CHAPTER 6 EXERCISES

6.1. A manufacturer of components for automobile transmissions wants to use control charts to monitor a process producing a shaft. The resulting data from 20 samples of 4 shaft diameters that have been measured are:

$$\sum_{i=1}^{20} \bar{x}_i = 10.275, \quad \sum_{i=1}^{20} R_i = 1.012$$

a. Find the control limits that should be used on the $\bar{x}$ and R control charts.

b. Assume that the 20 preliminary samples plot in control on both charts. Estimate the process mean and standard deviation.

SS **6.2.** Reconsider the situation described in Exercise 6.1. Suppose that several of the preliminary 20 samples plot out of control on the R chart. Does this have any impact on the reliability of the control limits on the $\bar{x}$ chart?

SS **6.3.** Discuss why it is important to establish control on the R chart first when using $\bar{x}$ and R control charts to bring a process into statistical control.

6.4. The data shown in Table 6E.1 are $\bar{x}$ and R values for 24 samples of size $n = 5$ taken from a process producing bearings. The measurements are made on the inside diameter of the bearing, with only the last three decimals recorded (i.e., 34.5 should be 0.50345).

a. Set up $\bar{x}$ and R charts on this process. Does the process seem to be in statistical control? If necessary, revise the trial control limits.

b. If specifications on this diameter are 0.5030 ± 0.0010, find the percentage of nonconforming bearings produced by this process. Assume that diameter is normally distributed.

Table 6E.1 Bearing Diameter Data

Sample Number	$\bar{x}$	R	Sample Number	$\bar{x}$	R
1	34.5	3	13	35.4	8
2	34.2	4	14	34.0	6
3	31.6	4	15	37.1	5
4	31.5	4	16	34.9	7
5	35.0	5	17	33.5	4
6	34.1	6	18	31.7	3
7	32.6	4	19	34.0	8
8	33.8	3	20	35.1	4
9	34.8	7	21	33.7	2
10	33.6	8	22	32.8	1
11	31.9	3	23	33.5	3
12	38.6	9	24	34.2	2

6.5. The data shown in Table 6E.2 are the deviations from nominal diameter for holes drilled in a carbon-fiber composite material used in aerospace manufacturing. The values reported are deviations from nominal in ten-thousandths of an inch. SS

a. Set up $\bar{x}$ and R charts on the process. Is the process in statistical control?

b. Estimate the process standard deviation using the range method.

c. If specifications are at nominal ± 100, what can you say about the capability of this process? Calculate the PCR C_p.

6.6. The fill volume of soft-drink beverage bottles is an important quality characteristic. The volume is measured (approximately) by placing a gauge over the crown and comparing the height of the liquid in the neck of the bottle against a coded scale. SS

Table 6E.2 Hole Diameter Data for Exercise 6.5

Sample Number	x_1	x_2	x_3	x_4	x_5
1	−30	+50	−20	+10	+30
2	0	+50	−60	−20	+30
3	−50	+10	+20	+30	+20
4	−10	−10	+30	−20	+50
5	+20	−40	+50	+20	+10
6	0	0	+40	−40	+20
7	0	0	+20	−20	−10
8	+70	−30	+30	−10	0
9	0	0	+20	−20	+10
10	+10	+20	+30	+10	+50
11	+40	0	+20	0	+20
12	+30	+20	+30	+10	+40
13	+30	−30	0	+10	+10
14	+30	−10	+50	−10	−30
15	+10	−10	+50	+40	0
16	0	0	+30	−10	0
17	+20	+20	+30	+30	−20
18	+10	−20	+50	+30	+10
19	+50	−10	+40	+20	0
20	+50	0	0	+30	+10

On this scale, a reading of zero corresponds to the correct fill height. Fifteen samples of size $n = 10$ have been analyzed, and the fill heights are shown in Table 6E.3.

