1.Round-off errors

$evalf\left(25^{\frac{1}{8}}\right);$

$$1.495348781 \tag{1}$$

$evalf\left(25^{\frac{1}{8}}\right)^{8};$

$$24.99999997 \tag{2}$$

Iterations: the logistic map

λ= 1 => f1(x) = x(1-x)

$f1 := x \rightarrow x \cdot (1 - x);$

$$x \rightarrow x\,(1 - x) \tag{3}$$

$f1 := x \cdot (1 - x) - x;$

$$x\,(1 - x) - x \tag{4}$$

$solve(f1, x);$

$$0, 0 \tag{5}$$

A fixed point for f1 is 0
Compute the first 200 iterations of f2 starting from x0 = 0.5

$f1 := x \rightarrow x - x^2;$

$x := x0;$

$$x0 \tag{6}$$

$x0 := 0.5;$

$$0.5 \tag{7}$$

$x := x0;$

Numerical methods
Ex.4 - Find the solution of the ivp

$eq := diff(y(x), x) = 2 \cdot x \cdot y(x);$

$$\frac{d}{dx}\, y(x) = 2\, x\, y(x) \tag{8}$$

$f := (x, y) \rightarrow 2 \cdot x \cdot y;$

$$(x, y) \rightarrow 2\, y\, x \tag{9}$$

$ic := y(0) = 1;$

$$y(0) = 1 \tag{10}$$

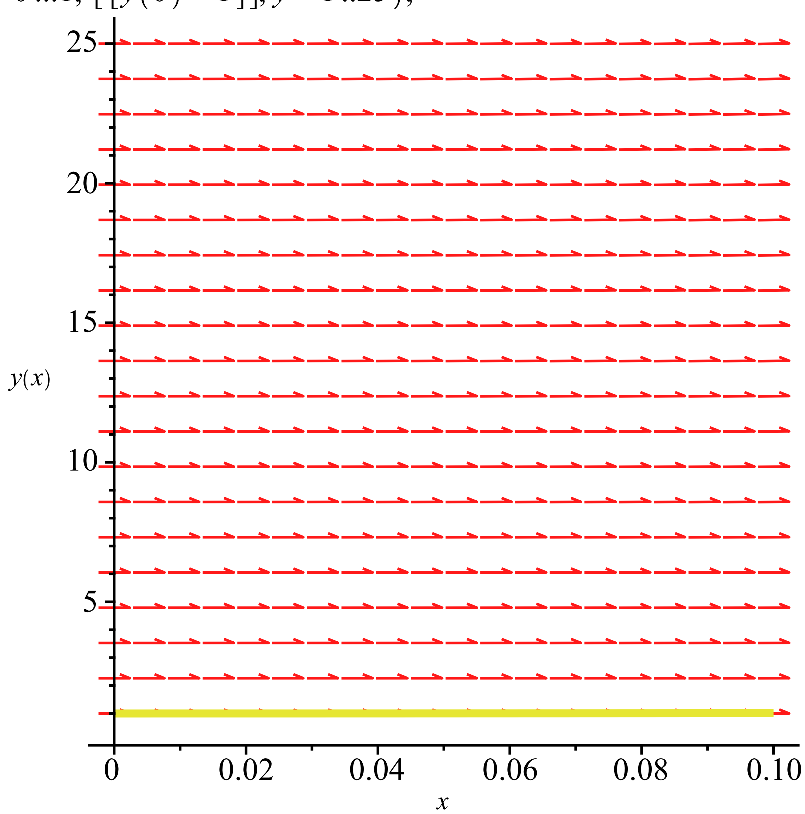
$dsolve(\{eq, ic\}, y(x));$

$$y(x) = e^{x^2} \tag{11}$$

$with(DEtools);$

[*AreSimilar, Closure, DEnormal, DEplot, DEplot3d, DEplot_polygon, DFactor, DFactorLCLM, DFactorsols, Dchangevar, Desingularize, FunctionDecomposition, GCRD, Gosper, Heunsols, Homomorphisms, IVPsol, IsHyperexponential, LCLM, MeijerGsols, MultiplicativeDecomposition, ODEInvariants, PDEchangecoords, PolynomialNormalForm, RationalCanonicalForm, ReduceHyperexp, RiemannPsols, Xchange, Xcommutator, Xgauge, Zeilberger, abelsol, adjoint, autonomous, bernoullisol, buildsol, buildsym, canoni, caseplot, casesplit, checkrank, chinisol, clairautsol, constcoeffsols, convertAlg, convertsys, dalembertsol, dcoeffs, de2diffop, dfieldplot, diff_table, diffop2de, dperiodic_sols, dpolyform, dsubs, eigenring, endomorphism_charpoly, equinv, eta_k, eulersols, exactsol, expsols, exterior_