Laboratory test

Exercise 1

$eq1 := diff(x(t), t) = x(t) + y(t) + t - 1;$

$$\frac{d}{dt}\, x(t) = x(t) + y(t) + t - 1 \tag{1}$$

$eq2 := diff(y(t), t) = -2\cdot x(t) + 4\cdot y(t) + \exp(1)^t;$

$$\frac{d}{dt}\, y(t) = -2\, x(t) + 4\, y(t) + (e)^t \tag{2}$$

$dsolve(\{eq1, eq2\}, \{x(t), y(t)\});$

$$\left\{x(t) = e^{3t}\,\_C2 + e^{2t}\,\_C1 + \frac{1}{2}\, e^t - \frac{2}{3}\, t + \frac{5}{18}, y(t) = 2\, e^{3t}\,\_C2 + e^{2t}\,\_C1 + \frac{1}{18} - \frac{1}{3}\, t\right\} \tag{3}$$

Exercise 2

$eq1 :=' eq1';$

$$eq1 \tag{4}$$

$eq1 := diff(x(t), t) = y(t) + x(t)^2;$

$$\frac{d}{dt}\, x(t) = y(t) + x(t)^2 \tag{5}$$

$eq2 :=' eq2';$

$$eq2 \tag{6}$$

$eq2 := diff(y(t), t) = -x(t) + x(t)\cdot y(t);$

$$\frac{d}{dt}\, y(t) = -x(t) + x(t)\, y(t) \tag{7}$$

a) Find its equilibria

$f1 := y + x^2;$

$$x^2 + y \tag{8}$$

$f2 := -x + x\cdot y;$

$$x\, y - x \tag{9}$$

$solve(\{f1, f2\}, \{x, y\});$

$$\{x = 0, y = 0\}, \{x = RootOf(\_Z^2 + 1), y = 1\} \tag{10}$$

b)Find the matrix of the linearized system around (0,0) and it's eigenvalues. Check if (0,0) is a hyperbolic equilibrium point

$f1 := (x, y) \rightarrow y + x^2;$

$$(x, y) \rightarrow y + x^2 \tag{11}$$

$f2 := (x, y) \rightarrow -x + x\cdot y;$

$$(x, y) \rightarrow -x + y\, x \tag{12}$$

$with(linalg) : with(VectorCalculus) :$
$Jm := Jacobian([f1(x, y), f2(x, y)], [x, y]);$

$$\begin{bmatrix} 2x & 1 \\ y-1 & x \end{bmatrix} \quad \textbf{(13)}$$

$A := subs([x=0, y=0], Jm);$

$$\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \quad \textbf{(14)}$$

$eigenvalues(A);$

$$\mathrm{I}, -\mathrm{I} \quad \textbf{(15)}$$

(0,0) is not a hyperbolic equilibrium as the real parts of both eigenvalues are 0

c) Find the general solution of the cartesian differential equation of the orbits
$eq := 'eq';$

$$eq \quad \textbf{(16)}$$

$eq := diff(y(x), x) = \frac{(-x + x \cdot y(x))}{y(x) + x^2};$

$$\frac{\mathrm{d}}{\mathrm{d}x}\, y(x) = \frac{-x + x\, y(x)}{y(x) + x^2} \quad \textbf{(17)}$$

$sol := dsolve(eq, y(x));$

$$y(x) = -\frac{1}{2}\,\frac{-2\,\_C1 - 1 + \sqrt{2\,\_C1\,x^2 + 2\,\_C1 + 1}}{\_C1}, y(x) = \frac{1}{2}\,\frac{2\,\_C1 + 1 + \sqrt{2\,\_C1\,x^2 + 2\,\_C1 + 1}}{\_C1} \quad \textbf{(18)}$$

$ysolexpr := rhs(sol[1]);$

$$-\frac{1}{2}\,\frac{-2\,\_C1 - 1 + \sqrt{2\,\_C1\,x^2 + 2\,\_C1 + 1}}{\_C1} \quad \textbf{(19)}$$

$solexpr := ysolexpr - y;$

$$-\frac{1}{2}\,\frac{-2\,\_C1 - 1 + \sqrt{2\,\_C1\,x^2 + 2\,\_C1 + 1}}{\_C1} - y \quad \textbf{(20)}$$