Table 6E.3 Fill Height Data for Exercise 6.6

Sample Number	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}
1	2.5	0.5	2.0	−1.0	1.0	−1.0	0.5	1.5	0.5	−1.5
2	0.0	0.0	0.5	1.0	1.5	1.0	−1.0	1.0	1.5	−1.0
3	1.5	1.0	1.0	−1.0	0.0	−1.5	−1.0	−1.0	1.0	−1.0
4	0.0	0.5	−2.0	0.0	−1.0	1.5	−1.5	0.0	−2.0	−1.5
5	0.0	0.0	0.0	−0.5	0.5	1.0	−0.5	−0.5	0.0	0.0
6	1.0	−0.5	0.0	0.0	0.0	0.5	−1.0	1.0	−2.0	1.0
7	1.0	−1.0	−1.0	−1.0	0.0	1.5	0.0	1.0	0.0	0.0
8	0.0	−1.5	−0.5	1.5	0.0	0.0	0.0	−1.0	0.5	−0.5
9	−2.0	−1.5	1.5	1.5	0.0	0.0	0.5	1.0	0.0	1.0
10	−0.5	3.5	0.0	−1.0	−1.5	−1.5	−1.0	−1.0	1.0	0.5
11	0.0	1.5	0.0	0.0	2.0	−1.5	0.5	−0.5	2.0	−1.0
12	0.0	−2.0	−0.5	0.0	−0.5	2.0	1.5	0.0	0.5	−1.0
13	−1.0	−0.5	−0.5	−1.0	0.0	0.5	0.5	−1.5	−1.0	−1.0
14	0.5	1.0	−1.0	−0.5	−2.0	−1.0	−1.5	0.0	1.5	1.5
15	1.0	0.0	1.5	1.5	1.0	−1.0	0.0	1.0	−2.0	−1.5

a. Set up $\bar{x}$ and s control charts on this process. Does the process exhibit statistical control? If necessary, construct revised control limits.

b. Set up an R chart, and compare it with the s chart in part (a).

c. Set up an s^2 chart and compare it with the s chart in part (a).

6.7. Rework Exercise 6.5 using the s chart. SS

6.8. Samples of $n = 6$ items each are taken from a process at regular intervals. A quality characteristic is measured, and $\bar{x}$ and R values are calculated for each sample. After 50 samples, we have SS

$$\sum_{i=1}^{50} \bar{x}_i = 2{,}000 \quad \text{and} \quad \sum_{i=1}^{50} R_i = 200$$

Assume that the quality characteristic is normally distributed.

a. Compute control limits for the $\bar{x}$ and R control charts.

b. All points on both control charts fall between the control limits computed in part (a). What are the natural tolerance limits of the process?

c. If the specification limits are 41 ± 5.0, what are your conclusions regarding the ability of the process to produce items within these specifications?

d. Assuming that if an item exceeds the upper specification limit it can be reworked, and if it is below the lower specification limit it must be scrapped, what percentage scrap and rework is the process producing?

e. Make suggestions as to how the process performance could be improved.

6.9. Table 6E.4 presents 20 subgroups of five measurements on the critical dimension of a part produced by a machining process. SS

a. Set up $\bar{x}$ and R control charts on this process. Verify that the process is in statistical control.

b. Following the establishment of control charts in part (a) above, 10 new samples in Table 6E.5 were collected. Plot the $\bar{x}$ and R values on the control chart you established in part (a) and draw conclusions.

c. Suppose that the assignable cause responsible for the action signals generated in part (b) has been identified and adjustments made to the process to correct its performance. Plot the $\bar{x}$

