*ARPA/ONR URI REVIEW:*

# Scaling Properties of Strongly Compressible Turbulence

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# Forced-Dissipative Burgers Equation

1-dimensional case:

$$\frac{\partial u}{\partial t} + u\,\frac{\partial u}{\partial x} = \nu\,(-1)^{p+1}\,\frac{\partial^{2p} u}{\partial x^{2p}} + f,$$

$$\overline{f(k,\omega)\,f(k',\omega')} = D(k)\,\delta(k+k')\,\delta(\omega+\omega') \quad (1)$$

3-dimensional case:

$$\frac{\partial \phi}{\partial t} + \frac{[\nabla\phi]^2}{2} = \nu\,(-1)^{p+1}\,\Delta^p\phi + g,$$

$$\overline{g(\vec{k},\omega)\,g(\vec{k'},\omega')} = k^{-2}\,D(k)\,\delta(\vec{k}+\vec{k'})\,\delta(\omega+\omega').(2)$$

Force types:

1. $D(k) \propto \delta(\vec{k})$, large-scale;

2. $D(k) \propto k^{-y}$, distributed.

# Velocity Structure Functions

$$\overline{\left|u_i(\vec{x}+\vec{r})-u_j(\vec{x})\right|^p} \propto r^{\zeta_p} \tag{3}$$

## Normal Scaling

$$R(r) \equiv \int \overline{(f(x+r)-f(x))^2}\, dx, \;\; \Delta u \propto R(r)^{1/3}. \tag{4}$$

## Questions

- What are the scaling exponents $\zeta_p$?
- Why are they "anomalous" (if they are)?
- Is "anomaly" related to structures?

# 1-D Case, Distributed Force, $y = 1$

$D(k) = D_0\, k^{-1}$, normal scaling $\overline{\Delta u} \propto r^{1/3}$.

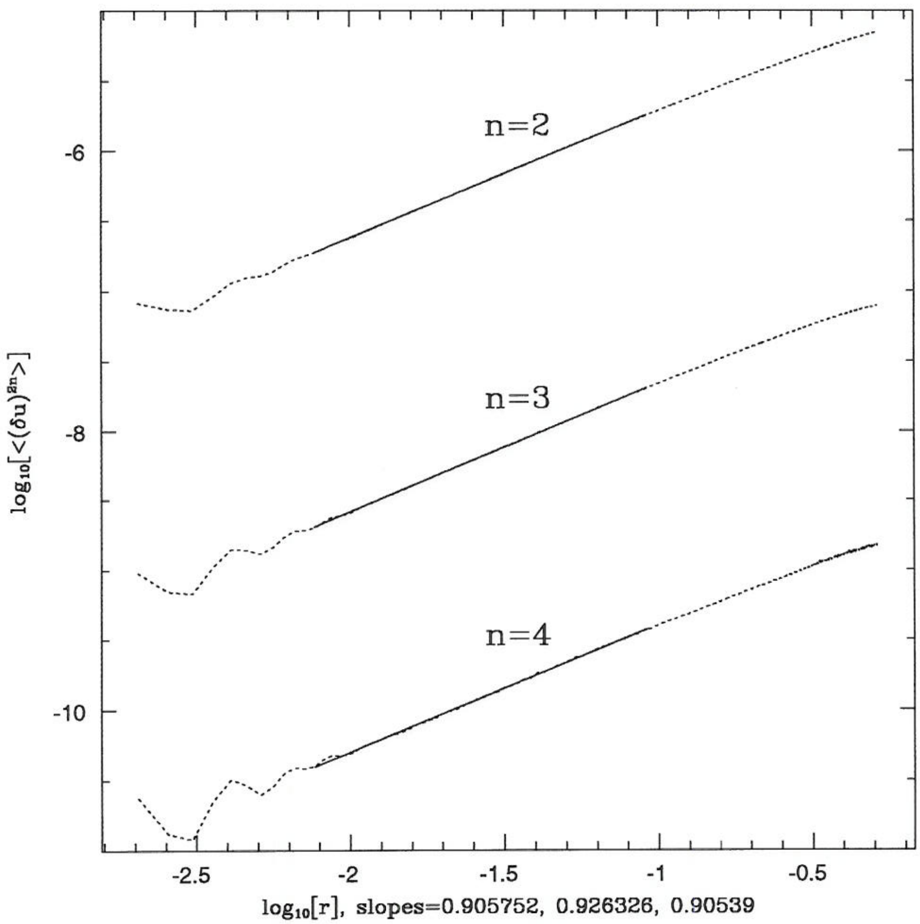


*Fig. 1* High-order velocity structure functions $\overline{(u(x+r)-u(x))^n}$ for $n = 2, 3, 4$ (dotted curves) with linear least-squares fits (solid lines).

