

Práctica 1

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2.1. Algoritmos iterativos

Algoritmo poco eficiente (vectores sin ordenar)

```
void EliminaRepetidos(int* v, int& N){
    ....int i = 0;
    ....for (int i = 0; i < N-1; i++){
        ....int j = i+1;
        ....while (j < N-1){
            ....if (v[i] == v[j]){
                ....for (int k = j; k < N-1; k++){
                    ....v[k] = v[k+1];
                ....}
            ....N--;
            ....}
            ....else{
                ....j++;
            ....}
        ....}
    ....}
}
```

Algoritmo eficiente (vectores ordenados)

```
void EliminaRepetidos(int* v, int& N){
    ....int i = 0;
    ....while (i < N){
        ....if (v[i] == v[i+1]){
            ....for (int j=i; j<N-1; j++){
                ....v[j] = v[j+1];
            ....}
            ....N--;
        ....}
        ....else{
            ....i++;
        ....}
    ....}
}
```

2.1. Algoritmos iterativos - Poco Eficiente

Mejor caso: No tener elementos repetidos

```
82 void EliminaRepetidos(int* v, int& N) {
83
84     int i = 0;  $\rightarrow \Omega(1)$ 
85     for (int i = 0; i < N-1; i++) {  $n^{\circ} \text{ veces} = n-1$ 
86
87         int j = i+1;  $\rightarrow \Omega(1)$ 
88
89         while (j < N-1) {  $n^{\circ} \text{ veces} = n-1$ 
90
91             if (v[i] == v[j]) {  $\rightarrow \Omega(1)$ 
92                 for (int k = j; k < N-1; k++) {
93                     v[k] = v[k+1];
94                 }
95                 N--;
96             }
97             j++;  $\rightarrow \Omega(1)$ 
98         }
99     }
100 }
101
102
103
104
105
106
107
108 }
```

$\Omega(1)$

$\Omega(n)$

$\Omega(n^2)$

nunca entra en el bucle

Peor caso: Todos los elementos son el mismo número

```
82 void EliminaRepetidos(int* v, int& N) {
83
84     int i = 0;  $\rightarrow O(1)$ 
85     for (int i = 0; i < N-1; i++) {  $n^{\circ} \text{ veces} = n-1$ 
86
87         int j = i+1;  $\rightarrow O(1)$ 
88
89         while (j < N-1) {  $n^{\circ} \text{ veces} = n-1$ 
90
91             if (v[i] == v[j]) {  $\rightarrow O(1)$ 
92                 for (int k = j; k < N-1; k++) {  $n^{\circ} \text{ veces} = n-1$ 
93                     v[k] = v[k+1];  $\rightarrow O(1)$ 
94                 }
95                 N--;  $\rightarrow O(1)$ 
96             }
97             else {
98                 j++;  $\rightarrow O(1)$ 
99             }
100         }
101     }
102 }
103
104
105
106
107
108 }
```

