# Lecture 17: Skiplists Conclusion.

⇒ Question: What is the expected max height of skiplist with $n$ nodes?

"Back of Envelope"

- $n$ nodes, on average how many have ...
  - height $\geq 0$?
    - $n$
  - height $\geq 1$
    - $n/2$
  - height $\geq 2$
    - $n/4$
  - height $\geq k$?
    - $n/2^k$

→ when is this $<1$?

$\Rightarrow n < 2^k$

$\Rightarrow \log n < k$

Hint that max. height may be $O(\log n)$.

## ⊛ Formalising things

→ $H$ = rand var = max height in list of length $n$.

→ For each $k = 0, 1, 2, \ldots$

$L_k$ = length of list at height $k$, # nodes with height $\geq k$.

Observe: $\boxed{E(L_k) = \frac{n}{2^k}}$

therefore, if $k = \log n + j$. $(j \geq 0)$

$$E(L_{\log n + j}) = \frac{n}{2^{\log n + j}} = \frac{n}{2^{\log n} \cdot 2^j} = \boxed{\frac{1}{2^j}}$$

To analyse $E(H)$, we can write ~~E(H)~~ $H$ as sum of simpler variables.

two facts
1) $J_k \le 1$ always
2) $J_k \le L_k$. (if $L_k = 0$, so is $J_k$)

$\Rightarrow$ <u>Def.</u> $J_k = \begin{cases} 1 & \text{, if } H \geq k \\ 0 & \text{, otherwise} \end{cases}$

Therefore..

$H = J_1 + J_2 + J_3 + \ldots$

$\Rightarrow E(H) = E(J_1 + J_2 + \ldots)$

$\Rightarrow E(H) = E(J_1) + E(J_2) + \ldots$

$= \underbrace{E(J_1) + E(J_2) + \ldots + E(J_{\log n})}_{\boxed{\text{each} \le 1}} + \underbrace{\ldots}_{}$

$\boxed{E(J_{\log n + k}) \le E(L_{\log n + k})}$

$\le \underbrace{1 + 1 + 1 + \ldots + 1}_{\log n \text{ times}} + \underbrace{\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \ldots}_{1}$

$\boxed{\le \log n + 1}$

<u>Trick</u>: Analyse search in reverse.

Rev. dir$^n$:

from a node at height h
if I can go up, go up.
if I can't go up, go left

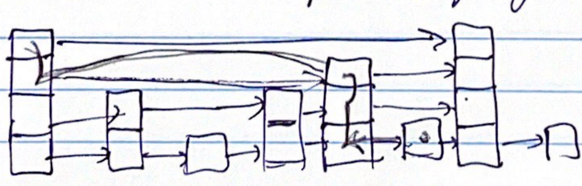
Each step costs O(1) in find procedure

total # of steps = $\underbrace{\#\text{up}}_{H} + \underbrace{\#\text{left}}_{??}$

to understand this:

Observe: probability that I can go up is 1/2.

$\Rightarrow$ prob of going left is 1/2.



$\Rightarrow$ probability of going left many times is small

Can show (See ODS §4.4)

$$E(\text{left steps}) = \log n + O(1)$$

So, expected time to find is

$$E(H) + E(\text{left steps}) = 2(\log n) + 1.$$

GOOGLE SEARCH $\Rightarrow$ "concentration of measure"