Random Walks, liquidity molasses and critical response in financial markets

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Introduction

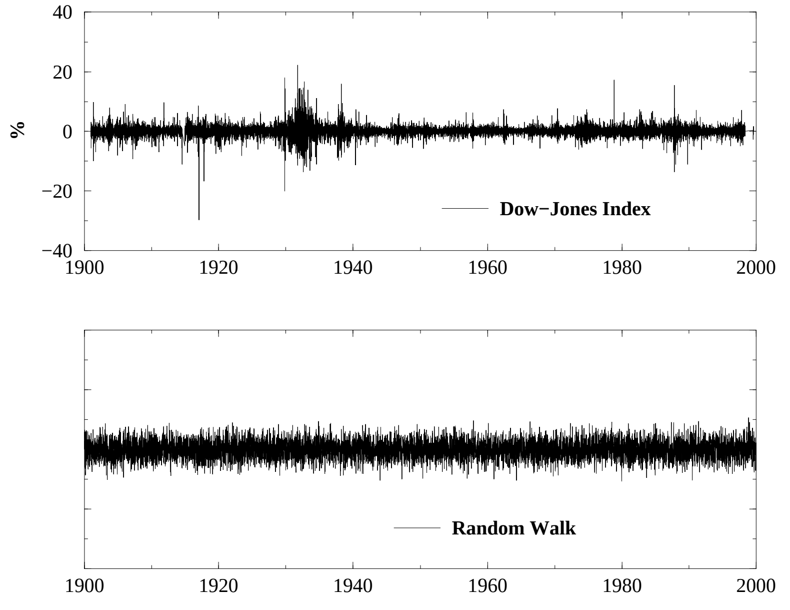
- Best known stylized fact in financial markets: price changes are weakly correlated $\longrightarrow$ approximate diffusion (Bachelier 1900)

- Efficient market theory: prices are fully rational and correspond to the best anticipation of future dividends $\longrightarrow$ price changes can only be due to unpredictable news; but: excess volatility, with long range, multiscale memory!

- More fundamentally: ambiguous information, psychological and cognitive biases, herding (cf. prediction of financial analysts!)

- 'Zero intelligence' investors: Each trade has a totally random motivation, but has a non zero impact on the price – each trade is considered by others as containing *some* information

Volatility clustering



Volatility clustering: comparison between the Dow Jones and a Brownian Random Walk

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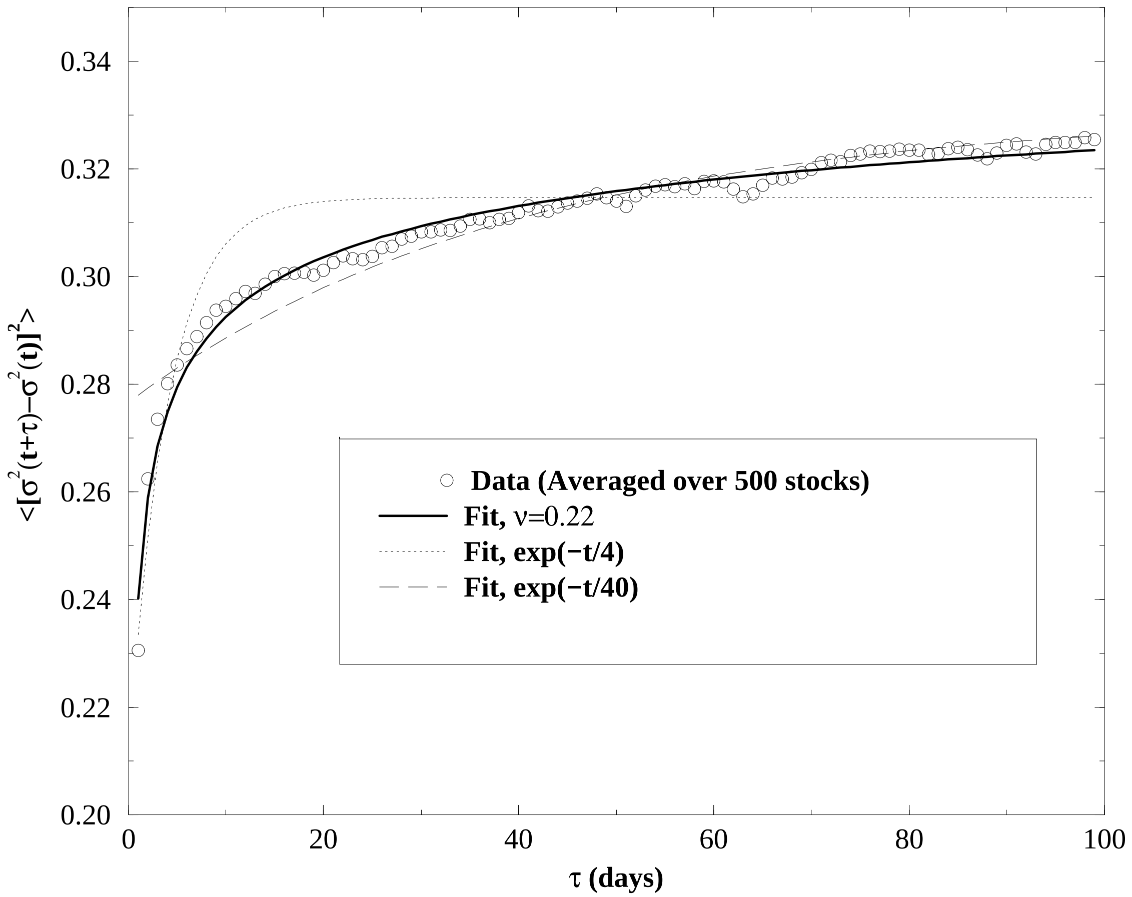
Volatility correlation



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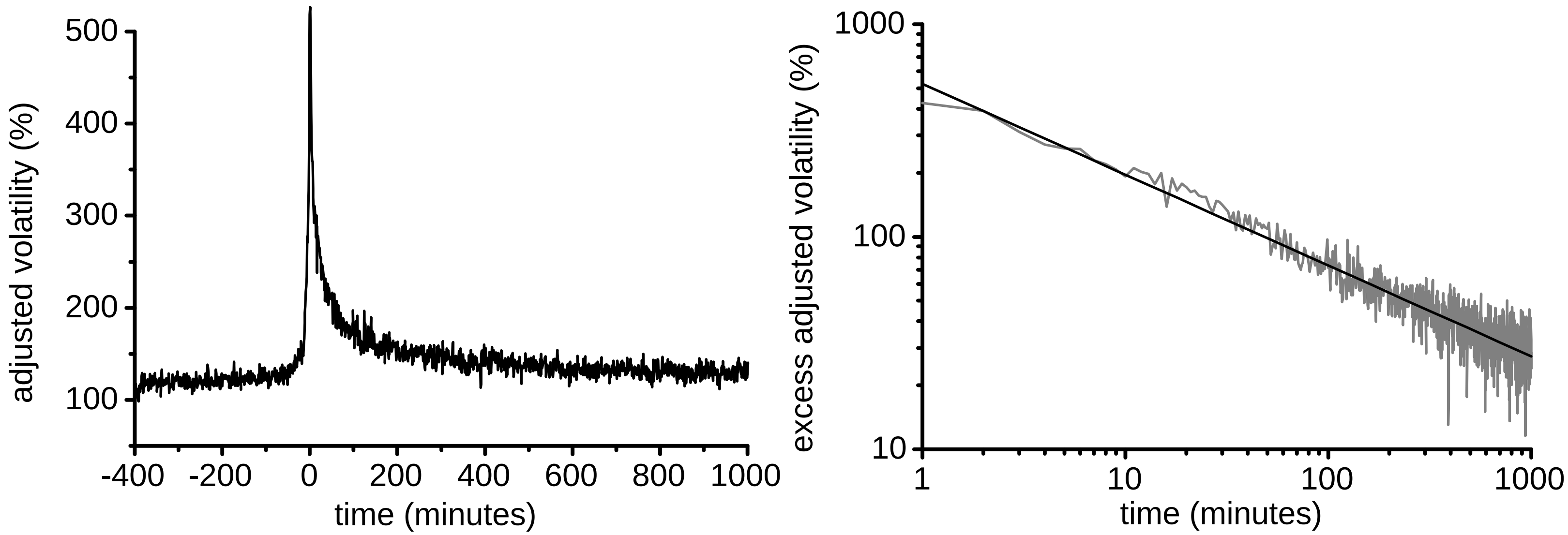
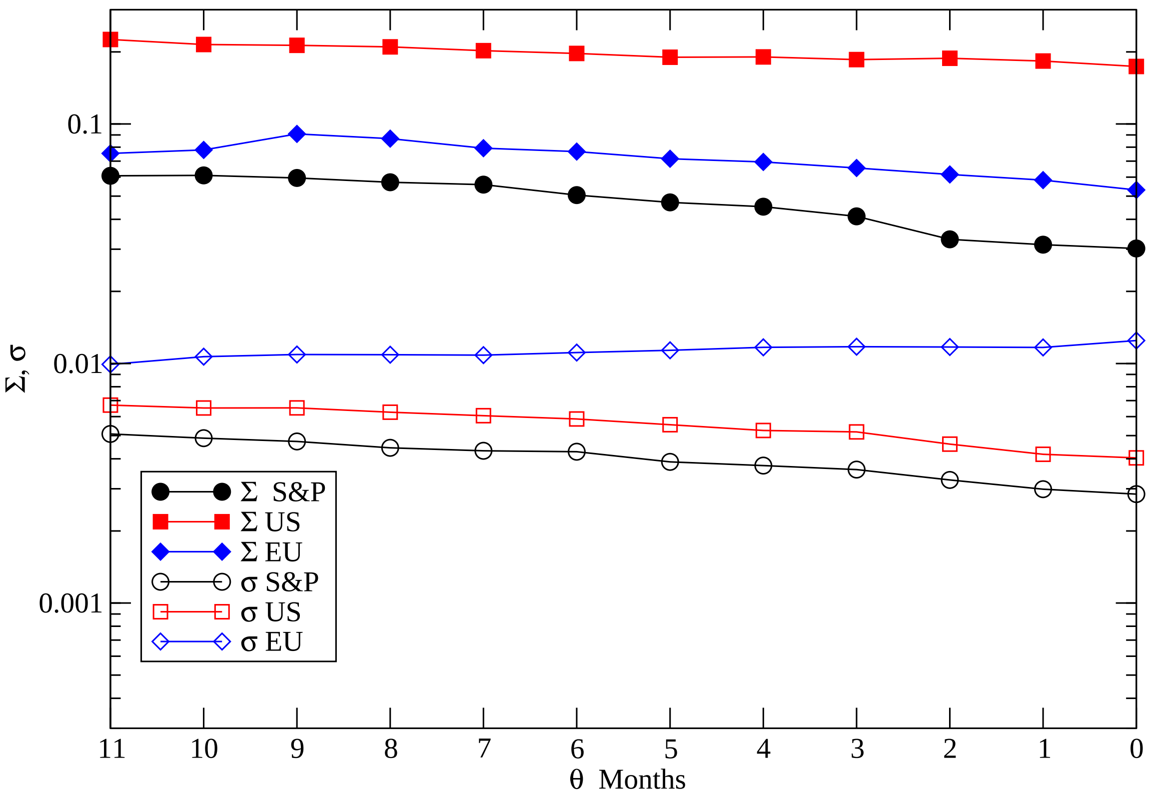
Power-law response to volatility shocks - HF



