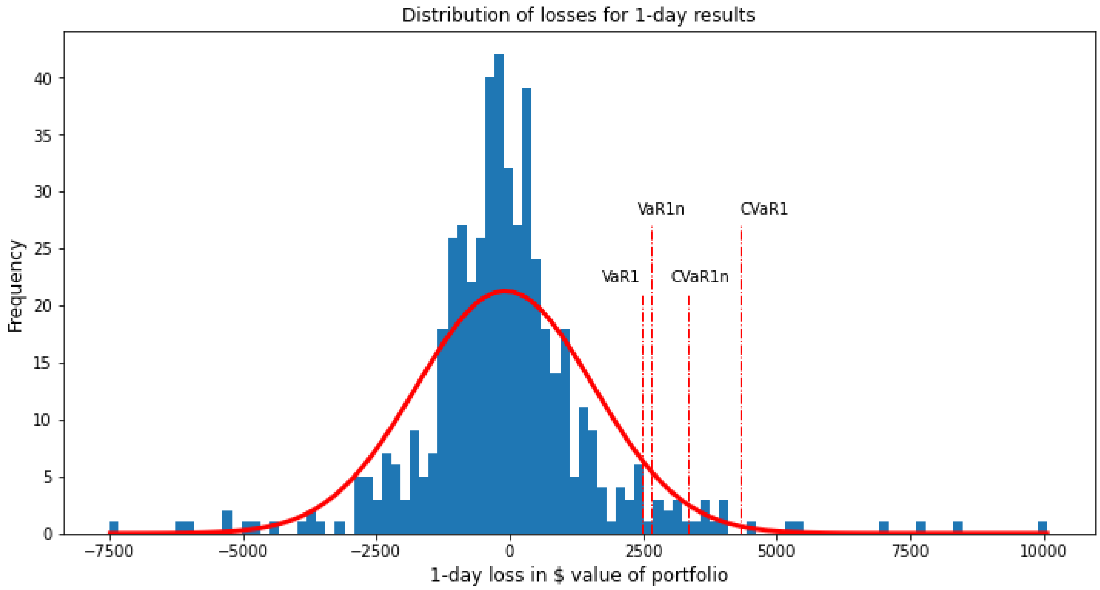
Question 1, part 1

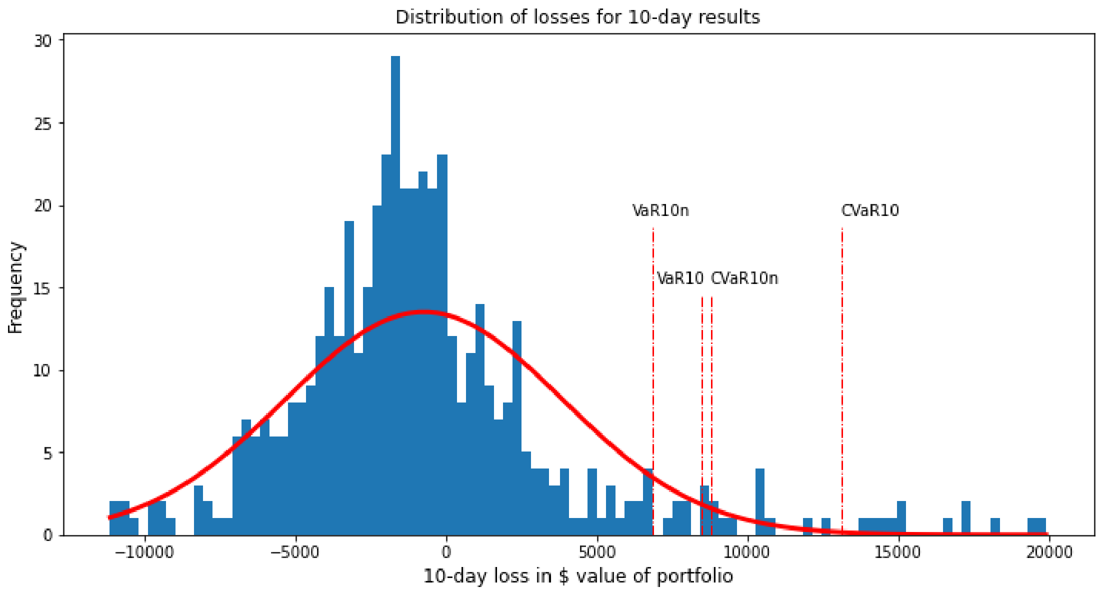
VaR and CVaR of portfolio value at 95% quantile for 1-day and 10-day time horizons:

```
Historical 1-day VaR 95.0% = $2477.25,    Historical 1-day CVaR 95.0% = $4326.98
    Normal 1-day VaR 95.0% = $2646.49,        Normal 1-day CVaR 95.0% = $3339.94
Historical 10-day VaR 95.0% = $8472.11,   Historical 10-day CVaR 95.0% = $13113.38
    Normal 10-day VaR 95.0% = $6842.63,       Normal 10-day CVaR 95.0% = $8764.32
```

Histogram of distribution of losses for 1-day time horizon:



Histogram of distribution of losses for 10-day time horizon:



Q: Is the following true for the given dataset? Please explain your answer.

