MIE1622 Assignment 3 Report

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Implementation of Portfolio Credit Risk Simulation Models in Python

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Portfolio 1:

Out-of-sample: VaR 99.0% = $38307840.66, CVaR 99.0% = $45932665.13
In-sample MC1: VaR 99.0% = $38113787.21, CVaR 99.0% = $45621046.64
In-sample MC2: VaR 99.0% = $37999819.50, CVaR 99.0% = $45811460.43
In-sample No: VaR 99.0% = $26637962.98, CVaR 99.0% = $29590272.82
In-sample N1: VaR 99.0% = $26531257.75, CVaR 99.0% = $29471222.09
In-sample N2: VaR 99.0% = $26585907.19, CVaR 99.0% = $29531695.05

Out-of-sample: VaR 99.9% = $56488627.28, CVaR 99.9% = $63397992.27
In-sample MC1: VaR 99.9% = $54614904.71, CVaR 99.9% = $61526674.80
In-sample MC2: VaR 99.9% = $55161243.49, CVaR 99.9% = $62830831.49
In-sample No: VaR 99.9% = $33293163.63, CVaR 99.9% = $35705235.32
In-sample N1: VaR 99.9% = $33158628.75, CVaR 99.9% = $35560614.02
In-sample N2: VaR 99.9% = $33226405.75, CVaR 99.9% = $35633148.90


Portfolio 2:

Out-of-sample: VaR 99.0% = $27969947.49, CVaR 99.0% = $33572245.08
In-sample MC1: VaR 99.0% = $27733280.68, CVaR 99.0% = $33386746.20
In-sample MC2: VaR 99.0% = $27712730.12, CVaR 99.0% = $33687637.07
In-sample No: VaR 99.0% = $21370487.40, CVaR 99.0% = $23573499.94
In-sample N1: VaR 99.0% = $21285391.87, CVaR 99.0% = $23478049.99
In-sample N2: VaR 99.0% = $21305494.84, CVaR 99.0% = $23500336.32