Table 6E.4 Data for Exercise 6.9

Sample Number	x_1	x_2	x_3	x_4	x_5	$\bar{x}$	R
1	138.1	110.8	138.7	137.4	125.4	130.1	27.9
2	149.3	142.1	105.0	134.0	92.3	124.5	57.0
3	115.9	135.6	124.2	155.0	117.4	129.6	39.1
4	118.5	116.5	130.2	122.6	100.2	117.6	30.0
5	108.2	123.8	117.1	142.4	150.9	128.5	42.7
6	102.8	112.0	135.0	135.0	145.8	126.1	43.0
7	120.4	84.3	112.8	118.5	119.3	111.0	36.1
8	132.7	151.1	124.0	123.9	105.1	127.4	46.0
9	136.4	126.2	154.7	127.1	173.2	143.5	46.9
10	135.0	115.4	149.1	138.3	130.4	133.6	33.7
11	139.6	127.9	151.1	143.7	110.5	134.6	40.6
12	125.3	160.2	130.4	152.4	165.1	146.7	39.8
13	145.7	101.8	149.5	113.3	151.8	132.4	50.0
14	138.6	139.0	131.9	140.2	141.1	138.1	9.2
15	110.1	114.6	165.1	113.8	139.6	128.7	54.8
16	145.2	101.0	154.6	120.2	117.3	127.6	53.3
17	125.9	135.3	121.5	147.9	105.0	127.1	42.9
18	129.7	97.3	130.5	109.0	150.5	123.4	53.2
19	123.4	150.0	161.6	148.4	154.2	147.5	38.3
20	144.8	138.3	119.6	151.8	142.7	139.4	32.2

Table 6E.5 Additional Data for Exercise 6.9, part (b)

Sample Number	x_1	x_2	x_3	x_4	x_5	$\bar{x}$	R
1	131.0	184.8	182.2	143.3	212.8	170.8	81.8
2	181.3	193.2	180.7	169.1	174.3	179.7	24.0
3	154.8	170.2	168.4	202.7	174.4	174.1	48.0
4	157.5	154.2	169.1	142.2	161.9	157.0	26.9
5	216.3	174.3	166.2	155.5	184.3	179.3	60.8
6	186.9	180.2	149.2	175.2	185.0	175.3	37.8
7	167.8	143.9	157.5	171.8	194.9	167.2	51.0
8	178.2	186.7	142.4	159.4	167.6	166.9	44.2
9	162.6	143.6	132.8	168.9	177.2	157.0	44.5
10	172.1	191.7	203.4	150.4	196.3	182.8	53.0

and R values from the new subgroups shown in Table 6E.6, which were taken following the adjustment, against the control chart limits established in part (a). What are your conclusions?

Table 6E.6 New Data for Exercise 6.9, part (c)

Sample Number	x_1	x_2	x_3	x_4	x_5	$\bar{x}$	R
1	131.5	143.1	118.5	103.2	121.6	123.6	39.8
2	111.0	127.3	110.4	91.0	143.9	116.7	52.8
3	129.8	98.3	134.0	105.1	133.1	120.1	35.7
4	145.2	132.8	106.1	131.0	99.2	122.8	46.0
5	114.6	111.0	108.8	177.5	121.6	126.7	68.7
6	125.2	86.4	64.4	137.1	117.5	106.1	72.6
7	145.9	109.5	84.9	129.8	110.6	116.1	61.0
8	123.6	114.0	135.4	83.2	107.6	112.8	52.2
9	85.8	156.3	119.7	96.2	153.0	122.2	70.6
10	107.4	148.7	127.4	125.0	127.5	127.2	41.3

6.10. Consider the $\bar{x}$ and R charts you established in Exercise 6.4 using $n = 5$.

a. Suppose that you wished to continue charting this quality characteristic using $\bar{x}$ and R charts based on a sample size of $n = 3$. What limits would be used on the $\bar{x}$ and R charts?

b. What would be the impact of the decision you made in part (a) on the ability of the $\bar{x}$ chart to detect a 2σ shift in the mean?

c. Suppose you wished to continue charting this quality characteristic using $\bar{x}$ and R charts based on a sample size of $n = 8$. What limits would be used on the $\bar{x}$ and R charts?

d. What is the impact of using $n = 8$ on the ability of the $\bar{x}$ chart to detect a 2σ shift in the mean?

