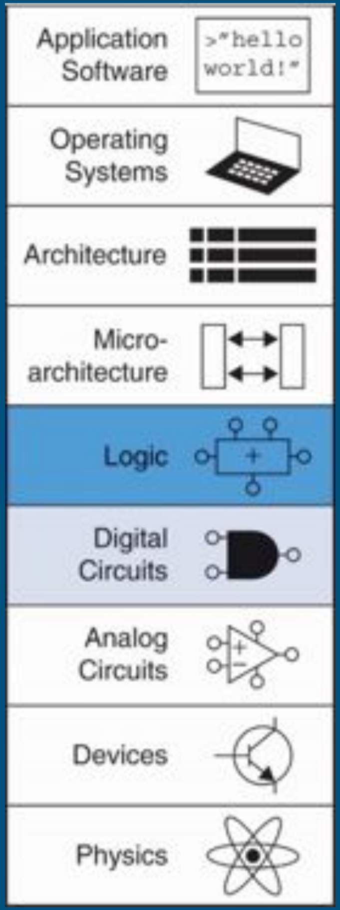
COSC175 (Systems I): Computer Organization & Design

Professor Lillian Pentecost
Fall 2024

Warm-Up September 5

- Where we were
  - Introducing ourselves, some goals of the course
  - Our first day in binary, working with boolean operators and gates
- Where we are going
  - Syllabus questions
  - More gates, boolean expressions
  - Introducing ***combinational logic***
  - Tools and techniques for constructing and simplifying our circuits
- Logistics, Reminders
  - Evening help sessions start TONIGHT!
  - Weekly exercises to be posted tomorrow (Friday), due next Friday
  - Lab 0 due Monday, 10PM, via Moodle
- Textbook Tags: 2.1, 2.2, 2.3, 2.4



Syllabus Annotation and Questions

- I'm looking for ***your*** thoughts and questions on our syllabus
- Go to THIS GOOGLE DOC with the section headings of the syllabus
- With your group:
    a. Assign a note-taker
    b. Discuss your assigned section of the syllabus, add at least 1 question/comment in the doc
    c. Move onto other sections if you have additional time
- I'll look through them all before next class and comment back on the google doc, modify the syllabus as needed

Operating on Binary Values: Two-Input ***Logic Gates***

- Pro tip: order truth table rows by increasing binary value of inputs
- A “bubble” negates (inverts 0→1, 1→0) the result of the gate

