STAT GU4261/GR5261 - Statistical Methods in Finance - Homework #8 Solutions

4/9/23

Question 1

Part (a)

$$\begin{aligned}
\alpha &= P(L > VaR(\alpha)) \\
&= P(-S_0 R > VaR(\alpha)) \\
&= P(R < -VaR(\alpha)/S_0) \\
&= P(\mu + \lambda Y < -VaR(\alpha)/S_0) \\
&= P(Y < -\frac{1}{\lambda}(VaR(\alpha)/S_0 + \mu))
\end{aligned}$$

Since $Y \sim t_\nu$, it follows that

$$-\frac{1}{\lambda}(VaR(\alpha)/S_0 + \mu) = t_{\alpha,\nu}.$$

Rearranging this gives $VaR(\alpha) = -S_0(\mu + \lambda t_{\alpha,\nu})$.

Part (b)

Plugging in the values we get $VaR(0.05) = 14124.6$.

Question 2

Part (a)

We have the formula

$$VaR(0.05) = -S_0(\mu_A + \Phi^{-1}(0.05)\sigma_A).$$

Now $\mu_A = 0.0002$ and $\sigma_A = \sqrt{0.0003}$ and $S_0 = 1000$. Thus, $VaR(0.05) = 28.29$.

Part (b)

Here $\mu_B = 0.0003$ and $\sigma_B = \sqrt{0.0004}$. Thus, $VaR(0.05) = 32.5$.

Part (c)

The returns are normally distributed. The mean is $\mu = 0.5(0.0002 + 0.0003) = 0.00025$. The variance is $\sigma^2 = 0.25(0.0003 + 0.0004 + 2 \cdot 0.0002) = 0.000275$. Thus, $VaR(0.05) = 27.03$.

Question 3

Firstly, $P(L \geq VaR(\alpha)) = \alpha$ that is $P(R \leq -VaR(\alpha)/S_0) = \alpha$. So, $VaR(\alpha) = -S_0 q_\alpha$. Then,

$$ES(\alpha) = E[L | L \geq VaR(\alpha)] = -S_0 E[R | R \leq q_\alpha].$$

By conditional probability,

$$E[R | R \leq q_\alpha] = \frac{1}{P(R \leq q_\alpha)} \int_{-\infty}^{q_\alpha} r f(r) dr = \int_{-\infty}^{q_\alpha} r f(r) dr / \alpha.$$

Therefore, $ES(\alpha) = -S_0 \int_{-\infty}^{q_\alpha} r f(r) dr / \alpha$.

Question 4

From Question 3, $ES(\alpha) = -S_0 \int_{-\infty}^{q_\alpha} r f(r) dr / \alpha$. To calculate $\int_{-\infty}^{q_\alpha} r f(r) dr$, make the substitution $u = (r - \mu)/\sigma$:

$$\int_{-\infty}^{q_\alpha} r f(r) dr = \int_{-\infty}^{z_\alpha} (\mu + \sigma u) \frac{e^{-u^2/2}}{\sqrt{2\pi}} du = \mu \Phi(z_\alpha) + \frac{\sigma}{\sqrt{2\pi}} \left[-e^{-u^2/2} \right] \Big|_{u=-\infty}^{z_\alpha} = \mu\alpha - \frac{\sigma}{\sqrt{2\pi}} e^{-z_\alpha^2/2}.$$

Thus, $ES(\alpha) = -S_0 \Big(\mu - \frac{\sigma}{\alpha\sqrt{2\pi}} e^{-z_\alpha^2/2} \Big)$.