Analysts herding behaviour



With O. Guedj

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Empirical facts on trades and quotes data

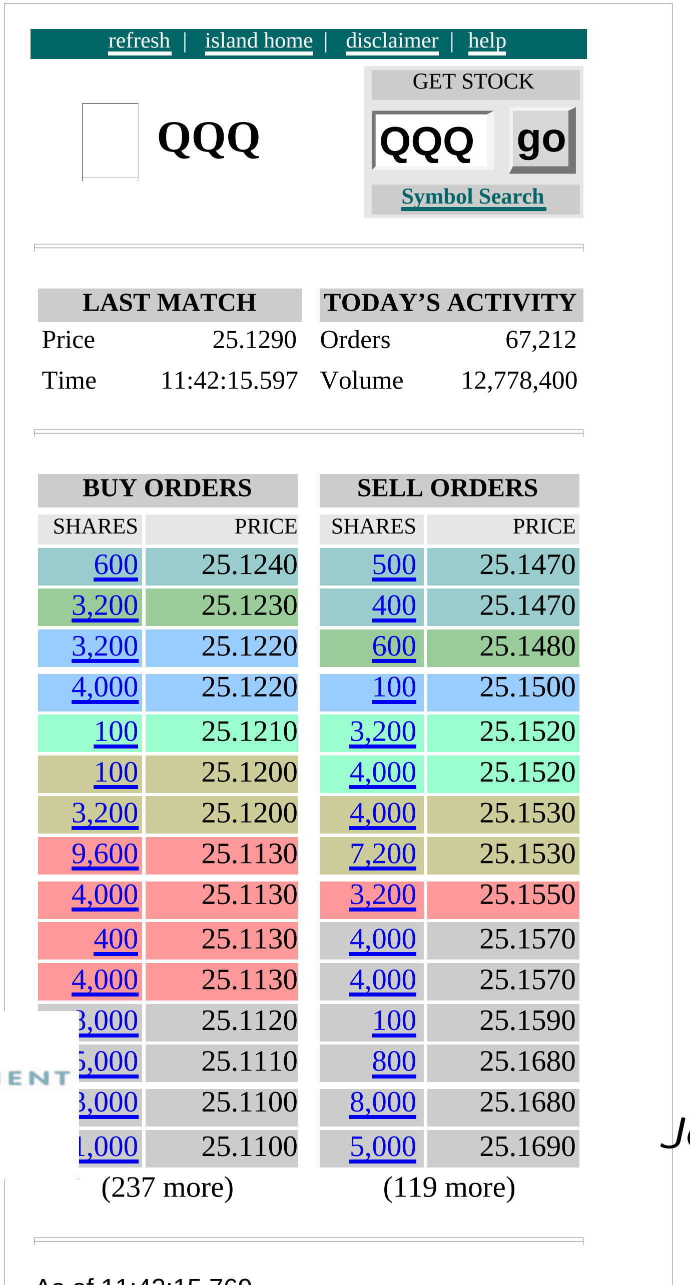
- Paris Bourse: fully electronic. Data form 2001-2002

- Example of a liquid stock: France Telecom – 5000 trades/day – 1.2 M-trades in 2002

- Quotes: Bid price + Ask price $\rightarrow$ midpoint $m =$ (Bid+Ask)/2

- Trades: At the Ask $\rightarrow \varepsilon = +1$, at the Bid $\rightarrow \varepsilon = -1$

The order book



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QQQ

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QQQ go

Symbol Search

LAST MATCH		TODAY'S ACTIVITY	
Price	25.1290	Orders	67,212
Time	11:42:15.597	Volume	12,778,400

BUY ORDERS		SELL ORDERS	
SHARES	PRICE	SHARES	PRICE
600	25.1240	500	25.1470
3,200	25.1230	400	25.1470
3,200	25.1220	600	25.1480
4,000	25.1220	100	25.1500
100	25.1210	3,200	25.1520
100	25.1200	4,000	25.1520
3,200	25.1200	4,000	25.1530
9,600	25.1130	7,200	25.1530
4,000	25.1130	3,200	25.1550
400	25.1130	4,000	25.1570
4,000	25.1130	4,000	25.1570
3,000	25.1120	100	25.1590
5,000	25.1110	800	25.1680
3,000	25.1100	8,000	25.1680
1,000	25.1100	5,000	25.1690
(237 more)		(119 more)	

As of 11:42:15.769

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Order flow and order book: New 'Stylized facts'

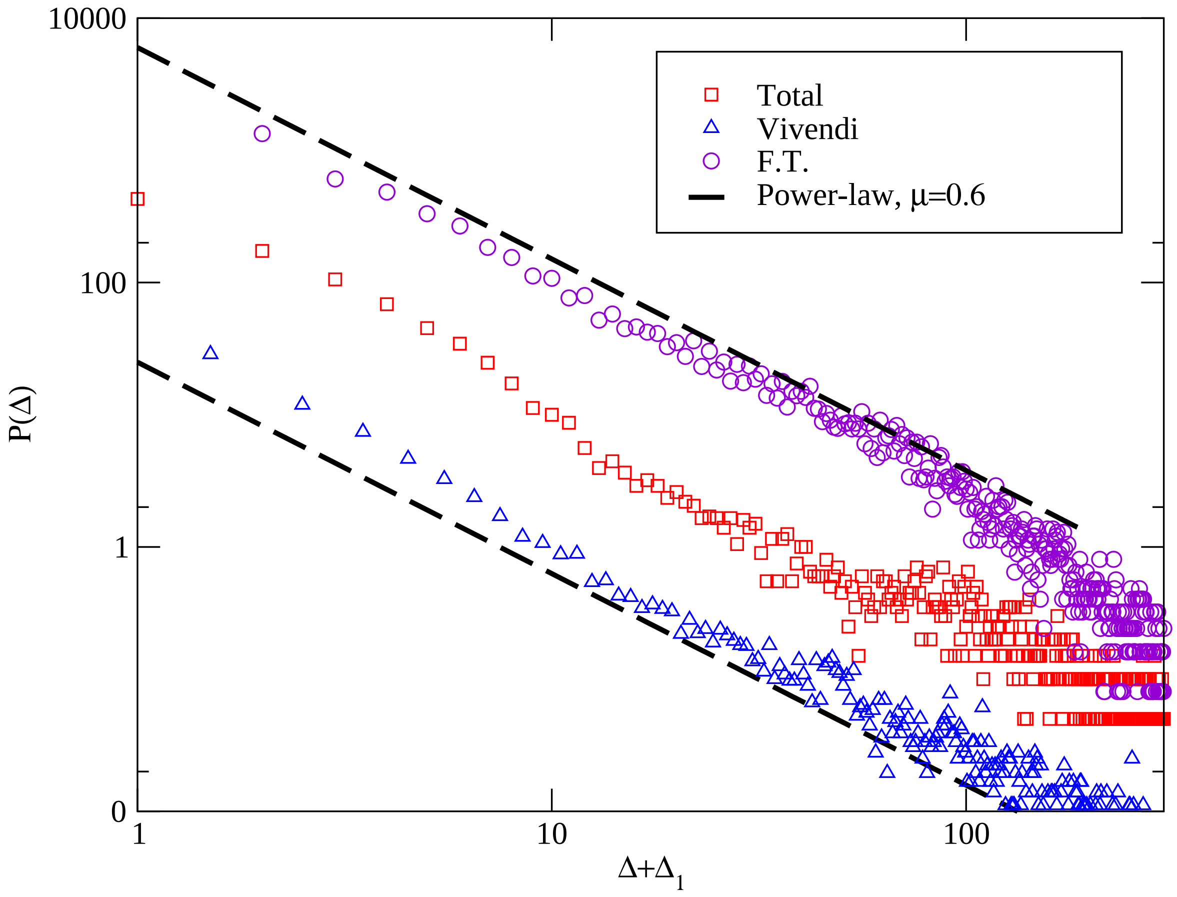
- Many new quantities can be analyzed
  - Statistics of the 'rain' of incoming orders as a function of distance from current bid/ask
  - Average size of the queue as a function of distance from current bid/ask
  - Probability distribution of the size of the queue
  - Collective modes of the order book

- Also: interaction between order book and price changes, between order flow and price changes ('Impact') – see below.

