Chapter -1 Integers

1. Determine each of the following products:

(i) 12 × 7

(ii) (-15) × 8

(iii) (-25) × (-9)

(iv) 125 × (-8)

Solution:

(i) Given 12 × 7

Here we have to find the products of given numbers

12 ×7 = 84

Because the product of two integers of like signs is equal to the product of their absolute values.

(ii) Given (-15) × 8

Here we have to find the products of given numbers

(-15) ×8 = -120

Because the product of two integers of opposite signs is equal to the additive inverse of the product of their absolute values.

(iii) Given (-25) × (-9)

Here we have to find the products of given numbers

(-25) × (-9) = + (25 ×9) = +225

Because the product of two integers of opposite signs is equal to the additive inverse of the product of their absolute values.

(iv) Given 125 × (-8)

Here we have to find the products of given numbers

125 × (-8) = -1000

Because the product of two integers of opposite signs is equal to the additive inverse of the product of their absolute values.

2. Find each of the following products:

(i) 3 × (-8) × 5

(ii) 9 × (-3) × (-6)

(iii) (-2) × 36 × (-5)

(iv) (-2) × (-4) × (-6) × (-8)

Solution:

(i) Given 3 × (-8) ×5

Here we have to find the product of given number.

3 × (-8) × 5 = 3 × (-8 × 5)

=3 × -40 = -120

Since the product of two integers of opposite signs is equal to the additive inverse of the product of their absolute values.

(ii) Given 9 × (-3) × (-6)

Here we have to find the product of given number.

9 × (-3) × (-6) = 9 × (-3 × -6) [∵ the product of two integers of like signs is equal to

the product of their absolute values.]

=9 × +18 = +162

(iii) Given (-2) × 36 × (-5)

Here we have to find the product of given number.

(-2) × 36 × (-5) = (-2 × 36) × -5 [∵ the product of two integers of like signs is equal to

the product of their absolute values.]

=-72 × -5 = +360

(iv) Given (-2) × (-4) × (-6) × (-8)

Here we have to find the product of given number.

(-2) × (-4) × (-6) × (-8) = (-2 × -4) × (-6 × -8) [∵ the product of two integers of like signs is

equal to the product of their absolute values.]

=-8 × -48 = +384

3. Find the value of:

(i) 1487 × 327 + (-487) × 327

(ii) 28945 × 99 – (-28945)

Solution:

(i) Given 1487 × 327 + (-487) × 327

By using the rule of multiplication of integers, we have

1487 × 327 + (-487) × 327 = 486249 – 159249

Since the product of two integers of opposite signs is equal to the additive inverse of the product of their absolute values.

=327000

(ii) Given 28945 × 99 – (-28945)

By using the rule of multiplication of integers, we have

28945 × 99 – (-28945) = 2865555 + 28945

Since the product of two integers of like signs is equal to the product of their absolute values.

=2894500

4. Complete the following multiplication table:

Second number

First number	**x**	**-4**	**-3**	**-2**	**-1**	**0**	**1**	**2**	**3**	**4**
	-4									
	-3									
	-2									
	-1									
	0									
	1									
	2									
	3									

	4								

Is the multiplication table symmetrical about the diagonal joining the upper left corner to the lower right corner?

Solution:

Second number

First number	x	-4	-3	-2	-1	0	1	2	3	4
	-4	16	12	8	4	0	-4	-8	-12	-16
	-3	12	9	6	3	0	-3	-6	-9	-12
	-2	8	6	4	2	0	-2	-4	-6	-8
	-1	4	3	2	1	0	-1	-2	-3	-4
	0	0	0	0	0	0	0	0	0	0
	1	-4	-3	-2	-1	0	1	2	3	4
	2	-8	-6	-4	-2	0	2	4	6	8
	3	-12	-9	-6	-3	0	3	6	9	12
	4	-16	-12	-8	-4	0	4	8	12	16

From the table it is clear that, the table is symmetrical about the diagonal joining the upper left corner to the lower right corner.

5. Determine the integer whose product with '-1' is

(i) 58

(ii) 0

(iii) -225

Solution:

(i) Given 58

Here we have to find the integer which is multiplied by -1

We get, 58 × -1 = -58

Since the product of two integers of opposite signs is equal to the additive inverse of the product of their absolute values.

(ii) Given 0

Here we have to find the integer which is multiplied by -1

We get, 0 × -1 = 0 [because anything multiplied with 0 we get 0 as their result]

(iii) Given -225

Here we have to find the integer which is multiplied by -1

We get, -225 × -1 = 225

Since the product of two integers of like signs is equal to the product of their absolute values.

1. Divide:

(i) 102 by 17

(ii) -85 by 5

(iii) -161 by -23

(iv) 76 by -19

(v) 17654 by -17654

(vi) (-729) by (-27)

(vii) 21590 by -10

(viii) 0 by -135

Solution:

(i) Given 102 by 17

We can write given question as 102 ÷ 17

102 ÷ 17 = l102/17l = l102l/l17l [by applying the mod]

= 102/17 = 6

(ii) Given -85 by 5

We can write given question as -85 ÷ 5

-85 ÷ 5 = l-85/5l = l-85l/l5l [by applying the mod]

= -85/5 = -17

(iii) Given -161 by -23

We can write given question as -161 ÷ -23

-161 ÷ -23 = l-161/-23l = l-161l/l-23l [by applying the mod]

= 161/23 = 7

(iv) Given 76 by -19

We can write given question as 76 ÷ -19

76 ÷ -19 = l76/-19l = l76l/l-19l [by applying the mod]

= 76/-19 = -4

(v) Given 17654 by -17654

We can write given question as 17654 ÷ -17654

17654 ÷ -17654 = l17654/-17654l = l17654l/l-17654l [by applying the mod]

= 17654/-17654 = -1

(vi) Given (-729) by (-27)

We can write given question as (-729) ÷ (-27)

(-729) ÷ (-27) = l-729/-27l = l-729l/l-27l [by applying the mod]

= 729/27 = 27

(vii) Given 21590 by -10

We can write given question as 21590 ÷ -10

21590 ÷ -10 = l21590/-10l = l21590l/l-10l [by applying the mod]

= 21590/-10 = -2159

(viii) Given 0 by -135

We can write given question as 0 ÷ -135

0 ÷ -135 = 0 [because anything divided by 0 we get the result as 0]

Find the value of

1. 36 ÷ 6 + 3

Solution:

Given 36 ÷ 6 + 3

According to BODMAS rule we have to operate division first then we have to do addition

Therefore 36 ÷ 6 + 3 = 6 + 3 = 9

2. 24 + 15 ÷ 3

Solution:

Given 24 + 15 ÷ 3

According to BODMAS rule we have to operate division first then we have to do addition

Therefore 24 + 15 ÷ 3 = 24 + 5 = 29

3. 120 – 20 ÷ 4

Solution:

Given 120 – 20 ÷ 4

According to BODMAS rule we have to operate division first then we have to do subtraction

Therefore 120 – 20 ÷ 4 = 120 – 5 = 115

4. 32 – (3 × 5) + 4

Solution:

Given 32 – (3 × 5) + 4

According to BODMAS rule we have to operate in brackets first then move to addition and subtraction.

Therefore 32 – (3 × 5) + 4 = 32 – 15 + 4

= 32 – 11 = 21

5. 3 – (5 – 6 ÷ 3)

Solution:

Given 3 – (5 – 6 ÷ 3)

According to BODMAS rule we have to operate in brackets first then we have move to subtraction.

Therefore 3 – (5 – 6 ÷ 3) = 3 – (5 – 2)

= 3 –3 = 0

6. 21 – 12 ÷ 3 × 2

Solution:

Given 21 – 12 ÷ 3 × 2

According to BODMAS rule we have to perform division first then move to multiplication and subtraction.

Therefore, 21 – 12 ÷ 3 × 2 = 21 – 4 × 2

= 21 – 8 = 13

7. 16 + 8 ÷ 4 – 2 × 3

Solution:

Given 16 + 8 ÷ 4 – 2 × 3

According to BODMAS rule we have to perform division first followed by multiplication, addition and subtraction.

Therefore, 16 + 8 ÷ 4 – 2 × 3 = 16 + 2 – 2 × 3

= 16 + 2 – 6

= 18 -6

= 12

8. 28 – 5 × 6 + 2

Solution:

Given 28 – 5 × 6 + 2

According to BODMAS rule we have to perform multiplication first followed by addition and subtraction.

Therefore, 28 – 5 × 6 + 2 = 28 – 30 +2

= 28 – 28 = 0

9. (-20) × (-1) + (-28) ÷ 7

Solution:

Given (-20) × (-1) + (-28) ÷ 7

According to BODMAS rule we have to perform division first followed by multiplication, addition and subtraction.

Therefore, (-20) × (-1) + (-28) ÷ 7 = (-20) × (-1) – 4

= 20 – 4 = 16

10. (-2) + (-8) ÷ (-4)

Solution:

Given (-2) + (-8) ÷ (-4)

According to BODMAS rule we have to perform division first followed by addition and subtraction.

Therefore, (-2) + (-8) ÷ (-4) = (-2) + 2

=0

11. (-15) + 4 ÷ (5 – 3)

Solution:

Given (-15) + 4 ÷ (5 – 3)

According to BODMAS rule we have to perform division first followed by addition and subtraction.

Therefore, (-15) + 4 ÷ (5 – 3) = (-15) + 4 ÷ 2

= -15 + 2

= -13

12. (-40) × (-1) + (-28) ÷ 7

Solution:

Given (-40) × (-1) + (-28) ÷ 7

According to BODMAS rule we have to perform division first followed by multiplication, addition and subtraction.

(-40) × (-1) + (-28) ÷ 7 = (-40) × (-1) – 4

= 40 – 4

= 36

13. (-3) + (-8) ÷ (-4) -2 × (-2)

Solution:

Given (-3) + (-8) ÷ (-4) -2 × (-2)

According to BODMAS rule we have to perform division first followed by multiplication, addition and subtraction.

(-3) + (-8) ÷ (-4) -2 × (-2) = -3 + 2 -2 × (-2)

= -3 + 2 + 4

= 6 – 3

=3

14. (-3) × (-4) ÷ (-2) + (-1)

Solution:

Given (-3) × (-4) ÷ (-2) + (-1)

According to BODMAS rule we have to perform division first followed by multiplication, addition and subtraction.

(-3) × (-4) ÷ (-2) + (-1) = -3 × 2 -1

= – 6 – 1

= -7

Simplify each of the following:

1. 3 – (5 – 6 ÷ 3)

Solution:

Given 3 – (5 – 6 ÷ 3)

According to removal of bracket rule firstly remove inner most bracket

We get 3 – (5 – 6 ÷ 3) = 3 – (5 – 2)

= 3 – 3

= 0

2. -25 + 14 ÷ (5 – 3)

Solution:

Given -25 + 14 ÷ (5 – 3)

According to removal of bracket rule firstly remove inner most bracket

We get -25 + 14 ÷ (5 – 3) = -25 + 14 ÷ 2

= -25 + 7

= -18

$$3.\ 25-\frac{1}{2}\left\{5+4-\left(3+2-\overline{1+3}\right)\right\}$$

Solution:

Given $25-\frac{1}{2}\left\{5+4-\left(3+2-\overline{1+3}\right)\right\}$

Solution:

$$3.\ 25-\frac{1}{2}\left\{5+4-\left(3+2-\overline{1+3}\right)\right\}$$

Solution:

Given $25-\frac{1}{2}\left\{5+4-\left(3+2-\overline{1+3}\right)\right\}$

According to removal of bracket rule first we have to remove vinculum we get

= 25 – ½ {5 + 4 – (5 – 4)}

Now by removing the innermost bracket we get

= 25 – ½ {5 + 4 – 1}

By removing the parentheses we get

= 25 – ½ (8)

Now simplifying we get

= 25 – 4

= 21

$$4.\ 27-\left[38-\left\{46-\left(15-\overline{13-2}\right)\right\}\right]$$

Solution:

Given $27-\left[38-\left\{46-\left(15-\overline{13-2}\right)\right\}\right]$

Solution:

$$4.\ 27 - \left[38 - \left\{46 - \left(15 - \overline{13 - 2}\right)\right\}\right]$$

Solution:
Given $27 - \left[38 - \left\{46 - \left(15 - \overline{13 - 2}\right)\right\}\right]$

According to removal of bracket rule first we have to remove vinculum we get

= 27 – [38 – {46 – (15 – 11)}]

Now by removing inner most bracket we get

= 27 – [38 – {46 – 4}]

By removing the parentheses we get

= 27 – [38 – 42]

Now by removing braces we get

= 27 – (-4)

= 27 + 4

= 31

$$5.\ 36 - [18 - \{14 - (15 - 4 \div 2 \times 2)\}]$$

Solution:
Given $36 - [18 - \{14 - (15 - 4 \div 2 \times 2)\}]$

Solution:

$$5.\ 36 - [18 - \{14 - (15 - 4 \div 2 \times 2)\}]$$

Solution:
Given $36 - [18 - \{14 - (15 - 4 \div 2 \times 2)\}]$

By removing innermost bracket we get

= 36 – [18 – {14 – (11 ÷ 2 × 2)}]

= 36 – [18 – {14 – 11}]

Now by removing the parentheses we get

= 36 – [18 – 3]

Now remove the braces we get

= 36 – 15

= 21

6. 45 – [38 – {60 ÷ 3 – (6 – 9 ÷ 3) ÷ 3}]

Solution:

Given 45 – [38 – {60 ÷ 3 – (6 – 9 ÷ 3) ÷ 3}]

First remove the inner most brackets

= 45 – [38 – {20 – (6 – 3) ÷ 3}]

= 45 – [38 – {20 – 3 ÷ 3}]

Now remove the parentheses we get

= 45 – [38 – 19]

Now remove the braces we get

= 45 – 19

= 26

7\. $23-\left[23-\left\{23-\left(23-\overline{23-23}\right)\right\}\right]$

Solution:

Given $23-\left[23-\left\{23-\left(23-\overline{23-23}\right)\right\}\right]$

Solution:

7. $23-\left[23-\left\{23-\left(23-\overline{23-23}\right)\right\}\right]$

Solution:

Given $23-\left[23-\left\{23-\left(23-\overline{23-23}\right)\right\}\right]$

Now first remove the vinculum we get

= 23 – [23 – {23 – (23 – 0)}]

Now remove the innermost bracket we get,

= 23 – [23 – {23 – 23}]

By removing the parentheses we get,

= 23 – [23 -0]

Now we have to remove the braces and on simplifying we get,

= 23 – 23

= 0

$$8.\ 2550 - \left[510 - \left\{270 - \left(90 - \overline{80 + 70}\right)\right\}\right]$$

Solution:

Given $2550 - \left[510 - \left\{270 - \left(90 - \overline{80 + 70}\right)\right\}\right]$

Solution:

$$8.\ 2550 - \left[510 - \left\{270 - \left(90 - \overline{80 + 70}\right)\right\}\right]$$

Solution:

Given $2550 - \left[510 - \left\{270 - \left(90 - \overline{80 + 70}\right)\right\}\right]$

First we have to remove the vinculum from the given equation we get,

= 2550 – [510 – {270 – (90 – 150)}]

We get,

= 2550 – [510 – {270 – (-60)}]

= 2550 – [510 – {270 + 60}]

Now remove the parentheses we get,

= 2550 – [510 – 330]

Now we have to remove braces

= 2550 – 180

= 2370

9. $4+\frac{1}{5}\left[\left\{-10\times\left(25-\overline{13-3}\right)\right\}\div(-5)\right]$

Solution:
Given
$4+\frac{1}{5}\left[\left\{-10\times\left(25-\overline{13-3}\right)\right\}\div(-5)\right]$

Solution:

9. $4+\frac{1}{5}\left[\left\{-10\times\left(25-\overline{13-3}\right)\right\}\div(-5)\right]$

Solution:
Given
$4+\frac{1}{5}\left[\left\{-10\times\left(25-\overline{13-3}\right)\right\}\div(-5)\right]$

First we have to remove vinculum from the given equation,

= 4 + 1/5 [{-10 × (25 – 10)} ÷ (-5)]

Now remove the innermost bracket, we get

= 4 + 1/5 [{-10 × 15} ÷ -5]

Now by removing the parentheses we get,

= 4 + 1/5 [-150 ÷ -5]

By removing the braces we get,

= 4 + 1/5 (30)

On simplifying we get,

= 4 + 6

= 10

$$10.\ 22 - \frac{1}{4}\{-5 - (-48) \div (-16)\}$$

Solution:

Given $22 - \frac{1}{4}\{-5 - (-48) \div (-16)\}$

Solution:

$$10.\ 22 - \frac{1}{4}\{-5 - (-48) \div (-16)\}$$

Solution:

Given $22 - \frac{1}{4}\{-5 - (-48) \div (-16)\}$

Now we have to remove innermost bracket

= 22 – ¼ {-5 – (- 48 ÷ – 16)}

After removing innermost bracket

= 22 – ¼ {-5 – 3}

Now remove the parentheses we get

= 22 – ¼ (-8)

On simplifying we get,

= 22 + 2

= 24

$$11.\ 63 - \left[(-3)\left\{-2 - \overline{8-3}\right\}\right] \div [3\{5 + (-2)(-1)\}]$$

Solution:

Given $63 - \left[(-3)\left\{-2 - \overline{8-3}\right\}\right] \div [3\{5 + (-2)(-1)\}]$

Solution:

11. $63 - \left[(-3)\left\{-2 - \overline{8-3}\right\}\right] \div [3\{5 + (-2)(-1)\}]$

Solution:

Given $63 - \left[(-3)\left\{-2 - \overline{8-3}\right\}\right] \div [3\{5 + (-2)(-1)\}]$

First we have to remove vinculum from the given equation then we get,

= 63 – [(-3) {-2 – 5}] ÷ [3 {5 + 2}]

Now remove the parentheses from the above equation

= 63 – [(-3) (-7)] ÷ [3 (7)]

= 63 – [21] ÷ [21]

= 63 – 1

= 62

12. $[29 - (-2)\{6 - (7-3)\}] \div [3 \times \{5 + (-3) \times (-2)\}]$

Solution:

Given $[29 - (-2)\{6 - (7-3)\}] \div [3 \times \{5 + (-3) \times (-2)\}]$

Solution:

12. $[29 - (-2)\{6 - (7-3)\}] \div [3 \times \{5 + (-3) \times (-2)\}]$

Solution:

Given $[29 - (-2)\{6 - (7-3)\}] \div [3 \times \{5 + (-3) \times (-2)\}]$

First we have to remove the innermost brackets then we get,

= [29 – (-2) {6 – 4}] ÷ [3 × {5 + 6}]

Now remove the parentheses in the above equation,

= [29 + 2 (2)] ÷ [3 × 11]

Now remove all braces present in the above equation,

= 33 ÷ 33

= 1

13. Using brackets, write a mathematical expression for each of the following:

(i) Nine multiplied by the sum of two and five.

(ii) Twelve divided by the sum of one and three.

(iii) Twenty divided by the difference of seven and two.

(iv) Eight subtracted from the product of two and three.

(v) Forty divided by one more than the sum of nine and ten.

(vi) Two multiplied by one less than the difference of nineteen and six.

Solution:

(i) $9\ (2 + 5)$

(ii) $12 \div (1 + 3)$

(iii) $20 \div (7 - 2)$

(iv) $2 \times 3 - 8$

(v) $40 \div [1 + (9 + 10)]$

(vi) $2 \times [(19 - 6) - 1]$

Chapter -2 Fractions

Exercise 2.1

1. Compare the following fractions by using the symbol > or < or =:

(i) (7/9) and (8/13)

(ii) (11/9) and (5/9)

(iii) (37/41) and (19/30)

(iv) (17/15) and (119/105)

Solution:

(i) Given (7/9) and (8/13)

Taking LCM for 9 and 13 we get,

$9 \times 13 = 117$

Now we convert the given fractions into its equivalent fractions, then it becomes

$(7 \times 13)/ (9 \times 13)$ and $(8 \times 9)/ (13 \times 9)$

Therefore $(91/117) > (72/117)$

Hence $(7/9) > (8/13)$

(ii) Given (11/9) and (5/9)

As the denominator is equal, they form equivalent fractions.

But we know that $11 > 5$

Hence $(11/9) > (5/9)$

(iii) Given (37/41) and (19/30)

Taking LCM for 41 and 30 we get,

$30 \times 41 = 1230$

Now we convert the given fractions into its equivalent fractions, then it becomes

$(37 \times 30)/ (41 \times 30)$ and $(19 \times 41)/ (30 \times 41)$

Therefore $(1110/1230) > (779/1230)$

Hence $(37/41) > (19/30)$

(iv) Given (17/15) and (119/105)

Taking LCM for 15 and 105 we get, $5 \times 3 \times 7 = 105$

Now we convert the given fractions into its equivalent fractions, then it becomes

(17 × 7)/ (15 × 7) and (119/105)

Therefore (119/105) = (119/105)

Hence (17/15) = (119/105)

2. Arrange the following fractions in ascending order:

(i) (3/8), (5/6), (6/8), (2/4), (1/3)

(ii) (4/6), (3/8), (6/12), (5/16)

Solution:

(i) Given (3/8), (5/6), (6/8), (2/4), (1/3)

Now we have to arrange these in ascending order, to arrange these in ascending order we have to make those as equivalent fractions by taking LCM's.

LCM of 8, 6, 4 and 3 is 24

Equivalent fractions are

(9/24), (20/24), (18/24), (12/24), (8/24)

We know that 8 < 9 < 12 < 18 < 20

Now arranging in ascending order

(8/24) < (9/24) < (12/24) < (18/24) < (20/24)

Hence (1/3) < (3/8) < (2/4) < (6/8) < (5/6)

(ii) Given (4/6), (3/8), (6/12), (5/16)

Now we have to arrange these in ascending order, to arrange these in ascending order we have to make those as equivalent fractions by taking LCM's.

LCM of 8, 6, 12 and 16 is 48

Equivalent fractions are

(32/48), (15/48), (18/48), (15/48)

We know that 12 < 15 < 18 < 32

Now arranging in ascending order

(12/48) < (15/48) < (18/48) < (32/48)

(6/12) < (5/16) < (3/8) < (4/6)

3. Arrange the following fractions in descending order:

(i) (4/5), (7/10), (11/15), (17/20)

(ii) (2/7), (11/35), (9/14), (13/28)

Solution:

(i) Given (4/5), (7/10), (11/15), (17/20)

Now we have to arrange these in descending order, to arrange these in descending order we have to make those as equivalent fractions by taking LCM's.

LCM of 5, 10, 15 and 20 is 60

Equivalent fractions are

(48/60), (42/60), (44/60), (51/60)

We know that 51 > 48 > 44 > 42

Now arranging in descending order

Hence (17/20) > (4/5) > (11/15) > (7/10)

(ii) Given (2/7), (11/35), (9/14), (13/28)

Now we have to arrange these in descending order, to arrange these in descending order we have to make those as equivalent fractions by taking LCM's.

LCM of 7, 35, 14 and 28 is 140

Equivalent fractions are

(40/140), (44/140), (90/140), (65/140)

We know that 90 > 65 > 44 > 40

Now arranging in descending order

Hence (9/14) > (13/28) > (11/35) > (2/7)

4. Write five equivalent fraction of (3/5).

Solution:

Given (3/5)

By multiplying or dividing both the numerator and denominator so that it keeps the same value by this we can get the equivalent fractions.

(3 × 2)/ (5 × 2), (3 × 3)/ (5 × 3), (3 × 4)/ (5 × 4), (3 × 5)/ (5 × 5), (3 × 6)/ (5 × 6)

Equivalent fractions are

(6/10), (9/15), (12/20), (15/25), (18/30)

5. Find the sum:

(i) (5/8) + (3/10)

(ii) 4 3/4 + 9 2/5

(iii) (5/6) + 3 + (3/4)

(iv) 2 3/5 + 4 7/10 + 2 4/15

Solution:

(i) Given (5/8) + (3/10)

Taking LCM for 8 and 10 we get 40

Now we have to convert the given fractions into equivalent fractions with denominator 40

(5/8) + (3/10) = (5 × 5)/ (8 × 5) + (3 × 4)/ (10 × 4)

= (25/40) + (12/40)

= (37/40)

(ii) Given 4 3/4 + 9 2/5

First convert given mixed fractions into improper fractions.

4 3/4 + 9 2/5 = (19/4) + (47/5)

Taking LCM for 4 and 5 we get 20

Now we have to convert the given fractions into equivalent fractions with denominator 20

4 3/4 + 9 2/5 = (19/4) + (47/5) = (19 × 5)/ (4 × 5) + (47 × 4)/ (5 × 4)

= (95/20) + (188/20)

= (283/20)

(iii) Given (5/6) + 3 + (3/4)

Taking LCM for 6, 1 and 4 we get 12

Now we have to convert the given fractions into equivalent fractions with denominator 12

(5/6) + 3 + (3/4) = (5 × 2)/ (6 × 2) + (3 × 12)/ (1 × 12) + (3 × 3)/ (4 × 3)

= (10/12) + (36/12) + (9/12)

= (55/12)

(iv) Given 2 3/5 + 4 7/10 + 2 4/15

First convert given mixed fractions into improper fractions

2 3/5 + 4 7/10 + 2 4/15 = (13/5) + (47/10) + (34/15)

Taking LCM for 5, 10 and 15 we get 30

Now we have to convert the given fractions into equivalent fractions with denominator 30

2 3/5 + 4 7/10 + 2 4/15 = (13/5) + (47/10) + (34/15) = (13 × 6)/ (5 × 6) + (47 × 3)/ (10 × 3) + (34 × 2)/ (15 × 2)

= (78/30) + (141/30) + (68/30)

= (287/30)

6. Find the difference of:

(i) (13/24) and (7/16)

(ii) 6 and (23/3)

(iii) (21/25) and (18/20)

(iv) 3 3/10 and 2 7/15

Solution:

(i) Given (13/24) and (7/16)

To find the difference we have to make it equivalent fractions.

LCM of 24 and 16 is 48.

Now converting the given fractions into equivalent fractions with denominator 48.

(13/24) – (7/16) = (26/48) – (21/48)

= (26 – 21)/48

= (5/48)

(ii) Given 6 and (23/3)

To find the difference we have to make it equivalent fractions.

LCM of 3 and 1 is 3.

Now converting the given fractions into equivalent fractions with denominator 3.

(23/3) – 6 = (23/3) – (18/3)

= (23 – 18)/3

= (5/3)

(iii) Given (21/25) and (18/20)

To find the difference we have to make it equivalent fractions.

LCM of 25 and 20 is 100.

Now converting the given fractions into equivalent fractions with denominator 100.

(18/20) – (21/25) = (90/100) – (84/100)

= (90 – 84)/100

= 6/100

By converting it into its simplest form we get

= 3/50

(iv) Given 3 3/10 and 2 7/15

First convert given mixed fractions into improper fractions.

(33/10) and (37/15)

To find the difference we have to make it equivalent fractions.

LCM of 10 and 15 is 30.

Now converting the given fractions into equivalent fractions with denominator 30.

(33/10) – (37/15) = (99/30) – (74/30)

= (99 – 74)/30

= (25/30)

By converting it into simplest form we get

= (5/6)

7. Find the difference:

(i) (6/7) – (9/11)

(ii) 8 – (5/9)

(iii) 9 – 5 2/3

(iv) 4 3/10 – 1 2/15

Solution:

(i) Given (6/7) – (9/11)

To find the difference we have to make it equivalent fractions.

LCM of 7 and 11 is 77.

Now converting the given fractions into equivalent fractions with denominator 77.

Equivalent fractions are (66/77) and (63/77)

(6/7) – (9/11) = (66/77) – (63/77)

= (66 – 63)/77

= (3/77)

(ii) Given 8 – (5/9)

To find the difference we have to make it equivalent fractions.

LCM of 1 and 9 is 9.

Now converting the given fractions into equivalent fractions with denominator 9.

Equivalent fractions are (72/9) and (5/9)

8 – (5/9) = (72/9) – (5/9)

= (72 -5)/9

= (67/9)

(iii) Given 9 – 5 2/3

First convert the given mixed fractions into improper fractions.

We get 5 2/3 = (17/3)

To find the difference we have to make it equivalent fractions.

LCM of 1 and 3 is 3.

Now converting the given fractions into equivalent fractions with denominator 3.

Equivalent fractions are (27/3) and (17/3)

9 – 5 2/3 = (27/3) – (17/3)

= (10/3)

(iv) Given 4 3/10 – 1 2/15

First convert the given mixed fractions into improper fractions.

We get (43/10) and (17/15)

To find the difference we have to make it equivalent fractions.

LCM of 10 and 15 is 30.

Now converting the given fractions into equivalent fractions with denominator 30.

Equivalent fractions are (129/30) and (34/30)

4 3/10 – 1 2/15 = (43/10) – (17/15)

= (129/30) – (34/30)

= (129 – 34)/30

= (95/30)

= (19/6)

8. Simplify:

(i) (2/3) + (1/6) – (2/9)

(ii) 12 – 3 1/2

(iii) 7 5/6 – 4 3/8 + 2 7/12

Solution:

(i) Given (2/3) + (1/6) – (2/9)

LCM of 3, 6 and 9 is 18

Now we have to convert the given fraction into equivalent fraction with denominator 18

(2/3) + (1/6) – (2/9) = (12/18) + (3/18) – (4/18)

= (12 + 3 – 4)/18

= 11/18

(ii) Given 12 – 3 (1/2)

First convert the given mixed fraction into improper fraction we get (7/2)

LCM of 2 and 1 is 2

Now we have to convert the given fraction into equivalent fraction with denominator 2

12 – 3 1/2 = (24/2) – (7/2)

= (24 -7)/2

= (17/2)

(iii) Given 7 5/6 – 4 3/8 + 2 7/12

First convert the given mixed fraction into improper fraction we get (47/6), (35/8) and (31/12)

LCM of 12, 6 and 8 is 48

Now we have to convert the given fraction into equivalent fraction with denominator 48

7 5/6 – 4 3/8 + 2 7/12 = (47/6) – (35/8) + (31/12)

= (376/48) – (210/48) + (124/48)

= (376 – 210 + 124)/48

= (290/48)

= (145/24)

9. What should be added to 5 3/7 to get 12?

Solution:

Given 5 3/7

First convert the given mixed fraction into improper fraction we get (38/7)

Let x be the number added to (38/7) to get 12

Therefore x + (38/7) = 12

x = 12 – (38/7)

LCM for 7 and 1 is 7

x = (12 × 7 – 38)/7

x = (84 – 38)/7

x = (46/7)

Hence (46/7) is the number which is added to 5 3/7 to get 12.

10. What should be added to 5 4/15 to get 12 3/5?

Solution:

Given 5 4/15

First convert the given mixed fraction into improper fraction we get (79/15)

Let x be the number added to (79/15) to get (63/5)

Therefore x + (79/15) = (63/5)

x = (63/5) – (79/15)

LCM for 15 and 5 is 15

x = (63 × 3 – 79)/15

x = (189 – 79)/15

x = (110/15) = (22/3)

Hence (22/3) is the number which is added to 5 4/15 to get 12 3/5.

11. Suman studies for 5 2/3 hours daily. She devotes 2 4/5 hours of her time for Science and Mathematics. How much time does she devote for other subject?

Solution:

Given Suman studies for 5 2/3 hours i.e. (17/3) hours

And she devotes 2 4/5 hours i.e. (14/5) hours for Science and Mathematics.

Let x be the hours she devotes for other subjects.

(17/3) = x + (14/5)

x = (17/3) – (14/5)

LCM of 3 and 5 is 15

x = (17 × 5 – 14× 3)/15

x = (85 – 42)/15

x = (43/15) = 2 13/15 hours

12. A piece of wire is of length 12 3/4 m. If it is cut into two pieces in such a way that the length of one piece is 5 1/4 m, what is the length of other piece?

Solution:

Given the total length of piece of wire is 12 3/4 i.e. (51/4) m

Length of one piece of wire is 5 1/4 i.e. ((21/4) m

Let the length of other piece be ‘x’ m

(51/4) = x + (21/4)

x = (51/4) – (21/4)

x = (51-21)/4

x = (30/4)

x = (15/2)

x = 7 1/2 m

13. A rectangular sheet of paper is 12 1/2 cm long and 10 2/3 cm wide. Find its perimeter.

Solution:

Given length of rectangular sheet of paper is 12 1/2 i.e. (25/2)

Breadth of rectangular sheet of paper is 10 2/3 i.e. (32/3)

But we know that perimeter of rectangle = 2 (length + breadth)

Perimeter of rectangular sheet = 2 [(25/2) + (32/3)]

LCM of 2 and 3 is 6, by taking this and simplifying we get

Perimeter = 2[(25 × 3)/6 + (32 × 2)/6]

= 2[(75/6) + (64/6)]

= 2(139/6)

= (139/3)

= 46 1/3 cm

14. In a "magic square", the sum of the numbers in each row, in each column and along the diagonal is the same. Is this a magic square?

(4/11)	**(9/11)**	**(2/11)**
(3/11)	**(5/11)**	**(7/11)**
(8/11)	**(1/11)**	**(6/11)**

Solution:

Along first column = (4/11) + (3/11) + (8/11) = (15/11)

Along second column = (9/11) + (5/11) + (7/11) = (15/11)

Along third column = (2/11) + (7/11) + (6/11) = (15/11)

Along first row = (4/11) + (9/11) + (2/11) = (15/11)

Along second row = (3/11) + (5/11) + (7/11) = (15/11)

Along third row = (8/11) + (1/11) + (6/11) = (15/11)

Along diagonal = (4/11) + (5/11) + (6/11) = (15/11)

= (2/11) + (5/11) + (8/11) = (15/11)

Therefore sum along all direction is same i.e. (15/11). Hence it is a magical square

15. The cost of Mathematics book is Rs 25 3/4 and that of Science book is Rs 20 1/2. Which cost more and by how much?

Solution:

Given the cost of Mathematics book is Rs 25 3/4 i.e. (103/4)

Cost of Science book is Rs 20 1/2 i.e. (41/2)

Now the LCM of 2 and 4 is 4

Now we have to convert the given fractions into its equivalent fractions with denominator 4

Mathematics book cost is Rs (103/4)

Science book cost is Rs (41 × 2/2 × 2) = (82/4)

(103 – 82)/4 = 21/4 = 5 1/4

Hence the cost of Mathematics book is more than cost of Science book by 5 1/4

16. (i) Provide the number in box [ ]and also give its simplest form in each of the following:

(i) (2/3) × [ ] = (10/30)

(ii) (3/5) × [ ] = (24/75)

Solution:

(i) (2/3) × [5/10] = (10/30)

(ii) (3/5) × [8/15] = (24/75)

Exercise 2.2

1. Multiply:

(i) (7/11) by (3/5)

(ii) (3/5) by 25

(iii) 3 4/15 by 24

(iv) 3 1/8 by 4 10/11

Solution:

(i) Given (7/11) by (3/5)

We have to multiply the given number

(7/11) × (3/5) = (21/55)

(ii) Given (3/5) by 25

(3/5) × 25 = 15 [dividing 25 by 5]

(iii) Given 3 4/15 by 24

First convert the given mixed fraction to improper fraction.

(49/15) × 24 = (1176/15)

= 78 2/5

(iv) Given 3 1/8 by 4 10/11

First convert the given mixed fractions to improper fractions.

(25/8) × (54/11) = (1350/88) = (675/44)

= 15 15/44

2. Find the product:

(i) (4/7) × (14/25)

(ii) 7 1/2 × 2 4/15

(iii) 3 6/7 × 4 2/3

(iv) 6 11/14 × 3 1/2

Solution:

(i) Given (4/7) × (14/25)

(4/7) × (14/25) = (4 × 14)/ (7 × 25)

= (56/175)

Converting above fractions into simplest form

= (8/25)

(ii) Given 7 1/2 × 2 4/15

We have to convert mixed fractions into improper fractions

Then we get (15/2) and (34/15)

7 (1/2) × 2 (4/15) = (15/2) × (34/15)

= (15 × 34)/ (2 × 15)

= (510/30)

= 17

(iii) Given 3 6/7 × 4 2/3

We have to convert mixed fractions into improper fractions

Then we get (27/7) and (14/3)

3 6/7 × 4 2/3 = (27/7) × (14/3)

On simplifying

= 9 × 2

= 18

(iv) Given 6 11/14 × 3 1/2

We have to convert mixed fractions into improper fractions

Then we get (95/14) and (7/2)

6 11/14 × 3 1/2 = (95/14) × (7/2)

= (95 × 7)/28

= (665/28)

= 23 3/4

3. Simplify:

(i) (12/25) × (15/28) × (35/36)

(ii) (10/27) × (39/56) × (28/65)

(iii) 2 2/17 × 7 2/9 × 1 33/52

Solution:

(i) Given (12/25) × (15/28) × (35/36)

= (12 × 15 × 35)/ (25 × 28 × 36)

= (6300/25200)

On simplifying we get

= (1/4)

(ii) Given (10/27) × (39/56) × (28/65)

= (10 × 39 × 28)/ (27 × 56 × 65)

= (10920/98280)

On simplifying we get

= (1/9)

(iii) Given 2 2/17 × 7 2/9 × 1 33/52

First convert the given mixed fractions into improper fractions then we get

= (36/17) × (65/9) × (85/52)

= (36 × 65 × 85)/ (17 × 9 × 52)

= (198900/7956)

On simplifying we get

= 25

4. Find:

(i) (1/2) of 4 2/9

(ii) (5/8) of 9 2/3

(iii) (2/3) of (9/16)

Solution:

(i) Given (1/2) of 4 2/9

First convert given mixed fraction into improper fraction then we get (38/9)

= (1/2) × (38/9)

= (1 × 38)/ (2 × 9)

= (38 /18)

= 2 1/9

(ii) Given (5/8) of 9 2/3

First convert given mixed fraction into improper fraction then we get (29/3)

= (5/8) × (29/3)

= (5 × 29)/ (8 × 3)

= (145 /24)

= 6 1/24

(iii) Given (2/3) of (9/16)

= (2/3) × (9/16)

= (2 × 9)/ (3 × 16)

= (18 /48)

= (3/8)

5. Which is greater? (1/2) of (6/7) or (2/3) of (3/7)

Solution:

Given (1/2) of (6/7)

= (1/2) × (6/7)

= (1 × 6)/ (2 × 7)

= (6 /14)

Also given that (2/3) of (3/7)

= (2/3) × (3/7)

= (2 × 3)/ (3 × 7)

= (6 /21)

While comparing two fractions, if numerators of both the fractions are same, then the denominator having higher value shows the fraction has lower value.

Therefore (6/14) is greater.

Hence (1/2) of (6/7) is greater.

6. Find:

(i) (7/11) of Rs 330

(ii) (5/9) of 108 meters

(iii) (3/7) of 42 liters

(iv) (1/12) of an hour

(v) (5/6) of an year

(vi) (3/20) of a kg

(vii) (7/20) of a liter

(viii) (5/6) of a day

(ix) (2/7) of a week

Solution:

(i) Given (7/11) of Rs 330

= (7/11) × 330

On dividing by 11 we get

= 7 × 30

= 210

(7/11) of Rs 330 is Rs 210

(ii) Given (5/9) of 108 meters

= (5/9) × 108

Dividing 108 by 9 we get

= 5 × 12

= 60

(5/9) of 108 meters is 60 meters

(iii) Given (3/7) of 42 liters

= (3/7) × 42

Dividing 42 by 7 we get

= 3 × 6

= 18

(3/7) of 42 liters is 18 liters

(iv) Given (1/12) of an hour

An hour = 60 minutes

= (1/12) × 60

Dividing 60 by 12 we get

= 1 × 5

= 5

(1/12) of an hour is 5 minutes

(v) Given (5/6) of an year

1 year = 12 months

= (5/6) × 12

Dividing 12 by 6 we get

= 5 × 2

= 10

(5/6) of an year is 10 months

(vi) Given (3/20) of a kg

1 kg = 1000 grams

= (3/20) × 1000

= 3 × 50

= 150

(3/20) of a kg is 150 grams

(vii) Given (7/20) of a liter

1 liter = 1000 ml

= (7/20) × 1000

= 7 × 50

= 350

(7/20) of a liter is 350ml

(viii) Given (5/6) of a day

1 day = 24 hours

= (5/6) × 24

= 5 × 4

= 20

(5/6) of a day is 20 hours

(ix) Given (2/7) of a week

1 week = 7 days

= (2/7) × 7

= 2 × 1

= 2

(2/7) of a week is 2 days

7. Shikha plants 5 saplings in a row in her garden. The distance between two adjacent saplings is ¾ m. Find the distance between the first and the last sapling.

Solution:

Given that the distance between two adjacent saplings is (3/4) m

There are 4 adjacent spacing for 5 sapling

Therefore, distance between the first and the last sapling is

= (3/4) × 4

= 3

The distance between them is 3m

8. Ravish reads (1/3) part of a book in 1 hour. How much part of the book will he read in 2 1/5 hours?

Solution:

Given Ravish takes 1 hour to read (1/3) part of the book

Then we have to calculate how much part he will read in 2 1/5 hours

First convert the given mixed fraction into improper fraction i.e. (11/5)

Now let x be the full part of book

1 hour = (1/3) x

Remaining part of the book, he will read in

= (11/5) × (1/3) x

= (11/15) part of the book

9. Lipika reads a book for 1 3/4 hours every day. She reads the entire book in 6 days. How many hours in all were required by her to read the book?

Solution:

Given time taken by Lipika to read a book per day = 1 3/4 = (7/4) hours

Time taken by Lipika to read a book in 6 days = (7/4) × 6

= (42/4)

= 10 ½ hours

10. Find the area of a rectangular park which is 41 2/3 m along and 18 3/5 m broad.

Solution:

Given length of rectangular park is = 41 2/3 = (125/3)

Breadth of rectangular park is = 18 3/5 = (93/5)

Area of rectangular park = length × breadth

= (125/3) × (93/5)

= (125 × 93)/15

= (11625/15)

= 775 m2

11. If milk is available at Rs 17 3/4 per liter, find the cost of 7 2/5 liters of milk.

Solution:

Given the cost of milk per liter is = 17 3/4 = Rs (71/4)

And the cost of 7 2/5 = (37/5) is

= (37/ 5) × (71/4)

= (37 × 71)/20

= (2627/20)

= Rs 131 7/20

12. Sharada can walk 8 1/3 km in one hour. How much distance will she cover in 2 2/5 hours?

Solution:

Given distance covered by Sharada in one hour = 8 1/3 = (25/3) km

Distance covered by her in 2 2/5 hours = (12/5) is

= (25/3) × (12/5)

= (25 × 12)/15

= (300/15)

= 20 km

13. A sugar bag contains 30kg of sugar. After consuming (2/3) of it, how much sugar is left in the bag?

Solution:

A sugar bag contains 30kg of sugar.

After consuming, the left sugar in the bag is = 30- (2/3) × 30

= 30 – 2 × 10

= 30 – 20

= 10kg

14. Each side of a square is 6 2/3 m long. Find its area.

Solution:

Side of a square = 6 2/3 = (20/3) m

Area of square = side × side

= (20/3) × (20/3)

= (400/9)

=44 4/9 m2

15. There are 45 students in a class and (3/5) of them are boys. How many girls are there in the class?

Solution:

Total number of students = 45

Number of boys out of 45 is = (3/5)

Number of girls = 45 – (3/5) × 45

= 45 – 3 × 9

= 45 – 27

= 18 girls

Exercise 2.3

1. Find the reciprocal of each of the following fractions and classify them as proper, improper and whole numbers:

(i) (3/7)

(ii) (5/8)

(iii) (9/7)

(iv) (6/5)

(v) (12/7)

(vi) (1/8)

Solution:

(i) Given (3/7)

Reciprocal of (3/7) is (7/3)

(7/3) is improper fraction

(ii) Given (5/8)

Reciprocal of (5/8) is (8/5)

It is improper fraction

(iii) Given (9/7)

Reciprocal of (9/7) is (7/9)

It is proper fraction

(iv) Given (6/5)

Reciprocal of (6/5) is (5/6)

It is proper fraction

(v) Given (12/7)

Reciprocal of (12/7) is (7/12)

It is proper fraction

(vi) Given (1/8)

Reciprocal of (1/8) is (8/1) = 8

It is whole number

2. Divide:

(i) (3/8) by (5/9)

(ii) 3 1/4 by (2/3)

(iii) (7/8) by 4 1/2

(iv) 6 1/4 by 2 3/5

Solution:

(i) Given (3/8) by (5/9)

From the rule of division of fraction we know that (a/b) ÷ (c/d) = (a/b) × (d/c)

(3/8)/ (5/9) = (3/8) × (9/5)

= (3 × 9) / (8 × 5)

= (27/40)

(ii) Given 3 1/4 by (2/3)

Converting 3 1/4 to improper fraction we get (13/4)

From the rule of division of fraction we know that (a/b) ÷ (c/d) = (a/b) × (d/c)

(13/4)/ (2/3) = (13/4) × (3/2)

= (13 × 3) / (4 × 2)

= (39/8)

= 4 7/8

(iii) Given (7/8) by 4 1/2

Converting 4 1/2 to improper fraction we get (9/2)

From the rule of division of fraction we know that (a/b) ÷ (c/d) = (a/b) × (d/c)

(7/8)/ (9/2) = (7/8) × (2/9)

= (7 × 2) / (8 × 9)

= (14/72)

= (7/36)

(iv) Given 6 1/4 by 2 3/5

Converting 6 1/4 and 2 3/5 to improper fraction we get (25/4) and (13/5)

From the rule of division of fraction we know that (a/b) ÷ (c/d) = (a/b) × (d/c)

(25/4)/ (13/5) = (25/4) × (5/13)

= (25 × 5) / (4 × 13)

= (125/52)

= 2 21/52

3. Divide:

(i) (3/8) by 4

(ii) (9/16) by 6

(iii) 9 by (3/16)

(iv) 10 by (100/3)

Solution:

(i) Given (3/8) by 4

= (3/8)/4

= (3/8 × 4)

= (3/32)

(ii) Given (9/16) by 6

= (9/16)/6

= (9/ 16 × 6)

= (9/ 96)

= (3/32)

(iii) Given 9 by (3/16)

= 9/ (3/16)

= (9 × 16)/3

= 16 × 3

= 48

(iv) Given 10 by (100/3)

= 10/ (100/3)

= (10 × 3)/100

= (3/10)

4. Simplify:

(i) (3/10) ÷ (10/3)

(ii) 4 3/5 ÷ (4/5)

(iii) 5 4/7 ÷ 1 3/10

(iv) 4 ÷ 2 2/5

Solution:

(i) Given (3/10) ÷ (10/3)

= (3 × 3)/ (10 × 10)

= (9/100)

(ii) Given 4 3/5 ÷ (4/5)

First convert the given mixed fraction into improper fractions

4 3/5 = (23/5)

(23/5) ÷ (4/5) = (23 × 5)/ (5 × 4)

= (23/4)

= 5 3/4

(iii) Given 5 4/7 ÷ 1 3/10

First convert the given mixed fractions into improper fractions

(39/7) and (13/10)

(39/7) ÷ (13/10) = (39 × 10)/ (7 × 13)

= (390/91)

= (30/7)

= 4 2/7

(iv) Given 4 ÷ 2 2/5

First convert the given mixed fraction into improper fraction

2 2/5 = (12/5)

4 ÷ (12/5) = (4 × 5)/12

= (20/12)

= 1 2/3

5. A wire of length 12 1/2 m is cut into 10 pieces of equal length. Find the length of each piece.

Solution:

Given total length of the wire is = 12 1/2 = (25/2) m

It is cut into 10 pieces, so length of one piece is

= (25/2)/10

= (25/20)

= (5/4)

= 1 1/4 m

6. The length of rectangular plot of area 65 1/3 m2 is 12 1/4 m. What is the width of the plot?

Solution:

Given area of rectangular plot is 65 1/3 m2 = (196/3) m2

Length of the same plot is 12 1/4 m = (49/4) m

Width of the plot is

Area = length × breadth

(196/3) = (49/4) × breadth

Breadth = (196/3)/ (49/4)

= (196 × 4)/ (49 × 3)

= (784/147)

= 5 1/3

7. By what number should 6 2/9 be multiplied to get 4 4/9?

Solution:

Let x be the number which needs to be multiplied by 6 2/9 = (56/9)

x × (56/9) = 4 4/9

x × (56/9) = (40/9)

x = (40/9) × (9/56)

x = (40/56)

x = (5/7)

8. The product of two numbers is 25 5/6. If one of the numbers is 6 2/3, find the other.

Solution:

Given product of two numbers is 25 5/6 = (155/6)

One of the number is 6 2/3 = (20/3)

Let the other number be x

(155/6) = x × (20/3)

x = (3/20) × (155/6)

x = (31/8)

x = 3 7/8

9. The cost of 6 1/4 kg of apples is Rs 400. At what rate per kg are the apples being sold?

Solution:

The cost of 6 1/4 kg = (25/4) of apples is Rs 400

Cost of apple per kg is = (25/4) /400

= (4/25) × 400

= Rs 64

10. By selling oranges at the rate of Rs 5 1/4 per orange, a fruit-seller gets Rs 630. How many dozens of oranges does he sell?

Solution:

Given cost of 1 orange is Rs 5 1/4 = (21/4)

He got Rs 630 by selling the oranges

Number of dozens of oranges sold by him for Rs 630 is = (4/21) × 630

= 120 apples

But we know that 1 dozen = 12

120 apples means 10 dozens

11. In mid-day meal scheme (3/10) liter of milk is given to each student of a primary school. If 30 liters of milk is distributed every day in the school, how many students are there in the school?

Solution:

Given (3/10) liter of milk is given to each student

Number of student given (3/10) liter of milk = 1

Number of students giving 1 liter of milk = (10/3)

Numbers of students giving 30 liters of milk = (10/3) × 30 = 100 students

12. In a charity show Rs 6496 were collected by selling some tickets. If the price of each ticket was Rs 50 3/4, how many tickets were sold?

Solution:

Given amount collected by selling tickets = Rs 6496

The price of each ticket is = 50 3/4 = (203/4)

Number of ticket bought at Rs (203/4) = 1

Number of tickets bought for Rs 6496 is = (4/203) × 6496

= 4 × 32

= 128 tickets

Chapter - 3 Decimals

Exercise 3.1

1. Write each of the following as decimals:

(i) (8/100)

(ii) 20 + (9/10) + (4/100)

(iii) 23 + (2/10) + (6/1000)

Solution:

(i) Given (8/100)

Mark the decimal point two places from right to left

(8/100) = 0.08

(ii) Given 20 + (9/10) + (4/100)

First convert the fractions (9/10) and (4/100) to decimals

Consider (9/10)

Mark the decimal point one place from right to left

(9/10) = 0.9

Now consider (4/100)

Mark the decimal point two places from right to left

(4/100) = 0.04

20 + (9/10) + (4/100) = 20 + 0.9 + 0.04

= 20.94

(iii) Given 23 + (2/10) + (6/1000)

First convert the fractions (2/10) and (6/1000) to decimals

Consider (2/10)

Mark the decimal point one place from right to left

(2/10) = 0.2

Now consider (6/1000)

Mark the decimal point three places from right to left

(6/1000) = 0.006

23 + (2/10) + (6/1000) = 23 + 0.2 + 0.006

= 23.206

2. Convert each of the following decimals as fractions:

(i) 0.04

(ii) 2.34

(iii) 0.342

(iv) 17.38

Solution:

(i) Given 0.04

Here we have to convert given decimals into fractions

0.04 can be written as (0.04/1)

Now multiply both numerator and denominator by 100 then we get

(0.04/1) = (0.04 × 100 /1 × 100)

= (4/100)

= (1/25)

(ii) Given 2.34

Here we have to convert given decimals into fractions

2.34 can be written as (2.34/1)

Now multiply both numerator and denominator by 100 then we get

(2.34/1) = (2.34 × 100 /1 × 100)

= (234/100)

= (117/50)

(iii) Given 0.342

Here we have to convert given decimals into fractions

0.342 can be written as (0.342/1)

Now multiply both numerator and denominator by 1000 then we get

(0.342/1) = (0.342 × 1000 /1 × 1000)

= (342/1000)

= (171/500)

(iv) Given 17.38

Here we have to convert given decimals into fractions

17.38 can be written as (17.38/1)

Now multiply both numerator and denominator by 100 then we get

(17.38/1) = (17.38 × 100 /1 × 100)

= (1738/100)

= (869/50)

3. Express the following fractions as decimals:

(i) (23/10)

(ii)

–

(iii)

—

(iv)

—

Solution:

(i) Given (23/10)

Divide 23 by 10 we get

(23/10) = 2.3

(ii) Given

–

can be written as

= 25 + (1/8)

Consider (1/8),

Now multiply both numerator and denominator by 125 to get 1000 as denominator

= 25 + (1/8) = 25 + (1 × 125 /8 × 125)

= 25 + (125/1000)

= 25 + 0.125

= 25.125

(iii) Given

First convert given mixed fraction

into improper fraction

= 1372/35

By dividing we get

= 39.2

(iv) Given

can be written as

= 15 + (1/25)

Consider (1/25),

Now multiply both numerator and denominator by 4 to get 100 as denominator

= 15 + (1/25) = 15 + (1 × 4 /25 × 4)

= 15 + (4/100)

= 15 + 0.04

= 15.04

4. Add the following:

(i) 41.8, 39.24, 5.01 and 62.6

(ii) 18.03, 146.3, .829 and 5.324

Solution:

(i) Given 41.8, 39.24, 5.01 and 62.6

41.8

39.24

5.01

+ 62.6

148.65

(ii) Given 18.03, 146.3, 0.829 and 5.324

18.03

146.3

0.829

+ 5.324

170.483

5. Find the value of:

(i) 9.756 – 6.28

(ii) 48.1 – 0.37

(iii) 108.032 – 86.8

(iv) 100 – 26.32

Solution:

(i) Given 9.756 – 6.28

9.756

– 6.28

3.476

(ii) Given 48.1 – 0.37

48.1

– 0.37

47.73

(iii) Given 108.032 – 86.8

108.032

– 86.8

21.232

(iv) Given 100 – 26.32

100.00

– 26.32

73.68

6. Take out of 3.547 from 7.2

Solution:

Given 3.547 from 7.2

7.2

– 3.547

3.653

7. What is to be added to 36.85 to get 59.41?

Solution:

Given 36.85 and 59.41

Let the unknown number be x

x + 36.85 = 59.41

x = 59.41 – 36.85

x = 22.56

Hence 22.56 is to be added to 36.85 to get 59.41

8. What is to be subtracted from 17.1 to get 2.051?

Solution:

Let the unknown number be x

Given that x is to be subtracted from 17.1 to get 2.051

17.1 – x = 2.051

17.1 – 2.051 = x

x = 17.1 – 2.051

x = 15.049

9. By how much should 34.79 be increased to get 70.15?

Solution:

Let x be the unknown number

x + 34.79 = 70.15

x = 70.15 – 34.79

x = 35.36

35.36 should be increased to 34.79 to get 70.15

10. By how much should 59.71 be decreased to get 34.58?

Solution:

Let x be the unknown number

59.71 – x =34.58

59.71 – 34.58 = x

x = 59.71 – 34.58

x = 25.13

25.13 should be decreased by 59.71 to get 34.58

Exercise 3.2

1. Find the product:

(i) 4.74 × 10

(ii) 0.45 × 10

(iii) 0.0215 × 10

(iv) 0.0054 × 10

Solution:

(i) Given 4.74 × 10

Here we have to do normal multiplication with shifting the decimal point by one place to the right

Therefore 4.74 × 10 = 47.4

(ii) Given 0.45 × 10

Here we have to do normal multiplication with shifting the decimal point by one place to the right

Therefore 0.45 × 10 = 4.5

(iii) Given 0.0215 × 10

Here we have to do normal multiplication with shifting the decimal point by one place to the right

Therefore 0.0215 × 10 = 0.215

(iv) Given 0.0054 × 10

Here we have to do normal multiplication with shifting the decimal point by one place to the right

Therefore 0.0054 × 10 = 0.054

2. Find the product:

(i) 35.853 × 100

(ii) 42.5 × 100

(iii) 12.075 × 100

(iv) 100 × 0.005

Solution:

(i) Given 35.853 × 100

Here we have to do normal multiplication with shifting the decimal point by two places to the right

Therefore 35.853 × 100 = 3585.3

(ii) Given 42.5 × 100

Here we have to do normal multiplication with shifting the decimal point by two places to the right

Therefore 42.5 × 100 = 4250

(iii) Given 12.075 × 100

Here we have to do normal multiplication with shifting the decimal point by two places to the right

Therefore 12.075 × 100 = 1207.50

(iv) Given 100 × 0.005

Here we have to do normal multiplication with shifting the decimal point by two places to the right

Therefore 100 × 0.005 = 0.5

3. Find the product:

(i) 2.506 × 1000

(ii) 20.708 × 1000

(iii) 0.0529 × 1000

(iv) 1000 × 0.1

Solution:

(i) Given 2.506 × 1000

Here we have to do normal multiplication with shifting the decimal point by three places to the right

Therefore 2.506 × 1000 = 2506

(ii) Given 20.708 × 1000

Here we have to do normal multiplication with shifting the decimal point by three places to the right

Therefore 20.708 × 1000 = 20708

(iii) Given 0.0529 × 1000

Here we have to do normal multiplication with shifting the decimal point by three places to the right

Therefore 0.0529 × 1000 = 52.9

(iv) Given 1000 × 0.1

Here we have to do normal multiplication with shifting the decimal point by three places to the right

Therefore 1000 × 0.1 = 100

4. Find the product:

(i) 3.14 × 17

(ii) 0.745 × 12

(iii) 28.73 × 47

(iv) 0.0415 × 59

Solution:

(i) Given 3.14 × 17

First multiply as usual without looking at the decimal point

3.14 × 17 = 578

Now mark the decimal point in the product to have one place of decimal as there in the given decimal

3.14 × 17 = 57.8

(ii) Given 0.745 × 12

First multiply as usual without looking at the decimal point

0.745 × 12 = 894

Now mark the decimal point in the product to have three places of decimal as there in the given decimal

0.745 × 12 = 8.94

(iii) Given 28.73 × 47

First multiply as usual without looking at the decimal point

28.73 × 47 = 135031

Now mark the decimal point in the product to have two places of decimal as there in the given decimal

28.73 × 47 = 1350.31

(iv) Given 0.0415 × 59

First multiply as usual without looking at the decimal point

0.0415 × 59 = 24485

Now mark the decimal point in the product to have four places of decimal as there in the given decimal

0.0415 × 59 = 2.4485

5. Find:

(i) 1.07 × 0.02

(ii) 211.9 × 1.13

(iii) 10.05 × 1.05

(iv) 13.01 × 5.01

Solution:

(i) Given 1.07 × 0.02

First multiply as usual without looking at the decimal point

1.07 × 0.02 = 00214

Sum of the decimals is 4

Now mark the decimal point in the product to have four places of decimal as there in the given decimal

1.07 × 0.02 = 0.0214

(ii) Given 211.9 × 1.13

First multiply as usual without looking at the decimal point

211.9 × 1.13 = 239447

Sum of the decimals is 3

Now mark the decimal point in the product to have three places of decimal as there in the given decimal

211.9 × 1.13 = 239.447

(iii) Given 10.05 × 1.05

First multiply as usual without looking at the decimal point

10.05 × 1.05 = 105525

Sum of the decimals is 4

Now mark the decimal point in the product to have four places of decimal as there in the given decimal

10.05 × 1.05 = 10.5525

(iv) Given 13.01 × 5.01

First multiply as usual without looking at the decimal point

13.01 × 5.01 = 651801

Sum of the decimals is 4

Now mark the decimal point in the product to have four places of decimal as there in the given decimal

13.01 × 5.01 = 65.1801

6. Find the area of a rectangle whose length is 5.5m and breadth is 3.4m.

Solution:

Given length of rectangle = 5.5m

Breadth of rectangle = 3.4 m

Area of rectangle = length × breadth

= 5.5 × 3.4

= 18.7 m^2

7. If the cost of a book is Rs 25.57, find the cost 0f 24 such books.

Solution:

Given cost of a book is Rs 25.57

Cost of 24 books = 25.57 × 24

= Rs 618.00

8. A car covers a distance of 14.75km in one liter of petrol. How much distance it will cover in 15.5 liters of petrol?

Solution:

Given that distance covered by car in 1 liter of petrol = 14.75 km

Distance covered by car in 15.5 liters of petrol = 14.75 × 15.5

= 228.625 km

9. One kg of rice costs Rs 42.65. What will be the cost of 18.25 kg of rice?

Solution:

Given cost of 1kg of rice = 42.65

Cost of 18.25kg of rice = 42.65 × 18.25

= Rs 778.3625

10. One meter of cloth costs Rs 152.50. What is the cost of 10.75 meters of cloth?

Solution:

Given that cost of 1m cloth = Rs 152.50

Cost of 10.75 m of cloth = 152.50 × 10.75

= Rs 1639.375

Exercise 13.3

Question: 1

Divide:

(i) 142.45 by 10

(ii) 54.25 by 10

(iii) 3.45 by 10

(iv) 0.57 by 10

(v) 0.043 by 10

(vi) 0.004 by 10

Solution:

(i) 142.45 by 10

Shifting the decimal point by one place to the left

142.45/10

= 14.245

(ii) 54.25 by 10

Shifting the decimal point by one place to the left

54.25/10

= 5.425

(iii) 3.45 by 10

Shifting the decimal point by one place to the left

3.45/10

0.345

(iv) 0.57 by 10

Shifting the decimal point by one place to the left

0.57/10

= 0.057

(v) 0.043 by 10

Shifting the decimal point by one place to the left

0.043/10

= 0.0043

(vi) 0.004 by 10

Shifting the decimal point by one place to the left

0.004/10

= 0.0004

Question: 2

Divide:

(i) 459.5 by 100

(ii) 74.3 by 100

(iii) 5.8 by 100

(iv) 0.7 by 100

(v) 0.48 by 100

(vi) 0.03 by 100

Solution:

(i) 459.5 by 100

Shifting the decimal point by two places to the left

459.5/100

= 4.595

(ii) 74.3 by 100

Shifting the decimal point by two places to the left

74.3/100

= 0.743

(iii) 5.8 by 100

Shifting the decimal point by two places to the left

5.8/100

= 0.058

(iv) 0.7 by 100

Shifting the decimal point by two places to the left

0.7/100

= 0.007

(v) 0.48 by 100

Shifting the decimal point by two places to the left

0.48/100

= 0.0048

(vi) 0.03 by 100

Shifting the decimal point by two places to the left

0.03/100

= 0.0003

Question: 3

Divide:

(i) 235.41 by 1000

(ii) 29.5 by 1000

(iii) 3.8 by 1000

(iv) 0.7 by 1000

Solution:

(i) 235.41 by 1000

Shifting the decimal point by three places to the left

235.41/1000

= 0.23541

(ii) 29.5 by 1000

Shifting the decimal point by three places to the left

29.51/000

= 0.0295

(iii) 3.8 by 1000

Shifting the decimal point by three places to the left

3.8/1000

= 0.0038

(iv) 0.7 by 1000

Shifting the decimal point by three places to the left

0.7/1000

= 0.007

Question: 4

Divide:

(i) 0.45 by 9

(ii) 217.44 by 18

(iii) 319.2 by 2.28

(iv) 40.32 by 9.6

(v) 0.765 by 0.9

(vi) 0.768 by 1.6

Solution:

(i) 0.45 by 9

= 0.45/9

= 0.05

(ii) 217.44 by 18

= 217.44/18

= 12.08

(iii) 319.2 by 2.28

= 319.2/2.28

$$= \frac{319.2 \times 100}{2.28 \times 100}$$

$$= \frac{31920}{228}$$

$$= 140$$

$$= \frac{319.2}{2.28} = 140$$

(iv) 40.32 by 9.6

= 40.32/9.6

$$= \frac{40.32 \times 10}{9.6 \times 10}$$

= 403.2/96

= 4.2

= 403.2/96 = 4.2

(v) 0.765 by 0.9

= 0.765/0.9

$$= \frac{0.765 \times 10}{0.9 \times 10}$$

= 7.65/9

= 0.85

= 7.65/0.9 = 0.85

(vi) 0.768 by 1.6

= 0.768/1.6

$$= \frac{0.768 \times 10}{1.6 \times 10}$$

= 7.68/16

= 0.48

Question: 5

Divide:

(i) 16.64 by 20

(ii) 0.192 by 12

(iii) 163.44 by 24

(iv) 403.2 by 96

(v) 16.344 by 12

(vi) 31.92 by 228

Solution:

(i) 16.64 by 20

= 16.64/20

$$= \frac{16.64}{2 \times 10}$$

$$= \frac{16.64}{10} \times \frac{1}{2}$$

= 1.664/2

= 0.832

(ii) 0.192 by 12

= 0.192/12

= 0.016

(iii) 163.44 by 24

= 163.44/24

= 6.81

(iv) 403.2 by 96

= 403.2/96

= 4.2

(v) 16.344 by 12

= 16.344/12

= 1.362

(vi) 31.92 by 228

= 31.92/228

= 0.14

Question: 6

Divide:

(i) 15.68 by 20

(ii) 164.6 by 200

(iii) 403.80 by 30

Solution:

(i) 15.68 by 20

=15.68/20

$$= \frac{15.68}{2 \times 10}$$

$$= \frac{15.68}{10} \times \frac{1}{2}$$

=1.568/2

= 0.784

(ii) 164.6 by 200

Question: 7

(i) 76 by 0.019

(ii) 88 by 0.08

(iii) 148 by 0.074

(iv) 7 by 0.014

(iii) 403.80 by 30

Solution:

(i) 76 by 0.019

= 76/0.019

$$= \frac{76 \times 1000}{0.019 \times 1000}$$

= 76000/19

= 4000

(ii) 88 by 0.08

= 88/0.08

$$= \frac{88 \times 100}{0.08 \times 100}$$

= 8800/8

=1100

(iii) 148 by 0.074

= 148/0.074

$$= \frac{148 \times 1000}{0.074 \times 1000}$$

= 148000/74

= 2000

(iv) 7 by 0.014

= 70.014

$$= \frac{7 \times 1000}{0.014 \times 1000}$$

= 7000/14

$$= \frac{164.6}{200}$$

$$= \frac{164.6}{2 \times 100}$$

$$= \frac{164.6}{100} \times \frac{1}{2}$$

$$= \frac{1.646}{2}$$

= 0.823

(iii) 403.80 by 30

= 403.80/30

$$= \frac{403.80}{3 \times 10}$$

$$= \frac{403.80}{10} \times \frac{1}{3}$$

$$= \frac{40.380}{3}$$

$$= 13.46$$

Question: 8

Divide:

(i) 20 by 50

(ii) 8 by 100

(iii) 72 by 576

(iv) 144 by 15

Solution:

(i) 20 by 50

= 20/50

= 0.4

(ii) 8 by 100

= 8/100

By shifting the decimal point to the left

= 8/100

= 0.08

(iii) 72 by 576

= 72/576

= 0.125

(iv)144 by 15

= 144/15

= 9.6

Question: 9

A vehicle covers a distance of 43.2 km in 2.4 litres of petrol. How much distance will it travel in 1 litre of petrol?

Solution:

Distance covered in 2.4 litres of petrol = 43.2 km

Distance covered in 1 litre of petrol = 43.2/2.4

= 18km

The distance travelled in 1 litre of petrol is 18 km

Question: 10

The total weight of some bags of wheat is 1743 kg. If each bag weights 49.8 kg, how many bags are there?

Solution:

Total weight of bags of wheat = 1743 kg

Each bag weight = 49.8 kg

No of bags = 1743/49.8

$$= \frac{1743 \times 10}{49.8 \times 10}$$

$$= \frac{17430}{498}$$

= 35

Therefore the total numbers of bags are 35

Question: 11

Shikha cuts 50 m of cloth into pieces of 1.25 m each. How many pieces does she get?

Solution:

Total length of cloth = 50 m

Length of each piece of cloth = 1.25 m

Number of pieces = 50/1.25

$$= \frac{50 \times 100}{1.25 \times 100}$$

= 5000/125

= 40 pieces

Therefore Shikha got 40 pieces

Question: 12

Each side of a rectangular polygon is 2.5 cm in length. The perimeter of the polygon is 12.5 cm. How many sides does the polygon have?

Solution:

Length of each side of rectangular polygon = 5.2 cm

Perimeter of polygon = 12.5 cm

No of sides polygon has = 12.5 cm

No of sides polygon have = 12.5/2.5

$$= \frac{12.5 \times 10}{2.5 \times 10}$$

= 5

Therefore the sides of the polygon is 5

Question: 13

The product of two decimals is 42.987. If one of them is 12.46, find the other.

Solution:
We have,

The product of the given decimals = 42.987

one decimal = 12.46

The other decimal = 42.987/12.46

= 3.45

The number is 3.45

Question: 14
The weight of 34 bags of sugar is 3483.3 kg. If all bags weigh equally, find the weight of each bag.

Solution:
Total weight of sugar = 3483.3kg

No of bags = 34

Weight of each bag = 3483.3/34

= 102.45 kg

Therefore weight of each bag is 102.45 kg

Question: 15
How many buckets of equal capacity can be filled from 586.5 litres of water, if each bucket has capacity of 8.5 litres?

Solution:
Capacity of each bucket = 805 litres

Total water available = 586.5 litres

Number of buckets = 805/586.5

$$= \frac{805 \times 10}{586.5 \times 10}$$

= 69

Total number of buckets is 69

Chapter 4 – Rational Numbers

Exercise 4.1

1. Write down the numerator of each of the following rational numbers:

(i) (-7/5)

(ii) (14/-4)

(iii) (-17/-21)

(iv) (8/9)

(v) 5

Solution:

(i) Given (-7/5)

Numerator of (-7/5) is -7

(ii) Given (14/-4)

Numerator of (14/-4) is 1

(iii) Given (-17/-21)

Numerator of (-17/-21) is -17

(iv) Given (8/9)

Numerator of (8/9) is 8

(v) Given 5

Numerator of 5 is 5

2. Write down the denominator of each of the following rational numbers:

(i) (-4/5)

(ii) (11/-34)

(iii) (-15/-82)

(iv) 15

(v) 0

Solution:

(i) Given (-4/5)

Denominator of (-4/5) is 5

(ii) Given (11/-34)

Denominator of (11/-34) is -43

(iii) Given (-15/-82)

Denominator of (15/-82) is -82

(iv) Given 15

Denominator of 15 is 1

(v) Given 0

Denominator of 0 is any non-zero integer

3. Write down the rational number whose numerator is (-3) × 4, and whose denominator is (34 – 23) × (7 – 4).

Solution:

Given numerator = (-3) × 4 = -12

Denominator = (34 – 23) × (7 – 4)

= 11 × 3 = 33

Therefore the rational number = (-12/33)

4. Write down the rational numbers as integers: (7/1), (-12/1), (34/1), (-73/1), (95/1)

Solution:

Given (7/1), (-12/1), (34/1), (-73/1), (95/1)

Integers of (7/1), (-12/1), (34/1), (-73/1), (95/1) are 7, -12, 34, -73, 95

5. Write the following integers as rational numbers: -15, 17, 85, -100

Solution:

Given -15, 17, 85, -100

The rational numbers of given integers are (-15/1), (17/1), (85/1) and (-100/1)

6. Write down the rational number whose numerator is the smallest three digit number and denominator is the largest four digit number.

Solution:

Smallest three digit number = 100

Largest four digit number = 9999

Therefore the rational number is = 100/9999

7. Separate positive and negative rational numbers from the following rational numbers:

(-5/-7), (12/-5), (7/4), (13/-9), 0, (-18/-7), (-95/116), (-1/-9)

Solution:

Given (-5/-7), (12/-5), (7/4), (13/-9), 0, (-18/-7), (-95/116), (-1/-9)

A rational number is said to be positive if its numerator and denominator are either positive integers or both negative integers.

Therefore positive rational numbers are: (-5/-7), (-18/-7), (7/4), (-1/-9)

A rational number is said to be negative integers if its numerator and denominator are such that one of them is positive integer and another one is a negative integer.

Therefore negative rational numbers are: (12/-5), (13/-9), (-95/116)

8. Which of the following rational numbers are positive:

(i) (-8/7)

(ii) (9/8)

(iii) (-19/-13)

(iv) (-21/13)

Solution:

Given (-8/7), (9/8), (-19/-13), (-21/13)

A rational number is said to be positive if its numerator and denominator are either positive integers or both negative integers.

Therefore the positive rational numbers are (9/8) and (-19/-13)

9. Which of the following rational numbers are negative:

(i) (-3/7)

(ii) (-5/-8)

(iii) (9/-83)

(iv) (-115/-197)

Solution:

Given (-3/7), (-5/-8), (9/-83), (-115/-197)

A rational number is said to be negative integers if its numerator and denominator are such that one of them is positive integer and another one is a negative integer.

Therefore negative rational numbers are (-3/7) and (9/-83)

Exercise 4.2

1. Express each of the following as a rational number with positive denominator.

(i) (-15/-28)

(ii) (6/-9)

(iii) (-28/-11)

(iv) (19/-7)

Solution:

(i) Given (-15/-28)

Multiplying both numerator and denominator we can rational number with positive denominator.

(-15/-28) = (-15/-28) × (-1/-1)

= (15/28)

(ii) Given (6/-9)

Multiplying both numerator and denominator we can rational number with positive denominator.

(6/-9) = (6/-9) × (-1/-1)

= (-6/9)

(iii) Given (-28/-11)

Multiplying both numerator and denominator we can rational number with positive denominator.

(-28/-11) = (-28/-11) × (-1/-1)

= (28/11)

(iv) Given (19/-7)

Multiplying both numerator and denominator we can rational number with positive denominator.

(19/-7) = (19/-7) × (-1/-1)

= (-19/7)

2. Express (3/5) as a rational number with numerator:

(i) 6

(ii) -15

(iii) 21

(iv) -27

Solution:

(i) Given (3/5)

To get numerator 6 we have to multiply both numerator and denominator by 2

Then we get, (3/5) × (2/2) = (6/10)

Therefore (3/5) as a rational number with numerator 6 is (6/10)

(ii) Given (3/5)

To get numerator -15 we have to multiply both numerator and denominator by -5

Then we get, (3/5) × (-5/-5)

= (-15/-25)

Therefore (3/5) as a rational number with numerator -15 is (-15/-25)

(iii) Given (3/5)

To get numerator 21 we have to multiply both numerator and denominator by 7

Then we get, (3/5) × (7/7)

= (21/35)

Therefore (3/5) as a rational number with numerator 21 is (21/35)

(iv) Given (3/5)

To get numerator -27 we have to multiply both numerator and denominator by -9

Then we get, (3/5) × (-9/-9)

= (-27/-45)

Therefore (3/5) as a rational number with numerator -27 is (-27/-45)

3. Express (5/7) as a rational number with denominator:

(i) -14

(ii) 70

(iii) -28

(iv) -84

Solution:

(i) Given (5/7)

To get denominator -14 we have to multiply both numerator and denominator by -2

Then we get, (5/7) × (-2/-2)

= (-10/-14)

Therefore (5/7) as a rational number with denominator -14 is (-10/-14)

(ii) Given (5/7)

To get denominator 70 we have to multiply both numerator and denominator by -2

Then we get, (5/7) × (10/10)

= (50/70)

Therefore (5/7) as a rational number with denominator 70 is (50/70)

(iii) Given (5/7)

To get denominator -28 we have to multiply both numerator and denominator by -4

Then we get, (5/7) × (-4/-4)

= (-20/-28)

Therefore (5/7) as a rational number with denominator -28 is (-20/-28)

(iv) Given (5/7)

To get denominator -84 we have to multiply both numerator and denominator by -12

Then we get, (5/7) × (-12/-12)

= (-60/-84)

Therefore (5/7) as a rational number with denominator -84 is (-60/-84)

4. Express (3/4) as a rational number with denominator:

(i) 20

(ii) 36

(iii) 44

(iv) -80

Solution:

(i) Given (3/4)

To get denominator 20 we have to multiply both numerator and denominator by 5

Then we get, (3/4) × (5/5)

= (15/20)

Therefore (3/4) as a rational number with denominator 20 is (15/20)

(ii) Given (3/4)

To get denominator 36 we have to multiply both numerator and denominator by 9

Then we get, (3/4) × (9/9)

= (27/36)

Therefore (3/4) as a rational number with denominator 36 is (27/36)

(iii) Given (3/4)

To get denominator 44 we have to multiply both numerator and denominator by 11

Then we get, (3/4) × (11/11)

= (33/44)

Therefore (3/4) as a rational number with denominator 44 is (33/44)

(iv) Given (3/4)

To get denominator -80 we have to multiply both numerator and denominator by -20

Then we get, (3/4) × (-20/-20)

= (-60/-80)

Therefore (3/4) as a rational number with denominator -80 is (-60/-80)

5. Express (2/5) as a rational number with numerator:

(i) -56

(ii) 154

(iii) -750

(iv) 500

Solution:

(i) Given (2/5)

To get numerator -56 we have to multiply both numerator and denominator by -28

Then we get, (2/5) × (-28/-28)

= (-56/-140)

Therefore (2/5) as a rational number with numerator -56 is (-56/-140)

(ii) Given (2/5)

To get numerator 154 we have to multiply both numerator and denominator by 77

Then we get, (2/5) × (77/77)

= (154/385)

Therefore (2/5) as a rational number with numerator 154 is (154/385)

(iii) Given (2/5)

To get numerator -750 we have to multiply both numerator and denominator by -375

Then we get, (2/5) × (-375/-375)

= (-750/-1875)

Therefore (2/5) as a rational number with numerator -750 is (-750/-1875)

(iv) Given (2/5)

To get numerator 500 we have to multiply both numerator and denominator by 250

Then we get, (2/5) × (250/250)

= (500/1250)

Therefore (2/5) as a rational number with numerator 500 is (500/1250)

6. Express (-192/108) as a rational number with numerator:

(i) 64

(ii) -16

(iii) 32

(iv) -48

Solution:

(i) Given (-192/108)

To get numerator 64 we have to divide both numerator and denominator by -3

Then we get, (-192/108) ÷ (-3/-3)

= (64/-36)

Therefore (-192/108) as a rational number with numerator 64 is (64/-36)

(ii) Given (-192/108)

To get numerator -16 we have to divide both numerator and denominator by 12

Then we get, (-192/108) ÷ (12/12)

= (-16/9)

Therefore (-192/108) as a rational number with numerator -16 is (-16/9)

(iii)) Given (-192/108)

To get numerator 32 we have to divide both numerator and denominator by -6

Then we get, (-192/108) ÷ (-6/-6)

= (32/-18)

Therefore (-192/108) as a rational number with numerator 32 is (32/-18)

(iv) Given (-192/108)

To get numerator -48 we have to divide both numerator and denominator by 4

Then we get, (-192/108) ÷ (4/4)

= (-48/27)

Therefore (-192/108) as a rational number with numerator -48 is (-48/27)

7. Express (169/-294) as a rational number with denominator:

(i) 14

(ii) -7

(iii) -49

(iv) 1470

Solution:

(i) Given (169/-294)

To get denominator 14 we have to divide both numerator and denominator by -21

Then we get, (169/-294) ÷ (-21/-21)

= (-8/14)

Therefore (169/-294) as a rational number with denominator 14 is (-8/14)

(ii) Given (169/-294)

To get denominator -7 we have to divide both numerator and denominator by 42

Then we get, (169/-294) ÷ (42/42)

= (4/-7)

Therefore (169/-294) as a rational number with denominator -7 is (4/-7)

(iii) Given (169/-294)

To get denominator -49 we have to divide both numerator and denominator by 6

Then we get, (169/-294) ÷ (6/6)

= (28/-49)

Therefore (169/-294) as a rational number with denominator -49 is (28/-49)

(iv) Given (169/-294)

To get denominator 1470 we have to multiply both numerator and denominator by -5

Then we get, (169/-294) × (-5/-5)

= (-840/1470)

Therefore (169/-294) as a rational number with denominator 1470 is (-840/1470)

8. Write (-14/42) in a form so that the numerator is equal to:

(i) -2

(ii) 7

(iii) 42

(iv) -70

Solution:

(i) Given (-14/42)

To get numerator -2 we have to divide both numerator and denominator by 7

Then we get, (-14/42) ÷ (7/7)

= (-2/6)

Therefore (-14/42) as a rational number with numerator -2 is (-2/6)

(ii) Given (-14/42)

To get numerator 7 we have to divide both numerator and denominator by -2

Then we get, (-14/42) ÷ (-2/-2)

= (7/-21)

Therefore (-14/42) as a rational number with numerator -14 is (-14/21)

(iii) Given (-14/42)

To get numerator 42 we have to multiply both numerator and denominator by -3

Then we get, (-14/42) × (-3/-3)

= (42/-126)

Therefore (-14/42) as a rational number with numerator 42 is (42/-126)

(iv) Given (-14/42)

To get numerator -70 we have to multiply both numerator and denominator by 5

Then we get, (-14/42) × (5/5)

= (-70/210)

Therefore (-14/42) as a rational number with numerator -70 is (-70/210)

9. Select those rational numbers which can be written as a rational number with numerator 6:

(1/22), (2/3), (3/4), (4/-5), (5/6), (-6/7), (-7/8)

Solution:

Given rational numbers that can be written as a rational number with numerator 6 are:

Consider (1/22)

On multiplying by 6, (1/22) can be written as

(1/22) = (6/132)

Consider (2/3)

On multiplying by 3, (2/3) can be written as

(2/3) = (6/9)

Consider (3/4)

On multiplying by 2, (3/4) can be written as

(3/4) = (6/8)

Consider (-6/7)

On multiplying by -1, (-6/7) can be written as

(-6/7) = (6/-7)

Therefore rational numbers that can be written as a rational number with numerator 6 are (1/22), (2/3), (3/4) and (-6/7)

10. Select those rational numbers which can be written as rational number with denominator 4:

(7/8), (64/16), (36/-12), (-16/17), (5/-4), (140/28)

Solution:

Given rational numbers that can be written as a rational number with denominator 4 are:

(7/8) = (3.5/4) (On dividing both denominator and denominator by 2)

(64/16) = (16/4) (On dividing both denominator and numerator by 4)

(36/-12) = (-12/4) (On dividing both denominator and numerator by -3)

(5/- 4) = (- 5/4) (On multiplying both denominator and numerator by -1)

(140/28) = (20/4) (On dividing both numerator and denominator by 7)

11. In each of the following, find an equivalent form of the rational number having a common denominator:

(i) (3/4) and (5/12)

(ii) (2/3), (7/6) and (11/12)

(iii) (5/7), (3/8), (9/14) and (20/21)

Solution:

(i) Given (3/4) and (5/12)

On multiplying both numerator and denominator by 3

(3/4) = (3 × 3)/ (4 × 3) = (9/12)

Equivalent forms with same denominators are (9/12) and (5/12)

(ii) Given (2/3), (7/6) and (11/12)

On multiplying both numerator and denominator by 4

(2/3) = (2 × 4)/ (3 × 4) = (8/12)

And (7/6) = (7 × 2)/ (6 × 2) = (14/12)

Equivalent forms are (8/12), (14/12) and (11/12) having same denominators

(iii) Given (5/7), (3/8), (9/14) and (20/21)

(5/7) = (5 × 24)/ (7 × 24) = (120/168) [on multiplying both numerator and denominator by 24]

(3/8) = (3 × 21)/ (8 × 21) = (63/168) [on multiplying both numerator and denominator by 21]

(9/14) = (9 × 12)/ (14 × 12) = (108/168) [on multiplying both numerator and denominator by 12]

(20/21) = (20 × 8)/ (21×8) = (160/168) [on multiplying both numerator and denominator by 8]

Forms are (120/168), (63/168), (108/168) and (160/168) having same denominators.

Exercise 4.3

1. Determine whether the following rational numbers are in the lowest form or not:

(i) (65/84)

(ii) (-15/32)

(iii) (24/128)

(iv) (-56/-32)

Solution:

(i) Given (65/84)

Here we can observe that 65 and 84 have no common factor their HCF is 1.

Thus, (65/84) is in its lowest form.

(ii) Given (-15/32)

Here we can observe that -15 and 32 have no common factor i.e., their HCF is 1.

Thus, (-15/32) is in its lowest form.

(iii) Given (24/128)

Here we can observe that HCF of 24 and 128 is not 1.

Thus, given rational number is not in its simplest form.

(iv) Given (-56/-32)

Here we can observe that HCF of 56 and 32 is 8 and also not equal to 1.

Therefore the given rational number is not in its simplest form.

2. Express each of the following rational numbers to the lowest form:

(i) (4/22)

(ii) (-36/180)

(iii) (132/-428)

(iv) (-32/-56)

Solution:

(i) Given (4/22)

We know that HCF of 4 and 22 is 2

By dividing the given number by its HCF we get

(4 ÷ 2/22 ÷ 2) = (2/11)

Therefore (2/11) is the simplest form of the given number

(ii) Given (-36/180)

We know that HCF of 36 and 180 is 36

By dividing the given number by its HCF we get

(-36 ÷ 36/180 ÷ 36) = (-1/5)

Therefore (-1/5) is the simplest form of the given number

(iii) Given (132/-428)

We know that HCF of 132 and 428 is 4

By dividing the given number by its HCF we get

(132 ÷ 4/-428 ÷ 4) = (33/-107)

Therefore (33/-107) is the simplest form of the given number

(iv) Given (-32/-56)

We know that HCF of 32 and 56 is 8

By dividing the given number by its HCF we get

(-32 ÷ 8/-56 ÷ 8) = (4/7)

Therefore (4/7) is the simplest form of the given number

3. Fill in the blanks:

(i) (-5/7) = (…/35) = (…/49)

(ii) (-4/-9) = (…/18) = (12/…)

(iii) (6/-13) = (-12/…) = (24/…)

(iv) (-6/…) = (3/11) = (…/-55)

Solution:

(i) (-5/7) = (-25/35) = (-35/49)

Explanation:

Given (-5/7) = (…/35) = (…/49)

Here (-5/7) × (5/5) = (-25/35)

And also (-5/7) × (7/7) = (-35/49)

(ii) (-4/-9) = (8/18) = (12/27)

Explanation:

Given (-4/-9) = (…/18) = (12/…)

On multiplying by -2 we get

(-4/-9) × (-2/-2) = (8/18)

Also on multiplying by -3

(-4/-9) × (-3/-3) = (12/27)

(iii) (6/-13) = (-12/26) = (24/-52)

Explanation:

Given (6/-13) = (-12/…) = (24/…)

On multiplying by -2

(6/-13) × (-2/-2) = (-12/26)

Also multiplying by 4

And also (6/-13) × (4/4) = (24/-52)

(iv) (-6/-22) = (3/11) = (-15/-55)

Explanation:

Given (-6/…) = (3/11) = (…/-55)

0n multiplying by -2

(3/11) × (-2/-2) = (-6/-22)

And also on multiplying by -5

(3/11) × (-5/-5) = (-15/-55)

Exercise 4.4

1. Write each of the following rational numbers in the standard form:

(i) (2/10)

(ii) (-8/36)

(iii) (4/-16)

(iv) (-15/-35)

(v) (299/-161)

(vi) (-63/-210)

(vii) (68/-119)

(viii) (-195/275)

Solution:

(i) Given (2/10)

We know that HCF of 2 and 10 is 2

Now dividing the numerator and denominator by HCF i.e. 2, we get:

(2/10) ÷ (2/2) = (1/5)

Therefore (1/5) is the standard form of given number

(ii) Given (-8/36)

We know that HCF of 8 and 36 is 4

Now dividing the numerator and denominator by HCF i.e. 4, we get:

(-8/36) ÷ (4/4) = (-2/9)

Therefore (-2/9) is the standard form of given number

(iii) Given (4/-16)

Here denominator is negative so we have multiply both numerator and denominator by -1

(4/-16) × (-1/-1) = (-4/16)

We know that HCF of 4 and 16 is 4

Now dividing the numerator and denominator by HCF i.e. 4, we get:

(-4/16) ÷ (4/4) = (-1/4)

Therefore (-1/4) is the standard form of given number

(iv) Given (-15/-35)

Here denominator is negative so we have multiply both numerator and denominator by -1

(-15/-35) × (-1/-1) = (15/35)

We know that HCF of 15 and 35 is 4

Now dividing the numerator and denominator by HCF i.e. 5, we get:

(15/35) ÷ (5/5) = (3/7)

Therefore (3/7) is the standard form of given number

(v) Given (299/-161)

Here denominator is negative so we have multiply both numerator and denominator by -1

(299/-161) × (-1/-1) = (-299/161)

The HCF of 299 and 161 is 23

Now dividing the numerator and denominator by HCF i.e. 23, we get:

(-299/161) ÷ (23/23) = (-13/7)

Therefore (-13/7) is the standard form of given number

(vi) Given (-63/-210)

The HCF of 63 and 210 is 21

Now dividing the numerator and denominator by HCF i.e. 21, we get:

(-63/-210) ÷ (21/21) = (-3/-10) = (3/10)

Therefore (3/10) is the standard form of given number

(vii) Given (68/-119)

Here denominator is negative so we have multiply both numerator and denominator by -1

(68/-119) × (-1/-1) = (-68/119)

The HCF of 68 and 119 is 17

Now dividing the numerator and denominator by HCF i.e. 17, we get:

(-68/119) ÷ (17/17) = (-4/7)

Therefore (-4/7) is the standard form of given number

(viii) Given (-195/275)

The HCF of 195 and 257 is 5

Now dividing the numerator and denominator by HCF i.e. 5, we get:

(-165/275) ÷ (5/5) = (-39/55)

Therefore (-39/55) is the standard form of given number

Exercise 4.5

1. Which of the following rational numbers are equal?

(i) (-9/12) and (8/-12)

(ii) (-16/20) and (20/-25)

(iii) (-7/21) and (3/-9)

(iv) (-8/-14) and (13/21)

Solution:

(i) Given (-9/12) and (8/-12)

The standard form of (-9/12) is (-3/4) [on diving the numerator and denominator of given number by their HCF i.e. by 3]

The standard form of (8/-12) = (-2/3) [on diving the numerator and denominator of given number by their HCF i.e. by 4]

Since, the standard forms of two rational numbers are not same. Hence, they are not equal.

(ii) Given (-16/20) and (20/-25)

Multiplying numerator and denominator of (-16/20) by the denominator of (20/-25)

i.e. -25.

(-16/20) × (-25/-25) = (400/-500)

Now multiply the numerator and denominator of (20/-25) by the denominator of

(-16/20) i.e. 20

(20/-25) × (20/20) = (400/-500)

Clearly, the numerators of the above obtained rational numbers are equal.

Hence, the given rational numbers are equal

(iii) Given (-7/21) and (3/-9)

Multiplying numerator and denominator of (-7/21) by the denominator of (3/-9)

i.e. -9.

(-7/21) × (-9/-9) = (63/-189)

Now multiply the numerator and denominator of (3/-9) by the denominator of

(-7/21) i.e. 21

(3/-9) × (21/21) = (63/-189)

Clearly, the numerators of the above obtained rational numbers are equal.

Hence, the given rational numbers are equal

(iv) Given (-8/-14) and (13/21)

Multiplying numerator and denominator of (-8/-14) by the denominator of (13/21)

i.e. 21

(-8/-14) × (21/21) = (-168/-294)

Now multiply the numerator and denominator of (13/21) by the denominator of

(-8/-14) i.e. -14

(13/21) × (-14/-14) = (-182/-294)

Clearly, the numerators of the above obtained rational numbers are not equal.

Hence, the given rational numbers are also not equal

2. In each of the following pairs represent a pair of equivalent rational numbers, find the values of x.

(i) (2/3) and (5/x)

(ii) (-3/7) and (x/4)

(iii) (3/5) and (x/-25)

(iv) (13/6) and (-65/x)

Solution:

(i) Given (2/3) and (5/x)

Also given that they are equivalent rational number so (2/3) = (5/x)

x = (5 × 3)/2

x = (15/2)

(ii) Given (-3/7) and (x/4)

Also given that they are equivalent rational number so (-3/7) = (x/4)

x = (-3 × 4)/7

x = (-12/7)

(iii) Given (3/5) and (x/-25)

Also given that they are equivalent rational number so (3/5) = (x/-25)

x = (3 × -25)/5

x = (-75)/5

x = -15

(iv) Given (13/6) and (-65/x)

Also given that they are equivalent rational number so (13/6) = (-65/x)

x = 6/13 x (- 65)

x = 6 x (-5)

x = -30

3. In each of the following, fill in the blanks so as to make the statement true:

(i) A number which can be expressed in the form p/q, where p and q are integers and q is not equal to zero, is called a ………..

(ii) If the integers p and q have no common divisor other than 1 and q is positive, then the rational number (p/q) is said to be in the ….

(iii) Two rational numbers are said to be equal, if they have the same …. form

(iv) If m is a common divisor of a and b, then (a/b) = (a ÷ m)/…..

(v) If p and q are positive Integers, then p/q is a ….. rational number and (p/-q) is a …… rational number.

(vi) The standard form of -1 is …

(vii) If (p/q) is a rational number, then q cannot be ….

(viii) Two rational numbers with different numerators are equal, if their numerators are in the same …. as their denominators.

Solution:

(i) Rational number

(ii) Standard form

(iii) Standard

(iv) b ÷ m

(v) Positive, negative

(vi) (-1/1)

(vii) Zero

(viii) Ratio

4. In each of the following state if the statement is true (T) or false (F):

(i) The quotient of two integers is always an integer.

(ii) Every integer is a rational number.

(iii) Every rational number is an integer.

(iv) Every traction is a rational number.

(v) Every rational number is a fraction.

(vi) If a/b is a rational number and m any integer, then (a/b) = (a x m)/ (b x m)

(vii) Two rational numbers with different numerators cannot be equal.

(viii) 8 can be written as a rational number with any integer as denominator.

(ix) 8 can be written as a rational number with any integer as numerator.

(x) (2/3) is equal to (4/6).

Solution:

(i) False

Explanation:

The quotient of two integers is not necessary to be an integer

(ii) True

Explanation:

Every integer can be expressed in the form of p/q, where q is not zero.

(iii) False

Explanation:

Every rational number is not necessary to be an integer

(iv) True

Explanation:

According to definition of rational number i.e. every integer can be expressed in the form of p/q, where q is not zero.

(v) False

Explanation:

It is not necessary that every rational number is a fraction.

(vi) True

Explanation:

If a/b is a rational number and m any integer, then (a/b) = (a x m)/ (b x m) is one of the rule of rational numbers

(vii) False

Explanation:

They can be equal, when simplified further.

(viii) False

Explanation:

8 can be written as a rational number but we can’t write 8 with any integer as denominator.

(ix) False

Explanation:

8 can be written as a rational number but we can’t with any integer as numerator.

(x) True

Explanation:

When convert it into standard form they are equal

Exercise 4.6

1**. Draw the number line and represent following rational number on it:**

(i) (2/3)

(ii) (3/4)

(iii) (3/8)

(iv) (-5/8)

(v) (-3/16)

(vi) (-7/3)

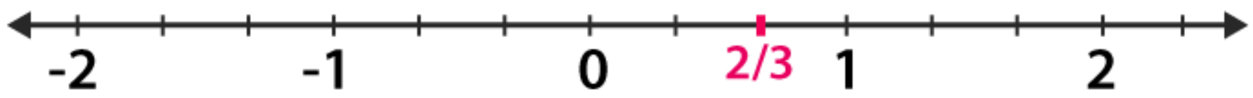
(vii) (22/-7)

(viii) (-31/3)

Solution:

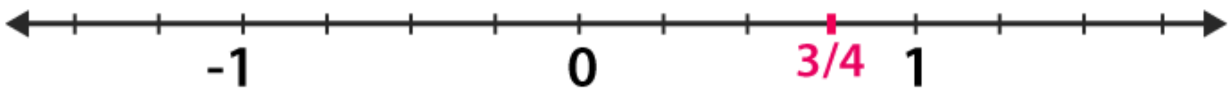
(i) We know that (2/3) is greater than 2 and less than 3.

∴ it lies between 2 and 3. It can be represented on number line as,



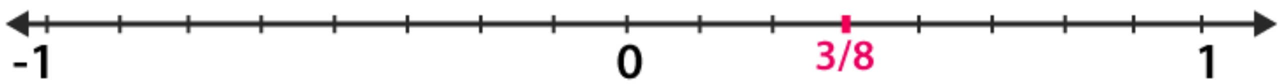
(ii) We know that (3/4) is greater than 0 and less than 1.

∴ it lies between 0 and 1. It can be represented on number line as,



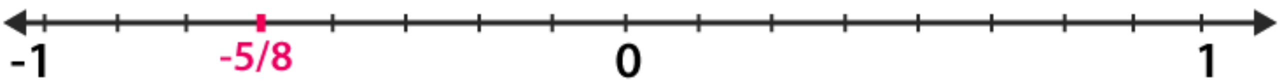
(iii) We know that (3/8) is greater than 0 and less than 1.

∴ it lies between 0 and 1. It can be represented on number line as,



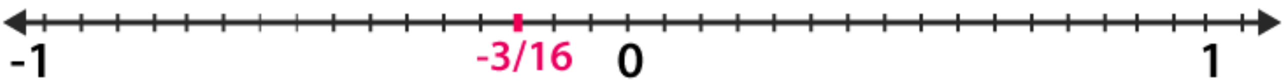
(iv) We know that (-5/8) is greater than -1 and less than 0.

∴ it lies between 0 and -1. It can be represented on number line as,



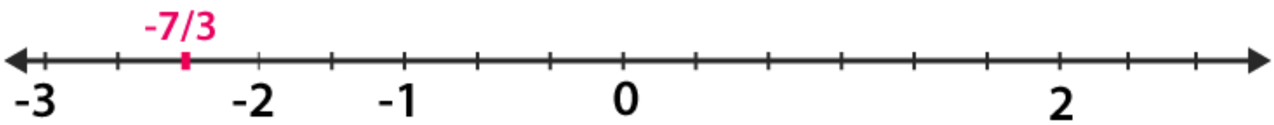
(v) We know that (-3/16) is greater than -1 and less than 0.

∴ it lies between 0 and -1. It can be represented on number line as,



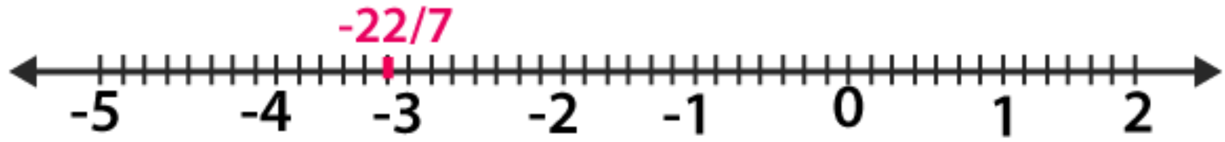
(vi) We know that (-7/3) is greater than -3 and less than -2.

∴ it lies between -3 and -2. It can be represented on number line as,



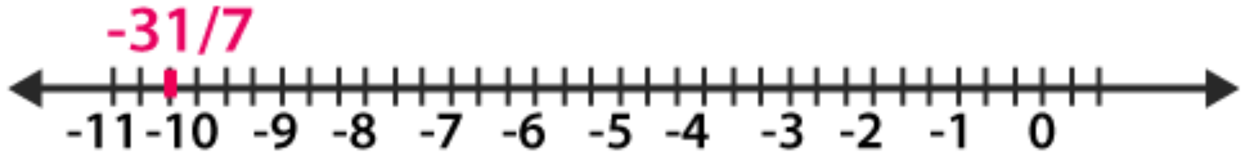
(vii) We know that (22/-7) is greater than -4 and less than -3.

∴ it lies between -3 and -4. It can be represented on number line as,



(Viii) We know that (-31/3) is greater than -11 and less than -10.

∴ it lies between -10 and -11. It can be represented on number line as,



2. Which of the two rational numbers in each of the following pairs of rational number is greater?
(i) (-3/8), 0

(ii) (5/2), 0

(iii) (– 4/11), (3/11)

(iv) (– 7/12), (5/- 8)

(v) (4/-9), (– 3/- 7)

(vi) (– 5/8), (3/- 4)

(vii) (5/9), (-3/- 8)

(viii) (5/- 8), (-7/12)

Solution:

(i) Given (-3/8), 0

We know that every positive rational number is greater than zero and every negative

rational number is smaller than zero. Thus, – (3/8) > 0

(ii) Given (5/2), 0

We know that every positive rational number is greater than zero and every negative rational number is smaller than zero. Thus, (5/2) > 0

(iii) Given (– 4/11), (3/11)

We know that every positive rational number is greater than zero and every negative rational number is smaller than zero, also the denominator is same in given question now we have to compare the numerator, thus – 4/11 < 3/11.

(iv) Given (– 7/12), (5/- 8)

Consider (– 7/12)

Multiply both numerator and denominator by 2 then we get

(-7/12) × (2/2) = (-14/24)…… (1)

Now consider (5/-8)

Multiply both numerator and denominator by 3 we get

(5/-8) × (3/3) = (15/-24)…… (2)

The denominator is same in equation (1) and (2) now we have to compare the numerator, thus (– 7/12) > (5/- 8)

(v) Given (4/-9), (– 3/- 7)

Consider (4/-9)

Multiply both numerator and denominator by 7 then we get

(4/-9) × (7/7) = (28/-63)…… (1)

Now consider (-3/-7)

Multiply both numerator and denominator by 9 we get

(-3/-7) × (9/9) = (-27/-63)…… (2)

The denominator is same in equation (1) and (2) now we have to compare the numerator, thus (4/-9) < (– 3/- 7)

(vi) Given (– 5/8), (3/- 4)

Now consider (3/-4)

Multiply both numerator and denominator by 2 we get

(3/-4) × (2/2) = (6/-8)

The denominator is same in above equation now we have to compare the numerator, thus (– 5/8) > (3/- 4)

(vii) Given (5/9), (-3/- 8)

Consider (5/9)

Multiply both numerator and denominator by 8 then we get

(5/9) × (8/8) = (40/72)…… (1)

Now consider (5/-8)

Multiply both numerator and denominator by 9 we get

(-3/-8) × (9/9) = (-27/-72)…… (2)

The denominator is same in equation (1) and (2) now we have to compare the numerator, thus (5/9) > (-3/- 8)

(viii) Given (5/- 8), (-7/12)

Consider (5/-8)

Multiply both numerator and denominator by 3 then we get

(5/-8) × (3/3) = (15/-24)…… (1)

Now consider (-7/12)

Multiply both numerator and denominator by 2 we get

(-7/12) × (2/2) = (-14/24)…… (2)

The denominator is same in equation (1) and (2) now we have to compare the numerator, thus (5/- 8) < (-7/12)

3. Which of the two rational numbers in each of the following pairs of rational numbers is smaller?

(i) (-6/-13), (7/13)

(ii) (16/-5), 3

(iii) (-4/3), (8/-7)

(iv) (-12/5), (-3)

Solution:

(i) Given (-6/-13), (7/13)

Here denominator is same therefore compare the numerator,

Thus (-6/-13) < (7/13)

(ii) Given (16/-5), 3

We know that 3 is a whole number with positive sign

Therefore (16/-5) < 3

(iii) Given (-4/3), (8/-7)

Consider (-4/3)

Multiply both numerator and denominator by 7 then we get

(-4/3) × (7/7) = (-28/21)…… (1)

Now consider (8/-7)

Multiply both numerator and denominator by 3 we get

(8/-7) × (3/3) = (-24/21)…… (2)

The denominator is same in equation (1) and (2) now we have to compare the numerator, thus (-4/3) < (8/-7)

(iv) Given (-12/5), (-3)

Now consider (-3/1)

Multiply both numerator and denominator by 5 we get

(-3/1) × (5/5) = (-15/5)

The denominator is same in above equation, now we have to compare the numerator, thus (-12/5) > (-3)

4. Fill in the blanks by the correct symbol out of >, =, or <:

(i) (-6/7) …. (7/13)

(ii) (-3/5) …. (-5/6)

(iii) (-2/3) …. (5/-8)

(iv) 0 …. (-2/5)

Solution:

(i) (- 6/7) < (7/13)

Explanation:

Because every positive number is greater than a negative number.

(ii) (-3/5) > (-5/6)

Explanation:

Consider (-3/5)

Multiply both numerator and denominator by 6 then we get

(-3/5) × (6/6) = (-18/30)…… (1)

Now consider (-5/6)

Multiply both numerator and denominator by 5 we get

(-5/6) × (5/5) = (-25/30)…… (2)

The denominator is same in equation (1) and (2) now we have to compare the numerator, thus (-3/5) > (-5/6)

(iii) (-2/3) < (5/-8)

Explanation:

Consider (-2/3)

Multiply both numerator and denominator by 8 then we get

(-2/3) × (8/8) = (-16/24)…… (1)

Now consider (5/-8)

Multiply both numerator and denominator by 3 we get

(5/-8) × (3/3) = (15/-24)…… (2)

The denominator is same in equation (1) and (2) now we have to compare the numerator, thus (-2/3) < (5/-8)

(iv) 0 > (-2/5)

Explanation:

Because every positive number is greater than a negative number

5. Arrange the following rational numbers in ascending order:

(i) (3/5), (-17/-30), (8/-15), (-7/10)

(ii) (-4/9), (5/-12), (7/-18), (2/-3)

Solution:

(i) Given (3/5), (-17/-30), (8/-15), (-7/10)

The LCM of 5, 30, 15 and 10 is 30

Multiplying the numerators and denominators to get the denominator equal to the LCM i.e. 30

Consider (3/5)

Multiply both numerator and denominator by 6, then we get

(3/5) × (6/6) = (18/30) ….. (1)

Consider (8/-15)

Multiply both numerator and denominator by 2, then we get

(8/-15) × (2/2) = (16/-30) ….. (2)

Consider (-7/10)

Multiply both numerator and denominator by 3, then we get

(-7/10) × (3/3) = (-21/30) ….. (3)

In the above equation, denominators are same

Now on comparing the ascending order is:

(-7/10) < (8/-15) < (-17/30) < (3/5)

(ii) Given (-4/9), (5/-12), (7/-18), (2/-3)

The LCM of 9, 12, 18 and 3 is 36

Multiplying the numerators and denominators to get the denominator equal to the LCM i.e. 36

Consider (-4/9)

Multiply both numerator and denominator by 4, then we get

(-4/9) × (4/4) = (-16/36) ….. (1)

Consider (5/-12)

Multiply both numerator and denominator by 3, then we get

(5/-12) × (3/3) = (15/-36) ….. (2)

Consider (7/-18)

Multiply both numerator and denominator by 2, then we get

(7/-18) × (2/2) = (14/-36) ….. (3)

Consider (2/-3)

Multiply both numerator and denominator by 12, then we get

(2/-3) × (12/12) = (24/-36) ….. (4)

In the above equation, denominators are same

Now on comparing the ascending order is:

(2/-3) < ((-4/9) < (5/-12) < (7/-18)

6. Arrange the following rational numbers in descending order:

(i) (7/8), (64/16), (39/-12), (5/-4), (140/28)

(ii) (-3/10), (17/-30), (7/-15), (-11/20)

Solution:

(i) Given (7/8), (64/16), (39/-12), (5/-4), (140/28)

The LCM of 8, 16, 12, 4 and 28 is 336

Multiplying the numerators and denominators to get the denominator equal to the LCM i.e. 336

Consider (7/8)

Multiply both numerator and denominator by 42, then we get

(7/8) × (42/42) = (294/336) ….. (1)

Consider (64/16)

Multiply both numerator and denominator by 21, then we get

(64/16) × (21/21) = (1344/336) ….. (2)

Consider (39/-12)

Multiply both numerator and denominator by 28, then we get

(39/-12) × (28/28) = (-1008/336) ….. (3)

Consider (5/-4)

Multiply both numerator and denominator by 84, then we get

(5/-4) × (84/84) = (-420/336) ….. (4)

In the above equation, denominators are same

Now on comparing the descending order is:

(140/28) > (64/16) > (7/8) > (5/-4) > (36/-12)

(ii) Given (-3/10), (17/-30), (7/-15), (-11/20)

The LCM of 10, 30, 15 and 20 is 60

Multiplying the numerators and denominators to get the denominator equal to the LCM i.e. 60

Consider (-3/10)

Multiply both numerator and denominator by 6, then we get

(-3/10) × (6/6) = (-18/60) ….. (1)

Consider (17/-30)

Multiply both numerator and denominator by 2, then we get

(17/-30) × (2/2) = (34/-60) ….. (2)

Consider (7/-15)

Multiply both numerator and denominator by 4, then we get

(7/-15) × (4/4) = (28/-60) ….. (3)

In the above equation, denominators are same

Now on comparing the descending order is:

(-3/10) > (7/-15) > (-11/20) > (17/-30)

7. Which of the following statements are true:

(i) The rational number (29/23) lies to the left of zero on the number line.

(ii) The rational number (-12/-17) lies to the left of zero on the number line.

(iii) The rational number (3/4) lies to the right of zero on the number line.

(iv) The rational number (-12/-5) and (- 7/17) are on the opposite side of zero on the number line.

(v) The rational number (-2/15) and (7/-31) are on the opposite side of zero on the number line.

(vi) The rational number (- 3/-5) is on the right of (- 4/7) on the number line.

Solution:

(i) False

Explanation:

It lies to the right of zero because it is a positive number.

(ii) False

Explanation:

It lies to the right of zero because it is a positive number.

(iii) True

Explanation:

Always positive number lie on the right of zero

(iv) True

Explanation:

Because they are of opposite sign

(v) False

Explanation:

Because they both are of same sign

(vi) True

Explanation:

They both are of opposite signs and positive number is greater than the negative number. Thus, it is on the right of the negative number.

