APS1022 PROJECT 2 REPORT

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<u>Estimation of parameters</u>

The monthly adjusted closing prices of each of the 20 stocks were downloaded from *Yahoo Finance*. The time period I used was from December 2004 through to September 2008. The 20 individual csv files of the closing prices were combined into a single file. I then calculated the sample mean (mu) of each of the stocks. After that, I calculated the covariance matrix of the 20 stocks. For my market capitalization portfolio (Model 4), I obtained the market cap values of the 20 stocks from *Google Finance*. I then normalized the 20 market cap values and used that as the portfolio weights for the market capitalization portfolio. For the annual risk-free rate, I took the average of the annual risk-free rates from 2004 to 2008. The individual risk-free rates for these years were obtained from *macrotrends.net*.

Sample mean of stocks:

```
mean return:

array([-2.47672167e-02,  2.05868694e-03,  1.43739778e-05,  1.63954745e-02,
        4.54341806e-03,  4.17431531e-03,  3.74537781e-03, -1.98470248e-02,
        7.82040636e-03,  7.83931502e-03,  3.15632257e-02,  3.08112537e-03,
       -1.94987945e-03,  1.40589894e-03,  1.14095581e-02,  1.66172102e-02,
        4.67715630e-03,  8.13844736e-03,  1.87990673e-03, -4.39309782e-03])
```

First 14 rows and columns of covariance matrix:

	F	CAT	DIS	MCD	KO	PEP	WMT	C	WFC	JPM	AAPL	IBM	PFE	JNJ
F	0.021957	0.005966	0.003265	0.003193	0.002823	0.002279	0.002748	0.008763	0.003159	0.004948	0.002881	0.004677	0.001585	0.001856
CAT	0.005966	0.007163	0.002322	0.001826	0.001839	0.001650	0.000381	0.002927	-0.000239	0.000640	0.004730	0.002547	0.001178	0.000683
DIS	0.003265	0.002322	0.002522	0.001118	0.001132	0.001054	0.000432	0.001672	-0.000248	0.000175	0.001547	0.001674	0.000220	0.000579
MCD	0.003193	0.001826	0.001118	0.002818	0.001361	0.000948	0.000268	0.001907	0.000525	0.001265	0.004143	0.001809	0.000684	0.000738
KO	0.002823	0.001839	0.001132	0.001361	0.001971	0.001550	0.000235	0.001880	0.000632	0.001276	0.001687	0.001323	0.000483	0.000967
PEP	0.002279	0.001650	0.001054	0.000948	0.001550	0.002249	0.000165	0.002044	0.000960	0.001426	0.001275	0.001069	0.000494	0.001194
WMT	0.002748	0.000381	0.000432	0.000268	0.000235	0.000165	0.002129	0.000844	0.000811	0.001184	-0.000468	0.000780	-0.000639	0.000448
C	0.008763	0.002927	0.001672	0.001907	0.001880	0.002044	0.000844	0.008502	0.004235	0.004881	0.001344	0.002489	0.001428	0.001538
WFC	0.003159	-0.000239	-0.000248	0.000525	0.000632	0.000960	0.000811	0.004235	0.005671	0.004895	-0.002409	0.000970	0.001383	0.000734
JPM	0.004948	0.000640	0.000175	0.001265	0.001276	0.001426	0.001184	0.004881	0.004895	0.005632	-0.000177	0.001196	0.001388	0.001028
AAPL	0.002881	0.004730	0.001547	0.004143	0.001687	0.001275	-0.000468	0.001344	-0.002409	-0.000177	0.016221	0.002846	0.001076	0.000712
IBM	0.004677	0.002547	0.001674	0.001809	0.001323	0.001069	0.000780	0.002489	0.000970	0.001196	0.002846	0.004144	-0.000175	0.000905
PFE	0.001585	0.001178	0.000220	0.000684	0.000483	0.000494	-0.000639	0.001428	0.001383	0.001388	0.001076	-0.000175	0.002892	0.000302
JNJ	0.001856	0.000683	0.000579	0.000738	0.000967	0.001194	0.000448	0.001538	0.000734	0.001028	0.000712	0.000905	0.000302	0.001277