- *VaR (10 day) = 10 * VaR (one day)*
- *CVaR(10 day) = 10 * CVaR (one day)*

Answer:

	VaR (10 day)	10 * VaR (one day)	CVaR(10 day)	10 * CVaR (one day)
Historical method	8472.113802	24772.521920	13113.384500	43269.814970
Normal method	6842.625427	26464.918309	8764.321591	33399.361398

As we can see from the output above, the first equation is false for both historical and normal methods. The second equation is also false for both historical and normal methods. The reason for this is because both equations assume that the tail risk of a portfolio increases by a linear amount as the number of days increases. However, this is simply not true, since tail risk can also decrease as the time period increases, and the change is not always linear.

Question 1, part 2

1-day 95% VaR for 100 MSFT, AAPL, and IBM stocks separately using historical scenarios:

```
100 MSFT stocks historical 1-day VaR 95.0% = $531.61

200 AAPL stocks historical 1-day VaR 95.0% = $564.25

500 IBM stocks historical 1-day VaR 95.0% = $1837.73
```

1-day 95% VaR for 100 MSFT, AAPL, and IBM stocks separately using normal distribution model:

```
100 MSFT stocks normal 1-day VaR 95.0% = $556.21

200 AAPL stocks normal 1-day VaR 95.0% = $592.79

500 IBM stocks normal 1-day VaR 95.0% = $1936.14
```

Q: Is the following true? Explain: VaR(Portfolio) = VaR(MSFT) + VaR(AAPL) + VaR(IBM)

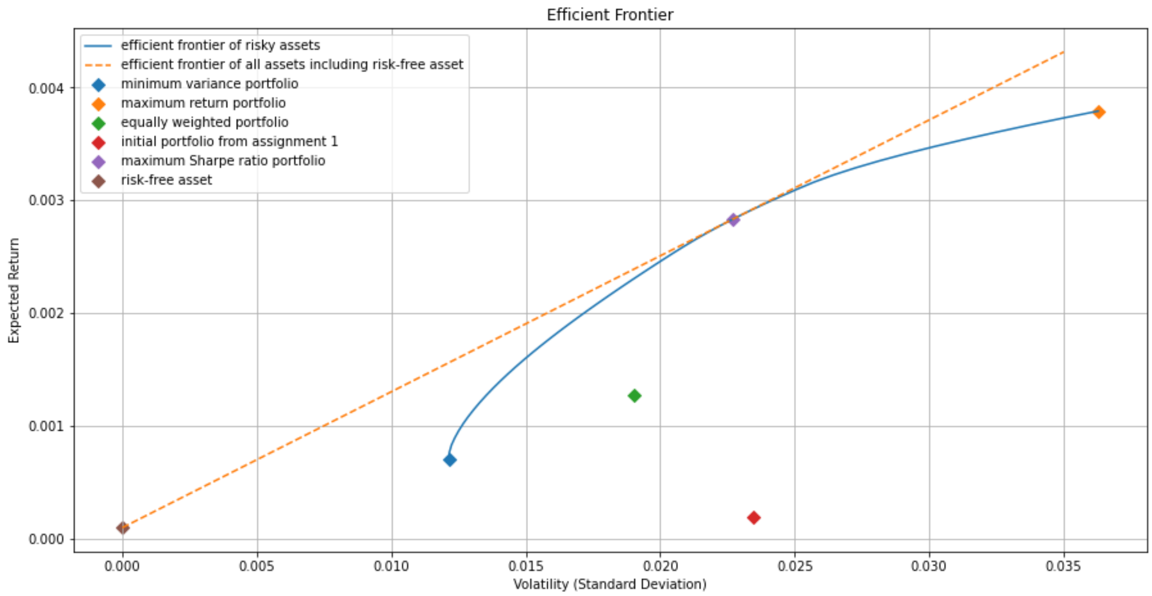
Answer:

	VaR(Portfolio)	VaR(MSFT) + VaR(AAPL) + VaR(IBM)
Historical method	2477.252192	2933.583838
Normal method	2646.491831	3085.143379

As we can see from the output above, the equation is false for both historical and normal methods. The reason for this is because the equation assumes that the tail risks of the three assets, MSFT, AAPL, and IBM are linearly separable. This is not the case, since there is correlation between the three assets. Thus, the tail risks of the individual assets added together will be different than a portfolio containing all three due to the correlation between these assets.