Chapter - 5 Operations On Rational Numbers

Exercise 5.1

1. Add the following rational numbers:

(i) (-5/7) and (3/7)

(ii) (-15/4) and (7/4)

(iii) (-8/11) and (-4/11)

(iv) (6/13) and (-9/13)

Solution:

(i) Given (-5/7) and (3/7)

= (-5/7) + (3/7)

Here denominators are same so add the numerator

= ((-5+3)/7)

= (-2/7)

(ii) Given (-15/4) and (7/4)

= (-15/4) + (7/4)

Here denominators are same so add the numerator

= ((-15 + 7)/4)

= (-8/4)

On simplifying

= -2

(iii) Given (-8/11) and (-4/11)

= (-8/11) + (-4/11)

Here denominators are same so add the numerator

= (-8 + (-4))/11

= (-12/11)

(iv) Given (6/13) and (-9/13)

= (6/13) + (-9/13)

Here denominators are same so add the numerator

= (6 + (-9))/13

= (-3/13)

2. Add the following rational numbers:

(i) (3/4) and (-3/5)

(ii) -3 and (3/5)

(iii) (-7/27) and (11/18)

(iv) (31/-4) and (-5/8)

Solution:

(i) Given (3/4) and (-3/5)

If p/q and r/s are two rational numbers such that q and s do not have a common factor other than one, then

(p/q) + (r/s) = (p × s + r × q)/ (q × s)

(3/4) + (-3/5) = (3 × 5 + (-3) × 4)/ (4 × 5)

= (15 – 12)/ 20

= (3/20)

(ii) Given -3 and (3/5)

If p/q and r/s are two rational numbers such that q and s do not have a common factor other than one, then

(p/q) + (r/s) = (p × s + r × q)/ (q × s)

(-3/1) + (3/5) = (-3 × 5 + 3 × 1)/ (1 × 5)

= (-15 + 3)/ 5

= (-12/5)

(iii) Given (-7/27) and (11/18)

LCM of 27 and 18 is 54

(-7/27) = (-7/27) × (2/2) = (-14/54)

(11/18) = (11/18) × (3/3) = (33/54)

(-7/27) + (11/18) = (-14 + 33)/54

= (19/54)

(iv) Given (31/-4) and (-5/8)

LCM of -4 and 8 is 8

(31/-4) = (31/-4) × (2/2) = (62/-8)

(31/-4) + (-5/8) = (-62 – 5)/8

= (-67/8)

3. Simplify:

(i) (8/9) + (-11/6)

(ii) (-5/16) + (7/24)

(iii) (1/-12) + (2/-15)

(iv) (-8/19) + (-4/57)

Solution:

(i) Given (8/9) + (-11/6)

The LCM of 9 and 6 is 18

(8/9) = (8/9) × (2/2) = (16/18)

(-11/6) = (-11/6) × (3/3) = (-33/18)

= (16 – 33)/18

= (-17/18)

(ii) Given (-5/16) + (7/24)

The LCM of 16 and 24 is 48

Now (-5/16) = (-5/16) × (3/3) = (-15/48)

Consider (7/24) = (7/24) × (2/2) = (14/48)

(-5/16) + (7/24) = (-15/48) + (14/48)

= (14 – 15) /48

= (-1/48)

(iii) Given (1/-12) + (2/-15)

The LCM of 12 and 15 is 60

Consider (-1/12) = (-1/12) × (5/5) = (-5/60)

Now (2/-15) = (-2/15) × (4/4) = (-8/60)

(1/-12) + (2/-15) = (-5/60) + (-8/60)

= (-5 – 8)/60

= (-13/60)

(iv) Given (-8/19) + (-4/57)

The LCM of 19 and 57 is 57

Consider (-8/57) = (-8/57) × (3/3) = (-24/57)

(-8/19) + (-4/57) = (-24/57) + (-4/57)

= (-24 – 4)/57

= (-28/57)

4. Add and express the sum as mixed fraction:

(i) (-12/5) + (43/10)

(ii) (24/7) + (-11/4)

(iii) (-31/6) + (-27/8)

Solution:

(i) Given (-12/5) + (43/10)

The LCM of 5 and 10 is 10

Consider (-12/5) = (-12/5) × (2/2) = (-24/10)

(-12/5) + (43/10) = (-24/10) + (43/10)

= (-24 + 43)/10

= (19/10)

Now converting it into mixed fraction

=

(ii) Given (24/7) + (-11/4)

The LCM of 7 and 4 is 28

Consider (24/7) = (24/7) × (4/4) = (96/28)

Again (-11/4) = (-11/4) × (7/7) = (-77/28)

(24/7) + (-11/4) = (96/28) + (-77/28)

= (96 – 77)/28

= (19/28)

(iii) Given (-31/6) + (-27/8)

The LCM of 6 and 8 is 24

Consider (-31/6) = (-31/6) × (4/4) = (-124/24)

Again (-27/8) = (-27/8) × (3/3) = (-81/24)

(-31/6) + (-27/8) = (-124/24) + (-81/24)

= (-124 – 81)/24

= (-205/24)

Exercise 5.2

1. Subtract the first rational number from the second in each of the following:

(i) (3/8), (5/8)

(ii) (-7/9), (4/9)

(iii) (-2/11), (-9/11)

(iv) (11/13), (-4/13)

Solution:

(i) Given (3/8), (5/8)

(5/8) – (3/8) = (5 – 3)/8

= (2/8)

= (1/4)

(ii) Given (-7/9), (4/9)

(4/9) – (-7/9) = (4/9) + (7/9)

= (4 + 7)/9

= (11/9)

(iii) Given (-2/11), (-9/11)

(-9/11) – (-2/11) = (-9/11) + (2/11)

= (-9 + 2)/ 11

= (-7/11)

(iv) Given (11/13), (-4/13)

(-4/13) – (11/13) = (-4 – 11)/13

= (-15/13)

2. Evaluate each of the following:

(i) (2/3) – (3/5)

(ii) (-4/7) – (2/-3)

(iii) (4/7) – (-5/-7)

(iv) -2 – (5/9)

Solution:

(i) Given (2/3) – (3/5)

The LCM of 3 and 5 is 15

Consider (2/3) = (2/3) × (5/5) = (10/15)

Now again (3/5) = (3/5) × (3/3) = (9/15)

(2/3) – (3/5) = (10/15) – (9/15)

= (1/15)

(ii) Given (-4/7) – (2/-3)

The LCM of 7 and 3 is 21

Consider (-4/7) = (-4/7) × (3/3) = (-12/21)

Again (2/-3) = (-2/3) × (7/7) = (-14/21)

(-4/7) – (2/-3) = (-12/21) – (-14/21)

= (-12 + 14)/21

= (2/21)

(iii) Given (4/7) – (-5/-7)

(4/7) – (5/7) = (4 -5)/7

= (-1/7)

(iv) Given -2 – (5/9)

Consider (-2/1) = (-2/1) × (9/9) = (-18/9)

-2 – (5/9) = (-18/9) – (5/9)

= (-18 -5)/9

= (-23/9)

3. The sum of the two numbers is (5/9). If one of the numbers is (1/3), find the other.

Solution:

Given sum of two numbers is (5/9)

And one them is (1/3)

Let the unknown number be x

x + (1/3) = (5/9)

x = (5/9) – (1/3)

LCM of 3 and 9 is 9

Consider (1/3) = (1/3) × (3/3) = (3/9)

On substituting we get

$x = (5/9) – (3/9)$

$x = (5 – 3)/9$

$x = (2/9)$

4. The sum of two numbers is (-1/3). If one of the numbers is (-12/3), find the other.

Solution:

Given sum of two numbers = (-1/3)

One of them is (-12/3)

Let the required number be x

$x + (-12/3) = (-1/3)$

$x = (-1/3) – (-12/3)$

$x = (-1/3) + (12/3)$

$x = (-1 + 12)/3$

$x = (11/3)$

5. The sum of two numbers is (– 4/3). If one of the numbers is -5, find the other.

Solution:

Given sum of two numbers = (-4/3)

One of them is -5

Let the required number be x

$x + (-5) = (-4/3)$

LCM of 1 and 3 is 3

$(-5/1) = (-5/1) \times (3/3) = (-15/3)$

On substituting

x + (-15/3) = (-4/3)

x = (-4/3) – (-15/3)

x = (-4/3) + (15/3)

x = (-4 + 15)/3

x = (11/3)

6. The sum of two rational numbers is – 8. If one of the numbers is (-15/7), find the other.

Solution:

Given sum of two numbers is -8

One of them is (-15/7)

Let the required number be x

x + (-15/7) = -8

The LCM of 7 and 1 is 7

Consider (-8/1) = (-8/1) × (7/7) = (-56/7)

On substituting

x + (-15/7) = (-56/7)

x = (-56/7) – (-15/7)

x = (-56/7) + (15/7)

x = (-56 + 15)/7

x = (-41/7)

7. What should be added to (-7/8) so as to get (5/9)?

Solution:

Given (-7/8)

Let the required number be x

x + (-7/8) = (5/9)

The LCM of 8 and 9 is 72

x = (5/9) – (-7/8)

x = (5/9) + (7/8)

Consider (5/9) = (5/9) × (8/8) = (40/72)

Again (7/8) = (7/8) × (9/8) = (63/72)

On substituting

x = (40/72) + (63/72)

x = (40 + 63)/72

x = (103/72)

8. What number should be added to (-5/11) so as to get (26/33)?

Solution:

Given (-5/11)

Let the required number be x

x + (-5/11) = (26/33)

x = (26/33) – (-5/11)

x = (26/33) + (5/11)

Consider (5/11) = (5/11) × (3/3) = (15/33)

On substituting

x = (26/33) + (15/33)

x = (41/33)

9. What number should be added to (-5/7) to get (-2/3)?

Solution:

Given (-5/7)

Let the required number be x

x + (-5/7) = (-2/3)

x = (-2/3) – (-5/7)

x = (-2/3) + (5/7)

LCM of 3 and 7 is 21

Consider (-2/3) = (-2/3) × (7/7) = (-14/21)

Again (5/7) = (5/7) × (3/3) = (15/21)

On substituting

x = (-14/21) + (15/21)

x = (-14 + 15)/21

x = (1/21)

10. What number should be subtracted from (-5/3) to get (5/6)?

Solution:

Given (-5/3)

Let the required number be x

(-5/3) – x = (5/6)

– x = (5/6) – (-5/3)

– x = (5/6) + (5/3)

Consider (5/3) = (5/3) × (2/2) = (10/6)

On substituting

– x = (5/6) + (10/6)

– x = (15/6)

x = (-15/6)

11. What number should be subtracted from (3/7) to get (5/4)?

Solution:

Given (3/7)

Let the required number be x

(3/7) – x = (5/4)

– x = (5/4) – (3/7)

The LCM of 4 and 7 is 28

Consider (5/4) = (5/4) × (7/7) = (35/28)

Again (3/7) = (3/7) × (4/4) = (12/28)

On substituting

-x = (35/28) – (12/28)

– x = (35 -12)/28

– x = (23/28)

x = (-23/28)

12. What should be added to ((2/3) + (3/5)) to get (-2/15)?

Solution:

Given ((2/3) + (3/5))

Let the required number be x

((2/3) + (3/5)) + x = (-2/15)

Consider (2/3) = (2/3) × (5/5) = (10/15)

Again $(3/5) = (3/5) \times (3/3) = (9/15)$

On substituting

$((10/15) + (9/15)) + x = (-2/15)$

$x = (-2/15) - ((10/15) + (9/15))$

$x = (-2/15) - (19/15)$

$x = (-2 - 19)/15$

$x = (-21/15)$

$x = (-7/5)$

13. What should be added to ((1/2) + (1/3) + (1/5)) to get 3?

Solution:

Given $((1/2) + (1/3) + (1/5))$

Let the required number be x

$((1/2) + (1/3) + (1/5)) + x = 3$

$x = 3 - ((1/2) + (1/3) + (1/5))$

LCM of 2, 3 and 5 is 30

Consider $(1/2) = (1/2) \times (15/15) = (15/30)$

$(1/3) = (1/3) \times (10/10) = (10/30)$

$(1/5) = (1/5) \times (6/6) = (6/30)$

On substituting

$x = 3 - ((15/30) + (10/30) + (6/30))$

$x = 3 - (31/30)$

$(3/1) = (3/1) \times (30/30) = (90/30)$

$x = (90/30) - (31/30)$

x = (90 – 31)/30

x = (59/30)

14. What should be subtracted from ((3/4) – (2/3)) to get (-1/6)?

Solution:

Given ((3/4) – (2/3))

Let the required number be x

((3/4) – (2/3)) – x = (-1/6)

– x = (-1/6) – ((3/4) – (2/3))

Consider (3/4) = (3/4) × (3/3) = (9/12)

(2/3) = (2/3) × (4/4) = (8/12)

On substituting

– x = (-1/6) – ((9/12) – ((8/12))

– x = (-1/6) – (1/12)

(1/6) = (1/6) × (2/2) = (2/12)

– x = (-2/12) – (1/12)

– x = (-2 – 1)/12

– x = (-3/12)

x = (3/12)

x = (1/4)

15. Simplify:

(i) (-3/2) + (5/4) – (7/4)

(ii) (5/3) – (7/6) + (-2/3)

(iii) (5/4) – (7/6) – (-2/3)

(iv) (-2/5) – (-3/10) – (-4/7)

Solution:

(i) Given (-3/2) + (5/4) – (7/4)

Consider (-3/2) = (-3/2) × (2/2) = (-6/4)

On substituting

(-3/2) + (5/4) – (7/4) = (-6/4) + (5/4) – (7/4)

= (-6 + 5 – 7)/4

= (-13 + 5)/4

= (-8/4)

= -2

(ii) Given (5/3) – (7/6) + (-2/3)

Consider (5/3) = (5/3) × (2/2) = (10/6)

(-2/3) = (-2/3) × (2/2) = (-4/6)

(5/3) – (7/6) + (-2/3) = (10/6) – (7/6) – (4/6)

= (10 – 7 – 4)/6

= (10 – 11)/6

= (-1/6)

(iii) Given (5/4) – (7/6) – (-2/3)

The LCM of 4, 6 and 3 is 12

Consider (5/4) = (5/4) × (3/3) = (15/12)

(7/6) = (7/6) × (2/2) = (14/12)

(-2/3) = (-2/3) × (4/4) = (-8/12)

(5/4) – (7/6) – (-2/3) = (15/12) – (14/12) + (8/12)

= (15 – 14 + 8)/12

= (9/12)

= (3/4)

(iv) Given (-2/5) – (-3/10) – (-4/7)

The LCM of 5, 10 and 7 is 70

Consider (-2/5) = (-2/5) × (14/14) = (-28/70)

(-3/10) = (-3/10) × (7/7) = (-21/70)

(-4/7) = (-4/7) × (10/10) = (-40/70)

On substituting

(-2/5) – (-3/10) – (-4/7) = (-28/70) + (21/70) + (40/70)

= (-28 + 21 + 40)/70

= (33/70)

16. Fill in the blanks:

(i) (-4/13) – (-3/26) = …..

(ii) (-9/14) + ….. = -1

(iii) (-7/9) + ….. = 3

(iv) ….. + (15/23) = 4

Solution:

(i) (-5/26)

Explanation:

Consider (-4/13) – (-3/26)

(-4/13) = (-4/13) × (2/2) = (-8/26)

(-4/13) – (-3/26) = (-8/26) – (-3/26)

= (-5/26)

(ii) (-5/14)

Explanation:

Given (-9/14) + ….. = -1

(-9/14) + 1 = ….

(-9/14) + (14/14) = (5/14)

(-9/14) + (-5/14) = -1

(iii) (34/9)

Explanation:

Given (-7/9) + ….. = 3

(-7/9) + x = 3

x = 3 + (7/9)

(3/1) = (3/1) × (9/9) = (27/9)

x = (27/9) + (7/9) = (34/9)

(iv) (77/23)

Explanation:

Given ….. + (15/23) = 4

x + (15/23) = 4

x = 4 – (15/23)

(4/1) = (4/1) × (23/23) = (92/23)

x = (92/23) – (15/23)

= (77/23)

Exercise 5.3

1. Multiply:

(i) (7/11) by (5/4)

(ii) (5/7) by (-3/4)

(iii) (-2/9) by (5/11)

(iv) (-3/13) by (-5/-4)

Solution:

(i) Given (7/11) by (5/4)

(7/11) × (5/4) = (35/44)

(ii) Given (5/7) by (-3/4)

(5/7) × (-3/4) = (-15/28)

(iii) Given (-2/9) by (5/11)

(-2/9) × (5/11) = (-10/99)

(iv) Given (-3/13) by (-5/-4)

(-3/13) × (-5/-4) = (-15/68)

2. Multiply:

(i) (-5/17) by (51/-60)

(ii) (-6/11) by (-55/36)

(iii) (-8/25) by (-5/16)

(iv) (6/7) by (-49/36)

Solution:

(i) Given (-5/17) by (51/-60)

(-5/17) × (51/-60) = (-225/- 1020)

= (225/1020)

= (1/4)

(ii) Given (-6/11) by (-55/36)

(-6/11) × (-55/36) = (330/ 396)

= (5/6)

(iii) Given (-8/25) by (-5/16)

(-8/25) × (-5/16) = (40/400)

= (1/10)

(iv) Given (6/7) by (-49/36)

(6/7) × (-49/36) = (-294/252)

= (-7/6)

3. Simplify each of the following and express the result as a rational number in standard form:

(i) (-16/21) × (14/5)

(ii) (7/6) × (-3/28)

(iii) (-19/36) × 16

(iv) (-13/9) × (27/-26)

Solution:

(i) Given (-16/21) × (14/5)

(-16/21) × (14/5) = (-224/105)

= (-32/15)

(ii) Given (7/6) × (-3/28)

(7/6) × (-3/28) = (-21/168)

= (-1/8)

(iii) Given (-19/36) × 16

(-19/36) × 16 = (-304/36)

= (-76/9)

(iv) Given (-13/9) × (27/-26)

(-13/9) × (27/-26) = (-351/234)

= (3/2)

4. Simplify:

(i) (-5 × (2/15)) – (-6 × (2/9))

(ii) ((-9/4) × (5/3)) + ((13/2) × (5/6))

Solution:

(i) Given (-5 × (2/15)) – (-6 × (2/9))

(-5 × (2/15)) – (-6 × (2/9)) = (-10/15) – (-12/9)

= (-2/3) + (12/9)

= (-6/9) + (12/9)

= (6/9)

= (2/3)

(ii) Given ((-9/4) × (5/3)) + ((13/2) × (5/6))

((-9/4) × (5/3)) + ((13/2) × (5/6)) = ((-3/4) × 5) + ((13/2) × (5/6))

= (-15/4) + (65/12)

= (-15/4) × (3/3) + (65/12)

= (-45/12) + (65/12)

= (65 – 45)/12

= (20/12)

= (5/3)

5. Simplify:

(i) ((13/9) × (-15/2)) + ((7/3) × (8/5)) + ((3/5) × (1/2))

(ii) ((3/11) × (5/6)) – ((9/12) × ((4/3)) + ((5/13) × (6/15))

Solution:

(i) Given ((13/9) × (-15/2)) + ((7/3) × (8/5)) + ((3/5) × (1/2))

((13/9) × (-15/2)) + ((7/3) × (8/5)) + ((3/5) × (1/2)) = (-195/18) + (56/15) + (3/10)

= (-65/6) + (56/15) + (3/10)

= (-65/6) × (5/5) + (56/15) × (2/2) + (3/10) × (3/3).

= (-325/30) + (112/30) + (9/30)

= (-325 + 112 + 9)/30

= (-204/30)

= (-34/5)

(ii) Given ((3/11) × (5/6)) – ((9/12) × ((4/3)) + ((5/13) × (6/15))

((3/11) × (5/6)) – ((9/12) × ((4/3)) + ((5/13) × (6/15)) = (15/66) – (36/36) + (30/195)

= (5/22) – (12/12) + (1/11)

= (5/22) – 1 + (2/13)

= (5/22) × (13/13) + (1/1) × (286/286) + (2/13) × (22/22)

= (65/286) – (286/286) + (44/286)

= (-177/286)

Exercise 5.4

1. Divide:

(i) 1 by (1/2)

(ii) 5 by (-5/7)

(iii) (-3/4) by (9/-16)

(iv) (-7/8) by (-21/16)

(v) (7/-4) by (63/64)

(vi) 0 by (-7/5)

(vii) (-3/4) by -6

(viii) (2/3) by (-7/12)

Solution:

(i) Given 1 by (1/2)

1 ÷ (1/2) = 1 × 2 = 2

(ii) Given 5 by (-5/7)

5 ÷ (-5/7) = 5 × (-7/5)

= -7

(iii) Given (-3/4) by (9/-16)

(-3/4) ÷ (9/-16) = (-3/4) × (-16/9)

= (-4/-3)

= (4/3)

(iv) Given (-7/8) by (-21/16)

(-7/8) ÷ (-21/16) = (-7/8) × (16/-21)

= (-2/-3)

= (2/3)

(v) Given (7/-4) by (63/64)

(7/-4) ÷ (63/64) = (7/-4) × (64/63)

= (-16/9)

(vi) Given 0 by (-7/5)

0 ÷ (-7/5) = 0 × (5/7)

= 0

(vii) Given (-3/4) by -6

(-3/4) ÷ -6 = (-3/4) × (1/-6)

= (-1/-8)

= (1/8)

(viii) Given (2/3) by (-7/12)

(2/3) ÷ (-7/12) = (2/3) × (12/-7)

= (8/-7)

2. Find the value and express as a rational number in standard form:

(i) (2/5) ÷ (26/15)

(ii) (10/3) ÷ (-35/12)

(iii) -6 ÷ (-8/17)

(iv) (40/98) ÷ (-20)

Solution:

(i) Given (2/5) ÷ (26/15)

(2/5) ÷ (26/15) = (2/5) × (15/26)

= (3/13)

(ii) Given $(10/3) \div (-35/12)$

$(10/3) \div (-35/12) = (10/3) \times (12/-35)$

$= (-40/35)$

$= (-8/7)$

(iii) Given $-6 \div (-8/17)$

$-6 \div (-8/17) = -6 \times (17/-8)$

$= (102/8)$

$= (51/4)$

(iv) Given $(40/98) \div -20$

$(40/98) \div -20 = (40/98) \times (1/-20)$

$= (-2/98)$

$= (-1/49)$

3. The product of two rational numbers is 15. If one of the numbers is -10, find the other.

Solution:

Let required number be x

$x \times -10 = 15$

$x = (15/-10)$

$x = (3/-2)$

$x = (-3/2)$

Hence the number is $(-3/2)$

4. The product of two rational numbers is (- 8/9). If one of the numbers is (- 4/15), find the other.

Solution:

Given product of two numbers = (-8/9)

One of them is (-4/15)

Let the required number be x

x × (-4/15) = (-8/9)

x = (-8/9) ÷ (-4/15)

x = (-8/9) × (15/-4)

x = (-120/-36)

x = (10/3)

5. By what number should we multiply (-1/6) so that the product may be (-23/9)?

Solution:

Given product = (-23/9)

One number is (-1/6)

Let the required number be x

x × (-1/6) = (-23/9)

x = (-23/9) ÷ (-1/6)

x = (-23/9) × (-6/1)

x = (138/9)

x = (46/3)

6. By what number should we multiply (-15/28) so that the product may be (-5/7)?

Solution:

Given product = $(-5/7)$

One number is $(-15/28)$

Let the required number be x

$x \times (-15/28) = (-5/7)$

$x = (-5/7) \div (-15/28)$

$x = (-5/7) \times (28/-15)$

$x = (-4/-3)$

$x = (4/3)$

7. By what number should we multiply (-8/13) so that the product may be 24?

Solution:

Given product = 24

One of the number is = $(-8/13)$

Let the required number be x

$x \times (-8/13) = 24$

$x = 24 \div (-8/13)$

$x = 24 \times (13/-8)$

$x = -39$

8. By what number should (-3/4) be multiplied in order to produce (-2/3)?

Solution:

Given product = $(-2/3)$

One of the number is = $(-3/4)$

Let the required number be x

$x \times (-3/4) = (-2/3)$

$x = (-2/3) \div (-3/4)$

$x = (-2/3) \times (4/-3)$

$x = (-8/-9)$

$x = (8/9)$

9. Find $(x + y) \div (x – y)$, if

(i) $x = (2/3)$, $y = (3/2)$

(ii) $x = (2/5)$, $y = (1/2)$

(iii) $x = (5/4)$, $y = (-1/3)$

Solution:

(i) Given $x = (2/3)$, $y = (3/2)$

$(x + y) \div (x – y) = ((2/3) + (3/2)) \div ((2/3) – (3/2))$

$= (4 + 9)/6 \div (4 – 9)/6$

$= (4 + 9)/6 \times (6/ (4 – 9)$

$= (4 + 9)/ (4 -9)$

$= (13/-5)$

(ii) Given $x = (2/5)$, $y = (1/2)$

$(x + y) \div (x – y) = ((2/5) + (1/2)) \div ((2/5) – (1/2))$

$= (4 + 5)/10 \div (4 -5)/10$

$= (4 + 5)/10 \times (10/ (4 – 5)$

$= (4 + 5)/ (4 -5)$

$= (9/-1)$

(iii) Given $x = (5/4)$, $y = (-1/3)$

(x + y) ÷ (x – y) = ((5/4) + (-1/3)) ÷ ((5/4) – (-1/3))

= (15 – 4)/12 ÷ (15 + 4)/12

= (15 – 4)/12 × (12/ (15 + 4)

= (15 – 4)/ (15 + 4)

= (11/19)

10. The cost of

meters of rope is Rs.

. Find its cost per meter.

Solution:

Given cost of

= (23/3) meters of rope is Rs.

= (51/4)

Cost per meter = (51/4) ÷ (23/3)

= (51/4) × (3/23)

= (153/92)

= Rs

11. The cost of

meters of cloth is Rs.

. Find the cost of cloth per meter.

Solution:

Given cost of

metres of rope = Rs.

Cost of cloth per meter =

÷

= (301/4) ÷ (7/3)

= (301/4) × (3/7)

= (129/4)

= Rs

12. By what number should (-33/16) be divided to get (-11/4)?

Solution:

Let the required number be x

(-33/16) ÷ x = (-11/4)

x = (-33/16) ÷ (-11/4)

x = (-33/16) × (4/-11)

x = (3/4)

13. Divide the sum of (-13/5) and (12/7) by the product of (-31/7) and (-1/2)

Solution:

Given

((-13/5) + (12/7)) ÷ (-31/7) x (-1/2)

= ((-13/5) × (7/7) + (12/7) × (5/5)) ÷ (31/14)

= ((-91/35) + (60/35)) ÷ (31/14)

= (-31/35) ÷ (31/14)

= (-31/35) × (14/31)

= (-14/35)

= (-2/5)

14. Divide the sum of (65/12) and (8/3) by their difference.

Solution:

((65/12) + (8/3)) ÷ ((65/12) – (8/3))

= ((65/12) + (32/12)) ÷ ((65/12) – (32/12))

= (65 + 32)/12 ÷ (65 -32)/12

= (65 + 32)/12 × (12/ (65 – 32)

= (65 + 32)/ (65 – 32)

= (97/33)

15. If 24 trousers of equal size can be prepared in 54 metres of cloth, what length of cloth is required for each trouser?

Solution:

Given material required for 24 trousers = 54m

Cloth required for 1 trouser = (54/24)

= (9/4) meters

Exercise 5.5

1. Find six rational numbers between (-4/8) and (3/8)

Solution:

We know that between -4 and -8, below mentioned numbers will lie

-3, -2, -1, 0, 1, 2.

According to definition of rational numbers are in the form of (p/q) where q not equal to zero.

Therefore six rational numbers between (-4/8) and (3/8) are

(-3/8), (-2/8), (-1/8), (0/8), (1/8), (2/8), (3/8)

2. Find 10 rational numbers between (7/13) and (- 4/13)

Solution:

We know that between 7 and -4, below mentioned numbers will lie

-3, -2, -1, 0, 1, 2, 3, 4, 5, 6.

According to definition of rational numbers are in the form of (p/q) where q not equal to zero.

Therefore six rational numbers between (7/13) and (-4/13) are

(-3/13), (-2/13), (-1/13), (0/13), (1/13), (2/13), (3/13), (4/13), (5/13), (6/13)

3. State true or false:

(i) Between any two distinct integers there is always an integer.

(ii) Between any two distinct rational numbers there is always a rational number.

(iii) Between any two distinct rational numbers there are infinitely many rational numbers.

Solution:

(i) False

Explanation:

Between any two distinct integers not necessary to be one integer.

(ii) True

Explanation:

According to the properties of rational numbers between any two distinct rational numbers there is always a rational number.

(iii) True

Explanation:

According to the properties of rational numbers between any two distinct rational numbers there are infinitely many rational numbers.

Chapter - 6 Exponents

Exercise 6.1

1. Find the values of each of the following:

(i) 13^2

(ii) 7^3

(iii) 3^4

Solution:

(i) Given 13^2

$13^2 = 13 \times 13 = 169$

(ii) Given 7^3

$7^3 = 7 \times 7 \times 7 = 343$

(iii) Given 3^4

$3^4 = 3 \times 3 \times 3 \times 3$

$= 81$

2. Find the value of each of the following:

(i) $(-7)^2$

(ii) $(-3)^4$

(iii) $(-5)^5$

Solution:

(i) Given $(-7)^2$

We know that $(-a)^{\text{even number}}$= positive number

$(-a)^{\text{odd number}}$ = negative number

We have, $(-7)^2 = (-7) \times (-7)$

$= 49$

(ii) Given $(-3)^4$

We know that $(-a)^{\text{even number}}$= positive number

$(-a)^{\text{odd number}}$ = negative number

We have, $(-3)^4 = (-3) \times (-3) \times (-3) \times (-3)$

$= 81$

(iii) Given $(-5)^5$

We know that $(-a)^{\text{even number}}$= positive number

$(-a)^{\text{odd number}}$ = negative number

We have, $(-5)^5 = (-5) \times (-5) \times (-5) \times (-5) \times (-5)$

$= -3125$

3. Simplify:

(i) 3×10^2

(ii) $2^2 \times 5^3$

(iii) $3^3 \times 5^2$

Solution:

(i) Given 3×10^2

$3 \times 10^2 = 3 \times 10 \times 10$

$= 3 \times 100$

$= 300$

(ii) Given $2^2 \times 5^3$

$2^2 \times 5^3 = 2 \times 2 \times 5 \times 5 \times 5$

$= 4 \times 125$

$= 500$

(iii) Given $3^3 \times 5^2$

$3^3 \times 5^2 = 3 \times 3 \times 3 \times 5 \times 5$

$= 27 \times 25$

$= 675$

4. Simply:

(i) $3^2 \times 10^4$

(ii) $2^4 \times 3^2$

(iii) $5^2 \times 3^4$

Solution:

(i) Given $3^2 \times 10^4$

$3^2 \times 10^4 = 3 \times 3 \times 10 \times 10 \times 10 \times 10$

$= 9 \times 10000$

$= 90000$

(ii) Given$2^4 \times 3^2$

$2^4 \times 3^2 = 2 \times 2 \times 2 \times 2 \times 3 \times 3$

$= 16 \times 9$

$= 144$

(iii) Given $5^2 \times 3^4$

$5^2 \times 3^4 = 5 \times 5 \times 3 \times 3 \times 3 \times 3$

$= 25 \times 81$

$= 2025$

5. Simplify:

(i) $(-2) \times (-3)^3$

(ii) $(-3)^2 \times (-5)^3$

(iii) $(-2)^5 \times (-10)^2$

Solution:

(i) Given $(-2) \times (-3)^3$

$(-2) \times (-3)^3 = (-2) \times (-3) \times (-3) \times (-3)$

$= (-2) \times (-27)$

$= 54$

(ii) Given $(-3)^2 \times (-5)^3$

$(-3)^2 \times (-5)^3 = (-3) \times (-3) \times (-5) \times (-5) \times (-5)$

$= 9 \times (-125)$

$= -1125$

(iii) Given $(-2)^5 \times (-10)^2$

$(-2)^5 \times (-10)^2 = (-2) \times (-2) \times (-2) \times (-2) \times (-2) \times (-10) \times (-10)$

$= (-32) \times 100$

$= -3200$

6. Simplify:

(i) $(3/4)^2$

(ii) $(-2/3)^4$

(iii) $(-4/5)^5$

Solution:

(i) Given $(3/4)^2$

$(3/4)^2 = (3/4) \times (3/4)$

$= (9/16)$

(ii) Given $(-2/3)^4$

$(-2/3)^4 = (-2/3) \times (-2/3) \times (-2/3) \times (-2/3)$

$= (16/81)$

(iii) Given $(-4/5)^5$

$(-4/5)^5 = (-4/5) \times (-4/5) \times (-4/5) \times (-4/5) \times (-4/5)$

$= (-1024/3125)$

7. Identify the greater number in each of the following:

(i) 2^5 or 5^2

(ii) 3^4 or 4^3

(iii) 3^5 or 5^3

Solution:

(i) Given 2^5 or 5^2

$2^5 = 2 \times 2 \times 2 \times 2 \times 2$

$= 32$

$5^2 = 5 \times 5$

$= 25$

Therefore, $2^5 > 5^2$

(ii) Given 3^4 or 4^3

$3^4 = 3 \times 3 \times 3 \times 3$

$= 81$

$4^3 = 4 \times 4 \times 4$

$= 64$

Therefore, $3^4 > 4^3$

(iii) Given 3^5 or 5^3

$3^5 = 3 \times 3 \times 3 \times 3 \times 3$

= 243

$5^3 = 5 \times 5 \times 5$

= 125

Therefore, $3^5 > 5^3$

8. Express each of the following in exponential form:

(i) (-5) × (-5) × (-5)

(ii) (-5/7) × (-5/7) × (-5/7) × (-5/7)

(iii) (4/3) × (4/3) × (4/3) × (4/3) × (4/3)

Solution:

(i) Given (-5) × (-5) × (-5)

Exponential form of (-5) × (-5) × (-5) = $(-5)^3$

(ii) Given (-5/7) × (-5/7) × (-5/7) × (-5/7)

Exponential form of (-5/7) × (-5/7) × (-5/7) × (-5/7) = $(-5/7)^4$

(iii) Given (4/3) × (4/3) × (4/3) × (4/3) × (4/3)

Exponential form of (4/3) × (4/3) × (4/3) × (4/3) × (4/3) = $(4/3)^5$

9. Express each of the following in exponential form:

(i) x × x × x × x × a × a × b × b × b

(ii) (-2) × (-2) × (-2) × (-2) × a × a × a

(iii) (-2/3) × (-2/3) × x × x × x

Solution:

(i) Given x × x × x × x × a × a × b × b × b

Exponential form of x × x × x × x × a × a × b × b × b = $x^4a^2b^3$

(ii) Given (-2) × (-2) × (-2) × (-2) × a × a × a

Exponential form of (-2) × (-2) × (-2) × (-2) × a × a × a = $(-2)^4a^3$

(iii) Given (-2/3) × (-2/3) × x × x × x

Exponential form of (-2/3) × (-2/3) × x × x × x = $(-2/3)^2 x^3$

10. Express each of the following numbers in exponential form:

(i) 512

(ii) 625

(iii) 729

Solution:

(i) Given 512

Prime factorization of 512 = 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2

= 2^9

(ii) Given 625

Prime factorization of 625 = 5 x 5 x 5 x 5

= 5^4

(iii) Given 729

Prime factorization of 729 = 3 x 3 x 3 x 3 x 3 x 3

$= 3^6$

11. Express each of the following numbers as a product of powers of their prime factors:
(i) 36
(ii) 675
(iii) 392

Solution:

(i) Given 36

Prime factorization of 36 = 2 x 2 x 3 x 3

$= 2^2 \times 3^2$

(ii) Given 675

Prime factorization of 675 = 3 x 3 x 3 x 5 x 5

$= 3^3 \times 5^2$

(iii) Given 392

Prime factorization of 392 = 2 x 2 x 2 x 7 x 7

$= 2^3 \times 7^2$

12. Express each of the following numbers as a product of powers of their prime factors:
(i) 450
(ii) 2800
(iii) 24000

Solution:

(i) Given 450

Prime factorization of 450 = 2 x 3 x 3 x 5 x 5

$= 2 \times 3^2 \times 5^2$

(ii) Given 2800

Prime factorization of 2800 = 2 x 2 x 2 x 2 x 5 x 5 x 7

$= 2^4 \times 5^2 \times 7$

(iii) Given 24000

Prime factorization of 24000 = 2 x 2 x 2 x 2 x 2 x 2 x 3 x 5 x 5 x 5

$= 2^6 \times 3 \times 5^3$

13. Express each of the following as a rational number of the form (p/q):

(i) $(3/7)^2$

(ii) $(7/9)^3$

(iii) $(-2/3)^4$

Solution:

(i) Given $(3/7)^2$

$(3/7)^2 = (3/7) \times (3/7)$

= (9/49)

(ii) Given $(7/9)^3$

$(7/9)^3 = (7/9) \times (7/9) \times (7/9)$

= (343/729)

(iii) Given $(-2/3)^4$

$(-2/3)^4 = (-2/3) \times (-2/3) \times (-2/3) \times (-2/3)$

= ((16/81)

14. Express each of the following rational numbers in power notation:

(i) (49/64)

(ii) (- 64/125)

(iii) (-12/16)

Solution:

(i) Given (49/64)

We know that $7^2 = 49$ and $8^2 = 64$

Therefore $(49/64) = (7/8)^2$

(ii) Given (- 64/125)

We know that $4^3 = 64$ and $5^3 = 125$

Therefore $(- 64/125) = (- 4/5)^3$

(iii) Given (-1/216)

We know that $1^3 = 1$ and $6^3 = 216$

Therefore $-1/216) = -(1/6)^3$

15. Find the value of the following:

(i) $(-1/2)^2 \times 2^3 \times (3/4)^2$

(ii) $(-3/5)^4 \times (4/9)^4 \times (-15/18)^2$

Solution:

(i) Given $(-1/2)^2 \times 2^3 \times (3/4)^2$

$(-1/2)^2 \times 2^3 \times (3/4)^2 = 1/4 \times 8 \times 9/16$

$= 9/8$

(ii) Given $(-3/5)^4 \times (4/9)^4 \times (-15/18)^2$

$(-3/5)^4 \times (4/9)^4 \times (-15/18)^2 = (81/625) \times (256/6561) \times (225/324)$

$= (64/18225)$

16. If a = 2 and b= 3, the find the values of each of the following:

(i) $(a + b)^a$

(ii) $(a\ b)^b$

(iii) $(b/a)^b$

(iv) $((a/b) + (b/a))^a$

Solution:

(i) Consider $(a + b)^a$

Given a = 2 and b= 3

$(a + b)^a = (2 + 3)^2$

$= (5)^2$

$= 25$

(ii) Given a = 2 and b = 3

Consider, $(a\ b)^b = (2 \times 3)^3$

$= (6)^3$

$= 216$

(iii) Given a =2 and b = 3

Consider, $(b/a)^b = (3/2)^3$

= 27/8

(iv) Given a = 2 and b = 3

Consider, $((a/b) + (b/a))^a = ((2/3) + (3/2))^2$

= (4/9) + (9/4)

LCM of 9 and 6 is 36

= 169/36

Exercise 6.2

1. Using laws of exponents, simplify and write the answer in exponential form

(i) $2^3 \times 2^4 \times 2^5$

(ii) $5^{12} \div 5^3$

(iii) $(7^2)^3$

(iv) $(3^2)^5 \div 3^4$

(v) $3^7 \times 2^7$

(vi) $(5^{21} \div 5^{13}) \times 5^7$

Solution:

(i) Given $2^3 \times 2^4 \times 2^5$

We know that first law of exponents states that $a^m \times a^n \times a^p = a^{(m+n+p)}$

Therefore above equation can be written as $2^3 \text{ x } 2^4 \text{ x } 2^5 = 2^{(3 + 4 + 5)}$

$= 2^{12}$

(ii) Given $5^{12} \div 5^3$

According to the law of exponents we have $a^m \div a^n = a^{m-n}$

Therefore given question can be written as $5^{12} \div 5^3 = 5^{12 - 3} = 5^9$

(iii) Given $(7^2)^3$

According to the law of exponents we have $(a^m)^n = a^{mn}$

Therefore given question can be written as $(7^2)^3 = 7^6$

(iv) Given $(3^2)^5 \div 3^4$

According to the law of exponents we have $(a^m)^n = a^{mn}$

Therefore $(3^2)^5 \div 3^4 = 3^{10} \div 3^4$

According to the law of exponents we have $a^m \div a^n = a^{m-n}$

$3^{10} \div 3^4 = 3^{(10 - 4)} = 3^6$

(v) Given $3^7 \times 2^7$

We know that law of exponents states that $a^m \text{ x } b^m = (a \text{ x } b)^m$

$3^7 \times 2^7 = (3 \text{ x } 2)^7 = 6^7$

(vi) Given $(5^{21} \div 5^{13}) \times 5^7$

According to the law of exponents we have $a^m \div a^n = a^{m-n}$

$= 5^{(21 - 13)} \text{ x } 5^7$

$= 5^8 \text{ x } 5^7$

According to the law of exponents we have $a^m \text{ x } a^n = a^{(m + n)}$

$= 5^{(8+7)} = 5^{15}$

2. Simplify and express each of the following in exponential form:

(i) $\{(2^3)^4 \times 2^8\} \div 2^{12}$

(ii) $(8^2 \times 8^4) \div 8^3$

(iii) $(5^7/5^2) \times 5^3$

(iv) $(5^4\times x^{10}y^5)/ (5^4 \times x^7y^4)$

Solution:

(i) Given $\{(2^3)^4 \times 2^8\} \div 2^{12}$

$\{(2^3)^4 \text{ x } 2^8\} \div 2^{12}= \{2^{12} \text{ x } 2^8\} \div 2^{12}$ [According to the law of exponents we have $(a^m)^n = a^{mn}$]

$= 2^{(12 + 8)} \div 2^{12}$[According to the law of exponents we have $a^m \text{ x } a^n = a^{(m +n)}$]

$= 2^{20} \div 2^{12}$ [According to the law of exponents we have $a^m \div a^n = a^{m-n}$]

$= 2^{(20 – 12)}$

$= 2^8$

(ii) Given $(8^2 \times 8^4) \div 8^3$

$(8^2 \times 8^4) \div 8^3$ [According to the law of exponents we have $a^m \text{ x } a^n = a^{(m +n)}$]

$= 8^{(2 + 4)} \div 8^3$

$= 8^6 \div 8^3$[According to the law of exponents we have $a^m \div a^n = a^{m-n}$]

$= 8^{(6-3)} = 8^3 = (2^3)^3 = 2^9$

(iii) Given $(5^7/5^2) \times 5^3$

$= 5^{(7-2)} \text{ x } 5^3$[According to the law of exponents we have $a^m \div a^n = a^{m-n}$]

$= 5^5 \text{ x } 5^3$[According to the law of exponents we have $a^m \text{ x } a^n = a^{(m +n)}$]

$= 5^{(5 + 3)} = 5^8$

(iv) Given $(5^4\times x^{10}y^5)/ (5^4 \times x^7y^4)$

$= (5^{4-4}\times x^{10-7}y^{5-4})$ [According to the law of exponents we have $a^m \div a^n = a^{m-n}$]

$= 5^0x^3y^1$ [since $5^0 = 1$]

$= 1x^3y$

3. Simplify and express each of the following in exponential form:

(i) $\{(3^2)^3 \times 2^6\} \times 5^6$

(ii) $(x/y)^{12} \times y^{24} \times (2^3)^4$

(iii)$(5/2)^6 \times (5/2)^2$

(iv) $(2/3)^5\times (3/5)^5$

Solution:

(i) Given $\{(3^2)^3 \times 2^6\} \times 5^6$

$= \{3^6 \times 2^6\} \times 5^6$[According to the law of exponents we have $(a^m)^n = a^{mn}$]

$= 6^6 \times 5^6$ [since law of exponents states that $a^m \text{ x } b^m = (a \text{ x } b)^m$]

$= 30^6$

(ii) Given $(x/y)^{12} \times y^{24} \times (2^3)^4$

$= (x^{12}/y^{12}) \times y^{24} \times 2^{12}$

$= x^{12} \times y^{24-12} \times 2^{12}$[According to the law of exponents we have $a^m \div a^n = a^{m-n}$]

$= x^{12} \times y^{12} \times 2^{12}$

$= (2xy)^{12}$

(iii) Given $(5/2)^6 \times (5/2)^2$

$= (5/2)^{6+2}$[According to the law of exponents we have $a^m \times a^n = a^{(m+n)}$]

$= (5/2)^8$

(iv) Given $(2/3)^5 \times (3/5)^5$

$= (2/5)^5$[since law of exponents states that $a^m \times b^m = (a \times b)^m$]

4. Write 9 × 9 × 9 × 9 × 9 in exponential form with base 3.

Solution:

Given $9 \times 9 \times 9 \times 9 \times 9 = (9)^5 = (3^2)^5$

$= 3^{10}$

5. Simplify and write each of the following in exponential form:

(i) $(25)^3 \div 5^3$

(ii) $(81)^5 \div (3^2)^5$

(iii) $9^8 \times (x^2)^5 / (27)^4 \times (x^3)^2$

(iv) $3^2 \times 7^8 \times 13^6 / 21^2 \times 91^3$

Solution:

(i) Given $(25)^3 \div 5^3$

$= (5^2)^3 \div 5^3$[According to the law of exponents we have $(a^m)^n = a^{mn}$]

$= 5^6 \div 5^3$ [According to the law of exponents we have $a^m \div a^n = a^{m-n}$]

$= 5^{6-3}$

$= 5^3$

(ii) Given $(81)^5 \div (3^2)^5$[According to the law of exponents we have $(a^m)^n = a^{mn}$]

$= (81)^5 \div 3^{10}$[$81 = 3^4$]

$= (3^4)^5 \div 3^{10}$ [According to the law of exponents we have $(a^m)^n = a^{mn}$]

$= 3^{20} \div 3^{10}$

$= 3^{20-10}$ [According to the law of exponents we have $a^m \div a^n = a^{m-n}$]

$= 3^{10}$

(iii) Given $9^8 \times (x^2)^5 / (27)^4 \times (x^3)^2$

$= (3^2)^8 \times (x^2)^5 / (3^3)^4 \times (x^3)^2$[According to the law of exponents we have $(a^m)^n = a^{mn}$]

$= 3^{16} \times x^{10} / 3^{12} \times x^6$

$= 3^{16-12} \times x^{10-6}$[According to the law of exponents we have $a^m \div a^n = a^{m-n}$]

$= 3^4 \times x^4$

$= (3x)^4$

(iv) Given $(3^2 \times 7^8 \times 13^6) / (21^2 \times 91^3)$

$= (3^2 \times 7^2 7^6 \times 13^6)/(21^2 \times 13^3 \times 7^3)$[According to the law of exponents we have $(a^m)^n = a^{mn}$]

$= (21^2 \times 7^6 \times 13^6)/(21^2 \times 13^3 \times 7^3)$

$= (7^6 \times 13^6)/(13^3 \times 7^3)$

$= 91^6/91^3$[According to the law of exponents we have $a^m \div a^n = a^{m-n}$]

$= 91^{6-3}$

$= 91^3$

6. Simplify:

(i) $(3^5)^{11} \times (3^{15})^4 – (3^5)^{18} \times (3^5)^5$

(ii) $(16 \times 2^{n+1} – 4 \times 2^n)/(16 \times 2^{n+2} – 2 \times 2^{n+2})$

(iii) $(10 \times 5^{n+1} + 25 \times 5^n)/(3 \times 5^{n+2} + 10 \times 5^{n+1})$

(iv) $(16)^7 \times(25)^5\times (81)^3/(15)^7 \times(24)^5\times (80)^3$

Solution:

(i) Given $(3^5)^{11} \times (3^{15})^4 – (3^5)^{18} \times (3^5)^5$

$= (3)^{55} \times (3)^{60} – (3)^{90} \times (3)^{25}$[According to the law of exponents we have $(a^m)^n = a^{mn}$]

$= 3^{55+60} – 3^{90+25}$

$= 3^{115} – 3^{115}$

$= 0$

(ii) Given $(16 \times 2^{n+1} – 4 \times 2^n)/(16 \times 2^{n+2} – 2 \times 2^{n+2})$

$= (2^4 \times 2^{(n+1)} -2^2 \times 2^n)/(2^4 \times 2^{(n+2)} -2^{n+1} \times 2^2)$ [According to the law of exponents we have $(a^m)^n = a^{mn}$]

$= 2^2 \times 2^{(n+3-2n)}/)2^2\times 2^{(n+4-2n+1)}$

$= 2^n \times 2^3 – 2^n/ 2^n \times 2^4 – 2^n \times 2$

$= 2^n(2^3 – 1)/ 2^n(2^4 – 1)$ [According to the law of exponents we have $a^m \div a^n = a^{m-n}$]

$= 8 -1 /16 -2$

$= 7/14$

$= (1/2)$

(iii) Given $(10 \times 5^{n+1} + 25 \times 5^n)/(3 \times 5^{n+2} + 10 \times 5^{n+1})$

$= (10 \times 5^{n+1} + 5^2 \times 5^n)/(3 \times 5^{n+2} + (2 \times 5) \times 5^{n+1})$

$= (10 \times 5^{n+1} + 5 \times 5^{n+1})/(3 \times 5^{n+2} + (2 \times 5) \times 5^{n+1})$ [According to the law of exponents we have $(a^m)^n = a^{mn}$]

$= 5^{n+1} (10+5)/ 5^{n+1} (10+15)$[According to the law of exponents we have $a^m \div a^n = a^{m-n}$]

$= 15/25$

$= (3/5)$

(iv) Given $(16)^7 \times(25)^5\times (81)^3/(15)^7 \times(24)^5\times (80)^3$

$= (16)^7 \times(5^2)^5\times (3^4)^3/(3 \times 5)^7 \times(3 \times 8)^5\times (16 \times 5)^3$

$= (16)^7 \times(5^2)^5\times (3^4)^3/3^7 \times 5^7 \times 3^5 \times 8^5\times 16^3\times 5^3$

$= (16)^7/ 8^5 \times 16^{3}$

$= (16)^4/8^5$

$= (2 \times 8)^4/8^5$

$= 2^4/8$

$= (16/8)$

$= 2$

7. Find the values of n in each of the following:

(i) $5^{2n} \times 5^3 = 5^{11}$

(ii) $9 \times 3^n = 3^7$

(iii) $8 \times 2^{n+2} = 32$

(iv) $7^{2n+1} \div 49 = 7^3$

(v) $(3/2)^4 \times (3/2)^5 = (3/2)^{2n+1}$

(vi) $(2/3)^{10} \times \{(3/2)^2\}^5 = (2/3)^{2n-2}$

Solution:

(i) Given $5^{2n} \times 5^3 = 5^{11}$

$= 5^{2n+3} = 5^{11}$

On equating the coefficients, we get

$2n + 3 = 11$

$\Rightarrow 2n = 11 - 3$

$\Rightarrow 2n = 8$

$\Rightarrow n = (8/2)$

$\Rightarrow n = 4$

(ii) Given $9 \times 3^n = 3^7$

$= (3)^2 \times 3^n = 3^7$

$= (3)^{2+n} = 3^7$

On equating the coefficients, we get

$2 + n = 7$

$\Rightarrow n = 7 - 2 = 5$

(iii) Given $8 \times 2^{n+2} = 32$

$= (2)^3 \times 2^{n+2} = (2)^5$ [since $2^3 = 8$ and $2^5 = 32$]

$= (2)^{3+n+2} = (2)^5$

On equating the coefficients, we get

$3 + n + 2 = 5$

$\Rightarrow n + 5 = 5$

$\Rightarrow n = 5 - 5$

$\Rightarrow n = 0$

(iv) Given $7^{2n+1} \div 49 = 7^3$

$= 7^{2n+1} \div 7^2 = 7^3$ [since $49 = 7^2$]

$= 7^{2n+1-2} = 7^3$

$= 7^{2n-1} = 7^3$

On equating the coefficients, we get

$2n - 1 = 3$

$\Rightarrow 2n = 3 + 1$

$\Rightarrow 2n = 4$

$\Rightarrow n = 4/2 = 2$

(v) Given $(3/2)^4 \times (3/2)^5 = (3/2)^{2n+1}$

$= (3/2)^{4+5} = (3/2)^{2n+1}$

$= (3/2)^9 = (3/2)^{2n+1}$

On equating the coefficients, we get

$2n + 1 = 9$

⇒ $2n = 9 – 1$

⇒ $2n = 8$

⇒ $n = 8/2 = 4$

(vi) Given $(2/3)^{10} \times \{(3/2)^2\}^5 = (2/3)^{2n-2}$

$= (2/3)^{10} \times (3/2)^{10} = (2/3)^{2n-2}$

$= 2^{10} \times 3^{10}/3^{10} \times 2^{10} = (2/3)^{2n-2}$

$= 1 = (2/3)^{2n-2}$

$= (2/3)^0 = (2/3)^{2n-2}$

On equating the coefficients, we get

$0 = 2n - 2$

$2n - 2 = 0$

$2n = 2$

$n = 1$

8. If $(9^n \times 3^2 \times 3^n – (27)^n)/ (3^3)^5 \times 2^3 = (1/27)$, find the value of n.

Solution:

Given $(9^n \times 3^2 \times 3^n – (27)^n)/ (3^3)^5 \times 2^3 = (1/27)$

$= (3^2)^n \times 3^3 \times 3^n – (3^3)^n/ (3^{15} \times 2^3) = (1/27)$

$= 3^{(2n+2+n)} – (3^3)^n/ (3^{15} \times 2^3) = (1/27)$

$= 3^{(3n+2)} – (3^3)^n/ (3^{15} \times 2^3) = (1/27)$

$= 3^{3n} \times 3^2 – 3^{3n}/ (3^{15} \times 2^3) = (1/27)$

$= 3^{3n} \times (3^2 – 1)/ (3^{15} \times 2^3) = (1/27)$

$= 3^{3n} \times (9 – 1)/ (3^{15} \times 2^3) = (1/27)$

$= 3^{3n} \times (8)/ (3^{15} \times 2^3) = (1/27)$

$= 3^{3n} \times 2^3/ (3^{15} \times 2^3) = (1/27)$

$= 3^{3n}/3^{15} = (1/27)$

$= 3^{3n-15} = (1/27)$

$= 3^{3n-15} = (1/3^3)$

$= 3^{3n-15} = 3^{-3}$

On equating the coefficients, we get

$3n - 15 = -3$

⇒ $3n = -3 + 15$

⇒ $3n = 12$

⇒ $n = 12/3 = 4$

Exercise 6.3

Express the following numbers in the standard form:
(i) 3908.78
(ii) 5,00,00,000
(iii) 3,18,65,00,000
(iv) 846×10^7
(v)723×10^9

Solution:

(i) Given 3908.78

3908.78 = 3.90878 x 10^3 [since the decimal point is moved 3 places to the left]

(ii) Given 5,00,00,000

5,00,00,000 = 5,00,00,000.00 = 5 x 10^7 [since the decimal point is moved 7 places to the left]

(iii) Given 3,18,65,00,000

3,18,65,00,000 = 3,18,65,00,000.00

= 3.1865 x 10^9 [since the decimal point is moved 9 places to the left]

(iv) Given846 × 10^7

846 × 10^7 = 8.46 x 10^2 x 10 [since the decimal point is moved 2 places to the left]

= 8.46 x 10^9 [since a^m x a^n = a^{m+n}]

(v) Given 723 × 10^9

723 × 10^9 = 7.23 x 10^2 x 10^9 [since the decimal point is moved 2 places to the left]

= 7.23 x 10^{11} [since a^m x a^n = a^{m+n}]

2. Write the following numbers in the usual form:
(i) 4.83 × 10^7
(ii) 3.21 × 10^5
(iii) 3.5 × 10^3

Solution:

(i) Given 4.83 × 10^7

4.83 × 10^7 = 483 × 10^{7-2} [since the decimal point is moved two places to the right]

= 483 × 10^5

= 4, 83, 00,000

(ii) Given 3.21 × 10^5

3.21 × 10^5 = 321 x 10^{5-2} [since the decimal point is moved two places to the right]

= 321 x 10^3

= 3, 21,000

(iii) Given 3.5 × 10^3

3.5 × 10^3 = 35 x 10^{3-1} [since the decimal point is moved one place to the right]

= 35 x 10^2

= 3,500

3. Express the numbers appearing in the following statements in the standard form:

(i) The distance between the Earth and the Moon is 384,000,000 meters.
(ii) Diameter of the Earth is 1, 27, 56,000 meters.
(iii) Diameter of the Sun is 1,400,000,000 meters.
(iv) The universe is estimated to be about 12,000,000,000 years old.

Solution:

(i) Given the distance between the Earth and the Moon is 384,000,000 meters.

The distance between the Earth and the Moon is 3.84 x 10^8 meters.

[Since the decimal point is moved 8 places to the left.]
(ii) Given diameter of the Earth is 1, 27, 56,000 meters.

The diameter of the Earth is 1.2756 x 10^7 meters.

[Since the decimal point is moved 7 places to the left.]
(iii) Given diameter of the Sun is 1,400,000,000 meters.

The diameter of the Sun is 1.4×10^9 meters.

[Since the decimal point is moved 9 places to the left.]
(iv) Given the universe is estimated to be about 12,000,000,000 years old.

The universe is estimated to be about 1.2×10^{10} years old.

[Since the decimal point is moved 10 places to the left.]

Exercise 6.4

1. Write the following numbers in the expanded exponential forms:
(i) 20068
(ii) 420719
(iii) 7805192
(iv) 5004132
(v) 927303

Solution:

(i) Given 20068

$20068 = 2 \times 10^4 + 0 \times 10^3 + 0 \times 10^2 + 6 \times 10^1 + 8 \times 10^0$

(ii) Given 420719

$420719 = 4 \times 10^5 + 2 \times 10^4 + 0 \times 10^3 + 7 \times 10^2 + 1 \times 10^1 + 9 \times 10^0$

(iii) Given 7805192

$7805192 = 7 \times 10^6 + 8 \times 10^5 + 0 \times 10^4 + 5 \times 10^3 + 1 \times 10^2 + 9 \times 10^1 + 2 \times 10^0$

(iv) Given 5004132

$5004132 = 5 \times 10^6 + 0 \times 10^5 + 0 \times 10^4 + 4 \times 10^3 + 1 \times 10^2 + 3 \times 10^1 + 2 \times 10^0$

(v) Given 927303

$927303 = 9 \times 10^5 + 2 \times 10^4 + 7 \times 10^3 + 3 \times 10^2 + 0 \times 10^1 + 3 \times 10^0$

2. Find the number from each of the following expanded forms:
(i) $7 \times 10^4 + 6 \times 10^3 + 0 \times 10^2 + 4 \times 10^1 + 5 \times 10^0$
(ii) $5 \times 10^5 + 4 \times 10^4 + 2 \times 10^3 + 3 \times 10^0$
(iii) $9 \times 10^5 + 5 \times 10^2 + 3 \times 10^1$
(iv) $3 \times 10^4 + 4 \times 10^2 + 5 \times 10^0$

Solution:

(i) Given $7 \times 10^4 + 6 \times 10^3 + 0 \times 10^2 + 4 \times 10^1 + 5 \times 10^0$

$= 7 \times 10000 + 6 \times 1000 + 0 \times 100 + 4 \times 10 + 5 \times 1$

$= 70000 + 6000 + 0 + 40 + 5$

$= 76045$

(ii) Given $5 \times 10^5 + 4 \times 10^4 + 2 \times 10^3 + 3 \times 10^0$

$= 5 \times 100000 + 4 \times 10000 + 2 \times 1000 + 3 \times 1$

$= 500000 + 40000 + 2000 + 3$

$= 542003$

(iii) Given $9 \times 10^5 + 5 \times 10^2 + 3 \times 10^1$

$= 9 \times 100000 + 5 \times 100 + 3 \times 10$

$= 900000 + 500 + 30$

$= 900530$

(iv) Given $3 \times 10^4 + 4 \times 10^2 + 5 \times 10^0$

$= 3 \times 10000 + 4 \times 100 + 5 \times 1$

$= 30000 + 400 + 5$

= 30405

Chapter - 7 Algebraic Expressions

Exercise 7.1

1. Identify the monomials, binomials, trinomials and quadrinomials from the following expressions:

(i) a^2

(ii) $a^2 - b^2$

(iii) $x^3 + y^3 + z^3$

(iv) $x^3 + y^3 + z^3 + 3xyz$

(v) 7 + 5

(vi) a b c + 1

(vii) 3x – 2 + 5

(viii) 2x – 3y + 4

(ix) x y + y z + z x

(x) $ax^3 + bx^2 + cx + d$

Solution:

(i) Given a^2

a^2 is a monomial expression because it contains only one term

(ii) Given $a^2 - b^2$

$a^2 - b^2$ is a binomial expression because it contains two terms

(iii) Given $x^3 + y^3 + z^3$

$x^3 + y^3 + z^3$ is a trinomial because it contains three terms

(iv) Given $x^3 + y^3 + z^3 + 3xyz$

$x^3 + y^3 + z^3 + 3xyz$ is a quadrinomial expression because it contains four terms

(v) Given 7 + 5

7 + 5 is a monomial expression because it contains only one term

(vi) Given a b c + 1

a b c + 1 is a binomial expression because it contains two terms

(vii) Given 3x – 2 + 5

3x – 2 + 5 is a binomial expression because it contains two terms

(viii) Given 2x – 3y + 4

2x – 3y + 4 is a trinomial because it contains three terms

(ix) Given x y + y z + z x

x y + y z + z x is a trinomial because it contains three terms

(x) Given $ax^3 + bx^2 + cx + d$

$ax^3 + bx^2 + cx + d$ is a quadrinomial expression because it contains four terms

2. Write all the terms of each of the following algebraic expressions:

(i) $3x$

(ii) $2x – 3$

(iii) $2x^2 – 7$

(iv) $2x^2 + y^2 – 3xy + 4$

Solution:

(i) Given $3x$

$3x$ is the only term of the given algebraic expression.

(ii) Given $2x – 3$

$2x$ and -3 are the terms of the given algebraic expression.

(iii) Given $2x^2 – 7$

$2x^2$ and $–7$ are the terms of the given algebraic expression.

(iv) Given $2x^2 + y^2 – 3xy + 4$

$2x^2$, y^2, $–3xy$ and 4 are the terms of the given algebraic expression.

3. Identify the terms and also mention the numerical coefficients of those terms:

(i) $4xy, -5x^2y, -3yx, 2xy^2$

(ii) $7a^2bc,-3ca^2b,-(5/2)\ abc^2,\ 3/2abc^2,(-4/3)cba^2$

Solution:

(i) Like terms $4xy$, $-3yx$ and Numerical coefficients 4, -3

(ii) Like terms ($7a^2bc$, $–3ca^2b$) and ($–4/3cba^2$) and their Numerical coefficients 7, -3,

(-4/3)

Like terms are ($–5/2abc^2$) and ($3/2\ abc^2$) and numerical coefficients are ($–5/2$) and ($3/2$)

4. Identify the like terms in the following algebraic expressions:

(i) $a^2 + b^2 -2a^2 + c^2 + 4a$

(ii) $3x + 4xy – 2yz + 52zy$

(iii) $abc + ab^2c + 2acb^2 + 3c^2ab + b^2ac – 2a^2bc + 3cab^2$

Solution:

(i) Given $a^2 + b^2 -2a^2 + c^2 + 4a$

The like terms in the given algebraic expressions are a^2 and $–2a^2$.

(ii) Given $3x + 4xy – 2yz + 52zy$

The like terms in the given algebraic expressions are $-2yz$ and $52zy$.

(iii) Given $abc + ab^2c + 2acb^2 + 3c^2ab + b^2ac – 2a^2bc + 3cab^2$

The like terms in the given algebraic expressions are ab^2c, $2acb^2$, b^2ac and $3cab^2$.

5. Write the coefficient of x in the following:

(i) –12x

(ii) –7xy

(iii) xyz

(iv) –7ax

Solution:

(i) Given -12x

The numerical coefficient of x is -12.

(ii) Given -7xy

The numerical coefficient of x is -7y.

(iii) Given xyz

The numerical coefficient of x is yz.

(iv) Given -7ax

The numerical coefficient of x is -7a.

6. Write the coefficient of x^2 in the following:

(i) $-3x^2$

(ii) $5x^2yz$

(iii) $5/7x^2z$

(iv) $(-3/2)\ ax^2 + yx$

Solution:

(i) Given $-3x^2$

The numerical coefficient of x^2 is -3.

(ii) Given $5x^2yz$

The numerical coefficient of x^2 is 5yz.

(iii) Given $5/7x^2z$

The numerical coefficient of x^2 is 5/7z.

(iv) Given $(-3/2)\ ax^2 + yx$

The numerical coefficient of x^2 is – (3/2) a.

7. Write the coefficient of:

(i) y in –3y

(ii) a in 2ab

(iii) z in –7xyz

(iv) p in –3pqr

(v) y^2 in $9xy^2z$

(vi) x^3 in $x^3 + 1$

(vii) x^2 in $-x^2$

Solution:

(i) Given $-3y$

The coefficient of y is -3.

(ii) Given 2ab

The coefficient of a is 2b.

(iii) Given -7xyz

The coefficient of z is -7xy.

(iv) Given -3pqr

The coefficient of p is -3qr.

(v) Given $9xy^2z$

The coefficient of y^2 is 9xz.

(vi) Given $x^3 + 1$

The coefficient of x^3 is 1.

(vii) Given $-x^2$

The coefficient of x^2 is -1.

8. Write the numerical coefficient of each in the following:

(i) xy

(ii) -6yz

(iii) 7abc

(iv) $-2x^3y^2z$

Solution:

(i) Given xy

The numerical coefficient in the term xy is 1.

(ii) Given -6yz

The numerical coefficient in the term – 6yz is – 6.

(iii) Given 7abc

The numerical coefficient in the term 7abc is 7.

(iv) Given $-2x^3y^2z$

The numerical coefficient in the term $-2x^3y^2z$ is -2.

9. Write the numerical coefficient of each term in the following algebraic expressions:

(i) $4x^2y - (3/2)xy + 5/2\ xy^2$

(ii) $(–5/3)x^2y + (7/4)xyz + 3$

Solution:

(i) Given $4x^2y – (3/2) xy + 5/2 xy^2$

Numerical coefficient of following algebraic expressions are given below

Term	Coefficient
$4x^2y$	4
$– (3/2) xy$	-(3/2)
$5/2 xy^2$	(5/2)

(ii) Given $(–5/3)x^2y + (7/4)xyz + 3$

Numerical coefficient of following algebraic expressions are given below

Term	Coefficient
$(–5/3)x^2y$	(-5/3)
$(7/4)xyz$	(7/4)
3	3

10. Write the constant term of each of the following algebraic expressions:

(i) $x^2y – xy^2 + 7xy – 3$

(ii) $a^3 – 3a^2 + 7a + 5$

Solution:

(i) Given $x^2y – xy^2 + 7xy – 3$

The constant term in the given algebraic expressions is -3.

(ii) Given $a^3 – 3a^2 + 7a + 5$

The constant term in the given algebraic expressions is 5.

11. Evaluate each of the following expressions for x = -2, y = -1, z = 3:

(i) (x/y) + (y/z) + (z/x)

(ii) $x^2 + y^2 + z^2 – xy – yz – zx$

Solution:

(i) Given x = -2, y = -1, z = 3

Consider (x/y) + (y/z) + (z/x)

On substituting the given values we get,

= (-2/-1) + (-1/3) + (3/-2)

The LCM of 3 and 2 is 6

= (12 – 2 – 9)/6

= (1/6)

(ii) Given x = -2, y = -1, z = 3

Consider $x^2 + y^2 + z^2 - xy - yz - zx$

On substituting the given values we get,

= $(-2)^2 + (-1)^2 + 3^2 - (-2)(-1) - (-1)(3) - (3)(-2)$

= 4 + 1 + 9 – 2 + 3 + 6

= 23 – 2

= 21

12. Evaluate each of the following algebraic expressions for x = 1, y = -1, z = 2, a = -2, b = 1, c = -2:

(i) ax + by + cz

(ii) $ax^2 + by^2 - cz$

(iii) axy + byz + cxy

Solution:

(i) Given x = 1, y = -1, z = 2, a = -2, b = 1, c = -2

Consider ax + by + cz

On substituting the given values

= (-2) (1) + (1) (-1) + (-2) (2)

= –2 – 1 – 4

= –7

(ii) Given x = 1, y = -1, z = 2, a = -2, b = 1, c = -2

Consider $ax^2 + by^2 - cz$

On substituting the given values

= $(-2) \times 1^2 + 1 \times (-1)^2 - (-2) \times 2$

= -2 + 1 – (-4)

= -1 + 4

= 3

(iii) Given x = 1, y = -1, z = 2, a = -2, b = 1, c = -2

Consider axy + byz + cxy

= (-2) × 1 × -1 + 1 × -1 × 2 + (-2) × 1 × (-1)

= 2 + (-2) + 2

= 4 – 2

= 2

Exercise 7.2

1. Add the following:

(i) 3x and 7x

(ii) -5xy and 9xy

Solution:

(i) Given 3x and 7x

$3x + 7x = (3 + 7)\ x$

$= 10x$

(ii) Given -5xy and 9xy

$-5xy + 9xy = (-5 + 9)\ xy$

$= 4xy$

2. Simplify each of the following:

(i) $7x^3y + 9yx^3$

(ii) $12a^2b + 3ba^2$

Solution:

(i) Given $7x^3y + 9yx^3$

$7x^3y + 9yx^3 = (7 + 9)\ x^3y$

$= 16x^3y$

(ii) Given

$12a^2b + 3ba^2 = (12 + 3)\ a^2b$

$= 15a^2b$

3. Add the following:

(i) 7abc, -5abc, 9abc, -8abc

(ii) $2x^2y$, $-4x^2y$, $6x^2y$, $-5x^2y$

Solution:

(i) Given 7abc, -5abc, 9abc, -8abc

Consider 7abc + (-5abc) + (9abc) + (-8abc)

= 7abc – 5abc + 9abc – 8abc

= (7 – 5 + 9 – 8) abc [by taking abc common]

= (16 – 13) abc

= 3abc

(ii) Given $2x^2y$, $-4x^2y$, $6x^2y$, $-5x^2y$

$2x^2y + (-4x^2y) + (6x^2y) + (-5x^2y)$

$= 2x^2y - 4x^2y + 6x^2y - 5x^2y$

$= (2 - 4 + 6 – 5)\ x^2y$ [by taking $x^2\ y$ common]

$= (8 – 9)\ x^2y$

$= -x^2y$

4. Add the following expressions:

(i) $x^3 - 2x^2y + 3xy^2 – y^3,\ 2x^3 – 5xy^2 + 3x^2y – 4y^3$

(ii) $a^4 – 2a^3b + 3ab^3 + 4a^2b^2 + 3b^4,\ – 2a^4 – 5ab^3 + 7a^3b – 6a^2b^2 + b^4$

Solution:

(i) Given $x^3 - 2x^2y + 3xy^2 – y^3,\ 2x^3 – 5xy^2 + 3x^2y – 4y^3$

Collecting positive and negative like terms together, we get

$= x^3 + 2x^3 – 2x^2y + 3x^2y + 3xy^2 – 5xy^2 – y^3 – 4y^3$

$= 3x^3 + x^2y – 2xy^2 – 5y^3$

(ii) Given $a^4 – 2a^3b + 3ab^3 + 4a^2b^2 + 3b^4,\ – 2a^4 – 5ab^3 + 7a^3b – 6a^2b^2 + b^4$

$= a^4 – 2a^3b + 3ab^3 + 4a^2b^2 + 3b^4 – 2a^4 – 5ab^3 + 7a^3b – 6a^2b^2 + b^4$

Collecting positive and negative like terms together, we get

$= a^4 – 2a^4 – 2a^3b + 7a^3b + 3ab^3 – 5ab^3 + 4a^2b^2 – 6a^2b^2 + 3b^4 + b^4$

$= – a^4 + 5a^3b – 2ab^3 – 2a^2b^2 + 4b^4$

5. Add the following expressions:

(i) $8a – 6ab + 5b,\ –6a – ab – 8b$ and $–4a + 2ab + 3b$

(ii) $5x^3 + 7 + 6x – 5x^2,\ 2x^2 – 8 – 9x,\ 4x – 2x^2 + 3x^3,\ 3x^3 – 9x – x^2$ and $x – x^2 – x^3 – 4$

Solution:

(i) Given $8a – 6ab + 5b,\ –6a – ab – 8b$ and $–4a + 2ab + 3b$

$= (8a – 6ab + 5b) + (–6a – ab – 8b) + (–4a + 2ab + 3b)$

Collecting positive and negative like terms together, we get

$= 8a – 6a – 4a – 6ab – ab + 2ab + 5b – 8b + 3b$

$= 8a – 10a – 7ab + 2ab + 8b – 8b$

$= –2a – 5ab$

(ii) Given $5x^3 + 7 + 6x – 5x^2,\ 2x^2 – 8 – 9x,\ 4x – 2x^2 + 3x^3,\ 3x^3 – 9x – x^2$ and $x – x^2 – x^3 – 4$

$= (5x^3 + 7 + 6x – 5x^2) + (2x^2 – 8 – 9x) + (4x – 2x^2 + 3x^3) + (3x^3 – 9x - x^2) + (x – x^2 – x^3 – 4)$

Collecting positive and negative like terms together, we get

$5x^3 + 3x^3 + 3x^3 – x^3 – 5x^2 + 2x^2 – 2x^2 – x^2 – x^2 + 6x – 9x + 4x – 9x + x + 7 – 8 – 4$

$= 10x^3 – 7x^2 – 7x – 5$

6. Add the following:

(i) $x – 3y – 2z$

$5x + 7y – 8z$

$3x - 2y + 5z$

(ii) $4ab - 5bc + 7ca$

$-3ab + 2bc - 3ca$

$5ab - 3bc + 4ca$

Solution:

(i) Given $x - 3y - 2z$, $5x + 7y - 8z$ and $3x - 2y + 5z$

$= (x - 3y - 2z) + (5x + 7y - 8z) + (3x - 2y + 5z)$

Collecting positive and negative like terms together, we get

$= x + 5x + 3x - 3y + 7y - 2y - 2z - 8z + 5z$

$= 9x - 5y + 7y - 10z + 5z$

$= 9x + 2y - 5z$

(ii) Given $4ab - 5bc + 7ca$, $-3ab + 2bc - 3ca$ and $5ab - 3bc + 4ca$

$= (4ab - 5bc + 7ca) + (-3ab + 2bc - 3ca) + (5ab - 3bc + 4ca)$

Collecting positive and negative like terms together, we get

$= 4ab - 3ab + 5ab - 5bc + 2bc - 3bc + 7ca - 3ca + 4ca$

$= 9ab - 3ab - 8bc + 2bc + 11ca - 3ca$

$= 6ab - 6bc + 8ca$

7. Add $2x^2 - 3x + 1$ to the sum of $3x^2 - 2x$ and $3x + 7$.

Solution:

Given $2x^2 - 3x + 1$, $3x^2 - 2x$ and $3x + 7$

sum of $3x^2 - 2x$ and $3x + 7$

$= (3x^2 - 2x) + (3x + 7)$

$= 3x^2 - 2x + 3x + 7$

$= (3x^2 + x + 7)$

Now, required expression $= 2x^2 - 3x + 1 + (3x^2 + x + 7)$

$= 2x^2 + 3x^2 - 3x + x + 1 + 7$

$= 5x^2 - 2x + 8$

8. Add $x^2 + 2xy + y^2$ to the sum of $x^2 - 3y^2$ and $2x^2 - y^2 + 9$.

Solution:

Given $x^2 + 2xy + y^2$, $x^2 - 3y^2$ and $2x^2 - y^2 + 9$.

First we have to find the sum of $x^2 - 3y^2$ and $2x^2 - y^2 + 9$

$= (x^2 - 3y^2) + (2x^2 - y^2 + 9)$

$= x^2 + 2x^2 - 3y^2 - y^2 + 9$

$= 3x^2 - 4y^2 + 9$

Now, required expression = $(x^2 + 2xy + y^2) + (3x^2 - 4y^2 + 9)$

$= x^2 + 3x^2 + 2xy + y^2 - 4y^2 + 9$

$= 4x^2 + 2xy - 3y^2 + 9$

9. Add $a^3 + b^3 - 3$ to the sum of $2a^3 - 3b^3 - 3ab + 7$ and $-a^3 + b^3 + 3ab - 9$.

Solution:

Given $a^3 + b^3 - 3$, $2a^3 - 3b^3 - 3ab + 7$ and $-a^3 + b^3 + 3ab - 9$.

First, we need to find the sum of $2a^3 - 3b^3 - 3ab + 7$ and $-a^3 + b^3 + 3ab - 9$.

$= (2a^3 - 3b^3 - 3ab + 7) + (-a^3 + b^3 + 3ab - 9)$

Collecting positive and negative like terms together, we get

$= 2a^3 - a^3 - 3b^3 + b^3 - 3ab + 3ab + 7 - 9$

$= a^3 - 2b^3 - 2$

Now, the required expression = $(a^3 + b^3 - 3) + (a^3 - 2b^3 - 2)$.

$= a^3 + a^3 + b^3 - 2b^3 - 3 - 2$

$= 2a^3 - b^3 - 5$

10. Subtract:

(i) $7a^2b$ from $3a^2b$

(ii) $4xy$ from $-3xy$

Solution:

(i) Given $7a^2b$ from $3a^2b$

$= 3a^2b - 7a^2b$

$= (3 - 7)\, a^2b$

$= -4a^2b$

(ii) Given $4xy$ from $-3xy$

$= -3xy - 4xy$

$= -7xy$

11. Subtract:

(i) $-4x$ from $3y$

(ii) $-2x$ from $-5y$

Solution:

(i) Given $-4x$ from $3y$

$= (3y) - (-4x)$

$= 3y + 4x$

(ii) Given $-2x$ from $-5y$

$= (-5y) - (-2x)$

$= -5y + 2x$

12. Subtract:

(i) $6x^3 - 7x^2 + 5x - 3$ from $4 - 5x + 6x^2 - 8x^3$

(ii) $- x^2 - 3z$ from $5x^2 - y + z + 7$

(iii) $x^3 + 2x^2y + 6xy^2 - y^3$ from $y^3 - 3xy^2 - 4x^2y$

Solution:

(i) Given $6x^3 - 7x^2 + 5x - 3$ and $4 - 5x + 6x^2 - 8x^3$

$= (4 - 5x + 6x^2 - 8x^3) - (6x^3 - 7x^2 + 5x - 3)$

$= 4 - 5x + 6x^2 - 8x^3 - 6x^3 + 7x^2 - 5x + 3$

$= - 8x^3 - 6x^3 + 7x^2 + 6x^2 - 5x - 5x + 3 + 4$

$= - 14x^3 + 13x^2 - 10x + 7$

(ii) Given $- x^2 - 3z$ and $5x^2 - y + z + 7$

$= (5x^2 - y + z + 7) - (- x^2 - 3z)$

$= 5x^2 - y + z + 7 + x^2 + 3z$

$= 5x^2 + x^2 - y + z + 3z + 7$

$= 6x^2 - y + 4z + 7$

(iii) Given $x^3 + 2x^2y + 6xy^2 - y^3$ and $y^3 - 3xy^2 - 4x^2y$

$= (y^3 - 3xy^2 - 4x^2y) - (x^3 + 2x^2y + 6xy^2 - y^3)$

$= y^3 - 3xy^2 - 4x^2y - x^3 - 2x^2y - 6xy^2 + y^3$

$= y^3 + y^3 - 3xy^2 - 6xy^2 - 4x^2y - 2x^2y - x^3$

$= 2y^3 - 9xy^2 - 6x^2y - x^3$

13. From

(i) $p^3 - 4 + 3p^2$, take away $5p^2 - 3p^3 + p - 6$

(ii) $7 + x - x^2$, take away $9 + x + 3x^2 + 7x^3$

(iii) $1 - 5y^2$, take away $y^3 + 7y^2 + y + 1$

(iv) $x^3 - 5x^2 + 3x + 1$, take away $6x^2 - 4x^3 + 5 + 3x$

Solution:

(i) Given $p^3 - 4 + 3p^2$, take away $5p^2 - 3p^3 + p - 6$

$= (p^3 - 4 + 3p^2) - (5p^2 - 3p^3 + p - 6)$

$= p^3 - 4 + 3p^2 - 5p^2 + 3p^3 - p + 6$

$= p^3 + 3p^3 + 3p^2 - 5p^2 - p - 4 + 6$

$= 4p^3 - 2p^2 - p + 2$

(ii) Given $7 + x - x^2$, take away $9 + x + 3x^2 + 7x^3$

$= (7 + x - x^2) - (9 + x + 3x^2 + 7x^3)$

$= 7 + x – x^2 – 9 – x – 3x^2 – 7x^3$

$= – 7x^3 – x^2 – 3x^2 + 7 – 9$

$= – 7x^3 – 4x^2 – 2$

(iii) Given $1 – 5y^2$, take away $y^3 + 7y^2 + y + 1$

$= (1 – 5y^2) – (y^3 + 7y^2 + y + 1)$

$= 1 – 5y^2 – y^3 – 7y^2 – y – 1$

$= – y^3 – 5y^2 – 7y^2 – y$

$= – y^3 – 12y^2 – y$

(iv) Given $x^3 – 5x^2 + 3x + 1$, take away $6x^2 – 4x^3 + 5 + 3x$

$= (x^3 – 5x^2 + 3x + 1) – (6x^2 – 4x^3 + 5 + 3x)$

$= x^3 – 5x^2 + 3x + 1 – 6x^2 + 4x^3 – 5 – 3x$

$= x^3 + 4x^3 – 5x^2 – 6x^2 + 1 – 5$

$= 5x^3 – 11x^2 – 4$

14. From the sum of $3x^2 – 5x + 2$ and $– 5x^2 – 8x + 9$ subtract $4x^2 – 7x + 9$.

Solution:

First we have to add $3x^2 – 5x + 2$ and $– 5x^2 – 8x + 9$ then from the result we have to subtract $4x^2 – 7x + 9$.

$= \{(3x^2 – 5x + 2) + (- 5x^2 – 8x + 9)\} – (4x^2 – 7x + 9)$

$= \{3x^2 – 5x + 2 – 5x^2 – 8x + 9\} – (4x^2 – 7x + 9)$

$= \{3x^2 – 5x^2 – 5x – 8x + 2 + 9\} – (4x^2 – 7x + 9)$

$= \{- 2x^2 – 13x + 11\} – (4x^2 – 7x + 9)$

$= – 2x^2 – 13x + 11 – 4x^2 + 7x – 9$

$= – 2x^2 – 4x^2 – 13x + 7x + 11 – 9$

$= – 6x^2 – 6x + 2$

15. Subtract the sum of $13x – 4y + 7z$ and $– 6z + 6x + 3y$ from the sum of $6x – 4y – 4z$ and $2x + 4y – 7$.

Solution:

First we have to find the sum of $13x – 4y + 7z$ and $– 6z + 6x + 3y$

Therefore, sum of $(13x – 4y + 7z)$ and $(–6z + 6x + 3y)$

$= (13x – 4y + 7z) + (–6z + 6x + 3y)$

$= (13x – 4y + 7z – 6z + 6x + 3y)$

$= (13x + 6x – 4y + 3y + 7z – 6z)$

$= (19x – y + z)$

Now we have to find the sum of $(6x – 4y – 4z)$ and $(2x + 4y – 7)$

$= (6x – 4y – 4z) + (2x + 4y – 7)$

$= (6x – 4y – 4z + 2x + 4y – 7)$

$= (6x + 2x – 4z – 7)$

$= (8x – 4z – 7)$

Now, required expression $= (8x – 4z – 7) – (19x – y + z)$

$= 8x – 4z – 7 – 19x + y – z$

$= 8x – 19x + y – 4z – z – 7$

$= –11x + y – 5z – 7$

16. From the sum of $x^2 + 3y^2 – 6xy$, $2x^2 – y^2 + 8xy$, $y^2 + 8$ and $x^2 – 3xy$ subtract $–3x^2 + 4y^2 – xy + x – y + 3$.

Solution:

First we have to find the sum of $(x^2 + 3y^2 – 6xy)$, $(2x^2 – y^2 + 8xy)$, $(y^2 + 8)$ and $(x^2 – 3xy)$

$=\{(x^2 + 3y^2 – 6xy) + (2x^2 – y^2 + 8xy) + (y^2 + 8) + (x^2 – 3xy)\}$

$=\{x^2 + 3y^2 – 6xy + 2x^2 – y^2 + 8xy + y^2 + 8 + x^2 – 3xy\}$

$= \{x^2+ 2x^2+ x^2 + 3y^2– y^2 + y^2– 6xy + 8xy – 3xy + 8\}$

$= 4x^2 + 3y^2 – xy + 8$

Now, from the result subtract the $–3x^2 + 4y^2 – xy + x – y + 3$.

Therefore, required expression $= (4x^2 + 3y^2 – xy + 8) – (- 3x^2 + 4y^2 – xy + x – y + 3)$

$= 4x^2 + 3y^2 – xy + 8 + 3x^2 – 4y^2 + xy – x + y – 3$

$= 4x^2 + 3x^2+ 3y^2– 4y^2– x + y – 3 + 8$

$= 7x^2 – y^2– x + y + 5$

17. What should be added to $xy – 3yz + 4zx$ to get $4xy – 3zx + 4yz + 7$?

Solution:

By subtracting $xy – 3yz + 4zx$ from $4xy – 3zx + 4yz + 7$, we get the required expression.

Therefore, required expression $= (4xy – 3zx + 4yz + 7) – (xy – 3yz + 4zx)$

$= 4xy – 3zx + 4yz + 7 – xy + 3yz – 4zx$

$= 4xy – xy – 3zx – 4zx + 4yz + 3yz + 7$

$= 3xy – 7zx + 7yz + 7$

18. What should be subtracted from $x^2 – xy + y^2 – x + y + 3$ to obtain $–x^2 + 3y^2 – 4xy + 1$?

Solution:

Let 'E' be the required expression. Then, we have

$x^2 – xy + y^2– x + y + 3 – E = – x^2 + 3y^2 – 4xy + 1$

Therefore, $E = (x^2 – xy + y^2– x + y + 3) – (- x^2 + 3y^2 – 4xy + 1)$

$= x^2 – xy + y^2– x + y + 3 + x^2 – 3y^2 + 4xy – 1$

Collecting positive and negative like terms together, we get

$= x^2 + x^2– xy + 4xy + y^2– 3y^2 – x + y + 3 – 1$

$= 2x^2 + 3xy - 2y^2 - x + y + 2$

19. How much is $x - 2y + 3z$ greater than $3x + 5y - 7$?

Solution:

By subtracting $x - 2y + 3z$ from $3x + 5y - 7$ we can get the required expression,

Required expression $= (x - 2y + 3z) - (3x + 5y - 7)$

$= x - 2y + 3z - 3x - 5y + 7$

Collecting positive and negative like terms together, we get

$= x - 3x - 2y + 5y + 3z + 7$

$= -2x - 7y + 3z + 7$

20. How much is $x^2 - 2xy + 3y^2$ less than $2x^2 - 3y^2 + xy$?

Solution:

By subtracting the $x^2 - 2xy + 3y^2$ from $2x^2 - 3y^2 + xy$ we can get the required expression,

Required expression $= (2x^2 - 3y^2 + xy) - (x^2 - 2xy + 3y^2)$

$= 2x^2 - 3y^2 + xy - x^2 + 2xy - 3y^2$

Collecting positive and negative like terms together, we get

$= 2x^2 - x^2 - 3y^2 - 3y^2 + xy + 2xy$

$= x^2 - 6y^2 + 3xy$

21. How much does $a^2 - 3ab + 2b^2$ exceed $2a^2 - 7ab + 9b^2$?

Solution:

By subtracting $2a^2 - 7ab + 9b^2$ from $a^2 - 3ab + 2b^2$ we get the required expression

Required expression $= (a^2 - 3ab + 2b^2) - (2a^2 - 7ab + 9b^2)$

$= a^2 - 3ab + 2b^2 - 2a^2 + 7ab - 9b^2$

Collecting positive and negative like terms together, we get

$= a^2 - 2a^2 - 3ab + 7ab + 2b^2 - 9b^2$

$= - a^2 + 4ab - 7b^2$

22. What must be added to $12x^3 - 4x^2 + 3x - 7$ to make the sum $x^3 + 2x^2 - 3x + 2$?

Solution:

Let 'E' be the required expression. Thus, we have

$12x^3 - 4x^2 + 3x - 7 + E = x^3 + 2x^2 - 3x + 2$

Therefore, $E = (x^3 + 2x^2 - 3x + 2) - (12x^3 - 4x^2 + 3x - 7)$

$= x^3 + 2x^2 - 3x + 2 - 12x^3 + 4x^2 - 3x + 7$

Collecting positive and negative like terms together, we get

$= x^3 - 12x^3 + 2x^2 + 4x^2 - 3x - 3x + 2 + 7$

$= - 11x^3 + 6x^2 - 6x + 9$

23. If $P = 7x^2 + 5xy – 9y^2$, $Q = 4y^2 – 3x^2 – 6xy$ and $R = –4x^2 + xy + 5y^2$, show that $P + Q + R = 0$.

Solution:

Given $P = 7x^2 + 5xy – 9y^2$, $Q = 4y^2 – 3x^2 – 6xy$ and $R = –4x^2 + xy + 5y^2$

Now we have to prove $P + Q + R = 0$,

Consider $P + Q + R = (7x^2 + 5xy – 9y^2) + (4y^2 – 3x^2 – 6xy) + (- 4x^2 + xy + 5y^2)$

$= 7x^2 + 5xy – 9y^2 + 4y^2 – 3x^2 – 6xy – 4x^2 + xy + 5y^2$

Collecting positive and negative like terms together, we get

$= 7x^2– 3x^2 – 4x^2 + 5xy – 6xy + xy – 9y^2 + 4y^2 + 5y^2$

$= 7x^2– 7x^2 + 6xy – 6xy – 9y^2 + 9y^2$

$= 0$

24. If $P = a^2 – b^2 + 2ab$, $Q = a^2 + 4b^2 – 6ab$, $R = b^2 + b$, $S = a^2 – 4ab$ and $T = –2a^2 + b^2 – ab + a$. Find $P + Q + R + S – T$.

Solution:

Given $P = a^2 – b^2 + 2ab$, $Q = a^2 + 4b^2 – 6ab$, $R = b^2 + b$, $S = a^2 – 4ab$ and $T = –2a^2 + b^2 – ab + a$.

Now we have to find $P + Q + R + S – T$

Substituting all values we get

Consider $P + Q + R + S – T = \{(a^2 – b^2 + 2ab) + (a^2 + 4b^2 – 6ab) + (b^2 + b) + (a^2 – 4ab)\} – (-2a^2 + b^2 – ab + a)$

$= \{a^2 – b^2 + 2ab + a^2 + 4b^2 – 6ab + b^2 + b + a^2 – 4ab\}- (- 2a^2 + b^2 – ab + a)$

$= \{3a^2 + 4b^2 – 8ab + b \} – (-2a^2 + b^2 – ab + a)$

$= 3a^2+ 4b^2 – 8ab + b + 2a^2 – b^2 + ab – a$

Collecting positive and negative like terms together, we get

$3a^2+ 2a^2 + 4b^2 – b^2 – 8ab + ab – a + b$

$= 5a^2 + 3b^2– 7ab – a + b$

Exercise 7.3

1. Place the last two terms of the following expressions in parentheses preceded by a minus sign:

(i) $x + y – 3z + y$

(ii) $3x – 2y – 5z – 4$

(iii) $3a – 2b + 4c – 5$

(iv) $7a + 3b + 2c + 4$

(v) $2a^2 – b^2 – 3ab + 6$

(vi) $a^2 + b^2 – c^2 + ab – 3ac$

Solution:

(i) Given $x + y – 3z + y$

$x + y – 3z + y = x + y – (3z – y)$

(ii) Given $3x – 2y – 5z – 4$

$3x – 2y – 5z – 4 = 3x – 2y – (5z + 4)$

(iii) Given $3a – 2b + 4c – 5$

$3a – 2b + 4c – 5 = 3a – 2b – (–4c + 5)$

(iv) Given $7a + 3b + 2c + 4$

$7a + 3b + 2c + 4 = 7a + 3b – (–2c – 4)$

(v) Given $2a^2 – b^2 – 3ab + 6$

$2a^2 – b^2 – 3ab + 6 = 2a^2 – b^2 – (3ab – 6)$

(vi) Given $a^2 + b^2 – c^2 + ab – 3ac$

$a^2 + b^2 – c^2 + ab – 3ac = a^2 + b^2 – c^2 – (- ab + 3ac)$

2. Write each of the following statements by using appropriate grouping symbols:

(i) The sum of $a – b$ and $3a – 2b + 5$ is subtracted from $4a + 2b – 7$.

(ii) Three times the sum of $2x + y – [5 – (x – 3y)]$ and $7x – 4y + 3$ is subtracted from $3x – 4y + 7$

(iii) The subtraction of $x^2 – y^2 + 4xy$ from $2x^2 + y^2 – 3xy$ is added to $9x^2 – 3y^2 – xy$.

Solution:

(i) Given the sum of $a – b$ and $3a – 2b + 5 = [(a – b) + (3a – 2b + 5)]$.

This is subtracted from $4a + 2b – 7$.

Thus, the required expression is $(4a + 2b – 7) – [(a – b) + (3a – 2b + 5)]$

(ii) Given three times the sum of $2x + y – \{5 – (x – 3y)\}$ and $7x – 4y + 3 = 3[(2x + y – \{5 – (x – 3y)\}) + (7x – 4y + 3)]$

This is subtracted from $3x – 4y + 7$.

Thus, the required expression is $(3x – 4y + 7) – 3[(2x + y – \{5 – (x – 3y)\}) + (7x – 4y + 3)]$

(iii) Given the product of subtraction of $x^2 – y^2 + 4xy$ from $2x^2 + y^2 – 3xy$ is given by $\{(2x^2 + y^2 – 3xy) – (x^2 - y^2 + 4xy)\}$

When the above equation is added to $9x^2 – 3y^2 – xy$, we get

$\{(2x^2 + y^2 – 3xy) – (x^2 – y^2 + 4xy)\} + (9x^2 – 3y^2 – xy))$

This is the required expression.

Exercise 7.4

Simplify each of the following algebraic expressions by removing grouping symbols.

1. $2x + (5x – 3y)$

Solution:

Given $2x + (5x – 3y)$

Since the ‘+’ sign precedes the parentheses, we have to retain the sign of each term in the parentheses when we remove them.

= 2x + 5x – 3y

On simplifying, we get

= 7x – 3y

2. 3x – (y – 2x)

Solution:

Given 3x – (y – 2x)

Since the ‘–’ sign precedes the parentheses, we have to change the sign of each term in the parentheses when we remove them. Therefore, we have

= 3x – y + 2x

On simplifying, we get

= 5x – y

3. 5a – (3b – 2a + 4c)

Solution:

Given 5a – (3b – 2a + 4c)

Since the ‘-‘sign precedes the parentheses, we have to change the sign of each term in the parentheses when we remove them.

= 5a – 3b + 2a – 4c

On simplifying, we get

= 7a – 3b – 4c

4. $-2(x^2 – y^2 + xy) – 3(x^2 + y^2 – xy)$

Solution:

Given $– 2(x^2 – y^2 + xy) – 3(x^2 + y^2 – xy)$

Since the ‘–’ sign precedes the parentheses, we have to change the sign of each term in the parentheses when we remove them. Therefore, we have

$= -2x^2 + 2y^2 – 2xy – 3x^2 – 3y^2 + 3xy$

On rearranging,

$= -2x^2 – 3x^2 + 2y^2 – 3y^2 – 2xy + 3xy$

On simplifying, we get

$= -5x^2 – y^2 + xy$

5. 3x + 2y – {x – (2y – 3)}

Solution:

Given 3x + 2y – {x – (2y – 3)}

First, we have to remove the parentheses. Then, we have to remove the braces.

Then we get,

= 3x + 2y – {x – 2y + 3}

= 3x + 2y – x + 2y – 3

On simplifying, we get

= 2x + 4y – 3

6. 5a – {3a – (2 – a) + 4}

Solution:

Given 5a – {3a – (2 – a) + 4}

First, we have to remove the parentheses. Then, we have to remove the braces.

Then we get,

= 5a – {3a – 2 + a + 4}

= 5a – 3a + 2 – a – 4

On simplifying, we get

= 5a – 4a – 2

= a – 2

7. a – [b – {a – (b – 1) + 3a}]

Solution:

Given a – [b – {a – (b – 1) + 3a}]

First we have to remove the parentheses, then the curly brackets, and then the square brackets.

Then we get,

= a – [b – {a – (b – 1) + 3a}]

= a – [b – {a – b + 1 + 3a}]

= a – [b – {4a – b + 1}]

= a – [b – 4a + b – 1]

= a – [2b – 4a – 1]

On simplifying, we get

= a – 2b + 4a + 1

= 5a – 2b + 1

8. a – [2b – {3a – (2b – 3c)}]

Solution:

Given a – [2b – {3a – (2b – 3c)}]

First we have to remove the parentheses, then the braces, and then the square brackets.

Then we get,

= a – [2b – {3a – (2b – 3c)}]

= a – [2b – {3a – 2b + 3c}]

= a – [2b – 3a + 2b – 3c]

= a – [4b – 3a – 3c]

On simplifying we get,

= a – 4b + 3a + 3c

= 4a – 4b + 3c

9. -x + [5y – {2x – (3y – 5x)}]

Solution:

Given -x + [5y – {2x – (3y – 5x)}]

First we have to remove the parentheses, then remove braces, and then the square brackets.

Then we get,

= – x + [5y – {2x – (3y – 5x)}]

= – x + [5y – {2x – 3y + 5x)]

= – x + [5y – {7x – 3y}]

= – x + [5y – 7x + 3y]

= – x + [8y – 7x]

On simplifying we get

= – x + 8y – 7x

= – 8x + 8y

10. 2a – [4b – {4a – 3(2a – b)}]

Solution:

Given 2a – [4b – {4a – 3(2a – b)}]

First we have to remove the parentheses, then remove braces, and then the square brackets.

Then we get,

= 2a – [4b – {4a – 3(2a – b)}]

= 2a – [4b – {4a – 6a + 3b}]

= 2a – [4b – {- 2a + 3b}]

= 2a – [4b + 2a – 3b]

= 2a – [b + 2a]

On simplifying, we get

= 2a – b – 2a

= – b

11. -a – [a + {a + b – 2a – (a – 2b)} – b]

Solution:

Given -a – [a + {a + b – 2a – (a – 2b)} – b]

First we have to remove the parentheses, then remove braces, and then the square brackets.

Then we get,

$= - a - [a + \{a + b - 2a - (a - 2b)\} - b]$

$= - a - [a + \{a + b - 2a - a + 2b\} - b]$

$= - a - [a + \{- 2a + 3b\} - b]$

$= - a - [a - 2a + 3b - b]$

$= - a - [- a + 2b]$

On simplifying, we get

$= - a + a - 2b$

$= - 2b$

12. $2x - 3y - [3x - 2y -\{x - z - (x - 2y)\}]$

Solution:

Given $2x - 3y - [3x - 2y -\{x - z - (x - 2y)\}]$

First we have to remove the parentheses, then remove braces, and then the square brackets.

Then we get,

$= 2x - 3y - [3x - 2y - \{x - z - (x - 2y)\})$

$= 2x - 3y - [3x - 2y - \{x - z - x + 2y\}]$

$= 2x - 3y - [3x - 2y - \{- z + 2y\}]$

$= 2x - 3y - [3x - 2y + z - 2y]$

$= 2x - 3y - [3x - 4y + z]$

On simplifying, we get

$= 2x - 3y - 3x + 4y - z$

$= - x + y - z$

13. $5 + [x - \{2y - (6x + y - 4) + 2x\} - \{x - (y - 2)\}]$

Solution:

Given $5 + [x - \{2y - (6x + y - 4) + 2x\} - \{x - (y - 2)\}]$

First we have to remove the parentheses, then remove braces, and then the square brackets.

Then we get,

$= 5 + [x - \{2y - (6x + y - 4) + 2x\} - \{x - (y - 2)\}]$

$= 5 + [x - \{2y - 6x - y + 4 + 2x\} - \{x - y + 2\}]$

$= 5 + [x - \{y - 4x + 4\} - \{x - y + 2\}]$

$= 5 + [x - y + 4x - 4 - x + y - 2]$

$= 5 + [4x - 6]$

$= 5 + 4x - 6$

$= 4x - 1$

14. $x^2 - [3x + [2x - (x^2 - 1)] + 2]$

Solution:

Given $x^2 - [3x + [2x - (x^2 - 1)] + 2]$

First we have to remove the parentheses, then remove braces, and then the square brackets.

Then we get,

$= x^2 - [3x + [2x - (x^2 - 1)] + 2]$

$= x^2 - [3x + [2x - x^2 + 1] + 2]$

$= x^2 - [3x + 2x - x^2 + 1 + 2]$

$= x^2 - [5x - x^2 + 3]$

On simplifying we get

$= x^2 - 5x + x^2 - 3$

$= 2x^2 - 5x - 3$

15. $20 - [5xy + 3[x^2 - (xy - y) - (x - y)]]$

Solution:

Given $20 - [5xy + 3[x^2 - (xy - y) - (x - y)]]$

First we have to remove the parentheses, then remove braces, and then the square brackets.

Then we get,

$= 20 - [5xy + 3[x^2 - (xy - y) - (x - y)]]$

$= 20 - [5xy + 3[x^2 - xy + y - x + y]]$

$= 20 - [5xy + 3[x^2 - xy + 2y - x]]$

$= 20 - [5xy + 3x^2 - 3xy + 6y - 3x]$

$= 20 - [2xy + 3x^2 + 6y - 3x]$

On simplifying we get

$= 20 - 2xy - 3x^2 - 6y + 3x$

$= - 3x^2 - 2xy - 6y + 3x + 20$

16. $85 - [12x - 7(8x - 3) - 2\{10x - 5(2 - 4x)\}]$

Solution:

Given $85 - [12x - 7(8x - 3) - 2\{10x - 5(2 - 4x)\}]$

First we have to remove the parentheses, then remove braces, and then the square brackets.

Then we get,

$= 85 - [12x - 7(8x - 3) - 2\{10x - 5(2 - 4x)\}]$

$= 85 - [12x - 56x + 21 - 2\{10x - 10 + 20x\}]$

$= 85 - [12x - 56x + 21 - 2\{30x - 10\}]$

= 85 – [12x – 56x + 21 – 60x + 20]

= 85 – [12x – 116x + 41]

= 85 – [- 104x + 41]

On simplifying, we get

= 85 + 104x – 41

= 44 + 104x

17. xy [yz – zx – {yx – (3y – xz) – (xy – zy)}]

Solution:

Given xy [yz – zx – {yx – (3y – xz) – (xy – zy)}]

First we have to remove the parentheses, then remove braces, and then the square brackets.

Then we get,

= xy – [yz – zx – {yx – (3y – xz) – (xy – zy)}]

= xy – [yz – zx – {yx – 3y + xz – xy + zy}]

= xy – [yz – zx – {- 3y + xz + zy}]

= xy – [yz – zx + 3y – xz – zy]

= xy – [- zx + 3y – xz]

On simplifying, we get

= xy – [- 2zx + 3y]

= xy + 2xz – 3y

Chapter - 8 Linear Equations in One variable

Exercise 8.1

1. Verify by substitution that:

(i) $x = 4$ is the root of $3x - 5 = 7$

(ii) $x = 3$ is the root of $5 + 3x = 14$

(iii) $x = 2$ is the root of $3x - 2 = 8x - 12$

(iv) $x = 4$ is the root of $(3x/2) = 6$

(v) $y = 2$ is the root of $y - 3 = 2y - 5$

(vi) $x = 8$ is the root of $(1/2)x + 7 = 11$

Solution:

(i) Given $x = 4$ is the root of $3x - 5 = 7$

Now, substituting $x = 4$ in place of 'x' in the given equation, we get

$= 3(4) - 5 = 7$

$= 12 - 5 = 7$

$7 = 7$

Since, LHS = RHS

Hence, $x = 4$ is the root of $3x - 5 = 7$.

(ii) Given $x = 3$ is the root of $5 + 3x = 14$.

Now, substituting $x = 3$ in place of 'x' in the given equation, we get

$= 5 + 3(3) = 14$

$= 5 + 9 = 14$

14 = 14

Since, LHS = RHS

Hence, x = 3 is the root of 5 + 3x = 14.

(iii) Given x = 2 is the root of 3x – 2 = 8x – 12.

Now, substituting x = 2 in place of 'x' in the given equation, we get

= 3(2) – 2 = 8(2) – 12

= 6 – 2 = 16 – 12

4 = 4

Since, LHS = RHS

Hence, x = 2 is the root of 3x – 2 = 8x – 12.

(iv) Given x = 4 is the root of 3x/2 = 6.

Now, substituting x = 4 in place of 'x' in the given equation, we get

= (3 × 4)/2 = 6

= (12/2) = 6

6 = 6

Since, LHS = RHS

Hence, x = 4 is the root of (3x/2) = 6.

(v) Given y = 2 is the root of y – 3 = 2y – 5.

Now, substituting y = 2 in place of 'y' in the given equation, we get

= 2 – 3 = 2(2) – 5

= -1 = 4 – 5

-1 = -1

Since, LHS = RHS

Hence, y = 2 is the root of y – 3 = 2y – 5.

(vi) Given x = 8 is the root of (1/2)x + 7 = 11.

Now, substituting x = 8 in place of ‘x’ in the given equation, we get

= (1/2) (8) + 7 =11

= 4 + 7 = 11

= 11 = 11

Since, LHS = RHS

Hence, x = 8 is the root of 12x + 7 = 11.

2. Solve each of the following equations by trial – and – error method:

(i) x + 3 =12

(ii) x -7 = 10

(iii) 4x = 28

(iv) (x/2) + 7 = 11

(v) 2x + 4 = 3x

(vi) (x/4) = 12

(vii) (15/x) = 3

(vii) (x/18) = 20

Solution:

(i) Given x + 3 =12

Here LHS = x +3 and RHS = 12

x	LHS	RHS	Is LHS = RHS
1	1 + 3 = 4	12	No
2	2 + 3 = 5	12	No
3	3 + 3 = 6	12	No
4	4 + 3 = 7	12	No
5	5 + 3 = 8	12	No
6	6 + 3 = 9	12	No
7	7 + 3 = 10	12	No
8	8 + 3 = 11	12	No
9	9 + 3 = 12	12	Yes

Therefore, if x = 9, LHS = RHS.

Hence, x = 9 is the solution to this equation.

(ii) Given x -7 = 10

Here LHS = x -7 and RHS = 10

x	LHS	RHS	Is LHS = RHS
9	9 – 7 = 2	10	No
10	10 -7 = 3	10	No
11	11 – 7 = 4	10	No
12	12 – 7 = 5	10	No
13	19 – 7 = 6	10	No
14	14 – 7 = 7	10	No
15	15 – 7 = 8	10	No
16	16 – 7 = 9	10	No
17	17 – 7 = 10	10	Yes

Therefore if x = 17, LHS = RHS

Hence, x = 17 is the solution to this equation.

(iii) Given 4x = 28

Here LHS = 4x and RHS = 28

x	LHS	RHS	Is LHS = RHS
1	4 × 1 = 4	28	No
2	4 × 2 = 8	28	No
3	4 × 3 = 12	28	No
4	4 × 4 = 16	28	No
5	4 × 5 = 20	28	No
6	4 × 6 = 24	28	No
7	4 × 7 = 28	28	Yes

Therefore if x = 7, LHS = RHS

Hence, x = 7 is the solution to this equation.

(iv) Given (x/2) + 7 = 11

Here LHS = (x/2) + 7 and RHS = 11

Since RHS is a natural number, (x/2) must also be a natural number, so we must substitute values of x that are multiples of 2.

x	LHS	RHS	Is LHS = RHS

2	(2/2) + 7 = 1 + 7 =8	11	No
4	(4/2) + 7 = 2 + 7 = 9	11	No
6	(6/2) + 7 = 3 + 7 = 10	11	No
8	(8/2) + 7 = 4 + 7 = 11	11	Yes

Therefore if x = 8, LHS = RHS

Hence, x = 8 is the solutions to this equation.

(v) Given 2x + 4 = 3x

Here LHS = 2x + 4 and RHS = 3x

x	LHS	RHS	Is LHS = RHS
1	2 (1) + 4 = 2 + 4 = 6	3 (1) = 3	No
2	2 (2) + 4 = 4 + 4 = 8	3 (2) = 6	No
3	2 (3) + 4 = 6 + 4 = 10	3 (3) = 9	No
4	2 (4) + 4 = 8 + 4 = 12	3 (4) = 12	Yes

Therefore if x = 4, LHS = RHS

Hence, x = 4 is the solutions to this equation.

(vi) Given (x/4) = 12

Here LHS = (x/4) and RHS = 12

Since RHS is a natural number, x/4 must also be a natural number, so we must substitute values of x that are multiples of 4.

x	LHS	RHS	Is LHS = RHS
16	(16/4) = 4	12	No
20	(20/4) = 5	12	No
24	(24/4) = 6	12	No
28	(28/4) = 7	12	No
32	(32/4) = 8	12	No
36	(36/4) = 9	12	No
40	(40/4) = 10	12	No
44	(44/4) = 11	12	No
48	(48/4) = 12	12	Yes

Therefore if x = 48, LHS = RHS

Hence, x = 48 is the solutions to this equation.

(vii) Given (15/x) = 3

Here LHS = (15/x) and RHS = 3

Since RHS is a natural number, 15x must also be a natural number, so we must substitute values of x that are factors of 15.

x	LHS	RHS	Is LHS = RHS
1	(15/1) = 15	3	No
3	(15/3) = 5	3	No
5	(15/5) = 3	3	Yes

Therefore if x = 5, LHS = RHS

Hence, x = 5 is the solutions to this equation.

(viii) Given (x/18) = 20

Here LHS = (x/18) and RHS = 20

Since RHS is a natural number, (x/18) must also be a natural number, so we must substitute values of x that are multiples of 18.

x	LHS	RHS	Is LHS = RHS
324	(324/18) = 18	20	No
342	(342/18) = 19	20	No
360	(360/18) = 20	20	Yes

Therefore if x = 360, LHS = RHS

Hence, x = 360 is the solutions to this equation.

Exercise 8.2

Solve each of the following equations and check your answers:

1. x – 3 = 5

Solution:

Given x – 3 = 5

Adding 3 to both sides we get,

x – 3 + 3 = 5 + 3

x = 8

Verification:

Substituting x = 8 in LHS, we get

LHS = x – 3 and RHS = 5

LHS = 8 – 3 = 5 and RHS = 5

LHS = RHS

Hence, verified.

2. x + 9 = 13

Solution:

Given x + 9 = 13

Subtracting 9 from both sides i.e. LHS and RHS, we get x + 9 − 9 = 13 − 9

x = 4

Verification:

Substituting x = 4 on LHS, we get

LHS = 4 + 9 = 13 = RHS

LHS = RHS

Hence, verified.

3. x – (3/5) = (7/5)

Solution:

Given x – (3/5) = (7/5)

Add (3/5) to both sides, we get

x – (3/5) + (3/5) = (7/5) + (3/5)

x = (7/5) + (3/5)

x = (10/5)

x = 2

Verification:

Substitute x = 2 in LHS of given equation, then we get

2 – (3/5) = (7/5)

(10 – 3)/5 = (7/5)

(7/5) = (7/5)

LHS = RHS

Hence, verified

4. 3x = 0

Solution:

Given $3x = 0$

On dividing both sides by 3 we get,

$(3x/3) = (0/3)$

$x = 0$

Verification:

Substituting $x = 0$ in LHS we get

$3 (0) = 0$

And RHS = 0

Therefore LHS = RHS

Hence, verified.

5. (x/2) = 0

Solution:

Given $x/2 = 0$

Multiplying both sides by 2, we get

$(x/2) \times 2 = 0 \times 2$

$x = 0$

Verification:

Substituting $x = 0$ in LHS, we get

LHS = $0/2 = 0$ and RHS = 0

LHS = 0 and RHS = 0

Therefore LHS = RHS

Hence, verified.

6. x – (1/3) = (2/3)

Solution:

Given x – (1/3) = (2/3)

Adding (1/3) to both sides, we get

x – (1/3) + (1/3) = (2/3) + (1/3)

x = (2 + 1)/3

x = (3/3)

x =1

Verification:

Substituting x = 1 in LHS, we get

1 – (1/3) = (2/3)

(3 – 1)/3 = (2/3)

(2/3) = (2/3)

Therefore LHS = RHS

Hence, verified.

7. x + (1/2) = (7/2)

Solution:

Given x + (1/2) = (7/2)

Subtracting (1/2) from both sides, we get

x + (1/2) – (1/2) = (7/2) – (1/2)

x = (7 – 1)/2

x = (6/2)

x = 3

Verification:

Substituting x = 3 in LHS we get

3 + (1/2) = (7/2)

(6 + 1)/2 = (7/2)

(7/2) = (7/2)

Therefore LHS = RHS

Hence, verified.

8. 10 – y = 6

Solution:

Given 10 – y = 6

Subtracting 10 from both sides, we get

10 – y – 10 = 6 – 10

-y = -4

Multiplying both sides by -1, we get

-y × -1 = – 4 × – 1

y = 4

Verification:

Substituting y = 4 in LHS, we get

10 – y = 10 – 4 = 6 and RHS = 6

Therefore LHS = RHS

Hence, verified.

9. 7 + 4y = -5

Solution:

Given 7 + 4y = -5

Subtracting 7 from both sides, we get

7 + 4y – 7 = -5 -7

4y = -12

Dividing both sides by 4, we get

y = -12/ 4

y = -3

Verification:

Substituting y = -3 in LHS, we get

7 + 4y = 7 + 4(-3) = 7 – 12 = -5, and RHS = -5

Therefore LHS = RHS

Hence, verified.

10. (4/5) – x = (3/5)

Solution:

Given (4/5) – x = (3/5)

Subtracting (4/5) from both sides, we get

(4/5) – x – (4/5) = (3/5) – (4/5)

– x = (3 -4)/5

$- x = (-1/5)$

$x = (1/5)$

Verification:

Substituting $x = (1/5)$ in LHS we get

$(4/5) - (1/5) = (3/5)$

$(4 - 1)/5 = (3/5)$

$(3/5) = (3/5)$

Therefore LHS =RHS

Hence, verified.

11. $2y - (1/2) = (-1/3)$

Solution:

Given $2y - (1/2) = (-1/3)$

Adding $(1/2)$ from both the sides, we get

$2y - (1/2) + (1/2) = (-1/3) + (1/2)$

$2y = (-1/3) + (1/2)$

$2y = (-2 + 3)/6$ [LCM of 3 and 2 is 6]

$2y = (1/6)$

Now divide both the side by 2, we get

$y = (1/12)$

Verification:

Substituting $y = (1/12)$ in LHS we get

$2 (1/12) - (1/2) = (-1/3)$

(1/6) – (1/2) = (-1/3)

(2 – 6)/12 = (-1/3) [LCM of 6 and 2 is 12]

(-4/12) = (-1/3)

(-1/3) = (-1/3)

Therefore LHS = RHS

Hence, verified.

12. 14 = (7x/10) – 8

Solution:

Given 14 = (7x/10) – 8

Adding 8 to both sides we get,

14 + 8 = (7x/10) – 8 + 8

22 = (7x/10)

Multiply both sides by 10 we get,

220 = 7x

x = (220/7)

Verification:

Substituting x = (220/7) in RHS we get,

14 = (7/10) × (220/7) – 8

14 = 22 -8

14 = 14

Therefore LHS = RHS.

Hence, verified.

13. 3 (x + 2) = 15

Solution:

Given 3 (x + 2) = 15

Dividing both sides by 3 we get,

3 (x + 2)/3 = (15/3)

(x + 2) = 5

Now subtracting 2 by both sides, we get

x + 2 -2 = 5 -2

x = 3

Verification:

Substituting x =3 in LHS we get,

3 (3 + 2) = 15

3 (5) = 15

15 = 15

Therefore LHS = RHS

Hence, verified.

14. (x/4) = (7/8)

Solution:

Given (x/4) = (7/8)

Multiply both sides by 4 we get,

(x/4) × 4 = (7/8) × 4

x = (7/2)

Verification:

Substituting x = (7/2) in LHS we get,

(7/2)/4 = (7/8)

(7/8) = (7/8)

Therefore LHS = RHS

Hence, verified.

15. (1/3) – 2x = 0

Solution:

Given (1/3) – 2x = 0

Subtract (1/3) from both sides we get,

(1/3) – 2x – (1/3) = 0 – (1/3)

– 2x = – (1/3)

2x = (1/3)

Divide both side by 2 we get,

2x/2 = (1/3)/2

x = (1/6)

Verification:

Substituting x = (1/6) in LHS we get,

(1/3) – 2 (1/6) = 0

(1/3) – (1/3) = 0

0 = 0

Therefore LHS = RHS

Hence, verified.

16. 3 (x + 6) = 24

Solution:

Given 3 (x + 6) = 24

Divide both the sides by 3 we get,

3 (x + 6)/3 = (24/3)

(x + 6) = 8

Now subtract 6 from both sides we get,

x + 6 − 6 = 8 − 6

x = 2

Verification:

Substituting x = 2 in LHS we get,

3 (2 + 6) = 24

3 (8) =24

24 = 24

Therefore LHS =RHS

Hence, verified.

17. 3 (x + 2) − 2 (x − 1) = 7

Solution:

Given 3 (x + 2) − 2 (x − 1) = 7

On simplifying the brackets, we get

3 × x + 3 × 2 − 2 × x + 2 × 1 = 7

$3x + 6 - 2x + 2 = 7$

$3x - 2x + 6 + 2 = 7$

$x + 8 = 7$

Subtracting 8 from both sides, we get

$x + 8 - 8 = 7 - 8$

$x = -1$

Verification:

Substituting $x = -1$ in LHS, we get

$3(x + 2) - 2(x - 1) = 7$

$3(-1 + 2) - 2(-1 - 1) = 7$

$(3 \times 1) - (2 \times -2) = 7$

$3 + 4 = 7$

Therefore LHS = RHS

Hence, verified.

18. $8(2x - 5) - 6(3x - 7) = 1$

Solution:

Given $8(2x - 5) - 6(3x - 7) = 1$

On simplifying the brackets, we get

$(8 \times 2x) - (8 \times 5) - (6 \times 3x) + (-6) \times (-7) = 1$

$16x - 40 - 18x + 42 = 1$

$16x - 18x + 42 - 40 = 1$

$-2x + 2 = 1$

Subtracting 2 from both sides, we get

$-2x + 2 - 2 = 1 - 2$

$-2x = -1$

Multiplying both sides by -1, we get

$-2x \times (-1) = -1 \times (-1)$

$2x = 1$

Dividing both sides by 2, we get

$2x/2 = (1/2)$

$x = (1/2)$

Verification:

Substituting $x = (1/2)$ in LHS we get,

$(8 \times (2 \times (1/2) - 5) - (6 \times (3 \times (1/2) - 7) = 1$

$8(1 - 5) - 6(3/2 - 7) = 1$

$8 \times (-4) - (6 \times 3/2) + (6 \times 7) = 1$

$-32 - 9 + 42 = 1$

$-41 + 42 = 1$

$1 = 1$

Therefore LHS = RHS

Hence, verified.