power, firint, firtest, formal_sol, gen_exp, generate_ic, genhomosol, gensys, hamilton_eqs, hypergeomsols, hyperode, indicialeq, infgen, initialdata, integrate_sols, intfactor, invariants, kovacicsols, leftdivision, liesol, line_int, linearsol, matrixDE, matrix_riccati, maxdimsystems, moser_reduce, muchange, mult, mutest, newton_polygon, normalG2, ode_int_y, ode_y1, odeadvisor, odepde, parametricsol, particularsol, phaseportrait, poincare, polysols, power_equivalent, rational_equivalent, ratsols, redode, reduceOrder, reduce_order, regular_parts, regularsp, remove_RootOf, riccati_system, riccatisol, rifread, rifsimp, rightdivision, rtaylor, separablesol, singularities, solve_group, super_reduce, symgen, symmetric_power, symmetric_product, symtest, transinv, translate, untranslate, varparam, zoom*] **(12)**

$DEplot(eq, y(x), x=0\ldots1, [[y(0)=1]], y=1..25);$



$dsolve(\{diff(y(x),x)=f(x,y(x)), y(0)=1\});$ phi $:= unapply(rhs(\%), x);$

$h := 0.1;$

$$0.1 \qquad (13)$$

$x := 0; y := 1;$

$$1 \qquad (14)$$

Error, invalid input: diff received 1.5, which is not valid for its 2nd argument

Error, invalid input: rhs received 1.5, which is not valid for its 1st argument, expr

for i **from** 1 **to** 15 **do** $y := y + h \cdot f(x, y) : \mathrm{psi}(i) := y : x := x + h : print(x, y, \mathrm{phi}(x), \mathrm{abs}(y - \mathrm{phi}(x)));$ **od**:

$$1.5, 6.835368997, \phi(1.5), |6.835368997 - \phi(1.5)| \qquad (15)$$

$restart :$

Exercise 2

$f := x \rightarrow x \cdot (1 - x);$

$$x \rightarrow x\,(1 - x) \qquad (16)$$

$x0 := 0.5;$

$$0.5 \qquad (17)$$

$x := x0;$

$$0.5 \qquad (18)$$

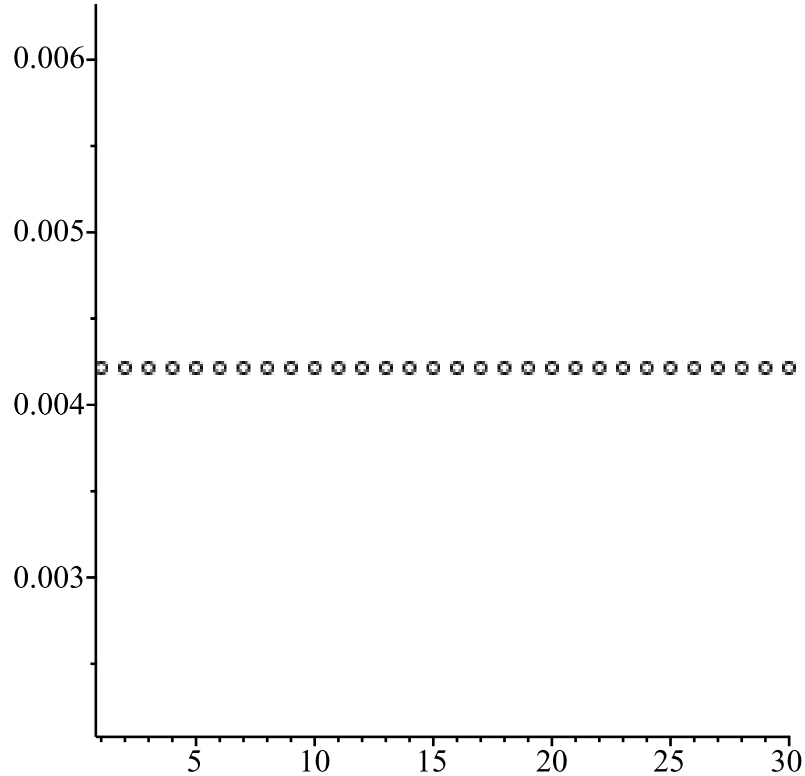
for i **from** 1 **to** 30 **do** $x := f(x) : \mathrm{psi}(i) := x : print(x);$**od**:

