

#Ex.3 -  $x' = 1 - x^2$

$eq := \text{diff}(x(t), t) = 1 - x(t)^2;$

$$\frac{d}{dt} x(t) = 1 - x(t)^2 \quad (1)$$

#Find the equilibrium points

$f := 1 - x^2;$

$$-x^2 + 1 \quad (2)$$

$\text{solve}(f, x);$

$$-1, 1 \quad (3)$$

#Equilibrium points are +1 and -1

$ic1 := x(0) = -1;$

$$x(0) = -1 \quad (4)$$

$\text{dsolve}(\{eq, ic1\}, x(t));$

$$x(t) = -1 \quad (5)$$

$ic2 := x(0) = 1;$

$$x(0) = 1 \quad (6)$$

$\text{dsolve}(\{eq, ic2\}, x(t));$

$$x(t) = 1 \quad (7)$$

#ii) Find the expression of each of the solutions for -2, 0, 2

$ic := x(0) = -2;$

$$x(0) = -2 \quad (8)$$

$sol1 := \text{dsolve}(\{eq, ic\}, x(t));$

$$x(t) = \tanh(\text{arctanh}(2) + t) \quad (9)$$

$\text{convert}(\text{convert}(\tanh(t - \text{arctanh}(2)), \text{exp}), \text{exp})$

$$\frac{e^{2t} + 3}{e^{2t} - 3} \quad (10)$$

$ic := x(0) = 0;$

$$x(0) = 0 \quad (11)$$

$sol2 := \text{dsolve}(\{eq, ic\}, x(t));$

$$x(t) = \tanh(\text{arctanh}(2) + t) \quad (12)$$

$\text{convert}(\text{convert}(\tanh(t), \text{exp}), \text{exp})$

$$\frac{e^{2t} - 1}{e^{2t} + 1} \quad (13)$$

$ic := x(0) = 2;$

$$x(0) = 2 \quad (14)$$

$sol3 := \text{dsolve}(\{eq, ic\}, x(t));$

$$x(t) = \tanh(\text{arctanh}(2) + t) \quad (15)$$

$\text{convert}(\text{convert}(\tanh(t + \text{arctanh}(2)), \text{exp}), \text{exp})$

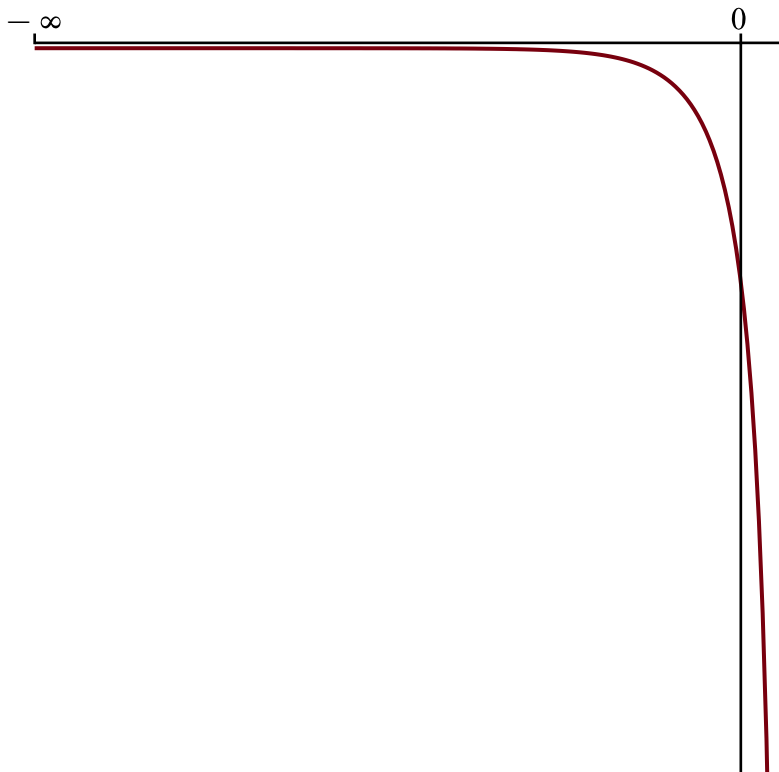
$$\frac{3e^{2t} + 1}{3e^{2t} - 1}$$

(16)

