

$$f := (x, y) \rightarrow 2 \cdot x - x^2 - x \cdot y; \quad (x, y) \rightarrow 2 x - x^2 - x y \quad (1)$$

$$f1 := (x, y) \rightarrow 2 \cdot x - x^2 - x \cdot y; \quad (x, y) \rightarrow 2 x - x^2 - x y \quad (2)$$

$$f2 := -y + x \cdot y; \quad x y - y \quad (3)$$

$$Jm := \text{Jacobian}([f1(x, y), f2(x, y)], [x, y]); \quad \text{Jacobian}([-x^2 - x y + 2 x, x(x, y) y(x, y) - y(x, y)], [x, y]) \quad (4)$$

$$\text{with}(\text{linalg}) : \text{with}(\text{VectorCalculus}) \\ [\&x, \text{'*'}, \text{'+'}, \text{'-'}, \text{'.'}, \text{'<'}, \text{'>'}, \text{'<|>'}, \text{About}, \text{AddCoordinates}, \text{ArcLength}, \text{BasisFormat}, \text{Binormal}, \text{Compatibility}, \text{ConvertVector}, \text{CrossProduct}, \text{Curl}, \text{Curvature}, \text{D}, \text{Del}, \text{DirectionalDiff}, \text{Divergence}, \text{DotProduct}, \text{Flux}, \text{GetCoordinateParameters}, \text{GetCoordinates}, \text{GetNames}, \text{GetPVDDescription}, \text{GetRootPoint}, \text{GetSpace}, \text{Gradient}, \text{Hessian}, \text{IsPositionVector}, \text{IsRootedVector}, \text{IsVectorField}, \text{Jacobian}, \text{Laplacian}, \text{LineInt}, \text{MapToBasis}, \text{Nabla}, \text{Norm}, \text{Normalize}, \text{PathInt}, \text{PlotPositionVector}, \text{PlotVector}, \text{PositionVector}, \text{PrincipalNormal}, \text{RadiusOfCurvature}, \text{RootedVector}, \text{ScalarPotential}, \text{SetCoordinateParameters}, \text{SetCoordinates}, \text{SpaceCurve}, \text{SurfaceInt}, \text{TNBFrame}, \text{Tangent}, \text{TangentLine}, \text{TangentPlane}, \text{TangentVector}, \text{Torsion}, \text{Vector}, \text{VectorField}, \text{VectorPotential}, \text{VectorSpace}, \text{Wronskian}, \text{diff}, \text{eval}, \text{evalVF}, \text{int}, \text{limit}, \text{series}] \quad (5)$$

$$Jm := \text{Jacobian}([f1(x, y), f2(x, y)], [x, y]); \\ \left[\left[-2 x - y + 2, -x \right], \right. \\ \left[\left(\frac{\partial}{\partial x} x(x, y) \right) y(x, y) + x(x, y) \left(\frac{\partial}{\partial x} y(x, y) \right) - \left(\frac{\partial}{\partial x} y(x, y) \right), \left(\frac{\partial}{\partial y} x(x, y) \right) y(x, y) \right. \\ \left. \left. + x(x, y) \left(\frac{\partial}{\partial y} y(x, y) \right) - \left(\frac{\partial}{\partial y} y(x, y) \right) \right] \right] \quad (6)$$

$$A := \text{subs}([x=p1, y=p2], Jm); \\ \left[\left[-2 p1 - p2 + 2, -p1 \right], \right. \\ \left[\left(\frac{\partial}{\partial p1} p1(p1, p2) \right) p2(p1, p2) + p1(p1, p2) \left(\frac{\partial}{\partial p1} p2(p1, p2) \right) - \left(\frac{\partial}{\partial p1} p2(p1, p2) \right), \right. \\ \left. \left(\frac{\partial}{\partial p2} p1(p1, p2) \right) p2(p1, p2) + p1(p1, p2) \left(\frac{\partial}{\partial p2} p2(p1, p2) \right) - \left(\frac{\partial}{\partial p2} p2(p1, p2) \right) \right] \quad (7)$$

$$\text{eigenvalues}(A); \\ \frac{1}{2} \left(\frac{\partial}{\partial p2} p1(p1, p2) \right) p2(p1, p2) + \frac{1}{2} p1(p1, p2) \left(\frac{\partial}{\partial p2} p2(p1, p2) \right) - \frac{1}{2} \frac{\partial}{\partial p2} p2(p1, p2) \\ - p1 - \frac{1}{2} p2 + 1 \quad (8)$$