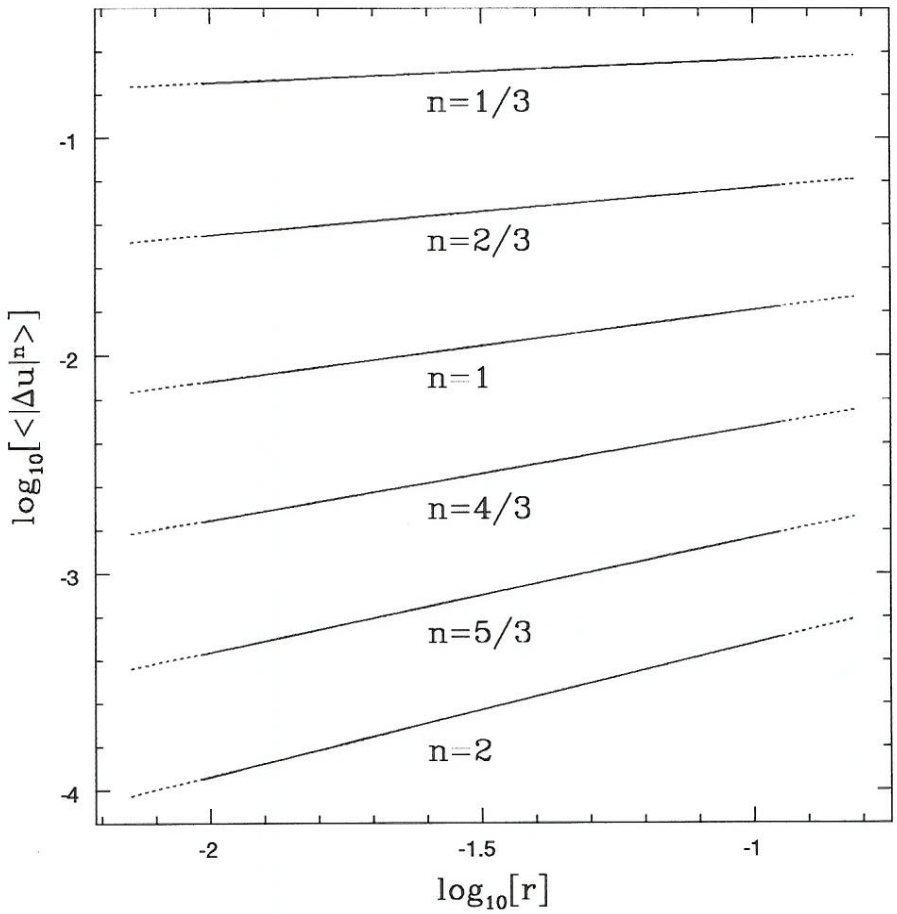


*Fig. 2* Low-order velocity structure functions $\overline{|u(x+r)-u(x)|^n}$ for $n = 1/3, 2/3, 1, 4/3, 5/3, 2$ (dotted curves). Slopes of the linear least-squares fits (solid lines) from top to bottom, respectively, are 0.111, 0.222, 0.330, 0.433, 0.531 and 0.620.

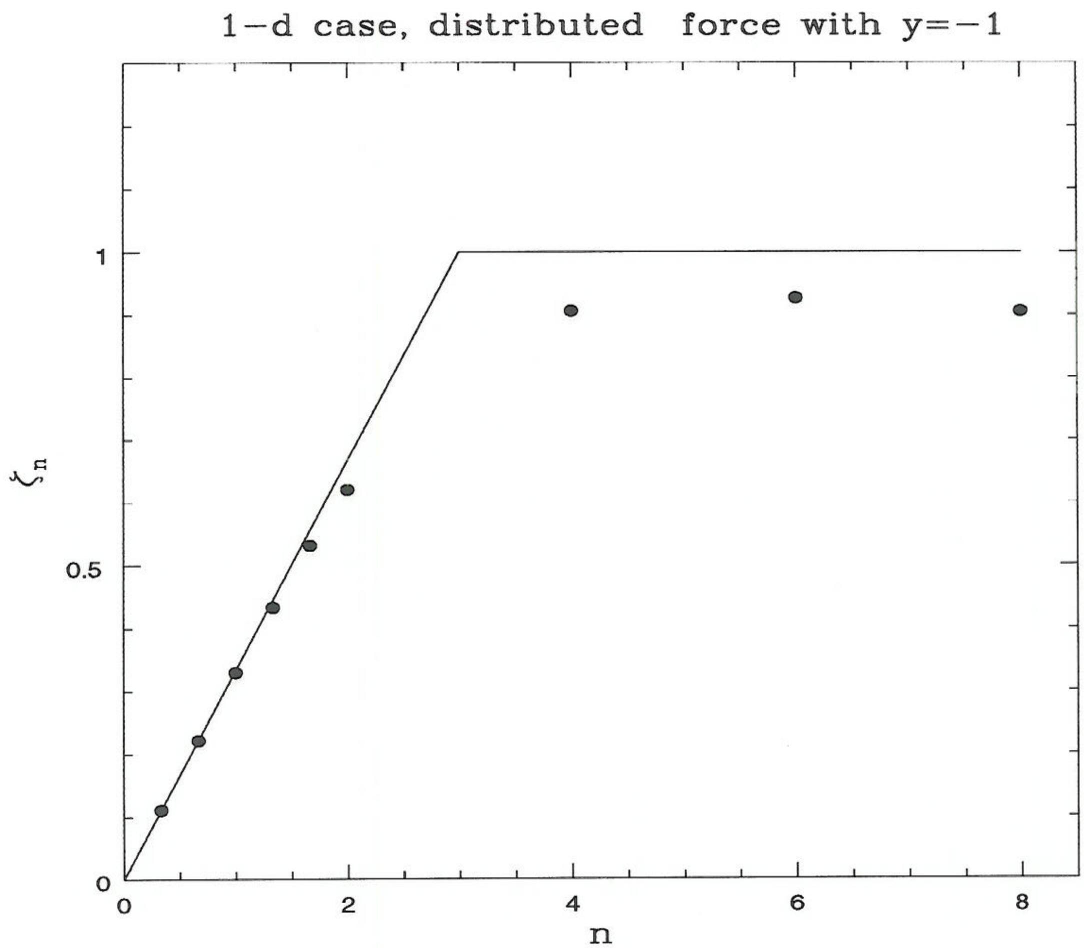


*Fig.* *3* Critical diagram for 1-d distributed force, $y = 1$ case.

## Phenomenological theory for $y = 1$ case

$$u(x,t) = -\sum_{i=0}^{N} U_i \tanh\left[\frac{(x-a_i)\,U_i}{2\nu_0}\right] + \phi(x),$$

$$\bar{\epsilon}_r \propto \sum_{i=0}^{N} \frac{U_i^3}{r} \propto \frac{\overline{U^3}}{r} \quad \text{for } \nu/U_0 < r \ll L. \qquad (5)$$

$$\bar{\epsilon}_r \propto D_0 \ln\left(\frac{r\,U_0}{\nu}\right). \qquad (6)$$

$$\mathcal{P}(U,r) \propto \frac{D_0\, r}{U^4} = \mathcal{P}(U)\frac{r}{L}, \text{ where}$$

$\mathcal{P}(U) \propto U^{-4}$ is a PDF of the shock amplitudes.

