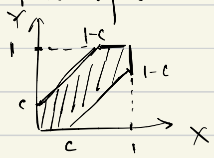
4. a) First of all, note that X and Y are symmetric.



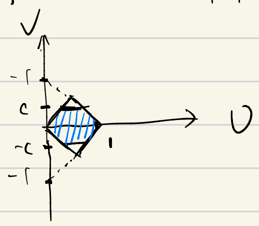
Given X, Y is a uniform distribution.

For $0 < X < c$, $f(Y|X) = \frac{1}{c+X} \mathbb{1}_{0<Y<c+X}$

For $c \le X < 1-c$, $f(Y|X) = \frac{1}{2c} \mathbb{1}_{X-c<Y<X+c}$

For $1-c \le X < 1$, $f(Y|X) = \frac{1}{1-X+c} \mathbb{1}_{X-c<Y<1}$

e) $f(U,V) \propto \mathbb{1}_{|V|<2c} \cdot \mathbb{1}_{U+V\in(0,1)} \cdot \mathbb{1}_{U-V\in(0,1)}$



The area is $\frac{1}{2}(1-(1-c)^2) = (2c-c^2)\frac{1}{2}$

So the constant is $\frac{2}{2c-c^2}$