Summer/Winter/Semester2 **Tutorial 12** 2015

This tutorial explores t tests.

1 Sample t Test

Context	n observations from a population with unknown mean μ and unknown variance σ^2
Hypothesis	$H_0 : \mu = \mu_0$
Test Statistic	$\tau = \dfrac{\bar{X} - \mu_0}{\frac{s}{\sqrt{n}}} \overset{H_0}{\sim} t_{n-1}$

2 Sample t Test

Context	2 populations with unknown means μ_X and μ_Y and unknown variances σ_X^2 and σ_Y^2 2 samples $\{X_i\}$ and $\{Y_j\}$ of size n_x and n_y
Hypothesis	$H_0 : \mu_X - \mu_Y = 0$
Test Statistic	$\tau = \dfrac{\bar{X} - \bar{Y} - 0}{s_p\sqrt{\frac{1}{n_x} + \frac{1}{n_y}}} \overset{H_0}{\sim} t_{n_x+n_y-2}$ where $s_p^2 = \dfrac{(n_x-1)s_x^2 + (n_y-1)s_y^2}{n_x+n_y-2}$

1. Using t Tables

(a) Find $P(t_5 \geq 3.365)$.

(b) Find $P(t_4 \leq 0.741)$.

(c) Find $P(t_7 \geq 1)$.

(d) Find a, if $P(t_{10} \geq a) = 0.005$.

(e) Find b, if $P(t_{10} \geq b) = 0.9$.

2. 1 Sample t Test

The breaking strengths of ropes produced by a manufacturer are normally distributed with mean 1800N and standard deviation 100N. By a new technique in the manufacturing process, it is claimed that the breaking strength can be increased, To test this claim a sample of 50 ropes is tested and it is found that the mean breaking strength is 1850N with standard deviation 110N. Can you support the manufacturer's claim? Justify your decision using a statistical argument.

Let X_i $i = 1, \ldots, 50$ be the breaking strengths and let μ be the population mean of the ropes produced by the new technique.

(a) Hypothesis: State the hypotheses to test the manufacturer's claim.

(b) Assumptions: Are the assumptions for a t test valid here?

(c) Test statistic: What is the test statistic and its distribution under H_0?

(d) P-value: Calculate the p-value using the t tables. Confirm using R.

```
> t=(1850-1800)/(110/sqrt(50))
> 1-pt(t,49)
```

(e) Conclusion: Draw your conclusion based on the p-value.

3. 1 Sample t Test

Suppose that $X_1, X_2, ..., X_{25}$ is a random sample from a normal population with unknown mean μ and unknown variance σ^2. Test the hypothesis $H_0 : \mu = 25$ vs $H_1 : \mu < 25$, given the sum and sum of squares of the sample take the values 587.5 and 14431.25 respectively. Write all your steps.

4. Paired t Test

In order to test the difference between two drugs A and B for treatment of high blood pressure, 24 patients are paired according to age. One of each pair is chosen at random to receive drug A and the other receives drug B. The resultant drops in blood pressure are set out below:

Pair	1	2	3	4	5	6	7	8	9	10	11	12
Drug A	5	4	2	6	9	1	1	5	6	3	7	14
Drug B	2	5	1	2	6	3	0	2	5	2	7	12
Difference												

(a) By hand, calculate Difference = Drug A - Drug B.

(b) By hand, calculate the mean and standard deviation of the differences.

(c) Perform a t test for $H_0 : \mu_d = 0$, where μ_d is the population mean of differences.

(d) Confirm your results using R.

```
> a =c(5,4,2,6,9,1,1,5,6,3,7,14)
> b=c(2,5,1,2,6,3,0,2,5,2,7,12)
> diff=a-b
> mean(diff)
> sd(diff)
> t=mean(diff)/( sd(diff)/sqrt(length(diff)) )
> pt(t,length(diff-1))
> boxplot(diff)
```

(e) Produce a boxplot of the differences. Comment.

```
> boxplot(diff)
```

(f) (Revision of Week 11) Perform a test for $H_0 : \mu_d = 0$, using a sign test.

5. Paired t Test

A new measuring technique is being considered to replace the standard technique. When 10 samples are measured by both techniques, the measurements are:

Sample	1	2	3	4	5	6	7	8	9	10
New Technique	2.5	2.2	2.6	2.6	1.9	3.3	3.3	2.8	3.0	2.9
Standard Technique	2.1	2.4	2.1	1.9	2.0	2.8	2.7	2.8	2.8	3.0
Difference										

(a) Show that the mean and the standard deviation of the differences (New -Standard) are 0.25 and 0.331. Assuming that the differences are normally distributed, test the hypothesis that the techniques give the same results.

(b) Produce a boxplot of the differences. Comment.

(c) (Revision of Week 11) Test the hypothesis that there is no long-run systematic difference between the two techniques using a sign test.

6. 2 Sample t Test

Two samples have been taken from two independent normal populations with equal variances. From these samples ($n_x = 12, n_y = 15$) we calculate $\bar{x} = 119.4$, $\bar{y} = 112.7$, $s_x = 9.2$, $s_y = 11.1$.

(a) Calculate the pooled standard deviation.

(b) Perform a test for $H_0 : \mu_x = \mu_y$ vs $H_1 : \mu_x \neq \mu_y$. Write down all your steps.

7. 2 Sample t Test

Using the following R output, test the hypothesis $H_0 : \mu_x = \mu_y$ vs $H_1 : \mu_x \neq \mu_y$. Write down all your steps.

```
> length(x)
[1] 12
> mean(x)
[1] 119.4
> sd(x)
[1] 9.2
> length(y)
[1] 15
> mean(y)
[1] 112.7
> sd(y)
[1] 11.1
```