Price Impact Models and Applications

Introduction to Algorithmic Trading

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Columbia University

Introduction

Class Lecturer

Kevin Webster

(a) Currently on garden leave – Adjunct Assistant Professor at Columbia University, Visiting Reader at Imperial.

(b) Previously doing quantitative trading at Citadel and alpha research at Deutsche Bank.

(c) PhD in Financial Engineering from Princeton, undergraduate at Ecole Polytechnique.

(d) Just completed a textbook (link) on which the course is based.

Teaching Assistants

Long Zhao

(a) Fifth-year PhD student at Columbia University working with Marcel Nutz.

(b) Previously interned at Bank of America and Hudson River Trading.

Shashank Choudhary

(a) Second year MAFN student.

(b) Previously worked at HSBC.

Recommended “Light” Reading (Optional)

My top four books

(a) *Trades, Quotes and Prices* by Bouchaud et al. (2018, link): empirical book on trading data.

(b) *Algorithmic Trading and Quantitative Strategies* by Velu, Hardy and Nehren (2020, link): generalist book, reviews trading infrastructure in depth.

(c) *Machine Learning and Big Data with kdb+/q* by Novotny et al. (2020, link): one of the few books on kdb+.

(d) *Causality* by Pearl (2008, link): *not* a finance book, but core to causal inference.

Kdb+ Documentation (Recommended)

Kx website

(a) references: `code.kx.com/q/ref`

(b) course: `kx.com/academy`

(c) guide for quants: `code.kx.com/q/learn/brief-introduction`

(d) Q for mortals: `code.kx.com/q4m3`

Recommended “Heavy” Reading (Very Optional)

A brief summary of research papers

(a) Almgren and Chriss (2001) first articulate algorithmic trading as a live optimization problem.

(b) Obizhaeva and Wang (2005) propose a price impact model with static parameters and solve the optimal trading strategy.

(c) Fruth, Schoeneborn, and Urusov (2013, 2019) generalize the model and strategy to allow dynamic parameters. Muhle-Karbe, Wang, and Webster (2022) analyze the model further.

(d) Bouchaud et al. (2004, 2006, 2009, 2015, 2022), Almgren et al. (2005), and Cont et al. (2014) fit price impact models on high-frequency data.

(e) Westray and Kolm (2022) and Cont et al. (2022) use machine learning to fit alpha signals on order flow imbalance.

(f) Lopez de Prado (2022) outlines applications of causal inference to finance.

What is Algorithmic Trading?

A general definition
Algorithmic Trading is an automated process taking input signals and producing trades as outputs.

Trading algorithms can be

(a) rule-based.

(b) model-based.

This class focuses on model-based strategies.

Signal Examples
Isichenko (Bloomberg, 2021) details "Event-based predictors":

> *"One can use material company news, mergers and acquisitions, earnings surprises, index rebalancing, or even dividends and splits."*

Why does Algorithmic Trading Matter?

The U.S. Securities and Exchange Commission (2020)

> *"A common theme echoed by nearly all market professionals, academic researchers, and other students of the securities markets is that algorithmic trading, in one form or another, is an integral and permanent part of our modern capital markets."*

Algorithmic trading affects all market professionals.

(a) High-frequency traders and statistical arbitrageurs specialize in algorithmic trading.

(b) Brokers use trading algorithms on behalf of clients.

(c) Portfolio managers trade sizable positions: trading algorithms allow them to scale up.

Rising Algorithmic Trading

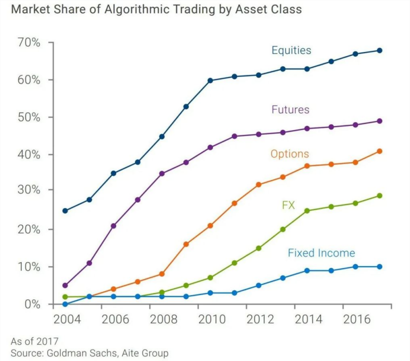


Figure 1: Algorithmic trading follows an asset's electronic trading: e.g., FX and Fixed Income are partly electronic and over the counter.

Electronification of Fixed Income is the new frontier,
but even mature asset classes see continued growth.