XOR

$Y = A \oplus B$

A	B	Y
0	0	0
0	1	1
1	0	1
1	1	0

NAND

$Y = \overline{AB}$

A	B	Y
0	0	1
0	1	1
1	0	1
1	1	0

NOR

$Y = \overline{A + B}$

A	B	Y
0	0	1
0	1	0
1	0	0
1	1	0

XNOR

$Y = \overline{A \oplus B}$

A	B	Y
0	0	1
0	1	0
1	0	0
1	1	1

Operating on Binary Values:
Defining behavior of ***<u>three-input</u>*** gates

NOR3

$$Y = \overline{A+B+C}$$

A	B	C	Y
0	0	0	1
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	0

AND3

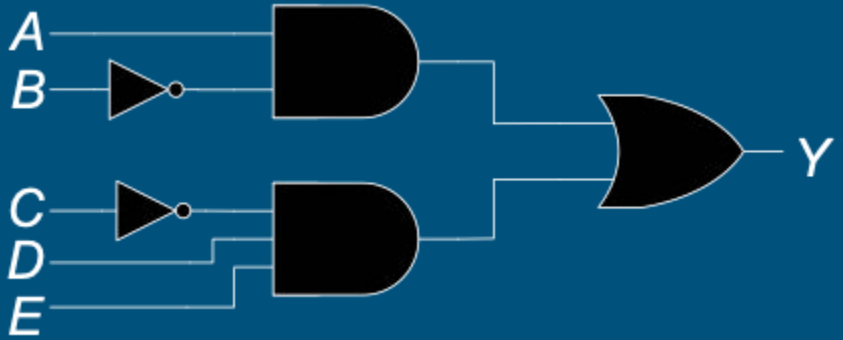
$$Y = ABC$$

A	B	C	Y
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	1

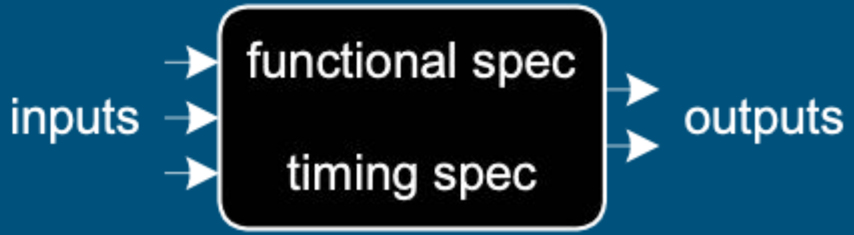
- Pro tip: order truth table rows by increasing binary value of inputs
- Translating from truth table to equation to gate schematics takes time and practice!
- ***<u>Let's learn some terms and tips that will improve your quality of life!</u>***

Some Circuit Drawing Tips!

- Inputs go on the left
- Outputs go on the right
- Gates flow from left → right
- Wires connect at a T
- Try for straight, non-overlapping wires where possible
- A dot means crossed wires connect
- No dot or a bump means crossed wires do not connect



What is a circuit?



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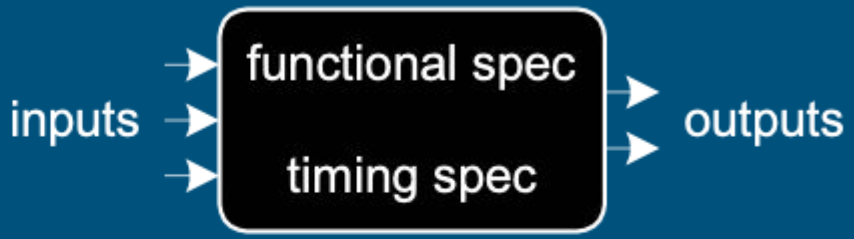
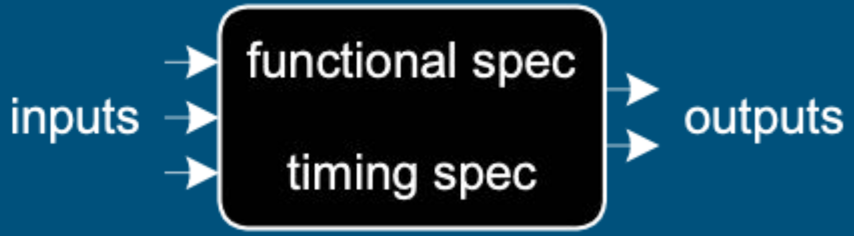


- It has one or more ***inputs***
- It has one or more ***outputs***
- It has a ***functional specification*** translating inputs → outputs
- It has a ***timing specification*** describing how long it takes between changing an input and observing the translated output
    - We will ignore this part, just for now!

What is a circuit?



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What's in the box?

What is a Boolean Equation?

- A functional specification of outputs in terms of inputs!
  - *Dictates what needs to happen in the box!*

- As with arithmetic, there is an ***order of operations*** for boolean operators
  - NOT before AND
  - AND before OR
  - (i.e., `multiplication’ resolved before `addition’)

- Example, ***draw the gate schematic*** for A + BC = Y

Key Definitions (also see textbook)

- The inverse of a variable is called its ***complement***
  - (variable with a bar over it)
- A variable or its complement is referred to as a ***literal***
- The "AND" of one or more literals is called a ***product*** or an ***implicant***
  - Example: ABC, AB, A
- The "OR" of one or more literals is called a ***sum***
  - Example: A + B, C + A + B
- When a ***product*** includes all inputs to a function, we call it a ***minterm***
- When a ***sum*** includes all inputs to a function, we call it a ***maxterm***

