$eq := diff(x(t), t) + x(t) = \frac{2}{\mathrm{sqrt}(\mathrm{Pi})} \cdot \exp(1)^{-t^2 - t};$

$$\frac{d}{dt} x(t) + x(t) = \frac{2\,(e)^{-t^2 - t}}{\sqrt{\pi}} \tag{1}$$

$dsolve(eq, x(t));$

$$x(t) = (\mathrm{erf}(t) + \_C1)\, e^{-t} \tag{2}$$

$int(\exp(t^2), t);$

$$\frac{1}{2} \sqrt{\pi}\, \mathrm{erfi}(t) \tag{3}$$

$int\left(\frac{2}{\mathrm{sqrt}(\mathrm{Pi})} \cdot \exp(1)^{-t^2}, t\right)$

$$\mathrm{erf}(t) \tag{4}$$

Ex2

$eq2 := diff(x(t), t\$2) + 3 \cdot diff(x(t), t) + x(t) = 1;$

$$\frac{d^2}{dt^2} x(t) + 3 \left(\frac{d}{dt} x(t)\right) + x(t) = 1 \tag{5}$$

$sol1 := dsolve(eq2, x(t));$

$$x(t) = e^{\frac{1}{2} (\sqrt{5} - 3)\, t}\, \_C2 + e^{-\frac{1}{2} (\sqrt{5} + 3)\, t}\, \_C1 + 1 \tag{6}$$

$limit(sol1, t = \mathrm{infinity});$

$$\lim_{t \to \infty} x(t) = 1 \tag{7}$$

Statement 2 is true

$eq3 := diff(x(t), t\$2) + 4 \cdot x(t) = 1;$

$$\frac{d^2}{dt^2} x(t) + 4\, x(t) = 1 \tag{8}$$

$sol2 := dsolve(eq3, x(t));$

$$x(t) = \sin(2\, t)\, \_C2 + \cos(2\, t)\, \_C1 + \frac{1}{4} \tag{9}$$

$ic1 := x(0) = \frac{5}{4}, \mathrm{D}(x)(0) = 0;$

$$x(0) = \frac{5}{4}, \mathrm{D}(x)(0) = 0 \tag{10}$$

$sol2 := dsolve(\{eq3, ic1\}, x(t));$

$$x(t) = \frac{1}{4} + \cos(2\, t) \tag{11}$$

$sol2(\mathrm{Pi});$

$$x(t)(\pi) = \frac{1}{4} + \cos(2\, t)(\pi) \tag{12}$$

$expr := rhs(sol2);$

$$\frac{1}{4} + \cos(2\,t) \tag{13}$$

$eval(expr, t = \mathrm{Pi});$

$$\frac{5}{4} \tag{14}$$

Statment 3 is true

$eq4 := diff(x(t), t) = 3 \cdot x(t) + t^3;$

$$\frac{\mathrm{d}}{\mathrm{d}t}\,x(t) = 3\,x(t) + t^3 \tag{15}$$

$dsolve(eq4, x(t));$

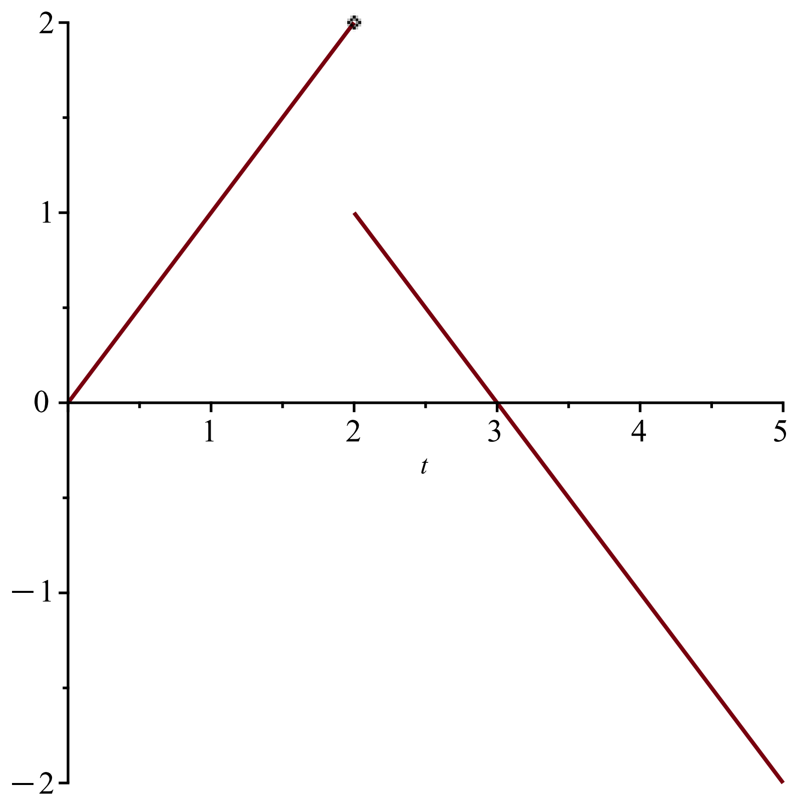
$$x(t) = -\frac{1}{3}\,t^2 - \frac{1}{3}\,t^3 - \frac{2}{9}\,t - \frac{2}{27} + \mathrm{e}^{3\,t}\,\_C1 \tag{16}$$

