HW2-2020 (10월 26일, 월요일)

1. Ex. 9.5

2. Ex. 9.10

3. Ex. 9.11

4. Ex. 9.19

5. Ex. 9.40

6. Ex. 9.41

7. First, we state the relation between ARL_0 and ARL_1 values in CUSUM chart as:

   Assume that the process distribution shifts at the initial time point from $\mathrm{N}\big(\mu_0, \sigma^2\big)$ *to* $\mathrm{N}\big(\mu_0 + \delta, \lambda^2\sigma^2\big)$*. Then, the* ARL_1 *value of the upper CUSUM chart with parameters* h *and* k *equals to the* ARL_0 *value of the chart when the allowance is chosen to be* $k^* = (k-\delta)/(\lambda\delta)$*, the control limit is chosen to be* $h^* = h/(\lambda\sigma)$*, and the process IC distribution is* $N(0,1)$*.*

   Assume that the IC process distribution is $N(0,1)$. Using the relationship between the ARL_0 and ARL_1 values that is summarized above and the approximation formula

   $$\mathrm{ARL}_0 \approx \frac{\exp(2k(h+1.166)) - 2k(h+1.166) - 1}{2k^2},$$

   compute the ARL_1 values of the chart with the charting statistic

   $$C_n^+ = \max(0, C_{n-1}^+ + (X_n - \mu_0) - k), \quad C_0^+ = 0$$

and the decision rule

$$C_n^+ > h$$

with $k = 0.5$ and $h = 4.095$ (note: its ARL_0 value is 370), in cases when the process distribution shifts at the initial observation time point to one of the following six distributions:

(i) $N(0.5, 1)$,

(ii) $N(1, 1)$,

(iii) $N(-1, 1)$,

(iv) $N(0, 2^2)$,

(v) $N(0, 0.5^2)$,

(vi) $N(1, 2^2)$.

Summarize your results in terms of the relationship between ARL_1 and the shift size in the process mean and/or the shift size in the process variance.

8. For the two-sided EWMA charts defined by

$$E_n = \lambda X_n + (1-\lambda)E_{n-1}, \quad E_0 = \mu_0$$

and

$$\begin{aligned} U &= \mu_0 + \rho\sigma\sqrt{\frac{\lambda}{2-\lambda}}, \\ C &= \mu_0, \\ L &= \mu_0 - \rho\sigma\sqrt{\frac{\lambda}{2-\lambda}}, \end{aligned}$$

compute its ARL_0 values in the following cases, using the function `xewma.arl()` in the `R`-package `spc`:

(i) $\lambda = 0.1,\ \rho = 1$,

(ii) $\lambda = 0.1,\ \rho = 2$,

(iii) $\lambda = 0.5,\ \rho = 1$,

(iv) $\lambda = 0.5,\ \rho = 2$.

Summarize your major findings about the relationship between ARL_0 and (λ, ρ).

5.7 For the two-sided EWMA charts defined by

$$E_n = \lambda X_n + (1 - \lambda)E_{n-1}, \quad E_0 = \mu_0$$

and

$$\begin{aligned} U &= \mu_0 + \rho\sigma\sqrt{\frac{\lambda}{2-\lambda}}, \\ C &= \mu_0, \\ L &= \mu_0 - \rho\sigma\sqrt{\frac{\lambda}{2-\lambda}}, \end{aligned}$$

compute its ρ values in the following cases, using the function `xewma.crit()` in the `R`-package `spc`:

(i) $\mathrm{ARL}_0 = 150,\ \lambda = 0.1$,

(ii) $\mathrm{ARL}_0 = 150,\ \lambda = 0.5$,

(iii) $\mathrm{ARL}_0 = 450,\ \lambda = 0.1$,

(iv) $\mathrm{ARL}_0 = 450,\ \lambda = 0.5$.

Summarize your major findings about the relationship between ρ and $(\mathrm{ARL}_0, \lambda)$.