Statistics of the rain of orders

- As a function of the distance Δ from the current bid/ask:
  - Probability that a new order is placed is *very broad* – up to 50% away from current price !
  - Power law distribution $P(\Delta) \approx \Delta^{-1-\mu}$ with:
    - $\mu \sim 0.6$ for (liquid) CAC40 stocks
    - $\mu \sim 1.$ for (liquid) NASDAQ stocks
    - $\mu \sim 1.5$ for LSE stocks (Farmer & Zovko)
  - Conditional average size of the order: $\langle \Phi \rangle \approx \Phi_0$ for $\Delta \leq \Delta^*$, $\langle \Phi \rangle \approx \Delta^{-\nu}$ for $\Delta \geq \Delta^*$, with $\nu \sim 1.5$

Statistics of the rain of orders

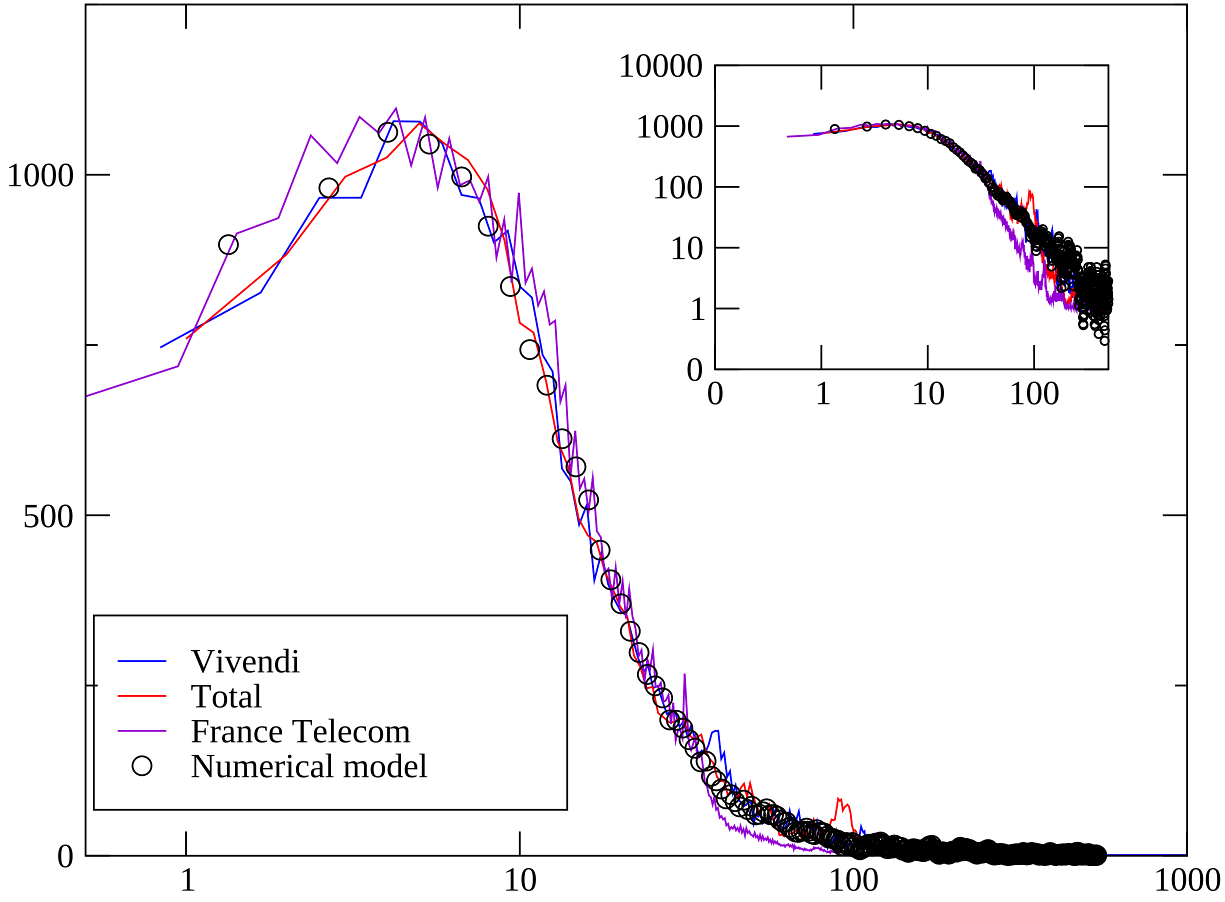


Note: same distribution for buy and sell orders

Shape of the order book

- As a function of the distance Δ from the current bid/ask:
  - The average size of the queue $\rho(\Delta)$ has a characteristic 'humped' shape, with a maximum away from the bid (ask)
  - Symmetric shape for buy and sell orders
  - The shape is found to be stock independent for French stocks
  - The shape can be different on NASDAQ stocks – but not a centralized market!

The shape of the order book



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A simple analytical model I

- Orders at distance Δ at time t are those which were placed there at a time $t' < t$, and have survived until time t, that is:
  - (i) have not been cancelled;
  - (ii) have not been touched by the price at any intermediate time t'' between t' and t.

- Therefore:

$$\rho(\Delta, t) = \int_{-\infty}^{t} \mathrm{d}t' \int du P(\Delta + u) \mathcal{P}\left(u|\mathcal{C}(t,t')\right) \mathrm{e}^{-\Gamma(t-t')},$$

where $\mathcal{P}(u|\mathcal{C}(t,t'))$ is the conditional probability for the ask difference $u = a(t) - a(t')$, such that $\Delta + a(t) - a(t'') \geq 0$, $\forall t'' \in [t', t]$.

A simple analytical model II

- Assuming that the price follows a Gaussian ranodm walk:

$$\rho_{\text{st}}(\Delta) = \mathrm{e}^{-\alpha\Delta} \int_0^{\Delta} \mathrm{d}u P(u) \sinh(\alpha u) + \sinh(\alpha\Delta) \int_{\Delta}^{\infty} \mathrm{d}u P(u) \mathrm{e}^{-\alpha u},$$

where $\alpha^{-1} = \sqrt{D/2\Gamma}$ measures the typical variation of price during the lifetime of an order.

- When $\mu < 1$, α can be rescaled away, and:

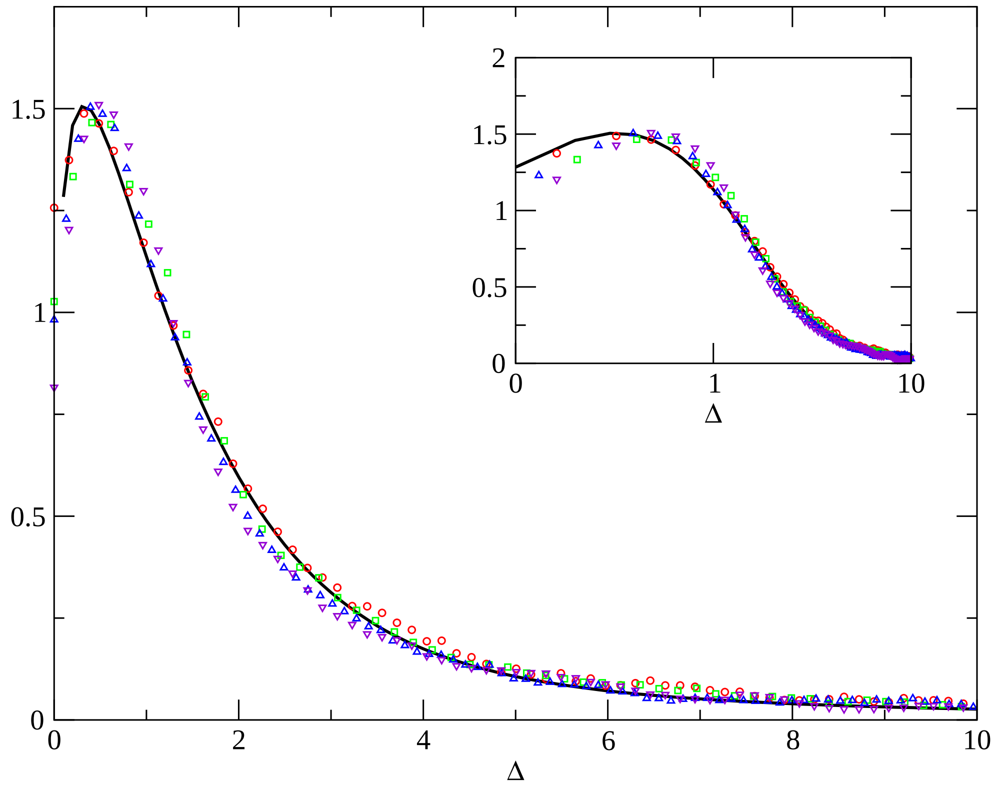
$$\rho_{\text{st}}(\hat{\Delta}) = \mathrm{e}^{-\hat{\Delta}} \int_0^{\hat{\Delta}} \mathrm{d}u\, u^{-1-\mu} \sinh(u) + \sinh(\hat{\Delta}) \int_{\hat{\Delta}}^{\infty} \mathrm{d}u\, u^{-1-\mu} \mathrm{e}^{-u}$$

with $\hat{\Delta} = \alpha\Delta$.

