**Practice Midterm**

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**Exercise 1 [11 points]**
Let $X_1, \dots, X_n$ be iid with pdf

$$f(x;\theta) = \frac{1}{\sqrt{2\pi x^3}} e^{-\frac{x}{2\theta^2} + \frac{1}{\theta} - \frac{1}{2x}}, \qquad x > 0,$$

where $\theta > 0$ is an unknown parameter.

(a) Find the MLE of $\theta$.

(b) Show that $T(X_1, \dots, X_n) = \frac{1}{n}\sum_{i=1}^n X_i$ is an MVUE of $\theta$. You may use without proof that $\mathbb{E}[X_1] = \theta$ and $\mathrm{Var}(X_1) = \theta^3$.

**Exercise 2 [14 points]**
Let $X_1, \dots, X_n$ be iid with pdf

$$f(x;\theta) = 3\theta^3 x^{-4} I_{[\theta,\infty)}(x), \qquad x > 0,$$

where $\theta > 0$ is an unknown parameter.

(a) Find the MLE of $\theta$. Include a sketch of the likelihood function in your argument.

(b) Find an MOM estimator of $\theta$.

(c) Find a sufficient statistic for $\theta$ and discuss its implications on the MOM estimator you found in b).

Note: You won't get points if you suggest something trivial like $T(X_1, \dots, X_n) = (X_1, \dots, X_n)$ as a sufficient statistic.

**Exercise 3 [8 points]**
Let $X_1, \dots, X_n$ be iid Gamma$(2, \beta)$-distributed for some unknown $\beta > 0$.

(a) For $\alpha \in (0,1)$, construct a one-sided $(1-\alpha)$-confidence interval for $\beta$ of the form $(A, \infty)$, where $A$ is some statistic of $X_1, \dots, X_n$.

(b) For $n = 10$, $\overline{x}_n = 4.2$ and $\alpha = 0.05$, compute the endpoint $A$.

**Exercise 4 [8 points]**
Let $X_1, \dots, X_n$ be iid and Uniform$([1, 1+\theta])$-distributed, where $\theta > 0$ is an unknown parameter. The variance of this distribution is $\frac{\theta^2}{12}$.

(a) How do we have to choose $g(\theta)$ such that $T(X_1, \dots, X_n) = \log(\overline{X}_n)$ is a consistent estimator for $g(\theta)$?

(b) Determine the asymptotic distribution of $T(X_1, \dots, X_n) = \log(\overline{X}_n)$.

# Sampling distributions

**Definition:** The **gamma distribution** $\mathrm{Gamma}(\alpha, \lambda)$ with **shape parameter** $\alpha > 0$ and **scale parameter** $\lambda > 0$ has density

$$f(x; \alpha, \lambda) = \frac{\lambda^\alpha}{\Gamma(\alpha)} e^{-\lambda x} x^{\alpha-1} I_{(0,\infty)}(x).$$

**Properties:**

- $\mathrm{Gamma}(1, \lambda) = \mathrm{Exp}(\lambda)$
- $X \sim \mathrm{Gamma}(\alpha, \lambda) \implies cX \sim \mathrm{Gamma}(\alpha, \frac{\lambda}{c})$
- $X_i \sim \mathrm{Gamma}(\alpha_i, \lambda)$ independent $\implies \displaystyle\sum_{i=1}^n X_i \sim \mathrm{Gamma}(\sum_{i=1}^n \alpha_i, \lambda)$
- If $X \sim \mathrm{Gamma}(\alpha, \lambda)$, then

$$\mathbb{E}[X] = \frac{\alpha}{\lambda}, \qquad \mathrm{Var}(X) = \frac{\alpha}{\lambda^2}, \qquad \mathbb{E}[X^k] = \frac{\prod_{i=1}^k (\alpha + i - 1)}{\lambda^k}.$$

