

$$eq := diff(x(t), t^2) + t^2 \cdot x(t) = 0;$$

$$\frac{d^2}{dt^2} x(t) + t^2 x(t) = 0 \quad (1)$$

$$dsolve(eq, x(t));$$

$$x(t) = _C1 \sqrt{t} \text{BesselJ}\left(\frac{1}{4}, \frac{1}{2} t^2\right) + _C2 \sqrt{t} \text{BesselY}\left(\frac{1}{4}, \frac{1}{2} t^2\right) \quad (2)$$

$$infolevel[dsolve] := 3$$

$$3 \quad (3)$$

$$dsolve(eq, x(t));$$

$$x(t) = _C1 \sqrt{t} \text{BesselJ}\left(\frac{1}{4}, \frac{1}{2} t^2\right) + _C2 \sqrt{t} \text{BesselY}\left(\frac{1}{4}, \frac{1}{2} t^2\right) \quad (4)$$

$$ic1 := x(0) = 0;$$

$$x(0) = 0 \quad (5)$$

$$dsolve(\{eq, ic1\}, x(t));$$

$$x(t) = \begin{cases} _C1 \sqrt{t} \text{BesselJ}\left(\frac{1}{4}, \frac{1}{2} t^2\right) & t < 0 \\ 0 & t = 0 \\ _C1 \sqrt{t} \text{BesselJ}\left(\frac{1}{4}, \frac{1}{2} t^2\right) & 0 < t \end{cases} \quad (6)$$

$$ic1 := 'ic1'$$

$$ic1 \quad (7)$$

$$ic1 := x(0) = 0, D(x)(0) = 0;$$

$$false \quad (8)$$

$$ic1 := x(0) = 0, D(x)(0) = 0;$$

$$x(0) = 0, D(x)(0) = 0 \quad (9)$$

$$dsolve(\{eq, ic1\}, x(t));$$

$$x(t) = 0 \quad (10)$$

$$ic1 := 'ic1'$$

$$ic1 \quad (11)$$

$$ic1 := x(0) = 0, D(x)(0) = 0, (D@@2)(x)(0) = 1;$$

$$x(0) = 0, D(x)(0) = 0, D^{(2)}(x)(0) = 1 \quad (12)$$

$$dsolve(\{eq, ic1\}, x(t));$$

Methods for second order ODEs:

--- Trying classification methods ---

trying a quadrature

checking if the LODE has constant coefficients

checking if the LODE is of Euler type

trying a symmetry of the form [xi=0, eta=F(x)]

checking if the LODE is missing 'y'

-> Trying a Liouvillian solution using Kovacic's algorithm

<- No Liouvillian solutions exists

```

-> Trying a solution in terms of special functions:
-> Bessel
<- Bessel successful
<- special function solution successful
expand(x^i);

```

$$x^i \quad (13)$$

```
expand(e^3*i);
```

$$(e^i)^3 \quad (14)$$

```
simplify(e^3*i)
```

$$e^{3i} \quad (15)$$

```
expand(e^3*i);
```

$$(e^i)^3 \quad (16)$$

```
eq := 'eq';
```

$$eq \quad (17)$$

```
eq := diff(x(t), t) - 2*x(t) = 0;
```

$$\frac{d}{dt} x(t) - 2 x(t) = 0 \quad (18)$$

```
dsolve(eq, x(t));
```

$$x(t) = _C1 e^{2t} \quad (19)$$

```
eq := 'eq'
```

$$eq \quad (20)$$

```
eq := diff(x(t), t) - x(t) = e^(t-1);
```

Error, invalid power

$$eq := diff(x(t), t) - x(t) = e^{t-1};$$

```
eq := diff(x(t), t) - x(t) = e^(t-1);
```

$$\frac{d}{dt} x(t) - x(t) = e^{t-1} \quad (21)$$

```
dsolve(eq, x(t));
```

$$x(t) = \frac{e^{t-1}}{\ln(e) - 1} + e^t _C1 \quad (22)$$

```
eq := diff(x(t), t) - x(t) = exp(1)^(t-1);
```

$$\frac{d}{dt} x(t) - x(t) = (e)^{t-1} \quad (23)$$

```
dsolve(eq, x(t));
```

$$x(t) = (e^{-1} t + _C1) e^t \quad (24)$$

```
eq := 'eq';
```

$$eq \quad (25)$$

```
eq := diff(x(t), t$2) - 2*diff(x(t), t) + 2*x(t) = 0;
```

$$\frac{d^2}{dt^2} x(t) - 2 \left(\frac{d}{dt} x(t) \right) + 2 x(t) = 0 \quad (26)$$