$O(1)$

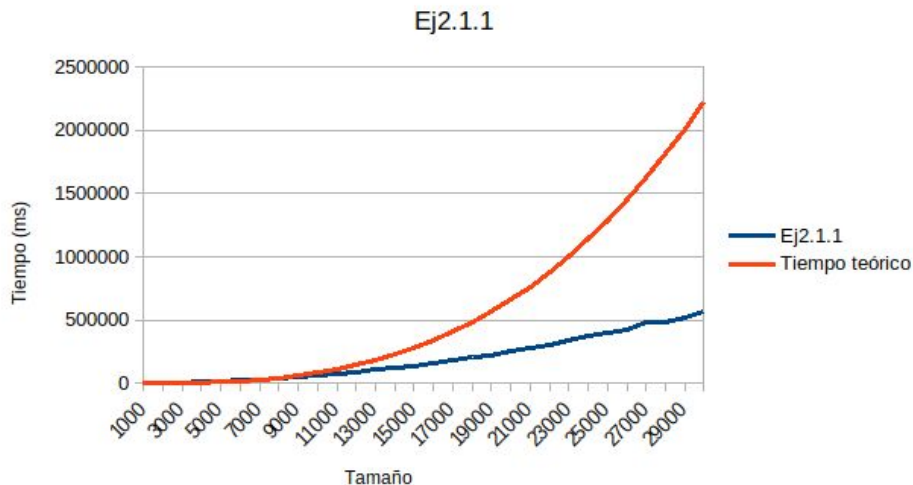
$O(n)$

$O(n^2)$

$O(n^3)$

2.1. Algoritmos iterativos - Poco Eficiente

Tamaño del vector	Ej2.1.1	$O(n^3)$	K	Tiempo teórico
1000	622	1000000000	0,000000622	82,55852812353
2000	2330	8000000000	2,9125E-07	660,46822498824
3000	5539	27000000000	2,05148E-07	2229,08025933531
4000	10363	64000000000	1,61922E-07	5283,74579990592
5000	16052	1,25E+11	1,28416E-07	10319,8160154413
6000	22069	2,16E+11	1,02171E-07	17832,6420746825
7000	30436	3,43E+11	8,87347E-08	28317,5751463708
8000	38535	5,12E+11	7,52637E-08	42269,9663992474
9000	51022	7,29E+11	6,9989E-08	60185,1670020534
10000	61251	1E+12	6,1251E-08	82558,52812353
11000	74064	1,331E+12	5,56454E-08	109885,400932418
12000	91233	1,728E+12	5,27969E-08	142661,13659746
13000	105231	2,197E+12	4,78976E-08	181381,086287395
14000	123952	2,744E+12	4,5172E-08	226540,601170966
15000	138857	3,375E+12	4,11428E-08	278635,032416914
16000	162038	4,096E+12	3,95601E-08	338159,731193979
17000	187266	4,913E+12	3,81164E-08	405610,048670903
18000	202679	5,832E+12	3,47529E-08	481481,336016427
19000	221457	6,859E+12	3,22871E-08	566268,944399292
20000	253206	8E+12	3,16508E-08	660468,22498824
21000	275860	9,261E+12	2,97873E-08	764574,528952011
22000	303654	1,0648E+13	2,85175E-08	879083,207459347
23000	337225	1,2167E+13	2,77164E-08	1004489,61167899
24000	374058	1,3824E+13	2,70586E-08	1141289,09277968
25000	393819	1,5625E+13	2,52044E-08	1289977,00193016
26000	425892	1,7576E+13	2,42315E-08	1451048,69029916
27000	488311	1,9683E+13	2,48088E-08	1624999,50905544
28000	481421	2,1952E+13	2,19306E-08	1812324,80936773
29000	522291	2,4389E+13	2,1415E-08	2013519,94240477
30000	564793	2,7E+13	2,09183E-08	2229080,2593354
			8,25585E-08	



2.1. Algoritmos iterativos - Eficiente

Mejor caso: No tener elementos repetidos

```
81 void EliminaRepetidos(int* v, int& N) {
82
83     int i = 0;  $\rightarrow O(1)$ 
84     while (i < N) {  $n^{\circ}$  veces =  $n-1$ 
85
86         if (v[i] == v[i+1]) {  $O(1)$ 
87
88             for (int j=i; j<N-1; j++) {
89                 v[j] = v[j+1];
90             }
91
92             N--;
93         }
94         else {
95             i++;  $\rightarrow O(1)$ 
96         }
97     }
98 }
```

$O(1)$

$O(n)$

\rightarrow nunca entra en el bucle

Peor caso: Todos los elementos son el mismo número

```
81 void EliminaRepetidos(int* v, int& N) {
82
83     int i = 0;  $\rightarrow O(1)$ 
84     while (i < N) {  $n^{\circ}$  veces =  $n-1$ 
85
86         if (v[i] == v[i+1]) {  $O(1)$ 
87
88             for (int j=i; j<N-1; j++) {  $n^{\circ}$  veces =  $n-1$ 
89                 v[j] = v[j+1];  $\rightarrow O(1)$ 
90             }
91
92             N--;  $\rightarrow O(1)$ 
93         }
94         else {
95             i++;  $\rightarrow O(1)$ 
96         }
97     }
98 }
```