Statement 4 is true

$f := piecewise(t \leq 2, t, t > 2, 3 - t);$

$$\begin{cases} t & t \leq 2 \\ 3 - t & 2 < t \end{cases} \tag{17}$$

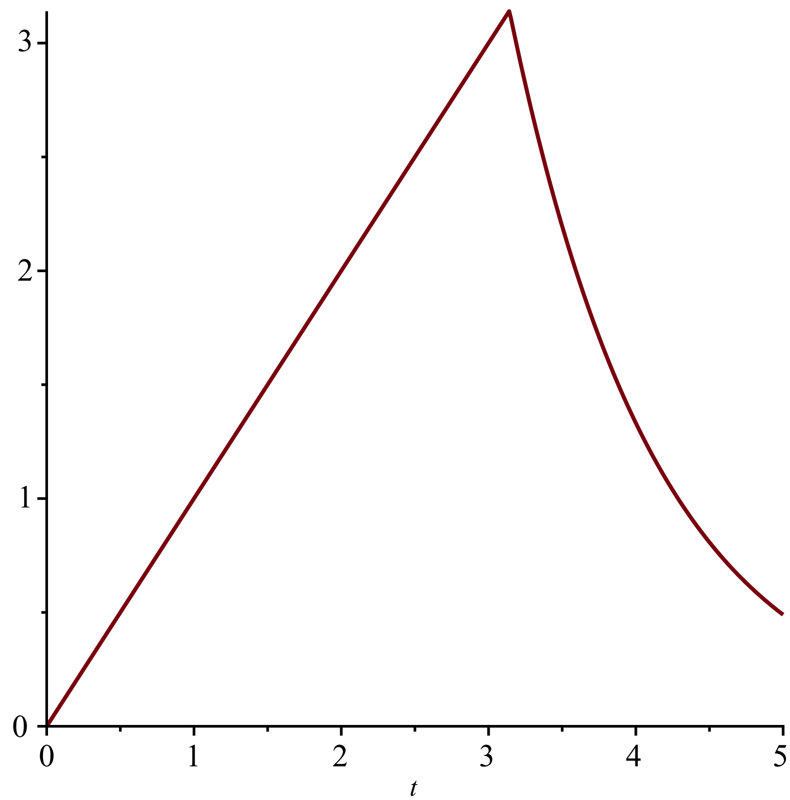
$plot(f, t = 0\,..5, discont = true);$



$f := piecewise\left(0 \leq t \leq \mathrm{Pi}, t, t > \mathrm{Pi}, \mathrm{Pi} \cdot \exp(1)^{\mathrm{Pi} - t}\right);$

$$\begin{cases} t & 0 \leq t \textbf{ and } t \leq \pi \\ \pi\,(\mathrm{e})^{\pi - t} & \pi < t \end{cases} \tag{18}$$

$plot(f, t = 0\,..5);$



Exercise 7

$eq := diff(x(t), t\$2) + x(t) = f;$

$$\frac{d^2}{dt^2}\,x(t) + x(t) = \begin{cases} t & 0 \le t \textbf{ and } t \le \pi \\ \pi\,(e)^{\pi - t} & \pi < t \end{cases} \tag{19}$$