$$0.02831421228 \qquad (19)$$

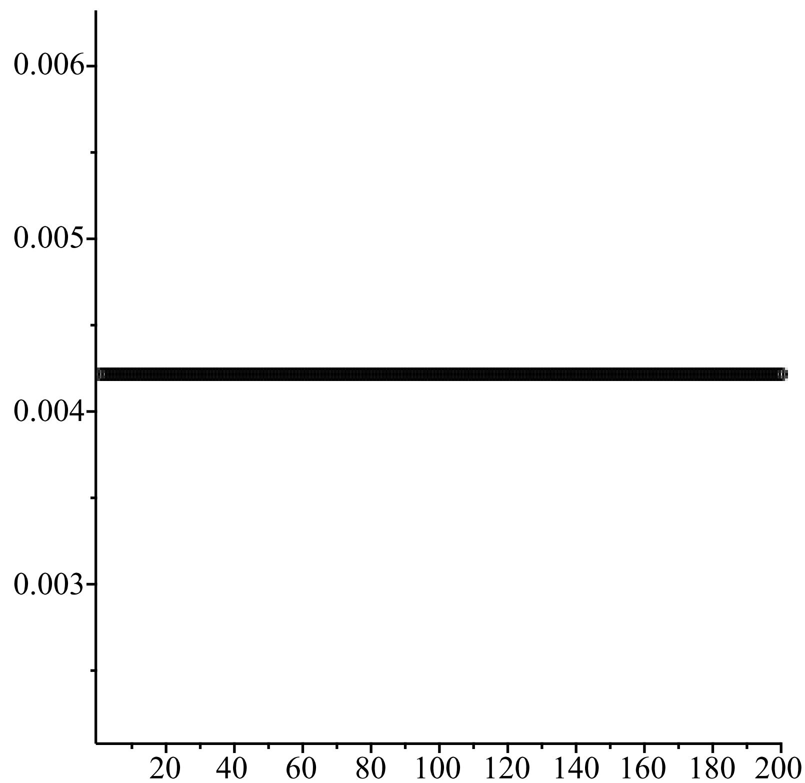
for i **from** 1 **to** 200 **do** $x := f(x) : \mathrm{psi}(i) := x : print(x);$**od**:

$$0.004215230853 \qquad (20)$$

$points := [[n, \mathrm{psi}(n)]\$n = 1..30] : with(plots) : pointplot(points, symbol = circle);$



$points := [[n, \mathrm{psi}(n)]\$n = 1..200] : with(plots) : pointplot(points, symbol = circle);$



$x0 := 0;$

$$0 \tag{21}$$

$x := x0;$

$$0 \tag{22}$$

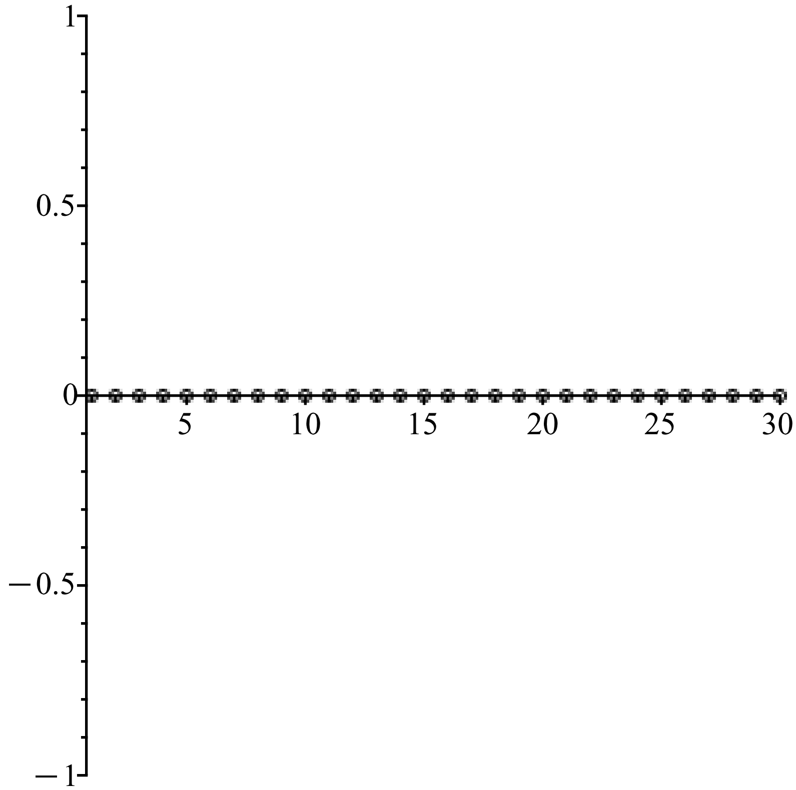
for i **from** 1 **to** 30 **do** $x := f(x)$: $\mathrm{psi}(i) := x$: $print(x)$;**od**:

$$0 \tag{23}$$

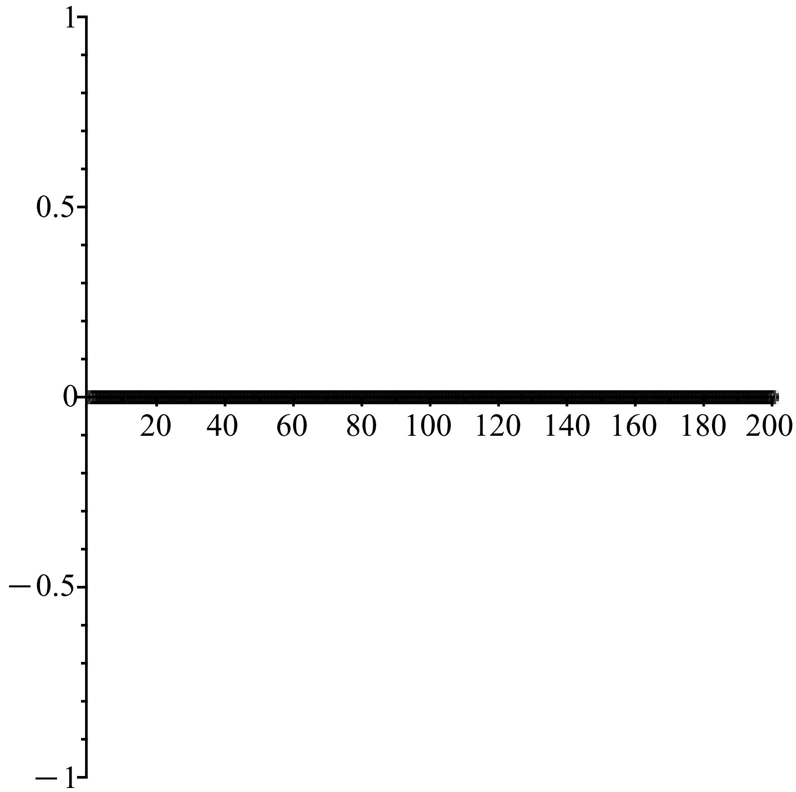
for i **from** 1 **to** 30 **do** $x := f(x)$: $\mathrm{psi}(i) := x$: $print(x)$;**od**:

$$0 \tag{24}$$

$points := [[n, \mathrm{psi}(n)]\$n = 1..30]$: $with(plots)$: $pointplot(points, symbol = circle)$;



$points := [[n, \mathrm{psi}(n)]\$n = 1..200]$: $with(plots)$: $pointplot(points, symbol = circle)$;



