

Mathematics of Finance. Homework 4 due December 16, 2022 11.59pm

Please write a pledge that homework solutions represent your own work and that you did not copy solutions from the work of other students. Each problem 20pts.

1. a) Create a Matlab code modeling trajectories of a constant elasticity of variance process

$dX_t = 0.02X_t dt + 0.3X_t^a dW_t$ where $a=0.75$. Process starts at 100. Consider a period $T=1$ years with 250 steps per trajectory and 8,000 trajectories. Plot the trajectories and submit the printout.

b) Assume that interest rate is 4% and a non-dividend paying stock in a risk neutral world follows such process. Write a Matlab code calculating price C of a European call maturing in 1 year with strike 99 with 100,000 trajectories. How long does the code runs on your computer. Write the number of seconds and milliseconds.

2. Explain the difference between VaR and CVaR. (CVaR is also called Expected Shortfall, Average Value at Risk or AVaR, Expected Tail Loss or ETL.)

3. On November 17, 2022 portfolio had 100,000 dollars in cash and was long 500 shares of MSFT, long 400 Shares of AAPL and short 100 shares of SPY at a closing prices on November 17, 2022. Look for closing prices at Yahoo finance.

a) What is the Net Asset Value (NAV) of portfolio on the close of November 17, 2022, i.e. value of money left if portfolio is liquidated.

b) What is the gross and net leverage of portfolio on close of November 17, 2022? (See leverage explanation example at the end of the homework, on page 2)

c) Using historical data from Yahoo and assuming cash earned 1% annually with daily compounding, simulate in spreadsheet portfolio NAV and leverage from January 4, 2010 to November 17, 2022. (Calculate portfolio NAV and portfolio leverage for every business day. Use split and dividend adjusted price, adj. close, last column on Yahoo.)

4. For portfolio described in the problem 3 make a spreadsheet: download data from January 4, 2010 to November 17, 2022. For each day from June 1, 2010 to November 17, 2022, calculate 90 days rolling volatility of portfolio for 1day and annualized: $\text{AnnualizedVol} = 1\text{dayVol} * \sqrt{250}$ Print a plot of AnnualizedVol, 1dayVol,

5. For portfolio described in the problem 4 based on rolling 1 day volatility calculate 95% VaR for each day from June 1, 2010 to November 17, 2022.

$$\text{VaR} = 1.645 * 1\text{dayVol}.$$

Print a plot of VaR as a function of time.

Leverage explanation and example.

Portfolio:

Long 1000 Shares AAPL with price 171.00

Long 1000 shares of IBM with price 154.00

Short 500 Shares of SPY with price 264.00 and have

Cash 150,000 USD (that includes cash received from short sale).

NAV of Portfolio is the value of portfolio if it is liquidated: Longs sold, Shorts bought back.

$$\text{NAV} = 1000 \times 171 + 1000 \times 154 - 500 \times 264 + 150,000 = 343,000$$

GROSS LEVERAGE is calculated as:

$$(\text{VALUE OF Long Stocks} + \text{ABS}(\text{VALUE of Short Stocks}) / \text{NAV}$$

$$(1000 \times 171 + 1000 \times 154 + \text{Abs}(- 500 \times 264)) / 343,000 = 1.33 = 133\%$$

NET LEVERAGE is calculated as:

$$(\text{VALUE OF Long Stocks} - \text{ABS}(\text{VALUE of Short Stocks}) / \text{NAV}$$

$$(1000 \times 171 + 1000 \times 154 - \text{Abs}(- 500 \times 264)) / 343,000 = 0.56 = 56\%$$