$with(Student[LinearAlgebra]):$
$with(LinearAlgebra);$

[&x, Add, Adjoint, BackwardSubstitute, BandMatrix, Basis, BezoutMatrix, BidiagonalForm, BilinearForm, CARE, CharacteristicMatrix, CharacteristicPolynomial, Column, ColumnDimension, ColumnOperation, ColumnSpace, CompanionMatrix, CompressedSparseForm, ConditionNumber, ConstantMatrix, ConstantVector, Copy, CreatePermutation, CrossProduct, DARE, DeleteColumn, DeleteRow, Determinant, Diagonal, DiagonalMatrix, Dimension, Dimensions, DotProduct, EigenConditionNumbers, Eigenvalues, Eigenvectors, Equal, ForwardSubstitute, FrobeniusForm, FromCompressedSparseForm, FromSplitForm, GaussianElimination, GenerateEquations, GenerateMatrix, Generic, GetResultDataType, GetResultShape, GivensRotationMatrix, GramSchmidt, HankelMatrix, HermiteForm, HermitianTranspose, HessenbergForm, HilbertMatrix, HouseholderMatrix, IdentityMatrix, IntersectionBasis, IsDefinite, IsOrthogonal, IsSimilar, IsUnitary, JordanBlockMatrix, JordanForm, KroneckerProduct, LA_Main, LUDecomposition, LeastSquares, LinearSolve, LyapunovSolve, Map, Map2, MatrixAdd, MatrixExponential, MatrixFunction, MatrixInverse, MatrixMatrixMultiply, MatrixNorm, MatrixPower, MatrixScalarMultiply, MatrixVectorMultiply, MinimalPolynomial, Minor, Modular, Multiply, NoUserValue, Norm, Normalize, NullSpace, OuterProductMatrix, Permanent, Pivot, PopovForm, ProjectionMatrix, QRDecomposition, RandomMatrix, RandomVector, Rank, RationalCanonicalForm, ReducedRowEchelonForm, Row, RowDimension, RowOperation, RowSpace, ScalarMatrix, ScalarMultiply, ScalarVector, SchurForm, SingularValues, SmithForm, SplitForm, StronglyConnectedBlocks, SubMatrix, SubVector, SumBasis, SylvesterMatrix, SylvesterSolve, ToeplitzMatrix, Trace, Transpose, TridiagonalForm, UnitVector, VandermondeMatrix, VectorAdd, VectorAngle, VectorMatrixMultiply, VectorNorm, VectorScalarMultiply, ZeroMatrix, ZeroVector, Zip] **(1)**

$with(linalg):$
$A := Matrix([[0,-2,0],[1,-2,0],[0,0,-2]]);$

$$\begin{bmatrix} 0 & -2 & 0 \\ 1 & -2 & 0 \\ 0 & 0 & -2 \end{bmatrix} \tag{2}$$

$det(A);$

$$-4 \tag{3}$$

$A^{(-1)};$

$$\begin{bmatrix} -1 & 1 & 0 \\ -\frac{1}{2} & 0 & 0 \\ 0 & 0 & -\frac{1}{2} \end{bmatrix} \tag{4}$$

CharacteristicPolynomial(A, p);

$$p^3 + 4p^2 + 6p + 4 \tag{5}$$

Eigenvalues(A);

$$\begin{bmatrix} -2 \\ -1 - \mathrm{I} \\ -1 + \mathrm{I} \end{bmatrix} \tag{6}$$

$x, P :=$ *Eigenvectors*(A);

$$\begin{bmatrix} -2 \\ -1 + \mathrm{I} \\ -1 - \mathrm{I} \end{bmatrix}, \begin{bmatrix} 0 & 1 + \mathrm{I} & 1 - \mathrm{I} \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix} \tag{7}$$

#Check that (0,0,1) is an eigenvector corresponding to the eigenvalue -2

$u1 := \langle 0, 0, 1 \rangle$;

$$\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \tag{8}$$

$A1 := A.u1$;

$$\begin{bmatrix} 0 \\ 0 \\ -2 \end{bmatrix} \tag{9}$$

#Check that (1+i,1,0) is an eigenvector corresponding to the eigenvalue -1+i;

$u2 := \langle 1 + i, 1, 0 \rangle$;

$$\begin{bmatrix} 1 + i \\ 1 \\ 0 \end{bmatrix} \tag{10}$$

$A2 := A.u2$;

$$\begin{bmatrix} -2 \\ -1 + i \\ 0 \end{bmatrix} \tag{11}$$

#Check that (1-i,1,0) is an eigenvector corresponding to the eigenvalue(1-i,1,0)