$ic1 := x(0) = 0, D(x)(0) = 1;$

$$x(0) = 0, D(x)(0) = 1 \tag{20}$$

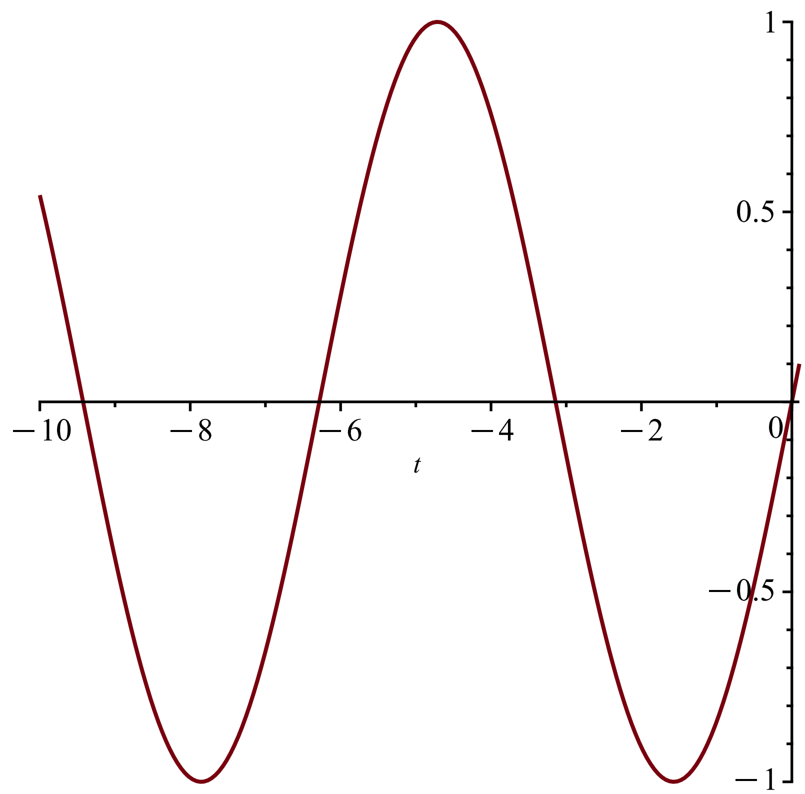
$sol := dsolve(\{eq, ic1\}, x(t));$

$$x(t) = \begin{cases} \sin(t) & t < 0 \\ t & t < \pi \\ \frac{1}{2}\,\pi\,e^{\pi - t} - \frac{1}{2}\,\sin(t)\,\pi - \frac{1}{2}\,\cos(t)\,\pi - \sin(t) & \pi \le t \end{cases} \tag{21}$$

$solfct := rhs(sol);$

$$\begin{cases} \sin(t) & t < 0 \\ t & t < \pi \\ \frac{1}{2}\,\pi\,e^{\pi - t} - \frac{1}{2}\,\sin(t)\,\pi - \frac{1}{2}\,\cos(t)\,\pi - \sin(t) & \pi \le t \end{cases} \tag{22}$$

$plot(solfct, t = -10\,..10);$



$eq2 := diff(x(t), t\$2) + x(t) = \mathrm{Pi} \cdot \exp(1)^{\mathrm{Pi}-t};$

$$\frac{d^2}{dt^2} x(t) + x(t) = \pi (e)^{\pi - t} \qquad \textbf{(23)}$$

$ic := x(0) = 0, D(x)(0) = 0;$

$$x(0) = 0, D(x)(0) = 0 \qquad \textbf{(24)}$$

$dsolve(\{eq2, ic\}, x(t));$

$$x(t) = \frac{1}{2} \sin(t) \pi e^{\pi} - \frac{1}{2} \cos(t) \pi e^{\pi} + \frac{1}{2} \pi e^{\pi - t} \qquad \textbf{(25)}$$

Exercise 8

$eq := diff(x(t), t\$2) + x(t) = \cos(\omega \cdot t);$

$$\frac{d^2}{dt^2} x(t) + x(t) = \cos(\omega t) \qquad \textbf{(26)}$$

$ic1 := x(0) = 0, D(x)(0) = 0;$

$$x(0) = 0, D(x)(0) = 0 \qquad \textbf{(27)}$$

$sol := dsolve(\{eq, ic1\}, x(t));$

$$x(t) = \frac{\cos(t)}{\omega^2 - 1} - \frac{\cos(\omega t)}{\omega^2 - 1} \qquad \textbf{(28)}$$

$\varphi := rhs(sol);$

$$\frac{\cos(t)}{\omega^2 - 1} - \frac{\cos(\omega t)}{\omega^2 - 1} \qquad \textbf{(29)}$$