Note: for $\Delta \to 0$, $\rho_{\text{st}}(\Delta) \propto \Delta^{1-\mu} \to 0 \longrightarrow$ hump !

- Reproduces the numerical results satisfactorily

Comparison numerical model - analytical approx.



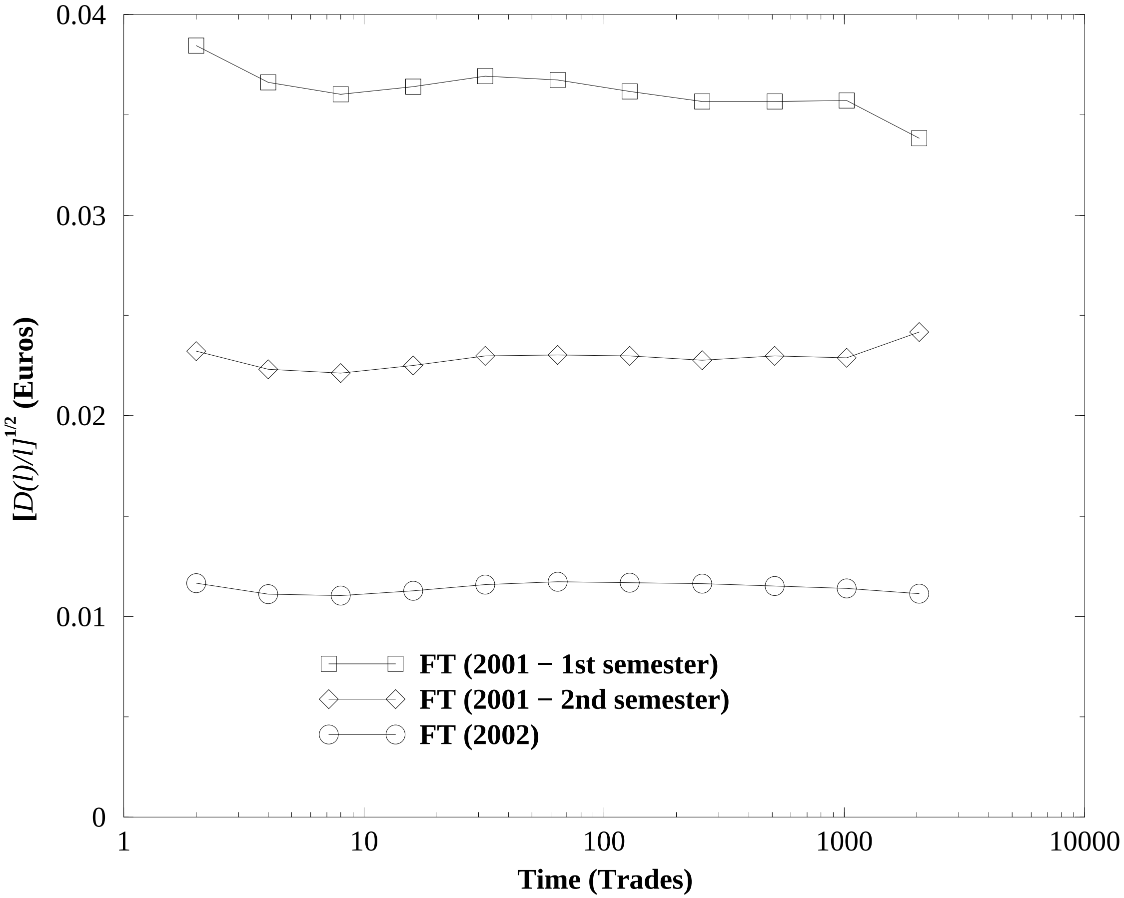
Price dynamics: Diffusion

- Price fluctuations in trade time:

$$\mathcal{D}(\ell) = \left\langle \left(p_{n+\ell} - p_n\right)^2 \right\rangle \approx D\ell$$

- Note : $\sqrt{\mathcal{D}(1)} \sim 0.01$ Euros: precisely the bid-ask spread. True for all stocks.

Price Diffusion



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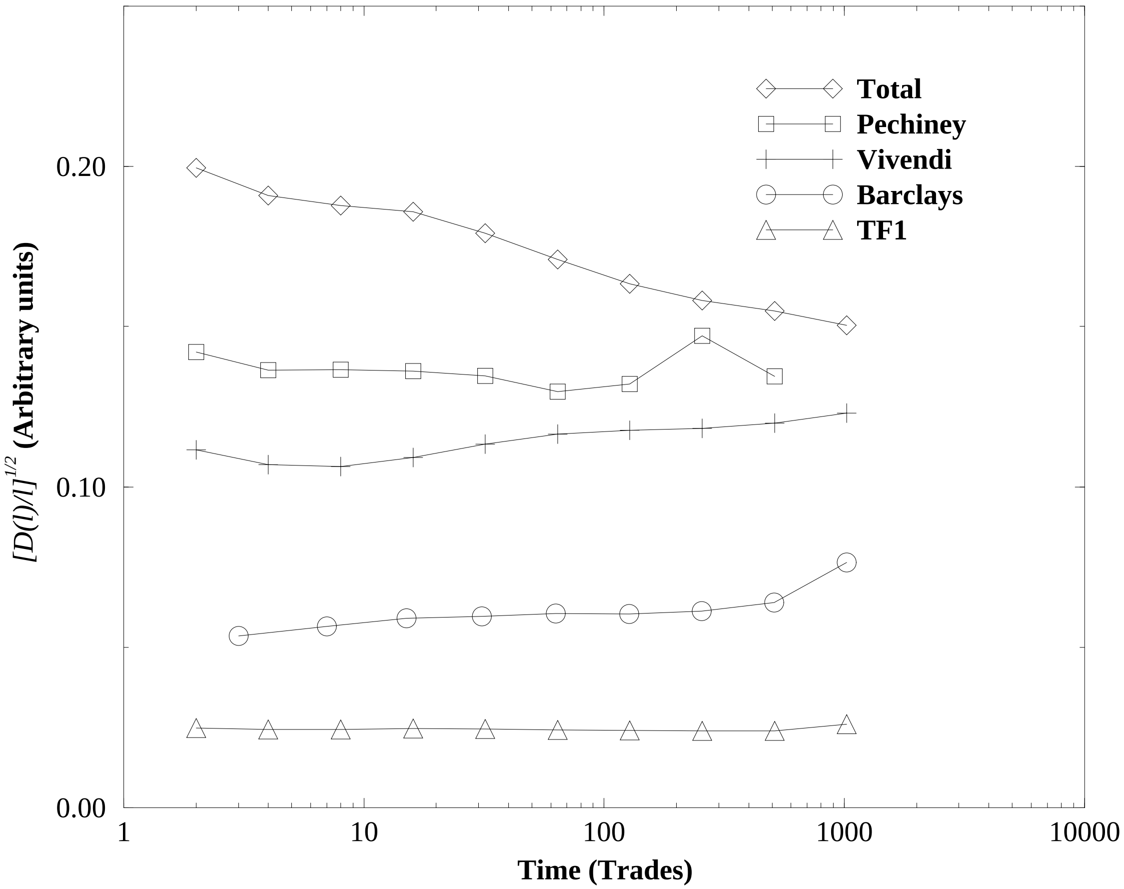
Price Diffusion



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Price dynamics: Response function/Market impact

- Average response function:

$$\mathcal{R}(\ell) = \left\langle \left(p_{n+\ell} - p_n\right) \cdot \varepsilon_n \right\rangle$$

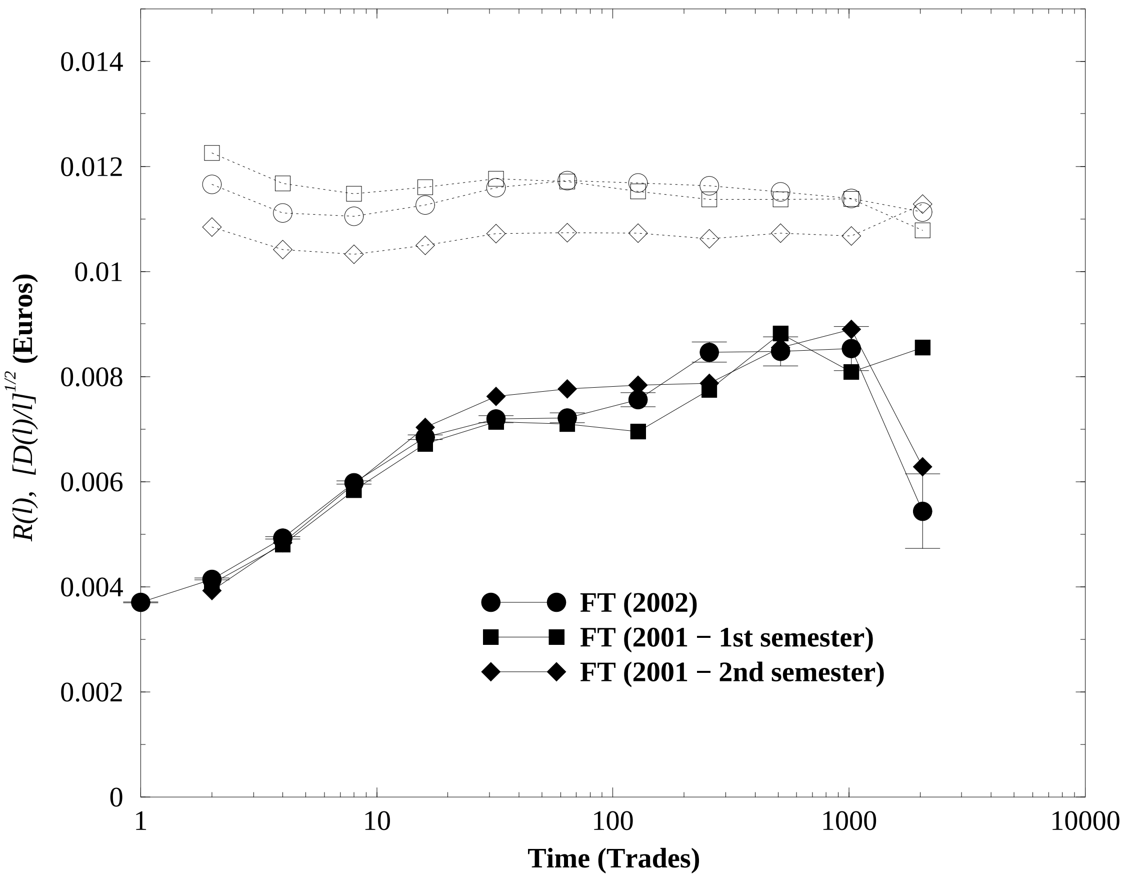
  Weak growth as a function of ℓ and then declines for $\ell > \ell^*$

