# Econ 361: Advanced Econometrics

Asymptotics:

When You Have No Other Option ...

# When Have We Seen Asymptotics ...

1. Hypothesis Testing with OLS
   when we were not given $Y|X \sim$ Multivariate Normal
2. OLS with Non-Spherical Errors and Unknown $\Omega$
3. Hypothesis Testing with GLS
   when we were not given $Y|X \sim$ Multivariate Normal
4. Feasible GLS
5. 2SLS and Instrumental Variables (IV) (To Be Discussed)
6. GMM (To Be Discussed)

# Consistency and Asymptotic Normality of OLS, GLS, 2SLS

$$
\begin{aligned}
b^{ols} &= (X'X)^{-1}X'Y \;=\; \beta + (X'X)^{-1}X'\epsilon \\
b^{gls} &= (\tilde{X}'\tilde{X})^{-1}\tilde{X}'\tilde{Y} \;=\; \beta + (\tilde{X}'\tilde{X})^{-1}\tilde{X}'\epsilon \\
&\quad \text{where } \tilde{X} = \Omega^{-\frac{1}{2}}X \\
b^{2sls} &= (\hat{X}'\hat{X})^{-1}\hat{X}'Y \;=\; \beta + (\hat{X}'\hat{X})^{-1}\hat{X}'\epsilon \\
&\quad \text{where } \hat{X} = Z(Z'Z)^{-1}Z'X
\end{aligned}
$$

# Consistency and Asymptotic Normality of OLS, GLS, 2SLS

$$
\begin{aligned}
b^{ols} &= \beta + \left(\frac{X'X}{N}\right)^{-1}\left(\frac{X'\epsilon}{N}\right) \\
b^{gls} &= \beta + \left(\frac{\tilde{X}'\tilde{X}}{N}\right)^{-1}\left(\frac{\tilde{X}'\epsilon}{N}\right) \\
&\quad \text{where } \tilde{X} = \Omega^{-\frac{1}{2}}X \\
b^{2sls} &= \beta + \left(\frac{\hat{X}'\hat{X}}{N}\right)^{-1}\left(\frac{\hat{X}'\epsilon}{N}\right) \\
&\quad \text{where } \hat{X} = Z(Z'Z)^{-1}Z'X
\end{aligned}
$$

# Consistency and Asymptotic Normality of OLS, GLS, 2SLS

$$b^{ols} = \beta + \left(\frac{X'X}{N}\right)^{-1}\left(\frac{X'\epsilon}{N}\right)$$

$$b^{gls} = \beta + \left(\frac{\tilde{X}'\tilde{X}}{N}\right)^{-1}\left(\frac{X'\Omega^{-\frac{1}{2}\prime}\epsilon}{N}\right)$$

where $\tilde{X} = \Omega^{-\frac{1}{2}}X$

$$b^{2sls} = \beta + \left(\frac{\hat{X}'\hat{X}}{N}\right)^{-1}\left(\frac{X'Z}{N}\right)\left(\frac{Z'Z}{N}\right)^{-1}\left(\frac{Z'\epsilon}{N}\right)$$

where $\hat{X} = Z(Z'Z)^{-1}Z'X$

# Consistency and Asymptotic Normality of OLS, GLS, 2SLS

$$\lim_{N\to\infty} b^{ols} = \beta + \lim_{N\to\infty}\left(\frac{X'X}{N}\right)^{-1}\lim_{N\to\infty}\left(\frac{X'\epsilon}{N}\right)$$

$$\lim_{N\to\infty} b^{gls} = \beta + \lim_{N\to\infty}\left(\frac{\tilde{X}'\tilde{X}}{N}\right)^{-1}\lim_{N\to\infty}\left(\frac{X'\Omega^{-\frac{1}{2}\prime}\epsilon}{N}\right)$$

$$\lim_{N\to\infty} b^{2sls} = \beta + \lim_{N\to\infty}\left(\frac{\hat{X}'\hat{X}}{N}\right)^{-1}$$

$$\lim_{N\to\infty}\left(\frac{X'Z}{N}\right)\lim_{N\to\infty}\left(\frac{Z'Z}{N}\right)^{-1}\lim_{N\to\infty}\left(\frac{Z'\epsilon}{N}\right)$$

Courtesy of Slutsky Theorem and Continuous Mapping (Mann-Wald) Theorem

# Consistency and Asymptotic Normality of OLS, GLS, 2SLS

$$b^{ols} \quad \text{“} \xrightarrow{d} \text{”} \quad \beta + \lim_{N\to\infty} \left( \frac{X'X}{N} \right)^{-1} \text{ asymp distrib of } \left( \frac{X'\epsilon}{N} \right)$$

$$b^{gls} \quad \text{“} \xrightarrow{d} \text{”} \quad \beta + \lim_{N\to\infty} \left( \frac{\tilde{X}'\tilde{X}}{N} \right)^{-1} \text{ asymp distrib of } \left( \frac{X'\Omega^{-\frac{1}{2}\prime}\epsilon}{N} \right)$$

$$b^{2sls} \quad \text{“} \xrightarrow{d} \text{”} \quad \beta + \lim_{N\to\infty} \left( \frac{\hat{X}'\hat{X}}{N} \right)^{-1}$$

$$\lim_{N\to\infty} \left( \frac{X'Z}{N} \right) \lim_{N\to\infty} \left( \frac{Z'Z}{N} \right)^{-1} \text{ asymp distrib of } \left( \frac{Z'\epsilon}{N} \right)$$

Courtesy of Slutsky Theorem and Continuous Mapping (Mann-Wald) Theorem