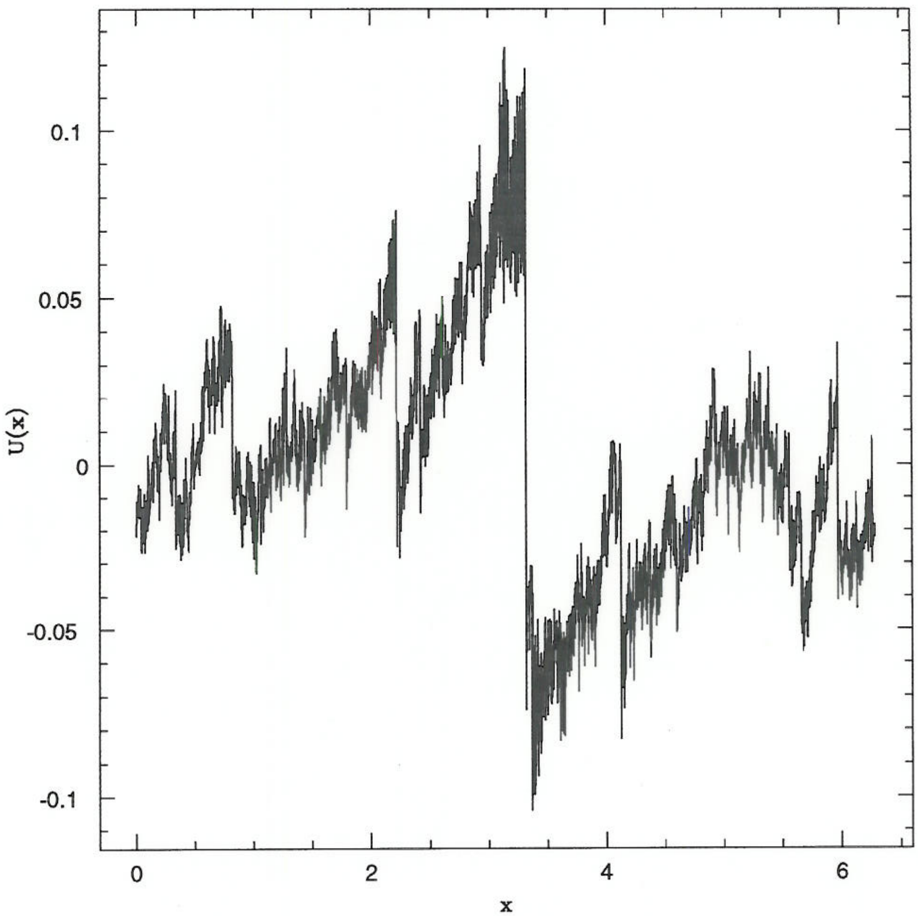


*Fig. 4* Solution at $t = 213.5$.

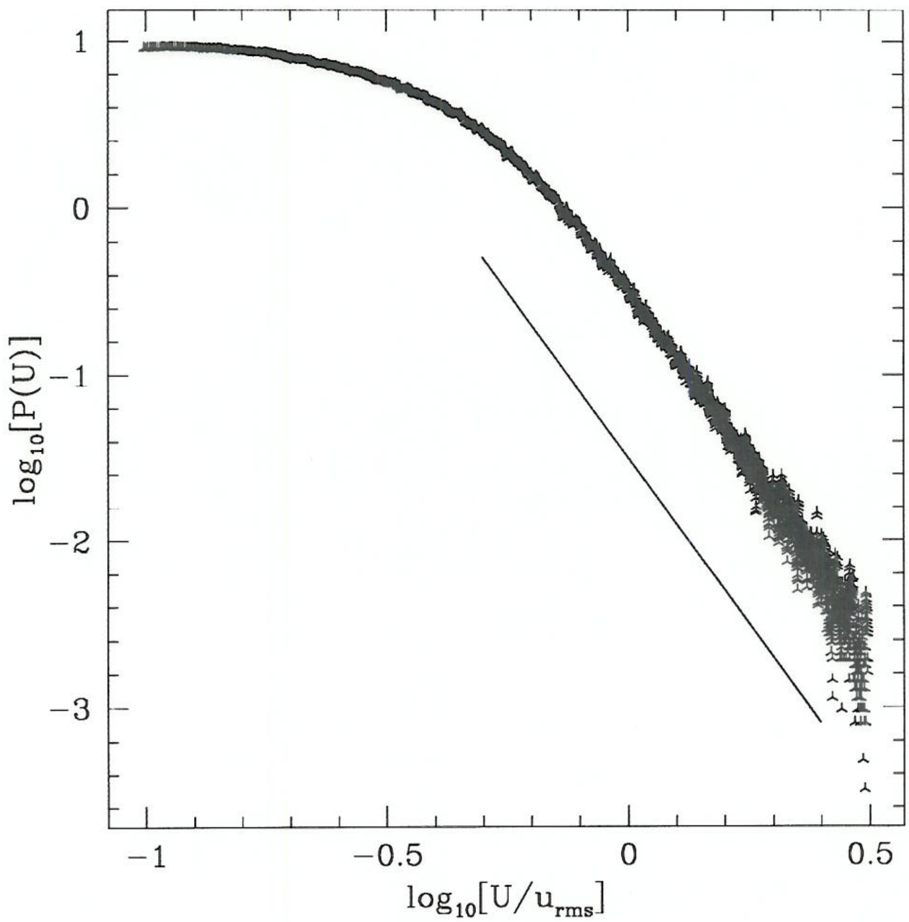


*Fig.* 5 PDF of the shock amplitudes, $\mathcal{P}(U)$, on a logarithmic-logarithmic scale (points). The exact slope of the solid line is −4.

# 3-D Case, Large-Scale Force

Normal Scaling: $\Delta u \propto r^{1/2}$. Implementation: quasi-spectral $128^3$ method on parallel 32-processor machine IBM PVS. Stochastic time-integration: strong second-order and first-order Euler.