19. 6 (1 – 4x) + 7 (2 + 5x) = 53

Solution:

Given $6(1 - 4x) + 7(2 + 5x) = 53$

On simplifying the brackets, we get

$(6 \times 1) - (6 \times 4x) + (7 \times 2) + (7 \times 5x) = 53$

$6 - 24x + 14 + 35x = 53$

$6 + 14 + 35x - 24x = 53$

$20 + 11x = 53$

Subtracting 20 from both sides, we get $20 + 11x - 20 = 53 - 20$

$11x = 33$

Dividing both sides by 11, we get

$11x/11 = 33/11$

$x = 3$

Verification:

Substituting $x = 3$ in LHS, we get

$6(1 - 4 \times 3) + 7(2 + 5 \times 3) = 53$

$6(1 - 12) + 7(2 + 15) = 53$

$6(-11) + 7(17) = 53$

$-66 + 119 = 53$

$53 = 53$

Therefore LHS = RHS

Hence, verified.

20. $5(2 - 3x) - 17(2x - 5) = 16$

Solution:

Given $5(2 - 3x) - 17(2x - 5) = 16$

On expanding the brackets, we get

(5 × 2) – (5 × 3x) – (17 × 2x) + (17 × 5) = 16

10 – 15x – 34x + 85 = 16

10 + 85 – 34x – 15x = 16

95 – 49x = 16

Subtracting 95 from both sides, we get

– 49x + 95 – 95 = 16 – 95

– 49x = -79

Dividing both sides by – 49, we get

– 49x/ -49 = -79/-49

x = 79/49

Verification:

Substituting x = (79/49) in LHS we get,

5 (2 – 3 × (79/49) – 17 (2 × (79/49) – 5) = 16

(5 × 2) – (5 × 3 × (79/49)) – (17 × 2 × (79/49)) + (17 × 5) = 16

10 – (1185/49) – (2686/49) + 85 = 16

(490 – 1185 – 2686 + 4165)/49 = 16

784/49 = 16

16 = 16

Therefore LHS = RHS

Hence, verified.

21. (x – 3)/5 -2 = -1

Solution:

Given ((x – 3)/5) -2 = -1

Adding 2 to both sides we get,

((x -3)/5) – 2 + 2 = -1 + 2

(x -3)/5 = 1

Multiply both sides by 5 we get

(x – 3)/ 5 × 5 = 1 × 5

x – 3 = 5

Now add 3 to both sides we get,

x – 3 + 3 = 5 + 3

x = 8

Verification:

Substituting x = 8 in LHS we get,

((8 – 3)/5) -2 = -1

(5/5) – 2 = -1

1 -2 = -1

-1 = -1

Therefore LHS = RHS

Hence, verified.

22. 5 (x – 2) + 3 (x +1) = 25

Solution:

Given 5 (x – 2) + 3 (x +1) = 25

On simplifying the brackets, we get

$(5 \times x) - (5 \times 2) + 3 \times x + 3 \times 1 = 25$

$5x - 10 + 3x + 3 = 25$

$5x + 3x - 10 + 3 = 25$

$8x - 7 = 25$

Adding 7 to both sides, we get

$8x - 7 + 7 = 25 + 7$

$8x = 32$

Dividing both sides by 8, we get

$8x/8 = 32/8$

$x = 4$

Verification:

Substituting $x = 4$ in LHS, we get

$5(4 - 2) + 3(4 + 1) = 25$

$5(2) + 3(5) = 25$

$10 + 15 = 25$

$25 = 25$

Therefore LHS = RHS

Hence, verified.

Exercise 8.3

Solve each of the following equations. Also, verify the result in each case.

1. $6x + 5 = 2x + 17$

Solution:

Given $6x + 5 = 2x + 17$

Transposing 2x to LHS and 5 to RHS, we get

$6x - 2x = 17 - 5$

$4x = 12$

Dividing both sides by 4, we get

$4x/4 = 12/4$

$x = 3$

Verification:

Substituting $x = 3$ in the given equation, we get

$6 \times 3 + 5 = 2 \times 3 + 17$

$18 + 5 = 6 + 17$

$23 = 23$

Therefore LHS = RHS

Hence, verified.

2. $2(5x - 3) - 3(2x - 1) = 9$

Solution:

Given $2(5x - 3) - 3(2x - 1) = 9$

Simplifying the brackets, we get

$2 \times 5x - 2 \times 3 - 3 \times 2x + 3 \times 1 = 9$

$10x - 6 - 6x + 3 = 9$

$10x - 6x - 6 + 3 = 9$

$4x - 3 = 9$

Adding 3 to both sides, we get

$4x - 3 + 3 = 9 + 3$

$4x = 12$

Dividing both sides by 4, we get

$4x/4 = 12/4$

Therefore $x = 3$.

Verification:

Substituting $x = 3$ in LHS, we get

$2(5 \times 3 - 3) - 3(2 \times 3 - 1) = 9$

$2 \times 12 - 3 \times 5 = 9$

$24 - 15 = 9$

$9 = 9$

Thus, LHS = RHS

Hence, verified.

3. $(x/2) = (x/3) + 1$

Solution:

Given $(x/2) = (x/3) + 1$

Transposing $(x/3)$ to LHS we get

$(x/2) - (x/3) = 1$

$(3x - 2x)/6 = 1$ [LCM of 3 and 2 is 6]

$x/6 = 1$

Multiplying 6 to both sides we get,

$x = 6$

Verification:

Substituting $x = 6$ in given equation we get

$(6/2) = (6/3) + 1$

$3 = 2 + 1$

$3 = 3$

Thus LHS = RHS

Hence, verified.

4. $(x/2) + (3/2) = (2x/5) - 1$

Solution:

Given $(x/2) + (3/2) = (2x/5) - 1$

Transposing $(2x/5)$ to LHS and $(3/2)$ to RHS, then we get

$(x/2) - (2x/5) = -1 - (3/2)$

$(5x - 4x)/10 = (-2 - 3)/2$ [LCM of 5 and 2 is 10]

$x/10 = -5/2$

Multiplying both sides by 10 we get,

$x/10 \times 10 = (-5/2) \times 10$

$x = (-50/2)$

$x = -25$

Verification:

Substituting $x = -25$ in given equation we get

$(-25/2) + (3/2) = (-50/5) - 1$

$(-25 + 3)/2 = -10 - 1$

$(-22/2) = -11$

$-11 = -11$

Thus LHS = RHS

Hence, verified.

5. (3/4) (x -1) = (x – 3)

Solution:

Given $(3/4)(x - 1) = (x - 3)$

On simplifying the brackets both sides we get,

$(3/4)x - (3/4) = (x - 3)$

Now transposing (3/4) to RHS and (x – 3) to LHs

$(3/4)x - x = (3/4) - 3$

$(3x - 4x)/4 = (3 - 12)/4$

$-x/4 = (-9/4)$

Multiply both sides by -4 we get

$-x/4 \times -4 = (-9/4) \times -4$

$x = 9$

Verification:

Substituting x = 9 in the given equation:

$(3/4)(9 - 1) = (9 - 3)$

$(3/4)(8) = 6$

$3 \times 2 = 6$

$6 = 6$

Thus LHS = RHS

Hence, verified.

6. 3 (x – 3) = 5 (2x + 1)

Solution:

Given 3 (x – 3) = 5 (2x + 1)

On simplifying the brackets we get,

3x – 9 = 10x + 5

Now transposing 10x to LHS and 9 to RHs

3x – 10x = 5 + 9

-7x = 14

Now dividing both sides by -7 we get

-7x/-7 = 14/-7

x = -2

Verification:

Substituting x = -2 in the given equation we get

3 (-2 – 3) = 5 (-4 + 1)

3 (-5) = 5 (-3)

-15 = – 15

Thus LHS = RHS

Hence, verified.

7. $3x - 2(2x - 5) = 2(x + 3) - 8$

Solution:

Given $3x - 2(2x - 5) = 2(x + 3) - 8$

On simplifying the brackets on both sides, we get

$3x - 2 \times 2x + 2 \times 5 = 2 \times x + 2 \times 3 - 8$

$3x - 4x + 10 = 2x + 6 - 8$

$-x + 10 = 2x - 2$

Transposing x to RHS and 2 to LHS, we get

$10 + 2 = 2x + x$

$3x = 12$

Dividing both sides by 3, we get

$3x/3 = 12/3$

$x = 4$

Verification:

Substituting $x = 4$ on both sides, we get

$3(4) - 2\{2(4) - 5\} = 2(4 + 3) - 8$

$12 - 2(8 - 5) = 14 - 8$

$12 - 6 = 6$

$6 = 6$

Thus LHS = RHS

Hence, verified.

8. $x - (x/4) - (1/2) = 3 + (x/4)$

Solution:

Given x – (x/4) – (1/2) = 3 + (x/4)

Transposing (x/4) to LHS and (1/2) to RHS

x – (x/4) – (x/4) = 3 + (1/2)

(4x – x – x)/4 = (6 + 1)/2

2x/4 = 7/2

x/2 = 7/2

x = 7

Verification:

Substituting x = 7 in the given equation we get

7 – (7/4) – (1/2) = 3 + (7/4)

(28 – 7 – 2)/4 = (12 + 7)/4

19/4 = 19/4

Thus LHS = RHS

Hence, verified.

9. (6x – 2)/9 + (3x + 5)/18 = (1/3)

Solution:

Given (6x – 2)/9 + (3x + 5)/18 = (1/3)

(6x (2) – 2 (2) + 3x + 5)/18 = (1/3)

(12x – 4 + 3x + 5)/18 = (1/3)

(15x + 1)/ 18 = (1/3)

Multiplying both sides by 18 we get

$(15x + 1)/18 \times 18 = (1/3) \times 18$

$15x + 1 = 6$

Transposing 1 to RHS, we get

$= 15x = 6 - 1$

$= 15x = 5$

Dividing both sides by 15, we get

$= 15x/15 = 5/15$

$=x = 1/3$

Verification:

Substituting $x = 1/3$ both sides, we get

$(6 (1/3) - 2)/9 + (3 (1/3) + 5)/18 = (1/3)$

$(2 - 2)/9 + (1 + 5)/ 18 = 1/3$

$(6/18) = (1/3)$

$(1/3) = (1/3)$

Thus LHS = RHS

Hence, verified.

10. $m - (m - 1)/2 = 1 - (m - 2)/3$

Solution:

Given $m - (m - 1)/2 = 1 - (m - 2)/3$

$(2m - m + 1)/2 = (3 - m + 2)/3$

$(m + 1)/2 = (5 - m)/3$

$(m + 1)/2 = (5/3) - (m/3)$

(m/2) + (1/2) = (5/3) – (m/3)

Transposing (m/3) to LHS and (1/2) to RHS

(m/2) + (m/3) = (5/3) – (1/2)

(3m + 2m)/6 = (10 – 3)/6

5m/6 = (7/6)

5m = 7

Dividing both sides by 5, we get

5m/5 = 7/5

m = 7/5

Verification:

Substituting m = 7/5 on both sides, we get

(7/5) – (7 – 5)/10 = 1 – (7 – 10)/15

(7/5) – (2/10) = (15 + 3)/15

(14 – 2)/10 = (15 + 3)/15

12/10 = 18/15

(6/5) = (6/5)

Thus LHS = RHS

Hence, verified.

11. (5x – 1)/3 – (2x – 2)/3 = 1

Solution:

Given (5x – 1)/3 – (2x – 2)/3 = 1

(5x – 1 – 2x + 2)/3 = 1

$(3x + 1)/3 = 1$

Multiplying both sides by 3 we get

$(3x + 1)/3 \times 3 = 1 \times 3$

$(3x + 1) = 3$

Subtracting 1 from both sides we get

$3x + 1 - 1 = 3 - 1$

$3x = 2$

Dividing both sides by 3, we get

$3x/3 = 2/3$

$x = 2/3$

Verification:

Substituting $x = 2/3$ in LHS, we get

$(5 (2/3) - 1)/3 - (2 (2/3) - 2)/3 = 1$

$(10/3 -1)/3 - (4/3 - 2)/3 = 1$

$(7/3)/3 - (-2/3)/3 = 1$

$(7/9) + (2/9) = 1$

$(9/9) = 1$

$1 = 1$

Thus LHS = RHS

Hence, verified.

12. $0.6x + 4/5 = 0.28x + 1.16$

Solution:

Given 0.6x + 4/5 = 0.28x + 1.16

Transposing 0.28x to LHS and 45 to RHS, we get

0.6x – 0.28x = 1.16 – 45

0.32x = 1.16 – 0.8

0.32x = 0.36

Dividing both sides by 0.32, we get

0.32 x 0.32 = 0.360.32

x = 9/8

Verification:

Substituting x = 9/8 on both sides, we get

0.6(9/8) + 45 = 0.28(9/8) + 1.16

5.4/8 + 4/5 = 2.52/8 + 1.16

0.675 + 0.8 = 0.315 + 1.16

1.475 = 1.475

Thus LHS = RHS

Hence, verified.

13. 0.5x + (x/3) = 0.25x + 7

Solution:

Given 0.5x + (x/3) = 0.25x + 7

(5/10) x + (x/3) = (25x/100) + 7

(x/2) + (x/3) = (x/4) + 7

Transposing (x/4) to LHS we get

$(x/2) + (x/3) – (x/4) = 7$

$(6x + 4x – 3x)/12 = 7$

$(7x/12) = 7$

Multiplying both sides by 12 we get

$(7x/12) \times 12 = 7 \times 12$

$7x = 84$

Dividing both sides by 7 we get

$(7x/7) = (84/7)$

$x = 12$

Verification:

Substituting x = 12 in given equation we get

$0.5 (12) + (12/3) = 0.25 (12) + 7$

$6 + 4 = 3 + 7$

$10 = 10$

Thus LHS = RHS

Hence, verified.

Exercise 8.4

1. If 5 is subtracted from three times a number, the result is 16. Find the number.

Solution:

Let the required number be x.

Then, given that 5 subtracted from 3 times x i.e. $3x – 5$

$\Rightarrow 3x - 5 = 16$

Adding 5 to both sides, we get

$\Rightarrow 3x - 5 + 5 = 16 + 5$

$\Rightarrow 3x = 21$

Dividing both sides by 3, we get

$\Rightarrow 3x/3 = 21/3$

$\Rightarrow x = 7$

Thus, the required number is $x = 7$.

2. Find the number which when multiplied by 7 is increased by 78.

Solution:

Let the required number be x.

Given that, when multiplied by 7, it gives 7x, and x increases by 78.

According to the question we can write as $7x = x + 78$

Transposing x to LHS, we get

$\Rightarrow 7x - x = 78$

$\Rightarrow 6x = 78$

Dividing both sides by 6, we get

$\Rightarrow 6x/6 = 78/6$

$\Rightarrow x = 13$

Thus, the required number is $x = 13$.

3. Find three consecutive natural numbers such that the sum of the first and second is 15 more than the third.

Solution:

Let first number be x.

According to the question second number is x + 1 and the third is x + 2

Sum of first and second numbers = (x) + (x + 1).

According to question:

$\Rightarrow (x) + (x + 1) = 15 + (x + 2)$

$\Rightarrow 2x + 1 = 17 + x$

Transposing x to LHS and 1 to RHS, we get

$\Rightarrow 2x - x = 17 - 1$

$\Rightarrow x = 16$

So, first number = x = 16,

Second number = x + 1 = 16 + 1 = 17

And third number = x + 2 = 16 + 2 = 18

Thus, the required consecutive natural numbers are 16, 17 and 18.

4. The difference between two numbers is 7. Six times the smaller plus the larger is 77. Find the numbers.

Solution:

Let the smaller number be ‘x’.

So, the larger number = x + 7.

According to question:

$\Rightarrow 6x + (x + 7) = 77$

$\Rightarrow 6x + x + 7 = 77$

On simplifying we get

⇒ $7x + 7 = 77$

Subtracting 7 from both sides, we get

⇒ $7x + 7 - 7 = 77 - 7$

⇒ $7x = 70$

Dividing both sides by 7, we get

⇒ $7x/7 = 70/7$

⇒ $x = 10$

Thus, the smaller number = $x = 10$

And the larger number = $x + 7 = 10 + 7 = 17$.

The two required numbers are 10 and 17.

5. A man says, "I am thinking of a number. When I divide it by 3 and then add 5, my answer is twice the number I thought of". Find the number.

Solution:

Let the required number be x.

So, according to question:

⇒ $x/3 + 5 = 2x$

Transposing x/3 to RHS, we get

⇒ $5 = 2x - (x/3)$

⇒ $5 = (6x - x)/3$

⇒ $5 = (5x/3)$

Multiplying both sides by 3 we get,

⇒ $5 \times 3 = (5x/3) \times 3$

⇒ $15 = 5x$

Dividing both sides by 5 we get

⇒ $15/5 = 5x/5$

⇒ $3 = x$

Thus the number thought of by the man is 3.

6. If a number is tripled and the result is increased by 5, we get 50. Find the number.

Solution:

Let the required number be ‘x’.

According to question:

⇒ $3x + 5 = 50$

Subtracting 5 from both sides, we get

⇒ $3x + 5 - 5 = 50 - 5$

⇒ $3x = 45$

Dividing both sides by 3, we get

⇒ $3x/3 = 45/3$

⇒ $x = 15$

Therefore, the required number is 15.

7. Shikha is 3 years younger to her brother Ravish. If the sum of their ages 37 years, what are their present age?

Solution:

Let the present age of Shikha be x years.

Therefore, the present age of Shikha’s brother Ravish = $(x + 3)$ years.

So, sum of their ages = $x + (x+ 3)$

$\Rightarrow x +(x + 3) = 37$

$\Rightarrow 2x + 3 = 37$

Subtracting 3 from both sides, we get

$\Rightarrow 2x+ 3 - 3 = 37 - 3$

$\Rightarrow 2x = 34$

Dividing both sides by 2, we get

$\Rightarrow 2x/2 = 34/2$

$\Rightarrow x = 17$

Therefore, the present age of Shikha = 17 years,

And the present age of Ravish = $x + 3 = 17 + 3 = 20$ years.

8. Mrs Jain is 27 years older than her daughter Nilu. After 8 years she will be twice as old as Nilu. Find their present ages?

Solution:

Let the present age of Nilu be x years

Therefore the present age of Nilu’s mother = $(x + 27)$ years

So, after 8 years,

Nilu’s age = $(x + 8)$, and Mrs Jain’s age = $(x + 27 + 8) = (x + 35)$ years

$\Rightarrow x + 35 = 2(x + 8)$

Expanding the brackets, we get

$\Rightarrow x + 35 = 2x + 16$

Transposing x to RHS and 16 to LHS, we get

$\Rightarrow 35 - 16 = 2x - x$

$\Rightarrow x = 19$

So, the present age of Nilu = x = 19 years,

And the present age of Nilu's mother that is Mrs Jain = x+ 27 = 19 + 27 = 46 years.

9. A man 4 times as old as his son. After 16 years, he will be only twice as old as his son. Find their present ages.

Solution:

Let the present age of the son = x years.

Therefore, the present age of his father = 4x years.

So, after 16 years,

Son's age = (x + 16) and father's age = (4x + 16) years

According to question:

$\Rightarrow 4x + 16 = 2(x + 16)$

$\Rightarrow 4x + 16 = 2x + 32$

Transposing 2x to LHS and 16 to RHS, we get

$\Rightarrow 4x - 2x = 32 - 16$

$\Rightarrow 2x = 16$

Dividing both sides by 2, we get

$\Rightarrow 2x/2 = 16/2$

$\Rightarrow x = 8$

So, the present age of the son = x = 8 years, And the present age of the father = $4x = 4(8) = 32$ years.

10. The difference in age between a girl and her younger sister is 4 years. The younger sister in turn is 4 years older than her brother. The sum of the ages of the younger sister and her brother is 16. How old are the three children?

Solution:

Let the age of the girl = x years.

So, the age of her younger sister = $(x - 4)$ years.

Thus, the age of the brother = $(x - 4 - 4)$ years = $(x - 8)$ years.

According to question:

$\Rightarrow (x - 4) + (x - 8) = 16$

$\Rightarrow x + x - 4 - 8 = 16$

$\Rightarrow 2x - 12 = 16$

Adding 12 to both sides, we get

$\Rightarrow 2x - 12 + 12 = 16 + 12$

$\Rightarrow 2x = 28$

Dividing both sides by 2, we get

$\Rightarrow 2x/2 = 28/2$

$\Rightarrow x = 14$

Thus, the age of the girl = x = 14 years,

The age of the younger sister = $x - 4 = 14 - 4 = 10$ years,

The age of the younger brother = $x - 8 = 14 - 8 = 6$ years.

11. One day, during their vacation at beach resort, Shella found twice as many sea shells as Anita and Anita found 5 shells more than sandy. Together sandy and Shella found 16 sea shells. How many did each of them find?

Solution:

Let the number of sea shells found by sandy = x

So, the number of sea shells found by Anita = (x + 5).

The number of sea shells found by Shella = 2 (x + 5).

According to the question,

$\Rightarrow x + 2(x + 5) = 16$

$\Rightarrow x + 2x + 10 = 16$

$\Rightarrow 3x + 10 = 16$

Subtracting 10 from both sides, we get

$\Rightarrow 3x + 10 - 10 = 16 - 10$

$\Rightarrow 3x = 6$

Dividing both sides by 3, we get

$\Rightarrow 3x/3 = 6/3$

$\Rightarrow x = 2$

Thus, the number of sea shells found by Sandy = x = 2,

The number of sea shells found by Anita = x + 5 = 2 + 5 = 7,

The number of sea shells found by Shelia = 2 (x + 5) = 2 (2 + 5) = 2(7) = 14.

12. Andy has twice as many marbles as Pandy, and Sandy has half as many has Andy and Pandy put together. If Andy has 75 marbles more than Sandy. How many does each of them have?

Solution:

Let the number of marbles with Pandy = x

So, the number of marbles with Andy = 2x

Thus, the number of marbles with Sandy = (x/2) + (2x/2) = (3x/2)

According to the question:

⇒ 2x = (3x/2) + 75

By transposing we get

⇒2x − (3x/2) = 75

⇒ (4x − 3x)/2 = 75

⇒ (x/2) = 75

⇒ x = 150

Since number of marbles cannot be negative

Therefore x = 150

So, Pandy has 150 marbles,

Andy has 2x = 2 (150) = 300 marbles,

Sandy has 3x/2 = 225 marbles.

13. A bag contains 25 paise and 50 paise coins whose total value is Rs 30. If the number of 25 paise coins is four times that of 50 paise coins, find the number of each type of coins.

Solution:

Let the number of 50 paise coins = x

So, the money value contribution of 50 paise coins in bag = 0.5x.

The number of 25 paise coins in bag = 4x

The money value contribution of 25 paise coins in bag = 0.25 (4x) = x.

According to the question,

⇒ 0.5x + x = 30

⇒ 1.5x = 30

Dividing both sides by 1.5, we get

⇒ 1.5x/1.5= 30/1.5

⇒ x = 20

Thus, the number of 50 paise coins = x = 20,

The number of 25 paise coins = 4x = 4 (20) = 80.

14. The length of a rectangular field is twice its breadth. If the perimeter of the field is 228 meters, find the dimensions of the field.

Solution:

Let the breadth of the rectangle = x metres.

According to the question,

Length of the rectangle = 2x metres

Perimeter of a rectangle = 2 (length + breadth)

So, 2 (2x + x) = 228

⇒ 2 (3x) = 228

⇒ 6x = 228

Dividing both sides by 6, we get

⇒ 6x/6 = 228/6

⇒ x = 38

So, the breadth of the rectangle = x = 38 metres,

The length of the rectangle = 2x = 2(38) = 76 metres.

15. There are only 25 paise coins in a purse. The value of money in the purse is Rs 17.50. Find the number of coins in the purse.

Solution:

Let the number of 25-paise coins in the purse be x

So, the value of money in the purse = 0.25x.

But 0.25x = 17.50.

Dividing both sides by 0.25, we get

⇒ 0.25x/0.25 = 17.5/0.25

⇒ x = 70

Thus, the number of 25 paise coins in the purse = 70.

16. In a hostel mess, 50kg rice are consumed every day. If each student gets 400gm of rice per day, find the number of students who take meals in the hostel mess.

Solution:

Let the number of students in the hostel be x

Quantity of rice consumed by each student = 400 gm.

So, daily rice consumption in the hostel mess = 400 (x).

But, daily rice consumption = 50 kg = 50 × 1000 = 50000gm [since 1 kg = 1000gm].

According to the question,

⇒ 400x = 50000

Dividing both sides by 400, we get

$\Rightarrow 400x/400 = 50000/400$

$\Rightarrow x = 125$

Thus, 125 students have their meals in the hostel mess.

Chapter - 9 Ratio and Proportion

Exercise 9.1

1. If x: y = 3: 5, find the ratio 3x + 4y: 8x + 5y

Solution:

Given x: y = 3: 5

We can write above equation as

$x/y = 3/5$

$5x = 3y$

$x = 3y/5$

By substituting the value of x in given equation 3x + 4y: 8x + 5y we get,

$3x + 4y: 8x + 5y = 3 (3y/5) + 4y: 8 (3y/5) + 5y$

$= (9y + 20y)/5: (24y + 25y)/5$

$= 29y/5: 49y/5$

$= 29y: 49y$

$= 29: 49$

2. If x: y = 8: 9, find the ratio (7x – 4y): 3x + 2y.

Solution:

Given x: y = 8: 9

We can write above equation as

$x/y = 8/9$

$9x = 8y$

$x = 8y/9$

By substituting the value of x in the given equation (7x – 4y): 3x + 2y we get,

(7x – 4y): 3x + 2y = 7 (8y/9) – 4y: 3 (8y/9) + 2y

= (56y – 36y)/9: 42y/9

= 20y/9: 42y/9

= 20y: 42y

= 20: 42

= 10: 21

3. If two numbers are in the ratio 6: 13 and their L.C.M is 312, find the numbers.

Solution:

Given two numbers are in the ratio 6: 13

Let the required number be 6x and 13x

The LCM of 6x and 13x is 78x

= 78x = 312

x = (312/78)

x = 4

Thus the numbers are 6x = 6 (4) = 24

13x = 13 (4) = 52

4. Two numbers are in the ratio 3: 5. If 8 is added to each number, the ratio becomes 2:3. Find the numbers.

Solution:

Let the required numbers be 3x and 5x

Given that if 8 is added to each other then ratio becomes 2: 3

That is $3x + 8: 5x + 8 = 2: 3$

$(3x + 8)/ (5x + 8) = 2/3$

$3 (3x + 8) = 2 (5x + 8)$

$9x + 24 = 10x + 16$

By transposing

$24 – 16 = 10x – 9x$

$x = 8$

Thus the numbers are $3x = 3 (8) = 24$

And $5x = 5 (8) = 40$

5. What should be added to each term of the ratio 7: 13 so that the ratio becomes 2: 3

Solution:

Let the number to be added is x

Then $(7 + x)/ (13 + x) = (2/3)$

$(7 + x) 3 = 2 (13 + x)$

$21 + 3x = 26 + 2x$

$3x – 2x = 26 – 21$

$x = 5$

Hence the required number is 5

6. Three numbers are in the ratio 2: 3: 5 and the sum of these numbers is 800. Find the numbers

Solution:

Given that three numbers are in the ratio 2: 3: 5 and sum of them is 800

Therefore sum of the terms of the ratio = 2 + 3 + 5 = 10

First number = (2/10) × 800

= 2 × 80

= 160

Second number = (3/10) × 800

= 3 × 80

= 240

Third number = (5/10) × 800

= 5 × 80

= 400

The three numbers are 160, 240 and 400

7. The ages of two persons are in the ratio 5: 7. Eighteen years ago their ages were in the ratio 8: 13. Find their present ages.

Solution:

Let present ages of two persons be 5x and 7x

Given ages of two persons are in the ratio 5: 7

And also given that 18 years ago their ages were in the ratio 8: 13

Therefore (5x – 18)/ (7x – 18) = (8/13)

13 (5x – 18) = 8 (7x – 18)

65x – 234 = 56x – 144

65x – 56x = 234 – 144

9x = 90

x = 90/9

x = 10

Thus the ages are 5x = 5 (10) = 50 years

And 7x = 7 (10) = 70 years

8. Two numbers are in the ratio 7: 11. If 7 is added to each of the numbers, the ratio becomes 2: 3. Find the numbers.

Solution:

Let the required numbers be 7x and 11x

If 7 is added to each of them then

(7x + 7)/ (11x + 7) = (2/3)

3 (7x + 7) = 2 (11x + 7)

21x + 21 = 22x + 14

22x – 21x = 21 – 14

x = 21 – 14 = 7

Thus the numbers are 7x = 7 (7) =49

And 11x = 11 (7) = 77

9. Two numbers are in the ratio 2: 7. 11 the sum of the numbers is 810. Find the numbers.

Solution:

Given two numbers are in the ratio 2: 7

And their sum = 810

Sum of terms in the ratio = 2 + 7 = 9

First number = (2/9) × 810

= 2 × 90

= 180

Second number = (7/9) × 810

= 7 × 90

= 630

10. Divide Rs 1350 between Ravish and Shikha in the ratio 2: 3.

Solution:

Given total amount to be divided = 1350

Sum of the terms of the ratio = 2 + 3 = 5

Ravish share of money = (2/5) × 1350

= 2 × 270

= Rs. 540

And Shikha's share of money = (3/5) × 1350

= 3 × 270

= Rs. 810

11. Divide Rs 2000 among P, Q, R in the ratio 2: 3: 5.

Solution:

Given total amount to be divided = 2000

Sum of the terms of the ratio = 2 + 3 + 5 = 10

P's share of money = (2/10) × 2000

= 2 × 200

= Rs. 400

And Q’s share of money = (3/10) × 2000

= 3 × 200

= Rs. 600

And R’s share of money = (5/10) × 2000

= 5 × 200

= Rs. 1000

12. The boys and the girls in a school are in the ratio 7:4. If total strength of the school be 550, find the number of boys and girls.

Solution:

Given that boys and the girls in a school are in the ratio 7:4

Sum of the terms of the ratio = 7 + 4 = 11

Total strength = 550

Boys strength = (7/11) × 550

= 7 × 50

= 350

Girls strength = (4/11) × 550

= 4 × 50

= 200

13. The ratio of monthly income to the savings of a family is 7: 2. If the savings be of Rs. 500, find the income and expenditure.

Solution:

Given that the ratio of income and savings is 7: 2

Let the savings be 2x

$2x = 500$

So, $x = 250$

Therefore,

Income = $7x$

Income = $7 \times 250 = 1750$

Expenditure = Income – savings

= 1750 – 500

= Rs.1250

14. The sides of a triangle are in the ratio 1: 2: 3. If the perimeter is 36 cm, find its sides.

Solution:

Given sides of a triangle are in the ratio 1: 2: 3

Perimeter = 36cm

Sum of the terms of the ratio = 1 + 2 + 3 = 6

First side = $(1/6) \times 36$

= 6cm

Second side = $(2/6) \times 36$

= 2×6

= 12cm

Third side = $(3/6) \times 36$

= 6×3

= 18cm

15. A sum of Rs 5500 is to be divided between Raman and Amen in the rate 2: 3. How much will each get?

Solution:

Given total amount to be divided = 5500

Sum of the terms of the ratio = 2 + 3 = 5

Raman's share of money = (2/5) × 5500

= 2 × 1100

= Rs. 2200

And Aman's share of money = (3/5) × 5500

= 3 × 1100

= Rs. 3300

16. The ratio of zinc and copper in an alloy is 7: 9. It the weight of the copper in the alloy is 11.7 kg, find the weight of the zinc in the alloy.

Solution:

Given that ratio of zinc and copper in an alloy is 7: 9

Let their ratio = 7x: 9x

Weight of copper = 11.7kg

9x = 11.7

x = 11.7/9

x = 1.3

Weight of the zinc in the alloy = 1.3 × 7

= 9.10kg

17. In the ratio 7: 8. If the consequent is 40, what a the antecedent

Solution:

Given ratio = 7: 8

Let the ratio of consequent and antecedent 7x: 8x

Consequent = 40

8x = 40

x = 40/8

x = 5

Antecedent = 7x = 7 × 5 = 35

18. Divide Rs 351 into two parts such that one may be to the other as 2: 7.

Solution:

Given total amount is to be divided = 351

Ratio 2: 7

The sum of terms = 2 + 7

= 9

First ratio of amount = (2/9) × 351

= 2 × 39

= Rs. 78

Second ratio of amount = (7/9) × 351

= 7 × 39

= Rs. 273

19. Find the ratio of the price of pencil to that of ball pen, if pencil cost Rs.16 per score and ball pen cost Rs.8.40 per dozen.

Solution:

One score contains 20 pencils

And cost per score = 16

Therefore pencil cost = 16/20

= Rs. 0.80

Cost of one dozen ball pen = 8.40

1 dozen = 12

Therefore cost of pen = 8.40/12

= Rs 0.70

Ratio of the price of pencil to that of ball pen = 0.80/0.70

= 8/7

= 8: 7

20. In a class, one out of every six students fails. If there are 42 students in the class, how many pass?

Solution:

Given, total number of students = 42

One out of 6 student fails

x out of 42 students

1/6 = x/42

x = 42/6

x = 7

Number of students who fail = 7 students

No of students who pass =Total students – Number of students who fail

= 42 – 7

= 35 students.

Exercise 9.2

1. Which ratio is larger in the following pairs?

(i) 3: 4 or 9: 16

(ii) 15: 16 or 24: 25

(iii) 4: 7 or 5: 8

(iv) 9: 20 or 8: 13

(v) 1: 2 or 13: 27

Solution:

(i) Given 3: 4 or 9: 16

LCM for 4 and 16 is 16

3: 4 can be written as = 3/4

3/4 × (4/4) = 12/16

And we have 9/16

Clearly 12 > 9

Therefore 3: 4 > 9: 16

(ii) Given 15: 16 or 24: 25

LCM for 16 and 25 is 400

15: 16 can be written as = 15/16

15/16 × (25/25) = 375/400

And we have 24/25

24/25 × (16/16) = 384/400

Clearly 384 > 375

Therefore 15: 16 < 24: 25

(iii) Given 4: 7 or 5: 8

LCM for 7 and 8 is 56

4: 7 can be written as = 4/7

4/7 × (8/8) = 32/56

And we have 5/8

5/8 × (7/7) = 35/56

Clearly 35 > 32

Therefore 4: 7 < 5: 8

(iv) Given 9: 20 or 8: 13

LCM for 20 and 13 is 260

9: 20 can be written as = 9/20

9/20 × (13/13) = 117/260

And we have 8/13

8/13 × (20/20) = 160/260

Clearly 160 > 117

Therefore 9: 20 < 8: 13

(v) Given 1: 2 or 13: 27

LCM for 2 and 27 is 54

1: 2 can be written as = 1/2

1/2 × (27/27) = 27/54

And we have 13/27

13/27 × (2/2) = 26/54

Clearly 27 > 26

Therefore 1: 2 > 13: 27

2. Give the equivalent ratios of 6: 8.

Solution:

Given 6: 8

By multiplying both numerator and denominator by 2 we equivalent ratios

6/8 × (2/2) = 12/16

And also by dividing both numerator and denominator by 2 we equivalent ratios

(6/2)/ (8/2) = 3/4

Two equivalent ratios are 3: 4 = 12: 16

3. Fill in the following blanks:

12/20 = …. /5 = 9/….

Solution:

12/20 = 3/5 = 9/15

Explanation:

Consider 12/20 = …. /5

Let unknown value be x

Therefore 12/20 = x/5

On cross multiplying

x = 60/20

x = 3

Consider 12/20 = 9/….

Let the unknown value be y

Therefore 12/20 = 9/y

On cross multiplying we get

y = 180/12

y = 15

Exercise 9.3

1. Find which of the following are in proportion?

(i) 33, 44, 66, 88

(ii) 46, 69, 69, 46

(iii) 72, 84, 186, 217

Solution:

(i) Given 33, 44, 66, 88

Product of extremes = 33 × 88 = 2904

Product of means = 44 × 66 = 2904

Therefore product of extremes = product of means

Hence given numbers are in proportion.

(ii) Given 46, 69, 69, 46

Product of extremes = 46 × 46 = 2116

Product of means = 69 × 69 = 4761

Therefore product of extremes is not equal to product of means

Hence given numbers are not in proportion.

(iii) Given 72, 84, 186, 217

Product of extremes = 72 × 217 = 15624

Product of means = 84 × 186 = 15624

Therefore product of extremes = product of means

Hence given numbers are in proportion.

2. Find x in the following proportions:

(i) 16: 18 = x: 96

(ii) x: 92 = 87: 116

Solution:

(i) Given 16: 18 = x: 96

In proportion we know that product of extremes = product of means

16/18 = x/96

On cross multiplying

x = (16 × 96)/ 18

x = 1536/18

Dividing both numerator and denominator by 6

$x = 256/3$

(ii) Given x: 92 = 87: 116

In proportion we know that product of extremes = product of means

$x/ 92 = 87/116$

On cross multiplying

$x = (87 \times 92)/ 116$

$x = 69$

3. The ratio of income to the expenditure of a family is 7: 6. Find the savings if the income is Rs.1400.

Solution:

Given that income = 1400

Given the ratio of income and expenditure = 7: 6

$7x = 1400$

Therefore $x = 200$

Expenditure = $6x = 6 \times 200$ = Rs.1200

Savings = Income – Expenditure

= 1400 -1200

= Rs.200

4. The scale of a map is 1: 4000000. What is the actual distance between the two towns if they are 5cm apart on the map?

Solution:

Given that the scale of map = 1: 4000000

Let us assume the actual distance between towns is x cm

1: 4000000 =5: x

x = 5 × 4000000

x = 20000000 cm

We know that 1km = 1000 m

1m = 100 cm

Therefore

x = 200 km

5. The ratio of income of a person to his savings is 10: 1. If his savings for one year is Rs.6000, what is his income per month?

Solution:

Given that the ratio of income of a person to his savings is 10: 1

Savings per year = 6000

Savings per month = 6000/12

= Rs.500

Then let income per month be x

x: 500 = 10:1

x = 500 × 10

x = 5000

Income per month is Rs. 5000

6. An electric pole casts a shadow of length 20 meters at a time when a tree 6 meters high casts a shadow of length 8 meters. Find the height of the pole.

Solution:

Given that length electric pole shadow is 20m

Height of the tree: Length of the shadow of tree

Height of the pole: Length of the shadow of pole

$x: 20 = 6: 8$

$x = 120/8$

$x = 15$

Therefore height of the pole is 15 meters

Chapter - 10 Unitary Method

Exercise 10.1

1. 20 chocolates cost Rs 320. Find the cost of 35 such chocolates.

Solution:

Given cost of 20 chocolates = Rs 320

Cost of 1 chocolate = (320/20)

Therefore, the cost of 35 chocolates = (320/20) × 35

= Rs 560

2. The cost of 40 meters of cloth is Rs 200. Find the cost of 50 meters of cloth.

Solution:

Given cost of 40 meters of cloth = Rs 200

Cost of 1 meter of cloth = (200/40)

Therefore, the cost of 50 chocolates = (200/40) × 50

= Rs 250

3. A car can cover a distance of 522 km on 36 litres of petrol. How far can it travel on 14 litres of petrol?

Solution:

Given that number of kilometres a car can cover by using 36 litres of petrol = 522 km

Number of kilometres a car can cover by using 1 litre of petrol = 522/36

Hence, the number of kilometres a car can cover by using 14 litres = (522/36) × 14

= 203 km

4. Travelling 900 km by rail costs Rs 280. What would be the fare for a journey of 360 km when a person travels by the same class?

Solution:

Given that cost of travelling 900 km by rail = Rs 280

Therefore cost of travelling 1 km by rail = (280/900)

Hence, Cost of travelling 360 km by rail = (280/900) × 360

= Rs 112

5. If 6 oil tankers can be filled by a pipe in 4(1/2) hours, how long does the pipe take to fill 4 such oil tankers?

Solution:

Given that time taken by 6 oil tankers to be filled by a pipe = 4 (1/2) hours = (9/2) hours

Time taken by 1 oil tanker to be filled by a pipe = (9/2)/6 hours = 9/ (2 x 6) = 9/12 hours

Hence time taken by 4 oil tankers to be filled by a pipe = (9/12) × 4

= 3 hours

6. 3/4 of the salary per month is Rs 600. What is the salary per month?

Solution:

Given that 3/4 of the salary per month = 600

Let the salary of the month be x

Therefore ¾ × x = 600

x = 600 × (4/3)

x = 800

Therefore salary per month is Rs 800

7. The cost of 32 tables is Rs 23520. Find the number of such tables that can be purchased for Rs 51450.

Solution:

Given that number of tables bought for Rs 23520 = 32

Number of tables bought for Rs 1 = 32/23520

Hence, number of tables bought for Rs 51450 = (32/23520) × 51450 = 70

8. The yield of wheat from 6 hectares is 280 quintals. Find the number of hectares required for a yield of 225 quintals.

Solution:

Given number of hectares required for a yield of 280 quintals = 6 hectares

Number of hectares required for a yield of 1 quintal = 6/280 hectares

Hence, the number of hectares required for a yield of 225 quintals = 6/280 × 225

= 4 23/28 hectares

9. Fifteen post cards cost Rs 2.25. What will be the cost of 36 post cards? How many postcards can we buy in Rs 45?

Solution:

Given cost of 15 post cards = Rs 2.25

Cost of 1 post card is = 2.25/15

Hence, the cost of 36 post cards = (2.25/15) × 36

= Rs 5.40

Number of post cards bought for Rs 2.25 = 15

Numbers of post cards bought for Rs 1 = 15/2.25

Hence number of post cards bought for Rs 45 = (15/2.25) × 45

= 300

10. A rail journey of 75 km costs Rs 215. How much will a journey of 120 km cost?

Solution:

Given cost of a rail journey of 75 km = Rs 215

Cost of a rail journey of 1 km = 215/75

Hence, cost of a rail journey of 120 km = (215/75) × 120

= Rs 344

11. If the sales tax on a purchase worth Rs 60 is Rs 4.20. What will be the sales tax on the purchase worth of Rs 150?

Solution:

Given sales tax on the purchase worth of Rs 60 = Rs 4.20

Sales tax on the purchase worth of Rs 1 = Rs 4.2060

Hence, sales tax on the purchase worth of Rs 150 = (4.20/60) × 150

= Rs 10.50

12. 52 packets of 12 pencils each, cost Rs 499.20. Find the cost of 65 packets of 10 pencils each.

Solution:

Given total number of pencils in 52 packets of 12 pencils each = 52 × 12

= 624 pencils

Also given that cost of 624 pencils = Rs 499.20

Cost of 1 pencil = (499.20/624)

Number of pencils in 65 packets of 10 pencils each = 65 x 10

= 650 pencils

Therefore, cost of 650 pencils = (499.20/624) × 650

= Rs 520.

Chapters-11 Percentage

Exercise 11.1

1. Express each of the following percents as fractions in the simplest forms:

(i) 45%

(ii) 0.25%

(iii) 150%

(iv) 6 1/4 %

Solution:

(i) Given 45%

= (45/100)

On simplifying the above fraction we get

= (9/20)

(ii) Given 0.25%

= (0.25/100)

= (25/10000)

On simplifying the above fraction we get

= (1/400)

(iii) Given 150%

= (150/100)

On simplifying the above fraction we get

= (3/2)

(iv) Given 6 1/4 %

We can write 6 1/4 as 6.25

= (6.25/100)

= (625/10000)

= (1/16)

2. Express each of the following fractions as a percent:

(i) (3/4)

(ii) (53/100)

(iii) 1 3/5

(iv) (7/20)

Solution:

(i) Given (3/4)

= (3/4) × 100

= 75%

(ii) Given (53/100)

= (53/100) × 100

= 53%

(iii) Given 1 3/5

Convert the given mixed fraction into improper fraction

1 3/5 = (8/5)

= (8/5) × 100

= 160%

(iv) Given (7/20)

= (7/20) × 100

= 35%

Exercise 11.2

1. Express each of the following ratios as per cents:

(i) 4: 5

(ii) 1: 5

(iii) 11: 125

Solution:

(i) Given 4: 5

4: 5 can be written as (4/5)

= (4/5) × 100

= 80%

(ii) Given 1: 5

1: 5 can be written as (1/5)

= (1/5) × 100

= 20%

(iii) Given 11: 125

11: 125 can be written as (11/125)

= (11/125) × 100

= (44/5) %

2. Express each of the following percents as ratios in the simplest form:

(i) 2.5%

(ii) 0.4%

(iii) 13 3/4 %

Solution:

(i) Given 2.5%

= (2.5/100)

= (25/1000)

= (1/40)

(ii) Given 0.4%

= (0.4/100)

= (4/1000)

= (1/250)

(iii) Given 13 3/4 %

13 3/4 = 13.75

= 13.75/100

= 1375/10000

= 11/80

Exercise 11.3

1. Express each of the following percents as decimals:

(i) 12.5%

(ii) 75%

(iii) 128.8%

(iv) 0.05%

Solution:

(i) Given 12.5%

= (12.5/100)

= 0.125

(ii) Given 75%

= (75/100)

= 0.75

(iii) Given 128.8%

= (128.8/100)

= 1.288

(iv) Given 0.05%

= (0.05/100)

= 0.0005

2. Express each of the following decimals as per cents:

(i) 0.004

(ii) 0.24

(iii) 0.02

(iv) 0.275

Solution:

(i) Given 0.004

0.004 can be written as 4/1000

= (4/1000) × 100

= 0.4%

(ii) Given 0.24

0.24 can be written as (24/100)

= (24/100) × 100

= 24%

(iii) Given 0.02

0.02 can be written as (2/100)

= (2/100) × 100

= 2%

(iv) Given 0.275

0.275 can be written as (275/1000)

= (275/1000) × 100

= 27.5%

3. Write each of the following as whole numbers or mixed numbers:

(i) 136%

(ii) 250%

(iii) 300%

Solution:

(i) Given 136%

= (136/100)

On simplifying we get

= (34/25)

(ii) Given 250%

= (250/100)

On simplifying

= (5/2)

(iii) Given 300%

= (300/100)

= 3

Exercise 11.4

1. Find each of the following:

(i) 7% of Rs 7150

(ii) 40% of 400kg

(iii) 20% of 15.125liters

(iv) 3 1/3 % of 90km

(v) 2.5% of 600meters

Solution:

(i) Given 7% of Rs 7150

= (7/100) × 7150

= Rs 500.50

(ii) Given 40% of 400kg

= (40/100) × 400

= 160kg

(iii) Given 20% of 15.125liters

= (20/100) × 15.125

= 3.025liters

(iv) Given 3 1/3 % of 90km

We know that 3 1/3 = (10/3)

= (10/300) × 90

= 3km

(v) Given 2.5% of 600 meters

= (2.5/100) x 600

= 15 meters

2. Find the number whose 12 ½ % is 64.

Solution:

Let the required number be x

Then according to the question, 12 ½ % × x = 64

= 12.5 % × x = 64

= (12.5/ 100) × x = 64

x = (64 x 100)/ 12.5

x = 64 × 8 = 512

Therefore 512 is the number whose 12 ½ % is 64.

3. What is the number, 6 ¼ % of which is 2?

Solution:

Let the required number be x

Then according to the question, 6 ¼ % × x = 64

= 6.25 % × x = 2

= (6.25/100) × x = 2

x = (2 × 100)/ 6.25

x = 2 × 16 = 32

Therefore 32 is the number whose 6 ¼ % is 32.

4. If 6 is 50% of a number, what is that number?

Solution:

Let the required number be x

Given that 50 % of x = 6

(50/100) × x = 6

x = (6 × 100)/ 50

x = 12

The required number is 12

Exercise 11.5

1. What percent of

(i) 24 is 6?

(ii) Rs 125 is Rs 10?

(iii) 4km is 160 meters?

(iv) Rs 8 is 25 paise?

(v) 2 days is 8 hours?

(vi) 1 liter is 175ml?

Solution:

(i) According to the question required percentage = (6/24) × 100

= (100/4)

= 25%

(ii) According to the question required percentage = (10/125) × 100

= (1000/125)

= 8%

(iii) According to the question required percentage = (160/4) × 100

We know that 1km = 1000 meters

Therefore 4km = 4000 meters

= (160/4000) × 100

= 4%

(iv) According to the question required percentage = (25/8) × 100

We know that 1Rs = 100 paise

Therefore 8Rs = 800 paise

= (25/800) × 100

= (25/8)

= 3.125%

(v) We know that 1day = 24 hours

1 hour = (1/24) day

8 hours = (8/24) day = (1/3) day

According to the question required percentage = [(1/3)/2] × 100

= 100/6

= 16 2/3 %

(vi) We know that 1liter = 1000ml

According to the question required percentage = (175/1000) × 100

= 17500/1000

= 17.50 %

2. What percent is equivalent to (3/8)?

Solution:

Given (3/8)

= (3/8) × 100

= 37.5%

3. Find the following:

(i) 8 is 4% of which number?

(ii) 6 is 60% of which number?

(iii) 6 is 30% of which number?

(iv) 12 is 25% of which number?

Solution:

(i) Let x be the required number

Given that 4% of x = 8

$(4/100) \times x = 8$

$x = (800/4)$

$x = 200$

(ii) Let the required number be x

Given that 60% of x = 6

$(60/100) \times x = 6$

$x = (60/6)$

$x = 10$

(iii) Let the required number be x

Given that 30% of x = 6

$(30/100) \times x = 6$

$x = (6 \times 100)/30$

$x = 20$

(iv) Let the required number be x

Given that 25% of x = 12

$(25/100) \times x = 12$

$x = (12 \times 100)/25$

$x = 48$

4. Convert each of the following pairs into percentages and find out which is more?

(i) 25 marks out of 30, 35 marks out of 40

(ii) 100 runs scored off 110 balls, 50 runs scored off 55 balls

Solution:

(i) Given 25 marks out of 30

Consider 25 marks out of 30 = $(25/30) \times 100$

= (250/3)

= 83.33%

Also given that 35 marks out of 40

Now consider 35 marks out of 40 = (35/40) × 100

= 87.5%

Clearly 87.5 > 83.33

After converting into percentage 35 marks out of 40 = 87.5% is more

(ii) Given 100 runs scored off 110 balls

Consider 100 runs scored off 110 balls = (100/110) × 100

= 90.91%

Also given that 50 runs scored off 55 balls

Consider 50 runs scored off 55 balls = (50/55) × 100

= 90.91%

Here both are equal

5. Find 20% more than Rs.200.

Solution:

Consider 20% of 200 = (20/100) × 200

= Rs 40

Therefore 20% more than Rs 200 = 200 + 40

= Rs 240

6. Find 10% less than Rs.150

Solution:

Consider 10% of 150 = (10/100) × 150

= Rs 15

Therefore 10% less than Rs 150 = 150 – 15

= Rs 135

Exercise 11.6

1. Ashu had 24 pages to write. By the evening, he had completed 25% of his work. How many pages were left?

Solution:

Given total number of pages Ashu had to write = 24

Number of pages Ashu completed by the evening = 25% of 24

= (25/100) × 24

= 600/100

= 6

Therefore number of pages left for completion = 24 – 6 = 18 pages

2. A box contains 60 eggs. Out of which 16 2/3 % are rotten ones. How many eggs are rotten?

Solution:

Given that total number of eggs = 60

Number of eggs rotten = 16 2/3% of 60 eggs

= 16.66 % of 60 eggs

= (16.66/100) × 60

= 10 eggs

Therefore number of eggs rotten = 10

3. Rohit obtained 45 marks out of 80. What percent marks did he get?

Solution:

Given total number of marks = 80

Marks scored by Rohit = 45

Percentage obtained by Rohit = (45/80) × 100

= 56.25%

4. Mr Virmani saves 12% of his salary. If he receives Rs 15900 per month as salary, find his monthly expenditure.

Solution:

Given Mr Virmani's salary per month = Rs. 15900

Mr Virmani's savings = 12% of Rs. 15900

= (12/100) × 15900

= Rs. 1908

Mr Virmani's monthly expenditure = salary – savings

= Rs. (15900 – 1908)

= Rs. 13992

5. A lawyer willed his 3 sons Rs 250000 to be divided into portions 30%, 45% and 25%. How much did each of them inherit?

Solution:

Given total amount with the lawyer = Rs. 250000

First son's inheritance = 30% of 250000

= (30/100) × 250000

= 7500000/100

= Rs. 75000

Second son’s inheritance = 45% of 250000

= (45/100) × 250000

= 11250000/100

= Rs. 112500

Third son’s inheritance = 25% of 250000

= (25/100) × 250000

= 6250000/100

= Rs. 62500

6. Rajdhani College has 2400 students, 40% of whom are girls. How many boys are there in the college?

Solution:

Given total number of students in Rajdhani College = 2400

Number of girls = 40% of 2400

= (40/100) × 2400

= 96000/100

= 960

Number of boys = total number of students – number of girls

= 2400 – 960 = 1440 boys

7. Aman obtained 410 marks out of 500 in CBSE XII examination while his brother Anish gets 536 marks out of 600 in IX class examination. Find whose performance is better?

Solution:

Given Aman’s marks in CBSE XII = 410/500

Percentage of marks obtained by Aman = (410/500) × 100

= 82%

Given that Anish’s marks in CBSE IX = 536/600

Percentage of marks obtained by Anish = (536/600) × 100

= 89.33%

Clearly 89.33 > 82

Therefore, Anish’s performance is better than Aman’s

8. Rahim obtained 60 marks out of 75 in Mathematics. Find the percentage of marks obtained by Rahim in Mathematics.

Solution:

Given marks obtained by Rahim in Mathematics = 60/75

Percentage of marks obtained by Rahim = (60/75) × 100

= 80%

9. In an orchard, 16 2/3 % of the trees are apple trees. If the number of trees in the orchard is 240, find the number of other type of trees in the orchard.

Solution:

Let the number of apple trees be x

Number of trees in the orchard = 240

Number of apple trees = 16 2/3 %

According to the given condition, 16 (2/3) % of 240 = x

= 16.66 % of 240 = x

x = (16.66/100) × 240

x = 40 trees

Number of other types of trees = Total number of trees – number of apple trees

= 240 – 40

= 200 trees

10. Ram scored 553 marks out of 700 and Gita scored 486 marks out of 600 in science. Whose performance is better?

Solution:

Given marks scored by Ram = 553/700

Percentage of marks scored by Ram = (553/700) × 100

= 0.79 × 100 = 79%

Also given that marks scored by Gita = (486/600)

Percentage of marks scored by Gita = (486/600) × 100

= 0.81 × 100 = 81

Gita’s performance (81%) is better than Ram’s (79%).

11. Out of an income of Rs 15000, Nazima spends Rs 10200. What percent of her income does she save?

Solution:

Given Nazima's total income = Rs 15000

Amount Nazima spends = Rs 10200

Amount Nazima saves = 15000 – 10200

= Rs 4800

Percentage of income Nazima saves = (4800/15000) × 100

= 480000/15000

= 32%

Nazima saves 32% of her income.

12. 45% of the students in a school are boys. If the total number of students in the school is 880, find the number of girls in the school.

Solution:

Given total number of students in the school = 880

Number of boys in the school = 45% of 880

= (45/100) × 880

= 39600/100

Number of boys = 396

Number of girls in the school = total number of students – number of boys

= 880 – 396

Number of girls = 484

13. Mr. Sidhana saves 28% of his income. If he saves as 840 per month, find his monthly income.

Solution:

Let Mr. Sidhana's monthly income be x

Monthly savings of Mr. Sidhana's = Rs 840

28% of x = Rs 840

⇒ (28/100) × x = Rs 840

⇒ 28x = Rs 84000

⇒ x = (84000/28) = Rs 3000

Mr. Sidhana's monthly income = Rs 3000

14. In an examination, 8% of the students fail. What percentage of the students pass? If 1650 students appeared in the examination, how many passed?

Solution:

Given total number of students who appeared for the examination = 1650

Number of students who failed = 8% of 1650

= (8/100) ×1650

= (8 × 1650)/100

= 13200/100

Number of students failed = 132

Number of students passed = 1650 – 132

= 1518

Percentage of students passed = (1518/1650) × 100

= 0.92 × 100 = 92%

92% of the students passed the examination.

15. In an examination, 92% of the candidates passed and 46 failed. How many candidates appeared?

Solution:

Let the total number of candidates be x

Number of candidates who failed = 46

Number of candidates who passed = 92% of x

According to the given condition

92% of x = x – 46

$\Rightarrow (92/100)\,x = x - 46$

$\Rightarrow 92x = 100x - 4600$

$\Rightarrow -8x = -4600$

$\Rightarrow x = 4600/8 = 575$

Number of candidates who appeared for the examination = 575

Chapter - 12 Profit And Loss

Exercise 12.1

1. Given the following values, find the unknown values:

(i) C.P. = Rs 1200, S.P. = Rs 1350 Profit/Loss?

(ii) C.P. = Rs 980, S.P. = Rs 940 Profit/Loss =?

(iii) C.P. = Rs 720, S.P. =?, Profit = Rs 55.50

(iv) C.P. =? S.P. = Rs 1254, Loss = Rs 32

Solution:

(i) Given CP = Rs. 1200, SP = Rs. 1350

Clearly CP < SP. So, profit.

Profit = SP – CP

= Rs. (1350 – 1200)

= Rs. 150

(ii) Given CP = Rs. 980, SP = Rs. 940

Clearly CP > SP. So, loss.

Loss = CP – SP

= Rs. (980 – 940)

= Rs. 40

(iii) CP = Rs. 720, SP =?, profit = Rs. 55.50

Profit = SP – CP

55.50 = SP – 720

SP = (55.50 + 720)

= Rs. 775.50

(iv) CP =?, SP = Rs. 1254, loss = Rs. 32

Loss = CP – SP

32 = CP – 1254

CP = (1254 + 32)

= Rs. 1286

2. Fill in the blanks in each of the following:

(i) C.P. = Rs 1265, S.P. = Rs 1253, Loss = Rs …….

(ii) C.P. = Rs……., S.P. = Rs 450, Profit = Rs 150

(iii) C.P. = Rs 3355, S.P. = Rs 7355,……. = Rs……

(iv) C.P. = Rs……., S.P. = Rs 2390, Loss = Rs 5.50

Solution:

(i) Loss = Rs 12

Explanation:

Given CP = Rs. 1265, SP = Rs. 1253

Loss = CP – SP

= Rs. (1265 – 1253)

= Rs. 12

(ii) C.P. = Rs 300

Explanation:

Given CP = ?, SP = Rs. 450, profit = Rs. 150

Profit = SP – CP

150 = 450 – CP

CP = Rs. (450 – 150)

= Rs. 300

(iii) Profit = Rs 4000

Explanation:

Given CP = Rs. 3355, SP = Rs. 7355,

Here SP > CP, so profit.

Profit = SP – CP

Profit = Rs. (7355 – 3355)

= Rs. 4000

(iv) C. P. = Rs 2395.50

Explanation:

Given CP = ?, SP = Rs. 2390, loss = Rs. 5.50

Loss = CP – SP

5.50 = CP – 2390

= Rs. (5.50 + 2390)

= Rs. 2395.50

3. Calculate the profit or loss and profit or loss percent in each of the following cases:

(i) C.P. = Rs 4560, S.P. = Rs 5000

(ii) C.P. = Rs 2600, S.P. = Rs 2470

(iii) C.P. = Rs 332, S.P. = Rs 350

(iv) C.P. = Rs 1500, S.P. = Rs 1500

Solution:

(i) Given CP = Rs. 4560, SP = Rs. 5000

Here, clearly SP > CP. So, profit.

Profit = SP – CP

= Rs. (5000 – 4560)

= Rs. 440

Profit % = {(Profit/CP) x 100} %

= {(440/4560) x 100} %

= {0.0965 x 100} %

Profit % = 9.65%

(ii) Given CP = Rs. 2600, SP = Rs. 2470.

Here, clearly CP > SP. So, loss.

Loss = CP – SP

= Rs. (2600 – 2470)

= Rs. 130

Loss % = {(Loss/CP) x 100} %

= {(130/2600) x 100} %

= {0.05 x 100} %

Loss % = 5%

(iii) Given CP = Rs. 332, SP= Rs. 350.

Here, clearly SP > CP. So, profit.

Profit = SP – CP

= Rs. (350 – 332)

= Rs. 18

Profit% = {(Profit/CP) x 100} %

= {(18/332) x 100} %

= {0.054 x 100} %

Profit % = 5.4%

(iv) Given CP = Rs. 1500, SP = Rs. 1500

Here clearly SP = CP.

So, neither profit nor loss.

4. Find the gain or loss percent, when:

(i) C.P. = Rs 4000 and gain = Rs 40.

(ii) S.P. = Rs 1272 and loss = Rs 328

(iii) S.P. = Rs 1820 and gain = Rs 420.

Solution:

(i) Given CP = Rs. 4000, gain = Rs. 40

Gain % = {(Gain/CP) x 100) %

= {(40/4000) x 100} %

= (0.01 x 100) %

Gain % = 1%

(ii) Given SP = Rs. 1272, loss = Rs. 328

Loss = CP – SP

Hence, CP = Loss+ SP

= Rs. 328 + Rs. 1272

= Rs. 1600

Loss % = {(Loss/CP) x 100} %

= {(328/1600) x 100%

Loss % = 20.5%

(iii) Given SP = Rs. 1820, gain = Rs. 420

Gain = SP – CP

CP = 1820 – 420

= Rs. 1400

Gain % = {(Gain/CP) x 100} %

= {(420/1400) x 100 %

Gain % = 30%

5. Find the gain or loss percent, when:

(i) C.P. = Rs 2300, Overhead expenses = Rs 300 and gain = Rs 260.

(ii) C.P. = Rs 3500, Overhead expenses = Rs 150 and loss = Rs 146

Solution:

(i) Given CP = Rs. 2300, overhead expenses = Rs. 300 and gain = Rs. 260

We know that Gain % = {(Gain/ (CP + overhead expenses)} x 100

= {260/ (2300 + 300} x 100

= {260/2600} x 100

Gain = 10%

(ii) Given CP = Rs. 3500, overhead expenses = Rs. 150 and loss = Rs. 146

We know that Loss % = {(Loss/ (CP + overhead expenses)} x 100

= {146/ (3500+ 150)} x 100

= {146/3650} x 100

= 14600/3650

Loss = 4%

6. A grain merchant sold 600 quintals of rice at a profit of 7%. If a quintal of rice cost him Rs 250 and his total overhead charges for transportation, etc. were Rs 1000 find his total profit and the selling price of 600 quintals of rice.

Solution:

Given Cost of 1 quintal of rice = Rs. 250

Cost of 600 quintals of rice = 600 x 250 = Rs. 150000

Overhead expenses = Rs. 1000

CP = Rs. (150000 + 1000) = Rs. 151000

Profit % = (Profit/CP) x 100

7 = (Profit /151000) x 100

Profit = 1510 x 7

Profit = Rs. 10570

Now SP = CP + profit

= Rs. (151000 + 10570)

SP = Rs. 161570

7. Naresh bought 4 dozen pencils at Rs 10.80 a dozen and sold them for 80 paise each. Find his gain or loss percent.

Solution:

Given Cost of 1 dozen pencils = Rs. 10.80

Therefore cost of 4 dozen pencils = 4 x 10.80

= Rs. 43.2

Also given that selling price of each pencil = 80 paise

Total number of pencils = 12 x 4 = 48

SP of 48 pencils = 48 x 80 paise

= 3840 paise

= Rs. 38.40

Here, clearly SP < CP.

Loss = CP – SP

= Rs. (43.2 – 38.4)

= Rs. 4.8

Loss % = (Loss/CP) x 100

= (4.8/43.2) x 100

= 480/43.2

Loss = 11.11%

8. A vendor buys oranges at Rs 26 per dozen and sells them at 5 for Rs 13. Find his gain percent.

Solution:

Given CP of 1 dozen oranges = Rs. 26

CP of 1 orange = 26/12

= Rs. 2.16

CP of 5 oranges = 2.16 x 5

= Rs. 10.8

Now, SP of 5 oranges = Rs. 13

Gain = SP – CP

= Rs. (13- 10.8)

= Rs. 2.2

Gain %= (Gain/CP) x 100

= (2.2/10.8) x 100

Gain = 20.3%

9. Mr Virmani purchased a house for Rs 365000 and spent Rs 135000 on its repairs. If he sold it for Rs 550000, find his gain percent.

Solution:

Given Mr. Virmani spent to purchase the house = Rs. 365000

Amount he spent on repair = Rs. 135000

Total amount he spent on the house (CP) = Rs. (365000 + 135000)

= Rs. 500000

Given SP of the house = Rs. 550000

Gain = SP – CP

= Rs. (550000 – 500000)

= Rs. 50000

Gain % = (Gain/CP) x 100

= (50000/500000) x 100

= 5000000/500000

Gain = 10%

10. Shikha purchased a wrist watch for Rs 840 and sold it to her friend Vidhi for Rs 910. Find her gain percent.

Solution:

Given CP of the wristwatch that Shikha purchased, CP = Rs. 840

The price at which she sold it, SP = Rs. 910

Gain = SP – CP

= (910 – 840)

= Rs. 70

Now Gain % = (Gain/CP) x 100

= (70/840) x 100

= 7000/840

Gain = 8.3%

11. A business man makes a 10% profit by selling a toy costing him Rs 120. What is the selling price?

Solution:

CP = Rs. 120

Profit % = 10

We now that

SP = {(100 + profit %) /100} x CP = {(100+ 10)/100} x 120

= {(110/100)} x 120 = 1.1 x 120

= Rs. 132

12. Harish purchased 50 dozen bananas for Rs 135. Five dozen bananas could not be sold because they were rotten. At what price per dozen should Harish sell the remaining bananas so that he makes a profit of 20%?

Solution:

Given cost price of 50 dozens bananas that Harish purchased, CP = Rs. 135

Bananas left after removing 5 dozen rotten bananas = 45 dozens

Effective CP of one dozen bananas = Rs. 135/45 = Rs. 3

Calculating the price at which Harish should sell each dozen bananas to make a profit of 20% (or 1/5), we get

Profit % = (Gain/CP) x100

To get a gain of 20% we give profit % = 20

And substitute 20 = (gain/135) x100

Gain = 270/10 = 27

We know; SP = CP + Gain

SP = 27 + 135

SP = 162

Now that SP is for 45 Dozens of bananas

Calculating for one dozen

= 162/45

= Rs. 3.6

Harish should sell the bananas at Rs. 3.60 a dozen in order to make a profit of 20%.

13. A woman bought 50 dozen eggs at Rs 6.40 a dozen. Out of these 20 eggs were found to be broken. She sold the remaining eggs at 55 paise per egg. Find her gain or loss percent.

Solution:

Given cost of one dozen eggs = Rs. 6.40

Cost of 50 dozen eggs = 50 x 6.40 = Rs. 320

Total number of eggs = 50 x 12 = 600

Number of eggs left after removing the broken ones = 600 – 20 = 580

SP of 1 egg = 55 paise

So, SP of 580 eggs = 580 x 55 = 31900 paise

= Rs. 31900/100

= Rs. 319

Loss = CP – SP

= Rs. (320 – 319) = Rs. 1

Loss % = (Loss/CP) x 100

= (1/320) x 100

Loss = 0.31%

14. Jyotsana bought 400 eggs at Rs 8.40 a dozen. At what price per hundred must she sell them so as to earn a profit of 15%?

Solution:

Given cost of eggs per dozen = Rs. 8.40

Cost of 1 egg = 8.40/12

= Rs. 0.7

Cost of 400 eggs = 400 x 0.7 = Rs. 280

Calculating the price at which Jyotsana should sell the eggs to earn a profit of 15%,

We get 15% of 280 + 280

= {(15/100) x 280} + 280

= {4200/100} + 280

= 42 + 280

= Rs. 322

So, Jyotsana must sell the 400 eggs for Rs. 322 in order to earn a profit of 15%.
Therefore, the SP per one hundred eggs = Rs. 322/4 = Rs. 80.50.

15. A shopkeeper makes a profit of 15% by selling a book for Rs 230. What is the C.P. and the actual profit?

Solution:

Given that the SP of a book = Rs. 230

Profit % = 15

Since

CP = (SP x 100)/ (100 + profit %)

CP = (230x 100)/ (100 + 15)

CP = 23000/ 115 = Rs. 200

Also, Profit = SP – CP = Rs. (230 – 200) = Rs. 30

Actual profit = Rs. 30

16. A bookseller sells all his books at a profit of 10%. If he buys a book from the distributor at Rs 200, how much does he sell it for?

Solution:

Given profit % = 10% CP = Rs. 200

Since SP = {(100 + profit %) /100} x CP

= {(100 + 10)/100} x 200

= {110/100} x 200 = Rs. 220

The bookseller sells the book for Rs. 220.

17. A floweriest buys 100 dozen roses at Rs 2 a dozen. By the time the flowers are delivered, 20 dozen roses are mutilated and are thrown away. At what price should he sell the rest if he needs to make a 20% profit on his purchase?

Solution:

Given cost of 1 dozen roses = Rs. 2

Number of roses bought by the floweriest = 100 dozens

Thus, cost price of 100 dozen roses = 2 x 100 = Rs. 200

Roses left after discarding the mutilated ones = 80 dozens

Calculating the price at which the floweriest should sell the 80 dozen roses in order to make a profit of 20%, we have

Profit % = ((SP-CP)/CP) x100 = ((SP-200)/200) x 100

40 = SP – 200

SP = Rs. 240

Therefore, the SP of the roses should be Rs. 240/80 = Rs. 3 per dozen.

18. By selling an article for Rs 240, a man makes a profit of 20%.What is his C.P.? What would his profit percent be if he sold the article for Rs 275?

Solution:

Let CP = Rs. x SP = Rs. 240

Let profit be Rs. P.

Now, profit % = 20%

Since Profit % = (Profit/CP) x 100

20 = (P/x) x 100

P = 20x/100 = x/5

Profit = SP – CP = 240 – x

P = 240 – x

x /5 = 240 – x

240 = x + x/5

240 = 6×/5

x = 1200/6

x = 200

So, CP = Rs. 200

New SP = Rs. 275 and CP = Rs. 200

Profit % = {(SP – CP)/CP} x 100

{(275 – 200)/200} x 100 = (75/200) x 100

= 7500/200

= 37.5%

Chapter - 13 Simple Interest

Exercise 13.1

1. Find the simple interest, when:
(i) Principal = Rs 2000, Rate of Interest = 5% per annum and Time = 5 years.
(ii) Principal = Rs 500, Rate of Interest = 12.5% per annum and Time = 4 years.
(iii) Principal = Rs 4500, Rate of Interest = 4% per annum and Time = 6 months.

(iv) Principal = Rs 12000, Rate of Interest = 18% per annum and Time = 4 months.
(v) Principal = Rs 1000, Rate of Interest = 10% per annum and Time = 73 days.

Solution:

(i) Given Principal = Rs 2000, Rate of Interest = 5% per annum and Time = 5 years.

We know that simple interest = $(P \times T \times R)/100$

On substituting these values in above equation we get

$SI = (2000 \times 5 \times 5)/100$

= Rs 500

(ii) Given Principal = Rs 500, Rate of Interest = 12.5% per annum and Time = 4 years.

We know that simple interest = $(P \times T \times R)/100$

On substituting these values in above equation we get

$SI = (500 \times 4 \times 12.5)/100$

= Rs 250

(iii) Given Principal = Rs 4500, Rate of Interest = 4% per annum and Time = 6 months = ½ years

We know that simple interest = $(P \times T \times R)/100$

On substituting these values in above equation we get

$SI = (4500 \times ½ \times 4)/100$

$SI = (4500 \times 1 \times 4)/100 \times 2$

= Rs 90

(iv) Given Principal = Rs 12000, Rate of Interest = 18% per annum and Time = 4 months = (4/12) = (1/3) years

We know that simple interest = $(P \times T \times R)/100$

On substituting these values in above equation we get

$SI = (12000 \times (1/3) \times 18)/100$

$SI = (12000 \times 1 \times 18)/100 \times 3$

= Rs 720

(v) Given Principal = Rs 1000, Rate of Interest = 10% per annum and

Time = 73 days = (73/365) days

We know that simple interest = $(P \times T \times R)/100$

On substituting these values in above equation we get

$SI = (1000 \times (73/365) \times 10)/100$

$SI = (1000 \times 73 \times 10)/100 \times 365$

= Rs 20

2. Find the interest on Rs 500 for a period of 4 years at the rate of 8% per annum. Also, find the amount to be paid at the end of the period.

Solution:

Given Principal amount P = Rs 500

Time period T = 4 years

Rate of interest R = 8% p.a.

We know that simple interest = (P × T × R)/100

On substituting these values in above equation we get

SI = (500 × 4 × 8)/100

= Rs 160

Amount = Principal amount + Interest

= Rs 500 + 160

= Rs 660

3. A sum of Rs 400 is lent at the rate of 5% per annum. Find the interest at the end of 2 years.

Solution:

Given Principal amount P = Rs 400

Time period T = 2 years

Rate of interest R = 5% p.a.

We know that simple interest = (P × T × R)/100

On substituting these values in above equation we get

SI = (400 × 2 × 5)/100

= Rs 40

4. A sum of Rs 400 is lent for 3 years at the rate of 6% per annum. Find the interest.

Solution:

Principal amount P = Rs 400

Time period T = 3 years

Rate of interest R = 6% p.a.

We know that simple interest = (P × T × R)/100

On substituting these values in above equation we get

SI = (400 × 3 × 6)/100

= Rs 72

5. A person deposits Rs 25000 in a firm who pays an interest at the rate of 20% per annum. Calculate the income he gets from it annually.

Solution:

Given Principal amount P = Rs 25000

Time period T = 1 year

Rate of interest R = 20% p.a.

We know that simple interest = (P × T × R)/100

On substituting these values in above equation we get

SI = (25000 × 1 × 20)/100

= Rs 5000

6. A man borrowed Rs 8000 from a bank at 8% per annum. Find the amount he has to pay after 4 ½ years.

Solution:

Given Principal amount P = Rs 8000

Time period T = 4 ½ years = 9/2 years

Rate of interest R = 8% p.a.

We know that simple interest = (P × T × R)/100

On substituting these values in above equation we get

SI = (8000 × (9/2) × 8)/100

= Rs 2880

Amount = Principal amount + Interest

= Rs 8000 + 2880

= Rs 10880

7. Rakesh lent out Rs 8000 for 5 years at 15% per annum and borrowed Rs 6000 for 3 years at 12% per annum. How much did he gain or lose?

Solution:

Given Principal amount P = Rs 8000

Time period T = 5 years

Rate of interest R = 15% p.a.

We know that simple interest = (P × T × R)/100

On substituting these values in above equation we get

SI = (8000 × 5 × 15)/100

= Rs 6000

Principal amount P = Rs 6000

Time period T = 3 years

Rate of interest R = 12% p.a.

We know that simple interest = (P × T × R)/100

On substituting these values in above equation we get

SI = (6000 × 3 × 12)/100

= Rs 2160

Amount gained by Rakesh = Rs 6000 – Rs 2160

= Rs 3840

8. Anita deposits Rs 1000 in a savings bank account. The bank pays interest at the rate of 5% per annum. What amount can Anita get after one year?

Solution:

Given Principal amount P = Rs 1000

Time period T = 1 year

Rate of interest R = 5% p.a.

We know that simple interest = (P × T × R)/100

On substituting these values in above equation we get

SI = (1000 × 1 × 5)/100

= Rs 50

Total amount paid after 1 year = Principal amount + Interest

= Rs 1000 + Rs 50

= Rs 1050

9. Nalini borrowed Rs 550 from her friend at 8% per annum. She returned the amount after 6 months. How much did she pay?

Solution:

Given Principal amount P = Rs 550

Time period T = ½ year

Rate of interest R = 8% p.a.

We know that simple interest = (P × T × R)/100

On substituting these values in above equation we get

SI = (550 × ½ × 8)/100

= Rs 22

Total amount paid after ½ year = Principal amount + Interest

= Rs 550 + Rs 22

= Rs 572

10. Rohit borrowed Rs 60000 from a bank at 9% per annum for 2 years. He lent this sum of money to Rohan at 10% per annum for 2 years. How much did Rohit earn from this transaction?

Solution:

Given Principal amount P = Rs 60000

Time period T = 2 years

Rate of interest R = 10% p.a.

We know that simple interest = (P × T × R)/100

On substituting these values in above equation we get

SI = (60000 × 2 × 10)/100

= Rs 12000

Principal amount P = Rs 60000

Time period T = 2 years

Rate of interest R = 9% p.a.

We know that simple interest = (P × T × R)/100

On substituting these values in above equation we get

SI = (60000 × 2 × 9)/100

= Rs 10800

Amount gained by Rohit = Rs 12000 – Rs 10800

= Rs 1200

11. Romesh borrowed Rs 2000 at 2% per annum and Rs 1000 at 5% per annum. He cleared his debt after 2 years by giving Rs 2800 and a watch. What is the cost of the watch?

Solution:

Given Principal amount P = Rs 2000

Time period T = 2 years

Rate of interest R = 2% p.a.

We know that simple interest = (P × T × R)/100

On substituting these values in above equation we get

SI = (2000 × 2 × 2)/100

= Rs 80

Principal amount P = Rs 1000

Time period T = 2 years

Rate of interest R = 5% p.a.

We know that simple interest = (P × T × R)/100

On substituting these values in above equation we get

SI = (1000 × 2 × 5)/100

= Rs 100

Total amount that he will have to return = Rs. 2000 + 1000 + 80 + 100 = Rs. 3180

Amount repaid = Rs. 2800

Value of the watch = Rs. 3180 – 2800 = Rs. 380

12. Mr Garg lent Rs 15000 to his friend. He charged 15% per annum on Rs 12500 and 18% on the rest. How much interest does he earn in 3 years?

Solution:

Given Principal amount P = Rs 12500

Time period T = 3 years

Rate of interest R = 15% p.a.

We know that simple interest = (P × T × R)/100

On substituting these values in above equation we get

SI = (12500 × 3 × 15)/100

= Rs 5625

Rest of the amount lent = Rs 15000 – Rs 12500 = Rs 2500

Rate of interest = 18 % p.a.

Time period = 3 years

We know that simple interest = (P × T × R)/100

On substituting these values in above equation we get

SI = (2500 × 3 × 18)/100

= Rs 1350

Total interest earned = Rs 5625 + Rs 1350 = Rs 6975

13. Shikha deposited Rs 2000 in a bank which pays 6% simple interest. She withdrew Rs 700 at the end of first year. What will be her balance after 3 years?

Solution:

Given Principal amount P = Rs 2000

Time period T = 1 year

Rate of interest R = 6% p.a.

We know that simple interest = (P × T × R)/100

On substituting these values in above equation we get

SI = (2000 × 1 × 6)/100

= Rs 120

So amount after 1 year = Principal amount + Interest = 2000 + 120 = Rs 2120

after 1 year, amount withdrawn = Rs 700

Principal amount left = Rs 2120 – Rs 700 = Rs 1420

Time period = 2 years

Rate of interest = 6% p.a.

We know that simple interest = (P × T × R)/100

On substituting these values in above equation we get

SI = (1420 × 2 × 6)/100

Interest after two years = Rs 170.40

Total amount after 3 years = Rs 1420 + Rs 170.40 = Rs 1590.40

14. Reema took a loan of Rs 8000 from a money lender, who charged interest at the rate of 18% per annum. After 2 years, Reema paid him Rs 10400 and wrist watch to clear the debt. What is the price of the watch?

Solution:

Given Principal amount P = Rs 8000

Time period T = 2 years

Rate of interest R = 18% p.a.

We know that simple interest = (P × T × R)/100

On substituting these values in above equation we get

SI = (8000 × 2 × 18)/100

= Rs 2880

Total amount payable by Reema after 2 years = Rs 8,000 + Rs 2,880

= Rs 10,880

Amount paid = Rs 10,400

Value of the watch = Rs 10,880 – Rs 10,400 = Rs 480

15. Mr Sharma deposited Rs 20000 as a fixed deposit in a bank at 10% per annual. If 30% is deducted as income tax on the interest earned, find his annual income.

Solution:

Given Principal amount P = Rs 20000

Time period T = 1 year

Rate of interest R = 10% p.a.

We know that simple interest = (P × T × R)/100

On substituting these values in above equation we get

SI = (20000 × 1 × 10)/100

= Rs 2000

Amount deducted as income tax = 30% of 2000 = (30 × 2000)/100

= Rs 600

Annual interest after tax deduction = Rs 2,000 – Rs 600 = Rs 1,400

Chapter - 14 Lines And Angles

Exercise 14.1

1. Write down each pair of adjacent angles shown in fig. 13.

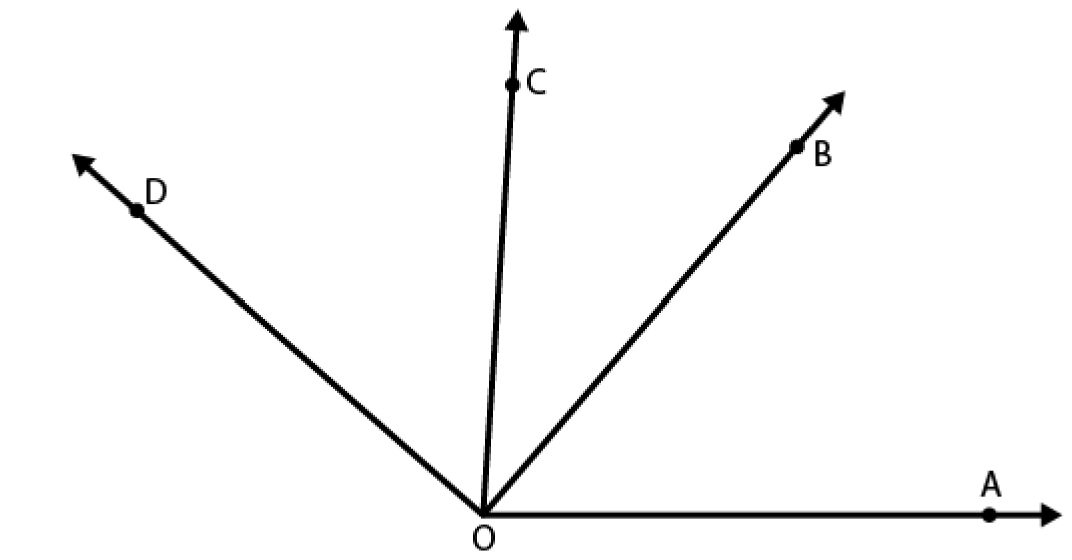


Fig 13

Solution:

The angles that have common vertex and a common arm are known as adjacent angles

Therefore the adjacent angles in given figure are:

∠DOC and ∠BOC

∠COB and ∠BOA

2. In Fig. 14, name all the pairs of adjacent angles.

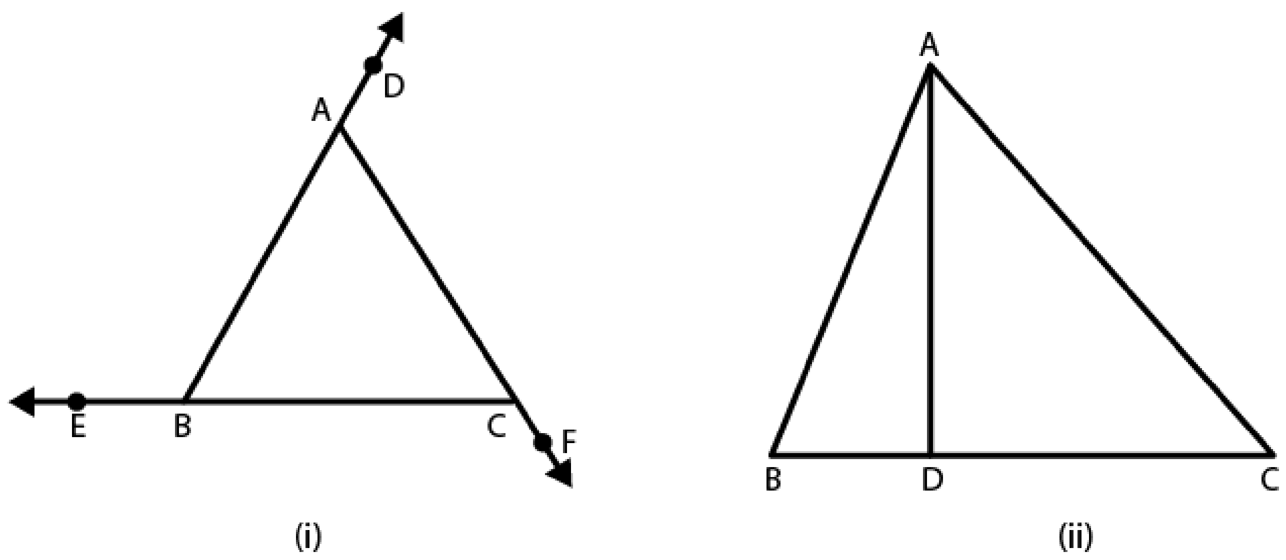


Fig 14

Solution:

The angles that have common vertex and a common arm are known as adjacent angles.

In fig (i), the adjacent angles are

∠EBA and ∠ABC

∠ACB and ∠BCF

∠BAC and ∠CAD

In fig (ii), the adjacent angles are

∠BAD and ∠DAC

∠BDA and ∠CDA

3. In fig. 15, write down

(i) Each linear pair

(ii) Each pair of vertically opposite angles.

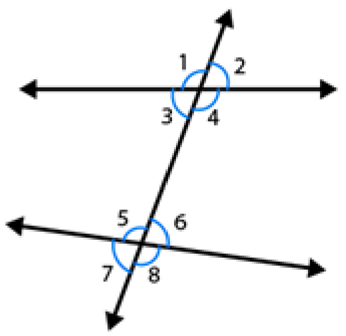


Fig 15

Solution:

(i) The two adjacent angles are said to form a linear pair of angles if their non – common arms are two opposite rays.

∠1 and ∠3

∠1 and ∠2

∠4 and ∠3

∠4 and ∠2

∠5 and ∠6

∠5 and ∠7

∠6 and ∠8

∠7 and ∠8

(ii) The two angles formed by two intersecting lines and have no common arms are called vertically opposite angles.

∠1 and ∠4

∠2 and ∠3

∠5 and ∠8

∠6 and ∠7

4. Are the angles 1 and 2 given in Fig. 16 adjacent angles?

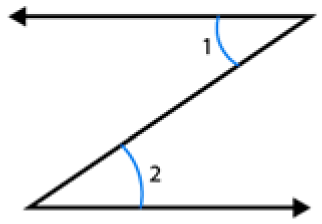


Fig 16

Solution:

No, because they don’t have common vertex.

5. Find the complement of each of the following angles:

(i) 35^o

(ii) 72^o

(iii) 45^o

(iv) 85^o

Solution:

(i) The two angles are said to be complementary angles if the sum of those angles is 90^o

Complementary angle for given angle is

$90^o – 35^o = 55^o$

(ii) The two angles are said to be complementary angles if the sum of those angles is 90^o

Complementary angle for given angle is

$90^0 – 72^o = 18^o$

(iii) The two angles are said to be complementary angles if the sum of those angles is 90^o

Complementary angle for given angle is

$90^o – 45^o = 45^o$

(iv) The two angles are said to be complementary angles if the sum of those angles is 90^o

Complementary angle for given angle is

$90^o – 85^o = 5^o$

6. Find the supplement of each of the following angles:

(i) 70^o

(ii) 120^o

(iii) 135^o

(iv) 90^o

Solution:

(i) The two angles are said to be supplementary angles if the sum of those angles is 180^o

Therefore supplementary angle for the given angle is

$180^o – 70^o = 110^o$

(ii) The two angles are said to be supplementary angles if the sum of those angles is 180^o

Therefore supplementary angle for the given angle is

$180^o – 120^o = 60^o$

(iii) The two angles are said to be supplementary angles if the sum of those angles is 180^o

Therefore supplementary angle for the given angle is

$180^\circ - 135^\circ = 45^\circ$

(iv) The two angles are said to be supplementary angles if the sum of those angles is 180°

Therefore supplementary angle for the given angle is

$180^\circ - 90^\circ = 90^\circ$

7. Identify the complementary and supplementary pairs of angles from the following pairs:

(i) 25°, 65°

(ii) 120°, 60°

(iii) 63°, 27°

(iv) 100°, 80°

Solution:

(i) $25^\circ + 65^\circ = 90^\circ$ so, this is a complementary pair of angle.

(ii) $120^\circ + 60^\circ = 180^\circ$ so, this is a supplementary pair of angle.

(iii) $63^\circ + 27^\circ = 90^\circ$ so, this is a complementary pair of angle.

(iv) $100^\circ + 80^\circ = 180^\circ$ so, this is a supplementary pair of angle.

8. Can two obtuse angles be supplementary, if both of them be

(i) Obtuse?

(ii) Right?

(iii) Acute?

Solution:

(i) No, two obtuse angles cannot be supplementary

Because, the sum of two angles is greater than 90° so their sum will be greater than 180°

(ii) Yes, two right angles can be supplementary

Because, $90^\circ + 90^\circ = 180^\circ$

(iii) No, two acute angle cannot be supplementary

Because, the sum of two angles is less than 90° so their sum will also be less than 90°

9. Name the four pairs of supplementary angles shown in Fig.17.

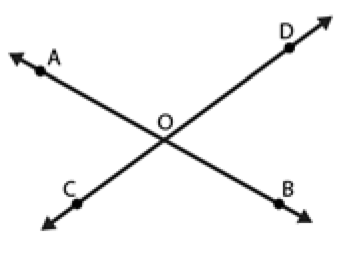


Fig 17

Solution:

The two angles are said to be supplementary angles if the sum of those angles is 180°

The supplementary angles are

$\angle AOC$ and $\angle COB$

$\angle BOC$ and $\angle DOB$

$\angle BOD$ and $\angle DOA$

$\angle AOC$ and $\angle DOA$

10. In Fig. 18, A, B, C are collinear points and ∠DBA = ∠EBA.

(i) Name two linear pairs.

(ii) Name two pairs of supplementary angles.

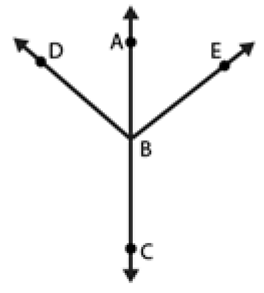


Fig 18

Solution:

(i) Two adjacent angles are said to be form a linear pair of angles, if their non-common arms are two opposite rays.

Therefore linear pairs are

∠ABD and ∠DBC

∠ABE and ∠EBC

(ii) We know that every linear pair forms supplementary angles, these angles are

∠ABD and ∠DBC

∠ABE and ∠EBC

11. If two supplementary angles have equal measure, what is the measure of each angle?

Solution:

Let p and q be the two supplementary angles that are equal

The two angles are said to be supplementary angles if the sum of those angles is 180^o

$\angle p = \angle q$

So,

$\angle p + \angle q = 180^o$

$\angle p + \angle p = 180^o$

$2\angle p = 180^o$

$\angle p = 180^o/2$

$\angle p = 90^o$

Therefore, $\angle p = \angle q = 90^o$

12. If the complement of an angle is 28^o, then find the supplement of the angle.

Solution:

Given complement of an angle is 28^o

Here, let x be the complement of the given angle 28^o

Therefore, $\angle x + 28^o = 90^o$

$\angle x = 90^o – 28^o$

$= 62^o$

So, the supplement of the angle $= 180^o – 62^o$

= 118^{o}

13. In Fig. 19, name each linear pair and each pair of vertically opposite angles:

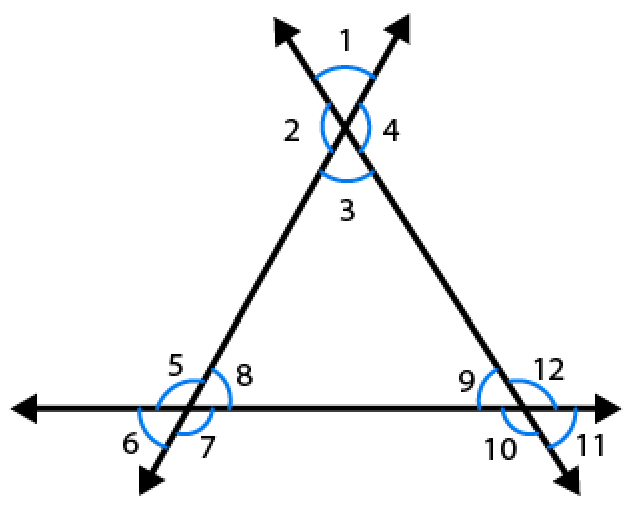


Fig 19

Solution:

Two adjacent angles are said to be linear pair of angles, if their non-common arms are two opposite rays.

Therefore linear pairs are listed below:

∠1 and ∠2

∠2 and ∠3

∠3 and ∠4

∠1 and ∠4

∠5 and ∠6

∠6 and ∠7

∠7 and ∠8

∠8 and ∠5

∠9 and ∠10

∠10 and ∠11

∠11 and ∠12

∠12 and ∠9

The two angles are said to be vertically opposite angles if the two intersecting lines have no common arms.

Therefore supplement of the angle are listed below:

∠1 and ∠3

∠4 and ∠2

∠5 and ∠7

∠6 and ∠8

∠9 and ∠11

∠10 and ∠12

14. In Fig. 20, OE is the bisector of ∠BOD. If ∠1 = 70°, find the magnitude of ∠2, ∠3 and ∠4.

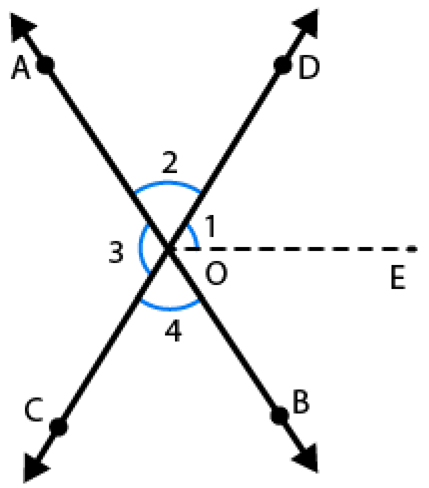


Fig 20

Solution:

Given, ∠1 = 70°

∠3 = 2(∠1)

= 2(70°)

∠3 = 140°

∠3 = ∠4

As, OE is the angle bisector,

∠DOB = 2(∠1)

= 2(70°)

= 140°

∠DOB + ∠AOC + ∠COB +∠AOD = 360° [sum of the angle of circle = 360°]

140° + 140° + 2(∠COB) = 360°

Since, ∠COB = ∠AOD

2(∠COB) = 360° – 280°

2(∠COB) = 80°

∠COB = 80°/2

∠COB = 40°

Therefore, ∠COB = ∠AOD = 40°

The angles are, ∠1 = 70°, ∠2 = 40°, ∠3 = 140° and ∠4 = 40°

15. One of the angles forming a linear pair is a right angle. What can you say about its other angle?

Solution:

Given one of the angle of a linear pair is the right angle that is 90°

We know that linear pair angle is 180°

Therefore, the other angle is

$180^\circ - 90^\circ = 90^\circ$

16. One of the angles forming a linear pair is an obtuse angle. What kind of angle is the other?

Solution:

Given one of the angles of a linear pair is obtuse, then the other angle should be acute, because only then their sum will be 180°.

17. One of the angles forming a linear pair is an acute angle. What kind of angle is the other?

Solution:

Given one of the Angles of a linear pair is acute, then the other angle should be obtuse, only then their sum will be 180°.

18. Can two acute angles form a linear pair?

Solution:

No, two acute angles cannot form a linear pair because their sum is always less than 180°.

19. If the supplement of an angle is 65°, then find its complement.

Solution:

Let x be the required angle

So, $x + 65^\circ = 180^\circ$

$x = 180^\circ - 65^\circ$

$x = 115^\circ$

The two angles are said to be complementary angles if the sum of those angles is 90° here it is more than 90° therefore the complement of the angle cannot be determined.

20. Find the value of x in each of the following figures.

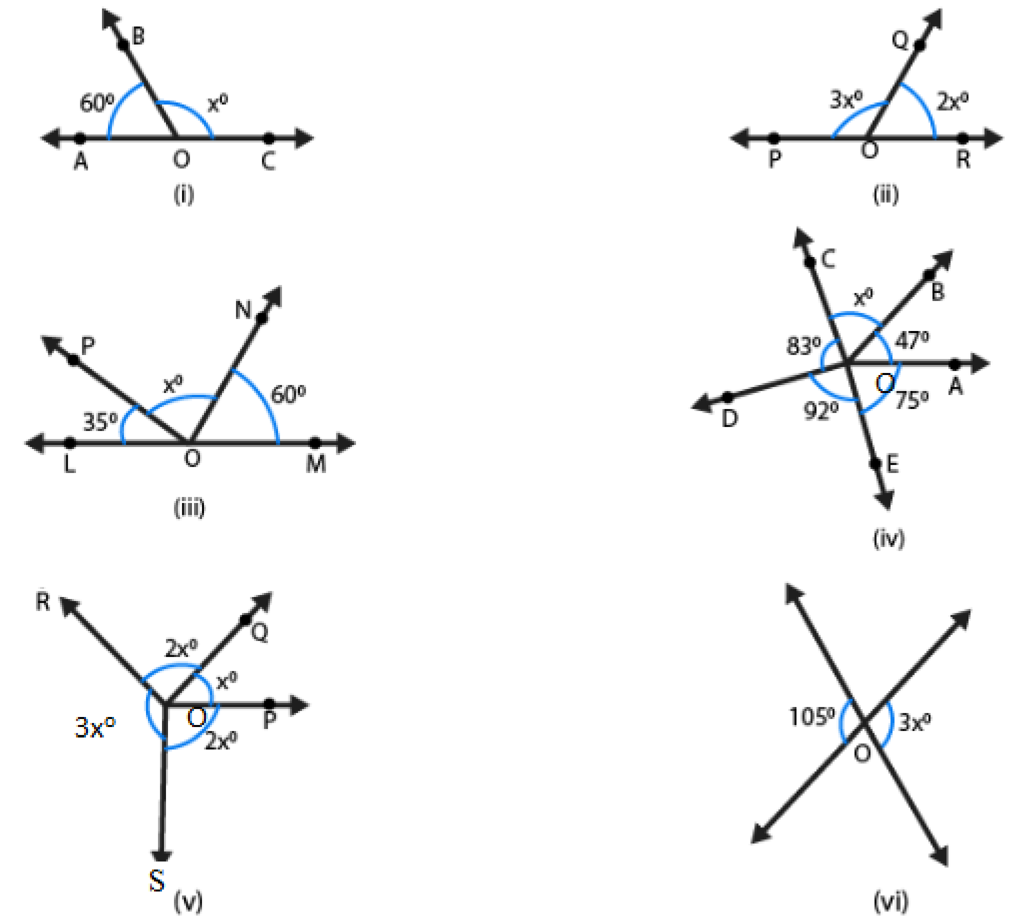


Fig 21

Solution:

(i) We know that ∠BOA + ∠BOC = 180^o

[Linear pair: The two adjacent angles are said to form a linear pair of angles if their non–common arms are two opposite rays and sum of the angle is 180^o]
$60^o + x^o = 180^o$

$x^o = 180^o – 60^o$

$x^o = 120^o$

(ii) We know that ∠POQ + ∠QOR = 180^o

[Linear pair: The two adjacent angles are said to form a linear pair of angles if their non–common arms are two opposite rays and sum of the angle is 180^o]
$3x^o + 2x^o = 180^o$

$5x^o = 180^o$

$x^o = 180^o/5$

$x^o = 36^o$

(iii) We know that ∠LOP + ∠PON + ∠NOM = 180^o

[Linear pair: The two adjacent angles are said to form a linear pair of angles if their non–common arms are two opposite rays and sum of the angle is 180^o]
Since, $35^o + x^o + 60^o = 180^o$

$x^o = 180^o – 35^o – 60^o$

$x^o = 180^o – 95^o$

$x^o = 85^o$

(iv) We know that ∠DOC + ∠DOE + ∠EOA + ∠AOB+ ∠BOC = 360^o

$83^o + 92^o + 47^o + 75^o + x^o = 360^o$

$x^o + 297^o = 360^o$

$x^o = 360^o – 297^o$

$x^o = 63^o$

(v) We know that ∠ROS + ∠ROQ + ∠QOP + ∠POS = 360^o

$3x^o + 2x^o + x^o + 2x^o = 360^o$

$8x^o = 360^o$

$x^o = 360^o/8$

$x^o = 45^o$

(vi) Linear pair: The two adjacent angles are said to form a linear pair of angles if their non–common arms are two opposite rays and sum of the angle is 180^o

Therefore $3x^o = 105^o$

$x^o = 105^o/3$

$x^o = 35^o$

21. In Fig. 22, it being given that ∠1 = 65^o, find all other angles.

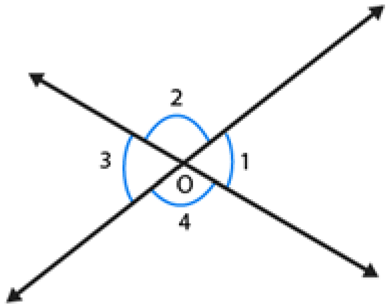


Fig 22

Solution:

Given from the figure 22, ∠1 = ∠3 are the vertically opposite angles

Therefore, ∠3 = 65^o

Here, ∠1 + ∠2 = 180° are the linear pair [The two adjacent angles are said to form a linear pair of angles if their non–common arms are two opposite rays and sum of the angle is 180^o]

Therefore, ∠2 = $180^o – 65^o$

= 115^o

∠2 = ∠4 are the vertically opposite angles [from the figure]

Therefore, ∠2 = ∠4 = 115^o

And ∠3 = 65^o

22. In Fig. 23, OA and OB are opposite rays:

(i) If x = 25^o, what is the value of y?

(ii) If y = 35^o, what is the value of x?

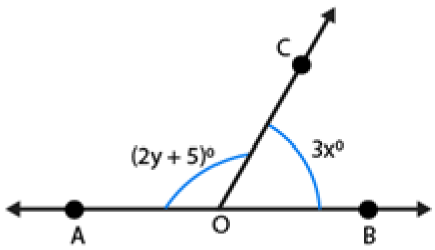


Fig 23

Solution:

(i) ∠AOC + ∠BOC = 180^o [The two adjacent angles are said to form a linear pair of angles if their non–common arms are two opposite rays and sum of the angle is 180^o]

$2y + 5^0 + 3x = 180^o$

$3x + 2y = 175^o$

Given If x = 25^o, then

$3(25^o) + 2y = 175^o$

$75^o + 2y = 175^o$

$2y = 175^o – 75^o$

$2y = 100^o$

$y = 100^o/2$

$y = 50^o$

(ii) ∠AOC + ∠BOC = 180° [The two adjacent angles are said to form a linear pair of angles if their non–common arms are two opposite rays and sum of the angle is 180°]

$2y + 5 + 3x = 180^\circ$

$3x + 2y = 175^\circ$

Given If $y = 35^\circ$, then

$3x + 2(35^\circ) = 175^\circ$

$3x + 70^\circ = 175^\circ$

$3x = 175^0 - 70^\circ$

$3x = 105^\circ$

$x = 105^\circ/3$

$x = 35^\circ$

23. In Fig. 24, write all pairs of adjacent angles and all the liner pairs.

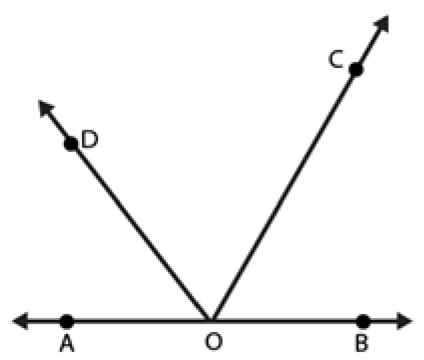


Fig 24

Solution:

Pairs of adjacent angles are:

∠DOA and ∠DOC

∠BOC and ∠COD

∠AOD and ∠BOD

∠AOC and ∠BOC

Linear pairs: [The two adjacent angles are said to form a linear pair of angles if their non–common arms are two opposite rays and sum of the angle is 180°]

∠AOD and ∠BOD

∠AOC and ∠BOC

24. In Fig. 25, find ∠x. Further find ∠BOC, ∠COD and ∠AOD.

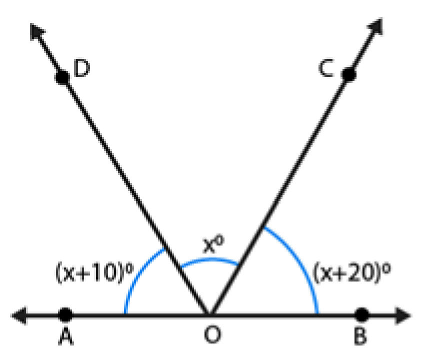


Fig 25

Solution:

$(x + 10)^o + x^o + (x + 20)^o = 180^o$[linear pair]

On rearranging we get

$3x^o + 30^o = 180^o$

$3x^o = 180^o – 30^o$

$3x^o = 150^o$

$x^o = 150^o/3$

$x^o = 50^o$

Also given that

∠BOC = $(x + 20)^o$

= $(50 + 20)^o$

= 70^o

∠COD = 50^o

∠AOD = $(x + 10)^o$

= $(50 + 10)^o$

= 60^o

25. How many pairs of adjacent angles are formed when two lines intersect in a point?

Solution:

If the two lines intersect at a point, then four adjacent pairs are formed and those are linear.

26. How many pairs of adjacent angles, in all, can you name in Fig. 26?

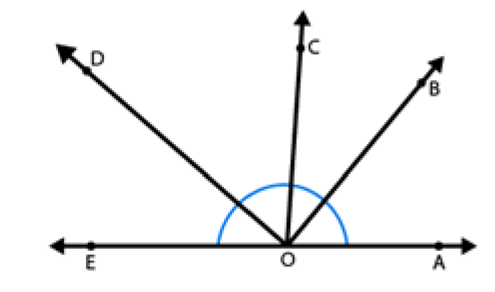


Fig 26

Solution:

There are 10 adjacent pairs formed in the given figure, they are

∠EOD and ∠DOC

∠COD and ∠BOC

∠COB and ∠BOA

∠AOB and ∠BOD

∠BOC and ∠COE

∠COD and ∠COA

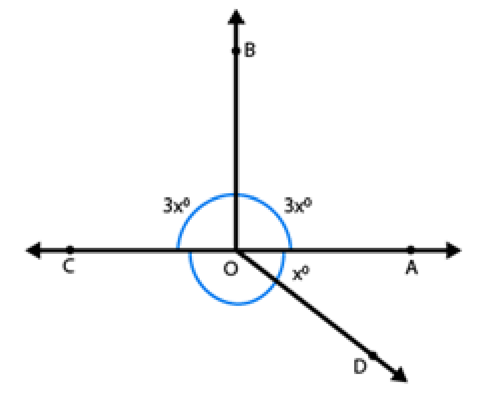
∠DOE and ∠DOB

∠EOD and ∠DOA

∠EOC and ∠AOC

∠AOB and ∠BOE

27. In Fig. 27, determine the value of x.



Solution:

From the figure we can write as ∠COB + ∠AOB = 180° [linear pair]

$3x^o + 3x^o = 180^o$

$6x^o = 180^o$

$x^o = 180^o/6$

$x^o = 30^o$

28. In Fig.28, AOC is a line, find x.

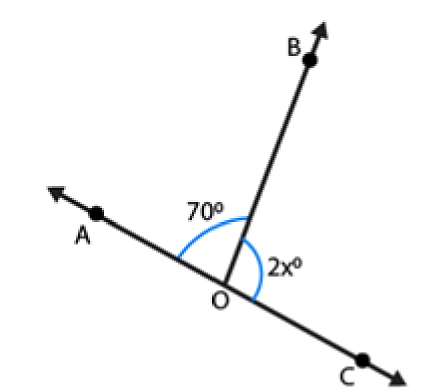


Fig 28

Solution:

From the figure we can write as

∠AOB + ∠BOC = 180° [linear pair]

Linear pair

$2x + 70^o = 180^o$

$2x = 180^o – 70^o$

$2x = 110^o$

$x = 110^o/2$

$x = 55^o$

29. In Fig. 29, POS is a line, find x.

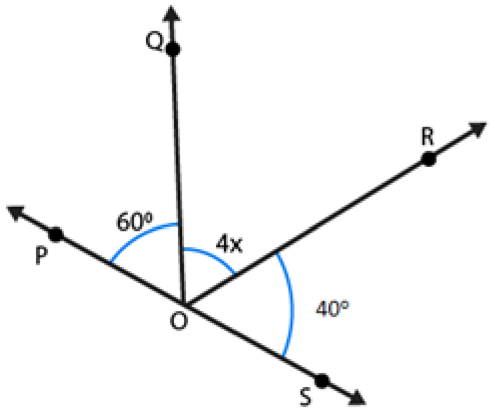


Fig 29

Solution:

From the figure we can write as angles of a straight line,

$\angle QOP + \angle QOR + \angle ROS = 180^\circ$

$60^\circ + 4x + 40^\circ = 180^\circ$

On rearranging we get, $100^\circ + 4x = 180^\circ$

$4x = 180^\circ - 100^\circ$

$4x = 80^\circ$

$x = 80^\circ/4$

$x = 20^\circ$

30. In Fig. 30, lines l_1 and l_2 intersect at O, forming angles as shown in the figure. If $x = 45^\circ$, find the values of y, z and u.

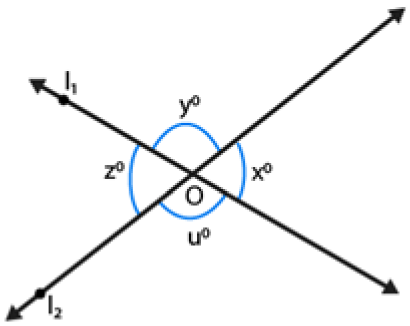


Fig 30

Solution:

Given that, $\angle x = 45^\circ$

From the figure we can write as

$\angle x = \angle z = 45^\circ$

Also from the figure, we have

$\angle y = \angle u$

From the property of linear pair we can write as

$\angle x + \angle y + \angle z + \angle u = 360^\circ$

$45^\circ + 45^\circ + \angle y + \angle u = 360^\circ$

$90^\circ + \angle y + \angle u = 360^\circ$

$\angle y + \angle u = 360^\circ - 90^\circ$

$\angle y + \angle u = 270^o$ (vertically opposite angles $\angle y = \angle u$)

$2\angle y = 270^o$

$\angle y = 135^o$

Therefore, $\angle y = \angle u = 135^o$

So, $\angle x = 45^o$, $\angle y = 135^o$, $\angle z = 45^o$ and $\angle u = 135^o$

31. In Fig. 31, three coplanar lines intersect at a point O, forming angles as shown in the figure. Find the values of x, y, z and u

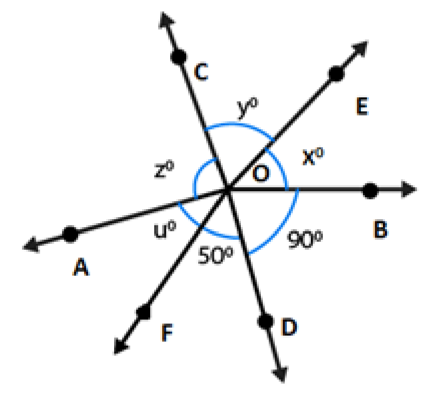


Fig 31

Solution:

Given that, $\angle x + \angle y + \angle z + \angle u + 50^o + 90^o = 360^o$

Linear pair, $\angle x + 50^o + 90^o = 180^o$

$\angle x + 140^o = 180^o$

On rearranging we get

$\angle x = 180^o - 140^o$

$\angle x = 40^o$

From the figure we can write as

$\angle x = \angle u = 40^o$ are vertically opposite angles

$\angle z = 90^o$ is a vertically opposite angle

$\angle y = 50^o$ is a vertically opposite angle

Therefore, $\angle x = 40^o$, $\angle y = 50^o$, $\angle z = 90^o$ and $\angle u = 40^o$

32. In Fig. 32, find the values of x, y and z.

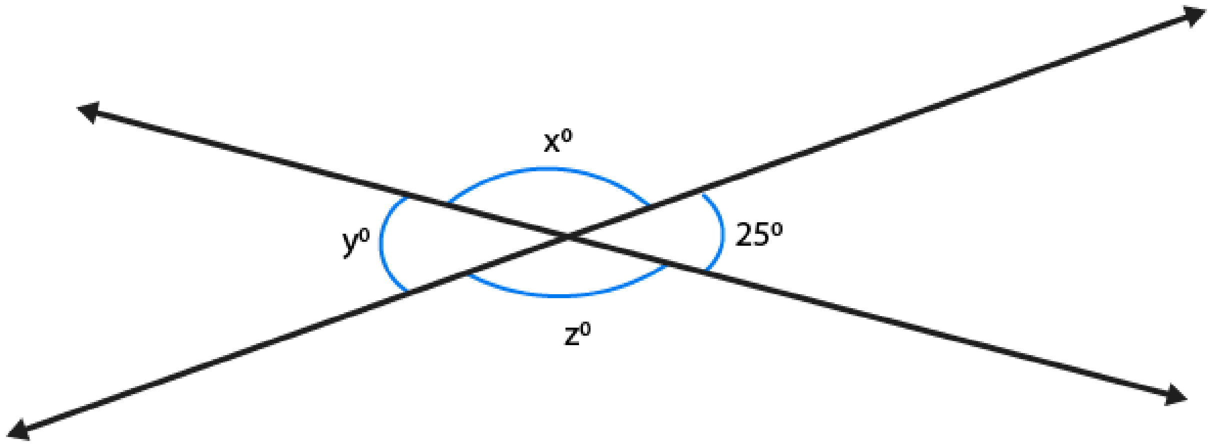


Fig 32

Solution:

$\angle y = 25^{o}$ vertically opposite angle

From the figure we can write as

$\angle x = \angle z$ are vertically opposite angles

$\angle x + \angle y + \angle z + 25^{o} = 360^{o}$

$\angle x + \angle z + 25^{o} + 25^{o} = 360^{o}$

On rearranging we get,

$\angle x + \angle z + 50^{o} = 360^{o}$

$\angle x + \angle z = 360^{o} - 50^{o}$ [$\angle x = \angle z$]

$2\angle x = 310^{o}$

$\angle x = 155^{o}$

And, $\angle x = \angle z = 155^{o}$

Therefore, $\angle x = 155^{o}$, $\angle y = 25^{o}$ and $\angle z = 155^{o}$

Exercise 14.2

1. In Fig. 58, line n is a transversal to line l and m. Identify the following:

(i) Alternate and corresponding angles in Fig. 58 (i)

(ii) Angles alternate to $\angle d$ and $\angle g$ and angles corresponding to $\angle f$ and $\angle h$ in Fig. 58 (ii)

(iii) Angle alternate to $\angle PQR$, angle corresponding to $\angle RQF$ and angle alternate to $\angle PQE$ in Fig. 58 (iii)

(iv) Pairs of interior and exterior angles on the same side of the transversal in Fig. 58 (ii)

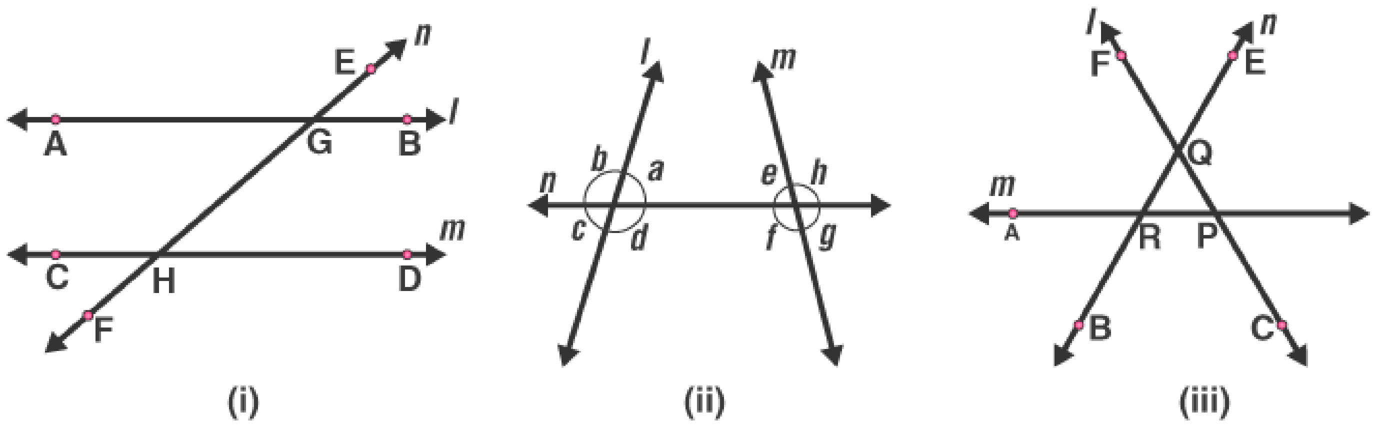


Fig.58

Solution:

(i) A pair of angles in which one arm of both the angles is on the same side of the transversal and their other arms are directed in the same sense is called a pair of corresponding angles.

