Mathematics of Finance. FINAL EXAM. TAKE-HOME PART
Due 23:59 December 23, 2022

Please write a pledge that final exam solutions represent your own work and that you did not copy solutions from the work of other students or other sources, and did not use any other forbidden means.

1. Give the definition of the stock beta. Estimate MSFT stock beta relative to Nasdaq 100 using last 250 days historical regression from November 21, 2022 back 250 trading days. You can use QQQ as a proxy for Nasdaq 100. Please use adjusted price for both and subtract risk free rate (you can use 3 month Libor)

2. Suppose that the spot interest rates with continuous compounding are

Maturity (years)	rate
1	4.0
2	4.1
3	4.3

Calculate forward continuously compounded interest rates for second and third years, and for a 2 year period between years 1 and 3. (See Hull's def. of Forward Rates or slides)

3. Using bootstrap method (Hull) calculate zero coupon yield curve from coupon bearing bonds.

Bond Principal	Maturity (years)	Coupon (Paid every 0.5year)	Bond Price
100	0.25	0	98.85
100	0.50	0	98.30
100	1.00	0	98
100	1.50	3	98.4
100	2.00	4	98.1

4. Give the definition of duration and convexity of a bond and write the relationship between the change in yield and the bond price.

5.Bond is maturing in 10.5 years and has a semiannual coupon of 4.3% and price 92-15+
a)Calculate its Yield to Maturity. (You can use Excel and try several Yields until you get required price.)
b) Suppose you have 8 Million market value of this bond. What is the Modified Duration, Macaulay Duration, DV01 and convexity of this portfolio.
c) Using Duration and Convexity formula approximation calculate bond price if the Yield to Maturity is increased 20 Basis Points.
d) Calculate the exact Bond price for 20 Basis points increased yield using full discount formula. Compare the two results c) and d)

6.Which of these bonds is cheaper on a relative value basis i.e. which one has higher Yield: Bond A: Maturity 10yr , Coupon 5.5%, Price 83-13, Or Bond B Maturity 9yr, Coupon 6.0% Price 82-15.

7. a) Create a Matlab code modeling trajectories of a stochastic volatility process

$$dX_t = (r-q)X_t dt + \sqrt{V_t}\ X_t\ dW_{1t}$$

$$dV_t = a(V_L - V_t)dt + bV_t^c dW_{2t}$$

where process starts at *Xo=100, Vo=0.3*, and parameters *r=0.02, q=0.01,a=0.4, b=0.1, c*=0.95, $V_L = 0.25$.

Brownian motions W_{1t} and W_{2t} are independent.

Consider a period T=2 years with 250 steps per trajectory and 50,000 trajectories. Plot the trajectories of X and V and submit the printouts.

8. For the process above write a Matlab code calculating price of a European put maturing in 1 year with strike 99. Find how put price depend on *b*. Make a graph of that put price as a function of *b* for *b= 0.04, 0.06, 0.08.*