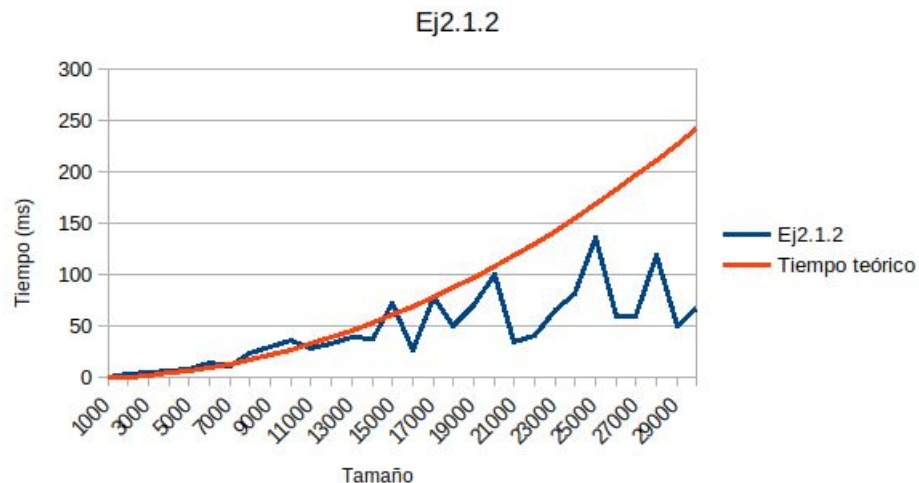
$O(n)$

$O(n)$

$O(n^2)$

2.1. Algoritmos iterativos - Eficiente

Tamaño del vector	Ej2.1.2	$O(n^2)$	K	Tiempo teórico
1000	1	1000000	0,000001	0,270418444256032
2000	3	4000000	0,00000075	1,08167377702412
3000	5	9000000	5,55556E-07	2,43376599830427
4000	7	16000000	4,375E-07	4,32669510809648
5000	8	25000000	0,00000032	6,76046110640075
6000	15	36000000	4,16667E-07	9,73506399321708
7000	11	49000000	2,2449E-07	13,2505037685455
8000	24	64000000	0,000000375	17,3067804323859
9000	30	81000000	3,7037E-07	21,9038939847384
10000	36	100000000	0,00000036	27,041844425603
11000	29	121000000	2,39669E-07	32,7206317549796
12000	33	144000000	2,29167E-07	38,9402559728683
13000	39	169000000	2,30769E-07	45,7007170792691
14000	38	196000000	1,93878E-07	53,0020150741819
15000	72	225000000	0,00000032	60,8441499576067
16000	27	256000000	1,05469E-07	69,2271217295437
17000	79	289000000	2,73356E-07	78,1509303899927
18000	51	324000000	1,57407E-07	87,6155759389537
19000	70	361000000	1,93906E-07	97,6210583764268
20000	100	400000000	0,00000025	108,167377702412
21000	35	441000000	7,93651E-08	119,254533916909
22000	41	484000000	8,47107E-08	130,882527019919
23000	66	529000000	1,24764E-07	143,05135701144
24000	82	576000000	1,42361E-07	155,761023891473
25000	137	625000000	2,192E-07	169,011527660019
26000	60	676000000	8,87574E-08	182,802868317076
27000	59	729000000	8,09328E-08	197,135045862646
28000	120	784000000	1,53061E-07	212,008060296727
29000	51	841000000	6,06421E-08	227,421911619321
30000	68	900000000	7,55556E-08	243,376599830429
			2,70418E-07	



2.2. Algoritmos recursivos - Hanoi

```
1  /**
2   Se trata del problema clásico de las torres de Hanoi.
3   Se tienen 3 barras, y hay que mover M anillos de la primera barra
4   a la segunda. Solo se puede mover un anillo en cada movimiento,
5   y ningún anillo de tamaño mayor puede ponerse sobre otro de tamaño
6   menor.
7   Los valores de "i" y "j" sólo pueden tomar los valores {1, 2, 3}
8
9   Si M=3, la llamada sería hanoi(3, 1, 2)
10
11  */
12  void hanoi (int M, int i, int j)
13  {
14      if (M > 0) →  $O(1)$ 
15      {
16          hanoi(M-1, i, 6-i-j); →  $O(n)$ 
17          cout << i << " -> " << j << endl; →  $O(1)$ 
18          hanoi (M-1, 6-i-j, j); →  $O(n)$ 
19      }
20  }
```


