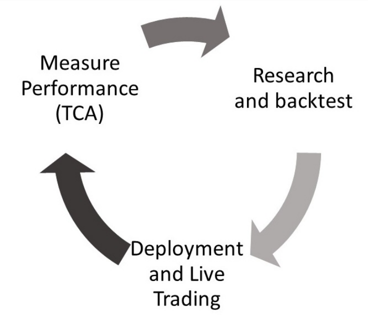


Figure 1: Improvement cycle for trading algorithms.

AB testing for TCA

Deutsche Bank TCA research

"It is important to acknowledge that when we run these analyses, it's done only for a subset of orders that are affected by a controlled A/B experiment with routine changes, so we can really quantify their impact." (Sotiropoulos and Battle, 2017)

Algorithm	Sample size	Average order size	Arrival Slippage
A	18k	5% ADV	-10bps (± 1bps)
B	1k	5% ADV	-40bps (± 5bps)
C	1k	5% ADV	-8bps (± 5bps)

Figure 2: Simple mock TCA report.

Motivation (2/3)

Prediction bias, Bouchaud et al. (CFM, 2018)
Prediction bias refers to the difficulty in establishing whether a trade

(a) *caused* a price move (impact) or

(b) *anticipated* a price move (alpha).

Separating impact from alpha at CFM

> *"The impact of a metaorder $\mathcal{I}(Q, T)$ can be affected by several artifacts and biases.* **One of the recurrent criticism is that metaorders are not exogenous, and possibly conditioned on trading signals.** *[...]*
> *CFM's proprietary data allows one to eliminate many of these biases, since the strength of the trading signal is known and can be factored in the regression." (Bouchaud, CFM 2021)*

Motivation (3/3)

Live trading experiments
are used in addition to backtests to increase trust in models used in live trading.

Testing the anonymity assumption

> *"We have actually shown that the short term impact of CFM's trades are indistinguishable from the trades of the rest of the market, or, for that matter, from* **purely random trades that were studied at CFM during a specifically designed experimental campaign in 2010-2011***" (Bouchaud, CFM 2021)*

Bouchaud's List of Trading Biases

(a) **Prediction bias:**
Trades triggered by or considering an alpha signal exhibit bias when estimating their price impact.

(b) **Synchronization bias:**
"The impact of a metaorder can change according to whether or not other traders are seeking to execute similar metaorders at the same time."

(c) **Implementation bias:**
Tactical deviations from the strategic trading trajectory introduce biases.

(d) **Issuer bias:**
"Another bias may occur if a trader submits several dependent metaorders successively."

Prediction Bias

Scenario

Trading with Alpha
One wants to estimate the parameter λ of an impact model I via a regression

$$\Delta P = \Delta I(Q, \lambda) + \epsilon.$$

However, $\epsilon \not\perp \Delta I(Q, \lambda)$ because trades have alpha.

Better description

$$\Delta P = \Delta I(Q, \lambda) + \alpha + \epsilon'.$$

where the order depends on a (hidden) alpha and $\epsilon' \perp Q, \alpha$. The (hidden) trade formula links trades with their alpha

$$Q = f(\alpha) + \eta$$

where $\eta \perp \alpha$.

Purpose of the graph
Describe (potentially hidden) dependencies between variables in a concise, non-parametric way.

Prediction bias graph

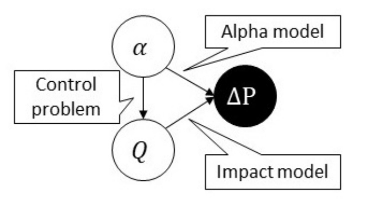


Figure 3: Trades triggered by an alpha signal exhibit bias when estimating their price impact.

Why it Matters For Brokers and TCA

A major hurdle, Bacidore (2020)

> *"Traders are understandably reluctant to pass their alphas over to a broker due to the potential for lost intellectual property, front-running, etc."*

Naively regressing returns against trades over-estimates price impact.
Consequently:

(a) Brokers cite high transaction costs to their clients.

(b) Brokers recommend down-sizing orders and trading slower to minimize the trading costs.

(c) Brokers over-estimate their contribution to P&L.

(d) Clients may lose out on additional alpha they could have captured.

Why it Matters For Alpha Research (1/2)

Clients outsource their trading infrastructure.
They may not have the technology to fit an impact model or may not trust the quality of the data their broker provides. For instance,

(a) Does the broker give fill-level data, or only order-level information?

(b) How precise are the timestamps on the fill-level data? Do they match fills on the public tape?

(c) Is sufficient meta-information, e.g. venue traded, provided with the fill?

(d) Was the fill affected by an internal alpha of the broker? How can the client control for the prediction bias from this unknown trading alpha?

(e) How does the broker decide if an order should cross, not fully fill, or switch algorithm? How can the client control for downstream decisions and interactions with other clients?

Why it Matters For Alpha Research (2/2)

Alpha researchers have three methods to deal with prediction bias.

(a) Use the broker's price impact model to detrend prices.

(b) Use the broker's trading data to co-fit an alpha and a price impact model.

(c) Use randomization to determine alpha, e.g., by randomly not submitting trades and measuring alpha.

Synchronization Bias

Scenario

Let M be correlated market flow
Traders decompose returns into alpha, their impact and the correlated flow's impact.

$$\Delta P = \Delta I(Q, \lambda) + \Delta I(M, \lambda) + \alpha + \epsilon.$$

where the order depends on a (hidden) alpha

$$Q = f(\alpha) + \eta$$

and ϵ, η are idiosyncratic noises.