- Response to a trade of volume V:

$$\mathcal{R}(\ell, V) = \left\langle \left(p_{n+\ell} - p_n\right) \cdot \varepsilon_n \right\rangle\Big|_{V_n = V}.$$

  Approximate factorisation: $\mathcal{R}(\ell, V) \approx \ln V \times \mathcal{R}(\ell)$ – large volumes affect prices less than small volumes ! (cf. Hasbrouck (1991), Gopikrishnan et al., Lillo et al.)

Average response



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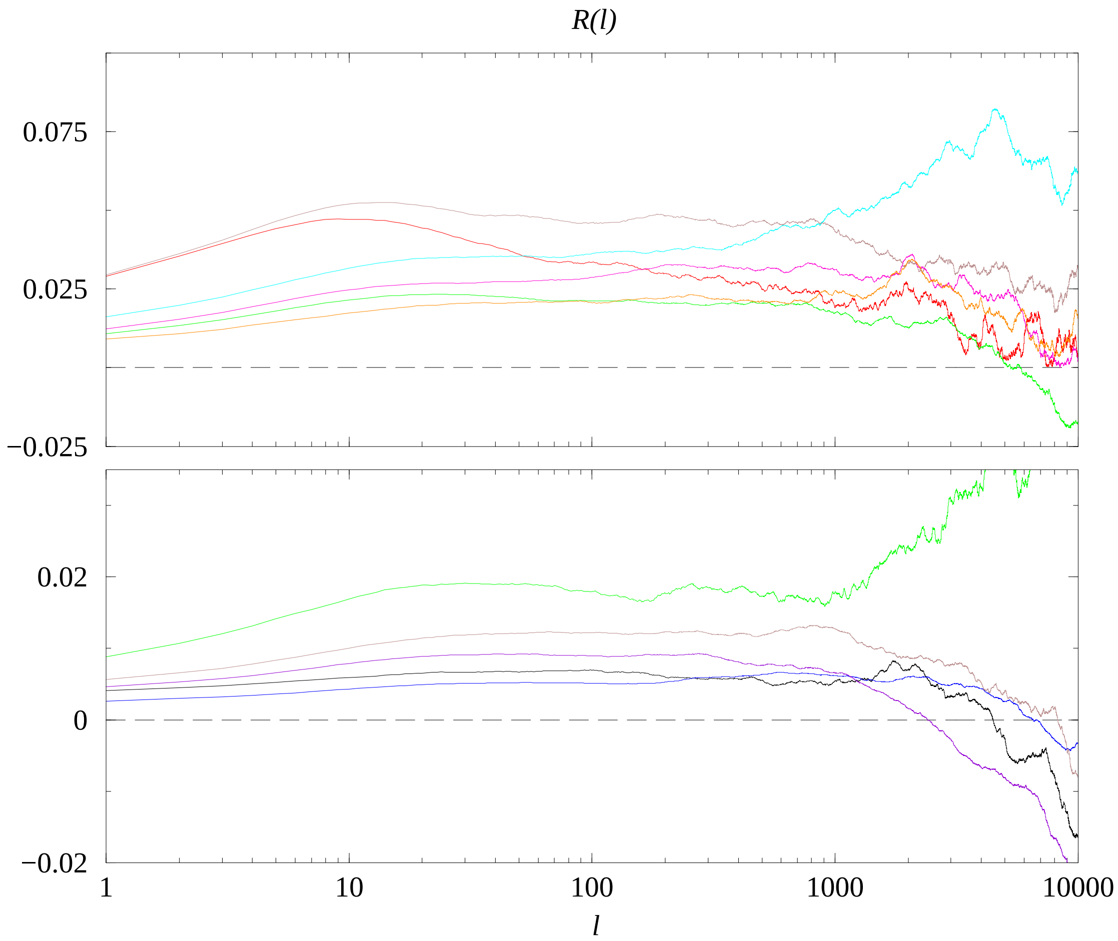
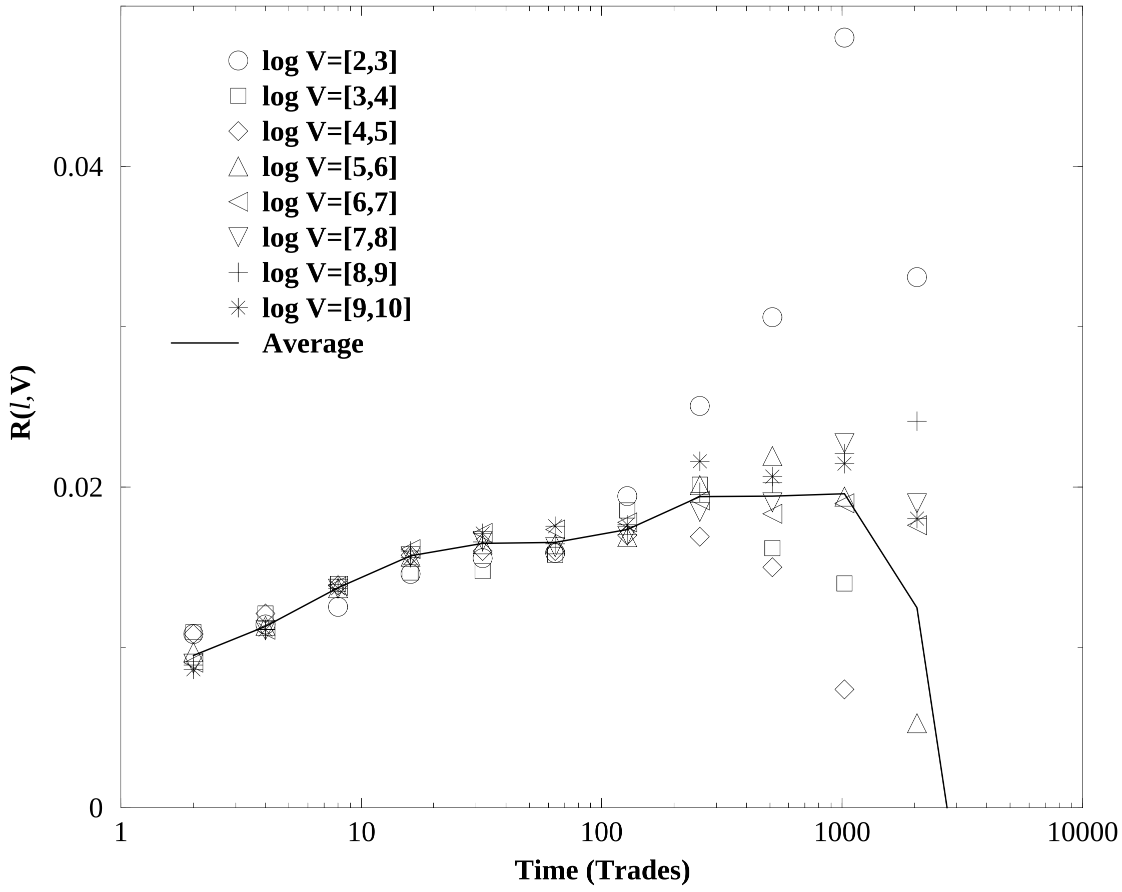
Average response



Response: factorisation



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Price dynamics: Fraction of informed trades

