ECON 111: Fall 2021
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Today, we'll consider a review problem on markets, elasticity, surplus and taxation.

1. Consider the following market supply and demand curves:

$$P = 15 + Q_s$$
$$P = 60 - 2Q_d$$

   (a) Sketch the supply and demand curves, labeling the axes. Compute the equilibrium price and quantity.

   (b) Compute consumer and producer surplus at the market equilibrium.

   (c) Compute the price elasticity of demand and price elasticity of supply at the equilibrium point.

   (d) Let us suppose that you as a policymaker are interested in designing a tax that would result in a market quantity of 10 units. What tax should you impose to achieve this?

   (e) For the tax that you found in part (d), what are the deadweight loss and revenue associated with the tax?

   (f) For the tax that you found in part (d), what is the incidence of the tax on each side of the market? Check this with your solution for part (c).

   (g) Suppose that instead of targeting a particular quantity, you were only interested in obtaining a specific amount of revenue from the tax. In particular, suppose that you wanted to achieve a revenue of $\$\frac{200}{3}$ from the tax. What taxes achieve this goal *exactly*. Is there one that achieves the goal with less deadweight loss?

   (h) For part (g), did it matter which side of the market you imposed the tax on?

1. (a) The following figure shows a sketch of the market supply and demand curves:

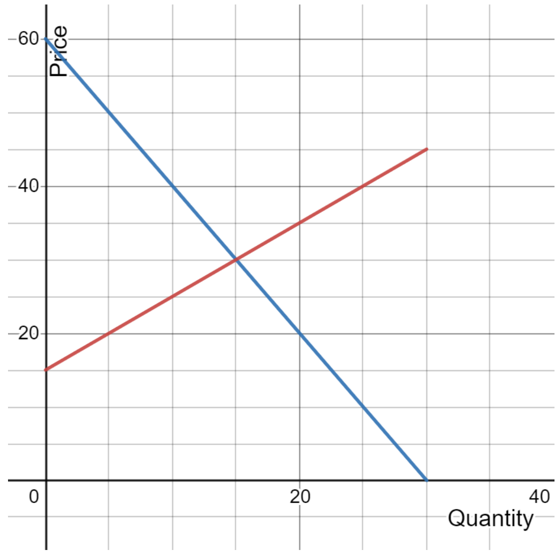


Figure 1: Demand (blue) and Supply (red)

The equilibrium quantity is 15 units and the price is 30.

(b) The following sketch shows the regions corresponding to consumer and producer surplus:

Figure 2: Consumer surplus (green) and producer surplus (red)

We can compute consumer surplus as: $(60 - 30) \cdot \frac{15}{2} = 225$

We can compute producer surplus as: $(30 - 15) \cdot \frac{15}{2} = \frac{225}{2}$

(c) To compute price elasticity of demand and supply, it's helpful to rewrite the demand and supply equations as:

$$Q_s = P - 15$$
$$Q_d = 30 - \frac{P}{2}$$

so that we have:

$$\frac{\Delta Q_s}{\Delta P} = 1$$
$$\frac{\Delta Q_d}{\Delta P} = -\frac{1}{2}$$

We can then compute price elasticity of supply and demand at the equilibrium point as:

$$\epsilon_{p,s} = \frac{\Delta Q_s}{\Delta P}\frac{P}{Q} = 1 \cdot \frac{30}{15} = 2$$
$$\epsilon_{p,d} = \frac{\Delta Q_d}{\Delta P}\frac{P}{Q} = -\frac{1}{2}\frac{30}{15} = -1$$

(d) There are two ways to approach this problem. The first is algebraically. In particular, if we impose a tax on one side of the market, then we can solve for the equilibrium quantity as a function of the tax. Let's begin by imposing a tax $\tau > 0$ on the demand side. The new relationship between the (pre-tax) market price and quantity demanded becomes:

$$P = 60 - \tau - 2Q_d$$

Supply is still given as:

$$P = 15 + Q_s$$

We can then solve for the market quantity as:

$$60 - \tau - 2Q = 15 + Q$$

So that quantity as a function of the tax τ is:

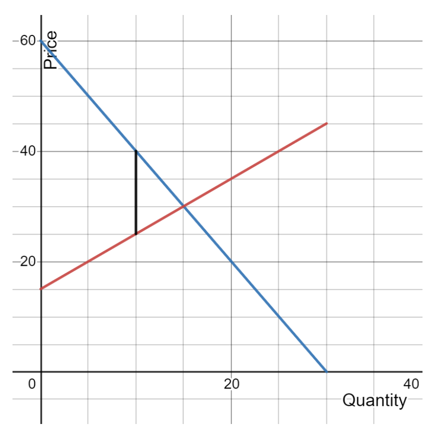
$$Q = \frac{45 - \tau}{3} = 15 - \frac{\tau}{3}$$

In order to get a quantity of 10, we see that we want to solve:

$$10 = 15 - \frac{\tau}{3}$$

which yields a tax of $\tau = \$15$.

