



Single Image Dehazing with Boundary Constraint and Contextual Regularization

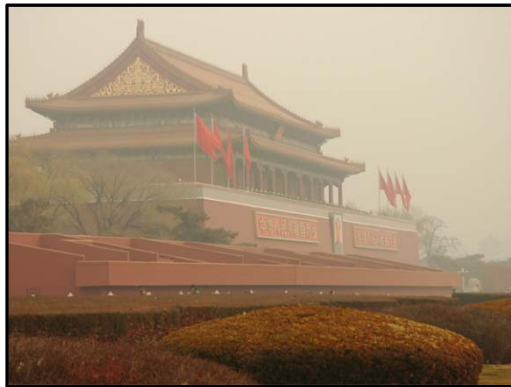
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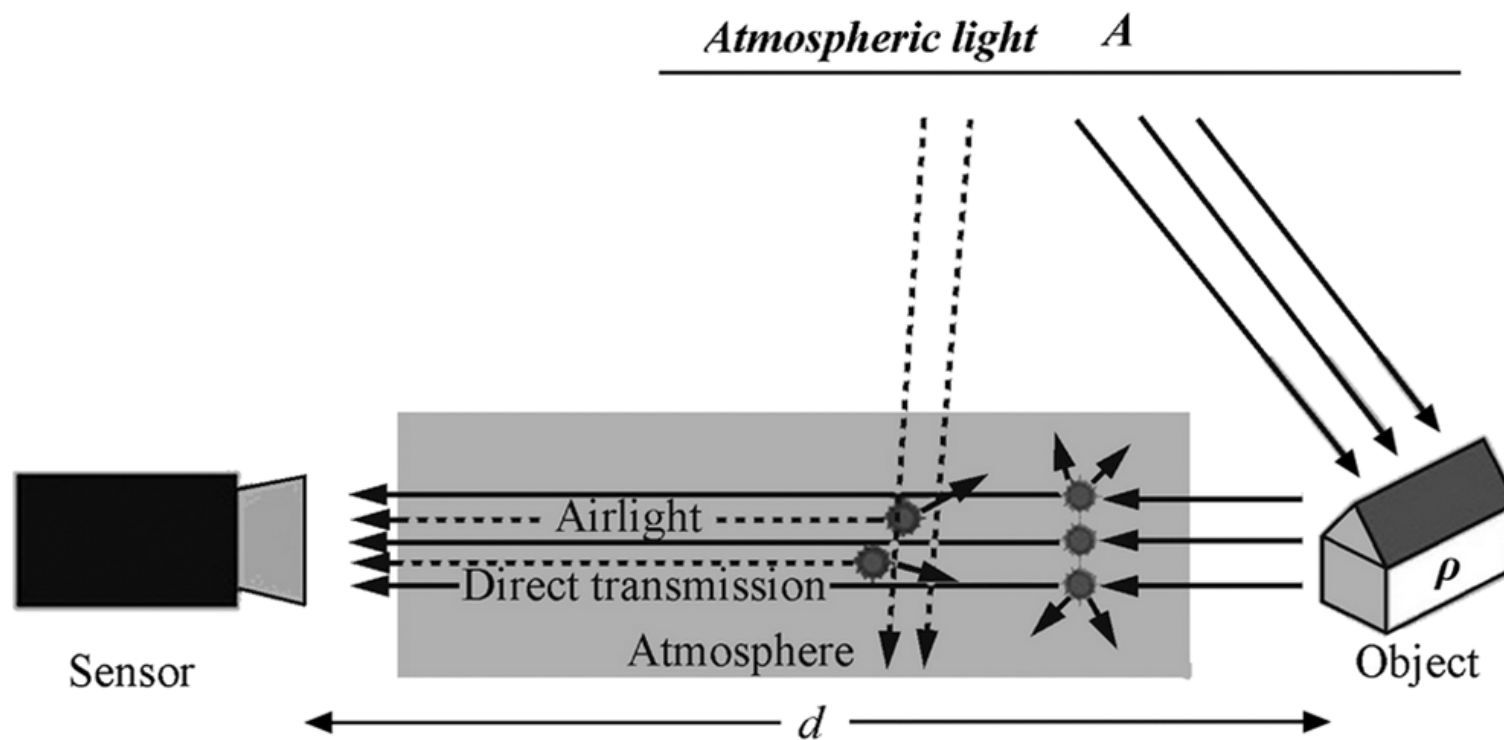
Institute of Automation, Chinese Academy of Sciences (CASIA)

Backgrounds

Images captured at foggy weather conditions often suffer from poor visibility (blurring, low contrasts, color fading).