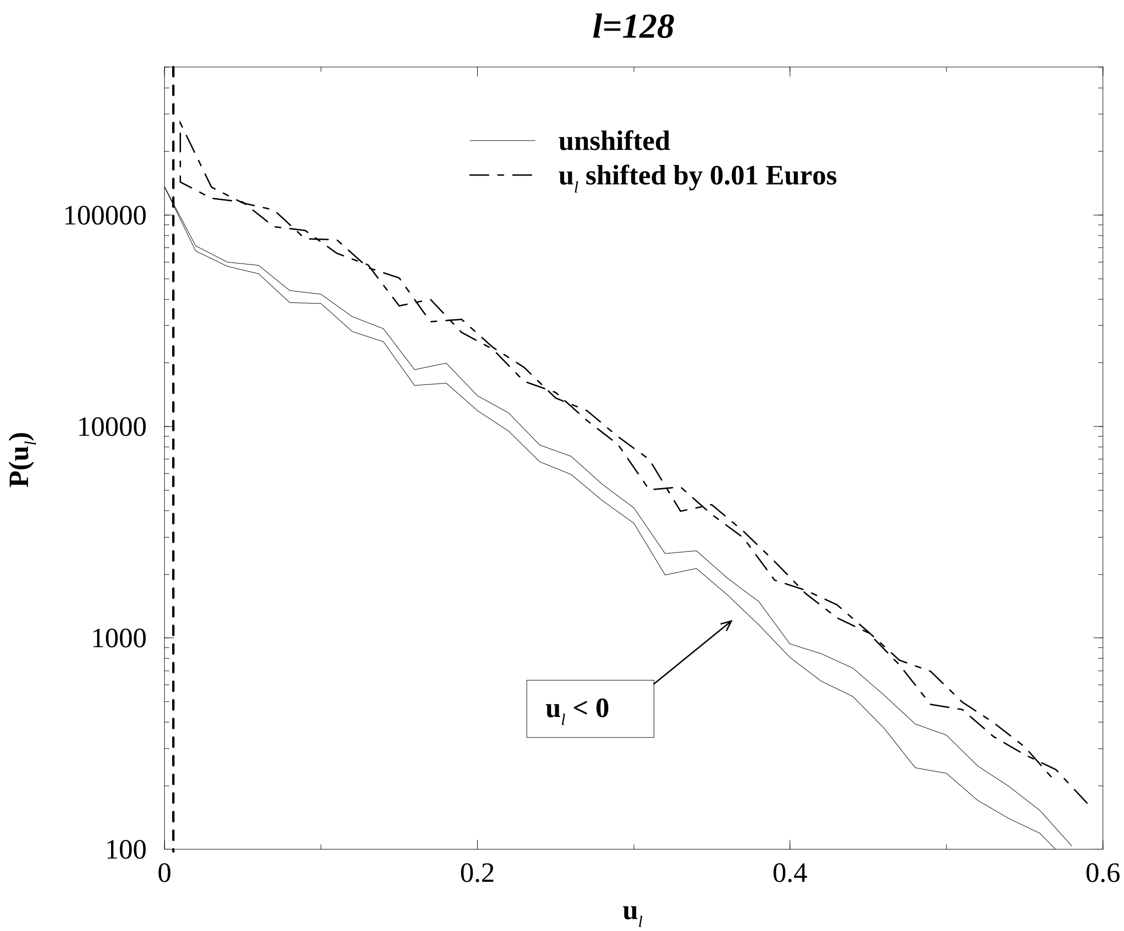
- Full distribution of $u_\ell = (p_{n+\ell} - p_n).\varepsilon_n$:

$$\mathcal{R}(\ell) = \langle u_\ell \rangle \qquad \mathcal{D}(\ell) = \langle u_\ell^2 \rangle$$

- Only very small assymetry that disappears when u_ℓ is shifted by 0.01 Euros; skewness decays as $\ell^{-1/2}$.

- Very few trades can be qualified as 'informed', i.e. correctly anticipating short term moves to at least cover minimal costs (cf. *Do investors trade too much ?* – Odean 1999)

Impact distribution



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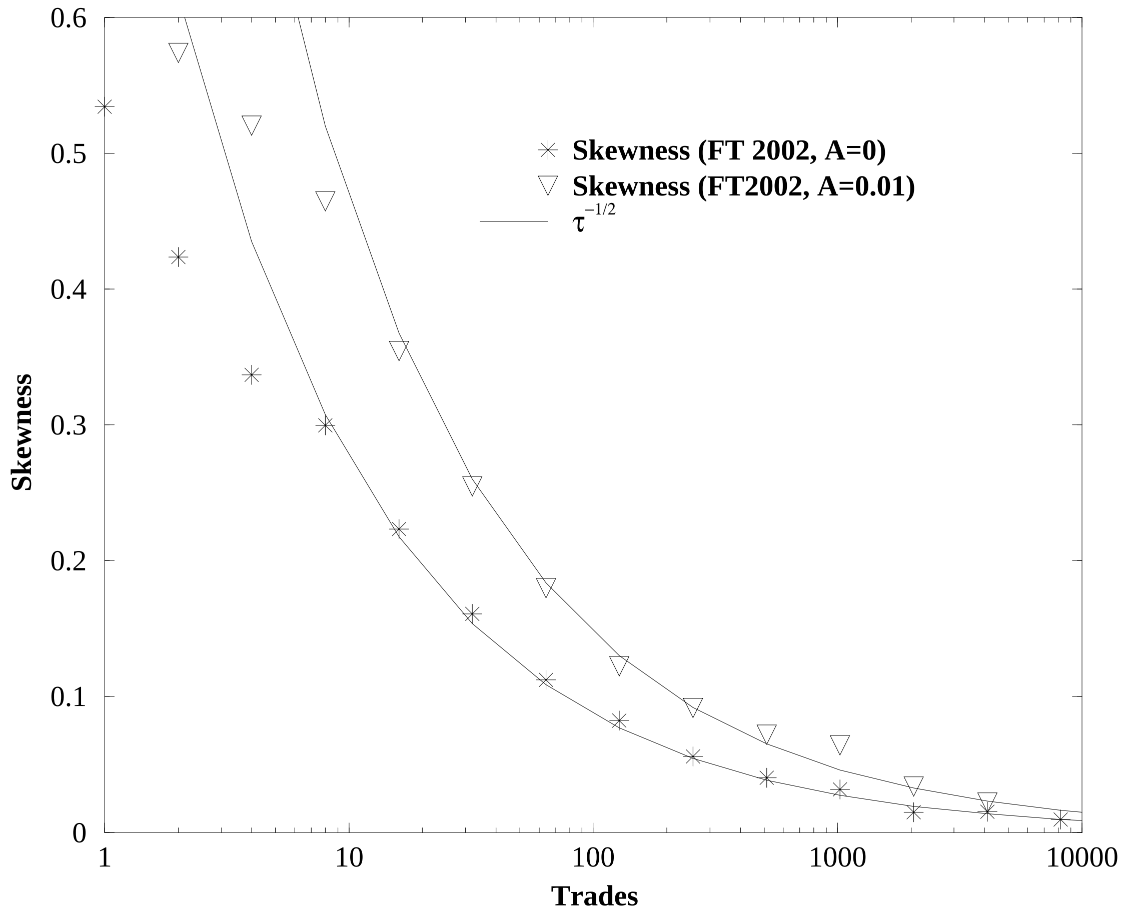
Skewness



Skewness (FT 2002, A=0)
Skewness (FT2002, A=0.01)
$\tau^{-1/2}$
Skewness
Trades
0.6
0.5
0.4
0.3
0.2
0.1
0
1
10
100
1000
10000

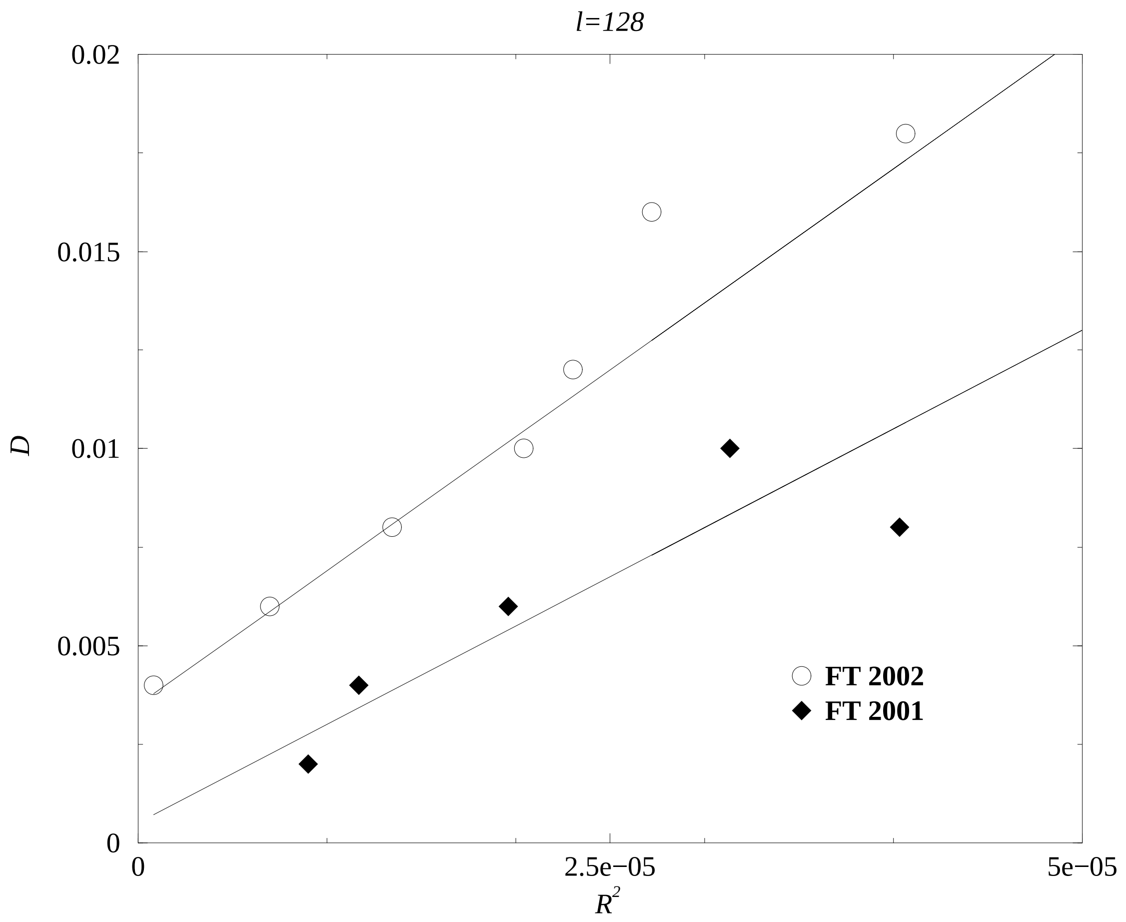
Price dynamics: A fluctuation-response relation