$dsolve(eq, x(t));$

$$x(t) = _C1 e^t \sin(t) + _C2 e^t \cos(t) \quad (27)$$

$eq := eq;$

$$eq \quad (28)$$

$eq := diff(x(t), t\$4) + 2 \cdot diff(x(t), t\$3) + diff(x(t), t\$2) = 0;$

Error, invalid input: diff received x(t)+t, which is not valid for its 2nd argument

$eq := diff(x(t), t\$4) + 2 \cdot diff(x(t), t\$3) + diff(x(t), t\$2) = 0;$

$$\frac{d^4}{dt^4} x(t) + 2 \left(\frac{d^3}{dt^3} x(t) \right) + \frac{d^2}{dt^2} x(t) = 0 \quad (29)$$

$dsolve(eq, x(t));$

$$x(t) = _C1 e^{-t} + _C2 e^{-t} t + _C3 + _C4 t \quad (30)$$

$eq := 'eq';$

$$eq \quad (31)$$

$eq := k \cdot (21 - x(t)) = diiff(x(t), t);$

$$k (21 - x(t)) = diiff(x(t), t) \quad (32)$$

$eq := k \cdot (21 - x(t)) = diff(x(t), t);$

$$k (21 - x(t)) = \frac{d}{dt} x(t) \quad (33)$$

$dsolve(eq, x(t));$

$$x(t) = 21 + e^{-kt} _C1 \quad (34)$$

$ic1 := x(0) = \eta;$

$$x(0) = \eta \quad (35)$$

$dsolve(\{eq, ic1\}, x(t));$

$$x(t) = 21 + e^{-kt} (\eta - 21) \quad (36)$$

$eq := diff(x(t), t\$2) - x(t) = t \cdot e;$

$$\frac{d^2}{dt^2} x(t) - x(t) = t e \quad (37)$$

$dsolve(eq, x(t));$

$$x(t) = e^t _C2 + _C1 e^{-t} - t e \quad (38)$$

$ic1 := x(0) = 0. D(x)(0) = 0;$

$$false \quad (39)$$

$dsolve(\{ec, ic1\}, x(t));$

Error, (in dsolve) not a system with respect to the unknowns [x(t)]

$dsolve(\{eq, ic1\}, x(t));$

Error, (in dsolve) found the following equations not depending on the unknowns of the input system: {false}

$ic1 := x(0) = 0, D(x)(0) = 0;$

$$x(0) = 0, D(x)(0) = 0 \quad (40)$$

$dsolve(\{eq, ic1\}, x(t));$

$$x(t) = \frac{1}{2} e^t e - \frac{1}{2} e e^{-t} - t e \quad (41)$$

$$eq := \text{diff}(x(t), t\$2) - x(t) = t \cdot (\exp)^{(-2 \cdot t)};$$

$$\frac{d^2}{dt^2} x(t) - x(t) = t \exp^{-2t} \quad (42)$$

$$eq := \text{diff}(x(t), t\$2) - x(t) = t \cdot \exp(1)^{(-2 \cdot t)}$$

$$\frac{d^2}{dt^2} x(t) - x(t) = t (e)^{-2t} \quad (43)$$

$$\text{dsolve}(\{eq, ic1\}, x(t));$$

Error, (in dsolve) found the following equations not depending on the unknowns of the input system: {ic1}

$$\text{dsolve}(eq, x(t));$$

$$x(t) = e^t _C2 + e^{-t} _C1 + \frac{1}{9} (3t + 4) e^{-2t} \quad (44)$$

$$\text{dsolve}(\{eq, ic1\}, x(t));$$

Error, (in dsolve) found the following equations not depending on the unknowns of the input system: {ic1}

$$eq := \text{diff}(x(t), t\$2) - x(t) = t \cdot e^{(-2 \cdot t)}$$

$$\frac{d^2}{dt^2} x(t) - x(t) = t e^{-2t} \quad (45)$$

$$\text{dsolve}(\{eq, ic1\}, x(t));$$

Error, (in dsolve) found the following equations not depending on the unknowns of the input system: {ic1}

$$eq := \text{diff}(x(t), t\$2) - 2 \cdot \text{diff}(x(t), t) + x(t) = 0;$$

$$\frac{d^2}{dt^2} x(t) - 2 \left(\frac{d}{dt} x(t) \right) + x(t) = 0 \quad (46)$$

