APS502 Project 1

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Problem 1

Decision variables:

- x_1 = amount of corporate bond
- x_2 = amount of government bond

Objective function:

Maximize $0.04x_1 + 0.03x_2$

Constraints:

1. $x_1 + x_2 <= 100,000$
2. $2x_1 + x_2 <= 150,000$
3. $3x_1 + 4x_2 <= 360,000$
4. $x_1, x_2 >= 0$

Solution:

x_1 = **50,000**

x_2 = **50,000**

maximum yield = **$3,500**

Matlab code:

```
format longG

f = [-0.04 -0.03]'; %multiply objective function by -1 to
convert into minimize problem
A = [1 1; 2 1; 3 4];
b = [100000; 150000; 360000];
lb = [0 0]';
ub = [];
Aeq = [];
beq = [];

[x, fval] = linprog(f,A,b,Aeq,beq,lb,ub);
fval = round(fval*-1); %multiply result by -1 since this
was initially a maximize problem
x = round(x); % assume we can only buy whole units of each
stock

x
fval
```

Matlab output:

```
>> Problem1_matlab
Optimization terminated.

x =

       50000
       50000


fval =

        3500
        3500
```

Problem 2, part 1

Decision variables:

- xi = amount of bond i, i ∈ [1, 13]
- zj = amount of cash carried over in year j, j ∈ [1, 5]

Objective function:

Minimize

$108x_1 + 94x_2 + 99x_3 + 92.7x_4 + 96.6x_5 + 95.9x_6 + 92.9x_7 + 110x_8 + 104x_9 + 101x_{10} + 107x_{11} + 102x_{12} + 95.2x_{13}$

Constraints

1. $10x_1 + 7x_2 + 8x_3 + 6x_4 + 7x_5 + 6x_6 + 5x_7 + 10x_8 + 8x_9 + 6x_{10} + 10x_{11} + 7x_{12} + 100x_{13} – z_1 >= 500$
2. $10x_1 + 7x_2 + 8x_3 + 6x_4 + 7x_5 + 6x_6 + 5x_7 + 10x_8 + 8x_9 + 6x_{10} + 110x_{11} + 107x_{12} + z_1 – z_2 >= 200$
3. $10x_1 + 7x_2 + 8x_3 + 6x_4 + 7x_5 + 6x_6 + 5x_7 + 110x_8 + 108x_9 + 106x_{10} + z_2 – z_3 >= 800$
4. $10x_1 + 7x_2 + 8x_3 + 6x_4 + 7x_5 + 106x_6 + 105x_7 + z_3 – z_4 >= 400$
5. $10x_1 + 7x_2 + 8x_3 + 106x_4 + 107x_5 + z_4 – z_5 >= 700$
6. $110x_1 + 107x_2 + 108x_3 + z_5 >= 900$

Solution:

x_1 = **8.1818**, x_2 = **0**, x_3 = **0**, x_4 = **0**, x_5 = **5.7774**, x_6 = **2.6202**, x_7 = **0**, x_8 = **0**, x_9 = **6.1298**, x_{10} = **0**, x_{11} = **0.118**, x_{12} = **0**, x_{13} = **3.118**, z_1 = **0**, z_2 = **0**, z_3 = **0**, z_4 = **0**, z_5 = **0**

minimum required cost = **$2,639.97**

Matlab code:

```
format longG

%first 13 elements of each row correspond to x (units of
each bond)
%last 5 elements correspond to z (amount carried over from
each period)
f = [108, 94, 99, 92.7, 96.6, 95.9, 92.9, 110, 104, 101,
107, 102, 95.2, 0, 0, 0, 0, 0]';

A = [-10, -7, -8, -6, -7, -6, -5, -10, -8, -6, -10, -7, -
100, 1, 0, 0, 0, 0
    -10, -7, -8, -6, -7, -6, -5, -10, -8, -6, -110, -107,
0, -1, 1, 0, 0, 0
    -10, -7, -8, -6, -7, -6, -5, -110, -108, -106, 0, 0, 0,
0, -1, 1, 0, 0
    -10, -7, -8, -6, -7, -106, -105, 0, 0, 0, 0, 0, 0, 0,
0, -1, 1, 0
    -10, -7, -8, -106, -107, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, -1, 1
    -110, -107, -108, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, -1];

b = [-500
    -200
    -800
    -400
    -700
    -900];

lb = [0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0]';

ub = [];
Aeq = [];
beq = [];

[x, fval] = linprog(f,A,b,Aeq,beq,lb,ub);
x = round(x,4); %round all outputs to 4 decimal places
fval = round(fval,4);

x
fval
```