6.11. Control charts for $\bar{x}$ and R are maintained for an important quality characteristic. The sample size is $n = 7$; $\bar{x}$ and R are computed for each sample. After 35 samples, we have found that

$$\sum_{i=1}^{35} \bar{x}_i = 7{,}805 \quad \text{and} \quad \sum_{i=1}^{35} R_i = 1{,}200$$

a. Set up $\bar{x}$ and R charts using these data.

b. Assuming that both charts exhibit control, estimate the process mean and standard deviation.

c. If the quality characteristic is normally distributed and if the specifications are 220 ± 35, can the process meet the specifications? Estimate the fraction nonconforming.

d. Assuming the variance to remain constant, state where the process mean should be located to minimize the fraction nonconforming. What would be the value of the fraction nonconforming under these conditions?

Table 6E.23 Diameter Data for Exercise 6.54

	Diameter				
Casting	**1**	**2**	**3**	**4**	**5**
1	11.7629	11.7403	11.7511	11.7474	11.7374
2	11.8122	11.7506	11.7787	11.7736	11.8412
3	11.7742	11.7114	11.7530	11.7532	11.7773
4	11.7833	11.7311	11.7777	11.8108	11.7804
5	11.7134	11.6870	11.7305	11.7419	11.6642
6	11.7925	11.7611	11.7588	11.7012	11.7611
7	11.6916	11.7205	11.6958	11.7440	11.7062
8	11.7109	11.7832	11.7496	11.7496	11.7318
9	11.7984	11.8887	11.7729	11.8485	11.8416
10	11.7914	11.7613	11.7356	11.7628	11.7070
11	11.7260	11.7329	11.7424	11.7645	11.7571
12	11.7202	11.7537	11.7328	11.7582	11.7265
13	11.8356	11.7971	11.8023	11.7802	11.7903
14	11.7069	11.7112	11.7492	11.7329	11.7289
15	11.7116	11.7978	11.7982	11.7429	11.7154
16	11.7165	11.7284	11.7571	11.7597	11.7317
17	11.8022	11.8127	11.7864	11.7917	11.8167
18	11.7775	11.7372	11.7241	11.7773	11.7543
19	11.7753	11.7870	11.7574	11.7620	11.7673
20	11.7572	11.7626	11.7523	11.7395	11.7884

d. Do you believe that the charts in part (c) are more informative than those in part (a)? Discuss why.

e. Provide a practical interpretation of the "within" chart.

6.55. In the semiconductor industry, the production of microcircuits involves many steps. The wafer fabrication process typically builds these microcircuits on silicon wafers, and there are many microcircuits per wafer. Each production lot consists of between 16 and 48 wafers. Some processing steps treat each wafer separately, so that the batch size for that step is one wafer. It is usually necessary to estimate several components of variation: within-wafer, between-wafer, between-lot, and the total variation.

a. Suppose that one wafer is randomly selected from each lot and that a single measurement on a critical dimension of interest is taken. Which components of variation could be estimated with these data? What type of control charts would you recommend?

b. Suppose that each wafer is tested at five fixed locations (say, the center and four points at the circumference). The average and range of these within-wafer measurements are $\bar{x}_{WW}$ and R_{WW}, respectively. What components of variability are estimated using control charts based on these data?

c. Suppose that one measurement point on each wafer is selected and that this measurement is recorded for five consecutive wafers. The average and range of these between-wafer measurements are $\bar{x}_{BW}$ and R_{BW}, respectively. What components of variability are estimated using control charts based on these data? Would it be necessary to run separate $\bar{x}$ and R charts for all five locations on the wafer?

d. Consider the question in part (c). How would your answer change if the test sites on each wafer were randomly selected and varied from wafer to wafer?

e. What type of control charts and rational subgroup scheme would you recommend to control the batch-to-batch variability?

