Summer/Winter/Semester2 **Tutorial 5** 2015

This tutorial explores probability.
Most of the questions involve hand calculations, not R.
There are lots of questions, so work carefully through some questions in your tutorial and complete the rest at home.
The 1st 20 minutes of class will be Online Quiz 1.

Probability

For 2 events A and B on the same sample space Ω

A and B are mutually exclusive	$P(A \cap B) = 0$
A and B are independent	$P(A \cap B) = P(A)P(B)$
the union of A and B	$P(A \cup B) = P(A) + P(B) - P(A \cap B)$
probability of A conditional on B	$P(A\|B) = \dfrac{P(A \cap B)}{P(B)}$

Combinatorial coefficients

$$\binom{N}{n} = \frac{N!}{n!(N-n)!} \qquad \text{where } N! = N(N-1)(N-2)\dots 1$$

Generalised Hypergeometric Distribution (Sampling without replacement)

Given an urn of size N with N_1 balls of type 1, N_2 balls of type 2, ... N_k balls of type k.
Draw a sample of size n without replacement.
The probability of selecting n_1 balls of type 1, n_2 balls of type 2, ... n_k balls of type k is

$$\frac{\binom{N_1}{n_1}\binom{N_2}{n_2}\dots\binom{N_k}{n_k}}{\binom{N}{n}}$$

1. Combinatorial coefficients

(a) By hand, show that $6! = 720$, $\binom{6}{0} = 1$ and $\binom{6}{3} = 20$.

(b) Confirm your answers by finding the correct button on your calculator, and by using R:

```
> factorial(6)
> choose(6,0)
```

2. Hypergeometric Distribution

A fish tank has 6 tropical fish, of which 3 are angel fish. Assume that each fish has an equal chance of being caught and that 3 fish are sampled without replacement.

(a) What is the probability of catching no angel fish?

(b) What is the probability of catching two angel fish?

3. Hypergeometric Distribution

There are 10 marbles in a jar: 7 marbles are red, 2 are blue and 1 is white. John picks out 2 marbles without replacement. What is the probability that they are the same colour?

4. Two dependent events

A smoke-detector system consists of two parts A and B. If smoke occurs then A detects it with probability 0.95, B detects it with probability 0.98 and both of them detect it with probability 0.94.

(a) Write down $P(A), P(B)$ and $P(A \cap B)$.

(b) Show that A and B are not independent.

(c) What is the probability that the smoke will not be detected?

(d) What is the probability that A will not detect the smoke, given that B did detect the smoke.

5. Three independent events

Three football players will attempt to kick a field goal. Let A_1, A_2, A_3 denote the events that the field goal is made by player $1, 2, 3$, respectively. Assume that A_1, A_2, A_3 are independent with $P(A_1) = 0.5$, $P(A_2) = 0.7$ and $P(A_3) = 0.6$. Compute the probability that exactly one player is successful.

6. Probability in R

Hospital data of discharged patients contains the following columns:

```
Column  Label
1       ID no.            2       Duration of hospital stay
3       Age               4       Sex  1=male  2=female
5       First temperature following admission
6       First WBC(x1000) following admission
7       Received antibiotic 1=yes  2=no
8       Received bacterial culture 1=yes  2=no
9       Service 1=med 2=surg.
```

(a) Read in the data and have a look at it.

```
> data=read.table(file=url("http://www.maths.usyd.edu.au/math1015/r/hospital.txt"),skip=1)
```

(b) How many patients were discharged?

(c) Isolate the data on treatments A (antibiotic) and B (bacterial culture).

```
> a = data[,7]
> b = data[,8]
```

(d) Count the number of discharged patients receiving each treatment and both treatments.

```
> number.a = length(a[a==1])
> number.b = length(b[b==1])
> number.both=length(a[a==1 & b==1])
```

(e) Use the data to estimate the following probabilities.

Event	A	B	$A \cap B$
Probability			

(f) Are the events A and B mutually exclusive? Are the events A and B independent?

7. Unfair die

A six-sided die is loaded in such a way that every even number is twice as likely to occur as every odd number. The die is tossed once.

