- f. For the case in (e), calculate the diagnostic quantities  $d_i$ , i = 1, 2, 3, 4 from equation 11.22. Does this information assist in identifying which process variables have shifted?
- Table 11.5. Calculate an estimate of the sample covariance matrix using both estimators  $\mathbf{S}_1$  and  $\mathbf{S}_2$  discussed in Section 11.3.2.
  - 11.7. Consider all 30 observations on the first two process variables in Table 11.6. Calculate an estimate of the sample covariance matrix using both estimators  $S_1$  and  $S_2$  discussed in Section 11.3.2. Are the estimates very different? Discuss your findings.
- Suppose that there are p = 4 quality characteristics, and in correlation form all four variables have variance unity and that all pairwise correlation coefficients are 0.9. The in-control value of the process mean vector is  $\mu' = [0, 0, 0, 0]$ , and we want to design a MEWMA control chart to provide good protection against a shift to a new mean vector of y' = [1, 1, 1, 1]. Suppose that an in-control ARL<sub>0</sub> of 500 is desired. What value of  $\lambda$  and what upper control limit would you recommend? Approximately, what is the ARL<sub>1</sub> for detecting the shift in the mean vector?
  - Suppose that there are p=2 quality characteristics, and in correlation form both variables have variance unity and the correlation coefficient is 0.8. The in-control value of the process mean vector is  $\mu' = [0,0]$ , and we want to design a MEWMA control chart to provide good protection against a shift to a new mean vector of  $\mathbf{y}' = [1,1]$ . If an in-control ARL<sub>0</sub> of 200 is satisfactory, what value of  $\lambda$  and what upper control limit should be used? Approximately, what is the ARL<sub>1</sub> for detecting the shift in the mean vector?
  - In fitting a regression model to  $y_1$  you will find that not all of the process variables are required to obtain a satisfactory regression model for the output variable. Remove the nonsignificant variables and obtain a subset regression model for  $y_1$ . Then, construct an individuals control chart for the residuals. Compare this chart to the residual control chart in the text (Fig. 11.10). Are there any substantial differences between the charts from the two different approaches to fitting the regression models?
  - **11.11. Continuation of Exercise 11.10.** Using the residuals from the regression model in Exercise 11.10, set

- up an EWMA control chart. Compare this EWMA control chart to the Shewhart chart for individuals constructed previously. What are the potential advantages of the EWMA control chart in this situation?
- 11.12. Consider the p = 9 process variables in Table 11.5.
  - **a.** Perform a PCA on the first 30 observations. Be sure to work with the standardized variables.
  - **b.** How much variability is explained if only the first r = 3 principal components are retained?
  - **c.** Construct an appropriate set of pairwise plots of the first r = 3 principal component scores.
  - **d.** Now consider the last 10 observations. Obtain the principal component scores and plot them on the chart in part (c). Does the process seem to be in control?
  - 1.13. Consider the blood pressure and pulse data in Table 4E.8. Set up the  $T^2$  control chart for the systolic and diastolic blood pressure data. Does the blood pressure for this individual seem to be in a state of statistical control? What are some potential assignable causes for this process?
    - 4. Consider the blood pressure and pulse data in Table 4E.8. Set up a regression control chart for systolic blood pressure data. Does the systolic blood pressure for this individual seem to be in a state of statistical control? What are some potential assignable causes for this process?
    - Table 4E.8. Set up a multivariate EWMA control chart for the systolic and diastolic blood pressure data. Does the blood pressure for this individual seem to be in a state of statistical control? What are some potential assignable causes for this process?
- Table 4E.8. Set up a multivariate control chart for all three recorded parameters. Do these variables for this individual seem to be in a state of statistical control? What are some potential assignable causes for this process?
- 11.17. Consider the blood pressure and pulse data in Table 4E.8. Set up the control chart for the generalized variance for the systolic and diastolic blood pressure data. Interpret this chart.
- 11.18. Consider the blood pressure and pulse data in Table 4E.8. Set up the control chart for the generalized variance for all three recorded parameters. Interpret this chart.



Consider a  $T^2$  control chart for monitoring p=8 quality characteristics. Suppose that the subgroup size is n=5 and there are 30 preliminary samples available to estimate the sample covariance matrix.

- **a.** Find the phase II control limits assuming that  $\alpha = 0.01$ .
- **b.** Compare the limits from part (a) to the chi-
- c. How many preliminary samples would be required to ensure that the chi-square limit is within 1% of the exact phase II control limit?
- 11.20. Consider the blood pressure and pulse data in Table 4E.8. Find the principal components for this data. How much variability is explained if only the first two principal components are retained. Construct an appropriate control chart for the first two principal components.