$$\begin{aligned}
& + \frac{1}{2} \left(\left(\frac{\partial}{\partial p_2} p_1(p_1, p_2) \right)^2 p_2(p_1, p_2)^2 + 2 \left(\frac{\partial}{\partial p_2} p_1(p_1, p_2) \right) p_2(p_1, \right. \\
& p_2) p_1(p_1, p_2) \left(\frac{\partial}{\partial p_2} p_2(p_1, p_2) \right) + p_1(p_1, p_2)^2 \left(\frac{\partial}{\partial p_2} p_2(p_1, p_2) \right)^2 - 2 \left(\frac{\partial}{\partial p_2} p_1(p_1, \right. \\
& p_2) \left. \right) p_2(p_1, p_2) \left(\frac{\partial}{\partial p_2} p_2(p_1, p_2) \right) + 4 \left(\frac{\partial}{\partial p_2} p_1(p_1, p_2) \right) p_2(p_1, p_2) p_1 \\
& + 2 \left(\frac{\partial}{\partial p_2} p_1(p_1, p_2) \right) p_2(p_1, p_2) p_2 - 4 p_2(p_1, p_2) \left(\frac{\partial}{\partial p_1} p_1(p_1, p_2) \right) p_1 - 2 p_1(p_1, \\
& p_2) \left(\frac{\partial}{\partial p_2} p_2(p_1, p_2) \right)^2 + 4 p_1(p_1, p_2) \left(\frac{\partial}{\partial p_2} p_2(p_1, p_2) \right) p_1 + 2 p_1(p_1, \\
& p_2) \left(\frac{\partial}{\partial p_2} p_2(p_1, p_2) \right) p_2 - 4 p_1(p_1, p_2) \left(\frac{\partial}{\partial p_1} p_2(p_1, p_2) \right) p_1 - 4 \left(\frac{\partial}{\partial p_2} p_1(p_1, \right. \\
& p_2) \left. \right) p_2(p_1, p_2) - 4 p_1(p_1, p_2) \left(\frac{\partial}{\partial p_2} p_2(p_1, p_2) \right) + \left(\frac{\partial}{\partial p_2} p_2(p_1, p_2) \right)^2 \\
& - 4 \left(\frac{\partial}{\partial p_2} p_2(p_1, p_2) \right) p_1 - 2 \left(\frac{\partial}{\partial p_2} p_2(p_1, p_2) \right) p_2 + 4 \left(\frac{\partial}{\partial p_1} p_2(p_1, p_2) \right) p_1 + 4 p_1^2 \\
& + 4 p_1 p_2 + p_2^2 + 4 \left(\frac{\partial}{\partial p_2} p_2(p_1, p_2) \right) - 8 p_1 - 4 p_2 + 4 \Big)^{1/2}, \frac{1}{2} \left(\frac{\partial}{\partial p_2} p_1(p_1, \right. \\
& p_2) \left. \right) p_2(p_1, p_2) + \frac{1}{2} p_1(p_1, p_2) \left(\frac{\partial}{\partial p_2} p_2(p_1, p_2) \right) - \frac{1}{2} \frac{\partial}{\partial p_2} p_2(p_1, p_2) - p_1 - \frac{1}{2} p_2 \\
& + 1 \\
& - \frac{1}{2} \left(\left(\frac{\partial}{\partial p_2} p_1(p_1, p_2) \right)^2 p_2(p_1, p_2)^2 + 2 \left(\frac{\partial}{\partial p_2} p_1(p_1, p_2) \right) p_2(p_1, \right. \\
& p_2) p_1(p_1, p_2) \left(\frac{\partial}{\partial p_2} p_2(p_1, p_2) \right) + p_1(p_1, p_2)^2 \left(\frac{\partial}{\partial p_2} p_2(p_1, p_2) \right)^2 - 2 \left(\frac{\partial}{\partial p_2} p_1(p_1, \right. \\
& p_2) \left. \right) p_2(p_1, p_2) \left(\frac{\partial}{\partial p_2} p_2(p_1, p_2) \right) + 4 \left(\frac{\partial}{\partial p_2} p_1(p_1, p_2) \right) p_2(p_1, p_2) p_1 \\
& + 2 \left(\frac{\partial}{\partial p_2} p_1(p_1, p_2) \right) p_2(p_1, p_2) p_2 - 4 p_2(p_1, p_2) \left(\frac{\partial}{\partial p_1} p_1(p_1, p_2) \right) p_1 - 2 p_1(p_1, \\
& p_2) \left(\frac{\partial}{\partial p_2} p_2(p_1, p_2) \right)^2 + 4 p_1(p_1, p_2) \left(\frac{\partial}{\partial p_2} p_2(p_1, p_2) \right) p_1 + 2 p_1(p_1, \\
& p_2) \left(\frac{\partial}{\partial p_2} p_2(p_1, p_2) \right) p_2 - 4 p_1(p_1, p_2) \left(\frac{\partial}{\partial p_1} p_2(p_1, p_2) \right) p_1 - 4 \left(\frac{\partial}{\partial p_2} p_1(p_1, \right. \\
& p_2) \left. \right) p_2(p_1, p_2) - 4 p_1(p_1, p_2) \left(\frac{\partial}{\partial p_2} p_2(p_1, p_2) \right) + \left(\frac{\partial}{\partial p_2} p_2(p_1, p_2) \right)^2 \\
& - 4 \left(\frac{\partial}{\partial p_2} p_2(p_1, p_2) \right) p_1 - 2 \left(\frac{\partial}{\partial p_2} p_2(p_1, p_2) \right) p_2 + 4 \left(\frac{\partial}{\partial p_1} p_2(p_1, p_2) \right) p_1 + 4 p_1^2
\end{aligned}$$

