

Exercise 1

$eq := \text{diff}(u(x), x^2) + 5 \cdot \text{diff}(u(x), x) - 7 \cdot u(x) = 5 \cdot \cos(x) - 7 \cdot \sin(x);$

$$\frac{d^2}{dx^2} u(x) + 5 \left(\frac{d}{dx} u(x) \right) - 7 u(x) = 5 \cos(x) - 7 \sin(x) \quad (1)$$

$sol := \text{dsolve}(eq, u(x));$

$$u(x) = e^{\frac{1}{2}(-5+\sqrt{53})x} {}_C2 + e^{-\frac{1}{2}(5+\sqrt{53})x} {}_CI - \frac{5}{89} \cos(x) + \frac{81}{89} \sin(x) \quad (2)$$

$solexpr := \text{rhs}(sol);$

$$e^{\frac{1}{2}(-5+\sqrt{53})x} {}_C2 + e^{-\frac{1}{2}(5+\sqrt{53})x} {}_CI - \frac{5}{89} \cos(x) + \frac{81}{89} \sin(x) \quad (3)$$

$\text{simplify}\left(\text{solexpr}\left(\frac{\text{Pi}}{2}\right)\right);$

$$e^{\frac{1}{2}(-5+\sqrt{53})x} \left(\frac{1}{2} \pi \right) {}_C2 \left(\frac{1}{2} \pi \right) + e^{-\frac{1}{2}(5+\sqrt{53})x} \left(\frac{1}{2} \pi \right) {}_CI \left(\frac{1}{2} \pi \right) - \frac{5}{89} \cos(x) \left(\frac{1}{2} \pi \right) + \frac{81}{89} \sin(x) \left(\frac{1}{2} \pi \right) \quad (4)$$

$\text{eval}\left(\text{D}(\text{solexpr})\left(\frac{\text{Pi}}{2}\right)\right);$

$$\begin{aligned} & \text{D}\left(e^{\frac{1}{2}(-5+\sqrt{53})x}\right) \left(\frac{1}{2} \pi \right) {}_C2 \left(\frac{1}{2} \pi \right) + e^{\frac{1}{2}(-5+\sqrt{53})x} \left(\frac{1}{2} \pi \right) \text{D}({}_C2) \left(\frac{1}{2} \pi \right) \\ & + \text{D}\left(e^{-\frac{1}{2}(5+\sqrt{53})x}\right) \left(\frac{1}{2} \pi \right) {}_CI \left(\frac{1}{2} \pi \right) + e^{-\frac{1}{2}(5+\sqrt{53})x} \left(\frac{1}{2} \pi \right) \text{D}({}_CI) \left(\frac{1}{2} \pi \right) \\ & - \frac{5}{89} \text{D}(\cos(x)) \left(\frac{1}{2} \pi \right) + \frac{81}{89} \text{D}(\sin(x)) \left(\frac{1}{2} \pi \right) \end{aligned} \quad (5)$$