$$\text{dsolve}(eq, x(t));$$

$$x(t) = _C1 e^t + _C2 e^t t \quad (47)$$

$$eq := \text{diff}(x(t), t\$2) - 2 \cdot \text{diff}(x(t), t) + x(t) = \cos(2 \cdot t);$$

$$\frac{d^2}{dt^2} x(t) - 2 \left(\frac{d}{dt} x(t) \right) + x(t) = \cos(2t) \quad (48)$$

$$\text{dsolve}(eq, x(t));$$

$$x(t) = e^t _C2 + e^t t _C1 - \frac{3}{25} \cos(2t) - \frac{4}{25} \sin(2t) \quad (49)$$

$$eq := \text{diff}(x(t), t\$2) - 2 \cdot \text{diff}(x(t), t) + x(t) = 3 \cdot e^{(2 \cdot t)} - 5 \cdot e^{(-2 \cdot t)};$$

$$\frac{d^2}{dt^2} x(t) - 2 \left(\frac{d}{dt} x(t) \right) + x(t) = 3 e^{2t} - 5 e^{-2t} \quad (50)$$

$$\text{dsolve}(eq, x(t));$$

$$x(t) = e^t_C2 + e^t t_C1 + \frac{-20 \left(\ln(e) - \frac{1}{2} \right)^2 e^{-2t} + 12 e^{2t} \left(\ln(e) + \frac{1}{2} \right)^2}{16 \ln(e)^4 - 8 \ln(e)^2 + 1} \quad (51)$$

$$eq := diff(x(t), t\$2) - 2 \cdot diff(x(t), t) + x(t) = 3 \cdot \exp(1)^{(2 \cdot t)} - 5 \cdot e^{xp(1)(-2 \cdot t)}$$

$$\frac{d^2}{dt^2} x(t) - 2 \left(\frac{d}{dt} x(t) \right) + x(t) = 3 (e)^{2t} - 5 e^{xp(1)(-2t)} \quad (52)$$

$$eq := diff(x(t), t\$2) - 2 \cdot diff(x(t), t) + x(t) = 3 \cdot \exp(1)^{(2 \cdot t)} - 5 \cdot \exp(1)^{(1)(-2 \cdot t)}$$

$$\frac{d^2}{dt^2} x(t) - 2 \left(\frac{d}{dt} x(t) \right) + x(t) = 3 (e)^{2t} - 5 (e)^{-2t} \quad (53)$$

$dsolve(eq, x(t));$

$$x(t) = e^t_C2 + e^t t_C1 + 3 e^{2t} - \frac{5}{9} e^{-2t} \quad (54)$$

$$eq := diff(x(t), t\$2) - 2 \cdot diff(x(t), t) + x(t) = \exp(1)^{(2 \cdot t)}$$

$$\frac{d^2}{dt^2} x(t) - 2 \left(\frac{d}{dt} x(t) \right) + x(t) = (e)^{2t} \quad (55)$$

$dsolve(eq, x(t));$

$$x(t) = e^t_C2 + e^t t_C1 + e^{2t} \quad (56)$$

$$eq := diff(x(t), t\$2) + 4 \cdot x(t) = \cos(2 \cdot t);$$

$$\frac{d^2}{dt^2} x(t) + 4 x(t) = \cos(2t) \quad (57)$$

$dsolve(eq, x(t));$

$$x(t) = \sin(2t)_C2 + \cos(2t)_C1 + \frac{1}{8} \cos(2t) + \frac{1}{4} \sin(2t) t \quad (58)$$

$$eq := t^2 \cdot diff(x(t), t\$2) + 2 \cdot t \cdot diff(x(t), t) - 2 \cdot x(t) = 0;$$

$$t^2 \left(\frac{d^2}{dt^2} x(t) \right) + 2 t \left(\frac{d}{dt} x(t) \right) - 2 x(t) = 0 \quad (59)$$