$$+ 4 p1 p2 + p2^2 + 4 \left(\frac{\partial}{\partial p2} p2(p1, p2) \right) - 8 p1 - 4 p2 + 4 \Big)^{1/2}$$

with(linalg) : with(VectorCalculus) :

Jm := Jacobian([f1(x, y), f2(x, y)], [x, y]);

$$\left[\begin{array}{c} -2x - y + 2, -x \end{array} \right], \quad (9)$$

$$\left[\left(\frac{\partial}{\partial x} x(x, y) \right) y(x, y) + x(x, y) \left(\frac{\partial}{\partial x} y(x, y) \right) - \left(\frac{\partial}{\partial x} y(x, y) \right), \left(\frac{\partial}{\partial y} x(x, y) \right) y(x, y) + x(x, y) \left(\frac{\partial}{\partial y} y(x, y) \right) - \left(\frac{\partial}{\partial y} y(x, y) \right) \right]$$

Jm1 := Jacobian([2·x - x² - x·y, -y + x·y], [x, y]);

$$\left[\begin{array}{cc} -2x - y + 2 & -x \\ y & x - 1 \end{array} \right] \quad (10)$$

A := subs([x=p1, y=p2], Jm1);

$$\left[\begin{array}{cc} -2p1 - p2 + 2 & -p1 \\ p2 & p1 - 1 \end{array} \right] \quad (11)$$

eigenvalues(A);

$$-\frac{1}{2} p1 - \frac{1}{2} p2 + \frac{1}{2} + \frac{1}{2} \sqrt{9 p1^2 + 2 p1 p2 + p2^2 - 18 p1 - 6 p2 + 9}, -\frac{1}{2} p1 - \frac{1}{2} p2 + \frac{1}{2} - \frac{1}{2} \sqrt{9 p1^2 + 2 p1 p2 + p2^2 - 18 p1 - 6 p2 + 9} \quad (12)$$

with(DEtools);

$$\begin{aligned} &[AreSimilar, Closure, DEnormal, DEplot, DEplot3d, DEplot_polygon, DFactor, DFactorLCLM, \\ &DFactorsols, Dchangevar, Desingularize, FunctionDecomposition, GCRD, Gosper, Heunsols, \\ &Homomorphisms, IVPsol, IsHyperexponential, LCLM, MeijerGsols, \\ &MultiplicativeDecomposition, ODEInvariants, PDEchangecoords, PolynomialNormalForm, \\ &RationalCanonicalForm, ReduceHyperexp, RiemannPsols, Xchange, Xcommutator, Xgauge, \\ &Zeilberger, abelsol, adjoint, autonomous, bernoullisol, buildsol, buildsym, canoni, caseplot, \\ &casesplit, checkrank, chinisol, clairautsol, constcoeffsols, convertAlg, convertsys, dalembertsol, \\ &dcoeffs, de2diffop, dfieldplot, diff_table, diffop2de, dperiodic_sols, dpolyform, dsubs, eigenring, \\ &endomorphism_charpoly, equinv, eta_k, eulersols, exactsol, expsols, exterior_power, firint, \\ &firtest, formal_sol, gen_exp, generate_ic, genhomosol, gensys, hamilton_eqs, hypergeomsols, \\ &hyperode, indicialeq, infgen, initialdata, integrate_sols, intfactor, invariants, kovaciesols, \\ &leftdivision, liesol, line_int, linearsol, matrixDE, matrix_riccati, maxdimsystems, moser_reduce, \\ &muchange, mult, mutest, newton_polygon, normalG2, ode_int_y, ode_y1, odeadvisor, odepde, \\ ¶metricsol, particularsol, phaseportrait, poincare, polysols, power_equivalent, \\ &rational_equivalent, ratsols, redode, reduceOrder, reduce_order, regular_parts, regularsp, \end{aligned} \quad (13)$$