Haze Imaging Model



$$\mathbf{I}(x) = t(x)\mathbf{J}(x) + (1 - t(x))\mathbf{A}$$

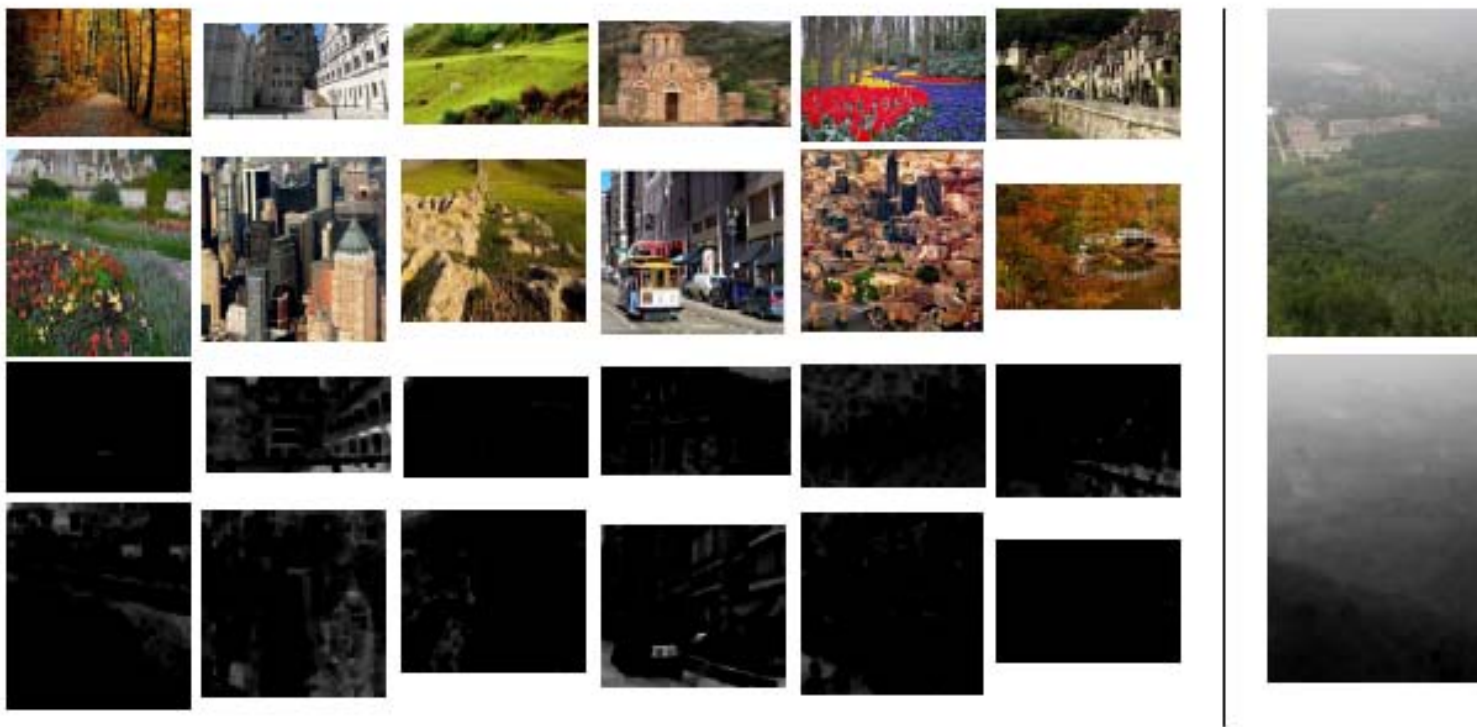
Related Work

Dark Channel Prior (Dr. Kaiming He, et al.)