**Definition:** If $Z_i \overset{\text{iid}}{\sim} N(0,1)$, the distribution of $W_m := \sum_{i=1}^m Z_i^2$ is called the $\chi^2$**-distribution with** $m$ **degrees of freedom**. We write

$$W_m \sim \chi^2_m.$$

**Theorem:** $\chi^2_m = \mathrm{Gamma}(\frac{m}{2}, \frac{1}{2})$.

**Consequences:**

- $X \sim \chi^2_m \implies \mathbb{E}[X] = m, \ \mathrm{Var}(X) = 2m$
- $X_i \sim \chi^2_{m_i}$ independent $\implies \sum_{i=1}^n X_i \sim \chi^2_{\sum_{i=1}^n m_i}$

**Proposition:** If $X_1, \ldots, X_n$ are iid with mean $\mu$ and variance $\sigma^2 < \infty$, then

$$X_i \sim N(\mu, \sigma^2) \quad \iff \quad \text{For all } n \in \mathbb{N}, \overline{X}_n \text{ and } s_n^2 \text{ are independent rv's.}$$

In this case,

$$\overline{X}_n \sim N(\mu, \tfrac{\sigma^2}{n}), \qquad \frac{n-1}{\sigma^2} s_n^2 \sim \chi^2_{n-1}.$$

**Definition:** If $Z \sim N(0,1)$ and $V \sim \chi^2_n$ are independent, then

$$T := \frac{Z}{\sqrt{V/n}}$$

is said to follow a **t-distribution with** $n$ **degrees of freedom**. We write $T \sim t_n$.

**Theorem:** If $X_1, \ldots, X_n \overset{\text{iid}}{\sim} N(\mu, \sigma^2)$, then

$$\frac{\overline{X}_n - \mu}{\sqrt{s_n^2/n}} \sim t_{n-1}.$$

### Table of the $\chi^2$ Distribution

If $X$ has a $\chi^2$ distribution with $m$ degrees of freedom, this table gives the value of $x$ such that $\Pr(X \le x) = p$, the $p$ quantile of $X$.

