Summer Mock AMC 8

2020

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Directions:

1. Do not continue onto the next page until you are ready to take the test.

2. This is a 25-question, multiple choice test. Each question is followed by answers marked A, B, C, D and E. Only one of these is correct.

3. You will receive 1 point for each correct answer, 0 points for each problem left unanswered, and 0 points for each incorrect answer.

4. No aids are permitted other than scratch paper, pencils, and erasers. In particular, calculators are NOT allowed.

5. Figures are not necessarily drawn to scale.

6. You will have **40 minutes** working time to complete the test.

1. Mount Everest, the tallest natural structure in the world, is 8848 meters tall. The Burj Khalifa, the tallest building in the world, is 829 meters tall. To the nearest integer, how many times as tall is Mount Everest's height compared to the Burj Khalifa?

(A) 8 **(B)** 9 **(C)** 10 **(D)** 11 **(E)** 12

2. Alice has 4 cats, and 60 cans of cat food. She gives the first cat 20% of the cans of food, 25% of what's left to the second cat, $\frac{4}{9}$ of what's left to the third cat, and the rest to the fourth cat. How many cans of cat food does Alice give to the fourth cat?

(A) 16 **(B)** $\frac{55}{3}$ **(C)** 20 **(D)** $\frac{182}{9}$ **(E)** 24

3. Ian is driving his bus when he notices that he has driven 2020 miles this month. After Ian drives for four more hours, he notices that his total number of miles for the month is a palindrome. What is the lowest possible speed, in miles per hour, that Ian could have averaged in those four hours?

Note: a palindrome is a number that is the same when read forwards and backwards, for example 1991.

(A) 4.5 **(B)** 18 **(C)** 23 **(D)** 72 **(E)** 92

4. Bob has a machine that takes a number, multiplies it by three, and then subtracts two. When Bob puts a number a into the machine, he gets b. When Bob puts b into the machine, he gets c. If $b = 28$, then what is $c - a$?

(A) 18 **(B)** $\frac{118}{3}$ **(C)** 54 **(D)** 72 **(E)** $\frac{232}{3}$

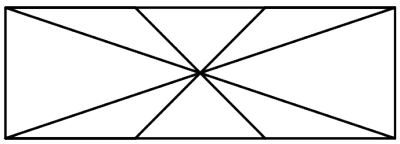
5. Freddy walks x miles at a rate of $y+7$ miles per hour, and it takes him 20 minutes. Then, Freddy walks $2y$ miles at a rate of $x + 5$ miles per hour, and it takes him 30 minutes. How long, in minutes, would it take Freddy to walk $8x + 10$ miles at a rate of $5y + 7$ miles per hour?

(A) 20 **(B)** 30 **(C)** 60 **(D)** 90 **(E)** 120

6. The surface area of a cube is 9 times the surface area of another cube. If the volume of the larger cube is 54, what is the volume of the smaller cube?

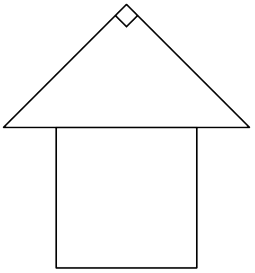
(A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 6

7. How many triangles are there in the figure below?



(A) 10 **(B)** 12 **(C)** 14 **(D)** 16 **(E)** 18

8. An isosceles right triangle is put on top of a square, as shown. The legs of the triangle are equal to each other and the side lengths of the square. If the side length of the square is 4, then the perimeter of the entire figure can be expressed as $a + b\sqrt{c}$, where a, b, and c are integers and c is not divisible by any perfect square other than 1. What is $a + b + c$?



(A) 18 **(B)** 20 **(C)** 22 **(D)** 24 **(E)** 26

9. Bill and Rob are counting cars. Bill counts blue cars, Rob counts red cars, and they both count purple cars. While Bill and Rob count cars, a total of 196 cars pass, and 121 of them are not blue, red, or purple. If Bill counts 36 cars and Rob counts 49 cars, how many of the cars that passed were purple?

(A) 10 **(B)** 13 **(C)** 15 **(D)** 16 **(E)** 18

10. On the number line, the distance between a number and 2 is equal to twice the distance between the number and 14. What is the sum of all possible values for the number?

(A) 10 **(B)** 16 **(C)** 26 **(D)** 36 **(E)** 52

11. Points P, Q, R, and S are placed on sides $\overline{AB}$, $\overline{BC}$, $\overline{CD}$, and $\overline{DA}$, respectively, of square $ABCD$ such that $PA = 3 \cdot PB$, $QB = 3 \cdot QC$, $RD = 3 \cdot RC$, and $SA = 3 \cdot SD$. In addition, lines $\overline{PR}$, $\overline{SQ}$, and $\overline{PS}$ are drawn. Let $\overline{PR}$ intersect $\overline{SQ}$ at X. The ratio of the area of triangle PXS to the area of square $ABCD$ can be expressed as $\frac{m}{n}$ for relatively prime positive integers m and n. What is $m + n$?

(A) 11 **(B)** 25 **(C)** 31 **(D)** 34 **(E)** 41

12. Bob has 7 pairs of white socks, 5 pairs of gray socks, and n pairs of black socks. In each pair, one sock is for his right foot, and one sock is for his left foot. If Bob needs to take at least 33 individual socks out of the drawer in order to guarantee that he gets one black sock for each of his feet, then what is the remainder when n is divided by 5?

