Exercise 1

$eq := diff(x(t), t\$4) - x(t) = 0;$

$$\frac{d^4}{dt^4} x(t) - x(t) = 0 \quad (1)$$

$eval(eq, x(t) = \sin(t));$

$$0 = 0 \quad (2)$$

#sin(t) is a solution

$eval(eq, x(t) = \cos(t));$

$$0 = 0 \quad (3)$$

#cos(t) is a solution

$eval(eq, x(t) = \sinh(t));$

$$0 = 0 \quad (4)$$

#sinh(t) is a solution

$eval(eq, x(t) = \cosh(t));$

$$0 = 0 \quad (5)$$

#cosh(t) is a solution

$eq1 := diff(x(t), t) + t \cdot x(t) = 0;$

$$\frac{d}{dt} x(t) + t\, x(t) = 0 \quad (6)$$

$dsolve(eq1, x(t));$

$$x(t) = \_C1\, e^{-\frac{1}{2} t^2} \quad (7)$$

$eq2 := diff(x(t), t\$2) + x(t) = 0;$

$$\frac{d^2}{dt^2} x(t) + x(t) = 0 \quad (8)$$

$dsolve(eq2, x(t));$

$$x(t) = \_C1 \sin(t) + \_C2 \cos(t) \quad (9)$$

$eq3 := 4 \cdot diff(x(t), t\$2) + 8 \cdot diff(x(t), t) + 5 \cdot x(t) = 0;$

$$4 \left(\frac{d^2}{dt^2} x(t) \right) + 8 \left(\frac{d}{dt} x(t) \right) + 5\, x(t) = 0 \quad (10)$$

$dsolve(eq3, x(t));$

$$x(t) = \_C1\, e^{-t} \sin\left(\frac{1}{2} t \right) + \_C2\, e^{-t} \cos\left(\frac{1}{2} t \right) \quad (11)$$

$eq4 := diff(x(t), t\$2) - 3 \cdot diff(x(t), t) + 2 \cdot x(t) = 0;$

$$\frac{d^2}{dt^2} x(t) - 3 \left(\frac{d}{dt} x(t) \right) + 2\, x(t) = 0 \quad (12)$$

$dsolve(eq4, x(t));$

$$x(t) = \_C1\, e^{t} + \_C2\, e^{2\,t} \tag{13}$$

Exercise 6

$eq := diff(x(t), t\$2) + x(t) = 0;$

$$\frac{d^2}{dt^2} x(t) + x(t) = 0 \tag{14}$$

$ic := x\left(\frac{Pi}{2}\right) = 1, D(x)\left(\frac{Pi}{2}\right) = -2;$

$$x\left(\frac{1}{2}\,\pi\right) = 1, D(x)\left(\frac{1}{2}\,\pi\right) = -2 \tag{15}$$

$sol := dsolve(\{eq, ic\}, x(t));$

$$x(t) = \sin(t) + 2\cos(t) \tag{16}$$

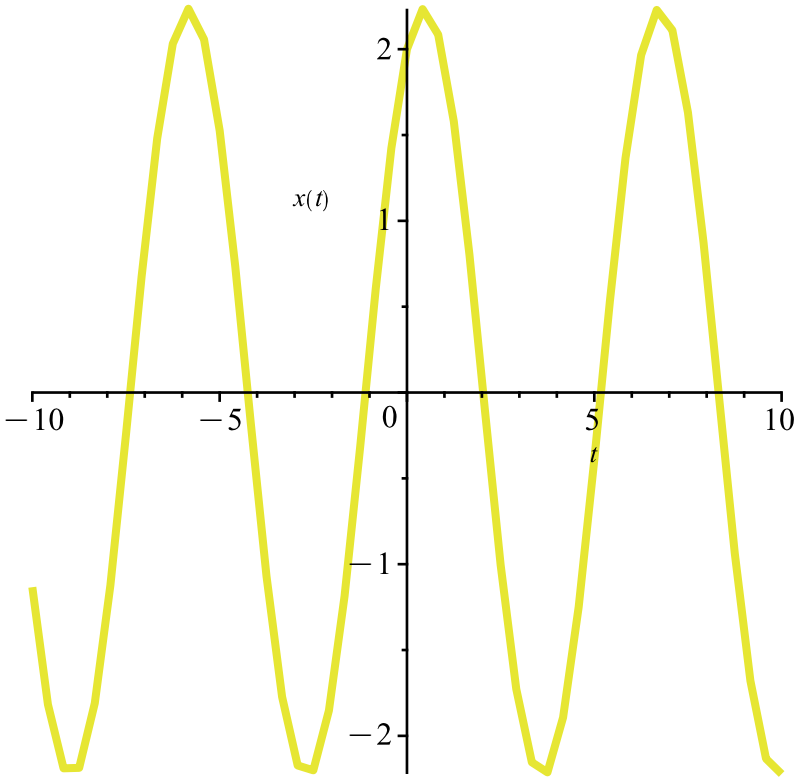
$expand(rhs(sol));$

$$\sin(t) + 2\cos(t) \tag{17}$$

$expand\left(\mathrm{sqrt}(5)\cdot\cos\left(t - \arctan\left(\frac{1}{2}\right)\right)\right);$

