STAT GU4261/GR5261 STATISTICAL METHODS IN FINANCE

HOMEWORK 3 SUGGESTED SOLUTIONS

Page 491: Exercise 16.11.1

(a) Solving $0.023w + 0.045(1 - w) = 0.03$ gives $w = 15/22$, i.e. invest $15/22$ amount of money in A and the rest in B.

(b) Solving $6w^2 + 11(1-w)^2 + 2w(1-w)(\sqrt{6})(\sqrt{11})(0.17) = 5.5$ gives $w = 0.94$ (i.e. invest 0.94 amount of money in A and the rest in B) or $w = 0.41$ (i.e. invest 0.41 amount of money in A and the rest in B). The expected return is largest when $w = 0.41$.

Page 492: Exercise 16.11.2

As $r_p = (1-w)r_f + wr_T$, we have $\sigma_p^2 = w^2\sigma_T^2$, and thus $w = \pm\frac{5}{7}$. Hence, the weights of risk-free asset, asset C and asset D in the portfolio are

$$\left(\frac{2}{7}, \frac{5}{7} \times 0.65, \frac{5}{7} \times 0.35\right) = \left(\frac{2}{7}, \frac{13}{28}, \frac{1}{4}\right) \quad \text{or} \quad \left(\frac{12}{7}, -\frac{5}{7} \times 0.65, -\frac{5}{7} \times 0.35\right) = \left(\frac{12}{7}, -\frac{13}{28}, -\frac{1}{4}\right).$$

Problem 2

(a)

$$\mu_P(cx, cy) = cx\mu_A + cy\mu_B = c(x\mu_A + y\mu_B) = c\mu_P(x, y).$$

$$\begin{aligned}
\sigma_P(cx, cy) &= \sqrt{c^2x^2\sigma_A^2 + c^2y^2\sigma_B^2 + 2c^2xy\sigma_{AB}} \\
&= \sqrt{c^2(x^2\sigma_A^2 + y^2\sigma_B^2 + 2xy\sigma_{AB})} \\
&= c\sqrt{x^2\sigma_A^2 + y^2\sigma_B^2 + 2xy\sigma_{AB}} \\
&= c\sigma_P(x, y).
\end{aligned}$$

(b) Marginal contribution to risk of asset A:

$$\frac{\partial}{\partial x}\sigma_P(x, y) = \frac{1}{2\sqrt{x^2\sigma_A^2 + y^2\sigma_B^2 + 2xy\sigma_{AB}}} \cdot (2x\sigma_A^2 + 2y\sigma_{AB}) = \frac{x\sigma_A^2 + y\sigma_{AB}}{\sqrt{x^2\sigma_A^2 + y^2\sigma_B^2 + 2xy\sigma_{AB}}}.$$

Marginal contribution to risk of asset B:

$$\frac{\partial}{\partial y}\sigma_P(x, y) = \frac{y\sigma_B^2 + x\sigma_{AB}}{\sqrt{x^2\sigma_A^2 + y^2\sigma_B^2 + 2xy\sigma_{AB}}}.$$

Contribution to risk of asset A:

$$x\frac{\partial}{\partial x}\sigma_P(x, y) = \frac{x^2\sigma_A^2 + xy\sigma_{AB}}{\sqrt{x^2\sigma_A^2 + y^2\sigma_B^2 + 2xy\sigma_{AB}}}.$$

Contribution to risk of asset B:

$$y\frac{\partial}{\partial y}\sigma_P(x, y) = \frac{y^2\sigma_B^2 + xy\sigma_{AB}}{\sqrt{x^2\sigma_A^2 + y^2\sigma_B^2 + 2xy\sigma_{AB}}}.$$

Problem 3

a) Let

$$L(w_1, w_2, \lambda) = w_1^2\sigma_B^2 + w_2^2\sigma_M^2 + 2w_1w_2\sigma_B\sigma_M\rho_{BM} - \lambda(w_1 + w_2 - 1).$$