In Figure (i) Corresponding angles are

∠EGB and ∠GHD

∠HGB and ∠FHD

∠EGA and ∠GHC

∠AGH and ∠CHF

A pair of angles in which one arm of each of the angle is on opposite sides of the transversal and whose other arms include the one segment is called a pair of alternate angles.

The alternate angles are:

∠EGB and ∠CHF

∠HGB and ∠CHG

∠EGA and ∠FHD

∠AGH and ∠GHD

(ii) In Figure (ii)

The alternate angle to ∠d is ∠e.

The alternate angle to ∠g is ∠b.

The corresponding angle to ∠f is ∠c.

The corresponding angle to ∠h is ∠a.

(iii) In Figure (iii)

Angle alternate to ∠PQR is ∠QRA.

Angle corresponding to ∠RQF is ∠ARB.

Angle alternate to ∠POE is ∠ARB.

(iv) In Figure (ii)

Pair of interior angles are

∠a is ∠e.

∠d is ∠f.

Pair of exterior angles are

∠b is ∠h.

∠c is ∠g.

2. In Fig. 59, AB and CD are parallel lines intersected by a transversal PQ at L and M respectively, If ∠CMQ = 60°, find all other angles in the figure.

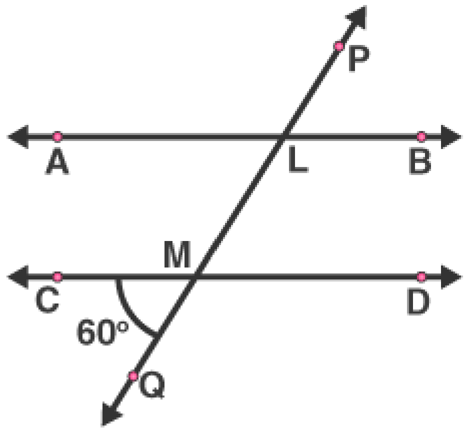


Fig. 59

Solution:

A pair of angles in which one arm of both the angles is on the same side of the transversal and their other arms are directed in the same sense is called a pair of corresponding angles.

Therefore corresponding angles are

∠ALM = ∠CMQ = 60° [given]

Vertically opposite angles are

∠LMD = ∠CMQ = 60° [given]

Vertically opposite angles are

∠ALM = ∠PLB = 60°

Here, ∠CMQ + ∠QMD = 180° are the linear pair

On rearranging we get

∠QMD = 180° – 60°

= 120°

Corresponding angles are

∠QMD = ∠MLB = 120°

Vertically opposite angles

∠QMD = ∠CML = 120°

Vertically opposite angles

∠MLB = ∠ALP = 120°

3. In Fig. 60, AB and CD are parallel lines intersected by a transversal by a transversal PQ at L and M respectively. If ∠LMD = 35° find ∠ALM and ∠PLA.

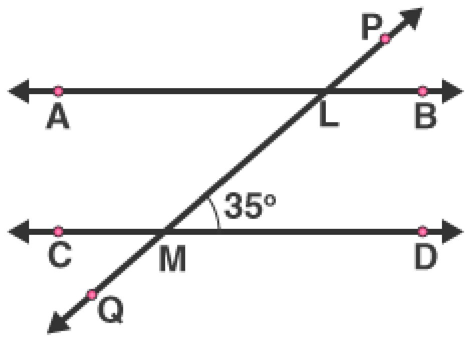


Fig. 60

Solution:

Given that, ∠LMD = 35^o

From the figure we can write

∠LMD and ∠LMC is a linear pair

∠LMD + ∠LMC = 180^o [sum of angles in linear pair = 180^o]

On rearranging, we get

∠LMC = $180^o – 35^o$

= 145^o

So, ∠LMC = ∠PLA = 145^o

And, ∠LMC = ∠MLB = 145^o

∠MLB and ∠ALM is a linear pair

∠MLB + ∠ALM = 180^o [sum of angles in linear pair = 180^o]

∠ALM = $180^o – 145^o$

∠ALM = 35^0

Therefore, ∠ALM = 35^o, ∠PLA = 145^o.

4. The line n is transversal to line l and m in Fig. 61. Identify the angle alternate to ∠13, angle corresponding to ∠15, and angle alternate to ∠15.

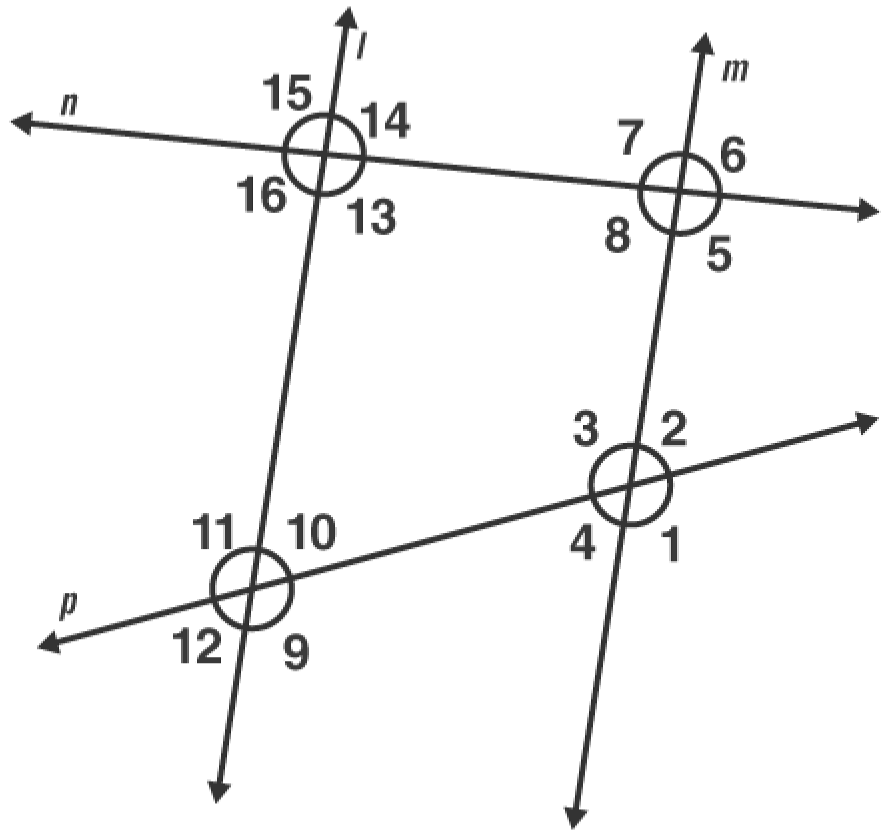


Fig.61

Solution:

Given that, l ∥ m

From the figure the angle alternate to ∠13 is ∠7

From the figure the angle corresponding to ∠15 is ∠7 [A pair of angles in which one arm of both the angles is on the same side of the transversal and their other arms are directed in the same sense is called a pair of corresponding angles.]

Again from the figure angle alternate to ∠15 is ∠5

5. In Fig. 62, line l ∥ m and n is transversal. If ∠1 = 40°, find all the angles and check that all corresponding angles and alternate angles are equal.

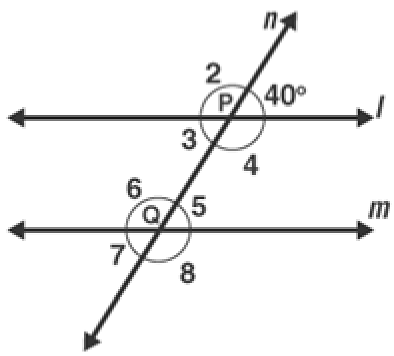


Fig. 62

Solution:

Given that, ∠1 = 40°

∠1 and ∠2 is a linear pair [from the figure]

∠1 + ∠2 = 180°

∠2 = 180° – 40°

$\angle 2 = 140^o$

Again from the figure we can say that

$\angle 2$ and $\angle 6$ is a corresponding angle pair

So, $\angle 6 = 140^o$

$\angle 6$ and $\angle 5$ is a linear pair [from the figure]

$\angle 6 + \angle 5 = 180^o$

$\angle 5 = 180^o - 140^o$

$\angle 5 = 40^o$

From the figure we can write as

$\angle 3$ and $\angle 5$ are alternate interior angles

So, $\angle 5 = \angle 3 = 40^o$

$\angle 3$ and $\angle 4$ is a linear pair

$\angle 3 + \angle 4 = 180^o$

$\angle 4 = 180^o - 40^o$

$\angle 4 = 140^o$

Now, $\angle 4$ and $\angle 6$ are a pair of interior angles

So, $\angle 4 = \angle 6 = 140^o$

$\angle 3$ and $\angle 7$ are a pair of corresponding angles

So, $\angle 3 = \angle 7 = 40^o$

Therefore, $\angle 7 = 40^o$

$\angle 4$ and $\angle 8$ are a pair of corresponding angles

So, $\angle 4 = \angle 8 = 140^o$

Therefore, $\angle 8 = 140^o$

Therefore, $\angle 1 = 40^o$, $\angle 2 = 140^o$, $\angle 3 = 40^o$, $\angle 4 = 140^o$, $\angle 5 = 40^o$, $\angle 6 = 140^o$, $\angle 7 = 40^o$ and $\angle 8 = 140^o$

6. In Fig.63, line l ∥ m and a transversal n cuts them P and Q respectively. If ∠1 = 75°, find all other angles.

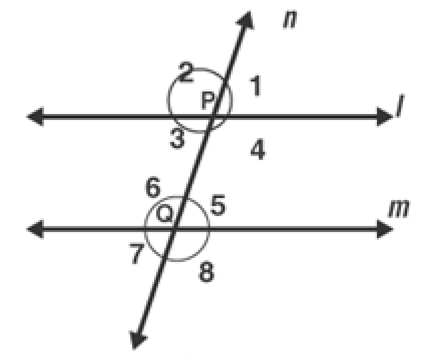


Fig. 63

Solution:

Given that, l ∥ m and $\angle 1 = 75^o$

$\angle 1 = \angle 3$ are vertically opposite angles

We know that, from the figure

$\angle 1 + \angle 2 = 180^o$ is a linear pair

$\angle 2 = 180^o - 75^o$

$\angle 2 = 105^o$

Here, $\angle 1 = \angle 5 = 75^o$ are corresponding angles

$\angle 5 = \angle 7 = 75^o$ are vertically opposite angles.

$\angle 2 = \angle 6 = 105^o$ are corresponding angles

$\angle 6 = \angle 8 = 105^o$ are vertically opposite angles

$\angle 2 = \angle 4 = 105^o$ are vertically opposite angles

So, $\angle 1 = 75^o$, $\angle 2 = 105^o$, $\angle 3 = 75^o$, $\angle 4 = 105^o$, $\angle 5 = 75^o$, $\angle 6 = 105^o$, $\angle 7 = 75^o$ and $\angle 8 = 105^o$

7. In Fig. 64, AB ∥ CD and a transversal PQ cuts at L and M respectively. If $\angle QMD = 100^o$, find all the other angles.

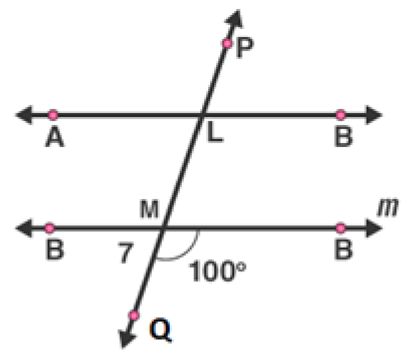


Fig. 64

Solution:

Given that, AB ∥ CD and $\angle QMD = 100^o$

We know that, from the figure $\angle QMD + \angle QMC = 180^o$ is a linear pair,

$\angle QMC = 180^o - \angle QMD$

$\angle QMC = 180^o - 100°$

$\angle QMC = 80^o$

Corresponding angles are

$\angle DMQ = \angle BLM = 100^o$

$\angle CMQ = \angle ALM = 80^o$

Vertically Opposite angles are

$\angle DMQ = \angle CML = 100^o$

$\angle BLM = \angle PLA = 100^o$

$\angle CMQ = \angle DML = 80^o$

$\angle ALM = \angle PLB = 80^o$

8. In Fig. 65, l ∥ m and p ∥ q. Find the values of x, y, z, t.

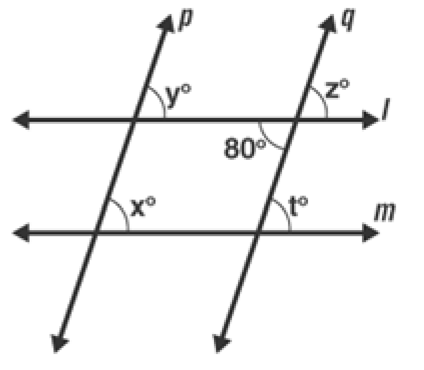


Fig. 65

Solution:

Given that one of the angle is 80^o

$\angle z$ and 80^o are vertically opposite angles

Therefore $\angle z = 80^o$

$\angle z$ and $\angle t$ are corresponding angles

$\angle z = \angle t$

Therefore, $\angle t = 80^o$

$\angle z$ and $\angle y$ are corresponding angles

$\angle z = \angle y$

Therefore, $\angle y = 80^o$

$\angle x$ and $\angle y$ are corresponding angles

$\angle y = \angle x$

Therefore, $\angle x = 80^o$

9. In Fig. 66, line l ∥ m, $\angle 1 = 120^o$ and $\angle 2 = 100^o$, find out $\angle 3$ and $\angle 4$.

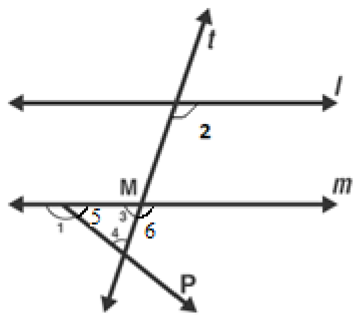


Fig. 66

Solution:

Given that, $\angle 1 = 120^o$ and $\angle 2 = 100^o$

From the figure $\angle 1$ and $\angle 5$ is a linear pair

$\angle 1 + \angle 5 = 180^o$

$\angle 5 = 180^o – 120^o$

$\angle 5 = 60^o$

Therefore, $\angle 5 = 60^o$

$\angle 2$ and $\angle 6$ are corresponding angles

$\angle 2 = \angle 6 = 100^o$

Therefore, $\angle 6 = 100^o$

$\angle 6$ and $\angle 3$ a linear pair

$\angle 6 + \angle 3 = 180^o$

$\angle 3 = 180^o - 100^o$

$\angle 3 = 80^o$

Therefore, $\angle 3 = 80^o$

By, angles of sum property

$\angle 3 + \angle 5 + \angle 4 = 180^o$

$\angle 4 = 180^o - 80^o - 60^o$

$\angle 4 = 40^o$

Therefore, $\angle 4 = 40^o$

10. In Fig. 67, l ∥ m. Find the values of a, b, c, d. Give reasons.

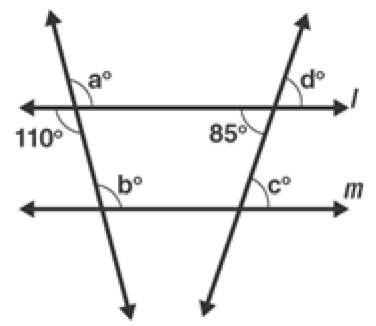


Fig. 67

Solution:

Given l ∥ m

From the figure vertically opposite angles,

$\angle a = 110^o$

Corresponding angles, $\angle a = \angle b$

Therefore, $\angle b = 110^o$

Vertically opposite angle,

$\angle d = 85^o$

Corresponding angles, $\angle d = \angle c$

Therefore, $\angle c = 85^o$

Hence, $\angle a = 110^o$, $\angle b = 110^o$, $\angle c = 85^o$, $\angle d = 85^o$

11. In Fig. 68, AB ∥ CD and $\angle 1$ and $\angle 2$ are in the ratio of 3: 2. Determine all angles from 1 to 8.

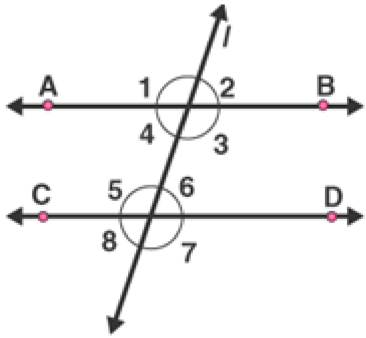


Fig. 68

Solution:

Given ∠1 and ∠2 are in the ratio 3: 2

Let us take the angles as 3x, 2x

∠1 and ∠2 are linear pair [from the figure]

$3x + 2x = 180^{o}$

$5x = 180^{o}$

$x = 180^{o}/5$

$x = 36^{o}$

Therefore, ∠1 = 3x = 3(36) = 108^{o}

∠2 = 2x = 2(36) = 72^{o}

∠1 and ∠5 are corresponding angles

Therefore ∠1 = ∠5

Hence, ∠5 = 108^{o}

∠2 and ∠6 are corresponding angles

So ∠2 = ∠6

Therefore, ∠6 = 72^{o}

∠4 and ∠6 are alternate pair of angles

∠4 = ∠6 = 72^{o}

Therefore, ∠4 = 72^{o}

∠3 and ∠5 are alternate pair of angles

∠3 = ∠5 = 108^{o}

Therefore, ∠3 = 108^{o}

∠2 and ∠8 are alternate exterior of angles

∠2 = ∠8 = 72^{o}

Therefore, ∠8 = 72^{o}

∠1 and ∠7 are alternate exterior of angles

∠1 = ∠7 = 108^{o}

Therefore, ∠7 = 108^{o}

Hence, ∠1 = 108^{o}, ∠2 = 72^{o}, ∠3 = 108^{o}, ∠4 = 72^{o}, ∠5 = 108^{o}, ∠6 = 72^{o}, ∠7 = 108^{o}, ∠8 = 72^{o}

12. In Fig. 69, l, m and n are parallel lines intersected by transversal p at X, Y and Z respectively. Find ∠1, ∠2 and ∠3.

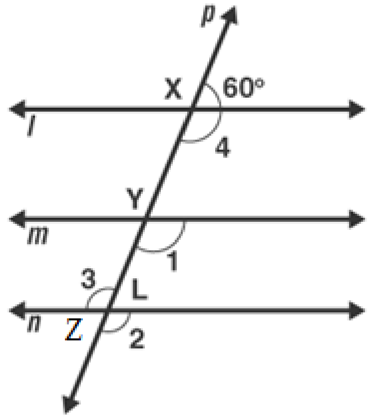


Fig. 69

Solution:

Given l, m and n are parallel lines intersected by transversal p at X, Y and Z

Therefore linear pair,

$\angle 4 + 60^\circ = 180^\circ$

$\angle 4 = 180^\circ - 60^\circ$

$\angle 4 = 120^\circ$

From the figure,

∠4 and ∠1 are corresponding angles

$\angle 4 = \angle 1$

Therefore, $\angle 1 = 120^\circ$

∠1 and ∠2 are corresponding angles

$\angle 2 = \angle 1$

Therefore, $\angle 2 = 120^\circ$

∠2 and ∠3 are vertically opposite angles

$\angle 2 = \angle 3$

Therefore, $\angle 3 = 120^0$

13. In Fig. 70, if l ∥ m ∥ n and ∠1 = 60°, find ∠2

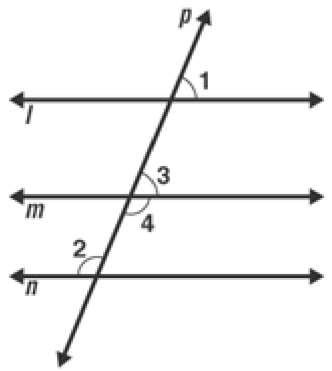


Fig. 70

Solution:

Given that l ∥ m ∥ n

From the figure Corresponding angles are

$\angle 1 = \angle 3$

$\angle 1 = 60^o$

Therefore, $\angle 3 = 60^o$

$\angle 3$ and $\angle 4$ are linear pair

$\angle 3 + \angle 4 = 180^o$

$\angle 4 = 180^o - 60^o$

$\angle 4 = 120^o$

$\angle 2$ and $\angle 4$ are alternate interior angles

$\angle 4 = \angle 2$

Therefore, $\angle 2 = 120^o$

14. In Fig. 71, if AB ∥ CD and CD ∥ EF, find ∠ACE

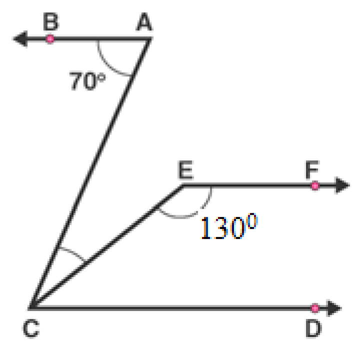


Fig. 71

Solution:

Given that, AB ∥ CD and CD ∥ EF

Sum of the interior angles,

$\angle CEF + \angle ECD = 180^o$

$130^o + \angle ECD = 180^o$

$\angle ECD = 180^o - 130^o$

$\angle ECD = 50^o$

We know that alternate angles are equal

$\angle BAC = \angle ACD$

$\angle BAC = \angle ECD + \angle ACE$

$\angle ACE = 70^o - 50^o$

$\angle ACE = 20^o$

Therefore, $\angle ACE = 20^o$

15. In Fig. 72, if l ∥ m, n ∥ p and ∠1 = 85°, find ∠2.

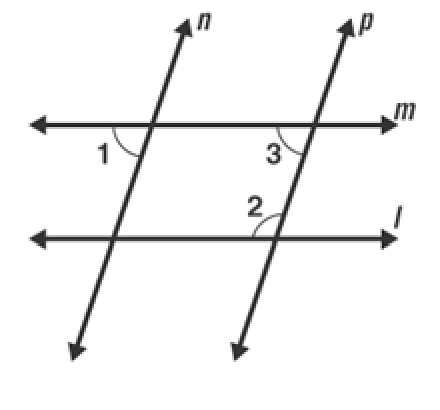


Fig. 72

Solution:

Given that, $\angle 1 = 85^o$

$\angle 1$ and $\angle 3$ are corresponding angles

So, $\angle 1 = \angle 3$

$\angle 3 = 85^o$

Sum of the interior angles is 180^o

$\angle 3 + \angle 2 = 180^o$

$\angle 2 = 180^o - 85^o$

$\angle 2 = 95^o$

16. In Fig. 73, a transversal n cuts two lines l and m. If $\angle 1 = 70^o$ and $\angle 7 = 80^o$, is l ı m?

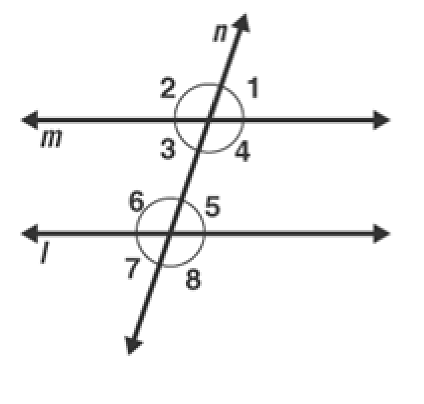


Fig. 73

Solution:

Given $\angle 1 = 70^o$ and $\angle 7 = 80^o$

We know that if the alternate exterior angles of the two lines are equal, then the lines are parallel.

Here, $\angle 1$ and $\angle 7$ are alternate exterior angles, but they are not equal

$\angle 1 \neq \angle 7$

17. In Fig. 74, a transversal n cuts two lines l and m such that $\angle 2 = 65^o$ and $\angle 8 = 65^o$. Are the lines parallel?

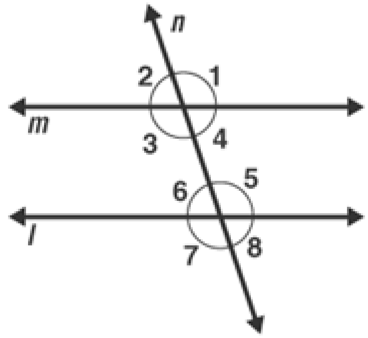


Fig. 74

Solution:

From the figure ∠2 = ∠4 are vertically opposite angles,

∠2 = ∠4 = 65^o

∠8 = ∠6 = 65^o

Therefore, ∠4 = ∠6

Hence, l ∥ m

18. In Fig. 75, Show that AB ∥ EF.

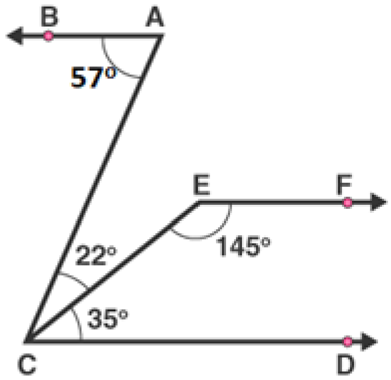


Fig. 75

Solution:

We know that,

∠ACD = ∠ACE + ∠ECD

∠ACD = 22^o + 35^o

∠ACD = 57^o = ∠BAC

Thus, lines BA and CD are intersected by the line AC such that, ∠ACD = ∠BAC

So, the alternate angles are equal

Therefore, AB ∥ CD ……1

Now,

∠ECD + ∠CEF = 35^o + 145^o = 180^o

This, shows that sum of the angles of the interior angles on the same side of the transversal CE is 180^o

So, they are supplementary angles

Therefore, EF ∥ CD …….2

From equation 1 and 2

We conclude that, AB ∥ EF

19. In Fig. 76, AB ∥ CD. Find the values of x, y, z.

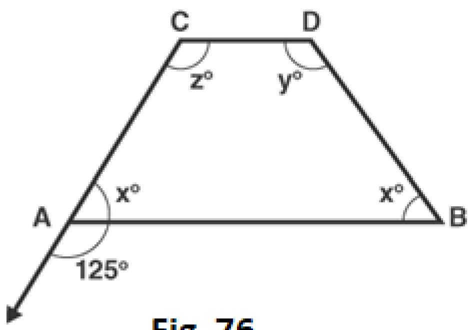


Fig. 76

Solution:

Given that AB ∥ CD

Linear pair,

$\angle x + 125^o = 180^o$

$\angle x = 180^o - 125^o$

$\angle x = 55^o$

Corresponding angles

$\angle z = 125^o$

Adjacent interior angles

$\angle x + \angle z = 180^o$

$\angle x + 125^o = 180^o$

$\angle x = 180^o - 125^o$

$\angle x = 55^o$

Adjacent interior angles

$\angle x + \angle y = 180^o$

$\angle y + 55^o = 180^o$

$\angle y = 180^o - 55^o$

$\angle y = 125^o$

20. In Fig. 77, find out ∠PXR, if PQ ∥ RS.

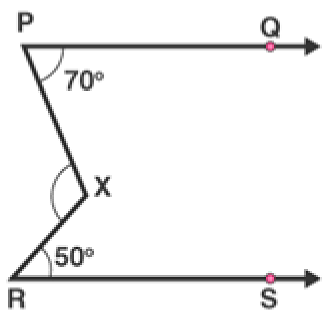


Fig. 77

Solution:

Given PQ ∥ RS

We need to find ∠PXR

∠XRS = 50^o

∠XPQ = 70^o

Given, that PQ ∥ RS

∠PXR = ∠XRS + ∠XPR

∠PXR = $50^o + 70^o$

∠PXR = 120^o

Therefore, ∠PXR = 120^o

21. In Figure, we have

(i) ∠MLY = 2∠LMQ

(ii) ∠XLM = $(2x – 10)^o$ and ∠LMQ = $(x + 30)^o$, find x.

(iii) ∠XLM = ∠PML, find ∠ALY

(iv) ∠ALY = $(2x – 15)^o$, ∠LMQ = $(x + 40)^o$, find x.

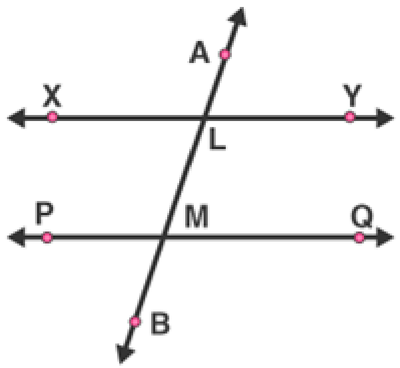


Fig. 78

Solution:

(i) ∠MLY and ∠LMQ are interior angles

∠MLY + ∠LMQ = 180^o

2∠LMQ + ∠LMQ = 180^o

3∠LMQ = 180^o

∠LMQ = $180^o/3$

∠LMQ = 60^o

(ii) ∠XLM = $(2x – 10)^o$ and ∠LMQ = $(x + 30)^o$, find x.

∠XLM = $(2x – 10)^o$ and ∠LMQ = $(x + 30)^o$

∠XLM and ∠LMQ are alternate interior angles

∠XLM = ∠LMQ

$(2x – 10)^o = (x + 30)^o$

$2x – x = 30^o + 10^o$

$x = 40^o$

Therefore, x = 40°

(iii) ∠XLM = ∠PML, find ∠ALY

∠XLM = ∠PML

Sum of interior angles is 180 degrees

∠XLM + ∠PML = 180^{o}

∠XLM + ∠XLM = 180^{o}

2∠XLM = 180^{o}

∠XLM = $180^{o}/2$

∠XLM = 90^{o}

∠XLM and ∠ALY are vertically opposite angles

Therefore, ∠ALY = 90^{o}

(iv) ∠ALY = $(2x – 15)^{o}$, ∠LMQ = $(x + 40)^{o}$, find x.

∠ALY and ∠LMQ are corresponding angles

∠ALY = ∠LMQ

$(2x – 15)^{o} = (x + 40)^{o}$

$2x – x = 40^{o} + 15^{o}$

$x = 55^{o}$

Therefore, $x = 55^{o}$

22. In Fig. 79, DE ∥ BC. Find the values of x and y.

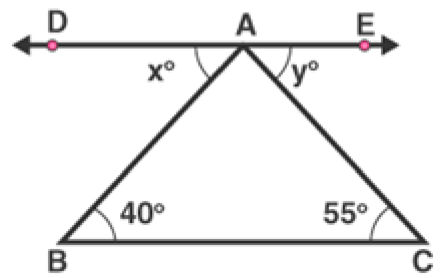


Fig. 79

Solution:

We know that,

ABC, DAB are alternate interior angles

∠ABC = ∠DAB

So, $x = 40^{o}$

And ACB, EAC are alternate interior angles

∠ACB = ∠EAC

So, $y = 55^{o}$

23. In Fig. 80, line AC ∥ line DE and ∠ABD = 32^{o}, Find out the angles x and y if ∠E = 122^{o}.

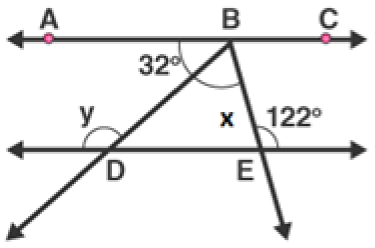


Fig. 80

Solution:

Given line AC ∥ line DE and ∠ABD = 32°

∠BDE = ∠ABD = 32° – Alternate interior angles

∠BDE + y = 180°– linear pair

32° + y = 180°

y = 180° – 32°

y = 148°

∠ABE = ∠E = 122° – Alternate interior angles

∠ABD + ∠DBE = 122°

32° + x = 122°

x = 122° – 32°

x = 90°

24. In Fig. 81, side BC of ΔABC has been produced to D and CE ∥ BA. If ∠ABC = 65°, ∠BAC = 55°, find ∠ACE, ∠ECD, ∠ACD.

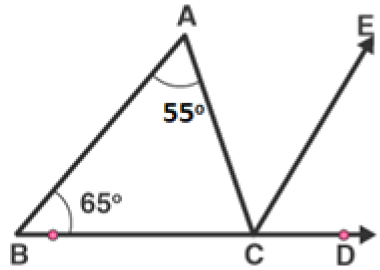


Fig. 81

Solution:

Given ∠ABC = 65°, ∠BAC = 55°

Corresponding angles,

∠ABC = ∠ECD = 65°

Alternate interior angles,

∠BAC = ∠ACE = 55°

Now, ∠ACD = ∠ACE + ∠ECD

∠ACD = 55° + 65°

= 120°

25. In Fig. 82, line CA ⊥ AB ∥ line CR and line PR ∥ line BD. Find ∠x, ∠y, ∠z.

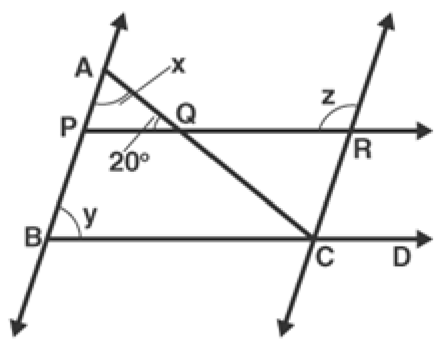


Fig. 82

Solution:

Given that, CA ⊥ AB

∠CAB = 90°

∠AQP = 20°

By, angle of sum property

In ΔABC

∠CAB + ∠AQP + ∠APQ = 180°

∠APQ = 180° – 90° – 20°

∠APQ = 70°

y and ∠APQ are corresponding angles

y = ∠APQ = 70°

∠APQ and ∠z are interior angles

∠APQ + ∠z = 180°

∠z = 180° – 70°

∠z = 110°

26. In Fig. 83, PQ ‖ RS. Find the value of x.

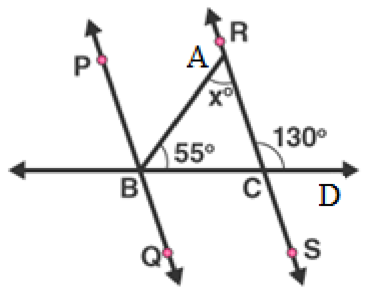


Fig. 83

Solution:

Given, linear pair,

∠RCD + ∠RCB = 180°

∠RCB = 180° – 130°

= 50°

In ΔABC,

∠BAC + ∠ABC + ∠BCA = 180°

By, angle sum property

∠BAC = 180° – 55° – 50°

∠BAC = 75°

27. In Fig. 84, AB ǁ CD and AE ǁ CF, ∠FCG = 90° and ∠BAC = 120°. Find the value of x, y and z.

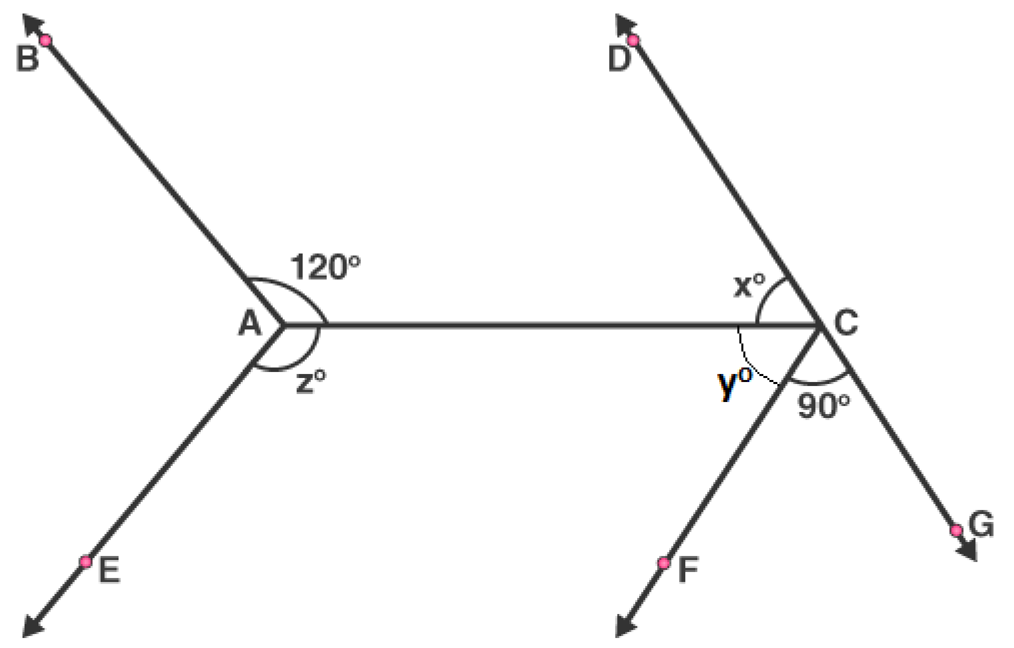


Fig. 84

Solution:

Alternate interior angle

∠BAC = ∠ACG = 120°

∠ACF + ∠FCG = 120°

So, ∠ACF = 120° – 90°

= 30°

Linear pair,

∠DCA + ∠ACG = 180°

∠x = 180° – 120°

= 60°

∠BAC + ∠BAE + ∠EAC = 360°

∠CAE = 360° – 120° – (60° + 30°)

= 150°

28. In Fig. 85, AB ǁ CD and AC ǁ BD. Find the values of x, y, z.

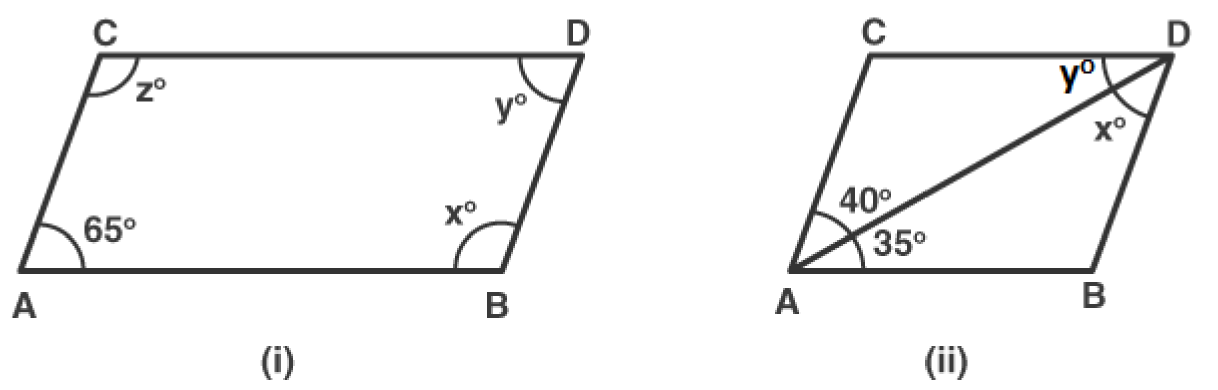


Fig. 85

Solution:

(i) Since, AC ∥ BD and CD ∥ AB, ABCD is a parallelogram

Adjacent angles of parallelogram,

∠CAB + ∠ACD = 180°

∠ACD = 180° – 65°

= 115°

Opposite angles of parallelogram,

∠CAB = ∠CDB = 65°

∠ACD = ∠DBA = 115°

(ii) Here,

AC ∥ BD and CD ∥ AB

Alternate interior angles,

∠CAD = x = 40°

∠DAB = y = 35°

29. In Fig. 86, state which lines are parallel and why?

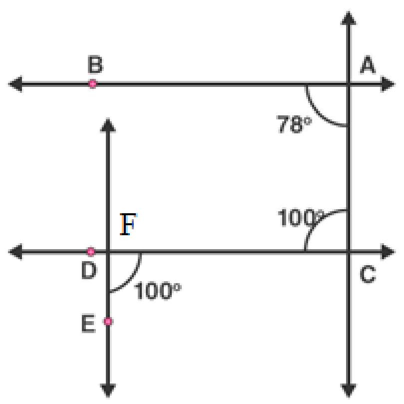


Fig. 86

Solution:

Let, F be the point of intersection of the line CD and the line passing through point E.

Here, ∠ACD and ∠CDE are alternate and equal angles.

So, ∠ACD = ∠CDE = 100°

Therefore, AC ∥ EF

30. In Fig. 87, the corresponding arms of ∠ABC and ∠DEF are parallel. If ∠ABC = 75°, find ∠DEF.

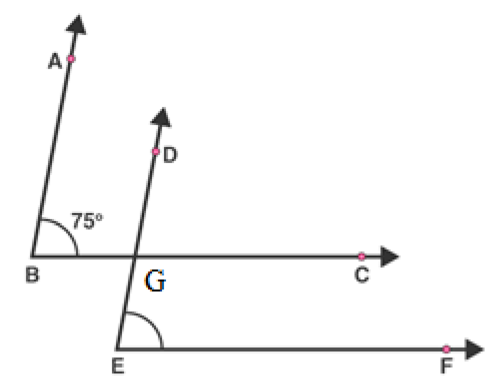


Fig. 87

Solution:

Let, G be the point of intersection of the lines BC and DE

Since, AB ∥ DE and BC ∥ EF

The corresponding angles are,

∠ABC = ∠DGC = ∠DEF = 75°

Chapter 15 – Properties Of Triangles

Exercise 15.1

1. Take three non-collinear points A. B and C on a page of your notebook. Join AB, BC and CA. What figure do you get? Name the triangle. Also, name

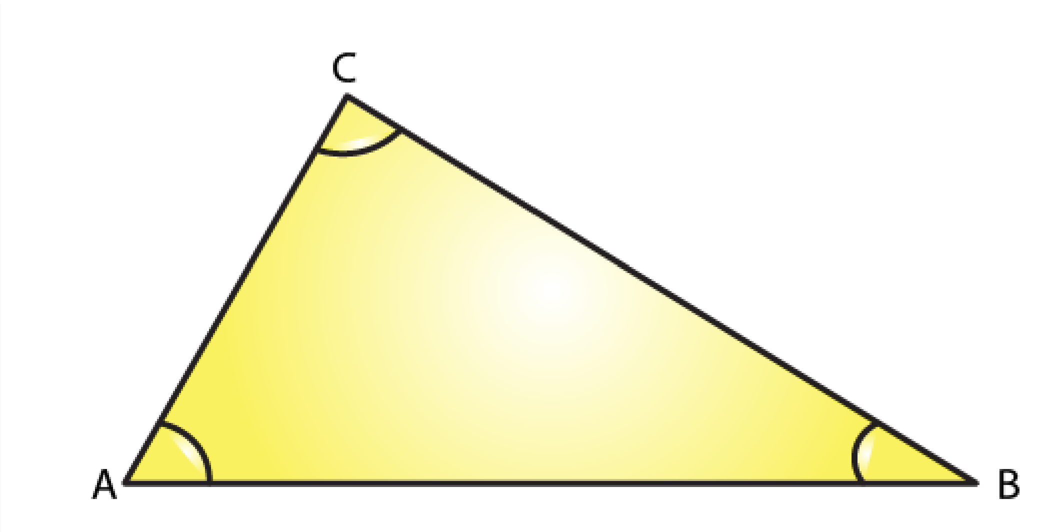
(i) The side opposite to ∠B

(ii) The angle opposite to side AB

(iii) The vertex opposite to side BC

(iv) The side opposite to vertex B.

Solution:



(i) The side opposite to ∠B is AC

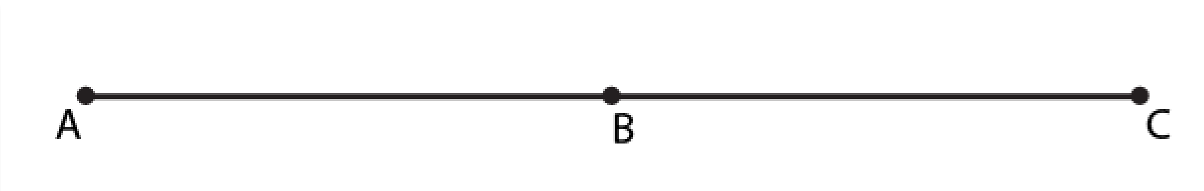
(ii) The angle opposite to side AB is ∠C

(iii) The vertex opposite to side BC is A

(iv) The side opposite to vertex B is AC

2. Take three collinear points A, B and C on a page of your note book. Join AB. BC and CA. Is the figure a triangle? If not, why?

Solution:



No, the figure is not a triangle. By definition a triangle is a plane figure formed by three non-parallel line segments

3. Distinguish between a triangle and its triangular region.

Solution:

Triangle:

A triangle is a plane figure formed by three non-parallel line segments.

Triangular region:

Whereas, it’s triangular region includes the interior of the triangle along with the triangle itself.

4. D is a point on side BC of a △CAD is joined. Name all the triangles that you can observe in the figure. How many are they?

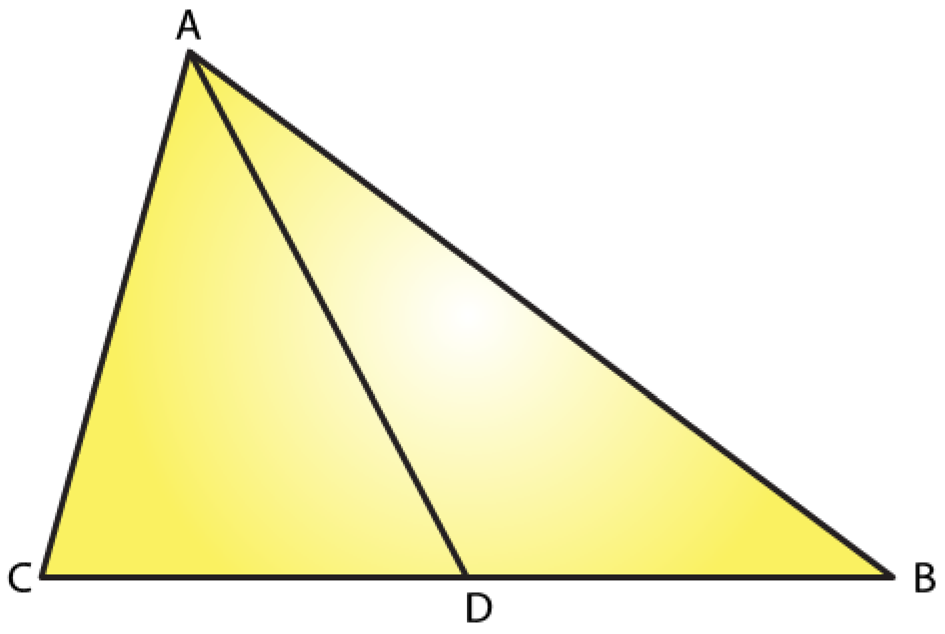


Fig. 9

Solution:

We can observe the following three triangles in the given figure

△ ABC

△ ACD

△ ADB

5. A, B, C and D are four points, and no three points are collinear. AC and BD intersect at O. There are eight triangles that you can observe. Name all the triangles

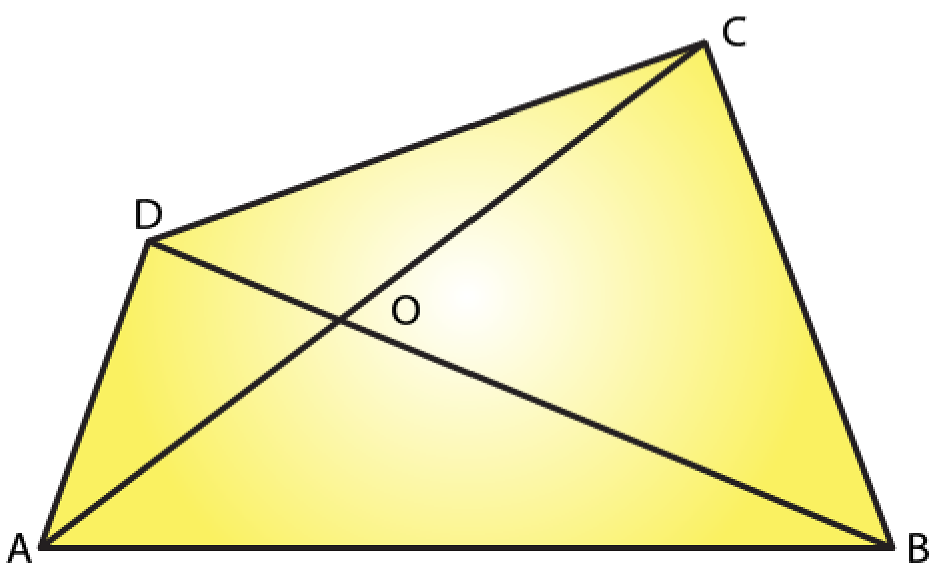


Fig. 10

Solution:

Given A, B, C and D are four points, and no three points are collinear

△ ABC

△ ACD

△ DBC

△ ABD

△ AOB

△ BOC

△ COD

△ AOD

6. What is the difference between a triangle and triangular region?

Solution:

Triangle:

A triangle is a plane figure formed by three non-parallel line segments.

Triangular region:

Whereas, it's triangular region includes the interior of the triangle along with the triangle itself.

7. Explain the following terms:

(i) Triangle

(a) Parts or elements of a triangle

(iii) Scalene triangle

(iv) Isosceles triangle

(v) Equilateral triangle

(vi) Acute triangle

(vii) Right triangle

(viii) Obtuse triangle

(ix) Interior of a triangle

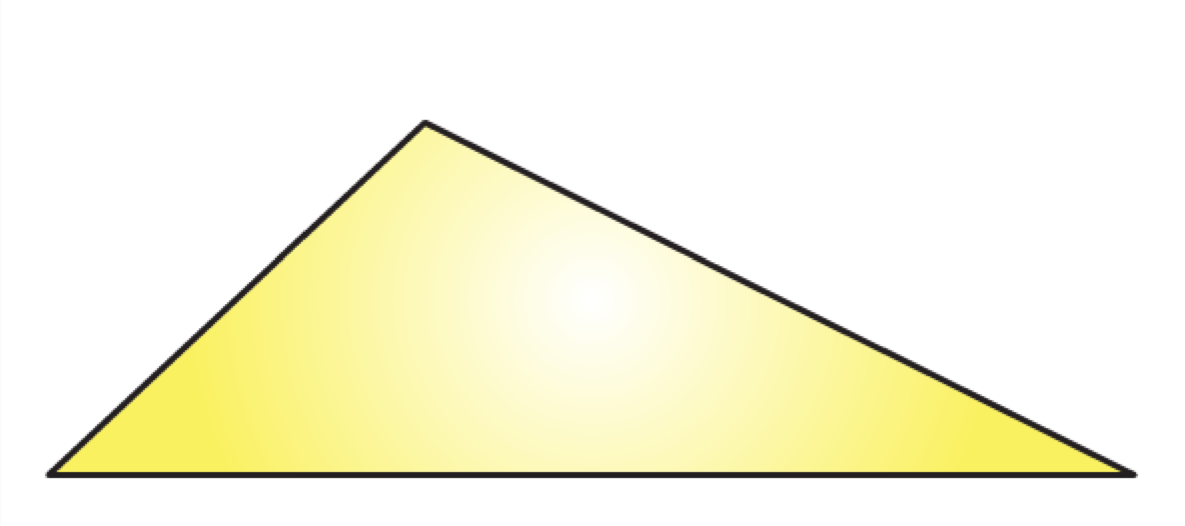
(x) Exterior of a triangle

Solution:

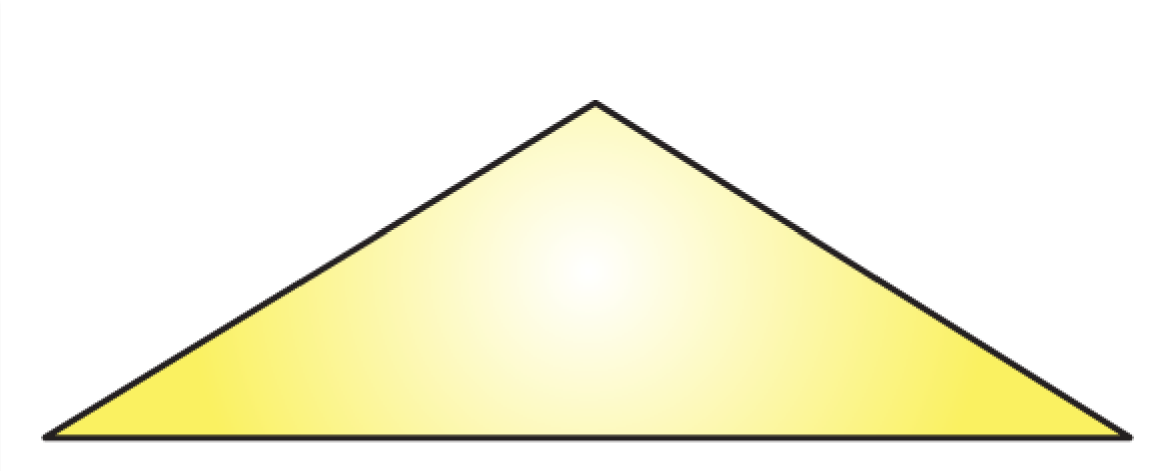
(i) A triangle is a plane figure formed by three non-parallel line segments.

(ii) The three sides and the three angles of a triangle are together known as the parts or elements of that triangle.

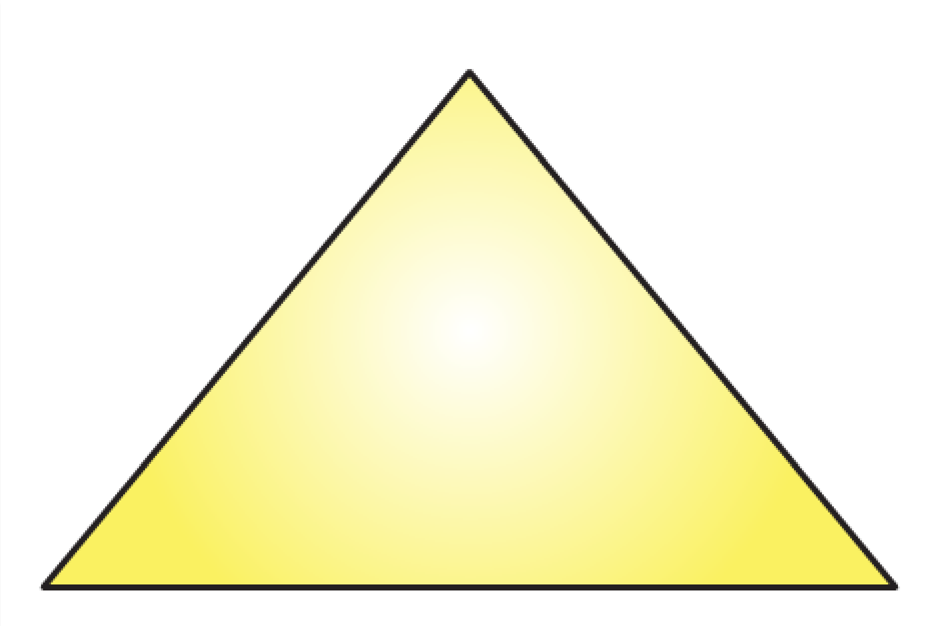
(iii) A scalene triangle is a triangle in which no two sides are equal.



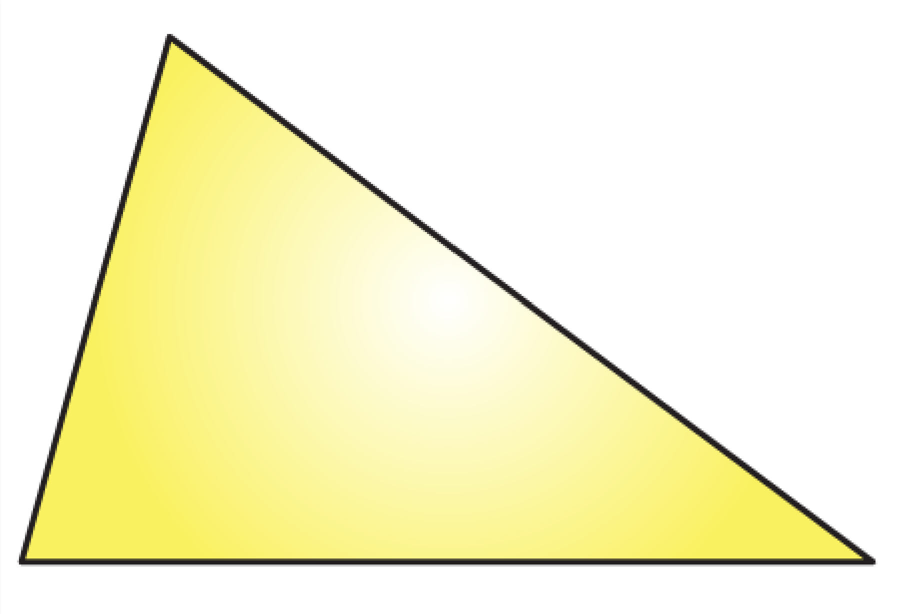
(iv) An isosceles triangle is a triangle in which two sides are equal.



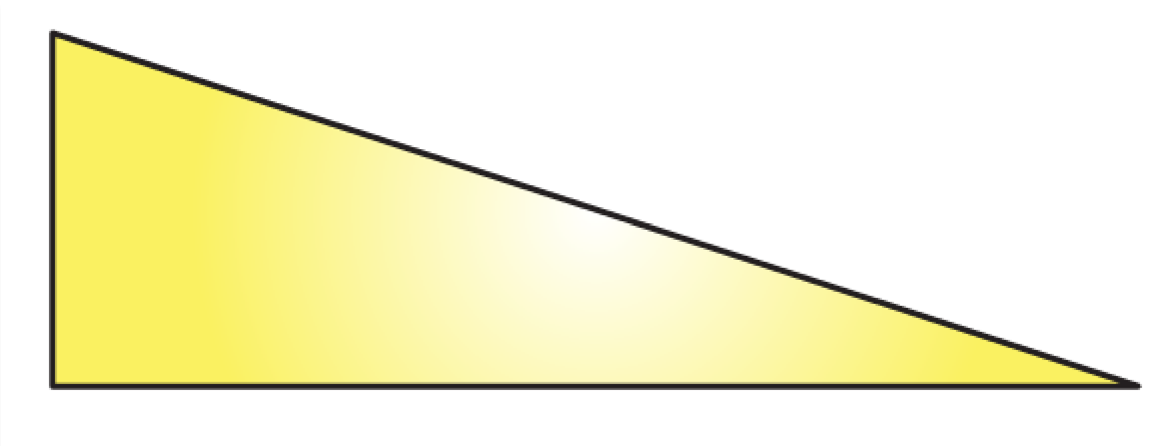
(v) An equilateral triangle is a triangle in which all three sides are equal.



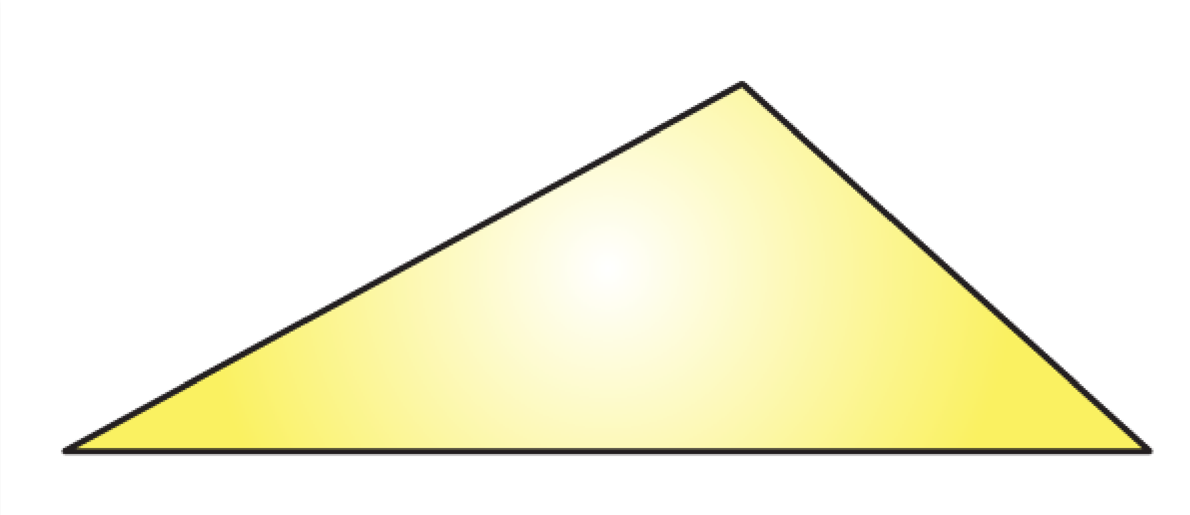
(vi) An acute triangle is a triangle in which all the angles are less than 90°.



(vii) A right angled triangle is a triangle in which one angle should be equal to 90°.



(viii) An obtuse triangle is a triangle in which one angle is more than 90°.



(ix) The interior of a triangle is made up of all such points that are enclosed within the triangle.

(x) The exterior of a triangle is made up of all such points that are not enclosed within the triangle.

8. In Fig. 11, the length (in cm) of each side has been indicated along the side. State for each triangle angle whether it is scalene, isosceles or equilateral:

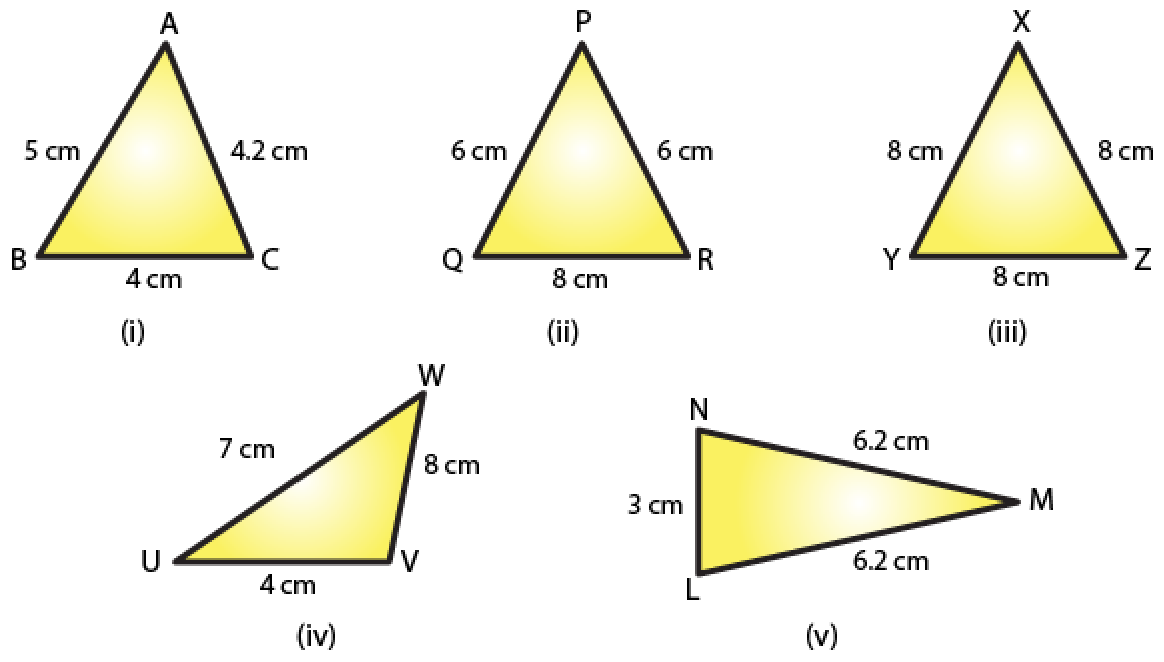


Fig. 11

Solution:

(i) The given triangle is a scalene triangle because no two sides are equal.

(ii) The given triangle is an isosceles triangle because two of its sides, viz. PQ and PR, are equal.

(iii) The given triangle is an equilateral triangle because all its three sides are equal.

(iv) The given triangle is a scalene triangle because no two sides are equal.

(v) The given triangle is an isosceles triangle because two of its sides are equal.

9. In Fig. 12, there are five triangles. The measures of some of their angles have been indicated. State for each triangle whether it is acute, right or obtuse.

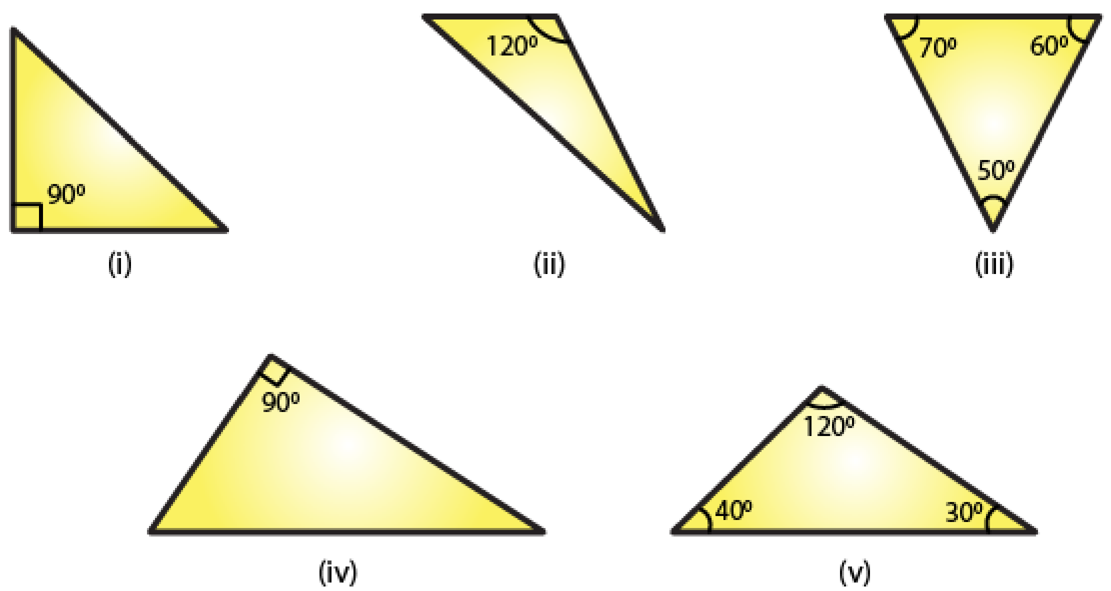


Fig. 12

Solution:

(i) The given triangle is a right triangle because one of its angles is 90^o.

(ii) The given triangle is an obtuse triangle because one of its angles is 120^o, which is greater than 90^o

(iii) The given triangle is an acute triangle because all its angles are less than 90^o

(iv) The given triangle is a right triangle because one of its angle is 90^o.

(v) The given triangle is an obtuse triangle because one of its angle is 120^o, which is greater than 90^o.

10. Fill in the blanks with the correct word/symbol to make it a true statement:

(i) A triangle has ……. sides.

(ii) A triangle has ……..vertices.

(iii) A triangle has ……..angles.

(iv) A triangle has ………parts.

(v) A triangle whose no two sides are equal is known as ………

(vi) A triangle whose two sides are equal is known as ……….

(vii) A triangle whose all the sides are equal is known as ………

(viii) A triangle whose one angle is a right angle is known as ……..

(ix) A triangle whose all the angles are of measure less than 90′ is known as ………

(x) A triangle whose one angle is more than 90′ is known as ……….

Solution:

(i) Three

(ii) Three

(iii) Three

(iv) Six

(v) A scalene triangle

(vi) An isosceles triangle

(vii) An equilateral triangle

(viii) A right triangle

(ix) An acute triangle

(x) An obtuse triangle

11. In each of the following, state if the statement is true (T) or false (F):

(i) A triangle has three sides.

(ii) A triangle may have four vertices.

(iii) Any three line-segments make up a triangle.

(iv) The interior of a triangle includes its vertices.

(v) The triangular region includes the vertices of the corresponding triangle.

(vi) The vertices of a triangle are three collinear points.

(vii) An equilateral triangle is isosceles also.

(viii) Every right triangle is scalene.

(ix) Each acute triangle is equilateral.

(x) No isosceles triangle is obtuse.

Solution:

(i) True

(ii) False

Explanation:

Any three non-parallel line segments can make up a triangle.

(iii) False.

Explanation:

Any three non-parallel line segments can make up a triangle.

(iv) False.

Explanation:

The interior of a triangle is the region enclosed by the triangle and the vertices are not enclosed by the triangle.

(v) True.

Explanation:

The triangular region includes the interior region and the triangle itself.

(vi) False.

Explanation:

The vertices of a triangle are three non-collinear points.

(vii) True.

Explanation:

In an equilateral triangle, any two sides are equal.

(viii) False.

Explanation:

A right triangle can also be an isosceles triangle.

(ix) False.

Explanation:

Each acute triangle is not an equilateral triangle, but each equilateral triangle is an acute triangle.

(x) False.

Explanation:

An isosceles triangle can be an obtuse triangle, a right triangle or an acute triangle

Exercise 15.2

1. Two angles of a triangle are of measures 150^o and 30^o. Find the measure of the third angle.

Solution:

Given two angles of a triangle are of measures 150^o and 30^o

Let the required third angle be x

We know that sum of all the angles of a triangle = 180^o

$150^o + 30^o + x = 180^o$

$135^o + x = 180^o$

$x = 180^o – 135^o$

$x = 45^o$

Therefore the third angle is 45^o

2. One of the angles of a triangle is 130^o, and the other two angles are equal. What is the measure of each of these equal angles?

Solution:

Given one of the angles of a triangle is 130^o

Also given that remaining two angles are equal

So let the second and third angle be x

We know that sum of all the angles of a triangle = 180^o

$130^o + x + x = 180^o$

$130^o + 2x = 180^o$

$2x = 180^o - 130^o$

$2x = 50^o$

$x = 50/2$

$x = 25^o$

Therefore the two other angles are 25^o each

3. The three angles of a triangle are equal to one another. What is the measure of each of the angles?

Solution:

Given that three angles of a triangle are equal to one another

So let the each angle be x

We know that sum of all the angles of a triangle = 180^o

$x + x + x = 180^o$

$3x = 180^o$

$x = 180/3$

$x = 60^o$

Therefore angle is 60^o each

4. If the angles of a triangle are in the ratio 1: 2: 3, determine three angles.

Solution:

Given angles of the triangle are in the ratio 1: 2: 3

So take first angle as x, second angle as 2x and third angle as 3x

We know that sum of all the angles of a triangle = 180^o

$x + 2x + 3x = 180^o$

$6x = 180^o$

$x = 180/6$

$x = 30^o$

$2x = 30^o \times 2 = 60^o$

$3x = 30^o \times 3 = 90^o$

Therefore the first angle is 30^o, second angle is 60^o and third angle is 90^o.

5. The angles of a triangle are $(x - 40)^o$, $(x - 20)^o$ and $(1/2 - 10)^o$. Find the value of x.

Solution:

Given the angles of a triangle are $(x - 40)^o$, $(x - 20)^o$ and $(1/2 - 10)^o$.

We know that sum of all the angles of a triangle = 180^o

$(x - 40)^o + (x - 20)^o + (1/2 - 10)^o = 180^o$

$x + x + (1/2) - 40^o - 20^o - 10^o = 180^o$

$x + x + (1/2) - 70^o = 180^o$

$(5x/2) = 180^o + 70^o$

$(5x/2) = 250^o$

$x = (2/5) \times 250^o$

$x = 100^o$

Hence the value of x is 100^o

6. The angles of a triangle are arranged in ascending order of magnitude. If the difference between two consecutive angles is 10^o. Find the three angles.

Solution:

Given that angles of a triangle are arranged in ascending order of magnitude

Also given that difference between two consecutive angles is 10^o

Let the first angle be x

Second angle be $x + 10^o$

Third angle be $x + 10^o + 10^o$

We know that sum of all the angles of a triangle = 180^o

$x + x + 10^o + x + 10^o + 10^o = 180^o$

$3x + 30 = 180$

$3x = 180 - 30$

$3x = 150$

$x = 150/3$

$x = 50^o$

First angle is 50^o

Second angle $x + 10^o = 50 + 10 = 60^o$

Third angle $x + 10^o + 10^o = 50 + 10 + 10 = 70^o$

7. Two angles of a triangle are equal and the third angle is greater than each of those angles by 30^o. Determine all the angles of the triangle

Solution:

Given that two angles of a triangle are equal

Let the first and second angle be x

Also given that third angle is greater than each of those angles by 30^o

Therefore the third angle is greater than the first and second by $30^o = x + 30^o$

The first and the second angles are equal

We know that sum of all the angles of a triangle = 180^o

$x + x + x + 30^o = 180^o$

$3x + 30 = 180$

$3x = 180 – 30$

$3x = 150$

$x = 150/3$

$x = 50^\circ$

Third angle $= x + 30^\circ = 50^\circ + 30^\circ = 80^\circ$

The first and the second angle is 50° and the third angle is 80°.

8. If one angle of a triangle is equal to the sum of the other two, show that the triangle is a right triangle.

Solution:

Given that one angle of a triangle is equal to the sum of the other two

Let the measure of angles be x, y, z

Therefore we can write above statement as $x = y + z$

$x + y + z = 180^\circ$

Substituting the above value we get

$x + x = 180^\circ$

$2x = 180^\circ$

$x = 180/2$

$x = 90^\circ$

If one angle is 90° then the given triangle is a right angled triangle

9. If each angle of a triangle is less than the sum of the other two, show that the triangle is acute angled.

Solution:

Given that each angle of a triangle is less than the sum of the other two

Let the measure of angles be x, y and z

From the above statement we can write as

$x > y + z$

$y < x + z$

$z < x + y$

Therefore triangle is an acute triangle

10. In each of the following, the measures of three angles are given. State in which cases the angles can possibly be those of a triangle:

(i) $63^\circ, 37^\circ, 80^\circ$

(ii) $45^\circ, 61^\circ, 73^\circ$

(iii) $59^\circ, 72^\circ, 61^\circ$

(iv) $45^\circ, 45^\circ, 90^\circ$

(v) $30^\circ, 20^\circ, 125^\circ$

Solution:

(i) $63^\circ + 37^\circ + 80^\circ = 180^\circ$

Angles form a triangle

(ii) $45^\circ, 61^\circ, 73^\circ$ is not equal to 180°

Therefore not a triangle

(iii) $59^o, 72^o, 61^o$ is not equal to 180^0

Therefore not a triangle

(iv) $45^o + 45^o + 90^o = 180^o$

Angles form a triangle

(v) $30^o, 20^o, 125^o$ is not equal to 180^o

Therefore not a triangle

11. The angles of a triangle are in the ratio 3: 4: 5. Find the smallest angle

Solution:

Given that angles of a triangle are in the ratio: 3: 4: 5

Therefore let the measure of the angles be 3x, 4x, 5x

We know that sum of the angles of a triangle $= 180^o$

$3x + 4x + 5x = 180^o$

$12x = 180^o$

$x = 180/12$

$x = 15^o$

Smallest angle = 3x

$= 3 \times 15^o$

$= 45^o$

Therefore smallest angle $= 45^o$

12. Two acute angles of a right triangle are equal. Find the two angles.

Solution:

Given that acute angles of a right angled triangle are equal

We know that Right triangle: whose one of the angle is a right angle

Let the measure of angle be x, x, 90^o

$x + x + 90^o = 180^o$

$2x = 90^o$

$x = 90/2$

$x = 45^o$

The two angles are 45^o and 45^o

13. One angle of a triangle is greater than the sum of the other two. What can you say about the measure of this angle? What type of a triangle is this?

Solution:

Given one angle of a triangle is greater than the sum of the other two

Let the measure of the angles be x, y, z

From the question we can write as

$x > y + z$ or

$y > x + z$ or

$z > x + y$

x or y or $z > 90^o$ which is obtuse

Therefore triangle is an obtuse angle

14. In the six cornered figure, (fig. 20), AC, AD and AE are joined. Find ∠FAB + ∠ABC + ∠BCD + ∠CDE + ∠DEF + ∠EFA.

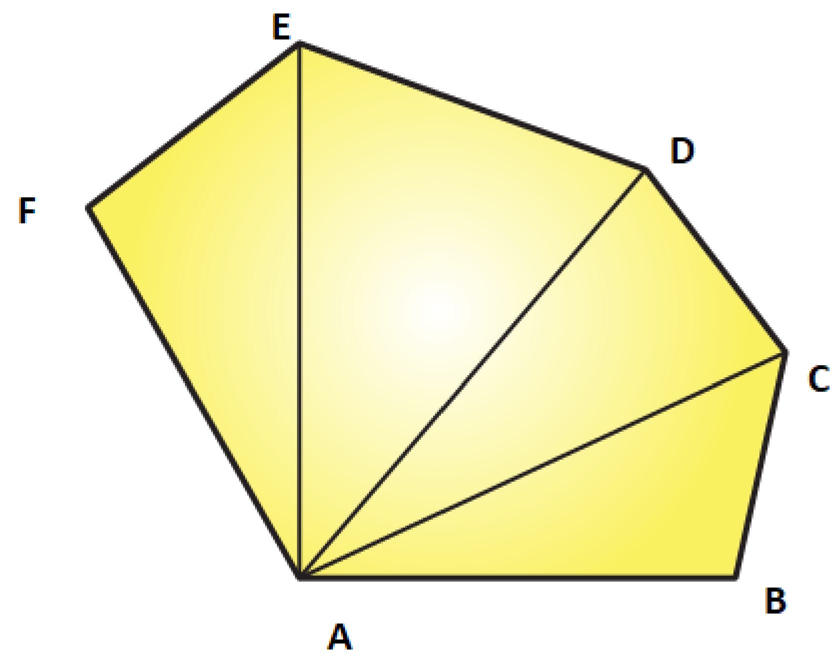


Fig. 20

Solution:

We know that sum of the angles of a triangle is 180^{o}

Therefore in ΔABC, we have

$\angle CAB + \angle ABC + \angle BCA = 180^{o}$ …….. (i)

In△ ACD, we have

$\angle DAC + \angle ACD + \angle CDA = 180^{o}$ …….. (ii)

In △ADE, we have

$\angle EAD + \angle ADE + \angle DEA = 180^{o}$ ………. (iii)

In △AEF, we have

$\angle FAE + \angle AEF + \angle EFA = 180^{o}$ ………. (iv)

Adding (i), (ii), (iii), (iv) we get

$\angle CAB + \angle ABC + \angle BCA + \angle DAC + \angle ACD + \angle CDA + \angle EAD + \angle ADE + \angle DEA + \angle FAE + \angle AEF + \angle EFA = 720^{o}$

Therefore $\angle FAB + \angle ABC + \angle BCD + \angle CDE + \angle DEF + \angle EFA = 720^{o}$

15. Find x, y, z (whichever is required) from the figures (Fig. 21) given below:

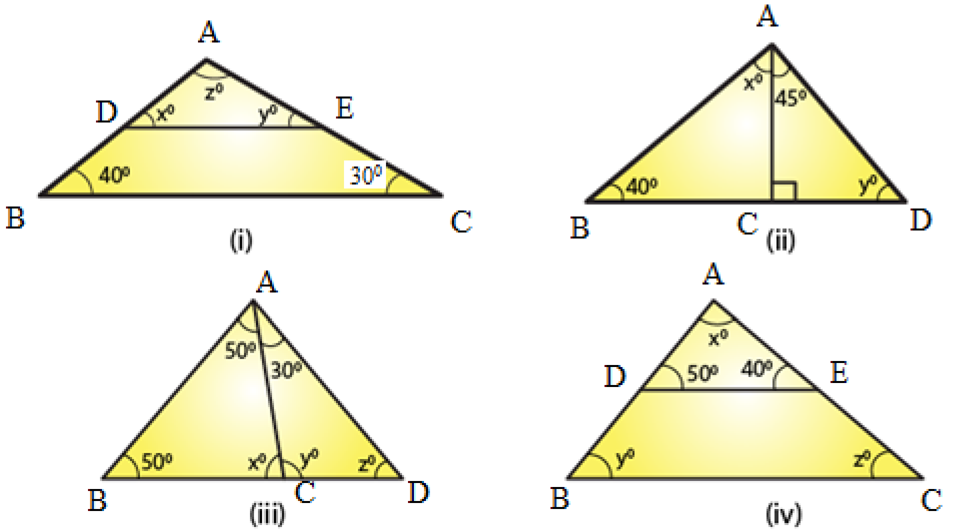


Fig.21

Solution:

(i) In $\triangle$ABC and $\triangle$ADE we have,

$\angle$ ADE = $\angle$ ABC [corresponding angles]

x = 40^o

$\angle$AED = $\angle$ACB (corresponding angles)

y = 30^o

We know that the sum of all the three angles of a triangle is equal to 180^o

x + y + z = 180^o (Angles of $\triangle$ADE)

Which means: 40^o + 30^o + z = 180^o

z = 180^o – 70^o

z = 110^o

Therefore, we can conclude that the three angles of the given triangle are 40^o, 30^o and 110^o

(ii) We can see that in $\triangle$ADC, $\angle$ADC is equal to 90^o.

($\triangle$ADC is a right triangle)

We also know that the sum of all the angles of a triangle is equal to 180^o.

Which means: 45^o + 90^o + y = 180^o (Sum of the angles of $\triangle$ADC)

135^o + y = 180^o

y = 180^o – 135^o.

y = 45^o.

We can also say that in $\triangle$ ABC, $\angle$ABC + $\angle$ACB + $\angle$BAC is equal to 180^o.

(Sum of the angles of $\triangle$ABC)

40^o + y + (x + 45^o) = 180^o

40^o + 45^o + x + 45^o = 180^o (y = 45^o)

x = 180^o –130^o

x = 50^o

Therefore, we can say that the required angles are 45^o and 50^o.

(iii) We know that the sum of all the angles of a triangle is equal to 180^o.

Therefore, for $\triangle ABD$:

$\angle ABD + \angle ADB + \angle BAD = 180°$ (Sum of the angles of $\triangle ABD$)

$50^o + x + 50^o = 180^o$

$100^o + x = 180^o$

$x = 180^o - 100^o$

$x = 80^o$

For $\triangle$ ABC:

$\angle ABC + \angle ACB + \angle BAC = 180^o$ (Sum of the angles of $\triangle ABC$)

$50^o + z + (50^o + 30^o) = 180^o$

$50^o + z + 50^o + 30^o = 180^o$

$z = 180^o - 130^o$

$z = 50^o$

Using the same argument for $\triangle ADC$:

$\angle ADC + \angle ACD + \angle DAC = 180^o$ (Sum of the angles of $\triangle ADC$)

$y + z + 30^o = 180^o$

$y + 50^o + 30^o = 180^o$ ($z = 50^o$)

$y = 180^o - 80^o$

$y = 100^0$

Therefore, we can conclude that the required angles are 80^o, 50^o and 100^o.

(iv) In $\triangle ABC$ and $\triangle ADE$ we have:

$\angle ADE = \angle ABC$ (Corresponding angles)

$y = 50^o$

Also, $\angle AED = \angle ACB$ (Corresponding angles)

$z = 40^o$

We know that the sum of all the three angles of a triangle is equal to 180^o.

We can write as $x + 50^o + 40^o = 180^o$ (Angles of $\triangle ADE$)

$x = 180^o - 90^o$

$x = 90^o$

Therefore, we can conclude that the required angles are 50^o, 40^o and 90^o.

16. If one angle of a triangle is 60^o and the other two angles are in the ratio 1: 2, find the angles.

Solution:

Given that one of the angles of the given triangle is 60^o.

Also given that the other two angles of the triangle are in the ratio 1: 2.

Let one of the other two angles be x.

Therefore, the second one will be 2x.

We know that the sum of all the three angles of a triangle is equal to 180^o.

$60^o + x + 2x = 180^o$

$3x = 180^o – 60^o$

$3x = 120^o$

$x = 120^o/3$

$x = 40^o$

$2x = 2 \times 40^o$

$2x = 80^o$

Hence, we can conclude that the required angles are 40^o and 80^o.

17. If one angle of a triangle is 100^o and the other two angles are in the ratio 2: 3. Find the angles.

Solution:

Given that one of the angles of the given triangle is 100^o.

Also given that the other two angles are in the ratio 2: 3.

Let one of the other two angle be 2x.

Therefore, the second angle will be 3x.

We know that the sum of all three angles of a triangle is 180^o.

$100^o + 2x + 3x = 180^o$

$5x = 180^o – 100^o$

$5x = 80^o$

$x = 80/5$

$x = 16$

$2x = 2 \times 16$

$2x = 32^o$

$3x = 3\times16$

$3x = 48^o$

Thus, the required angles are 32^o and 48^o.

18. In $\triangle ABC$, if $3\angle A = 4\angle B = 6\angle C$, calculate the angles.

Solution:

We know that for the given triangle, $3\angle A = 6\angle C$

$\angle A = 2\angle C$ ……. (i)

We also know that for the same triangle, $4\angle B = 6\angle C$

$\angle B = (6/4) \angle C$ …… (ii)

We know that the sum of all three angles of a triangle is 180^o.

Therefore, we can say that:

$\angle A + \angle B + \angle C = 180^o$ (Angles of $\triangle ABC$)…… (iii)

On putting the values of $\angle A$ and $\angle B$ in equation (iii), we get:

$2\angle C + (6/4) \angle C + \angle C = 180^o$

$(18/4) \angle C = 180^o$

$\angle C = 40^o$

From equation (i), we have:

$\angle A = 2\angle C = 2 \times 40$

$\angle A = 80^o$

From equation (ii), we have:

$\angle B = (6/4)\ \angle C = (6/4) \times 40^o$

$\angle B = 60^o$

$\angle A = 80^o$, $\angle B = 60^o$, $\angle C = 40^o$

Therefore, the three angles of the given triangle are 80^o, 60^o, and 40^o.

19. Is it possible to have a triangle, in which

(i) Two of the angles are right?

(ii) Two of the angles are obtuse?

(iii) Two of the angles are acute?

(iv) Each angle is less than 60^o?

(v) Each angle is greater than 60^o?

(vi) Each angle is equal to 60^o?

Solution:

(i) No, because if there are two right angles in a triangle, then the third angle of the triangle must be zero, which is not possible.

(ii) No, because as we know that the sum of all three angles of a triangle is always 180^o. If there are two obtuse angles, then their sum will be more than 180^o, which is not possible in case of a triangle.

(iii) Yes, in right triangles and acute triangles, it is possible to have two acute angles.

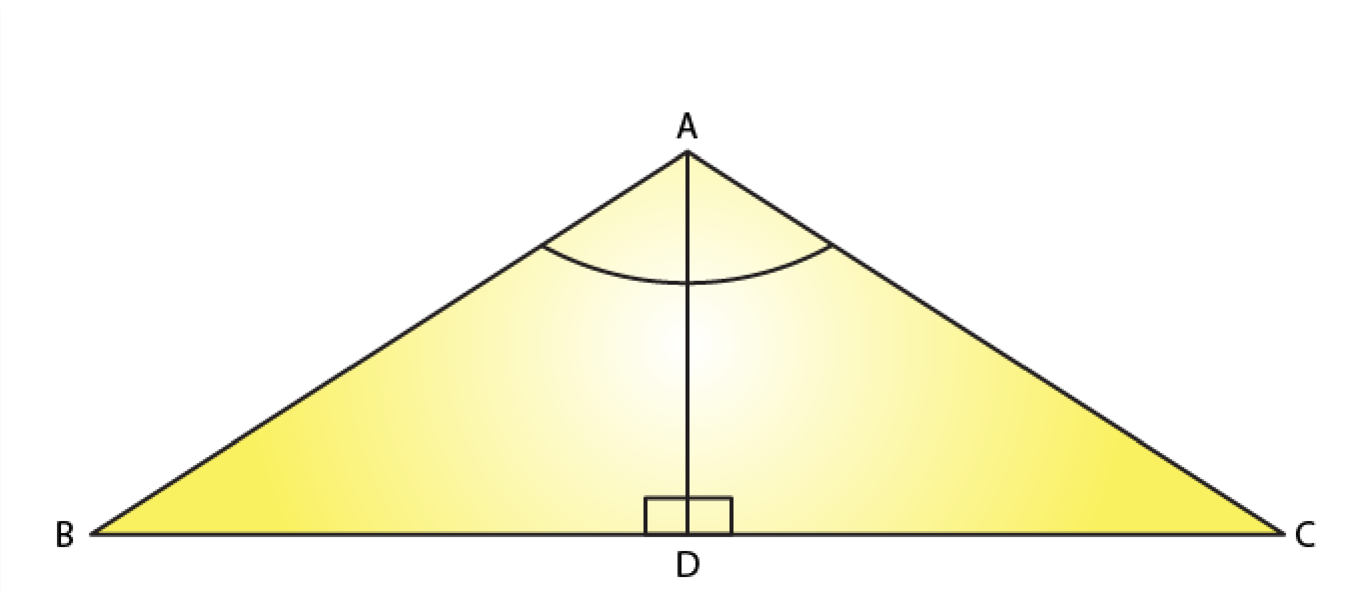
(iv) No, because if each angle is less than 60^o, then the sum of all three angles will be less than 180^o, which is not possible in case of a triangle.

(v) No, because if each angle is greater than 60^0, then the sum of all three angles will be greater than 180^0, which is not possible.

(vi) Yes, if each angle of the triangle is equal to 60^0, then the sum of all three angles will be 180^0, which is possible in case of a triangle.

20. In $\triangle$ ABC, $\angle A = 100^o$, AD bisects $\angle A$ and AD $\perp$ BC. Find $\angle B$

Solution:



Given that in $\triangle ABC$, $\angle A = 100^o$

Also given that $AD \perp BC$

Consider $\triangle ABD$

$\angle BAD = 100/2$ (AD bisects $\angle A$)

$\angle BAD = 50^o$

$\angle ADB = 90^o$ (AD perpendicular to BC)

We know that the sum of all three angles of a triangle is 180^o.

Thus,

$\angle ABD + \angle BAD + \angle ADB = 180^o$ (Sum of angles of $\triangle ABD$)

Or,

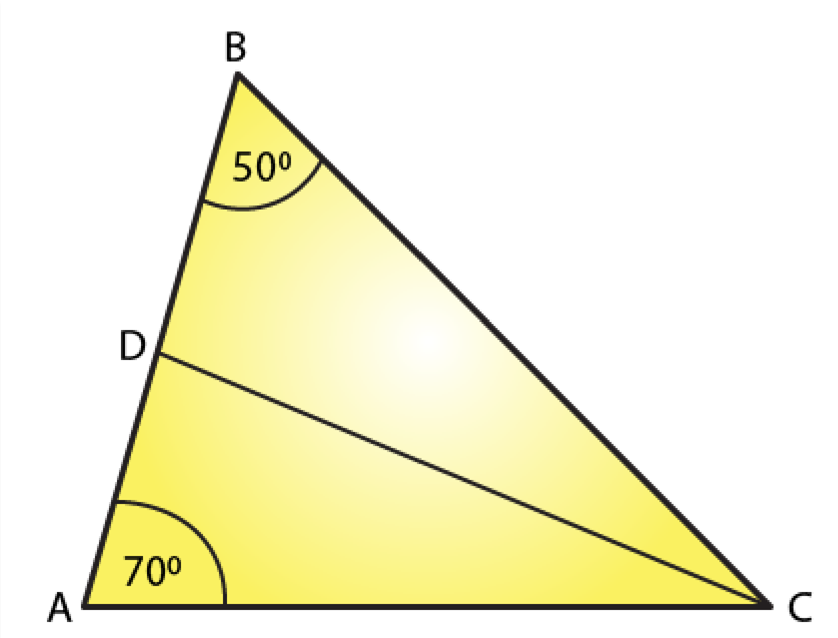
$\angle ABD + 50^o + 90^o = 180^o$

$\angle ABD = 180^o - 140^o$

$\angle ABD = 40^o$

21. In $\triangle ABC$, $\angle A = 50^o$, $\angle B = 70^o$ and bisector of $\angle C$ meets AB in D. Find the angles of the triangles ADC and BDC

Solution:



We know that the sum of all three angles of a triangle is equal to 180^o.

Therefore, for the given $\triangle ABC$, we can say that:

$\angle A + \angle B + \angle C = 180^o$ (Sum of angles of $\triangle ABC$)

$50^o + 70^o + \angle C = 180^o$

$\angle C = 180^o - 120^o$

$\angle C = 60^o$

$\angle ACD = \angle BCD = \angle C2$ (CD bisects $\angle C$ and meets AB in D.)

$\angle ACD = \angle BCD = 60/2 = 30^o$

Using the same logic for the given $\triangle ACD$, we can say that:

$\angle DAC + \angle ACD + \angle ADC = 180^o$

$50^o + 30^o + \angle ADC = 180^o$

$\angle ADC = 180^o - 80^o$

$\angle ADC = 100^o$

If we use the same logic for the given $\triangle BCD$, we can say that

$\angle DBC + \angle BCD + \angle BDC = 180^o$

$70^o + 30^o + \angle BDC = 180^o$

$\angle BDC = 180^o - 100^o$

$\angle BDC = 80^o$

Thus,

For $\triangle ADC$: $\angle A = 50^o$, $\angle D = 100^o$ $\angle C = 30^o$

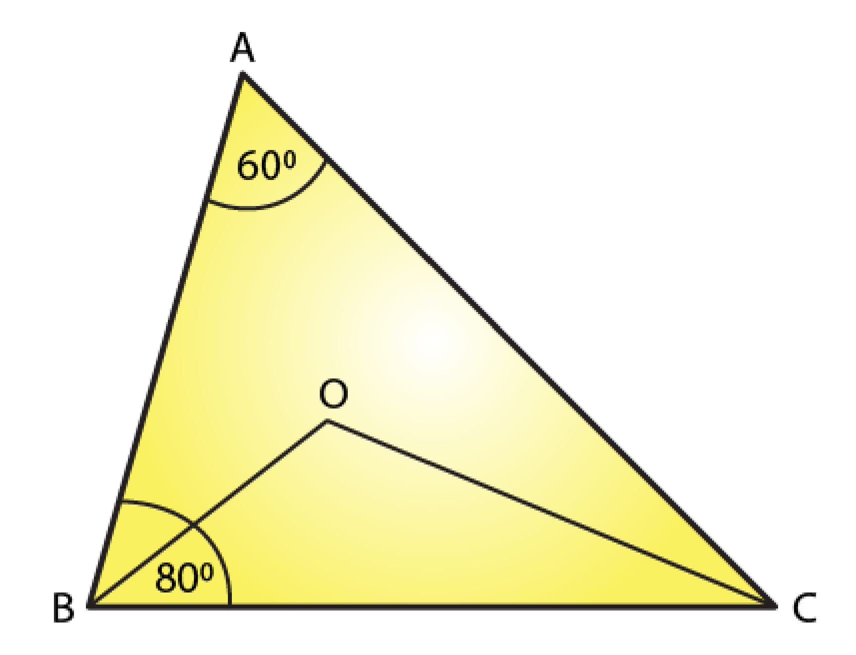
$\triangle BDC$: $\angle B = 70^o$, $\angle D = 80^o$ $\angle C = 30^o$

22. In ΔABC, ∠A = 60º, ∠B = 80º, and the bisectors of ∠B and ∠C, meet at O. Find

(i) ∠C

(ii) ∠BOC

Solution:



(i) We know that the sum of all three angles of a triangle is 180^o.

Hence, for $\triangle ABC$, we can say that:

$\angle A + \angle B + \angle C = 180^o$ (Sum of angles of ΔABC)

$60^o + 80^o + \angle C = 180^o$.

$\angle C = 180^o - 140^o$

$\angle C = 40^o$.

(ii)For $\triangle OBC$,

$\angle OBC = \angle B/2 = 80/2$ (OB bisects $\angle B$)

$\angle OBC = 40^o$

$\angle OCB = \angle C/2 = 40/2$ (OC bisects $\angle C$)

$\angle OCB = 20^o$

If we apply the above logic to this triangle, we can say that:

$\angle OCB + \angle OBC + \angle BOC = 180^o$ (Sum of angles of $\triangle OBC$)

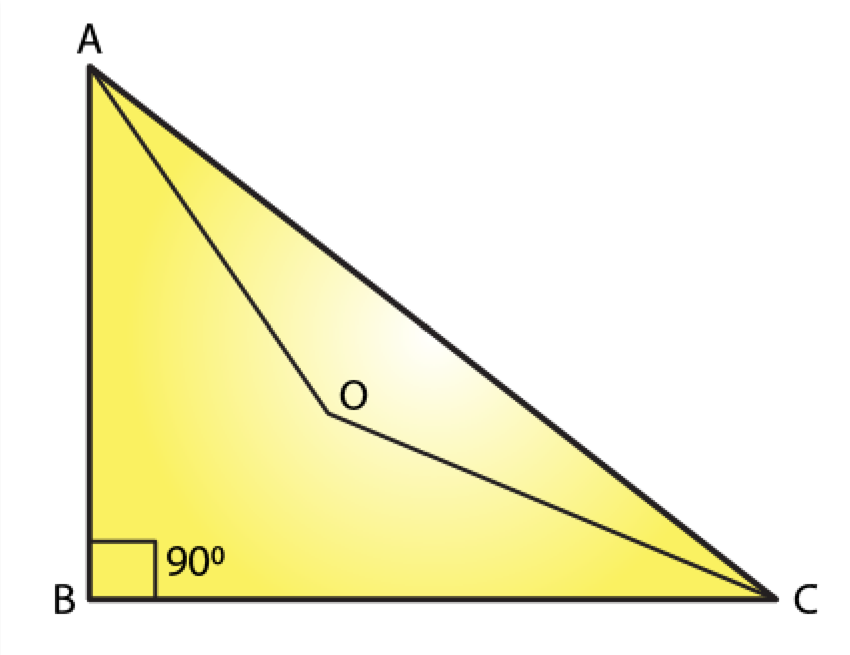
$20^o + 40^o + \angle BOC = 180^o$

$\angle BOC = 180^o - 60^o$

$\angle BOC = 120^o$

23. The bisectors of the acute angles of a right triangle meet at O. Find the angle at O between the two bisectors.

Solution:



Given bisectors of the acute angles of a right triangle meet at O

We know that the sum of all three angles of a triangle is 180^o.

Hence, for $\triangle ABC$, we can say that:

$\angle A + \angle B + \angle C = 180^o$

$\angle A + 90^o + \angle C = 180^o$

$\angle A + \angle C = 180^o - 90^o$

$\angle A + \angle C = 90^o$

For $\triangle OAC$:

$\angle OAC = \angle A/2$ (OA bisects $\angle A$)

$\angle OCA = \angle C/2$ (OC bisects $\angle C$)

On applying the above logic to $\triangle OAC$, we get

$\angle AOC + \angle OAC + \angle OCA = 180^o$ (Sum of angles of $\triangle AOC$)

$\angle AOC + \angle A/2 + \angle C/2 = 180^o$

$\angle AOC + (\angle A + \angle C)/2 = 180^o$

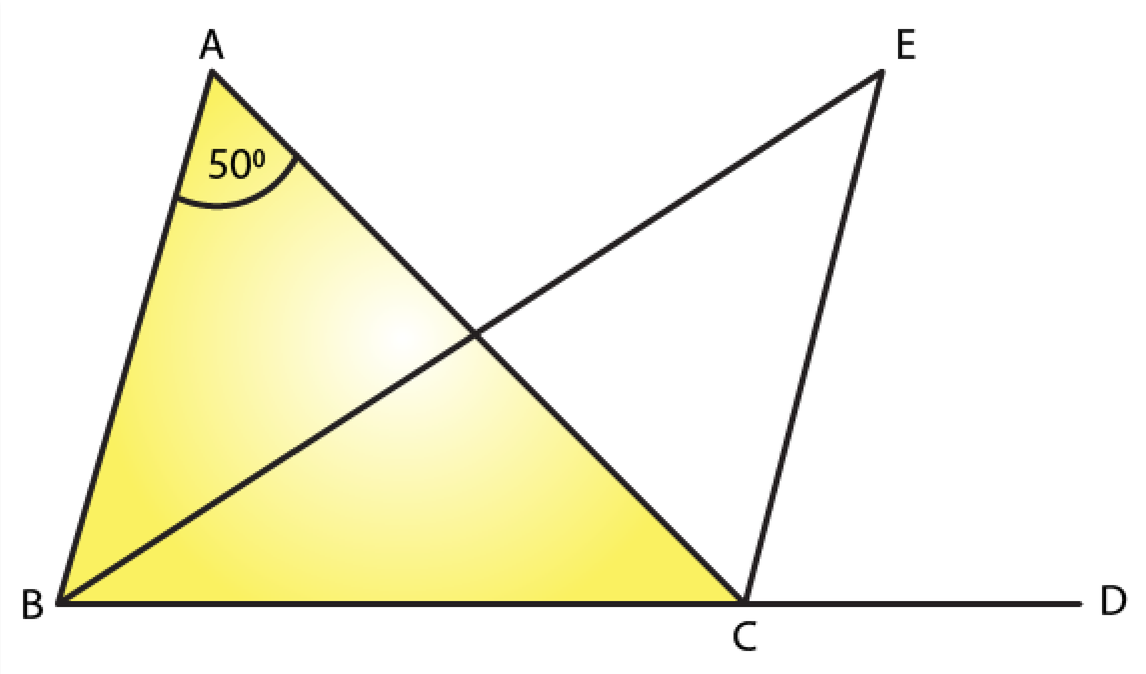
$\angle AOC + 90/2 = 180^o$

$\angle AOC = 180^o - 45^o$

$\angle AOC = 135^o$

24. In $\triangle ABC$, $\angle A = 50^o$ and BC is produced to a point D. The bisectors of $\angle ABC$ and $\angle ACD$ meet at E. Find $\angle E$.

Solution:



In the given triangle,

$\angle ACD = \angle A + \angle B$. (Exterior angle is equal to the sum of two opposite interior angles.)

We know that the sum of all three angles of a triangle is 180^o.

Therefore, for the given triangle, we know that the sum of the angles = 180^o

$\angle ABC + \angle BCA + \angle CAB = 180^o$

$\angle A + \angle B + \angle BCA = 180^o$

$\angle BCA = 180^o – (\angle A + \angle B)$

But we know that EC bisects $\angle ACD$

Therefore $\angle ECA = \angle ACD/2$

$\angle ECA = (\angle A + \angle B)/2$ [$\angle ACD = (\angle A + \angle B)$]

But EB bisects $\angle ABC$

$\angle EBC = \angle ABC/2 = \angle B/2$

$\angle EBC = \angle ECA + \angle BCA$

$\angle EBC = (\angle A + \angle B)/2 + 180^o – (\angle A + \angle B)$

If we use same steps for $\triangle EBC$, then we get,

$\angle B/2 + (\angle A + \angle B)/2 + 180^o – (\angle A + \angle B) + \angle BEC = 180^o$

$\angle BEC = \angle A + \angle B – (\angle A + \angle B)/2 – \angle B/2$

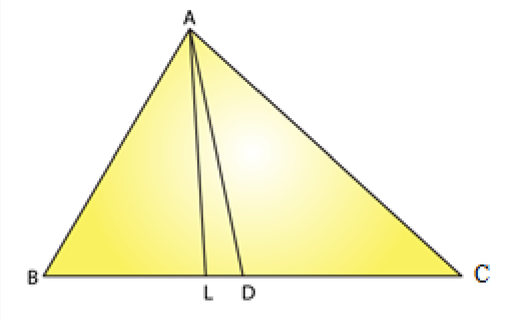
$\angle BEC = \angle A/2$

$\angle BEC = 50^o/2$

$= 25^o$

25. In ΔABC, $\angle B = 60°$, $\angle C = 40°$, $AL \perp BC$ and AD bisects $\angle A$ such that L and D lie on side BC. Find $\angle LAD$

Solution:



We know that the sum of all angles of a triangle is 180°

Consider △ABC, we can write as

$\angle A + \angle B + \angle C = 180^o$

$\angle A + 60^o + 40^o = 180^o$

$\angle A = 80^0$

But we know that ∠DAC bisects ∠A

$\angle DAC = \angle A/2$

$\angle DAC = 80^o/2$

If we apply same steps for the △ADC, we get

We know that the sum of all angles of a triangle is 180°

$\angle ADC + \angle DCA + \angle DAC = 180^o$

$\angle ADC + 40^o + 40^o = 180^o$

$\angle ADC = 180^o - 80^o = 100^0$

We know that exterior angle is equal to the sum of two interior opposite angles

Therefore we have

$\angle ADC = \angle ALD + \angle LAD$

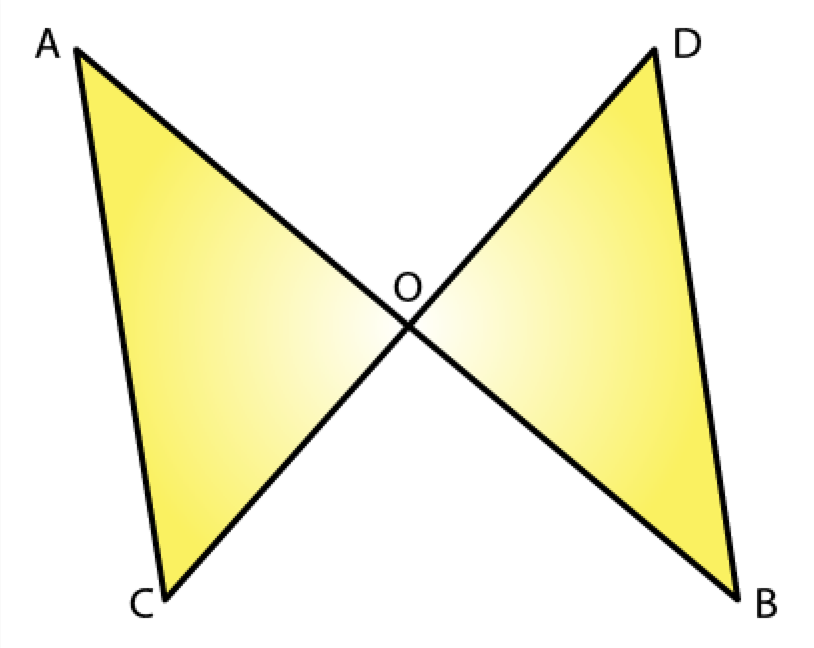
But here AL perpendicular to BC

$100^o = 90^o + \angle LAD$

$\angle LAD = 10^o$

26. Line segments AB and CD intersect at O such that AC ǀ DB. It ∠CAB = 35º and ∠CDB = 55º. Find ∠BOD.

Solution:



We know that AC parallel to BD and AB cuts AC and BD at A and B, respectively.

∠CAB = ∠DBA (Alternate interior angles)

∠DBA = 35^o

We also know that the sum of all three angles of a triangle is 180^o.

Hence, for △OBD, we can say that:

∠DBO + ∠ODB + ∠BOD = 180°

$35^o + 55^o$ + ∠BOD = 180^o (∠DBO = ∠DBA and ∠ODB = ∠CDB)

∠BOD = $180^o - 90^o$

∠BOD = 90^o

27. In Fig. 22, ΔABC is right angled at A, Q and R are points on line BC and P is a point such that QP ǁ AC and RP ǁ AB. Find ∠P

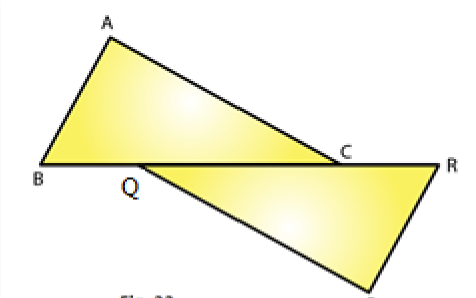


Fig. 22

Solution:

In the given triangle, AC parallel to QP and BR cuts AC and QP at C and Q, respectively.

∠QCA = ∠CQP (Alternate interior angles)

Because RP parallel to AB and BR cuts AB and RP at B and R, respectively,

∠ABC = ∠PRQ (alternate interior angles).

We know that the sum of all three angles of a triangle is 180^o.

Hence, for ΔABC, we can say that:

∠ABC + ∠ACB + ∠BAC = 180^o

∠ABC + ∠ACB + 90^o = 180^o (Right angled at A)

∠ABC + ∠ACB = 90°

Using the same logic for △PQR, we can say that:

∠PQR + ∠PRQ + ∠QPR = 180°

∠ABC + ∠ACB + ∠QPR = 180° (∠ACB = ∠PQR and ∠ABC = ∠PRQ)

Or,

90°+ ∠QPR =180° (∠ABC+ ∠ACB = 90°)

∠QPR = 90°

Exercise 15.3

1. In Fig. 35, ∠CBX is an exterior angle of ΔABC at B. Name

(i) The interior adjacent angle

(ii) The interior opposite angles to exterior ∠CBX

Also, name the interior opposite angles to an exterior angle at A.

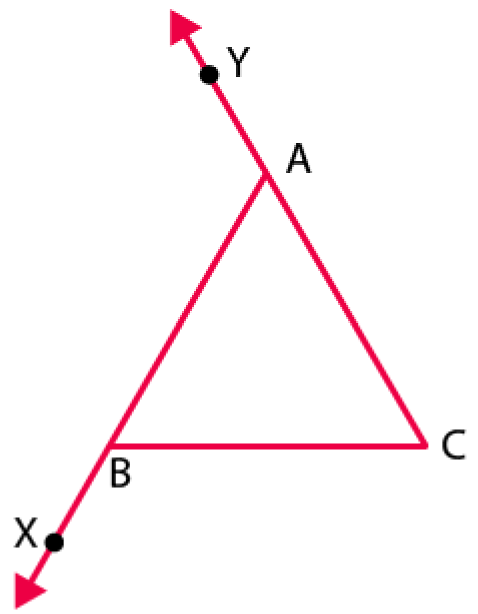


Fig. 35

Solution:

(i) The interior adjacent angle is ∠ABC

(ii) The interior opposite angles to exterior ∠CBX is ∠BAC and ∠ACB

Also the interior angles opposite to exterior are ∠ABC and ∠ACB

2. In the fig. 36, two of the angles are indicated. What are the measures of ∠ACX and ∠ACB?

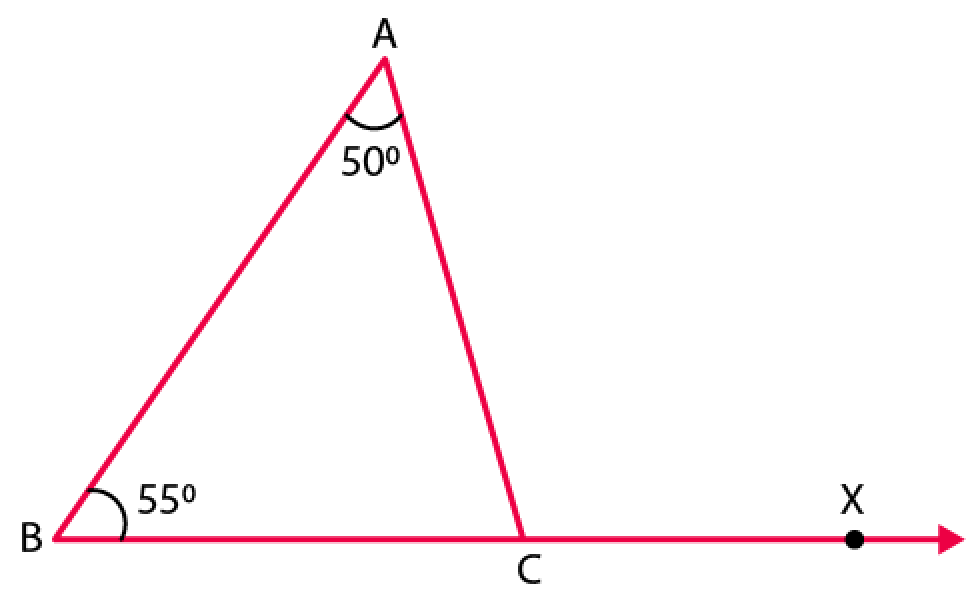


Fig. 36

Solution:

Given that in $\triangle ABC$, $\angle A = 50^o$ and $\angle B = 55^o$

We know that the sum of angles in a triangle is 180^o

Therefore we have

$\angle A + \angle B + \angle C = 180^o$

$50^o + 55^o + \angle C = 180^o$

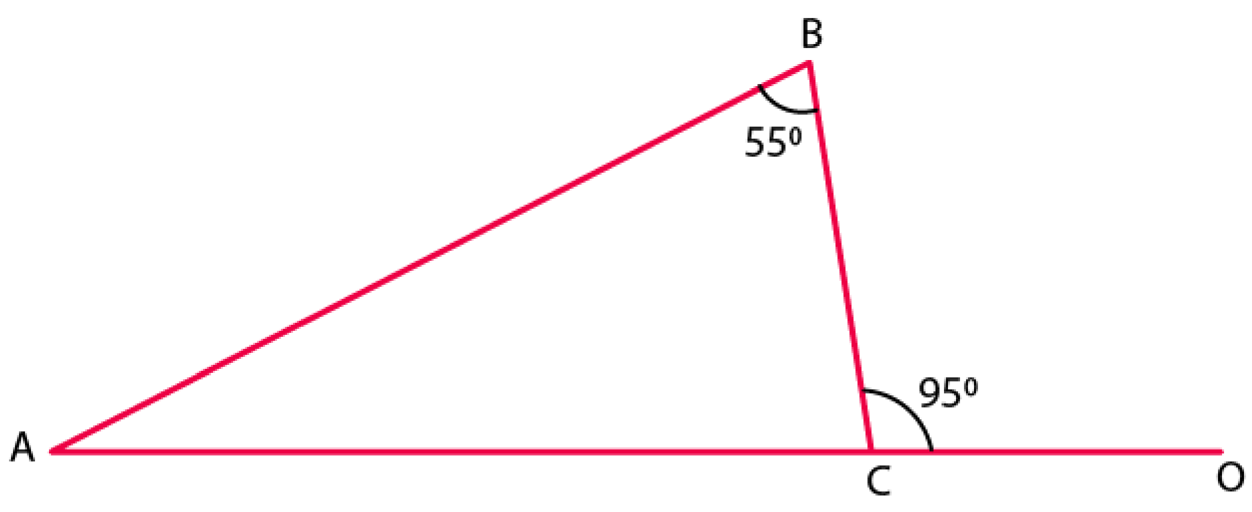
$\angle C = 75^o$

$\angle ACB = 75^o$

$\angle ACX = 180^o - \angle ACB = 180^o - 75^o = 105^o$

3. In a triangle, an exterior angle at a vertex is 95^o and its one of the interior opposite angles is 55^o. Find all the angles of the triangle.

