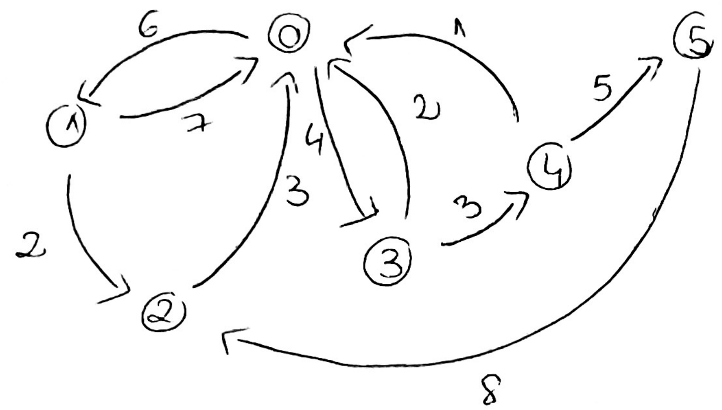
Problem 7. Floyd-Warshall algorithm.

D_i = cost(distance) matrix
P_i = path matrix



$K=0$: $D_0 = \begin{pmatrix} 0 & 6 & \infty & 4 & \infty & \infty \\ 7 & 0 & 2 & \infty & \infty & \infty \\ 3 & \infty & 0 & \infty & \infty & \infty \\ 2 & \infty & \infty & 0 & 3 & \infty \\ 1 & \infty & \infty & \infty & 0 & 5 \\ \infty & \infty & 8 & \infty & \infty & 0 \end{pmatrix}$; $P_0 = \begin{pmatrix} 0 & 1 & 0 & 3 & 0 & 0 \\ 0 & 1 & 2 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 3 & 4 & 0 \\ 0 & 0 & 0 & 0 & 4 & 5 \\ 0 & 0 & 2 & 0 & 0 & 5 \end{pmatrix}$

$K=1$: Using vertex 1 as an intermediate vertex

$D_1 = \begin{pmatrix} 0 & 6 & \infty & 4 & \infty & \infty \\ 7 & 0 & 2 & 11 & \infty & \infty \\ 3 & 9 & 0 & 7 & \infty & \infty \\ 2 & 8 & \infty & 0 & 3 & \infty \\ 1 & 7 & \infty & 5 & 0 & 5 \\ \infty & \infty & 8 & \infty & \infty & 0 \end{pmatrix}$; $P_1 : \begin{pmatrix} 0 & 1 & 0 & 3 & 0 & 0 \\ 0 & 1 & 2 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 3 & 4 & 0 \\ 0 & 0 & 0 & 0 & 4 & 5 \\ 0 & 0 & 2 & 0 & 0 & 5 \end{pmatrix}$

$K=2$: Using vertex 2 as an intermediate vertex

$D_2 = \begin{pmatrix} 0 & 6 & 8 & 4 & \infty & \infty \\ 7 & 0 & 2 & 11 & \infty & \infty \\ 3 & 9 & 0 & 7 & \infty & \infty \\ 2 & 8 & 10 & 0 & 3 & \infty \\ 1 & 7 & 9 & 5 & 0 & 5 \\ \infty & \infty & 8 & \infty & \infty & 0 \end{pmatrix}$; $P_2 = \begin{pmatrix} 0 & 1 & 1 & 3 & 0 & 0 \\ 0 & 1 & 2 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 3 & 4 & 0 \\ 0 & 0 & 0 & 0 & 4 & 5 \\ 0 & 0 & 2 & 0 & 0 & 5 \end{pmatrix}$

k=3: Using vertex 3 as an intermediate vertex

$$D_3 = \begin{pmatrix} 0 & 6 & 8 & 4 & \infty & \infty \\ 5 & 0 & 2 & 9 & \infty & \infty \\ 3 & 9 & 0 & 7 & \infty & \infty \\ 2 & 8 & 10 & 0 & 3 & \infty \\ 1 & 7 & 9 & 5 & 0 & 5 \\ 11 & 17 & 8 & 15 & \infty & 0 \end{pmatrix} ; \quad P_3 = \begin{pmatrix} 0 & 1 & 1 & 3 & 0 & 0 \\ 2 & 1 & 2 & 2 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 3 & 4 & 0 \\ 0 & 0 & 0 & 0 & 4 & 5 \\ 2 & 2 & 2 & 2 & 0 & 5 \end{pmatrix}$$

k=4. Using vertex 4 as an intermediate vertex

$$D_4 = \begin{pmatrix} 0 & 6 & 8 & 4 & 7 & \infty \\ 5 & 0 & 2 & 9 & 12 & \infty \\ 3 & 9 & 0 & 7 & 10 & \infty \\ 2 & 8 & 10 & 0 & 3 & \infty \\ 1 & 7 & 9 & 5 & 0 & 5 \\ 11 & 17 & 8 & 15 & 18 & 0 \end{pmatrix} ; \quad P_4 = \begin{pmatrix} 0 & 1 & 1 & 3 & 3 & 0 \\ 2 & 1 & 2 & 2 & 2 & 0 \\ 0 & 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 3 & 4 & 0 \\ 0 & 0 & 0 & 0 & 4 & 5 \\ 2 & 2 & 2 & 2 & 2 & 5 \end{pmatrix}$$

k=5: Using vertex 5 as an intermediate vertex

$$D_5 = \begin{pmatrix} 0 & 6 & 8 & 4 & 7 & 12 \\ 5 & 0 & 2 & 9 & 12 & 17 \\ 3 & 9 & 0 & 7 & 10 & 15 \\ 2 & 8 & 10 & 0 & 3 & 8 \\ 1 & 7 & 9 & 5 & 0 & 5 \\ 11 & 17 & 8 & 15 & 18 & 0 \end{pmatrix} ; \quad P_5 = \begin{pmatrix} 0 & 1 & 1 & 3 & 3 & 3 \\ 2 & 1 & 2 & 2 & 2 & 2 \\ 0 & 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 3 & 4 & 4 \\ 0 & 0 & 0 & 0 & 4 & 5 \\ 2 & 2 & 2 & 2 & 2 & 5 \end{pmatrix}$$

$D_5(3,5)=8 \Rightarrow$ The minimum cost walk from 3 to 5 has the cost $D_5(3,5)=8$ and it is obtained from P_5.

$s=3$: $P_5(3,5)=4$, $P_5(4,5)=5=t$

The minimum cost walk: $3 \xrightarrow{3} 4 \xrightarrow{5} 5$