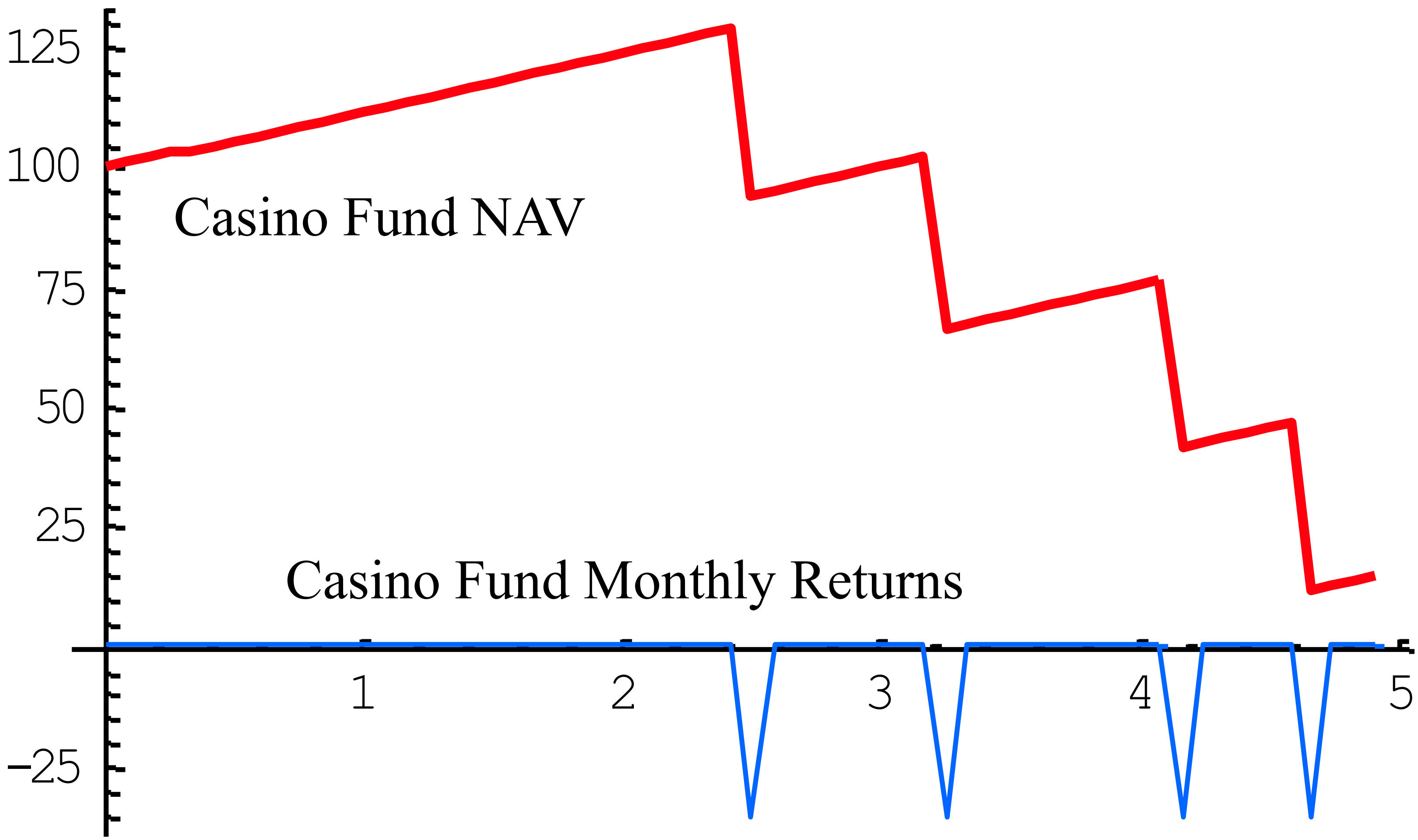
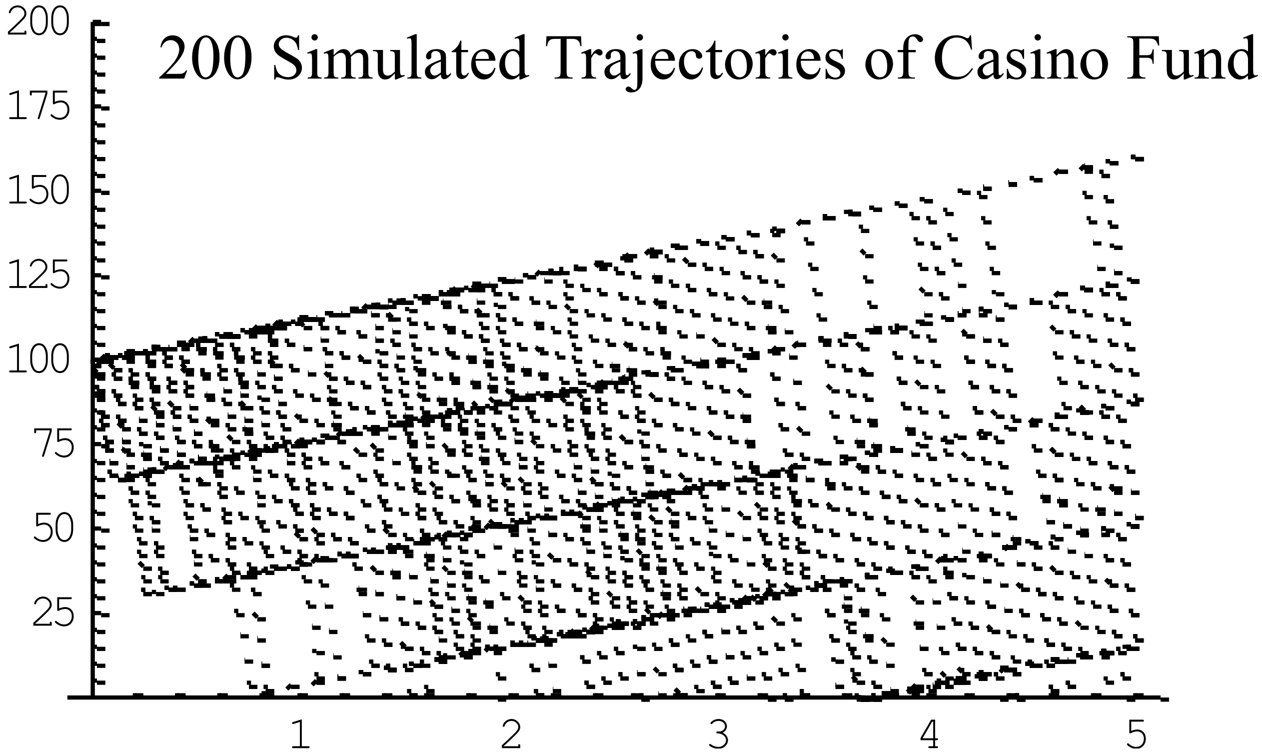
