

Economics 220, Jan 2022
Homework 6

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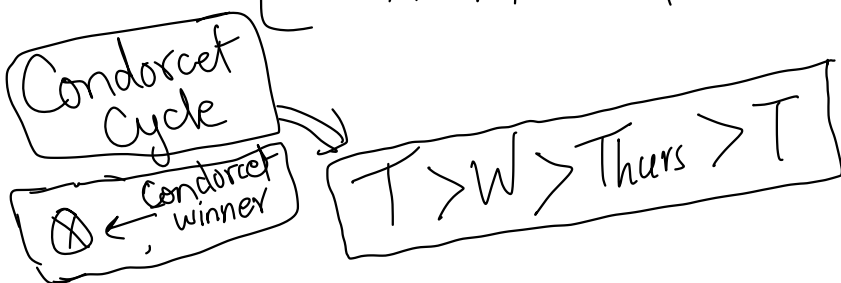
A professor allows the seven students in her class to choose the day of the week for their midterm. Their preferences are as follows:

| Aleah | Bobby | Cynthia | Darius | Ella | Faye | Gabby |
|-------|-------|---------|--------|-------|-------|-------|
| M | M | M | W | W | T | F |
| T | Thurs | Thurs | T | Thurs | W | W |
| W | T | T | Thurs | T | Thurs | Thurs |
| Thurs | F | F | F | M | M | T |
| F | W | W | M | F | F | M |

Does this social choice situation have a Condorcet winner? If so, what is it? If not, explain why not.

$M > F$
 $T > F$
 $W > F$
 $Thurs > F$
 $T > M$
 $W > M$
 $Thurs > M$
 $T > W$
 $W > Thurs$
 $Thurs > T$

No, there is no condorcet winner because there is a condorcet cycle in the pairwise comparisons of preferences over the weekdays.



What option will be chosen if they use....

- Plurality rule?

| Plurality voting | Aleah | Bobby | Cynthia | Darius | Ella | Faye | Gabby |
|------------------|-------|-------|---------|--------|-------|-------|-------|
| 1 | M | M | M | W | W | T | F |
| 0 | T | Thurs | Thurs | T | Thurs | W | W |
| 0 | W | T | T | Thurs | T | Thurs | Thurs |
| 0 | Thurs | F | F | F | M | M | T |
| 0 | F | W | W | M | F | F | M |

$M: 3$
 $W: 2$
 $T: 1$
 $F: 1$

Thus, under Plurality rule, **M** will be chosen.

- A Borda Count?

| | Amy | Bill | Cynthia | Darius | Ella | Fynn | Gabby |
|---|-------|-------|---------|--------|-------|-------|-------|
| 4 | M | M | M | W | W | T | F |
| 3 | T | Thurs | Thurs | T | Thurs | W | W |
| 2 | W | T | T | Thurs | T | Thurs | Thurs |
| 1 | Thurs | F | F | F | M | M | T |
| 0 | F | W | W | M | F | F | M |

| | | | | |
|---------|----|-----------|--------------------------------------|---|
| Points: | M | <u>14</u> | $= 4(3) + 3(0) + 2(0) + 1(2) + 0(2)$ | |
| | T | <u>17</u> | $= 4(1) + 3(2) + 2(3) + 1(1) + 0(0)$ | |
| | W | <u>16</u> | $= 4(2) + 3(2) + 2(1) + 1(0) + 0(2)$ | Winner: <u>T</u> is <u>Borda Winner</u> |
| | Th | <u>16</u> | $= 4(0) + 3(3) + 2(3) + 1(1) + 0(0)$ | |
| | F | <u>7</u> | $= 4(1) + 3(0) + 2(0) + 1(3) + 0(3)$ | |

Assuming all other students will vote honestly, would any student have an incentive to misrepresent their preferences to change the outcome of the Borda count? If so, give an example.

Ella likes W more than T, so she can lie in the following two steps for example.


She can first switch F and T, then she can switch F and Thurs, which leads the new Borda counts to be:

| | |
|-------|----|
| M | 14 |
| T | 15 |
| W | 16 |
| Thurs | 15 |
| F | 10 |

which makes W the Borda winner by serving the Ella's purpose of lying.

- A Single Transferrable Vote/runoff system? (every submits a ranked list; sequentially eliminate the options with the fewest first-preference votes and redistribute those votes to their second choices) (assume everyone votes honestly)

| | 1 st | 2 nd | 3 rd | 4 th | 5 th | |
|-------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------------------|
| M | 3 | 3 | 0 | 2 | 2 | 2 nd elimination |
| T | 1 | 2 | 3 | 1 | 0 | |
| W | 2 | 4 | 1 | 0 | 2 | 1 st elimination |
| Thurs | 0 | 3 | 3 | 1 | 0 | 2 nd elimination |
| F | 1 | 0 | 0 | 3 | 3 | |


 is STV winner

- Sequential Pairwise voting that begins by voting between Monday and Tuesday, and then compares the winner with Wednesday (the next day of the week in turn), and so on? (assuming everyone votes honestly).

$M > T : 3$
 $T > M : 4 \Rightarrow T > M$

$T > W : 4$
 $W > T : 3 \Rightarrow T > W$

$T > \text{Thurs} : 3$
 $\text{Thurs} > T : 4 \Rightarrow \text{Thurs} > T$

$\text{Thurs} > F : 6$
 $F > \text{Thurs} : 1 \Rightarrow \text{Thurs} > F$

\Rightarrow Thursday is Seq Pair-Wise winner

Consider Cynthia's preferences over the preference aggregation ('voting') method used. Please rank Cynthia's preferences over Plurality, Borda Count, Sequential Pairwise voting, and STV.

$M > \text{Thurs} > T > F > W$: Cynthia's Preferences

Based on the calculated answers of previous parts.

Plurality $>$ Sequential Pairwise Voting $>$ Borda Count $>$ STV

Ella wants the exam to be on Wednesday. Suppose she will get to set the agenda for a sequential pairwise-voting process in the students vote (honestly) between two days; then for the winner against another specified day; and so on until all options have been exhausted. Can Ella design an agenda to get her preferred outcome? Explain.

| | | |
|--------------------|--------------------|--------------------|
| $M > F$ | | |
| $T > F$ | $T > M$ | $T > W$ |
| $W > F$ | $W > M$ | $W > \text{Thurs}$ |
| $\text{Thurs} > F$ | $\text{Thurs} > M$ | $\text{Thurs} > T$ |

head-to-head comparisons from part (i)

Ella's First move:
 (Thurs) Winner plays this:
 (W) Winner plays this:
 (W) Winner plays this:
 ALL OPTIONS EXHAUSTED

Ella can design an agenda by setting the first comparison between Thurs & T

Thus Wednesday will be the outcome in sequential pairwise voting process