Math 280, Spring 2022 Professor Rob Benedetto

Homework #4

Due **Wednesday, March 2** in Gradescope by **11:59 pm ET**

READ Textbook Sections 1.3.1–1.3.3 and start 1.3.4

WRITE AND SUBMIT solutions to the following problems.

---

1. (20 points) Textbook Section 1.3.1, Problem 1:
Draw all unlabeled trees of order 7.
(More precisely: Find a set of trees of order 7 so that *every* tree of order 7 is isomorphic to one in your set, and so that no two in your set are isomorphic to each other.)

(*Hint*: there are 11 of them. Careful not to draw the same one twice in a different way! You don't need to give a formal proof that your set is complete; just draw 11 truly different trees of order 7. Make sure to draw **clearly**; unclear graphs will be marked wrong.)

2. (6 points) Textbook Section 1.3.1, Problem 3:
Let T be a tree of order $n \geq 2$. Prove that T is bipartite.

(*Hint*: Do we know any theorems about when a graph is bipartite?)

3. (6 points) Textbook Section 1.3.2, Problem 2:
Let T be a tree that has an even number of edges. Prove that at least one vertex of T has even degree.

4. (18 points) Textbook Section 1.3.2, (part of) Problem 5:
Let T be a tree, and let $u, v \in V(T)$. Prove that there is *exactly one* path from u to v.

5. (14 points) Textbook Section 1.3.2, (part of) Problem 6:
Let T be a tree, and let $u, v \in V(T)$ be two distinct vertices that are *not* adjacent. Define a new graph G with the same vertex set $V(G) = V(T)$ but with one extra edge $e = uv$. That is, $E(G) = E(T) \cup \{e\}$, where the new edge e runs between u and v.
Prove that the new graph G has exactly one cycle.

(*Suggestion*: Use the result of the previous problem.)

6. (10 points) Let T be a tree of order $n \geq 2$, and suppose that none of the vertices of T have degree 2. Prove that T has more than $n/2$ leaves.

7. (14 points) Textbook Section 1.3.3, Problem 1:
Let G be a connected graph. Prove that G contains at least one spanning tree.

(*Suggestion*: for any subtree T that is missing at least one vertex, show that there is a larger subtree T' of G that contains all of T and one more vertex.)

Optional Challenges (do NOT hand in):
Textbook Section 1.3.2, Problems 10, 12

Questions? You can ask in:

Class: MWF 11:00–11:50am, SMUD 205
Tu 9:00–9:50am, SMUD 205

My office hours: Mon 2:30–3:30pm, Tue 2–3:30pm, and Thu, 1–2:30pm,
SMUD 406

Anna's Math Fellow office hours:
Sundays, 7:30–9:00pm, and Tuesdays, 6:00–7:30pm,
SMUD 007

Also, you may email me any time at `rlbenedetto@amherst.edu`