remove_RootOf, riccati_system, riccatisol, rifread, rifsimp, righthdivision, rtaylor, separablesol, singularities, solve_group, super_reduce, symgen, symmetric_power, symmetric_product, symtest, transinv, translate, untranslate, varparam, zoom]

$$eq1 := 2 \cdot x(t) - x(t)^2 - x(t) \cdot y(t) = 0;$$

$$2x(t) - x(t)^2 - x(t)y(t) = 0 \quad (14)$$

$$eq2 := -y(t) + x(t) \cdot y(t) = 0;$$

$$-y(t) + x(t)y(t) = 0 \quad (15)$$

`dfieldplot([eq1, eq2], [x(t), y(t)], t=-3..3, x=-1..-1, y=2.2..1.2);`

Error, unable to match delimiters

`dfieldplot([eq1, eq2], [x(t), y(t)], t=-3..3, x=-1..-1, y=2.2..1.2);`

`dfieldplot([eq1, eq2], [x(t), y(t)], t=-3..3, x=-1..-1, y=2.2..1.2);`

Error, (in DEtools/dfieldplot) Invalid range for x

`dfieldplot([eq1, eq2], [x(t), y(t)], t=-3..3, x=-1..2.2, y=-1..1.2);`

Error, (in DEtools/dfieldplot) cannot produce plot, non-autonomous DE (s) require initial conditions.

`dfieldplot([eq1, eq2], [x(t), y(t)], t=-3..3, x=-1..2.2, y=-1..1.2);`

Error, (in DEtools/dfieldplot) cannot produce plot, non-autonomous DE (s) require initial conditions.

`?dfieldplot`

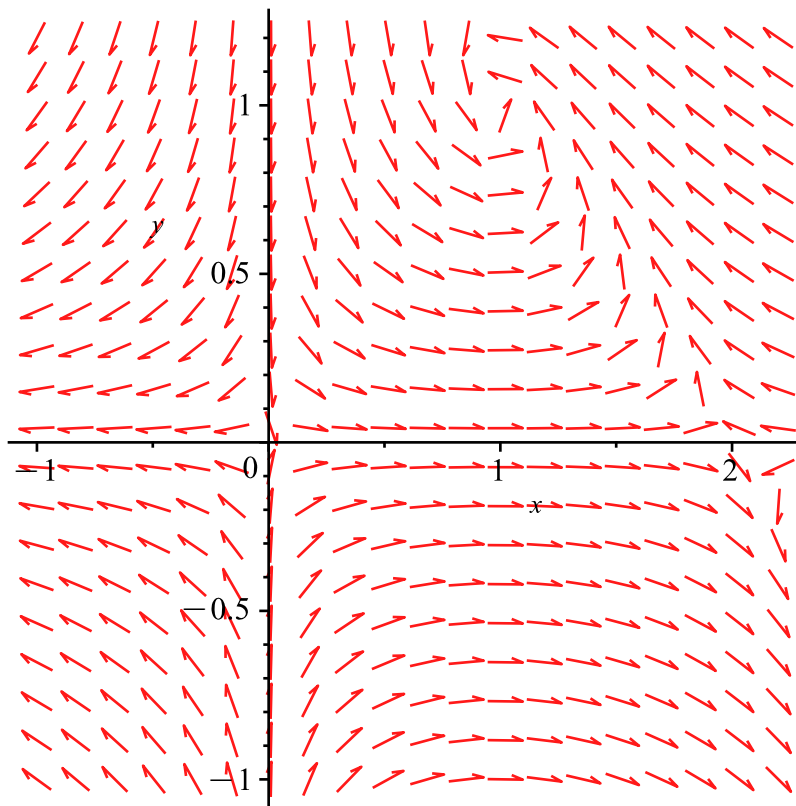
$$eq1 := \text{diff}(x(t), t) = 2 \cdot x(t) - x(t)^2 - x(t) \cdot y(t);$$