$dsolve(eq, x(t));$

$$x(t) =_C1 t + \frac{_C2}{t^2} \quad (60)$$

$$ic1 := x(1) = 0, D(x)(1) = 0;$$

$$x(1) = 0, D(x)(1) = 0 \quad (61)$$

$$dsolve(\{eq, ic1\}, x(t));$$

$$x(t) = 0 \quad (62)$$

$$eq := diff(x(t), t\$2) + 4 \cdot x(t) = \cos(2 \cdot t);$$

$$\frac{d^2}{dt^2} x(t) + 4 x(t) = \cos(2t) \quad (63)$$

$dsolve(eq, x(t));$

$$x(t) = \sin(2 t) _C2 + \cos(2 t) _C1 + \frac{1}{8} \cos(2 t) + \frac{1}{4} \sin(2 t) t \quad (64)$$

$$eq := \text{diff}(\Theta(t), t^2) + \text{diff}(\Theta(t), t) + \Theta(t) = 0;$$

$$\frac{d^2}{dt^2} \Theta(t) + \frac{d}{dt} \Theta(t) + \Theta(t) = 0 \quad (65)$$

$$\text{dsolve}(eq, \Theta(t));$$

$$\Theta(t) = _C1 e^{-\frac{1}{2} t} \sin\left(\frac{1}{2} \sqrt{3} t\right) + _C2 e^{-\frac{1}{2} t} \cos\left(\frac{1}{2} \sqrt{3} t\right) \quad (66)$$

$$eq := \text{diff}(x(t), t^2) + 25 \cdot x(t) = 0;$$

$$\frac{d^2}{dt^2} x(t) + 25 x(t) = 0 \quad (67)$$

$$ic1 := x(0) = 0, D(x)(0) = 1;$$

$$x(0) = 0, D(x)(0) = 1 \quad (68)$$

$$\text{dsolve}(\{eq, ic1\}, x(t));$$

$$x(t) = \frac{1}{5} \sin(5 t) \quad (69)$$

$$\text{plot}(eq, t = -5 .. 5);$$

Error, (in plot) unexpected options: [diff(diff(x(t), t), t)+25*x(t) = 0, t = -5 .. 5]

$$\text{with}(DEplot);$$

Error, invalid input: with expects its 1st argument, pname, to be of type {\`module`, package}, but received DEplot

$$\text{with}(DEtools);$$

$$[\text{AreSimilar}, \text{Closure}, \text{DEnormal}, \text{DEplot}, \text{DEplot3d}, \text{DEplot_polygon}, \text{DFactor}, \text{DFactorLCLM}, \quad (70)$$

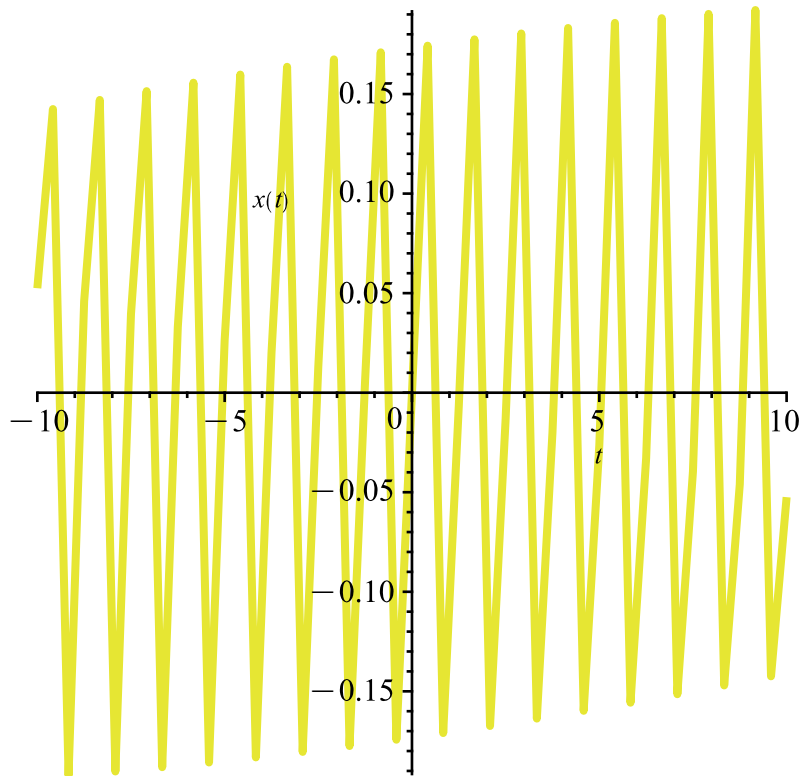
DFactorsols, Dchangevar, Desingularize, FunctionDecomposition, GCRD, Gosper, Heunsols, Homomorphisms, IVPsol, IsHyperexponential, LCLM, MeijerGsols, MultiplicativeDecomposition, ODEInvariants, PDEchangecoords, PolynomialNormalForm, RationalCanonicalForm, ReduceHyperexp, RiemannPsols, Xchange, Xcommutator, Xgauge, Zeilberger, abelsol, adjoint, autonomous, bernoullisol, buildsol, buildsym, canoni, caseplot, casesplit, checkrank, chinisol, clairautsol, constcoeffsols, convertAlg, convertsys, dalembertsol, dcoeffs, de2diffop, dfieldplot, diff_table, diffop2de, dperiodic_sols, dpolyform, dsubs, eigenring, endomorphism_charpoly, equinv, eta_k, eulersols, exactsol, expsols, exterior_power, firint, firtest, formal_sol, gen_exp, generate_ic, genhomosol, gensys, hamilton_eqs, hypergeomsols, hyperode, indicialeq, infgen, initialdata, integrate_sols, intfactor, invariants, kovaciesols, leftdivision, liesol, line_int, linearsol, matrixDE, matrix_riccati, maxdimsystems, moser_reduce, muchange, mult, mutest, newton_polygon, normalG2, ode_int_y, ode_y1, odeadvisor, odepde, parametricsol, particularsol, phaseportrait, poincare, polysols, power_equivalent, rational_equivalent, ratsols, redode, reduceOrder, reduce_order, regular_parts, regularsp, remove_RootOf, riccati_system, riccatisol, rifread, rifsimp, righdivision, rtaylor, separablesol,