Trading Strategy Architecture

Trading infrastructures are complex,
and especially for mature asset classes trading algorithms are *nested* within other processes.

Why the complexity?

(a) *Operations*, also called *production:* making sure the algorithm works as intended.

(b) *Technology:* sizable data, latency requirements and feedback loops require care.

(c) *Scientific rigor:* model-driven trading requires backtests and live trading tests to maintain confidence in the algorithm.

Trading Algorithms (1/3)

The Schema

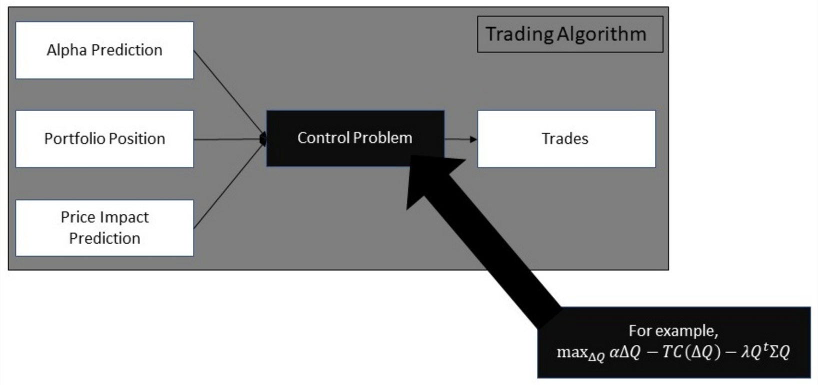


Figure 2: Trading algorithm schema. The algorithm runs in *simulation* (backtest) and *live* (production).

Trading Algorithms (2/3)

The Production Schema

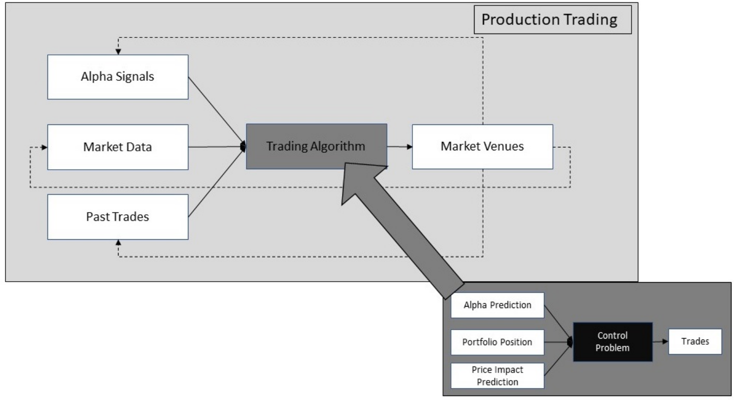


Figure 3: Schema in production trading: live trading connects inputs and outputs in a *feedback loop.*

The Research Schema

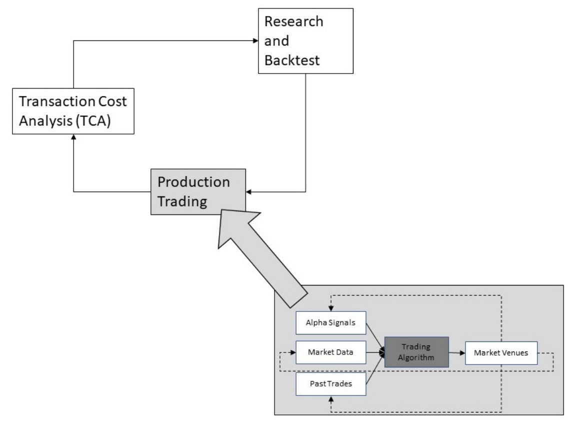


Figure 4: Research cycle schema.

Class scope (1/2)

What you will learn?
How to

(a) measure a trading strategy's performance.

(b) build an algorithmic trading strategy.

(c) test a trading algorithm.

(d) fit trading signals: both *alpha* and *impact.*

(e) handle trading data.

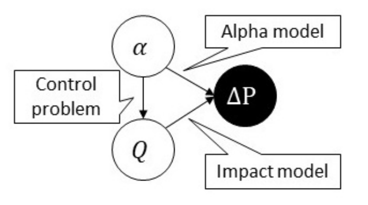
What roles need this?
Quantitative developers build production and backtest tools.