| | $p$ | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| $m$ | .005 | .01 | .025 | .05 | .10 | .20 | .25 | .30 | .40 |
| 1 | .0000 | .0002 | .0010 | .0039 | .0158 | .0642 | .1015 | .1484 | .2750 |
| 2 | .0100 | .0201 | .0506 | .1026 | .2107 | .4463 | .5754 | .7133 | 1.022 |
| 3 | .0717 | .1148 | .2158 | .3518 | .5844 | 1.005 | 1.213 | 1.424 | 1.869 |
| 4 | .2070 | .2971 | .4844 | .7107 | 1.064 | 1.649 | 1.923 | 2.195 | 2.753 |
| 5 | .4117 | .5543 | .8312 | 1.145 | 1.610 | 2.343 | 2.675 | 3.000 | 3.655 |
| 6 | .6757 | .8721 | 1.237 | 1.635 | 2.204 | 3.070 | 3.455 | 3.828 | 4.570 |
| 7 | .9893 | 1.239 | 1.690 | 2.167 | 2.833 | 3.822 | 4.255 | 4.671 | 5.493 |
| 8 | 1.344 | 1.647 | 2.180 | 2.732 | 3.490 | 4.594 | 5.071 | 5.527 | 6.423 |
| 9 | 1.735 | 2.088 | 2.700 | 3.325 | 4.168 | 5.380 | 5.899 | 6.393 | 7.357 |
| 10 | 2.156 | 2.558 | 3.247 | 3.940 | 4.865 | 6.179 | 6.737 | 7.267 | 8.295 |
| 11 | 2.603 | 3.053 | 3.816 | 4.575 | 5.578 | 6.989 | 7.584 | 8.148 | 9.237 |
| 12 | 3.074 | 3.571 | 4.404 | 5.226 | 6.304 | 7.807 | 8.438 | 9.034 | 10.18 |
| 13 | 3.565 | 4.107 | 5.009 | 5.892 | 7.042 | 8.634 | 9.299 | 9.926 | 11.13 |
| 14 | 4.075 | 4.660 | 5.629 | 6.571 | 7.790 | 9.467 | 10.17 | 10.82 | 12.08 |
| 15 | 4.601 | 5.229 | 6.262 | 7.261 | 8.547 | 10.31 | 11.04 | 11.72 | 13.03 |
| 16 | 5.142 | 5.812 | 6.908 | 7.962 | 9.312 | 11.15 | 11.91 | 12.62 | 13.98 |
| 17 | 5.697 | 6.408 | 7.564 | 8.672 | 10.09 | 12.00 | 12.79 | 13.53 | 14.94 |
| 18 | 6.265 | 7.015 | 8.231 | 9.390 | 10.86 | 12.86 | 13.68 | 14.43 | 15.89 |
| 19 | 6.844 | 7.633 | 8.907 | 10.12 | 11.65 | 13.72 | 14.56 | 15.35 | 16.85 |
| 20 | 7.434 | 8.260 | 9.591 | 10.85 | 12.44 | 14.58 | 15.45 | 16.27 | 17.81 |
| 21 | 8.034 | 8.897 | 10.28 | 11.59 | 13.24 | 15.44 | 16.34 | 17.18 | 18.77 |
| 22 | 8.643 | 9.542 | 10.98 | 12.34 | 14.04 | 16.31 | 17.24 | 18.10 | 19.73 |
| 23 | 9.260 | 10.20 | 11.69 | 13.09 | 14.85 | 17.19 | 18.14 | 19.02 | 20.69 |
| 24 | 9.886 | 10.86 | 12.40 | 13.85 | 15.66 | 18.06 | 19.04 | 19.94 | 21.65 |
| 25 | 10.52 | 11.52 | 13.12 | 14.61 | 16.47 | 18.94 | 19.94 | 20.87 | 22.62 |
| 30 | 13.79 | 14.95 | 16.79 | 18.49 | 20.60 | 23.36 | 24.48 | 25.51 | 27.44 |
| 40 | 20.71 | 22.16 | 24.43 | 26.51 | 29.05 | 32.34 | 33.66 | 34.87 | 36.16 |
| 50 | 27.99 | 29.71 | 32.36 | 34.76 | 37.69 | 41.45 | 42.94 | 44.31 | 46.86 |
| 60 | 35.53 | 37.48 | 40.48 | 43.19 | 46.46 | 50.64 | 52.29 | 53.81 | 56.62 |
| 70 | 43.27 | 45.44 | 48.76 | 51.74 | 55.33 | 59.90 | 61.70 | 63.35 | 66.40 |
| 80 | 51.17 | 53.54 | 57.15 | 60.39 | 64.28 | 69.21 | 71.14 | 72.92 | 76.19 |
| 90 | 59.20 | 61.75 | 65.65 | 69.13 | 73.29 | 78.56 | 80.62 | 82.51 | 85.99 |
| 100 | 67.33 | 70.06 | 74.22 | 77.93 | 82.86 | 87.95 | 90.13 | 92.13 | 95.81 |

"Table of the X2 Distribution" adapted in part from "A new table of percentage points of the chi-square distribution" by H. Leon Harter. From BIOMETRIKA, vol 51(1964), pp. 231–239.
"Table of the X2 Distribution" adapted in part from the BIOMETRIKA TABLES FOR STATISTICIANS, Vol. 1, 3rd ed., Cambridge University Press, © 1966, edited by E.S. Pearson and H.O. Hartley.

