Laboratory 1

0. Read carefully and try the commands written in the file Lab1-Sage-tutorial.pdf or in the first 3 pages of the file Lab-Maple8-tutorial.pdf

1. Evaluate the number using floating point arithmetic: $\frac{1}{2}$ (1./2) ; e=exp(1) ; $\sqrt{3}$ (sqrt(3.));

π (evalf(Pi) in Maple, numerical_approx(pi) in Sage) ; $\sin(0.1)$

2. Assign the following expression to a variable and then expand it:

a) $(x^2+2\cdot x-1)^3\cdot(x^2-2)$ b*) $(x+n)^5$. Unassign the used variables (a:='a' in Maple, a=var('a') in Sage).

3. Factorize: x^8-1

4. Add the following rational expressions by applying the factor command: $\frac{2\cdot x^2}{x^3-1}+\frac{3\cdot x}{x^2-1}$

5. Simplify the trigonometric expression (In Maple using simplify(expression, trig) and in Sage using expr.trig_simplify()): $\sin(x)^2+\cos(x)^2$

6. Evaluate using both subs and eval the expression $e^x+\ln(x)$ in x=1.

7. Solve: a) the equation $x^2-4\cdot x+3=0$ where x is the unknown;

b) the equation $x^2\cdot y+2\cdot y-x=0$ where x is a parameter and y is the unknown;

c*) the equation $x^2\cdot y+2\cdot y-x=0$ where y is a parameter and x is the unknown;

d) the equation $x-\cos(x)=0$ where x is the unknown; (for this equation one can find only approximate roots. So, use fsolve in Maple, and find_root in Sage. Hint: it has 1 real root, and it belongs to the interval [-1,1])

e*) the equation $x^5-3\cdot x^3-1=0$ where x is the unknown; (for this equation one can find only approximate roots. So, use fsolve in Maple, and find_root in Sage. Hint: it has 3 real roots, all in the interval [-2,2])

f) the system of two equations $4x+3y=10,\ 3x-y=1$ where x and y are the unknowns.

8* (only in Maple). Assign to a variable f the function (not the expression) $f: R\to R,\ f(x)=e^x-\sin(x)$. Evaluate $f(0), f(-1), f'(0), f'(1)$. Calculate the first and second order derivatives of f (using both D and diff) and a primitive of f. Evaluate $\int_{-1}^{1} f(x)\,dx$. Unassign f. Do not forget that for f'(0) (the derivative of a function in a given point you just have to write D(f)(0)).

9. Assign to a variable g the expression $e^x-\sin(x)$. Evaluate this expression in $x=0$. Compute its first order derivative and then evaluate it in $x=0$. Find a primitive. Evaluate $\int_{-1}^{1} g\,dx$. Assign to a variable f the second order derivative of g and evaluate it in $x=0$.

10. Find $\lim_{x\to 0}\frac{\sin(x)}{x}$ and $\lim_{x\to\pi}\frac{\cos(x)+1}{x-\pi}$.

11. First plot the graph of $f(x)=\sin(x)$ by hand on paper. Then plot it using Sage in the interval $[0, 2\cdot\pi]$, or using Maple, in each of the intervals: $[0, 2\cdot\pi]$, $[-4\pi, 4\cdot\pi]$, $[-100, 100]$, $(-\infty, \infty)$.

12. First plot the graph of $f(x)=\frac{1}{x}$ by hand on paper. Then plot it using Maple or Sage in the interval [-1,1]. In Sage use the option detect_poles=True and specify a range for the values of this function.

13. Plot the planar curve of parametric equations $x=2-t^2$, $y=t-t^3$, $t\in[-2, 2]$.

14. First plot by hand on paper the planar curve of parametric equations $x=\cos(t)$, $y=\sin(t)$ in each of the intervals: $\left[0, \frac{\pi}{6}\right]$, $\left[0, \frac{\pi}{3}\right]$, $\left[0, \frac{\pi}{2}\right]$, $[0, \pi]$, $\left[0, \frac{3\cdot\pi}{2}\right]$, $[0, 2\cdot\pi]$, $[0, 4\cdot\pi]$.
Then do the same using Maple or Sage. What do you notice?

15*. Plot the planar curves of parametric equations a) $x=2\cdot\cos\left(\frac{t}{3}\right)$, $y=2\cdot\sin\left(\frac{t}{3}\right)$
b) $x=\cos(4\cdot t)$, $y=\sin(4\cdot t)$ in different intervals at your choice. What do you notice?

16. Write by hand on paper the implicit equations of a circle centered in (9,10), an ellipse and a parabola. Then plot using Maple or Sage at leats a circle, an ellipse and a parabola.

17. Plot the planar curves of implicit equations a) $x^2-2\cdot x\cdot y-y^2=1$ b*)
$y^3+y^2-5\cdot y-x^2=-4$. You have to choose properly a rectangle where to see the curve.

18. First draw by hand on paper the graph of the function $H(x, y)=x^2+y^2$. Then plot it in 3d using Maple or Sage (using the command plot3d). For the variable (x,y) choose a rectangle centered at (0,0). What remarkable planar curves are the level curves of H?