MIE1622 Assignment 4 Report

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Implementation of Pricing Functions in Python

Number of time-steps used for multi-step Monte Carlo pricing procedures: 12

Number of scenarios used for Monte Carlo pricing procedures: 100000

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Black-Scholes price of an European call option is 8.021352235143176
Black-Scholes price of an European put option is 7.9004418077181455
One-step MC price of an European call option is 8.060450530055613
One-step MC price of an European put option is 7.840920573848785
Multi-step MC price of an European call option is 8.048620648976666
Multi-step MC price of an European put option is 7.885570669161491
One-step MC price of an Barrier call option is 7.835060419154218
One-step MC price of an Barrier put option is 0.0
Multi-step MC price of an Barrier call option is 7.919458557641565
Multi-step MC price of an Barrier put option is 1.2074092098027458
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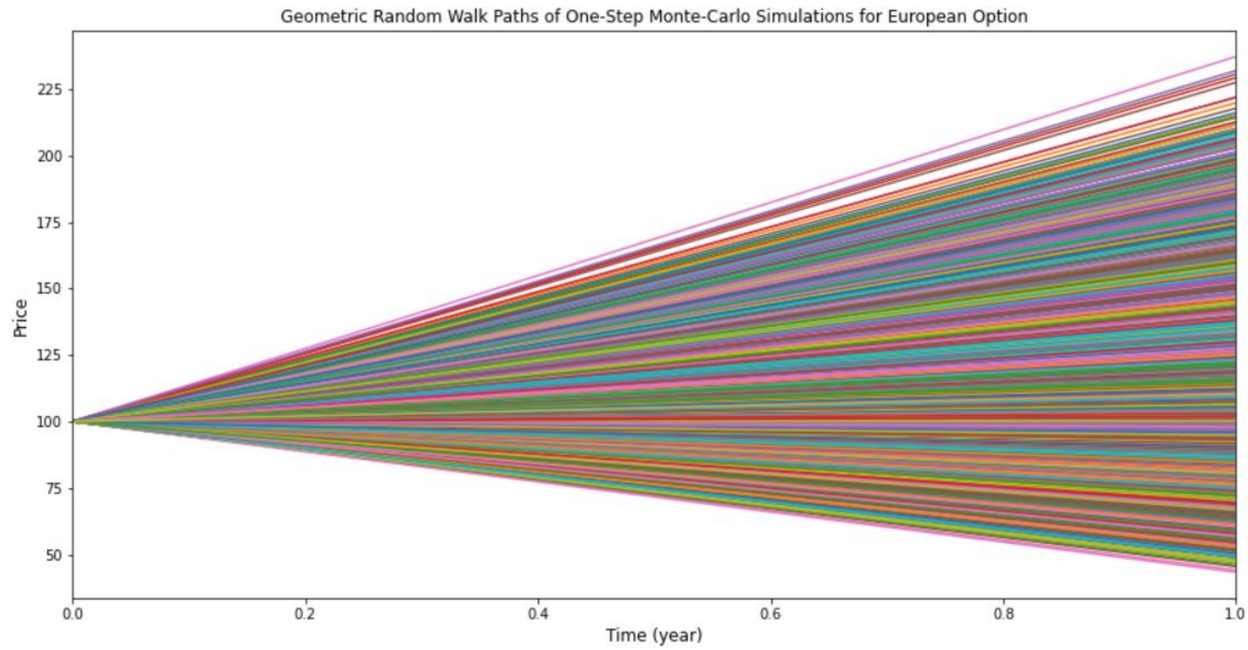
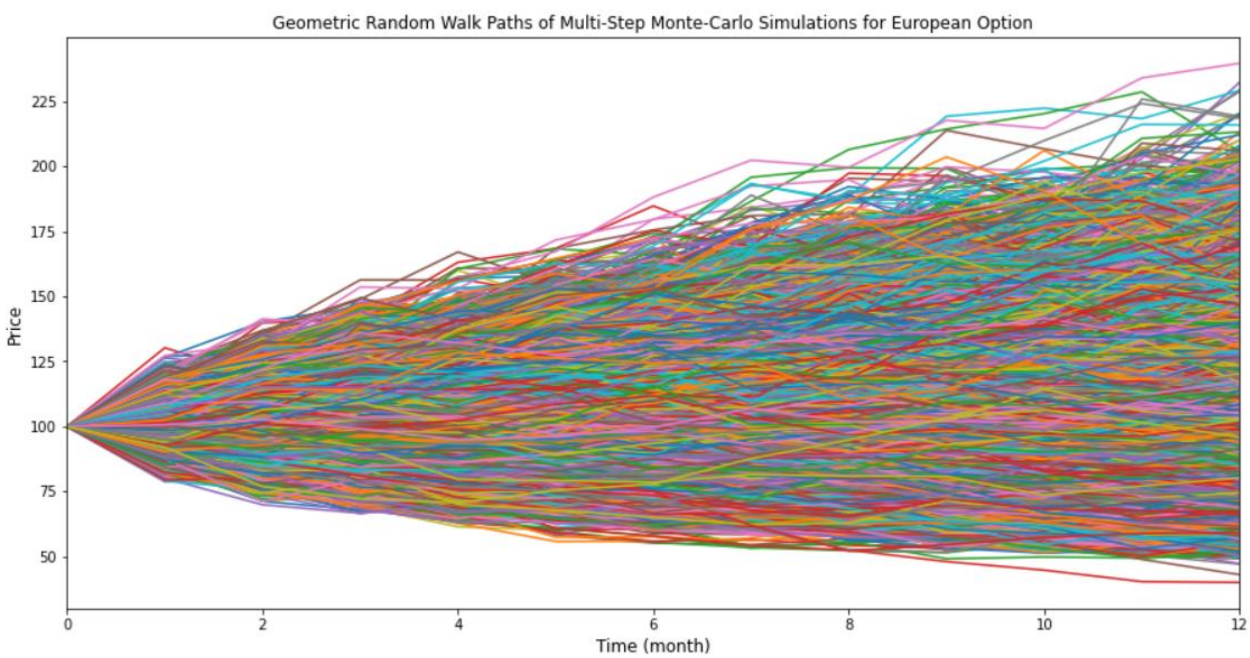
Analysis of Results:

For one-step MC price of European call and put options and barrier options, I used 100000 paths because it gives a result that is very close to the closed-form result while having a fairly short computation time. For multi-step MC price of European call and put options and barrier options, I used 12 steps to represent the 12 months in a given year. Similar to the one-step MC, I also used 100000 paths for the multi-step MC prices since it gives a result that is very close to the closed-form result while having a fairly short computation time.

Plot of geometric random walk paths for one-step Monte Carlo simulations of European option:



Plot of geometric random walk paths for multi-step Monte Carlo simulations of European option:



A comparison of the three pricing strategies for European options can be seen in the chart below:

	Black-Scholes price	One-step MC price	Multi-step MC
European call option	8.021352	8.060451	8.047648
European put option	7.900442	7.840921	7.901873

For European call options, the one-step MC strategy returns the highest price while the Black-Scholes strategy returns the lowest price. For European put options, the multi-step MC strategy returns the highest price while the one-step MC strategy returns the lowest price. In terms of overall performance, both the Black-Scholes strategy and multi-step MC strategy return very similar prices for both call and put options compared to each other, while the prices returned by the one-step MC strategy is slightly different.

There is a very big difference between the call and put prices of the barrier option. From the output results on page 1, we can see that the price of the Barrier call option is only slightly lower than the European call price (around 7.9). In contrast, the price of the Barrier put option is substantially lower than the price of the European put option (around 1.2). There are two reasons for this.

For barrier call options, since the barrier is at 110 and the strike price is at 105, when the barrier call option knocks in the payoff of the call option is already at 5 dollars. This means that when the barrier call option knocks in, it will almost always have a positive payoff. This is why the barrier call option is only slightly lower than the price of the regular European call option.

The opposite case applies for barrier put options. Since the barrier is at 110 and the strike price is at 105, when the barrier put option knocks in the payoff will be 0 since nobody is willing to sell at 105 when the market price is around 110. Therefore, the barrier put option is almost worthless since by the time the barrier knocks in the market price will already be higher than strike price. This is the reason that the barrier put option is much cheaper than the regular European put option.

Prices of Barrier options with volatility increased and decreased by 10%:

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One-step MC price of an Barrier call option with 10% increase in volatility:8.661143616993991
One-step MC price of an Barrier put option with 10% increase in volatility:0.0
Multi-step MC price of an Barrier call option with 10% increase in volatility:8.778848379464545
Multi-step MC price of an Barrier put option with 10% increase in volatility:1.5516151658236839
One-step MC price of an Barrier call option with 10% decrease in volatility:7.025451125349454
One-step MC price of an Barrier put option with 10% decrease in volatility:0.0
Multi-step MC price of an Barrier call option with 10% decrease in volatility:7.165087310424786
Multi-step MC price of an Barrier put option with 10% decrease in volatility:0.9394746531769422
```

From the output above, we can see that the price of both barrier call and put options increases when the volatility increases. The reason for this is because due to the increase in volatility the price of the stock will have more chances to breach to barrier and reach a higher price, thus causing the price of the call and put options to increase.

In contrast, the price of both barrier call and put options decrease when the volatility decreases. The reason for this is because due to the decrease in volatility the price of the stock will have less chances to breach to barrier and reach a higher price, thus causing the price of the call and put options to decrease.

Discussion of possible strategies to obtain the same prices from two procedures:

Procedure for optimal number of time steps:

I used a for loop to iterate through a set of Monte-Carlo simulations with pre-defined time-step values to see what step amount is the closest to the Black-Scholes strategy. I set the error threshold to be 0.01 – if the difference in price between a Monte Carlo simulation using a specific number of time-steps and the Black-Scholes strategy is less than 0.01, then I assume the two prices to be the same.

Procedure for optimal number of scenarios:

I used a for loop to iterate through a set of Monte-Carlo simulations with pre-defined path (scenario) values to see what scenario amount is the closest to the Black-Scholes strategy. I set the error threshold to be 0.01 – if the difference in price between a Monte Carlo simulation using a specific number of paths and the Black-Scholes strategy is less than 0.01, then I assume the two prices to be the same.

After running the loop, I obtained the following optimal number of scenarios and time-steps as well as the price error of the optimal model compared to the Black-Scholes strategy:

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optimal number of time-steps: 24
optimal number of scenarios: 1000000
Error in price: 0.005458679244505049
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