Quantitative researchers use the tools to build strategies and signals on massive datasets.

Quantitative traders measure and monitor algorithms and signals daily.

Class scope (2/2)

A mental model of quant research



(a) Alpha researchers predict returns ΔP using signals α.

(b) Trading algorithms turn signals α into trades Q.

(c) Trading researchers estimate and backtest the effect of trades Q on returns ΔP.

This class details $\alpha \to Q$ and $Q \to \Delta P$. $\alpha \to Q$ is assumed given.

Three Modules

(1) Introduction to Algorithmic Trading ($\approx$ 3 weeks)

(2) Using Price Impact Models ($\approx$ 6 weeks)

(3) Estimating Price Impact Models ($\approx$ 5 weeks)

Weekly homework accounts for 100% of the grade.
Homework is both proof- and code-based. Two weeks have light reading but do not require you to hand in an assignment.

(1) Introduction

(a) What is price impact? Overview of finance applications and glossary of trading terms.

(b) Preview of the two modules' mathematics and data.
Homework: read "direct estimation of equity market impact" by Almgren (2005).

(c) Primer on the database kdb+ and the programming language q.
Homework: first coding steps in kdb+/q.

(2) Using Price Impact Models

(a) Mathematical foundation of price impact. Example on the Obizhaeva and Wang (OW) model. Homework: proof-based exercises.

(b) The generalized OW model. Absence of price manipulation. Homework: empirical estimation of a price impact model (code, 1/2).

(c) Empirical review of price impact models. Homework: empirical estimation of a price impact model (code, 2/2).

(d) Optimal execution. Homework: proof-based exercises.

(e) Back testing and statistical arbitrage. Homework: backtesting rule-based strategies (code).

(f) Risk management. Homework: backtesting a model-based strategy (code).

(3) Estimating Price Impact Models

(a) Bouchaud's list of four causal trading biases. Introduction to live trading experiments. Homework: simulate a live trading experiment (code).

(b) The Mathematics of causal inference (1/2). Homework: proof-based exercises.

(c) The Mathematics of causal inference (2/2). Application to prediction bias. Homework: implement causal regularization for prediction bias (code).

(d) Transaction Cost Analysis (TCA). Homework: read "The Non-Linear Market Impact of Large Trades" by Bershova and Rakhlin (2013) and "Causal Factor Investing: Can Factor Investing Become Scientific?" by Lopez de Prado (2022), proof-based exercises.

(e) Further applications of causal inference and causal regularization. Cross impact. Homework: proof-based exercises.

Plan

For this week

(a) Price impact.

(b) Trader terminology: alpha signals and arrival slippage.

Next week
Preview of the two modules, trading data.

Price Impact

What is Price Impact?

Bouchaud et al. (CFM, 2018)

"Price impact is a reaction to order-flow imbalance."

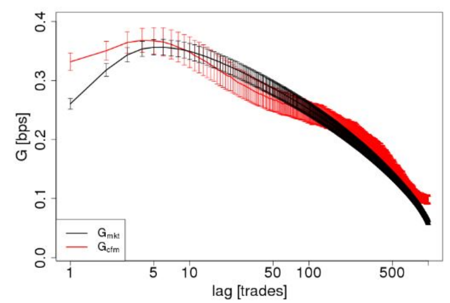


Figure 5: Average return profile after a single public (black) or Capital Fund Management (CFM, red) trade fill (Toth, CFM 2018).

Price Impact as a Causal Model

Price impact and "what-if" scenarios:

Trading of stock cause price moves for the stock that otherwise would not have happened.

Figure 6: Simulated "what-if" scenarios.

