Optimization Techniques
Time allowed: 2 hours

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الزمن: ساعتان

Answer the following questions

Question (1):

(a) Determine the stationary points of the function $f(x,y)=\frac{1}{3}x^3-\frac{2}{3}y^3+\frac{1}{2}x^2-6x+32y+4$ and classify them.

(b) We want to construct a box with a square base and we have only 10 m^2 of material to use in the construction of the box. Assuming that all the material is used in the construction process, determine the maximum volume that the box can have.

(c) Consider the following multi-objective optimization problem:

Minimize $f_1(x)=(x-4)^2$ and $f_2(x)=(x-1)^2$

(i) Determine the set of Pareto optimal solutions.
(ii) Solve this problem using the weighted function method.
(iii) Re-work the problem using the bounded function method, where $f_2(x)$ should not exceed 1.
(iv) Re-formulate the problem as a goal programming problem and hence solve it.

Question (2)

(a) Use three iterations of Newton's method to locate the maximum of $f(x)=x^2+3x+7\cos x$ starting with an initial guess of $x_0=0.1$.

(b) Maximize $f(x,y)=4xy-2x^2-3y^2$ by applying two iterations of Fletcher-Reeves method and starting from the point (1,1).

Question (3):

(a) Minimize $f(x,y)=x^2+2y^2-5x-8y$ subject to $3x+2y\le 6$ using two iterations of the exterior penalty function method studied in class with calculus method of unconstrained minimization. Start with $r_1=1$ and set c = 10. Compare your results with the exact solution by taking the limit $r_k\to\infty$.

(b) Minimize $f(x,y)=3x^2+4y^2$ subject to $x+2y=8$ using the augmented Lagrange multipliers method with a fixed value of the penalty parameter $r_k=1$. Perform two iterations.

Question (4)

(a) Given the fitness function $f(x,y)=4xy-2x^2-3y^2$ to be maximized and the initial population consisting of the following solutions (0.2,0.3), (0.5,0), (0.3,-0.4) and (-0.5,-0.25)
(i) Evaluate the fitness of each individual in the population.
(ii) Select two pairs of parents and apply the crossover operator to each of these pairs.
(iii) Apply the mutation operator to the fittest of the resulting offspring by adding 0.01 to each dimension. Re-evaluate its fitness.

(b) Use two iterations of the PSO algorithm to find the maximum point of the function $f(x,y) = 1 - 2x^2 - y^2$ starting with two particles one located at $(-0.25, 0.15)$ and the other at $(0.2, 0)$. The velocity vectors are initialized to $(0.5, 0.5)$ and $(0.25, 0.25)$ Set the algorithm parameters as follows: $c_1 = 1, c_2 = 0.5, \Delta t = 1.0$. Assume that the output of the random number generator is given by the following sequence which is repeated in cyclic order: [0.5 0.2 0.1 0.4].

(c) The knowledge acquired by the artificial neural network (ANN) is stored in the synaptic weights. Using the back-propagation algorithm, adjust the weights of the following ANN. Assume the activation function of the hidden neurons and the output neurons is a sigmoid function. Use a learning rate $\eta = 0.5$. (Hint: The delta rules for adjusting the weights of synapses connecting to the output neuron is $\delta_3 = e_3 \Phi_3'(v_3)$ and that for the weights of synapses connecting to the hidden neurons is $\delta_j = \Phi_j'(v_j) \delta_3 w_{3j}$).