- Best paper on CVPR'09
- Very simple and elegant
- It comes from a surprising observation that for haze-free outdoor images, in most of the non-sky patches, at least one channel has very low intensity at some pixels.

Dark Channel Prior


$$J^{dark}(\mathbf{x}) = \min_{c \in \{r, g, b\}} \left(\min_{\mathbf{y} \in \Omega(\mathbf{x})} (J^c(\mathbf{y})) \right)$$




Dehazing Using Dark Channel Prior

$$\mathbf{I}(x) = t(x)\mathbf{J}(x) + (1 - t(x))\mathbf{A}$$

$$\min_{\mathbf{y} \in \Omega(\mathbf{x})} (I^c(\mathbf{y})) = \tilde{t}(\mathbf{x}) \min_{\mathbf{y} \in \Omega(\mathbf{x})} (J^c(\mathbf{y})) + (1 - \tilde{t}(\mathbf{x}))A^c$$

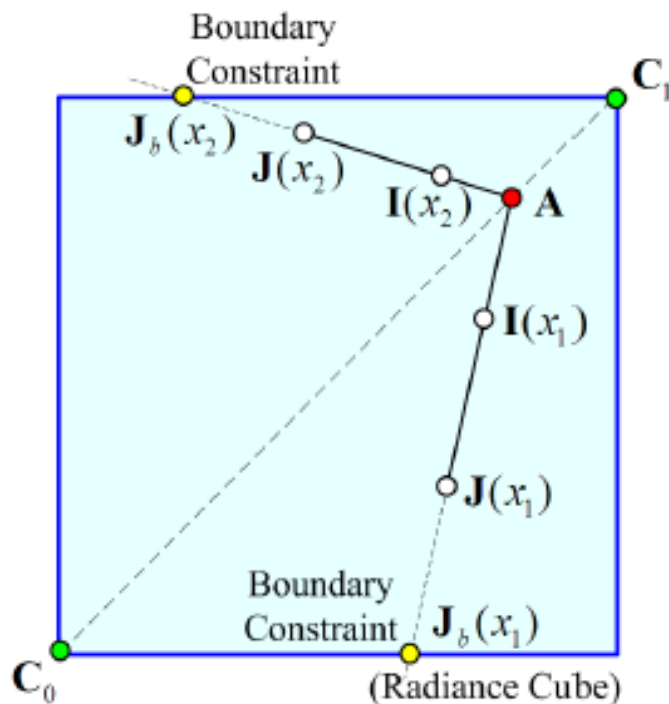
 Dark Channel of Clear Image

$$\min_c \left(\min_{\mathbf{y} \in \Omega(\mathbf{x})} \left(\frac{I^c(\mathbf{y})}{A^c} \right) \right) = \tilde{t}(\mathbf{x}) \min_c \left(\min_{\mathbf{y} \in \Omega(\mathbf{x})} \left(\frac{J^c(\mathbf{y})}{A^c} \right) \right) + (1 - \tilde{t}(\mathbf{x})).$$



$$\tilde{t}(\mathbf{x}) = 1 - \min_c \left(\min_{\mathbf{y} \in \Omega(\mathbf{x})} \left(\frac{I^c(\mathbf{y})}{A^c} \right) \right)$$