6.56. Consider the situation described in Exercise 6.55. A critical dimension (measured in μm) is of interest to the process engineer. Suppose that five fixed positions are used on each wafer (position 1 is the center) and that two consecutive wafers are selected from each batch. The data that result from several batches are shown in Table 6E.24.

a. What can you say about overall process capability?

b. Can you construct control charts that allow within-wafer variability to be evaluated?

c. What control charts would you establish to evaluate variability between wafers? Set up these charts and use them to draw conclusions about the process.

d. What control charts would you use to evaluate lot-to-lot variability? Set up these charts and use them to draw conclusions about lot-to-lot variability.

6.57. Consider the blood pressure and pulse data in Table 4E.8. Set up individuals and moving range control charts for the systolic blood pressure data. Does the blood pressure for this individual seem to be in a state of statistical control? What are some potential assignable causes for this process?

Table 6E.24 Data for Exercise 6.56

Lot Number	Wafer Number	Position 1	2	3	4	5	Lot Number	Wafer Number	Position 1	2	3	4	5
1	1	2.15	2.13	2.08	2.12	2.10	11	1	2.15	2.13	2.14	2.09	2.08
	2	2.13	2.10	2.04	2.08	2.05		2	2.11	2.13	2.10	2.14	2.10
2	1	2.02	2.01	2.06	2.05	2.08	12	1	2.03	2.06	2.05	2.01	2.00
	2	2.03	2.09	2.07	2.06	2.04		2	2.04	2.08	2.03	2.10	2.07
3	1	2.13	2.12	2.10	2.11	2.08	13	1	2.05	2.03	2.05	2.09	2.08
	2	2.03	2.08	2.03	2.09	2.07		2	2.08	2.01	2.03	2.04	2.10
4	1	2.04	2.01	2.10	2.11	2.09	14	1	2.08	2.04	2.05	2.01	2.08
	2	2.07	2.14	2.12	2.08	2.09		2	2.09	2.11	2.06	2.04	2.05
5	1	2.16	2.17	2.13	2.18	2.10	15	1	2.14	2.13	2.10	2.10	2.08
	2	2.17	2.13	2.10	2.09	2.13		2	2.13	2.10	2.09	2.13	2.15
6	1	2.04	2.06	1.97	2.10	2.08	16	1	2.06	2.08	2.05	2.03	2.09
	2	2.03	2.10	2.05	2.07	2.04		2	2.03	2.01	1.99	2.06	2.05
7	1	2.04	2.02	2.01	2.00	2.05	17	1	2.05	2.03	2.08	2.01	2.04
	2	2.06	2.04	2.03	2.08	2.10		2	2.06	2.05	2.03	2.05	2.00
8	1	2.13	2.10	2.10	2.15	2.13	18	1	2.03	2.08	2.04	2.00	2.03
	2	2.10	2.09	2.13	2.14	2.11		2	2.04	2.03	2.05	2.01	2.04
9	1	1.95	2.03	2.08	2.07	2.08	19	1	2.16	2.13	2.10	2.13	2.12
	2	2.01	2.03	2.06	2.05	2.04		2	2.13	2.15	2.18	2.19	2.13
10	1	2.04	2.08	2.09	2.10	2.01	20	1	2.06	2.03	2.04	2.09	2.10
	2	2.06	2.04	2.07	2.04	2.01		2	2.01	1.98	2.05	2.08	2.06

6.58. Consider the blood pressure and pulse data in Table 4E.8. Set up individuals and moving range control charts for the diastolic blood pressure data. Does the blood pressure for this individual seem to be in a state of statistical control? What are some potential assignable causes for this process?

6.59. Consider the blood pressure and pulse data in Table 4E.8. Set up individuals and moving range control charts for the pulse rate data. Does the pulse for this individual seem to be in a state of statistical control? Does this variable appear to exhibit more variability than either of the blood pressure measurements? What are some potential assignable causes for this process?

6.60. Consider the casting diameter data in Table 6E.23. Set up individuals and moving range control charts for the average diameter on each casting. Do these charts convey similar information as either of the control charts constructed in Exercise 6.54 parts (a) and (c)?