Solution:



We know that the sum of interior opposite angles is equal to the exterior angle.

Hence, for the given triangle, we can say that:

$\angle ABC + \angle BAC = \angle BCO$

$55^o + \angle BAC = 95^o$

$\angle BAC = 95^o - 55^o$

$\angle BAC = 40^o$

We also know that the sum of all angles of a triangle is 180^o.

Hence, for the given $\triangle ABC$, we can say that:

$\angle ABC + \angle BAC + \angle BCA = 180^o$

$55^o + 40^o + \angle BCA = 180^o$

$\angle BCA = 180^o - 95^o$

$\angle BCA = 85^o$

4. One of the exterior angles of a triangle is 80^o, and the interior opposite angles are equal to each other. What is the measure of each of these two angles?

Solution:

Let us assume that A and B are the two interior opposite angles.

We know that $\angle A$ is equal to $\angle B$.

We also know that the sum of interior opposite angles is equal to the exterior angle.

Therefore from the figure we have,

$\angle A + \angle B = 80^o$

$\angle A + \angle A = 80^o$ (because $\angle A = \angle B$)

$2\angle A = 80^o$

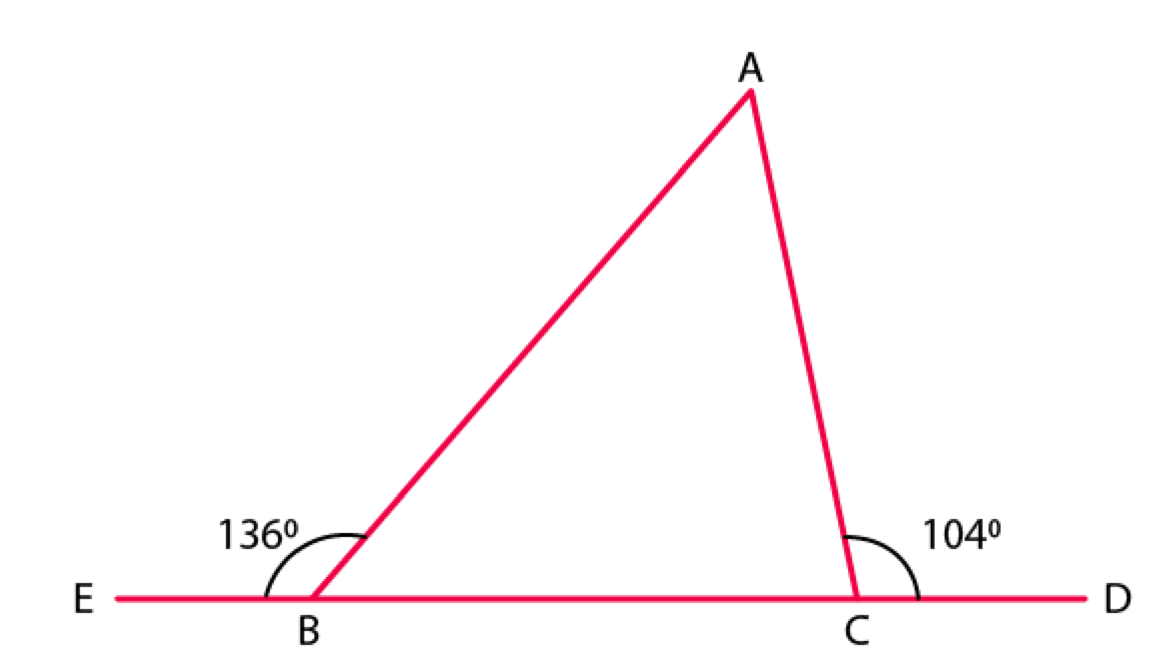
$\angle A = 80/2 = 40^o$

$\angle A = \angle B = 40^o$

Thus, each of the required angles is of 40^o.

5. The exterior angles, obtained on producing the base of a triangle both ways are 104^o and 136^o. Find all the angles of the triangle.

Solution:



In the given figure, $\angle ABE$ and $\angle ABC$ form a linear pair.

$\angle ABE + \angle ABC = 180^o$

$\angle ABC = 180^o - 136^o$

$\angle ABC = 44^o$

We can also see that $\angle ACD$ and $\angle ACB$ form a linear pair.

$\angle ACD + \angle ACB = 180^o$

$\angle ACB = 180^o - 104^o$

$\angle ACB = 76^o$

We know that the sum of interior opposite angles is equal to the exterior angle.

Therefore, we can write as

$\angle BAC + \angle ABC = 104^o$

$\angle BAC = 104^o - 44^o = 60^o$

Thus,

$\angle ACE = 76^o$ and $\angle BAC = 60^o$

6. In Fig. 37, the sides BC, CA and BA of a $\triangle ABC$ have been produced to D, E and F respectively. If $\angle ACD = 105^o$ and $\angle EAF = 45^o$; find all the angles of the $\triangle ABC$.

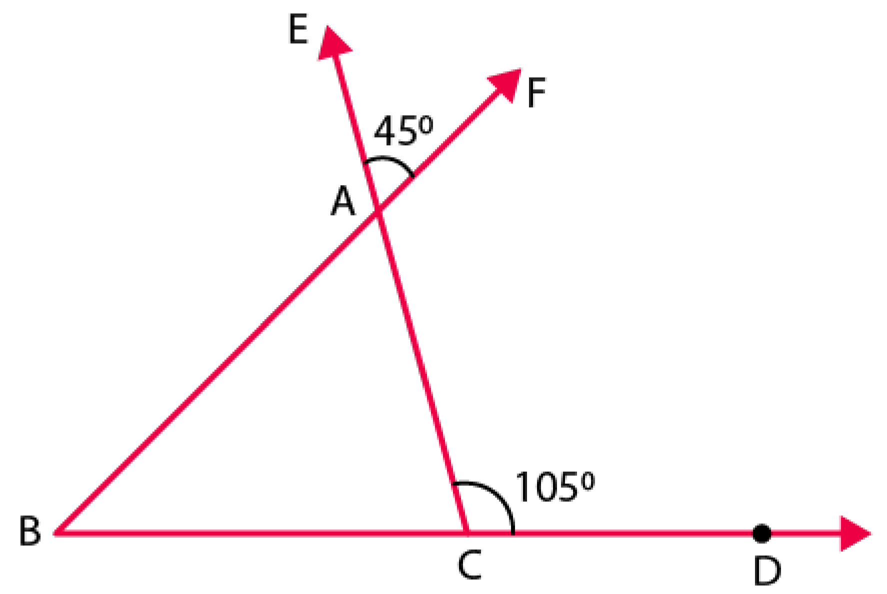


Fig. 37

Solution:

In a $\triangle ABC$, $\angle BAC$ and $\angle EAF$ are vertically opposite angles.

Hence, we can write as

$\angle BAC = \angle EAF = 45^o$

Considering the exterior angle property, we have

$\angle BAC + \angle ABC = \angle ACD = 105^o$

On rearranging we get

$\angle ABC = 105^o - 45^o = 60^o$

We know that the sum of angles in a triangle is 180^o

$\angle ABC + \angle ACB + \angle BAC = 180°$

$\angle ACB = 75^o$

Therefore, the angles are 45^o, 60^o and 75^o.

7. In Fig. 38, AC perpendicular to CE and C ∠A: ∠B: ∠C= 3: 2: 1. Find the value of ∠ECD.

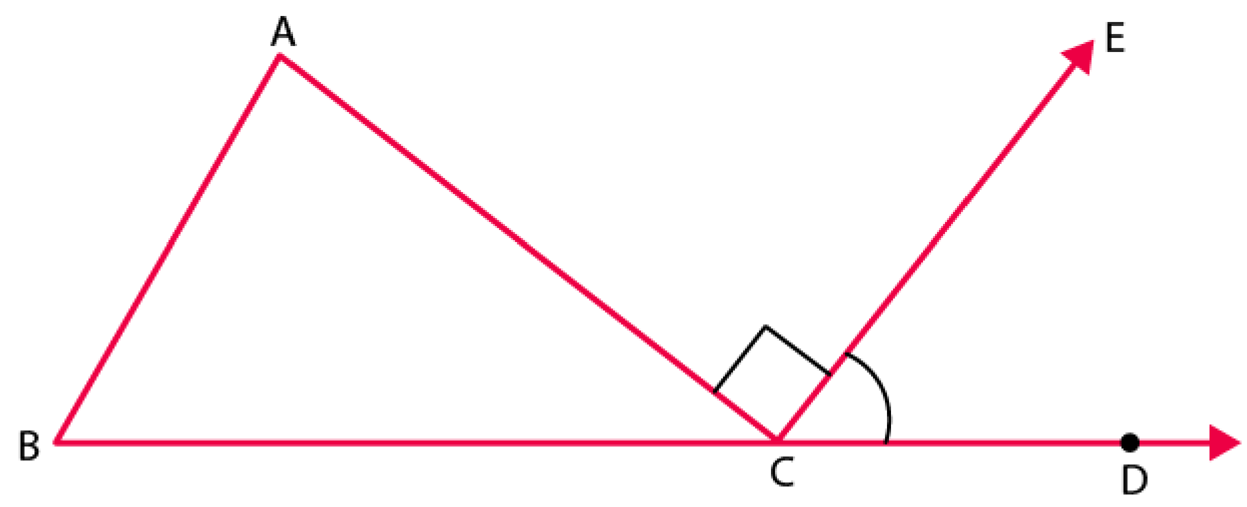


Fig. 38

Solution:

In the given triangle, the angles are in the ratio 3: 2: 1.

Let the angles of the triangle be 3x, 2x and x.

We know that sum of angles in a triangle is 180^o

$3x + 2x + x = 180^o$

$6x = 180^o$

$x = 30^o$

Also, $\angle ACB + \angle ACE + \angle ECD = 180^o$

$x + 90^o + \angle ECD = 180^o$ ($\angle ACE = 90^o$)

We know that $x = 30^o$

Therefore

$\angle ECD = 60^o$

8. A student when asked to measure two exterior angles of △ABC observed that the exterior angles at A and B are of 103^o and 74^o respectively. Is this possible? Why or why not?

Solution:

We know that sum of internal and external angle is equal to 180^o

Internal angle at A + External angle at A = 180^o

Internal angle at A + 103^o =180^o

Internal angle at A = 77^o

Internal angle at B + External angle at B = 180^o

Internal angle at B + 74^o = 180^o

Internal angle at B = 106^o

Sum of internal angles at A and B = $77^\circ + 106^\circ = 183^\circ$

It means that the sum of internal angles at A and B is greater than 180°, which cannot be possible.

9. In Fig.39, AD and CF are respectively perpendiculars to sides BC and AB of $\triangle ABC$. If $\angle FCD = 50^\circ$, find $\angle BAD$

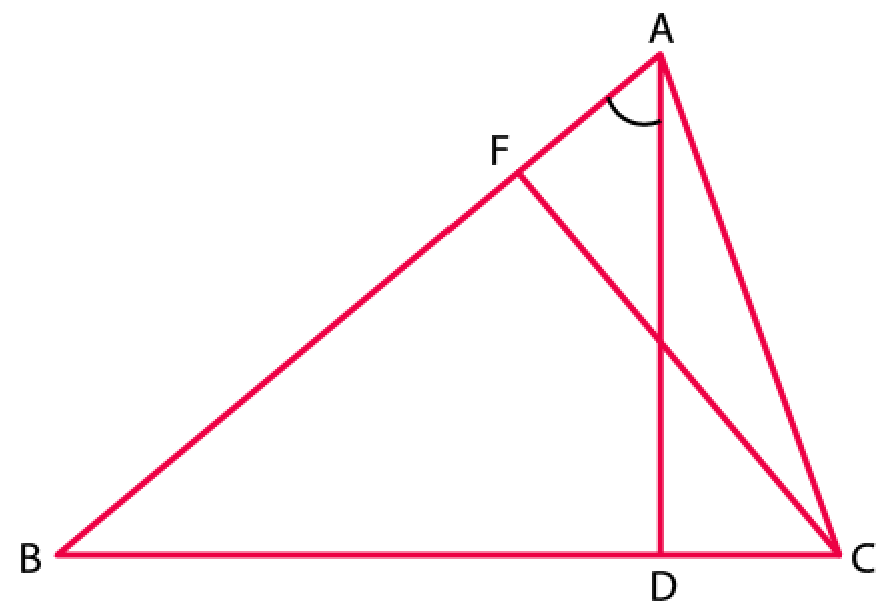


Fig. 39

Solution:

We know that the sum of all angles of a triangle is 180°

Therefore, for the given $\triangle FCB$, we have

$\angle FCB + \angle CBF + \angle BFC = 180^\circ$

$50^\circ + \angle CBF + 90^\circ = 180^\circ$

$\angle CBF = 180^\circ - 50^\circ - 90^\circ = 40^\circ$

Using the above steps for $\triangle ABD$, we can say that:

$\angle ABD + \angle BDA + \angle BAD = 180^\circ$

$\angle BAD = 180^\circ - 90^\circ - 40^\circ = 50^\circ$

10. In Fig.40, measures of some angles are indicated. Find the value of x.

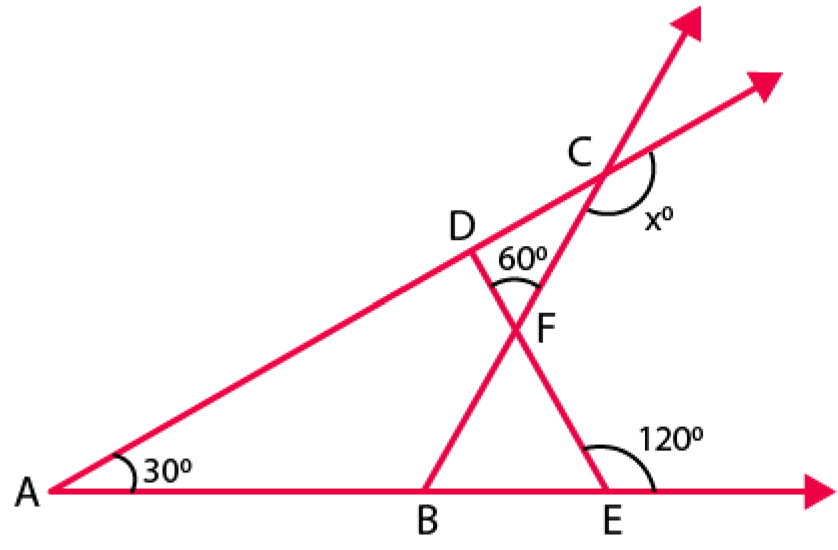


Fig. 40

Solution:

We know that the sum of the angles of a triangle is 180°

From the figure we have,

$\angle AED + 120^\circ = 180^\circ$ (Linear pair)

$\angle AED = 180^\circ - 120^\circ = 60^\circ$

We know that the sum of all angles of a triangle is 180°.

Therefore, for $\triangle ADE$, we have

$\angle ADE + \angle AED + \angle DAE = 180^\circ$

$60^\circ + \angle ADE + 30^\circ = 180^\circ$

$\angle ADE = 180^\circ - 60^\circ - 30^\circ = 90^\circ$

From the given figure, we have

$\angle FDC + 90^\circ = 180^\circ$ (Linear pair)

$\angle FDC = 180^\circ - 90^\circ = 90^\circ$

Using the same steps for $\triangle CDF$, we get

$\angle CDF + \angle DCF + \angle DFC = 180^\circ$

$90^\circ + \angle DCF + 60^\circ = 180^\circ$

$\angle DCF = 180^\circ - 60^\circ - 90^\circ = 30^\circ$

Again from the figure we have

$\angle DCF + x = 180^\circ$ (Linear pair)

$30^\circ + x = 180^\circ$

$x = 180^\circ - 30^\circ = 150^\circ$

11. In Fig. 41, ABC is a right triangle right angled at A. D lies on BA produced and DE perpendicular to BC intersecting AC at F. If $\angle AFE = 130^\circ$, find

(i) $\angle BDE$

(ii) $\angle BCA$

(iii) $\angle ABC$

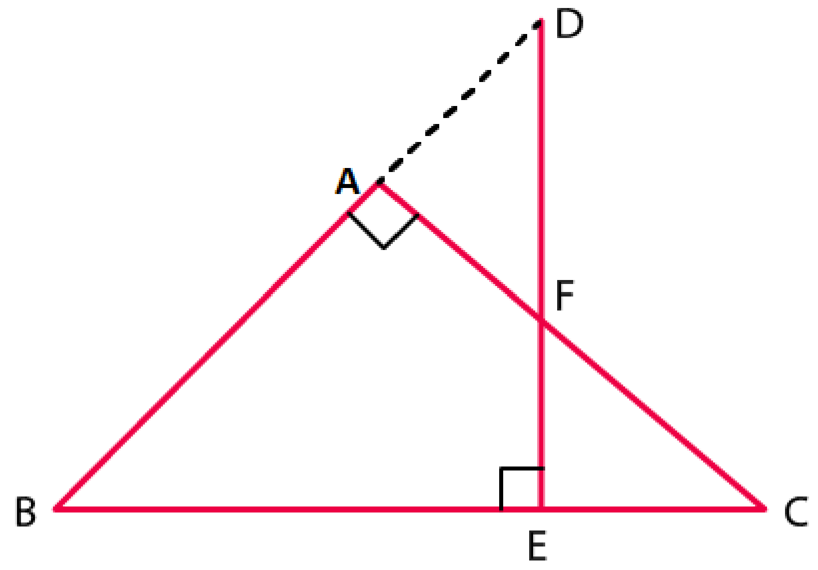


Fig. 41

Solution:

(i) Here,

$\angle BAF + \angle FAD = 180^o$ (Linear pair)

$\angle FAD = 180^o - \angle BAF = 180^o - 90^o = 90^o$

Also from the figure,

$\angle AFE = \angle ADF + \angle FAD$ (Exterior angle property)

$\angle ADF + 90^o = 130^o$

$\angle ADF = 130^o - 90^o = 40^o$

$\angle BDE = 40^o$

(ii) We know that the sum of all the angles of a triangle is 180^o.

Therefore, for $\triangle BDE$, we have

$\angle BDE + \angle BED + \angle DBE = 180^o$

$\angle DBE = 180^o - \angle BDE - \angle BED$

$\angle DBE = 180^o - 40^o - 90^o = 50^o$ …. Equation (i)

Again from the figure we have,

$\angle FAD = \angle ABC + \angle ACB$ (Exterior angle property)

$90^o = 50^o + \angle ACB$

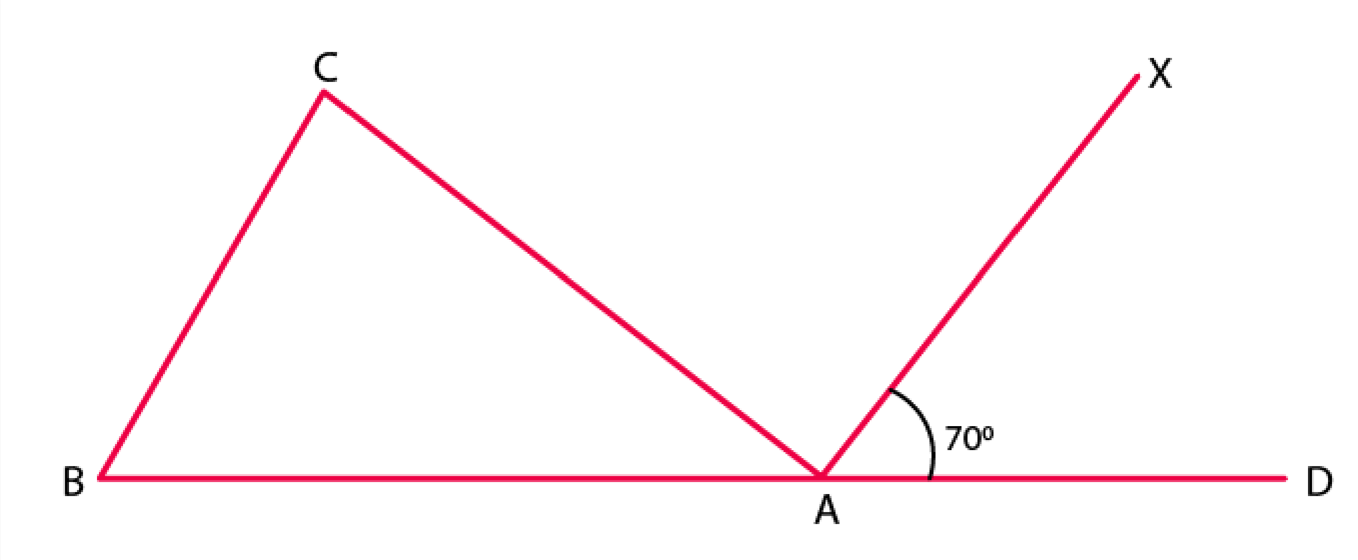
$\angle ACB = 90^o - 50^o = 40^o$

(iii) From equation we have

$\angle ABC = \angle DBE = 50^o$

12. ABC is a triangle in which $\angle B = \angle C$ and ray AX bisects the exterior angle DAC. If $\angle DAX = 70^o$. Find $\angle ACB$.

Solution:



Given that ABC is a triangle in which ∠B = ∠C

Also given that AX bisects the exterior angle DAC

∠CAX = ∠DAX (AX bisects ∠CAD)

∠CAX =70° [given]

∠CAX +∠DAX + ∠CAB =180°

70°+ 70° + ∠CAB =180°

∠CAB =180° –140°

∠CAB =40°

∠ACB + ∠CBA + ∠CAB = 180° (Sum of the angles of ΔABC)

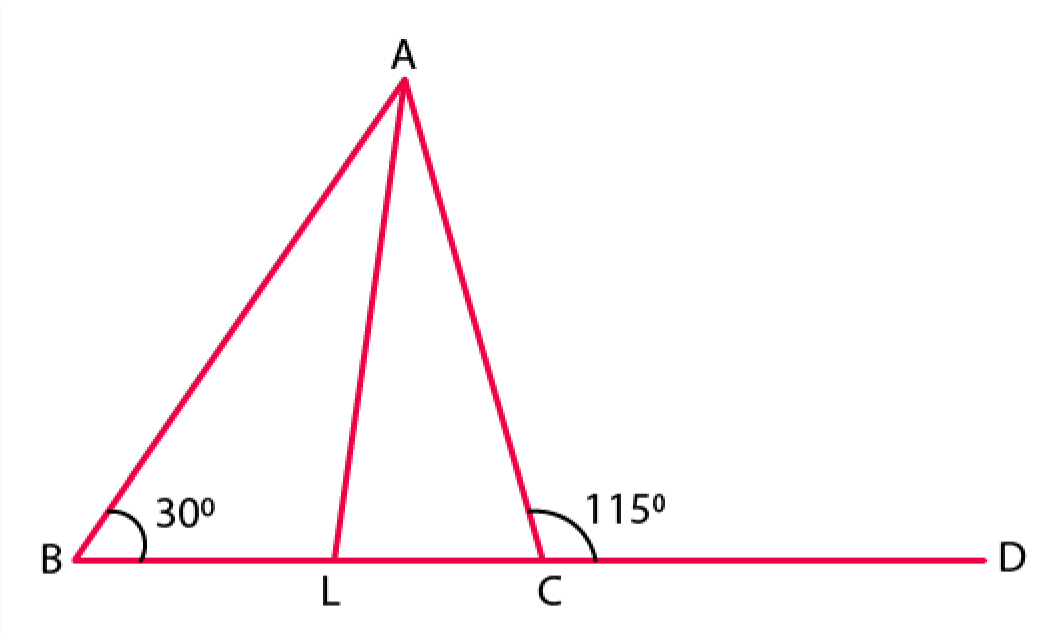
∠ACB + ∠ACB+ 40° = 180° (∠C = ∠B)

2∠ACB = 180° – 40°

∠ACB = 140/2

∠ACB = 70°

13. The side BC of △ABC is produced to a point D. The bisector of ∠A meets side BC in L. If ∠ABC= 30° and ∠ACD = 115°, find ∠ALC



Solution:

Given that ∠ABC= 30° and ∠ACD = 115°

From the figure, we have

∠ACD and ∠ACL make a linear pair.

∠ACD+ ∠ACB = 180°

115° + ∠ACB =180°

∠ACB = 180° – 115°

∠ACB = 65°

We know that the sum of all angles of a triangle is 180°.

Therefore, for △ ABC, we have

∠ABC + ∠BAC + ∠ACB = 180°

30° + ∠BAC + 65° = 180°

∠BAC = 85°

∠LAC = ∠BAC/2 = 85/2

Using the same steps for △ALC, we get

∠ALC + ∠LAC + ∠ACL = 180°

∠ALC + 85/2 + 65° = 180°

We know that ∠ACL = ∠ACB

∠ALC = 180° – 85/2 – 65°

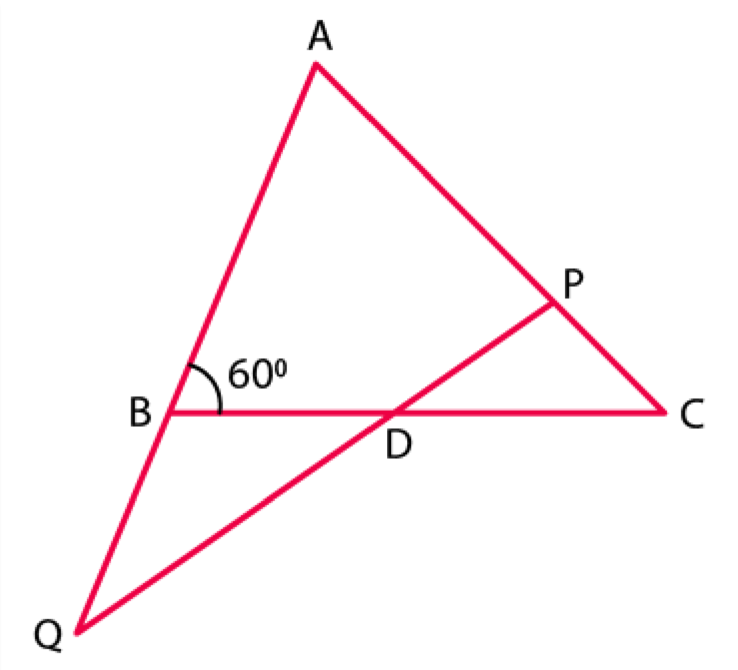
∠ALC = 72 ½°

14. D is a point on the side BC of △ABC. A line PDQ through D, meets side AC in P and AB produced at Q. If ∠A = 80°, ∠ABC = 60° and ∠PDC = 15°, find

(i) ∠AQD

(ii) ∠APD

Solution:



From the figure we have

∠ABD and ∠QBD form a linear pair.

$\angle ABC + \angle QBC = 180^o$

$60^o + \angle QBC = 180^o$

$\angle QBC = 120^o$

$\angle PDC = \angle BDQ$ (Vertically opposite angles)

$\angle BDQ = 15^o$

(i) In $\triangle QBD$:

$\angle QBD + \angle QDB + \angle BQD = 180^o$ (Sum of angles of $\triangle QBD$)

$120^o + 15^o + \angle BQD = 180^o$

$\angle BQD = 180^o - 135^o$

$\angle BQD = 45^o$

$\angle AQD = \angle BQD = 45^o$

(ii) In $\triangle AQP$:

$\angle QAP + \angle AQP + \angle APQ = 180^o$ (Sum of angles of $\triangle AQP$)

$80^o + 45^o + \angle APQ = 180^o$

$\angle APQ = 55^o$

$\angle APD = \angle APQ = 55^o$

15. Explain the concept of interior and exterior angles and in each of the figures given below. Find x and y (Fig. 42)

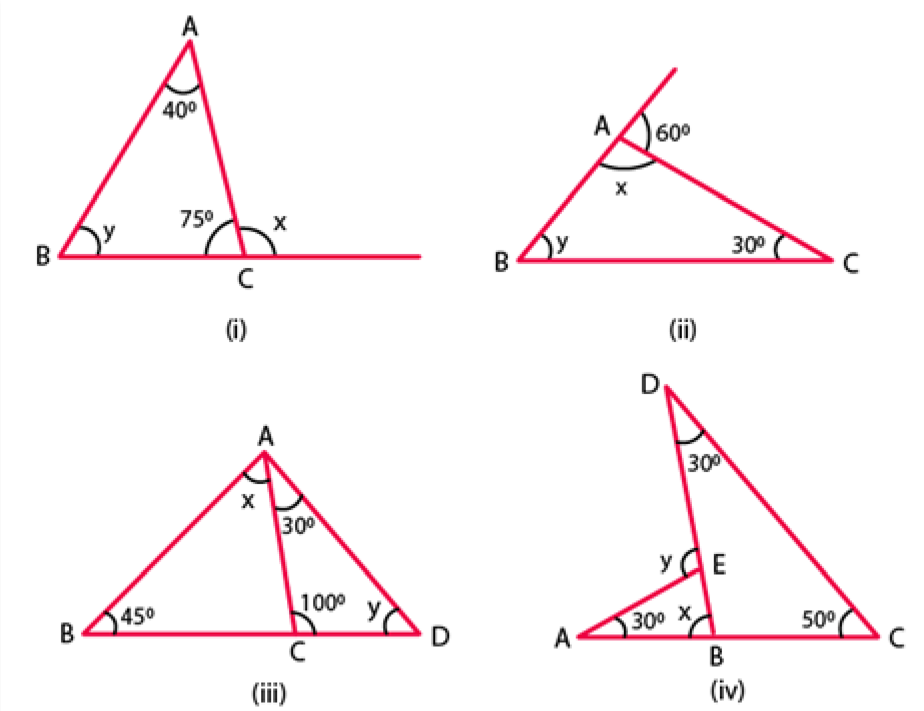


Fig. 42

Solution:

The interior angles of a triangle are the three angle elements inside the triangle.

The exterior angles are formed by extending the sides of a triangle, and if the side of a triangle is produced, the exterior angle so formed is equal to the sum of the two interior opposite angles.

Using these definitions, we will obtain the values of x and y.

(i) From the given figure, we have

$\angle ACB + x = 180^o$ (Linear pair)

$75^\circ + x = 180^\circ$

$x = 105^\circ$

We know that the sum of all angles of a triangle is 180°

Therefore, for $\triangle ABC$, we can say that:

$\angle BAC + \angle ABC + \angle ACB = 180^\circ$

$40^\circ + y + 75^\circ = 180^\circ$

$y = 65^\circ$

(ii) From the figure, we have

$x + 80^\circ = 180^\circ$ (Linear pair)

$x = 100^\circ$

In $\triangle ABC$, we have

We also know that the sum of angles of a triangle is 180°

$x + y + 30^\circ = 180^\circ$

$100^\circ + 30^\circ + y = 180^\circ$

$y = 50^\circ$

(iii) We know that the sum of all angles of a triangle is 180°.

Therefore, for $\triangle ACD$, we have

$30^\circ + 100^\circ + y = 180^\circ$

$y = 50^\circ$

Again from the figure we can write as

$\angle ACB + 100^\circ = 180^\circ$

$\angle ACB = 80^\circ$

Using the above rule for $\triangle ACB$, we can say that:

$x + 45^\circ + 80^\circ = 180^\circ$

$x = 55^\circ$

(iv) We know that the sum of all angles of a triangle is 180°.

Therefore, for $\triangle DBC$, we have

$30^\circ + 50^\circ + \angle DBC = 180^\circ$

$\angle DBC = 100^\circ$

From the figure we can say that

$x + \angle DBC = 180^\circ$ is a Linear pair

$x = 80^\circ$

From the exterior angle property we have

$y = 30^\circ + 80^\circ = 110^\circ$

16. Compute the value of x in each of the following figures:

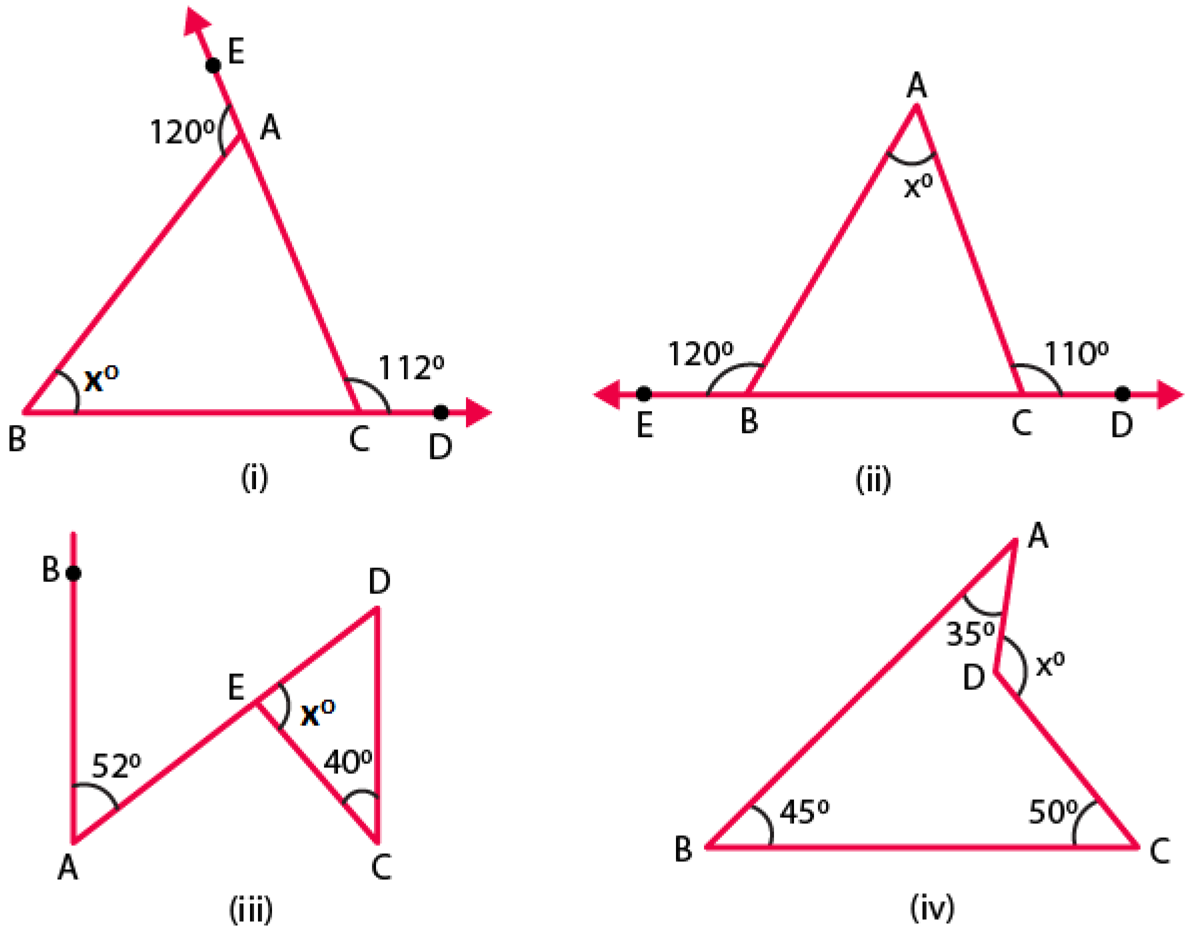


Fig. 43

Solution:

(i) From the given figure, we can write as

$\angle ACD + \angle ACB = 180^o$ is a linear pair

On rearranging we get

$\angle ACB = 180^o - 112^o = 68^o$

Again from the figure we have,

$\angle BAE + \angle BAC = 180^o$ is a linear pair

On rearranging we get,

$\angle BAC = 180^o - 120^o = 60^o$

We know that the sum of all angles of a triangle is 180^o.

Therefore, for $\triangle ABC$:

$x + \angle BAC + \angle ACB = 180^o$

$x = 180^o - 60^o - 68^o = 52^o$

$x = 52^o$

(ii) From the given figure, we can write as

$\angle ABC + 120^o = 180^o$ is a linear pair

$\angle ABC = 60^o$

Again from the figure we can write as

$\angle ACB + 110^o = 180^o$ is a linear pair

$\angle ACB = 70^o$

We know that the sum of all angles of a triangle is 180^o.

Therefore, consider $\triangle ABC$, we get

$x + \angle ABC + \angle ACB = 180^o$

$x = 50^o$

(iii) From the given figure, we can write as

$\angle BAD = \angle ADC = 52^o$ are alternate angles

We know that the sum of all the angles of a triangle is 180^o.

Therefore, consider $\triangle DEC$, we have

$x + 40^o + 52^o = 180^o$

$x = 88^o$

(iv) In the given figure, we have a quadrilateral and also we know that sum of all angles in a quadrilateral is 360^o.

Thus,

$35^o + 45^o + 50^o +$ reflex $\angle ADC = 360^o$

On rearranging we get,

Reflex $\angle ADC = 230^o$

$230^o + x = 360^o$ (A complete angle)

$x = 130^o$

Exercise 15.4

1. In each of the following, there are three positive numbers. State if these numbers could possibly be the lengths of the sides of a triangle:

(i) 5, 7, 9

(ii) 2, 10, 15

(iii) 3, 4, 5

(iv) 2, 5, 7

(v) 5, 8, 20

Solution:

(i) Given 5, 7, 9

Yes, these numbers can be the lengths of the sides of a triangle because the sum of any two sides of a triangle is always greater than the third side.

Here, $5 + 7 > 9$, $5 + 9 > 7$, $9 + 7 > 5$

(ii) Given 2, 10, 15

No, these numbers cannot be the lengths of the sides of a triangle because the sum of any two sides of a triangle is always greater than the third side, which is not true in this case.

Here, $2 + 10 < 15$

(iii) Given 3, 4, 5

Yes, these numbers can be the lengths of the sides of a triangle because the sum of any two sides of triangle is always greater than the third side.

Here, $3 + 4 > 5$, $3 + 5 > 4$, $4 + 5 > 3$

(iv) Given 2, 5, 7

No, these numbers cannot be the lengths of the sides of a triangle because the sum of any two sides of a triangle is always greater than the third side, which is not true in this case.

Here, $2 + 5 = 7$

(v) Given 5, 8, 20

No, these numbers cannot be the lengths of the sides of a triangle because the sum of any two sides of a triangle is always greater than the third side, which is not true in this case.

Here, $5 + 8 < 20$

2. In Fig. 46, P is the point on the side BC. Complete each of the following statements using symbol '=',' > 'or '< 'so as to make it true:

(i) AP… AB+ BP

(ii) AP… AC + PC

(iii) AP…. ½ (AB + AC + BC)

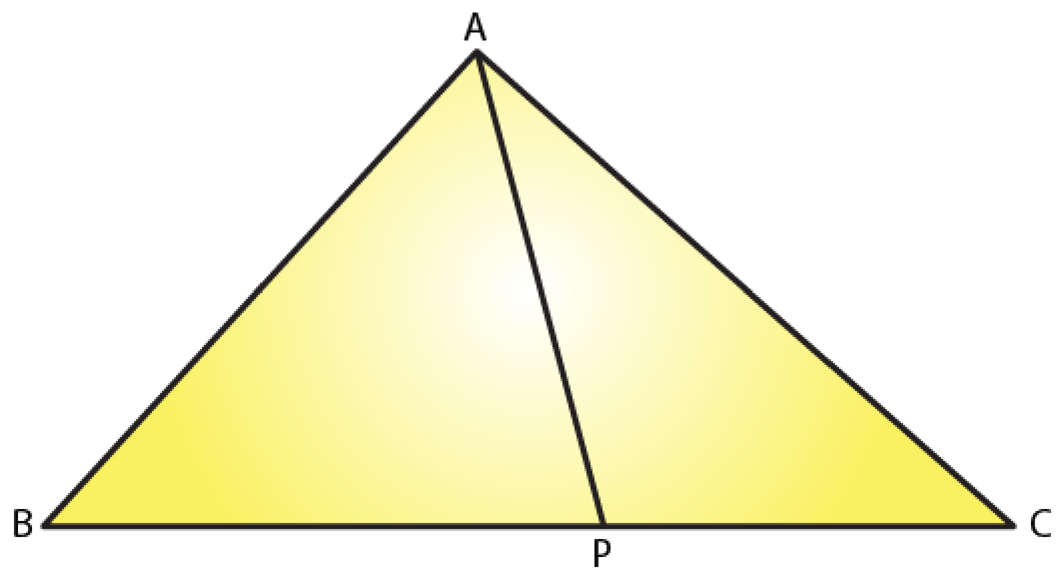


Fig. 46

Solution:

(i) In △APB, AP < AB + BP because the sum of any two sides of a triangle is greater than the third side.

(ii) In △APC, AP < AC + PC because the sum of any two sides of a triangle is greater than the third side.

(iii) AP < ½ (AB + AC + BC)

In △ABP and △ACP, we can write as

AP < AB + BP… (i) (Because the sum of any two sides of a triangle is greater than the third side)

AP < AC + PC … (ii) (Because the sum of any two sides of a triangle is greater than the third side)

On adding (i) and (ii), we have:

AP + AP < AB + BP + AC + PC

2AP < AB + AC + BC (BC = BP + PC)

AP < ½ (AB + AC + BC)

3. P is a point in the interior of △ABC as shown in Fig. 47. State which of the following statements are true (T) or false (F):

(i) AP + PB < AB

(ii) AP + PC > AC

(iii) BP + PC = BC

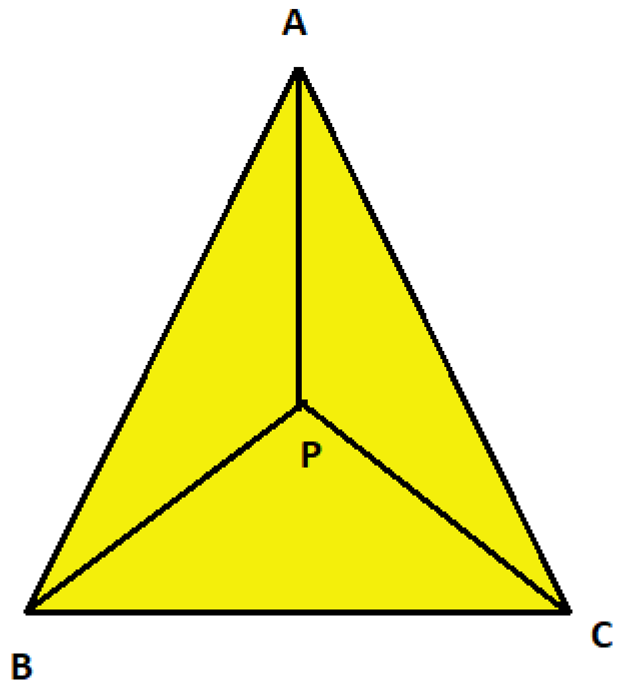


Fig. 47

Solution:

(i) False

Explanation:

We know that the sum of any two sides of a triangle is greater than the third side, it is not true for the given triangle.

(ii) True

Explanation:

We know that the sum of any two sides of a triangle is greater than the third side, it is true for the given triangle.

(iii) False

Explanation:

We know that the sum of any two sides of a triangle is greater than the third side, it is not true for the given triangle.

4. O is a point in the exterior of △ABC. What symbol '>','<' or '=' will you see to complete the statement OA+OB….AB? Write two other similar statements and show that OA + OB + OC > ½ (AB + BC +CA)

Solution:

We know that the sum of any two sides of a triangle is always greater than the third side, in △OAB, we have,

OA + OB > AB ….. (i)

In △OBC we have

OB + OC > BC …… (ii)

In △OCA we have

OA + OC > CA ….. (iii)

On adding equations (i), (ii) and (iii) we get:

OA + OB + OB + OC + OA + OC > AB + BC + CA

2(OA + OB + OC) > AB + BC + CA

OA + OB + OC > (AB + BC + CA)/2

Or

OA + OB + OC > ½ (AB + BC +CA)

Hence the proof.

5. In △ABC, ∠A = 100º, ∠B = 30º, ∠C = 50º. Name the smallest and the largest sides of the triangle.

Solution:

We know that the smallest side is always opposite to the smallest angle, which in this case is 30°, it is AC.

Also, because the largest side is always opposite to the largest angle, which in this case is 100°, it is BC.

Exercise 15.5 Page No: 15.30

1. State Pythagoras theorem and its converse.

Solution:

The Pythagoras Theorem:

In a right triangle, the square of the hypotenuse is always equal to the sum of the squares of the other two sides.

Converse of the Pythagoras Theorem:

If the square of one side of a triangle is equal to the sum of the squares of the other two sides, then the triangle is a right triangle, with the angle opposite to the first side as right angle.

2. In right △ABC, the lengths of the legs are given. Find the length of the hypotenuse

(i) a = 6 cm, b = 8 cm

(ii) a = 8 cm, b = 15 cm

(iii) a = 3 cm, b = 4 cm

(iv) a = 2 cm, b =1.5 cm

Solution:

(i) According to the Pythagoras theorem, we have

$(\text{Hypotenuse})^2 = (\text{Base})^2 + (\text{Height})^2$

Let c be hypotenuse and a and b be other two legs of right angled triangle

Then we have

$c^2 = a^2 + b^2$

$c^2 = 6^2 + 8^2$

$c^2 = 36 + 64 = 100$

c = 10 cm

(ii) According to the Pythagoras theorem, we have

$(\text{Hypotenuse})^2 = (\text{Base})^2 + (\text{Height})^2$

Let c be hypotenuse and a and b be other two legs of right angled triangle

Then we have

$c^2 = a^2 + b^2$

$c^2 = 8^2 + 15^2$

$c^2 = 64 + 225 = 289$

c = 17cm

(iii) According to the Pythagoras theorem, we have

$(\text{Hypotenuse})^2 = (\text{Base})^2 + (\text{Height})^2$

Let c be hypotenuse and a and b be other two legs of right angled triangle

Then we have

$c^2 = a^2 + b^2$

$c^2 = 3^2 + 4^2$

$c^2 = 9 + 16 = 25$

c = 5 cm

(iv) According to the Pythagoras theorem, we have

$(\text{Hypotenuse})^2 = (\text{Base})^2 + (\text{Height})^2$

Let c be hypotenuse and a and b be other two legs of right angled triangle

Then we have

$c^2 = a^2 + b^2$

$c^2 = 2^2 + 1.5^2$

$c^2 = 4 + 2.25 = 6.25$

c = 2.5 cm

3. The hypotenuse of a triangle is 2.5 cm. If one of the sides is 1.5 cm. find the length of the other side.

Solution:

Let c be hypotenuse and the other two sides be b and a

According to the Pythagoras theorem, we have

$c^2 = a^2 + b^2$

$2.5^2 = 1.5^2 + b^2$

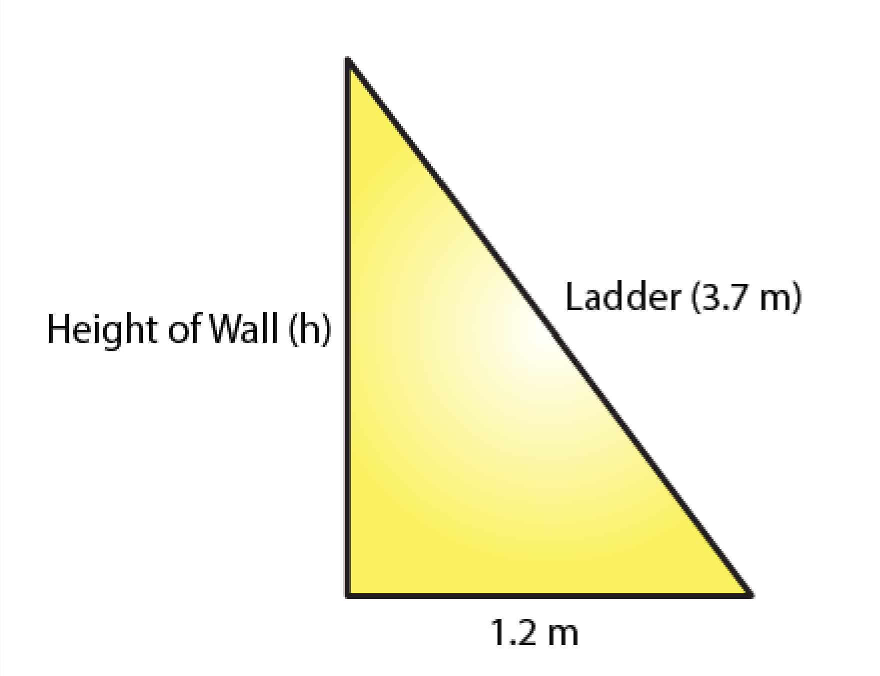
$b^2 = 6.25 - 2.25 = 4$

b = 2 cm

Hence, the length of the other side is 2 cm.

4. A ladder 3.7 m long is placed against a wall in such a way that the foot of the ladder is 1.2 m away from the wall. Find the height of the wall to which the ladder reaches.

Solution:



Let the height of the ladder reaches to the wall be h.

According to the Pythagoras theorem, we have

$(\text{Hypotenuse})^2 = (\text{Base})^2 + (\text{Height})^2$

$3.7^2 = 1.2^2 + h^2$

$h^2 = 13.69 – 1.44 = 12.25$

$h = 3.5$ m

Hence, the height of the wall is 3.5 m.

5. If the sides of a triangle are 3 cm, 4 cm and 6 cm long, determine whether the triangle is right-angled triangle.

Solution:

In the given triangle, the largest side is 6 cm.

We know that in a right angled triangle, the sum of the squares of the smaller sides should be equal to the square of the largest side.

Therefore,

$3^2 + 4^2 = 9 + 16 = 25$

But, $6^2 = 36$

$3^2 + 4^2 = 25$ which is not equal to 6^2

Hence, the given triangle is not a right angled triangle.

6. The sides of certain triangles are given below. Determine which of them are right triangles.

(i) a = 7 cm, b = 24 cm and c= 25 cm

(ii) a = 9 cm, b = 16 cm and c = 18 cm

Solution:

(i) We know that in a right angled triangle, the square of the largest side is equal to the sum of the squares of the smaller sides.

Here, the larger side is c, which is 25 cm.

$c^2 = 625$

Given that,

$a^2 + b^2 = 7^2 + 24^2$

$= 49 + 576$

$= 625$

$= c^2$

Thus, the given triangle is a right triangle.

(ii) We know that in a right angled triangle, the square of the largest side is equal to the sum of the squares of the smaller sides.

Here, the larger side is c, which is 18 cm.

$c^2 = 324$

Given that

$a^2 + b^2 = 9^2 + 16^2$

$= 81 + 256$

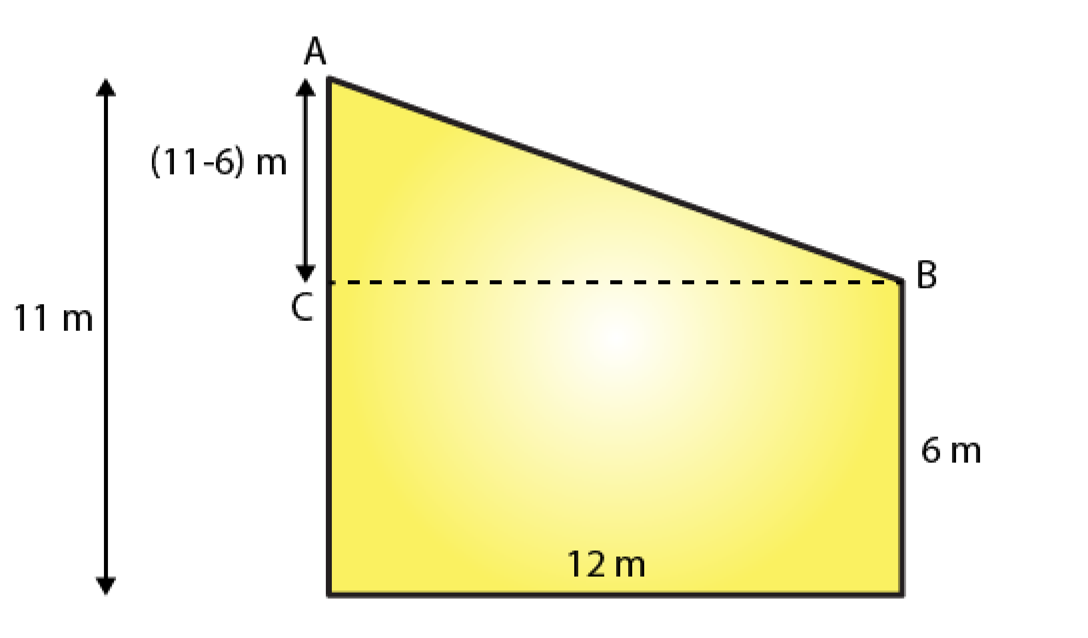
$= 337$ which is not equal to c^2

Thus, the given triangle is not a right triangle.

7. Two poles of heights 6 m and 11 m stand on a plane ground. If the distance between their feet is 12 m. Find the distance between their tops.

(Hint: Find the hypotenuse of a right triangle having the sides (11 – 6) m = 5 m and 12 m)

Solution:



Let the distance between the tops of the poles is the distance between points A and B.

We can see from the given figure that points A, B and C form a right triangle, with AB as the hypotenuse.

By using the Pythagoras Theorem in $\triangle ABC$, we get

$(11-6)^2 + 12^2 = AB^2$

$AB^2 = 25 + 144$

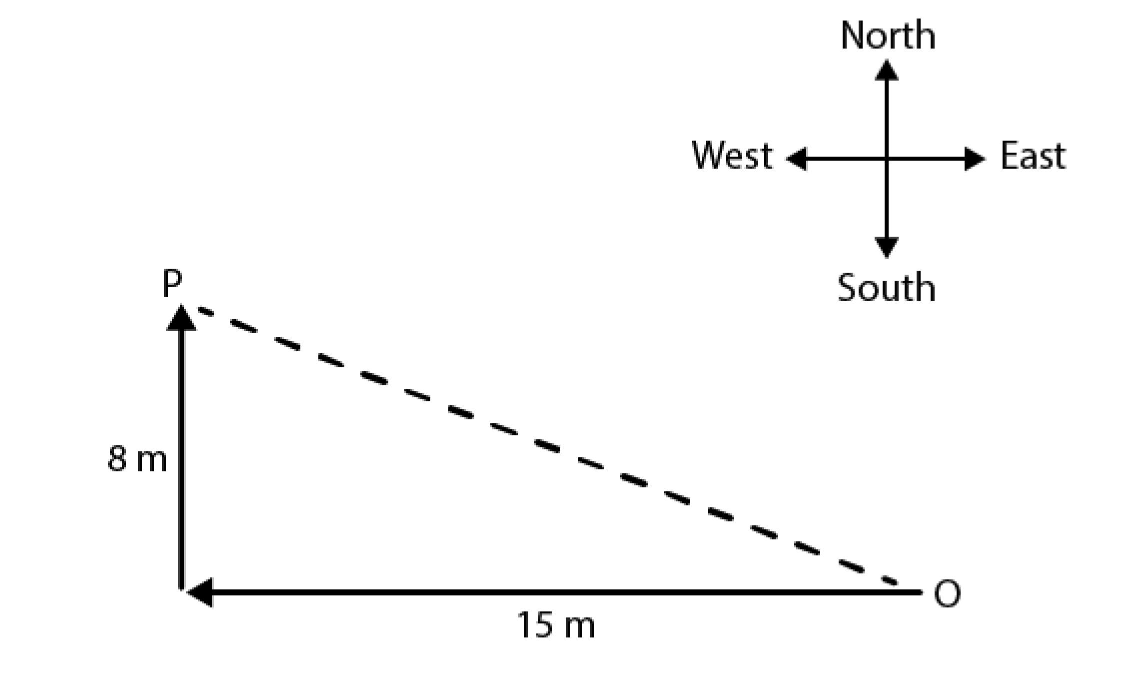
$AB^2 = 169$

$AB = 13$

Hence, the distance between the tops of the poles is 13 m.

8. A man goes 15 m due west and then 8 m due north. How far is he from the starting point?

Solution:



Given a man goes 15 m due west and then 8 m due north

Let O be the starting point and P be the final point.

Then OP becomes the hypotenuse in the triangle.

So by using the Pythagoras theorem, we can find the distance OP.

$OP^2 = 15^2 + 8^2$

$OP^2 = 225 + 64$

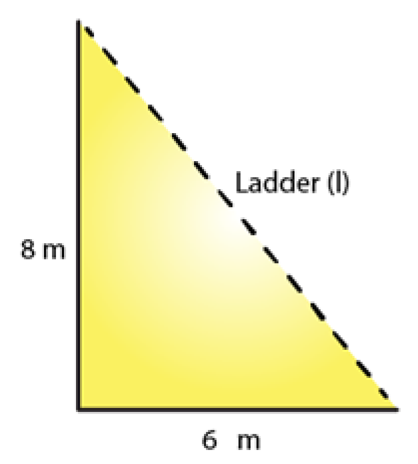
$OP^2 = 289$

OP = 17

Hence, the required distance is 17 m.

9. The foot of a ladder is 6 m away from a wall and its top reaches a window 8 m above the ground. If the ladder is shifted in such a way that its foot is 8 m away from the wall, to what height does its top reach?

Solution:



Given Let the length of the ladder be L m.

By using the Pythagoras theorem, we can find the length of the ladder.

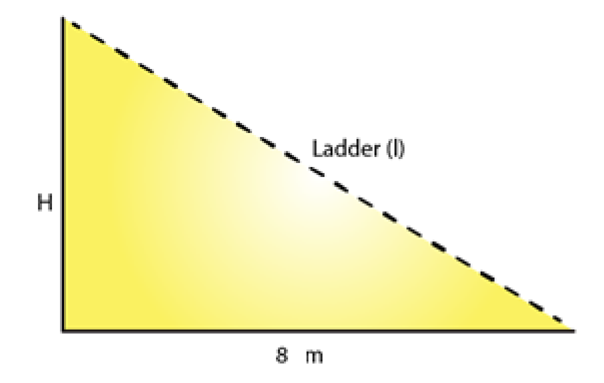
$6^2 + 8^2 = L^2$

$L^2 = 36 + 64 = 100$

L = 10

Thus, the length of the ladder is 10 m.

When ladder is shifted,



Let the height of the ladder after it is shifted be H m.

By using the Pythagoras theorem, we can find the height of the ladder after it is shifted.

$8^2 + H^2 = 10^2$

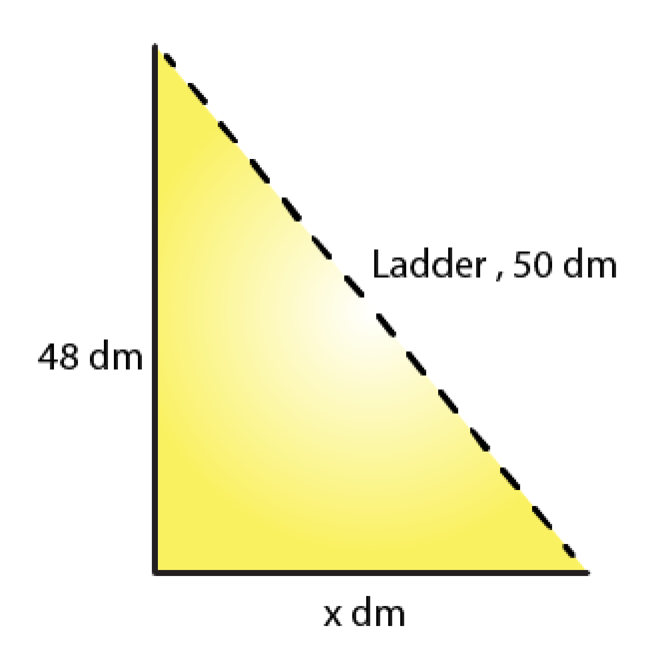
$H^2 = 100 – 64 = 36$

H = 6

Thus, the height of the ladder is 6 m.

10. A ladder 50 dm long when set against the wall of a house just reaches a window at a height of 48 dm. How far is the lower end of the ladder from the base of the wall?

Solution:



Given that length of a ladder is 50dm

Let the distance of the lower end of the ladder from the wall be x dm.

By using the Pythagoras theorem, we get

$x^2 + 48^2 = 50^2$

$x^2 = 50^2 – 48^2$

= 2500 – 2304

= 196

H = 14 dm

Hence, the distance of the lower end of the ladder from the wall is 14 dm.

11. The two legs of a right triangle are equal and the square of the hypotenuse is 50. Find the length of each leg.

Solution:

According to the Pythagoras theorem, we have

$(\text{Hypotenuse})^2 = (\text{Base})^2 + (\text{Height})^2$

Given that the two legs of a right triangle are equal and the square of the hypotenuse, which is 50

Let the length of each leg of the given triangle be x units.

Using the Pythagoras theorem, we get

$x^2 + x^2 = (\text{Hypotenuse})^2$

$x^2 + x^2 = 50$

$2x^2 = 50$

$x^2 = 25$

$x = 5$

Hence, the length of each leg is 5 units.

12. Verity that the following numbers represent Pythagorean triplet:

(i) 12, 35, 37

(ii) 7, 24, 25

(iii) 27, 36, 45

(iv) 15, 36, 39

Solution:

(i) The condition for Pythagorean triplet is the square of the largest side is equal to the sum of the squares of the other two sides.

$37^2 = 1369$

$12^2 + 35^2 = 144 + 1225 = 1369$

$12^2 + 35^2 = 37^2$

Yes, they represent a Pythagorean triplet.

(ii) The condition for Pythagorean triplet is the square of the largest side is equal to the sum of the squares of the other two sides.

$25^2 = 625$

$7^2 + 24^2 = 49 + 576 = 625$

$7^2 + 24^2 = 25^2$

Yes, they represent a Pythagorean triplet.

(iii) The condition for Pythagorean triplet is the square of the largest side is equal to the sum of the squares of the other two sides.

$45^2 = 2025$

$27^2 + 36^2 = 729 + 1296 = 2025$

$27^2 + 36^2 = 45^2$

Yes, they represent a Pythagorean triplet.

(iv) The condition for Pythagorean triplet is the square of the largest side is equal to the sum of the squares of the other two sides.

$39^2 = 1521$

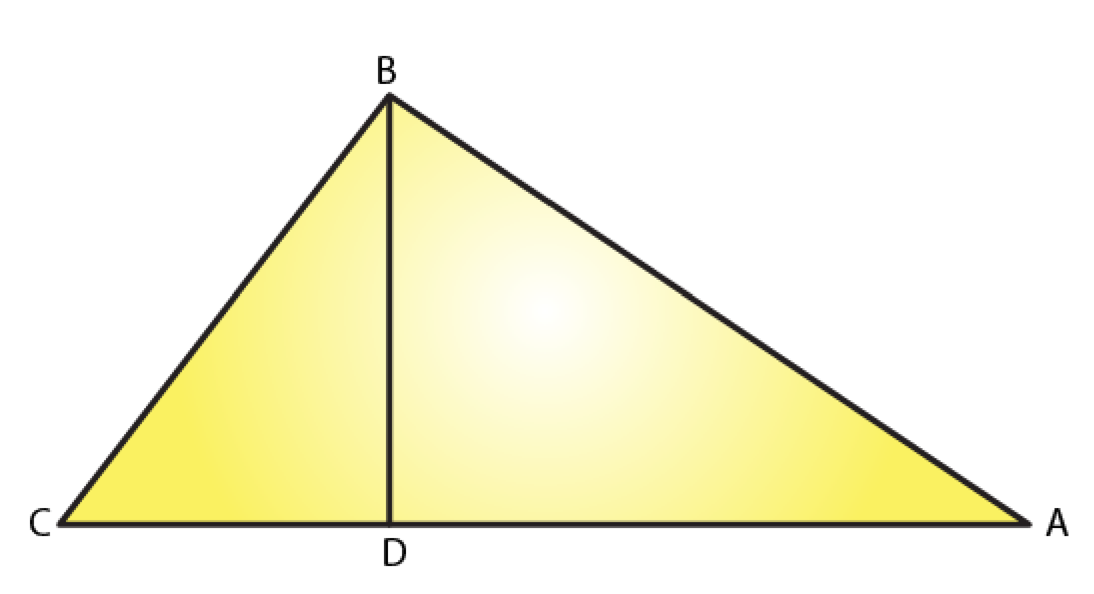
$15^2 + 36^2 = 225 + 1296 = 1521$

$15^2 + 36^2 = 39^2$

Yes, they represent a Pythagorean triplet.

13. In △ABC, ∠ABC = 100°, ∠BAC = 35° and BD ⊥ AC meets side AC in D. If BD = 2 cm, find ∠C, and length DC.

Solution:



We know that the sum of all angles of a triangle is 180°

Therefore, for the given △ABC, we can say that:

∠ABC + ∠BAC + ∠ACB = 180°

100° + 35° + ∠ACB = 180°

∠ACB = 180° –135°

∠ACB = 45°

∠C = 45°

On applying same steps for the △BCD, we get

∠BCD + ∠BDC + ∠CBD = 180°

45° + 90° + ∠CBD = 180° (∠ACB = ∠BCD and BD is perpendicular to AC)

∠CBD = 180° – 135°

∠CBD = 45°

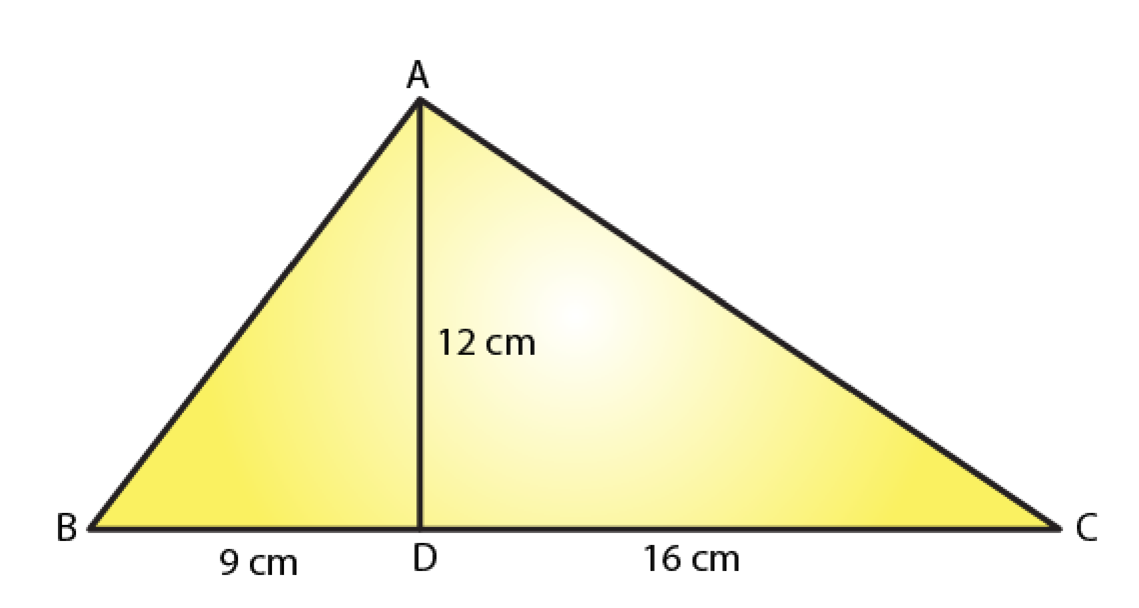
We know that the sides opposite to equal angles have equal length.

Thus, BD = DC

DC = 2 cm

14. In a △ABC, AD is the altitude from A such that AD = 12 cm. BD = 9 cm and DC = 16 cm. Examine if △ABC is right angled at A.

Solution:



Consider $\triangle ADC$,

$\angle ADC = 90^o$ (AD is an altitude on BC)

Using the Pythagoras theorem, we get

$12^2 + 16^2 = AC^2$

$AC^2 = 144 + 256$

$= 400$

AC = 20 cm

Again consider $\triangle ADB$,

$\angle ADB = 90^o$ (AD is an altitude on BC)

Using the Pythagoras theorem, we get

$12^2 + 9^2 = AB^2$

$AB^2 = 144 + 81 = 225$

AB = 15 cm

Consider $\triangle ABC$,

$BC^2 = 25^2 = 625$

$AB^2 + AC^2 = 15^2 + 20^2 = 625$

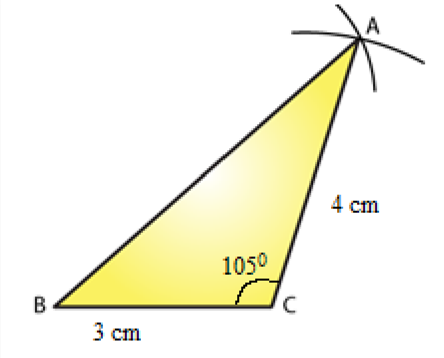
$AB^2 + AC^2 = BC^2$

Because it satisfies the Pythagoras theorem, therefore $\triangle ABC$ is right angled at A.

15. Draw a triangle ABC, with AC = 4 cm, BC = 3 cm and $\angle C = 105^o$. Measure AB. Is $(AB)^2 = (AC)^2 + (BC)^2$? If not which one of the following is true:

$(AB)^2 > (AC)^2 + (BC)^2$ or $(AB)^2 < (AC)^2 + (BC)^2$?

Solution:



Draw △ABC as shown in the figure with following steps.

Draw a line BC = 3 cm.

At point C, draw a line at 105° angle with BC.

Take an arc of 4 cm from point C, which will cut the line at point A.

Now, join AB, which will be approximately 5.5 cm.

$AC^2 + BC^2 = 4^2 + 3^2$

$= 9 + 16$

$= 25$

$AB^2 = 5.5^2 = 30.25$

AB^2 is not equal to $AC^2 + BC^2$

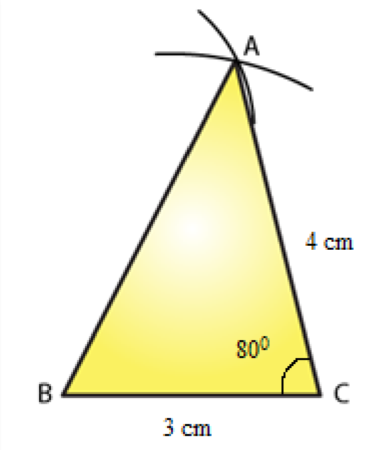
Therefore we have

$AB^2 > AC^2 + BC^2$

16. Draw a triangle ABC, with AC = 4 cm, BC = 3 cm and ∠C = 80°. Measure AB. Is $(AB)^2 = (AC)^2 + (BC)^2$? If not which one of the following is true:

$(AB)^2 > (AC)^2 + (BC)^2$ or $(AB)^2 < (AC)^2 + (BC)^2$?

Solution:



Draw △ABC as shown in the figure with following steps.

Draw a line BC = 3 cm.

At point C, draw a line at 80° angle with BC.

Take an arc of 4 cm from point C, which will cut the line at point A.

Now, join AB, it will be approximately 4.5 cm.

$AC^2 + BC^2 = 4^2 + 3^2$

$= 9 + 16$

$= 25$

$AB^2 = (4.5)^2$

$= 20.25$

AB^2 not equal to $AC^2 + BC^2$

Therefore here $AB^2 < AC^2 + BC^2$

Chapter 16 – Congruence

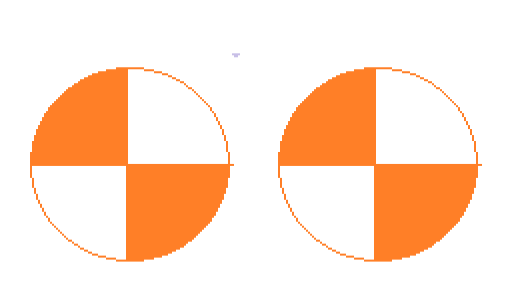
Exercise 16.1

1. Explain the concept of congruence of figures with the help of certain examples.

Solution:

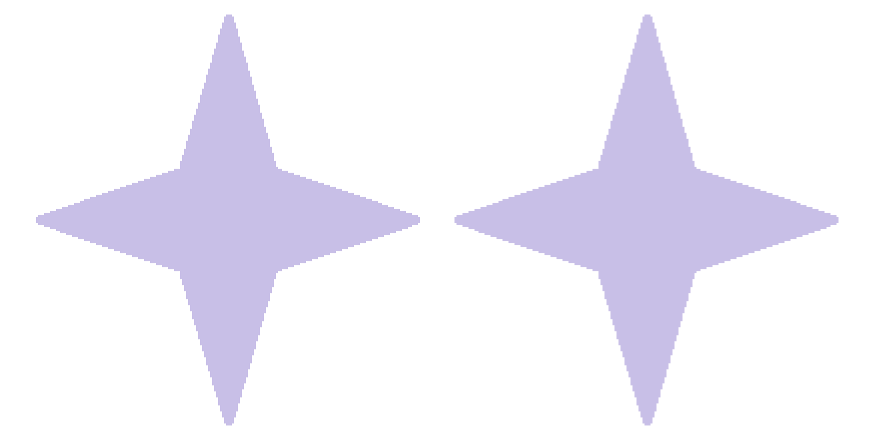
Congruent objects or figures are exact copies of each other or we can say mirror images of each other. The relation of two objects being congruent is called congruence.

Consider Ball 1 and Ball 2. These two balls are congruent.



Ball 1 Ball 2

Now consider the two stars below. Star A and Star B are exactly the same in size, colour and shape. These are congruent stars



Star A Star B

2. Fill in the blanks:

(i) Two line segments are congruent if ……..

(ii) Two angles are congruent if ……..

(iii) Two square are congruent if ………

(iv) Two rectangles are congruent if ………

(v) Two circles are congruent if …….

Solution:

(i) They are of equal lengths

(ii) Their measures are the same or equal.

(iii) Their sides are equal or they have the same side length

(iv) Their dimensions are same that is lengths are equal and their breadths are also equal.

(v) They have same radii

3. In Fig. 6, $\angle POQ \cong \angle ROS$, can we say that $\angle POR \cong \angle QOS$

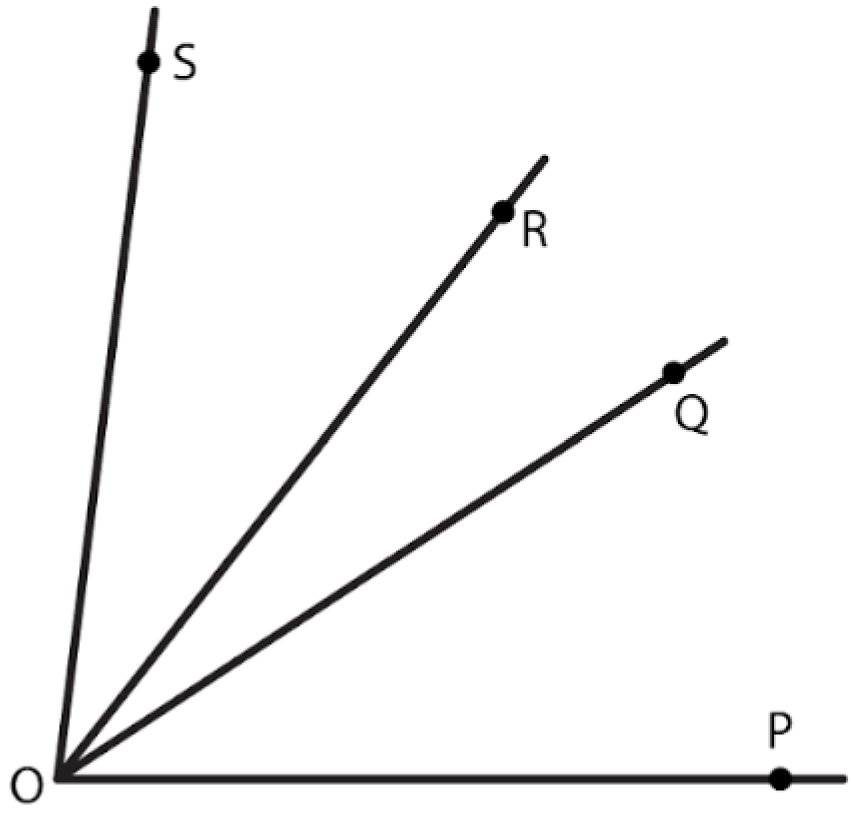


Fig. 6

Solution:

Given that

∠POQ ≅ ∠ROS

Also ∠ROQ ≅ ∠ROQ

Therefore adding ∠ROQ to both sides of ∠POQ ≅ ∠ROS,

We get, ∠POQ + ∠ROQ ≅ ∠ROQ + ∠ROS

Therefore, ∠POR ≅ ∠QOS

4. In fig. 7, a = b = c, name the angle which is congruent to ∠AOC

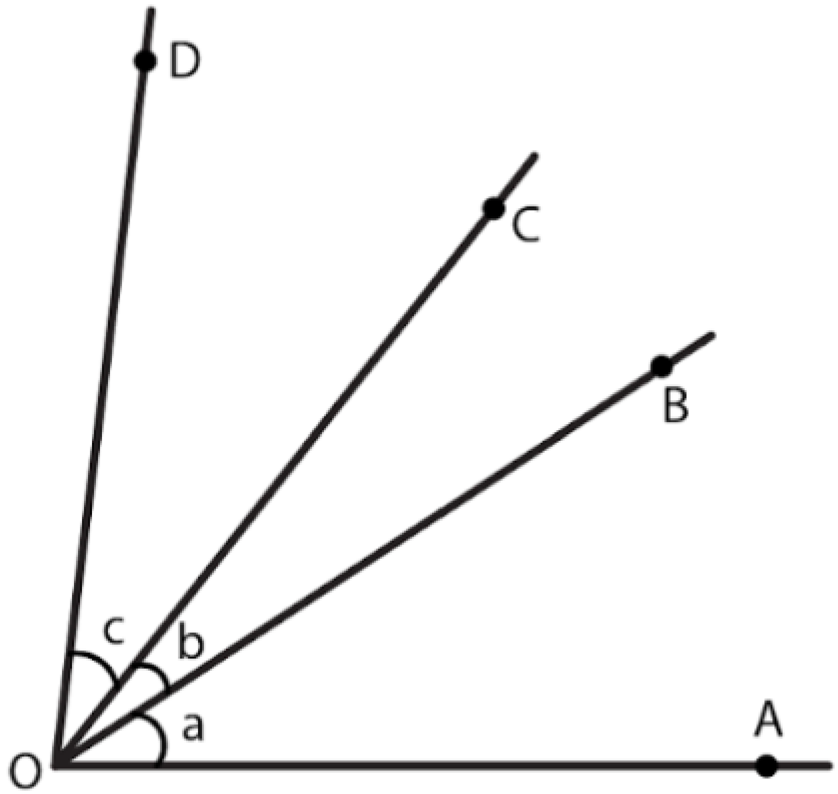


Fig. 7

Solution:

From the figure we have

∠ AOB = ∠ BOC = ∠ COD

Therefore, ∠ AOB = ∠COD

Also, ∠ AOB + ∠ BOC = ∠ BOC + ∠ COD

∠ AOC = ∠ BOD

Hence, ∠ BOD ≅ ∠ AOC

5. Is it correct to say that any two right angles are congruent? Give reasons to justify your answer.

Solution:

Two right angles are congruent to each other because they both measure 90^o.

We know that two angles are congruent if they have the same measure.

6. In fig. 8, ∠AOC ≅∠PYR and ∠BOC ≅∠QYR. Name the angle which is congruent to ∠AOB.

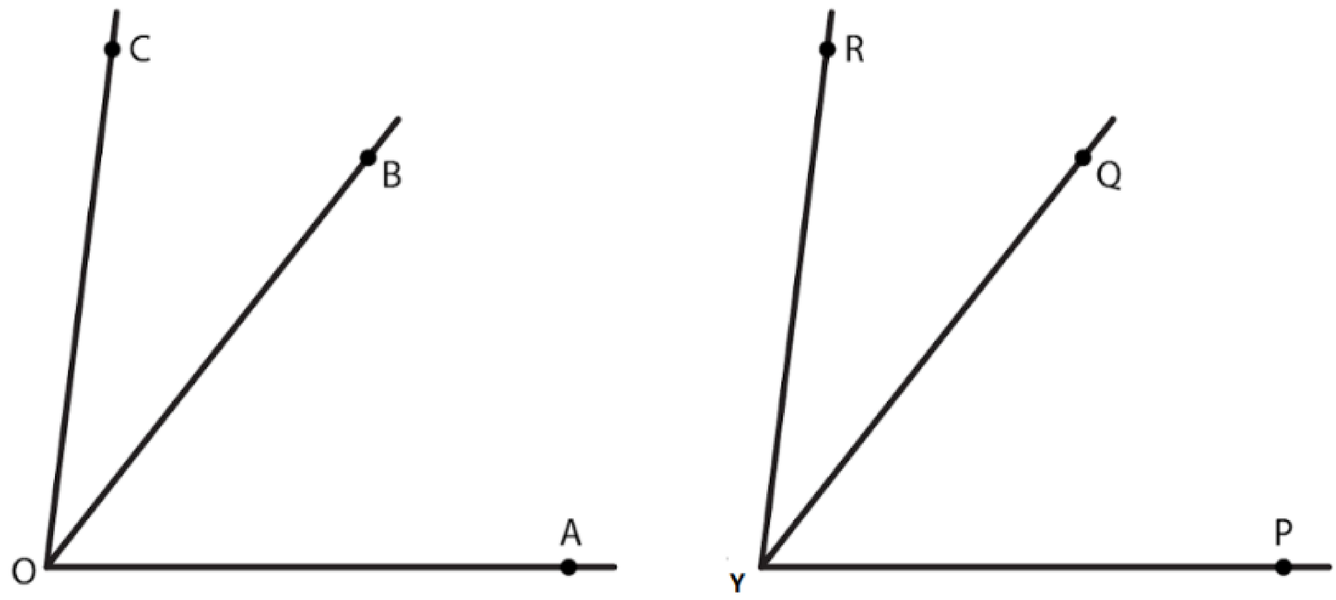


Fig. 8

Solution:

Given that ∠AOC ≅ ∠PYR

Also given that ∠BOC ≅ ∠QYR

Now, ∠AOC = ∠AOB + ∠BOC and ∠PYR = ∠PYQ + ∠QYR

By putting the value of ∠AOC and ∠PYR in ∠AOC ≅ ∠PYR

We get, ∠AOB + ∠BOC ≅ ∠PYQ + ∠QYR

∠AOB ≅ ∠PYQ (∠BOC ≅ ∠QYR)

Hence, ∠AOB ≅ ∠PYQ

7. Which of the following statements are true and which are false;

(i) All squares are congruent.

(ii) If two squares have equal areas, they are congruent.

(iii) If two rectangles have equal areas, they are congruent.

(iv) If two triangles have equal areas, they are congruent.

Solution:

(i) False.

Explanation:

All the sides of a square are of equal length. However, different squares can have sides of different lengths. Hence all squares are not congruent.

(ii) True.

Explanation:

Two squares that have the same area will have sides of the same lengths. Hence they will be congruent.

(iii) False

Explanation:

Area of a rectangle = length x breadth

Two rectangles can have the same area. However, the lengths of their sides can vary and hence they are not congruent.

(iv) False

Explanation:

Area of a triangle = 1/2 x base x height

Two triangles can have the same area but the lengths of their sides can vary and hence they cannot be congruent.

Exercise 16.2

1. In the following pairs of triangle (Fig. 12 to 15), the lengths of the sides are indicated along sides. By applying SSS condition, determine which are congruent. State the result in symbolic form.

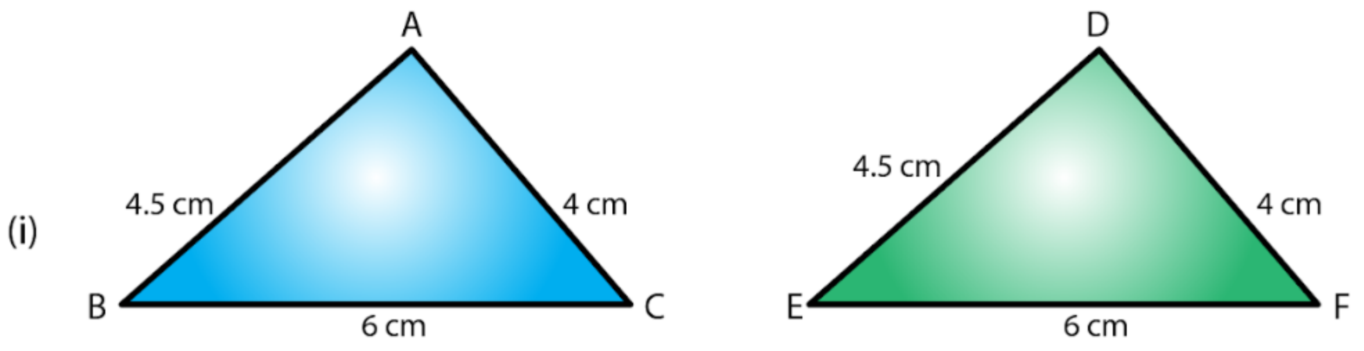


Fig. 12

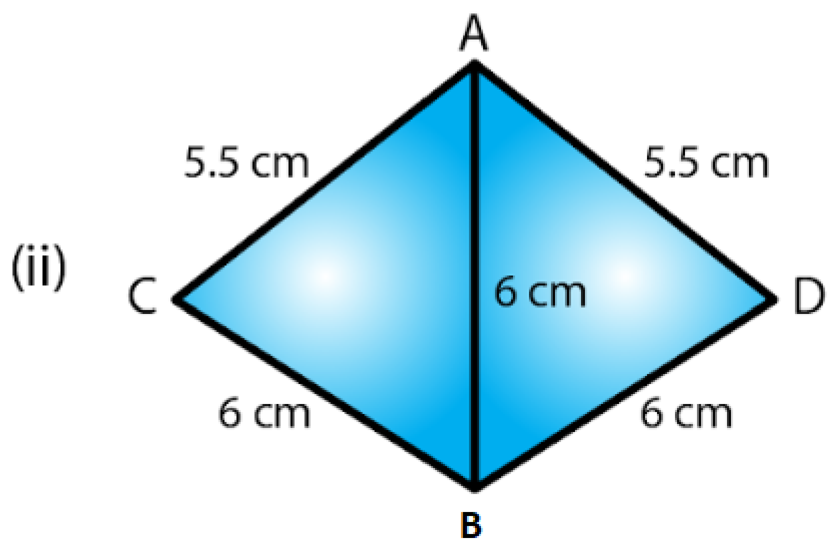


Fig. 13

(iii)

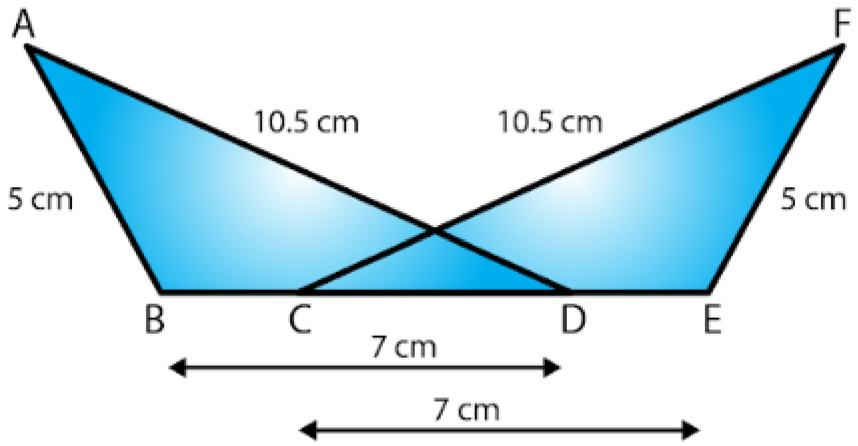


Fig. 14

(iv)

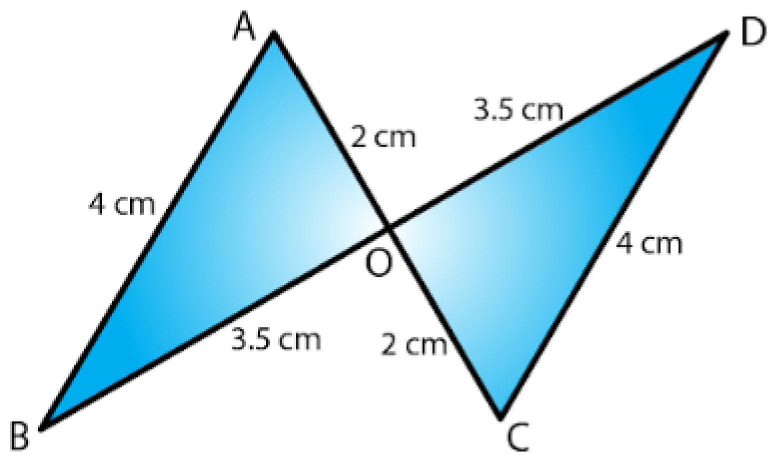


Fig. 15

Solution:

(i) In Δ ABC and Δ DEF

AB = DE = 4.5 cm (Side)

BC = EF = 6 cm (Side) and

AC = DF = 4 cm (Side)

SSS criterion is two triangles are congruent, if the three sides of triangle are respectively equal to the three sides of the other triangle.

Therefore, by SSS criterion of congruence, ΔABC ≅ ΔDEF

(ii) In Δ ACB and Δ ADB

AC = AD = 5.5cm (Side)

BC = BD = 5cm (Side) and

AB = AB = 6cm (Side)

SSS criterion is two triangles are congruent, if the three sides of triangle are respectively equal to the three sides of the other triangle.

Therefore, by SSS criterion of congruence, ΔACB ≅ ΔADB

(iii) In Δ ABD and Δ FEC,

AB = FE = 5cm (Side)

AD = FC = 10.5cm (Side)

BD = CE = 7cm (Side)

SSS criterion is two triangles are congruent, if the three sides of triangle are respectively equal to the three sides of the other triangle.

Therefore, by SSS criterion of congruence, ΔABD ≅ ΔFEC

(iv) In Δ ABO and Δ DOC,

AB = DC = 4cm (Side)

AO = OC = 2cm (Side)

BO = OD = 3.5cm (Side)

SSS criterion is two triangles are congruent, if the three sides of triangle are respectively equal to the three sides of the other triangle.

Therefore, by SSS criterion of congruence, ΔABO ≅ ΔODC

2. In fig.16, AD = DC and AB = BC

(i) Is ΔABD ≅ ΔCBD?

(ii) State the three parts of matching pairs you have used to answer (i).

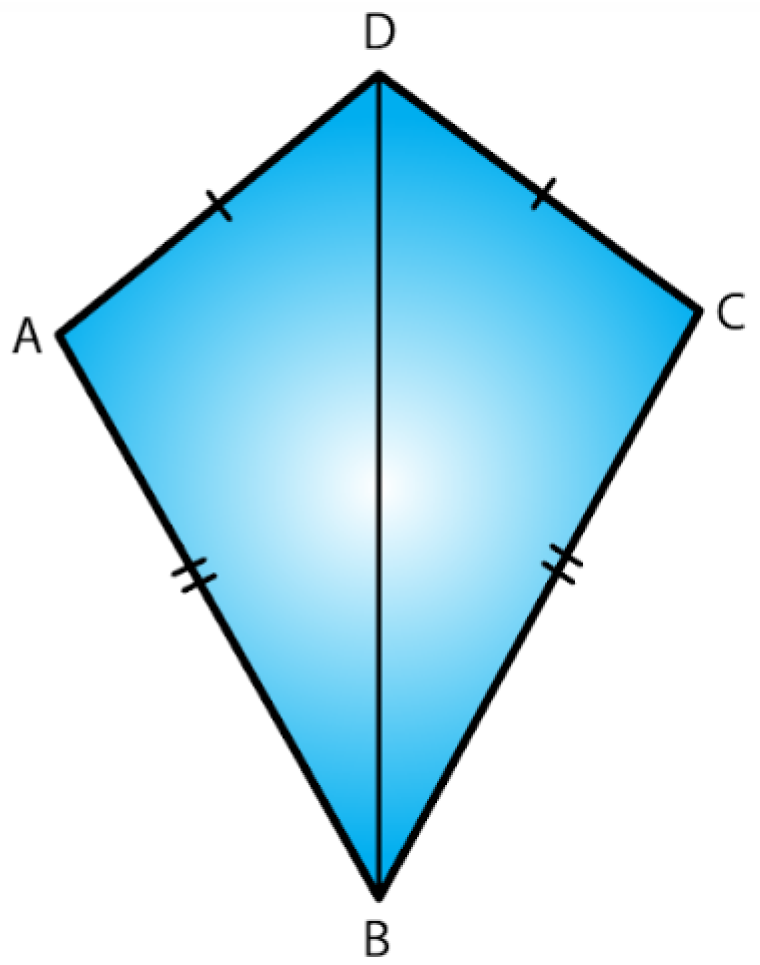


Fig. 16

Solution:

(i) Yes ΔABD ≅ΔCBD by the SSS criterion.

SSS criterion is two triangles are congruent, if the three sides of triangle are respectively equal to the three sides of the other triangle.

Hence ΔABD ≅ΔCBD

(ii) We have used the three conditions in the SSS criterion as follows:

AD = DC

AB = BC and

DB = BD

3. In Fig. 17, AB = DC and BC = AD.

(i) Is ΔABC ≅ ΔCDA?

(ii) What congruence condition have you used?

(iii) You have used some fact, not given in the question, what is that?

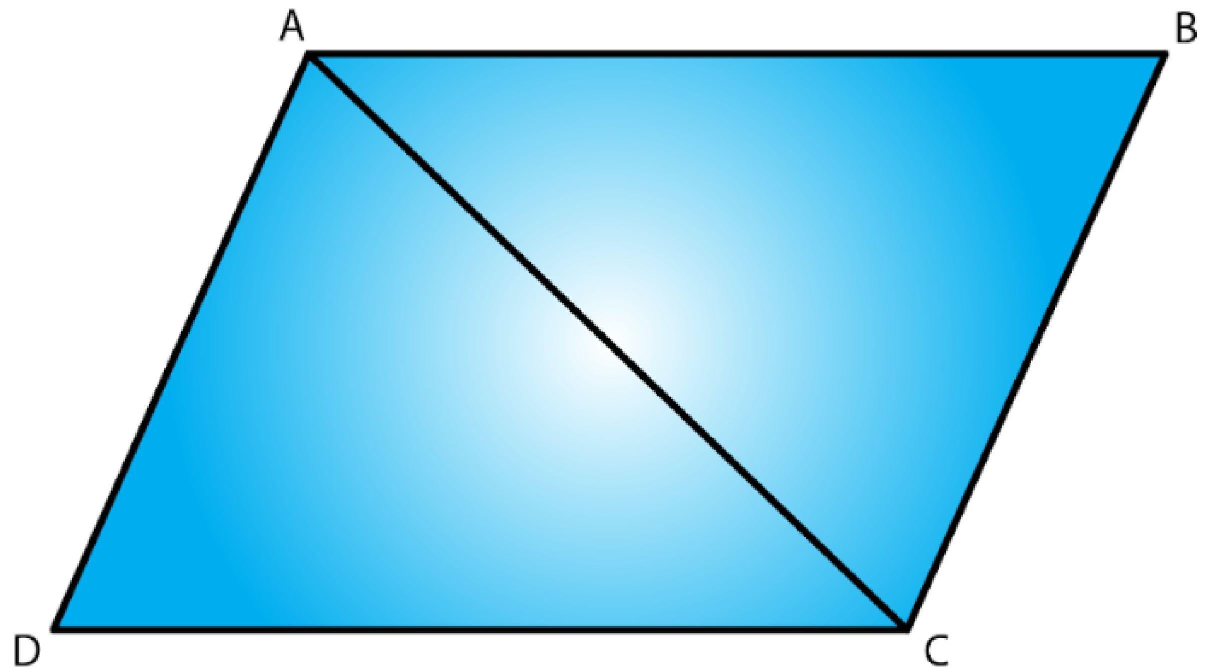


Fig. 17

Solution:

(i) From the figure we have AB = DC

BC = AD

And AC = CA

SSS criterion is two triangles are congruent, if the three sides of triangle are respectively equal to the three sides of the other triangle.

Therefore by SSS criterion ΔABC ≅ ΔCDA

(ii) We have used Side Side Side congruence condition with one side common in both the triangles.

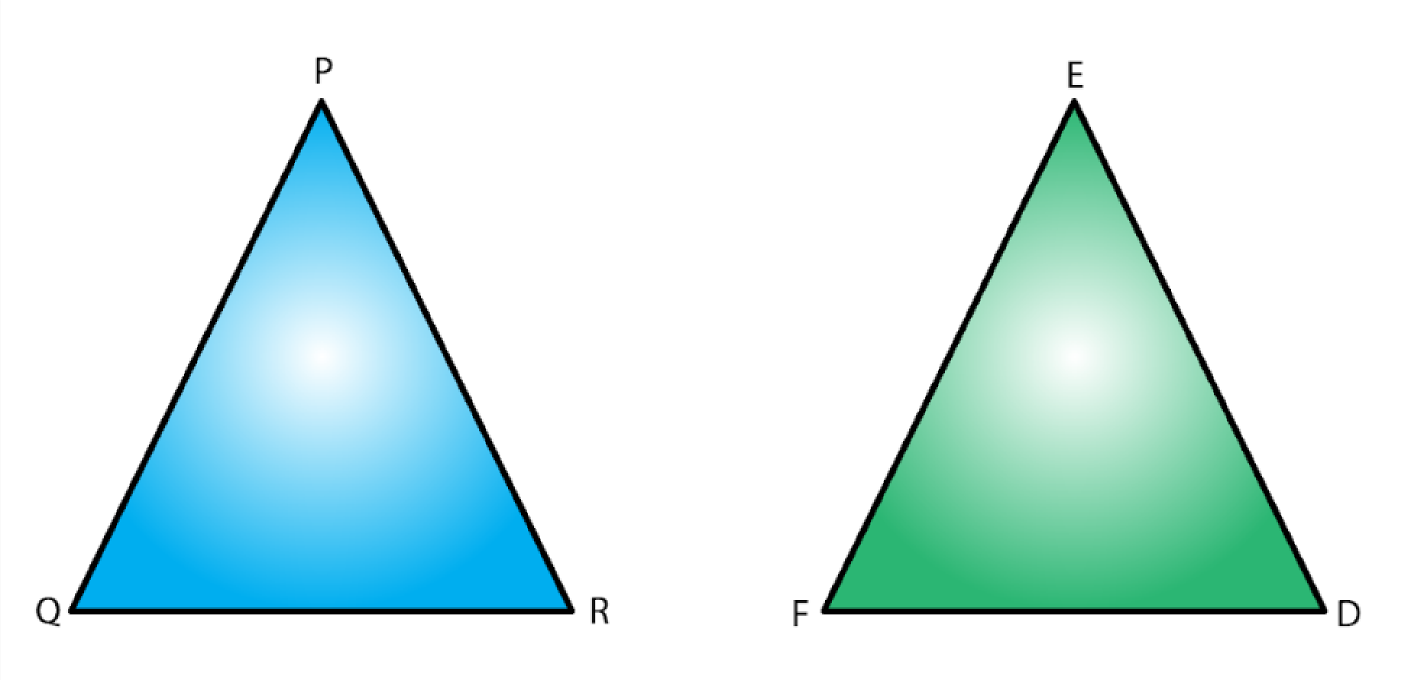
(iii)Yes, we have used the fact that AC = CA.

4. In ΔPQR ≅ ΔEFD,

(i) Which side of ΔPQR equals ED?

(ii) Which angle of ΔPQR equals angle E?

Solution:



(i) PR = ED

Since the corresponding sides of congruent triangles are equal.

(ii) ∠QPR = ∠FED

Since the corresponding angles of congruent triangles are equal.

5. Triangles ABC and PQR are both isosceles with AB = AC and PQ = PR respectively. If also, AB = PQ and BC = QR, are the two triangles congruent? Which condition do you use?

It ∠B = 50°, what is the measure of ∠R?

Solution:

Given that AB = AC in isosceles ΔABC

And PQ = PR in isosceles ΔPQR.

Also given that AB = PQ and QR = BC.

Therefore, AC = PR (AB = AC, PQ = PR and AB = PQ)

Hence, ΔABC ≅ ΔPQR

Now

∠ABC = ∠PQR (Since triangles are congruent)

However, ΔPQR is isosceles.

Therefore, ∠PRQ = ∠PQR = ∠ABC = 50^o

6. ABC and DBC are both isosceles triangles on a common base BC such that A and D lie on the same side of BC. Are triangles ADB and ADC congruent? Which condition do you use? If ∠BAC = 40° and ∠BDC = 100°, then find ∠ADB.

Solution:

Given ABC and DBC are both isosceles triangles on a common base BC

∠BAD = ∠CAD (corresponding parts of congruent triangles)

∠BAD + ∠CAD = 40^o/ 2

∠BAD = $40^o/2 = 20^o$

∠ABC + ∠BCA + ∠BAC = 180^o (Angle sum property)

Since ΔABC is an isosceles triangle,

$\angle ABC = \angle BCA$

$\angle ABC + \angle ABC + 40^o = 180^o$

$2\ \angle ABC = 180^o - 40^o = 140^o$

$\angle ABC = 140^o/2 = 70^o$

$\angle DBC + \angle BCD + \angle BDC = 180^o$ (Angle sum property)

Since ΔDBC is an isosceles triangle, $\angle DBC = \angle BCD$

$\angle DBC + \angle DBC + 100^o = 180^o$

$2\ \angle DBC = 180^\circ - 100^o = 80^o$

$\angle DBC = 80^o/2 = 40^o$

In Δ BAD,

$\angle ABD + \angle BAD + \angle ADB = 180^o$ (Angle sum property)

$30^o + 20^o + \angle ADB = 180^o$ ($\angle ADB = \angle ABC - \angle DBC$),

$\angle ADB = 180^o - 20^o - 30^o$

$\angle ADB = 130^o$

7. Δ ABC and ΔABD are on a common base AB, and AC = BD and BC = AD as shown in Fig. 18. Which of the following statements is true?

(i) ΔABC ≅ ΔABD

(ii) ΔABC ≅ ΔADB

(iii) ΔABC ≅ ΔBAD

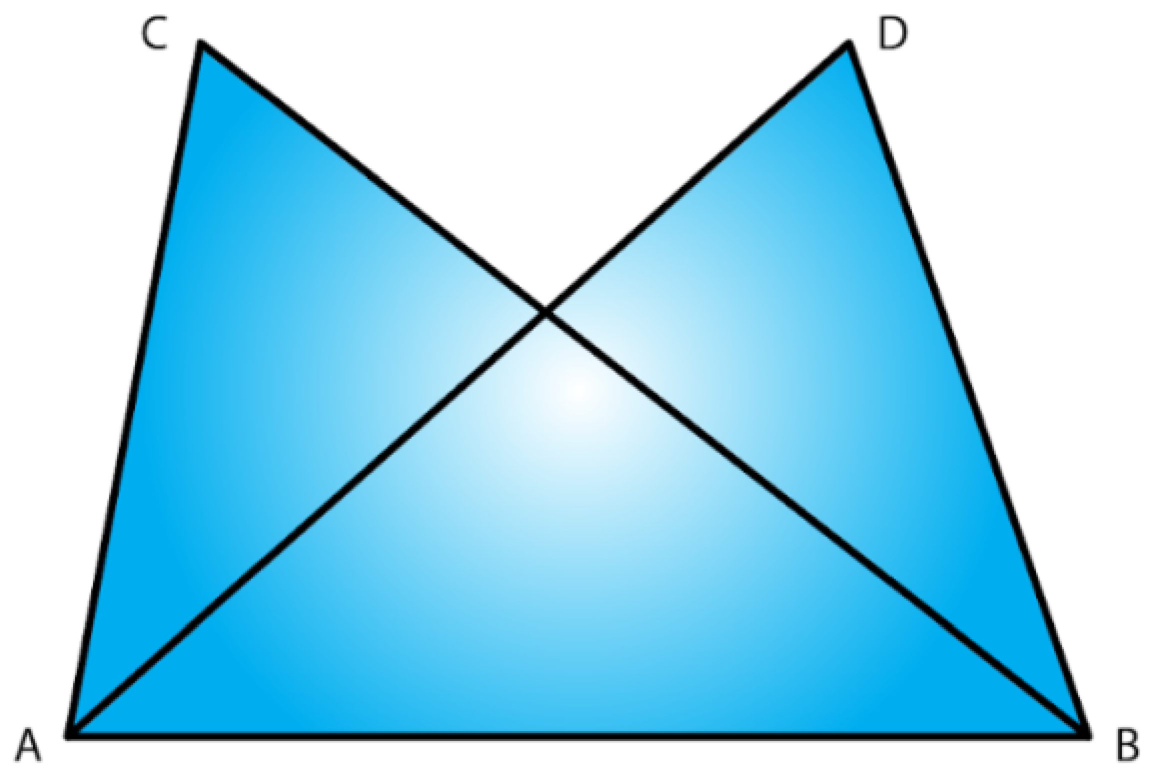


Fig. 18

Solution:

In ΔABC and ΔBAD we have,

AC = BD (given)

BC = AD (given)

And AB = BA (corresponding parts of congruent triangles)

Therefore by SSS criterion of congruency, ΔABC ≅ ΔBAD

Therefore option (iii) is true.

8. In Fig. 19, ΔABC is isosceles with AB = AC, D is the mid-point of base BC.

(i) Is ΔADB ≅ ΔADC?

(ii) State the three pairs of matching parts you use to arrive at your answer.

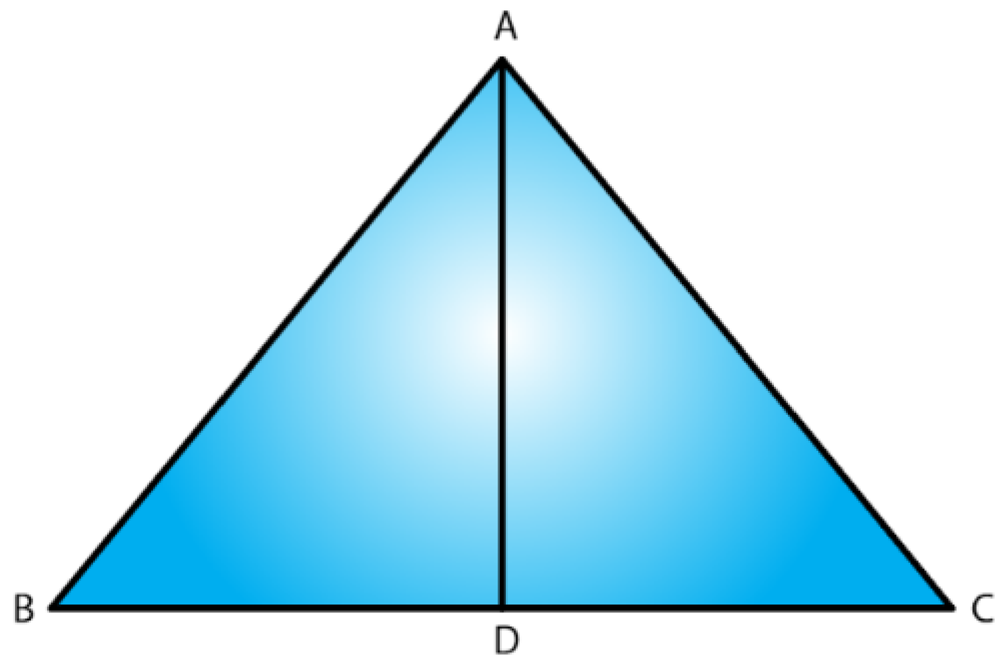


Fig. 19

Solution:

(i) Given that AB = AC.

Also since D is the midpoint of BC, BD = DC

Also, AD = DA

Therefore by SSS condition,

ΔADB ≅ ΔADC

(ii)We have used AB, AC; BD, DC and AD, DA

9. In fig. 20, ΔABC is isosceles with AB = AC. State if ΔABC ≅ ΔACB. If yes, state three relations that you use to arrive at your answer.

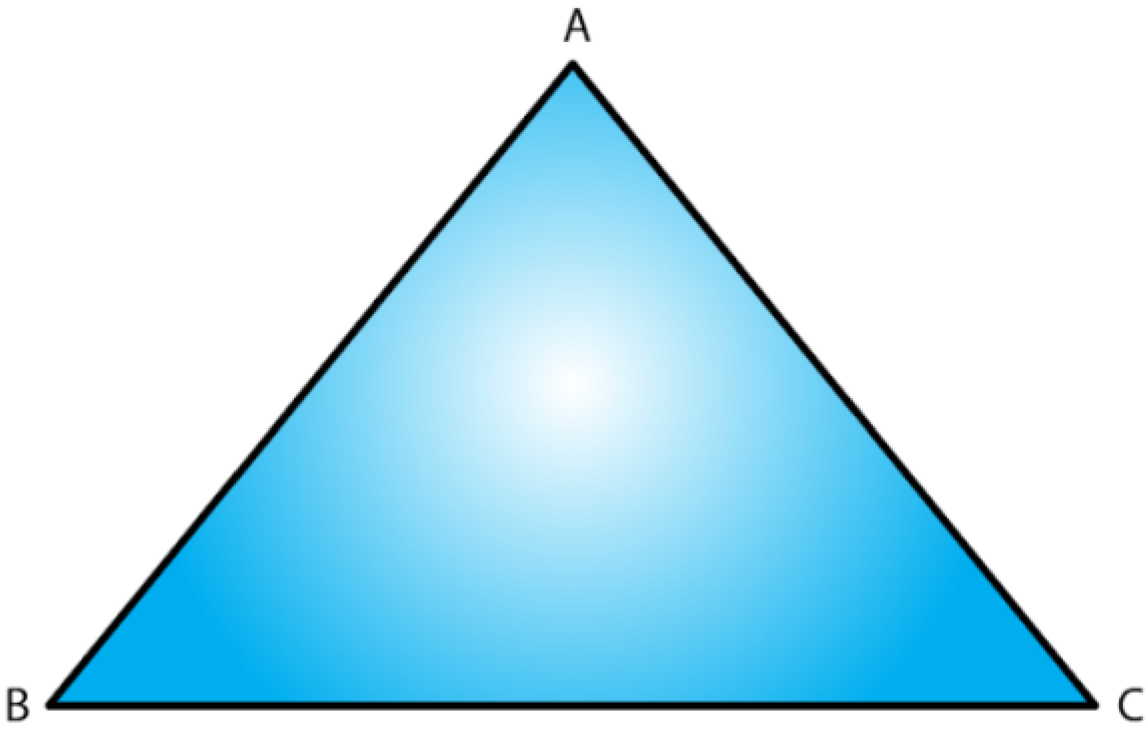


Fig. 20

Solution:

Given that ΔABC is isosceles with AB = AC

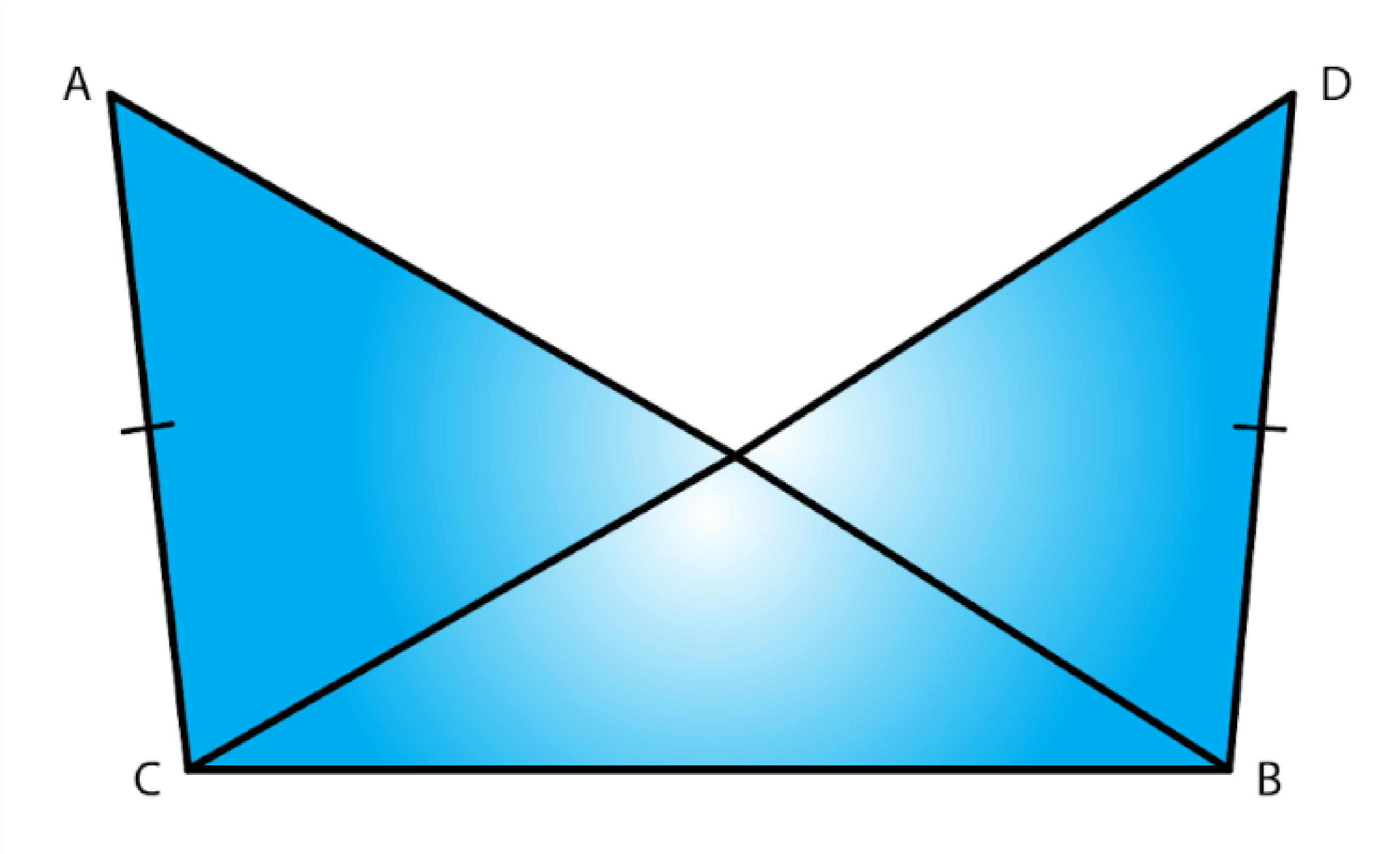
SSS criterion is two triangles are congruent, if the three sides of triangle are respectively equal to the three sides of the other triangle.