Model 1: Standard MVO

The main advantage of the standard mean-variance optimization method is that it is simple and easy to formulate and solve. The only required data to solve this optimization problem are the stock returns (mu) and the covariance matrix (Q) both of which we already have. The main disadvantage of this optimization method is that even small changes in the stock returns can cause very big differences in the optimal weights. Therefore, this model is fairly unstable and is very sensitive to changes in the stock market. The optimization model can be seen below:

$$\min_{x} \mu^T x - \lambda x^T Q x$$
$$\text{subject to } e^T x = 1$$

The lambda value (risk aversion coefficient) I used was calculated from the market capitalization returns of the 20 stocks as well as the annual-risk free rate obtained earlier. The resulting value for lambda is the following:

```
lambda value: 1.724853003659088
```

Optimal weights when using the standard MVO model:

```
optimal weights:

array([-0.04787524, -0.27615912,  1.43066752,  2.57924857, -1.72953239,
        1.59885866, -0.09976808, -2.87766163,  3.71492839,  0.8032586 ,
        0.40970341, -2.30184976, -3.20081513, -0.27213403,  1.5227508 ,
        1.05474703, -2.05166964,  1.75612452, -1.03203264,  0.01921016])
```

Model 2: Robust MVO

The main advantage of the robust MVO method is that unlike the MVO method in model 1, the robust MVO model is much more stable since it is less sensitive to changes in the stock market. Additionally, the robust MVO model provides an efficient frontier that is generally closer to the true frontier, therefore the model provides a better estimate of the real-word conditions compared to the non-robust estimate. The disadvantage of this model is that it is a bit more difficult to formulate since we must incorporate a specific confidence interval into the optimization problem. The robust MVO optimization model can be seen below:

$$\begin{aligned} \min_{\boldsymbol{x}} \quad & \boldsymbol{x}^T \boldsymbol{Q} \boldsymbol{x} \\ \text{s.t.} \quad & \left.\mu^T \boldsymbol{x} - \varepsilon_2 \|\Theta^{1/2} \boldsymbol{x}\|_2 \geq R \right\} \text{ Penalize our return constraint} \\ & \mathbf{1}^T \boldsymbol{x} = 1 \\ & (\boldsymbol{x} \geq \mathbf{0}) \end{aligned}$$

Optimal weights when using the robust MVO model:

```
optimal weights:

array([-0.71777204, -0.55249513,  1.35364844,  4.50197202, -2.87735193,
        3.09349214,  0.98032363, -2.60736622,  3.1925638 ,  1.98402768,
        0.4328779 , -2.86476344, -4.00570973, -1.68524945,  2.39447886,
        1.76549221, -3.09986383,  2.42470173, -2.45169026, -0.26131638])
```

Model 3: Risk parity optimization with no short-selling

The main advantage of the risk parity optimization method is that, as the name suggests, it helps to maintain a portfolio that has a very high amount of risk diversification while still meeting the required return. In the equal risk contributions portfolio, each stock should have the same amount of risk contribution to the portfolio, meaning that a significant change in one of the stocks will not cause a significant change in the portfolio. This generally results in a higher expected return compared to other optimization methods. A disadvantage of this model is that an equal risk contributions model commonly leverages low-risk assets to counterbalance the higher-risk assets, thus creating an imbalance in the resulting portfolio.

Optimal weights when using risk parity optimization:

```
optimal weights for risk parity optimization with no short-selling:

array([0.01448362, 0.02629311, 0.05523552, 0.04005757, 0.04790042,
       0.05155798, 0.12613948, 0.02288095, 0.05597793, 0.0362749 ,
       0.02315996, 0.03759066, 0.0641025 , 0.07050151, 0.05143452,
       0.02855864, 0.10397183, 0.04990735, 0.04961795, 0.04435361])
```

Model 4: Market capitalization

The main advantage of using market capitalization is that compared to the optimization approaches above, the market cap allows us to determine which stocks are worth more in the eyes of actual investors. A diversified portfolio that contains a wide variety of market caps can help reduce investment risk. A disadvantage of using this method is that since no optimization is performed in obtaining the portfolio, we are essentially just following market trends and it is unlikely that we will obtain a favorable portfolio return compared to optimization approaches.

Normalized weights when using market capitalization (market cap data obtained from *Google Finance*):

```
array([0.00978498, 0.02042391, 0.05299149, 0.02891391, 0.03981877,
       0.03368417, 0.06474205, 0.02625294, 0.03078534, 0.08020574,
       0.34686778, 0.02221771, 0.03761203, 0.07267541, 0.04387877,
       0.00175416, 0.00436228, 0.03441424, 0.03921093, 0.00940342])
```

Question A

Portfolio quantities for October 2008 using standard MVO model:

```
Using standard MVO:
Return for October 2008 = 0.4140118073934632
Variance for October 2008 = 0.016916682902129988
Standard deviation for October 2008 = 0.13006414918081766
Sharpe Ratio for October 2008 = 3.1552966489604795
```

