1 Sample Z Test

Context	n observations from a population with unknown mean μ and known variance σ^2
Hypothesis	$H_0 : \mu = \mu_0$
Test Statistic	$\tau = \dfrac{\bar{X} - \mu_0}{\frac{\sigma}{\sqrt{n}}} \overset{H_0}{\sim} N(0, 1)$

1 Sample t Test

Context	n observations from a population with unknown mean μ and unknown variance σ^2
Hypothesis	$H_0 : \mu = \mu_0$
Test Statistic	$\tau = \dfrac{\bar{X} - \mu_0}{\frac{s}{\sqrt{n}}} \overset{H_0}{\sim} t_{n-1}$

2 Sample t Test

Context	2 populations with unknown means μ_X and μ_Y and unknown variances σ_X^2 and σ_Y^2 2 samples $\{X_i\}$ and $\{Y_j\}$ of size n_x and n_y
Hypothesis	$H_0 : \mu_X - \mu_Y = 0$
Test Statistic	$\tau = \dfrac{\bar{X} - \bar{Y} - 0}{s_p\sqrt{\frac{1}{n_x} + \frac{1}{n_y}}} \overset{H_0}{\sim} t_{n_x+n_y-2}$ where $s_p^2 = \dfrac{(n_x - 1)s_x^2 + (n_y - 1)s_y^2}{n_x + n_y - 2}$

1. Breaking strenth of rope

The breaking strengths of ropes produced by a manufacturer have mean 1800N and standard deviation 100N. By a new technique in the manufacturing process, it is claimed that the breaking strength can be increased without changing the variability. To test this claim a sample of 50 ropes is tested and it is found that the mean breaking strength is 1850N. Can you support the manufacturer's claim? Justify your decision using a statistical argument.

Let X_i $i = 1, \ldots, 50$ be the breaking strengths and let μ be the population mean of the ropes produced by the new technique.

(a) Hypothesis: Explain why a one-sided test of $H_0 : \mu = 1800$ *vs* $H_1 : \mu > 1800$ is suitable to test the manufacturer's claim.

(b) Assumptions: Are the assumptions for a Z test valid here?

(c) Test statistic: What is the test statistic and its distribution under H_0?

(d) P-value: Calculate the p-value using the Normal tables and R.

```
1-pnorm(1850,1800,100/sqrt(50))
```

(e) Conclusion: Draw your conclusion based on the p-value.

(f) It is found that the sample of breaking strengths has standard deviation 110N. Now use a t Test, and compare your results.

```
t=(1850-1800)/(110/sqrt(50))
1-pt(t,49)
```

2. Paired t Test

In order to test the difference between two drugs A and B for treatment of high blood pressure, 24 patients are paired according to age. One of each pair is chosen at random to receive drug A and the other receives drug B. The resultant drops in blood pressure are set out below:

Pair	1	2	3	4	5	6	7	8	9	10	11	12
Drug A	5	4	2	6	9	1	1	5	6	3	7	14
Drug B	2	5	1	2	6	3	0	2	5	2	7	12
Difference												

(a) Fill in the row: Difference = Drug A - Drug B.

```
a =c(5,4,2,6,9,1,1,5,6,3,7,14)
b=c(2,5,1,2,6,3,0,2,5,2,7,12)
diff=a-b
```

(b) Calculate the mean and standard deviation of the differences.

```
mean(diff)
sd(diff)
```

(c) Perform a t test for $H_0 : \mu_d = 0$, where μ_d is the population mean of differences.

```
t=mean(diff)/( sd(diff)/sqrt(length(diff)) )
2*(1-pt(t,length(diff)-1))
```

(d) Produce a boxplot of the differences. Comment.

```
boxplot(diff)
```

(e) Perform a test for $H_0 : \mu_d = 0$, using a sign test.

3. 2 Sample t Test

Two samples have been taken from two independent normal populations with equal variances. From these samples ($n_x = 12, n_y = 15$) we calculate $\bar{x} = 119.4$, $\bar{y} = 112.7$, $s_x = 9.2$, $s_y = 11.1$.

(a) Calculate the pooled standard deviation.

(b) Perform a test for $H_0 : \mu_x = \mu_y$ vs $H_1 : \mu_x \neq \mu_y$. Write down all your steps.

Extra Questions

4. In what contexts can you use the Z test? Is this likely?

5. The following data represents marks for a group of 18 students, for a test out of 10.

```
1 2 3 3 4 5 5 5 6 6 6 7 7 8 9 9 9 10
```

(a) Find the mean and standard deviation of the data, and produce a boxplot. Comment

```r
x=c(1,2,3,3,4,5,5,5,6,6,6,7,7,8,9,9,9,10)
mean(x)
sd(x)
boxplot(x, horizontal=T)
```

(b) If appropriate, perform a Z test for $H_0 : \mu = 5$ vs $H_1 : \mu \neq 5$, showing all your steps.

6. A standard manufacturing process produces items whose lengths in mm are normally distributed with mean $\mu = 10$ and standard deviation $\sigma = 0.051$. A new, cheaper manufacturing process is being tested, but a manager is worried that the new process makes the items too short. A sample of 100 items produced by the new process has average length $\bar{x} = 9.9921$. Assume the variability in lengths of the new process is the same as the old. Perform a statistical test to address the manager's concerns.

7. A random sample of size 64 from a $N(\mu, 16)$ population yields a sample mean of 19.1. Test $H_0 \colon \mu = 20$ against $H_1 \colon \mu < 20$. Write all your steps.

8. A sample of size 22, from a normal population with known variance of 2.25, yields a total of 250. Find an estimate of the population mean μ. Test the hypothesis that the population mean μ is 10 against the alternative that the mean is greater than 10. Write all your steps.

9. Suppose that $X_1, X_2, ..., X_{49}$ are a $N(\mu, 10^2)$ random sample, for some unknown μ. Test the hypothesis $H_0 : \mu = 50$ against the alternative $H_1 : \mu < 50$ based on an observed value $\bar{x} = 48.8$, being sure to give a sensible interpretation of your p-value. Write down all your steps.

10. In accordance with the standards established for a reading comprehension test, year 8 students should average a score of 84.3 on a standard test with a standard deviation of 8.6. If 36 randomly selected eighth graders (from a school district in a low socio-economic area) averaged only 80.8, is there strong evidence that the average in that area is below standard, if it is assumed that the standard deviation for this area is 8.6, in accordance with the standard for eighth graders?

11. (a) Find $P(t_5 \geq 3.365)$.

(b) Find $P(t_4 \leq 0.741)$.

(c) Find $P(t_7 \geq 1)$.

(d) Find a, if $P(t_{10} \geq a) = 0.005$.

(e) Find b, if $P(t_{10} \geq b) = 0.9$.

12. A new measuring technique is being considered to replace the standard technique. When 10 samples are measured by both techniques, the measurements are:

Sample	1	2	3	4	5	6	7	8	9	10
New Technique	2.5	2.2	2.6	2.6	1.9	3.3	3.3	2.8	3.0	2.9
Standard Technique	2.1	2.4	2.1	1.9	2.0	2.8	2.7	2.8	2.8	3.0
Difference										

(a) Show that the mean and the standard deviation of the differences (New -Standard) are 0.25 and 0.331. Assuming that the differences are normally distributed, test the hypothesis that the techniques give the same results.

(b) Produce a boxplot of the differences. Comment.

(c) (Revision of Week 9) Test the hypothesis that there is no long-run systematic difference between the two techniques using a sign test.