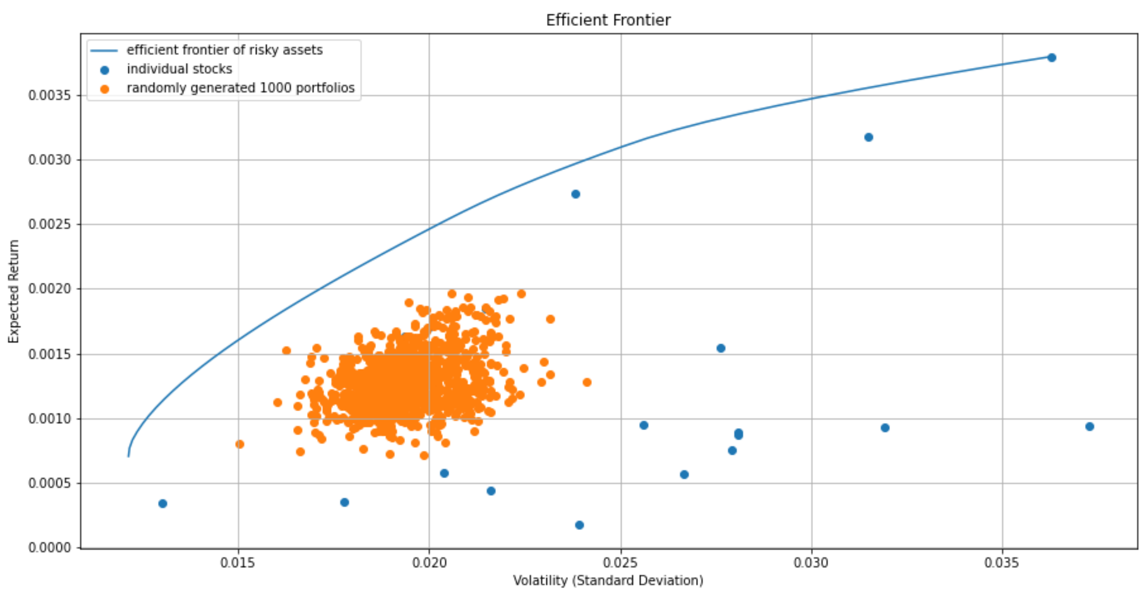
Question 2, part 1

Plot of efficient frontier with the different portfolios:



Question 2, part 2

Plot of efficient frontier with the individual assets and randomly generated portfolios:



I used the exponential distribution from *numpy* for the generation of the random portfolios. The reason I chose the exponential distribution instead of the standard normal distribution is because the exponential distribution returns weights that result in more spread-out portfolios due to the nature of the distribution. The standard normal distribution, on the other hand, mostly produces weights that have very similar expected return and variance, thus resulting in most of the points being clumped together. Therefore, I decided to use the exponential distribution for both better visualization purposes as well as better approximation of real-world portfolios.

Question 3, part 1

Factor loadings and R-squared value using data from different time periods:

```
factor loadings and R-squared values for 2019
```

	alpha	beta_m	beta_s	beta_v	R-squared
MSFT	-0.008105	0.012508	-0.004930	-0.005248	0.735844
F	-0.008084	0.010279	0.005270	0.005360	0.322625
JPM	-0.007736	0.011404	-0.003175	0.009947	0.738317
GOOG	-0.008718	0.012082	-0.000519	-0.004448	0.470290
HPQ	-0.009261	0.012507	0.001227	0.002370	0.275263
C	-0.007787	0.015460	-0.001642	0.010711	0.762569
HOG	-0.008723	0.014393	0.004718	0.010647	0.489093
VZ	-0.008341	0.003494	-0.002588	-0.000269	0.078964
AAPL	-0.007391	0.014815	-0.001891	-0.002669	0.547084
IBM	-0.008663	0.010834	-0.002832	0.001893	0.447541
T	-0.007534	0.005684	-0.000003	0.003081	0.184845
CSCO	-0.009066	0.012499	-0.001569	-0.000298	0.440500
BAC	-0.007696	0.013064	0.000163	0.012626	0.720036
INTC	-0.008620	0.013279	-0.004117	0.001244	0.377338
AMD	-0.006900	0.024581	0.002106	-0.008286	0.397062
SNE	-0.008068	0.011044	0.001884	-0.001128	0.298445
NVDA	-0.008009	0.021348	0.004142	-0.000932	0.506705
AMZN	-0.009014	0.012321	0.000035	-0.005480	0.575158
MS	-0.008306	0.014044	-0.001578	0.010495	0.692826
BK	-0.008657	0.009905	0.000136	0.008827	0.415195

```
factor loadings and R-squared values for 2020
```