Doing the same for λ=3.5

$f := x \rightarrow 3.5 \cdot x \cdot (1 - x);$

$$x \rightarrow 3.5\, x\, (1 - x) \tag{25}$$

$x0 := 0.5$

$$0.5 \tag{26}$$

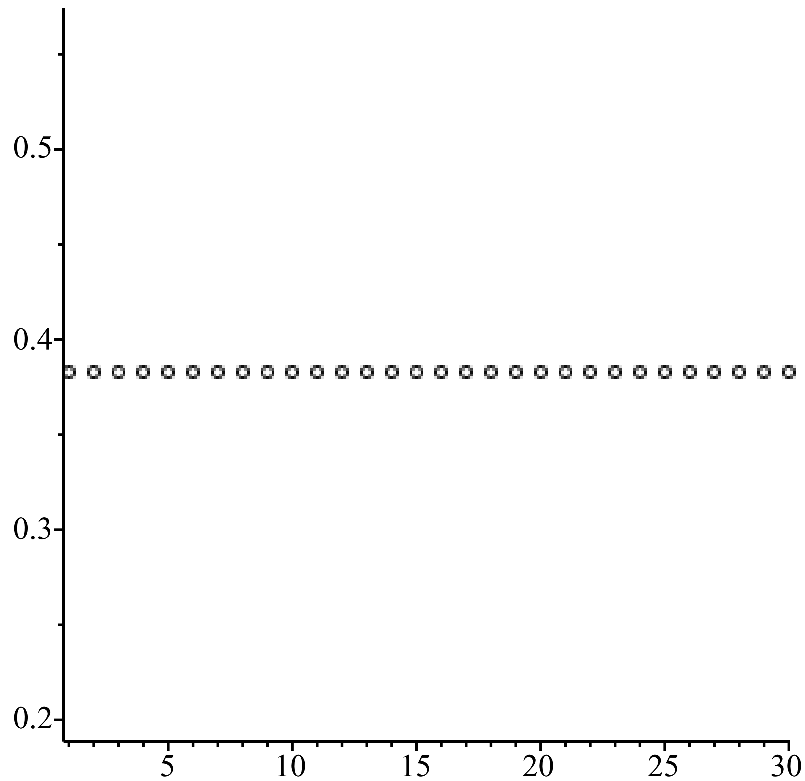
$x := x0;$

$$0.5 \tag{27}$$

for i **from** 1 **to** 30 **do** $x := f(x) : \mathrm{psi}(i) := x : print(x);$**od**:

$$0.3828196839 \tag{28}$$

$points := [[n, \mathrm{psi}(n)]\$n = 1..30] : with(plots) : pointplot(points, symbol = circle);$



$x0 := 0.7$

$$0.7 \tag{29}$$

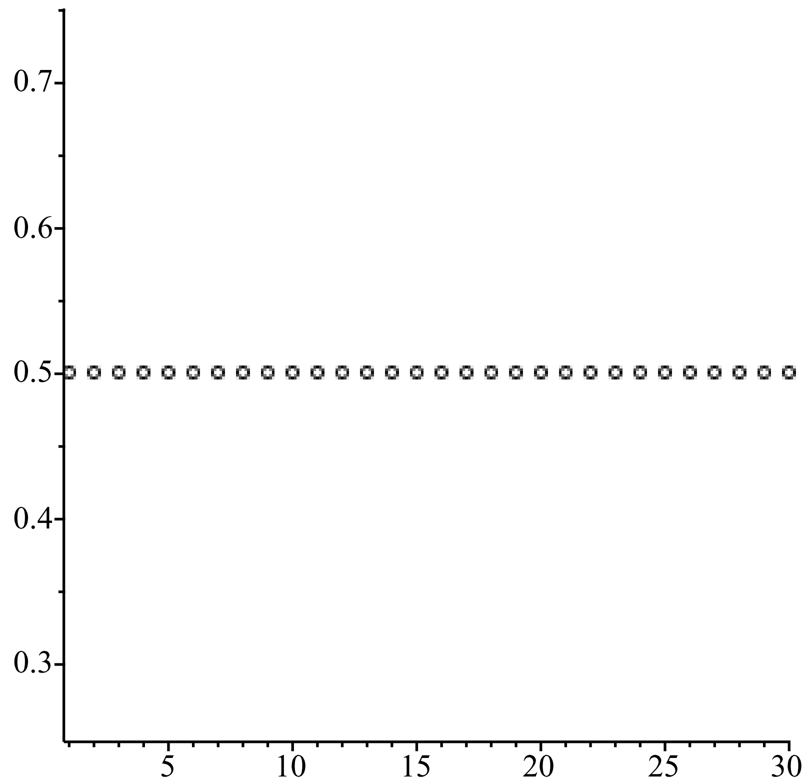
$x := x0;$

$$0.7 \tag{30}$$

for i **from** 1 **to** 30 **do** $x := f(x) : \mathrm{psi}(i) := x : print(x);$**od**:

$$0.5008840409 \tag{31}$$

$points := [[n, \mathrm{psi}(n)]\$n = 1 ..30] : with(plots) : pointplot(points, symbol = circle);$