$$\frac{d}{dt} x(t) = 2x(t) - x(t)^2 - x(t)y(t) \quad (16)$$

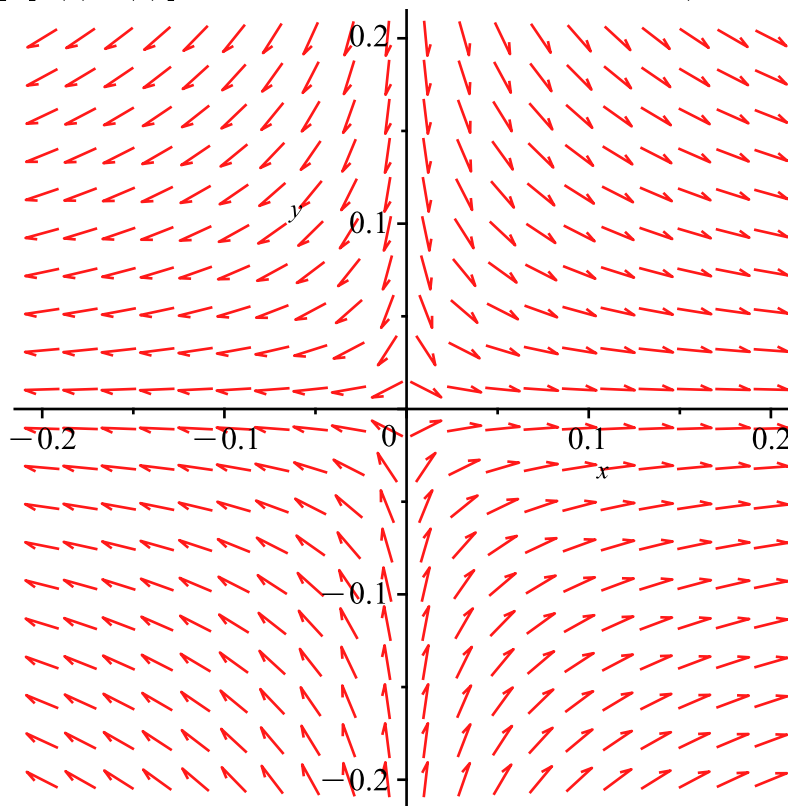
$$eq2 := \text{diff}(y(t), t) = -y(t) + x(t) \cdot y(t);$$

$$\frac{d}{dt} y(t) = -y(t) + x(t)y(t) \quad (17)$$

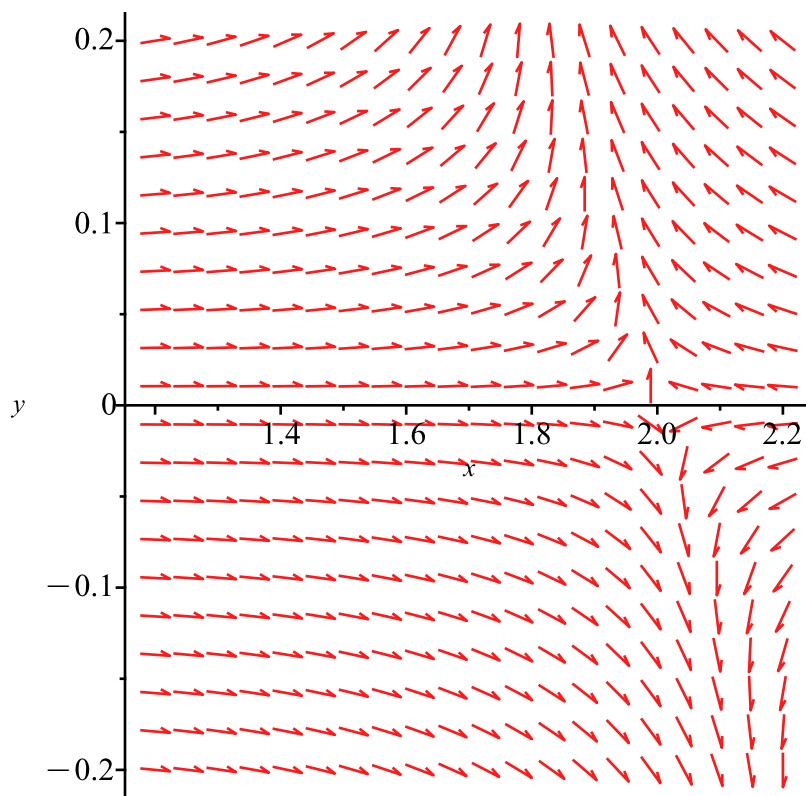
`dfieldplot([eq1, eq2], [x(t), y(t)], t=-3..3, x=-1..2.2, y=-1..1.2);`



