Homework #3

Due **Wednesday, February 23** in Gradescope by **11:59 pm ET**

READ Textbook Section 1.2.1–1.2.3

WRITE AND SUBMIT solutions to the following problems.

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1. (8 points) Suppose G is a graph that has 10 edges and 6 vertices, and suppose that the degrees of five of those vertices are $2, 2, 3, 4, 4$, and the sixth has some degree n.

(a) Find the integer n, i.e., the degree of the sixth vertex.

(b) Is G connected? (Yes, no, or maybe?) If "yes" or "no", prove it; if "maybe", draw two examples of such a graph G: one that is connected and one that is not.

2. (15 points) For each of the graphs P_5, C_5, and K_5:

(a) draw the graph

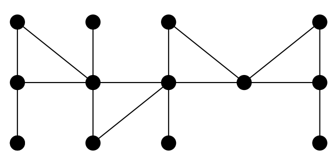
(b) find the eccentricity of each vertex

(c) find the radius and diameter of the graph

(d) find its adjacency matrix.

(For P_5, number the vertices 1 to 5 from one end to the other; for C_5, label them consecutively around the cycle.)

3. (8 points) Textbook, Section 1.2.1, Problem 1: Find the radius, diameter, and center of the following graph:



4. (10 points) Textbook, Section 1.2.1, Problem 5:
Let G be a graph, and let $u, v \in V(G)$ be adjacent vertices. Prove that their eccentricities $\mathrm{ecc}(u)$ and $\mathrm{ecc}(v)$ differ by at most 1.

5. (12 points) Textbook, Section 1.2.1, Problem 8(a,b,c):

(a) Draw a graph of order 7 that has radius 3 and diameter 6.

(b) Draw a graph of order 7 that has radius 3 and diameter 5.

(c) Draw a graph of order 7 that has radius 3 and diameter 4.

In all three cases, don't forget to (briefly) justify that your graph has the correct order, radius, and diameter.

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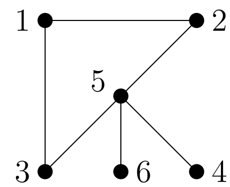
6. (18 points) Let G be the following graph:



(a) Find the adjacency matrix A of G.

(b) Find all the walks of length 3 from vertex 1 to vertex 4. What is the total number of such walks, and (without computing A^3) what does this say about the matrix A^3?

(c) How many closed walks of length 3 are there in G? Without computing A^3, how would this number be related to the matrix A^3?

(d) Find the eccentricities of all the vertices of G.

7. (10 points) Textbook, Section 1.2.2, Problem 3:
Let G be a graph with $V(G) = \{v_1, \ldots, v_n\}$ and with adjacency matrix A. For each $j = 1, \ldots, n$, prove that the (j, j) entry of A^2 is $\deg(v_j)$.

8. (15 points) Let $A = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}$, and let G be the graph with adjacency matrix A.

(a) Compute A^2 and A^3.

(b) How many walks are there in G from vertex 1 to vertex 2 of length exactly 3?

(c) Find the radius and the diameter of G.

(d) Draw the graph G.

Optional Challenges (do NOT hand in):
Textbook Section 1.2.1, Problems 8(d), 10, 11; Section 1.2.2, Problems 4, 5

Questions? You can ask in:

Class: MWF 11:00–11:50am, SMUD 205
Tu 9:00–9:50am, SMUD 205

My office hours: Mon 2:30–3:30pm, Tue 2–3:30pm, and Thu, 1–2:30pm, SMUD 406

Anna's Math Fellow office hours:
Sundays, 7:30–9:00pm, and Tuesdays, 6:00–7:30pm, SMUD 007

Also, you may email me any time at `rlbenedetto@amherst.edu`