Universitatea Babeş-Bolyai, Facultatea de Matematică şi Informatică
Secţia: Informatică engleză, Curs: Dynamical Systems

Laboratory 2. Solving Linear Homogeneous Differential Equations

1. Decide whether the functions $\sin t,\ \cos t,\ \sinh t,\ \cosh t$ are solutions to $x^{(4)} - x = 0$.

Find the general solution to each of the following differential equations. Write each equation and its corresponding general solution in your notebooks.

2. $x' + tx = 0$; 3. $x'' + x = 0$; 4. $4x'' + 8x' + 5x = 0$; 5. $x'' - 3x' + 2x = 0$.

Find the solution to each of the following IVPs and then represent its graph on a large interval. Write everything in your notebooks. If Maple/Sage does not return you directly an accurate graph, then use Maple/Sage, for example, to find the zeros of the function, or its limit at infinity, or portions of graphs on small intervals.

Describe the long-term behavior of the solution using some of the words: exponential increasing or decreasing, periodic with minimal period ..., bounded between ..., oscillatory around zero, with constant amplitude, with amplitude decreasing to zero as $t \to \infty$, with amplitude increasing to infinity as $t \to \infty$... . Write everything in your notebooks.

6. $x'' + x = 0, \quad x(\pi/2) = 1, \quad x'(\pi/2) = -2$. Show that $x(t) = \sqrt{5}\cos(t - \arctan\frac{1}{2})$ (In Maple you can use "expand", in Sage "*trig_simplify*"). What is the amplitude of the oscillations?

7. $4x'' + 8x' + 5x = 0, \quad x(0) = 0, \quad x'(0) = 0.5$ Here it is quite difficult to see the oscillations...

8. $x'' - 3x' + 2x = 0, \quad x(0) = 2, \quad x'(0) = 3$

9. Find the general solution to the second order linear homogeneous differential equation $x'' + a(t)x = 0$ taking first $a(t) = 5$, then $a(t) = t$ and finally $a(t) = t^5$. Pay attention that Sage does this for $a(t) = t$ and $a(t) = t^5$ only if we add the option "*contrib_ode = true*" in the command "desolve".

In order to find details on how Maple "thinks" when trying to solve it, set infolevel[dsolve] to 3. I do not know if something similar is possible in Sage. Only that in Sage we can add the option "*show_method = true*" in the command "desolve".

Ask Wikipedia what are the Bessel functions. Use Maple/Sage to plot on different intervals at your choice (preferably large intervals) the Bessel function denoted $BesselJ(1, t)$ in Maple, respectively $bessel\_J(1, t)$ in Sage. Describe its long-term behavior.

If you want, set back infolevel[dsolve] to 1 (in Maple).

Now let $a \in C(\mathbb{R})$ be an arbitrary function and consider the following IVP:
$x'' + a(t)x = 0, \quad x(0) = 0, \quad x'(0) = 0.$

Notice that the Existence and Uniqueness Theorem applies in this case and yields that this IVP has a unique solution. We also notice that the null solution, $x = 0$, verifies all the conditions, hence it is the unique solution of this IVP.

Now find with Maple/Sage the solution of this IVP taking first $a(t) = 5$, then $a(t) = t$ and finally $a(t) = t^5$. What is your opinion about what Maple/Sage returns?

Look for solutions to each of the following BVPs (boundary value problems). Notice that, indeed, these are not IVPs. Why?

10. $x'' + x = 0, \quad x(0) = x(\pi) = 0$ 11. $x'' + x = 0, \quad x(0) = x(1) = 0.$

12. $x'' + x = 1, \quad x(0) = x(\pi) = 0.$

We have here 3 BVP's, apparently very similar, with very few differences. But they have 3 different behaviours. The first one has as many solution as real numbers are. The second one has a unique solution $x = 0$. The last one has no solution. Check by hand that Maple/Sage returned the correct answer.

Find the general solution to each of the following linear nonhomogeneous differential equations. Notice that, taking $c_1 = 0$, we obtain a particular solution of the equation which has the same form as the nonhomogeneous term. Note that the proposition written after each equation is true and write down the corresponding particular solution.

13. $x' + x = 15$. "The right-hand side is a constant, a particular solution is also constant." This particular solution is $x_p =$

14. $x' + x = 2e^t - 7e^{-3t}$. "The right-hand side is a linear combination of e^t and e^{-3t}, the same for a particular solution." This particular solution is $x_p =$

15. $x' + x = -t^2 + 3t - 7$. "The right-hand side is a second degree polynomial, the same for a particular solution." This particular solution is $x_p =$

16. $x' + x = \sin t + 3\cos t$. "The right-hand side is a linear combination of $\sin t$ and $\cos t$, the same for a particular solution." This particular solution is $x_p =$

17. $x' + x = \sin t$. "The right-hand side is a linear combination of $\sin t$ and $\cos t$, the same for a particular solution." This particular solution is $x_p =$

18. $x' + x = 3\cos t$. "The right-hand side is a linear combination of $\sin t$ and $\cos t$, the same for a particular solution." This particular solution is $x_p =$