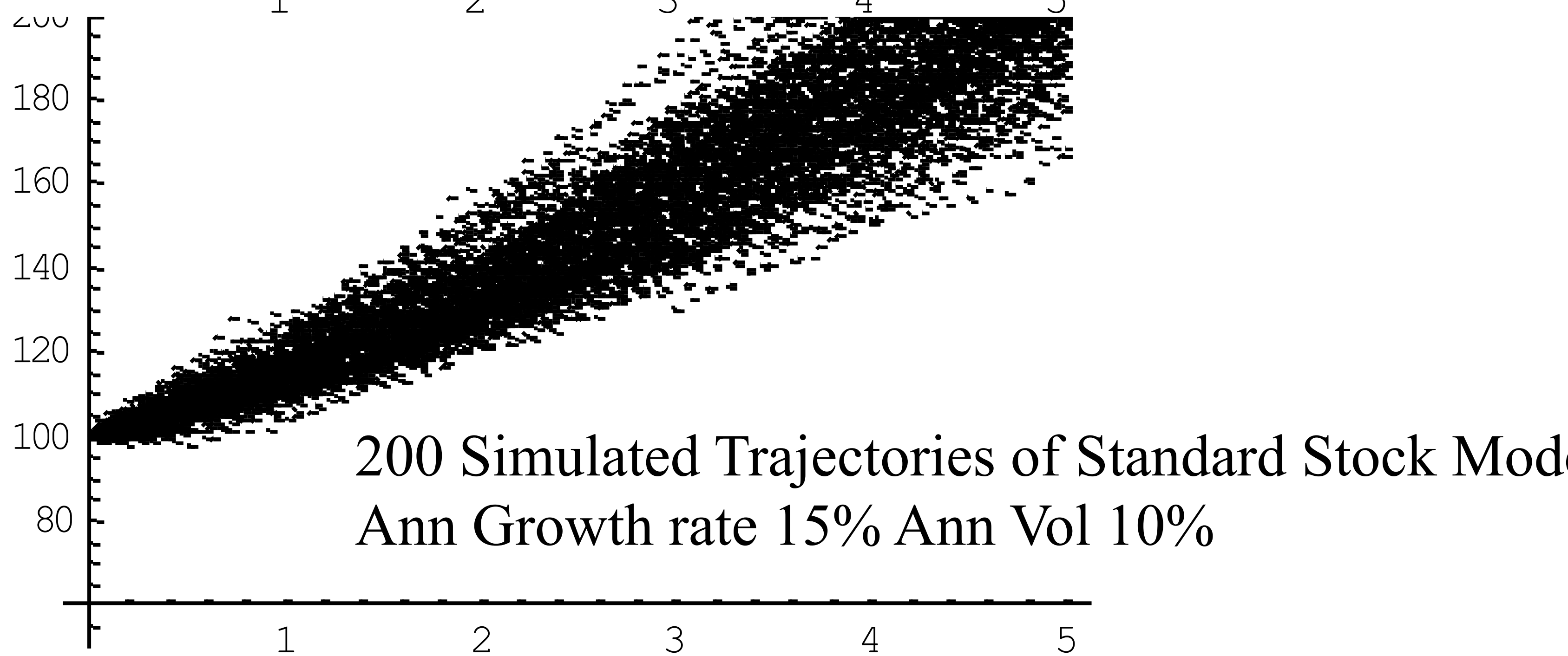
RISK MANAGEMENT
and
VALUE AT RISK