$$\sin(t) + 2\cos(t) \tag{18}$$

$with(DEtools):$
$DEplot(eq, x(t), t = -10..10, [[ic]]);$



$limit(rhs(sol), t = infinity);$

$$-3..3 \tag{19}$$

$limit(rhs(sol), t = -infinity);$

$$-3..3 \tag{20}$$

$solve(rhs(sol), t);$

$$-\arctan(2) \tag{21}$$

Exercise 7

$eq := 4 \cdot diff(x(t), t\$2) + 8 \cdot diff(x(t), t) + 5 \cdot x(t) = 0;$

$$4\left(\frac{d^2}{dt^2} x(t)\right) + 8\left(\frac{d}{dt} x(t)\right) + 5\,x(t) = 0 \tag{22}$$

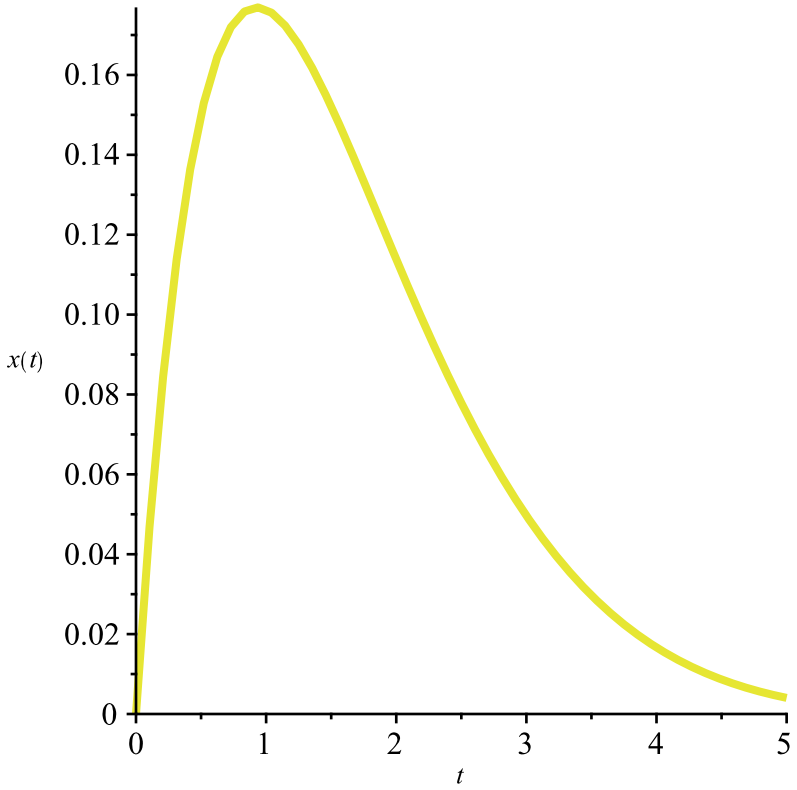
$ic := x(0) = 0, D(x)(0) = 0.5;$

$$x(0) = 0, D(x)(0) = 0.5 \tag{23}$$

$sol := dsolve(\{eq, ic\}, x(t));$

$$x(t) = e^{-t} \sin\left(\frac{1}{2} t\right) \tag{24}$$

$DEplot(eq, x(t), t = 0..5, [[ic]]);$



$limit(rhs(sol), t = infinity);$

$$0 \tag{25}$$

$limit(rhs(sol), t = -infinity);$

$$undefined \tag{26}$$

$solve(rhs(sol), t);$

$$0 \tag{27}$$

Exercise 8

$eq := diff(x(t), t\$2) - 3 \cdot diff(x(t), t) + 2 \cdot x(t) = 0;$

$$\frac{d^2}{dt^2} x(t) - 3\left(\frac{d}{dt} x(t)\right) + 2\,x(t) = 0 \tag{28}$$

