Alexandria University **Faculty of Engineering** **Specialized Scientific Programs** **June 2020**	**SSP**	**جامعة الإسكندرية** **كلية الهندسة** **البرامج العلمية المتخصصة** **يونيو 2020**

Optimization Techniques — **طرق الحل الأمثل**
Time allowed: 4 hours — **الزمن: 4 ساعات**

Answer the following questions

Question (1)

(a) Determine the stationary points of the function $f(x) = x^4 - 8x^2$ and classify them. Also, determine the absolute maximum and absolute minimum values of the function on the interval [- 3 , 3] and sketch the function over this interval.

(b) Determine the stationary points of the function $f(x,y) = \frac{1}{3}x^3 - \frac{2}{3}y^3 + \frac{1}{2}x^2 - 6x + 32y + 4$ and classify them.

(c) We want to construct a box with a square base and we have only 10 m^2 of material to use in the construction of the box. Assuming that all the material is used in the construction process, determine the maximum volume that the box can have.

(d) Using the method of Lagrange multipliers, solve the following non-linear constrained optimization problem: Maximize $f = x + y$ subject to $x + y + z = 1$ and $x^2 + 2y^2 + z^2 = 1$.

Question (2)

(a) Using the simplex method, solve the following linear programming problem:
Maximize $z = x_1 + 3x_2 + 2x_3$ subject to

$$3x_1 - x_2 + 2x_3 \le 7, \quad -2x_1 + 4x_2 \le 12, \quad -4x_1 + 3x_2 + 8x_3 \le 10, \quad x_1, x_2, x_3 \ge 0$$

(b) A company produces two types of products, A and B. Note the number of units produced from either type A or type B must be an integer. The unit revenues are \$ 1 and \$ 5, respectively. Two raw materials, M1 and M2, are used in the manufacture of the two products. The daily availabilities and requirements of each raw material in each product unit are summarized in the table below.

Resource	A	B	Availability
M1	1 tons	10 tons	20 ton/ day
M2	1 tons	0 tons	2 ton / day

(i) Formulate the problem as an integer linear programming problem with the objective to maximize the daily profit.
(ii) Solve it using branch-and-bound technique.

Question (3)

(a) Use three iterations of Newton's method to locate the maximum of $f(x) = x^2 + 3x + 7\cos x$ starting with an initial guess of $x_0 = 0.1$.

(b) Perform three iterations of the golden section search method to locate the maximum of the function $f(x) = x - \frac{x^2}{1-x}$ in the interval [0 ,0.5]. Give a bound for the error in your result.

(c) Maximize $f(x,y) = 4xy - 2x^2 - 3y^2$ by applying two iterations of Fletcher-Reeves method and starting from the point (1,1).

(d) Use two iterations of Newton's method to locate the minimum of the function $f(x,y) = x^4 + 2x^2y^2 + y^4$, starting from the point (1,1).

Question (4)

(a) Maximize $f(x,y) = 54\,x - 9\,x^2 + 78\,y - 13\,y^2$ subject to

$x \le 4,\ \ y \le 6,\ \ 3\,x + 2\,y \le 18,\ \ x \ge 0,\ \ y \ge 0$

(i) Solve the problem using calculus-based method ignoring the constraints.
(ii) Graph the feasible region and show that the obtained optimal solution is feasible.
(iii) Apply an iteration of Frank-Wolfe algorithm to solve the given linearly-constrained problem starting from the origin.

(b) Consider the following constrained optimization problem:

Minimize $f(x,y) = (x-6)^2 + \ (y-7)^2$ subject to $x + y - 7 \le 0$

Reformulate the problem as an unconstrained problem using the exterior point penalty function method studied in class. Solve the resulting problem using the classical calculus-based method for unconstrained minimization. Perform 3 iterations and set the initial penalty parameter to one and use a scaling factor of 10 to update it. Compare your results with the exact solution by taking the limit $r_k \to \infty$.

(c) Minimize $f(x,y) = 3\,x^2 + 4\,y^2$ subject to $x + 2\,y = 8$ using the augmented Lagrange multipliers method with a fixed value of the penalty parameter $r_k = 1$. Perform two iterations.

Question (5)

(a) Given the fitness function $f(x,y) = -2\,x^2 - 2\,x\,y - 2\,y^2 + 4\,x + 6\,y$ to be maximized and the initial population consisting of the following solutions (0,0), (1,0.5), (1.5,1) and (0.8, 1.2).
i) Evaluate the fitness of each individual in the population.
ii) Select two pairs of parents and apply the crossover operator to each of these pairs.
iii) Apply the mutation operator to the fittest of the resulting offspring by adding the vector (0.03,0.02) to it. Re-evaluate its fitness.
iv) Compare the fitness of the best offspring to the exact solution and verify the nature of the stationary point.

(b) Use two iterations of the PSO algorithm to find the maximum point of the function $f(x,y) = 1 - 2\,x^2 - y^2$ starting with two particles one located at $(-0.25, 0.15)$ and the other at $(0.2, 0)$. The velocity vectors are initialized to $(0.5, 0.5)$ and $(0.25, 0.25)$ Set the algorithm parameters as follows: $c_1 = 1, c_2 = 0.5, \Delta\,t = 1.0$. Assume that the output of the random number generator is given by the following sequence which is repeated in cyclic order: [0.5 0.2 0.1 0.4].

(c) In simulated annealing, the sampling step yielded the following set of energies: 4, 6, 3, 7, 5. Determine a suitable value for the initial temperature.

Question (6)

(a) Consider the following multi-objective optimization problem:

Minimize $f_1(x) = (x-4)^2$ and $f_2(x) = (x-1)^2$

(i) Solve this problem using the weighted function method.
(ii) Re-work the problem using the bounded function method, where $f_2(x)$ should not exceed 1.
(iii) Re-formulate the problem as a goal programming problem and hence solve it.

(b) The knowledge acquired by the artificial neural network (ANN) is stored in the synaptic weights. Using the back-propagation algorithm, adjust the weights of the following ANN. Assume the activation function of the hidden neurons and the output neurons is a sigmoid function. Use a learning rate $\eta = 0.5$. (Hint: The delta rules for adjusting the weights of

synapses connecting to the output neuron is $\delta_3 = e_3\,\Phi_3'(v_3)$ and that for the weights of synapses connecting to the hidden neurons is $\delta_j = \Phi_j'(v_j)\,\delta_3\,w_{3j}$).