A Geometric View

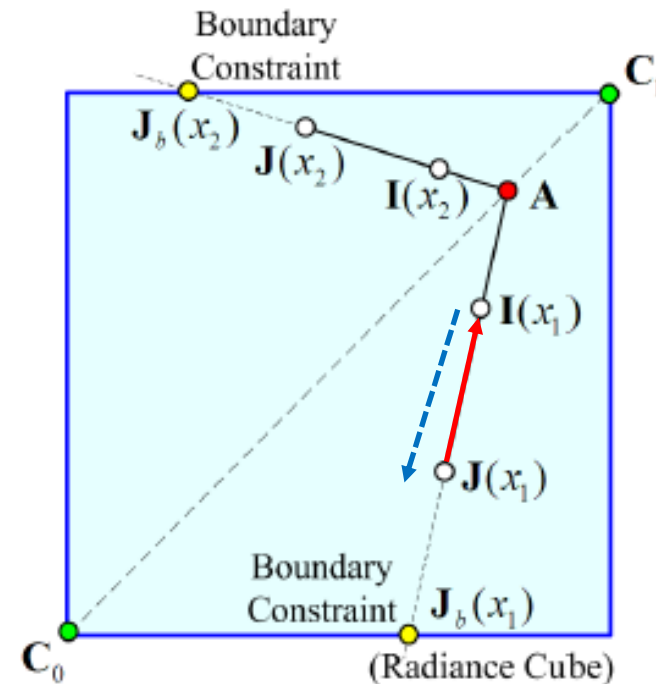


$$\mathbf{I}(x) = t(x)\mathbf{J}(x) + (1 - t(x))\mathbf{A}$$

The value of a hazy pixel is the linear interpolation between the clear pixel and the atmospheric light.

Boundary Constraints

Thus, to dehaze $I(x)$, an intuitive idea is to push it back. This process has to stop when $I(x)$ is pushed to the boundary of the radiance cube.



$$t_b(x) = \min \left\{ \max_{c \in \{r, g, b\}} \left(\frac{A^c - I^c(x)}{A^c - C_0^c}, \frac{A^c - I^c(x)}{A^c - C_1^c} \right), 1 \right\}$$

Boundary Constraints (cont.)

Now, instead of pushing a single pixel, we push an image patch. This process stops when any pixel in the patch touches the boundary. This gives the estimate of $t(x)$ equivalent to that using the dark channel prior.