## Table of the $\chi^2$ Distribution *(continued)*

| | | | | | $p$ | | | | |
|---|---|---|---|---|---|---|---|---|---|
| .50 | .60 | .70 | .75 | .80 | .90 | .95 | .975 | .99 | .995 |
| .4549 | .7083 | 1.074 | 1.323 | 1.642 | 2.706 | 3.841 | 5.024 | 6.635 | 7.879 |
| 1.386 | 1.833 | 2.408 | 2.773 | 3.219 | 4.605 | 5.991 | 7.378 | 9.210 | 10.60 |
| 2.366 | 2.946 | 3.665 | 4.108 | 4.642 | 6.251 | 7.815 | 9.348 | 11.34 | 12.84 |
| 3.357 | 4.045 | 4.878 | 5.385 | 5.989 | 7.779 | 9.488 | 11.14 | 13.28 | 14.86 |
| 4.351 | 5.132 | 6.064 | 6.626 | 7.289 | 9.236 | 11.07 | 12.83 | 15.09 | 16.75 |
| 5.348 | 6.211 | 7.231 | 7.841 | 8.558 | 10.64 | 12.59 | 14.45 | 16.81 | 18.55 |
| 6.346 | 7.283 | 8.383 | 9.037 | 9.803 | 12.02 | 14.07 | 16.01 | 18.48 | 20.28 |
| 7.344 | 8.351 | 9.524 | 10.22 | 11.03 | 13.36 | 15.51 | 17.53 | 20.09 | 21.95 |
| 8.343 | 9.414 | 10.66 | 11.39 | 12.24 | 14.68 | 16.92 | 19.02 | 21.67 | 23.59 |
| 9.342 | 10.47 | 11.78 | 12.55 | 13.44 | 15.99 | 18.31 | 20.48 | 23.21 | 25.19 |
| 10.34 | 11.53 | 12.90 | 13.70 | 14.63 | 17.27 | 19.68 | 21.92 | 24.72 | 26.76 |
| 11.34 | 12.58 | 14.01 | 14.85 | 15.81 | 18.55 | 21.03 | 23.34 | 26.22 | 28.30 |
| 12.34 | 13.64 | 15.12 | 15.98 | 16.98 | 19.81 | 22.36 | 24.74 | 27.69 | 29.82 |
| 13.34 | 14.69 | 16.22 | 17.12 | 18.15 | 21.06 | 23.68 | 26.12 | 29.14 | 31.32 |
| 14.34 | 15.73 | 17.32 | 18.25 | 19.31 | 22.31 | 25.00 | 27.49 | 30.58 | 32.80 |
| 15.34 | 16.78 | 18.42 | 19.37 | 20.47 | 23.54 | 26.30 | 28.85 | 32.00 | 34.27 |
| 16.34 | 17.82 | 19.51 | 20.49 | 21.61 | 24.77 | 27.59 | 30.19 | 33.41 | 35.72 |
| 17.34 | 18.87 | 20.60 | 21.60 | 22.76 | 25.99 | 28.87 | 31.53 | 34.81 | 37.16 |
| 18.34 | 19.91 | 21.69 | 22.72 | 23.90 | 27.20 | 30.14 | 32.85 | 36.19 | 38.58 |
| 19.34 | 20.95 | 22.77 | 23.83 | 25.04 | 28.41 | 31.41 | 34.17 | 37.57 | 40.00 |
| 20.34 | 21.99 | 23.86 | 24.93 | 26.17 | 29.62 | 32.67 | 35.48 | 38.93 | 41.40 |
| 21.34 | 23.03 | 24.94 | 26.04 | 27.30 | 30.81 | 33.92 | 36.78 | 40.29 | 42.80 |
| 22.34 | 24.07 | 26.02 | 27.14 | 28.43 | 32.01 | 35.17 | 38.08 | 41.64 | 44.18 |
| 23.34 | 25.11 | 27.10 | 28.24 | 29.55 | 33.20 | 36.42 | 39.36 | 42.98 | 45.56 |
| 24.34 | 26.14 | 28.17 | 29.34 | 30.68 | 34.38 | 37.65 | 40.65 | 44.31 | 46.93 |
| 29.34 | 31.32 | 33.53 | 34.80 | 36.25 | 40.26 | 43.77 | 46.98 | 50.89 | 53.67 |
| 39.34 | 41.62 | 44.16 | 45.62 | 47.27 | 51.81 | 55.76 | 59.34 | 63.69 | 66.77 |
| 49.33 | 51.89 | 54.72 | 56.33 | 58.16 | 63.17 | 67.51 | 71.42 | 76.15 | 79.49 |
| 59.33 | 62.13 | 65.23 | 66.98 | 68.97 | 74.40 | 79.08 | 83.30 | 88.38 | 91.95 |
| 69.33 | 72.36 | 75.69 | 77.58 | 79.71 | 85.53 | 90.53 | 95.02 | 100.4 | 104.2 |
| 79.33 | 82.57 | 86.12 | 88.13 | 90.41 | 96.58 | 101.9 | 106.6 | 112.3 | 116.3 |
| 89.33 | 92.76 | 96.52 | 98.65 | 101.1 | 107.6 | 113.1 | 118.1 | 124.1 | 128.3 |
| 99.33 | 102.9 | 106.9 | 109.1 | 111.7 | 118.5 | 124.3 | 129.6 | 135.8 | 140.2 |