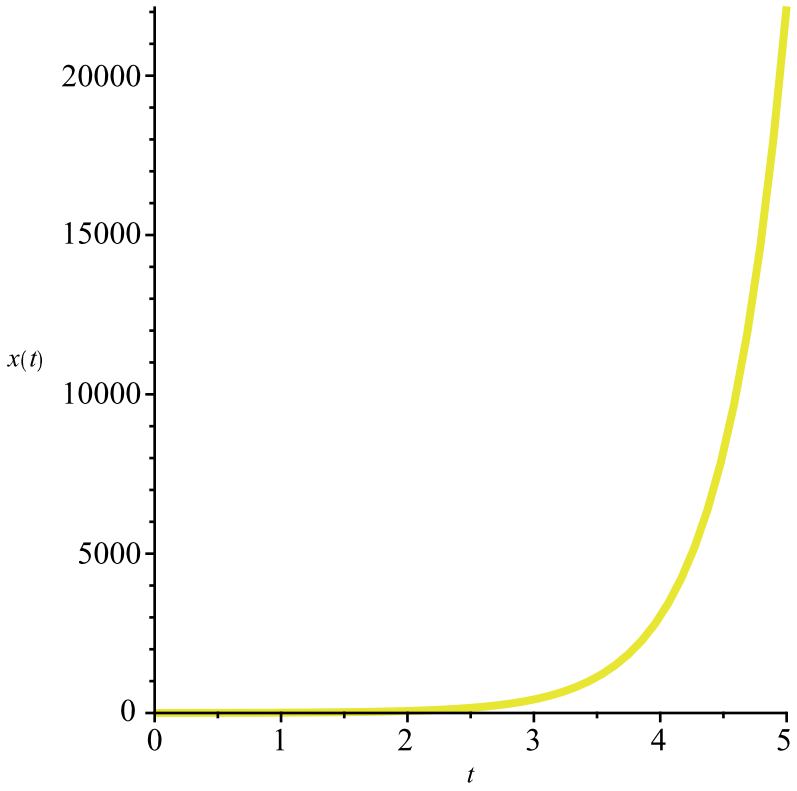
$ic := x(0) = 2, D(x)(0) = 3;$

$$x(0) = 2, D(x)(0) = 3 \tag{29}$$

```
sol := dsolve( {eq, ic}, x(t) );
```

$$x(t) = e^{t} + e^{2\,t} \tag{30}$$

```
DEplot(eq, x(t), t = 0 ..5, [[ic]]);
```



Exercise 9

```
infolevel[dsolve] := 3;
```

$$3 \tag{31}$$

```
eq := diff(x(t), t$2) + x(t)·5 = 0;
```

$$\frac{d^2}{dt^2} x(t) + 5\,x(t) = 0 \tag{32}$$

```
dsolve(eq, x(t) );
```

$$x(t) = \_C1 \sin\left(\sqrt{5}\ t\right) + \_C2 \cos\left(\sqrt{5}\ t\right) \tag{33}$$

```
eq := diff(x(t), t$2 ) + x(t)·t = 0;
```

$$\frac{d^2}{dt^2} x(t) + t\,x(t) = 0 \tag{34}$$

```
dsolve(eq, x(t) );
```

$$x(t) = \_C1\ \mathrm{AiryAi}(-t) + \_C2\ \mathrm{AiryBi}(-t) \tag{35}$$

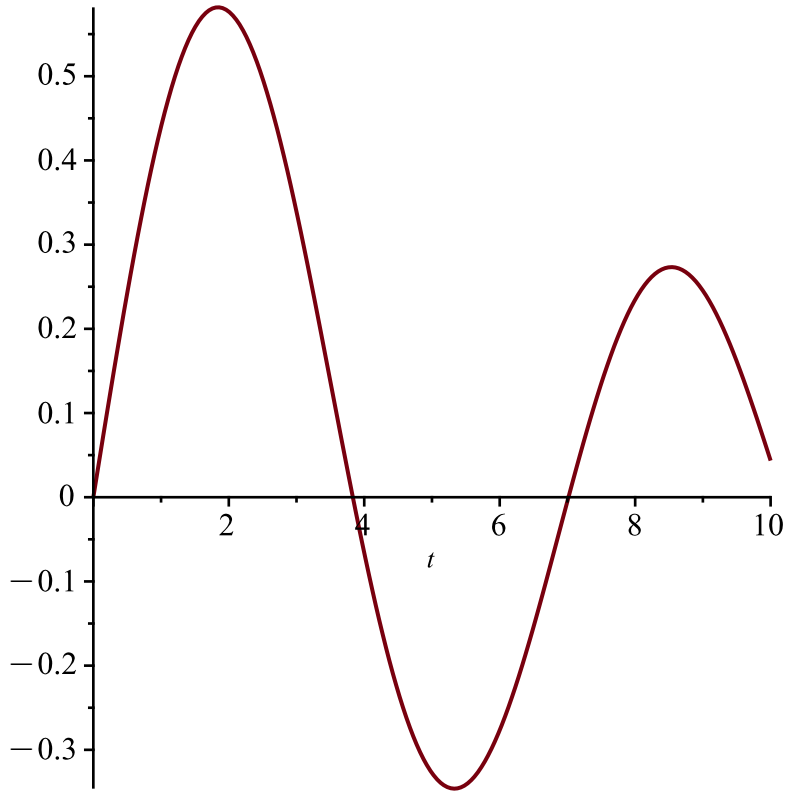
```
eq := diff(x(t), t$2) + t^5·x(t);
```