- For Brownian random walks: Mobility = Diffusion/Temperature

- Similar relation in financial markets ? Rosenow 2001

$$\frac{\mathcal{D}(\ell)}{\ell} = A\mathcal{R}^2(\ell) + B$$

Fluctuation-Response Relation



Market order flow: Long term memory

- Trade correlations:

$$\mathcal{C}(\ell) = \langle \varepsilon_{n+\ell}\, \varepsilon_n \rangle \simeq \frac{C_0}{\ell^\gamma}.$$

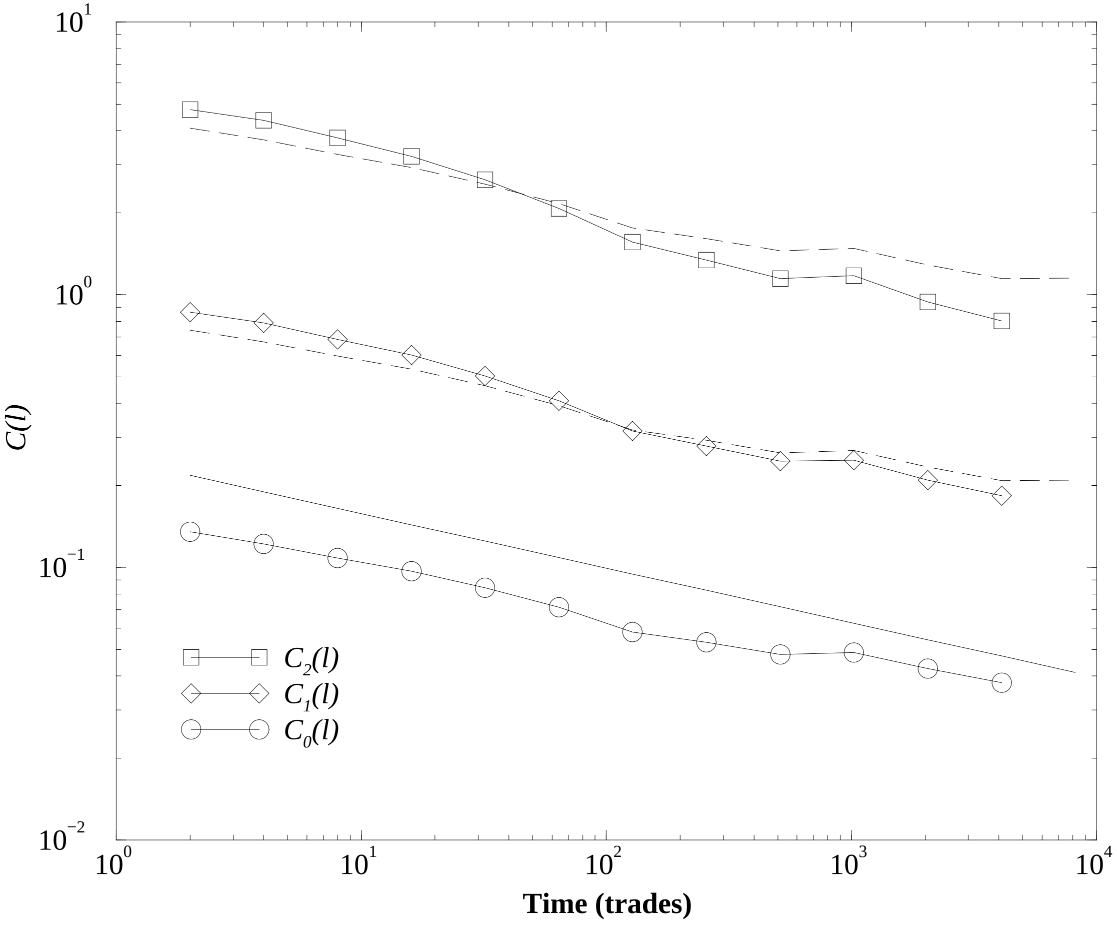
with $\gamma < 1$ ($\gamma \approx 1/4$ for FT, $\approx 1/2$ for Vodafone – see Lillo-Farmer)

- Paradox: The effective number of identical trades grows with ℓ:

$$N_e \simeq 1 + \sum_{\ell=1}^{1000} \mathcal{C}_0(\ell) \approx 1 + \frac{C_0}{1-\gamma} 1000^{1-\gamma} \approx 50$$

- $\mathcal{R}(\ell)$ should increase by a large factor and one should observe superdiffusion.

Trade correlations



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A micro-model of price fluctuations

- Linear superposition of impacts:

$$p_n = \sum_{n'<n} G_0(n-n')\varepsilon_{n'} \ln V_{n'} + \sum_{n'<n} \eta_{n'},$$

where $G_0(.)$ is the 'bare', non permanent response function (or propagator) of a single trade.

- Alternative model – Lillo-Farmer:

$$p_n = \sum_{n'<n} \frac{\varepsilon_{n'} V_{n'}^{\beta}}{\lambda_{n'}} + \sum_{n'<n} \eta_{n'} :$$

permanent, but fluctuating impact depending on instantaneous liquidity – see discussion and comparison in cond-mat/0406224

A simple case first

- Simple case: no correlation in signs

$$\mathcal{R}(\ell) \sim G_0(\ell)$$

$$\mathcal{D}(\ell) \sim \left(\sum_{0<n\leq\ell} G_0^2(n) + \sum_{n>0} \left[G_0(\ell+n) - G_0(n)\right]^2 \right),$$

- For a permanent impact: Constant response and pure diffusion

Role of correlations

- More generally:

$$\mathcal{R}(\ell) = \langle \ln V \rangle G_0(\ell) + \sum_{0<n<\ell} G_0(\ell - n)\mathcal{C}_1(n) + \sum_{n>0} \left[G_0(\ell + n) - G_0(n)\right] \mathcal{C}_1(n)$$

(and a more complicated equation for $\mathcal{D}(\ell)$).

- If G_0 were constant, then $\mathcal{R}(\ell) \propto \ell^{1-\gamma}$ and $\mathcal{D}(\ell) \propto \ell^{2-\gamma}$

- Only way out: the impact of single trades is itself non-permanent

$$G_0(n) = \frac{R_0}{(n_0 + n)^{\beta}}$$

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Role of correlations

- Asymptotic behaviour:

$$\mathcal{D}(\ell) \sim \ell^{2-2\beta-\gamma}, \qquad \mathcal{R}(\ell) \sim \ell^{1-\beta-\gamma}$$

- For diffusion to be normal: $\beta = (1-\gamma)/2 \approx 3/8$

- but $\mathcal{R}(\ell) \sim \ell^{1-3/8-1/4} \sim \ell^{3/8}$ incompatible with data ??

- In fact:

$$\mathcal{R}(\ell) \sim \frac{\Gamma(1-\gamma)}{\Gamma(\beta)\Gamma(2-\beta-\gamma)} \left[\frac{\pi}{\sin \pi\beta} - \frac{\pi}{\sin \pi(1-\beta-\gamma)} \right] \ell^{1-\beta-\gamma}$$

