# STAT GU4261/GR5261 - Statistical Methods in Finance - Homework #4 Solutions

February 19th, 2023

## Question 1 pp. 488

### Problem 1

```r
stock_bond <- read.csv("Stock_Bond.csv")
dat <- read.csv("Stock_Bond.csv", header = TRUE)
prices <- cbind(
    dat$GM_AC, dat$F_AC, dat$CAT_AC, dat$UTX_AC, dat$MRK_AC, dat$IBM_AC
)
n <- dim(prices)[1]
num_assets <- dim(prices)[2]
returns <- 100 * (prices[2:n, ] / prices[1:(n - 1), ] - 1)
mean_vect <- colMeans(returns)
cov_mat <- cov(returns)
sd_vect <- sqrt(diag(cov_mat))
library(quadprog)
Amat <- cbind(
    rep(1, num_assets), mean_vect, diag(num_assets), -diag(num_assets)
)
muP <- seq(
    min(mean_vect) + 0.0001, max(mean_vect) - 0.0001, length = 300
)
sdP <- muP
weights <- matrix(0, nrow = 300, ncol = num_assets)
for (i in seq_along(muP)){
    bvec <- c(
        1, muP[i], rep(-0.1, num_assets), rep(-0.5, num_assets)
    )
    result  <- solve.QP(
        Dmat = 2 * cov_mat, dvec = rep(0, num_assets), Amat = Amat,
        bvec = bvec,
        meq = 2
    )
    sdP[i] <- sqrt(result$value)
    weights[i, ] <- result$solution
```

```
32 }
33 mufree <- 3 / 365
34 sharpe <- (muP - mufree) / sdP
35 ind <- (sharpe == max(sharpe))
36 ind2 <- (sdP == min(sdP))
37 ind3 <- (muP > muP[ind2])
38 library(ggplot2)
39 plot_df <- data.frame(sdP = sdP, muP = muP)
40 plot_eff_df <- data.frame(sdP = sdP[ind3], muP = muP[ind3])
41 mean_df <- data.frame(
42     label = c("GM", "F", "CAT", "UTX", "MRK", "IBM"),
43     x = sd_vect,
44     y = mean_vect
45 )
46 ggplot(plot_df, aes(sdP, muP)) +
47     geom_line(orientation = "y", linetype = 2) +
48     xlim(0, 2.5) +
49     ylim(0, 0.1) +
50     geom_point(aes(x = 0, y = mufree), shape = 8, size = 4) +
51     geom_abline(
52         slope = (muP[ind] - mufree) / sdP[ind], intercept = mufree,
53         color = "blue",
54         linewidth = 2
55     ) +
56     geom_line(
57         data = plot_eff_df, aes(x = sdP, y = muP), colour = "red",
58         linewidth = 1.2
59     ) +
60     geom_point(aes(x = sdP[ind], y = muP[ind]), shape = 8, size = 4) +
61     geom_point(aes(x = sdP[ind2], y = muP[ind2]), shape = 17, size = 4) +
62     geom_text(
63         data = mean_df, aes(x = x, y = y, label = label), colour = "black"
64     ) +
65     ggtitle("Reward-risk Space") +
66     theme_bw()
```

Lines 1 to 37 are adapted from the textbook to solve this problem. Line 38 onwards uses package `ggplot2` - one could also follow the code given on Page 487 to produce the same figure.

Weights of the tangency portfolio is (-0.0921, -0.0032, 0.3364, 0.3845, 0.3196, 0.0548) and weights of the minimum variance portfolio is (0.0831, 0.0578, 0.1285, 0.2351, 0.296, 0.1995).