	alpha	beta_m	beta_s	beta_v	R-squared
MSFT	-0.001582	0.012518	-0.002615	-0.005145	0.880827
F	-0.000723	0.008948	0.001273	0.010016	0.646842
JPM	-0.000616	0.010523	-0.002929	0.011202	0.887016
GOOG	-0.001512	0.010155	-0.001221	-0.002578	0.752035
HPQ	-0.001058	0.010589	0.007337	0.003553	0.606785
C	-0.001064	0.012895	-0.000289	0.013293	0.870440
HOG	-0.000656	0.013524	0.003125	0.009001	0.562776
VZ	-0.001542	0.005112	-0.002930	0.001184	0.550817
AAPL	-0.000375	0.012391	-0.004109	-0.004340	0.763150
IBM	-0.001718	0.009248	-0.000535	0.003232	0.723028
T	-0.002031	0.007339	-0.003951	0.005035	0.736744
CSCO	-0.002145	0.009938	-0.003321	0.000240	0.658712
BAC	-0.000746	0.011508	-0.003549	0.012039	0.898547
INTC	-0.002891	0.012094	-0.003062	-0.001615	0.576110
AMD	-0.000605	0.013408	-0.000428	-0.007498	0.532429
SNE	-0.000690	0.007719	-0.000511	-0.001504	0.560590
NVDA	-0.000622	0.015458	0.002396	-0.009109	0.788197
AMZN	-0.000705	0.008540	-0.001877	-0.006814	0.590911
MS	0.000278	0.012766	-0.002803	0.008049	0.842088
BK	-0.001316	0.009330	-0.002745	0.008290	0.754725

```
factor loadings and R-squared values for 2019-2020
```

	alpha	beta_m	beta_s	beta_v	R-squared
MSFT	-0.004818	0.012503	-0.002833	-0.005240	0.836645
F	-0.004436	0.009341	0.002752	0.009034	0.571292
JPM	-0.004158	0.010692	-0.002539	0.010818	0.854627
GOOG	-0.005081	0.010496	-0.000499	-0.003147	0.650872
HPQ	-0.005003	0.010821	0.006537	0.003249	0.510710
C	-0.004329	0.013317	0.000015	0.012657	0.846797
HOG	-0.004725	0.013625	0.003922	0.008913	0.545291
VZ	-0.005008	0.004959	-0.002654	0.000951	0.380165
AAPL	-0.003882	0.012705	-0.003073	-0.004534	0.693281
IBM	-0.005112	0.009488	-0.000553	0.002850	0.648105
T	-0.004912	0.007241	-0.002908	0.004665	0.606403
CSCO	-0.005579	0.010334	-0.002339	-0.000231	0.585631
BAC	-0.004268	0.011759	-0.002236	0.011728	0.858923
INTC	-0.005732	0.012159	-0.002975	-0.001440	0.527248
AMD	-0.003480	0.014977	0.001523	-0.008650	0.442763
SNE	-0.004348	0.008198	0.000694	-0.001945	0.434783
NVDA	-0.004237	0.016024	0.003543	-0.008725	0.668146
AMZN	-0.004813	0.009046	-0.000714	-0.007173	0.555235
MS	-0.004037	0.012892	-0.002043	0.008031	0.799401
BK	-0.005048	0.009441	-0.001695	0.008061	0.677046

Q: Compare the R-squared score in 2019 and 2020, suggest possible reason for the difference.

Answer:

	2019 R-squared value	2020 R-squared value	difference (2020-2019)
MSFT	0.735844	0.880827	0.144982
F	0.322625	0.646842	0.324217
JPM	0.738317	0.887016	0.148698
GOOG	0.470290	0.752035	0.281746
HPQ	0.275263	0.606785	0.331522
C	0.762569	0.870440	0.107871
HOG	0.489093	0.562776	0.073683
VZ	0.078964	0.550817	0.471853
AAPL	0.547084	0.763150	0.216066
IBM	0.447541	0.723028	0.275487
T	0.184845	0.736744	0.551899
CSCO	0.440500	0.658712	0.218212
BAC	0.720036	0.898547	0.178511
INTC	0.377338	0.576110	0.198772
AMD	0.397062	0.532429	0.135366
SNE	0.298445	0.560590	0.262145
NVDA	0.506705	0.788197	0.281492
AMZN	0.575158	0.590911	0.015753
MS	0.692826	0.842088	0.149262
BK	0.415195	0.754725	0.339530