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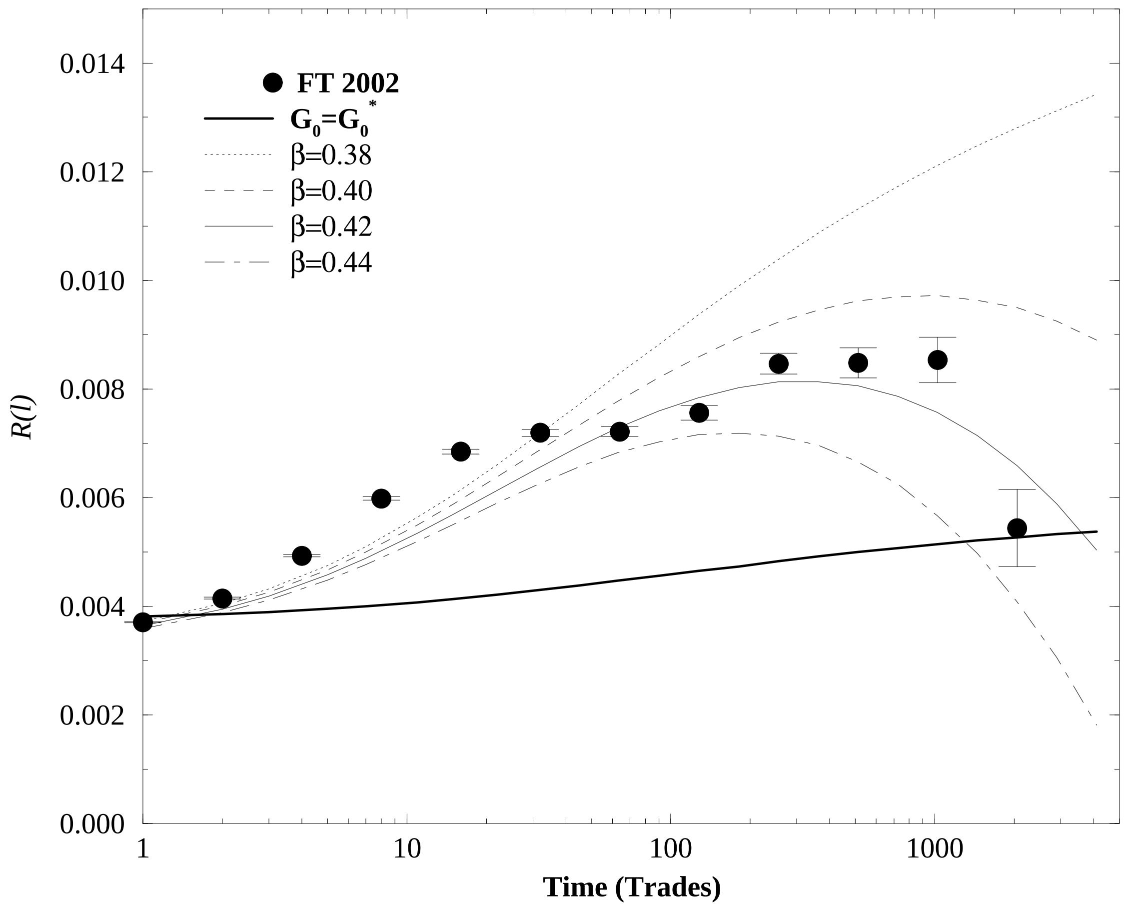
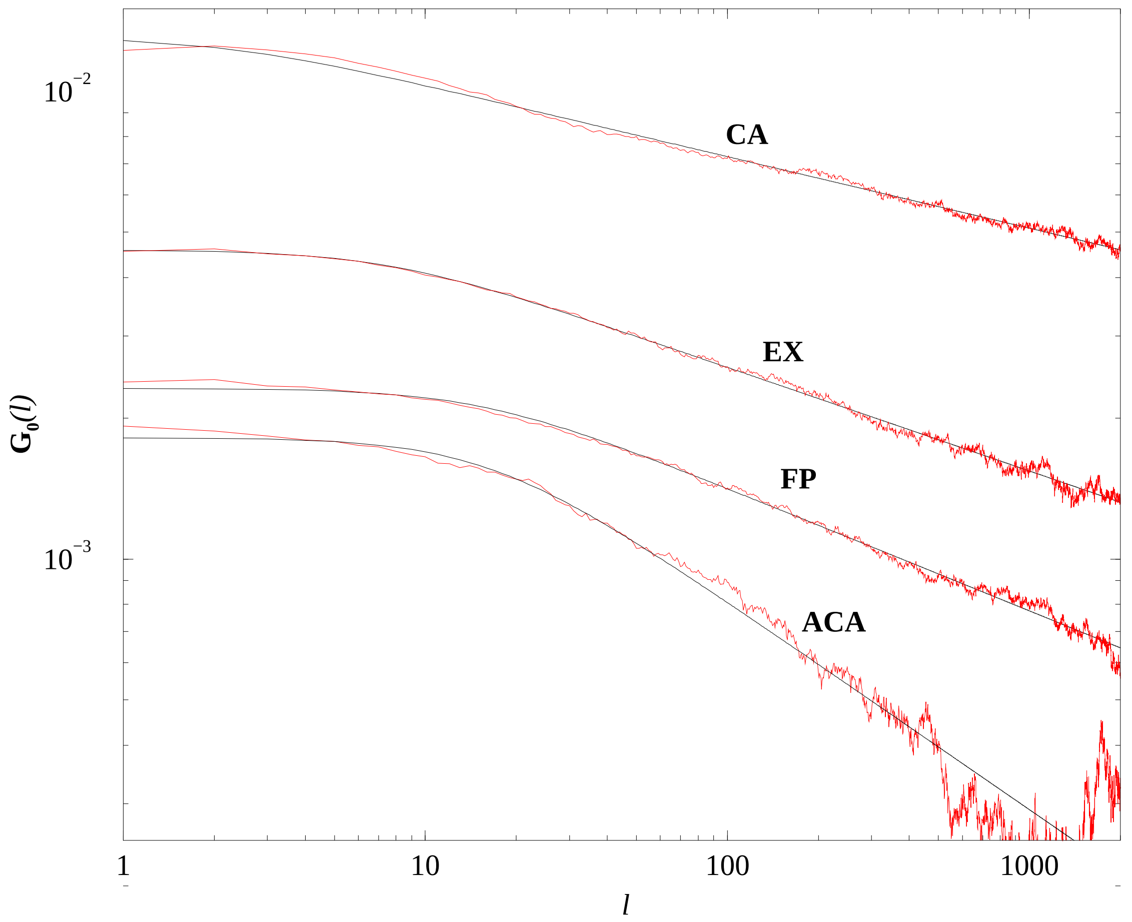
Theoretical and empirical response function



Theoretical and empirical response function



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Interpretation: two antagonist categories of traders

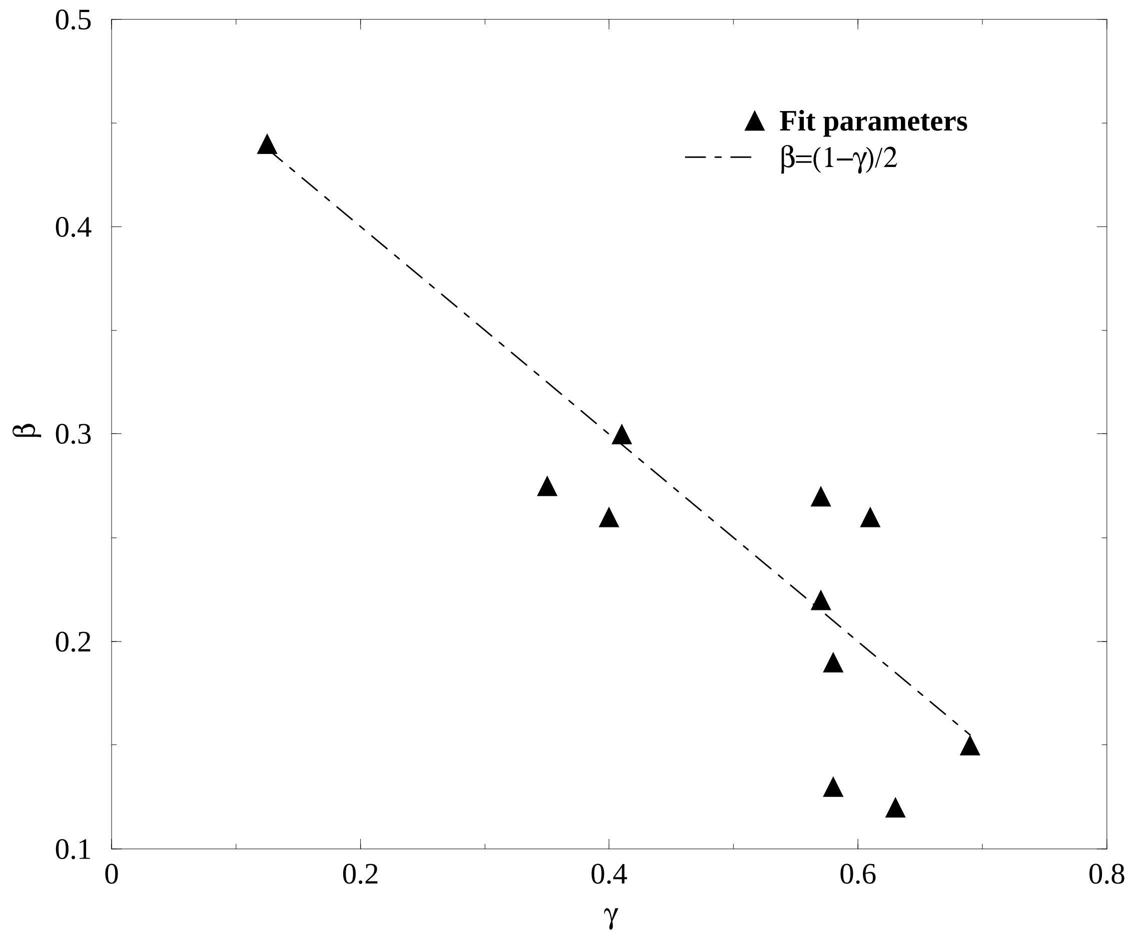
- Liquidity takers: place market orders, as a result of true/putative information, or urge to buy/sell. Must limit their impact $\rightarrow$ orders are cut in small pieces and create serial correlations due to their size.

- Liquidity providers: place limit orders, but no long term positions in markets. Must limit the fluctuations of the price $\rightarrow$ *slow* mean reversal force: *liquidity molasses*. How: order 'barrier' at the ask + anticorrelated quotes.

- Both populations compete such as to remove arbitrage opportunities (linear correlations), and impose $\beta \approx (1-\gamma)/2$.

- Volatility may come from these trading rules alone, and only weakly from external news.

Proximity of the critical line



Conclusion: a critical dynamical equilibrium

- Price diffusion: result from a subtle competition (compensation) between persistent effects (liquidity takers, correlated orders) and antipersistent effects (liquidity providers, mean reverting forces).

- Both effects are characterized by scale-less, power-law functions of time

- Dynamical equilibrium between the two can be temporarily broken $\rightarrow$ large, intermittent fluctuations and crashes.

- cf. Regulation of heart beats and anomalous statistics [H. E. Stanley et al.]; On-Off Intermittency in stick balancing task, [J. L. Cabrera and J. G. Milton, Phys. Rev. Lett. 89, 158702 (2002)]