Differentiate L with respect to w_1, w_2 and λ and set the derivatives to 0, we have

$$\frac{\partial L}{\partial w_1} = 2w_1\sigma_B^2 + 2w_2\sigma_B\sigma_B\rho_{BM} - \lambda = 0,$$
$$\frac{\partial L}{\partial w_2} = 2w_2\sigma_M^2 + 2w_1\sigma_B\sigma_M\rho_{BM} - \lambda = 0,$$
$$\frac{\partial L}{\partial \lambda} = w_1 + w_2 - 1 = 0.$$

Subtract the second equation from the first equation, we have

$$w_1\sigma_B^2 - w_2\sigma_M^2 + w_2\sigma_B\sigma_M\rho_{BM} - w_1\sigma_B\sigma_M\rho_{BM} = 0.$$

Substitute $w_2 = 1 - w_1$ into the above equation, we have

$$w_1\sigma_B^2 - \sigma_M^2 + w_1\sigma_M^2 + (1 - 2w_1)\sigma_B\sigma_M\rho_{BM} = 0,$$

which gives

$$w_1 = \frac{\sigma_M^2 - \sigma_B\sigma_M\rho_{BM}}{\sigma_B^2 + \sigma_M^2 - 2\sigma_B\sigma_M\rho_{BM}} = 0.662.$$

The minimum variance portfolio is to invest 0.662 amount of money in B and 0.338 in M.
Remark: the λ term in L could be added or subtracted, it does not matter.

b) Tangency portfolio:

$V_1 = \mu_1 - \mu_f = .1492 - .06 = .0892$

$V_2 = \mu_2 - \mu_f = .3308 - .06 = .2708$

0.3968

$$w_T = \frac{V_1\sigma_M^2 - V_2\rho_{12}\sigma_1\sigma_2}{V_1\sigma_M^2 + V_2\sigma_B^2 - (V_1 + V_2)\rho_{BM}\sigma_B\sigma_M} \simeq$$

$\Rightarrow$ Tangency portfolio = (.0382, .9618)

That is $R_T = \cancel{.0382}\ R_B + \cancel{.9618}\ R_M$

0.3968 0.6032

$$\mu_T = E(R_T) = .\overset{0.3968}{\cancel{0382}}\,\mu_B + \overset{0.6032}{\cancel{0.618}}\,\mu_M$$

$$= \cancel{0.2239} \quad \boxed{0.2587}$$

$$\sigma_T = \sqrt{w_T^2\sigma_B^2 + 2w_T(1-w_T)\rho_{BM}\sigma_B\sigma_M + (1-w_T)^2\sigma_M^2}$$

$$\approx \cancel{0.2557} \quad \boxed{0.2457}$$

c) $.20 = w\,\mu_B + (1-w)\,\mu_M$

$$= w(.1492) + (1-w)(.3308)$$

$$\Rightarrow \quad w = \frac{.20 - .3308}{0.1492 - .3308} \approx 0.7203.$$

$$\Rightarrow \quad R_p = .7203\,R_B + .2797$$

$$\sigma_p = \sqrt{w^2\sigma_B^2 + 2w(1-w)\rho_{BM}\sigma_B\sigma_m + (1-w)^2\sigma_m^2}$$

$$\cancel{= 0.0444} \quad \boxed{= 0.2155}$$

d) $R_p = (1-w)\mu_f + w R_T$

$$\Rightarrow \quad \sigma_p = w\,\sigma_T \quad \Rightarrow \quad w = \frac{\sigma_p}{\sigma_T} = \frac{.20}{\cancel{.4444}\ 0.2457} = \cancel{0.5620}\ \boxed{0.814052}$$

$$\cancel{\Rightarrow R_p = .4380\,r_f + .5620\,R_T}$$

$$\cancel{\mu_p = .438(.06) + .5620(.3277) = .20\ldots3\%}$$

$$\Rightarrow \mu_p = 0.814052(0.2587) + (1-0.814)(0.06)$$

$$\boxed{= 0.22178}$$