3.Sensitivity with respect to small errors

$f := x \to 4 \cdot x \cdot (1 - x);$

$$x \to 4\, x\, (1 - x) \tag{32}$$

$x0 := 0.67;$

$$0.67 \tag{33}$$

$x := x0;$

$$0.67 \tag{34}$$

for i **from** 1 **to** 30 **do** $x := f(x) : \mathrm{psi}(i) := x : print(x);$**od**:

$$0.2956936545 \tag{35}$$

$x0 := 0.67001$

$$0.67001 \tag{36}$$

$x := x0;$

$$0.67001 \tag{37}$$

for i **from** 1 **to** 40 **do** $x := f(x) : \mathrm{psi}(i) := x : print(x);$**od**:

$$0.9733598924 \tag{38}$$

$f1 := x \to 4 \cdot x - 4 \cdot x^2;$

$$x \to 4\, x - 4\, x^2 \tag{39}$$

$x0 := 0.67;$

$$0.67 \tag{40}$$

$x = x0;$

$$0.9733598924 = 0.67 \tag{41}$$

for i **from** 1 **to** 40 **do** $x := f1(x) : \mathrm{psi}(i) := x : print(x)\ ;$**od**:

$$0.112974260 \qquad (42)$$

Numerical methods
Exercise 4

$x := 'x';$

$$x \qquad (43)$$

$\text{phi} := '\text{phi}'; eq := diff(y(x), x) = 2 \cdot x \cdot y(x);$

$$\frac{d}{dx} y(x) = 2\, x\, y(x) \qquad (44)$$

$ic := y(0) = 1;$

$$y(0) = 1 \qquad (45)$$

a) Euler's method with step size h = 0.1

$restart:$

$with(DEtools):$

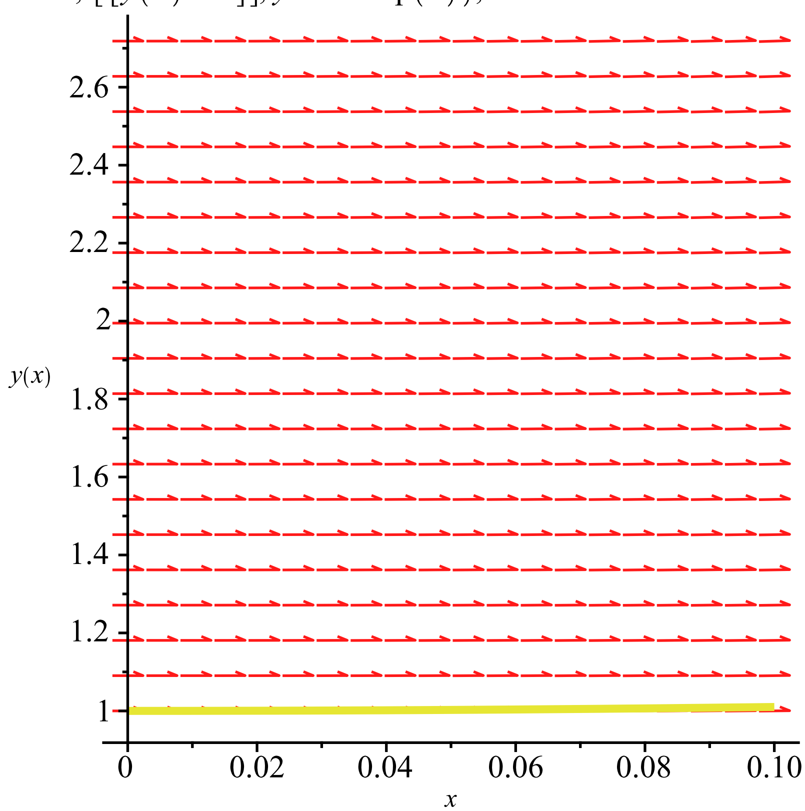
$f := (x, y) \rightarrow 2 \cdot x \cdot y;$

$$(x, y) \rightarrow 2\, x\, y \qquad (46)$$

$dsolve(\{diff(y(x), x) = f(x, y(x)), y(0) = 1\}, y(x)); \text{phi} := unapply(rhs(\%), x);$

$$x \rightarrow e^{x^2} \qquad (47)$$

$DEplot(eq, y(x), x = 0 ...1, [[y(0) = 1]], y = 1 ...\exp(1));$



$h := 0.1;$

$$0.1 \qquad (48)$$

$x := 0; y := 1;$

$$1 \qquad (49)$$

for i **from** 1 **to** 15 **do** $y := y + h \cdot f(x, y) : \text{psi}(i) := y : x := x + h : print(x, y, \text{phi}(x), \text{abs}(y$