Fund strategy: get 100$ from investors
Every Month put 1$ bets on numbers 1 to 35 leaving 36 and 0 empty
win = 1 $ Probability of win = 35/37=94.59% (put 35$ on the table win36$)
loss = 35 $ Probability of loss = 2/37=5.41% (lose 35$ when 36 or 0 come)





200 Simulated Trajectories of Casino Fund
200
175
150
125
100
75
50
25
1
2
3
4
5



180
160
140
120
100
80
1
2
3
4
5
200 Simulated Trajectories of Standard Stock Mod
Ann Growth rate 15% Ann Vol 10%

Ed Thorpe, Beat the Dealer

Ralph Vince, Mathematics of Money Management

Robert Pardo, Development and Testing of Trading Strategies

Nassim Taleb, Fooled by Randomness

Philipp Jorion, Value at Risk

Value at Risk (VAR) measures the maximum potential loss on a group of securities over some time period T, given a specified probability A%.

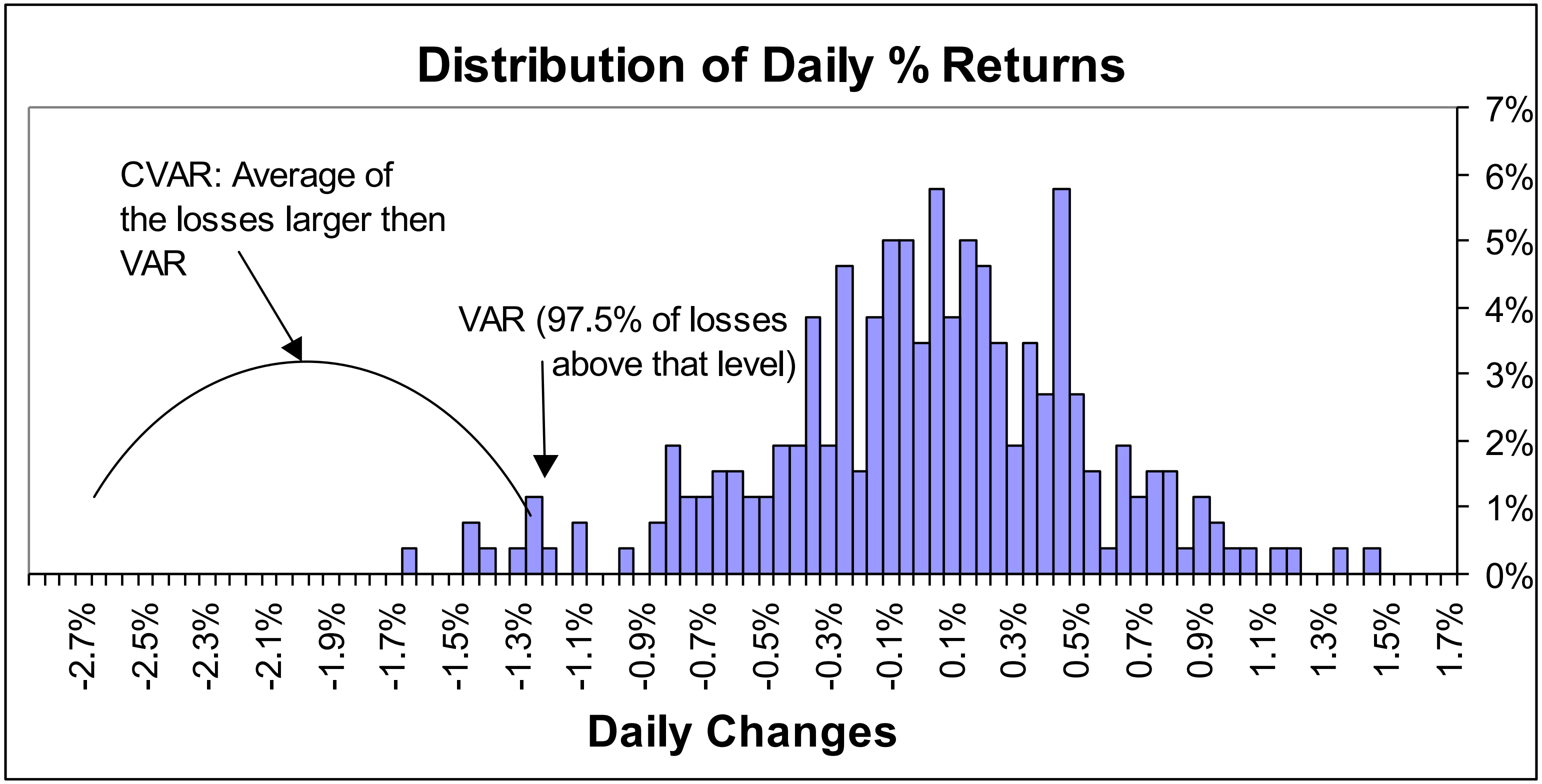
Once a probability, or degree of confidence has been set, VAR is the amount which represents the statistical maximum loss for a single security or group of securities.

For example "95% of the time losses will not exceed $10 million over a one week period" means 95% one week VAR is $10 Million

Roughly interpreted one would not expect a loss of more than the $10 Million in 95 one week periods out of 100 on average. In 5 periods out of 100 one would expect the loss to be more severe.

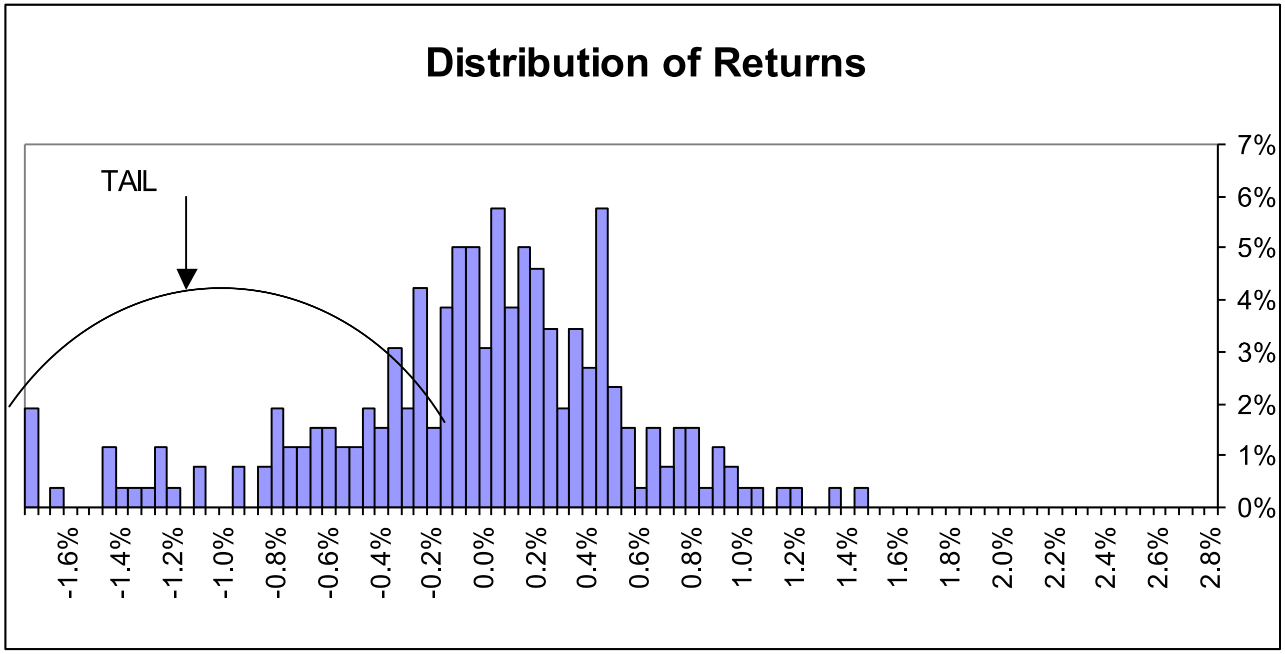
The average of these 5 more severe losses is called **Conditional VAR or CVAR** (for the same A=95% probability T=one week)

Measures of Market Risk
Value at Risk and Conditional Value at Risk



97.5% VAR and CVAR for 1 day returns

CVAR measures fatness of negative tail



VAR, Conditional VAR (Average loss exceeding VAR)

Methods of Calculation

1. Historical
2. Covariance Matrix (or parametric) with or without factor mapping
3. Simulation (including Monte-Carlo Simulations)

Software Vendors: RiskMetrics, Algorithmics, Bloomberg, other.

