Proof-Based Exercises for MATH 5440

April 14, 2023

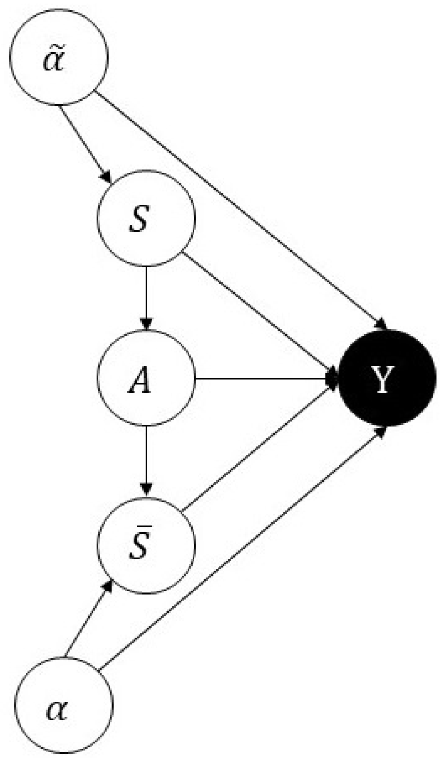

Figure 1: Causal graph for exercises.

Exercise 1 Examples of do-calculus

Consider the causal structure outlined in Figure 1, with the same interpretation as last week. This exercise defines counterfactuals of interest and establishes identifiability equations. Consider the following counterfactuals:

(a) *What if I had submitted a larger order?*

$$\mathbb{E}\left[Y \,|\, \mathrm{do}(S)\right]$$

(b) *What if I had chosen algorithm A?*

$$\mathbb{E}\left[Y \,|\, \mathrm{do}(A)\right]$$

(c) *What if I had traded algorithm A faster?*

$$\mathbb{E}\left[Y \,|\, \mathrm{do}(A), \mathrm{do}(\bar{S})\right]$$

1. Identify each counterfactual succinctly.

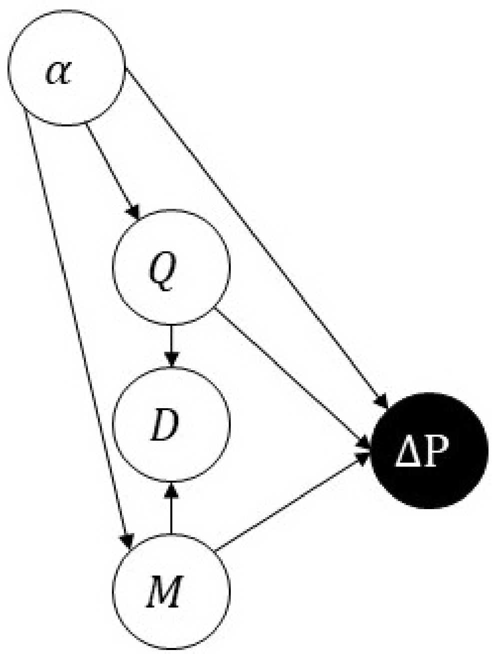

Figure 2: Causal graph $\mathcal{D}$.

Exercise 2 Causal structure for dark pools

This exercise presents a causal structure for trading in dark pools. Consider the causal structure $\mathcal{D}$ from Figure 2. The nodes have the following interpretation.

(a) $Q, \alpha, \Delta P$ are like the prediction bias causal graph.

(b) An *unobserved* external flow M exhibits *crowding* with the trading process Q.

(c) D captures trades between M and Q that cross, making these trades observable.

This graph differs from the synchronization trading graph in two ways: first, the external flow M interacting with Q over the dark pool is *unobserved*. Second, there is no leakage: Q does not trigger M. The only source of confounding is crowding, represented by node α.

1. Show that Q and M are independent conditional on α.
2. Show that Q and M are dependent conditional on α, D. Explain the reason for this dependence.
3. Identify $\mathbb{E}\left[\Delta P \middle| \mathrm{do}(Q), D\right]$, the impact of Q conditional on observing a cross. How does it differ from the unconditional impact $\mathbb{E}\left[\Delta P \middle| \mathrm{do}(Q)\right]$? Comment.