6.61. Suppose that someone suggests that you decrease the width of the control limits on Shewhart control charts from the traditional 3-sigma to 2.5-sigma. The effect of this is to (select all that are correct):

a. increase the ability of the control chart to detect assignable causes.
b. increase the ability of the control chart to detect only assignable causes that result in large shifts.
c. increase the rate of false alarms.
d. decrease the rate of false alarms.

6.62. Consider the hospital patient satisfaction data in Table 4E.9. Set up individuals and moving range control charts for the patient satisfaction variable. Does variable seem to be in a state of statistical control? What are some potential assignable causes for this process?

6.63. Consider the wine quality data in Table 4E.10. Set up individuals and moving range control charts for the wine quality variable. Does variable seem to

Table 4E.8 Blood Pressure Data for an Adult Male

Observation	Systolic	Diastolic	Pulse Rate
2/5/2018	81	56	82
2/6/2018	112	73	66
2/7/2018	108	62	76
2/8/2018	95	61	66
2/9/2018	111	68	78
2/11/2018	98	65	71
2/13/2018	98	64	84
2/16/2018	115	72	61
3/14/2018	112	72	77
3/15/2018	103	63	74
3/16/2018	117	72	59
3/17/2018	103	71	85
3/30/2018	100	65	64
3/31/2018	105	64	71
4/1/2018	100	71	85
4/2/2018	115	66	65
4/3/2018	113	84	78
4/4/2018	107	71	76
4/6/2018	117	74	90
4/8/2018	96	68	81
4/9/2018	107	60	72
4/10/2018	90	57	89
4/12/2018	119	69	61
4/13/2018	85	59	77
4/21/2018	107	70	83

4.47. You have fit a regression model to a response variable that required transformation to stabilize the variance. Specifically, you have used the natural logarithm of y as the response. The user of the model wants to predict the response in the original units at a particular set of values for the predictor variables. You obtain the prediction on the log scale and take the antilog as the final prediction. The model user asks if this is an estimate of the mean response at that set of predictors. How do you answer the question?

4.48. Table 4E.8 also contains data of the pulse rate of the subject. Fit a regression model to the systolic blood pressure data using diastolic pressure and pulse rate as the predictor variables.

a. Test for significance of regression.

b. Are both variables statistically significant?

c. Analyze the residuals from this model. Has adding the pulse rate to the original model improved the fit? How comfortable are you using this model as a predictor of systolic blood pressure?

4.49. A hospital conducts surveys of patients when they are discharged to determine their satisfaction with the care and service they have received. Table 4E.9 contains a recent sample of this data. The Surgical-Medical variable is an indicator variable taking on the value 0 if the patient is on the surgical service and 1 if the patient is on the medical service. The Severity and Anxiety variables are determined based on examination of the patient upon admission and increase as either anxiety or severity of illness increase.

Table 4E.9 Patient Satisfaction Data

Observation	Age	Severity	Surgical-Medical	Anxiety	Satisfaction
1	55	50	0	2.1	[illegible]
2	46	24	1	2.8	[illegible]
3	30	46	1	3.3	[illegible]
4	35	48	1	4.5	[illegible]
5	59	58	0	2.0	[illegible]
6	61	60	0	5.1	[illegible]
7	74	65	1	5.5	[illegible]
8	38	42	1	3.2	[illegible]
9	27	42	0	3.1	[illegible]
10	51	50	1	2.4	[illegible]
11	53	38	1	2.2	[illegible]
12	41	30	0	2.1	[illegible]
13	37	31	00	1.9	[illegible]
14	24	34	0	3.1	[illegible]
15	42	30	0	3.0	[illegible]
16	50	48	1	4.2	[illegible]
17	58	61	1	4.6	[illegible]
18	60	71	1	5.3	[illegible]
19	62	62	0	7.2	[illegible]
20	68	38	0	7.8	[illegible]
21	70	41	1	7.0	[illegible]
22	79	66	1	6.2	[illegible]
23	63	31	1	4.1	[illegible]
24	39	42	0	3.5	[illegible]
25	49	40	1	2.1	[illegible]