$solve(solexpr, \_C1);$

$$\frac{1}{2}\,\frac{x^2 + 2\,y - 1}{y^2 - 2\,y + 1} \quad \textbf{(21)}$$

$H := (x, y) \rightarrow \frac{1}{2}\,\frac{x^2 + 2\,y - 1}{y^2 - 2\,y + 1};$

$$(x, y) \rightarrow \frac{1}{2}\,\frac{x^2 + 2\,y - 1}{y^2 - 2\,y + 1} \quad \textbf{(22)}$$

Check whether H(x,y) is a first integral
$x := 'x'; y := 'y'$

$$y \quad \textbf{(23)}$$

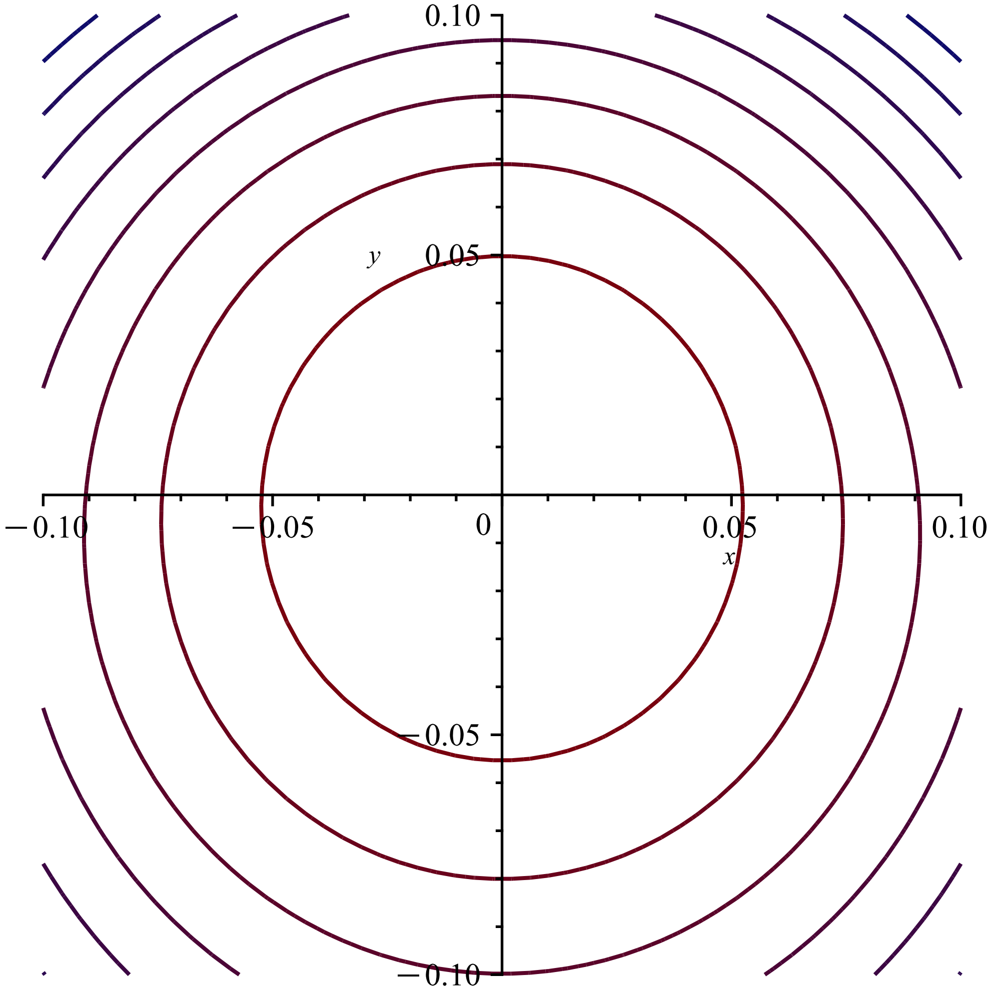
$simplify\big(diff(H(x, y), x) \cdot (y + x^2) + diff(H(x, y), y) \cdot (-x + x \cdot y)\big);$

$$0 \qquad (24)$$

e) Plot the level curves of H in the box [-0.1,0.1] x [-0.1,0.1]

$with(plots):$

$contourplot(H(x, y), x = -0.1 ..0.1, y = -0.1 ..0.1);$



*e) Plot the level curves of H in the box [-0.1,0