Lab test 2, exercise 4

$eq := diff(y(x), x) = y(x)^2 + x^2;$

$$\frac{d}{dx} y(x) = y(x)^2 + x^2 \tag{1}$$

$ic := x(0) = 0;$

$$x(0) = 0 \tag{2}$$

Apply the Euler's method and the improved Euler's method in the interval [0,1] with step size h=0.1

restart :

with(*DEtools*);

[*AreSimilar*, *Closure*, *DEnormal*, *DEplot*, *DEplot3d*, *DEplot_polygon*, *DFactor*, *DFactorLCLM*, *DFactorsols*, *Dchangevar*, *Desingularize*, *FunctionDecomposition*, *GCRD*, *Gosper*, *Heunsols*, *Homomorphisms*, *IVPsol*, *IsHyperexponential*, *LCLM*, *MeijerGsols*, *MultiplicativeDecomposition*, *ODEInvariants*, *PDEchangecoords*, *PolynomialNormalForm*, *RationalCanonicalForm*, *ReduceHyperexp*, *RiemannPsols*, *Xchange*, *Xcommutator*, *Xgauge*, *Zeilberger*, *abelsol*, *adjoint*, *autonomous*, *bernoullisol*, *buildsol*, *buildsym*, *canoni*, *caseplot*, *casesplit*, *checkrank*, *chinisol*, *clairautsol*, *constcoeffsols*, *convertAlg*, *convertsys*, *dalembertsol*, *dcoeffs*, *de2diffop*, *dfieldplot*, *diff_table*, *diffop2de*, *dperiodic_sols*, *dpolyform*, *dsubs*, *eigenring*, *endomorphism_charpoly*, *equinv*, *eta_k*, *eulersols*, *exactsol*, *expsols*, *exterior_power*, *firint*, *firtest*, *formal_sol*, *gen_exp*, *generate_ic*, *genhomosol*, *gensys*, *hamilton_eqs*, *hypergeomsols*, *hyperode*, *indicialeq*, *infgen*, *initialdata*, *integrate_sols*, *intfactor*, *invariants*, *kovacicsols*, *leftdivision*, *liesol*, *line_int*, *linearsol*, *matrixDE*, *matrix_riccati*, *maxdimsystems*, *moser_reduce*, *muchange*, *mult*, *mutest*, *newton_polygon*, *normalG2*, *ode_int_y*, *ode_y1*, *odeadvisor*, *odepde*, *parametricsol*, *particularsol*, *phaseportrait*, *poincare*, *polysols*, *power_equivalent*, *rational_equivalent*, *ratsols*, *redode*, *reduceOrder*, *reduce_order*, *regular_parts*, *regularsp*, *remove_RootOf*, *riccati_system*, *riccatisol*, *rifread*, *rifsimp*, *rightdivision*, *rtaylor*, *separablesol*, *singularities*, *solve_group*, *super_reduce*, *symgen*, *symmetric_power*, *symmetric_product*, *symtest*, *transinv*, *translate*, *untranslate*, *varparam*, *zoom*] **(3)**

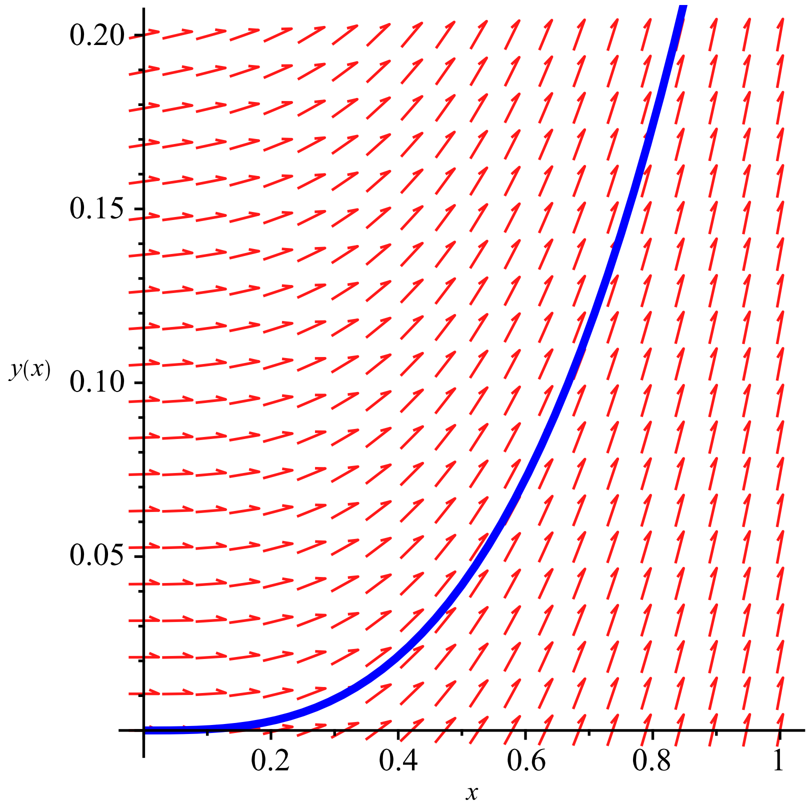
$f := (x, y) \to x^2 + y^2;$

$$(x, y) \to x^2 + y^2 \tag{4}$$

$dsolve(\{diff(y(x), x) = f(x, y(x)), y(0) = 0\});$ phi $:= unapply(rhs(\%), x);$

