

HW 3

1. (5 points each) Do Problems 1 and 2 on pages 491 and 492.
2. (6 pts) Let $f(x, y)$ be a continuous and differentiable function of x and y . the function f is said to be homogeneous of degree one if

$$f(cx, cy) = cf(x, y)$$

Euler's theorem states that if f is homogeneous of degree one, then

$$f(x, y) = x \frac{\partial}{\partial x} f(x, y) + y \frac{\partial}{\partial y} f(x, y)$$

Let $\mu_P(x, y) = x\mu_A + y\mu_B$ and $\sigma_P(x, y) = \sqrt{x^2\sigma_A^2 + y^2\sigma_B^2 + 2xy\sigma_{AB}}$

- (a) (3 pt) Show that $\mu_P(x, y)$ and $\sigma_P(x, y)$ are homogeneous of degree 1 (for $\sigma_P(x, y)$ assume $c \geq 0$).
- (b) (3 pt) In a portfolio where the portion x is invested in asset A and the portion y is invested in asset B, the partial derivatives

$$\frac{\partial}{\partial x} \sigma_P(x, y) \quad \text{and} \quad \frac{\partial}{\partial y} \sigma_P(x, y)$$

are called the marginal contributions to risk of A and B, respectively. The contributions to risk of assets A and B are given by

$$x \frac{\partial}{\partial x} \sigma_P(x, y) \quad \text{and} \quad y \frac{\partial}{\partial y} \sigma_P(x, y),$$

respectively. Find the expression of the marginal risks and the contribution to risk of assets A and B.

3. (14pt) The annual estimates of the parameters for Boeing (B) and Microsoft (M) stocks are given below:

$$\mu_B = 0.1492, \mu_M = 0.3308, \sigma_B^2 = 0.0695, \sigma_M^2 = 0.1369, \rho_{BM} = -0.0083$$

Assume a risk-free rate of 6% per year for the T-bill (risk free rate).

- (a) (3pt) Use the Lagrange multiplier method to derive the minimum variance portfolio.
- (b) (3pt) Find the tangency portfolio and compute its mean and risk.
- (c) (4pt) Suppose you desire a portfolio with an expected return of 20%. What should be the weights of this portfolio if you only use risky assets? What is its risk equal to?
- (d) (4pt) Suppose you desire a portfolio with a risk of 20%. What should be the weights of this portfolio if you use risky and risk free assets? What is its expected return equal to?