	Turbulence	Financial Markets
Elementary	Velocity difference at the same	Price differences between
fluctuating	moment of time between two	two moments of time,
quantity that	locations in	$\delta p = p(t + \delta t) - p(t).$
needs to be	space, $\delta \vec{v} = \vec{v}(\vec{x} + \delta \vec{x}, t) - \vec{v}(\vec{x}, t)$.	
described or		
studied		
statistically		
Independent	Location is space \vec{x} and moment of	Time t.
variable(s)	time t .	
Energy transfer	In 3-dimensional (and 1-	"Information" is injected
or distribution	dimensional) case the energy is	into the system at a Large
across various	injected at the Large or Box Scale	time Scale and through
scales	and through non-linear interactions	individual traders-agents
scares	gets distributed across all scales all	interactions with one
	the way through the smallest or	another via trading, gets
	Dissipation or Viscous Scale. 2-	transmitted across all time
	dimensional case is having an	scales, all the way to the
	inverse energy transfer.	smallest Limit Order Book
	inverse energy transfer.	Event time Scale.
Relationship to	The exact equations of motion are	The exact equations of
some exact	known, however, that does not help	motion are not known and
"equations of	in solving the problem, which is too	the exact analytical
motion"	complex to be solved precisely.	description is not yet exactly
monon	Despite that knowledge, strictly	known (may never be).
	speaking, this physics problem	known (may never be).
	remains to be analytically unsolved.	
Non-Gaussian	Strongly pronounced for strong	Strongly pronounced for
behavior of fat	turbulence regimes. Algebraic decay	highly liquid, mature
tails of two-point	of the probability density function	markets. Algebraic decay of
probability	tails.	the probability density
distribution	tans.	function tails.
functions		Tunction tails.
Auto-correlation	$\delta \vec{v}$ is anti-correlated with the	For δp - nearly absent
and energy		correlations, and a very
spectrum	power law energy spectrum:	close to a Random Walk
Special and	$E(k) \propto k^{-5/3}.$	
		power law energy spectrum: $E(x) = x^{-2}$
		$E(\omega) \propto \omega^{-2}$.
Meaning of the	(Kinetic) energy of the turbulent	Volatility.
second order	liquid.	
moment of the		
fluctuating		
quantity		
Large scale	Deviations from Gaussian manifest	Deviations from Gaussian
structures and	themselves through coherent	manifest themselves in

intermittency	structures, shock-waves for 1-dimensional case and coherent vortices for 2- and 3-dimensional cases.	higher than normal frequency of gaps or shocks (up- or down-).
Stationary process?	Non-stationary at small time scales but asymptotically stationary at long time scales.	Non-stationary at small time scales but asymptotically stationary at long time scales.
Convergence to a	As the time separation δt becomes	As the time separation
Gaussian	increasingly larger.	δt becomes increasingly larger.
Power law behavior	Yes.	Yes.