

**Homework #1**Due **Friday, February 11** in Gradescope by **11:59 pm ET****READ** Textbook Sections 1.1.1 and 1.1.2 and start 1.1.3**WATCH** Videos 1–8**WRITE AND SUBMIT** solutions to the following problems.

1. (8 points) Textbook, Section 1.1.2, Problem 1:

If  $G$  is a graph of order  $n$ , what is the maximum possible size of  $G$ ? (That is, what's the maximum possible number of edges  $G$  could have?)

**Don't forget to justify your answer!** You don't need to give a formal proof on this more computational problem, but you need to explain why you know your answers are correct.

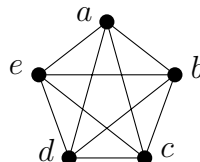
2. (12 points) Textbook, Section 1.1.2, Problem 2:

Let  $G$  be a graph of order  $n \geq 2$ . Prove that the degree sequence of  $G$  has at least one pair of repeated entries.

(*Suggestion:* What degrees are possible in such a graph  $G$ ? And use the pigeonhole principle.)

3. (15 points) Textbook, Section 1.1.2, Problem 3(c,d):

For this graph:



(c) What is the maximum length of a circuit in this graph? Give an example of such a circuit.

(d) What is the maximum length of a circuit that does not include vertex  $c$ ? Give an example of such a circuit.

**Don't forget to justify your answers!** You don't need to give a formal proof on this more computational problem, but you need to explain why you know your answers are correct.

4. (7 points) Let  $G$  be a graph of odd order. Suppose that all the vertices of  $G$  have the same degree  $r$ . Prove that  $r$  is an even number.

5. (12 points) Textbook, Section 1.1.2, Problem 6:

Prove that every closed walk of odd length in a graph contains a cycle of odd length.

6. (8 points) Let  $G$  be a graph of order  $n$  and size  $t$ . Let  $\overline{G}$  be the complement graph of  $G$ . (See the textbook, Section 1.1.3, item 3.) Find the order and size of  $\overline{G}$ .

**Don't forget to justify your answers!**

**Optional Challenges (do NOT hand in):** Textbook Section 1.1.2, Problems 5, 7, 8

**Questions?** You can ask in:

**Class:** MWF 11:00–11:50am; Tu 9:00–9:50am, on zoom this week.

**My office hours:** Mon 2:30–3:30pm, Tue 2–3:30pm, and Thu, 1–2:30pm, on zoom this week.

(Next week: Math Fellow office hours start, too!)

Also, you may email me any time at [rlbenedetto@amherst.edu](mailto:rlbenedetto@amherst.edu)