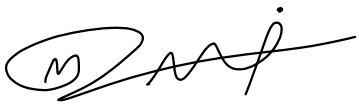
Quiz 01

COSC 373: Distributed Algorithms, Spring 2022

Instructions. This quiz is open book and open note—you may freely use your notes, lecture notes, or textbook while working on it. You may *not* consult any living resources such as other students or web forums. The quiz should be submitted through Gradescope by 5:00pm on Friday, March 4th.

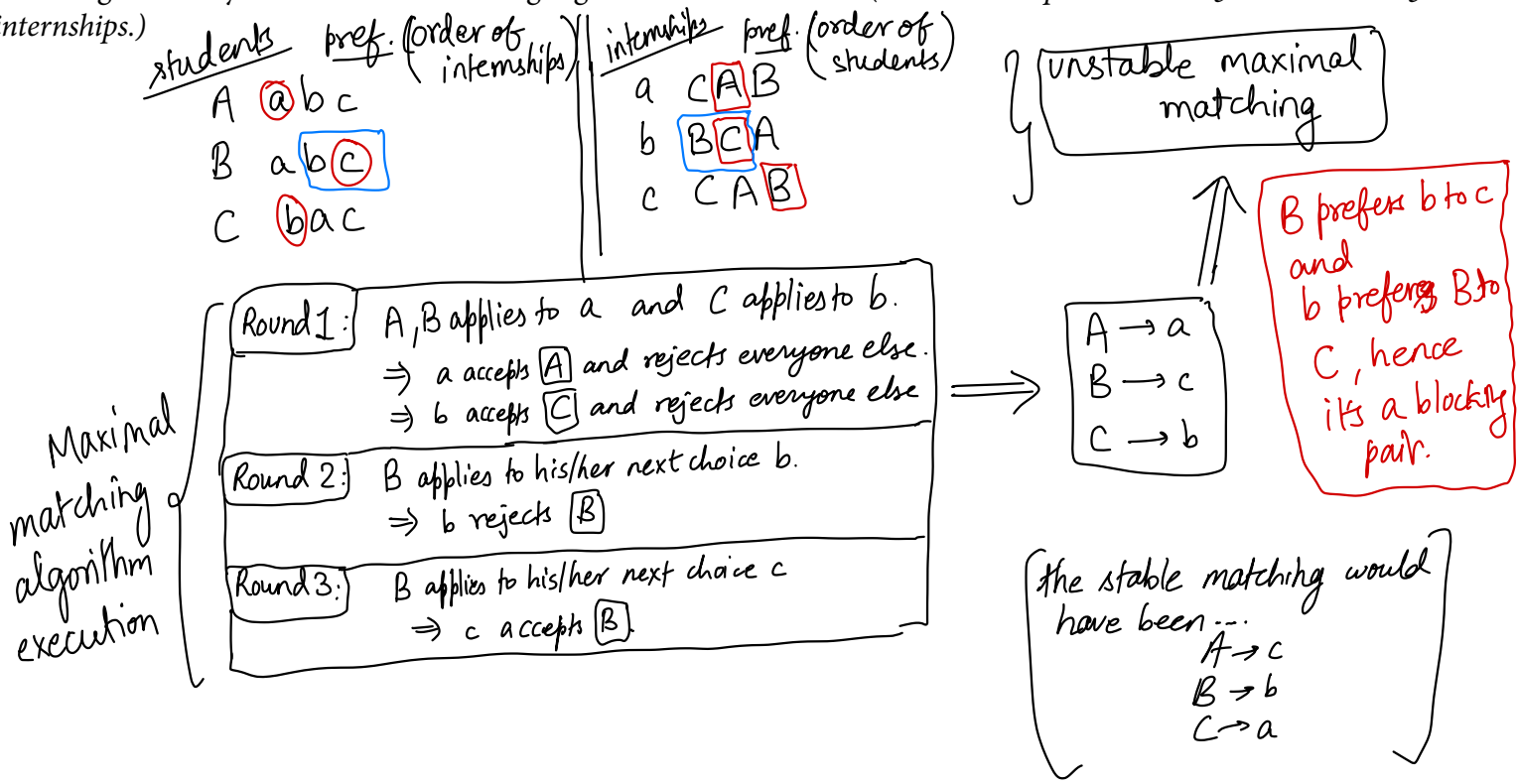
Affirmation. I attest that that work presented here is mine and mine alone. I have not consulted any disallowed resources while taking this quiz.

Name: Dhyey Dharmendrakumar Mavani

Signature: 

Background. In class, we described two algorithms for finding matchings between students and internships: the Gale-Shapley algorithm (which finds a *stable* matching) and a maximal matching algorithm.

Question 1. Describe a stable matching instance (i.e., a set of students, internships, and rankings) for which the matching found by the maximal matching algorithm is not stable. *(Hint: this is possible with 3 students and 3 internships.)*



Question 2. Find a stable matching instance with 3 students and 3 internships in which the maximal matching algorithm produces a perfect matching (i.e., all students/internships are matched), but for which any stable matching only matches 2 students/internships. *(Hint: this is only possible with incomplete preference lists.)*

students	pref. (order of internships)
A	a b
B	a b c
C	b a c

internships	pref. (order of students)
a	C A B
b	B C A
c	C A B

STABLE MATCHING ALGORITHM

MAXIMAL MATCHING ALGORITHM

Round 1: A & B applies to a and C applies to b. ⇒ B is rejected and A, C on waitlist

Round 2: B applies to b and now, b keeps B on waitlist while rejecting C.

Round 3: C applies to its next choice a, a keeps C on waitlist, while rejecting A.

Round 4: A applies to b, but gets rejected since b already has application from B.

Round 5: ~~A applies to his last preference c, and since c hasn't received any application yet, hence it would keep A on waitlist. Hence, everyone is matched and the algorithm will be terminated.~~

A doesn't have any more preferences, and c has not received any applications. Hence, everyone is matched except A and c. Because the preferences are exhausted, the algorithm will terminate, resulting in an incomplete stable matching.

students	pref. (order of internships)
A	a b
B	~~a~~ ~~b~~ c
C	b a c

internships	pref. (order of students)
a	C A B
b	B C A
c	C A B

Round 1: A, B applies to a and C applies to b.
⇒ a accepts A and rejects everyone else.
⇒ b accepts C and rejects everyone else

Round 2: B applies to his/her next choice b.
⇒ b rejects B

Round 3: B applies to his/her next choice c
⇒ c accepts B