Math 5440: Week 9 Assignment

Due Date: March 31, 2023 at 10am

Exercise 1 Creating Synthetic Alphas

Consider a synthetic signal

$$\alpha_t = a\,(R_t - R_T) + b\,(W_t - W_T)$$

for parameters a and b, where R and W are assumed to be two independent Wiener processes with volatility σ_R and σ_W, respectively. We call $R_t - R_T$ the signal's *realized alpha*, and the synthetic alpha is constructed by adding noise to the realized alpha.

1. Compute the correlation ρ between α_t and $R_t - R_T$ for given a, b.
2. Pick values of a, b such that $\alpha_t = \mathbb{E}\left[R_t - R_T \middle| \alpha_t\right]$ and $\rho = 0.30$.
3. Load the binned stock data using

```
\l pathToHdbFolder\columbiaHdb
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   Load in memory the table for the date 2019.01.03. For each stock, let $S = (S_t)_{0 \le t \le T}$ be its mid prices during the trading day and define the end-of-day return starting at t by

$$R_t := \frac{S_T}{S_t} - 1, \quad R_T = 0.$$

   Calculate R_t and treat $(R_t)_{0 \le t \le T}$ as a realization of a Wiener process (that starts with $R_T = 0$ and propagates backward in time). Estimate σ_R by the volatility of the returns over 10-second bins. (Why this is a reasonable estimate of σ_R?)
   The following function simulates n independent samples, each of which is a sum of twelve random numbers uniformly generated in $[0, 1]$. Its distribution is close to a standard normal distribution. You can treat the samples as if they are truly drawn from a standard Gaussian distribution.

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u12: {[n] -6f+sum n cut (12*n)?1f}
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Simulate a Wiener process W with volatility $\sigma_W = \sigma_R$, and then calculate the synthetic alpha α_t and α'_t. When calculating α'_t, you can use the approximation

$$\alpha'_t \approx \frac{1}{\Delta t}(\alpha_t - \alpha_{t-\Delta t})$$

for $\Delta t = 10$mins. In the first ten minutes of the day, we can assume $\alpha'_t = 0$.

Exercise 2 Simulating Trading Strategies

Given a trading signal α_t, α'_t and a standard OW price impact model, this exercise implements the corresponding optimal trading strategy. Let α_t, α'_t stem from the previous exercise. Furthermore, assume the price impact model

$$dI_t = -\beta I_t dt + 8 \cdot \frac{\sigma}{\text{adv}} dQ_t$$

with $\log(2)/\beta = 60$ minutes.

1. Simulate the target impact state for the optimal trading strategy.
2. Given an impact state, simulate the corresponding trades.
3. For each stock, compute the final order size as a percent of adv. Provide a scatter-plot of alpha strength against optimal order size across the whole universe of stocks and the full year of 2019. Here, alpha strength means the alpha at the beginning of the day, i.e., α_0.

The following is a bonus problem. It is not mandatory, but we will award two extra points to students who solve it.

Exercise 3 (Bonus 2pts) Backtesting Trading Strategies

Assume given the same alpha signals, the price impact model, and the previously computed optimal trading strategy.

1. Simulate a VWAP strategy for an order of the same size as the optimal trading strategy.
2. Backtest the trading costs and P&L of the optimal and the VWAP strategy across all stocks and dates.
3. Provide summary statistics: average P&L, sharpe ratio, and alpha-impact ratios. Then, repeat the analysis bucketing by vol.