(a) Fill out the following probability distribution.

x	1	2	3	4	5	6	Total
P(X=x)							1

(b) What is the probability that a number (strictly) less than 4 occurs?

(c) Let A be the event that an even number occurs and let B be the event that a number divisible by 3 occurs. Find $P(A \cup B)$ and $P(A \cap B)$.

8. Table of frequencies

In 1988 the Physicians' Health Study Research Group released the results of a five-year experiment based on 22,071 people between the ages of 40 and 84. The purpose was to determine whether taking aspirin reduces the risk of a heart attack. The people had been randomly assigned to one of two treatment groups: one group took aspirin, the other took the placebo (non drug). The following information was obtained.

	people with heart attack	people without heart attack
Aspirin	104	$10,933$
Placebo	189	$10,845$

One person is selected by chance.

(a) Let A be the event that the selected person had a heart attack. Find $P(A)$.

(b) Let B be the event that the selected person took aspirin. Find $P(B)$.

(c) Find the probability that the selected person had a heart attack given that they were in the Aspirin group.

(d) Are A and B independent?

9. Total of three dice

Suppose three fair six-sided dice are rolled independently and the numbers on the 3 top faces are added together.

(a) What is the total number of possible sequences $\{1,1,1\} \ldots \{6,6,6\}$?

(b) How many different ways can the numbers $\{1,1,1\}$, $\{1,1,2\}$, $\{1,1,3\}$, $\{1,2,2\}$, $\{1,1,4\}$, $\{1,2,3\}$, and $\{2,2,2\}$ can be arranged in order?

(c) What is the probability that the total is at most 6?

10. Deck of cards

A standard deck of cards consists of 52 different cards, with 13 cards of each of the 4 suits (hearts, diamonds, spades and clubs). 5 cards are taken out without replacement and placed face-down on a table.

(a) What is the probability that all 5 cards belong to the suit of hearts?

(b) What is the probability that all 5 cards belong to the same suit?

(c) What is the probability that exactly 4 cards belong to the suit of hearts?

(d) If 4 of the cards are turned face-up on the table, and are all hearts, what is the probability that the 5th card also belongs to the suit of hearts?

11. Events on two dice

Two 6-sided dice (one red, one green) are rolled, the red first and then the green. The set of all possible outcomes may be represented as follows:

$$\Omega = \{ \begin{array}{ccc} (1,1),(1,2), & \cdots & ,(1,6), \\ (2,1),(2,2), & \cdots & ,(2,6), \\ \vdots & \ddots & \vdots \\ (6,1),(6,2), & \cdots & ,(6,6) \end{array} \}$$

Let $A =$ 'first die shows 1', $B =$ 'sum of rolls is 7', $C =$ 'both rolls have the same number'.

(a) The event A can be written as $A = \{(1,1),(1,2),\ldots,(1,6)\}$. In a similar way, write out B, C, $A \cap B$, $A \cap C$ and $B \cap C$.

(b) Assuming all 36 possible outcomes are equally likely, determine the probabilities of A, B and C, and of the three pairwise intersections.

(c) Are any of the pairs (A and B, A and C or B and C) independent? Explain.

12. Probability with 3 tosses of a coloured die

A six-sided die has two of its faces white, one red and three green. It is thrown three times in such a way that each face is equally likely to land facing upwards and that each throw is independent. Compute the probability that

(a) a white face is uppermost at each throw

(b) the same colour is uppermost at each throw.

13. Extension ('Simulation' in R of Q9)

The following commands are not examinable,

```
> rolls = sample(3000,x=c(1,2,3,4,5,6),replace=T)     # 3000 random die rolls
> mat=matrix(rolls,ncol=3)                # puts rolls into a 1000-by-3 matrix
> getsums=apply(mat,1,sum)                          # gets the sums of each row
> table(getsums)                      # displays a frequency table of the sums
> sum(getsums<=6)                         # counts how many sums are at most 6
> mean(getsums<=6)        # same as above but divides by length(getsums)=1000
```

Note: You can copy and paste the above block of commands into the R command line, as the #'s "comment out" the trailing text, so R ignores the rest of the line.