Truth Tables are Your New Best Friend

- Truth table of N inputs contains 2^N rows, one per possible value of inputs, ordered by value
- The index of the row in the truth table also refers to the value of inputs when read as a binary value, and we will use these indices

A_3	A_2	A_1	A_0	Y_3	Y_2	Y_1	Y_0
0	0	0	0				
0	0	0	1				
0	0	1	0				
0	0	1	1				
0	1	0	0				
0	1	0	1				
0	1	1	0				
0	1	1	1				
1	0	0	0				
1	0	0	1				
1	0	1	0				
1	0	1	1				
1	1	0	0				
1	1	0	1				
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A	B	Y
0	0	
0	1	
1	0	
1	1	

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- Order truth table rows by increasing binary value of inputs
- Now, the index of the row in the truth table also refers to the value of inputs when read as a binary value, and we will use these indices
- What ***minterm*** would make each row (and only that row) a 1?

A	B	Y	minterm	minterm name
0	0			m_0
0	1			m_1
1	0			m_2
1	1			m_3

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Sum-of-Products (SOP) form

All Boolean equations can be written as a SOP!!

- Identify minterms for the rows that result in an output of 1, and sum them
- You can transcribe this directly from your truth table!

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1	0	0	$A\ \bar{B}$	m_2
1	1	1	$A\ B$	m_3

$$Y = \mathrm{F}(A, B) = \bar{A}B + AB = \Sigma(1, 3)$$

…. What about Product-of-Sums (POS)?

- All Boolean equations can be written as a POS, as well!
- Starting with same truth table…

what ***maxterm*** would make each row (and only that row) a 0?

A	B	Y	maxterm	maxterm name
0	0	0	$A + B$	M_0
0	1	1	$A + \overline{B}$	M_1
1	0	0	$\overline{A} + B$	M_2
1	1	1	$\overline{A} + \overline{B}$	M_3

$$Y = F(A, B) = (A + B) \bullet (\overline{A} + B) = \Pi(0, 2)$$

Building Circuits from Logic!
Check-In activity on notecards, with your pod

1. For the given truth table below, produce a SOP and POS equation:

C	M	E
0	0	0
0	1	0
1	0	1
1	1	0

2. For each of the given equations below, draw the gate schematic:
   a. $Y = A\overline{B} + \overline{C}DE$
   b. $E = HS + H\overline{S}$

Simplify your life!!

A	B	Y	minterm	minterm name
0	0	0	$\overline{A}\ \overline{B}$	m_0
0	1	1	$\overline{A}\ B$	m_1
1	0	0	$A\ \overline{B}$	m_2
1	1	1	$A\ B$	m_3

- SOP and POS are mega-useful, canonical forms, but not necessarily the simplest equation! We care about the size, energy, and time of our circuits, so we want to use simpler equations, fewer gates when we can!
- *Guideline:* SOP for when most rows are 0's, POS for when most rows are 1's!
- …. But we can do much better by exploiting **Boolean Algebra**
  - Simpler than "regular" algebra, leveraging the knowledge that we only have 2 values!

Simplification!!

- There are ***axioms*** and ***theorems*** that we will review today and Tuesday
- The ***Dual*** simply replaces * with + and 0 with 1!
- Things you already know but put into a table:

Number	Axiom	Dual	Name
A1	B = 0 if B ≠ 1	B = 1 if B ≠ 0	Binary Field
A2	$\overline{0}$ = 1	$\overline{1}$ = 0	NOT
A3	0 • 0 = 0	1 + 1 = 1	AND/OR
A4	1 • 1 = 1	0 + 0 = 0	AND/OR
A5	0 • 1 = 1 • 0 = 0	1 + 0 = 0 + 1 = 1	AND/OR

Simplification!!