## Explicit Order 2 Strong Scheme (E. Platen)

$$\phi_{n+1} = \phi + b\, f\, \tau +$$

$$\frac{1}{2\sqrt{\tau}}\left[N_- - N_+ - \nu\, k^2 \left(\phi_+ - \phi_-\right)\right] \Delta Z +$$

$$\frac{\tau}{4}\left[-N_+ - 2\, N - N_- - \nu\, k^2 \left(\phi_+ + 2\,\phi + \phi_-\right)\right]$$

$$f = \frac{U_1}{\sqrt{\tau}}, \quad b = A_f\, k^{-y/2}, \quad \Delta Z = \frac{\tau^{3/2}}{2}\left(U_1 + \frac{U_2}{\sqrt{3}}\right)$$

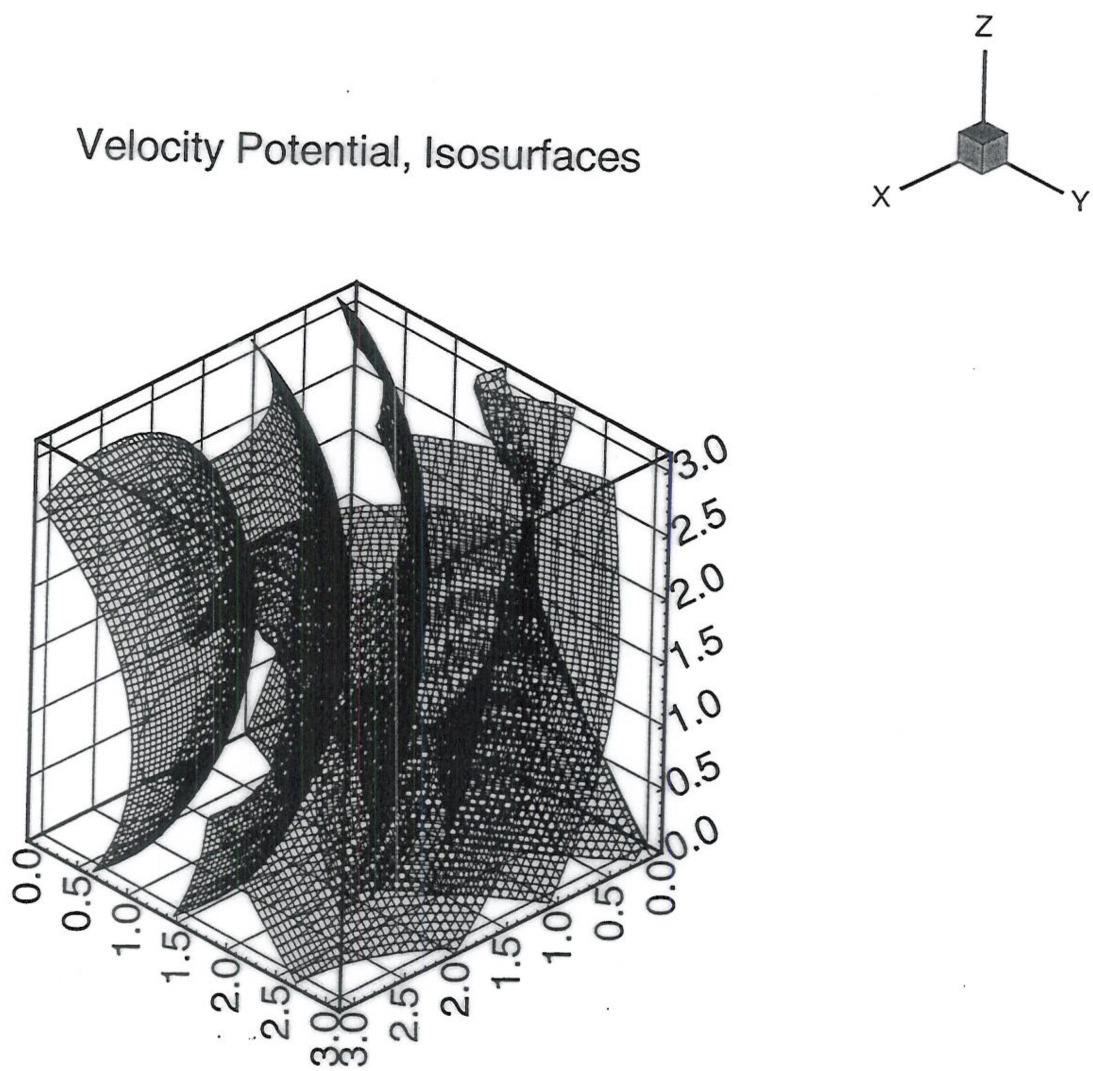


*Fig. 6* Velocity Potential Isosurfaces in 1/8 of domain.