Two correlation sources

(a) **Crowding:** M and Q may share a common alpha α.

(b) **Leakage:** M may directly react to Q.

Graph Representation

Synchronization bias graph

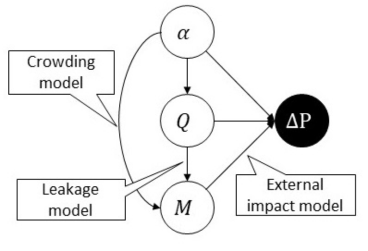


Figure 4: "The impact of a metaorder can change according to whether or not other traders are seeking to execute similar metaorders at the same time."

Why it Matters to Traders

How to deal with crowding?
Mathematically, one solves for a Nash equilibrium in the control problem. The solution

(a) Sends a smaller order.

(b) Trades the order faster.

How to deal with leakage?
Ideally, find the source of the leak and avoid that trading venue, e.g., a poorly designed dark pool. Otherwise, send a smaller, slower order.

Why it Matters to Alpha Researchers

How to deal with crowding?
Ideally, decompose your alpha into an orthogonal and a crowded alpha. The orthogonal alpha can be traded at full size and a slower speed using a standard control problem.

How to deal with leakage?
Leakage leads to feedback loops that are outside the alpha researcher's domain. Avoid at all costs and leave it to the traders.

Implementation Bias

Scenario

Distinguishing between intended and realized trades
An algorithm submits a trading schedule Q based on a signal α. A micro-alpha α^{LL} leads to realized trades Q^r. Which regression estimates price impact?

$$\Delta P = \Delta I(Q, \lambda) + \alpha + \epsilon,$$

$$\Delta P = \Delta I(Q^r, \lambda) + \alpha^{LL} + \epsilon,$$

or

$$\Delta P = \Delta I(Q^r, \lambda) + \alpha + \alpha^{LL} + \epsilon?$$

Graph Representation

Implementation bias graph

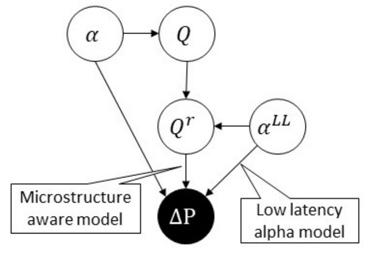


Figure 5: Tactical deviations from the strategic trading trajectory introduce biases.

Why it Matters

If the downstream micro-strategies are frozen then implementation bias does not matter.
Assuming nothing changes downstream, the implementation bias remains the same and can implicitly be baked into the price impact model.

But algo-traders continuously improve their micro-alphas.
Two solutions:

(a) Refit the price impact model after every new release. Batch releases.

(b) Build a causal model that allows for incremental downstream changes. Continuously monitor and update the models.

Issuer Bias

Scenario

Sequential trading
Consider two successive orders Q^1, Q^2 with alphas α^1, α^2. To decompose the second order's returns ΔP^2, should one regress

$$\Delta P = \Delta I(Q^2, \lambda) + \alpha^2 + \epsilon$$

or

$$\Delta P = \Delta I(Q^2, \lambda) + \Delta I(Q^1, \lambda) + \alpha^1 + \alpha^2 + \epsilon?$$

Broker scenario
What if the alphas are correlated and hidden?

Graph Representation

Issuer bias graph

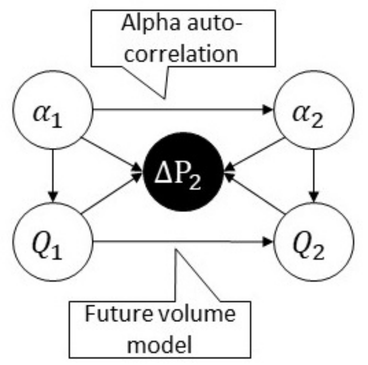


Figure 6: "Another bias may occur if a trader submits several dependent metaorders successively."

Why it Matters

How to trade follow-up orders?
We know the answer once the price impact model is given: trading is myopic in impact space. However, this only moved the goal post: now we must disentangle two consecutive orders' price impact during the fitting process.

General Causal Inference Goals and Template

Causal Inference Goals

Document dependencies in a graph
There are two types of dependencies to document for stakeholders.

(a) Model assumptions. E.g., our trades do not leak.

(b) Internal assumptions. E.g., our trades depend on the new alpha signal.

Articulate counterfactuals
Traders also call counterfactuals "what-if" scenarios. E.g.,

(a) What would the price have been if I hadn't traded? (alpha)

(b) What would the arrival slippage had been with algo B? (algo wheel)

(c) What would the market have traded if I hadn't trade? (leakage)

Identify and rectify trading biases
Many regressions are ill-posed due to trading biases. Causal inference identifies bias-free formulas.

Weekly Summary

(a) **Prediction bias:**
Trades triggered by or considering an alpha signal exhibit bias when estimating their price impact.

(b) **Synchronization bias:**
"The impact of a metaorder can change according to whether or not other traders are seeking to execute similar metaorders at the same time."

(c) **Implementation bias:**
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(d) **Issuer bias:**
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Questions?

Next week
The Mathematics of Causal Inference (1/2)

(a) Causal graphs
(b) *d*-separation criterion
(c) do() operator