**Introduction to the Mathematics of Finance. HOMEWORK 3. Due November 28, 2022, 11.59pm**

Please write a pledge that the homework solutions represent your own work and that you did not copy solutions from the work of other students.

1. Suppose that the price $X_t$ of Euro in terms of USD follows $d X_t = 0.02 X_t dt + 0.08 X_t dW_t$
Write the equation for $Y_t$ the price of USD in terms of Euro.

2. Suppose that $X_t$ follows the process $dX_t = 0.1 dt + 0.18 dW_t$. Using Ito's Lemma find the equation for the
process for $Y_t = 4 \sin(X_t)$.

3. Suppose that $X_t$ follows the process $dX_t = 0.05 X_t dt + 0.20 X_t dW_t$. Using Ito's Lemma find the equation for the process for $Y_t = (X_t)^n$ for all integers n.

4. Suppose that $X_t$ follows the process $dX_t = 0.02 X_t dt + 0.25 X_t dW_t$. Using Ito's Lemma find the equation for the process for $Y_t = \ln X_t$.

5. A stock price is $20 now. In 1 month it can go to $22 or $18. The annual interest rate is 11% with continuous compounding. Using risk-free portfolios, determine the value of the one-month European put with strike price 20 and with strike price 19.

6. Use risk-neutral valuation to calculate the probabilities that will give you the correct put prices in problem 5.

7. Construct trading strategies in stock only that replicate each of the two puts of problem 5. That means construct a) synthetic long put strategy with strike price 20. b) synthetic long put strategy with strike price 19
What is the cost of each synthetic trading strategy.

8. A stock price is $50 now. In one month it can go 10% up or down. In the second month it can go 10% up or down. The annual interest rate is 11% with continuous compounding. Use risk-free portfolios to determine the value of the a
a)Two-month European call with strike price 50.
b)Two-month European call with strike price 51

9.Use risk-neutral valuation to calculate the probabilities in the model that will give you the correct call prices in the previous problem parts a) and b).

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10. Create a spreadsheet modeling trajectories of geometric Brownian motion starting at 100 with growth rate 4 percent (it is also risk free rate) and volatility 15 percent. Make a spreadsheet that calculates European calls maturing in 1 year with strikes 100 and 99 on non-dividend paying stock using Monte-Carlo method and using 20,000 trajectories with 250 steps in each trajectory. Compare Monte-Carlo price with 20,000 trajectories to theoretical model price. Calculate with 40,000 trajectories. Compare Monte-Carlo price with 40,000 trajectories to theoretical model price. Calculate with 60,000 trajectories. Compare Monte-Carlo price with 60,000 trajectories to theoretical model price.

11.Create Matlab code doing the same thing as problem 10. Compare Matlab and spreadsheet results.