Out-of-sample: VaR 99.9% = $41042155.68, CVaR 99.9% = $46215826.94
In-sample MC1: VaR 99.9% = $40178074.22, CVaR 99.9% = $45503047.02
In-sample MC2: VaR 99.9% = $40818834.27, CVaR 99.9% = $47218035.18
In-sample No: VaR 99.9% = $26336595.73, CVaR 99.9% = $28136482.76
In-sample N1: VaR 99.9% = $26228158.90, CVaR 99.9% = $28019586.25
In-sample N2: VaR 99.9% = $26253183.67, CVaR 99.9% = $28046394.84
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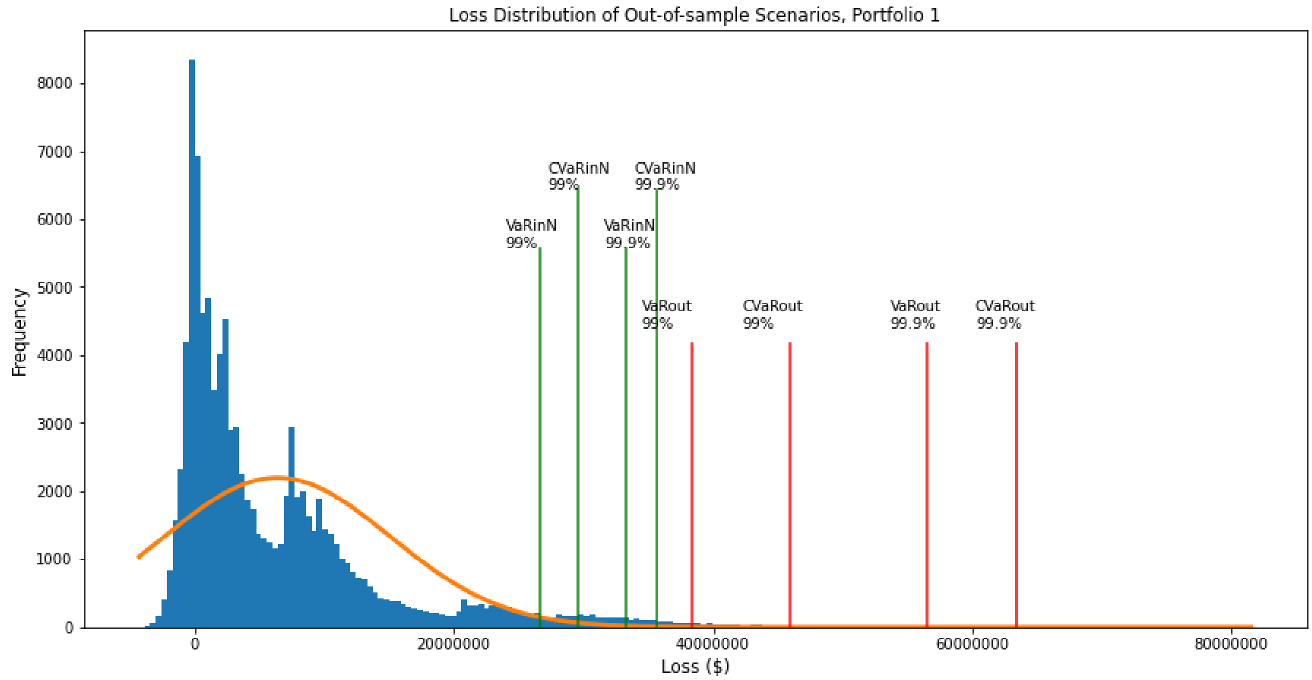
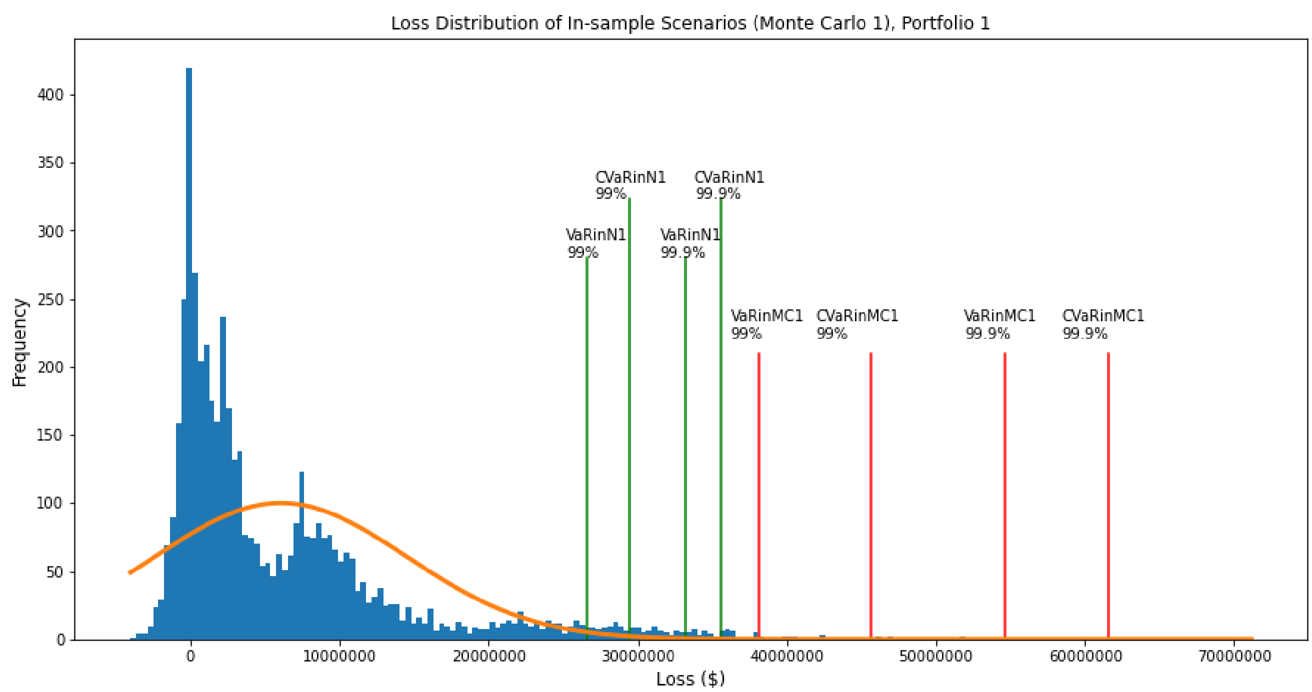
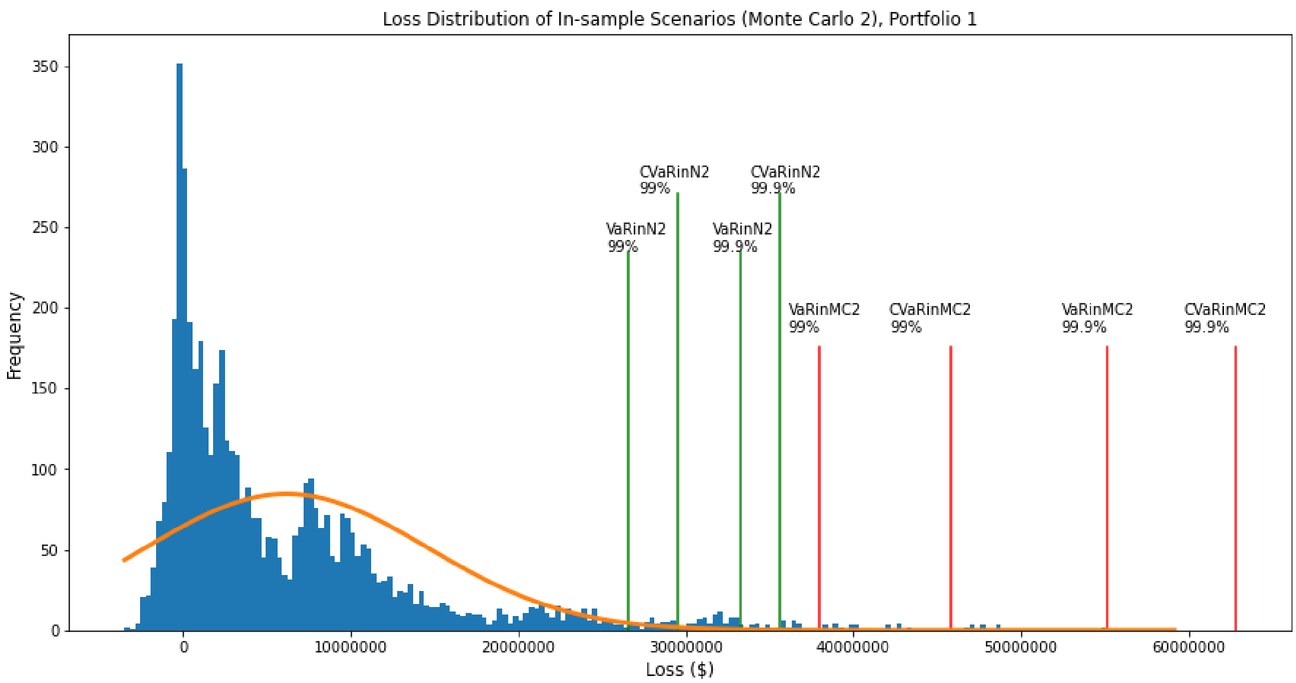
Analysis of results