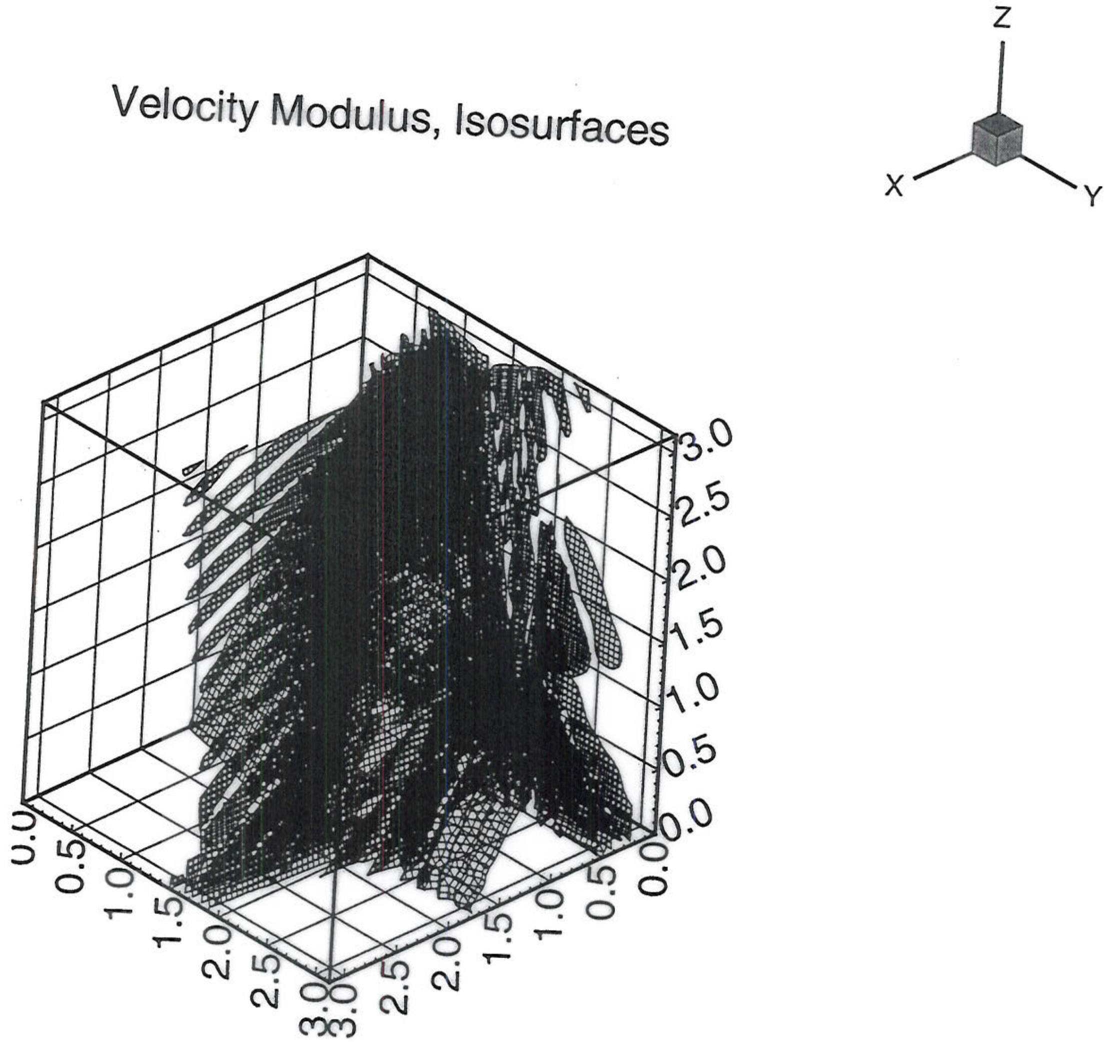


*Fig. 7* Velocity Modulus Isosurfaces.

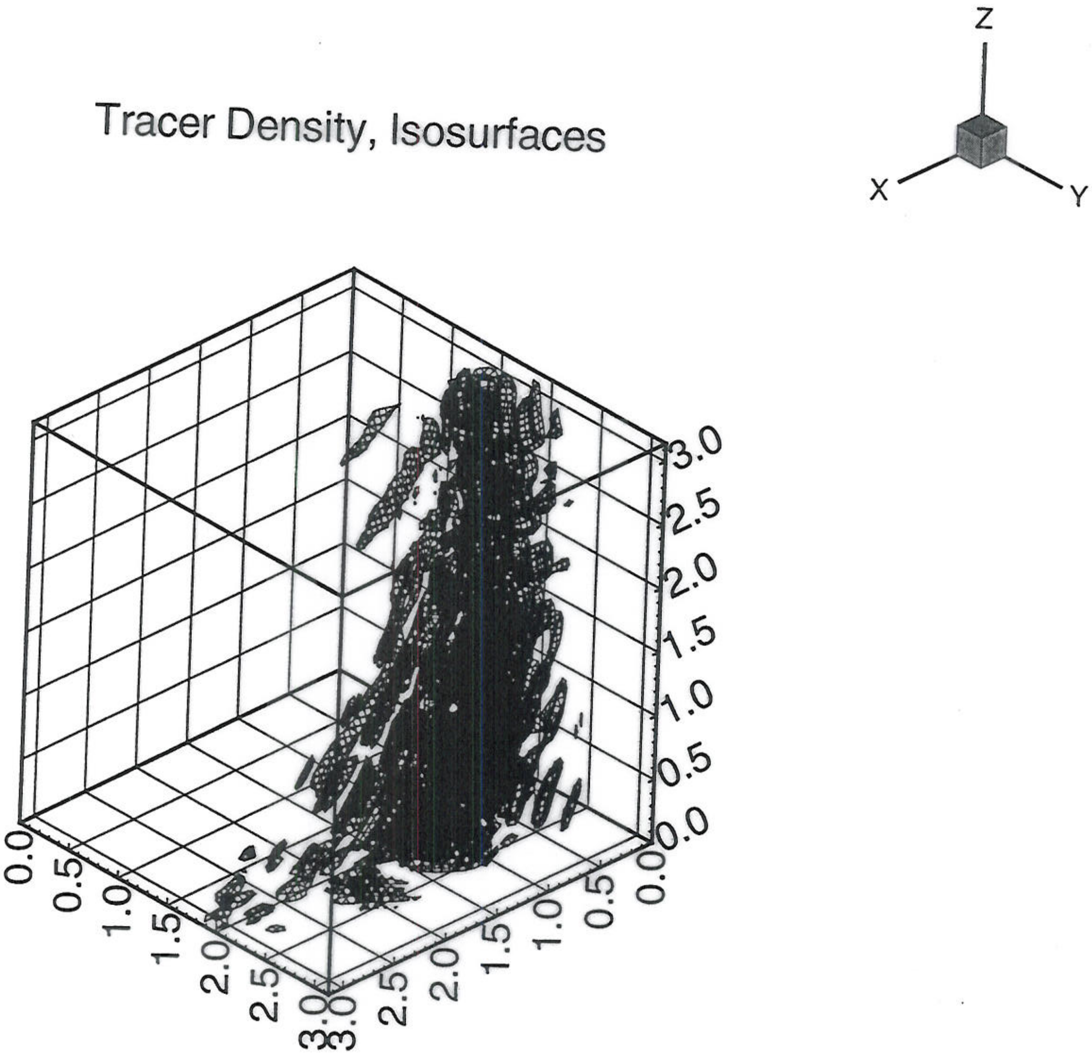


*Fig. 8* Tracer Isosurfaces.