$$x \to piecewise\left(x = 0, 0, -\frac{\left(-\mathrm{BesselJ}\left(-\frac{3}{4}, \frac{1}{2} x^2\right) + \mathrm{BesselY}\left(-\frac{3}{4}, \frac{1}{2} x^2\right)\right) x}{-\mathrm{BesselJ}\left(\frac{1}{4}, \frac{1}{2} x^2\right) + \mathrm{BesselY}\left(\frac{1}{4}, \frac{1}{2} x^2\right)}\right) \tag{5}$$

$DEplot(diff(y(x), x) = f(x, y(x)), y(x), x = 0..1, [[y(0) = 0]], y = 0...20, linecolor = blue);$



$h := 0.1$

$$0.1 \tag{6}$$

$x := 0;\, y := 0;$

$$0 \tag{7}$$

for i **from** 1 **to** 11 **do** $y := y + h \cdot f(x, y) : \mathrm{psi}(i) := y : x := x + h : print(x, y, \mathrm{phi}(x), \mathrm{abs}(y - \mathrm{phi}(x)));$**od**:

$$1.1,\ 0.4011001929,\ 0.4776170219,\ 0.0765168290 \tag{8}$$

The approximate value for x=1 is

1.0, 0.2925421046, 0.3502318440, 0.0576897394

$$1.0,\ 0.2925421046,\ 0.3502318440,\ 0.0576897394 \tag{9}$$

Improved Euler's method

restart :

with(*DEtools*) :

$f := (x, y) \rightarrow x^2 + y^2;$

$$(x, y) \rightarrow x^2 + y^2 \tag{10}$$

$dsolve(\{diff(y(x), x) = f(x, y(x)), y(0) = 0\});\ \mathrm{phi} := unapply(rhs(\%), x);$

$$x \rightarrow piecewise\left(x = 0, 0, -\frac{\left(-\mathrm{BesselJ}\left(-\frac{3}{4}, \frac{1}{2}x^2\right) + \mathrm{BesselY}\left(-\frac{3}{4}, \frac{1}{2}x^2\right)\right)x}{-\mathrm{BesselJ}\left(\frac{1}{4}, \frac{1}{2}x^2\right) + \mathrm{BesselY}\left(\frac{1}{4}, \frac{1}{2}x^2\right)}\right) \tag{11}$$

$h := 0.1;\ x := 0;\ y := 0;$

$$0 \tag{12}$$

for i **from** 1 **to** 11 **do** $y := y + \frac{h}{2} \cdot f(x, y) + \frac{h}{2} \cdot f(x + h, y + h \cdot f(x, y)) : \mathrm{psi}(i) := y : x := x + h :$
$print(x, y, \mathrm{phi}(x), \mathrm{abs}(y - \mathrm{phi}(x)))$;**od**:

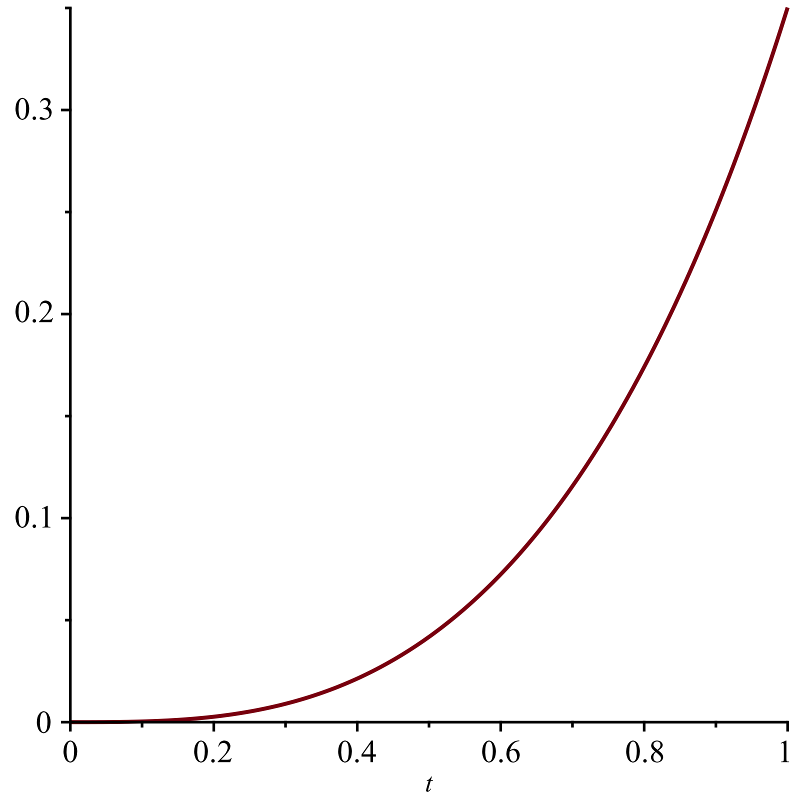
$$1.1, 0.4792938348, 0.4776170219, 0.0016768129 \tag{13}$$