ΔABC ≅ ΔACBby SSS condition.

Since, ABC is an isosceles triangle, AB = AC and BC = CB

10. Triangles ABC and DBC have side BC common, AB = BD and AC = CD. Are the two triangles congruent? State in symbolic form, which congruence do you use? Does ∠ABD equal ∠ACD? Why or why not?

Solution:



Yes, the two triangles are congruent because given that ABC and DBC have side BC common, AB = BD and AC = CD

Also from the above data we can say

By SSS criterion of congruency, ΔABC ≅ ΔDBC

No, ∠ABD and ∠ACD are not equal because AB not equal to AC

Exercise 16.3

1. By applying SAS congruence condition, state which of the following pairs (Fig. 28) of triangle are congruent. State the result in symbolic form

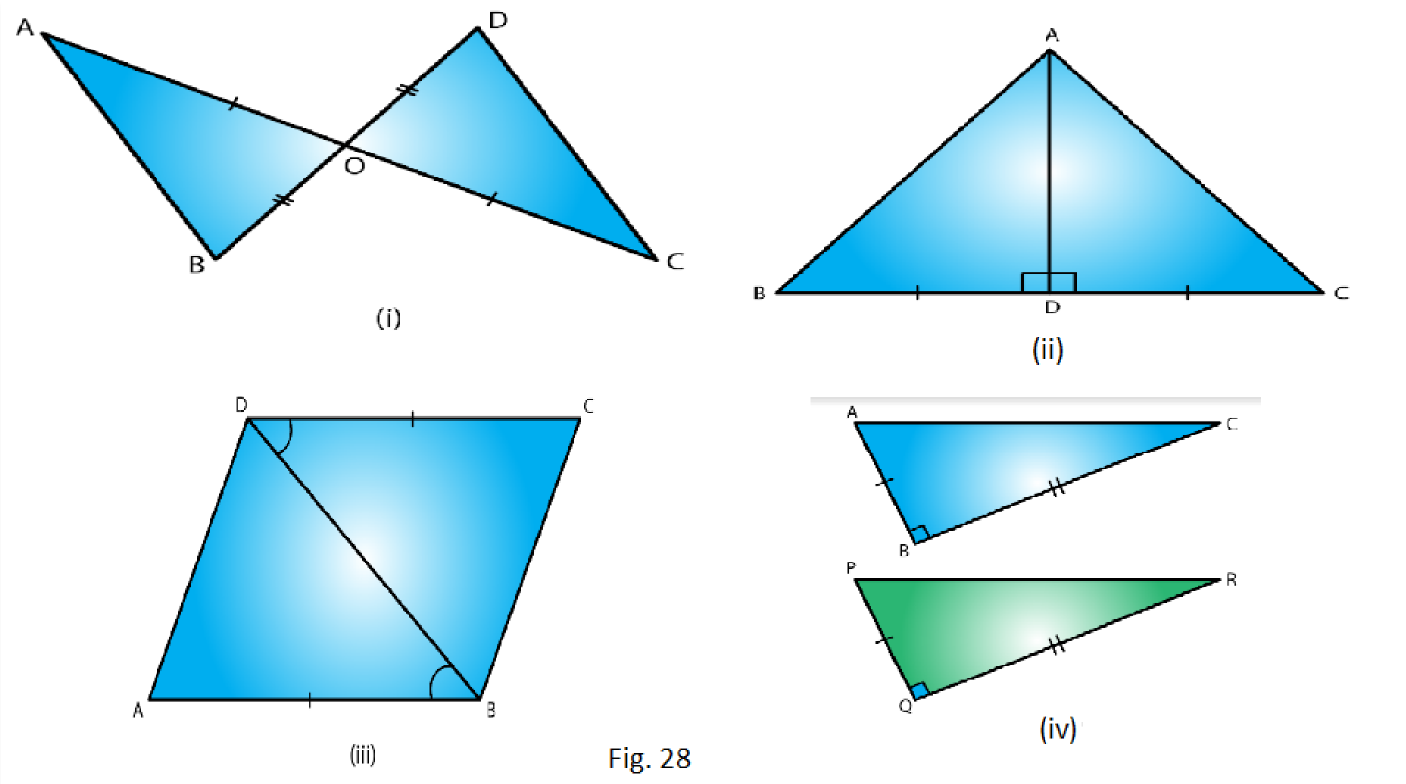


Fig. 28

Solution:

(i) From the figure we have OA = OC and OB = OD and

∠AOB = ∠COD which are vertically opposite angles.

Therefore by SAS condition, ΔAOB ≅ΔCOD

(ii) From the figure we have BD = DC

∠ADB = ∠ADC = 90^o and AD = DA

Therefore, by SAS condition, ΔADB ≅ΔADC.

(iii) From the figure we have AB = DC

∠ABD = ∠CDB and BD = DB

Therefore, by SAS condition, ΔABD ≅ΔCBD

(iv) We have BC = QR

ABC = PQR = 90^o

And AB = PQ

Therefore, by SAS condition, ΔABC≅ ΔPQR.

2. State the condition by which the following pairs of triangles are congruent.

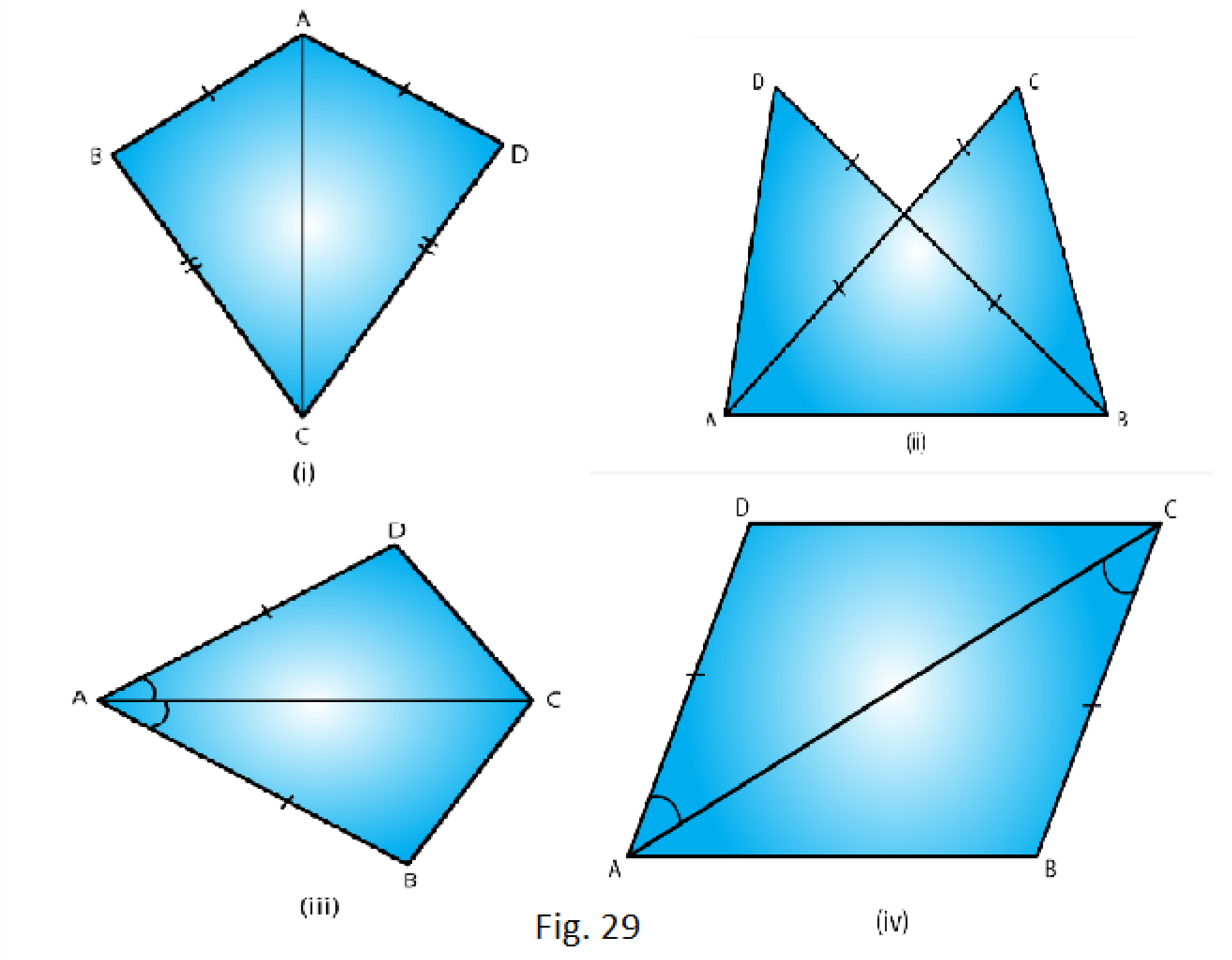


Fig. 29

Solution:

(i) AB = AD

BC = CD and AC = CA

Therefore by SSS condition, ΔABC≅ ΔADC

(ii) AC = BD

AD = BC and AB = BA

Therefore, by SSS condition, $\Delta ABD \cong \Delta BAC$

(iii) AB = AD

$\angle BAC = \angle DAC$ and AC = CA

Therefore by SAS condition, $\Delta BAC \cong \Delta DAC$

(iv) AD = BC

$\angle DAC = \angle BCA$ and AC = CA

Therefore, by SAS condition, $\Delta ABC \cong \Delta ADC$

3. In fig. 30, line segments AB and CD bisect each other at O. Which of the following statements is true?

(i) $\Delta AOC \cong \Delta DOB$

(ii) $\Delta AOC \cong \Delta BOD$

(iii) $\Delta AOC \cong \Delta ODB$

State the three pairs of matching parts, you have used to arrive at the answer.

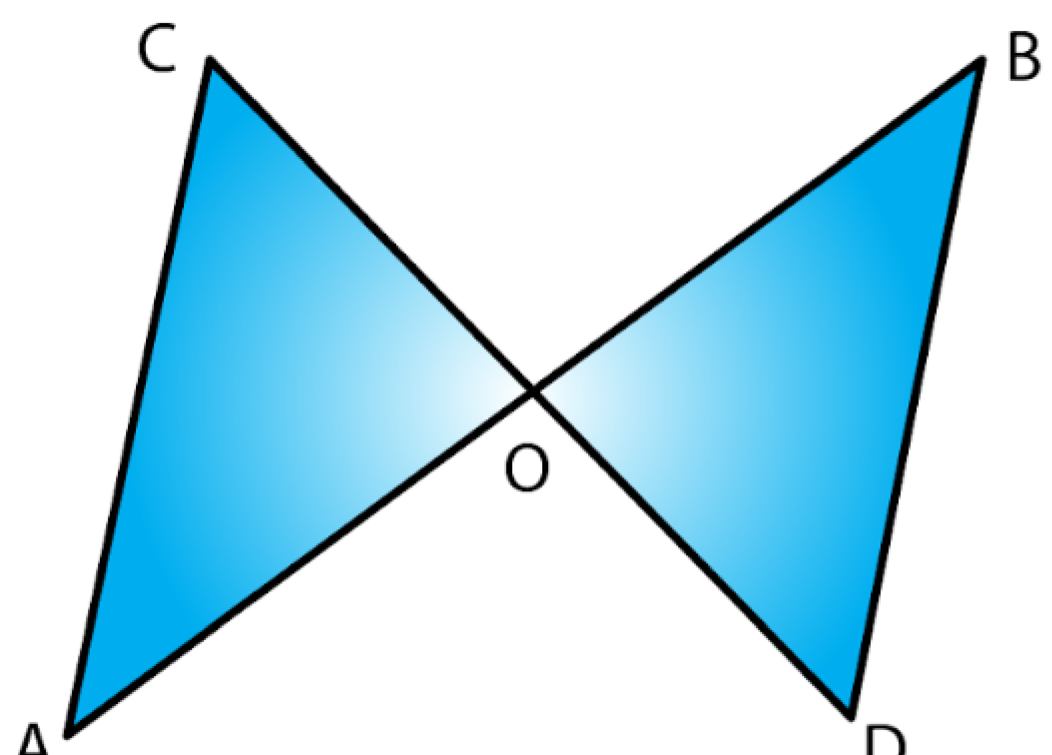


Fig. 30

Solution:

From the figure we have,

AO = OB

And, CO = OD

Also, AOC = BOD

Therefore, by SAS condition, $\Delta AOC \cong \Delta BOD$

Hence, (ii) statement is true.

4. Line-segments AB and CD bisect each other at O. AC and BD are joined forming triangles AOC and BOD. State the three equality relations between the parts of the two triangles that are given or otherwise known. Are the two triangles congruent? State in symbolic form, which congruence condition do you use?

Solution:

We have AO = OB and CO = OD

Since AB and CD bisect each other at 0.

Also ∠AOC = ∠BOD

Since they are opposite angles on the same vertex.

Therefore by SAS congruence condition, ΔAOC ≅ ΔBOD

5. ΔABC is isosceles with AB = AC. Line segment AD bisects ∠A and meets the base BC in D.

(i) Is ΔADB ≅ ΔADC?

(ii) State the three pairs of matching parts used to answer (i).

(iii) Is it true to say that BD = DC?

Solution:

(i) We have AB = AC (Given)

∠BAD = ∠CAD (AD bisects ∠BAC)

Therefore by SAS condition of congruence, ΔADB ≅ ΔADC

(ii) We have used AB, AC; ∠BAD = ∠CAD; AD, DA.

(iii) Now, ΔADB≅ΔADC

Therefore by corresponding parts of congruent triangles

BD = DC.

6. In Fig. 31, AB = AD and ∠BAC = ∠DAC.

(i) State in symbolic form the congruence of two triangles ABC and ADC that is true.

(ii) Complete each of the following, so as to make it true:

(a) ∠ABC =

(b) ∠ACD =

(c) Line segment AC bisects ….. And ……..

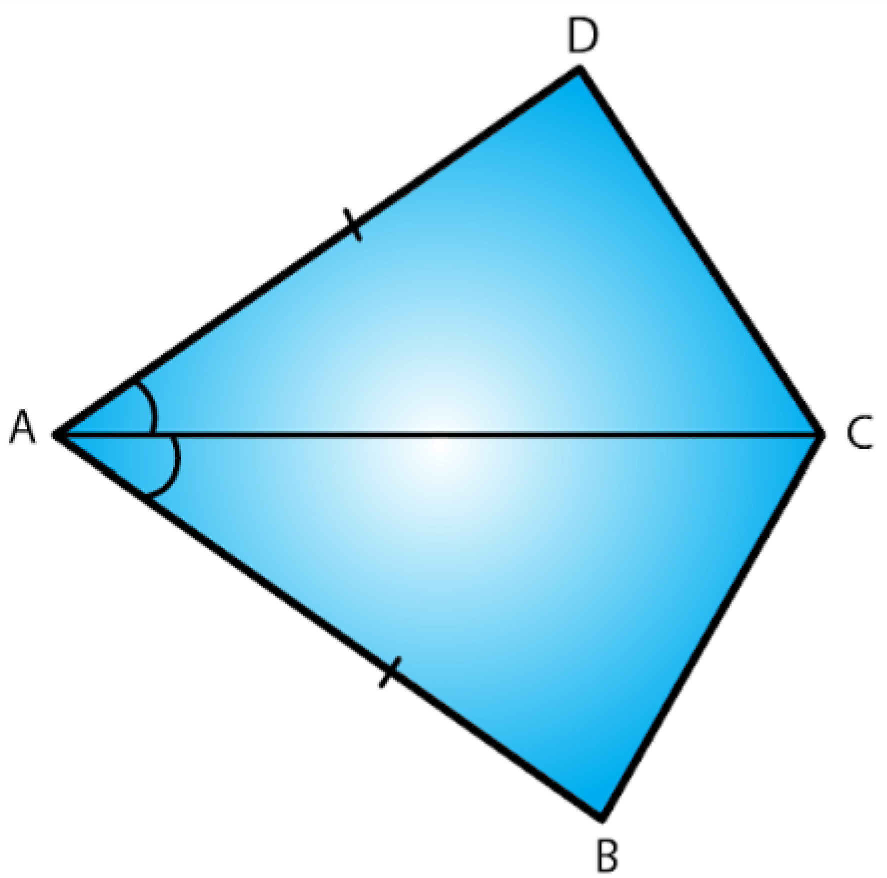


Fig. 31

Solution:

i) AB = AD (given)

∠BAC = ∠DAC (given)

AC = CA (common)

Therefore by SAS condition of congruency, ΔABC ≅ ΔADC

ii) ∠ABC = ∠ADC (corresponding parts of congruent triangles)

∠ACD = ∠ACB (corresponding parts of congruent triangles)

Line segment AC bisects ∠A and ∠C.

7. In fig. 32, AB II DC and AB = DC.

(i) Is ΔACD ≅ ΔCAB?

(ii) State the three pairs of matching parts used to answer (i).

(iii) Which angle is equal to ∠CAD?

(iv) Does it follow from (iii) that AD II BC?

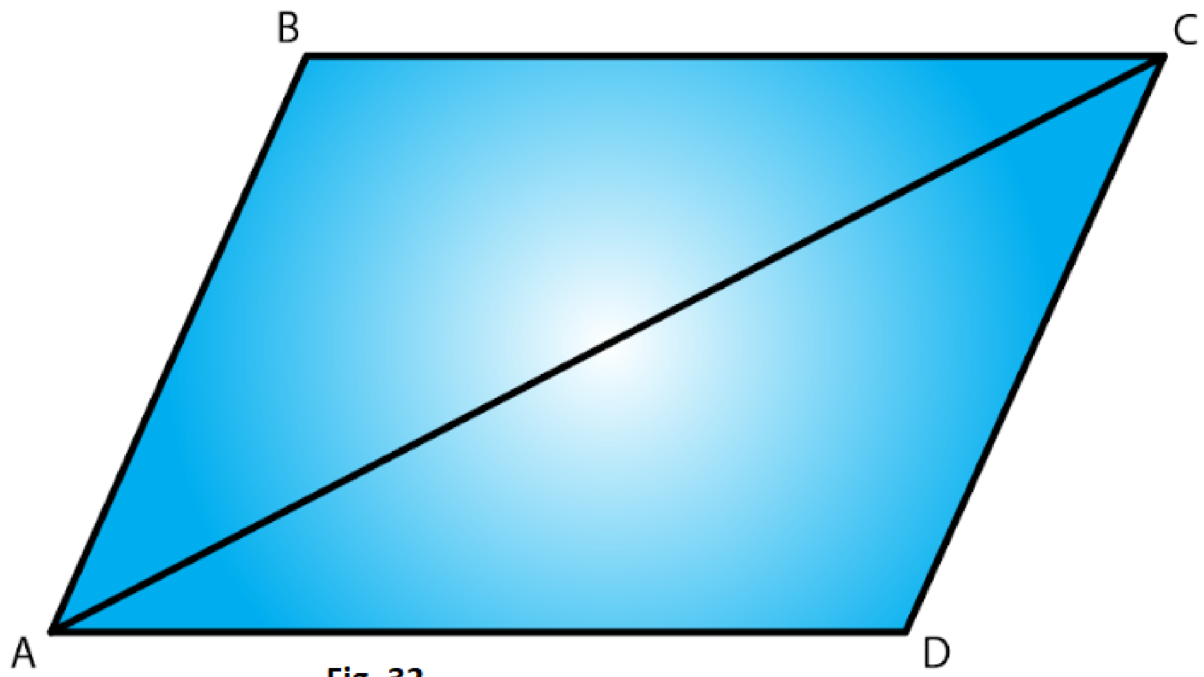


Fig. 32

Solution:

(i) Yes by SAS condition of congruency, ΔACD ≅ ΔCAB.

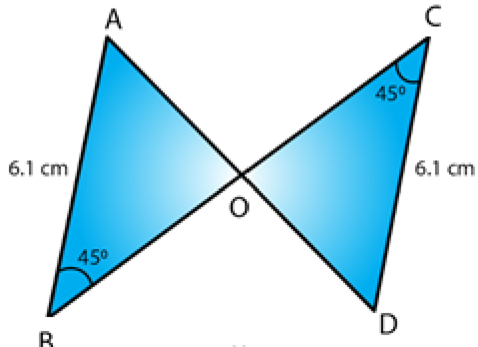
(ii) We have used AB = DC, AC = CA and ∠DCA = ∠BAC.

(iii) ∠CAD = ∠ACB since the two triangles are congruent.

(iv) Yes this follows from AD parallel to BC as alternate angles are equal. If alternate angles are equal then the lines are parallel

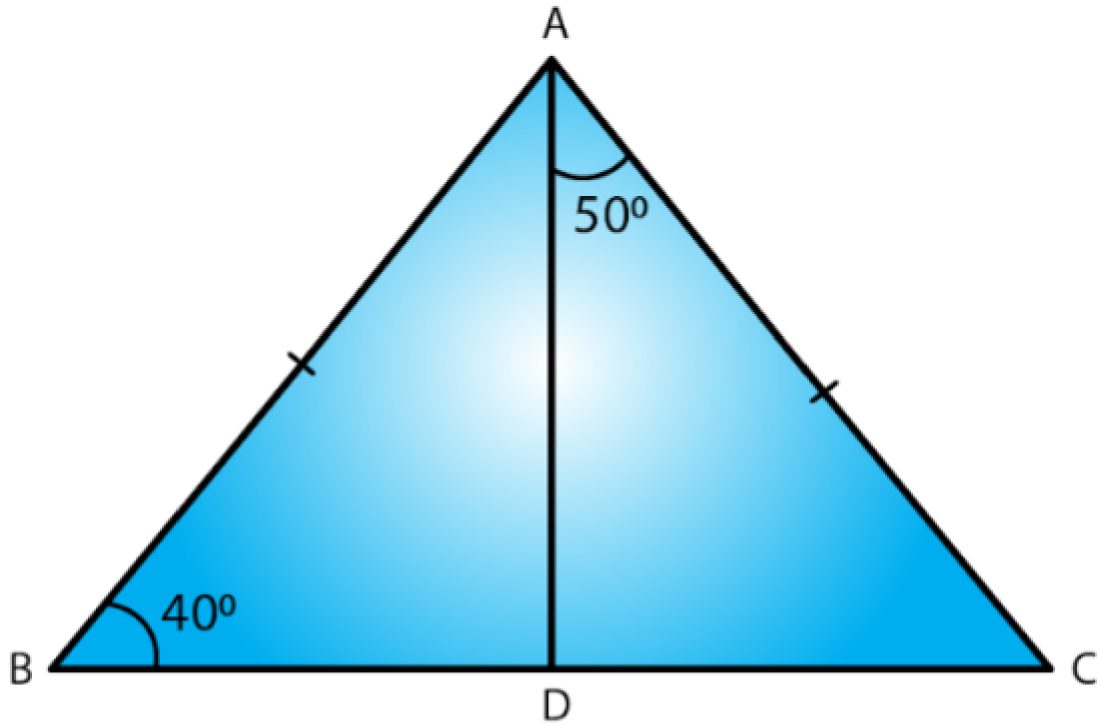
Exercise 16.4

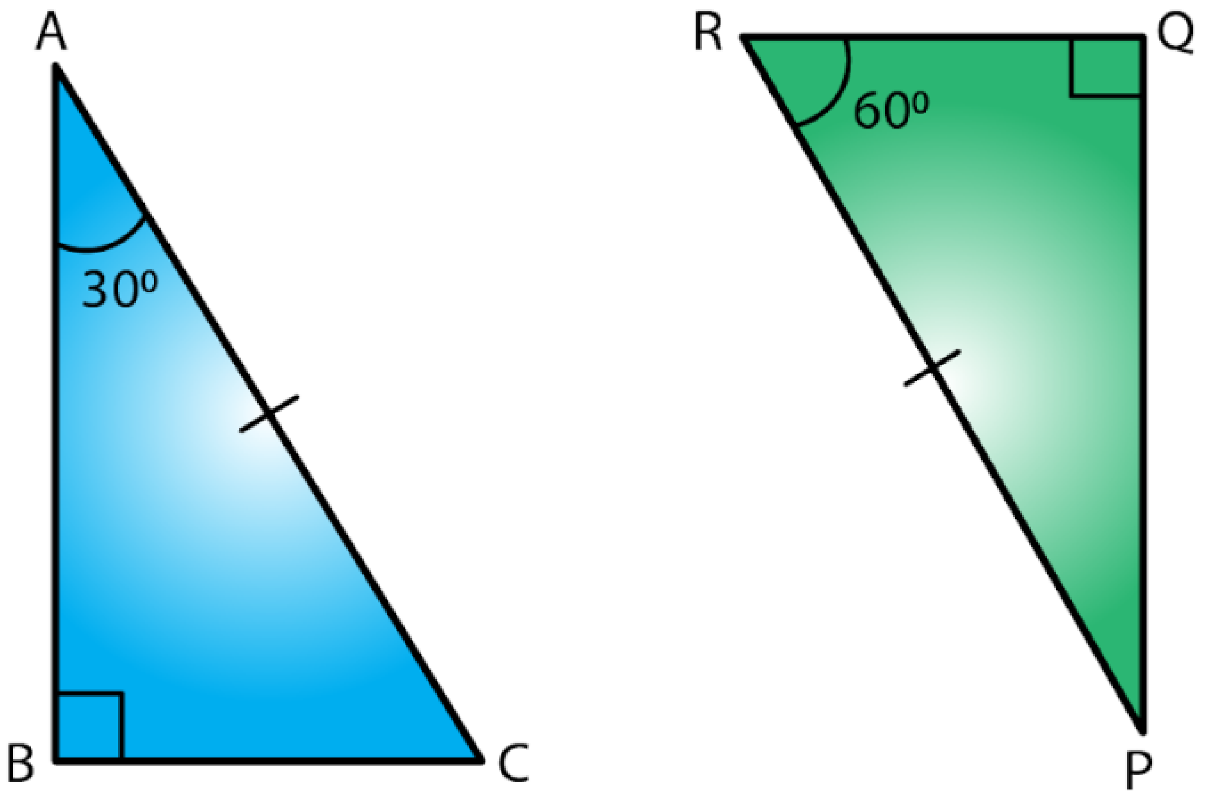
1. Which of the following pairs of triangle are congruent by ASA condition?



(i)

(ii)





(iii)

(iv)

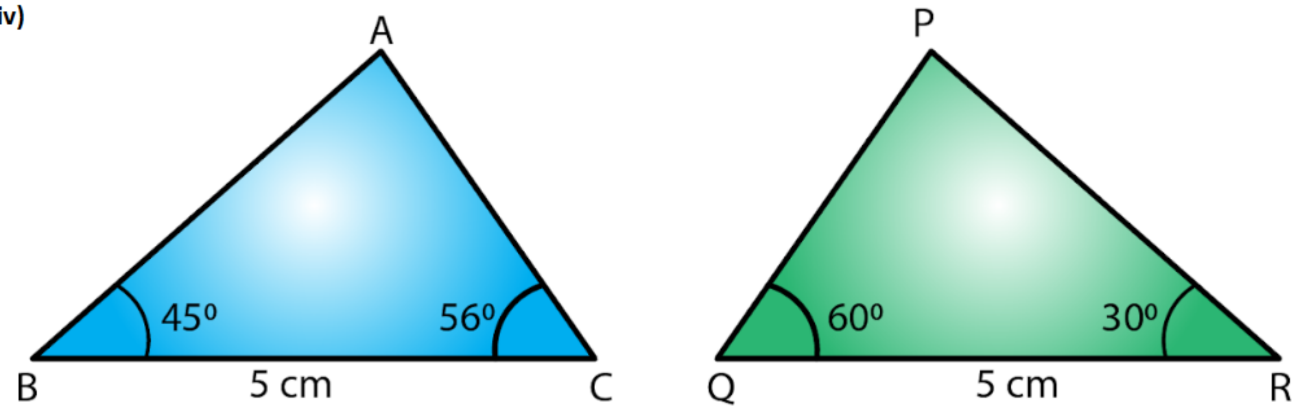


Fig. 36

Solution:

(i) We have,

Since ∠ABO = ∠CDO = 45° and both are alternate angles, AB parallel to DC, ∠BAO = ∠DCO (alternate angle, AB parallel to CD and AC is a transversal line)

∠ABO = ∠CDO = 45° (given in the figure) Also,

AB = DC (Given in the figure)

Therefore, by ASA ΔAOB ≅ ΔDOC

(ii) In ABC,

Now AB =AC (Given)

∠ABD = ∠ACD = 40° (Angles opposite to equal sides)

$\angle ABD + \angle ACD + \angle BAC = 180^\circ$ (Angle sum property)

$40^\circ + 40^\circ + \angle BAC = 180^\circ$

$\angle BAC = 180^\circ - 80^\circ = 100^\circ$

$\angle BAD + \angle DAC = \angle BAC$

$\angle BAD = \angle BAC - \angle DAC = 100^\circ - 50^\circ = 50^\circ$

$\angle BAD = \angle CAD = 50°$

Therefore, by ASA, $\Delta ABD \cong \Delta ACD$

(iii) In Δ ABC,

$\angle A + \angle B + \angle C = 180^\circ$ (Angle sum property)

$\angle C = 180^\circ - \angle A - \angle B$

$\angle C = 180^\circ - 30^\circ - 90^\circ = 60^\circ$

In PQR,

$\angle P + \angle Q + \angle R = 180^\circ$ (Angle sum property)

$\angle P = 180^\circ - \angle R - \angle Q$

$\angle P = 180^\circ - 60^\circ - 90^\circ = 30^\circ$

$\angle BAC = \angle QPR = 30^\circ$

$\angle BCA = \angle PRQ = 60^\circ$ and AC = PR (Given)

Therefore, by ASA, $\Delta ABC \cong \Delta PQR$

(iv) We have only

BC = QR but none of the angles of ΔABC and ΔPQR are equal.

Therefore, ΔABC is not congruent to ΔPQR

2. In fig. 37, AD bisects A and AD $\perp$ BC.

(i) Is $\Delta ADB \cong \Delta ADC$?

(ii) State the three pairs of matching parts you have used in (i)

(iii) Is it true to say that BD = DC?

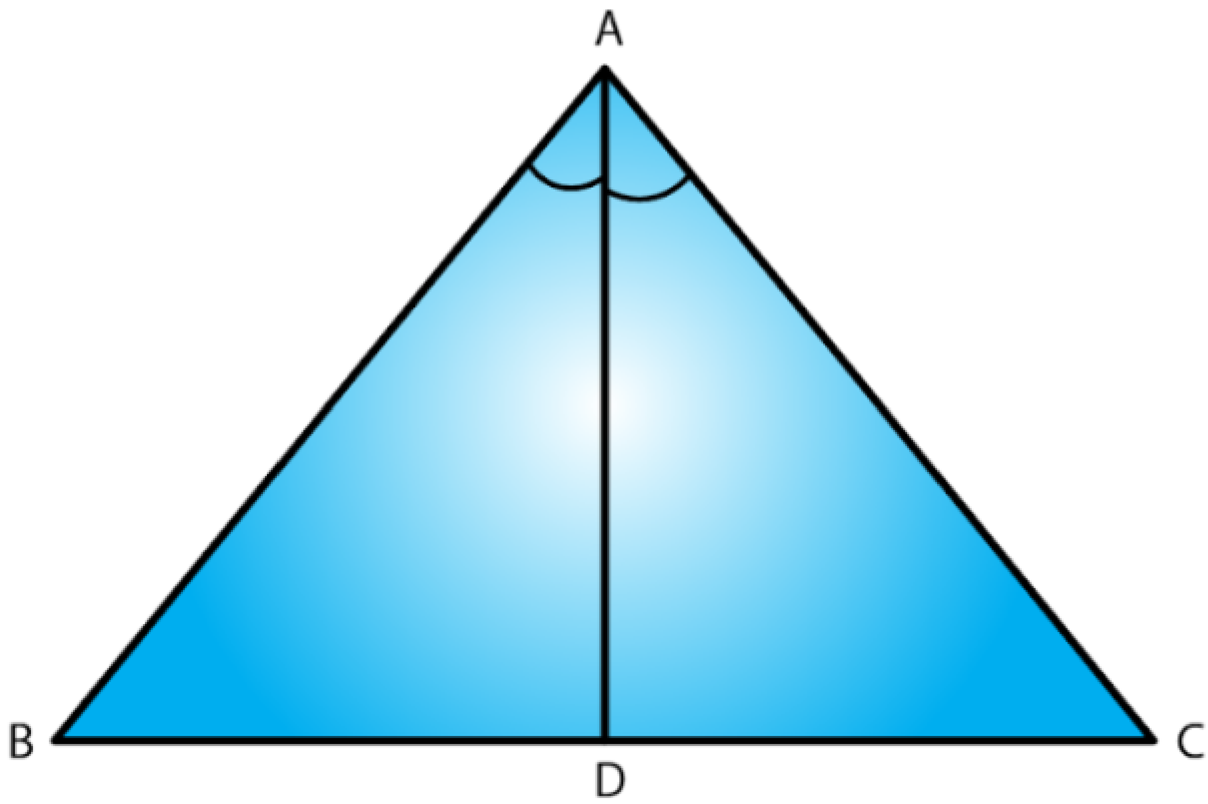


Fig. 37

Solution:

(i) Yes, ΔADB≅ΔADC, by ASA criterion of congruency.

(ii) We have used ∠BAD = ∠CAD ∠ADB = ∠ADC = 90°

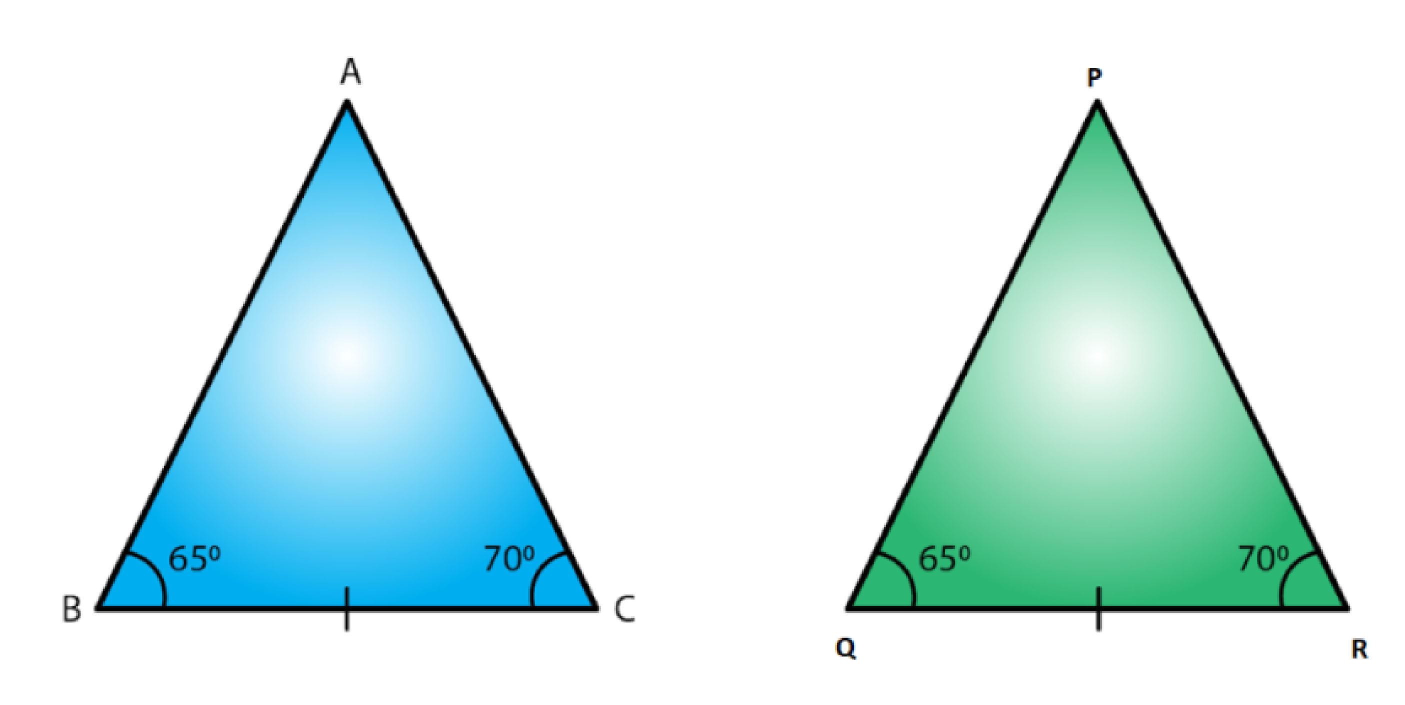
Since, AD ⊥ BC and AD = DA

ADB = ADC

(iii) Yes, BD = DC since, ΔADB ≅ ΔADC

3. Draw any triangle ABC. Use ASA condition to construct other triangle congruent to it.

Solution:



We have drawn

Δ ABC with ∠ABC = 65° and ∠ACB = 70°

We now construct ΔPQR ≅ ΔABC where ∠PQR = 65° and ∠PRQ = 70°

Also we construct ΔPQR such that BC = QR

Therefore by ASA the two triangles are congruent

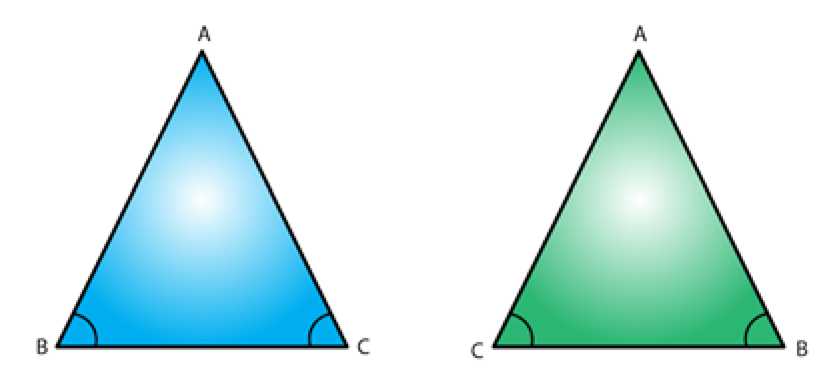
4. In Δ ABC, it is known that ∠B = C. Imagine you have another copy of Δ ABC

(i) Is ΔABC ≅ ΔACB

(ii) State the three pairs of matching parts you have used to answer (i).

(iii) Is it true to say that AB = AC?

Solution:



(i) Yes ΔABC ≅ ΔACB

(ii) We have used ∠ABC = ∠ACB and ∠ACB = ∠ABC again.

Also BC = CB

(iii) Yes it is true to say that AB = AC since ∠ABC = ∠ACB.

5. In Fig. 38, AX bisects ∠BAC as well as ∠BDC. State the three facts needed to ensure that ΔACD ≅ ΔABD

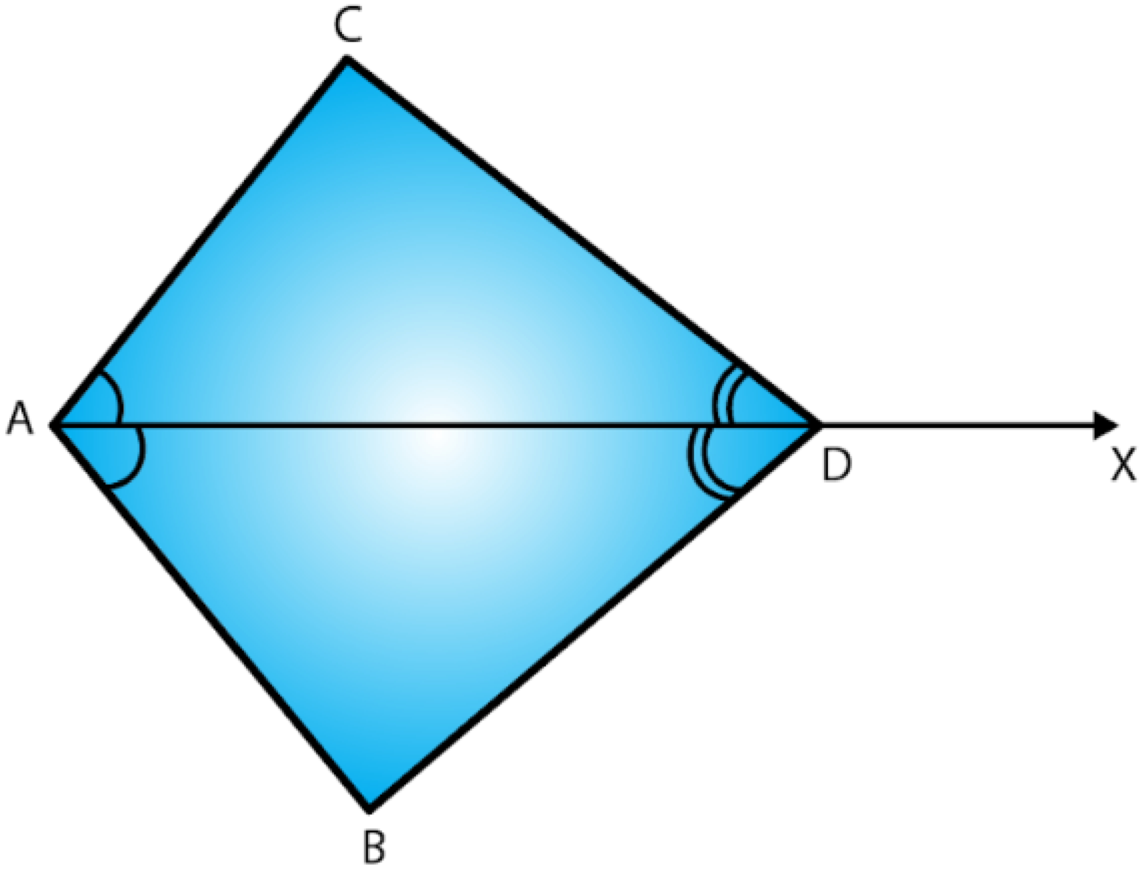


Fig. 38

Solution:

As per the given conditions,

∠CAD = ∠BAD and ∠CDA = ∠BDA (because AX bisects ∠BAC)

AD = DA (common)

Therefore, by ASA, ΔACD ≅ ΔABD

6. In Fig. 39, AO = OB and ∠A = ∠B.

(i) Is ΔAOC ≅ ΔBOD

(ii) State the matching pair you have used, which is not given in the question.

(iii) Is it true to say that ∠ACO = ∠BDO?

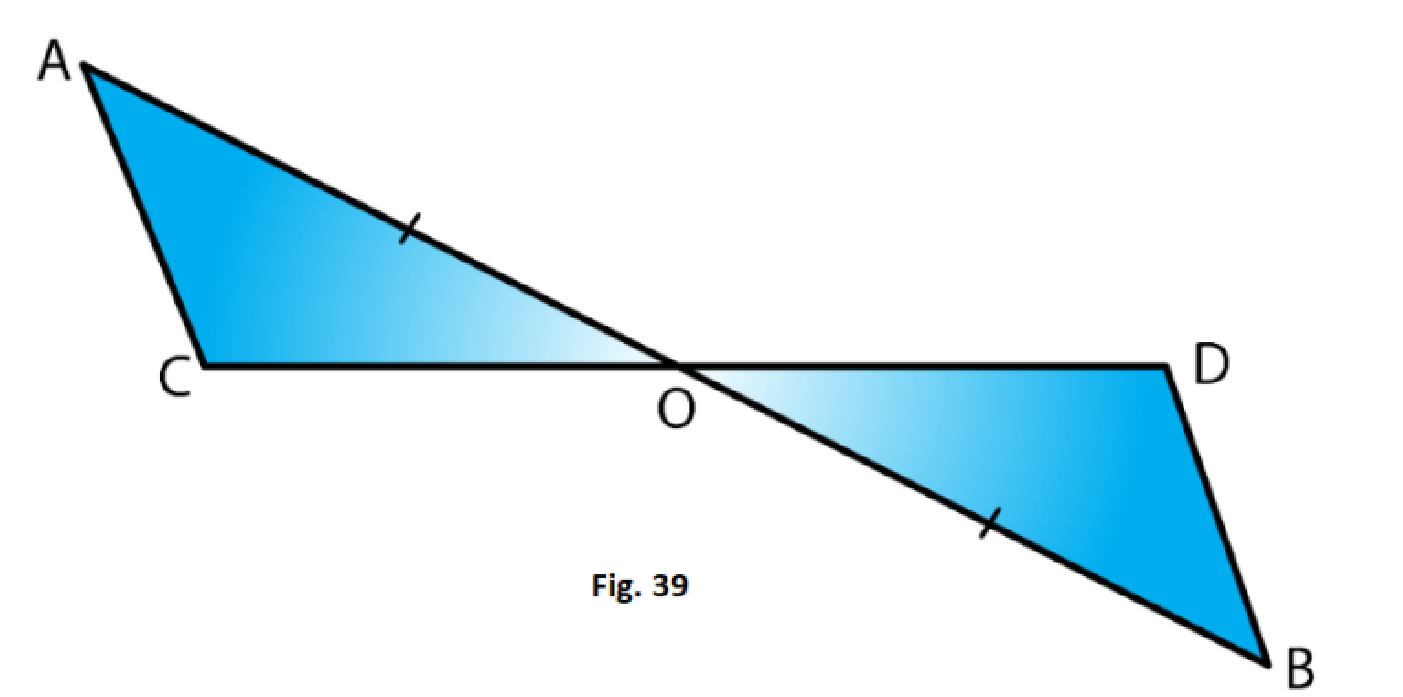


Fig. 39

Solution:

We have

∠OAC = ∠OBD,

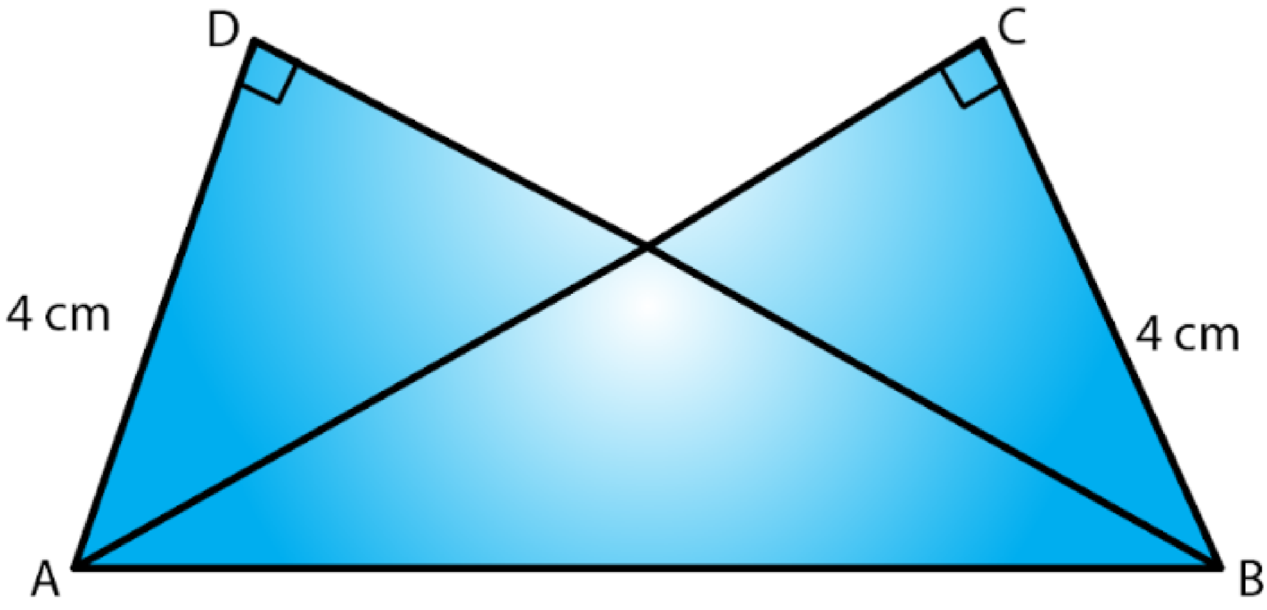
AO = OB

Also, ∠AOC = ∠BOD (Opposite angles on same vertex)

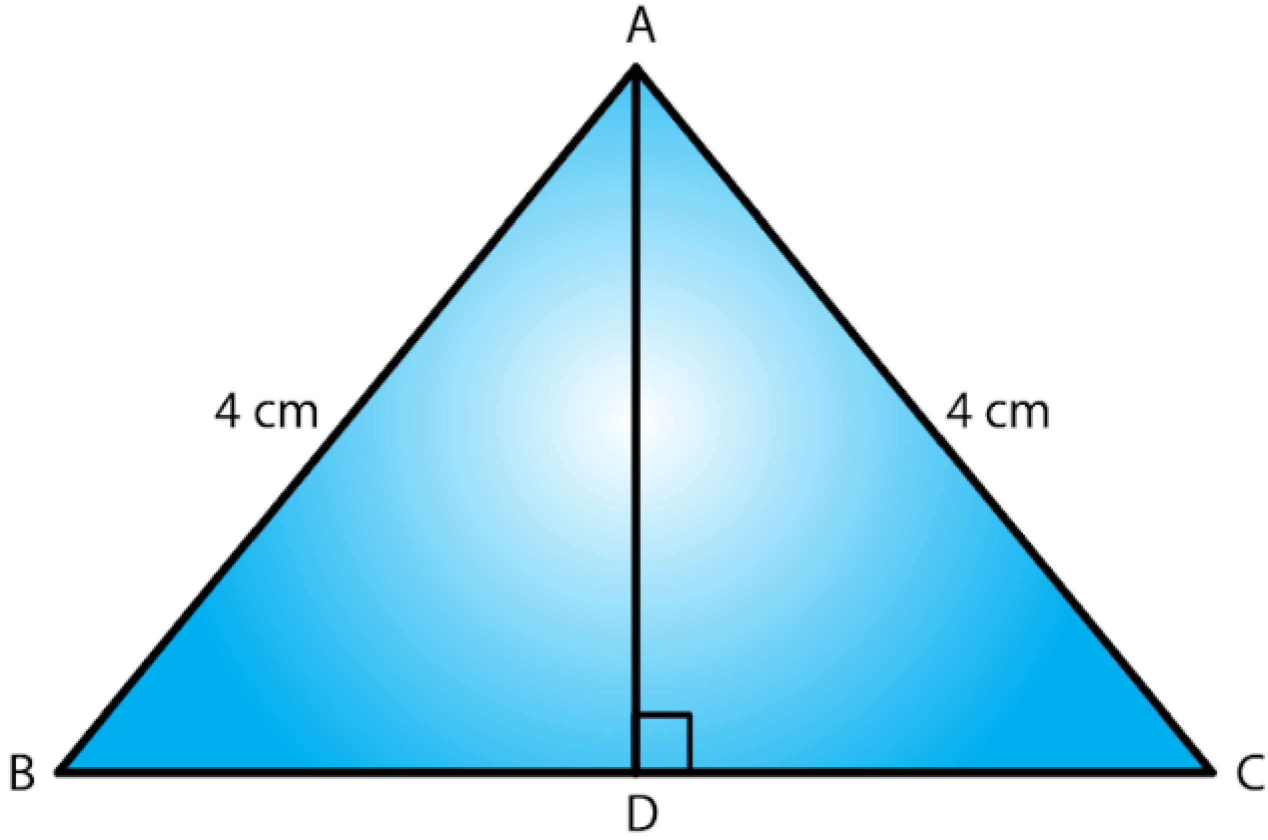
Therefore, by ASA ΔAOC ≅ ΔBOD

Exercise 16.5

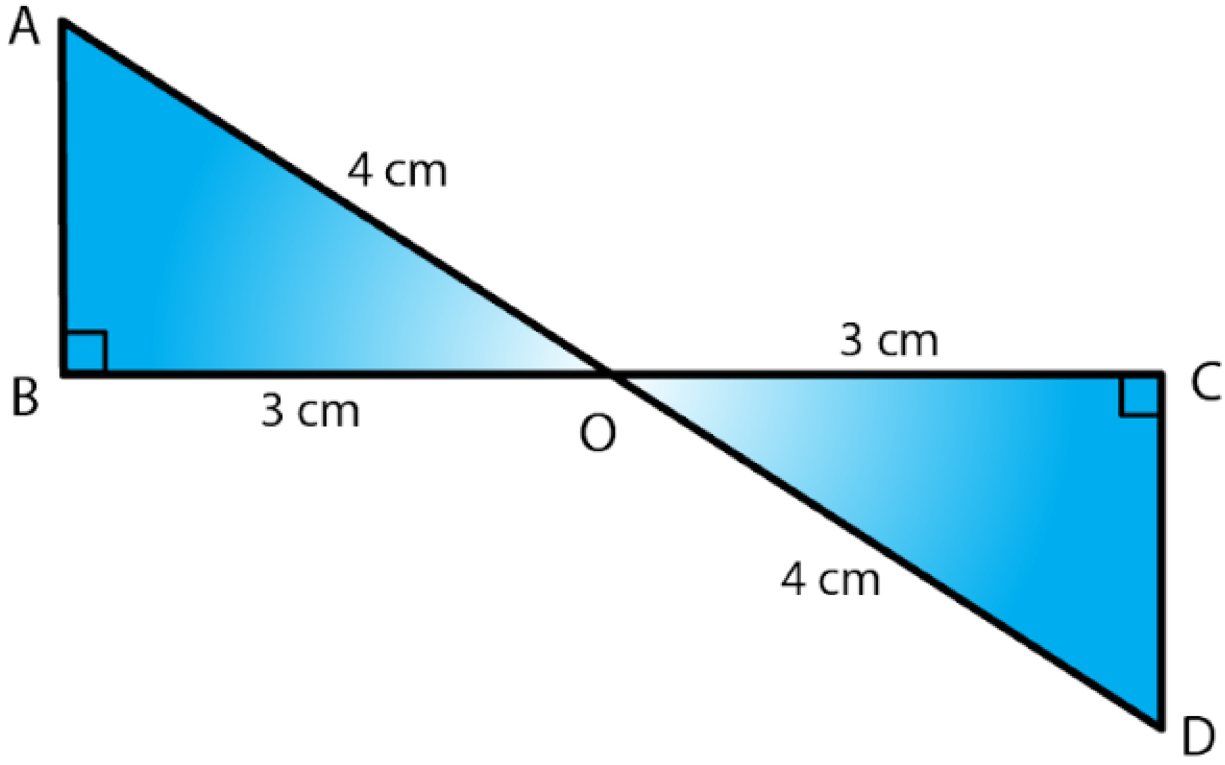
1. In each of the following pairs of right triangles, the measures of some parts are indicated alongside. State by the application of RHS congruence condition which are congruent, and also state each result in symbolic form. (Fig. 46)



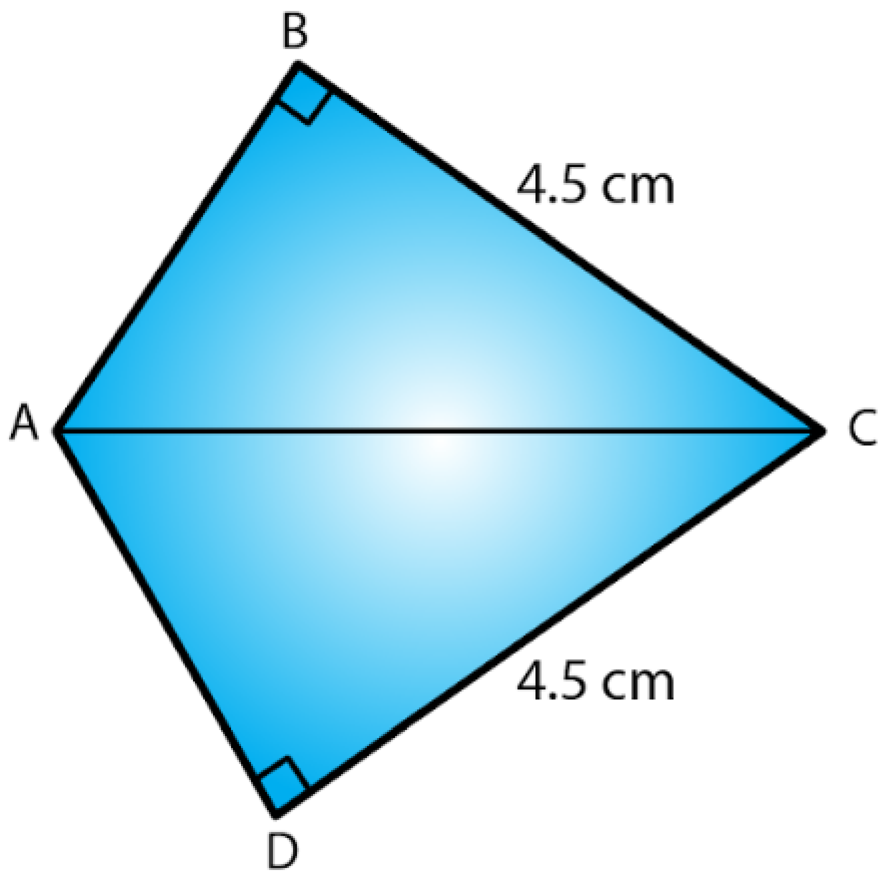
(i)



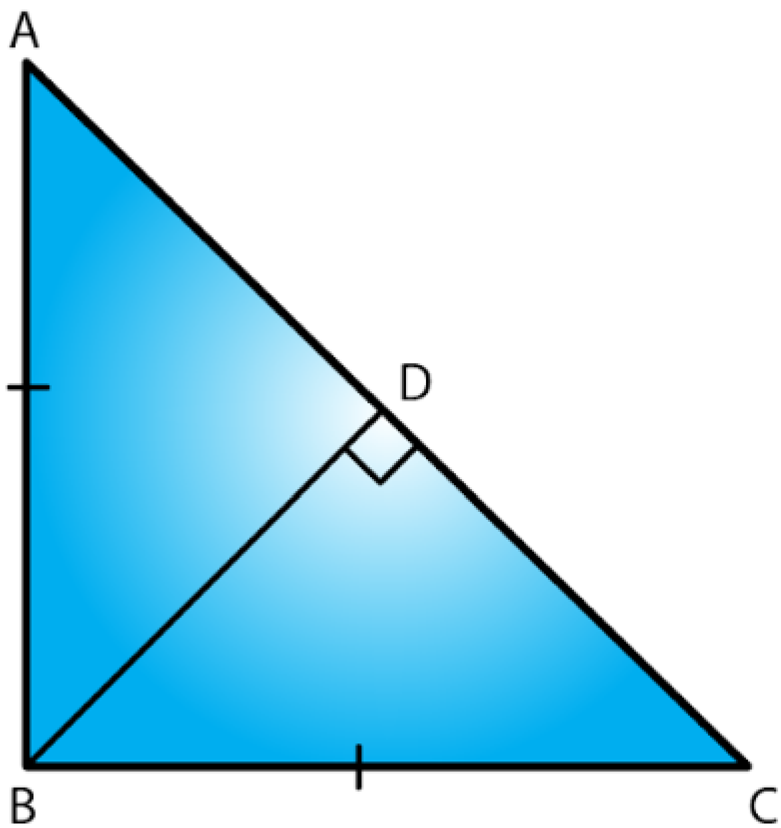
(ii)



(iii)



(iv)



(v)

Fig. 46

Solution:

(i) ∠ADB = ∠BCA = 90°

AD = BC and hypotenuse AB = hypotenuse AB

Therefore, by RHS ΔADB ≅ ΔACB

(ii) AD = AD (Common)

Hypotenuse AC = hypotenuse AB (Given)

∠ADB + ∠ADC = 180° (Linear pair)

∠ADB + 90° = 180°

∠ADB = 180° – 90° = 90°

∠ADB = ∠ADC = 90°

Therefore, by RHS Δ ADB ≅ Δ ADC

(iii) Hypotenuse AO = hypotenuse DO

BO = CO

∠B = ∠C = 90°

Therefore, by RHS, ΔAOB≅ΔDOC

(iv) Hypotenuse AC = Hypotenuse CA

BC = DC

∠ABC = ∠ADC = 90°

Therefore, by RHS, ΔABC ≅ ΔADC

(v) BD = DB

Hypotenuse AB = Hypotenuse BC, as per the given figure,

$\angle BDA + \angle BDC = 180^o$

$\angle BDA + 90^o = 180^o$

$\angle BDA = 180^o - 90^o = 90^o$

$\angle BDA = \angle BDC = 90^o$

Therefore, by RHS, $\Delta ABD \cong \Delta CBD$

2. Δ ABC is isosceles with AB = AC. AD is the altitude from A on BC.

(i) Is ΔABD ≅ ΔACD?

(ii) State the pairs of matching parts you have used to answer (i).

(iii) Is it true to say that BD = DC?

Solution:

(i) Yes, $\Delta ABD \cong \Delta ACD$ by RHS congruence condition.

(ii) We have used Hypotenuse AB = Hypotenuse AC

AD = DA

$\angle ADB = \angle ADC = 90^o$ ($AD \perp BC$ at point D)

(iii)Yes, it is true to say that BD = DC (corresponding parts of congruent triangles)

Since we have already proved that the two triangles are congruent.

3. ΔABC is isosceles with AB = AC. Also. AD ⊥ BC meeting BC in D. Are the two triangles ABD and ACD congruent? State in symbolic form. Which congruence condition do you use? Which side of ADC equals BD? Which angle of Δ ADC equals ∠B?

Solution:

We have AB = AC …… (i)

AD = DA (common) …… (ii)

And, $\angle ADC = \angle ADB$ ($AD \perp BC$ at point D) …… (iii)

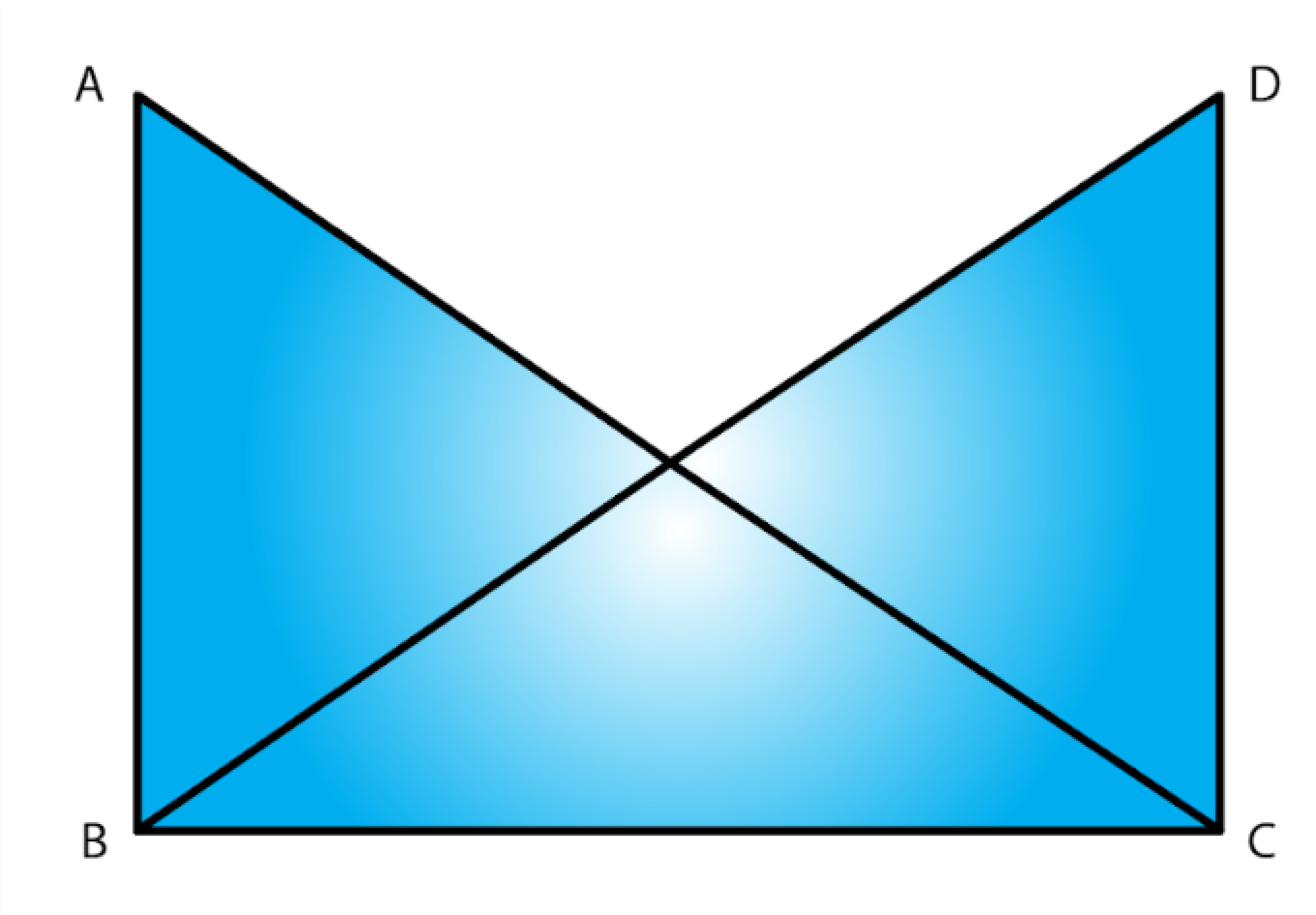
Therefore, from (i), (ii) and (iii), by RHS congruence condition, $\Delta ABD \cong \Delta ACD$, the triangles are congruent.

Therefore, BD = CD.

And $\angle ABD = \angle ACD$ (corresponding parts of congruent triangles)

4. Draw a right triangle ABC. Use RHS condition to construct another triangle congruent to it.

Solution:



Consider

Δ ABC with ∠B as right angle.

We now construct another triangle on base BC, such that ∠C is a right angle and AB = DC

Also, BC = CB

Therefore by RHS, ΔABC ≅ ΔDCB

5.In fig. 47, BD and CE are altitudes of Δ ABC and BD = CE.

(i) Is ΔBCD ≅ ΔCBE?

(ii) State the three pairs or matching parts you have used to answer (i)

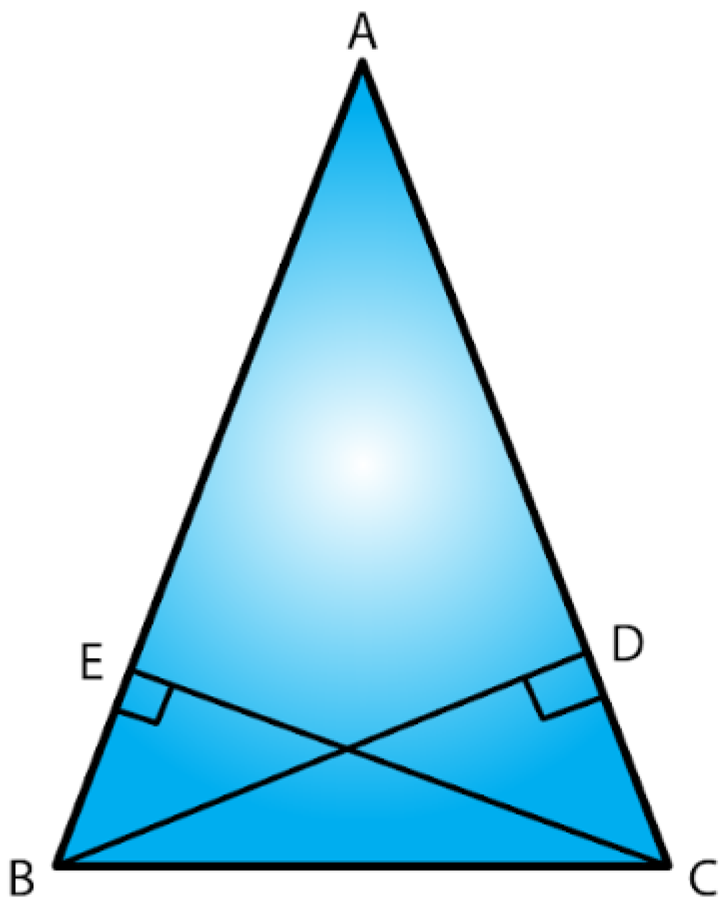


Fig. 47

Solution:

(i) Yes, ΔBCD ≅ ΔCBE by RHS congruence condition.

(ii) We have used hypotenuse BC = hypotenuse CB

BD = CE (Given in question)

And ∠BDC = ∠CEB = 90^o

Construction

Exercise 17.1

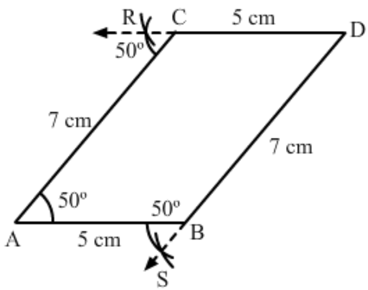
Question 1:

Draw an $\angle BAC$ of measure $50°$ such that $AB = 5$ cm and $AC = 7$ cm. Through C draw a line parallel to AB and through B draw a line parallel to AC, intersecting each other at D. Measure BD and CD.

Answer:

Steps of construction:

1. Draw angle BAC = 50° such that AB = 5 cm and AC = 7 cm.
2. Cut an arc through C at an angle of 50°.
3. Draw a straight line passing through C and the arc. This line will be parallel to AB since $\angle CAB = \angle RCA = 50^{\circ}$.
4. Alternate angles are equal; therefore the line is parallel to AB.
5. Again through B, cut an arc at an angle of 50° and draw a line passing through B and this arc and say this intersects the line drawn parallel to AB at D.
6. $\angle SBA = \angle BAC = 50^{\circ}$, since they are alternate angles. Therefore BD $\parallel$ AC.
7. Also we can measure BD = 7 cm and CD = 5 cm.

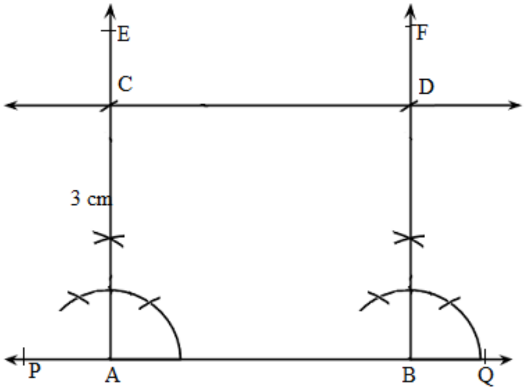


Question 2:

Draw a line *PQ*. Draw another line parallel to *PQ* at a distance of 3 cm from it.

Answer:

1. Draw a line PQ.
2. Take any two points A and B on the line.
3. Construct $\angle PBF = 90^{\circ}$ and $\angle QAE = 90^{\circ}$.
4. With A as centre and radius 3 cm cut AE at C.
5. With B as centre and radius 3 cm cut BF at D.
6. Join CD and produce it on either side to get the required line parallel to AB and at a distance of 5 cm from it.



Question 3:

Take any three non-collinear points *A*, *B*, *C* and draw ΔABC. Through each vertex of the triangle, draw a line parallel to the opposite side.

Answer:

Steps of construction:

1. Mark three non collinear points A, B and C such that none of them lie on the same line.
2. Join AB, BC and CA to form triangle ABC.

Parallel line to AC

1. With A as centre, draw an arc cutting AC and AB at T and U, respectively.
2. With centre B and the same radius as in the previous step, draw an arc on the opposite side of AB to cut AB at X.
3. With centre X and radius equal to TU, draw an arc cutting the arc drawn in the previous step at Y.
4. Join BY and produce in both directions to obtain the line parallel to AC.

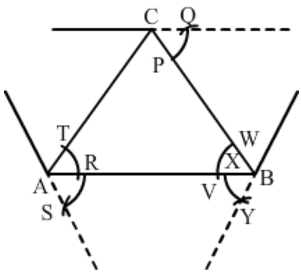
Parallel line to AB

1. With B as centre, draw an arc cutting BC and BA at W and V, respectively.
2. With centre C and the same radius as in the previous step, draw an arc on the opposite side of BC to cut BC at P.
3. With centre P and radius equal to WV, draw an arc cutting the arc drawn in the previous step at Q.
4. Join CQ and produce in both directions to obtain the line parallel to AB.

Parallel line to BC

1. With B as centre, draw an arc cutting BC and BA at W and V, respectively (already drawn).
2. With centre A and the same radius as in the previous step, draw an arc on the opposite side of AB to cut AB at R.
3. With centre R and radius equal to WV, draw an arc cutting the arc drawn in the previous step at S.

4. Join AS and produce in both directions to obtain the line parallel to BC.



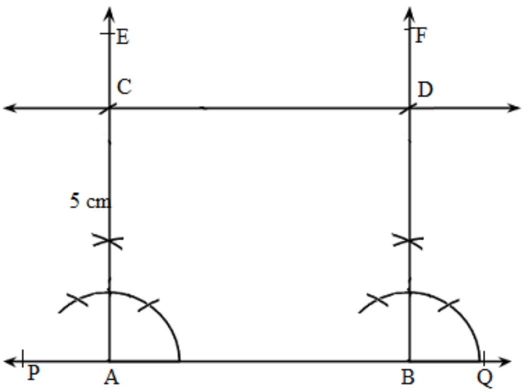
Question 4:

Draw two parallel lines at a distance 5 cm apart.

Answer:

Steps of construction:

1. Draw a line PQ.
2. Take any two points A and B on the line.
3. Construct $\angle PBF = 90^\circ$ and $\angle QAE = 90^\circ$ ∠PBF=90° and ∠QAE=90°.
4. With A as centre and radius 5 cm cut AE at C.
5. With B as centre and radius 5 cm cut BF at D.
6. Join CD and produce it on either side to get the required line parallel to AB and at a distance of 5 cm from it.



Exercise 17.2

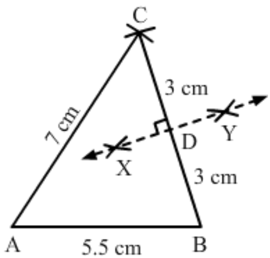
Question 1:

Draw ΔABC in which $AB = 5.5$ cm, $BC = 6$ cm and $CA = 7$ cm. Also, draw perpendicular bisector of side BC.

Answer:

Steps of construction:

1. Draw a line segment AB of length 5.5 cm.
2. From B, cut an arc of radius 6 cm.
3. With centre A, draw an arc of radius 7 cm intersecting the previously drawn arc at say, C.
4. Join AC and BC to obtain the desired triangle.
5. With centre B and radius more than $\frac{1}{2}BC$12BC, draw two arcs on both sides of BC.
6. With centre C and the same radius as in the previous step, draw two arcs intersecting the arcs drawn in the previous step at X and Y.
7. Join XY to get the perpendicular bisector of BC.



Question 2:

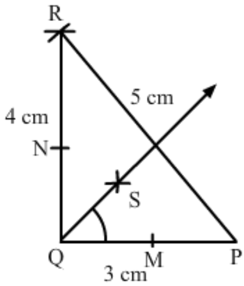
Draw ΔPQR in which $PQ = 3$ cm, $QR = 4$ cm and $RP = 5$ cm. Also, draw the bisector of $\angle Q$.

Answer:

Steps of construction:

1. Draw a line segment PQ of length 3 cm.
2. With Q as centre and radius 4 cm, draw an arc.
3. With P as centre and radius 5 cm, draw an arc intersecting the previously drawn arc at R.
4. Join PR and QR to obtain the required triangle.
5. From Q, cut arcs of equal radius intersecting PQ and QR at M and N, respectively.
6. From M and N, cut arcs of equal radius intersecting at point S.

7. Join QS and extend to produce the angle bisector of angle PQR.
8. Verify that angle PQS and angle SQR are equal to 45$^{\circ\circ}$ each.



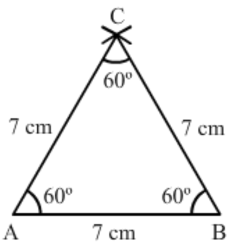
Question 3:

Draw an equilateral triangle one of whose sides is of length 7 cm.

Answer:

Steps of construction:

1. Draw a line segment AB of length 7 cm.
2. With centre A, draw an arc of radius 7 cm.
3. With centre B, draw an arc of radius 7 cm intersecting the previously drawn arc at C.
4. Join AC and BC to get the required triangle.



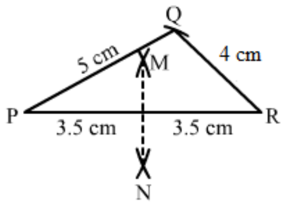
Question 4:

Draw a triangle whose sides are of lengths 4 cm, 5 cm and 7 cm. Draw the perpendicular bisector of the largest side.

Answer:

Steps of construction:

1. Draw a line segment PR of length 7 cm.
2. With centre P, draw an arc of radius 5 cm.
3. With centre R, draw an arc of radius 4 cm intersecting the previously drawn arc at Q.
4. Join PQ and QR to obtain the required triangle.
5. From P, draw arcs with radius more than half of PR on either sides.
6. With the same radius as in the previous step, draw arcs from R on either sides of PR intersecting the arcs drawn in the previous step at M and N.
7. MN is the required perpendicular bisector of the largest side.



Question 5:

Draw a triangle *ABC* with *AB* = 6 cm, *BC* = 7 cm and *CA* = 8 cm. Using ruler and compass alone, draw (i) the bisector *AD* of $\angle A$ and (ii) perpendicular *AL* from *A* on *BC*. Measure *LAD*.

Answer:

Steps of construction:

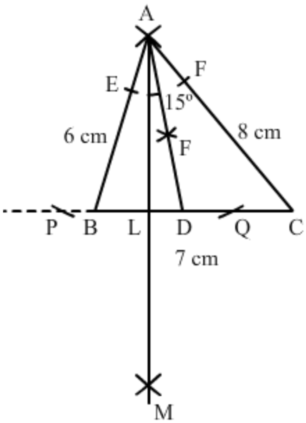
1. Draw a line segment BC of length 7 cm.
2. With centre B, draw an arc of radius 6 cm.
3. With centre C, draw an arc of radius 8 cm intersecting the previously drawn arc at A.
4. Join AC and BC to get the required triangle.

Angle bisector steps:

1. From A, cut arcs of equal radius intersecting AB and AC at E and F, respectively.
2. From E and F, cut arcs of equal radius intersecting at point H.
3. Join AH and extend to produce the angle bisector of angle A, meeting line BC at D.

Perpendicular from Point A to line BC steps:

1. From A, cut arcs of equal radius intersecting BC at P and Q, respectively (Extend BC to draw these arcs).
2. From P and Q, cut arcs of equal radius intersecting at M.
3. Join AM cutting BC at L.
4. AL is the perpendicular to the line BC.
5. Angle LAD is 15^{o}.



Question 6:

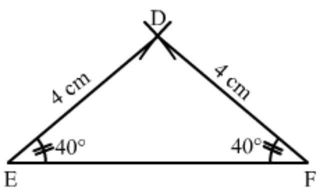
Draw ΔDEF such that $DE = DF = 4$ cm and EF = 6 cm. Measure $\angle E$ and $\angle F$.

Answer:

Steps of construction:

1. Draw a line segment EF of length 6 cm.
2. With E as centre, draw an arc of radius 4 cm.
3. With F as centre, draw an arc of radius 4 cm intersecting the previous arc at D.
4. Join DE and DF to get the desired triangle.DF, .

By measuring we get,$\angle E = \angle F = 40°$∠E=∠F=40°



Question 7:

Draw any triangle *ABC*. Bisect side *AB* at *D*. Through *D*, draw a line parallel to *BC*, meeting *AC* in *E*. Measure *AE* and *EC*.

Answer:

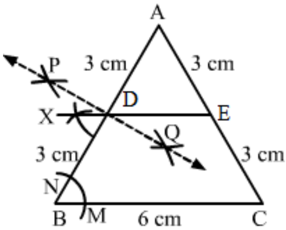
We first draw a triangle ABC with each side = 6 cm.

Steps to bisect line AB:
1. Draw an arc from A on either side of line AB.

2. With the same radius as in the previous step, draw an arc from B on either side of AB intersecting the arcs drawn in the previous step at P and Q.
3. Join PQ cutting AB at D. PQ is the perpendicular bisector of AB.

Parallel line to BC:

1. With B as centre, draw an arc cutting BC and BA at M and N, respectively.
2. With centre D and the same radius as in the previous step, draw an arc on the opposite side of AB to cut AB at Y.
3. With centre Y and radius equal to MN, draw an arc cutting the arc drawn in the previous step at X.
4. Join XD and extend it to intersect AC at E.
5. DE is the required parallel line.



Exercise 17.3

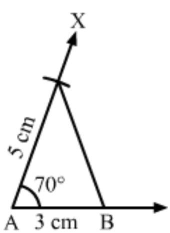
Question 1:

Draw ΔABC in which $AB = 3$ cm, $BC = 5$ cm and $\angle B = 70°$.

Answer:

Steps of construction:

1. Draw a line segment AB of length 3 cm.
2. Draw $\angle XBA = 70°$ ∠XBA = 70°.
3. Cut an arc on BX at a distance of 5 cm at C.
4. Join AC to get the required triangle.



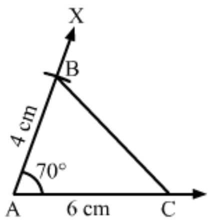
Question 2:

Draw ΔABC in which $\angle A = 70°$, $AB = 4$ cm and $AC = 6$ cm. Measure BC.

Answer:

Steps of construction:

1. Draw a line segment AC of length 6 cm.
2. Draw $\angle$∠XAC = 70°°.
3. Cut an arc on AX at a distance of 4 cm at B.
4. Join BC to get the desired triangle.
5. We see that BC = 6 cm.



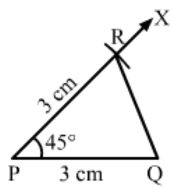
Question 3:

Draw an isosceles triangle in which each of the equal sides is of length 3 cm and the angle between them is 45°.

Answer:

Steps of construction:

1. Draw a line segment PQ of length 3 cm.
2. Draw $\angle QPX = 45°$∠QPX = 45°.
3. Cut an arc on PX at a distance of 3 cm at R.
4. Join QR to get the required triangle.



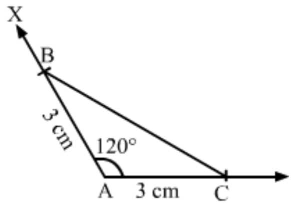
Question 4:

Draw ΔABC in which $\angle A = 120°$, $AB = AC = 3$ cm. Measure $\angle B$ and $\angle C$.

Answer:

Steps of construction:

1. Draw a line segment AC of length 3 cm.
2. Draw $\angle XAC \ = \ 120°$∠XAC = 120°.
3. Cut an arc on AX at a distance of 3 cm at B.
4. Join BC to get the required triangle.



By measuring,we get
$\angle B = \angle C = 30°$∠B=∠C=30°.

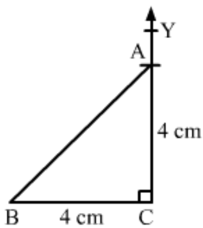
Question 5:

Draw ΔABC in which $\angle C = 90°$ and $AC = BC = 4$ cm.

Answer:

Steps of construction:

1. Draw a line segment BC of length 4 cm.
2. AT C, draw $\angle BCY \ = \ 90°$∠BCY = 90°.
3. Cut an arc on CY at a distance of 4 cm at A.
4. Join AB.
5. ABC is the required triangle.



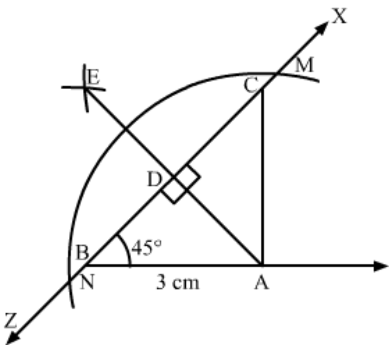
Question 6:

Draw a triangle *ABC* in which *BC* = 4 cm, *AB* = 3 cm and $\angle B = 45°$. Also, draw a perpendicular from A on BC.

Answer:

Steps of construction:

1. Draw a line segment AB of length 3 cm.
2. Draw an angle of 45°° and cut an arc at this angle at a radius of 4 cm at C.
3. Join AC to get the required triangle.
4. With A as centre, draw intersecting arcs at M and N.
5. With centre M and radius more that $\frac{1}{2}MN$12MN, cut an arc on the opposite side of A.
6. With N as centre and radius the same as in the previous step, cut an arc intersecting the previous arc at E.
7. Join AE, it meets BC at D, then AE is the required perpendicular.



Question 7:

Draw a triangle *ABC* with *AB* = 3 cm, *BC* = 4 cm and $\angle B = 60°$. Also, draw the bisector of angles C and A of the triangle, meeting in a point O. Measure $\angle COA$.

Answer:

Steps of construction:

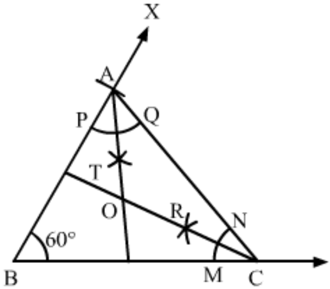
1. Draw a line segment BC = 4 cm.
2. Draw $\angle CBX = 60°$∠CBX = 60°.
3. Draw an arc on BX at a radius of 3 cm cutting BX at A.
4. Join AC to get the required triangle.

Angle bisector for angle A:
1. With A as centre, cut arcs of the same radius cutting AB and AC at P an Q, respectively.
2. From P and Q cut arcs of same radius intersecting at R.
3. Join AR to get the angle bisector of angle A.

Angle bisector for angle C:
1. With A as centre, cut arcs of the same radius cutting CB and CA at M an N, respectively.
2. From M and N, cut arcs of the same radius intersecting at T.
3. Join CT to get the angle bisector of angle C.

Mark the point of intersection of CT and AR as O.

Angle ∠∠COA = 120°



Exercise 17.4

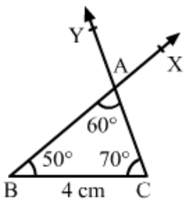
Question 1:

Construct ΔABC in which $BC = 4$ cm, $\angle B = 50°$ and $\angle C = 70°$.

Answer:

Steps of construction:

1. Draw a line segment BC of length 4 cm.
2. Draw $\angle CBX$ such that $\angle CBX = 50°$∠CBX such that ∠CBX =50°.
3. Draw $\angle BCY$∠BCY with Y on the same side of BC as X such that ∠∠BCY = 70°°.
4. Let CY and BX intersect at A.
5. ABC is the required triangle.



Question 2:

Draw ΔABC in which $BC = 8$ cm, $\angle B = 50°$ and $\angle A = 50°$.

Answer:

$\angle ABC + \angle BCA + \angle CAB = 180°$ ∠ABC+∠BCA+∠CAB=180°∠BCA=180°-∠ABC-∠CAB∠BCA=180

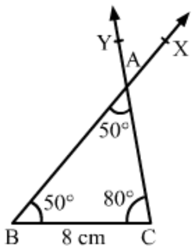
$\angle BCA = 180° - \angle ABC - \angle CAB$

$\angle BCA = 180° - 100° = 80°$

Steps of construction:

1. Draw a line segment BC of length 8 cm.
2. Draw $\angle CBX$ such that $\angle CBX = 50°$∠CBX such that ∠CBX = 50°.

3. Draw ∠∠BCY with Y on the same side of BC as X such that ∠∠BCY = 80°°.
4. Let CY and BX intersect at A.



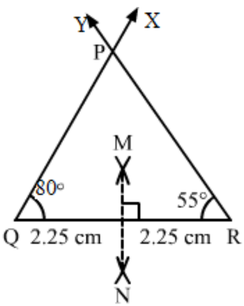
Question 3:

Draw ΔPQR in which $\angle Q = 80°$, $\angle R = 55°$ and $QR = 4.5$ cm. Draw the perpendicular bisector of side QR.

Answer:

Steps of construction:

1. Draw a line segment QR = 4.5 cm.
2. Draw $\angle RQX = 80°$ *and* $\angle QRY = 55°$ ∠RQX = 80° and ∠QRY =55°.
3. Let QX and RY intersect at P so that PQR is the required triangle.
4. With Q as centre and radius more that 2.25 cm, draw arcs on either sides of QR.
5. With R as centre and radius more than 2.25 cm, draw arcs intersecting the previous arcs at M and N.
6. Join MN; MN is the required perpendicular bisector of QR.



Question 4:

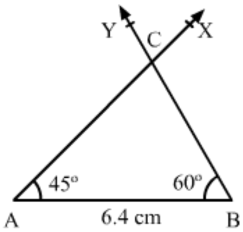
Construct ΔABC in which $AB = 6.4$ cm, $\angle A = 45°$ and $\angle B = 60°$.

Answer:

Steps of construction:

1. Draw a line segment AB = 6.4 cm.
2. Draw $\angle BAX = 45°$ ∠BAX = 45°.
3. Draw ∠∠ABY with Y on the same side of AB as X such that ∠∠ABY = 60°°.

Let AX and BY intersect at C; ABC is the required triangle.



Question 5:

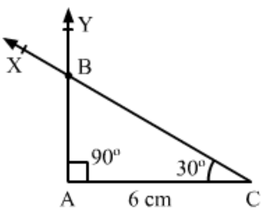
Draw Δ ABC in which AC = 6 cm, $\angle A = 90°$ and $\angle B = 60°$.

Answer:

We can see that ∠A+∠B+∠C = 180°. Therefore ∠C = 180°° − 60°° − 90°° = 30°.

Steps of construction:

1. Draw a line segment AC = 6 cm.
2. Draw $\angle ACX = 30°$ ∠ACX=30°.
3. Draw ∠∠CAY with Y on the same side of AC as X such that ∠∠CAY = 90°°.
4. Join CX and AY. Let these intersect at B.
5. ABC is the required triangle where angle ∠∠ABC = 60°°.



Exercise 17.5

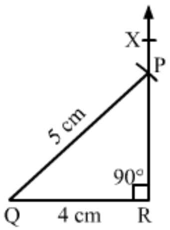
Question 1:

Draw a right triangle with hypotenuse of length 5 cm and one side of length 4 cm.

Answer:

Steps of construction:

1. Draw a line segment QR = 4 cm.
2. Draw ∠∠QRX of measure 90°°.
3. With centre Q and radius PQ = 5 cm, draw an arc of the circle to intersect ray RX at P.
4. Join PQ to obtain the desired triangle PQR.
5. PQR is the required triangle.



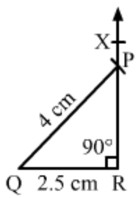
Question 2:

Draw a right triangle whose hypotenuse is of length 4 cm and one side is of length 2.5 cm.

Answer:

Steps of construction:

1. Draw a line segment QR = 2.5 cm.
2. Draw ∠∠QRX of measure 90°°.
3. With centre Q and radius PQ = 4 cm, draw an arc of the circle to intersect ray RX at P.
4. Join PQ to obtain the desired triangle PQR.
5. PQR is the required triangle.



Question 3:

Draw a right triangle having hypotenuse of length 5.4 cm, and one of the acute angles of measure 30°.

Answer:

Let ABC be the right triangle at A such that hypotenuse BC = 5.4 cm. Let $\angle C = 30°$∠C = 30°.
Therefore $\angle A + \angle B + \angle C = 180°$ ∠A+∠B+∠C =180°∠B =180-30-90 = 60°

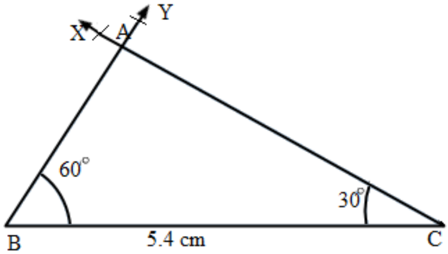
$$\angle B = 180 - 30 - 90 = 60°$$

Steps of construction:

1. Draw a line segment BC = 5.4 cm.

2. Draw angle CBY = 60^{o}.
3. Draw angle BCX of measure 30^{o} with X on the same side of BC as Y.
4. Let BY and CX intersect at A.

Then ABC is the required triangle.



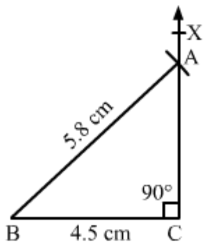
Question 4:

Construct a right triangle *ABC* in which *AB* = 5.8 cm, *BC* = 4.5 cm and $\angle C = 90°$.

Answer:

Steps of construction:

1. Draw a line segment BC = 4.5 cm.
2. Draw ∠∠BCX of measure 90°°.
3. With centre B and radius AB =5.8 cm, draw an arc of the circle to intersect ray BX at A.
4. Join AB to obtain the desired triangle ABC.
5. ABC is the required triangle.



Question 5:

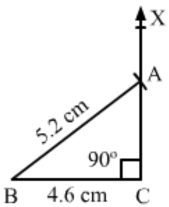
Construct a right triangle, right angled at C in which *AB* = 5.2 cm and *BC* = 4.6 cm.

Answer:

Steps of construction:

1. Draw a line segment BC = 4.6 cm.
2. Draw ∠∠BCX of measure 90°°.
3. With centre B and radius AB = 5.2 cm, draw an arc of the circle to intersect ray CX at A.

4. Join AB to obtain the desired triangle ABC.
5. ABC is the required triangle.



Symmetry

Exercise 18.1

Question 1:

State the number of lines of symmetry for the following figures:
(i) An equilateral triangle
(ii) An isosceles triangle
(iii) A scalene triangle
(iv) A rectangle
(v) A rhombus
(vi) A square
(vii) A parallelogram
(viii) A quadrilateral
(ix) A regular pentagon
(x) A regular hexagon
(xi) A circle
(xii) A semicircle

Answer:

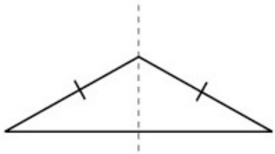
(i) An equilateral triangle has 3 lines of symmetry.
(ii) An isosceles triangle has 1 line of symmetry.
(iii) A scalene triangle has no line of symmetry.
(iv) A rectangle has 2 lines of symmetry.
(v) A rhombus has 2 lines of symmetry.
(vi) A square has 4 lines of symmetry.
(vii) A parallelogram has no line of symmetry.
(viii) A quadrilateral has no line of symmetry.
(ix) A regular pentagon has 5 lines of symmetry.
(x) A regular hexagon has 6 lines of symmetry.
(xi) A circle has an infinite number of lines of symmetry all along the diameters.
(xii) A semicircle has only one line of symmetry.

Question 2:

What other name can you give to the line of symmetry of
(i) An isosceles triangle?
(ii) A circle?

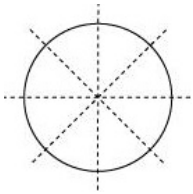
Answer:

(i) An isosceles triangle has only 1 line of symmetry.



This line of symmetry is also known as the **altitude** of an isosceles triangle.

(ii) A circle has an infinite number of lines of symmetry all along its **diameters.**

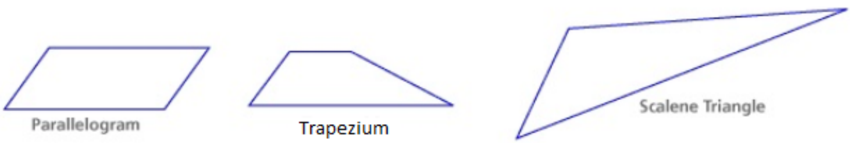


Question 3:

Identify three examples of shapes with no line of symmetry.

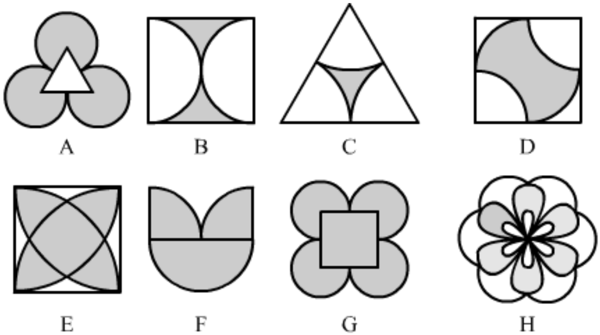
Answer:

A scalene triangle, a parallelogram and a trapezium do not have any line of symmetry.



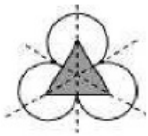
Question 4:

Identify multiple lines of symmetry, if any, in each of the following figures:

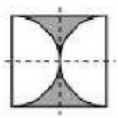


Answer:

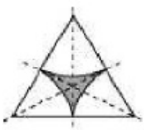
(A) The given figure has 3 lines of symmetry. Therefore it has multiple lines of symmetry.



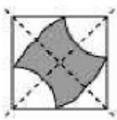
(B) The given figure has 2 lines of symmetry. Therefore it has multiple lines of symmetry.



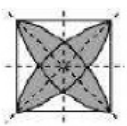
(C) The given figure has 3 lines of symmetry. Therefore it has multiple lines of symmetry.



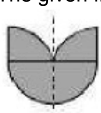
(D) The given figure has 2 lines of symmetry. Therefore it has multiple lines of symmetry.



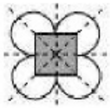
(E) The given figure has 4 lines of symmetry. Therefore it has multiple lines of symmetry.



(F) The given figure has only 1 line of symmetry.



(G) The given figure has 4 lines of symmetry. Therefore it has multiple lines of symmetry.

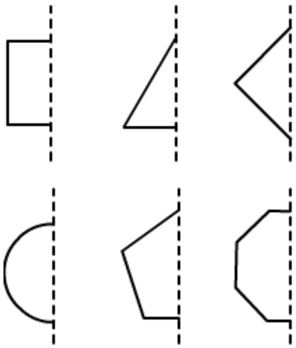


(H) The given figure has 6 lines of symmetry. Therefore it has multiple lines of symmetry.

Exercise 18.2

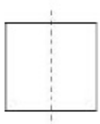
Question 1:

In the following figures, the mirror line (i.e. the line of symmetry) is given as dotted line. Complete each figure performing reflection in the dotted (mirror) line. Also, try to recall the name of the complete figure.

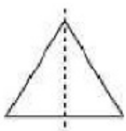


Answer:

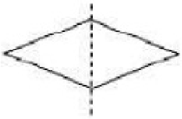
(a) It will be a square.



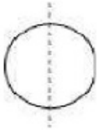
(b) It will be a triangle.



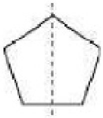
(c) It will be a rhombus.



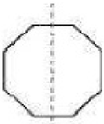
(d) It will be a circle.



(e) It will be a pentagon.

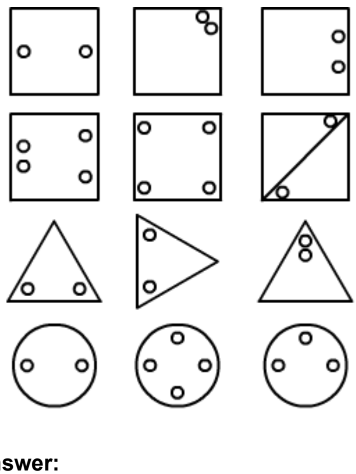


(f) It will be an octagon.



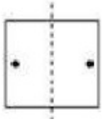
Question 2:

Each of the following figures shows paper cuttings with punched holes. Copy these figures on a plane sheet and mark the axis of symmetry so that if the paper is folded along it, then the wholes on one side of it coincide with the holes on the other side.



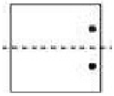
Answer:

The lines of symmetry in the given figures are as follows:
(a)

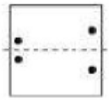


(b)

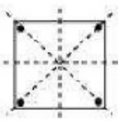
(c)



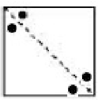
(d)



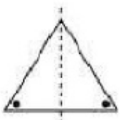
(e)



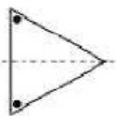
(f)



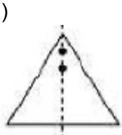
(g)



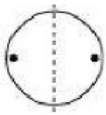
(h)



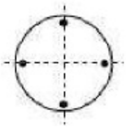
(i)



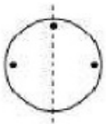
(j)



(k)

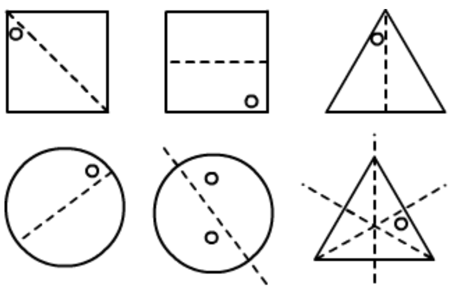


(l)



Question 3:

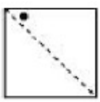
In the following figures if the dotted lines represent the lines of symmetry, find the other hole (s).



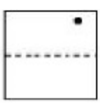
Answer:

The other holes in the given figures are as follows:

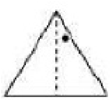
(a)



(b)



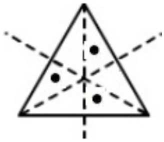
(c)



(d)

(e)

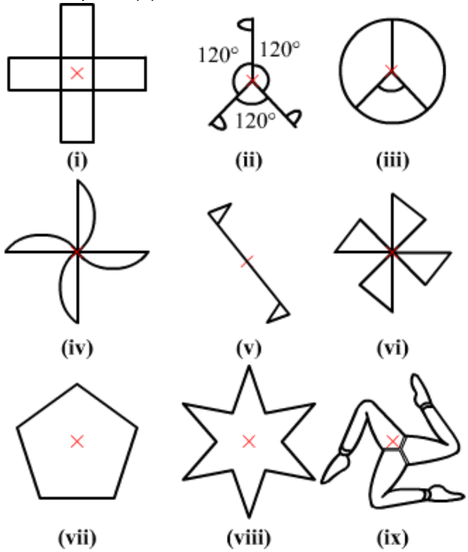
(f)



Exercise 18.3

Question 1:

Give the order of rotational symmetry for each of the following figures when rotated about the marked point (*x*):



Answer:

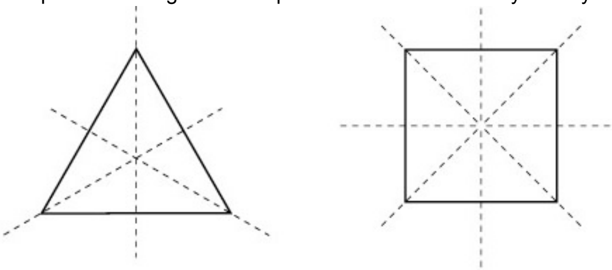
(i) The given figure has its rotational symmetry as 4.
(ii) The given figure has its rotational symmetry as 3.
(iii) The given figure has its rotational symmetry as 3.
(iv) The given figure has its rotational symmetry as 4.
(v) The given figure has its rotational symmetry as 2.
(vi) The given figure has its rotational symmetry as 4.
(vii) The given figure has its rotational symmetry as 5.
(viii) The given figure has its rotational symmetry as 6.
(ix) The given figure has its rotational symmetry as 3.

Question 2:

Name any two figures that have both line symmetry and rotational symmetry.

Answer:

An equilateral triangle and a square have both lines of symmetry and rotational symmetry.



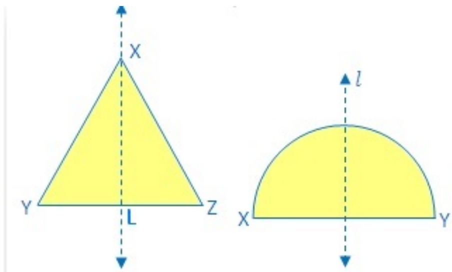
Equilateral triangle   Square

Question 3:

Give an example of a figure that has a line of symmetry but does not have rotational symmetry.

Answer:

A semicircle and an isosceles triangle have a line of symmetry but do not have rotational symmetry.



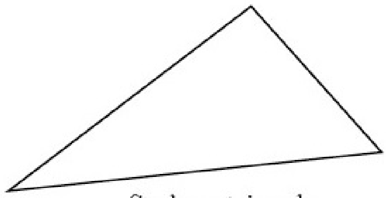
An isosceles triangle   Semi-circle

Question 4:

Give an example of a geometrical figure which has neither a line of symmetry nor a rotational symmetry.

Answer:

A scalene triangle has neither a line of symmetry nor a rotational symmetry.


Scalene triangle

Question 5:

Give an example of a letter of the English alphabet which has (i) no line of symmetry and (ii) rotational symmetry of order 2.

Answer:

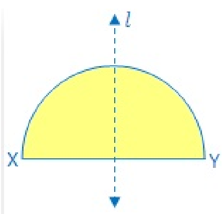
(i) The letter of the English alphabet which has no line of symmetry is Z.
(ii) The letter of the English alphabet which has rotational symmetry of order 2 is N.

Question 6:

What is the line of symmetry of a semi-circle? Does it have rotational symmetry?

Answer:

A semicircle (half of a circle) has only one line of symmetry .
In the figure, there is one line of symmetry. The figure is symmetric along the perpendicular bisector l of the diameter XY.



Semi-circle

A semi-circle does not have any rotational symmetry.

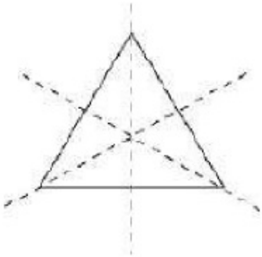
Question 7:

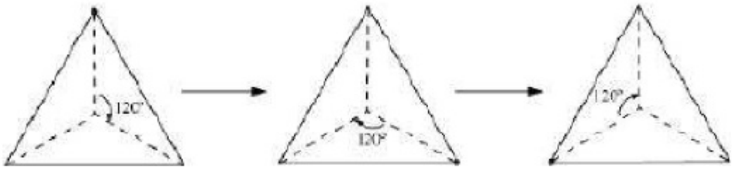
Draw, whenever possible, a rough sketch of
(i) a triangle with both line and rotational symmetries.
(ii) a triangle with only line symmetry and no rotational symmetry.

(iii) a quadrilateral with a rotational symmetry but not a line of symmetry.
(iv) a quadrilateral with line symmetry but not a rotational symmetry.

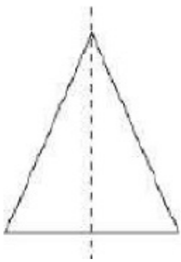
Answer:

(i) An equilateral triangle has 3 lines of symmetry and a rotational symmetry of order 3.

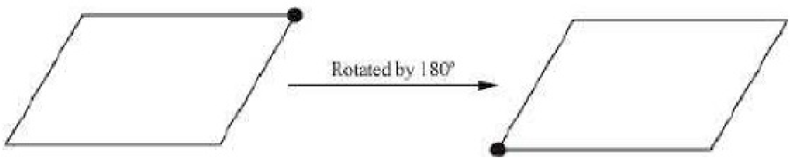




(ii) An isosceles triangle has only 1 line of symmetry and no rotational symmetry.



(iii) A parallelogram is a quadrilateral which has no line of symmetry but a rotational symmetry of order 2.



(iv) A kite is a quadrilateral which has only one line of symmetry and no rotational symmetry.

Question 8:

Fill in the blanks:

Figures	*Centre of rotation*	*Order of rotation*	*Angle of rotation*
Square			
Rectangle			
Rhombus			
Equilateral triangle			
Regular hexagon			
Circle			
Semi-circle			

Answer:

Figures	*Centre of rotation*	*Order of rotation*	*Angle of rotation*
Square	Intersection point of diagonals	4	90°
Rectangle	Intersection point of diagonals	2	180°
Rhombus	Intersection point of diagonals	2	180°
Equilateral triangle	Intersection point of diagonals	3	120°
Regular hexagon	Intersection point of diagonals	6	60°
Circle	Centre	Infinite	Any angle
Semi-circle	Centre	Nil	Nil

Question 9:

Fill in the blanks:

English alphabet Letter	*Line Symmetry*	*Number of Lines of symmetry*	*Rotational Symmetry*	*Order of rotational Sy*
Z	Nil	0	Yes	2
S	–	–	–	–
H	Yes	–	Yes	–
O	Yes	–	Yes	–
E	Yes	–	–	–
N	–	–	Yes	–
C	–	–	–	–

Answer:

English Alphabet Letter	*Line Symmetry*	*Number of Lines of Symmetry*	*Rotational Symmetry*	*Order of Rotational S*
Z	No	0	Yes	2
S	No	0	Yes	2
H	Yes	2	Yes	2
O	Yes	Infinite	Yes	Infinite
E	Yes	1	No	0
N	No	0	Yes	2
C	Yes	1	No	0

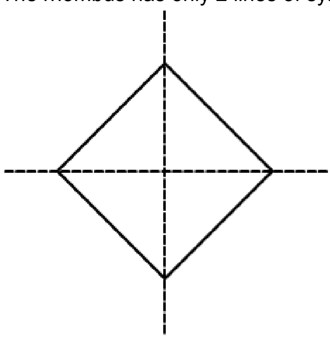
Objective Type Questions

Question 1:

Which of the following has only 2 lines of symmetry?
(a) Equilateral triangle
(b) Rhombus
(c) Circle
(d) None of these of rotation

Answer:

The rhombus has only 2 lines of symmetry.



Hence, the correct answer is option (b)

Question 2:

Which of the following is/are point symmetric?
(a) Rectangle
(b) Square
(c) Parallelogram
(d) All of these

Answer:

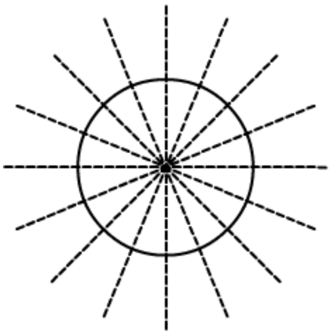
A point of symmetry is a point that represents a "center" of sorts for the figure.
Hence, the correct answer is option (d)

Question 3:

Which of the following has an infinite number of lines of symmetry?
(a) Equilateral triangle
(b) Isosceles triangle
(c) Regular hexagon
(d) Circle

Answer:

A circle has infinite number of lines of symmetry



Hence, the correct answer is option (d).

Question 4:

Which of the following is point symmetric?
(a) Equilateral triangle
(b) Trapezium
(c) Rectangle
(d) None of these

Answer:

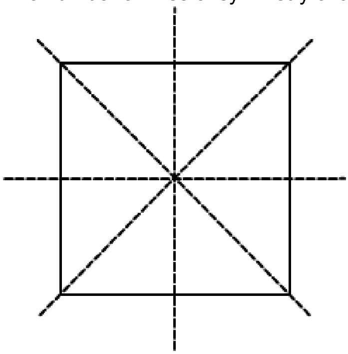
A point of symmetry is a point that represents a "center" of sorts for the figure.
Hence, the correct answer is option (c)

Question 5:

The number of lines of symmetry of a square is
(a) 2
(b) 3
(c) 4
(d) Infinite

Answer:

The number of lines of symmetry of a square is 4.



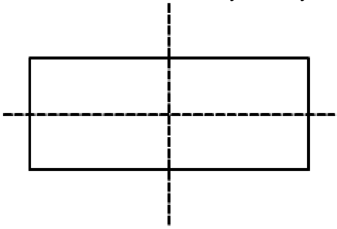
Hence, the correct answer is option (c).

Question 6:

The number of lines of symmetry of a rectangle is
(a) 2
(b) 3
(c) 4
(d) 1

Answer:

The number of lines of symmetry of a rectangle is 2



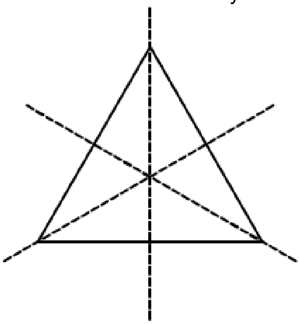
Hence, the correct answer is option (a).

Question 7:

The number of lines of symmetry of an equilateral triangle is
(a) 1
(b) 2
(c) 3
(d) 0

Answer:

The number of lines of symmetry of an equilateral triangle is 3



Hence, the correct answer is option (c).

Question 8:

The order of rotational symmetry of an equilateral triangle is
(a) 0
(b) 1
(c) 2
(d) 3

Answer:

The order of rotational symmetry of an equilateral triangle is 3 i.e., 120°, 240° and 360°.
Hence, the correct answer is option (d).

Question 9:

A rectangle has rotational symmetry of order
(a) 1
(b) 2
(c) 3
(d) 4

Answer:

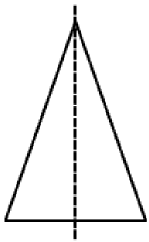
A rectangle has order of rotational symmetry of 2 i.e., 180° and 360°
Hence, the correct answer is option (b).