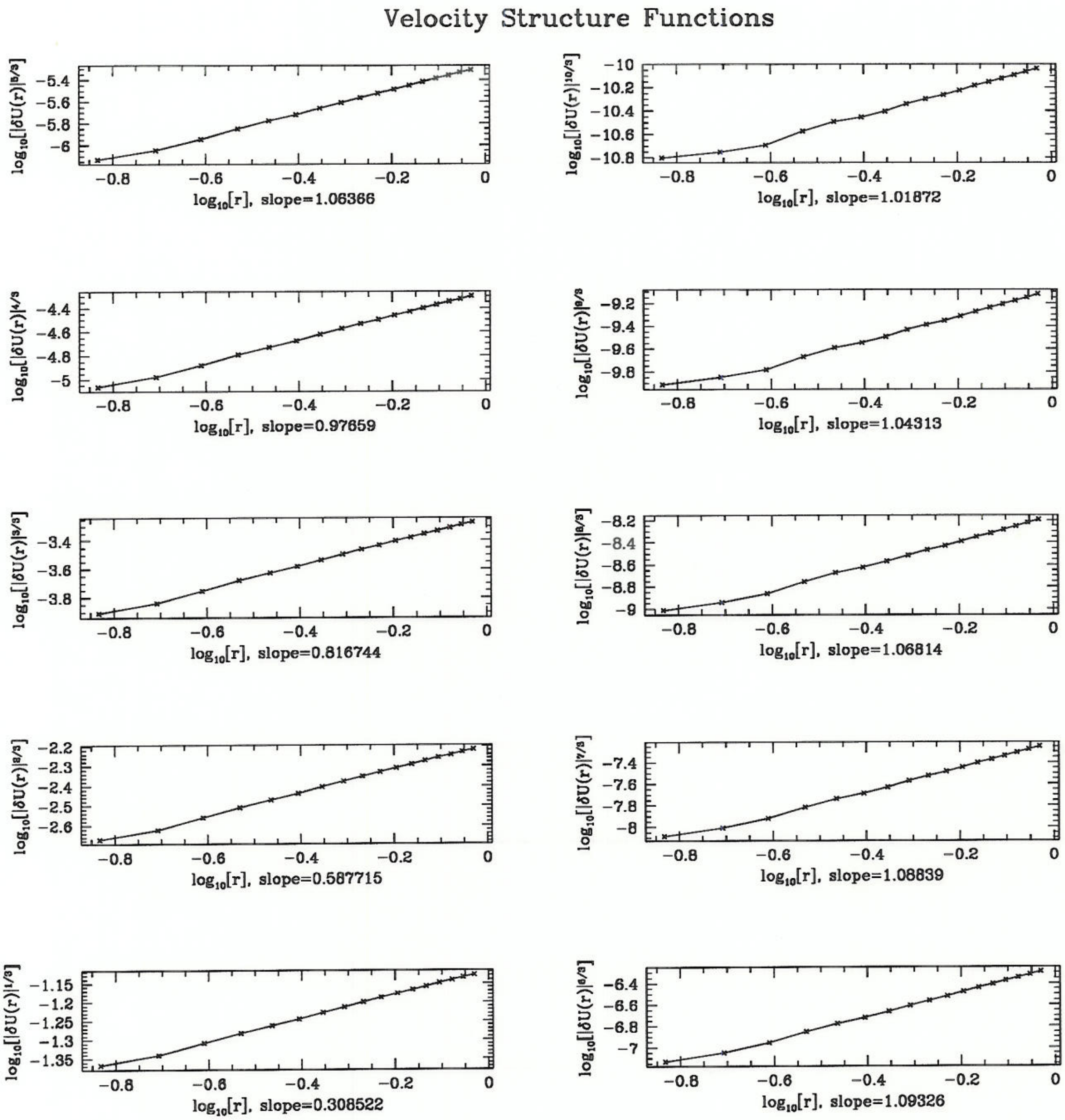


*Fig. 9* Lower-order moments of velocity differences $\overline{|\vec{u}(\vec{x}+\vec{r})-\vec{u}(\vec{x})|^{p}}$.