$$\tilde{t}(x) = \max_{y \in \omega_x} t_b(y)$$

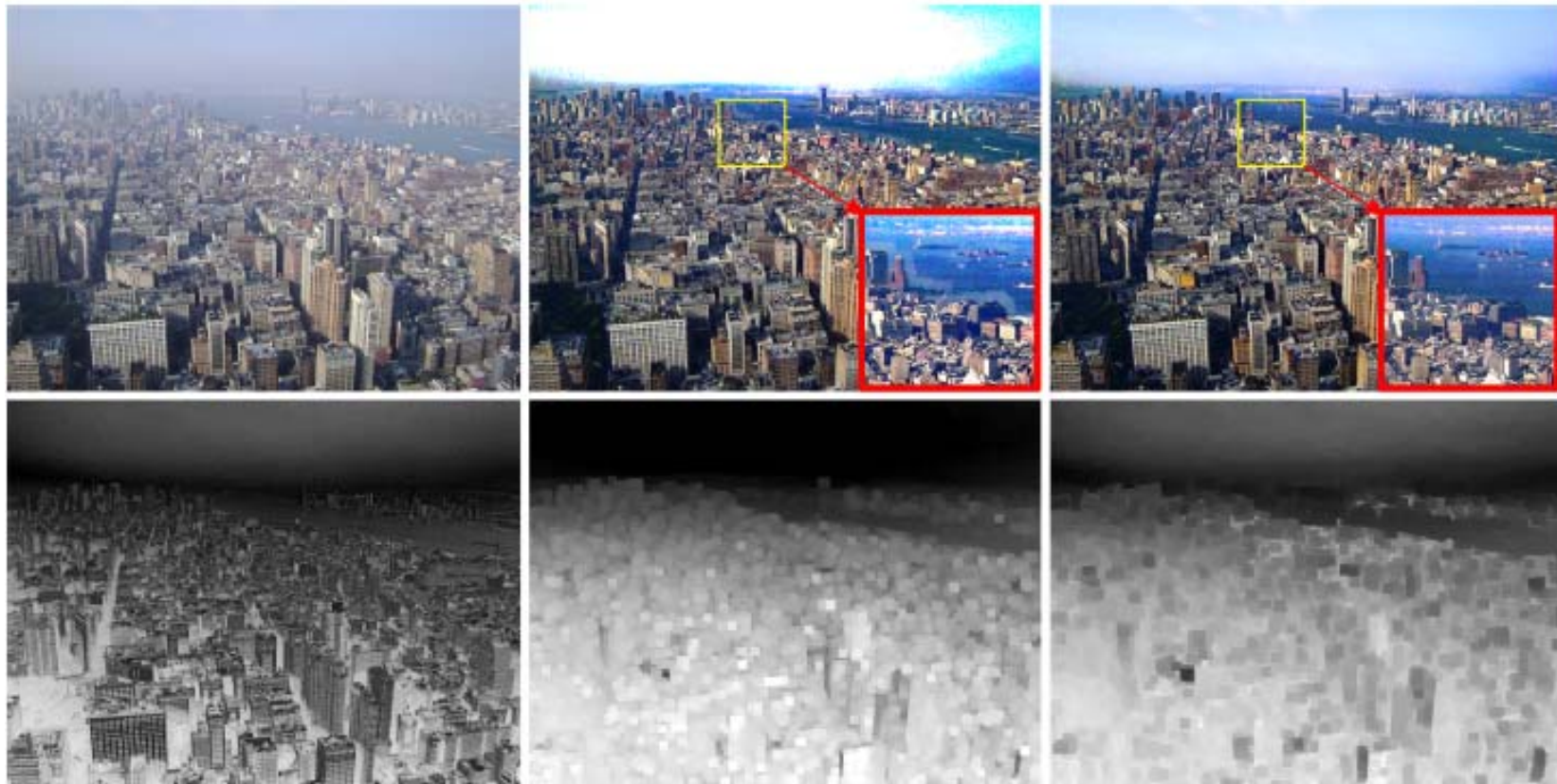
Dehazing using Boundary Constraints

For pushing an image patch, the above stopping condition is strict. We allow some of the pixels can be pushed outside and propose the following estimate of $t(x)$:

$$\hat{t}(x) = \min_{y \in \omega_x} \max_{z \in \omega_y} t_b(z)$$

This is equivalent to applying a morphological closing operation on $t_b(x)$

Dehazing using Boundary Constraints



Contextual Regularization

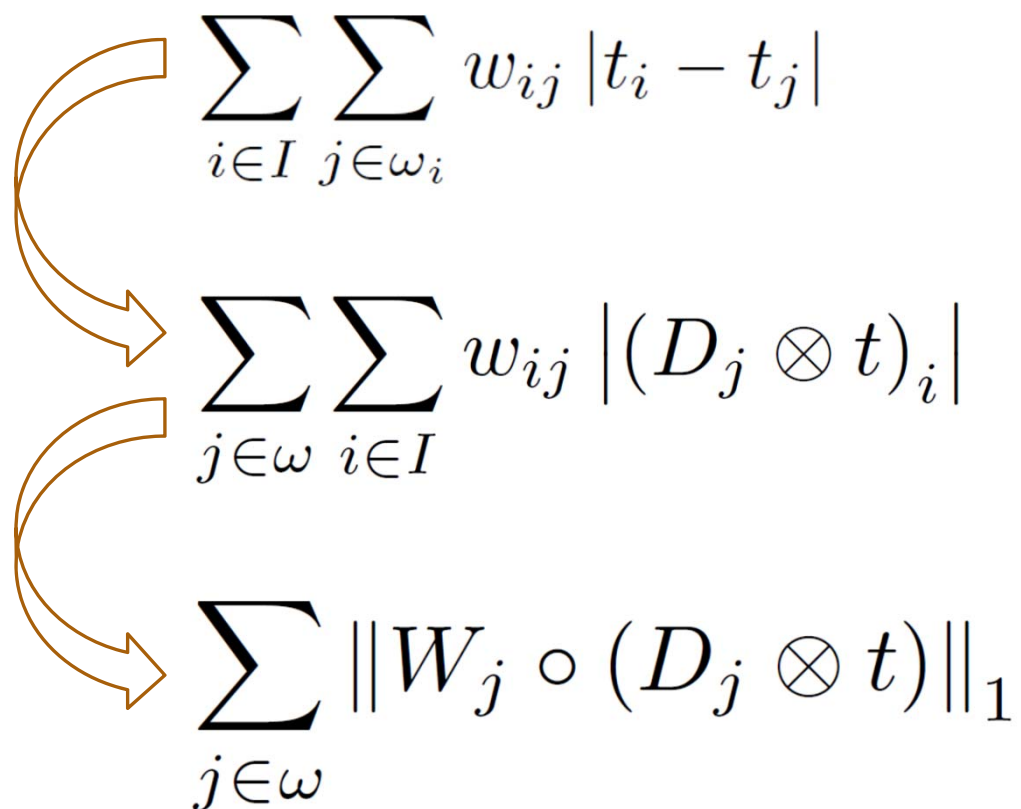
Assumptions

- Pixels in a local image patch share a similar transmission value (Not exactly true)
- Similar Pixels in a local image patch share a similar transmission value