The other way to solve this problem is graphically. Let us look at the supply and demand curves, and think about the difference between the price which yields a quantity demanded of 10 and the price which yields a quantity supplied of 10. In particular, examine the following figure:



Looking at where $Q = 10$, we see that the difference in these prices is $15.

(e) The deadweight loss associated with a tax of $15 (on either side of the market) is shaded in the following figure:

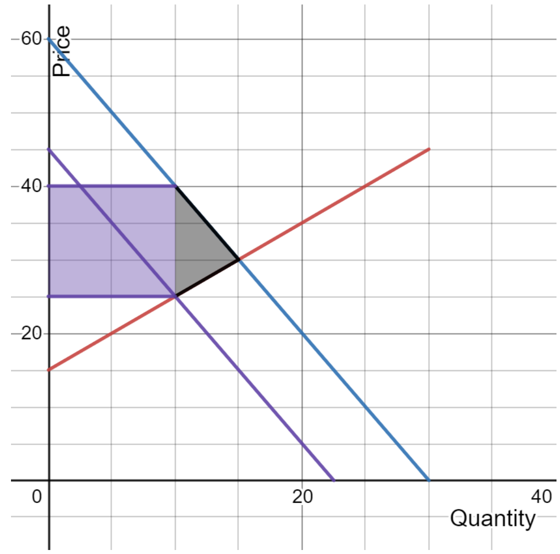


Figure 3: Deadweight loss (gray) and revenue (purple)

Note that we already know the dimensions of this triangle. In particular, the tax was $15, the old market quantity was 15, and the new quantity was targeted at 10. We then see that the amount of deadweight loss is:

$$DWL = 15 \cdot \frac{15 - 10}{2} = \frac{75}{2}$$

The revenue associated with the tax is given by:

$$R = \tau \cdot Q = 15 \cdot 10 = 250$$

(f) Recall that consumer incidence is the difference between the new total (after-tax) price that they pay minus the original price. In this case, consumer incidence is:

$$CI = 40 - 30 = 10$$

Producer incidence is the difference between the old price that producers receive per transaction minus the new (after-tax) amount that producers receive per transaction. This is given by:

$$PI = 30 - 25 = 5$$

Notice that the sum of the incidences is equal to the tax. The following figure shows the consumer and producer incidences graphically:

Figure 4: Consumer incidence (green) and producer incidence (yellow)

(g) For this problem, the first step should be to write tax revenue as a function of the tax. Recall that tax revenue is given by:

$$R = \tau \cdot Q$$

where τ is the amount of the tax and Q is the equilibrium quantity after the tax is imposed. Recall from part (d) that quantity as a function of the tax was computed as:

$$Q = 15 - \frac{\tau}{3}$$

Then we can write revenue as a function of the tax as:

$$R(\tau) = \tau(15 - \frac{\tau}{3})$$

Now, we're interested in finding a tax τ such that $R(\tau) = \frac{200}{3}$. Using this equation, we see that we want to find τ such that:

$$-\tau^2 + 45\tau - 200 = 0$$

Recall that for an equation of the form:

$$ax^2 + bx + c = 0$$

the solutions can be found by applying the quadratic equation:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Using $a = 1$, $b = 45$ and $c = -200$, this becomes:

$$\tau = \frac{-45 \pm \sqrt{2025 - 4(-1)(-200)}}{-2} = \frac{-45 \pm \sqrt{2025 - 800}}{-2} = \frac{-45 \pm 35}{-2}$$

This gives us two possible taxes of $\tau = 5$ and $\tau = 40$. Given our intuition about taxes, the deadweight loss arising from the tax of \$5 is much lower than the deadweight loss associated with the tax of \$40.

(h) It in fact did not matter which side of the market received the tax in the previous problem. Importantly: Quantity, incidence, revenue, deadweight loss, and surplus are the same regardless of whether the tax is imposed on the demand or supply side of the market.