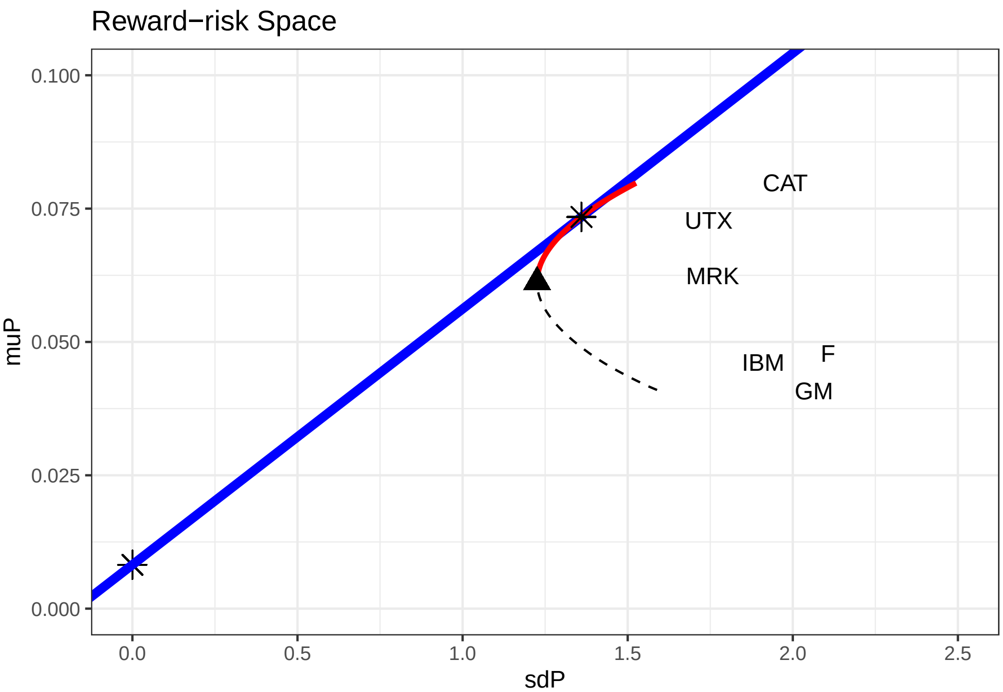


Figure 1: Efficient frontier (solid), line of efficient portfolios (dashed) connecting the risk-free asset and tangency portfolio (asterisks), and the minimum variance portfolio (plus) with three stocks (GM, F, CAT, UTX, MRK, and IBM). The six stocks are also shown on reward-risk space

**Problem 2**

Solve for $w$ in $0.07 = E(R_p) = wE(R_T) + (1-w)\mu_F$, where $E(R_T) = 0.07344$ and $\mu_F = 0.0082192$. This yields $w = \frac{0.07-\mu_F}{E(R_T)-\mu_F} = 0.9473$. Hence the proportion of capital to invest in stocks is (-0.0872, -0.0031, 0.3186, 0.3642, 0.3027, 0.0519).

**Problem 3**

Yes.

## Question 2

$$\begin{aligned}
0.01 &= P(R_p < -0.2) \\
&= P(wR_A + (1-w)0.05 < -0.2) \\
&= P(\mathcal{N}(0.12w + (1-w)0.05, 0.25^2 w^2) < -0.2) \\
&= \Phi\left(\frac{-0.2 - (0.05 + 0.07w)}{0.25w}\right)
\end{aligned}$$

Hence,

$$\left(\frac{-0.2 - (0.05 + 0.07w)}{0.25w}\right) = \Phi^{-1}(0.01) = -2.326,$$

so that $w = 0.4888$.

## Question 3

Risk here is defined as the standard deviation of the portfolio's return.

### (a)

$$\begin{aligned}
w_{\text{MVP}} &= \frac{\Omega^{-1}\mathbf{1}}{\mathbf{1}^T\Omega^{-1}\mathbf{1}} = (0.441, 0.366, 0.193)^T, \\
\mu_{\text{MVP}} &= w_{\text{MVP}}^T\mu = 0.02489, \\
\sigma^2_{\text{MVP}} &= w_{\text{MVP}}\Omega w_{\text{MVP}} = 0.005282.
\end{aligned}$$

**(b)**

The required weight is given by $w^* = \theta w_1 + (1-\theta) w_2$, where

$$\begin{aligned} w_1 &= \frac{\Omega^{-1}\mathbf{1}}{\mathbf{1}^T\Omega^{-1}\mathbf{1}}, \\ w_2 &= \frac{\Omega^{-1}\mu}{\mathbf{1}^T\Omega^{-1}\mu}, \\ \theta &= \frac{\mu_p - w_2^T\mu}{w_1^T\mu - w_2^T\mu}. \end{aligned}$$

Hence, $w^* = (0.828, -0.091, 0.263)^T$ and $\sigma^* = \sqrt{(w^*)^T\Sigma w^*} = 0.09166$.

**(c)**

$$w_T = \frac{\Omega^{-1}(\mu - \mu_f\mathbf{1})}{\mathbf{1}^T\Omega^{-1}(\mu - \mu_f\mathbf{1})} = (0.9093, -0.1873, 0.2780)^T.$$

**(d)**

First, $\mu_T = w_T^T\mu = 0.046469$. Then we solve $w$ from $0.0427 = w\mu_T + (1-w)\mu_f$. Hence, $w = 0.9188$. The risk is 0.09125034.