MATH 5440: Week 4 Assignement

Due Date: February 17, 2023 at 10am

Exercise 1 Example impact computations

This exercise provides impact curve examples for two impact models. Assume that $(0, 1)$ represents a full trading day.

Closed-form examples I

Consider the original OW model

$$dI_t = -\beta I_t dt + \lambda dQ_t$$

with $\beta, \lambda > 0$.

1. Assume the trader sends in a Time Weighted Average Price (TWAP) order for the day. Hence, one has $Q_t = Qt$ for a constant Q for all $t \in (0, 1)$. Furthermore, assume that $I_0 = 0$. Compute and plot I_t.
2. Assume the unperturbed price S is a martingale. Compute and plot the expected fundamental P&L Y and the mark-to-market P&L X for $t \in (0, 1)$.
3. Assume the impact state halves by the start of the next day

   $$I_{1+} = \frac{1}{2} I_{1-}.$$

   The trader does not trade on the second day. Compute and plot I, X, and Y for $t \in (1, 2)$.

Numerical examples II

Consider the impact model

$$dI_t = -\beta I_t dt + \frac{\lambda}{v_t} dQ_t$$

with v_t the intraday market activity captured by the volatility of market trades.

Assume the intraday volume profile v follows the deterministic curve

$$v_t = e^{4\cdot(t-0.7)^2}.$$

1. Plot the function v_t to visualize the intraday volume profile.
2. Numerically solve questions 1-3 for this model.

Exercise 2 Globally concave AFS model

This exercise establishes an order-based impact formula from a globally concave AFS model. Consider a globally concave impact model

$$I_t = \text{sign}(J_t)|J_t|^c$$

with local dynamics

$$dJ_t = -\beta J_t dt + \lambda dQ_t$$

for $\beta, \lambda > 0$ and $c \in (0, 1]$.

Consider an order of size $Q > 0$ traded over $[0, T]$. Let the unperturbed price S be a martingale, $J_0 = 0$, and $Q_0 = 0$.

1. Map the objective function in impact space.
2. Derive the optimal execution strategy. What is I_T as a function of Q?
3. Consider a TWAP execution. What is I_T as a function of Q?