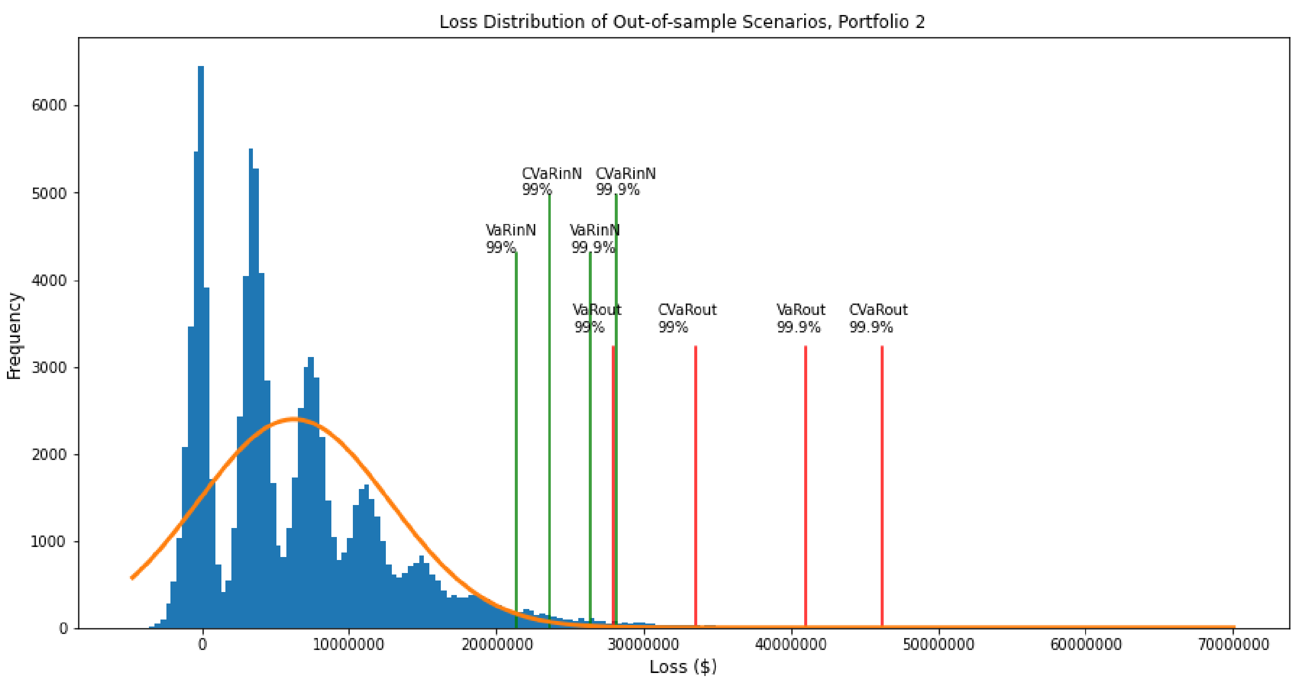
Loss distribution plots:



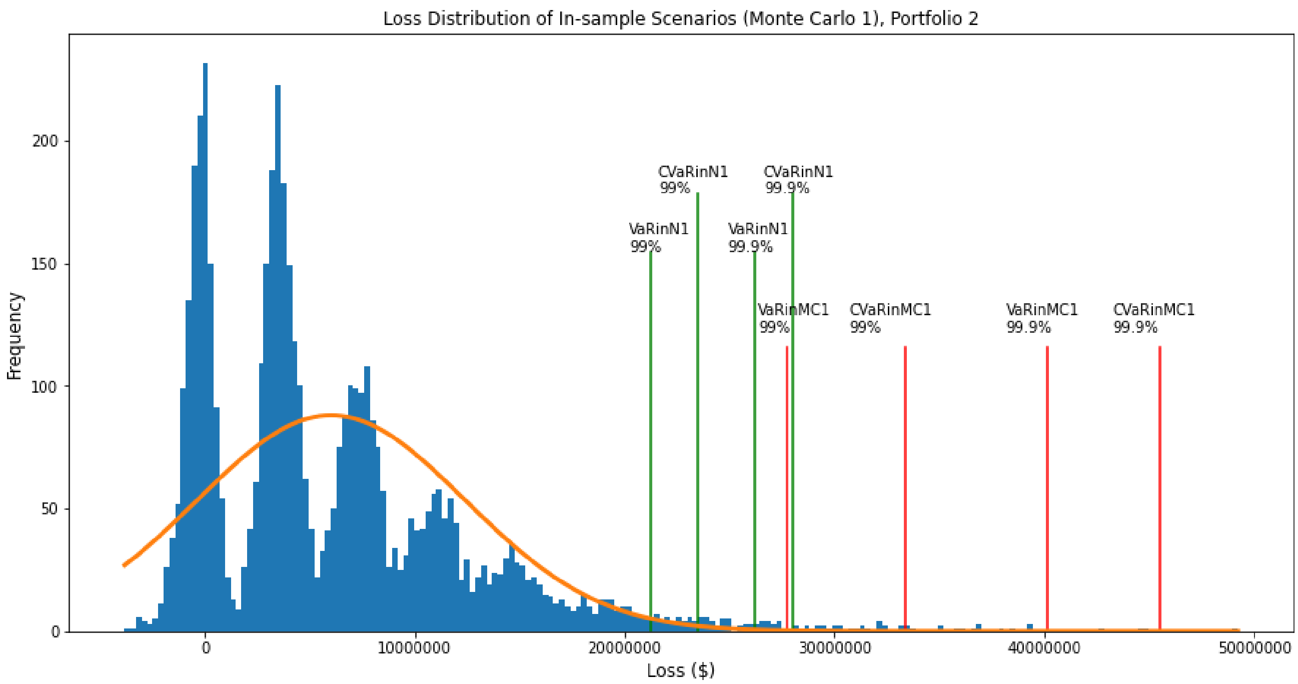




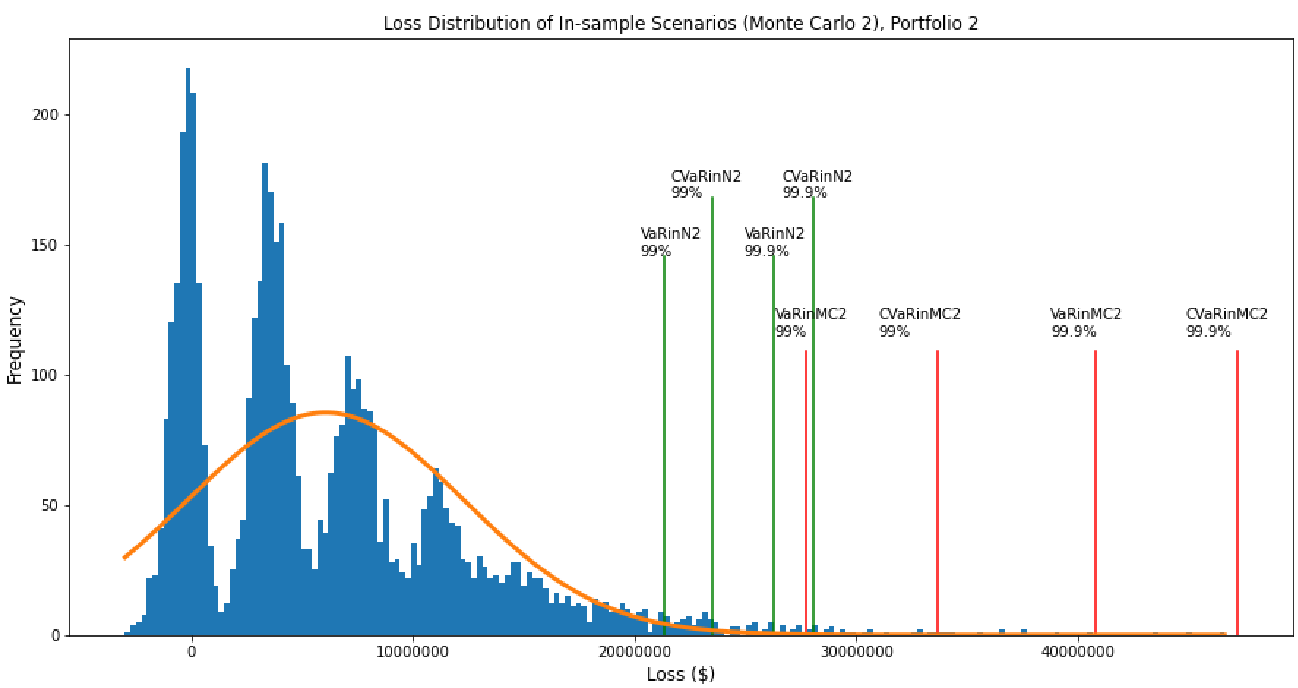
Loss Distribution of In-sample Scenarios (Monte Carlo 2), Portfolio 1
Frequency
Loss ($)
VaRinN2 99%
CVaRinN2 99%
VaRinN2 99.9%
CVaRinN2 99.9%
VaRinMC2 99%
CVaRinMC2 99%
VaRinMC2 99.9%
CVaRinMC2 99.9%