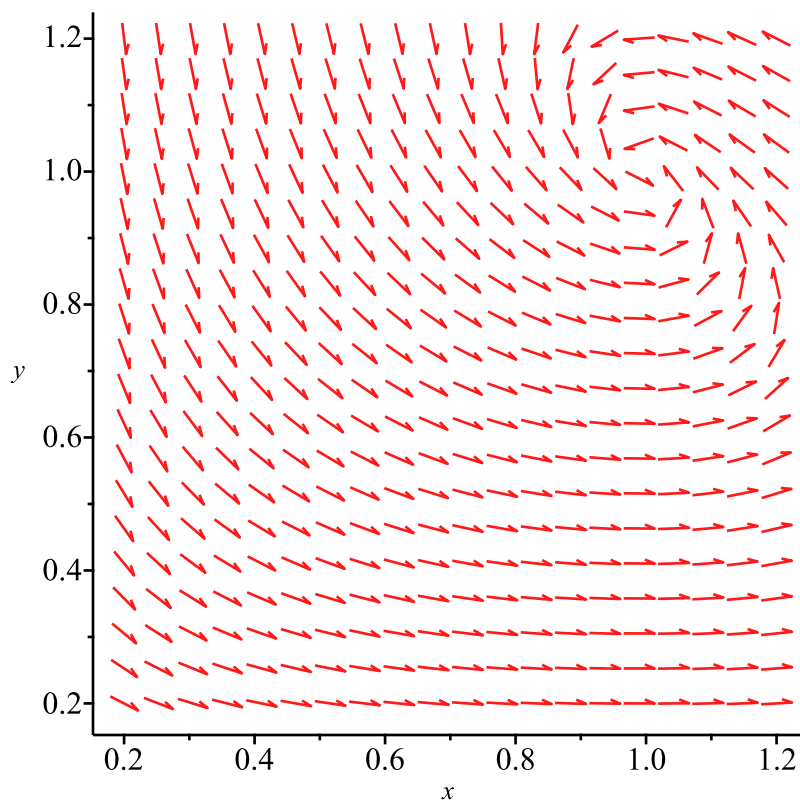
`dfieldplot([eq1, eq2], [x(t), y(t)], t=-3..3, x=-0.2..0.2, y=-0.2..0.2);`



`dfieldplot([eq1, eq2], [x(t), y(t)], t=-3..3, x=1.2..2.2, y=-0.2..0.2);`



`dfieldplot([eq1, eq2], [x(t), y(t)], t=-3..3, x=0.2..1.2, y=0.2..1.2);`

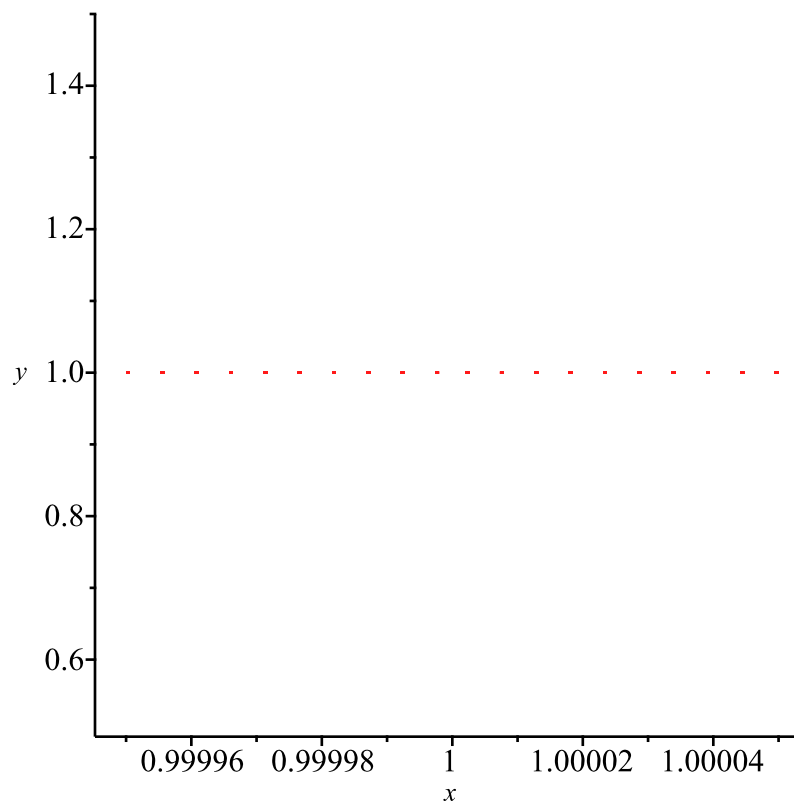


`ic1 := x(0) = 1, y(0) = 1;`

$x(0) = 1, y(0) = 1$

(18)