The Question Being Asked in VaR

"What loss level is such that we are *X*% confident it will not be exceeded in *N* business days?"

VaR and Regulatory Capital

- Regulators base the capital they require banks to keep on VaR
- The market-risk capital is k times the 10-day 99% VaR where k is at least 3.0

VaR vs. C-VaR

- VaR is the loss level that will not be exceeded with a specified probability
- C-VaR is the expected loss given that the loss is greater than the VaR level
- Although C-VaR is theoretically more appealing, it is not widely used

Advantages of VaR

- It captures an important aspect of risk in a single number
- It is easy to understand
- It asks the simple question: "How bad can things get?"

Time Horizon

- Instead of calculating the 10-day, 99% VaR directly analysts usually calculate a 1-day 99% VaR and assume

$$\text{10-day VaR} = \sqrt{10} \times \text{1-day VaR}$$

- This is exactly true when portfolio changes on successive days come from independent identically distributed normal distributions

Historical Simulation

- Create a database of the daily movements in all market variables.
- The first simulation trial assumes that the percentage changes in all market variables are as on the first day
- The second simulation trial assumes that the percentage changes in all market variables are as on the second day
- and so on

Historical Simulation continued

- Suppose we use m days of historical data
- Let v_i be the value of a variable on day i
- There are m-1 simulation trials
- The ith trial assumes that the value of the market variable tomorrow (i.e., on day m+1) is

$$v_m \frac{v_i}{v_{i-1}}$$

The Model-Building Approach

- The main alternative to historical simulation is to make assumptions about the probability distributions of return on the market variables and calculate the probability distribution of the change in the value of the portfolio analytically
- This is known as the model building approach or the variance-covariance approach

Daily Volatilities

- In option pricing we express volatility as volatility per year
- In VaR calculations we express volatility as volatility per day

$$\sigma_{\text{day}} = \frac{\sigma_{\text{year}}}{\sqrt{252}}$$

Daily Volatility continued

- Strictly speaking we should define σ_{day} as the standard deviation of the continuously compounded return in one day
- In practice we assume that it is the standard deviation of the percentage change in one day

Microsoft Example

- We have a position worth \$10 million in Microsoft shares
- The volatility of Microsoft is 2% per day (about 32% per year)
- We use N=10 and X=99

Microsoft Example continued

- The standard deviation of the change in the portfolio in 1 day is \$200,000
- The standard deviation of the change in 10 days is

$$200{,}000\sqrt{10} = \$632{,}456$$

Microsoft Example continued

- We assume that the expected change in the value of the portfolio is zero (This is OK for short time periods)
- We assume that the change in the value of the portfolio is normally distributed
- Since $N(-2.33)=0.01$, the VaR is

$$2.33 \times 632{,}456 = \$1{,}473{,}621$$

AT&T Example

- Consider a position of $5 million in AT&T
- The daily volatility of AT&T is 1% (approx 16% per year)
- The S.D per 10 days is
- The VaR is $$50{,}000\sqrt{10} = \$158{,}144$$

$$158{,}114 \times 2.33 = \$368{,}405$$

Portfolio

- Now consider a portfolio consisting of both Microsoft and AT&T
- Suppose that the correlation between the returns is 0.3

S.D. of Portfolio

- A standard result in statistics states that

$$\sigma_{X+Y} = \sqrt{\sigma_X^2 + \sigma_Y^2 + 2\rho\sigma_X\sigma_Y}$$

- In this case $\sigma_X = 200{,}000$ and $\sigma_Y = 50{,}000$ and $\rho = 0.3$. The standard deviation of the change in the portfolio value in one day is therefore 220,227

VaR for Portfolio

- The 10-day 99% VaR for the portfolio is

$$220{,}227 \times \sqrt{10} \times 2.33 = \$1{,}622{,}657$$

- The benefits of diversification are

$$(1{,}473{,}621+368{,}405)-1{,}622{,}657=\$219{,}369$$

- What is the incremental effect of the AT&T holding on VaR?

The Linear Model

We assume

- The daily change in the value of a portfolio is linearly related to the daily returns from market variables
- The returns from the market variables are normally distributed

The General Linear Model continued

$$\text{Change}P = \sum_{i=1}^{n} \alpha_i \text{Change } x_i$$

$$\sigma_P^2 = \sum_{i=1}^{n} \sum_{j=1}^{n} \alpha_i \alpha_j \sigma_i \sigma_j \rho_{ij}$$

$$\sigma_P^2 = \sum_{i=1}^{n} \alpha_i^2 \sigma_i^2 + 2 \sum_{i<j} \alpha_i \alpha_j \sigma_i \sigma_j \rho_{ij}$$

where σ_i is the volatility of variable i

and σ_P is the portfolio's standard deviation

Skewness

The linear model fails to capture skewness in the probability distribution of the portfolio value.

Monte Carlo Simulation

To calculate VaR using M.C. simulation we

- Value portfolio today
- Sample once from the multivariate distributions of the Change x_i
- Use the Change x_i to determine market variables at end of one day
- Revalue the portfolio at the end of day

Monte Carlo Simulation

- Calculate Change*P*
- Repeat many times to build up a probability distribution for Change *P*
- VaR is the appropriate fractile of the distribution times square root of *N*
- For example, with 10,000 trial the 1 percentile is the 100th worst case.

