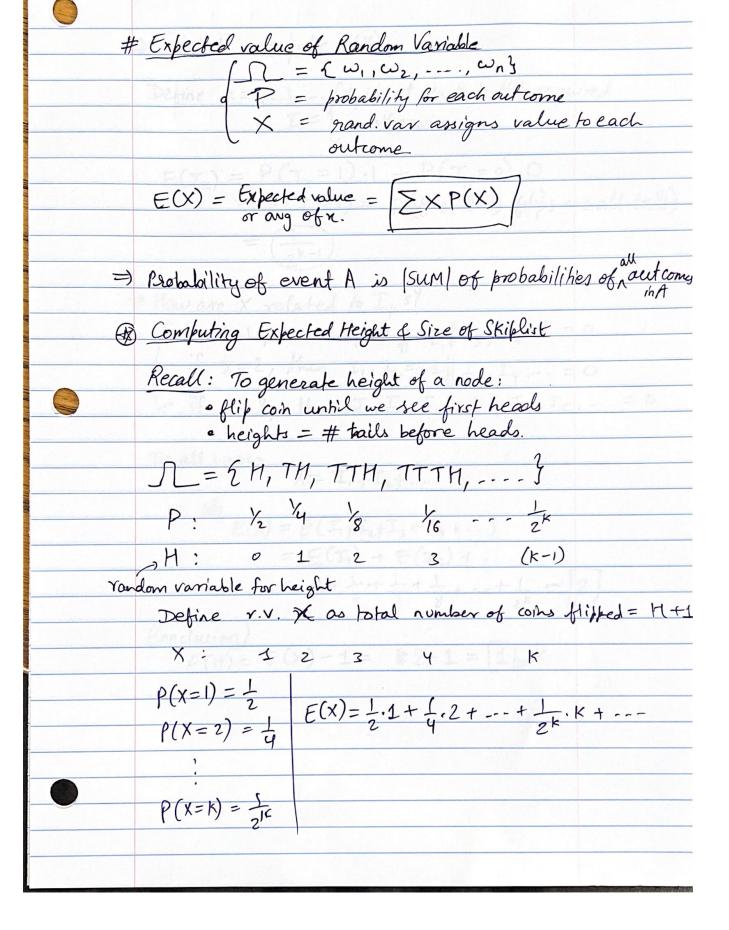
	Lecture 16: Analysis of Skiplists
a Ex	사람들은 기계 보고 있습니다. 그는 10mm 전 전 10mm
Section 1	- Basic Perabability
	- Height and Max. Pleight
	- Expected Running Time
⇒Basi	c Porobability
5000	We - the lated where - 1 % it \$ 6 % I have
Groa	1: formalize vocabulary and concepts to reason about
	random brocesses
	Concrete Examples [SAMPLE SPACE] (J = Omega)
	1) flipping coin - 7 2H, T3
	2) rolling dice -> {1,2,3,4,5,6}
1600	3) Shuffly picking card -> £10,27,, A & 4
Also	o, each element in sample space has an associated
prol	sability.
l l	P(w) = likelihood that w happens
	out come in ?
	h in s
# Pro	berties
(i	) $P(\omega) > 0$
Y 130 - (3	) $P(\omega) 70$ 2) $P(\omega_i) + P(\omega_2) + - + P(\omega_n) = 1 \Rightarrow \begin{bmatrix} n \\ \sum_{i=1}^{n} P(\omega_i) = 1 \end{bmatrix}$
	with a way the as bothed number of their districts
An	andom variable associates a real value/number to
	h possible outcome.
eg	Head $\rightarrow$ win \$1 $\rightarrow$ Random Var X: Tail $\rightarrow$ lost \$1 $\times$ (H) = 1, $\times$ (T) = -1.
	Tail $\rightarrow$ lost \$1 $X(H) = 1$ , $X(T) = -1$ .
In	tuitively: expect to win as much as lose, so "expected
	winnings $1 = 0$
	$\frac{1}{2} \cdot (1) + \frac{1}{2} \cdot (-1) = 6$
	L , L



> Trick to compute E(X) Define k=1,2,3,--f1, if at least k flips required  $I_{k} = \begin{cases} 0, \text{ otherwise} \end{cases}$  $E(I_k) = P(I_k = 1) \cdot 1 + P(I_k = 0) \cdot 0$ = P(Ik=1) = P(first (K-1) filips are all tails)  $=\left(\frac{1}{2k-1}\right).$ > How are X related to Ix 59  $\begin{cases} \text{if } X=1 \text{, then } I_1=1 \text{ } || I_2 \text{, } I_3 \text{, } I_4 \text{ ...} = 0 \\ \text{o if } X=2 \text{, then } I_1, I_2=1 \text{ } || I_3, I_4 \text{ ...} = 0 \\ \text{b if } X=3 \text{, then } I_1, I_2, I_3=1 \text{ } || I_4, I_7, \dots = 0. \end{cases}$ In all cases, X= I, +Izt . --So, E(X) = E(I+I+I+I+I+I+--) = E(I) + E(I) + ... = 1+ 1+ 1+ 1+ -+ 1 = 2 (Conclusion) E(H) = E(X) - 1 = (2 - 1 = 1)