`DEplot([eq1, eq2], [x(t), y(t)], t=0..1, [[ic1]]);`

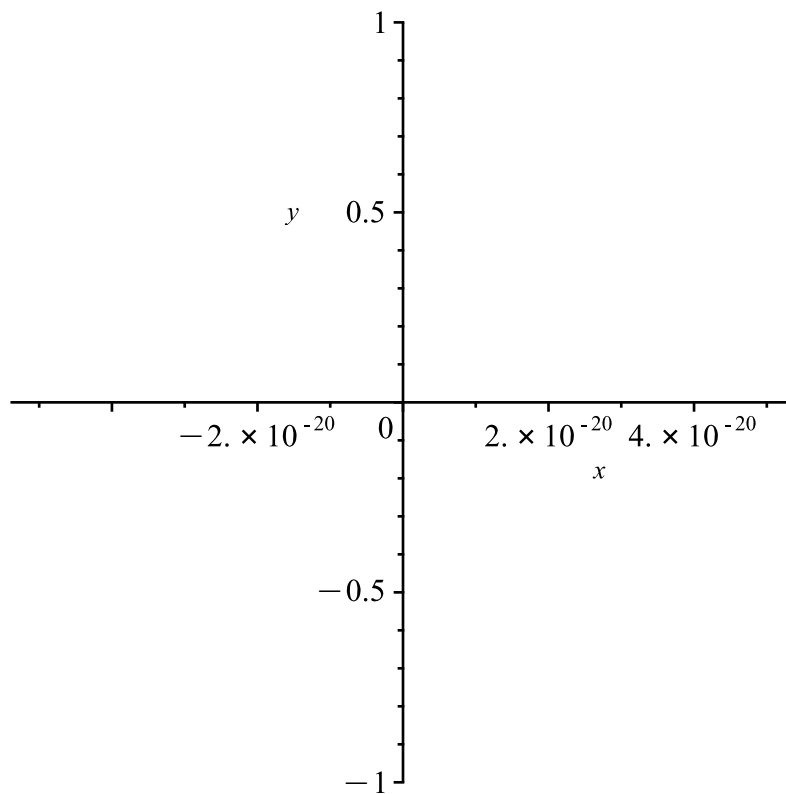


$ic1 := x(0) = 0, y(0) = 0;$

$x(0) = 0, y(0) = 0$

(19)

$DEplot([eq1, eq2], [x(t), y(t)], t = 0 .. 1, [[ic1]]);$

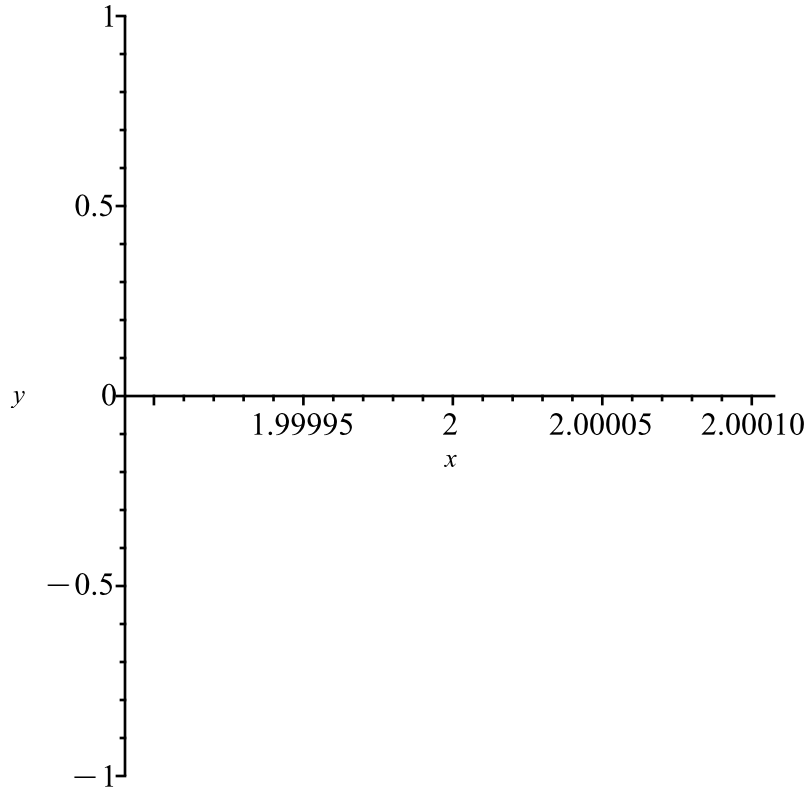


$ic1 := x(0) = 2, y(0) = 0;$

$x(0) = 2, y(0) = 0$

(20)