When ω=1

$eq1 := diff(x(t), t\$2) + x(t) = \cos(t);$

$$\frac{d^2}{dt^2}\, x(t) + x(t) = \cos(t) \tag{30}$$

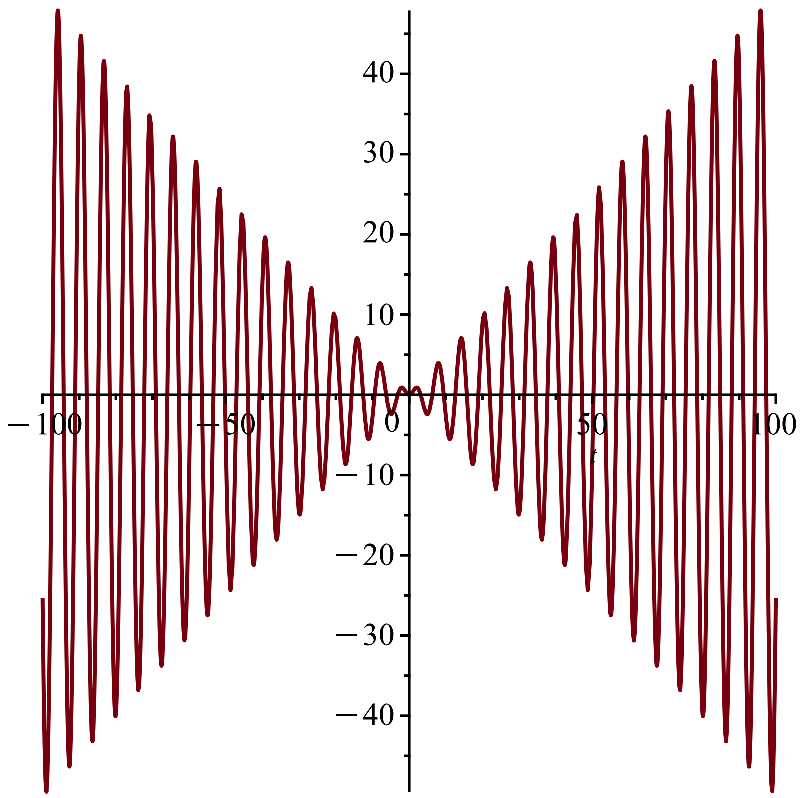
$sol2 := dsolve(\{eq1, ic1\}, x(t));$

$$x(t) = \frac{1}{2}\, t \sin(t) \tag{31}$$

$limit(\varphi, \omega = 1);$

$$\frac{1}{2}\, t \sin(t) \tag{32}$$

$plot(rhs(sol2), t = -100\,..100);$



Exercise 9

$eq := diff(x(t), t\$2) - 4 \cdot x(t) = \exp(1)^{\alpha \cdot t};$

$$\frac{d^2}{dt^2}\, x(t) - 4\, x(t) = (e)^{\alpha t} \tag{33}$$

$ic := x(0) = 0, D(x)(0) = 0;$

$$x(0) = 0, D(x)(0) = 0 \tag{34}$$

$sol := dsolve(\{eq, ic\}, x(t));$

$$x(t) = \frac{1}{4}\, \frac{e^{-2t}}{\alpha + 2} - \frac{1}{4}\, \frac{e^{2t}}{\alpha - 2} + \frac{e^{\alpha t}}{\alpha^2 - 4} \tag{35}$$

$limit(rhs(sol), \alpha = 2);$

$$\frac{1}{16}\, \frac{4\, t\, (e^t)^4 - (e^t)^4 + 1}{(e^t)^2} \tag{36}$$

Find separately the solution when α=2

$eq2 := diff(x(t), t\$2) - 4\cdot x(t) = \exp(1)^{2\cdot t};$

$$\frac{d^2}{dt^2}x(t) - 4\,x(t) = (e)^{2t} \tag{37}$$

$sol2 := dsolve(\{eq2, ic\}, x(t));$

$$x(t) = \frac{1}{4}\,t\,e^{2t} + \frac{1}{16}\,e^{-2t} - \frac{1}{16}\,e^{2t} \tag{38}$$

Exercise 10

$f := piecewise\left(0 \le t \le \frac{Pi}{2}, t, \frac{Pi}{2} \le t \le Pi, Pi - t, t > Pi, 0\right);$

$$\begin{cases} t & 0 \le t \textbf{ and } t \le \frac{1}{2}\pi \\ \pi - t & \frac{1}{2}\pi \le t \textbf{ and } t \le \pi \\ 0 & \pi < t \end{cases} \tag{39}$$