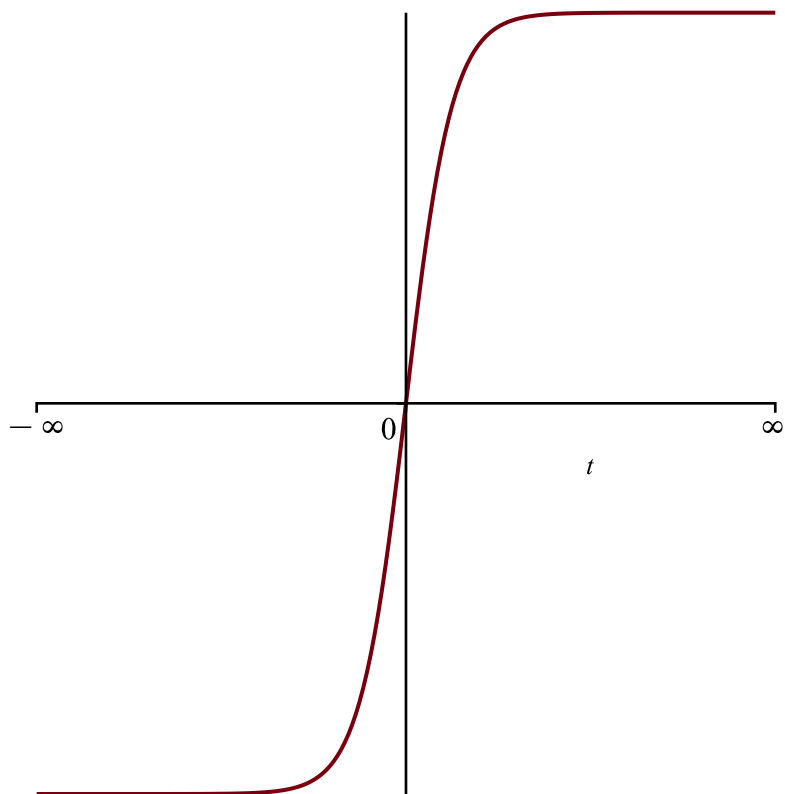
#iii) Represent the graph for 0,-2,2

#for -2

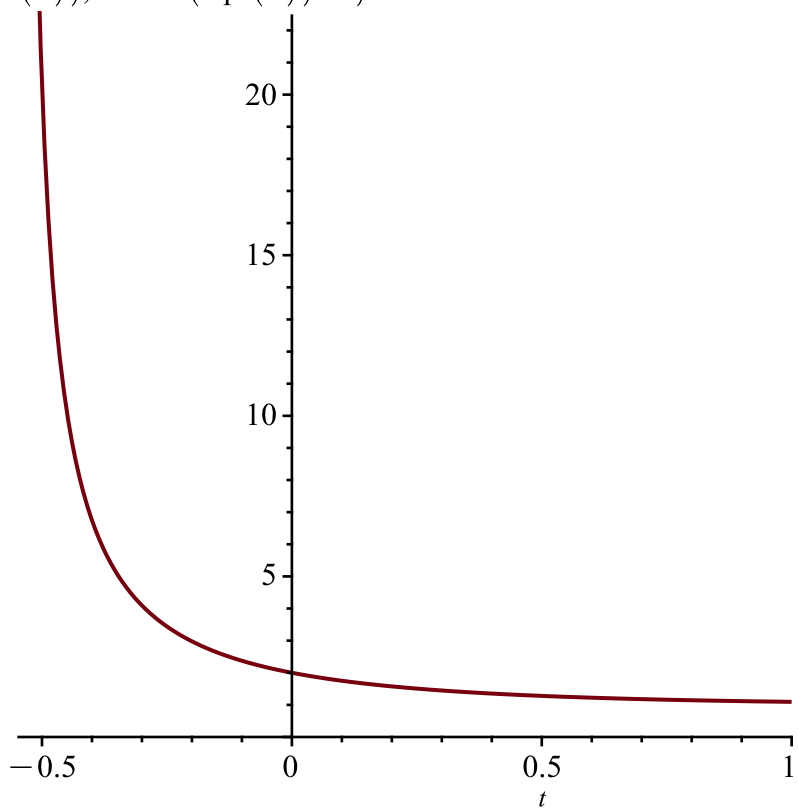
plot(tanh(t - arctanh(2)), t = -infinity ... ln(sqrt(3)));



plot(tanh(t), t = -infinity ... infinity);



`plot(tanh(t + arctanh(2)), t = -ln(sqrt(3)) .. 1)`



`limit(sol1, t = -infinity);`

$$\lim_{t \rightarrow -\infty} x(t) = -1$$

(17)

$$\text{limit}(\tanh(t - \operatorname{arctanh}(2)), t = \ln(\operatorname{sqrt}(3)), \text{left}) \qquad \qquad \qquad - \infty \qquad \qquad \qquad \textbf{(18)}$$

$$\text{limit}(\text{sol2}, t = -\infty); \qquad \qquad \qquad \lim_{t \rightarrow -\infty} x(t) = -1 \qquad \qquad \qquad \textbf{(19)}$$

$$\text{limit}(\text{sol2}, t = \infty); \qquad \qquad \qquad \lim_{t \rightarrow \infty} x(t) = 1 \qquad \qquad \qquad \textbf{(20)}$$

$$\text{limit}(\text{sol3}, t = \infty); \qquad \qquad \qquad \lim_{t \rightarrow \infty} x(t) = 1 \qquad \qquad \qquad \textbf{(21)}$$

$$\text{limit}(\text{sol3}, t = -\infty); \qquad \qquad \qquad \lim_{t \rightarrow -\infty} x(t) = -1 \qquad \qquad \qquad \textbf{(22)}$$