2.2. Algoritmos recursivos - Hanoi

$$\begin{aligned} T(n) &= 2 \cdot T(n-1) + 1 \\ T(n) - 2 \cdot T(n-1) &= 1 \\ \text{PH} \left(\begin{array}{l} X^n - 2 \cdot X^{n-1} = 0 \\ X \cdot X^{n-1} - 2 \cdot X^{n-1} = 0 \end{array} \right) \\ 1 &= b_1^m \cdot g_1(n) \left\{ \begin{array}{l} b = 1 \\ g_1(n) = 1 \end{array} \right. \end{aligned}$$

$$\begin{aligned} X^{n-1} \cdot (X-2) &= 1 \\ P_H(X) &= (X-2) \end{aligned}$$

Hanoi

$$\begin{aligned} p(x) &= p_H(x) \cdot (x-b_1)^{d_1} \\ p(x) &= (x-2) \cdot (x-1)^1 \\ r &= 2 \\ R_1 &= 2 \quad m=1 \\ R_2 &= 1 \quad m=1 \end{aligned}$$

$$\begin{aligned} T(n) &= c_{10} \cdot R_1^n \cdot n^m + c_{20} \cdot R_2^n \cdot n^m \\ &= c_{10} \cdot 2^n + c_{20} \cdot 1^n \Rightarrow \\ &\Rightarrow O(2^n) \end{aligned}$$

2.2. Algoritmos recursivos - InsertarEnPos y reestructurarRaiz

```
void insertarEnPos(double *apo, int pos){  
    int idx = pos-1;  
    int padre;  
    if (idx > 0) {  
        if (idx%2==0) {  
            padre=(idx-2)/2;  
        }else{  
            padre=(idx-1)/2;  
        }  
  
        if (apo[padre] > apo[idx]) {  
            double tmp=apo[idx];  
            apo[idx]=apo[padre];  
            apo[padre]=tmp;  
            insertarEnPos(apo, padre+1);  
        }  
    }  
}
```

$\rightarrow O(\log_2 n)$

$\rightarrow \text{padre} = (n-2)/2$

$T(n/2) + 1$

```
void reestructurarRaiz(double *apo, int pos, int tamapo){  
    int minhijo;  
    if (2*pos+1 < tamapo) {  
        minhijo=2*pos+1;  
        if ((minhijo+1 < tamapo) && (apo[minhijo]>apo[minhijo+1])) minhijo++;  
        if (apo[pos]>apo(minhijo)) {  
            double tmp = apo[pos];  
            apo[pos]=apo(minhijo);  
            apo(minhijo)=tmp;  
            reestructurarRaiz(apo, minhijo, tamapo);  
        }  
    }  
}
```

$\rightarrow O(1)$

$\rightarrow O(1)$

$\rightarrow O(1)$

$\rightarrow O(1)$

$\rightarrow T(n/2)$

2.2. Algoritmos recursivos - InsertarEnPos y reestructurarRaiz

$$T(n) = T(n/2) + 1$$

$$T(n) - T(n/2) = 1$$

cambio var: $n = 2^m \rightarrow m = \log_2(n)$

$$T(2^m) - T(2^{m/2}) = 1$$

PH \swarrow

$$x^m - x^{m-1} = 0$$

$$x^{m-1} \cdot (x-1) = 0$$

$$P_H(x) = (x-1)$$

PNH \searrow

$$1 = \begin{cases} b_1 = 1 \\ q_1(n) = 1 \end{cases}$$

$$P(x) = (x-1) \cdot (x-1) = (x-1)^2$$

$$r=1$$

$$R_1 = 1 \quad m=2$$

$$T(2^m) = c_p \cdot 1^{\log_2(n)} + c_{11} \cdot 1^{\log_2(n)} \cdot \log_2(n) \Rightarrow \Rightarrow O(\log_2(n))$$