$$\frac{d^2}{dt^2} x(t) + t^5\,x(t) \tag{36}$$

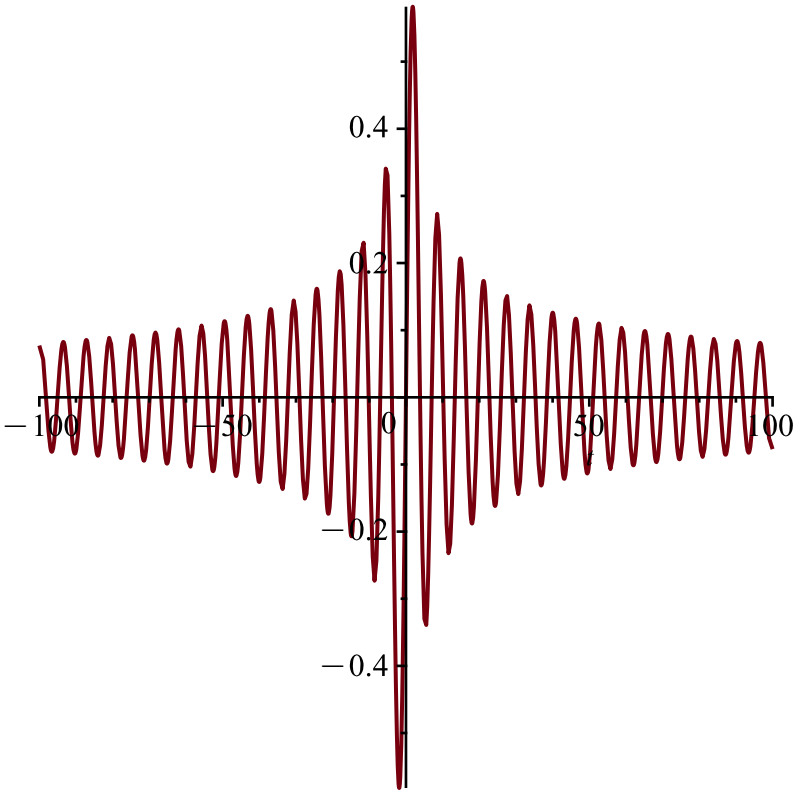
```
dsolve(eq, x(t) );
```

$$x(t) = \_C1 \sqrt{t}\ \mathrm{BesselJ}\left(\frac{1}{7}, \frac{2}{7}\ t^{7/2}\right) + \_C2 \sqrt{t}\ \mathrm{BesselY}\left(\frac{1}{7}, \frac{2}{7}\ t^{7/2}\right) \tag{37}$$

$plot(\text{BesselJ}(1, t), t = 0..10)$



$plot(\text{BesselJ}(1, t), t = -100..100)$



$ic := x(0) = 0, \text{D}(x)(0) = 0;$

$$x(0) = 0, \text{D}(x)(0) = 0 \tag{38}$$

$eq := diff(x(t), t\$2) + 5 \cdot x(t) = 0;$

$$\frac{d^2}{dt^2}x(t) + 5\,x(t) = 0 \tag{39}$$

```
dsolve({eq, ic}, x(t));
```

$$x(t) = 0 \tag{40}$$

```
eq := diff(x(t), t$2) + t·x(t) = 0;
```

$$\frac{d^2}{dt^2}x(t) + t\,x(t) = 0 \tag{41}$$

```
dsolve({eq, ic}, x(t));
```

$$x(t) = 0 \tag{42}$$

```
eq := diff(x(t), t$2) + t^5·x(t) = 0;
```

$$\frac{d^2}{dt^2}x(t) + t^5\,x(t) = 0 \tag{43}$$

```
dsolve({eq, ic}, x(t));
```

$$x(t) = \begin{cases} \_C1\,\sqrt{t}\ \mathrm{BesselJ}\left(\frac{1}{7}, \frac{2}{7}\,t^{7/2}\right) & t < 0 \\ 0 & t = 0 \\ \_C1\,\sqrt{t}\ \mathrm{BesselJ}\left(\frac{1}{7}, \frac{2}{7}\,t^{7/2}\right) & 0 < t \end{cases} \tag{44}$$