- Boolean Theorems of one Variable:

Number	**Theorem**	**Dual**	**Name**
T1	$B \bullet 1 = B$	$B + 0 = B$	Identity
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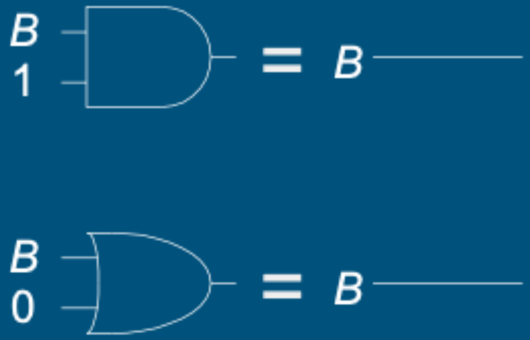

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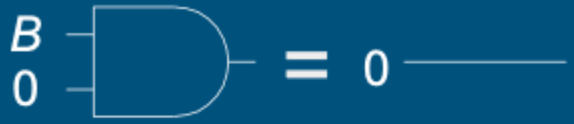

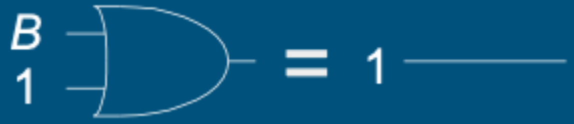

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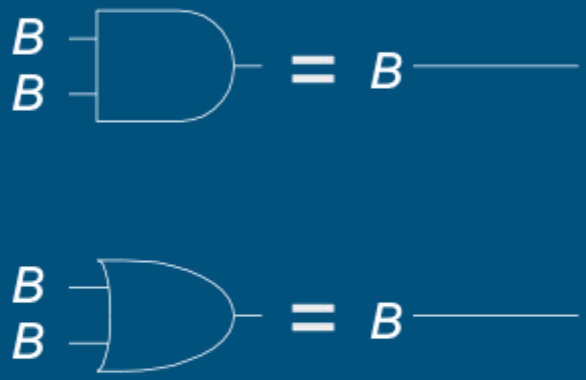

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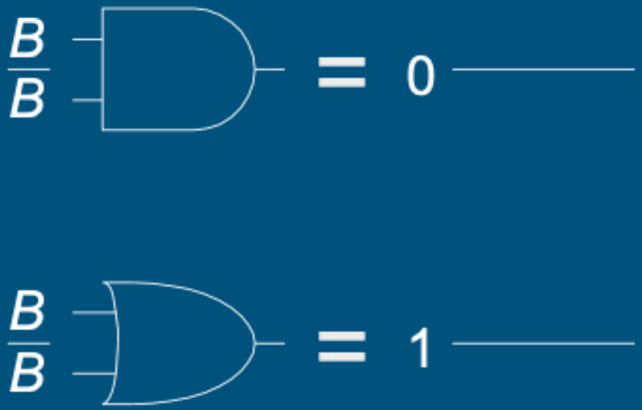

How do we ***prove a theorem***?

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- **Method 1: Perfect Induction**
  - I.e., exhaustively list every input value; if two expressions produce the same output for every possible input, they are equivalent
  - I.e., use a truth table
- Method 2: Use other theorems and axioms to make two sides of an equation look like one another
  - More on this next time!

Wrap-Up September 5

- Coming up next!
  - Introducing ***hardware description languages***
  - More tools & techniques with boolean expressions
- Logistics, Reminders
  - Get in the habit now of staying up-to-date with Moodle page!
  - TA hours start tonight, 7-9PM, C107
  - *Weekly Exercises* will be posted Friday
  - Lab 0 due Monday, 10PM via Moodle
- FEEDBACK
  - https://forms.gle/5Aafcm3iJthX78jx6

SCAN ME