$eq := diff(x(t), t\$2) + x(t) = f;$

$$\frac{d^2}{dt^2}x(t) + x(t) = \begin{cases} t & 0 \le t \textbf{ and } t \le \frac{1}{2}\pi \\ \pi - t & \frac{1}{2}\pi \le t \textbf{ and } t \le \pi \\ 0 & \pi < t \end{cases} \tag{40}$$

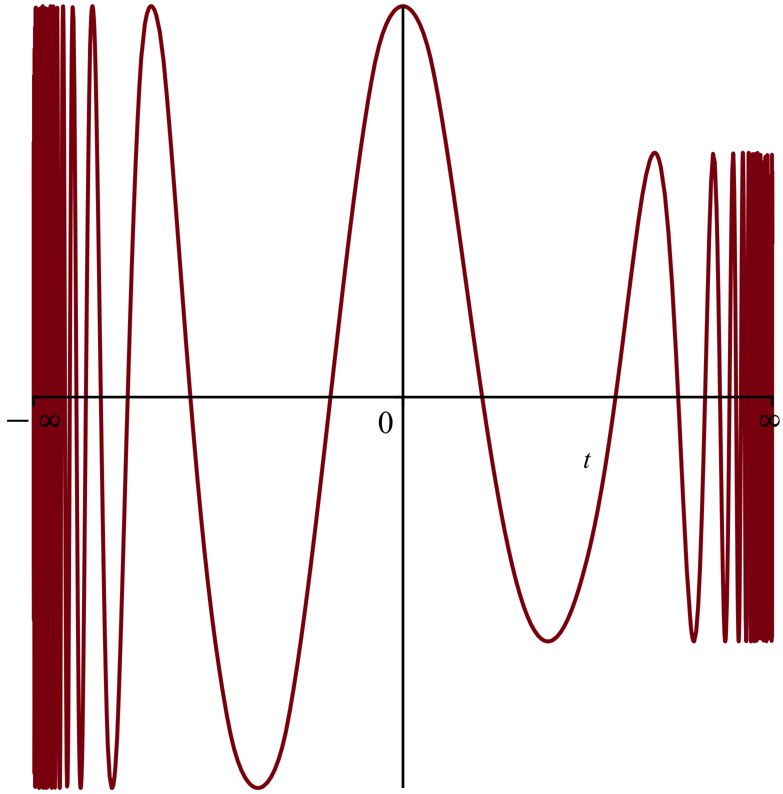
$ic := x(0) = 5, D(x)(0) = 0;$

$$x(0) = 5, D(x)(0) = 0 \tag{41}$$

$sol := dsolve(\{eq, ic\}, x(t));$

$$x(t) = \begin{cases} 5\cos(t) & t < 0 \\ 5\cos(t) + t - \sin(t) & t < \frac{1}{2}\pi \\ 3\cos(t) + \pi - t - \sin(t) & t < \pi \\ 3\cos(t) & \pi \le t \end{cases} \tag{42}$$

