STAT GU4261/GR5261 - Statistical Methods in Finance - Homework #5 Solutions

Question #1

Part (a)
According to the SML we have

$$\mu_j - 0.03 = 0.75(0.1 - 0.03)$$
$$\implies \boxed{\mu_j = 0.0825}$$

Part (b)
Since the true expected return is larger than this quantity, the asset is underpriced.

Question #2

Page 513, Problem 3
Part (a)
Read it directly off the CML:

$$\mu_R = 0.023 + \frac{0.1 - 0.023}{0.12}(0.05)$$
$$\implies \boxed{\mu_R = 0.0551}$$

Part (b)

$$\beta_A = \frac{\sigma_{AM}}{\sigma_M^2} = \frac{0.004}{0.12^2} = 0.278$$

Part (c)
Plug in directly to the SML equation to obtain

$$\mu_B = 0.023 + (.1 - .023)(1.5) = 0.1385$$
$$\mu_C = 0.023 + (.1 - .023)(1.8) = 0.1616$$

so the expected return is given by

$$(0.1385 + 0.1616)/2 = 0.15005$$

and volatility given by

$$\frac{1}{2}\sqrt{\sigma_B^2+\sigma_C^2+2\sigma_{BC}}=\frac{1}{2}\sqrt{(\beta_B+\beta_C)^2\sigma_M^2+\sigma_{\epsilon,B}^2+\sigma_{\epsilon,C}^2}=\frac{1}{2}\sqrt{(1.5+1.8)^2(0.12)^2+0.08^2+0.1^2}=0.2081$$

Page 513, Problem 6
Part (a)

$$\beta=.0165/(0.11)^2=1.364$$

Part (b)
Calculate using SML.

$$=0.04+1.364(.12-.04)=0.1491$$

Part (c)
The contribution to the variance of the asset by the market is $\beta^2\sigma_M^2$, so our answer is

$$=\beta^2\sigma_M^2/\sigma_A^2=(0.0165)^2/((0.11)^2(0.03))=0.75$$

which means 75% of the variance is due to market risk.

Question #3

```
library(readxl)
```

```
## Warning: package 'readxl' was built under R version 4.2.2
```

```
dat.q3 = as.data.frame(read_xlsx("CAPM-DATA-1.xlsx"))
```

We run the corresponding four linear models to get estimates for β in each case.

```
#get difference between columns and risk free rate
diff.mtx = dat.q3[,1:5] - dat.q3[,6]

MSOFT.lm = lm(diff.mtx$MSOFT ~ diff.mtx$MPORT)
GE.lm = lm(diff.mtx$GE ~ diff.mtx$MPORT)
GM.lm = lm(diff.mtx$GM ~ diff.mtx$MPORT)
IBM.lm = lm(diff.mtx$IBM ~ diff.mtx$MPORT)

summary(MSOFT.lm)
```

```
##
## Call:
## lm(formula = diff.mtx$MSOFT ~ diff.mtx$MPORT)
##
```

```
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.28112 -0.05494 -0.00580  0.04319  0.34337
##
## Coefficients:
##                Estimate Std. Error t value Pr(>|t|)
## (Intercept)    0.010192   0.008819   1.156     0.25
## diff.mtx$MPORT 1.429866   0.188159   7.599  7.8e-12 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.09542 on 118 degrees of freedom
## Multiple R-squared:  0.3286, Adjusted R-squared:  0.3229
## F-statistic: 57.75 on 1 and 118 DF,  p-value: 7.799e-12
```

```
summary(GE.lm)
```

```
##
## Call:
## lm(formula = diff.mtx$GE ~ diff.mtx$MPORT)
##
## Residuals:
##       Min        1Q    Median        3Q       Max
## -0.114633 -0.033966 -0.001445  0.037915  0.182273
##
## Coefficients:
##                Estimate Std. Error t value Pr(>|t|)
## (Intercept)    0.005886   0.004908   1.199    0.233
## diff.mtx$MPORT 0.983041   0.104718   9.388 5.65e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.05311 on 118 degrees of freedom
## Multiple R-squared:  0.4275, Adjusted R-squared:  0.4227
## F-statistic: 88.13 on 1 and 118 DF,  p-value: 5.65e-16
```

```
summary(GM.lm)
```

```
##
## Call:
## lm(formula = diff.mtx$GM ~ diff.mtx$MPORT)
##
## Residuals:
##       Min        1Q    Median        3Q       Max
```

```
## -0.230794 -0.041190 -0.006428  0.035515  0.215521
##
## Coefficients:
##                  Estimate Std. Error t value Pr(>|t|)
## (Intercept)    -0.002314   0.007301  -0.317    0.752
## diff.mtx$MPORT  1.074415   0.155765   6.898  2.8e-10 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.079 on 118 degrees of freedom
## Multiple R-squared:  0.2873, Adjusted R-squared:  0.2813
## F-statistic: 47.58 on 1 and 118 DF,  p-value: 2.8e-10
```

```
summary(IBM.lm)
```

```
##
## Call:
## lm(formula = diff.mtx$IBM ~ diff.mtx$MPORT)
##
## Residuals:
##       Min        1Q    Median        3Q       Max
## -0.272521 -0.050467 -0.009385  0.043036  0.262101
##
## Coefficients:
##                Estimate Std. Error t value Pr(>|t|)
## (Intercept)    0.006849   0.007285    0.94    0.349
## diff.mtx$MPORT 1.268299   0.155431    8.16 4.13e-13 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.07883 on 118 degrees of freedom
## Multiple R-squared:  0.3607, Adjusted R-squared:  0.3553
## F-statistic: 66.58 on 1 and 118 DF,  p-value: 4.129e-13
```

Part (a)
The betas are given as the slope parameter in each of the above regressions. From this we note that MSOFT, GM and IBM are "aggressive", while GE is relatively non-aggressive.
Part (b)
We would fail to reject the null hypothesis that the intercept is 0 in all four cases, hence the assumption that α is zero is statistically reasonable (all p-values are quite large).
Part (c)

```
confint(MSOFT.lm)[2,]
```

```
##    2.5 %   97.5 %
## 1.057261 1.802472
```

```
confint(GE.lm)[2,]
```

```
##     2.5 %    97.5 %
## 0.7756721 1.1904105
```

```
confint(GM.lm)[2,]
```

```
##     2.5 %    97.5 %
## 0.7659588 1.3828721
```

```
confint(IBM.lm)[2,]
```

```
##     2.5 %    97.5 %
## 0.9605042 1.5760938
```

Part (d)
Our test statistic in each case is the slope estimate minus 1/Std.Error of the slope estimate, and this follows a student's t-distribution on 118 degrees of freedom in all cases. We can obtain the result of the test by forming 95% confidence intervals for the slope parameters and checking if 1 is in the interval or not (see part (c) for the intervals). We note that we fail to reject the null hypothesis that $\beta = 1$ for GE, GM, IBM but do reject it for MSOFT. i.e. 1 is a realistic value for β in the former case, and not so in the latter case, at the 0.05 testing level.

Part (e)
This is discussed in part (d).