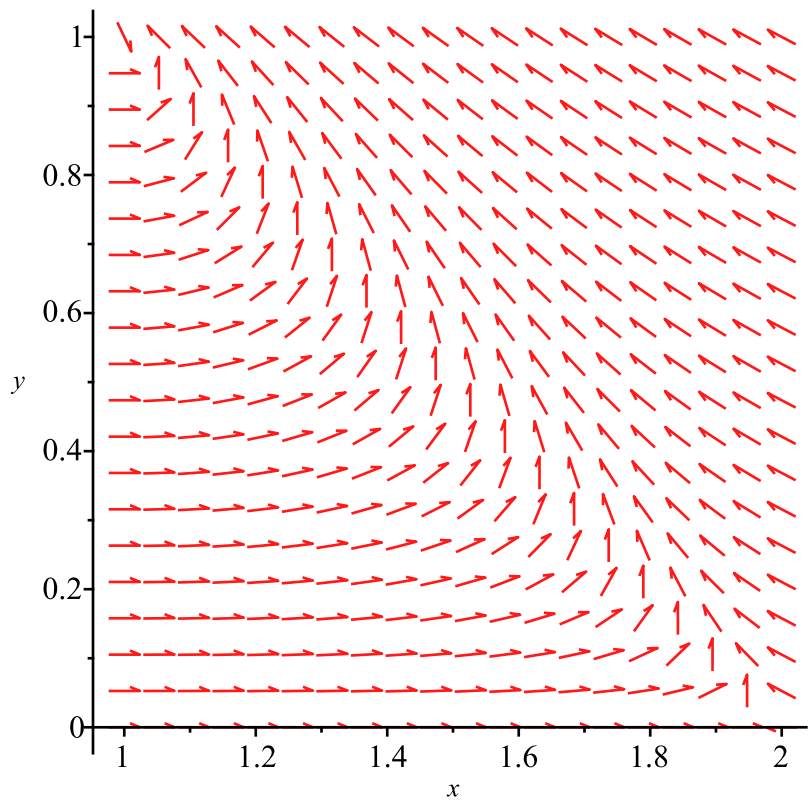
```
DEplot([eq1, eq2], [x(t), y(t)], t=0..10, [[ic1]]);
```



```
DEplot([eq1, eq2], [x(t), y(t)], t=0..1, [x(0)=1, y(0)=1]);
```

Error, (in DEtools/DEplot/CheckInitial) too few initial conditions:
[x(0) = 1]

```
DEplot([eq1, eq2], [x(t), y(t)], t=0..1, [[x(0)=1, y(0)=1], [x(0)=2, y(0)=0]]);
```



$$eq1 := 'eq1'$$

eq1 (21)

$$eq1 := diff(x(t), t) = x(t) - 2 \cdot x(t) \cdot y(t);$$

$$\frac{d}{dt} x(t) = x(t) - 2 x(t) y(t)$$

(22)

$$eq2 := 'eq2'$$

eq2 (23)

$$eq2 := diff(y(t), t) = \frac{x(t)^2}{2} - y(t);$$

$$\frac{d}{dt} y(t) = \frac{1}{2} x(t)^2 - y(t)$$

(24)

$$dsolve(\{eq1, eq2\}, \{x(t), y(t)\});$$

$$[\{x(t) = 0\}, \{y(t) = _Cl\ e^{-t}\}], \left[\left\{ x(t) = _a \ \&where \left[\left\{ \left(\frac{d}{d_a} _b(_a) \right) _b(_a) \right. \right. \right. \right.$$

(25)

$$\left. \left. \left. - \frac{_a^4 + _b(_a)^2 - _a _b(_a) + _a^2}{_a} = 0 \right\}, \left\{ _a = x(t), _b(_a) = \frac{d}{dt} x(t) \right\}, \left\{ t = \right. \right.$$

$$\left. \left. \int \frac{1}{_b(_a)} \ d_a + _Cl, x(t) = _a \right\} \right] \right], \left\{ y(t) = \frac{1}{2} \frac{- \left(\frac{d}{dt} x(t) \right) + x(t)}{x(t)} \right\} \right]$$