The approximate value for x=1 is

1.0, 0.3518301326, 0.3502318440, 0.0015982886

$$1.0, 0.3518301326, 0.3502318440, 0.0015982886 \tag{14}$$

$points := [[n, \mathrm{psi}(n)]\$n = 1..11] : with(plots) : pointplot(points, style = point); plot(\mathrm{phi}(t), t = 0..1);$



Exercise 2

$x :=' x';$

$$x \tag{15}$$

$y :=' y';$

$$y \tag{16}$$

$eq := diff(y(x), x\$2) \cdot x^2 - 2 \cdot x \cdot diff(y(x), x) + y(x) = 0;$

$$\left(\frac{d^2}{dx^2} y(x)\right) x^2 - 2x \left(\frac{d}{dx} y(x)\right) + y(x) = 0 \tag{17}$$

$dsolve(eq, y(x));$

$$y(x) = \_C1\, x^{\frac{1}{2}\sqrt{5} + \frac{3}{2}} + \_C2\, x^{-\frac{1}{2}\sqrt{5} + \frac{3}{2}} \tag{18}$$

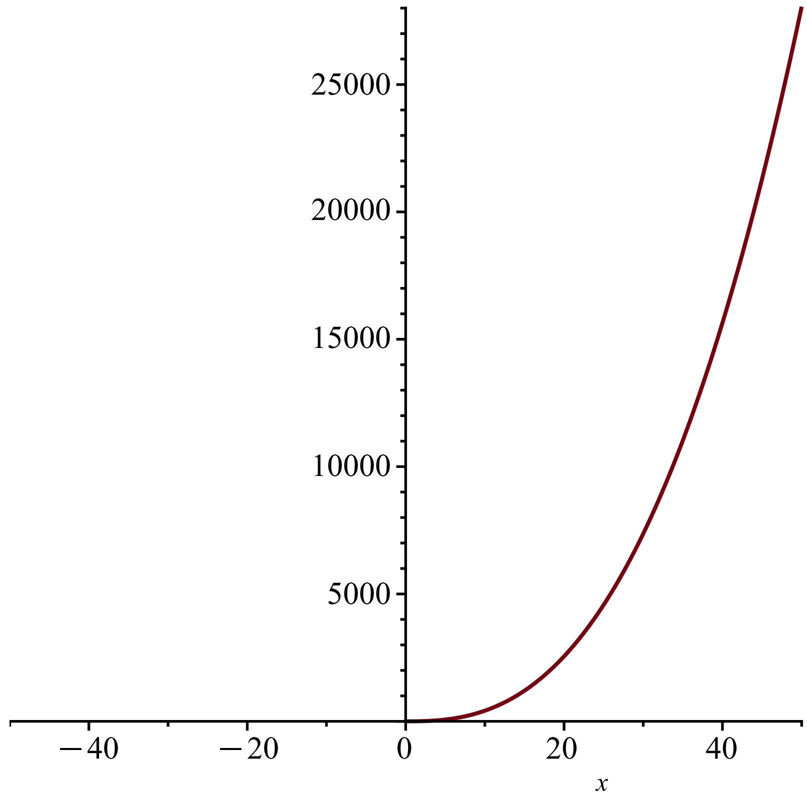
$ic := y(1) = 2, \mathrm{D}(y)(1) = 3;$

$$y(1) = 2, \mathrm{D}(y)(1) = 3 \tag{19}$$

$sol := dsolve(\{eq, ic\}, y(x));$

$$y(x) = x^{\frac{1}{2}\sqrt{5}+\frac{3}{2}} + x^{-\frac{1}{2}\sqrt{5}+\frac{3}{2}} \quad \textbf{(20)}$$

$plot(rhs(sol), x = -50..50);$



$eq :=\text{'}eq\text{'};$

$$eq \quad \textbf{(21)}$$

$eq := 2\cdot x + 5\cdot\sin(3\cdot x) + \ln(7\cdot x^2 + 1) = 10$

$$2\,x + 5\,\sin(3\,x) + \ln(7\,x^2 + 1) = 10 \quad \textbf{(22)}$$

$solve(eq, x);$

$$\frac{1}{3}\,RootOf\left(15\,\sin(\_Z) + 2\,\_Z + 3\,\ln\left(\frac{7}{9}\,\_Z^2 + 1\right) - 30\right) \quad \textbf{(23)}$$

$eq := 2\cdot x + 5\cdot\sin(3\cdot x) + \ln(7\cdot x^2 + 1) - 10;$

$$2\,x + 5\,\sin(3\,x) + \ln(7\,x^2 + 1) - 10 \quad \textbf{(24)}$$

$fsolve(eq, x);$

$$2.228999467 \quad \textbf{(25)}$$