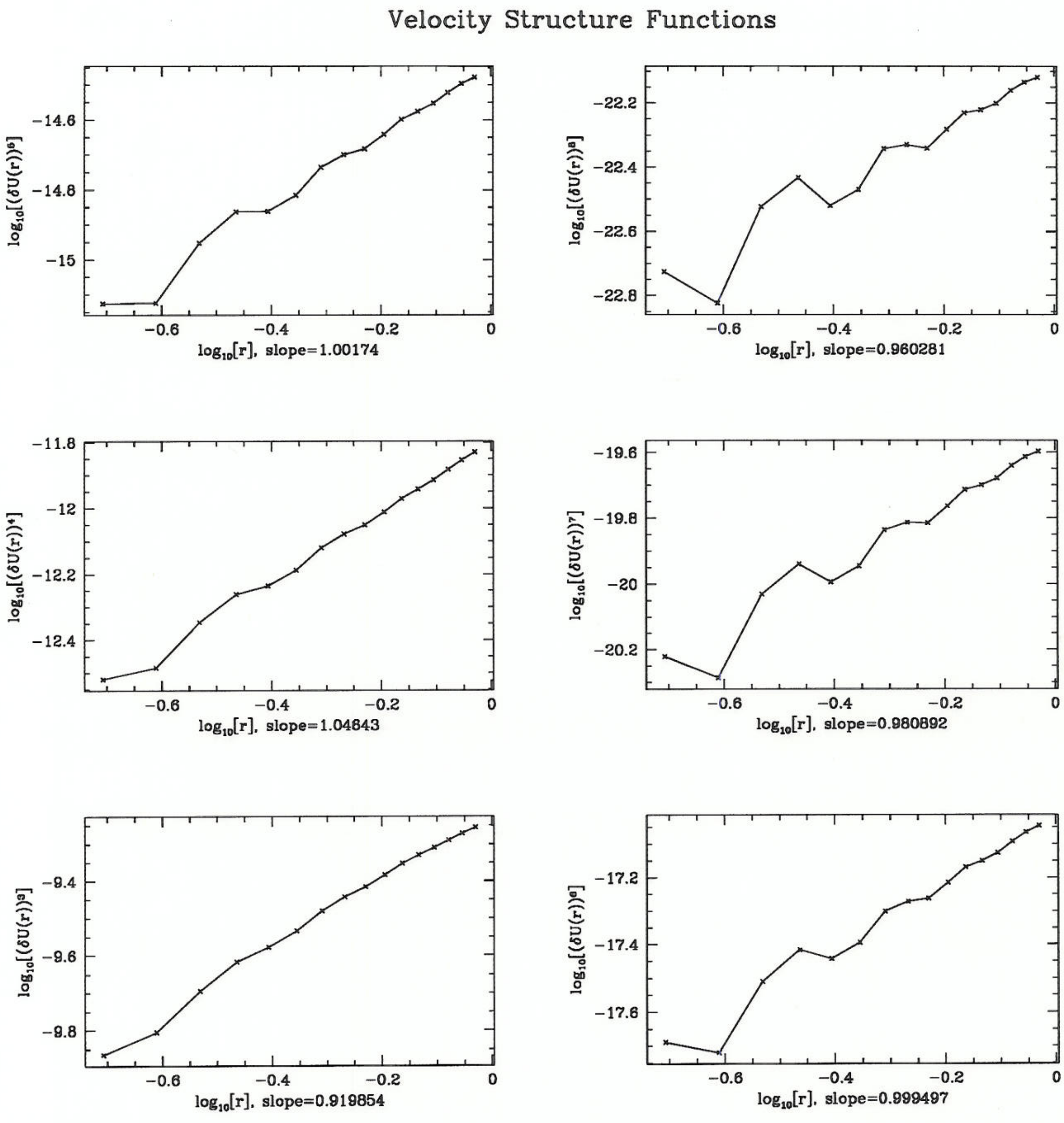


*Fig. 10* Higher-order moments of velocity differences $\overline{|\vec{u}(\vec{x}+\vec{r})-\vec{u}(\vec{x})|^p}$.

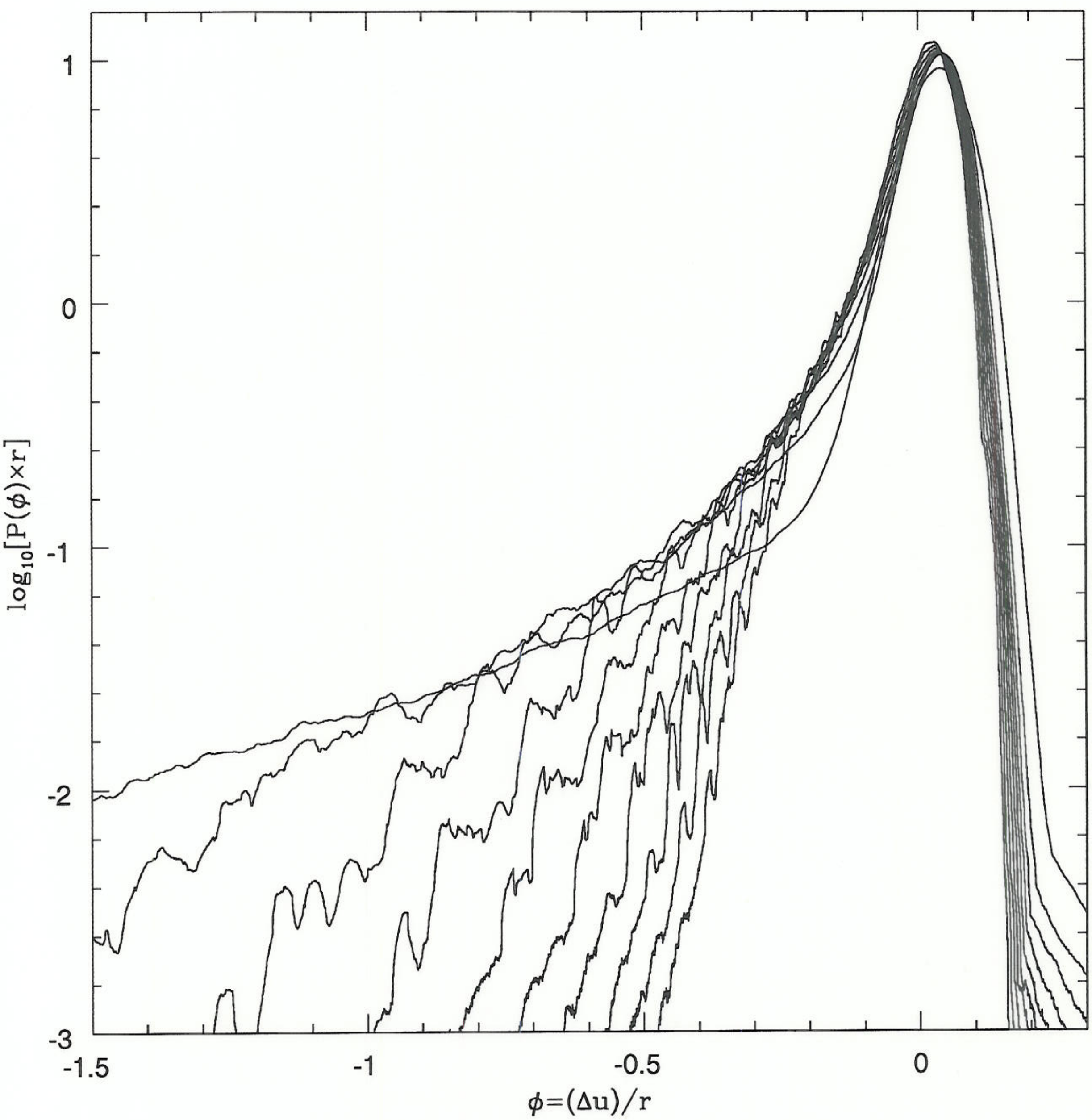


*Fig. 11* PDFs of velocity differences for large-scale forced case in 1-d.