Why do Traders Model Price Impact? (1/3)

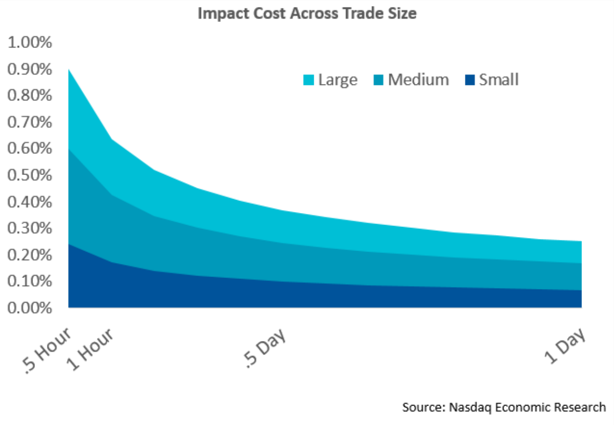
To reduce their trading costs.

> *"Market impact costs are typically used and considered by large financial institutions when determining the viability of a security purchase." (Capital.com)*

Figure 7: If I trade faster, I make my future fills more expensive.

Why do Traders Model Price Impact? (2/3)

To determine their order size and speed
See Nasdaq's "How Fast Should You Trade?" (`www.nasdaq.com/articles/how-fast-should-you-trade-2019-11-07`)



Why do Traders Model Price Impact? (3/3)

To detect other traders.

> *"The idea that it is order-flow that must be predicted, even if uninformed, resonates well with the intuition of finance professionals and allows one to understand why statistical regularities might exist and be exploited by quant firms. Indeed, flow data is quite popular among statistical arbitrage funds" (Bouchaud (CFM, 2021))*

One trader's impact is another trader's alpha.
If I predict someone else trading, I know the price will go up and then revert.

A Thought Experiment

Bouchaud (CFM, 2018)

"One would ideally like to assess the impact of a market order by somehow measuring the difference between the mid-price in a world where the order is executed and the mid-price in a world where all else is equal but where the given order is not executed [...] **this definition cannot be implemented in a real financial system**, *because the two situations (i.e. the market order arriving or not arriving) are mutually exclusive, and history cannot be replayed to repeat the experiment in the very same conditions"*

The Causality Challenge in Research

Quants continuously improve strategies.
The following principle is a significant hurdle:

> *A trader cannot directly observe two strategies' outcomes for a single trade.*

Why causality?
The causality challenge states that traders cannot achieve "what-if" scenarios in live trading: one can only estimate novel scenarios using models.

(a) A-B tests can *statistically* model a what-if scenario.

(b) Back tests can *simulate* a what-if scenario under a given model.

Trading Experiments Measure Scenarios

What is an A-B test?
A-B tests are *live trading experiments* where a coin flip determines the live trading scenario.

For example, one could randomize the decision to use strategy A or B to determine which performs best.

A-B test results are *statistical*: competing scenarios never happen on identical data.

What is a back test?
Back tests simulate competing scenarios on identical data (historical or simulated).

For example, one may simulate strategy A and B's P&L for past orders.

Simulations rely on a *data generation model* to simulate data for novel scenarios.

Three Hazards of the Causality Challenge

Fundamental measurement uncertainty
It is impossible to conclude with certainty whether the trade successfully *predicted* the price move or whether the trade *caused* it.

The Hedge Fund Journal interviews Rob Almgren

> *"Impact cannot actually be disentangled from alpha due to the endogeneity issue – it is not possible to work out if the order caused the move or the reverse."*

Price manipulation paradoxes
Many models that statistically describe price impact present price manipulation paradoxes.

Hidden assumptions
Trading infrastructures are technologically complex and can harbor hidden assumptions.

Example
At the start of the day, a portfolio manager asks a trader to buy 1000 shares by end of the day.

(a) The price begins at \$100.

(b) The trader expects the price to end at \$101: the expected *paper* profits are

$$1\% = 100bps.$$

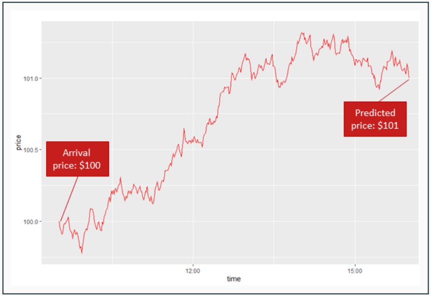
This is the trade's predicted alpha.

What the trader controls:

(a) The execution speed.

(b) Execution location, e.g., venue.

Order Slippage



Trades don't execute instantly.
The execution price *slips* the \$100 arrival price.