$$W(x, y) (t(y) - t(x)) \approx 0$$

$$W(x, y) = e^{-\|\mathbf{I}(x) - \mathbf{I}(y)\|^2 / 2\sigma^2}$$

Contextual Regularization



The diagram illustrates the transformation of a contextual regularization term into a matrix norm form. It consists of three equations arranged vertically, connected by curved arrows on the left side. The first equation is $\sum_{i \in I} \sum_{j \in \omega_i} w_{ij} |t_i - t_j|$. A curved arrow points from this equation to the second equation, $\sum_{j \in \omega} \sum_{i \in I} w_{ij} |(D_j \otimes t)_i|$. Another curved arrow points from the second equation to the third equation, $\sum_{j \in \omega} \|W_j \circ (D_j \otimes t)\|_1$.

$$\sum_{i \in I} \sum_{j \in \omega_i} w_{ij} |t_i - t_j|$$
$$\sum_{j \in \omega} \sum_{i \in I} w_{ij} |(D_j \otimes t)_i|$$
$$\sum_{j \in \omega} \|W_j \circ (D_j \otimes t)\|_1$$

Dehazing using Boundary Constraints

Further considering the contextual constraints between neighboring pixels, we finally have the following optimization model for refining the estimate $t(x)$:

$$\frac{\lambda}{2} \|t - \hat{t}\|_2^2 + \sum_{j \in \omega} \|W_j \circ (D_j \otimes t)\|_1$$

Data term

Smoothing term

Optimization

Half-quadric splitting method

$$\frac{\lambda}{2} \|t - \hat{t}\|_2^2 + \sum_{j \in \omega} \|W_j \circ (D_j \otimes t)\|_1$$



$$\frac{\lambda}{2} \|t - \hat{t}\|_2^2 + \sum_{j \in \omega} \|W_j \circ u_j\|_1 + \frac{\beta}{2} \left(\sum_{j \in \omega} \|u_j - D_j \otimes t\|_2^2 \right)$$

As $\beta \rightarrow \infty$, the solution of below problem converges to that of the above problem