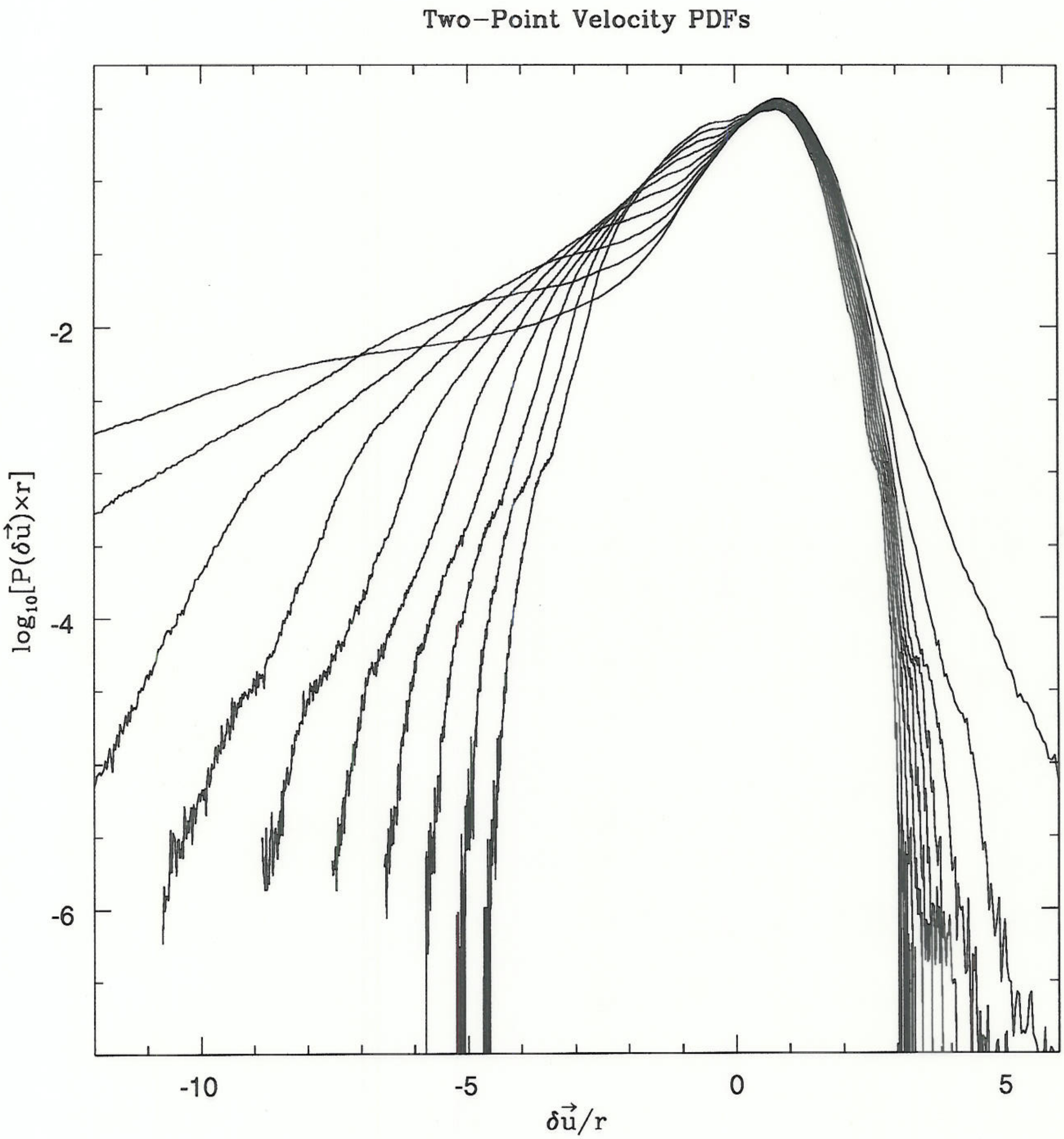


*Fig. 12* PDFs of velocity differences for large-scale forced case in 3-d.

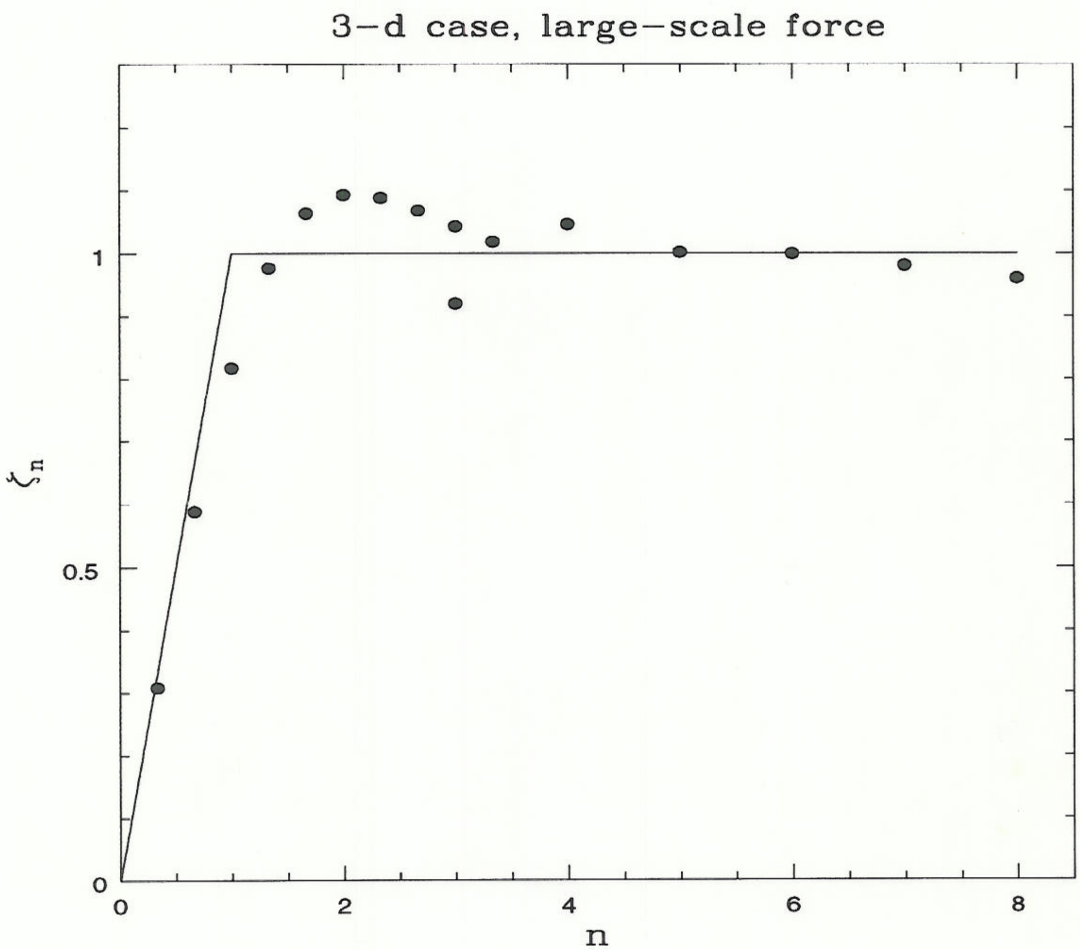


*Fig. 13* Critical diagram for 3-d large-scale forced case.