2.2. Algoritmos recursivos - HeapSort

```
void HeapSort(int *v, int n){  
    double *apo=new double [n];  
    int tamapo=0;  
  
    for (int i=0; i<n; i++){  
        apo[tamapo]=v[i];  
        tamapo++;  
        insertarEnPos(apo,tamapo);  
    }  
  
    for (int i=0; i<n; i++){  
        v[i]=apo[0];  
        tamapo--;  
        apo[0]=apo[tamapo];  
        reestructurarRaiz(apo, 0, tamapo);  
    }  
    delete [] apo;  
}
```

$O(n \cdot \log_2(n))$

$O(n \cdot \log_2(n))$

$\rightarrow O(n \cdot \log_2(n))$

$u^o \text{ veces} = n$

$\rightarrow O(\log_2(n))$

$u^o \text{ veces} = n$

$\rightarrow O(1)$

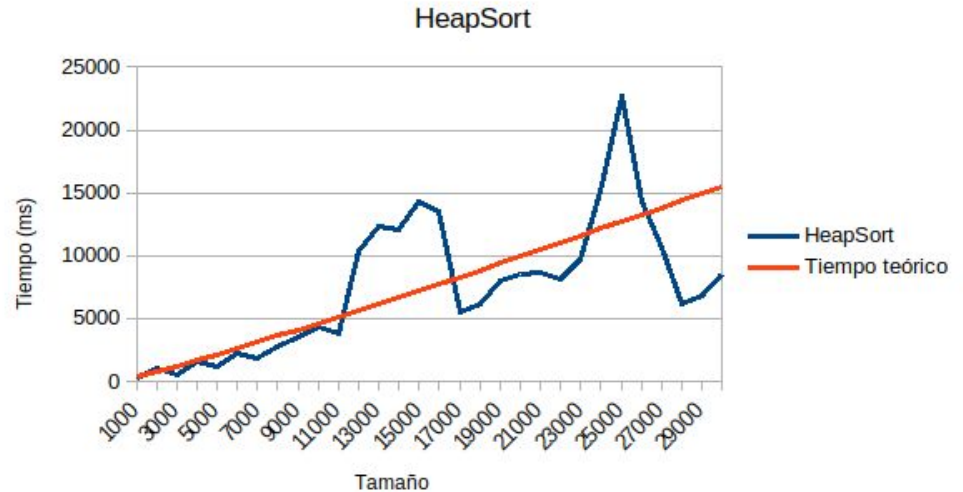
$\rightarrow O(1)$

$\rightarrow O(1)$

$\rightarrow O(\log_2(n))$

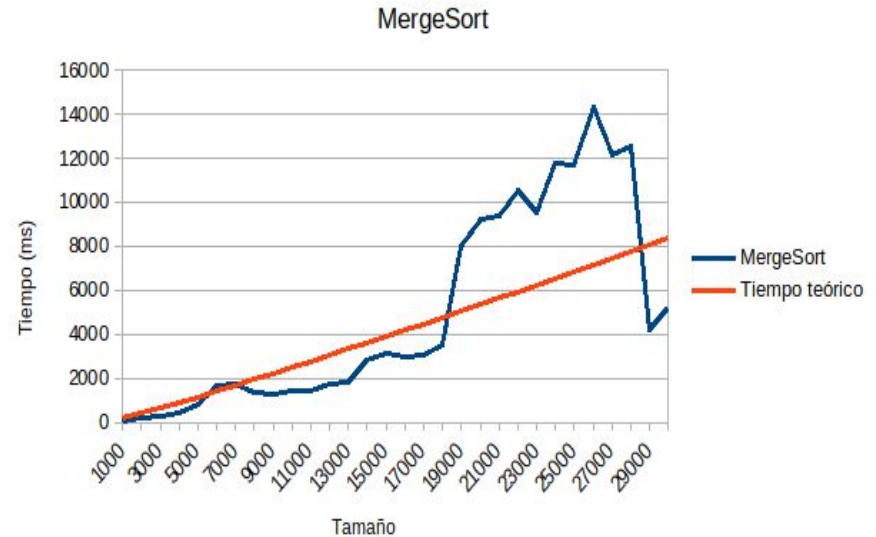
2.2. Algoritmos recursivos - HeapSort

Tamaño del vector	HeapSort	$O(n \cdot \log_2(n))$	K	Tiempo teórico
1000	319	9965,784285	0,032009523	346,817580394768
2000	1000	21931,56857	0,045596374	763,236823937824
3000	505	34652,24036	0,014573372	1205,92678030109
4000	1623	47863,13714	0,033909186	1665,67697417222
5000	1226	61438,5619	0,019954894	2138,11304476107
6000	2200	75304,48071	0,029214729	2620,65855004705
7000	1809	89411,97445	0,020232189	3111,61106344296
8000	2721	103726,2743	0,026232505	3609,76060093758
9000	3545	118221,3836	0,029986115	4114,20245825365
10000	4282	132877,1238	0,032225261	4624,23440526358
11000	3833	147677,3749	0,025955228	5139,29545246954
12000	10310	162608,9614	0,06340364	5658,92707898382
13000	12281	177660,912	0,069126066	6182,74747708887
14000	12043	192823,9489	0,062455935	6710,43376892392
15000	14324	208090,1232	0,068835559	7241,70932986717
16000	13492	223452,5486	0,06037971	7776,33450706146
17000	5535	238905,2011	0,023168185	8314,09966707774
18000	6195	254442,7671	0,024347322	8854,81988484188
19000	7999	270060,5242	0,029619286	9398,33081625647
20000	8542	285754,2476	0,029892819	9944,48544201002
21000	8661	301520,1359	0,02872445	10493,1514641464
22000	8186	317354,7499	0,025794478	11044,2091995702
23000	9763	333254,9635	0,029295888	11597,5498572823
