Homework 8 (Wednesday April 12 2023)
Due Monday April 24 2023

Problem 1. **(30 points)**

Consider the risk-neutral stochastic volatility process

$$d\sigma = b\sigma dW$$
$$dS = \sigma S dZ$$
$$\rho = 0$$

where ρ is the correlation between the stock price innovations and the volatility innovations.

Assume interest rates and dividend yields are zero, and that the initial volatility $\sigma = 0.2$ and the volatility of volatility $b = 0.6$. The initial stock price is 100.

Write a Monte Carlo program to simulate this stochastic volatility process and estimate the quantities below in a risk-neutral world. Show your programs and your results.

(i) Find the Black-Scholes implied volatility of an at-the-money call option with one year to expiration. [10 points]

(ii) Find the difference in Black-Scholes implied volatility between the at-the-money call option and an option of the same expiration with strike 85. [10 points]

(iii) Find the difference in Black-Scholes implied volatility between the at-the-money call option and an option of the same expiration with strike 115. [10 points]

Problem 2: **[30points]**

Assume a risk-neutral geometric Brownian motion for volatility, with zero drift, an initial value of 0.5 and a volatility of volatility of 1.0 (i.e. 100%), i.e. where

$$d\sigma = \sigma dW$$
$$dS = \sigma S dZ$$
$$\rho = 0.5$$
$$\sigma_0 = 0.5$$

so that the correlation between the stock price and the volatility is 0.5. Use Monte Carlo simulation to compute and plot the implied volatility skew as a function of strike for a call option with 1 year time to expiration, for strikes between 85% and 115%.

Problem 3: **[40 points]**

Consider the risk-neutral stochastic volatility mean-reverting model

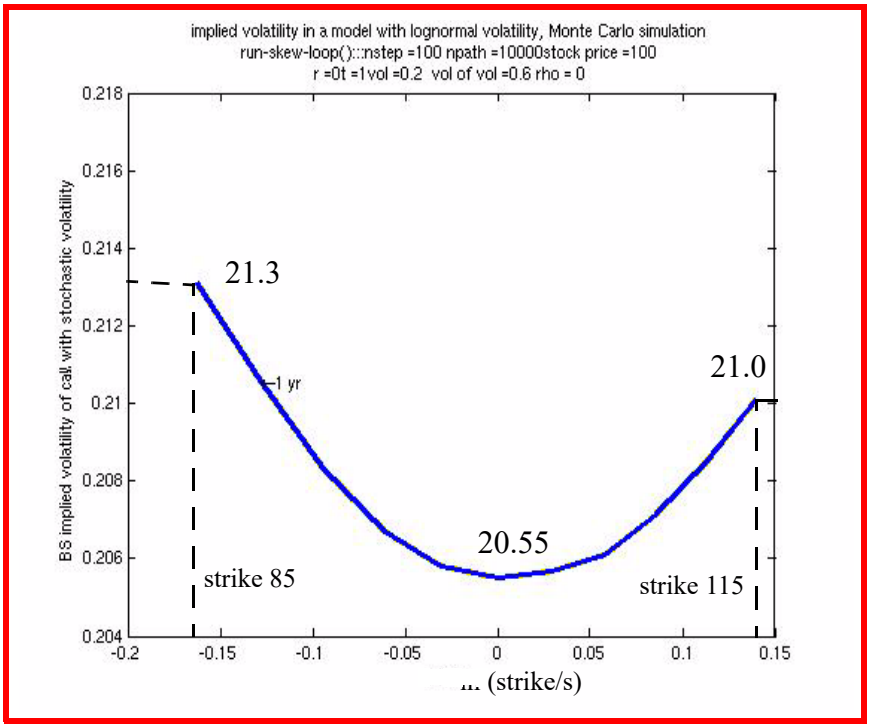
$$dS = rSdt + \sigma SdZ$$
$$d\sigma = \alpha(m - \sigma)dt + \xi\sigma dW$$
$$dZdW = \rho dt$$

Choose interest rates zero and correlation zero, with $\alpha = 1$, volatility of volatility of 1.0, initial volatility of 0.3 and a target volatility *m* of 0.3. Write a Monte Carlo program to simulate the process. Don't allow the volatility to go below 1/2 a volatility point in the simulation. If it does, set it back to 1/2 a volatility point.

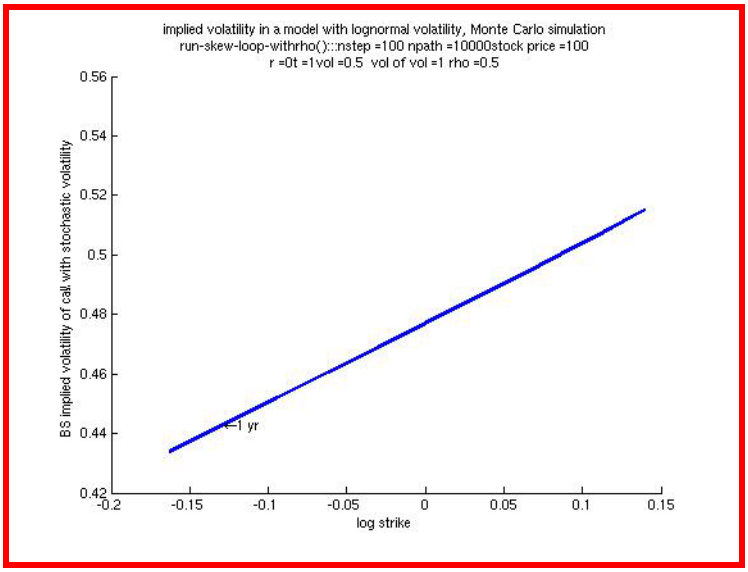
(i) Calculate the approximate difference between the BS implied volatility at a moneyness of 100 and a moneyness of 115 for an expiration of 1.5 years. [20]

(ii) Calculate the approximate difference between the BS implied volatility at a moneyness of 100 and a moneyness of 115 for an expiration of 0.25 years. [20]

Solution 1



Solution 2



Solution 3