Question 10:

The number of lines of symmetry of an isosceles triangle is
(a) 0
(b) 1
(c) 2
(d) 3

Answer:

The number of lines of symmetry of an isosceles triangle is 1



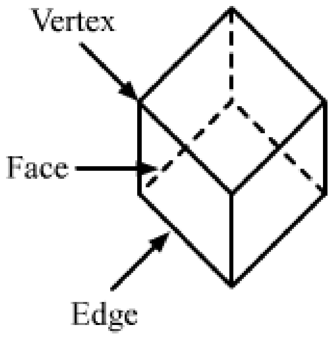
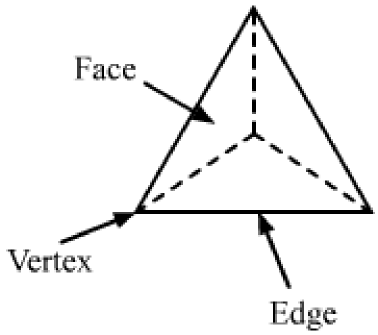
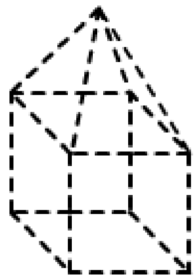
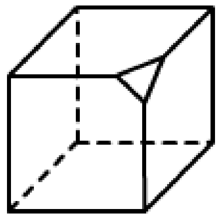
Hence, the correct answer is option (b).

Visualising Solid Shapes

Exericse 19.1

Question 1:

Complete the following table and verify Euler's formula in each case.

				
Faces (F)	6	4	9	7
Edges (E)	12	6	16	15
Vertex (V)	8	4	9	10

Answer:

	I	II	III	IV
Faces (F)	6	4	9	7
Edges (E)	12	6	16	15
Vertices (V)	8	4	9	10
Euler's formula (F − E + V)	6 − 12 + 8 = 2	4 − 6 + 4 = 2	9 − 16 + 9 = 2	7 − 15 + 10 = 2

Hence Euler's formula is verified for these figures.

Question 2:

Give three examples from our daily life which are in the form of (i) a cone (ii) a sphere (iii) a cuboid (iv) a cylinder (v) a pyramid.

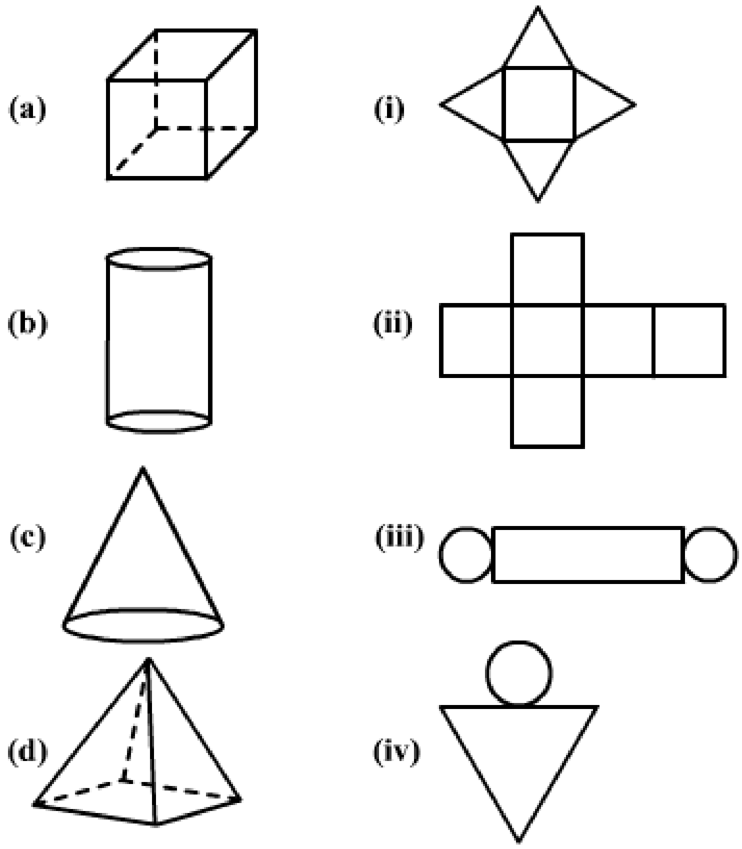
Answer:

Examples of

(i) Cone: Ice-cream cone, clown cap, rocket
(ii) Sphere: Football, a round apple, an orange
(iii) Cuboid: book, brick, duster
(iv) Cylinder: circular pipe, glass, circular pole
(v) Christmas decorations, cheese and patio umbrellas.

Question 1:

Match the following nets with appropriate solids:

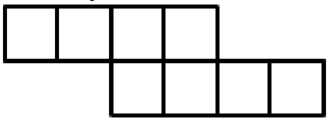


Answer:

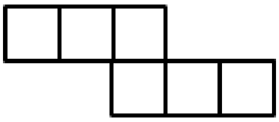
Here
(a) → (ii)
(b) → (iii)
(c) → (iv)
(d) → (i)

Question 2:

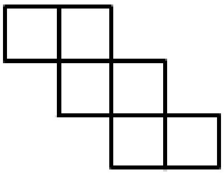
Identify the nets which can be used to make cubes (cut-out the nets and try it):



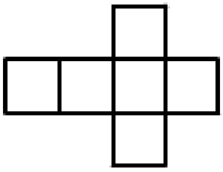
(i)



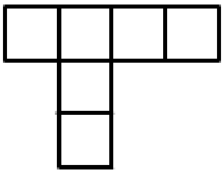
(ii)



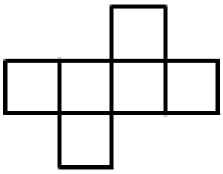
(iii)



(iv)



(v)



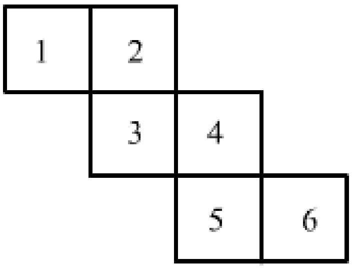
(vi)

Answer:

Only (ii), (iv) and (vi) form a cube.

Question 3:

Can the following be a net for a die? Explain your answer.

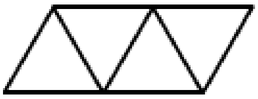


Answer:

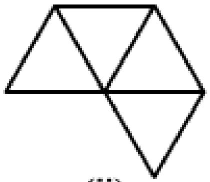
Since, in a die, the sum of the number of opposite faces of a die is 7. In the given figure, it is not possible to get the sum as 7. Hence the given net is not suitable for a die.

Question 4:

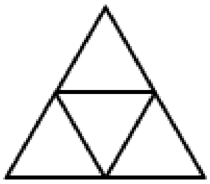
Out of the following four nets there are two correct nets to make a tetrahedron. Identify them.



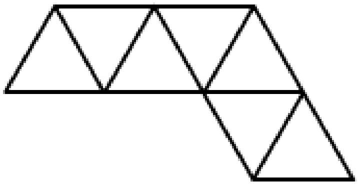
(i)



(ii)



(iii)



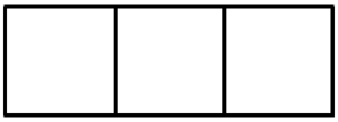
(iv)

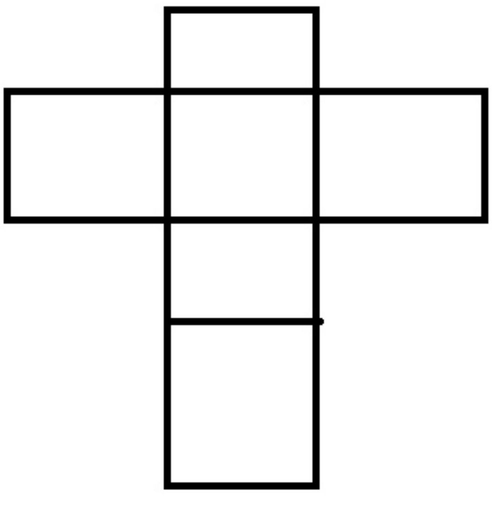
Answer:

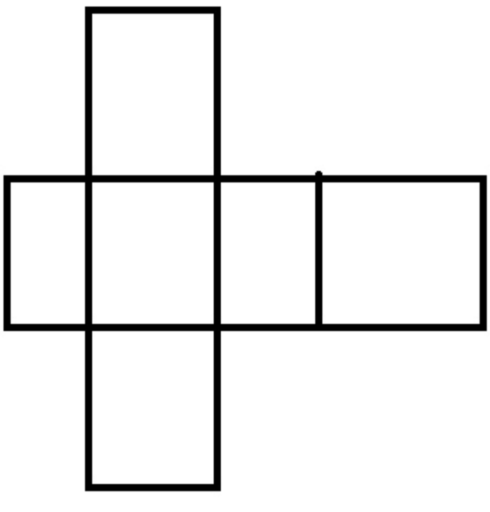
For making a tetrahedron, only (i) and (iii) are suitable nets.

Question 5:

Here is an incomplete net for making a cube. Complete it in atleast two different ways.







Mensuration I

Excercise 20.1

Question 1:

Find the area, in square metres, of a rectangle whose
(i) Length = 5.5 m, breadth = 2.4 m
(ii) Length = 180 cm, breadth = 150 cm

Answer:

We have,
(i) Length = 5.5 m, Breadth = 2.4 m
Therefore,
Area of rectangle = Length x Breadth
= 5.5 m x 2.4 m
= 13.2 m^2

(ii) Length = 180 cm = 1.8 m, Breadth = 150 cm = 1.5 m [Since 100 cm = 1 m]
Therefore,
Area of rectangle = Length x Breadth
= 1.8 m x 1.5 m
= 2.7 m^2

Question 2:

Find the area, in square centimetres, of a square whose side is
(i) 2.6 cm
(ii) 1.2 dm

Answer:

We have,
(i) Side of the square = 2.6 cm
Therefore, area of the square = $(\text{Side})^2$
= $(2.6 \text{ cm})^2$ = 6.76 cm^2

(ii) Side of the square = 1.2 dm = 1.2 x 10 cm = 12 cm [Since 1 dm = 10 cm]
Therefore, area of the square = $(\text{Side})^2$
= $(12 \text{ cm})^2$ = 144 cm^2

Question 3:

Find in square metres, the area of a square of side 16.5 dam.

Answer:

We have,
Side of the square = 16.5 dam = 16.5 x 10 m = 165 m   [Since 1 dam = 10 m]
Area of the square = $(\text{Side})^2 = (165\ \text{m})^2$
$= 27225\ \text{m}^2$

Question 4:

Find the area of a rectangular feild in ares whose sides are:
(i) 200 m and 125 m
(ii) 75 m 5 dm and 125 m

Answer:

We have,
(i) Length of the rectangular field = 200 m
Breadth of the rectangular field = 125 m
Therefore,
Area of the rectangular field = Length x Breadth
= 200 m x 125 m
= 25000 m^2 = 250 ares   [Since 100 m^2 = 1 are]

(ii) Length of the rectangular field =75 m 5 dm = (75 + 0.5) m = 75.5 m   [Since 1 dm = 10 cm = 0.1 m]
Breadth of the rectangular field = 120 m
Therefore,
Area of the rectangular field = Length x Breadth
= 75.5 m x 120 m
= 9060 m^2 = 90.6 ares   [Since 100 m^2 = 1 are]

Question 5:

Find the area of a rectangular field in hectares whose sides are:
(i) 125 m and 400 m
(ii) 75 m 5 dm and 120 m

Answer:

We have,
(i) Length of the rectangular field = 125 m
Breadth of the rectangular field = 400 m
Therefore,
Area of the rectangular field = Length x Breadth
= 125 m x 400 m
= 50000 m^2 = 5 hectares   [Since 10000 m^2 = 1 hectare]

(ii) Length of the rectangular field =75 m 5 dm = (75 + 0.5) m = 75.5 m   [Since 1 dm = 10 cm = 0.1 m]

Breadth of the rectangular field = 120 m
Therefore,
Area of the rectangular field = Length x Breadth
= 75.5 m x 120 m
= 9060 m^2 = 0.906 hectares [Since 10000 m^2 = 1 hectare]

Question 6:

A door of dimensions 3 m × 2m is on the wall of dimension 10 m × 10 m. Find the cost of painting the wall if rate of painting is Rs 2.50 per sq. m.

Answer:

We have,
Length of the door = 3 m
Breadth of the door = 2 m
Side of the wall = 10 m
Area of the wall = Side x Side = 10 m x 10 m = 100 m^2
Area of the door = Length x Breadth = 3 m x 2 m = 6 m^2
Thus,
Required area of the wall for painting = Area of the wall − Area of the door = (100 − 6) m^2 = 94 m^2
Rate of painting per square metre = Rs. 2.50
Hence, the cost of painting the wall = Rs. (94 x 2.50) = Rs. 235

Question 7:

A wire is in the shape of a rectangle. Its length is 40 cm and breadth is 22 cm. If the same wire is bent in the shape of a square, what will be the measure of each side. Also, find which side encloses more area?

Answer:

We have,
Perimeter the of rectangle = 2(Length + Breadth)
= 2(40 cm + 22 cm) = 124 cm
It is given that the wire which was in the shape of a rectangle is now bent into a square.
Therefore, the perimeter of the square = Perimeter of the rectangle
=> Perimeter of the square = 124 cm
⇒ 4 x side = 124 cm
∴ Side = $\frac{124}{4} = 31 \text{ cm}$1244=31 cm
Now,
Area of the rectangle = 40 cm x 22 cm = 880 cm^2
Area of the square = $(\text{Side})^2$ = $(31 \text{ cm})^2$ = 961 cm^2
Therefore, the square-shaped wire encloses more area.

Question 8:

How many square metres of glass will be required for a window, which has 12 panes, each pane measuring 25 cm by 16 cm?

Answer:

We have,
Length of the glass pane = 25 cm
Breadth of the glass pane = 16 cm
Area of one glass pane = 25 cm x 16 cm = 400 cm^2 = 0.04 m^2 [Since 1 m^2 = 10000 cm^2]
Thus,
Area of 12 such panes = 12 x 0.04 = 0.48 m^2

Question 9:

A marble tile measures 10 cm × 12 cm. How many tiles will be required to cover a wall of size 3 m × 4 m? Also, find the total cost of the tiles at the rate of Rs 2 per tile.

Answer:

We have,
Area of the wall = 3 m x 4 m = 12 m^2
Area of one marble tile = 10 cm x 12 cm = 120 cm^2 = 0.012 m^2 [Since 1 m^2 = 10000 cm^2]
Thus,
Number of tiles =
$\frac{\text{Area of wall}}{\text{Area of one tile}} = \frac{12\text{ m}^2}{0.012\text{ m}^2} = 1000$Area of wallArea of one tile=12 m20.012 m2=1000
Cost of one tile = Rs. 2
Total cost = Number of tiles x Cost of one tile
= Rs. (1000 x 2) = Rs. 2000

Question 10:

A table top is 9 dm 5 cm long 6 dm 5 cm broad. What will be the cost to polish it at the rate of 20 paise per square centimetre?

Answer:

We have,
Length of the table top = 9 dm 5 cm = (9 x 10 + 5) cm = 95 cm [Since 1 dm = 10 cm]
Breadth of the table top = 6 dm 5 cm = (6 x 10 + 5) cm = 65 cm
∴ Area of the table top = Length x Breadth = (95 cm x 65 cm) = 6175 cm^2
Rate of polishing per square centimetre = 20 paise = Rs. 0.20
Total cost = Rs. (6175 x 0.20) = Rs. 1235

Question 11:

A room is 9.68 m long and 6.2 m wide. Its floor is to be covered with rectangular tiles of size 22 cm by 10 cm. Find the total cost of the tiles at the rate of Rs 2.50 per tile.

Answer:

We have,
Length of the floor of the room = 9.68 m
Breadth of the floor of the room = 6.2 m
Area of the floor = 9.68 m x 6.2 m = 60.016 m^2
Length of the tile = 22 cm
Breadth of the tile = 10 cm
Area of one tile = 22 cm x 10 cm = 220 cm^2 = 0.022 m^2 [Since 1 m^2 = 10000 cm^2]
Thus,
Number of tiles = $\frac{60.016 \text{ m}^2}{0.022 \text{ m}^2} = 2728$60.016 m20.022 m2=2728
Cost of one tile = Rs. 2.50
Total cost = Number of tiles x Cost of one tile
= Rs. (2728 x 2.50) = Rs. 6820

Question 12:

One side of a square field is 179 m. Find the cost of raising a lown on the field at the rate of Rs 1.50 per square metre.

Answer:

We have,
Side of the square field = 179 m
Area of the field = $(\text{Side})^2$ = $(179 \text{ m})^2$ = 32041 m^2
Rate of raising a lawn on the field per square metre = Rs. 1.50
Thus,
Total cost of raising a lawn on the field = Rs.(32041 x 1.50) = Rs. 48061.50

Question 13:

A rectangular field is measured 290 m by 210 m. How long will it take for a girl to go two times round the field, if she walks at the rate of 1.5 m/sec?

Answer:

We have,
Length of the rectangular field = 290 m
Breadth of the rectangular field = 210 m
Perimeter of the rectangular field = 2(Length + Breadth)
= 2(290 + 210) = 1000 m
Distance covered by the girl = 2 x Perimeter of the rectangular field
= 2 x 1000 = 2000 m

The girl walks at the rate of 1.5 m/sec.
or,
Rate = 1.5 x 60 m/min = 90 m/min
Thus,
Required time to cover a distance of 2000 m = $\frac{2000 \text{ m}}{90 \text{ m/ min}} = 22\frac{2}{9}\text{min}$2000 m90 m/min=2229min
Hence, the girl will take $22\frac{2}{9}\text{min}$2229min to go two times around the field.

Question 14:

A corridor of a school is 8 m long and 6 m wide. It is to be covered with convas sheets. If the available canvas sheets have the size 2 m × 1 m, find the cost of canvas sheets required to cover the corridor at the rate of Rs 8 per sheet.

Answer:

We have,
Length of the corridor = 8 m
Breadth of the corridor = 6 m
Area of the corridor of a school = Length x Breadth = (8 m x 6 m) = 48 m^2
Length of the canvas sheet = 2 m
Breadth of the canvas sheet = 1 m
Area of one canvas sheet = Length x Breadth = (2 m x 1 m) = 2 m^2
Thus,
Number of canvas sheets = $\frac{48 \text{ m}^2}{2 \text{ m}^2} = 24$48 m22 m2=24
Cost of one canvas sheet = Rs. 8
∴ Total cost of the canvas sheets = Rs. (24 x 8) = Rs. 192

Question 15:

The length and breadth of a playground are 62 m 60 cm and 25 m 40 cm respectively. Find the cost of turfing it at Rs 2.50 per square metre. How long will a man take to go three times round the field, if he walks at the rate of 2 metres per second.

Answer:

We have,
Length of a playground = 62 m 60 cm = 62.6 m [Since 10 cm = 0.1 m]
Breadth of a playground = 25 m 40 cm = 25.4 m
Area of a playground = Length x Breadth= 62.6 m x 25.4 m = 1590.04 m^2
Rate of turfing = Rs. 2.50/m^2
∴ Total cost of turfing = Rs. (1590.04 x 2.50) = Rs. 3975.10

Again,
Perimeter of a rectangular field = 2(Length + Breadth)
= 2(62.6 + 25.4) = 176 m
Distance covered by the man in 3 rounds of a field = 3 x Perimeter of a rectangular field
= 3 x 176 m = 528 m

The man walks at the rate of 2 m/sec.
or,
Rate = 2 x 60 m/min = 120 m/min

Thus,
Required time to cover a distance of 528 m = $\frac{528 \text{ m}}{120 \text{ m/ min}} = 4.4 \text{ min}$528 m120 m/min=4.4 min
= 4 minutes 24 seconds [since 0.1 minutes = 6 seconds]

Question 16:

A lane 180 m long and 5 m wide is to be paved with bricks of length 20 cm and breadth 15 cm. Find the cost of bricks that are required, at the rate of Rs 750 per thousand.

Answer:

We have,
Length of the lane = 180 m
Breadth of the lane = 5 m
Area of a lane = Length x Breadth = 180 m x 5 m = 900 m^2
Length of the brick = 20 cm
Breadth of the brick = 15 cm
Area of a brick = Length x Breadth = 20 cm x 15 cm = 300 cm^2 = 0.03 m^2 [Since 1 m^2 = 10000 cm^2]

Required number of bricks = $\frac{900 \text{ m}^2}{0.03 \text{ m}^2} = 30000$900 m20.03 m2=30000

Cost of 1000 bricks = Rs. 750

∴ Total cost of 30,000 bricks = Rs. $\left(\frac{750 \times 30{,}000}{1000}\right) = \text{Rs. } 22{,}500$750×30,0001000=Rs. 22,500

Question 17:

How many envelopes can be made out of a sheet of paper 125 cm by 85 cm; supposing one envelope requires a piece of paper of size 17 cm by 5 cm?

Answer:

We have,
Length of the sheet of paper = 125 cm
Breadth of the sheet of paper = 85 cm
Area of a sheet of paper = Length x Breadth = 125 cm x 85 cm = 10,625 cm^2
Length of sheet required for an envelope = 17 cm
Breadth of sheet required for an envelope = 5 cm
Area of the sheet required for one envelope = Length x Breadth = 17 cm x 5 cm = 85 cm^2

Thus,
Required number of envelopes = $\frac{10,625\ \text{cm}^2}{85\ \text{cm}^2} = 125$10,625 cm285 cm2=125

Question 18:

The width of a cloth is 170 cm. Calculate the length of the cloth required to make 25 diapers, if each diaper requires a piece of cloth of size 50 cm by 17 cm.

Answer:

We have,
Length of the diaper = 50 cm
Breadth of the diaper = 17 cm
Area of cloth to make 1 diaper = Length x Breadth = 50 cm x 17 cm = 850 cm^2
Thus,
Area of 25 such diapers = (25 x 850) cm^2 = 21,250 cm^2
Area of total cloth = Area of 25 diapers
= 21,250 cm^2
It is given that width of a cloth = 170 cm
∴ Length of the cloth =
$\frac{\text{Area of cloth}}{\text{Width of a cloth}} = \frac{21,250\ \text{cm}^2}{170\ \text{cm}} = 125\ \text{cm}$Area of clothWidth of a cloth=21,250 cm2170 cm=125 cm
Hence, length of the cloth will be 125 cm.

Question 19:

The carpet for a room 6.6 m by 5.6 m costs Rs 3960 and it was made from a roll 70 cm wide. Find the cost of the carpet per metro.

Answer:

We have,
Length of a room = 6.6 m
Breadth of a room = 5.6 m
Area of a room = Length x Breadth = 6.6 m x 5.6 m = 36.96 m^2
Width of a carpet = 70 cm = 0.7 m [Since 1 m = 100 cm]

Length of a carpet =
$\frac{\text{Area of a room}}{\text{Width of a carpet}} = \frac{36.96\ \text{m}^2}{0.7\ \text{m}} = 52.8\ \text{m}$Area of a roomWidth of a carpet=36.96 m20.7 m=52.8 m
Cost of 52.8 m long roll of carpet = Rs. 3960
Therefore,
Cost of 1 m long roll of carpet = Rs. $\frac{3960}{52.8} = \text{Rs. } 75$396052.8= Rs. 75

Question 20:

A room is 9 m long, 8 m broad and 6.5 m high. It has one door of dimensions 2 m × 1.5 m and three windows each of dimensions 1.5 m × 1 m. Find the cost of white washing the walls at Rs

3.80 per square metre.

Answer:

We have,
Length of a room = 9 m
Breadth of a room = 8 m
Height of a room = 6.5 m
Area of 4 walls = 2(l + b)h
= 2(9 m + 8 m) x 6.5 m = 2 x 17 m x 6.5 m = 221 m^2
Length of a door = 2 m
Breadth of a door = 1.5 m
Area of a door = Length x Breadth = 2 m x 1.5 m = 3 m^2
Length of a window = 1.5 m
Breadth of a window = 1 m
Since, area of one window = Length x Breadth = 1.5 m x 1 m = 1.5 m^2
Thus,
Area of 3 such windows = 3 x 1.5 m^2 = 4.5 m^2

Area to be white-washed = Area of 4 walls − (Area of one door + Area of 3 windows)
Area to be white-washed = [221 − (3 + 4.5)] m^2
= (221 − 7.5) m^2 = 213.5 m^2
Cost of white-washing for 1 m^2 area = Rs. 3.80
∴ Cost of white-washing for 213.5 m^2 area = Rs. (213.5 x 3.80) = Rs. 811.30

Question 21:

A hall 36 m long and 24 m broad allowing 80 m^2 for doors and windows, the cost of papering the walls at Rs 8.40 per m^2 is Rs 9408. Find the height of the hall.

Answer:

We have,
Length of the hall = 36 m
Breadth of the hall = 24 m
Let h be the height of the hall.
Now, in papering the wall, we need to paper the four walls excluding the floor and roof of the hall.
So, the area of the wall which is to be papered = Area of 4 walls
= 2h(l + b)
= 2h (36 + 24) = 120h m^2
Now, area left for the door and the windows = 80 m^2
So, the area which is actually papered = (120h − 80) m^2
Again,
The cost of papering the walls at Rs 8.40 per m^2 = Rs. 9408.
⇒ (120h − 80) m^2 x Rs. 8.40 per m^2 = Rs. 9408

⇒ (120h − 80) m^2 = $\frac{\text{Rs. } 9408}{\text{Rs. } 8.40}$ Rs. 9408Rs. 8.40
⇒ (120h − 80) m^2 = 1120 m^2

⇒ $120h\ m^2 = (1120 + 80)\ m^2$
⇒ $120h\ m^2 = 1200\ m^2$

∴ $h = \frac{1200\ \text{m}^2}{120\ \text{m}} = 10\ \text{m}$1200 m2120 m=10 m
Hence, the height of the wall would be 10 m.

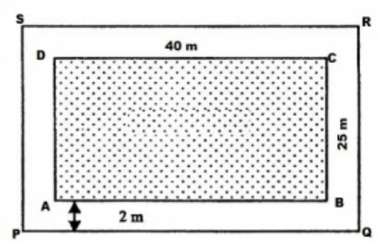
Exercise 20.2

Question 1:

A rectangular grassy lawn measuring 40 m by 25 m is to be surrounded externally by a path which is 2 m wide. Calculate the cost of levelling the path at the rate of Rs 8.25 per square metre.

Answer:

We have,
Length *AB* = 40 m and breadth *BC* = 25 m



∴ Area of lawn *ABCD* = 40 m x 25 m = 1000 m^2

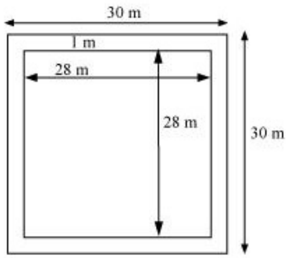
Length *PQ* = (40 + 2 + 2) m = 44 m
Breadth *QR* = (25 + 2 + 2) m = 29 m
∴ Area of *PQRS* = 44 m x 29 m = 1276 m^2
Now,
Area of the path = Area of *PQRS* − Area of the lawn *ABCD*
= 1276 m^2 − 1000 m^2
= 276 m^2
Rate of levelling the path = Rs. 8.25 per m^2
∴ Cost of levelling the path = Rs. (8.25 x 276)
= Rs. 2277

Question 2:

One metre wide path is built inside a square park of side 30 m along its sides. The remaining part of the park is covered by grass. If the total cost of covering by grass is Rs 1176, find the rate per square metre at which the park is covered by the grass.

Answer:

We have,
The side of the square garden (*a*) = 30 m



∴ Area of the square garden including the path = $a^2 = (30)^2 = 900\ m^2$
From the figure, it can be observed that the side of the square garden, when the path is not included, is 28 m.
Area of the square garden not including the path = $(28)^2 = 784\ m^2$
Total cost of covering the park with grass = Area of the park covering with green grass x Rate per square metre

1176 = 784 x Rate per square metre

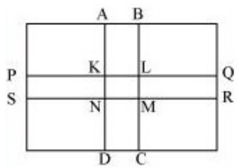
∴ Rate per square metre at which the park is covered with grass = Rs. (1176 ÷ 784)
= Rs. 1.50

Question 3:

Through a rectangular field of sides 90 m × 60 m, two roads are constructed which are parallel to the sides and cut each other at right angles through the centre of the field. If the which of the road is 3 m, find the total area cobered by the two roads.

Answer:

We have,
Length of the rectangular field = 90 m and breadth of the rectangular field = 60 m



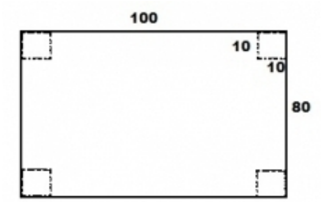
∴ Area of the rectangular field = 90 m x 60 m = $5400\ m^2$
Area of the road *PQRS* = 90 m x 3 m = $270\ m^2$
Area of the road *ABCD* = 60 m x 3 m = $180\ m^2$
Clearly, area of *KLMN* is common to the two roads.
Thus, area of KLMN = 3 m x 3 m = $9\ m^2$
Hence,
Area of the roads = Area (*PQRS*) + Area (*ABCD*) − Area (*KLMN*)
= $(270 + 180)m^2 - 9\ m^2 = 441\ m^2$

Question 4:

From a rectangular sheet of tin, of size 100 cm by 80 cm, are cut four squares of side 10 cm from each corner. Find the area of the remaining sheet.

Answer:

We have,



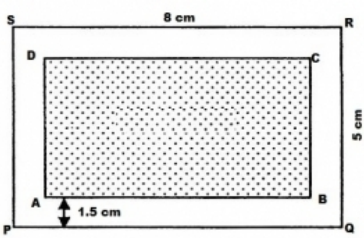
Length of the rectangular sheet = 100 cm
Breadth of the rectangular sheet = 80 cm
Area of the rectangular sheet of tin = 100 cm x 80 cm = 8000 cm^2
Side of the square at the corner of the sheet = 10 cm
Area of one square at the corner of the sheet = $(10\ cm)^2$ = 100 cm^2
∴ Area of 4 squares at the corner of the sheet = 4 x 100 cm^2 = 400 cm^2
Hence,
Area of the remaining sheet of tin =Area of the rectangular sheet − Area of the 4 squares
Area of the remaining sheet of tin = (8000 − 400) cm^2
= 7600 cm^2

Question 5:

A painting 8 cm long and 5 cm wide is painted on a cardboard such that there is a margin of 1.5 cm along each of its sides. Fund the total area of the margin.

Answer:

We have,
Length of the cardboard = 8 cm and breadth of the cardboard = 5 cm



∴ Area of the cardboard including the margin = 8 cm x 5 cm = 40 cm^2
From the figure, it can be observed that,
New length of the painting when the margin is not included = 8 cm − (1.5 cm + 1.5 cm) = (8 − 3) cm = 5 cm
New breadth of the painting when the margin is not included = 5 cm − (1.5 cm + 1.5 cm) = (5 − 3) cm = 2 cm
∴ Area of the painting not including the margin = 5 cm x 2 cm = 10 cm^2
Hence,

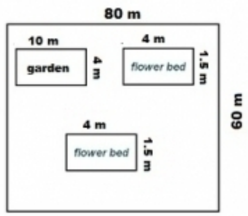
Area of the margin = Area of the cardboard including the margin − Area of the painting
$= (40 - 10)\ cm^2$
$= 30\ cm^2$

Question 6:

Rakesh has a rectangular field of length 80 m and breadth 60 m. In it, he wants to make a garden 10 m long and 4 m broad at one of the corners and at another corner, he wants to grow flowers in two floor-beds each of size 4 m by 1.5 m. In the remaining part of the field, he wants to apply mansures. Find the cost of applying the manures at the rate of Rs 300 per are.

Answer:

Length of the rectangular field = 80 m
Breadth of the rectangular field = 60 m



∴ Area of the rectangular field = 80 m x 60
$= 4800\ m^2$
Again,
Area of the garden = 10 m x 4 m = $40\ m^2$
Area of one flower bed = 4 m x 1.5 m = $6\ m^2$
Thus,
Area of two flower beds = 2 x 6 m^2 = 12 m^2
Remaining area of the field for applying manure = Area of the rectangular field − (Area of the garden + Area of the two flower beds)
Remaining area of the field for applying manure = $4800\ m^2 - (40 + 12)\ m^2$
$= (4800 - 52)\ m^2$
$= 4748\ m^2$
Since 100 m^2 = 1 are
∴ 4748 m^2 = 47.48 ares

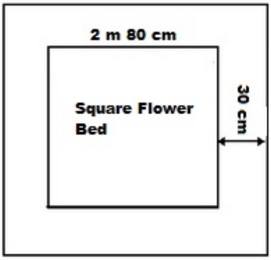
So, cost of applying manure at the rate of Rs. 300 per are will be Rs. (300 x 47.48) = Rs. 14244

Question 7:

Each side of a square flower bed is 2 m 80 cm long. It is extended by digging a strip 30 cm wide all around it. Find the area of the enlarged flower bed and also the increase in the area of the flower bed.

Answer:

We have,
Side of the flower bed = 2 m 80 cm = 2.80 m   [Since 100 cm = 1 m]



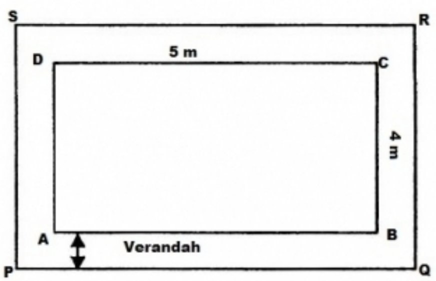
$\therefore$ Area of the square flower bed = $(\text{Side})^2 = (2.80 \text{ m})^2 = 7.84 \text{ m}^2$
Side of the flower bed with the digging strip = 2.80 m + 30 cm + 30 cm
= (2.80 + 0.3 + 0.3) m = 3.4 m
Area of the enlarged flower bed with the digging strip = $(\text{Side})^2 = (3.4)^2 = 11.56 \text{ m}^2$
Thus,
Increase in the area of the flower bed = $11.56 \text{ m}^2 - 7.84 \text{ m}^2$
$= 3.72 \text{ m}^2$

Question 8:

A room 5 m long and 4 m wide is surrounded by a verandah. If the verandah occupies an area of 22 m^2, find the width of the varandah.

Answer:

Let the width of the verandah be x m.
Length of the room AB = 5 m and BC = 4 m



$\therefore$ Area of the room = 5 m x 4 m = 20 m^2
Length of the verandah $PQ = (5 + x + x) = (5 + 2x)$ m
Breadth of the verandah $QR = (4 + x + x) = (4 + 2x)$ m
Area of verandah $PQRS = (5 + 2x) \times (4 + 2x) = (4x^2 + 18x + 20)$ m^2
$\therefore$ Area of verandah = Area of $PQRS$ − Area of $ABCD$
$\Rightarrow 22 = 4x^2 + 18x + 20 - 20$
$\Rightarrow 22 = 4x^2 + 18x$
$\Rightarrow 11 = 2x^2 + 9x$
$\Rightarrow 2x^2 + 9x - 11 = 0$
$\Rightarrow 2x^2 + 11x - 2x - 11 = 0$
$\Rightarrow x(2x + 11) - 1(2x + 11) = 0$

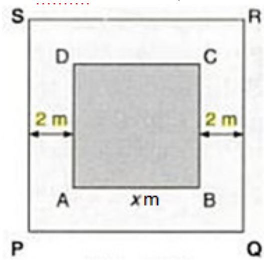
$\Rightarrow (x - 1)(2x + 11) = 0$
When $x - 1 = 0$, $x = 1$
When $2x + 11 = 0$, $x = -\frac{11}{2}$-112
The width cannot be a negative value.
So, width of the verandah = x = 1 m.

Question 9:

A square lawn has a 2 m wide path surrounding it. If the area of the path is 136 m^2, find the area of the lawn.

Answer:

We have,
Let *ABCD* be the square lawn and *PQRS* be the outer boundary of the square path.



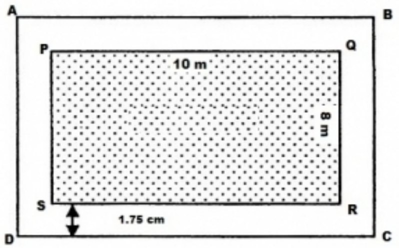
Let side of the lawn *AB* be x m.
Area of the square lawn = x^2
Length *PQ* = (x m + 2 m + 2 m) = (x + 4) m
$\therefore$ Area of *PQRS* = $(x + 4)^2 = (x^2 + 8x + 16)\ m^2$
Now,
Area of the path = Area of *PQRS* − Area of the square lawn
$\Rightarrow 136 = x^2 + 8x + 16 - x^2$
$\Rightarrow 136 = 8x + 16$
$\Rightarrow 136 - 16 = 8x$
$\Rightarrow 120 = 8x$
$\therefore x = 120 \div 8 = 15$
$\therefore$ Side of the lawn = 15 m
Hence,
Area of the lawn = $(\text{Side})^2 = (15\ m)^2 = 225\ m^2$

Question 10:

A poster of size 10 cm by 8 cm is pasted on a sheet of cardboard such that there is a margin of width 1.75 cm along each side of the poster. Find (i) the total area of the margin (ii) the cost of the cardboard used at the rate of Re 0.60 per cm^2.

Answer:

We have,
Length of the poster = 10 cm and breadth of the poster = 8 cm



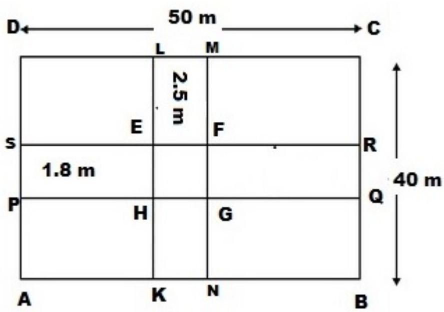
$\therefore$ Area of the poster = Length x Breadth = 10 cm x 8 cm = 80 cm^2
From the figure, it can be observed that,
Length of the cardboard when the margin is included = 10 cm + 1.75 cm + 1.75 cm = 13.5 cm
Breadth of the cardboard when the margin is included = 8 cm + 1.75 cm + 1.75 cm = 11.5 cm
$\therefore$ Area of the cardboard = Length x Breadth = 13.5 cm x 11.5 cm = 155.25 cm^2
Hence,
(i) Area of the margin = Area of cardboard including the margin − Area of the poster
= 155.25 cm^2 − 80 cm^2
= 75.25 cm^2
(ii) Cost of the cardboard = Area of the cardboard x Rate of the cardboard Rs. 0.60 per cm^2
= Rs. (155.25 x 0.60)
= Rs. 93.15

Question 11:

A rectangulr field is 50 m by 40 m. It has two roads through its centre, running parallel to its sides. The width of the longer and shorter roads are 1.8 m and 2.5 m respectively. Find the area of the roads and the erea of the remaining portion of the field.

Answer:

Let *ABCD* be the rectangular field and *KLMN* and *PQRS* the two rectangular roads with width 1.8 m and 2.5 m, respectively.



Length of the rectangular field *CD* = 50 cm and breadth of the rectangular field *BC* = 40 m
$\therefore$ Area of the rectangular field *ABCD* = 50 m x 40 m = 2000 m^2
Area of the road *KLMN* = 40 m x 2.5 m = 100 m^2
Area of the road *PQRS* = 50 m x 1.8 m = 90 m^2

Clearly area of *EFGH* is common to the two roads.
Thus, Area of *EFGH* = 2.5 m x 1.8 m = 4.5 m^2
Hence,
Area of the roads = Area (*KLMN*) + Area (*PQRS*) − Area (*EFGH)*
= (100 m^2 + 90 m^2) − 4.5 m^2 = 185.5 m^2

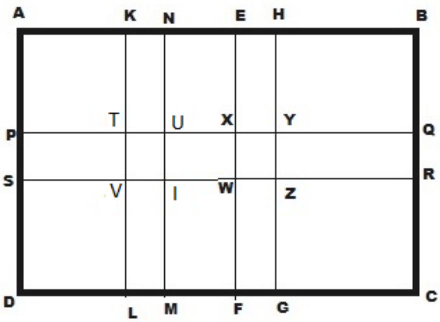
Area of the remaining portion of the field = Area of the rectangular field *ABCD* − Area of the roads
= (2000 − 185.5) m^2
= 1814.5 m^2

Question 12:

There is a rectangular field of size 94 m × 32 m. Three roads each of 2 m width pass through the field such that two roads are parallel to the breadth of the field and the third is parallel to the length. Calculate: (i) area of the field covered by the three roads (ii) area of the field not covered by the roads.

Answer:

Let *ABCD* be the rectangular field.
Here,
Two roads which are parallel to the breadth of the field *KLMN* and *EFGH* with width 2 m each.
One road which is parallel to the length of the field *PQRS* with width 2 m.



Length of the rectangular field *AB* = 94 m and breadth of the rectangular field *BC* = 32 m
∴ Area of the rectangular field = Length x Breadth = 94 m x 32 m = 3008 m^2
Area of the road *KLMN* = 32 m x 2 m = 64 m^2
Area of the road *EFGH* = 32 m x 2 m = 64 m^2
Area of the road *PQRS* = 94 m x 2 m = 188 m^2

Clearly area of *TUVI* and *WXYZ* is common to these three roads.
Thus,
Area of *TUVI* = 2 m x 2 m = 4 m^2
Area of *WXYZ* = 2 m x 2 m = 4 m^2
Hence,
(i) Area of the field covered by the three roads:
= Area (*KLMN*) + Area (*EFGH)* + Area (*PQRS*) − {Area (*TUVI*) + Area (*WXYZ*)}
= [64+ 64 + 188 − (4 + 4)] m^2
= 316 m^2 − 8 m^2
= 308 m^2

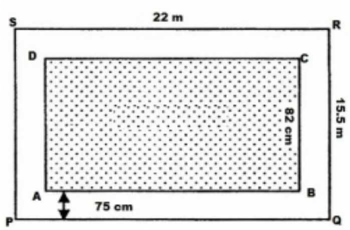
(ii) Area of the field not covered by the roads:
= Area of the rectangular field *ABCD* − Area of the field covered by the three roads
= 3008 m^2 − 308 m^2
= 2700 m^2

Question 13:

A school has a hall which is 22 m long and 15.5 m broad. A carpet is laid inside the hall leaving all around a margin of 75 cm from the walls. Find the area of the carpet and the area of the strip left uncoverd. If the width of the carpet is 82 cm, find the cost at the rate of Rs 18 per metre.

Answer:

We have,
Length of the hall *PQ* = 22 m and breadth of the hall *QR* = 15.5 m



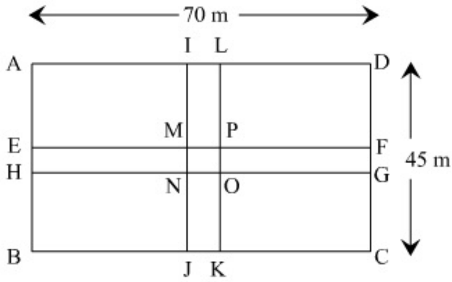
∴ Area of the school hall *PQRS* = 22 m x 15.5 m = 341 m^2
Length of the carpet *AB* = 22 m − (0.75 m + 0.75 m) = 20.5 m [Since 100 cm = 1 m]
Breadth of the carpet *BC* = 15.5 m − (0.75 m + 0.75 m) = 14 m
∴ Area of the carpet *ABCD* = 20.5 m x 14 m = 287 m^2
Area of the strip = Area of the school hall *PQRS* − Area of the carpet *ABCD*
= 341 m^2 − 287 m^2
= 54 m^2
Again,
Area of the 1 m length of carpet = 1 m x 0.82 m = 0.82 m^2
Thus,
Length of the carpet whose area is 287 m^2 = 287 m^2 ÷ 0.82 m^2 = 350 m
Cost of the 350 m long carpet = Rs. 18 x 350 = Rs. 6300

Question 14:

Two cross roads, each of width 5 m, run at right angles through the centre of a rectangular park of length 70 m and breadth 45 m parallel to its sides. Find the area of the roads. Also, find the cost of constructing the roads at the rate of Rs 105 per m^2.

Answer:

Let *ABCD* be the rectangular park then *EFGH* and *IJKL* the two rectangular roads with width 5 m.



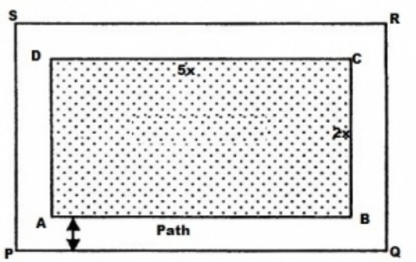
Length of the rectangular park *AD* = 70 cm
Breadth of the rectangular park *CD* = 45 m
∴ Area of the rectangular park = Length x Breadth = 70 m x 45 m = 3150 m^2
Area of the road *EFGH* = 70 m x 5 m = 350 m^2
Area of the road *JKIL* = 45 m x 5 m = 225 m^2
Clearly area of *MNOP* is common to the two roads.
Thus, Area of *MNOP* = 5 m x 5 m = 25 m^2
Hence,
Area of the roads = Area (*EFGH*) + Area (*JKIL*) − Area (*MNOP*)
= (350 + 225) m^2− 25 m^2 = 550 m^2
Again, it is given that the cost of constructing the roads = Rs. 105 per m^2
Therefore,
Cost of constructing 550 m^2 area of the roads = Rs. (105 × 550)
= Rs. 57750.

Question 15:

The length and breadth of a rectangular park are in the ratio 5 : 2. A 2.5 m wide path running all around the outside the park has an area 305 m^2. Find the dimensiions of the park.

Answer:

We have,
Area of the path = 305 m^2



Let the length of the park be 5*x* m and the breadth of the park be 2*x* m
Thus,
Area of the rectangular park = 5*x* x 2*x* = $10x^2$ m^2
Width of the path = 2.5 m
Outer length *PQ* = 5*x* m + 2.5 m + 2.5 m = (5*x* + 5) m

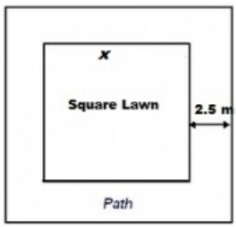
Outer breadth QR = $2x$ + 2.5 m + 2.5 m = $(2x + 5)$ m
Area of $PQRS$ = $(5x + 5)$ m x $(2x + 5)$ m = $(10x^2 + 25x + 10x + 25)$ m^2= $(10x^2 + 35x + 25)$ m^2
∴ Area of the path = $[(10x^2 + 35x + 25) - 10x^2]$ m^2
⇒ $305 = 35x + 25$
⇒ $305 - 25 = 35x$
⇒ $280 = 35x$
⇒ $x = 280 \div 35 = 8$
Therefore,
Length of the park = $5x$ = 5 x 8 = 40 m
Breadth of the park = $2x$ = 2 x 8 = 16 m

Question 16:

A square lawn is surrounded by a path 2.5 m wide. If the area of the path is 165 m^2, find the area of the lawn.

Answer:

Let the side of the lawn be x m.



Given that width of the path = 2.5 m
Side of the lawn including the path = $(x + 2.5 + 2.5)$ m = $(x + 5)$ m
So, area of lawn = (Area of the lawn including the path) − (Area of the path)
We know that the area of a square = (Side)^2
∴ Area of lawn $(x^2) = (x + 5)^2 - 165$
⇒ $x^2 = (x^2 + 10x + 25) - 165$
⇒ $165 = 10x + 25$
⇒ $165 - 25 = 10x$
⇒ $140 = 10x$
Therefore $x = 140 \div 10 = 14$
Thus the side of the lawn = 14 m
Hence,
The area of the lawn = (14 m)^2 = 196 m^2

Exericse 20.3

Question 1:

Find the area of a parallogram with base 8 cm and altitude 4.5 cm.

Answer:

We have,
Base = 8 cm and altitude = 4.5 cm
Thus,
Area of the parallelogram = Base x Altitude
= 8 cm x 4.5 cm
= 36 cm^2

Question 2:

Find the area in square metres of the parallelogram whose base and altitudes are as under:
(i) Base = 15 dm, altitude = 6.4 dm
(ii) Base = 1 m 40 cm, altitude = 60 cm

Answer:

We have,
(i) Base = 15 dm = (15 x 10) cm = 150 cm = 1.5 m [Since 100 cm = 1 m]
Altitude = 6.4 dm = (6.4 x 10) cm = 64 cm = 0.64 m
Thus,
Area of the parallelogram = Base x Altitude
= 1.5 m x 0.64 m
= 0.96 m^2

(ii) Base = 1 m 40 cm = 1.4 m [Since 100 cm = 1 m]
Altitude = 60 cm = 0.6 m
Thus,
Area of the parallelogram = Base x Altitude
= 1.4 m x 0.6 m
= 0.84 m^2

Question 3:

Find the altitude of a parallelogram whose area is 54 dm^2 and base is 12 dm.

Answer:

We have,
Area of the given parallelogram = 54 dm^2
Base of the given parallelogram = 12 dm
∴ Altitude of the given parallelogram = $\frac{\text{Area}}{\text{Base}} = \frac{54}{12}\text{dm} = 4.5\ \text{dm}$AreaBase=5412dm =4.5 dm

Question 4:

The area of a rhombus is 28 m^2. If its perimeter be 28 m, find its altitude.

Answer:

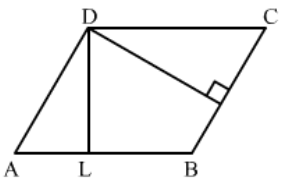
We have,
Perimeter of a rhombus = 28 m
$\therefore$ 4(Side) = 28 m [Since perimeter = 4(Side)]

$\Rightarrow$ Side = $\frac{28\text{ m}}{4} = 7\text{ m}$28 m4=7 m
Now,
Area of the rhombus = 28 m^2
$\Rightarrow$ (Side x Altitude) = 28 m^2
$\Rightarrow$ (7 m x Altitude) = 28 m^2

$\Rightarrow$ Altitude = $\frac{28\text{ m}^2}{7\text{m}} = 4\text{ m}$28 m27m=4 m

Question 5:

In Fig. 20, *ABCD* is a parallelogram, *DL* $\perp$ *AB* and *DM* $\perp$ *BC*. If AB = 18 cm, *BC* = 12 cm and *DM* = 9.3 cm, find *DL*.



Answer:

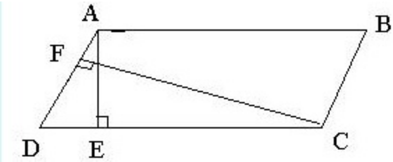
We have,
Taking *BC* as the base,
BC = 12 cm and altitude *DM* = 9.3 cm
$\therefore$ Area of parallelogram *ABCD* = Base x Altitude
= (12 cm x 9.3 cm) = 111.6 cm^2 .... (i)
Now,
Taking *AB* as the base, we have,
Area of the parallelogram *ABCD* = Base x Altitude = (18 cm x *DL*).................(ii)
From (i) and (ii), we have
18 cm x *DL* = 111.6 cm^2
$\Rightarrow$ *DL* = $\frac{111.6\ \text{cm}^2}{18\ \text{cm}} = 6.2\ \text{cm}$111.6 cm218 cm=6.2 cm

Question 6:

The longer side of a parallelogram is 54 cm and the corresponding altitude is 16 cm. If the altitude corresponding to the shorter side is 24 cm, find the length of the shorter side.

Answer:

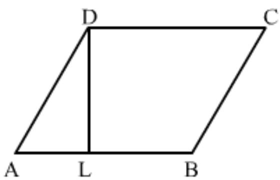
We have,
ABCD is a parallelogram with the longer side *AB* = 54 cm and corresponding altitude *AE* = 16 cm.
The shorter side is *BC* and the corresponding altitude is *CF* = 24 cm.



Area of a parallelogram = base × height.We have two altitudes and two corresponding bases. So,
$\frac{1}{2} \times BC \times CF = \frac{1}{2} \times AB \times AE$12×BC×CF =12×AB×AE
⇒ *BC* x *CF* = *AB* x *AE*
⇒ *BC* x 24 = 54 x 16
⇒ BC = $\frac{54 \times 16}{24} = 36 \text{ cm}$54×1624=36 cm
Hence, the length of the shorter side *BC* = *AD* = 36 cm.

Question 7:

In Fig. 21, *ABCD* is a parallelogram, *DL* ⊥ *AB*. If *AB* = 20 cm, *AD* = 13 cm and area of the parallelogram is 100 cm^2, find *AL*.

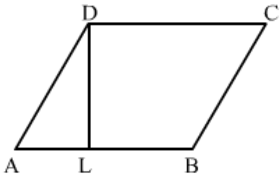


Answer:

We have,
ABCD is a parallelogram with base *AB* = 20 cm and corresponding altitude *DL.*
It is given that the area of the parallelogram *ABCD* = 100 cm^2
Now,
Area of a parallelogram = Base x Height
100 cm^2 = *AB* x *DL*
100 cm^2 = 20 cm x *DL*
∴ *DL* = $\frac{100 \text{ cm}^2}{20 \text{ cm}} = 5 \text{ cm}$100 cm220 cm= 5 cm
Again by Pythagoras theorem, we have,
$(AD)^2 = (AL)^2 + (DL)^2$
⇒ $(13)^2 = (AL)^2 + (5)^2$
⇒ $(AL)^2 = (13)^2 - (5)^2$
= 169 − 25 = 144
⇒ $(AL)^2 = (12)^2$
⇒ *AL* = 12 cm
Hence. length of *AL* is 12 cm.

Question 8:

In Fig. 21, if AB = 35 cm, AD = 20 cm and area of the parallelogram is 560 cm^2, find LB.



Answer:

We have,
$ABCD$ is a parallelogram with base AB = 35 cm and corresponding altitude DL. The adjacent side of the parallelogram AD = 20 cm.
It is given that the area of the parallelogram $ABCD$ = 560 cm^2
Now,
Area of the parallelogram = Base x Height
560 cm^2 = AB x DL
560 cm^2 = 35 cm x DL

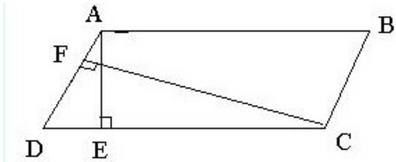
$\therefore DL = \frac{560\ \text{cm}^2}{35\ \text{cm}} = 16\ \text{cm}$560 cm235 cm= 16 cm
Again by Pythagoras theorem, we have,
$(AD)^2 = (AL)^2 + (DL)^2$
$\Rightarrow (20)^2 = (AL)^2 + (16)^2$
$\Rightarrow (AL)^2 = (20)^2 - (16)^2$
$= 400 - 256 = 144$
$\Rightarrow (AL)^2 = (12)^2$
$\Rightarrow AL = 12$ cm
From the figure,
$AB = AL + LB$
35 cm = 12 cm + LB
$\therefore LB$ = 35 cm − 12 cm
= 23 cm
Hence, length of LB is 23 cm.

Question 9:

The adjacent sides of a parallelogram are 10 m and 8 m. If the distance between the longer sides is 4 m, find the distance between the shorter sides.

Answer:

We have,
$ABCD$ is a parallelogram with side AB = 10 m and corresponding altitude AE = 4 m.
The adjacent side AD = 8 m and the corresponding altitude is CF.



Area of a parallelogram = Base × Height
We have two altitudes and two corresponding bases. So,
AD x *CF* = *AB* x *AE*
⇒ 8 m x *CF* = 10 m x 4 m

⇒ *CF* = $\frac{10\times4}{8} = 5\ \text{m}$10×48=5 m
Hence, the distance between the shorter sides is 5 m.

Question 10:

The base of a parallelogram is twice its height. If the area of the parallelogram is 512 cm^2, find the base and height.

Answer:

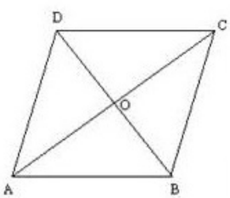
Let the height of the parallelogram be x cm.
Then the base of the parallelogram is $2x$ cm.
It is given that the area of the parallelogram = 512 cm^2
So,
Area of a parallelogram = Base x Height
512 cm^2 = $2x \times x$
512 cm^2 = $2x^2$
⇒ $x^2 = \frac{512\ \text{cm}^2}{2} = 256\ \text{cm}^2$512 cm22= 256 cm2
⇒ $x^2 = (16\ \text{cm})^2$
⇒ x = 16 cm
Hence, base = $2x$ = 2 x 16 = 32 cm and height = x = 16 cm.

Question 11:

Find the area of a rhombus having each side equal to 15 cm and one of whose diagonals is 24 cm.

Answer:

Let *ABCD* be the rhombus where diagonals intersect at *O*.



Then *AB* = 15 cm and *AC* = 24 cm.

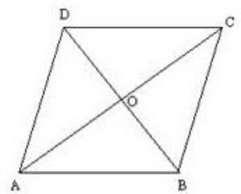
The diagonals of a rhombus bisect each other at right angles.
Therefore, Δ *AOB* is a right-angled triangle, right angled at *O* such that
OA = $\frac{1}{2}AC$ 12AC = 12 cm and *AB* = 15 cm.
By Pythagoras theorem, we have,
$(AB)^2 = (OA)^2 + (OB)^2$
$\Rightarrow (15)^2 = (12)^2 + (OB)^2$
$\Rightarrow (OB)^2 = (15)^2 - (12)^2$
$\Rightarrow (OB)^2 = 225 - 144 = 81$
$\Rightarrow (OB)^2 = (9)^2$
$\Rightarrow OB = 9$ cm
$\therefore BD = 2 \times OB = 2 \times 9$ cm = 18 cm
Hence,
Area of the rhombus *ABCD* =
$\left(\frac{1}{2} \times AC \times BD\right) = \left(\frac{1}{2} \times 24 \times 18\right) = 216\ \text{cm}^2$ 12×AC×BD=12×24×18=216 cm2

Question 12:

Find the area of a rhombus, each side of which measures 20 cm and one of whose diagonals is 24 cm.

Answer:

Let *ABCD* be the rhombus whose diagonals intersect at *O*.



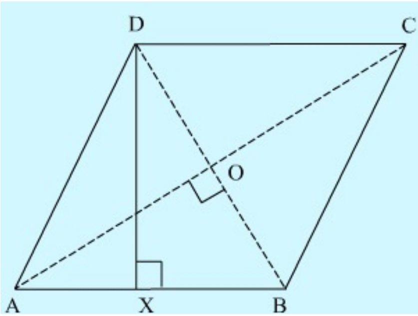
Then *AB* = 20 cm and *AC* = 24 cm.
The diagonals of a rhombus bisect each other at right angles.
Therefore Δ *AOB* is a right-angled triangle, right angled at *O* such that
OA = $\frac{1}{2}AC$ 12AC = 12 cm and *AB* = 20 cm
By Pythagoras theorem, we have,
$(AB)^2 = (OA)^2 + (OB)^2$
$\Rightarrow (20)^2 = (12)^2 + (OB)^2$
$\Rightarrow (OB)^2 = (20)^2 - (12)^2$
$\Rightarrow (OB)^2 = 400 - 144 = 256$
$\Rightarrow (OB)^2 = (16)^2$
$\Rightarrow OB = 16$ cm
$\therefore BD = 2 \times OB = 2 \times 16$ cm = 32 cm
Hence,
Area of the rhombus *ABCD* =
$\left(\frac{1}{2} \times AC \times BD\right) = \left(\frac{1}{2} \times 24 \times 32\right) = 384\ \text{cm}^2$ 12×AC×BD=12×24×32=384 cm2

Question 13:

The length of a side of a square field is 4 m. What will be the altitude of the rhombus, if the area of the rhombus is equal to the square field and one of its diagonals is 2 m?

Answer:

We have,
Side of a square = 4 m and one diagonal of a square = 2 m



Area of the rhombus = Area of the square of side 4 m

⇒ $\left(\frac{1}{2} \times AC \times BD\right) = (4\text{ m})^2$ 12×AC×BD= 4 m2
⇒ $\left(\frac{1}{2} \times AC \times 2\text{ m}\right) = 16\text{ m}^2$ 12×AC×2 m=16 m2
⇒ *AC* = 16 m

We know that the diagonals of a rhombus are perpendicular bisectors of each other.

⇒ $AO = \frac{1}{2}AC = 8\text{ m}$AO=12AC=8 m and $BO = \frac{1}{2}BD = 1\text{ m}$BO =12BD=1 m
By Pythagoras theorem, we have:
$AO^2 + BO^2 = AB^2$
⇒ $AB^2 = (8\text{ m})^2 + (1\text{m})^2 = 64\text{ m}^2 + 1\text{ m}^2 = 65\text{ m}^2$
⇒ Side of a rhombus = *AB* = $\sqrt{65}$65 m.
Let *DX* be the altitude.
Area of the rhombus = *AB* × *DX*
16 m^2 = $\sqrt{65}$65 m x *DX*
∴ *DX* = $\frac{16}{\sqrt{65}}\text{m}$1665m
Hence, the altitude of the rhombus will be $\frac{16}{\sqrt{65}}\text{m}$1665m.

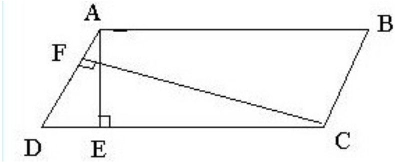
Question 14:

Two sides of a parallelogram are 20 cm and 25 cm. If the altitude corresponding to the sides of length 25 cm is 10 cm, find the altitude corresponding to the other pair of sides.

Answer:

We have,
ABCD is a parallelogram with longer side *AB* = 25 cm and altitude *AE* = 10 cm.

As ABCD is a parallelogram .hence *AB=CD* (opposite sides of parallelogram are equal)
The shorter side is *AD* = 20 cm and the corresponding altitude is *CF*.



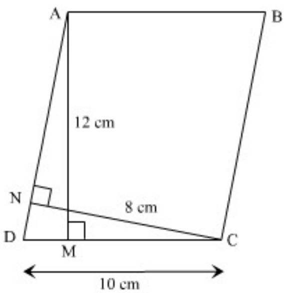
Area of a parallelogram = Base × Height
We have two altitudes and two corresponding bases.
So,
⇒ *AD* x *CF* = *CD* x *AE*
⇒ 20 x *CF* = 25 x 10
∴ *CF* = $\frac{25\times10}{20} = 12.5\ \text{cm}$25×1020=12.5 cm
Hence, the altitude corresponding to the other pair of the side *AD* is 12.5 cm.

Question 15:

The base and corresponding altitude of a parallelogram are 10 cm and 12 cm respectively. If the other altitude is 8 cm, find the length of the other pair of parallel sides.

Answer:

We have,
ABCD is a parallelogram with side *AB* = *CD* = 10 cm (Opposite sides of parallelogram are equal) and corresponding altitude *AM* = 12 cm.
The other side is *AD* and the corresponding altitude is *CN* = 8 cm



Area of a parallelogram = Base × Height
We have two altitudes and two corresponding bases.
So,
⇒ *AD* x *CN* = *CD* x *AM*
⇒ *AD* x 8 = 10 x 12

⇒ *AD* = $\frac{10\times12}{8} = 15\ \text{cm}$10×128=15 cm
Hence, the length of the other pair of the parallel side = 15 cm.

Question 16:

A floral design on the floor of a building consists of 280 tiles. Each tile is in the shape of a parallelogram of altitude 3 cm and base 5 cm. Find the cost of polishing the design at the rate of 50 paise per cm^2.

Answer:

We have,
Altitude of a tile = 3 cm
Base of a tile = 5 cm
Area of one tile = Altitude x Base = 5 cm x 3 cm = 15 cm^2
Area of 280 tiles = 280 x 15 cm^2 = 4200 cm^2
Rate of polishing the tiles at 50 paise per cm^2 = Rs. 0.5 per cm^2
Thus,
Total cost of polishing the design = Rs. (4200 x 0.5) = Rs. 2100

Exercise 20.4

Question 1:

Find the area in square centimetres of a triangle whose base and altitude are as under:
(i) base = 18 cm, altitude = 3.5 cm
(ii) base = 8 dm, altitude = 15 cm

Answer:

We know that the area of a triangle = $\frac{1}{2} \times \text{Base} \times \text{Height}$12×Base×Height
(i) Here, base = 18 cm and height = 3.5 cm
∴ Area of the triangle = $\left(\frac{1}{2} \times 18 \times 3.5\right) = 31.5 \text{ cm}^2$ 12×18×3.5=31.5 cm2
(ii) Here, base = 8 dm = (8 x 10) cm = 80 cm [Since 1 dm = 10 cm]
and height = 3.5 cm
∴ Area of the triangle = $\left(\frac{1}{2} \times 80 \times 15\right) = 600 \text{ cm}^2$ 12×80×15=600 cm2

Question 2:

Find the altitude of a triangle whose area is 42 cm^2 and base is 12 cm.

Answer:

We have,
Altitude of a triangle = $\frac{2 \times \text{Area}}{\text{Base}}$2×AreaBase
Here, base = 12 cm and area = 42 cm^2

∴ Altitude = $\frac{2 \times 42}{12} = 7 \text{ cm}$2×4212= 7 cm

Question 3:

The area of a triangle is 50 cm^2. If the altitude is 8 cm, what is its base?

Answer:

We have,
Base of a triangle = $\frac{2\times\text{Area}}{\text{Altitude}}$2×AreaAltitude
Here, altitude = 8 cm and area = 50 cm^2

∴ Altitude = $\frac{2\times50}{8} = 12.5\ \text{cm}$2×508=12.5 cm

Question 4:

Find the area of a right angled triangle whose sides containing the right angle are of lengths 20.8 m and 14.7 m.

Answer:

In a right-angled triangle, the sides containing the right angles are of lengths 20.8 m and 14.7 m.
Let the base be 20.8 m and the height be 14.7 m.
Then,
Area of a triangle = $\frac{1}{2}\times\text{Base}\times\text{Height}$12×Base×Height

= $\frac{1}{2}\times20.8\times14.7 = 152.88\ \text{m}^2$ 12×20.8×14.7 = 152.88 m2

Question 5:

The area of a triangle, whose base and the corresponding altitude are 15 cm and 7 cm, is equal to area of a right triangle whose one of the sides containing the right angle is 10.5 cm. Find the other side of this triangle.

Answer:

For the first triangle, we have,
Base = 15 cm and altitude = 7 cm
Thus, area of a triangle = $\frac{1}{2}\times\text{Base}\times\text{Altitude}$12×Base×Altitude

= $\frac{1}{2}\times15\times7 = 52.5\ \text{cm}^2$ 12×15×7=52.5 cm2
It is given that the area of the first triangle and the second triangle are equal.
Area of the second triangle = 52.5 cm^2
One side of the second triangle = 10.5 cm
Therefore,
The other side of the second triangle = $\frac{2\times\text{Area}}{\text{One side of a triangle}}$2×AreaOne side of a triangle

= $\frac{2\times52.5}{10.5} = 10\ \text{cm}$2×52.510.5=10 cm
Hence, the other side of the second triangle will be 10 cm.

Question 6:

A rectangular field is 48 m long and 20 m wide. How many right triangular flower beds, whose sides containing the right angle measure 12 m and 5 m can be laid in this field?

Answer:

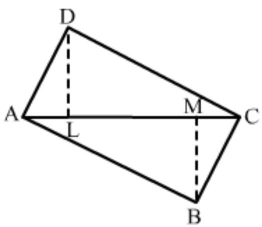
We have,
Length of the rectangular field = 48 m
Breadth of the rectangular field = 20 m
Area of the rectangular field = Length x Breadth = 48 m x 20 m = 960 m^2

Area of one right triangular flower bed = $\frac{1}{2} \times 12\ \text{m} \times 5\ \text{m} = 30\ \text{m}^2$ 12×12 m×5 m = 30 m2
Therefore,

Required number of right triangular flower beds = $\frac{960\ \text{m}^2}{30\ \text{m}^2} = 32$960 m230 m2= 32

Question 7:

In Fig. 29, *ABCD* is a quadrilateral in which diagonal AC = 84 cm; $DL \perp AC$, $BM \perp AC$, DL = 16.5 cm and BM = 12 cm. Find the area of quadrilateral *ABCD*.

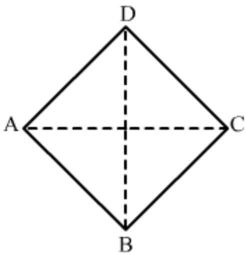


Answer:

We have,
AC = 84 cm, DL = 16.5 cm and BM = 12 cm
Area of Δ *ADC* = $\frac{1}{2}$12x AC x DL
= $\frac{1}{2}$12x 84 cm x 16.5 cm = 693 cm^2
Area of Δ *ABC* = $\frac{1}{2}$12x AC x BM
= $\frac{1}{2}$12x 84 cm x 12 cm = 504 cm^2
Hence,
Area of quadrilateral *ABCD* = Area of Δ *ADC* + Area of Δ *ABC*
= (693 + 504) cm^2
= 1197 cm^2

Question 8:

Find the area of the quadrilateral ABCD given in Fig. 30. The diagonals *AC* and BD measure 48 m and 32 m respectively and are perpendicular to each other.

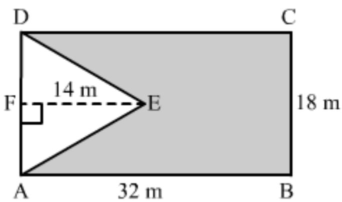


Answer:

We have,
Diagonal *AC* = 48 cm and diagonal *BD* = 32 m
∴ Area of a quadrilateral = $\frac{1}{2}$12x Product of diagonals
= $\frac{1}{2}$12x *AC* x *BD*
= ($\frac{1}{2}$12 x 48 x 32) m^2 = (24 x 32) m^2 = 768 m^2

Question 9:

In Fig 31, *ABCD* is a rectangle with dimensions 32 m by 18 m. *ADE* is a triangle such that *EF* ⊥ *AD* and *EF* = 14 cm. Calculate the area of the shaded region.



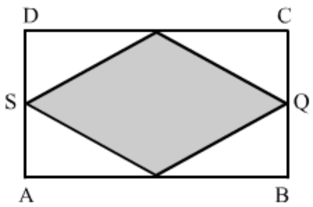
Answer:

We have,
Area of the rectangle = *AB* x *BC*
= 32 m x 18 m
= 576 m^2
Area of the triangle = $\frac{1}{2}$12x *AD* x *FE*
= $\frac{1}{2}$12x BC x FE [Since AD = BC]
= $\frac{1}{2}$12x 18 m x 14 m
= 9 m x 14 m = 126 m^2
∴ Area of the shaded region = Area of the rectangle − Area of the triangle

$=(576 - 126)\ m^2$
$= 450\ m^2$

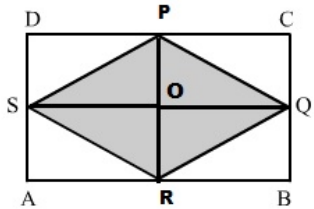
Question 10:

In Fig. 32, *ABCD* is a rectangle of length *AB* = 40 cm and breadth *BC* = 25 cm. If *P*, *Q*, *R*, *S* be the mid-points of the sides *AB*, *BC*, *CD* and *DA* respectively, find the area of the shaded region.



Answer:

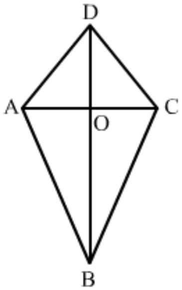
We have,
Join points *PR* and *SQ*.
These two lines bisect each other at point *O.*



Here, *AB* = *DC* = *SQ* = 40 cm and *AD* = *BC* =*RP* = 25 cm
Also *OP* = *OR* = $\frac{RP}{2} = \frac{25}{2} = 12.5$ cmRP2=252=12.5 cm
From the figure we observed that,
Area of Δ *SPQ* = Area of Δ *SRQ*
Hence, area of the shaded region = 2 x (Area of Δ *SPQ*)
= 2 x ($\frac{1}{2}$12 x *SQ* x *OP)*
= 2 x ($\frac{1}{2}$12 x 40 cm x 12.5 cm)
= 500 cm^2

Question 11:

Calculate the area of the quadrilateral *ABCD* as shown in Fig. 33, given that *BD* = 42 cm, *AC* = 28 cm, *OD* = 12 cm and *AC* ⊥ *BD*.



Answer:

We have,
BD = 42 cm, AC = 28 cm, OD = 12 cm
Area of ΔABC = $\frac{1}{2}$12 x AC x OB
= $\frac{1}{2}$12 x AC x ($BD - OD$)
= $\frac{1}{2}$12 x 28 cm x (42 cm − 12 cm) = $\frac{1}{2}$12 x 28 cm x 30 cm = 14 cm x 30 cm = 420 cm^2
Area of Δ ADC = $\frac{1}{2}$12 x AC x OD
= $\frac{1}{2}$12 x 28 cm x 12 cm = 14 cm x 12 cm = 168 cm^2
Hence,
Area of the quadrilateral $ABCD$ = Area of Δ ABC + Area of Δ ADC

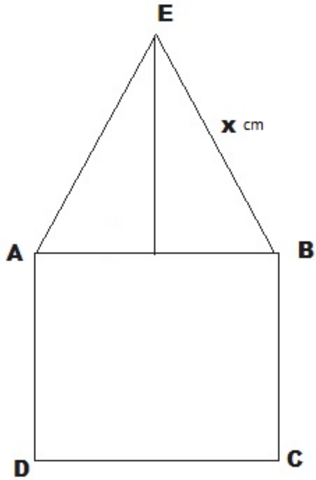
= (420 + 168) cm^2 = 588 cm^2

Question 12:

Find the area of a figure formed by a square of side 8 cm and an isosceles triangle with base as one side of the square and perimeter as 18 cm.

Answer:

Let x cm be one of the equal sides of an isosceles triangle.



Given that the perimeter of the isosceles triangle = 18 cm
Then,
$x + x + 8 = 18$
$\Rightarrow 2x = (18 - 8)$ cm = 10 cm
$\Rightarrow x = 5$ cm
Area of the figure formed = Area of the square + Area of the isosceles triangle

= (Side of square)^2 + $\frac{1}{2} \times \text{Base} \times \sqrt{(\text{Equal side})^2 - \frac{1}{4} \times (\text{Base})^2}$ 12×Base×Equal side2 -14×Base2

= $(8)^2 + \frac{1}{2} \times 8 \times \sqrt{(5)^2 - \frac{1}{4}(8)^2}$ 12×8×52 -1482

= $64 + 4 \times \sqrt{25 - 16}$ 64+4 ×25-16

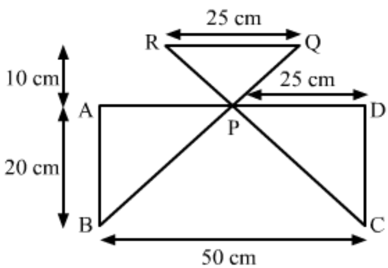
= $64 + 4 \times \sqrt{9}$ 64 +4 ×9

= $64 + 4 \times 3$ 64+4×3

= 64 + 12 = 76 cm^2

Question 13:

Find the area of Fig. 34 in the following ways:
(i) Sum of the areas of three triangles
(ii) Area of a rectangle − sum of the areas of five triangles



Answer:

We have,

(i) *P* is the midpoint of *AD*.

Thus *AP* = *PD* = 25 cm and *AB* = *CD* = 20 cm

From the figure, we observed that,

Area of Δ *APB* = Area of Δ *PDC*

Area of Δ APB = $\frac{1}{2}$12x *AB* x *AP*

= $\frac{1}{2}$12x 20 cm x 25 cm = 250 cm^2

Area of Δ *PDC* = Area of Δ *APB* = 250 cm^2

Area of Δ *RPQ* = $\frac{1}{2}$12x Base x Height

= $\frac{1}{2}$12x 25 cm x 10 cm = 125 cm^2

Hence,

Sum of the three triangles = (250 + 250 + 125) cm^2

= 625 cm^2

(ii) Area of the rectangle *ABCD* = 50 cm x 20 cm = 1000 cm^2

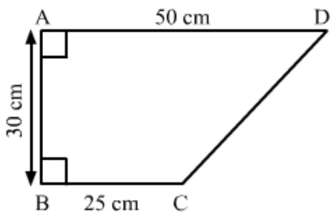
Thus,

Area of the rectangle − Sum of the areas of three triangles (There is a mistake in the question; it should be area of three triangles)

= (1000 − 625) cm^2 = 375 cm^2

Question 14:

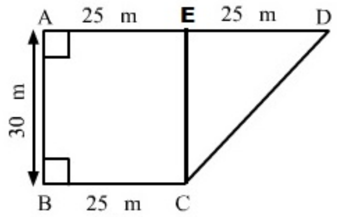
Calculate the area of quadrilateral field *ABCD* as shown in Fig. 35, by dividing it into a rectangle and a triangle.



Answer:

We have,

Join *CE*, which intersect *AD* at point *E.*



Here, $AE = ED = BC$ = 25 m and $EC = AB$ = 30 m

Area of the rectangle $ABCE = AB \times BC$

= 30 m x 25 m

= 750 m^2

Area of Δ CED = $\frac{1}{2}$12x $EC \times ED$

= $\frac{1}{2}$12x 30 m x 25 m

= 375 m^2

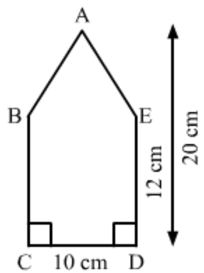
Hence,

Area of the quadrilateral $ABCD$ = (750 + 375) m^2

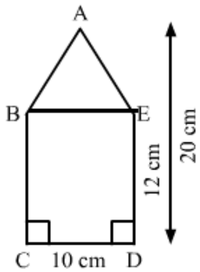
= 1125 m^2

Question 15:

Calculate the area of the pentagon *ABCDE*, where $AB = AE$ and with dimensions as shown in Fig. 36.



Answer:



Join *BE*.
Area of the rectangle *BCDE* = *CD* x *DE*
= 10 cm x 12 cm = 120 cm^2
Area of Δ*ABE* = $\frac{1}{2}$12x *BE* x height of the triangle
= $\frac{1}{2}$12x 10 cm x (20 − 12) cm
= $\frac{1}{2}$12x 10 cm x 8 cm = 40 cm^2
Hence,
Area of the pentagon *ABCDE* = (120 + 40) cm^2 = 160 cm^2

Question 16:

The base of a triangular field is three times its altitude. If the cost of cultivating the field at Rs 24.60 per hectare is Rs 332. 10, find its base and height.

Answer:

Let altitude of the triangular field be *h* m
Then base of the triangular field is *3h* m.

Area of the triangular field = $\frac{1}{2} \times h \times 3h = \frac{3h^2}{2} \mathrm{m}^2$ 12×h×3h=3h22m2..........(i)

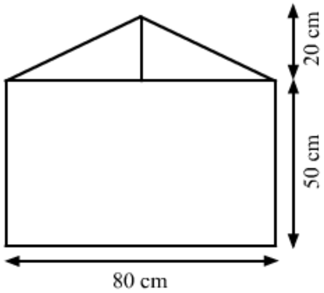
The rate of cultivating the field is Rs 24.60 per hectare.
Therefore,
Area of the triangular field = $\frac{332.10}{24.60} = 13.5$ hectare 332.10 24.60=13.5 hectare
= 135000 m^2 [Since 1 hectare = 10000 m^2]..........(ii)
From equation (i) and (ii) we have,
$\frac{3h^2}{2} = 135000 \mathrm{~m}^2$ 3h22=135000 m2
3h^2 = 135000 x 2 = 270000 m^2

h^2 = $\frac{270000}{3}\mathrm{m}$270000 3m^2=90000 m2 = (300 m)^2
⇒ *h* = 300 m

Hence,
Height of the triangular field = 300 m and base of the triangular field = 3 x 300 m = 900 m

Question 17:

A wall is 4.5 m long and 3 m high. It has two equal windows, each having form and dimensions as shown in Fig. 37. Find the cost of painting the wall (leaving windows) at the rate of Rs 15 per m_2.



Answer:

We have,
Length of a wall = 4.5 m
Breadth of the wall =3 m
Area of the wall = Length x Breadth = 4.5 m x 3 m = 13.5 m^2
From the figure we observed that,
Area of the window = Area of the rectangle + Area of the triangle
= (0.8 m x 0.5 m) + ($\frac{1}{2}$12 x 0.8 m x 0.2 m)   [Since 1 m = 100 cm]
= 0.4 m^2 + 0.08 m^2 = 0.48 m^2
Area of two windows = 2 x 0.48 = 0.96 m^2
Area of the remaining wall (leaving windows) = (13.5 − 0.96)m^2 = 12.54 m^2
Cost of painting the wall per m^2 = Rs. 15
Hence, the cost of painting on the wall = Rs. (15 x 12.54) = Rs. 188.1
(In the book, the answer is given for one window, but we have 2 windows.)

Objective Type Questions

Question 1:

If the area of a square is 225 m^2, then its perimeter is

(a) 15 m   (b) 60 m   (c) 225 m   (d) 30 m

Answer:

Let a be the side of the square. Then
Area of square = a^2
$\Rightarrow\Rightarrow$ 225 = a^2
$\Rightarrow\Rightarrow$ $a^2 = 15^2$
$\Rightarrow\Rightarrow$ a = 15 m
Perimeter of the square = 4a = 4 $\times\times$ 15 = 60 m
Hence, the correct option is (b).

Question 2:

If the perimeter of a square is 16 cm, then its area is

(a) 4 cm^2 (b) 8 cm^2 (c) 16 cm^2 (d) 12 cm^2

Answer:

Let *a* be the side of the square. Then
Perimeter = 4*a*
$\Rightarrow\Rightarrow$ 16 = 4*a*
$\Rightarrow\Rightarrow$ *a* = 4 cm
Area of the square = $a^2 = 4^2 = 16$ cm^2
Hence, the correct option is (c).

Question 3:

The length of a rectangle is 8 cm and its area is 48 cm^2. The perimeter of the rectangle is

(a) 14 cm (b) 24 cm (c) 12 cm (d) 28 cm

Answer:

Let *a* and *b* be the length and breadth of the rectangle respectively. Then
Area of the rectangle = *ab*
$\Rightarrow\Rightarrow$ 48 = *a* $\times$× 8 ($\because$ *b* = 8 cm)
$\Rightarrow\Rightarrow$ *a* = 6 cm
Perimeter of the rectangle = 2(*a* + *b*) = 2(6 + 8) = 28 cm
Hence, the correct option is (d).

Question 4:

The area of a square and that of a square drawn on its diagonal are in the ratio

(a) 1 : $\sqrt{2}$2 (b) 1 : 2 (c) 1 : 3 (d) 1 : 4

Answer:

Let *a* be the side of the square. Then
Area of the square = a^2
Area of the square drawn on the diagona = $\left(a\sqrt{2}\right)^2 = 2a^2$ a22=2a2
Required ratio = $a^2 : 2a^2 = 1 : 2$
Hence, the correct option is (b).

Question 5:

The length of the diagonal of a square is *d*. the area of the square is

(a) d^2 (b) $\frac{1}{2}d^2$ 12d2 (c) $\frac{1}{4}d^2$ 14d2 (d) $2d^2$

Answer:

Let *a* be the side of the square. Then
Diagonal of the square = $a\sqrt{2}$a2
Therefore
$a\sqrt{2} = d \Rightarrow a = \frac{d}{\sqrt{2}}$ a2=d⇒a=d2∴ Area of the square =d22=d22
$\therefore$ Area of the square $= \left(\frac{d}{\sqrt{2}}\right)^2 = \frac{d^2}{2}$
Hence, the correct option is (b).

Question 6:

The ratio of the areas of two squares, one having its diagonal double that of the other, is

(a) 2 : 1 (b) 3 : 1 (c) 3 : 2 (d) 4 : 1

Answer:

Let *d* be the diagonal of the second square. Then, the diagonal of the first square will be 2*d*.
$\because$ Side of a square $= \frac{\text{Diagonal}}{\sqrt{2}}$ Side of a square=Diagonal2
$\therefore$ Required ratio $= \frac{\left(\frac{2d}{\sqrt{2}}\right)^2}{\left(\frac{d}{\sqrt{2}}\right)^2} = \frac{2d^2}{\frac{d^2}{2}} = 4 : 1$ Required ratio=2d22d22=2d2d22=4:1
Hence, the correct option is (d).

Question 7:

If the ratio of the areas of two squares is 9 : 1, then the ratio of their perimeters is

(a) 2 : 1 (b) 3 : 1 (c) 3 : 2 (d) 4 : 1

Answer:

Let *a* and *b* be the sides of the squares, then as per the question
$\frac{a^2}{b^2} = \frac{9}{1} \Rightarrow \left(\frac{a}{b}\right)^2 = \frac{3^2}{1} \Rightarrow \frac{a}{b} = \frac{3}{1}$ a2b2=91⇒ab2=321⇒ab=31
Therefore
Ratio of the perimeters of the squares $= \frac{4a}{4b} = \frac{a}{b} = \frac{3}{1}$ Ratio of the perimeters of the square

Thus, the of the required ratio is 3 : 1.
Hence, the correct option is (b).

Question 8:

The ratio of the area of a square of side *a* and that of an equilateral triangle of side *a* is

(a) 2 : 1 (b) 2 : $\sqrt{3}$3 (c) 4 : 3 (d) 4 : $\sqrt{3}$3

Answer:

Area of the square = a^2
Area of the equilateral triangle = $\frac{a^2\sqrt{3}}{4}$a234
$\frac{\text{Area of square}}{\text{Area of equilateral triangle}} = \frac{a^2}{\frac{a^2\sqrt{3}}{4}} = \frac{4}{\sqrt{3}}$Area of squareArea of equilateral triangle=a2a234=43
Thus, the required ratio is 4 : $\sqrt{3}$3.
Hence, the correct option is (d).

Question 9:

On increasing each side of a square by 25%, the increase in area will be

(a) 25% (b) 55% (c) 55.5%
(d) 56.25%

Answer:

Let *a* be the side of the square. Then
Side of the new square = *a* + 25% of *a* = $a + a \times \frac{25}{100} = \frac{5a}{4}$a+a×25100=5a4
Old area = a^2
New area = $\left(\frac{5a}{4}\right)^2 = \frac{25a^2}{16}$5a42=25a216
% increase in the area =
$\frac{\text{Change in area}}{\text{Old area}} \times 100 = \frac{\frac{25a^2}{16} - a^2}{a^2} \times 100 = \frac{9}{16} \times 100 = 56.25$Change in areaOld area×100=25a2
Hence, the correct option is (d).

Question 10:

The area of a square is 50 cm^2. The length of its diagonal is

(a) 5$\sqrt{2}$2 cm (b) 10 cm (c) 10$\sqrt{2}$2 cm (d) 8 cm

Answer:

Let *a* be the side of the square. Then
Area of the square = a^2 = 50 cm^2 ⇒ $a = \sqrt{50} = 5\sqrt{2}$ cm⇒a=50=52 cm

Now
Diagonal of the square = $a\sqrt{2} = 5\sqrt{2} \times \sqrt{2} = 5 \times 2 = 10 \text{ cm}$a2=52×2=5×2=10 cm
Hence,the correct option is (b).

Question 11:

Each diagonal of a square is 14 cm. Its area is

(a) 196 cm^2 (b) 88 cm^2 (c) 98 cm^2 (d) 148 cm^2

Answer:

Let *a* be the side of the square. Then
Diagonal of the square =$a\sqrt{2} = 14 \Rightarrow a = \frac{14}{\sqrt{2}} \text{ cm}$a2=14⇒a=142 cm
Now
Area of the square = $a^2 = \left(\frac{14}{\sqrt{2}}\right)^2 = 98 \text{ cm}^2$a2=1422=98 cm2
Hence,the correct option is (c).

Question 12:

The area of a square filed is 64 m^2. A path of uniform width is laid around and outside of it.
If the area of the path is 17 m^2, then the width of the path is

(a) 1 m (b) 1.5 m (c) 0.5 m (d) 2 m

Answer:

Let *a* be the side of inner square. Then
$\text{Area of inner square} = a^2$ Area of inner square=a2⇒64=a2⇒a=64=8 cm
$\Rightarrow 64 = a^2 \Rightarrow a = \sqrt{64} = 8 \text{ cm}$
Let *x* be the width of the path, then
Side of outer square = (a + x) cm = (8 + x) cm
Now
Area of path = Area of outer square − Area of inner square
17 = $(8 + x)^2$ − 64
$\Rightarrow (8 + x)^2 = 64 + 17 = 81$⇒8+x2=64+17=81⇒8+x=9⇒x=1 m
$\Rightarrow 8 + x = 9 \Rightarrow x = 1 \text{ m}$
Thus, the width of the path is 1 m.
Hence, the correct option is (a).

Question 13:

A path of 1 m runs around and inside a square garden of side of 20 m. The cost of levelling the path
at the rate of ₹2.25 per square metre is

(a) ₹154 (b) ₹164 (c) ₹182 (d) ₹171

Answer:

Width of the path = 1 m
Side of the square garden = 20 m
Side of the inner square = (20 − 2) m = 18 m
∴ Area of the path = Area of square garden − Area of inner square

$= 20^2 - 18^2$ =202-182=400-324=76 m2

$= 400 - 324$

$= 76 \text{ m}^2$

Cost of levelling = ₹2.25 $\times$× 76 = ₹171
Thus, the required cost is ₹171.

Question 14:

The length of and breadth of a rectangle are (3*x* + 4) cm and (4*x* − 13) cm. If the perimeter of the rectangle is 94 cm, then *x* =

(a) 4 (b) 8 (c) 12 (d) 6

Answer:

Here, *l* = (3*x* + 4) cm and *b* = (4*x* − 13) cm.
Perimeter of rectangle = 2 (*l* + *b*)
= 2[(3*x* + 4) + (4*x* − 13)]
= 2(7*x* − 9) = 14*x* − 18
Now, as per the question
Perimeter of rectangle = 94 cm

$\therefore\ 14x - 18 = 94$ ∴ 14x-18=94⇒14x=94+18=112⇒x=11214=8

$\Rightarrow 14x = 94 + 18 = 112$

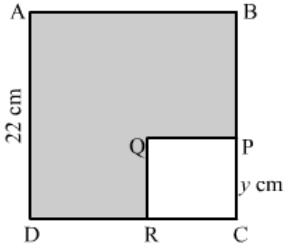
$\Rightarrow x = \frac{112}{14} = 8$

Hence, the correct option is (b).

Question 15:

In Fig. 38, *ABCD* and *PQRC* are squares such that *AD* = 22 cm and *PC* = *y* cm. If the area of the shaded region is 403 cm^2,
then the value of *y* is

(a) 3 (b) 6 (c) 9 (d) 10



Answer:

Here, *AD* = 22 cm.
Area of square *ABCD* = $(22)^2$ cm^2 = 484 cm^2
Area of square *PQRC* = y^2 cm^2
Now, as per the question
Area of shaded region = Area of square ABCD − Area of square *PQRC*
$403 = 484 - y^2$ 403=484-y2⇒y2=484-403=81⇒y=9
$\Rightarrow y^2 = 484 - 403 = 81$
$\Rightarrow y = 9$
Hence, the correct option is (c).

Question 16:

The length and breadth of a rectangle are (3*x* + 4) cm and (4*x* − 13) cm respectively. If the perimeter of the rectangle is 94 cm, then its area is
(a) 432 cm^2 (b) 512 cm^2 (c) 542 cm^2
(d) 532 cm^2

Answer:

Here, *l* = (3*x* + 4) cm, *b* = (4*x* − 13) cm and Perimeter of rectangle = 94 cm.
Perimeter of rectangle = 2(*l* + *b*) = 2[(3*x* + 4) + (4*x* − 13)] = (14*x* − 18) cm
As per the question
14*x* − 18 = 94 ⇒ $x = \frac{94+18}{14} = 8$⇒x=94+1814=8
Now
l = 3 ×× 8 + 4 = 28 cm
b = 4 ×× 8 − 13 = 19 cm
Area of rectangle = *l* ×× *b* = 28 ×× 19 = 532 cm^2
Hence, the correct option is (d).

Question 17:

The length and breadth of a rectangle are in the ratio 3 : 2. If the area is 216 cm^2, then its perimeter is

(a) 60 cm (b) 30 cm (c) 40 cm (d) 120 cm

Answer:

Here, l = 3x cm, b = 2x cm and area of rectangle = 216 cm^2.
Area of rectangle = l ×× b = 3x ×× 2x = 6x^2 cm^2
As per the question
216 = 6x^2 $\Rightarrow x^2 = 36 \Rightarrow x = 6$⇒x2=36⇒x=6
Now
l = 3x = 3 ×× 6 = 18 cm
b = 2x = 2 ×× 6 = 12 cm
Perimeter of rectangle = 2(l + b) = 2(18 + 12) = 60 cm
Hence, the correct option is (a).

Question 18:

If the length of a diagonal of a rectangle of length 16 cm is 20 cm, then its area is

(a) 192 cm^2 (b) 320 cm^2 (c) 160 cm^2
(d) 156 cm^2

Answer:

Here, l = 16 cm, Length of diagonal = 20 cm. Let b be the breadth of the rectangle.
In the right-angled triangle formed with the adjacent sides and the diagonal, using Pythagoras theorem, we get
$l^2 + b^2 = (\text{Diagonal})^2$ l2+b2=Diagonal2⇒162+b2=202⇒b2=202-162=144⇒b=12 cm
$\Rightarrow 16^2 + b^2 = 20^2$
$\Rightarrow b^2 = 20^2 - 16^2 = 144$
$\Rightarrow b = 12\ \text{cm}$
Area of rectangle = l ×× b = 16 ×× 12 = 192 cm^2
Hence, the correct option is (a).

Question 19:

The area of a rectangle 144 cm long is same as that of a square of side 84 cm. The width of the rectangle is

(a) 7 cm (b) 14 cm (c) 49 cm (d) 28 cm

Answer:

Here, Length of rectangle = 144 cm, Area of square = 84 cm^2.
Let b be the breadth of the rectangle, then as per the question

Area of rectangle = Area of square

$\Rightarrow 144 \times b = 84^2$ ⇒144×b=842⇒b=84×84144=49 cm

$\Rightarrow b = \frac{84 \times 84}{144} = 49 \text{ cm}$

Thus, the breadth of the rectangle is 49 cm.

Hence, the correct option is (c).

Question 20:

The length and breadth of a rectangular field are in the ratio 5 : 3 and its perimeter is 480 m. The area of the field is

(a) 7200 m^2 (b) 13500 m^2 (c) 15000 m^2
(d) 54000 m^2

Answer:

Let *l* = 5*x* and *b* = 3*x* be the length and breadth of the rectangular field. Here, perimeter = 480 m.
So, as per the question
Perimeter = 2 (*l* + *b*)

$\Rightarrow 480 = 2(5x + 3x)$⇒480=25x+3x⇒16x=480⇒x=30

$\Rightarrow 16x = 480$

$\Rightarrow x = 30$

l = 5 ×× 30 = 150 m
b = 3 ×× 30 = 90 m
Now
Area of the rectangular filed = *l* ×× *b* = 150 ×× 90 = 13500 m^2
Hence, the correct option is (b).

Question 21:

The length of a rectangular field is thrice its breadth and its perimeter is 240 m. The length of the filed is

(a) 30 m (b) 120 m (c) 90 m (d) 80 m

Answer:

Let *l* and *b* be the length and breadth of the rectangular field, then *l* = 3*b*.
So, as per the question
Perimeter = 2 (*l* + *b*)

$\Rightarrow 240 = 2(3b + b)$⇒240=23b+b⇒8b=240⇒b=30 m

$\Rightarrow 8b = 240$

$\Rightarrow b = 30 \text{ m}$

$l = 3b = 3 \times 30 = 90 \text{ m}$ l=3b=3×30=90 m

Hence, the correct option is (c).

Question 22:

If the diagonal of a rectangle is 17 cm and its perimeter is 46 cm, the area of the rectangle is

(a) 100 cm^2 (b) 110 cm^2 (c) 120 cm^2 (d) 240 cm^2

Answer:

Let *l* and *b* be the length and breadth of the rectangle, where diagonal = 17 cm and perimeter = 46 cm.
So, as per the question
Perimeter = 2 (*l* + *b*)
$\Rightarrow\Rightarrow 46 = 2\,(l + b)$
$\Rightarrow\Rightarrow l + b = 23$ ..... (*i*)
Now, in the triangle formed by the adjacent sides and one diagonal of the rectangle, using Pythagoras theorem, we have
$l^2 + b^2 = (\text{diagonal})^2$
$\Rightarrow\Rightarrow l^2 + b^2 = 17^2$
$\Rightarrow\Rightarrow l^2 + (23 - l)^2 = 17^2$ [From (*i*)]
$\Rightarrow\Rightarrow l^2 + l^2 + 23^2 - 46l = 289$
$\Rightarrow\Rightarrow 2l^2 + 529 - 46l = 289$
$\Rightarrow\Rightarrow 2l^2 - 46l + 240 = 0$
$\Rightarrow\Rightarrow l^2 - 23l + 120 = 0$
$\Rightarrow\Rightarrow l^2 - 15l - 8l + 120 = 0$
$\Rightarrow\Rightarrow l(l - 15) - 8(l - 15) = 0$
$\Rightarrow\Rightarrow (l - 15)\,(l - 8) = 0$
$\Rightarrow\Rightarrow l = 15$ cm or $l = 8$ cm
If $l = 15$ cm, then from (*i*), $b = 23 - 15 = 8$ cm.
If $l = 8$ cm, then from (*i*), $b = 23 - 8 = 23$ cm.
Therefore
Area of the rectangle = $l \times\times b = 15 \times\times 8 = 120$ cm2
Hence, the correct option is (c).

Question 23:

The length and breadth of a rectangular field are 4 m and 3 m respectively. The field is divided into two
parts by fencing diagonally. The cost of fencing at the rate of ₹10 per metre is

(a) ₹50 (b) ₹30 (c) ₹190 (d) ₹240

Answer:

Let *l* and *b* be the length and breadth of the rectangle respectively. Then
$l = 4$ m and $b = 3$ m

Now, in the triangle formed by the adjacent sides and one diagonal of the rectangle, using Pythagoras theorem, we have
$l^2 + b^2 = (\text{Diagonal})^2$
$\Rightarrow\Rightarrow (\text{Diagonal})^2 = 4^2 + 3^2 = 16 + 9 = 25$
$\Rightarrow\Rightarrow$ Diagonal = 5 m
Length of fencing = $2(l + b)$ + Length of diagonal
= 2(4 + 3) + 5
= 14 + 5
= 19 m
Cost of fencing = ₹10 ×× 19 = ₹190
Hence, the correct option is (c).

Question 24:

The area of a parallelogram is 100 cm^2. If the base is 25 cm, then the corresponding height is

(a) 4 cm (b) 6 cm (c) 10 cm (d) 5 cm

Answer:

Let b = 25 cm and h be the base and the corresponding height of the parallelogram. Then
Area of parallelogram = $b \times\times h$
$\Rightarrow\Rightarrow 100 = 25 \times\times h$
$\Rightarrow\Rightarrow h = 4$ cm
Hence, the correct option is (a).

Question 25:

The base of a parallelogram is twice of its height. If its area is 512 cm^2, then the length of base is

(a) 16 cm (b) 32 cm (c) 48 cm (d) 64 cm

Answer:

Let b and h be the base and height, then $b = 2h$.
Area of parallelogram = $b \times\times h$
$\Rightarrow\Rightarrow 512 = 2h \times\times h$
$\Rightarrow\Rightarrow 2h^2 = 512$
$\Rightarrow\Rightarrow h^2 = 256$
$\Rightarrow\Rightarrow h = 16$ cm
$\Rightarrow\Rightarrow b = 2 \times\times 16 = 32$ cm
Hence, the correct option is (b).

Question 26:

The lengths of the diagonals of a rhombus are 36 cm and 22.5 cm. Its area is

(a) 8.10 cm^2 (b) 405 cm^2 (c) 202.5 cm^2 (d) 1620 cm^2

Answer:

Here, d_1 = 36 cm and $d2$ = 22.5 cm.
Area of parallelogram =
$\frac{1}{2}(d_1 \times d_2) = \frac{1}{2}(36 \times 22.5) = 405\ \text{cm}^2$ 12d1×d2=1236×22.5=405 cm2
Hence, the correct option is (b).

Question 27:

The length of a diagonal of a rhombus is 16 cm. If its area is 96 cm^2, then the length of other diagonal is

(a) 6 cm (b) 8 cm (c) 12 cm (d) 18 cm

Answer:

Let d_1 and d_2 be the diagonals of the rhombus, where d_1 = 16 and area of rhombus = 96 cm^2.
Area of parallelogram = $\frac{1}{2}(d_1 \times d_2)$12d1×d2
$\Rightarrow 96 = \frac{1}{2}(16 \times d_2)$ ⇒96=1216×d2⇒d2=96×216=12 cm
$\Rightarrow d_2 = \frac{96\times2}{16} = 12\ \text{cm}$
Thus, the length of other diagonal is 12 cm.
Hence, the correct option is (c).

Question 28:

The length of the diagonals of a rhombus of a rhombus are 8 cm and 14 cm. The area of one of the 4 triangles formed by the diagonals is

(a) 12 cm^2 (b) 8 cm^2 (c) 16 cm^2 (d) 14 cm^2

Answer:

Let d_1 = 8 cm and d_2 = 14 cm.
Area of parallelogram = $\frac{1}{2}(d_1 \times d_2)$12d1×d2
$= \frac{1}{2}(8 \times 14)$=128×14=56 cm2
$= 56\ \text{cm}^2$
Since, the diagonals of a rhombus divides it into 4 equal parts, so
Area of the required triangle = $\frac{56}{4} = 14\ \text{cm}^2$564=14 cm2
Hence, the correct option is (d).

Question 29:

The length of a rectangle 8 cm more than the breadth. If the perimeter of the rectangle is 80 cm, then the length of the rectangle is
(a) 16 cm (b) 24 cm (c) 28 cm (d) 18 cm

Answer:

Let *l* and *b* be the length and breadth of the rectangle, then *l* = *b* + 8.
Perimeter of rectangle = 2 (*l* + *b*)
= 2(*l* + *l* − 8)
= 4*l* − 16
$\Rightarrow 80 = 4l - 16$ ⇒80=4l-16⇒4l=80+16=96⇒l=24 cm
$\Rightarrow 4l = 80 + 16 = 96$
$\Rightarrow l = 24\ \text{cm}$
Hence, the correct option is (b).

Question 30:

The length of a rectangle 8 cm more than the breadth. If the perimeter of the rectangle is 80 cm, then the area of the rectangle is
(a) 192 cm^2 (b) 364 cm^2 (c) 384 cm^2 (d) 382 cm^2

Answer:

Let *l* and *b* be the length and breadth of the rectangle, then *l* = *b* + 8.
Perimeter of rectangle = 2 (*l* + *b*)
= 2(*l* + *l* − 8)
= 4*l* − 16
$\Rightarrow 80 = 4l - 16$ ⇒80=4l-16⇒4l=80+16=96⇒l=24 cm⇒b=l-8=24-8=16 cm
$\Rightarrow 4l = 80 + 16 = 96$
$\Rightarrow l = 24\ \text{cm}$
$\Rightarrow b = l - 8 = 24 - 8 = 16\ \text{cm}$
Area of rectangle = $l \times b = 24 \times 16 = 384\ \text{cm}^2$ l×b=24×16=384 cm2
Hence, the correct option is (c).

Question 31:

The area of a rectangle is 11.6 m^2. If its breadth is 46.4 cm, then the perimeter is
(a) 25.464 m (b) 50.928 m (c) 101.856 m (d) None of these

Answer:

Here, area of rectangle(*A*) = 11.6 m^2, breadth(*b*) = 46.4 cm = 0.464 m.
Let *l* be the length of the rectangle, then

Area of rectangle = *l* ×× *b*

= *l* ×× 0.464

Area of rectangle = 11.6

$\Rightarrow l \times 0.464 = 11.6$ ⇒l×0.464=11.6⇒l=11.60.464=11600464=25 m

$\Rightarrow l = \frac{11.6}{0.464} = \frac{11600}{464} = 25 \text{ m}$

Now

Perimeter = 2(*l* + *b*)

$= 2(25 + 0.464)$=225+0.464=2(25.464)=50.928 m

$= 2(25.464)$

$= 50.928 \text{ m}$

Hence, the correct option is (b).

Question 32:

The area of a rhombus is 119 cm^2 and its perimeter is 56 cm. The height of the rhombus is

(a) 7.5 cm (b) 6.5 cm (c) 8.5 cm (d) 9.5 cm

Answer:

Let *b* the side of the rhombus and *h* be its height.

Perimeter of rhombus = 56 cm

$\Rightarrow 4b = 56$ ⇒4b=56⇒b=14 cm

$\Rightarrow b = 14 \text{ cm}$

Now

Area of rhombus = 119 cm^2

$\Rightarrow b \times h = 119$ ⇒b×h=119⇒14×h=119⇒h=172=8.5 cm

$\Rightarrow 14 \times h = 119$

$\Rightarrow h = \frac{17}{2} = 8.5 \text{ cm}$

Hence, the correct option is (c).

Question 33:

Each side of an equilateral triangle is 8 cm. Its area is

(a) $16\sqrt{3}$3 cm^2 (b) $32\sqrt{3}$3 cm^2 (c) $24\sqrt{3}$3 cm^2 (d) $8\sqrt{3}$3 cm^2

Answer:

$\text{Area of equilateral triangle} = \frac{(\text{Side})^2\sqrt{3}}{4}$ Area of equilateral triangle=Side234

$= \frac{(8)^2\sqrt{3}}{4}$ =8234=163 cm2

$= 16\sqrt{3} \text{ cm}^2$

Hence, the correct option is (a).

Question 34:

The area of an equilateral triangle is $4\sqrt{3}$3 cm^2. The length of each of its side is

(a) 3 cm (b) 4 cm (c) $2\sqrt{3}$3 cm (d) $\frac{\sqrt{3}}{2}$32 cm

Answer:

$\text{Area of equilateral triangle} = \frac{(\text{Side})^2\sqrt{3}}{4}$ Area of equilateral triangle=Side234⇒43=Side234⇒Side

$\Rightarrow 4\sqrt{3} = \frac{(\text{Side})^2\sqrt{3}}{4}$

$\Rightarrow (\text{Side})^2 = 16$

$\Rightarrow \text{Side} = 4 \text{ cm}$

Hence, the correct option is (b).

Question 35:

The height of an equilateral triangle is $\sqrt{6}$6 cm. Its area is

(a) $3\sqrt{3}$3 cm^2 (b) $2\sqrt{3}$3 cm^2 (c) $2\sqrt{2}$2 cm^2 (d) $6\sqrt{2}$2 cm^2

Answer:

Let *a* and *h* respectively be the side and height of the equilateral triangle.

$\text{Area of equilateral triangle} = \frac{a^2\sqrt{3}}{4}$ Area of equilateral triangle=a234⇒a234=12×a×h⇒a34=12×

$\Rightarrow \frac{a^2\sqrt{3}}{4} = \frac{1}{2} \times a \times h$

$\Rightarrow \frac{a\sqrt{3}}{4} = \frac{1}{2} \times \sqrt{6}$

$\Rightarrow a = 2\sqrt{2}$

Therefore

$\text{Area of equilateral triangle} = \frac{(2\sqrt{2})^2\sqrt{3}}{4} = 2\sqrt{3} \text{ cm}^2$ Area of equilateral triangle=22234=23 cm2

Hence, the correct option is (b).

Question 36:

If *A* is the area of an equilateral triangle of height *h*, then

(a) *A* =$\sqrt{3}$3 h^2 (b) $\sqrt{3}$3*A* = *h* (c) $\sqrt{3}$3*A* = h^2 (d) 3*A* = h^2

Answer:

Let *a* and *h* be the side and height of the equilateral triangle respectively. Then

$\frac{a^2\sqrt{3}}{4} = \frac{1}{2} \times a \times h$a234=12×a×h⇒a34=12×h⇒a=2h3

$\Rightarrow \frac{a\sqrt{3}}{4} = \frac{1}{2} \times h$

$\Rightarrow a = \frac{2h}{\sqrt{3}}$

Therefore

$\text{Area of equilateral triangle } (A) = \frac{a^2\sqrt{3}}{4}$Area of equilateral triangle A=a234⇒A=2h3234=h23⇒

$\Rightarrow A = \frac{\left(\frac{2h}{\sqrt{3}}\right)^2\sqrt{3}}{4} = \frac{h^2}{\sqrt{3}}$

$\Rightarrow \sqrt{3}A = h^2$

Hence, the correct option is (c).

Question 37:

If area of an equilateral triangle is $3\sqrt{3}$3 cm^2, then its height is

(a) 3 cm (b) $\sqrt{3}$3 cm (c) 6 cm (d) $2\sqrt{3}$3 cm

Answer:

Let *a* and *h* be respectively the side and height of the equilateral triangle. Then

$\frac{a^2\sqrt{3}}{4} = \frac{1}{2} \times a \times h$a234=12×a×h⇒a34=12×h⇒a=2h3

$\Rightarrow \frac{a\sqrt{3}}{4} = \frac{1}{2} \times h$

$\Rightarrow a = \frac{2h}{\sqrt{3}}$

Therefore

$\text{Area of equilateral triangle } = \frac{1}{2} \times a \times h$Area of equilateral triangle =12×a×h⇒33=12×2h3×h

$\Rightarrow 3\sqrt{3} = \frac{1}{2} \times \frac{2h}{\sqrt{3}} \times h$

$\Rightarrow h^2 = 9$

$\Rightarrow h = 3 \text{ cm}$

Hence, the correct option is (a).

Question 38:

The area of a rhombus is 144 cm^2 and one of its diagonals is double the other. The length of the longer diagonal is

(a) 12 cm (b) 16 cm (c) 18 cm (d) 24 cm

Answer:

Let d_1 and d_2 be the diagonals of the rhombus, where $d_1 = 2d_2$.

$\text{Area of rhombus} = \frac{1}{2} \times d_1 \times d_2$ Area of rhombus=12×d1×d2⇒144=12×d1×

$\Rightarrow 144 = \frac{1}{2} \times d_1 \times \frac{d_1}{2}$ $(\because\ d_1 = 2d_2)$

$\Rightarrow {d_1}^2 = 4 \times 144$

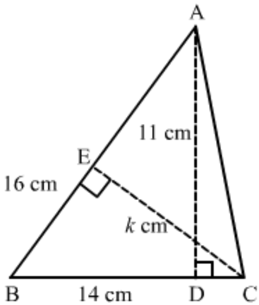
$\Rightarrow d_1 = 2 \times 12 = 24\ \text{cm}$

Hence, the correct option is (d).

Question 39:

In fig. 38, the value of *k* is

(a) $\frac{77}{8}$778 (b) $\frac{73}{8}$738 (c) $\frac{71}{8}$718
(d) $\frac{75}{8}$758



Answer:

In triangle *ABC*, we have

$\text{Area of } \Delta ABC = \frac{1}{2} \times BC \times AD = \frac{1}{2} \times AB \times CE$ Area of ΔABC=12×BC×AD=12×AB×CE⇒B

$\Rightarrow BC \times AD = AB \times CE$

$\Rightarrow 14 \times 11 = 16 \times k$

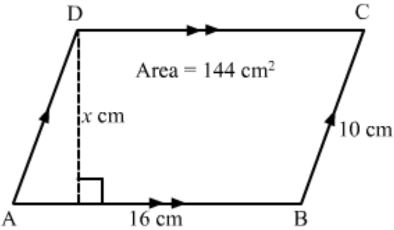
$\Rightarrow k = \frac{14 \times 11}{16} = \frac{77}{8}$

Hence, the correct option is (a).

Question 40:

In fig. 40, *ABCD* is a parallelogram of area 144 cm^2, the value of *x* is

(a) 8 (b) 6 (c) 9 (d) 10



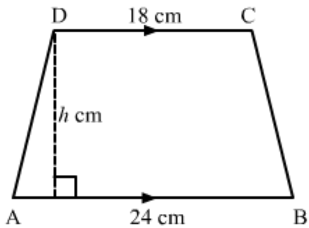
Answer:

$\text{Area of parallelogram} = \text{Base} \times \text{Height} =$ Area of parallelogram=Base×Height=⇒144=16×x⇒x
$\Rightarrow 144 = 16 \times x$
$\Rightarrow x = \frac{144}{16} = 9$
Hence, the correct option is (c).

Question 41:

In fig. 41, if *ABCD* is a parallelogram of area 273 cm^2, the value of *h* is

(a) 13 (b) 12 (c) 8 (d) 14



Answer:

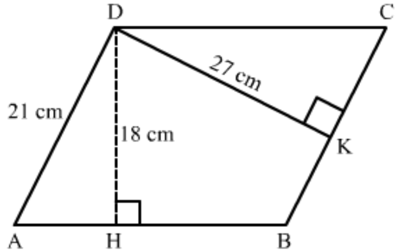
The quadrilateral *ABCD* is a trapezium whose area is 273 cm^2. So
$\text{Area of trapezium} = \frac{1}{2}(\text{Sum of parallel sides}) \times \text{Height}$Area of trapezium=12Sum of parall
$\Rightarrow 273 = \frac{1}{2}(24 + 18) \times h$
$\Rightarrow h = \frac{273 \times 2}{42} = 13$
Hence, the correct option is (a).

Question 42:

In Fig. 42, *ABCD* is a parallelogram in which *AD* = 21 cm, *DH* =18 cm and *DK* = 27 cm.
The length of *AB* is

(a) 63 cm (b) 63.5 cm (c) 31.5 cm

(d) 31 cm



Answer:

Area of a parallelogram = Base ×× Height

$AB \times DH = AD \times DK$ AB×DH=AD×DK⇒AB×18=21×27⇒AB=21×2718=632=31.5 cm

$\Rightarrow AB \times 18 = 21 \times 27$

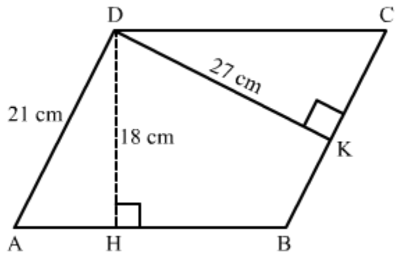
$\Rightarrow AB = \frac{21 \times 27}{18} = \frac{63}{2} = 31.5 \text{ cm}$

Hence, the correct option is (c).

