CHAPTER 11 EXERCISES

SS **11.1.** The data shown in Table 11E.1 come from a production process with two observable quality characteristics: x_1 and x_2. The data are sample means of each quality characteristic, based on samples of size $n = 25$. Assume that mean values of the quality characteristics and the covariance matrix were computed from 50 preliminary samples:

$$\bar{\bar{\mathbf{x}}} = \begin{bmatrix} 55 \\ 30 \end{bmatrix} \quad \mathbf{S} = \begin{bmatrix} 200 & 130 \\ 130 & 120 \end{bmatrix}$$

Construct a T^2 control chart using these data. Use the phase II limits.

Table 11E.1 Data for Exercise 11.1

Sample Number	$\bar{x}_1$	$\bar{x}_2$
1	58	32
2	60	33
3	50	27
4	54	31
5	63	38
6	53	30
7	42	20
8	55	31
9	46	25
10	50	29
11	49	27
12	57	30
13	58	33
14	75	45
15	55	27

11.2. Reconsider the situation in Exercise 11.1. Suppose that the sample mean vector and sample covariance matrix provided were the actual population parameters. What control limit would be appropriate for phase II for the control chart? Apply this limit to the data and discuss any differences in results that you find in comparison to the original choice of control limit.

11.3. SS Consider a T^2 control chart for monitoring $p = 10$ quality characteristics. Suppose that the subgroup size is $n = 3$ and there are 25 preliminary samples available to estimate the sample covariance matrix.

a. Find the phase II control limits assuming that $\alpha = 0.005$.

b. Compare the control limits from part (a) to the chi-square control limit. What is the magnitude of the difference in the two control limits?

c. How many preliminary samples would have to be taken to ensure that the chi-square control limit is within 1% of the exact phase II control limit?

11.4. SS Rework Exercise 11.3, assuming that the subgroup size is $n = 5$.

11.5. Suppose that we have $p = 4$ quality characteristics, and in correlation form all four variables have variance unity and all pairwise correlation coefficients are 0.7. The in-control value of the process mean vector is $\mu' = [0, 0, 0, 0]$.

a. Write out the covariance matrix Σ.

b. What is the chi-square control limit for the chart, assuming that $\alpha = 0.01$?

c. Suppose that a sample of observations results in the standardized observation vector $\mathbf{y}' = [3.5, 3.5, 3.5, 3.5]$. Calculate the value of the T^2 statistic. Is an out-of-control signal generated?

d. Calculate the diagnostic quantities d_i, $i = 1, 2, 3, 4$ from equation 11.22. Does this information assist in identifying which process variables have shifted?

e. Suppose that a sample of observations results in the standardized observation vector $\mathbf{y}' = [2.5, 2, -1, 0]$. Calculate the value of the T^2 statistic. Is an out-of-control signal generated?

f. For the case in (e), calculate the diagnostic quantities d_i, $i = 1, 2, 3, 4$ from equation 11.22. Does this information assist in identifying which process variables have shifted?

SS **11.6.** Consider the first two process variables in Table 11.5. Calculate an estimate of the sample covariance matrix using both estimators $\mathbf{S}_1$ and $\mathbf{S}_2$ discussed in Section 11.3.2.

11.7. Consider all 30 observations on the first two process variables in Table 11.6. Calculate an estimate of the sample covariance matrix using both estimators $\mathbf{S}_1$ and $\mathbf{S}_2$ discussed in Section 11.3.2. Are the estimates very different? Discuss your findings.

SS **11.8.** Suppose that there are $p = 4$ quality characteristics, and in correlation form all four variables have variance unity and that all pairwise correlation coefficients are 0.9. The in-control value of the process mean vector is $\boldsymbol{\mu}' = [0, 0, 0, 0]$, and we want to design a MEWMA control chart to provide good protection against a shift to a new mean vector of $\mathbf{y}' = [1, 1, 1, 1]$. Suppose that an in-control ARL_0 of 500 is desired. What value of λ and what upper control limit would you recommend? Approximately, what is the ARL_1 for detecting the shift in the mean vector?

11.9. Suppose that there are $p = 2$ quality characteristics, and in correlation form both variables have variance unity and the correlation coefficient is 0.8. The in-control value of the process mean vector is $\boldsymbol{\mu}' = [0, 0]$, and we want to design a MEWMA control chart to provide good protection against a shift to a new mean vector of $\mathbf{y}' = [1, 1]$. If an in-control ARL_0 of 200 is satisfactory, what value of λ and what upper control limit should be used? Approximately, what is the ARL_1 for detecting the shift in the mean vector?

11.10. Consider the cascade process data in Table 11.5. In fitting a regression model to y_1 you will find that not all of the process variables are required to obtain a satisfactory regression model for the output variable. Remove the nonsignificant variables and obtain a subset regression model for y_1. Then, construct an individuals control chart for the residuals. Compare this chart to the residual control chart in the text (Fig. 11.10). Are there any substantial differences between the charts from the two different approaches to fitting the regression models?

11.11. **Continuation of Exercise 11.10.** Using the residuals from the regression model in Exercise 11.10, set up an EWMA control chart. Compare this EWMA control chart to the Shewhart chart for individuals constructed previously. What are the potential advantages of the EWMA control chart in this situation?

11.12. Consider the $p = 9$ process variables in Table 11.5. SS

a. Perform a PCA on the first 30 observations. Be sure to work with the standardized variables.

b. How much variability is explained if only the first $r = 3$ principal components are retained?

c. Construct an appropriate set of pairwise plots of the first $r = 3$ principal component scores.

d. Now consider the last 10 observations. Obtain the principal component scores and plot them on the chart in part (c). Does the process seem to be in control?

11.13. Consider the blood pressure and pulse data in Table 4E.8. Set up the T^2 control chart for the systolic and diastolic blood pressure data. Does the blood pressure for this individual seem to be in a state of statistical control? What are some potential assignable causes for this process?

11.14. Consider the blood pressure and pulse data in Table 4E.8. Set up a regression control chart for systolic blood pressure data. Does the systolic blood pressure for this individual seem to be in a state of statistical control? What are some potential assignable causes for this process?

11.15. Consider the blood pressure and pulse data in Table 4E.8. Set up a multivariate EWMA control chart for the systolic and diastolic blood pressure data. Does the blood pressure for this individual seem to be in a state of statistical control? What are some potential assignable causes for this process?

11.16. Consider the blood pressure and pulse data in Table 4E.8. Set up a multivariate control chart for all three recorded parameters. Do these variables for this individual seem to be in a state of statistical control? What are some potential assignable causes for this process?

11.17. Consider the blood pressure and pulse data in Table 4E.8. Set up the control chart for the generalized variance for the systolic and diastolic blood pressure data. Interpret this chart.

11.18. Consider the blood pressure and pulse data in Table 4E.8. Set up the control chart for the generalized variance for all three recorded parameters. Interpret this chart.

11.19. Consider a T^2 control chart for monitoring $p = 8$ quality characteristics. Suppose that the subgroup size is $n = 5$ and there are 30 preliminary samples available to estimate the sample covariance matrix.

a. Find the phase II control limits assuming that $\alpha = 0.01$.

b. Compare the limits from part (a) to the chi-square limit. Discuss the difference.

c. How many preliminary samples would be required to ensure that the chi-square limit is within 1% of the exact phase II control limit?

11.20. Consider the blood pressure and pulse data in Table 4E.8. Find the principal components for this data. How much variability is explained if only the first two principal components are retained. Construct an appropriate control chart for the first two principal components.