Stress Testing

- This involves testing how well a portfolio performs under some of the most extreme market moves seen in the last 10 to 20 years

Stress Tests

- Imperfect Storm (October 1987,
  Fixed Income 1994
  Russia and Credit Fall 1998,
  Equity Meltdown 2000
  September 2001)
  Subprime Crisis Fall 2008
- Artificial Factor Scenarios
- Perfect Storm (everything moving in right direction)

When Linear Model Can be Used

- Portfolio of stocks
- Portfolio of bonds
- Forward contract on foreign currency
- Interest-rate swap

DRAWDOWN AS ANOTHER MEASURE OF RISK

P(t) be a portfolio value at time *t*.

M(t) be a maximal value of a portfolio prior to *t*.

If portfolio value is below its maximal prior value, portfolio is said to be in ***drawdown.***

Unless portfolio is at its up to date peak it is in drawdown.

Absolute drawdown $ADD(t)=P(t)-M(t)$ where $M(t)=Max(P(s), s<t)$.

Percentage Drawdown ***PDD(t)*** is absolute drawdown as percentage of ***M(t).***

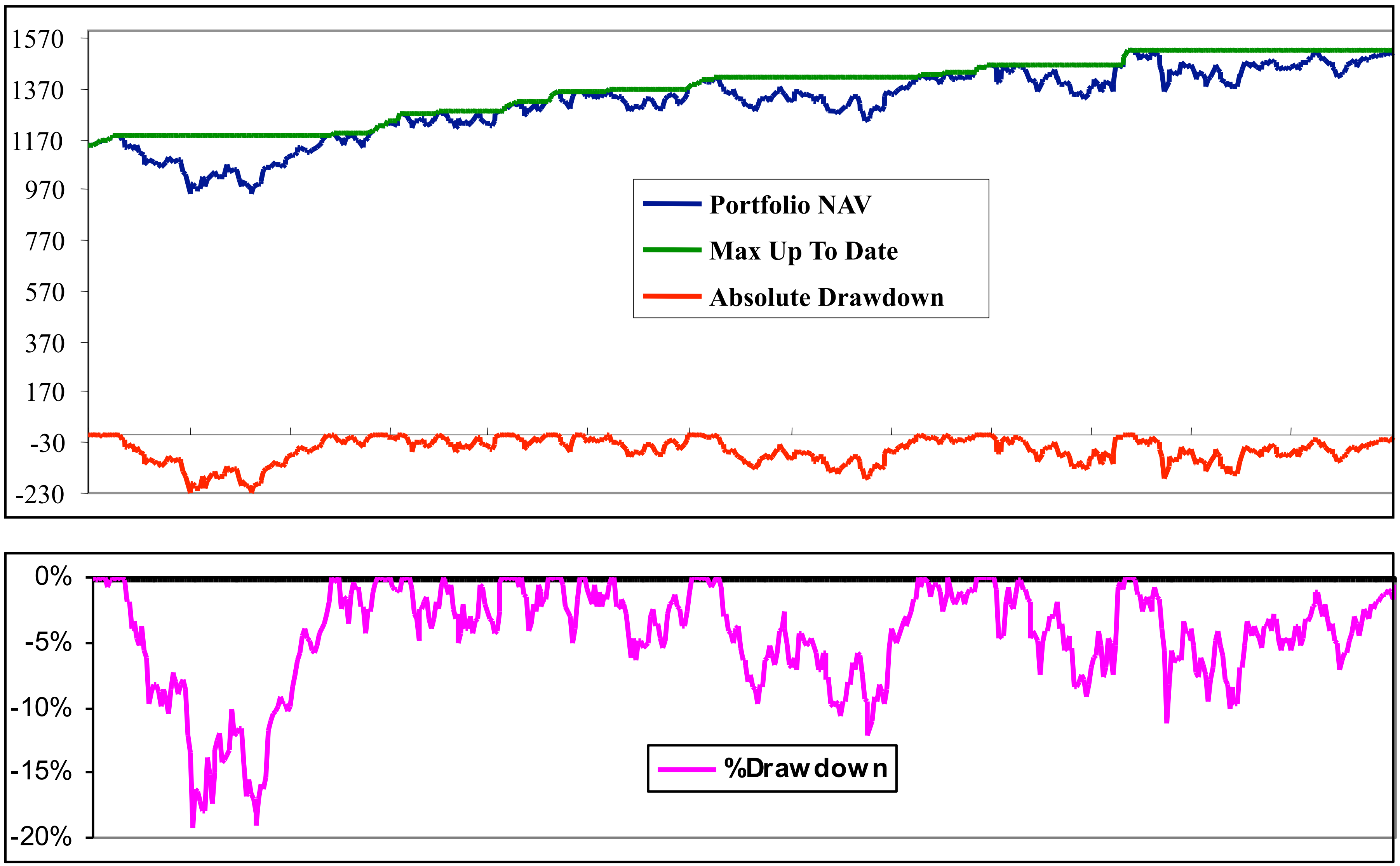
$$\text{Sharpe Ratio} = \frac{\text{(Annual Return - Risk Free Rate)}}{\text{Ann Vol of Excess Retrns over R Free rate}}$$

$$\text{Sortino Ratio} = \frac{\text{(Annual Return – Minimal Accepted Return)}}{\text{Annual Downside Deviation}}$$

Downside deviation =Square root of 1/n of the sum of the squared distances between the returns and the Min Accepted Return, where the sum is restricted to those returns that are less than the Min Accepted Return, n- is the total number of returns.

Minimal Accepted Return isusually a constant like 0% to 5% or risk free rate

Percentage and Absolute Drawdowns of a Sample Portfolio



Sharpe ratio was introduced in 1966 by William Sharpe (1990 Nobel Prize in Economics for the capital asset pricing model). It is a risk/return measure and one of the simplest of such measures.