Question 43:

In Fig. 42, *ABCD* is a parallelogram in which *AD* = 21 cm, *DH* =18 cm and *DK* = 27 cm.
The perimeter of the parallelogram is

(a) 105 cm (b) 84.5 cm (c) 169 cm (d) 52.5 cm



Answer:

Area of a parallelogram = Base ×× Height

$AB \times DH = AD \times DK$ AB×DH=AD×DK⇒AB×18=21×27⇒AB=21×2718=632=31.5 cm

$\Rightarrow AB \times 18 = 21 \times 27$

$\Rightarrow AB = \frac{21 \times 27}{18} = \frac{63}{2} = 31.5 \text{ cm}$

Here, *ABCD* is a parallelogram, so *AB* = *CD* and *AD* = *BC*.

Therefore

Perimeter of parallelogram *ABCD* = 2(*AB* + *AD*)

= 2(31.5 + 21)

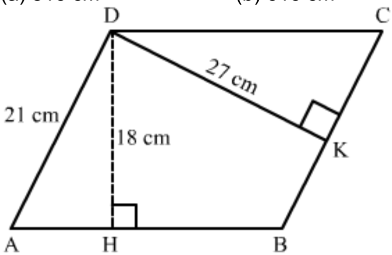
= 105 cm

Hence, the correct option is (a).

Question 44:

In Fig. 42, the area of the parallelogram is

(a) 516 cm^2 (b) 616 cm^2 (c) 416 cm^2 (d) 606 cm^2



Answer:

Here, *ABCD* is a parallelogram, so *AD* = *BC* = 21 cm.
Therefore
Area of parallelogram = Base ×× Height
= *BC* ×× *DK*
= 21 ×× 27
= 567 cm^2

Question 45:

A piece of wire of length 12 cm is bent to form a square. The area of the square is

(a) 36 cm^2 (b) 144 cm^2 (c) 9 cm^2 (d) 12 cm^2

Answer:

Let *a* be the length of the side of the square. Then as per he question, we have
$4a = 12$
$a = 3$ cm
Therefore
Area of square = a^2
= 3^2
= 9 cm^2
Thus, the area of the square is 9 cm^2.
Hence, the correct option is (c)

Question 46:

The area of a right isosceles triangle whose hypotenuse is $16\sqrt{2}2$ cm is

(a) 125 cm^2 (b) 158 cm^2 (c) 128 cm^2 (d) 144 cm^2

Answer:

Let a be the length of the equal sides of the right isosceles triangle whose hypotenuse is 16 $\sqrt{2}2$ cm.
Then using Pythagoras theorem in the triangle, we get
$a^2 + a^2 = (16\sqrt{2}2)^2$
$2a^2 = 512$
$a^2 = 256$
Therefore
Area of the triangle $= \frac{1}{2} \times \text{Base} \times \text{Height}$=12×Base×Height
$= \frac{1}{2} \times a \times a$ =12×a×a=12×a2=12×256=128 cm2
$= \frac{1}{2} \times a^2$
$= \frac{1}{2} \times 256 = 128\ \text{cm}^2$
Thus, the area of the square is 128 cm^2.
Hence, the correct option is (c)

Question 47:

A wire is in the form of a square of side 18 m. It is bent in the form of a rectangle, whose length and breadth are in the ratio 3 : 1. The area of the rectangle is

(a) 81 m^2 (b) 243 m^2 (c) 144 m^2 (d) 324 m^2

Answer:

Side of square (a) = 18 m
Let $l = 3x$ and $b = x$ be the length and breadth of the rectangle. Then
Perimeter of rectangle = Perimeter of square
$2(l + b) = 4a$
$2(3x + x) = 4 \times$× 18
$8x = 72$
$x = 9$ m
Thus
Length (l) = $3x = 3 \times$× 9 = 27 m
Breadth (b) = x = 9 m
Therefore
Area of the rectangle = $l \times$× b
= 27 $\times$× 9 = 243 m^2
Thus, the area of the rectangle is 243 m^2.
Hence, the correct option is (b)

Mensuration – II (Area of circle)

Exercise 21.1

1. Find the circumference of the circle whose radius is
(i) 14cm

(ii) 10m

(iii) 4km

Answer:

(i) Given radius = 14cm

We know that the circumference of the circle = 2 π r

π value is 22/7

C = 2 × (22/7) × 14

C = 88cm

(ii) Given radius = 10m

We know that the circumference of the circle = 2 π r

C = 2 × (22/7) × 10

C = 62.86 m

(iii) Given radius = 4km

We know that the circumference of the circle = 2 π r

C = 2 × (22/7) × 4

C = 25.142 km

2. Find the circumference of a circle whose diameter is

(i) 7 cm

(ii) 4.2 cm

(iii) 11.2 km

Answer:

(i) Given diameter = 7cm

We know that radius = diameter/2

Therefore, r = d/2

r = 7/2

We know that the circumference of the circle = 2 π r

$C = 2 \times (22/7) \times 7/2$

$C = 22$ cm

(ii) Given diameter = 4.2 cm

We know that radius = diameter/2

Therefore, $r = 4.2/2$

$r = 2.1$

We know that the circumference of the circle = $2 \pi r$

$C = 2 \times (22/7) \times 2.1$

$C = 13.2$ cm

(iii) Given diameter = 11.2 km

We know that radius = diameter/2

Therefore, $r = 11.2/2$

$r = 5.6$

We know that the circumference of the circle = $2 \pi r$

$C = 2 \times (22/7) \times 5.6$

$C = 35.2$ km

3. Find the radius of a circle whose circumference is

(i) 52.8 cm

(ii) 42 cm

(iii) 6.6 km

Answer:

(i) Given circumference, C = 52.8 cm

We know that the circumference of the circle = $2 \pi r$

Therefore radius, $r = C/2 \pi$

$r = (52.8 \times 7)/ (2 \times 22)$

$r = 369.6/ 44$

$r = 8.4$ cm

(ii) Given circumference, C = 42 cm

We know that the circumference of the circle = $2 \pi r$

Therefore radius, $r = C/2 \pi$

r = (42 × 7)/ (2 × 22)

r = 294/ 44

r = 6.68 cm

(iii) Given circumference, C = 6.6 km

We know that the circumference of the circle = 2 π r

Therefore radius, r = C/2 π

r = (6.6 × 7)/ (2 × 22)

r = 46.2/ 44

r = 1.05 km

4. Find the diameter of a circle whose circumference is

(i) 12.56 cm

(ii) 88 m

(iii) 11.0 km

Answer:

(i) Given circumference, C = 12.56 cm

We know that the circumference of the circle = 2 π r

Therefore radius, r = C/2 π

r = (12.56 × 7)/ (2 × 22)

r = 87.92/ 44

r = 1.99 cm

But diameter = 2r

= 2 × 1.99 = 3.99 cm

(ii) Given circumference, C = 88 m

We know that the circumference of the circle = 2 π r

Therefore radius, r = C/2 π

r = (88 × 7)/ (2 × 22)

r = 616/ 44

r = 14 m

But diameter = 2r

= 2 × 14 = 28 m

(iii) Given circumference, C = 11.0 km

We know that the circumference of the circle = 2 π r

Therefore radius, r = C/2 π

r = (11 × 7)/ (2 × 22)

r = 77/ 44

r = 1.75 km

But diameter = 2r

= 2 × 1.75 = 3.5 km

5. The ratio of the radii of two circles is 3: 2. What is the ratio of their circumferences?

Answer:

Given that the ratio of the radii = 3: 2

So, let the radii of the two circles be 3r and 2r respectively.

And let C1 and C2 be the circumference of the two circles of radii 3r and 2r respectively.

C1 = 2 π × 3r = 6 π r … (i)

Now C2 = 2 × 2 π r = 4 π r … (ii)

Consider, C1/C2 = (6 π r)/ 4 π r = 6/4 = 3/2

C1: C2 = 3: 2

6. A wire in the form of a rectangle 18.7 cm long and 14.3 cm wide is reshaped and bent into the form of a circle. Find the radius of the circle so formed.

Answer:

Given length of rectangular wire = 18.7 cm

Breadth of rectangular wire = 14.3 cm

According to the question length of wire = perimeter of the rectangle

= 2 (l + b) = 2 × (18.7 + 14.3)

= 2 (33)

= 66 cm

Let the wire bent in the form of circle of radius r cm then we have

Circumference = 66cm

2 π r = 66

2 × (22/7) × r = 66

$(44/7)\ r = 66$

$r = (66 \times 7)/44$

$r = 462/44$

$= 10.5$ cm

7. A piece of wire is bent in the shape of an equilateral triangle of each side 6.6 cm. It is re-bent to form a circular ring. What is the diameter of the ring?

Answer:

Given side of equilateral triangle = 6.6 cm

Length of the wire = the perimeter of equilateral triangle

Perimeter of equilateral triangle = 3 × side

$= 3 \times 6.6 = 19.8$ cm

Therefore circumference = 19.8 cm

$C = 2 \pi r$

$19.8 = 2 \times (22/7) \times r$

$19.8 = (44/7)\ r$

$r = (19.8 \times 7)/44$

$r = 138.6/44$

$= 3.15$ cm

Diameter = 2r

Therefore diameter of ring = $2 \times 3.15 = 6.3$ cm

8. The diameter of a wheel of a car is 63 cm. Find the distance travelled by the car during the period, the wheel makes 1000 revolutions.

Answer:

It may be noted that in one revolution, the cycle covers a distance equal to the circumference of the wheel.

Given the diameter of the wheel = 63 cm

We know that circumference of the wheel = πd

$= 22/7 \times 63$

$= 198$ cm.

Thus, the cycle covers 198 cm in one revolution.

Therefore the distance covered by the cycle in 1000 revolutions = (198×1000)

$= 198000$ cm

= 1980 m.

9. The diameter of a wheel of a car is 98 cm. How many revolutions will it make to travel 6160 meters.

Answer:

In one revolution of the wheel, the car travels a distance equal to the circumference of the wheel.

Given diameter of the wheel of a car = 98 cm

Circumference of the wheel of the car = π d

= 22/7 x 98

= 308 cm

The distance travelled by the car in one revolution = 308 cm

Total distance travelled by the car = 6160 m = 616000 cm

Therefore number of revolution = total distance travelled by the car/ distance travelled by the car in one revolution

Number of revolution = 616000/308 = 2000

10. The moon is about 384400 km from the earth and its path around the earth is nearly circular. Find the circumference of the path described by the moon in lunar month.

Answer:

From the question it is given that,

The radius of the path described by the moon around the earth = 384400 km

The circumference of the path described by the moon,

C = 2 π r

C = 2 × (22/7) × 384400

C = 2416228.57 km

11. How long will John take to make a round of a circular field of radius 21 m cycling at the speed of 8 km/hr.?

Answer:

Given the radius of the circular field = 21 m

Circumference of the circular field = 2 π r

C = 2 × (22/7) × 21

C = 132 m

If John cycles at a speed of 8 km/hr then John covers 8000 m in 1 hour.

(In 1 hour John covers 8 km = 8000 m)

So, time required to cover 132 m = 132/8000 = 0.0165 hours

As, 1 hour = 3600 seconds

By converting 0.0165 hours into minutes we get

0.0615 hours = 0.0165 x 3600 = 59.4 seconds.

12. The hour and minute hands of a clock are 4 cm and 6 cm long respectively. Find the sum of the distances travelled by their tips in 2 days.

Answer:

Length of the hour hand is 4 cm, which describes the radius of the path inscribed by the hour hand.

Length of the minute hand is 6 cm, which describes the radius of the path inscribed by the minute hand.

The circumference of the path inscribed by the hour hand = $2 \pi r$

$C = 2 \times (22/7) \times 4$

$C = 176/7$ cm

The hour hand makes 2 revolutions in one day.

Therefore distance covered by the hour hand in 2 days = $(176/7) \times 2 \times 2$

= 100.57 cm

The distance covered by the minute hand in 1 revolution = $2 \pi r$

$C = 2 \times (22/7) \times 6$

$C = 264/7$

As we know, the minute hand makes 1 revolution in one hour.

In 1 day, it makes 24 revolutions.

In 2 days, it makes 2×24 revolutions.

The distance covered by the minute hand in 2 days = $2 \times 24 \times (264/7)$

= 12672/7

= 1810.28 cm

The sum of the distances travelled by the hour and minute hands in 2 days = 1810.28 + 100.57

= 1910.85 cm

13. A rhombus has the same perimeter as the circumference of the circle. If the side of the rhombus is 2.2m, find the radius of the circle.

Answer:

Given the side of a rhombus = 2.2 m

We know that the perimeter of the rhombus = 4 x side

= 4 x 2.2 m

= 8.8 m.

According to the question it is clear that,

Perimeter of the rhombus = Circumference of the circle

$8.8 = 2 \pi r$

$8.8 = 2 \times (22/7) \times r$

$r = (8.8 \times 7)/ 44$

$r = 61.6/44$

r = 1.4 m

Therefore radius of the circle = 1.4m

14. A wire is looped in the form of a circle of radius 28 cm. It is re-bent into a square form. Determine the length of the side of the square.

Answer:

Given the radius of the circle = 28 cm

Using the circumference of the circle formula, we have

Circumference = $2 \pi r$

$C = 2 \times (22/7) \times 28$

C = 176 cm

Let x cm be the side of the square. Then,

The circumference of the circle = the perimeter of the square

Perimeter of square = 4x

$176 = 4 \times x$

$x = 176/4$

$x = 44$

Therefore the side of the square = 44 cm

15. A bicycle wheel makes 5000 revolutions in moving 11 km. Find the diameter of the wheel.

Answer:

Given total distance covered by bicycle in 5000 revolutions = 11 km = 11000 m

Therefore distance covered in 1 revolution = 11000/5000 = 2.2 m = 11/5

Distance covered in 1 revolution = Circumference of the wheel

$C = \pi d$

$11/5 = (22/7) d$

$d = (11 \times 7)/ (5 \times 22)$

$d = 77/110$

$d = 0.7$ m

Therefore diameter of wheel = 0.7 m = 70 cm

16. A boy is cycling such that the wheels of the cycle are making 140 revolutions per minute. If the diameter of the wheel is 60 cm, calculate the speed per hour with which the boy is cycling.

Answer:

Given the diameter of the wheel = 60 cm

Distance covered by the wheel in 1 revolution = Circumference of the wheel

Distance covered by the wheel in 1 revolution = πd

$= 22/7 \times 60$ cm

Distance covered by the wheel in 140 revolutions = $22/7 \times 60 \times 140$

= 26400 cm

Thus, the wheel covers 26400 cm in 1 minute. Then,

Speed = $26400/100 \times 60$ m/hr

$= 264 \times 60$ m/hr

$= 264 \times 60/1000$ km/hr

= 15.84 km/hr

The speed with which the boy is cycling is 15.84 km/hr.

17. The diameter of the driving wheel of a bus is 140 cm. How many revolutions per minute must the wheel make in order to keep a speed of 66 km per hour?

Answer:

Given diameter of the wheel = 140 cm

Desired speed of the bus = 66 km/hr

Distance covered by the wheel in 1 revolution = Circumference of the wheel

Circumference of the wheel = πd

C = 22/7 × 140

C = 440 cm

Now, the desired speed of the bus = 66 km/hr = (66 × 1000 × 100)/60 = 110000 cm/min

Number of revolution per minute = 110000/440

= 250

Therefore, the bus must make 250 revolutions per minute to keep the speed at 66 km/hr.

18. A water sprinkler in a lawn sprays water as far as 7 m in all directions. Find the length of the outer edge of wet grass.

Answer:

From the question it is clear that, a water sprinkler in a lawn sprays water as far as 7 m in all directions. So wet area shows a circular region of radius 7 m.

The length of the outer edge of the wet grass = Circumference of circle

Circumference of the circle = 2 π r

C = 2 × 22/7 × 7

C = 44 m

19. A well of diameter 150 cm has a stone parapet around it. If the length of the outer edge of the parapet is 660 cm. then find the width of the parapet.

Answer:

Given diameter of the well = 150 cm

Length of the outer edge of the parapet = 660 cm

Now we have to find width of the parapet

Now, radius of well = half of diameter = 150/2 = 75 cm

Consider the width of the stone parapet be x cm. then, according to the question outer edge of the parapet forms a circular region of radius (x + 75) cm

So, 660 = 2 × 22/7 × (x + 75)

(660 × 7)/ 2 × 22 = x + 75

4620/44 = x + 75

105 = x + 75

x = 105 – 75

x = 30

The width of the parapet = x = 30cm

20. An ox in a kolhu (an oil processing apparatus) is tethered to a rope 3 m long. How much distance does it cover in 14 rounds?

Answer:

Radius of the circular path traced by the ox = 3 m and

Distance covered by an ox in 1 round = Circumference of the circular path

Circumference = $2 \pi r$

$C = 2 \times (22/7) \times 3$ m

Distance covered in 14 rounds = $2 \times (22/7) \times 3 \times 14$

$C = 264$ m

Data Handling I (Collection And Organisation Of Data)

Exercise 22.1

Question 1:

Define the following terms:
(i) Observations
(ii) data
(iii) Frequency of an observation
(iv) Frequency distribution

Answer:

(i)Observation is the activity of paying close attention to someone or something in order to get information in numerical form.

(ii)Data
The collection of observations is known as data.

(iii)Frequency of an observation
The number of times an observation occurs in a given data is called the frequency of an observation.

(iv)Frequency Distribution
It is a method of presenting raw data in a form that can be easily understood.

Question 2:

The final marks in mathematics of 30 students are as follows:

53,	61,	48,	60,	78,	68,	55,	100,	67,	90
75,	88,	77,	37,	84,	58,	60,	48,	62,	56
44,	58,	52,	64,	98,	59,	70,	39,	50,	60

(i) Arrange these marks in the ascending order. 30 to 39 one group, 40 to 49 second group, etc.
(ii) What is the highest score?
(iii) What is the lowest score?
(iv) What is the range?
(v) If 40 is the pass mark how many have failed?
(vi) How many have scored 75 or more?
(vii) Which observations between 50 and 60 have not actually appeared?
(viii) How many have scored less than 50?

Answer:

(i) Ascending order of the numbers in groups :
(30-39): 37, 39
(40 - 49): 44, 48, 48
(50 - 59) : 50,52, 53, 55, 56, 58, 58, 59
(60 - 69) : 60, 60, 60, 61, 62, 64, 67, 68
(70 - 79) : 70, 75, 77, 78
(80 - 89) : 84, 88
(90 - 99) : 90, 98

(100-109): 100

(ii) The highest score is 100.
(iii) The lowest score is 37.
(iv) Range is = Maximum observation - Minimum observation.
= 100 - 37 = 63.
(v) If 40 is the pass mark, then only 2 students have failed.
(vi) 8 students have scored 75 or more.
(vii) 51, 54 and 57 are not there between 50 and 60.
(viii) 5 students scored less than 50.

Question 3:

The weights of new born babies (in kg) in a hospital on a particular day are as follows:

2.3, 2.2, 2.1, 2.7, 2.6, 3.0, 2.5, 2.9, 2.8, 3.1, 2.5, 2.8, 2.7, 2.9, 2.4

(i) Rearrange the weights in descending order.
(ii) Determine the highest weight.
(iii) Determine the lowest weight.
(iv) Determine the range.
(v) How many babies were born on that day?
(vi) How many babies weigh below 2.5 kg?
(vii) How many babies weigh more than 2.8?
(viii) How many babies weigh 2.8 kg?

Answer:

(i) Weights in descending order :
3.1, 3.0, 2.9, 2.9, 2.8, 2.8, 2.7, 2.7, 2.6, 2.5, 2.5, 2.4, 2.3, 2.2, 2.1
(ii) Highest weight: 3.1 Kg.
(iii) Lowest weight: 2.1 Kg.
(iv) Range = Maximum observation - Minimum observation
= (3.1-2.1) kg = 1.0 Kg.
(v) A total of 15 babies were born on that day.
(vi) 4 babies weigh below 2.5 kg.
(vii) 4 babies weigh more than 2.8kg.
(viii) 2 babies weigh 2.8 kg.

Question 4:

Following data gives the number of children in 40 families:

1, 2, 6, 5, 1, 5, 1, 3, 2, 6, 2, 3, 4, 2, 0, 0, 4, 4, 3, 2
2, 0, 0, 1, 2, 2, 4, 3, 2, 1, 0, 5, 1, 2, 4, 3, 4, 1, 6, 2

Represent it in the form of a frequency distribution.

Answer:

Required frequency-distribution table :

Number of children	Frequency
0	5

1	7
2	11
3	5
4	6
5	3
6	3

Question 5:

Prepare a frequency table of the following scores obtained by 50 students in a test:

42	51	21	42	37	37	42	49	38	52
7	33	17	44	39	7	14	27	39	42
42	62	37	39	67	51	53	53	59	41
29	38	27	31	54	19	53	51	22	61
42	39	59	47	33	34	16	37	57	43

Answer:

Frequency Table is :

Marks	No. of Students
7	2
14	1
16	1
17	1
19	1
21	1
22	1
27	2
29	1
31	1
33	2
34	1
37	4
38	2
39	4
41	1
42	6
43	1
44	1
47	1
49	1
51	3
52	1
53	3
54	1
57	1
59	2
61	1
62	1
67	1

Question 6:

A die was thrown 25 times and following scores were obtained:

1	5	2	4	3
6	1	4	2	5
1	6	2	6	3
5	4	1	3	2
3	6	1	5	2

Prepare a frequency table of the scores.

Answer:

Frequency Table is :

Score	Number of Times
1	5
2	5
3	4
4	3
5	4
6	4

Question 7:

In a study of number of accidents per day, the observations for 30 days were obtained as follows:

6	3	5	6	4	3	2	5	4	2
4	2	1	2	2	0	5	4	6	1
6	0	5	3	6	1	5	5	2	6

Prepare a frequency distribution table.

Answer:

Required frequency-distribution table:

Number of Accidents	Number of Days
0	2
1	3
2	6
3	3
4	4
5	6
6	6

Question 8:

Prepare a frequency table of the following ages (in years) of 30 students of class VIII in your school:

13, 14, 13, 12, 14, 13, 14, 15, 13, 14, 13, 14, 16, 12, 14
13, 14, 15, 16, 13, 14, 13, 12, 17, 13, 12, 13, 13, 13, 14

Answer:

Frequency Distribution Table is :

Ages (in years)	Number of Students
12	4
13	12
14	9
15	2
16	2
17	1

Question 9:

Following figures relate the weekly wages (in Rs.) of 15 workers in a factory:

300, 250, 200, 250, 200, 150, 350, 200, 250, 200, 150, 300, 150, 200, 250

Prepare a frequency table.
(i) What is the range in wages (in Rs)?
(ii) How many Workers are getting Rs 350?
(iii) How many workers are getting the minimum wages?

Answer:

Frequency Distribution Table is

Wages (in Rs.)	No. of Workers
150	3
200	5
250	4
300	2
350	1

(i) The range in wages (in Rs.) = 350 -150 = 200.
(ii) Only 1 worker is getting Rs. 350.
(iii) 3 workers are getting the minimum wages, i.e, Rs. 150.

Question 10:

Construct a frequency distribution table for the following marks obtained by 25 students in a history test in class VI of a school:

9, 17, 12, 20, 9, 18, 25, 17, 19, 9, 12, 9, 12, 18, 17, 19, 20, 25, 9, 12, 17, 19, 19, 20, 9
(i) What is the range of marks?
(ii) What is the highest mark?
(iii) Which mark is occurring more frequently?

Answer:

Required frequency-distribution table:

Marks	Frequency
9	6
12	4
17	4
18	2
19	4

20	3
25	2

(i) Range of marks: 25-9=16.
(ii) The highest mark is 25.
(iii) 9 is occurring most frequently.

Question 11:

In a mathematis test following marks were obtained by 40 students of class VI. Arrange these marks in a table using, tally marks.

8	1	3	7	6	5	5	4	4	2
4	9	5	3	7	1	6	5	2	7
7	3	8	4	2	8	9	5	8	6
7	4	5	6	9	6	4	4	6	6

(i) Find how many students obtained marks equal to or more than 7?
(ii) How many students obtaned marks below 4?

Answer:

The Frequency Distribution Table is :

Marks	Tally Marks	Frequency
1	II	2
2	III	3
3	III	3
4	~~IIII~~ II	7
5	~~IIII~~ I	6
6	~~IIII~~ II	7
7	~~IIII~~	5
8	IIII	4
9	III	3

(i) 12 students obtained marks equal to or more than 7.
(ii) Only 8 students obtained marks below 4.

Question 12:

Following is the choice of sweets of 30 students of class VI: Ladoo, Barfi, Ladoo, Jalebi, Ladoo, Rasgulla, Jalebi, Ladoo, Barfi, Rasgulla, Ladoo, Jalebi, Jalebi Rasgulla, Ladoo, Rasgulla, Jalebi, Ladoo, Rasgulla, Ladoo, Rasgulla, Jalebi, Ladoo, Rasgulla, Ladoo, Ladoo, Barfi, Rasgulla, Rasgulla, Ladoo.
(i) Arrange the names of sweets in a table using tally marks.
(ii) Which sweet is preferred by most of the students.

Answer:

Sweet	*Tally Marks*	*Frequency*
Ladoo	~~IIII~~ ~~IIII~~ II	12
Burfi	III	3
Jalebi	~~IIII~~ I	6
Rasgulla	~~IIII~~ IIII	9

(ii) Ladoo is preferred by most of the students, 12 students.

Chapter - 23 Data Handling – II (Central Values)

Exercise 23.1

1. Ashish studies for 4 hours, 5 hours and 3 hours on three consecutive days. How many hours does he study daily on an average?

Solution:

Given Ashish studies for 4 hours, 5 hours and 3 hours on three consecutive days

Average number of study hours = sum of hours/ number of days

Average number of study hours = (4 + 5 + 3) ÷ 3

= 12 ÷ 3

= 4 hours

Thus, Ashish studies for 4 hours on an average.

2. A cricketer scores the following runs in 8 innings: 58, 76, 40, 35, 48, 45, 0, 100.

Find the mean score.

Solution:

Given runs in 8 innings: 58, 76, 40, 35, 48, 45, 0, 100

Mean score = total sum of runs/number of innings

The mean score = (58 + 76 + 40 + 35 + 48 + 45 + 0 + 100) ÷ 8

= 402 ÷ 8

= 50.25 runs.

3. The marks (out of 100) obtained by a group of students in science test are 85, 76, 90, 84, 39, 48, 56, 95, 81 and 75. Find the

(i) Highest and the lowest marks obtained by the students.

(ii) Range of marks obtained.

(iii) Mean marks obtained by the group.

Solution:

In order to find the highest and lowest marks, we have to arrange the marks in ascending order as follows:

39, 48, 56, 75, 76, 81, 84, 85, 90, 95

(i) Clearly, the highest mark is 95 and the lowest is 39.

(ii) The range of the marks obtained is: (95 – 39) = 56.

(iii) From the following data, we have

Mean marks = Sum of the marks/ Total number of students

Mean marks = (39 + 48 + 56 + 75 + 76 + 81 + 84 + 85 + 90 + 95) ÷ 10

= 729 ÷ 10

= 72.9.

Hence, the mean mark of the students is 72.9.

4. The enrolment of a school during six consecutive years was as follows:

1555, 1670, 1750, 2019, 2540, 2820

Find the mean enrolment of the school for this period.

Solution:

Given enrolment of a school during six consecutive years as follows

1555, 1670, 1750, 2019, 2540, 2820

The mean enrolment = Sum of the enrolments in each year/ Total number of years

The mean enrolment = (1555 + 1670 + 1750 + 2019 + 2540 + 2820) ÷ 6

= 12354 ÷ 6

= 2059.

Thus, the mean enrolment of the school for the given period is 2059.

5. The rainfall (in mm) in a city on 7 days of a certain week was recorded as follows:

Day	Mon	Tue	Wed	Thu	Fri	Sat	Sun
Rainfall (in mm)	0.0	12.2	2.1	0.0	20.5	5.3	1.0

(i) Find the range of the rainfall from the above data.

(ii) Find the mean rainfall for the week.

(iii) On how many days was the rainfall less than the mean rainfall.

Solution:

(i) The range of the rainfall = Maximum rainfall – Minimum rainfall

= 20.5 – 0.0

= 20.5 mm.

(ii) The mean rainfall = (0.0 + 12.2 + 2.1 + 0.0 + 20.5 + 5.3 + 1.0) ÷ 7

= 41.1 ÷ 7

= 5.87 mm.

(iii) Clearly, there are 5 days (Mon, Wed, Thu, Sat and Sun), when the rainfall was less than the mean, i.e., 5.87 mm.

6. If the heights of 5 persons are 140 cm, 150 cm, 152 cm, 158 cm and 161 cm respectively, find the mean height.

Solution:

The mean height = Sum of the heights /Total number of persons

= (140 + 150 + 152 + 158 + 161) ÷ 5

= 761 ÷ 5

= 152.2 cm.

7. Find the mean of 994, 996, 998, 1002 and 1000.

Solution:

Mean = Sum of the given numbers/Total number of given numbers

Mean = (994 + 996 + 998 + 1002 + 1000) ÷ 5

= 4990 ÷ 5

= 998.

8. Find the mean of first five natural numbers.

Solution:

We know that first five natural numbers = 1, 2, 3, 4 and 5

Mean of first five natural numbers = (1 + 2 + 3 + 4 + 5) ÷ 5

= 15 ÷ 5

= 3

9. Find the mean of all factors of 10.

Solution:

We know that factors of 10 are 1, 2, 5 and 10

Arithmetic mean of all factors of 10 = (1 + 2 + 5 + 10) ÷ 4

= 18 ÷ 4

= 4.5

10. Find the mean of first 10 even natural numbers.

Solution:

The first 10 even natural numbers are 2, 4, 6, 8, 10, 12, 14, 16, 18 and 20.

Mean of first 10 even natural numbers = (2 + 4 + 6 + 8 + 10 + 12 + 14 + 16 + 18 + 20) ÷ 10

= 110 ÷ 10

= 11

11. Find the mean of x, x + 2, x + 4, x + 6, x + 8

Solution:

Mean = Sum of observations ÷ Number of observations

Mean = (x + x + 2 + x + 4 + x + 6 + x + 8) ÷ 5

Mean = (5x + 20) ÷ 5

Mean = 5 (x + 4) ÷ 5

Mean = x + 4

12. Find the mean of first five multiples of 3.

Solution:

The first five multiples of 3 are 3, 6, 9, 12 and 15.

Mean of first five multiples of 3 are = (3 + 6 + 9 + 12 + 15) ÷ 5

= 45 ÷ 5

= 9

13. Following are the weights (in kg) of 10 new born babies in a hospital on a particular day: 3.4, 3.6, 4.2, 4.5, 3.9, 4.1, 3.8, 4.5, 4.4, 3.6 Find the mean

Solution:

We know that

= sum of observations/ number of observations

= sum of weights of babies/ number of babies

= (3.4 + 3.6 + 4.2 + 4.5 + 3.9 + 4.1 + 3.8 + 4.5 + 4.4 + 3.6) ÷ 10

= (40) ÷ 10

= 4 kg

14. The percentage of marks obtained by students of a class in mathematics are:

64, 36, 47, 23, 0, 19, 81, 93, 72, 35, 3, 1 Find their mean.

Solution:

Mean = sum of the marks obtained/ total number of students

= (64 + 36 + 47 + 23 + 0 + 19 + 81 + 93 + 72 + 35 + 3 + 1) ÷ 12

= 474 ÷ 12

= 39.5%

15. The numbers of children in 10 families of a locality are:

2, 4, 3, 4, 2, 3, 5, 1, 1, 5 Find the mean number of children per family.

Solution:

Mean number of children per family = sum of total number of children / total number of families

= (2 + 4 + 3 + 4 + 2 + 3 + 5 + 1 + 1 + 5) ÷ 10

= 30 ÷ 10

= 3

Thus, on an average there are 3 children per family in the locality.

16. The mean of marks scored by 100 students was found to be 40. Later on it was discovered that a score of 53 was misread as 83. Find the correct mean.

Solution:

Given n = the number of observations = 100, Mean = 40

Mean = sum of observations/total number of observations

40 = sum of the observations/ 100

Sum of the observations = 40 x 100

Thus, the incorrect sum of the observations = 40 x 100 = 4000.

Now,

The correct sum of the observations = Incorrect sum of the observations – Incorrect observation + Correct observation

The correct sum of the observations = 4000 – 83 + 53

The correct sum of the observations = 4000 – 30 = 3970

Correct mean = correct sum of the observations/ number of observations

= 3970/100

= 39.7

17. The mean of five numbers is 27. If one number is excluded, their mean is 25. Find the excluded number.

Solution:

We know that

Mean = sum of five numbers/5 = 27

So, sum of the five numbers = 5 x 27 = 135.

Now,

The mean of four numbers = sum of the four numbers/4 = 25

So, sum of the four numbers = 4 x 25 = 100.

Therefore, the excluded number = Sum of the five number – Sum of the four numbers

The excluded number = 135 – 100

= 35.

18. The mean weight per student in a group of 7 students is 55 kg. The individual weights of 6 of them (in kg) are 52, 54, 55, 53, 56 and 54. Find the weight of the seventh student.

Solution:

We know that

Mean = sum of weights of students/ number of students

Let the weight of the seventh student be x kg.

Mean = (52 + 54 + 55 + 53 + 56 + 54 + x)/ 7

55 = (52 + 54 + 55 + 53 + 56 + 54 + x)/ 7

55 x 7 = 324 + x

385 = 324 + x

x = 385 – 324

x = 61 kg.

Therefore weight of seventh student is 61kg.

19. The mean weight of 8 numbers is 15 kg. If each number is multiplied by 2, what will be the new mean?

Solution:

Let x_1, x_2, x_3…x_8 be the eight numbers whose mean is 15 kg. Then,

$15 = x_1 + x_2 + x_3 + \ldots\ldots + x_8 /8$

$x_1 + x_2 + x_3 + \ldots + x_8 = 15 \times 8$

$x_1 + x_2 + x_3 + \ldots + x_8 = 120$.

Let the new numbers be $2x_1$, $2x_2$, $2x_3$ …$2x_8$.

Let M be the arithmetic mean of the new numbers.

Then,

$M = 2x_1 + 2x_2 + 2x_3 + \ldots + 2x_8/8$

$M = 2\ (x_1 + x_2 + x_3 + \ldots + x_8)/8$

$M = (2 \times 120)/8$

= 30

20. The mean of 5 numbers is 18. If one number is excluded, their mean is 16. Find the excluded number.

Solution:

Let x_1, x_2, x_3, x_4 and x_5 be five numbers whose mean is 18. Then,

18 = Sum of five numbers ÷ 5

Hence, sum of five numbers = 18 × 5 = 90

Now, if one number is excluded, then their mean is 16.

So,

16 = Sum of four numbers ÷ 4

Therefore sum of four numbers = 16 × 4 = 64.

The excluded number = Sum of five observations – Sum of four observations

The excluded number = 90 – 64

Therefore The excluded number = 26.

21. The mean of 200 items was 50. Later on, it was discovered that the two items were misread as 92 and 8 instead of 192 and 88. Find the correct mean.

Solution:

Given n = Number of observations = 200

Mean = sum of observations/ number of observations

50 = sum of observations/ 200

Sum of the observations = 50 x 200 = 10,000.

Thus, the incorrect sum of the observations = 50 x 200

Now,

The correct sum of the observations = Incorrect sum of the observations – Incorrect observations + Correct observations

Correct sum of the observations = 10,000 – (92 + 8) + (192 + 88)

Correct sum of the observations = 10,000 – 100 + 280

Correct sum of the observations = 9900 + 280

Correct sum of the observations = 10,180.

Therefore correct mean = correct sum of the observations/ number of observations

= 10180/200

= 50.9

22. The mean of 5 numbers is 27. If one more number is included, then the mean is 25. Find the included number.

Solution:

Given Mean = Sum of five numbers ÷ 5

Sum of the five numbers = 27 × 5 = 135.

Now, New mean = 25

25 = Sum of six numbers ÷ 6

Sum of the six numbers = 25 × 6 = 150.

The included number = Sum of the six numbers – Sum of the five numbers

The included number = 150 – 135

Therefore the included number = 15.

23. The mean of 75 numbers is 35. If each number is multiplied by 4, find the new mean.

Solution:

Let x_1, x_2, $x_3 \ldots x_{75}$ be 75 numbers with their mean equal to 35. Then,

$35 = x_1 + x_2 + x_3 + \ldots.. + x_{75} /75$

$x_1 + x_2 + x_3 + \ldots.. + x_{75} = 35 \times 75$

$x_1 + x_2 + x_3 + \ldots + x_{75} = 2625$

The new numbers are 4 x 1, 4 x 2, 4 x 3…4 x 75

Let M be the arithmetic mean of the new numbers. Then,

$M = 4x_1 + 4x_2 + 4x_3 + \ldots + 4x_{75}/75$

$M = 4\ (x_1 + x_2 + x_3 + \ldots + x_{75})/75$

$M = (4 \times 2625)/75$

= 140

Exercise 23.2

1. A die was thrown 20 times and the following scores were recorded:

5, 2, 1, 3, 4, 4, 5, 6, 2, 2, 4, 5, 5, 6, 2, 2, 4, 5, 5, 1

Prepare the frequency table of the scores on the upper face of the die and find the mean score.

Solution:

The frequency table for the given data is as follows:

x:	1	2	3	4	5	6
f:	2	5	1	4	6	2

To compute arithmetic mean we have to prepare the following table:

Scores (x_i)	Frequency (f_i)	$x_i f_i$
1	2	2
2	5	10
3	1	3
4	4	16
5	6	30
6	2	12
Total	$\Sigma f_i = 20$	$\Sigma f_i x_i$

Mean score = $\Sigma f_i x_i / \Sigma f_i$

= 73/20

= 3.65

2. The daily wages (in Rs) of 15 workers in a factory are given below:

200, 180, 150, 150, 130, 180, 180, 200, 150, 130, 180, 180, 200, 150, 180

Prepare the frequency table and find the mean wage.

Solution:

Wages (x_i)	130	150	180	200
Number of workers (f_i)	2	4	6	3

To compute arithmetic mean we have to prepare the following table:

x_i	f_i	$x_i f_i$
130	2	260
150	4	600
180	6	1080
200	3	600

Total	$\Sigma f_i = N = 15$	$\Sigma f_i x_i = 2540$

Mean score = $\Sigma f_i x_i / \Sigma f_i$

= 2540/15

= 169.33

3. The following table shows the weights (in kg) of 15 workers in a factory:

Weight (in Kg)	**60**	**63**	**66**	**72**	**75**
Number of workers	**4**	**5**	**3**	**1**	**2**

Calculate the mean weight.

Solution:

Calculation of mean:

x_i	f_i	$x_i f_i$
60	4	240
63	5	315
66	3	198
72	1	72
75	2	150
Total	$\Sigma f_i = N = 15$	$\Sigma f_i x_i = 975$

Mean score = $\Sigma f_i x_i / \Sigma f_i$

= 975/15

= 65 kg

4. The ages (in years) of 50 students of a class in a school are given below:

Age (in years)	**14**	**15**	**16**	**17**	**18**
Number of students	**15**	**14**	**10**	**8**	**3**

Find the mean age.

Solution:

Calculation of mean:

x_i	f_i	$x_i f_i$
14	15	210
15	14	210
16	10	160

17	8	136
18	3	54
Total	$\Sigma f_i = N = 50$	$\Sigma f_i x_i = 770$

Mean score = $\Sigma f_i x_i / \Sigma f_i$

= 770/50

= 15.4 years

5. Calculate the mean for the following distribution:

x:	**5**	**6**	**7**	**8**	**9**
f:	**4**	**8**	**14**	**11**	**3**

Solution:

x_i	f_i	$x_i f_i$
5	4	20
6	8	48
7	14	98
8	11	88
9	3	27
Total	$\Sigma f_i = N = 40$	$\Sigma f_i x_i = 281$

Mean score = $\Sigma f_i x_i / \Sigma f_i$

= 281/40

= 7.025

6. Find the mean of the following data:

x:	**19**	**21**	**23**	**25**	**27**	**29**	**31**
f:	**13**	**15**	**16**	**18**	**16**	**15**	**13**

Solution:

x_i	f_i	$x_i f_i$
19	13	247
21	15	315
23	16	368

25	18	450
27	16	432
29	15	435
31	13	403
Total	$\Sigma f_i = N = 106$	$\Sigma f_i x_i = 2650$

Mean score = $\Sigma f_i x_i / \Sigma f_i$

= 2650/106

= 25

7. The mean of the following data is 20.6. Find the value of p.

x:	**10**	**15**	**p**	**25**	**35**
f:	**3**	**10**	**25**	**7**	**5**

Solution:

x_i	f_i	$x_i f_i$
10	3	30
15	10	150
P	25	25p
25	7	175
35	5	175
Total	$\Sigma f_i = N = 50$	$\Sigma f_i x_i = 530 + 25p$

Mean score = $\Sigma f_i x_i / \Sigma f_i$

20.6 = 530 + 25p/50

530 + 25 p = 20.6 x 50

25 p = 1030 – 530

p = 500/25

p = 20

8. If the mean of the following data is 15, find p.

x:	**5**	**10**	**15**	**20**	**25**
f:	**6**	**p**	**6**	**10**	**5**

Solution:

x_i	f_i	$x_i f_i$
5	6	30
10	P	10p
15	6	90
20	10	200
25	5	125
Total	$\Sigma f_i = 27 + p$	$\Sigma f_i x_i = 445 + 10p$

Mean score = $\Sigma f_i x_i / \Sigma f_i$

$15 = 445 + 10p/27 + p$

$445 + 10 p = 405 + 15p$

$5 p = 445 – 405$

$p = 40/5$

$p = 8$

9. Find the value of p for the following distribution whose mean is 16.6

x:	**8**	**12**	**15**	**p**	**20**	**25**	**30**
f:	**12**	**16**	**20**	**24**	**16**	**8**	**4**

Solution:

x_i	f_i	$x_i f_i$
8	12	96
12	16	192
15	20	300
P	24	24p
20	16	320
25	8	200
30	4	120
Total	$\Sigma f_i = N = 100$	$\Sigma f_i x_i = 1228 + 24p$

Mean score = $\Sigma f_i x_i / \Sigma f_i$

$16.6 = 1228 + 24p/100$

$1228 + 24 p = 16.6 \times 100$

$24 p = 1660 – 1228$

p = 432/24

p = 18

10. Find the missing value of p for the following distribution whose mean is 12.58

x:	**5**	**8**	**10**	**12**	**p**	**20**	**25**
f:	**2**	**5**	**8**	**22**	**7**	**4**	**2**

Solution:

x_i	f_i	$x_i f_i$
5	2	10
8	5	40
10	8	80
12	22	264
P	7	7p
20	4	80
25	2	50
Total	$\Sigma f_i = N = 50$	$\Sigma f_i x_i = 524 + 7p$

Mean score = $\Sigma f_i x_i / \Sigma f_i$

12.58 = 524 + 7p/50

524 + 7 p = 12.58 x 50

7 p = 629 – 524

p = 105/7

p = 15

11. Find the missing frequency (p) for the following distribution whose mean is 7.68

x:	**3**	**5**	**7**	**9**	**11**	**13**
f:	**6**	**8**	**15**	**p**	**8**	**4**

Solution:

x_i	f_i	$x_i f_i$
3	6	18
5	8	40
7	15	105
9	P	9p

11	8	88
13	4	52
Total	$\Sigma f_i = N = 41 + p$	$\Sigma f_i x_i = 303 + 9p$

Mean score = $\Sigma f_i x_i / \Sigma f_i$

7.68 = 303 + 9p/41 + p

303 + 9 p = 314.88 + 7.68p

1.32 p = 314.88 – 303

p = 11.88/1.32

p = 9

12. Find the value of p, if the mean of the following distribution is 20

x:	**15**	**17**	**19**	**20 + p**	**23**
f:	**2**	**3**	**4**	**5p**	**6**

Solution:

x_i	f_i	$x_i f_i$
15	2	30
17	3	51
19	4	76
20 + p	5P	(20 + p) 5p
23	6	138
Total	$\Sigma f_i = 15 + 5p$	$\Sigma f_i x_i = 295 + (20 + p) 5p$

Mean score = $\Sigma f_i x_i / \Sigma f_i$

20 = [(295 + (20 + p) 5p)]/ 15 + 5p

295 + 100 p + $5p^2$ = 300 + 100p

$5p^2$ = 300 – 295

$5p^2$= 5

p^2 = 1

p = 1

Exercise 23.3

Find the median of the following data (1 – 8)

1. 83, 37, 70, 29, 45, 63, 41, 70, 34, 54

Solution:

First we have to arrange given data into ascending order, 29, 34, 37, 41, 45, 54, 63, 70, 70, 83

Given number of observations, n = 10 (even)

Therefore median = $(n/2)^{th}$ term + $((n + 1)/2)^{th}$ term

Median = (value of 5^{th} term + value of 6^{th} term)/2

= (45 + 54)/2

= 49.5

Hence median for given data = 49.5

2. 133, 73, 89, 108, 94,104, 94, 85, 100, 120

Solution:

First we have to arrange given data into ascending order,

73, 85, 89, 94, 100, 104, 108, 120, 133

Given number of observations, n = 10 (even)

Therefore median = $(n/2)^{th}$ term + $((n + 1)/2)^{th}$ term

Median = (value of 5^{th} term + value of 6^{th} term)/2

= (94 + 100)/2

= 97

Hence median for given data = 97

3. 31, 38, 27, 28, 36, 25, 35, 40

Solution:

First we have to arrange given data into ascending order

25, 27, 28, 31, 35, 36, 38, 40

Given number of observations, n = 8 (even)

Therefore median = $(n/2)^{th}$ term + $((n + 1)/2)^{th}$ term

Median = (value of 4^{th} term + value of 5^{th} term)/2

= (31 + 35)/2

= 33

Hence median for given data = 33

4. 15, 6, 16, 8, 22, 21, 9, 18, 25

Solution:

First we have to arrange given data into ascending order

6, 8, 9, 15, 16, 18, 21, 22, 25

Given number of observations, n = 9 (odd)

Therefore median = $((n+1)/2)^{th}$ term

Median = value of 5^{th} term

= 16

5. 41, 43,127, 99, 71, 92, 71, 58, 57

Solution:

First we have to arrange given data into ascending order

41, 43, 57, 58, 71, 71, 92, 99, 127

Given number of observations, n = 9 (odd)

Therefore median = $((n+1)/2)^{th}$ term

Median = value of 5^{th} term

= 71

6. 25, 34, 31, 23, 22, 26, 35, 29, 20, 32

Solution:

First we have to arrange given data into ascending order,

20, 22, 23, 25, 26, 29, 31, 32, 34, 35

Given number of observations, n = 10 (even)

Therefore median = $(n/2)^{th}$ term + $((n + 1)/2)^{th}$ term

Median = (value of 5^{th} term + value of 6^{th} term)/2

= (26 + 29)/2

= 27.5

Hence median for given data = 27.5

7. 12, 17, 3, 14, 5, 8, 7, 15

Solution:

First we have to arrange given data into ascending order,

3, 5, 7, 8, 12, 14, 15, 17

Given number of observations, n = 8 (even)

Therefore median = $(n/2)^{th}$ term + $((n +1)/2)^{th}$ term

Median = (value of 4^{th} term + value of 5^{th} term)/2

= (8 + 12)/2

= 10

Hence median for given data = 10

8. 92, 35, 67, 85, 72, 81, 56, 51, 42, 69

Solution:

First we have to arrange given data into ascending order,

35, 42, 51, 56, 67, 69, 72, 81, 85, 92

Given number of observations, n = 10 (even)

Therefore median = $(n/2)^{th}$ term + $((n + 1)/2)^{th}$ term

Median = (value of 5^{th} term + value of 6^{th} term)/2

= (67 + 69)/2

= 68

Hence median for given data = 68

9. Numbers 50, 42, 35, 2x +10, 2x – 8, 12, 11, 8, 6 are written in descending order and their median is 25, find x.

Solution:

Here, the number of observations n is 9.

Since n is odd, the median is the n+12th observation, i.e., the 5^{th} observation.

As the numbers are arranged in the descending order, we therefore observe from the last.

Median = 5^{th} observation.

=> 25 = 2x – 8

=> 2x = 25 + 8

=> 2x = 33

=> x = (33/2)

x = 16.5

10. Find the median of the following observations: 46, 64, 87, 41, 58, 77, 35, 90, 55, 92, 33. If 92 is replaced by 99 and 41 by 43 in the above data, find the new median?

Solution:

Arranging the given data in ascending order, we have:

33, 35, 41, 46, 55, 58, 64, 77, 87, 90, 92

Here, the number of observations n is 11 (odd).

Since the number of observations is odd, therefore,

Therefore median = $((n+1)/2)^{th}$ term

Median = value of 5^{th} term

= 58.

Hence, median = 58.

If 92 is replaced by 99 and 41 by 43, then the new observations arranged in ascending order are:

33, 35, 43, 46, 55, 58, 64, 77, 87, 90, 99

New median = Value of the 6^{th} observation = 58.

11. Find the median of the following data: 41, 43, 127, 99, 61, 92, 71, 58, 57, If 58 is replaced by 85, what will be the new median?

Solution:

Arranging the given data in ascending order, we have:

41, 43, 57, 58, 61, 71, 92, 99,127

Here, the number of observations, n, is 9(odd).

Therefore median = $((n+1)/2)^{th}$ term

Median = value of 5^{th} term

Hence, the median = 61.

If 58 is replaced by 85, then the new observations arranged in ascending order are:

41, 43, 57, 61, 71, 85, 92, 99, 12

New median = Value of the 5^{th} observation = 71.

12. The weights (in kg) of 15 students are: 31, 35, 27, 29, 32, 43, 37, 41, 34, 28, 36, 44, 45, 42, 30. Find the median. If the weight 44 kg is replaced by 46 kg and 27 kg by 25 kg, find the new median.

Solution:

Arranging the given data in ascending order, we have:

27, 28, 29, 30, 31, 32, 34, 35, 36, 37, 41, 42, 43, 44, 45

Here, the number of observations n is 15(odd).

Since the number of observations is odd, therefore,

Therefore median = $((n+1)/2)^{th}$ term

Median = value of 8^{th} term

Hence, median = 35 kg.

If 44 kg is replaced by 46 kg and 27 kg by 25 kg, then the new observations arranged in ascending order are:

25, 28, 29, 30, 31, 32, 34, 35, 36, 37, 41, 42, 43, 45, 46

∴ New median = Value of the 8^{th} observation = 35 kg.

13. The following observations have been arranged in ascending order. If the median of the data is 63, find the value of x: 29, 32, 48, 50, x, x + 2, 72, 78, 84, 95

Solution:

Here, the number of observations n is 10. Since n is even,

Therefore median = $(n/2)^{th}$ term + $((n + 1)/2)^{th}$ term

Median = (value of 5^{th} term + value of 6^{th} term)/2

63 = x + (x + 2)/2

63 = (2x + 2)/2

63 = 2 (x + 1)/2

63 = x + 1

x = 63 – 1

x = 62

Exercise 23.4

1. Find the mode and median of the data: 13, 16, 12, 14, 19, 12, 14, 13, 14

By using the empirical relation also find the mean.

Solution:

Arranging the data in ascending order such that same numbers are put together, we get:

12, 12, 13, 13, 14, 14, 14, 16, 19

Here, n = 9.

Therefore median = $((n+1)/2)^{th}$ term

Median = value of 5^{th} term

Median = 14

Here, 14 occurs the maximum number of times, i.e., three times. Therefore, 14 is the mode of the data.

Now,

Mode = 3 Median – 2 Mean

14 = 3 x 14 – 2 Mean

2 Mean = 42 – 14 = 28

Mean = 28 ÷ 2

= 14.

2. Find the median and mode of the data: 35, 32, 35, 42, 38, 32, 34

Solution:

Arranging the data in ascending order such that same numbers are put together, we get:

32, 32, 34, 35, 35, 38, 42

Here, n = 7

Therefore median = $((n+1)/2)^{th}$ term

Median = value of 4^{th} term

Median = 35

Here, 32 and 35, both occur twice. Therefore, 32 and 35 are the two modes.

3. Find the mode of the data: 2, 6, 5, 3, 0, 3, 4, 3, 2, 4, 5, 2, 4

Solution:

Arranging the data in ascending order such that same values are put together, we get:

0, 2, 2, 2, 3, 3, 3, 4, 4, 4, 5, 5, 6

Here, 2, 3 and 4 occur three times each. Therefore, 2, 3 and 4 are the three modes.

4. The runs scored in a cricket match by 11 players are as follows:

6, 15, 120, 50, 100, 80, 10, 15, 8, 10, 10

Find the mean, mode and median of this data.

Solution:

Arranging the data in ascending order such that same values are put together, we get:

6, 8, 10, 10, 15, 15, 50, 80, 100, 120

Here, n = 11

Therefore median = $((n+1)/2)^{th}$ term

Median = value of 6^{th} term

Median = 15

Here, 10 occur three times. Therefore, 10 is the mode of the given data.

Now,

Mode = 3 Median – 2 Mean

10 = 3 x 15 – 2 Mean

2 Mean = 45 – 10 = 35

Mean = 35 ÷ 2

= 17.5

5. Find the mode of the following data:

12, 14, 16, 12, 14, 14, 16, 14, 10, 14, 18, 14

Solution:

Arranging the data in ascending order such that same values are put together, we get:

10, 12, 12, 14, 14, 14, 14, 14, 14, 16, 18

Here, clearly, 14 occurs the most number of times.

Therefore, 14 is the mode of the given data.

6. Heights of 25 children (in cm) in a school are as given below:

168, 165, 163, 160, 163, 161, 162, 164, 163, 162, 164, 163, 160, 163, 163, 164, 163, 160, 165, 163, 162

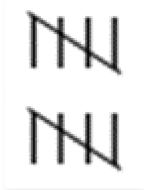
What is the mode of heights?

Also, find the mean and median.

Solution:

Arranging the data in tabular form, we get:

Height of Children (cm)	Tally marks	Frequency
160	III	3
161	I	1

162	IIII	4
163		10
164	III	3
165	III	3
168	I	1
Total		25

Therefore median = $((n+1)/2)^{th}$ term

Median = value of 13^{th} term

Median = 163 cm

Here, clearly, 163 cm occurs the most number of times. Therefore, the mode of the given data is 163 cm.

Mode = 3 Median – 2 Mean

163 = 3 x 163 – 2 Mean

2 Mean = 326

Mean = 163 cm.

7. The scores in mathematics test (out of 25) of 15 students are as follows:

19, 25, 23, 20, 9, 20, 15, 10, 5, 16, 25, 20, 24, 12, 20

Find the mode and median of this data. Are they same?

Solution:

Arranging the data in ascending order such that same values are put together, we get:

5, 9, 10, 12, 15, 16, 19, 20, 20, 20, 20, 23, 24, 25, 25

Here, n = 15

Therefore median = $((n+1)/2)^{th}$ term

Median = value of 8^{th} term

Median = 20

Here, clearly, 20 occurs most number of times, i.e., 4 times. Therefore, the mode of the given data is 20.

Yes, the median and mode of the given data are the same.

8. Calculate the mean and median for the following data:

Marks	**10**	**11**	**12**	**13**	**14**	**16**	**19**	**20**
Number of students	**3**	**5**	**4**	**5**	**2**	**3**	**2**	**1**

Using empirical formula, find its mode.

Solution:

Calculation of mean

Mean = $\Sigma f_i x_i / \Sigma f_i$

= 332/25

= 13.28

Here, n = 25, which is an odd number. Therefore,

Therefore median = $((n+1)/2)^{th}$ term

Median = value of 13^{th} term

Median = 13

Now, by using empirical formula we have,

Mode = 3Median – 2 Mean

Mode = 3 (13) – 2 (13.28)

Mode = 39 – 26.56

Mode = 12.44.

9. The following table shows the weights of 12 persons.

Weight (in kg)	**48**	**50**	**52**	**54**	**58**
Number of persons	**4**	**3**	**2**	**2**	**1**

Find the median and mean weights. Using empirical relation, calculate its mode.

Solution:

x_i	f_i	$x_i f_i$
48	4	192
50	3	150
52	2	104
54	2	108
58	1	58
Total	$\Sigma f_i = 12$	$\Sigma f_i x_i = 612$

Calculation of mean

Mean = $\Sigma f_i x_i / \Sigma f_i$

= 612/12

= 51 kg

Here n = 12

Therefore median = $(n/2)^{th}$ term + $((n + 1)/2)^{th}$ term

Median = (value of 6^{th} term + value of 7^{th} term)/2

= (50 + 50)/2

= 50

Now by empirical formula we have,

Now,

Mode = 3 Median – 2 Mean

Mode = 3 x 50 – 2 x 51

Mode = 150 – 102

Mode = 48 kg.

Thus, Mean = 51 kg, Median = 50 kg and Mode = 48 kg.

Data Handling III (Construction of Bar Graphs)

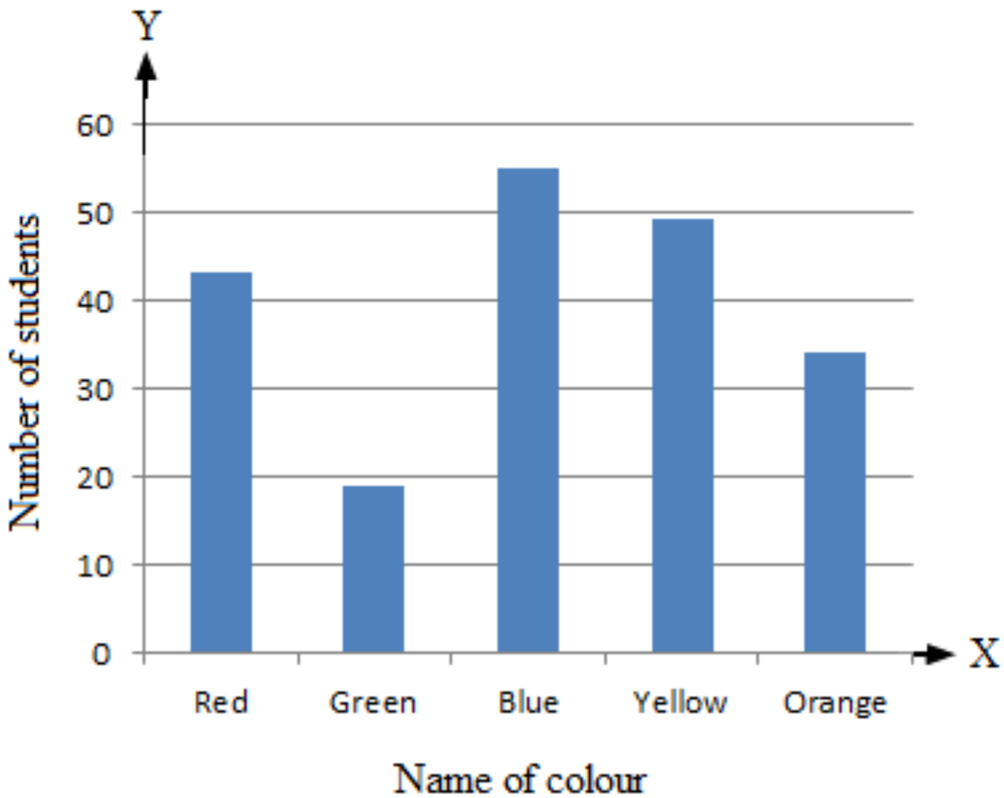
Exercise 24.1

Question 1:

Two hundred students of class VI and VII were asked to name their favourate colours so as to decide upon what should be the colour of their school house. The results are shown in the following table.

Colour:	*Red*	*Green*	*Blue*	*Yellow*	*Orange*
Number of students:	43	19	55	49	34

Represent the given data on a bar graph.
(i) Which is the most preferred colour and which is the least?
(ii) How many colours are there in all?
Solution:



Mark the horizontal axis OX as “Name of the Colour” and the vertical axis OY as “Number Of Students”.

- Along the horizontal axis OX, choose bars of uniform (equal) width, with a uniform gap between them.
- Choose a suitable scale to determine the heights of the bars, according to the space available for the graph. Here, we choose 1 small division to represent 10 student.

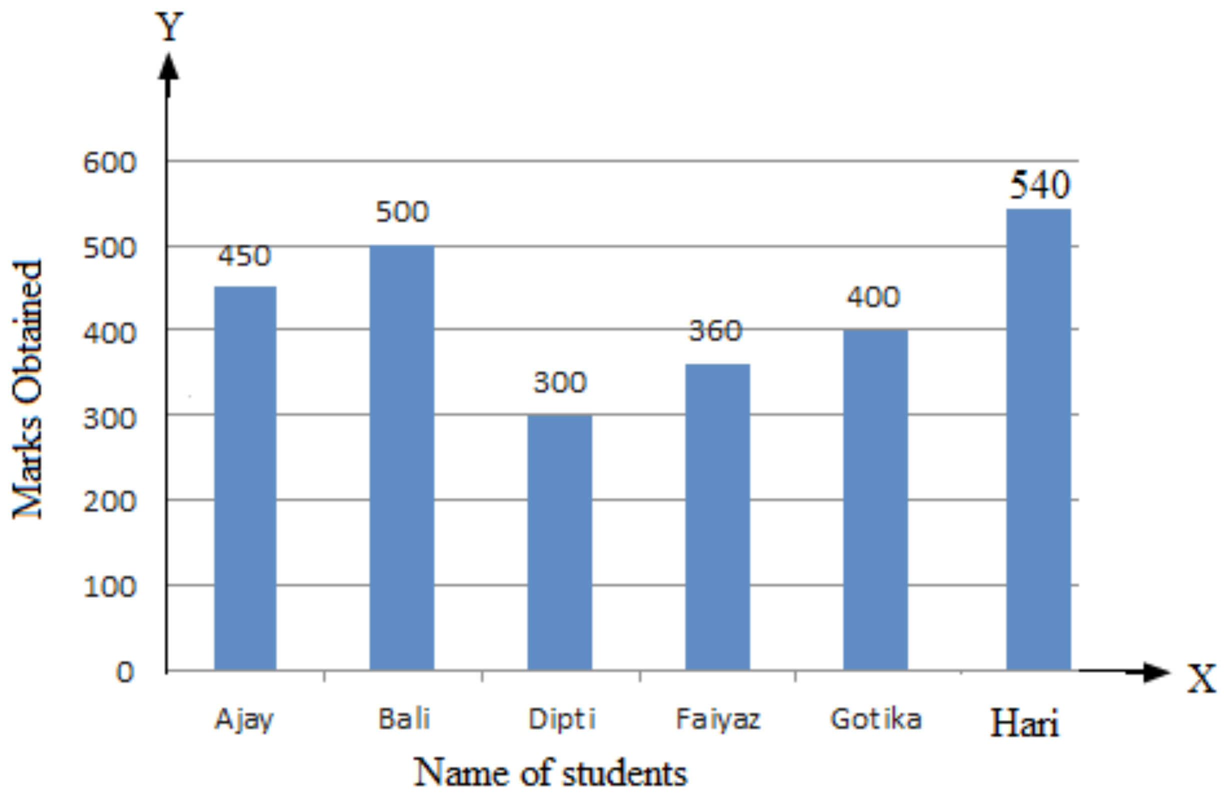
(i) The most preferred colour is blue and the least preferred is green.
(ii) In all, there are 5 colours.

Question 2:

Following data gives total marks (out of 600) obtained by six children of a particular class.

Student:	*Ajay*	*Bali*	*Dipti*	*Faiyaz*	*Gotika*	*Hari*
Marks obtained:	450	500	300	360	400	540

Represent the data by a bar graph
Solution:



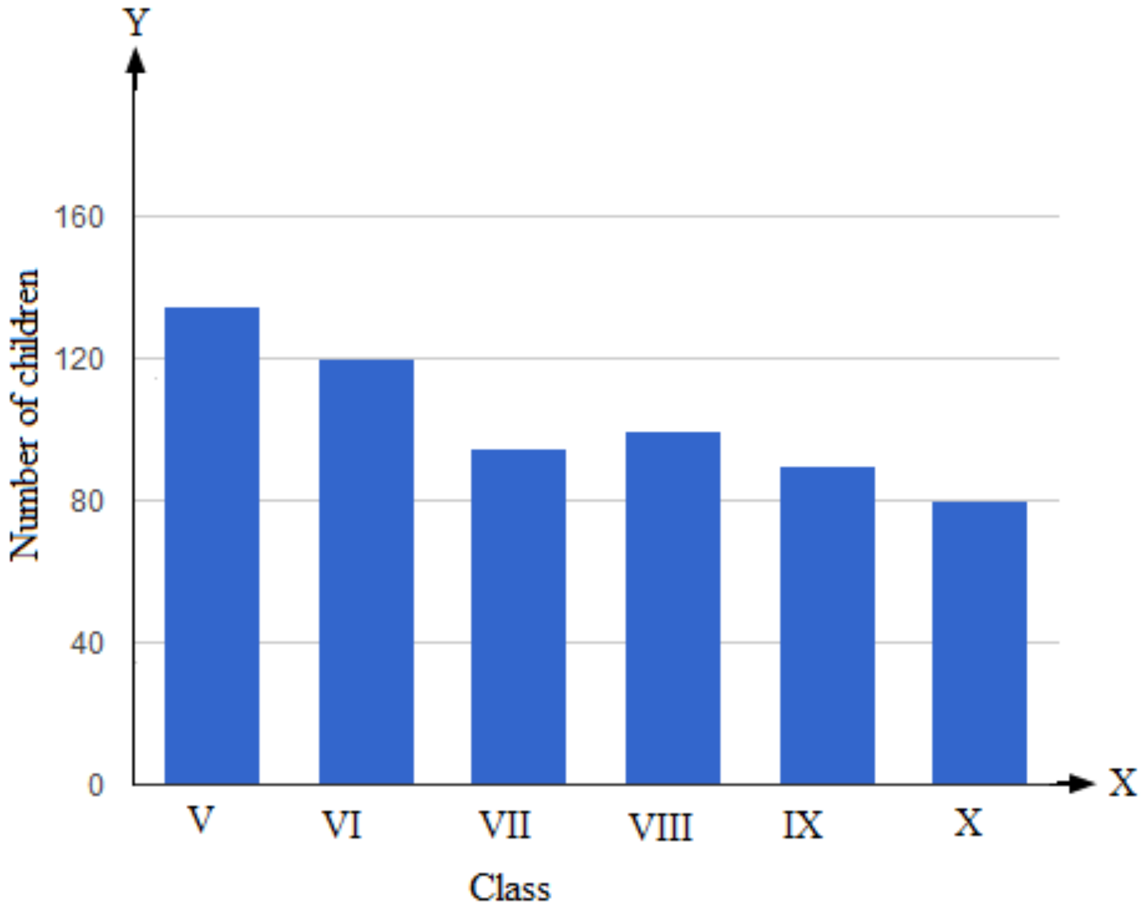
- Mark the horizontal axis OX as “Name of the Students” and the vertical axis OY as “Marks Obtained”.
- Along the horizontal axis OX, choose bars of uniform (equal) width, with a uniform gap between them.
- Choose a suitable scale to determine the heights of the bars,according to the space available for the graph.Here, we choose 1 small division to represent 100 marks.

Question 3:

Number of children in six different classes are given below. Represent the data on a bar graph.

Class:	V	VI	VII	VIII	IX	X
Number of children:	135	120	95	100	90	80

(i) How do you choose the scale.
(ii) Which class has the maximum number of children?
(iii) Which class has the minimum number of children?
Solution:



- Mark the horizontal axis OX as "Class " and the vertical axis OY as "Number of Children".
- Along the horizontal axis OX, choose bars of uniform (equal) width, with a uniform gap between them.
- Choose a suitable scale to determine the heights of the bars,according to the space available for the graph.Here, we choose 1 big division to represent 40 children.

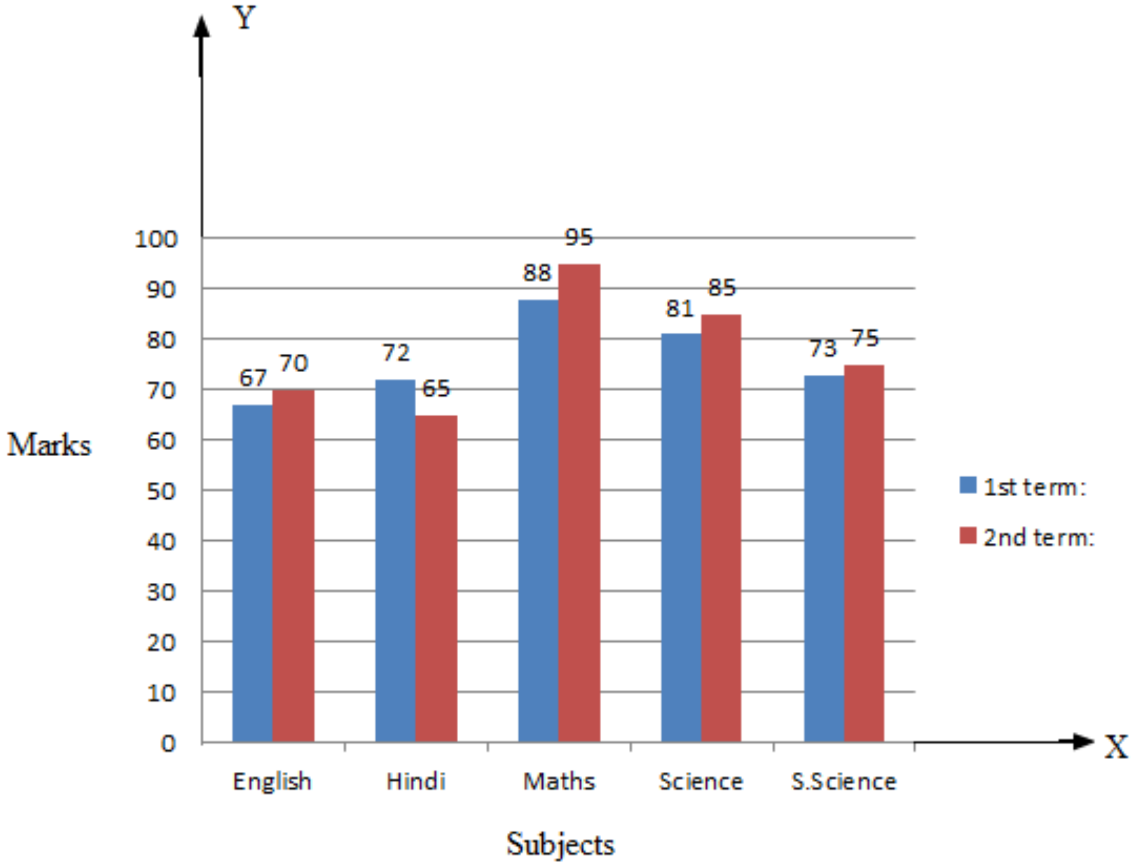
(i) We choose 1 big to represent 40 children.
(ii)The maximum number of students are in class V.
(iii) The minimum number of students are in class X.
Question 4:

The performance of students in 1st term and 2nd term is as given below. Draw a double bar graph choosing appropriate scale and answer the following:

Subject:	English	Hindi	Maths	Science	S.Science
1st term:	67	72	88	81	73
2nd term:	70	65	95	85	75

(i) In which subject, has the children improved their performance the most?
(ii) Has the performance gone down in any subject?

Solution:



We choose 1 small division to represent 1 child in the graph.

- Mark the horizontal axis OX as “Subject” and the vertical axis OY as “Marks”.
- Along the horizontal axis OX, choose bars of uniform (equal) width, with a uniform gap between them.
- Choose a suitable scale to determine the heights of the bars, according to the space available for the graph. Here, we choose 1 big division to represent 10 marks.

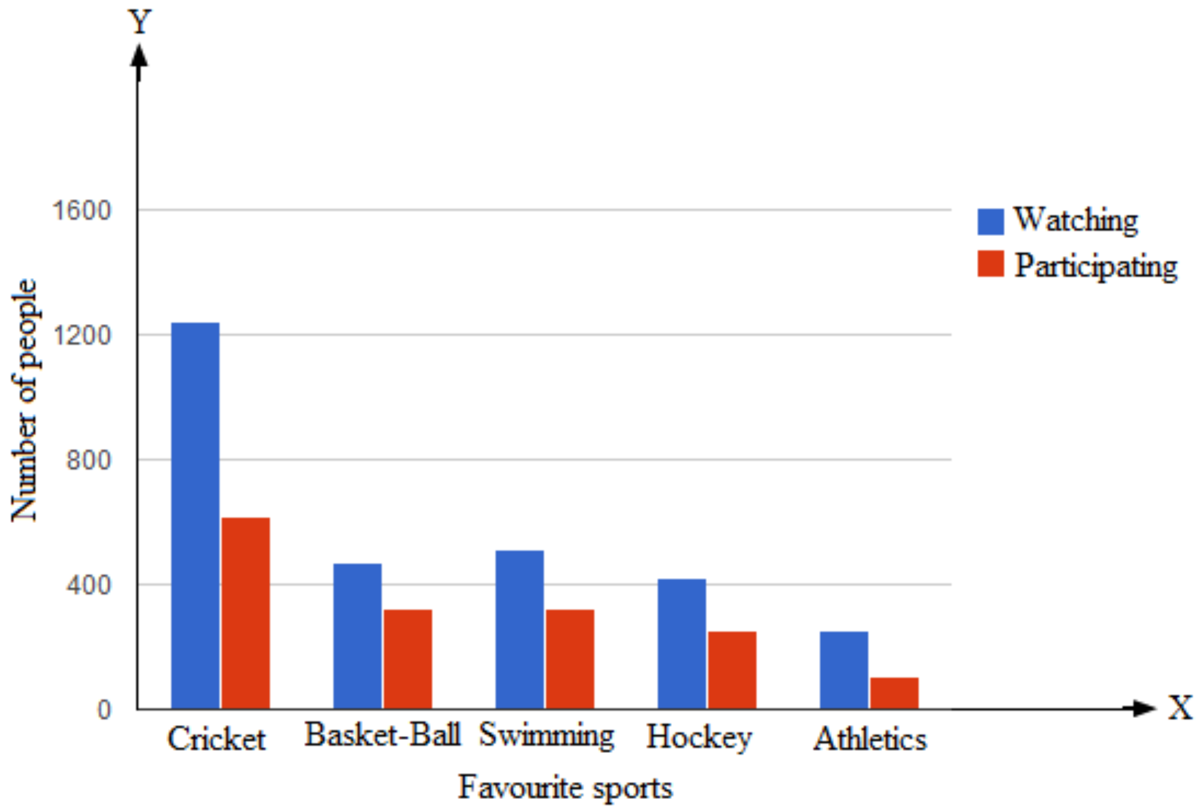
(i) In Maths, the students showed their greatest improvement.
(ii) The students performed worst in Hindi.

Question 5:

Consider the following data gathered from a survey of a colony:

Favourate Sport:	Cricket	Basket-Ball	Swimmin g	Hockey	Athletics
Watching	1240	470	510	423	250
Participating	620	320	320	250	105

Draw a double bar graph choosing an appropriate scale. What do you infer from the bar graph?
(i) Which sport is most popular?
(ii) What is more preferred watching or participating in sports?
Solution:



- Mark the horizontal axis OX as “Favourite Sports” and the vertical axis OY as “Number of People”.
- Along the horizontal axis OX, choose bars of uniform (equal) width, with a uniform gap between them.
- Choose a suitable scale to determine the heights of the bars,according to the space available for the graph.Here, we choose 2 big division to represent 400 people .

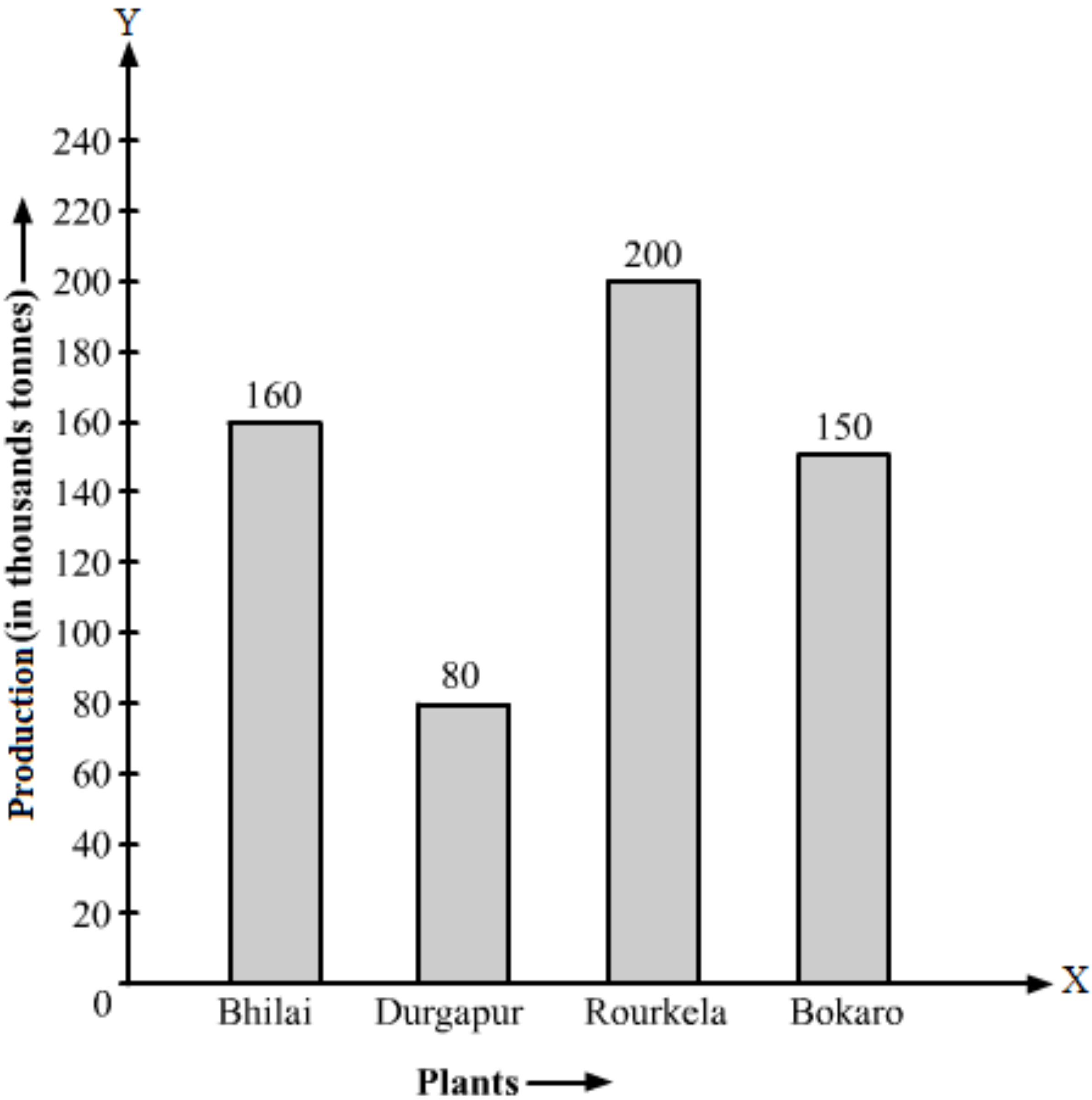
(i) Cricket is the most popular sport.
(ii) Watching is preferred over participation.
Question 6:

The production of saleable steel in some of the steel plants of our country during 1999 is given below:

Plant	Bhilai	Durgapur	Rourkela	Bokaro
Production (In thousand	160	80	200	150

Construct a bar graph to represent the above data on a graph paper by using the scale 1 big division = 20 thousand tonnes.

Solution:



- Mark the horizontal axis OX as “Name of the Steel Plant” and the vertical axis OY as “Production (in thousand tonnes)”.
- Along the horizontal axis OX, choose bars of uniform (equal) width,with a uniform gap between them.
- Choose a suitable scale to determine the heights of the bars,according to the space available for the graph.Here, we choose 1 big division to represent 20 thousand tonnes.

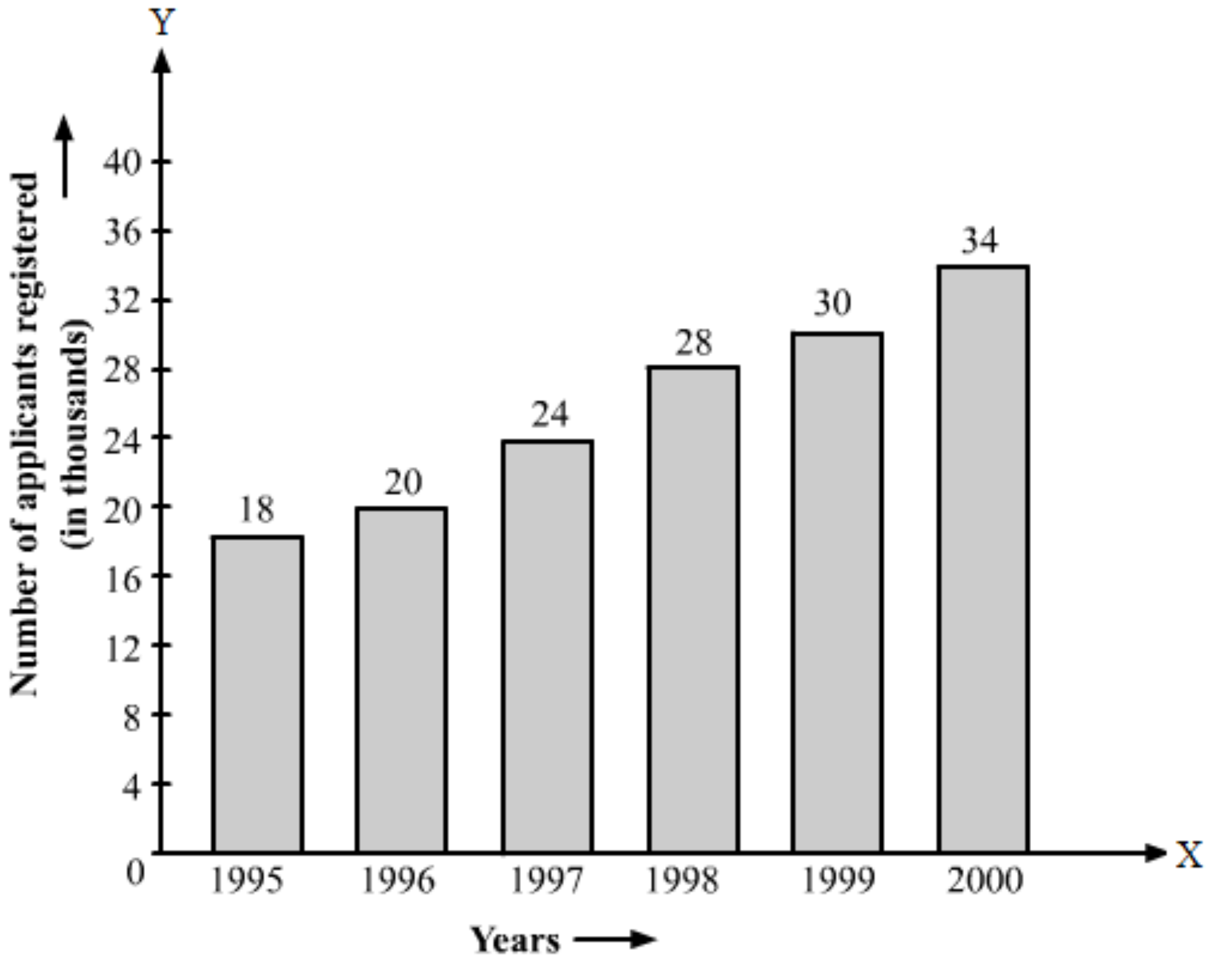
Question 7:

The following data gives the number (in thousands) of applicants registered with an Employment Exchange during, 1995-2000:

Year	1995	1996	1997	1998	1999	2000
Number of applicants registered (in	18	20	24	28	30	34

Construct a bar graph to represent the above data.

Solution:



- Mark the horizontal axis OX as "Years" and the vertical axis OY as "Number of Applicants Registered (in thousands)".
- Along the horizontal axis OX, choose bars of uniform (equal) width, with a uniform gap between them.
- Choose a suitable scale to determine the heights of the bars, according to the space available for the graph. Here, we choose 1 big divisions to represent 4 thousand applicants.

Question 8:

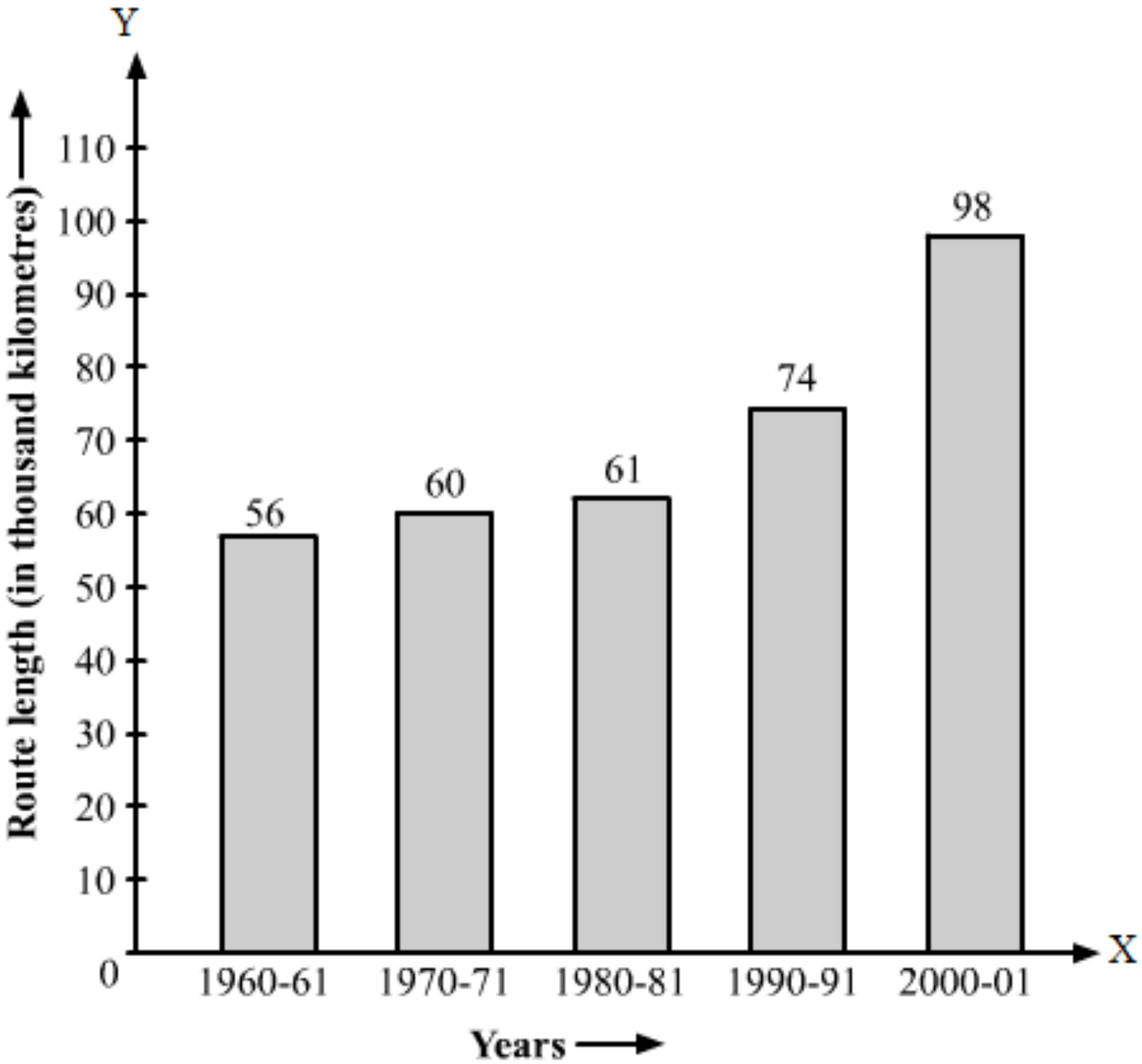
The following table gives the route length (in thousand kilometres) of the Indian Railways in some of the years:

Year	1960-61	1970-71	1980-81	1990-91	2000-20 01

Route length (in thousand	56	60	61	74	98

Represent the above data with the help of a bar graph.

Solution:



- Mark the horizontal axis OX as "Years" and the vertical axis OY as "Route Length (in thousand kilometres)".
- Along the horizontal axis OX, choose bars of uniform (equal) width, with a uniform gap between them.
- Choose a suitable scale to determine the heights of the bars, according to the space available for the graph. Here, we choose 1 big division to represent 1000 Km.

Question 9:

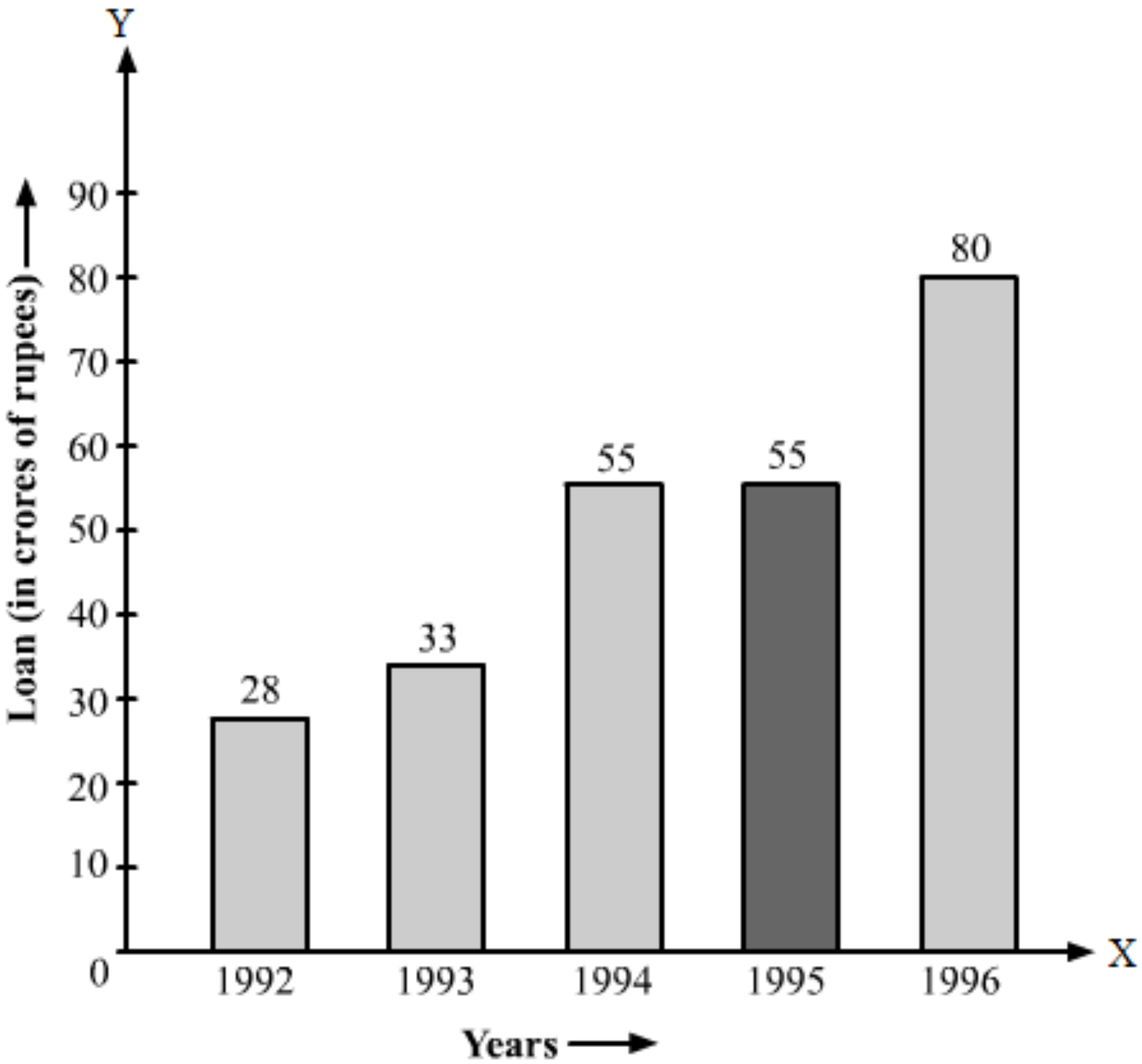
The following data gives the amount of loans (in crores of rupees) disbursed by a bank during some years:

Year	1992	1993	1994	1995	1996
Loan (in crores of rupees)	28	33	55	55	80

(i) Represent the above data with the help of a bar graph.

(ii) With the help of the bar graph, indicate the year in which amount of loan is not increased over that of the preceding year.

Solution:



- Mark the horizontal axis OX as "Years" and the vertical axis OY as "Loan (in crores of rupees)".
- Along the horizontal axis OX, choose bars of uniform (equal) width, with a uniform gap between them.
- Choose a suitable scale to determine the heights of the bars, according to the space available for the graph. Here, we choose 1 big division to represent 10 crore of rupees.

In 1995, the loan amount was not increased over that of the preceding year.

Question 10:

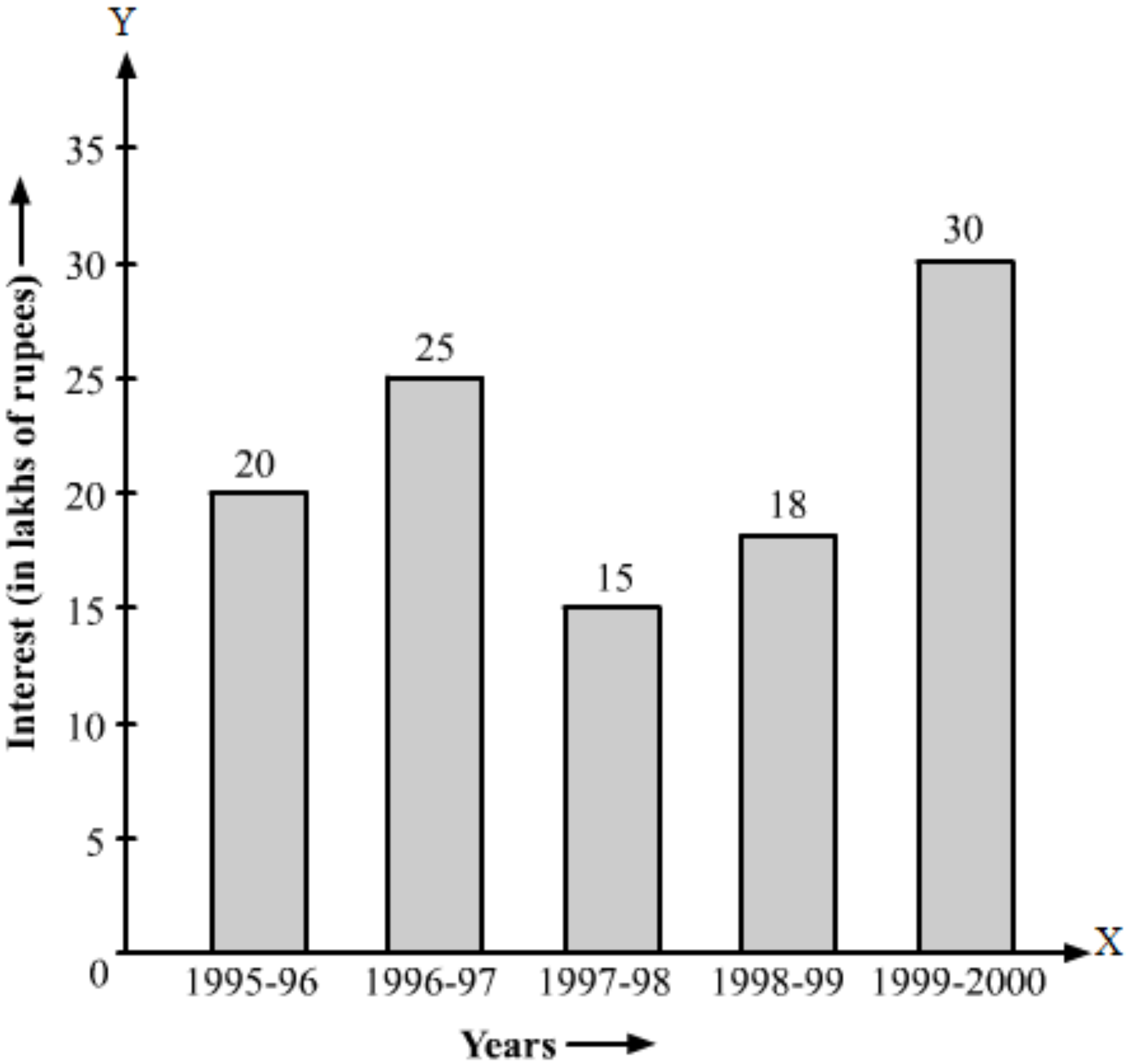
The following table shows the interest paid by a company (in lakhs):

Year	1995-96	1996-97	1997-98	1998-99	1999-2000

Interest (in lakhs of	20	25	15	18	30

Draw the bar graph to represent the above information.

Solution:



- Mark the horizontal axis OX as “Years” and the vertical axis OY as “Interest (in lakhs of rupees)”.
- Along the horizontal axis OX, choose bars of uniform (equal) width, with a uniform gap between them.
- Choose a suitable scale to determine the heights of the bars, according to the space available for the graph. Here, we choose 1 big divisions to represent 5 lakhs rupees.

Question 11:

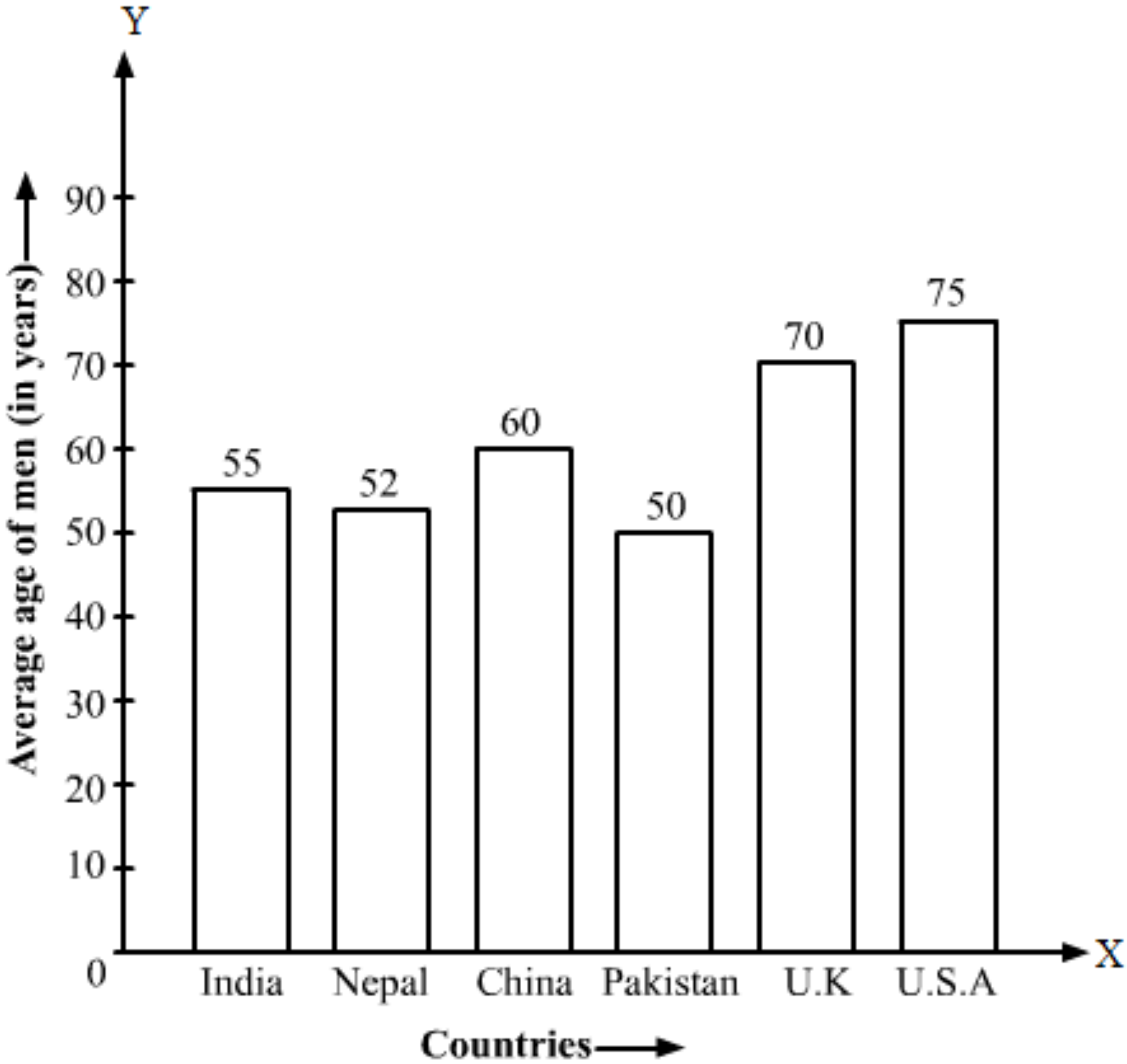
The following data shows the average age of men in various countries in a certain year:

Country	India	Nepal	China	Pakistan	U.K.	U.S.A.

Average age (in years)	55	52	60	50	70	75

Represent the above information by a bar graph.

Solution:



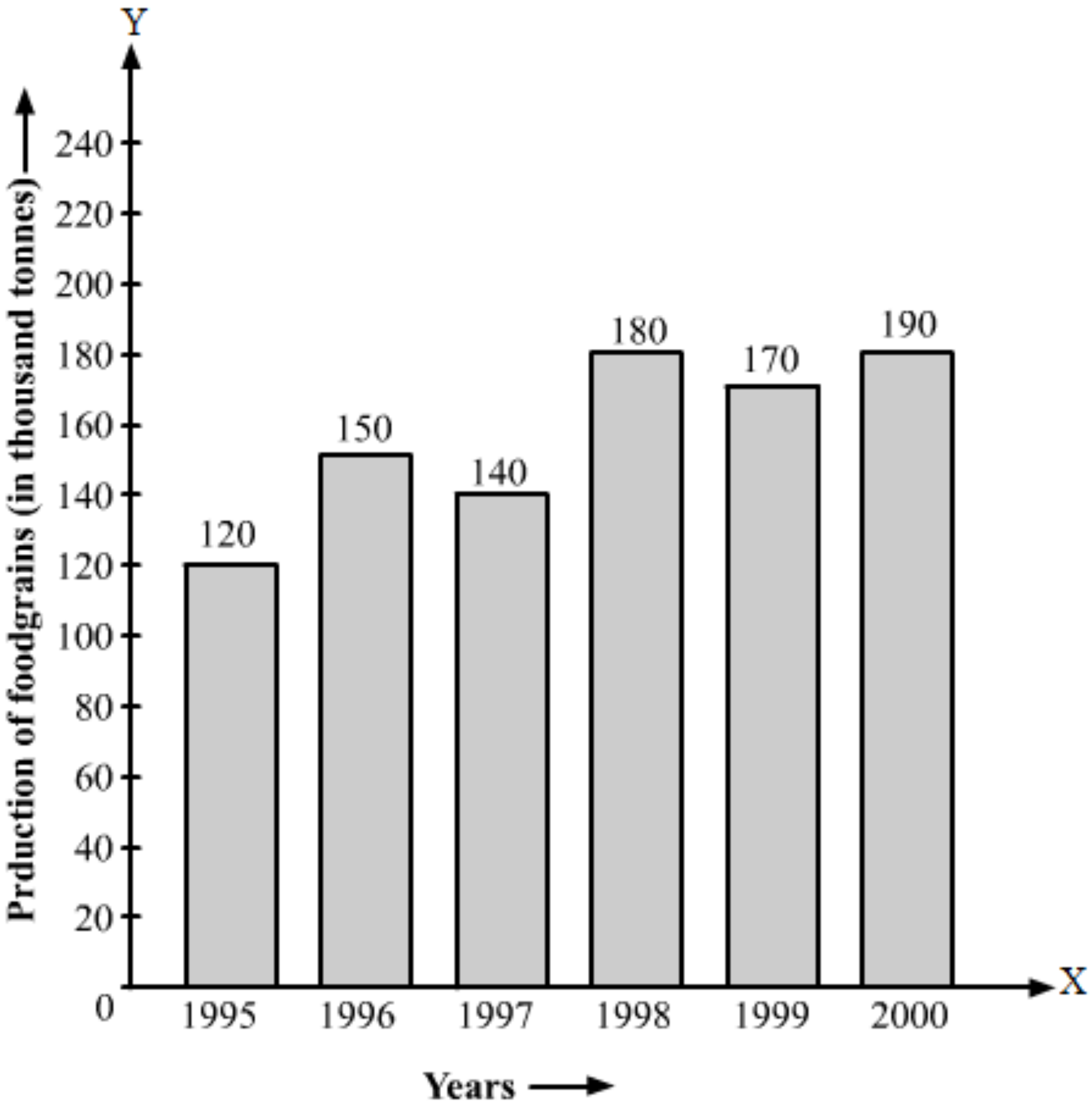
- Mark the horizontal axis OX as "Countries" and the vertical axis OY as "Average Age of men (in years)".
- Along the horizontal axis OX, choose bars of uniform (equal) width, with a uniform gap between them.
- Choose a suitable scale to determine the heights of the bars, according to the space available for the graph. Here, we choose 1 big division to represent 10 year.

Question 12:

The following data gives the production of foodgrains (in thousand tonnes) for some years:

Year	1995	1996	1997	1998	1999	2000
Production (in thousand	120	150	140	180	170	190

Represent the above data with the help of a bar graph.
Solution:



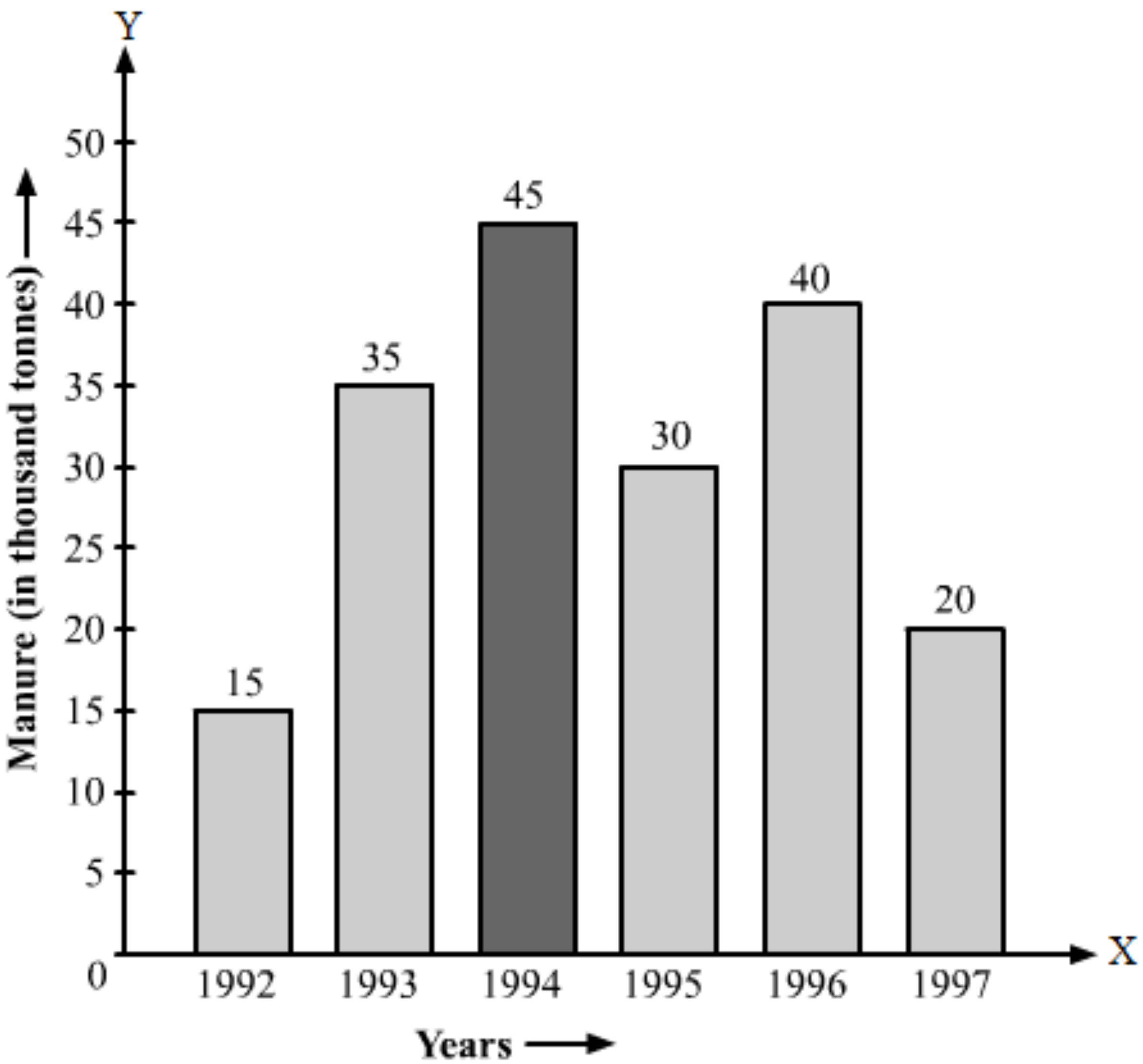
- Mark the horizontal axis OX as “Years” and the vertical axis OY as “Production of foodgrains (in thousand tonnes)”.
- Along the horizontal axis OX, choose bars of uniform (equal) width, with a uniform gap between them.
- Choose a suitable scale to determine the heights of the bars, according to the space available for the graph.Here, we choose 1 big division to represent 20 thousand tonnes.

Question 13:

The following data gives the amount of manure (in thousand tonnes) manufactured by a company during some years:

Year	1992	1993	1994	1995	1996	1997
Manure (in thousand	15	35	45	30	40	20

(i) Represent the above data with the help of a bar graph.
(ii) Indicate with the help of the bar graph the year in which the amount of manure manufactured by the company was maximum.
(iii) Choose the correct alternative:
The consecutive years during which there was maximum decrease in manure production are:
(a) 1994 and 1995
(b) 1992 and 1993
(c) 1996 and 1997
(d) 1995 and 1996
Solution:



- Mark the horizontal axis OX as “Years” and the vertical axis OY as “Manure (in thousand tonnes)”.

- Along the horizontal axis OX, choose bars of uniform (equal) width, with a uniform gap between them.
- Choose a suitable scale to determine the heights of the bars, according to the space available for the graph. Here, we choose 1 big divisions to represent 5 thousand tonnes.

(ii) In the year 1994 , the amount of manure manufactured by the company was maximum.
(iii) 1996 and 1997.

Probability

Exercise 25.1

Question :1. A coin is tossed 1000 times with the following frequencies:
Head: 445, Tail: 555
When a coin is tossed at random, what is the probability of obtaining?
(i) A head?
(ii) A tail?
Solution :

Specified below that total number of times a coin is tossed = 1000

Head comes in coin = 445

Tail comes in a coin = 555

(i) Probability of obtaining head = ((number of heads)/(total number of trails))

= (445/100) = 0.445

(ii) Probability of obtaining tail = ((number of tail)/(total number of trails))

= ((.555)/1000) = 0.555

Question :2. A die is thrown 100 times and outcomes are noted as Specified below that below:

Outcome	**1**	**2**	**3**	**4**	**5**	**6**
Frequency	**21**	**9**	**14**	**23**	**18**	**15**

If a die is thrown at random, find the probability of obtaining a/an:
(i) 3
(ii) 5
(iii) 4
(iv) Even number
(v) Odd number
(vi) Number less than 3.

Solution :

Specified below that total number of trials = 100

(i) From the table, no. of times 3 obtain = 14

Probability of obtaining 3 = ((frequency of 3)/(total number of trails))

= (14/100) = (7/50)

(ii) From the table, no. of times 5 occur = 18

Probability of obtaining 5 = ((frequency of 5)/(total number of trails))

= (18/100) = (5/50)

(iii) From the table, no. of times 4 occur = 23

Probability of obtaining 4 = ((frequency of 4)/(total number of trails))

= (23/100)

(iv) Frequency of obtaining an even number

= Frequency of 2 + Frequency of 4 + Frequency of 6

= 9 + 23 + 15

= 47

An even number's obtaining probability = ((frequency of an even number)/(total number of trails))

= (47/100)

(v) Frequency of obtaining an even number

= Frequency of 1 + Frequency of 3 + Frequency of 5

= 21 + 14 + 18

= 53

An odd number's obtaining probability = ((frequency of an odd number)/(total number of trails))

= (53/100)

(vi) Frequency of obtaining number less than 3

= Frequency of 1 + Frequency of 2

= 21 + 9

= 30

Probability of obtaining number less than 3 = ((frequency of number less than 3)/(total number of trails))

= (30/100)

= (3/10)

Question :3. A box contains two pair of socks of two colures (black and white). I have picked out a white sock. I pick out one more with my eyes closed. What is the probability that I will make a pair?

Solution :

Specified below that number of socks in the box = 4

Let B denote black and W denote white socks. Then

S = {B, B, W, W}

If a white sock is picked out, then the total no. of socks left in the box = 3

Number of white socks left = 2 – 1 = 1

Probability of obtaining white socks = ((number of white socks left in the box)/(total number of socks left in the box))

= (1/3)

Question :4. Two coins are tossed simultaneously 500 times and the outcomes are noted as Specified below that below:

Outcome:	**Two heads (HH)**	**One head (HT or TH)**	**No head (TT)**
Frequency:	**105**	**275**	**120**

If same pair of coins is tossed at random, find the probability of obtaining:

(i) Two heads

(ii) One head

(iii) No head.

Solution :

Specified below that number of trials = 500

From the Specified below that table, it is clear that,

Number of outcomes of two heads (HH) = 105

Number of outcomes of one head (HT or TH) = 275

Number of outcomes of no head (TT) = 120

(i) Probability of obtaining two heads = ((frequency of obtaining 2 heads)/(total number of trials))

= (105/500) = (21/100)

(ii) Probability of obtaining one head = ((frequency of obtaining 1 heads)/(total number of trials))

= (275/500) = (11/20)

(iii) Probability of obtaining no head = ((frequency of obtaining no heads)/(total number of trials))

= (120/500) = (6/25)