### Table of the *t* Distribution

If $X$ has a $t$ distribution with $m$ degrees of freedom, the table gives the value of $x$ such that $\Pr(X \le x) = p$.

| $m$ | $p = .55$ | .60 | .65 | .70 | .75 | .80 | .85 | .90 | .95 | .975 | .99 | .995 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | .158 | .325 | .510 | .727 | 1.000 | 1.376 | 1.963 | 3.078 | 6.314 | 12.706 | 31.821 | 63.657 |
| 2 | .142 | .289 | .445 | .617 | .816 | 1.061 | 1.386 | 1.886 | 2.920 | 4.303 | 6.965 | 9.925 |
| 3 | .137 | .277 | .424 | .584 | .765 | .978 | 1.250 | 1.638 | 2.353 | 3.182 | 4.541 | 5.841 |
| 4 | .134 | .271 | .414 | .569 | .741 | .941 | 1.190 | 1.533 | 2.132 | 2.776 | 3.747 | 4.604 |
| 5 | .132 | .267 | .408 | .559 | .727 | .920 | 1.156 | 1.476 | 2.015 | 2.571 | 3.365 | 4.032 |
| 6 | .131 | .265 | .404 | .553 | .718 | .906 | 1.134 | 1.440 | 1.943 | 2.447 | 3.143 | 3.707 |
| 7 | .130 | .263 | .402 | .549 | .711 | .896 | 1.119 | 1.415 | 1.895 | 2.365 | 2.998 | 3.499 |
| 8 | .130 | .262 | .399 | .546 | .706 | .889 | 1.108 | 1.397 | 1.860 | 2.306 | 2.896 | 3.355 |
| 9 | .129 | .261 | .398 | .543 | .703 | .883 | 1.100 | 1.383 | 1.833 | 2.262 | 2.821 | 3.250 |
| 10 | .129 | .260 | .397 | .542 | .700 | .879 | 1.093 | 1.372 | 1.812 | 2.228 | 2.764 | 3.169 |
| 11 | .129 | .260 | .396 | .540 | .697 | .876 | 1.088 | 1.363 | 1.796 | 2.201 | 2.718 | 3.106 |
| 12 | .128 | .259 | .395 | .539 | .695 | .873 | 1.083 | 1.356 | 1.782 | 2.179 | 2.681 | 3.055 |
| 13 | .128 | .259 | .394 | .538 | .694 | .870 | 1.079 | 1.350 | 1.771 | 2.160 | 2.650 | 3.012 |
| 14 | .128 | .258 | .393 | .537 | .692 | .868 | 1.076 | 1.345 | 1.761 | 2.145 | 2.624 | 2.977 |
| 15 | .128 | .258 | .393 | .536 | .691 | .866 | 1.074 | 1.341 | 1.753 | 2.131 | 2.602 | 2.947 |
| 16 | .128 | .258 | .392 | .535 | .690 | .865 | 1.071 | 1.337 | 1.746 | 2.120 | 2.583 | 2.921 |
| 17 | .128 | .257 | .392 | .534 | .689 | .863 | 1.069 | 1.333 | 1.740 | 2.110 | 2.567 | 2.898 |
| 18 | .127 | .257 | .392 | .534 | .688 | .862 | 1.067 | 1.330 | 1.734 | 2.101 | 2.552 | 2.878 |
| 19 | .127 | .257 | .391 | .533 | .688 | .861 | 1.066 | 1.328 | 1.729 | 2.093 | 2.539 | 2.861 |
| 20 | .127 | .257 | .391 | .533 | .687 | .860 | 1.064 | 1.325 | 1.725 | 2.086 | 2.528 | 2.845 |
| 21 | .127 | .257 | .391 | .532 | .686 | .859 | 1.063 | 1.323 | 1.721 | 2.080 | 2.518 | 2.831 |
| 22 | .127 | .256 | .390 | .532 | .686 | .858 | 1.061 | 1.321 | 1.717 | 2.074 | 2.508 | 2.819 |
| 23 | .127 | .256 | .390 | .532 | .685 | .858 | 1.060 | 1.319 | 1.714 | 2.069 | 2.500 | 2.807 |
| 24 | .127 | .256 | .390 | .531 | .685 | .857 | 1.059 | 1.318 | 1.711 | 2.064 | 2.492 | 2.797 |
| 25 | .127 | .256 | .390 | .531 | .684 | .856 | 1.058 | 1.316 | 1.708 | 2.060 | 2.485 | 2.787 |
| 26 | .127 | .256 | .390 | .531 | .684 | .856 | 1.058 | 1.315 | 1.706 | 2.056 | 2.479 | 2.779 |
| 27 | .127 | .256 | .389 | .531 | .684 | .855 | 1.057 | 1.314 | 1.703 | 2.052 | 2.473 | 2.771 |
| 28 | .127 | .256 | .389 | .530 | .683 | .855 | 1.056 | 1.313 | 1.701 | 2.048 | 2.467 | 2.763 |
| 29 | .127 | .256 | .389 | .530 | .683 | .854 | 1.055 | 1.311 | 1.699 | 2.045 | 2.462 | 2.756 |
| 30 | .127 | .256 | .389 | .530 | .683 | .854 | 1.055 | 1.310 | 1.697 | 2.042 | 2.457 | 2.750 |
| 40 | .126 | .255 | .388 | .529 | .681 | .851 | 1.050 | 1.303 | 1.684 | 2.021 | 2.423 | 2.704 |
| 60 | .126 | .254 | .387 | .527 | .679 | .848 | 1.046 | 1.296 | 1.671 | 2.000 | 2.390 | 2.660 |
| 120 | .126 | .254 | .386 | .526 | .677 | .845 | 1.041 | 1.289 | 1.658 | 1.980 | 2.358 | 2.617 |
| $\infty$ | .126 | .253 | .385 | .524 | .674 | .842 | 1.036 | 1.282 | 1.645 | 1.960 | 2.326 | 2.576 |

