Homework #6

Due **Wednesday, March 30** in Gradescope by **11:59 pm ET**

READ Textbook Sections 1.3.4 and 1.4.2, and start 1.4.3

WRITE AND SUBMIT solutions to the following problems.

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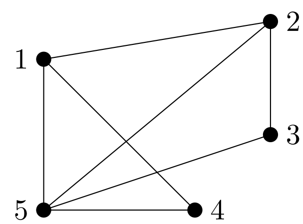
1. (24 points) Textbook Section 1.3.4, Problem 5 (expanded a bit):

(a) Use Prüfer's method to draw and label the trees with Prüfer sequences 1,2,3 and 3,4,1,2.

(b) Inspired by your answers in part (a), make a conjecture about which trees have Prüfer sequences consisting of all distinct terms.

(c) Prove your conjecture from part (b).

2. (8 points) Use the Matrix Tree Theorem to find the number of spanning trees of this graph:



3. (10 points) Let G be a graph with Laplacian matrix Δ. Prove that $\det(\Delta) = 0$.

(**Suggestion**: Remember that invertible matrices have trivial nullspace, and that nonzero determinant implies the matrix is invertible.)

4. (10 points) Textbook Section 1.4.2, Problem 7(b):
Determine the values of $m, n \geq 1$ such that the complete bipartite graph $K_{m,n}$ is Eulerian. Prove your answer.

5. (14 points) Textbook Section 1.4.2, Problem 7(a):
Determine the precise set of values of $m, n \geq 1$ such that the complete bipartite graph $K_{m,n}$ has an Eulerian trail. Prove your answer.

6. (16 points) Textbook Section 1.4.2, Problem 1:
For each of the following, draw an Eulerian graph the satisfies the conditions, or prove that no such graph exists.

(a) An even number of vertices, and an even number of edges.

(b) An even number of vertices, and an odd number of edges.

(c) An odd number of vertices, and an even number of edges.

(d) An odd number of vertices, and an odd number of edges.

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7. (8 points) For the graph $G = K_5$, determine:

   (a) is it Eulerian? (b) is it Hamiltonian?

   (c) is it traceable? (d) what is its independence number $\alpha(G)$?

As always, be sure to (briefly) justify your answers.

8. (10 points) For the graph $G = P_7$, determine:

   (a) is it Eulerian? (b) is it Hamiltonian?

   (c) is it traceable? (d) what is its independence number $\alpha(G)$?

As always, be sure to (briefly) justify your answers.

Optional Challenges (do NOT hand in): Textbook Section 1.4.2, Problem 5.

Questions? You can ask in:

Class: MWF 11:00–11:50am, SMUD 205
Tu 9:00–9:50am, SMUD 205

My office hours: Mon 2:30–3:30pm, Tue 2–3:30pm, and Thu, 1–2:30pm, SMUD 406

Anna's Math Fellow office hours:
Sundays, 7:30–9:00pm, and Tuesdays, 6:00–7:30pm,
SMUD 007

Also, you may email me any time at `rlbenedetto@amherst.edu`