Math 280, Spring 2022

Review Sheet for Midterm Exam 1

Briefly, you should know Sections 1.1.2–3, 1.2.1–2, and 1.3.1–3. More details are below.
THIS IS NOT A COMPREHENSIVE LIST, BUT MERELY AN AID.

You may bring one standard size (8.5x11") "cheat sheet" of notes to the exam

- 1.1.2: Formal definition of graph, and basic terminology. $\Delta(G)$ and $\delta(G)$ and the degree sequence. Neighborhood of a vertex. Theorem 1.1, on edges and degrees.

  Walk, path, trail, cycle, circuit. Theorem 1.2, that every $u-v$ walk contains a $u-v$ path. Connected and disconnected. Vertex deletion $G-v$ and edge deletion $G-e$. Cut vertex, vertex cut set, bridge. The connectivity $\kappa(G)$ of a graph, and basic facts about it.

- 1.1.3: Examples: Complete graph K_n. Empty graph E_n. Cycle graph C_n. Path graph P_n. Bipartite graphs, including the complete bipartite graph $K_{m,n}$. Theorem 1.3 characterizing bipartite graphs (no odd cycles). Regular graph (or r-regular graph). Subgraph. Subgraph $\langle S \rangle$ induced by a set of vertices $S \subseteq V(G)$. Complement $\overline{G}$ of G.

  Isomorphic graphs. Properties preserved by isomorphism, such as order, size, connectivity, degree sequence, existence of cycles of particular size(s), etc.

- 1.2.1: The distance function $d(u,v)$ on a graph G. The eccentricity $\mathrm{ecc}(v)$ of a vertex v. Radius and diameter of G, and their relation given by Theorem 1.4. Center of a graph.

- 1.2.2: The adjacency matrix A of a graph G. (Be able to produce A from G, and vice versa.) Theorem 1.7, that the number of walks of length k from v_i to v_j is the (i,j)-entry of A^k. The matrix S_k and its properties in Theorem 1.9 and the associated discussion in the book and in class.

- 1.3.1: Definition of tree (connected and acyclic). Leaf of a tree.

- 1.3.2: Theorem 1.10, that a tree has one more vertex than it has edges. Theorems 1.12 and 1.13, giving related characterizations of trees. Theorem 1.14, that a tree of order at least 2 has at least 2 leaves.

- 1.3.3: A spanning tree of a graph. Weighted graphs. Minimum weight spanning tree (also known as a minimum spanning tree). Kruskal's Algorithm for finding a minimum spanning tree of a graph G. Theorem 1.17, that Kruskal's Algorithm works.

Some proof techniques we've seen

- Direct proof of "for all" or "there exists" or set containment or set equality: know the first and last line of you proof based on the **goal** you're trying to prove, and then have the hypotheses ready to use to build from one end of the proof to the other.

- Induction and strong induction, when trying to prove things about all integers $k \geq 1$.

- Contradiction (but only if all else fails)

- Breaking into cases (but only if all else fails)

Some things you don't need to know

- 1.1.1: Other kinds of (non)graphs, like multigraphs, digraphs, and pseudographs.
- 1.2.1: Periphery.
- 1.2.1: Theorems 1.5 and 1.6, on when a graph is isomorphic to the center or periphery of another graph.
- 1.2.2: The distance matrix M defined near the end of this section. (You also don't need to know the degree matrix D or Laplacian matrix defined in class as part of this discussion.)
- 1.2.3: You don't need to know anything from this informal section on graph models.
- 1.3.1: Forests and stumps and twigs. The applied examples of trees (Figures 1.31 to 1.35).
- 1.3.2: Theorem 1.11 about vertices and edges in a forest.
- 1.3.2: Theorems 1.15 and 1.16, about the center of a tree, and about a tree being contained in a graph with $\delta(G)$ big enough.

Tip and other thoughts

- To "know" the definition of a term means three things:
  - You know the formal definition (not necessarily word for word, but close enough).
  - You have an intuitive, informal understanding of the concept.
  - You can use the definition correctly in a rigorous proof.

  So make sure that applies to all of the (many) terms on the first page of this handout. In particular, be ready to use the definition of various terms in a short proof.
- Yes, there will be proofs. But most of the problems on this timed, closed-book exam will be about analyzing examples. And the whole thing will be appropriate to a 50-minute, closed book exam. So don't panic; just use the week to prepare.
- Besides reviewing your notes and the book, and familiarizing yourself again with the definitions and theorems, also do as many practice problems as you can: from old homework, and from the practice problems.
- Use office hours, both mine and Anna's.
- Spend the week studying so that you can be sure to get a good night's sleep the night before the exam.