## Table of the Standard Normal Distribution Function

$$\Phi(x) = \int_{-\infty}^{x} \frac{1}{(2\pi)^{1/2}} \exp\left(-\tfrac{1}{2}u^2\right) du$$

| $x$ | $\Phi(x)$ | $x$ | $\Phi(x)$ | $x$ | $\Phi(x)$ | $x$ | $\Phi(x)$ | $x$ | $\Phi(x)$ |
|---|---|---|---|---|---|---|---|---|---|
| 0.00 | 0.5000 | 0.60 | 0.7257 | 1.20 | 0.8849 | 1.80 | 0.9641 | 2.40 | 0.9918 |
| 0.01 | 0.5040 | 0.61 | 0.7291 | 1.21 | 0.8869 | 1.81 | 0.9649 | 2.41 | 0.9920 |
| 0.02 | 0.5080 | 0.62 | 0.7324 | 1.22 | 0.8888 | 1.82 | 0.9656 | 2.42 | 0.9922 |
| 0.03 | 0.5120 | 0.63 | 0.7357 | 1.23 | 0.8907 | 1.83 | 0.9664 | 2.43 | 0.9925 |
| 0.04 | 0.5160 | 0.64 | 0.7389 | 1.24 | 0.8925 | 1.84 | 0.9671 | 2.44 | 0.9927 |
| 0.05 | 0.5199 | 0.65 | 0.7422 | 1.25 | 0.8944 | 1.85 | 0.9678 | 2.45 | 0.9929 |
| 0.06 | 0.5239 | 0.66 | 0.7454 | 1.26 | 0.8962 | 1.86 | 0.9686 | 2.46 | 0.9931 |
| 0.07 | 0.5279 | 0.67 | 0.7486 | 1.27 | 0.8980 | 1.87 | 0.9693 | 2.47 | 0.9932 |
| 0.08 | 0.5319 | 0.68 | 0.7517 | 1.28 | 0.8997 | 1.88 | 0.9699 | 2.48 | 0.9934 |
| 0.09 | 0.5359 | 0.69 | 0.7549 | 1.29 | 0.9015 | 1.89 | 0.9706 | 2.49 | 0.9936 |
| 0.10 | 0.5398 | 0.70 | 0.7580 | 1.30 | 0.9032 | 1.90 | 0.9713 | 2.50 | 0.9938 |
| 0.11 | 0.5438 | 0.71 | 0.7611 | 1.31 | 0.9049 | 1.91 | 0.9719 | 2.52 | 0.9941 |
| 0.12 | 0.5478 | 0.72 | 0.7642 | 1.32 | 0.9066 | 1.92 | 0.9726 | 2.54 | 0.9945 |
| 0.13 | 0.5517 | 0.73 | 0.7673 | 1.33 | 0.9082 | 1.93 | 0.9732 | 2.56 | 0.9948 |
| 0.14 | 0.5557 | 0.74 | 0.7704 | 1.34 | 0.9099 | 1.94 | 0.9738 | 2.58 | 0.9951 |
| 0.15 | 0.5596 | 0.75 | 0.7734 | 1.35 | 0.9115 | 1.95 | 0.9744 | 2.60 | 0.9953 |
| 0.16 | 0.5636 | 0.76 | 0.7764 | 1.36 | 0.9131 | 1.96 | 0.9750 | 2.62 | 0.9956 |