24000	15055	349217,9228	0,043110617	12153,0741157471
25000	22652	365241,0119	0,062019322	12710,6909377415
26000	14282	381321,8241	0,037453928	13270,3165751055
27000	10613	397458,1382	0,026702183	13831,8737268121
28000	6182	413647,8978	0,014945078	14395,2908219239
29000	6817	429889,1931	0,015857575	14960,5014051012
30000	8479	446180,2464	0,019003531	15527,4436069586
			0,034800832	



2.2. Algoritmos recursivos - MergeSort

Tamaño del vector	MergeSort	$O(n \log_2(n))$	K	Tiempo teórico
1000	77	9965,784285	0,007726437	186,292585736891
2000	160	21931,56857	0,00729542	409,971608991521
3000	250	34652,24036	0,007214541	647,761909462429
4000	461	47863,13714	0,009631629	894,71609301852
5000	824	61438,5619	0,013411772	1148,48447778493
6000	1659	75304,48071	0,022030562	1407,68313147807
7000	1741	89411,97445	0,019471665	1671,39759800092
8000	1318	103726,2743	0,01270652	1938,977936108
9000	1250	118221,3836	0,010573383	2209,93818514256
10000	1403	132877,1238	0,010558627	2483,90114315855
11000	1395	147677,3749	0,009446268	2760,56547541971
12000	1733	162608,9614	0,010657469	3039,68488806258
13000	1829	177660,912	0,010294893	3321,05429359745
14000	2832	192823,9489	0,014686972	3604,50025862601
15000	3098	208090,1232	0,01488778	3889,87419461356
16000	2973	223452,5486	0,013304838	4177,0473723579
17000	3073	238905,2011	0,012862843	4465,90718755127
18000	3509	254442,7671	0,013790921	4756,35430794476
19000	8013	270060,5242	0,029671127	5048,30045633273
20000	9205	285754,2476	0,032212994	5341,66666149449
21000	9360	301520,1359	0,031042703	5636,38185976505
22000	10508	317354,7499	0,033111211	5932,38176353454
23000	9554	333254,9635	0,02866874	6229,60793586757
24000	11766	349217,9228	0,033692429	6528,00702633803
25000	11698	365241,0119	0,032028167	6827,530134427
26000	14328	381321,8241	0,037574561	7128,13227492551
27000	12129	397458,1382	0,030516421	7429,77192569347
28000	12549	413647,8978	0,030337396	7732,41064250036
29000	4170	429889,1931	0,009700174	8036,01272895199
30000	5215	446180,2464	0,011688101	8340,54495199321
			0,018693219	



2.2. Algoritmos recursivos - MergeSort y HeapSort