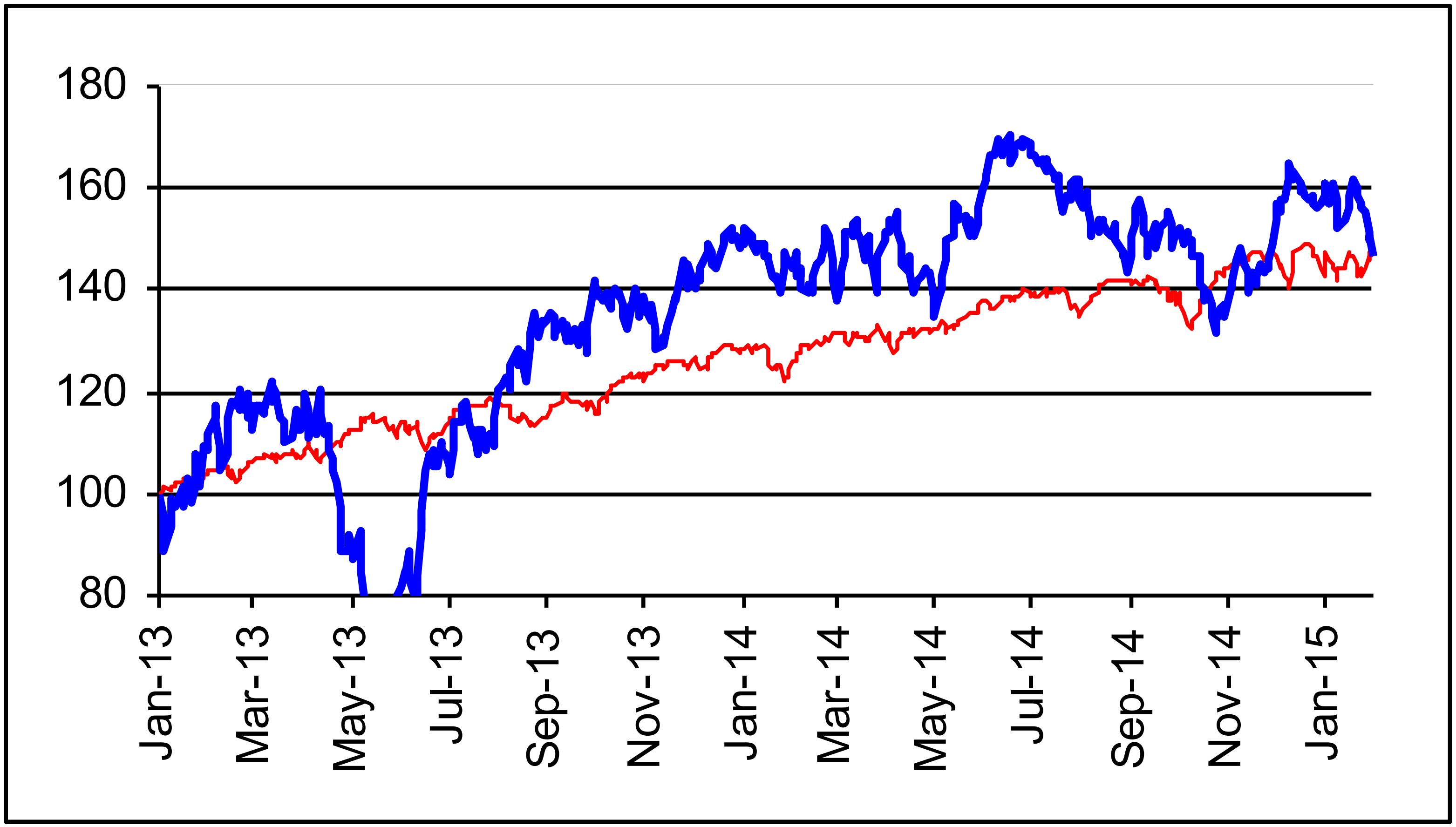
$$\textbf{Sharpe Ratio} = \frac{\textbf{(Annual Expected Portfolio Return — Ann Risk Free Rate)}}{\textbf{Annual Volatility of Excess Returns over Risk Free rate}}$$

Sharpe ratio measures fund's risk-adjusted performance.

The higher Sharpe ratio is the better is the fund manager.

Typical range 0.6 to 2.5. Sharpe 2.5 is VERY VERY good.

Negative Sharpe ratio implies that a cash would perform better than the fund. Cash would have no volatility.

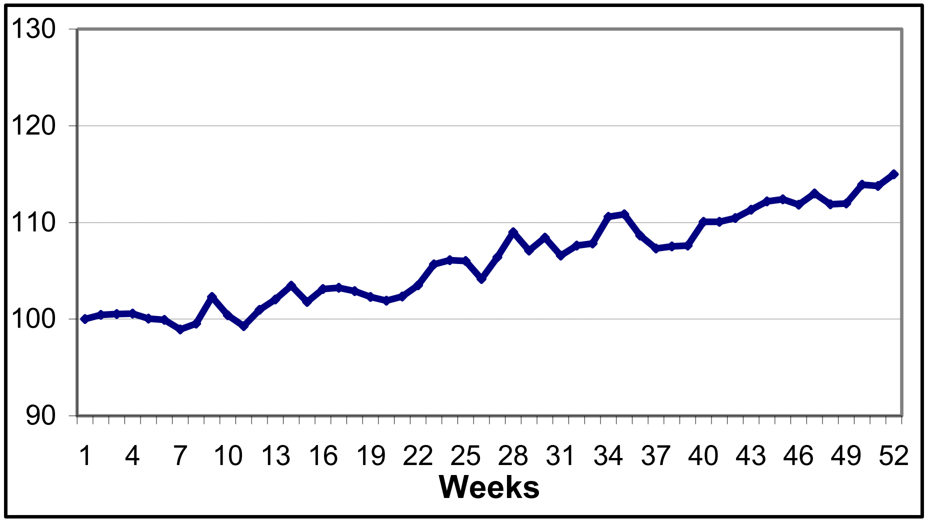


High Sharpe ratio fund (red thin line) Sharpe=1.6
Low Sharpe ratio fund (blue thick line) Sharpe=0.7

