Math 280, Spring 2022 Professor Rob Benedetto

Homework #2

Due **Wednesday, February 16** in Gradescope by **11:59 pm ET**

READ Textbook Section 1.1.3 and start 1.2.1

WRITE AND SUBMIT solutions to the following problems.

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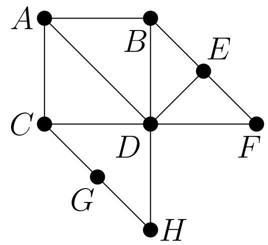
1. (12 points) Let X be the following graph:



(a) Find the degree sequence of X.
(b) Find a trail in X of longest possible length. Justify why there are no longer trails.
(c) Find a path in X of longest possible length. Justify why there are no longer paths.

2. (12 points) Textbook, Section 1.1.2, Problem 14:
Let G be a 2-connected graph. Prove that G contains at least one cycle.

(*Suggestion*: Remember that 2-connected means that if you delete any one vertex v, the subgraph $G-v$ is still connected, which means for any two of the remaining vertices, there's a path between them that avoids v.)

3. (20 points) Textbook, Section 1.1.2, Problem 16:

(a) Let G be a graph of order n such that $\delta(G) \geq (n-1)/2$. Prove that G is connected.

(b) For any positive even integer $n = 2m \geq 2$, find a graph G of order n such that $\delta(G) \geq (n-2)/2$ but G is *not* connected.

(*Suggestion*: For both (a) and (b), think about what the two separate components of G would have to look like.)

4. (12 points) Textbook, Section 1.1.3, Problem 3:
Is K_4 a subgraph of $K_{4,4}$? (More precisely, is there a subgraph of $K_{4,4}$ isomorphic to K_4, i.e., that is a complete graph on 4 vertices?) If yes, exhibit one explicitly. If no, prove no such subgraph exists.

5. (8 points) Textbook, Section 1.1.3, Problem 8:
Let G and H be isomorphic graphs. Prove that their complements $\overline{G}$ and $\overline{H}$ are also isomorphic.

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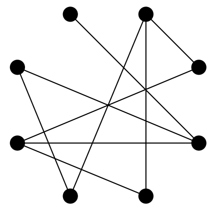
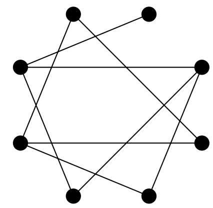
6. (12 points) Textbook, Section 1.1.3, Problem 9 (plus a little extra):
Consider the following two graphs:

G  H 

Verify that G and H have the same order, same size, and same degree sequence.
Then prove that in spite of that, G and H are *not* isomorphic.

Optional Challenges (do NOT hand in):
A. Textbook Section 1.1.3, Problem 10.
B. Let G be a graph with 10 vertices such that among any three vertices of G, at least two are adjacent. What is the minimum possible size (i.e., number of edges) that such a graph G can have? Find such a graph with this minimum number, and prove that no smaller number is possible.

Questions? You can ask in:

Class: MWF 11:00–11:50am, SMUD 205
Tu 9:00–9:50am, SMUD 205

My office hours: Mon 2:30–3:30pm, Tue 2–3:30pm, and Thu, 1–2:30pm,
SMUD 406

Anna's Math Fellow office hours: Tuesday, 6:00–7:30pm, SMUD 007

(Starting next week, Anna's hours will be Sundays, 7:30–9:00pm, and Tuesdays, 6:00–7:30pm, in SMUD 007.)

Also, you may email me any time at `rlbenedetto@amherst.edu`