Optimization

Optimizing u_j

$$\|W_j \circ u_j\|_1 + \frac{\beta}{2} \|u_j - D_j \otimes t\|_2^2$$

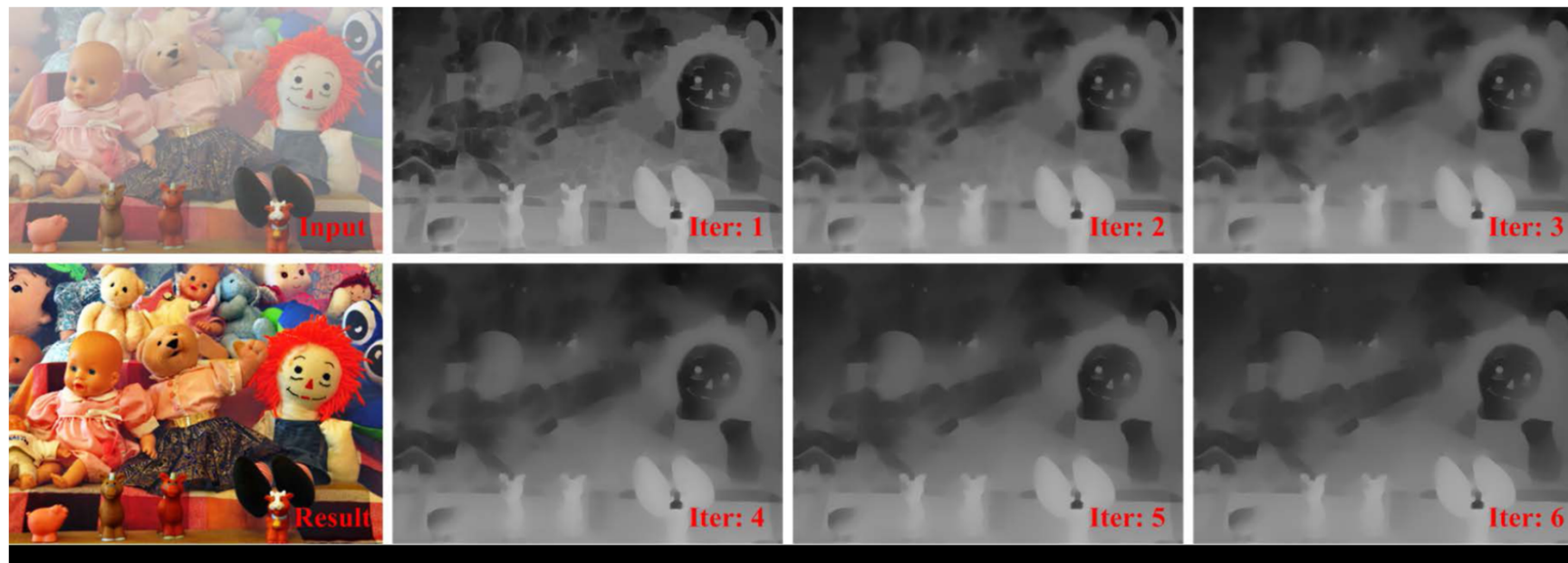
$$\gg \min_x |w \cdot x| + \frac{\beta}{2} (x - a)^2$$

Optimizing t

$$\frac{\lambda}{2} \|t - \hat{t}\|_2^2 + \frac{\beta}{2} \left(\sum_{j \in \omega} \|u_j - D_j \otimes t\|_2^2 \right)$$

$$\gg \frac{\lambda}{\beta} (t - \hat{t}) + \sum_{j \in \omega} D_j^T \otimes (D_j \otimes t - u_j) = 0$$