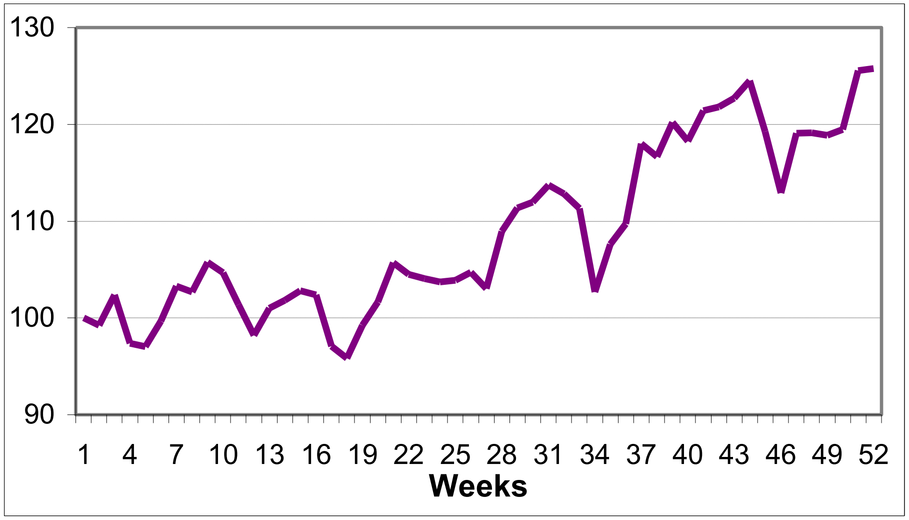
Measuring of Performance of Leveraged Investment

Example.

Fund A, Annual Return 15%



Fund B, Annual Return 25%



Which one is better to choose if one can leverage the investment?

Postulate:

If two funds X and Y have the same return

then the fund that has less risk as measured by "oscillation" or standard deviation of returns is better.

Note that risk can be measured by other measures like drawdown, downside deviation etc.

Answer. The one with higher Sharpe ratio.

Suppose that the risk free rate is 5% (one can borrow and lend money at 5%). Define Sharpe Ratio, named after economist W.Sharpe as

$$\textbf{Sharpe Ratio} = \frac{\textbf{(Annual Return — Risk Free Rate)}}{\textbf{Annual Volatility of Excess Returns over Risk Free rate}}$$

Fund A.

Annual Return 15%,

Annual Volatility 10%

(of returns over 5% risk free)

$$\text{Sharpe Ratio} = \frac{(15\% - 5\%\text{Risk Free Rate})}{10\%} = \mathbf{1}$$

Fund B.

Annual Return 25%,

Annual Volatility 30%

(of returns over 5% risk free)

$$\text{Sharpe Ratio} = \frac{(25\% - 5\%\text{Risk Free Rate})}{30\%} = \mathbf{0.66}$$

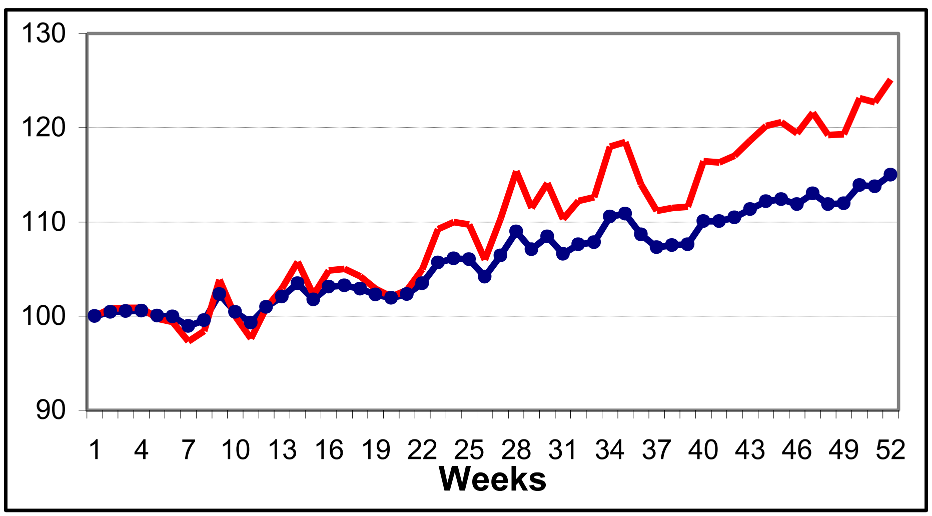
Fund A Leveraged 2 to 1.

Annual Return 25%=2×15% - 5%(borrowing),

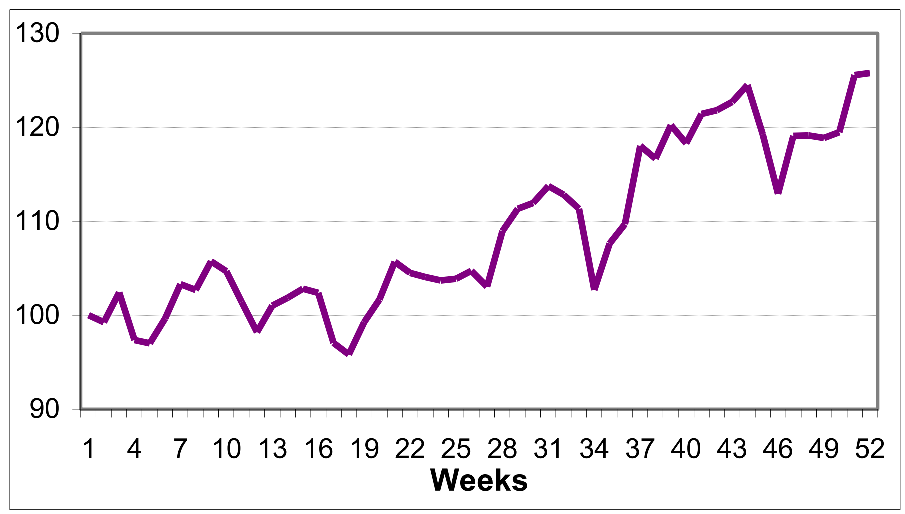
Annual Volatility 20%=2×10%
(of returns over 5% risk free)

$$\text{Sharpe Ratio} = \frac{(25\% - 5\%\text{Risk Free Rate})}{20\%} = 1$$

Fund A Leveraged 2 to1, Return 25%, Vol.20%



Fund B, Return 25%, Volatility 30%



RISK MANAGEMENT

BANKS
(Difficult)

INVESTMENT FUNDS	NON FINANCIAL COMPANIES
(Easier)	(More Difficult)

Financial risk management *are practices by which a firm optimizes the manner in which it takes financial risk.*
It includes monitoring of risk taking activities, upholding relevant policies and procedures, and distributing risk-related reports.