Matlab output:

```
>> problem2part1_matlab
Optimization terminated.

x =

                    8.1818
                         0
                         0
                         0
                    5.7774
                    2.6202
                         0
                         0
                    6.1298
                         0
                     0.118
                         0
                     3.118
                         0
                         0
                         0
                         0
                         0


fval =

                  2639.9694
```

Problem 2, part 2

Formulation is the same as part 1, except with one additional constraint:

$108x_1 + 94x_2 + 99x_3 + 92.7x_4 + 96.6x_5 + 95.9x_6 <= 0.5\ (108x_1 + 94x_2 + 99x_3 + 92.7x_4 + 96.6x_5 + 95.9x_6 + 92.9x_7 + 110x_8 + 104x_9 + 101x_{10} + 107x_{11} + 102x_{12} + 95.2x_{13})$

This can be simplified to the following:

$54x_1 + 47x_2 + 49.5x_3 + 46.35x_4 + 48.3x_5 + 47.95x_6 – 46.45x_7 – 55x_8 – 52x_9 – 50.5x_{10} – 53.5x_{11} – 51x_{12} – 47.6x_{13} <= 0$

Solution:

x_1 = **0**, x_2 = **8.4112**, x_3 = **0**, x_4 = **5.7422**, x_5 = **0**, x_6 = **0**, x_7 = **3.2297**, x_8 = **0**, x_9 = **6.3937**, x_{10} = **0**, x_{11} = **0.3579**, x_{12} = **0**, x_{13} = **3.3579**, z_1 = **0**, z_2 = **0**, z_3 = **0**, z_4 = **32.4504**, z_5 = **0**

minimum required cost = **$ 2645.91**

This is an increase of approximately $5.94 compared to the minimum cost in part 1.

Below is a table comparing the two different bond portfolios from part 1 and part 2.

Bond	**Amount in bond portfolio in part 1**	**Amount in bond portfolio in part 2**
1	8.1818	0
2	0	8.4112
3	0	0
4	0	5.7422
5	5.7774	0
6	2.6202	0
7	0	3.2297
8	0	0
9	6.1298	6.3937
10	0	0
11	0.118	0.3579
12	0	0
13	3.118	3.3579

Matlab code:

```
format longG

f = [108, 94, 99, 92.7, 96.6, 95.9, 92.9, 110, 104, 101,
107, 102, 95.2, 0, 0, 0, 0, 0]';

A = [-10, -7, -8, -6, -7, -6, -5, -10, -8, -6, -10, -7, -
100, 1, 0, 0, 0, 0
    -10, -7, -8, -6, -7, -6, -5, -10, -8, -6, -110, -107,
0, -1, 1, 0, 0, 0
    -10, -7, -8, -6, -7, -6, -5, -110, -108, -106, 0, 0, 0,
0, -1, 1, 0, 0
    -10, -7, -8, -6, -7, -106, -105, 0, 0, 0, 0, 0, 0, 0,
0, -1, 1, 0
    -10, -7, -8, -106, -107, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, -1, 1
    -110, -107, -108, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, -1
    54, 47, 49.5, 46.35, 48.3, 47.95, -46.45, -55, -52, -
50.5, -53.5, -51, -47.6, 0, 0, 0, 0, 0];

b = [-500; -200; -800; -400; -700; -900; 0];

lb = [0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0]';

ub = [];
Aeq = [];
beq = [];

[x, fval] = linprog(f,A,b,Aeq,beq,lb,ub);
x = round(x,4);
fval = round(fval,4);

x
fval
```

Matlab output:

```
>> problem2part2_matlab
Optimization terminated.

x =

                         0
                    8.4112
                         0
                    5.7422
                         0
                         0
                    3.2297
                         0
                    6.3937
                         0
                    0.3579
                         0
                    3.3579
                         0
                         0
                         0
                   32.4504
                         0


fval =

                 2645.9085
```