singularities, solve_group, super_reduce, symgen, symmetric_power, symmetric_product, symtest, transinv, translate, untranslate, varparam, zoom]

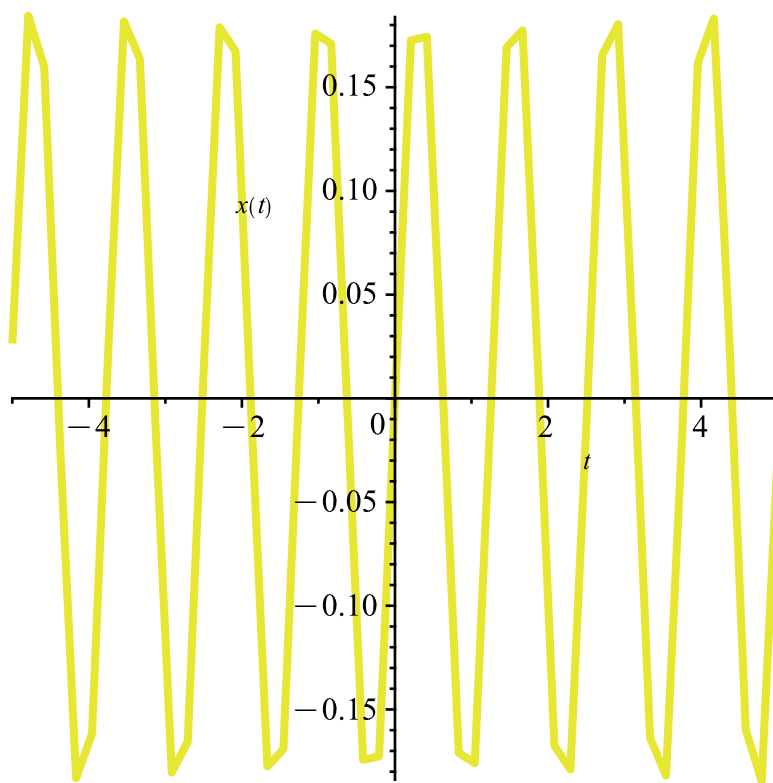
```
DEplot(eq, x(t), t=-10..10);
```

Error, (in DEtools/DEplot) cannot produce plot, non-autonomous DE(s) require initial conditions.

