Economics 220, Jan 2022 **Your Name:** Dhyey Dharmendrakumar Mavani

Homework 5

1. The ten dog owners on Pleasant St. all take their puppies for daily walks during which they greet their puppy friends and poop on the sidewalk. The neighbors follow a 'social norm' (an informally enforced rule) according to which they all pick up their dog's poop. If anyone ever fails to do so, the rest of the community will see the poop, and starting the next day, will cease picking up their own poop (a grim trigger strategy). The cost of picking up poop is 5 units of utility. Living on a poop-covered street costs each resident 8 units of utility each day (so overall, they will be 3 utility units per day worse off if the norm breaks down). All residents discount future utility (payoffs) at a discount rate δ.

For what values of δ can the norm of picking up poop be sustained as an equilibrium?

$$\pi_{cheat} = 0 - 8\delta - 8\delta^2 - \cdots = 5\,\frac{-8\delta}{1-\delta}$$

$$\pi_{collab} = -5 - 5\delta - 5\delta^2 + \cdots = \frac{-5}{1-\delta}$$

or $\pi_{cheat} = 5$

$\pi_{collab} = 3\delta + 3\delta^2 + \cdots$

$$\pi_{collab} > \pi_{cheat} : -8\delta < -5 \Rightarrow 8\delta > 5 \Rightarrow \boxed{\delta > \frac{5}{8}}$$

Suppose that if a dog owner fails to pick up their poop, there is only a 50% chance that it will be observed by others (say, a 50% chance that a sudden rainstorm will wash the poop away before anyone sees it). For what values of δ can the norm of picking up poop be sustained as an equilibrium?

$$\pi_{cheat} = 0 + \frac{1}{2}(-8\delta - 8\delta^2 - \cdots) + \frac{1}{2}(-5\delta - 5\delta^2 + \cdots) = \frac{-4\delta}{1-\delta} - \frac{5\delta}{2(1-\delta)}$$

$$\pi_{collab} : (-5 - 5\delta - 5\delta^2 + \cdots) = \left(\frac{-5}{1-\delta}\right)$$

$$\pi_{collab} > \pi_{cheat} : \frac{-5}{1-\delta} > \frac{-6.5\delta}{1-\delta} \Rightarrow \boxed{\delta > \frac{10}{13}}$$

2. The five professors in a department each drink 10 cups of coffee a week. They used to have to buy (environmentally unfriendly) individual plastic coffee pods, which cost $1 per cup of coffee. To save money (and the planet…), they decided to purchase a communal drip-coffee pot and organize their coffee-drinking on an honor system. Each time they drink a cup of coffee from the pot, they are supposed to add 30 cents to a jar (to cover the cost of ground coffee, filters, and milk). Naturally, since they don't always have the correct change, they tend to add $3 every week or so, and this is considered perfectly fine. However, the honor system makes it difficult for each to determine whether the others are paying their share. Assume that if someone cheats (by not paying their $3 weekly dues), the others figure it out (ie., cheating is detected) with probability *p*. If the cheating is not detected, there is no consequence.

cheat → $10/week
collab → $3/week

For what values of the weekly discount rate, δ, can this honor system be sustained as an equilibrium, assuming a "grim trigger" strategy, such that if anyone is detected cheating, they will be excluded from the communal coffee system and will need to go back to purchasing individual coffee pods?

$$\Pi_{cheat} = 0 - (3+7p)\delta - (3+7p)\delta^2 - \cdots$$

$$\Pi_{collab} = -3 - 3\delta - 3\delta^2 - \cdots$$

$$\underline{\underline{\Pi_{collab} > \Pi_{cheat}}}: \quad \frac{-3}{1-\delta} > \frac{-(3+7p)\delta}{1-\delta}$$

$$\Rightarrow -3 > -(3+7p)\delta$$

$$\Rightarrow \boxed{\delta > \frac{3}{3+7p}} \text{ Ans}$$

3. Consider an instrumental 'friendship' between two individuals, Amy and Bernie. Each period, one of them has an opportunity to do the other a 'favor'. Doing a favor costs the person doing it a cost *c*, and yields a benefit *b* to the recipient, where $b > c$. These opportunities arrive at random, so that in each person is equally likely to be chosen as the potential recipient of a favor in each period. Of course, they are free to decline to perform a favor when the opportunity arises, in which case, the friendship will end (and they receive zero payoffs thereafter).

As a function of b and c, for what values of the discount rate, δ, can the friendship be sustained?

$$\boxed{E(\text{benefits}) = \tfrac{1}{2}(b-c)}$$

$$\Pi_{cheat} = c + (0\delta + 0\delta^2 + \cdots) = c$$

$$\Pi_{collab} = 0 + \delta E(\text{benefits}) + \delta^2 E(\text{benefits}) + \cdots$$

To sustain the friendship $\quad \Pi_{collab} > \Pi_{cheat}: \quad \frac{\frac{1}{2}(b-c)\delta}{1-\delta} > c$

$$\Rightarrow \left(\frac{b}{2c} - \frac{1}{2}\right)\delta > 1 - \delta$$

$$\Rightarrow \frac{b\delta}{2c} + \frac{\delta c}{2c} > 1$$

$$\boxed{\delta > \frac{2c}{b+c}}$$

Suppose now that the friendship is somewhat asymmetric; each period, as before, one person has the opportunity to do the other a favor. But now, the probability that Amy can do a favor for Bernie is 0.6 (and with probability 0.4, Bernie has an opportunity to do a favor for Amy).

not obey

As a function of *b* and *c*, for what values of the discount rate, δ, can the friendship be sustained? Can you explain why it changed in the direction that it did?

$$E(\text{benefits}) = 0.4b - 0.6c$$

$$\Pi_{cheat} = c + (0\delta + 0\delta^2 + \cdots) = c$$

$$\Pi_{collab} = 0 + \delta E(\text{benefits}) + \delta^2 E(\text{benefits}) + \cdots$$

$$= \left(\frac{\delta E(\text{benefits})}{1-\delta}\right)$$

$$\frac{\delta(0.4b - 0.6c)}{1-\delta} > c \quad \Rightarrow \quad \delta(0.4b - 0.6c) > c - \delta c$$

$$\Rightarrow \delta(0.4b) + \delta(0.4c) > c$$

$$\boxed{\delta > \frac{5c}{2(b+c)}}$$