Portfolio quantities for October 2008 using robust MVO model:

```
Using robust MVO:
Return for October 2008 (90% CL) = 0.2685947640335115
Return for October 2008 (95% CL) = 0.15689180695145954
Variance for October 2008 (90% CL) = 0.03413310565198897
Variance for October 2008 (95% CL) = 0.036303574885277023
Standard deviation for October 2008 (90% CL) = 0.18475146995893962
Standard deviation for October 2008 (95% CL) = 0.190534970242412 5
Sharpe Ratio for October 2008 (90% CL) = 1.4342182541717676
Sharpe Ratio for October 2008 (95% CL) = 0.8044243711436473
```

Portfolio quantities for October 2008 using risk parity optimization:

```
Using risk parity optimization with no short-selling:
Return for October 2008 = -0.022701204942354853
Variance for October 2008 = 0.0009151974679887562
Standard deviation for October 2008 = 0.030252230793592003
Sharpe Ratio for October 2008 = -0.8700858609495897
```

Portfolio quantities for October 2008 using market capitalization:

```
Using market capitalization:
Return for October 2008 = -0.07666716076850223
Variance for October 2008 = 0.0031247839094572256
Standard deviation for October 2008 = 0.05589976663150952
Sharpe Ratio for October 2008 = -1.4362849603844128
```

The model with the highest return was the standard MVO model (model 1). The reason for this is because the standard MVO model directly optimizes for the highest possible return, thus it is natural for this model to have the highest return. The model with the lowest variance and standard deviation was the risk parity model (model 3). This is also expected, since the risk parity model assumes that all the assets have the same risk contribution to the overall portfolio, thus it is natural for this model to have a low volatility compared to other models. The model with the highest Sharpe ratio was the standard MVO model (model 1). I believe the reason for this is due the fact that the lambda value used in the optimization problem might have been too small. A small lambda value means that an increase in risk has little effect on the objective function values, meaning that the optimization problem is free to obtain the highest return and Sharpe ratio value possible with very little constraint to the amount of risk that is in the portfolio. In contrast, models such as the equal risk contributions model are much more conservative about the amount of risk in the portfolio, thus leading to smaller returns and Sharpe ratio values.

<u>Question B</u>

Portfolio quantities for November 2008 using standard MVO model:

```
Using standard MVO:
Return for November 2008 = 0.273479778228532
Variance for November 2008 = 0.016916682902129988
Standard deviation for November 2008 = 0.13006414918081766
Sharpe Ratio for November 2008 = 2.0748142097176645
```

Portfolio quantities for November 2008 using robust MVO model:

```
Using robust MVO:
Return for November 2008 (90% CL) = 0.26794248638057977
Return for November 2008 (95% CL) = 0.2295049167911477
Variance for November 2008 (90% CL) = 0.03413310565198897
Variance for November 2008 (95% CL) = 0.036303574885277023
Standard deviation for November 2008 (90% CL) = 0.18475146995893962
Standard deviation for November 2008 (95% CL) = 0.1905349702424125
Sharpe Ratio for November 2008 (90% CL) = 1.4306876860356834
Sharpe Ratio for November 2008 (95% CL) = 1.185525592338394
```

Portfolio quantities for November 2008 using risk parity optimization:

```
Using risk parity optimization with no short-selling:
Return for November 2008 = 0.016216964569248263
Variance for November 2008 = 0.000915197467988756
Standard deviation for November 2008 = 0.030252230793592003
Sharpe Ratio for November 2008 = 0.4163703272613876
```

Portfolio quantities for October 2008 using market capitalization:

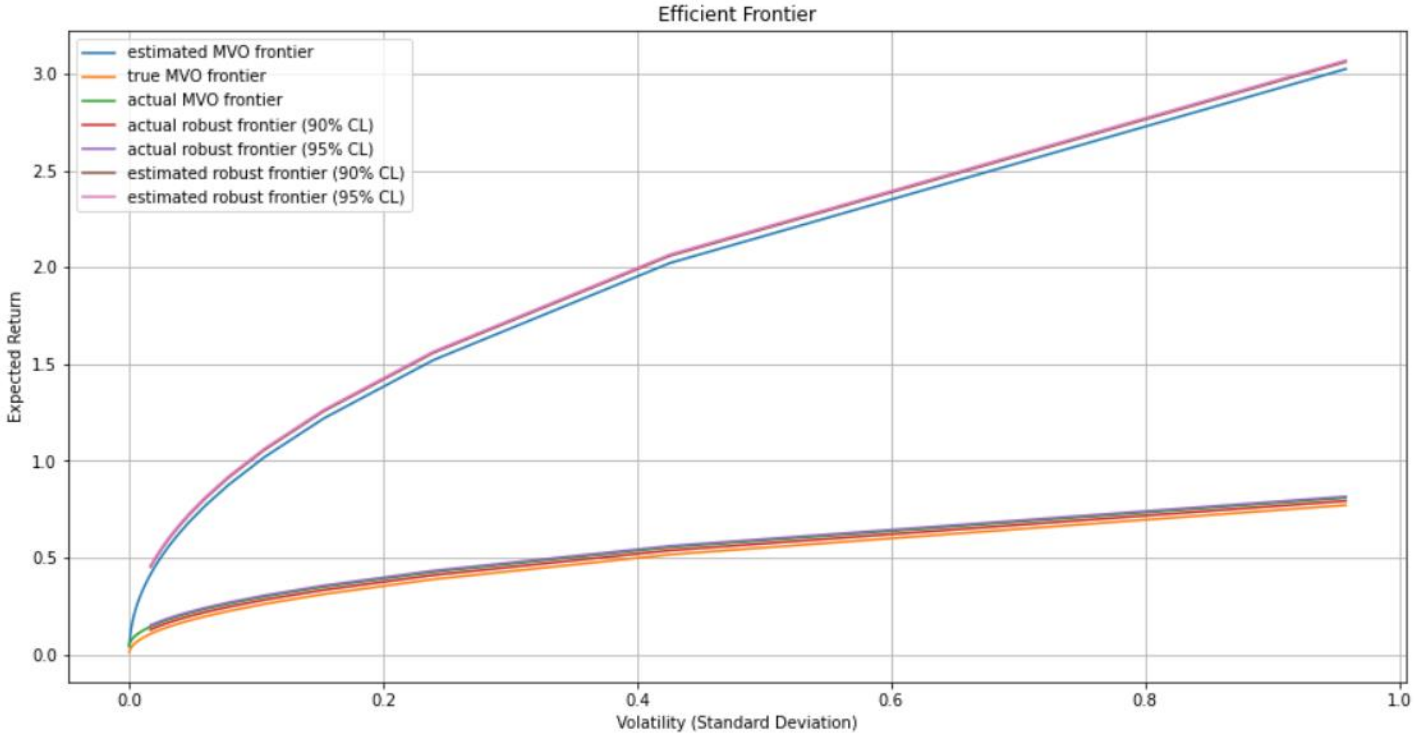
```
Using market capitalization:
Return for November 2008 = -0.020910651885338048
Variance for November 2008 = 0.003124783909457225
Standard deviation for November 2008 = 0.05589976663150952
Sharpe Ratio for November 2008 = -0.4388477215009248
```

The results for November 2008 are fairly similar to October 2008: the model with the highest return was the standard MVO model, the model with the lowest variance and standard deviation was the risk parity model, and the model with the highest Sharpe ratio was the standard MVO model. I believe that the reason for such a high return and Sharpe ratio for the standard MVO model is the same reason as before: the lambda value may have been set to be too small, thus causing a large amount of the stock risks to not be accounted for during optimization.

Question C

For our efficient frontier plots, we used 250 different lambda values ranging from 0.2 to 5. We know that the estimated MVO frontiers use both the estimated return as well as the estimated covariance matrix for calculation while the true MVO frontiers use the real return from the market as well as the real covariance matrix for calculation. The actual MVO frontiers use the real return as well as the estimated weights. We used two confidence levels (90% and 95%) when computing the robust frontiers.

The plot of the efficient frontiers can be seen below:



As we can see from the above plot, the estimated robust frontier for both confidence levels is slightly higher than the estimated non-robust frontier. The actual robust frontier for both confidence levels is fairly close to the true MVO frontier. Furthermore, we can see that the estimated frontiers for both robust and non-robust models seem to be higher than the actual frontiers, which is expected.

Discussion

Overall, we determined that the standard MVO model performs fairly well in terms of obtaining the best possible portfolio return and Sharpe ratio. The risk parity model, on the hand, is much better at reducing the portfolio risk. In practice, different optimizations strategies should be used depending on the situation. Investors who are extremely risk averse should use the risk parity model, while investors who are more indifferent to risk can find better results using the standard MVO and robust MVO models. Finally, through computing the efficient frontiers of the different models, we discovered that the real and actual efficient frontier of a model is generally lower than the estimated efficient frontier.

References

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