(A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** 4

13. Detective Holmes is interviewing four suspects in the same room. Watson recorded the following conversation, knowing that two of the suspects will lie and two will tell the truth. Given that there is only one criminal, and that the criminal always lies, who is the criminal?

Bob: It was Chad!
Chad: I didn't do it.
Alice: Bob is lying.
Dan: Then Bob did it.

(A) Alice **(B)** Bob **(C)** Chad **(D)** Dan **(E)** Impossible Scenario

14. 54 students plan to split the cost of a field trip equally. When another k students are added and the cost is redistributed evenly, each student's share is 10% less. If k students are instead removed from the original 54 and the cost is redistributed evenly, the cost of each student's share increases by h%. If the cost of the field trip is positive, then what is $h + k$?

(A) 16 **(B)** 18.5 **(C)** 21.5 **(D)** 26 **(E)** 29

15. Tanya is selling cookies. She has 2020 cookies to sell in either red or yellow bags, but not every cookie must be sold. Each red bag holds exactly 35 cookies, and each yellow bag holds exactly 40 cookies. Each bag of red cookies sells for \$21, and each bag of yellow cookies sells for \$24. What is the maximum amount of money Tanya can make from selling red and yellow bags of cookies?

(A) 1197 **(B)** 1200 **(C)** 1208 **(D)** 1212 **(E)** 1215

16. Given that $\frac{1}{101} = 0.\overline{0099}$, what is the 2020th digit to the right of the decimal point in the expansion of $\frac{1}{2020}$?

(A) 0 **(B)** 2 **(C)** 4 **(D)** 5 **(E)** 9

17. A sequence follows the pattern

$$1, 0, 2, 1, 3, 2, 4, 3, 5, 4, 6, 5, \ldots,$$

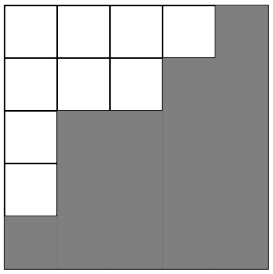
where the nth term in the sequence is one less than the term before it if n is even, and two more than the term before it if n is odd. A second sequence is formed by multiplying each consecutive pair of terms of the first sequence:

$$0, 2, 6, 12, 20, 30, \ldots,$$

where the nth term in the sequence is the product of the $2n - 1$th and $2n$th terms of the first sequence. What is the result when the 2020th term in the second sequence is divided by 2020?

(A) 2018 **(B)** 2019 **(C)** 2020 **(D)** 2021 **(E)** 2022

18. Mario is building stairs in a 5 by 5 grid. The first stair can be any number less than 5 units tall, and each stair must be taller or the same height as the previous stair. If the last stair is 5 units tall, how many possible staircases are there? One possible arrangement is shown.



(A) 70 **(B)** 84 **(C)** 126 **(D)** 210 **(E)** 252

19. A right triangle has hypotenuse 6 and area 9. The perimeter of the triangle can be expressed as $a + b\sqrt{c}$, where a, b, and c are integers and c is not divisible by any perfect square other than 1. What is $a + b + c$?

(A) 14 **(B)** 15 **(C)** 16 **(D)** 17 **(E)** 18

20. A cow stands on vertex A of trapezoid $ABCD$. Each minute, the cow moves to a random adjacent vertex of the trapezoid. What is the probability that after 1001 minutes, the cow is on vertex D?

(A) $\frac{1}{4}$ **(B)** $\frac{2}{7}$ **(C)** $\frac{1}{3}$ **(D)** $\frac{2}{5}$ **(E)** $\frac{1}{2}$

21. Larry and Harry each have a stack of cards. Larry has four more cards than Harry. Larry can make 960 more sets of cards from his stack than Harry can, where order does not matter. What is the remainder when the number of cards Larry has is divided by 5?

For example, if Larry has cards A, B, and C, he could make sets (A), (B), (C), (A, B), (A, C), (B, C), (A, B, C), and an empty set, for a total of 8.

(A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** 4

22. How many positive integers less than or equal to 2020 leave a remainder of 2 when divided by 3, a remainder of 3 when divided by 5, and a remainder of 2 when divided by 7?

(A) 9 **(B)** 10 **(C)** 18 **(D)** 19 **(E)** 20

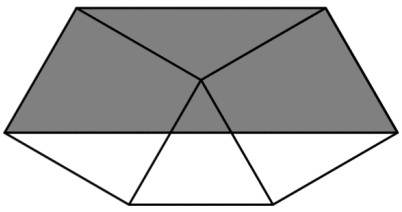
23. Bob's calendar is incorrect, and it advances two days of the week every day, instead of just one. For example, if Bob's calendar says it is Monday, then a day later, it will say that it is Wednesday, and a day after that, it will say that it is Friday. Bob's calendar says the correct day of the week on March 1. If there are five Saturdays in March, but Bob's calendar says that there are four Saturdays in March, then what day of the week is March 31?

(A) Wednesday **(B)** Thursday **(C)** Friday **(D)** Saturday **(E)** Sunday

24. The equation $x^2 - bx + c = 0$, where b and c are constants, has two solutions for x which are both positive integers. If $b + c = 80$, then what is the remainder when bc is divided by 5?

(A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** 4

25. Two squares and an equilateral triangle, each with side length 4, are put together to form the figure below. The area of the shaded region can be expressed as $a + b\sqrt{c}$, where a, b, and c are integers and c is not divisible by any perfect square other than 1. What is $a + b + c$?



(A) 19 **(B)** 23 **(C)** 27 **(D)** 31 **(E)** 35