swap rates

1M R_1 $\longrightarrow$ $DF(1M) = x_1$

3M R_2 $\longrightarrow$ $DF(3M) = x_2$

6M R_3 $\longrightarrow$ $DF(6M) = x_3$

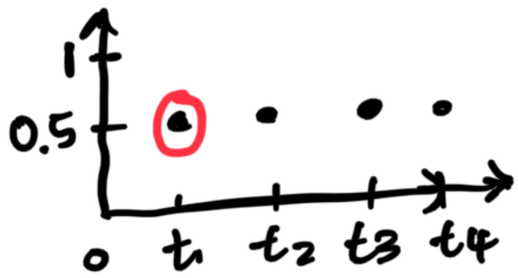
1Y R_4 $\longrightarrow$ $DF(1Y) = x_4$

① DF curve : $\vec{t} = \{0, t_1, t_2, t_3, t_4\}$

$\vec{x} = \{1.0, x_1, x_2, x_3, x_4\}$

② make a guess for $\vec{x}$, $x_i = 0.5 \ \forall i$

③ Solve for x_1



$\boxed{PV(1M\ \text{swap}) = f(x_1)} = 0$

Define $f(x_1)$: $\leftarrow$ function

input $= x_1$

$\vec{x} = \{1.0, x_1, 0.5, 0.5, 0.5\}$

swap price $(\vec{t}, \vec{x}) \rightarrow$ output

---

model on DF

- parametrize DF curve

$DF(t; \vec{x})$ $\qquad \vec{x} = (x_1, \cdots, x_K)$

$x_i = DF(t_i)$

- solve for x_i's iteratively

model on forward rates

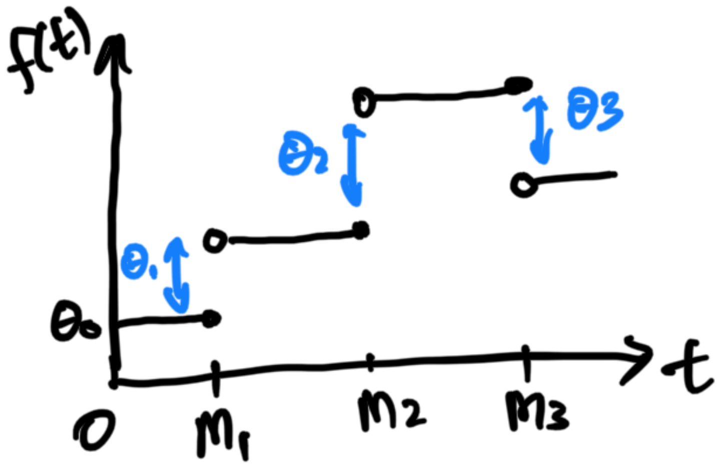
- parametrize fwd rate curve

$f(t; \vec{\theta})$ $\qquad \vec{\theta} = (\theta_1, \cdots, \theta_K)$

- solve for $\vec{\theta}$

CME methodology for SOFR term rates

$f(t)$ = SOFR forward rate on day t



FOMC meeting

Fed Fund target rate

$f(t;\vec{\theta})$

$\vec{\theta}=(\theta_0,\theta_1,\cdots,\theta_K)$

m_i = ith FOMC policy rate announcement date

- market instruments.

  13 1M SOFR futures P_i^{1M}, $i=0,1,\cdots,12$

  5 3m " " P_j^{3M}, $j=0,1,\cdots,5$

- given $f(t;\vec{\theta})$, find implied futures prices $\hat{P}_i^{1M}(\vec{\theta})$, $\hat{P}_j^{3M}(\vec{\theta})$

- Find $\vec{\theta}$ that minimizes pricing error.

$$\text{error}=\left(\sum_{i=0}^{12} w_i^{1M}\left(P_i^{1M}-\hat{P}_i^{1M}(\vec{\theta})\right)^2\right)^{1/2}$$

$$+\left(\sum_{j=0}^{4} w_j^{3M}\left(P_j^{3M}-\hat{P}_j^{3M}(\vec{\theta})\right)^2\right)^{1/2}$$

$$+\boxed{\lambda\left(\sum_{s=0}^{K}\theta_s^2\right)^{1/2}}$$

penalty term to minimize jump sizes.

λ = user-defined input parameter

<u>CME</u>: $\lambda=0.1/\sqrt{K}$

$w_i^{1M}=w_j^{3M}=0.05$

jumps

$P_0^{1M}, P_1^{1M}, P_2^{1M}, \cdots P_{12}^{1M}$

$\hat{P}_0^{1M}, \cdots\cdots \hat{P}_{12}^{1M}$

$$\left[w_0\left(P_0^{1M}-\hat{P}_0^{1M}\right)^2+w_1\left(P_1^{1M}-\hat{P}_1^{1M}\right)^2+\cdots\right]^{1/2}$$

1M SOFR futures

$P = 99.5$

$100 - P = 0.5 \leftarrow$ rate%

$P^{1m} = 100 -$ arith. avg of SOFR rates over contract month.

Feb 2023 $\leftarrow$ 1m future

28 SOFR rates

Feb 1 — Mar 1

Imply price from $f(t)$ curve:

$$\hat{P}^{1m} = 100 - \frac{1}{N} \sum_{T_1 \le t \le T_2} f(t)$$

T_1 — T_2

3M SOFR futures

$$P^{3m} = 100 - \underbrace{\text{daily compound avg}}_{}(R) \text{ of SOFR rates over contract period.}$$

$100 - R$

T_1 — T_2

$r_1, r_2, \ldots, r_k$ $\quad$ k business days in 3m period.

t_1 — t_2 — t_3 $\quad$ 1 day (t_2 to t_3), 3 calendar days (t_1 to t_2).

$$\underbrace{\boxed{\left(1 + r_1 \times \frac{3}{360}\right)\left(1 + r_2 \times \frac{1}{360}\right) \cdots}}$$

$$= 1 + R \times \frac{\boxed{T_2 - T_1}}{360}$$

$T_2 - T_1 \leftarrow 90$

eg. contract period = 91 days.

$$\boxed{1.05\ldots} = 1 + R \times \frac{90}{360}$$

$$R = \left(\boxed{} - 1\right) \times \frac{360}{90}$$

Implied price $\hat{P}^{3m}$ from $f(t)$

$$\hat{P}^{3m} = 100 - \left(\boxed{\prod_{i=1}^{n} \left(1 + f(t_i)\Delta t_i\right)} - 1\right) \times \frac{360}{N}$$

n = # business days.

$N \leftarrow$ # calendar days