Arrival slippage
If the average execution price is \$100.2, then the arrival slippage is

$$0.2\% = 20bps.$$

The order *realizes* a $100bps - 20bps = 80bps$ profit.

Why Traders Optimize Order Slippage

Minimizing order slippage equals maximizing the portfolio's P&L.
The trader doesn't need to know the portfolio position to maximize its P&L.

Informal Formulas
If Q is a portfolio position, ΔQ a day order, and δQ its execution, then

$$P\&L = \underbrace{Q\Delta P}_{\text{position P\&L}} + \underbrace{\Delta P \Delta Q}_{\text{order's paper P\&L}} - \underbrace{TC(\delta Q)}_{\text{order's slippage}}$$

where ΔP is a daily return.

This separation of objectives
only works because the portfolio manager, not the trader, controls ΔQ and Q. The trader only controls δQ and may not even know Q.

Impact and Alpha Slippage (1/3)

The objective function isn't enough.
For example, it doesn't answer the fundamental question:

Should the trader have traded faster or slower?

Informal Formulas
In addition to the objective

$$TC(\delta Q)$$

one wants the gradient

$$\nabla TC(\delta Q)$$

to optimize the trading decision.

Impact and Alpha Slippage (2/3)

Two opposing slippage drivers:

(a) If the price moves regardless of trading, one must speed up.

(b) If the price moves *because* of trading, one must slow down.

Traders refer to the former as *alpha slippage* and the latter as *price impact.*

Informal Formulas
Later slides will provide explicit formulas, but informally:

$$TC(\delta Q) = \alpha(\delta Q) + I(\delta Q)$$

with

$$\nabla\alpha(\delta Q) < 0; \quad \nabla I(\delta Q) > 0$$

when δQ speeds up.

Impact and Alpha Slippage (3/3)

A good practitioner quote from Velu, Hardy, and Nehren (ADIA, 2020)

> *"Large scale trading will often occur in the presence of market drift (alpha) and the realized execution cost is a combination of alpha and the price impact.*
> *[...]*
> *If we find that most of the cost is due to price drift, the best option would be to accelerate the trading and pay more impact to capture more attractive prices. Conversely, if the cost is mostly driven by impact then it would behoove us to slow down our trading to minimize the impact."*

Transaction Cost Analysis (TCA, 1/3)

Bershova and Rahklin (Alliance & Bernstein 2013)

> *"Practitioners use impact models as a pre-trade tool to estimate the expected transaction cost of an order and to optimize the execution strategy."*

TCA answers "what-if" questions

(a) What if we submitted a larger order?

(b) What if we traded the order faster?

(c) What if we traded passively in a dark pool?

Mock TCA report

Grouping	Order group	Nb of orders traded	Notional traded	Order slippage	Realized alpha	Predicted alpha	Price impact	Alpha slippage	Spread and fees
By trading strategy	Aggressive strategy	70k	7bil	**35bps (±15bps)**	**50bps**	50bps	25bps	10bps	4bps
	Passive strategy	30k	4bil	**25bps (±10bps)**	**40bps**	50bps	10bps	15bps	0bps
A-B test (experiment)	A: keep current speed	50k	5.5bil	**35bps (±10bps)**	**45bps**	50bps	20bps	15bps	3bps
	B: trade slower	50k	5.5bil	**25bps (±10bps)**	**45bps**	50bps	8bps	17bps	1bps

Unbiased A-B test confirms that, overall, trading slower outperforms trading faster

Aggressive strategies have more slippage but also trade when we have more alpha: Alpha bias present in dataset

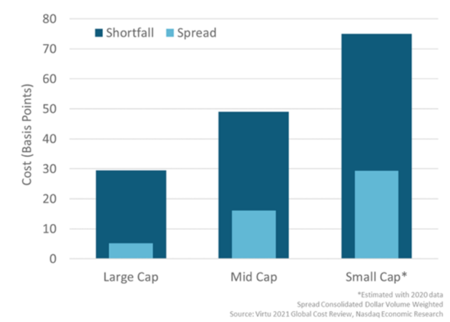
Regular TCA reports (e.g., monthly)

Enable traders to improve trading algorithms and find inconsistencies with back tests.