$u3 := \langle 1 - i, 1, 0 \rangle$

$$\begin{bmatrix} 1 - i \\ 1 \\ 0 \end{bmatrix} \tag{12}$$

$A3 := A.u3$;

$$\begin{bmatrix} -2 \\ -1-i \\ 0 \end{bmatrix} \quad (13)$$

$P := Matrix([\langle 0, 0, 1\rangle, \langle 1+I, 1, 0\rangle, \langle 1-I, 1, 0\rangle]);$

$$\begin{bmatrix} 0 & 1+\mathrm{I} & 1-\mathrm{I} \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix} \quad (14)$$

$J := DiagonalMatrix([-2, -1+I, -1-I]);$

$$\begin{bmatrix} -2 & 0 & 0 \\ 0 & -1+\mathrm{I} & 0 \\ 0 & 0 & -1-\mathrm{I} \end{bmatrix} \quad (15)$$

$P.J.P^{-1} = A;$

$$\begin{bmatrix} 0 & -2 & 0 \\ 1 & -2 & 0 \\ 0 & 0 & -2 \end{bmatrix} = \begin{bmatrix} 0 & -2 & 0 \\ 1 & -2 & 0 \\ 0 & 0 & -2 \end{bmatrix} \quad (16)$$

#compute e^{tJ} **and** e^{tA}

$expJ := MatrixExponential(t \cdot J);$

$$\begin{bmatrix} \mathrm{e}^{-2t} & 0 & 0 \\ 0 & \mathrm{e}^{-t}\cos(t) + \mathrm{I}\,\mathrm{e}^{-t}\sin(t) & 0 \\ 0 & 0 & \mathrm{e}^{-t}\cos(t) - \mathrm{I}\,\mathrm{e}^{-t}\sin(t) \end{bmatrix} \quad (17)$$

$expA := MatrixExponential(t \cdot A);$

$$\begin{bmatrix} \mathrm{e}^{-t}\sin(t) + \mathrm{e}^{-t}\cos(t) & -2\,\mathrm{e}^{-t}\sin(t) & 0 \\ \mathrm{e}^{-t}\sin(t) & \mathrm{e}^{-t}\cos(t) - \mathrm{e}^{-t}\sin(t) & 0 \\ 0 & 0 & \mathrm{e}^{-2t} \end{bmatrix} \quad (18)$$

#Compute the limit as t->inf for each entry of e^{tA}

$limit(expA[1, 1], t = \text{infinity});$

$$0 \quad (19)$$

$limit(expA[1, 2], t = \text{infinity});$

$$0 \quad (20)$$

$limit(expA[1, 3], t = \text{infinity});$

$$0 \quad (21)$$

$limit(expA[2, 1], t = \text{infinity});$

$$0 \quad (22)$$

$limit(expA[2, 2], t = \text{infinity});$

$$0 \quad (23)$$

$limit(expA[2, 3], t = \text{infinity});$

$$0 \tag{24}$$

$limit(expA[3, 1], t = \text{infinity});$

$$0 \tag{25}$$

$limit(expA[3, 2], t = \text{infinity});$

$$0 \tag{26}$$

$limit(expA[3, 3], t = \text{infinity});$

$$0 \tag{27}$$

#TRUE: Each solution of the differential system X' = AX satisfied lim as t->inf X(t) = 03

#EX.2

$P := Matrix([\langle 2, 5, 3, 1\rangle, \langle 4, 8, 3, 1\rangle, \langle 4, 7, 9, 8\rangle, \langle 0, 4, 5, 6\rangle]);$

$$\begin{bmatrix} 2 & 4 & 4 & 0 \\ 5 & 8 & 7 & 4 \\ 3 & 3 & 9 & 5 \\ 1 & 1 & 8 & 6 \end{bmatrix} \tag{28}$$

$P^{-1};$

$$\begin{bmatrix} -\frac{27}{50} & \frac{2}{75} & \frac{68}{75} & -\frac{58}{75} \\ \frac{13}{50} & \frac{4}{25} & -\frac{14}{25} & \frac{9}{25} \\ \frac{13}{50} & -\frac{13}{75} & \frac{8}{75} & \frac{2}{75} \\ -\frac{3}{10} & \frac{1}{5} & -\frac{1}{5} & \frac{1}{5} \end{bmatrix} \tag{29}$$

$J := DiagonalMatrix([2, 2, -1, 0]);$

$$\begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{30}$$

$A := P.J.P^{-1};$

$$\begin{bmatrix} -\frac{28}{25} & \frac{52}{25} & -\frac{32}{25} & -\frac{8}{25} \\ -\frac{153}{50} & \frac{101}{25} & -\frac{16}{25} & -\frac{54}{25} \\ -\frac{201}{50} & \frac{67}{25} & \frac{28}{25} & -\frac{68}{25} \\ -\frac{66}{25} & \frac{44}{25} & -\frac{4}{25} & -\frac{26}{25} \end{bmatrix} \tag{31}$$