Exercise 10

```
eq := diff(x(t), t$2) + x(t) = 0;
```

$$\frac{d^2}{dt^2}x(t) + x(t) = 0 \tag{45}$$

```
bc := x(0) = 0, x(Pi) = 0;
```

$$x(0) = 0, x(\pi) = 0 \tag{46}$$

```
dsolve({eq, bc}, x(t));
```

$$x(t) = \_C1\,\sin(t) \tag{47}$$

Exercise 11

```
bc := x(0) = 0, x(1) = 0;
```

$$x(0) = 0, x(1) = 0 \tag{48}$$

```
dsolve({eq, bc}, x(t));
```

$$x(t) = 0 \tag{49}$$

Exercise 12

```
eq := diff(x(t), t$2 ) + x(t) = 1;
```

$$\frac{d^2}{dt^2}x(t) + x(t) = 1 \tag{50}$$

```
bc := x(0) = 0, x(Pi) = 0;
```

$$x(0) = 0, x(\pi) = 0 \tag{51}$$

```
dsolve({eq, bc}, x(t));
Methods for second order ODEs:
```

```
--- Trying classification methods ---
trying a quadrature
trying high order exact linear fully integrable
trying differential order: 2; linear nonhomogeneous with symmetry [0,
1]
trying a double symmetry of the form [xi=0, eta=F(x)]
-> Try solving first the homogeneous part of the ODE
   checking if the LODE has constant coefficients
   <- constant coefficients successful
   -> Determining now a particular solution to the non-homogeneous ODE
      trying a rational particular solution
      <- rational particular solution successful
<- solving first the homogeneous part of the ODE successful
trying a rational particular solution
<- rational particular solution successful
```

Exercise 13

$eq := diff(x(t), t) + x(t) = 15;$

$$\frac{d}{dt} x(t) + x(t) = 15 \tag{52}$$

$dsolve(eq, x(t));$

$$x(t) = 15 + e^{-t} \_C1 \tag{53}$$

$\#xp=15$

Exercise 14

$eq := diff(x(t), t) + x(t) = 2 \cdot \exp(1)^{t} - 7 \cdot \exp(1)^{(-3 \cdot t)};$

$$\frac{d}{dt} x(t) + x(t) = 2\ (e)^{t} - 7\ (e)^{-3\,t} \tag{54}$$

$\mathrm{dsolve}(\mathrm{eq}, \mathrm{x(t)});$

$$x(t) = e^{t} + \frac{7}{2} e^{-3\,t} + e^{-t} \_C1 \tag{55}$$

$\#xp= e^{t} + \frac{7}{2} \cdot e^{(-3 \cdot t)}$

Exercise 15

$eq := diff(x(t), t) + x(t) = -t^2 + 3 \cdot t - 7;$

$$\frac{d}{dt} x(t) + x(t) = -t^2 + 3\,t - 7 \tag{56}$$

$dsolve(eq, x(t));$

$$x(t) = -t^2 + 5\,t - 12 + e^{-t} \_C1 \tag{57}$$

```
#xp:= -t^2+5*t-12
```

Exercise 16

$eq := diff(x(t), t) + x(t) = \sin(t) + 3 \cdot \cos(t);$

$$\frac{d}{dt} x(t) + x(t) = \sin(t) + 3\cos(t) \tag{58}$$

$dsolve(eq, x(t))$;

$$x(t) = 2\sin(t) + \cos(t) + e^{-t}\_C1 \quad \textbf{(59)}$$

```
xp:=2*sin(t)+cos(t)
Exercise 17
```

$dsolve(diff(x(t), t) + x(t) = \sin(t), x(t))$;

$$x(t) = -\frac{1}{2}\cos(t) + \frac{1}{2}\sin(t) + e^{-t}\_C1 \quad \textbf{(60)}$$

```
xp:=-1/2*cos(t)+1/2*sin(t)
Exercise 18
```

$dsolve(diff(x(t), t) + x(t) = 3\cdot\cos(t), x(t))$;

$$x(t) = \frac{3}{2}\cos(t) + \frac{3}{2}\sin(t) + e^{-t}\_C1 \quad \textbf{(61)}$$

$\#xp = 3:2\cdot cos(t) + 3:2\cdot sin(t)$