The 2022 Intern's Guide to Trading, Nasdaq (2022)
`www.nasdaq.com/articles/the-2022-interns-guide-to-trading`.

> *"Even though average trade costs are reported at just 0.31%, that adds to around $70 billion each year in trading costs."*

Mutual fund and hedge fund total trading costs and spread costs:



Trader Terminology

Trading Signals

Quantitative developers, researchers, and traders
spend time building and analyzing signals. A trading algorithm may have hundreds of signals, thousands of features, and millions of model parameters.

Trading signal teams are massive
Consequently, teams standardize signals to facilitate their communication and re-usability. For example, the next five slides asks questions another researcher may have about your signal.

Trading Signal Questions (1/5)

Is it directional or non-directional?
For instance, a return prediction is directional, and a volatility prediction is non-directional. Traders use non-directional signals to *re-scale* directional signals.

Example: scaling for the time of day
Let α be signal fitted on the whole day. One wishes to change the prediction by the time of day, e.g., every 30 min. Instead of calibrating a separate model per time of day, one re-scales

$$\alpha_t = \frac{\sigma_t}{\sigma}\alpha$$

where σ_t is a volatility prediction per time of day and σ a daily forecast. For instance, if predicting from t to T and assuming constant volatility, use the rescaling

$$\sigma_t = \sigma\sqrt{T - t}.$$

Intuition: alpha scales with volatility.

Trading Signal Questions (2/5)

What horizon does it predict?
For example, one predicts the price for the next tick, the next hour or at the day's close. Researchers re-use the same model features and architecture across horizons but with different parameters.

Example: decay kernel
Let f_t be a feature used in a *decay kernel* signal

$$\alpha_t = \sum_{s<t} k(t-s) f_s$$

where every minute, α_t predicts a forward 30 min return. This model generalizes to horizons $h \in \{1, 5, 15, 90\}$ min:

$$\alpha_t^h = \sum_{s<t} k^h(t-s) f_s.$$

The model parameters $k^h(0), k^h(1), \ldots$ are horizon-specific but re-use the feature f.

Trading Signal Questions (3/5)

How frequently does it trigger?
For example, a signal triggers every hour, every second, every time a trade prints on the tape. Trigger times drive production latency requirements.

Example: conditional on news
Consider a signal α_t that is too weak to deploy on its own: it captures less alpha than the bid-ask spread.

Let $\{t_i\}_{i \geq 0}$ be the set of Bloomberg news events on the stock. α_{t_i} may be larger than the bid-ask spread.

Trading Signal Questions (4/5)

Is it endogenous or exogenous?
Endogenous signals are *path-dependent*: the signal depends on one's past trades. Path-dependent signals are *challenging* to reproduce in simulation.

Example: de-trended volume signal
Let v_t be the trades on the public tape and q_t our algorithm's trades. A volume signal

$$\alpha_t = f(v)$$

is endogenous, as v includes q. A de-trended signal

$$\tilde{\alpha}_t = f(v - q)$$

is exogenous. Some signals are *approximately* exogenous for small trades.

Trading Signal Questions (5/5)

Is it cross-sectional?
A signal is cross-sectional if the stock prediction depends on other stocks. They can (a) capture causal effects between stocks (b) increase statistical robustness.

Cross-sectional predictions *change* when the stock universe changes, reflecting a *relative* bet: a *growth* stock for the S&P 500 is not a growth stock for the Russell 3000.

Example: supply chains
Consider a stock-specific signal α and a supply chain graph $\mathcal{G}$ across the S&P 500. One may update the alpha based on the supply chain:

$$\tilde{\alpha}^i = \alpha^i + \rho \sum_{(i,j)\in\mathcal{G}} \alpha^j.$$

for $\rho \in (0, 1)$. $\tilde{\alpha}$ is a cross-sectional alpha on the S&P 500.

Why all these questions?

Questions reflect "tricks" and recipes.
The answers often boost signals, but may hamper the signal in a different context:

Standardization and communication drives re-usability.

Example
A trader uses a stock-specific signal for AAPL in any portfolio. However, a cross-sectional signal for AAPL should only be used on a portfolio with the same universe as the cross-section.