Loss Distribution of Out-of-sample Scenarios, Portfolio 2
Frequency
Loss ($)
VaRinN 99%
CVaRinN 99%
VaRinN 99.9%
CVaRinN 99.9%
VaRout 99%
CVaRout 99%
VaRout 99.9%
CVaRout 99.9%



Loss Distribution of In-sample Scenarios (Monte Carlo 1), Portfolio 2
Frequency
Loss ($)
CVaRinN1
99%
CVaRinN1
99.9%
VaRinN1
99%
VaRinN1
99.9%
VaRinMC1
99%
CVaRinMC1
99%
VaRinMC1
99.9%
CVaRinMC1
99.9%
0
50
100
150
200
0
10000000
20000000
30000000
40000000
50000000



Loss Distribution of In-sample Scenarios (Monte Carlo 2), Portfolio 2
Frequency
Loss ($)
CVaRinN2
99%
CVaRinN2
99.9%
VaRinN2
99%
VaRinN2
99.9%
VaRinMC2
99%
CVaRinMC2
99%
VaRinMC2
99.9%
CVaRinMC2
99.9%
0
50
100
150
200
0
10000000
20000000
30000000
40000000

Mean and standard deviation of in-sample scenarios across 100 iterations:

	Mean	Standard deviation
Portfolio 1, MC1	$6,064,159.01	$8,440,120.30
Portfolio 1, MC2	$6,171,424.10	$8,413,430.16
Portfolio 2, MC1	$6,018,425.97	$6,389,034.50
Portfolio 2, MC2	$6,053,689.85	$6,240,550.83

Analysis of sampling error:

The sampling error for each of the approximations can be obtained by subtracting the VaR and CVaR values of the non-normal in-sample scenarios from the VaR and CVaR values of the non-normal out-of-sample scenarios (true distribution). These calculations can be seen in the following figure:

	In-sample value	True distribution value	Percent difference (sampling error)
99% VaR, MC1, Portfolio 1	$38,113,787.21	$38,307,840.66	0.51%
99% CVaR, MC1, Portfolio 1	$45,621,046.64	$45,932,665.13	0.81%
99% VaR, MC2, Portfolio 1	$37,999,819.50	$38,307,840.66	0.80%
99% CVaR, MC2, Portfolio 1	$45,811,460.43	$45,932,665.13	0.32%
99.9% VaR, MC1, Portfolio 1	$54,614,904.71	$56,488,627.28	3.32%
99.9% CVaR, MC1, Portfolio 1	$61,526,674.80	$63,397,992.27	3.31%
99.9% VaR, MC2, Portfolio 1	$55,161,243.49	$56,488,627.28	2.35%
99.9% CVaR, MC2, Portfolio 1	$62,830,831.49	$63,397,992.27	1.00%
99% VaR, MC1, Portfolio 2	$27,733,280.68	$27,969,947.49	0.85%
99% CVaR, MC1, Portfolio 2	$33,386,746.20	$33,572,245.08	0.66%
99% VaR, MC2, Portfolio 2	$27,712,730.12	$27,969,947.49	0.92%
99% CVaR, MC2, Portfolio 2	$33,687,637.07	$33,572,245.08	0.41%
99.9% VaR, MC1, Portfolio 2	$40,178,074.22	$41,042,155.68	2.11%
99.9% CVaR, MC1, Portfolio 2	$45,503,047.02	$46,215,826.94	1.74%
99.9% VaR, MC2, Portfolio 2	$40,818,834.27	$41,042,155.68	0.54%
99.9% CVaR, MC2, Portfolio 2	$47,218,035.18	$46,215,826.94	2.44%

From the above figure, it can be seen that the sampling error (percent difference column) of the non-normal in-sample scenarios can be anywhere between 0.3% to 3.5%.