| 0.17 | 0.5675 | 0.77 | 0.7794 | 1.37 | 0.9147 | 1.97 | 0.9756 | 2.64 | 0.9959 |
| 0.18 | 0.5714 | 0.78 | 0.7823 | 1.38 | 0.9162 | 1.98 | 0.9761 | 2.66 | 0.9961 |
| 0.19 | 0.5753 | 0.79 | 0.7852 | 1.39 | 0.9177 | 1.99 | 0.9767 | 2.68 | 0.9963 |
| 0.20 | 0.5793 | 0.80 | 0.7881 | 1.40 | 0.9192 | 2.00 | 0.9773 | 2.70 | 0.9965 |
| 0.21 | 0.5832 | 0.81 | 0.7910 | 1.41 | 0.9207 | 2.01 | 0.9778 | 2.72 | 0.9967 |
| 0.22 | 0.5871 | 0.82 | 0.7939 | 1.42 | 0.9222 | 2.02 | 0.9783 | 2.74 | 0.9969 |
| 0.23 | 0.5910 | 0.83 | 0.7967 | 1.43 | 0.9236 | 2.03 | 0.9788 | 2.76 | 0.9971 |
| 0.24 | 0.5948 | 0.84 | 0.7995 | 1.44 | 0.9251 | 2.04 | 0.9793 | 2.78 | 0.9973 |
| 0.25 | 0.5987 | 0.85 | 0.8023 | 1.45 | 0.9265 | 2.05 | 0.9798 | 2.80 | 0.9974 |
| 0.26 | 0.6026 | 0.86 | 0.8051 | 1.46 | 0.9279 | 2.06 | 0.9803 | 2.82 | 0.9976 |
| 0.27 | 0.6064 | 0.87 | 0.8079 | 1.47 | 0.9292 | 2.07 | 0.9808 | 2.84 | 0.9977 |
| 0.28 | 0.6103 | 0.88 | 0.8106 | 1.48 | 0.9306 | 2.08 | 0.9812 | 2.86 | 0.9979 |
| 0.29 | 0.6141 | 0.89 | 0.8133 | 1.49 | 0.9319 | 2.09 | 0.9817 | 2.88 | 0.9980 |
| 0.30 | 0.6179 | 0.90 | 0.8159 | 1.50 | 0.9332 | 2.10 | 0.9821 | 2.90 | 0.9981 |
| 0.31 | 0.6217 | 0.91 | 0.8186 | 1.51 | 0.9345 | 2.11 | 0.9826 | 2.92 | 0.9983 |
| 0.32 | 0.6255 | 0.92 | 0.8212 | 1.52 | 0.9357 | 2.12 | 0.9830 | 2.94 | 0.9984 |
| 0.33 | 0.6293 | 0.93 | 0.8238 | 1.53 | 0.9370 | 2.13 | 0.9834 | 2.96 | 0.9985 |
| 0.34 | 0.6331 | 0.94 | 0.8264 | 1.54 | 0.9382 | 2.14 | 0.9838 | 2.98 | 0.9986 |
| 0.35 | 0.6368 | 0.95 | 0.8289 | 1.55 | 0.9394 | 2.15 | 0.9842 | 3.00 | 0.9987 |
| 0.36 | 0.6406 | 0.96 | 0.8315 | 1.56 | 0.9406 | 2.16 | 0.9846 | 3.05 | 0.9989 |
| 0.37 | 0.6443 | 0.97 | 0.8340 | 1.57 | 0.9418 | 2.17 | 0.9850 | 3.10 | 0.9990 |
