under GBM.

at time 0, $S(0)$ known $\longrightarrow$ $S(T)$ log-normal dist.

$\downarrow$

discounted option payoff.

under other stochastic processes.

at $t=0$, $S(0) \longrightarrow S(T)$ unknown

Divide $[0,T]$ into very small subintervals

* $S_0 = S(0) \to S_1 \to S_2 \to \cdots \to S_n \leftarrow$ sample of $S(T)$

$$dS(t) = a(t, S(t))\,dt + b(t, S(t))\,dW(t)$$

Integrate over $[t_1, t_2]$

$$S(t_2) - S(t_1)$$

$$= \int_{t_1}^{t_2} a(t, S(t))\,dt + \int_{t_1}^{t_2} b(t, S(t))\,dW(t)$$

$$\approx \int_{t_1}^{t_2} a(t_1, S(t_1))\,dt + \int_{t_1}^{t_2} b(t_1, S(t_1))\,dW(t) \leftarrow \text{Euler scheme}$$

$$= a(t_1, S(t_1))(t_2 - t_1) + b(t_1, S(t_1))\underbrace{(W(t_2) - W(t_1))}_{\sim N(0,\, t_2 - t_1)}$$

$\sqrt{t_2 - t_1}\, Z$

$Z \sim N(0,1)$

S_i = sample of $S(t_i)$

$$\boxed{\begin{aligned} S_{i+1} &= S_i + a(t_i, S(t_i))(t_{i+1} - t_i) \\ &\quad + b(t_i, S(t_i))\sqrt{t_{i+1} - t_i}\, Z_{i+1} \\ &Z_1, Z_2, \ldots, Z_n \quad \text{iid. } N(0,1) \end{aligned}}$$