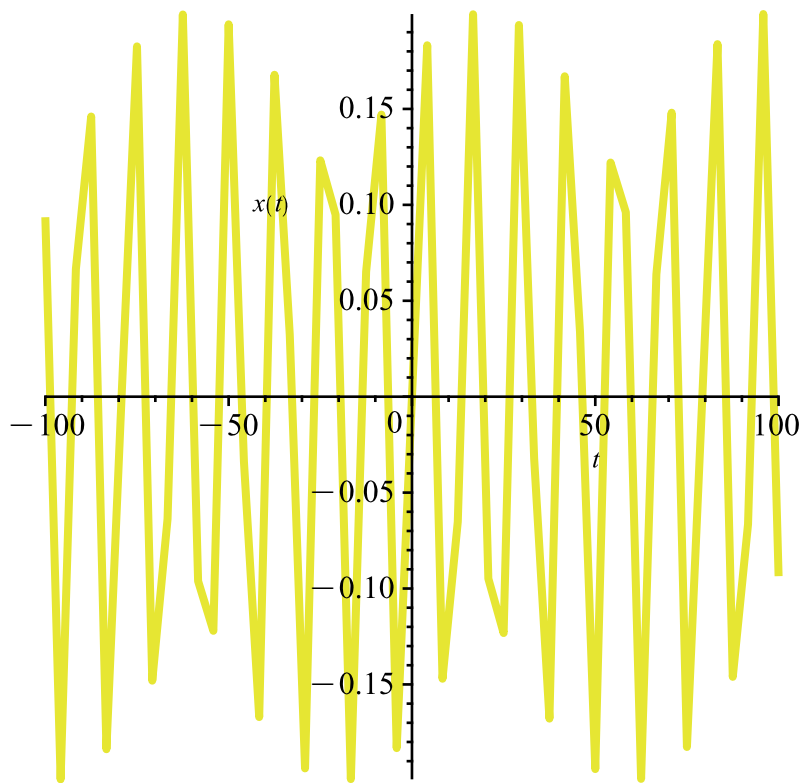
```
DEplot(eq, x(t), t=-10..10, [[ic1]]);
```



```
DEplot(eq, x(t), t=-5..5, [[ic1]]);
```



$DEplot(eq, x(t), t = -100..100, [[ic1]]);$



$eq := diff(x(t), t\$2) + 25 \cdot x(t) = 25 - 25 \cdot \sin(5 \, t);$

$$\frac{d^2}{dt^2} x(t) + 25 x(t) = 25 - 25 \sin(5 \, t)$$

(71)

$dsolve(eq, x(t))l$

$$lx(t) = l \left(\sin(5t) _C2 + \cos(5t) _C1 + 1 - \frac{1}{2} \sin(5t) + \frac{5}{2} \cos(5t) t \right) \quad (72)$$

$dsolve(eq, x(t));$

$$x(t) = \sin(5t) _C2 + \cos(5t) _C1 + 1 - \frac{1}{2} \sin(5t) + \frac{5}{2} \cos(5t) t \quad (73)$$

$eq := diff(x(t), t^2) + 25 \cdot x(t) = \sin(5t);$

$$\frac{d^2}{dt^2} x(t) + 25 x(t) = \sin(5t) \quad (74)$$

$dsolve(eq, x(t));$

$$x(t) = \sin(5t) _C2 + \cos(5t) _C1 - \frac{1}{10} \cos(5t) t \quad (75)$$

$eq := diff(x(t), t) + \frac{1}{t^2} \cdot x(t) = 0;$

$$\frac{d}{dt} x(t) + \frac{x(t)}{t^2} = 0 \quad (76)$$

$ic1 := x(-1) = 1;$

$$x(-1) = 1 \quad (77)$$

$dsolve(\{eq, ic1\}, x(t))l$

$$lx(t) = \frac{l e^{\frac{1}{t}}}{e^{-1}} \quad (78)$$

$dsolve(\{eq, ic1\}, x(t));$

$$x(t) = \frac{e^{\frac{1}{t}}}{e^{-1}} \quad (79)$$

$dsolve(eq, x(t));$

$$x(t) = _C1 e^{\frac{1}{t}} \quad (80)$$

$eq := diff(x(t), t) + \frac{1}{t^2} \cdot x(t) = 1 + \frac{1}{t};$

$$\frac{d}{dt} x(t) + \frac{x(t)}{t^2} = 1 + \frac{1}{t} \quad (81)$$

$dsolve(eq, x(t));$

$$x(t) = t + _C1 e^{\frac{1}{t}} \quad (82)$$

$eq := diff(x(t), t^2) - diff(x(t), t) - 20 \cdot x(t) = 0;$

$$\frac{d^2}{dt^2} x(t) - \left(\frac{d}{dt} x(t) \right) - 20 x(t) = 0 \quad (83)$$

$dsolve(eq, x(t))l$

$$l x(t) = l \left({}_C1 \, e^{5 \, t} + {}_C2 \, e^{-4 \, t} \right) \tag{84}$$

$$dsolve(eq, x(t));$$

$$x(t) = {}_C1 \, e^{5 \, t} + {}_C2 \, e^{-4 \, t} \tag{85}$$