Where Alpha Research and Algorithms Intersect

What's the statistical loss function?

(a) Standard (naive) answer: least-square loss function.

(b) Ideal answer: algo strategy's P&L.

(a) is independent of the trading algorithm and uses off-the-shelf implementations. (b) specializes the alpha for a given algorithm but requires a *model* and custom implementations (see Week 4).

Practical compromises include least squares with algorithm-derived weights and classification classes considering historical algorithm outcomes.

Informal definition of an alpha signal

Definition

An alpha signal is a directional, exogenous signal predicting returns.

Therefore, an alpha signal ignores one's impact and impact decay. It can span multiple prediction horizons and trigger frequencies. It could be cross-sectional or stock-specific.

Example trader lingo:

"This is a stock-specific, forward 30min alpha triggering every 5 minutes. Maybe cross-sectionally normalizing it will make it work over a daily horizon?"

Intended, Predicted, and Realized

Predicted and realized alpha
Traders refer to the *realized alpha* as the realization of the return the alpha signal predicts and the alpha signal as the *predicted alpha*.

Intended and realized trades
Similarly, practitioners refer to the trades an algorithm outputs as *intended* trades and compare them to the *realized* trades on the market.

Example
A strategy intends to trade ten thousand shares in the hour but only realizes nine thousand due to adverse market conditions.

Predicted and Realized Alpha

An example from Almgren's Quantitative Brokers
See `quantitativebrokers.com/whitepapers/2021-11`.

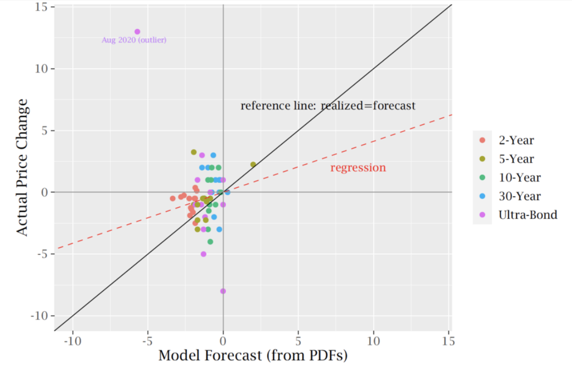


Figure 8: Scatterplot of realized against predicted alpha across trading instruments for QB models.

Intended and Realized Trading Data

Refresher: The Production Schema

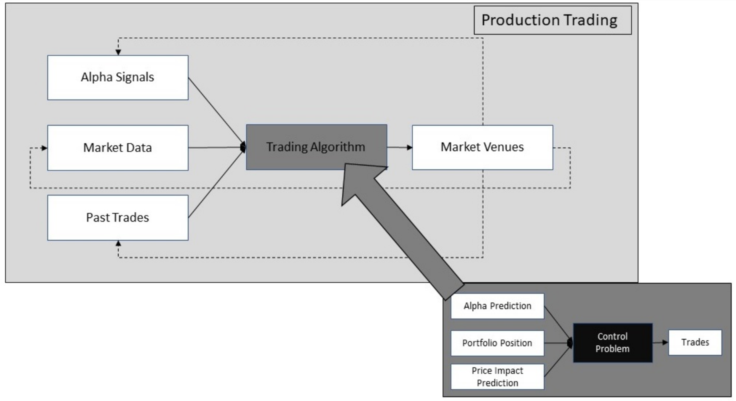


Figure 9: Production data distinguishes between *intended* and *realized* trades.

Weekly Summary

Price impact captures price moves caused by trading.
Price impact introduces feedback loops both in simulation and production. Furthermore, impact primarily drives trading costs.

Alpha signals predict price moves independent of trading.
Directionality, prediction horizon, trigger frequency, exogeneity, and cross-sectionality are core signal characteristics.

Trades capture alpha and pay impact.
Therefore, alpha increases trading speed, and price impact decreases trading speed.

Model-driven trading algorithms
Quantify the trade-off between alpha and impact to submit orders on the market.

Questions?

Next week
The mathematics of price impact.

(a) Implementing trading algorithms as stochastic control problems.

(b) Measuring trading performance using causal inference.

We also briefly describe trading data.