Analysis of model error:

The model error for when a normal distribution is assumed can be obtained by subtracting the VaR and CVaR values of the normal model scenarios from the VaR and CVaR values of the non-normal out-of-sample scenarios (true distribution). These calculations can be seen in the following figure:

	Normal model value	True distribution value	Percent difference (model error)
99% VaR, N1, Portfolio 1	$26,531,257.75	$38,307,840.66	30.74%
99% CVaR, N1, Portfolio 1	$29,471,222.09	$45,932,665.13	42.97%
99% VaR, N2, Portfolio 1	$26,585,907.19	$38,307,840.66	30.60%
99% CVaR, N2, Portfolio 1	$29,531,695.05	$45,932,665.13	42.81%
99.9% VaR, N1, Portfolio 1	$33,158,628.75	$56,488,627.28	41.30%
99.9% CVaR, N1, Portfolio 1	$35,560,614.02	$63,397,992.27	49.28%
99.9% VaR, N2, Portfolio 1	$33,226,405.75	$56,488,627.28	41.18%
99.9% CVaR, N2, Portfolio 1	$35,633,148.90	$63,397,992.27	49.15%
99% VaR, N1, Portfolio 2	$21,285,391.87	$27,969,947.49	23.90%
99% CVaR, N1, Portfolio 2	$23,478,049.99	$33,572,245.08	36.09%
99% VaR, N2, Portfolio 2	$21,305,494.84	$27,969,947.49	23.83%
99% CVaR, N2, Portfolio 2	$23,500,336.32	$33,572,245.08	36.01%
99.9% VaR, N1, Portfolio 2	$26,228,158.90	$41,042,155.68	36.09%
99.9% CVaR, N1, Portfolio 2	$28,019,586.25	$46,215,826.94	44.34%
99.9% VaR, N2, Portfolio 2	$26,253,183.67	$41,042,155.68	36.03%
99.9% CVaR, N2, Portfolio 2	$28,046,394.84	$46,215,826.94	44.27%

From the above figure, it can be seen that the model error (percent difference column) of the normal model scenarios can be anywhere between 23% to 50%.

Compared to the sampling error of the non-normal approximations, the model error of the models for when a normal distribution is assumed is much larger. Therefore, I can make the conclusion that for portfolio credit risk simulations in general, a large error can be expected if a normal distribution is assumed for the counterparty losses.

Discussion

Q: If you report the in-sample VaR and CVaR to decision-makers in your bank, what consequences for the bank capital requirements it may have?

Answer:

If the in-sample VaR and CVaR are reported, then the bank will be underestimating the tail risk of the portfolios since the in-sample VaR and CVaR for both portfolios are lower than the VaR and CVaR of their respective true distributions (out-of-sample). This underestimation of the tail risks will in turn cause the bank to underestimate its capital requirements for the year. Generally, a bank's capital requirements should be an amount for which if the worst-case scenarios as defined by the VaR and CVaR were to actually happen, the bank will still be able to function properly by using the pre-allocated capital to cover the various cash flow problems that the worse-case scenario would have caused. However, since the VaR and CVaR values of the in-sample scenarios are undervalued compared to the true distribution, the bank will not have enough of this emergency capital prepared to handle this worst-case scenario. This in turn can cause various cash flow problems that may eventually cascade further into other much bigger financial problems for the bank and may even cause the bank to default.

Q: Can you suggest techniques for minimizing impacts of sampling and model errors?

Answer:

One technique for minimizing the impact of sampling error is to increase the number of scenarios. In the case of portfolio credit risk simulation, increasing the number of in-sample scenarios is inversely-proportional to the sampling error. Therefore, as the number of in-sample scenarios are increased, the sampling error should decrease.

One technique for minimizing the impact of model error is to use a distribution model that very closely approximates the true distribution of short-term counterparty losses. We already know from earlier analysis that the normal distribution should never be used due to the high model error compared to the true distribution. Therefore, we should always perform simulations using a non-normal distribution by assuming tail risks are higher than normal measurements in order to minimize the model error.

A final technique to reduce errors is to generate scenarios based on historical data instead of performing Monte-Carlo simulations. The historical simulation method assumes that past performance is also an indication of future performance, meaning that whatever financial trends that happened in the past will again be repeated in the future. Compared to Monte-Carlo simulations which are completely random, the historical simulation method may reduce the errors associated with the randomness aspects of Monte-Carlo simulations.