Math 5440: Week 10 Assignment

Due Date: April 14, 2023 at 10am

Exercise 1 Simulating an AB-Test

Consider a synthetic signal

$$\alpha_t = a\,(R_t - R_T) + b\,(W_t - W_T)$$

for parameters a and b, where R and W are assumed to be two independent Wiener processes with volatility σ_R and σ_W, respectively. We suppose that $\sigma_R = \sigma_W$ for simplicity. Then, one can verify that

$$\rho := \mathrm{corr}(\alpha_t, R_t - R_T) = \frac{a}{\sqrt{a^2 + b^2}}.$$

We will create three alpha signals with correlations $\rho_0 = 0$, $\rho_1 = 0.05$, and $\rho_2 = 0.3$ in the form above with the same b but different a.

1. Let $b = \rho_2\sqrt{1-\rho_2^2}$ in all alpha signals and set

$$a = \frac{\rho\, b}{\sqrt{1-\rho^2}} \qquad \text{for } \rho = \rho_0, \rho_1, \rho_2.$$

   In particular, we set $a = 0$ when $\rho = \rho_0$; we set $a = \rho_2^2$ when $\rho = \rho_2$. Create the synthetic alphas and store the output in long format (duplicate each table row for each alpha). For example, one can design the codes in the following way

```
dt:  2019.01.03
raw: select from bin10 where date = dt
raw: raw cross ([]rho: 0, .05, .3, name: 0, 1, 2)
createAlpha:{[rho]} / For You To Do
tbl: update alpha: createAlpha[rho] by id, rho from raw
```

2. For each synthetic alpha, use your work in Assignment 9 to backtest the optimal trading strategy. The result should be in long format (duplicate each table row for each strategy).

3. The next step is to simulate an A-B testing engine. The engine is a function with signature

```
abTest:{[tbl; strat1; strat2; prob1]}
```

where tbl is a table with separate rows for each strategy. Strat1 and strat2 are the strategy names (i.e., from 0, 1, 2) and prob1 is the probability assigned to strat1 (the rest being assigned to strat2). The function returns a table randomly selecting a strategy. Implement an abTest that randomly assigns each (stock, day) pair to one of two strategies.

4. For each day, bucket all stocks into three equal-size groups: low, medium, and high volatility. Implement the version of abTest that uses stratified sampling and randomizes within each volatility bucket. That is, for each day and within each volatility bucket, we require that the number of stocks that use strat1 is precisely (number-of-stock * prob1), up to rounding to the nearest integer. Note that this part is not used in Exercise 2.

Exercise 2 Analyzing an AB-Test

The baseline scenario is strat1 follows the alpha signal with $\rho_2 = 0.3$ (name = 2) and strat2 follows the alpha signal with $\rho_0 = 0$ (name = 0).

1. Simulate an AB-test with prob1 at 80%. What is the average daily P&L of this randomized strategy? (That is, the average over all stocks.) What is the average daily P&L for each strategy? (That is, the average of stocks following strat1, and the average of stocks following strat2.)
2. For every day, compute the daily t-stat (mean/sdev) of each strategy's P&L across all stocks. Use suitably normalized units.
3. For every month, compute the t-stat (mean/sdev) of each strategy's P&L across all (stock,days). Use suitably normalized units.
4. Repeat Questions 1-3 for prob1 $= 0.1, 0.2, ..., 0.9$. Comment on the P&L and uncertainty trade-off across A-B allocations. Uncertainty trade-off refers to whether we can statistically tell one strategy is better than the other by looking at the t-stat.
5. Repeat Questions 1-4 for every strategy pair. Comment on the amount of A-B testing needed based on an alpha's strength and the trader's waiting time. Waiting time refers to the duration of experiments that is needed for a trader to tell one strategy is better than the other.