| 0.38 | 0.6480 | 0.98 | 0.8365 | 1.58 | 0.9429 | 2.18 | 0.9854 | 3.15 | 0.9992 |
| 0.39 | 0.6517 | 0.99 | 0.8389 | 1.59 | 0.9441 | 2.19 | 0.9857 | 3.20 | 0.9993 |
| 0.40 | 0.6554 | 1.00 | 0.8413 | 1.60 | 0.9452 | 2.20 | 0.9861 | 3.25 | 0.9994 |
| 0.41 | 0.6591 | 1.01 | 0.8437 | 1.61 | 0.9463 | 2.21 | 0.9864 | 3.30 | 0.9995 |
| 0.42 | 0.6628 | 1.02 | 0.8461 | 1.62 | 0.9474 | 2.22 | 0.9868 | 3.35 | 0.9996 |
| 0.43 | 0.6664 | 1.03 | 0.8485 | 1.63 | 0.9485 | 2.23 | 0.9871 | 3.40 | 0.9997 |
| 0.44 | 0.6700 | 1.04 | 0.8508 | 1.64 | 0.9495 | 2.24 | 0.9875 | 3.45 | 0.9997 |
| 0.45 | 0.6736 | 1.05 | 0.8531 | 1.65 | 0.9505 | 2.25 | 0.9878 | 3.50 | 0.9998 |
| 0.46 | 0.6772 | 1.06 | 0.8554 | 1.66 | 0.9515 | 2.26 | 0.9881 | 3.55 | 0.9998 |
| 0.47 | 0.6808 | 1.07 | 0.8577 | 1.67 | 0.9525 | 2.27 | 0.9884 | 3.60 | 0.9998 |
| 0.48 | 0.6844 | 1.08 | 0.8599 | 1.68 | 0.9535 | 2.28 | 0.9887 | 3.65 | 0.9999 |
| 0.49 | 0.6879 | 1.09 | 0.8621 | 1.69 | 0.9545 | 2.29 | 0.9890 | 3.70 | 0.9999 |
| 0.50 | 0.6915 | 1.10 | 0.8643 | 1.70 | 0.9554 | 2.30 | 0.9893 | 3.75 | 0.9999 |
| 0.51 | 0.6950 | 1.11 | 0.8665 | 1.71 | 0.9564 | 2.31 | 0.9896 | 3.80 | 0.9999 |
| 0.52 | 0.6985 | 1.12 | 0.8686 | 1.72 | 0.9573 | 2.32 | 0.9898 | 3.85 | 0.9999 |
| 0.53 | 0.7019 | 1.13 | 0.8708 | 1.73 | 0.9582 | 2.33 | 0.9901 | 3.90 | 1.0000 |
| 0.54 | 0.7054 | 1.14 | 0.8729 | 1.74 | 0.9591 | 2.34 | 0.9904 | 3.95 | 1.0000 |
| 0.55 | 0.7088 | 1.15 | 0.8749 | 1.75 | 0.9599 | 2.35 | 0.9906 | 4.00 | 1.0000 |
| 0.56 | 0.7123 | 1.16 | 0.8770 | 1.76 | 0.9608 | 2.36 | 0.9909 | | |
| 0.57 | 0.7157 | 1.17 | 0.8790 | 1.77 | 0.9616 | 2.37 | 0.9911 | | |
| 0.58 | 0.7190 | 1.18 | 0.8810 | 1.78 | 0.9625 | 2.38 | 0.9913 | | |
| 0.59 | 0.7224 | 1.19 | 0.8830 | 1.79 | 0.9633 | 2.39 | 0.9916 | | |