Looking at the output above, we can see that all 20 stocks have a higher R-squared value in 2020 compared to 2019. The three stocks with the biggest increase are T (increase of 0.55), VZ (increase of 0.47), and HPQ (increase of 0.33). I believe that the main reason for this increase is due to the COVID-19 pandemic. Most of the 20 stocks that are shown are stocks related to technology, otherwise known as tech stocks. The COVID-19 pandemic started in early 2020, which caused many people around the world to stay in their homes and gather in cyberspace for most of the year. As a result, many tech stocks saw a surge in their price, such as AT&T, Verizon, and HP.

Q: If some investor select the Fama-French three-factor model to model asset return during COVID-19, would the performance of the model be affected? Explain your reason.

Answer:

If the Fama-French three-factor model is used to model asset return during COVID-19, I believe the performance of the model will be greatly affected. The first reason is that, as stated earlier, the pandemic has caused the value of many technology-related stocks to increase substantially, including the 20 stocks that we are dealing with in this project. However, at the same time the pandemic has also caused other industries such as tourism and food service to slow down, thus decreasing the value of many stocks that are related to these industries. This means that using the Fama-French model during the pandemic will cause technology-related stocks like Verizon and IBM to be overvalued and cause stocks that are related to industries such as tourism and food service to be undervalued. The second reason is that the Fama-French model is country-specific. This means that the model will have different performance depending on which country the model is used in. Different countries are affected differently by the pandemic, and depending on which country the model is used in the performance may be greatly affected.

Question 3, part 2

1-day 95% VaR, Monte Carlo simulation:

```
100 MSFT stocks Monte Carlo 1-day VaR 95.0% = $644.15

200 AAPL stocks Monte Carlo 1-day VaR 95.0% = $666.41

500 IBM stocks Monte Carlo 1-day VaR 95.0% = $2679.15

Whole portfolio Monte Carlo 1-day VaR 95.0% = $2881.98
```

1-day 95% VaR, Monte Carlo simulation using normal distribution model:

```
100 MSFT stocks normal distribution model 1-day VaR 95.0% = $648.08

200 AAPL stocks normal distribution model 1-day VaR 95.0% = $645.34

500 IBM stocks normal distribution model 1-day VaR 95.0% = $2694.18

Whole portfolio normal distribution model 1-day VaR 95.0% = $2869.84
```

I used 1000 scenarios and 504 steps for my Monte Carlo simulations. The reason I chose 1000 scenarios is because 1000 scenarios provide a fairly accurate approximation while also having a fairly reasonable runtime. The reason I chose 504 steps for each scenario is because we are dealing with a 2-year period. Since the data provided has 505 daily values across 2019 and 2020, by setting the number of steps to 504 we are able to accurately simulate the daily change of our portfolio during the 2-year period. Thus, to obtain the most accurate VaR values, I set the number of steps to be 504 which is the same as the number of work days in a 2-year period.

Q: Compare the results with Part 2 in Question 1, analyze the reasons for the differences.

Answer:

	MSFT	AAPL	IBM	Whole portfolio	MSFT (normal)	AAPL (normal)	IBM (normal)	Whole portfolio (normal)
Historical method (Q1)	531.608580	564.248658	1837.72660	2477.252192	556.210053	592.791219	1936.142107	2646.491831
Monte Carlo method (Q3)	644.150028	666.412185	2679.15496	2881.981961	648.078728	645.336575	2694.180894	2869.844882

Compared with question 1, the VaR from the Monte Carlo method are substantially different for both individual stocks as well as the whole portfolio. This is also true if the normal distribution method is used in the simulations. I believe the reason for this is due to the fact that I limited the number of scenarios to 1000. Using only 1000 scenarios may have introduced a large amount of sampling error to the simulations, thus causing the VaR results to be different than the historical method.

Q: Is the following true? Explain: VaR(Portfolio) = VaR(MSFT) + VaR(AAPL) + VaR(IBM)

Answer:

	VaR(Portfolio)	VaR(MSFT) + VaR(AAPL) + VaR(IBM)
Historical method	2881.981961	3989.717173
Normal method	2869.844882	3987.596197

As we can see from the output above, the equation is false for both historical and normal methods. The reason for this once again has do with the fact that the individual stocks have correlation amongst them, thus are not linearly separable. Additionally, the amount of scenarios that I used may have also introduced some amount of sampling error to the simulations, thus causing the VaR values to be different.