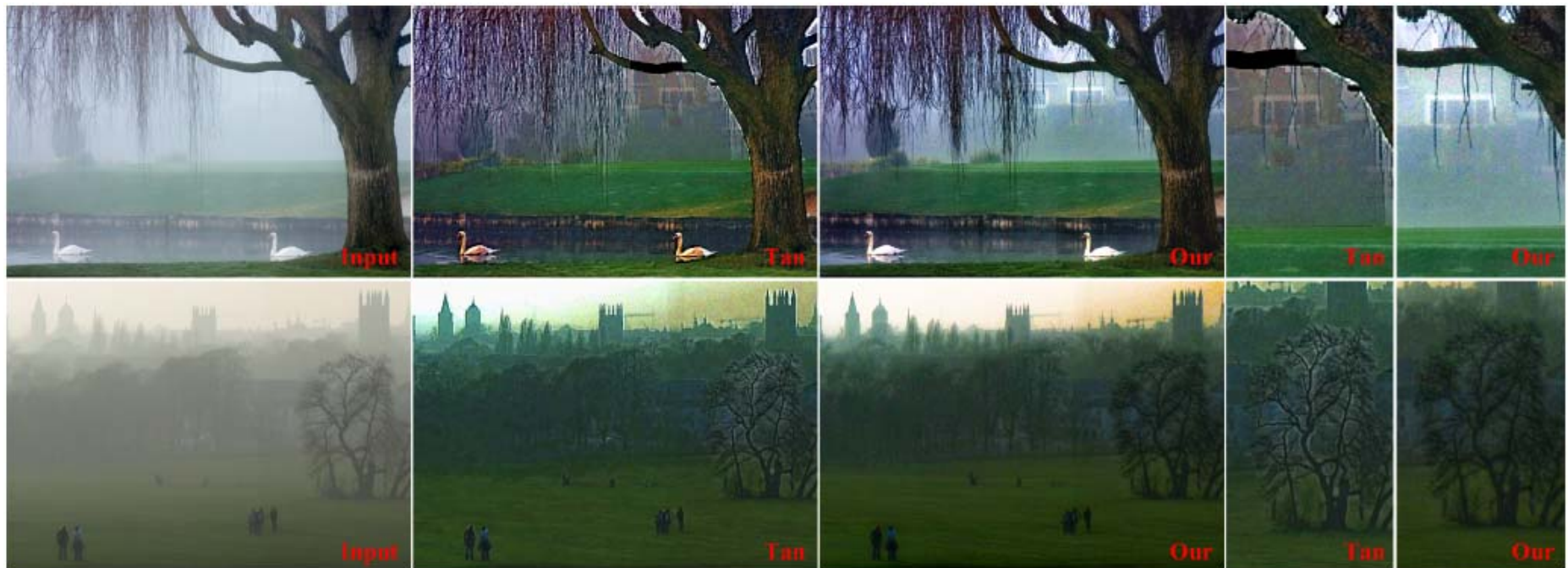
Optimization



Results



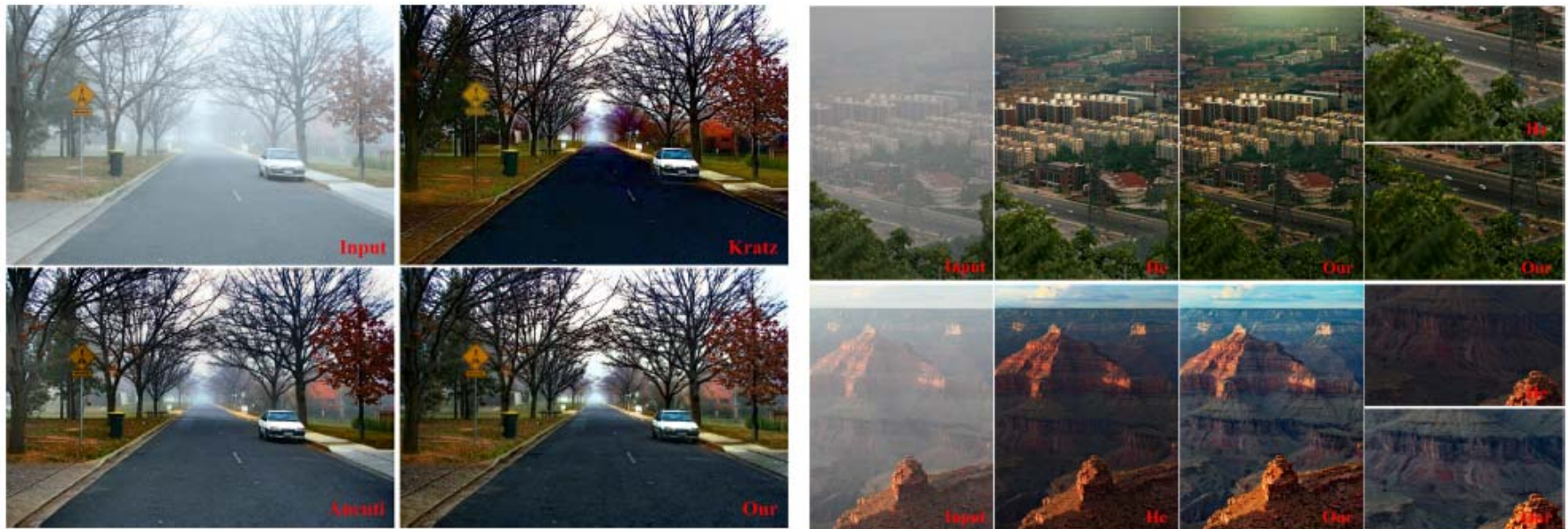
Results (Cont.)



Results (Cont.)



Results (Cont.)



Issues Remained

- Ambiguity between White objects and Haze
- Robust Estimation of Atmospheric light A
- Non-constant Atmospheric light
- Noises Reduction during Dehazing

Conclusions

- ✓ Deriving a new boundary constraint directly from the haze imaging model
- ✓ Equivalent to the famous dark channel prior in some conditions
- ✓ An intuitive yet effective haze removal method

Thanks for your attention!
Questions?