Organizationally, financial risk management is implemented in different ways. There may be, within the board of directors, a **risk committee**.
Usually, there is some sort of **risk oversight committee**, comprising senior managers. In practice, various names are given to these two committees.

A senior manager, called the **head of risk management** or **chief risk officer** (CRO), reports to the risk oversight committee.
This head of risk management may oversee a single department called the **risk management department**.

Professionals are responsible for facilitating the taking of applicable financial risks—market risks, credit risks and operational risks—by other departments within the firm.

In larger organizations, there may be more specialization. The head of risk management might oversee three professionals:

- •a head of market risk management,
- •a head of credit risk management, and
- •a head or operational risk management.

Each would oversee a respective department. Other arrangements are also possible.

Functionally, there are four aspects of financial risk management. Success depends upon

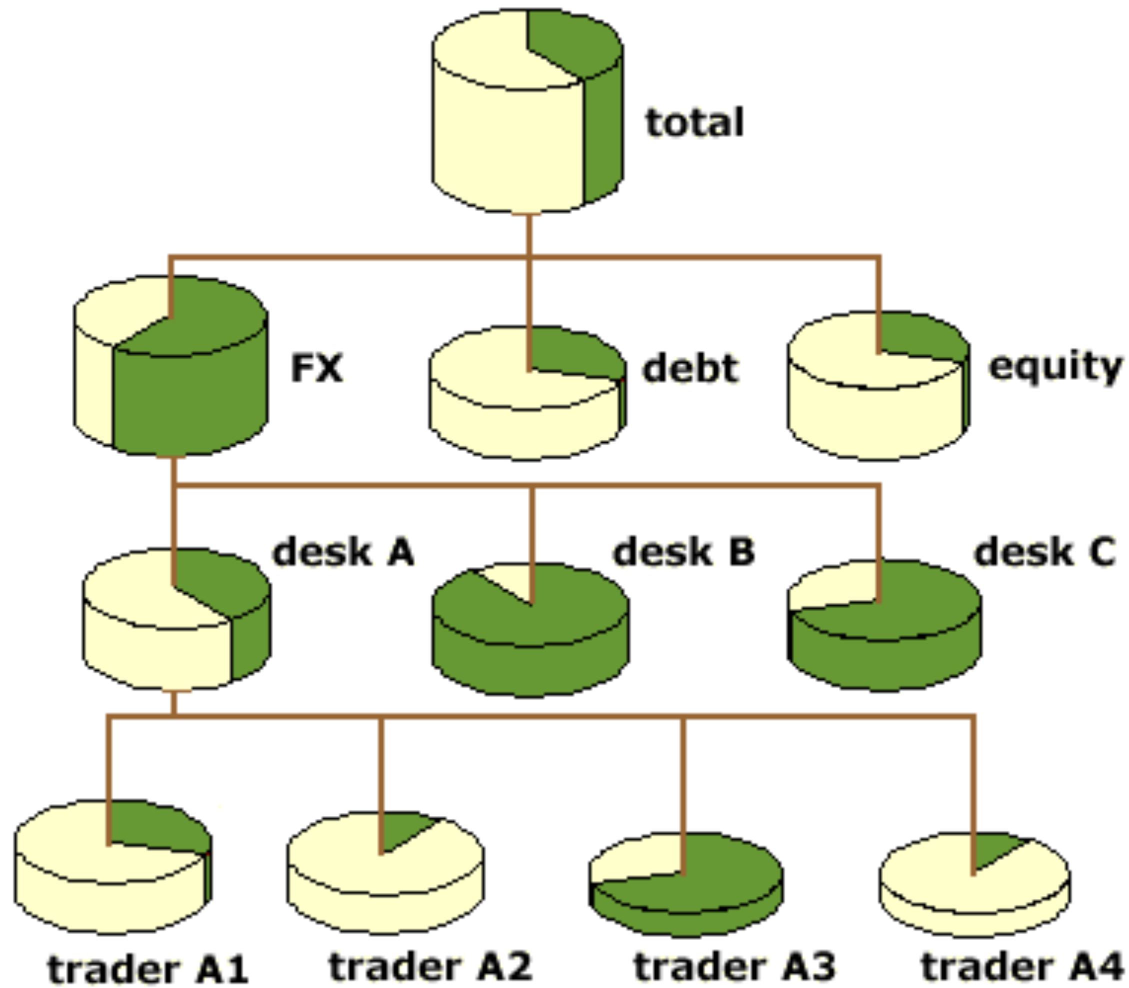
1. a positive corporate culture,
2. actively observed policies and procedures,
3. effective use of technology,
4. independence or risk management professionals.

A more staged approach starts off by recognizing that risk management is primarily about people—how they think and how they interact with one another. **Technology is just a tool.** In the wrong hands, it is worse than useless, but applied appropriately, it can transform an organization.

A good approach to implementing an enterprise risk management initiative is:

- Initially allocate minimal funding for the initiative, but ensure that board members, senior management or other supervisors are involved in the process.
- Start by planning a risk management strategy that involves no technology at all. This can be an empowering exercise. It focuses participants on the procedural and cultural issues of risk management. Ultimately, it is these which determine the success of an initiative.
- Once you have decided on a strategy for managing risk, then determine where technology needs to be incorporated or where it can enhance the strategy.

A hierarchy of Market Risk Limits



Liquidity risk is financial risk due to uncertain liquidity. An institution might lose liquidity if its credit rating falls, it experiences sudden, unexpected cash outflows, or some other event causes counterparties to avoid trading with or lending to the institution.

A firm is also exposed to liquidity risk if markets on which it depends are subject to loss of liquidity.

Liquidity risk tends to compound other risks. If a trading organization has a position in an illiquid asset, its limited ability to liquidate that position at short notice will compound its market risk. Suppose a firm has offsetting cash flows with two different counterparties on a given day. If the counterparty that owes it a payment defaults, the firm will have to raise cash from other sources to make its payment. Should it be unable to do so, it too we default. Here, liquidity risk is compounding credit risk.

A position can be hedged against market risk but still entail liquidity risk.