HW1-2020 (10월 7일, 수요일)

1. Ex. 6.6

2. Ex. 6.9

3. Ex. 6.57

4. Ex. 6.58

5. Ex. 6.59

6. Assume that we have collected 10 samples of size 5 each from a process producing bearings. The quality characteristics of interest X is the inside diameter measurements of the bearings. The original observed data of X along with the sample means and sample ranges are given in the table below.

Sample	x_1	x_2	x_3	x_4	x_5	$\bar{X}$	R
1	34.09	36.30	35.76	35.01	36.50	35.53	2.41
2	36.11	34.39	35.15	36.76	37.63	36.01	3.24
3	33.43	35.41	34.00	35.20	35.67	34.74	2.24
4	36.79	35.96	35.62	34.48	33.63	35.30	3.16
5	36.46	35.89	35.83	35.43	35.40	35.80	1.06
6	33.59	34.76	33.98	34.35	35.39	34.41	1.80
7	36.17	36.20	34.60	34.97	34.83	35.35	1.60
8	34.66	35.05	36.08	34.99	35.15	35.19	1.42
9	35.95	34.18	35.02	35.32	34.77	35.05	1.77
10	35.62	35.18	34.93	36.35	36.24	35.66	1.42

(i) Construct the $\bar{X}$ and R chart using $\alpha = 0.0027$. Does the process seem to be in statistical control?

(ii) Assume that the manufacturing process is IC, and the observed dataset shown in the above table is an IC dataset. Provide estimates for the IC mean μ_0 and the IC standard deviation σ of X.

(iii) Using the results in parts (i) and (ii), compute the probability that the sample mean of a new sample of size 5 would give a signal of mean shift in the $\bar{X}$ chart constructed in part (i) in cases when the true process mean is not shifted.

(iv) Compute the ARL_0 value of the $\bar{X}$ chart constructed in part (i), and its ARL_1 value when detecting a mean shift of size 1 (i.e., the shifted mean $\mu_1 = \mu_0 + 1$).

7. The sample standard deviations of the observed data presented in Problem 1 are computed to be

$$0.99, 1.28, 0.97, 1.25, 0.43, 0.70, 0.77, 0.53, 0.66, 0.63.$$

(i) Construct the $\bar{X}$ and s chart using $\alpha = 0.0027$. Does the process seem to be in statistical control?

(ii) Compare the control charts in part (i) and the control charts obtained in part (i) of Problem 1, and summarize your findings.

(iii) The standard deviation σ of the quality characteristic X can be estimated by

$$\widehat{\sigma} = \frac{\bar{R}}{d_1(m)}$$

based on the sample ranges. It can also be estimated by

$$\widehat{\sigma} = \frac{\bar{s}}{d_3(m)}$$

based on the sample standard deviations. Further, it can be estimated by the sample standard deviation of the combined sample (i.e., all samples are combined into a single sample). Discuss the strengths and limitations of the three estimators of σ for the purpose of SPC, and compute their values using the observed data given in Problem 6 above.

8. Assume that we have collected 14 samples of size 5 each from an injection molding process. The sample means and sample standard deviations of the compressive strength measurements of the sampled parts a listed below.

Sample	1	2	3	4	5	6	7
$\bar{X}_i$	80.22	78.31	81.40	78.53	81.32	80.54	77.33
s_i	3.99	5.35	4.79	4.68	3.79	5.78	3.52
$\bar{X}_i$	79.24	81.44	77.76	79.48	76.74	81.12	86.79
s_i	4.51	5.16	4.81	4.56	7.35	3.91	6.31

(i) Use the first 10 samples to construct the $\bar{X}$ and s charts. Does the process seem to be in statistical control? If the answer is positive, then use the constructed control charts to monitor the last 4 samples, which are treated as phase II data. Describe your findings of the phase II SPC.

(ii) Use the first 10 samples to construct the s^2 chart. If the process variability seems to be in statistical control at the first 10 time points, then use the constructed control chart to monitor the process variability at the last 4 time points by treating the last 4 samples as phase II data. Describe your findings of the phase II SPC.

(iii) Note that by taking the squared root of the lower and upper control limits of the s^2 chart, we obtain the lower and upper control limits of a new version of the s chart.

Compare this version of the s chart with the s chart constructed in part (i) for both phase I and phase II analyses.

9. The data given below are measurements of the tensile strength of the sampled papers manufactured by a production process. This is an individual observation dataset. At each time point, only on observation is obtained, and different observations are obtained at equally space time points.

$$25, 24, 39, 26, 25, 22, 24, 21, 28, 24, 24, 22, 16, 26, 25, 26, 21, 25, 23, 24$$

Construct the $\bar{X}$ and R charts using moving windows of size $\tilde{m}$ with (i) $\tilde{m} = 2$, and (ii) $\tilde{m} = 5$. Compare the two sets of control charts, and summarize your major findings.