$plot(rhs(sol), t = -\text{infinity}..\text{infinity});$



Exercise 11

$eq1 := diff(x(t), t) = -2 \cdot x(t);$

$$\frac{d}{dt}\, x(t) = -2\, x(t) \tag{43}$$

$eq2 := diff(y(t), t) = -3 \cdot y(t);$

$$\frac{d}{dt}\, y(t) = -3\, y(t) \tag{44}$$

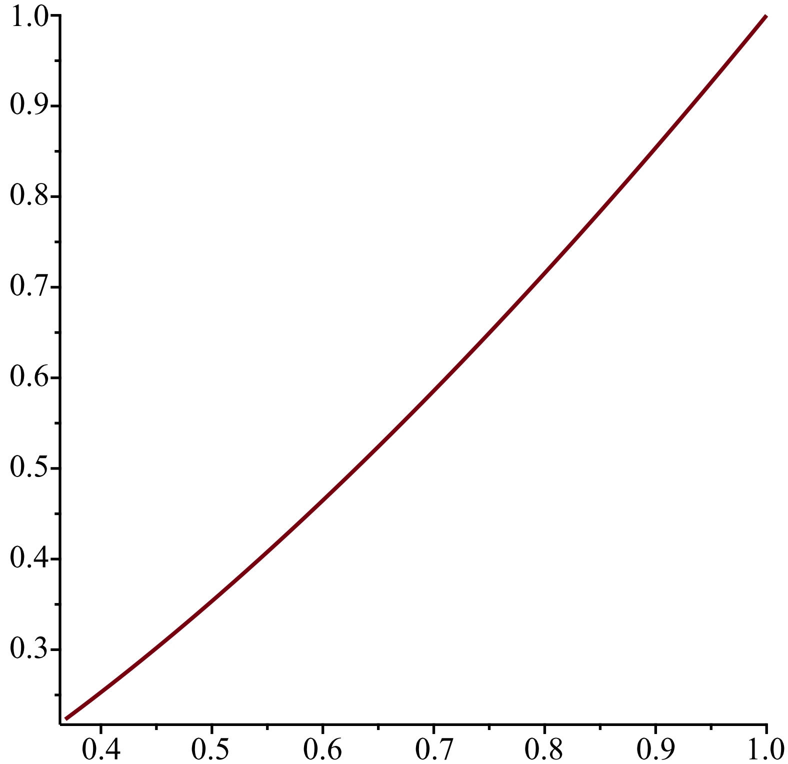
$ic := x(0) = 1, y(0) = 1;$

$$x(0) = 1, y(0) = 1 \tag{45}$$

$dsolve(\{eq1, eq2, ic\}, \{x(t), y(t)\});$

$$\{x(t) = e^{-2\, t}, y(t) = e^{-3\, t}\} \tag{46}$$

$plot([\exp(1)^{-2 \cdot t}, \exp(1)^{-3 \cdot t}, t = 0\, ...5]);$



Exercise 14

$eq1 := diff(x(t), t) = -x(t) + 3\cdot y(t);$

$$\frac{d}{dt} x(t) = -x(t) + 3\,y(t) \tag{47}$$

$eq2 := diff(y(t), t) = -3\cdot x(t) - y(t);$

$$\frac{d}{dt} y(t) = -3\,x(t) - y(t) \tag{48}$$

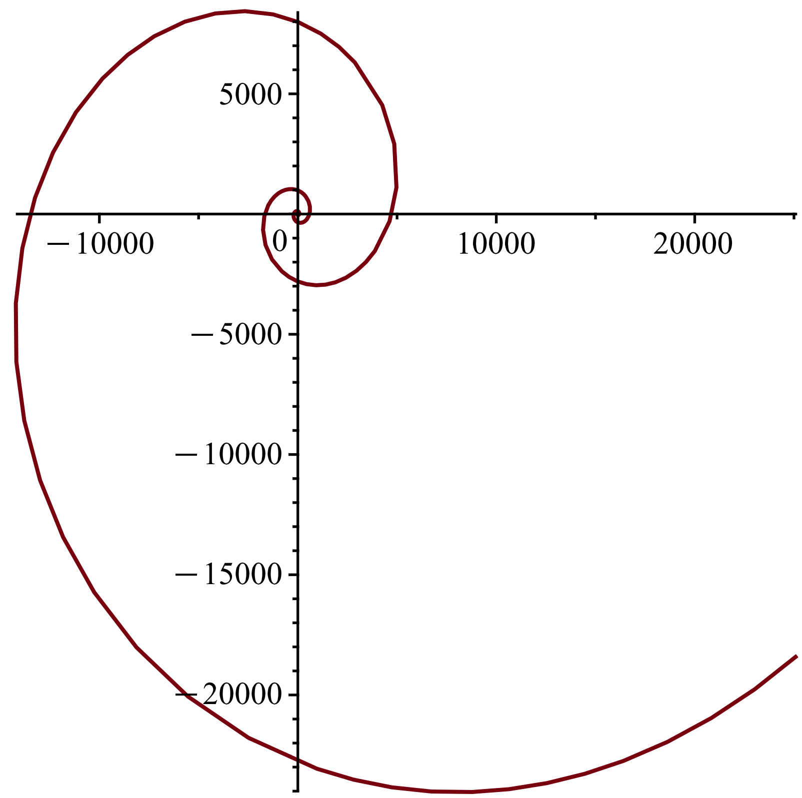
$ic := x(0) = 1, y(0) = 1;$

$$x(0) = 1, y(0) = 1 \tag{49}$$

$dsolve(\{eq1, eq2, ic\}, \{x(t), y(t)\});$

$$\{x(t) = e^{-t}(\cos(3\,t) + \sin(3\,t)), y(t) = e^{-t}(\cos(3\,t) - \sin(3\,t))\} \tag{50}$$

$plot([\exp(1)^{-t}(\cos(3\,t) + \sin(3\,t)), \exp(1)^{-t}(\cos(3\,t) - \sin(3\,t)), t = -10..10]);$


5000
−10000
0
10000
20000
−5000
−10000
−15000
−20000