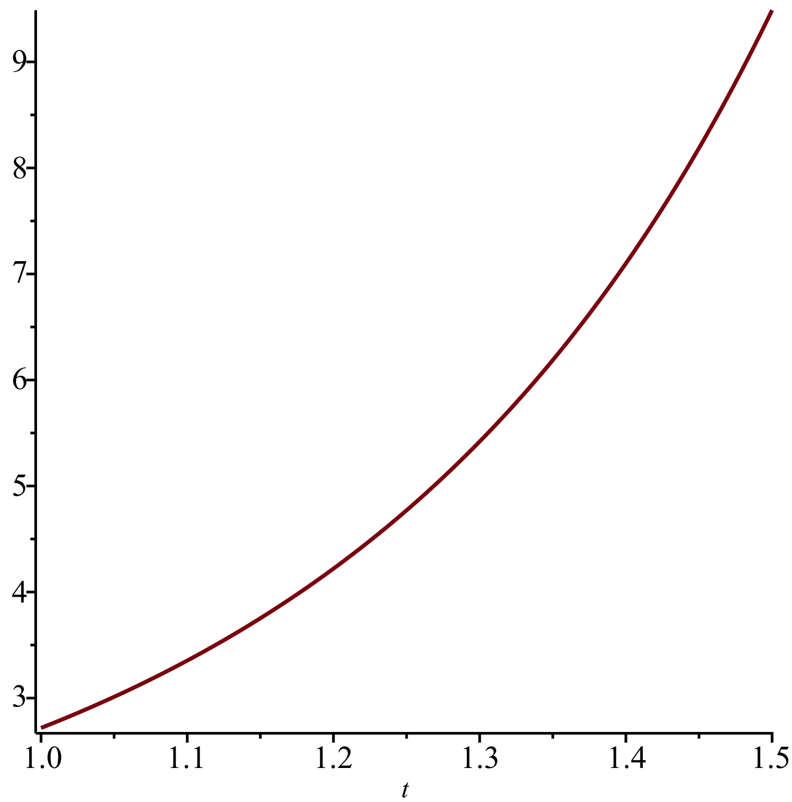
$- \text{phi}(x)));$ **od**:

$$1.5, 6.835368997, 9.487735836, 2.652366839 \tag{50}$$

The approximate value at x=0.5 is
0.5, 1.21440384, 1.284025417, 0.069621577

$$0.5, 1.21440384, 1.284025417, 0.069621577 \tag{51}$$

$points := [[n, \text{psi}(n)]\$n = 1..15] : with(plots) : pointplot(points, symbol = point); plot(\text{phi}(t), t = 1..1.5);$



Applying Euler's improved numerical method
restart :
with(*DEtools*) :
$f := (x, y) \rightarrow 2 \cdot x \cdot y;$

$$(x, y) \rightarrow 2\, x\, y \tag{52}$$

$dsolve(\{diff(y(x), x) = f(x, y(x)), y(0) = 1\}, y(x)); \text{phi} := unapply(rhs(\%), x);$

$$x \rightarrow e^{x^2} \tag{53}$$

$h := 0.1; x := 0; y := 1;$

$$1 \tag{54}$$

for i **from** 1 **to** 15 **do** $y := y + \frac{h}{2} \cdot f(x, y) + \frac{h}{2} \cdot f(x + h, y + h \cdot f(x, y)) : \text{psi}(i) := y : x := x + h :$
$print(x, y, \text{phi}(x), \text{abs}(y - \text{phi}(x)));$**od**:

$$1.5, 9.348762067, 9.487735836, 0.138973769 \tag{55}$$

The approximate value at x=0.5 is 0.5, 1.283472900, 1.284025417, 0.000552517