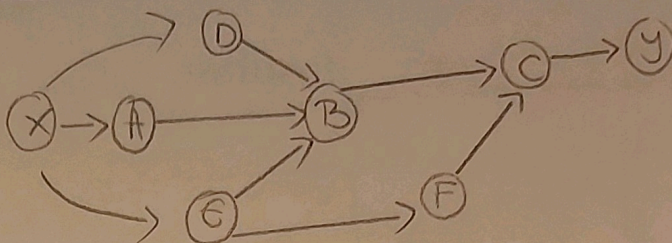


## ACTIVITY SCHEDULING

Considering the following activities, determine the earliest and the latest scheduling, and the critical activities. Show the step by step computations for topological sorting, then for starting and ending times.

Activity	Duration	Prerequisites
A	1	—
B	4	A, D, E
C	1	B, F
D	3	—
E	2	—
F	5	E



Perform a topological sorting using predecessor counting.

Predecessor count for each vertex: A B C D E F  
0 3 2 0 0 1

Queue  $\leftarrow A | D | E \leftarrow$

	X, y	Count: dict	Q: queue	sorted: list
init		A B C D E F 0 3 2 0 0 1	$\leftarrow A   D   E \leftarrow$	[ ]
it. 1	X=A y=B	A B C D E F 0 2 2 0 0 1	$\leftarrow D   E \leftarrow$	[A]
it. 2	X=D y=B	A B C D E F 0 1 2 0 0 1	$\leftarrow E \leftarrow$	[A, D]
it. 3	X=E y=B y=F	A B C D E F 0 0 2 0 0 1 0 0 2 0 0 0	$\leftarrow \leftarrow$ $\leftarrow B \leftarrow$ $\leftarrow B   F \leftarrow$	[A, D, E]
it. 4	X=B y=C	A B C D E F 0 0 1 0 0 0	$\leftarrow F \leftarrow$	[A, D, E, B]
it. 5	X=F y=C	A B C D E F 0 0 0 0 0 0	$\leftarrow \leftarrow$ $\leftarrow C \leftarrow$	[A, D, E, B, F]
it. 6	X=C	—	$\leftarrow \leftarrow$	[A, D, E, B, F, C]

Topological sorting: X A D E B F C Y



## ACTIVITY SCHEDULING

→ Computing the earliest starting ( $t_{em}$ ) and ending ( $t_{em}^*$ ) times

$$t_{em}(x) = t_{em}^*(x) = 0$$

$$t_{em}(A) = \max\{t_{em}^*(x)\} = 0; \quad t_{em}^*(A) = t_{em}(A) + 1 = 1$$

$$t_{em}(D) = \max\{t_{em}^*(x)\} = 0; \quad t_{em}^*(D) = t_{em}(D) + 3 = 3$$

$$t_{em}(E) = \max\{t_{em}^*(x)\} = 0; \quad t_{em}^*(E) = t_{em}(E) + 2 = 2$$

$$t_{em}(B) = \max\{t_{em}^*(A), t_{em}^*(D), t_{em}^*(E)\} = 3; \quad t_{em}^*(B) = t_{em}(B) + 4 = 7$$

$$t_{em}(F) = \max\{t_{em}^*(E)\} = 2; \quad t_{em}^*(F) = t_{em}(F) + 5 = 7$$

$$t_{em}(C) = \max\{t_{em}^*(B), t_{em}^*(F)\} = 7; \quad t_{em}^*(C) = t_{em}(C) + 1 = 8$$

$$t_{em}(Y) = \max\{t_{em}^*(C)\} = 8 = t_{em}^*(Y)$$

→ Computing the latest starting ( $t_M$ ) and ending ( $t_M^*$ ) times

$$t_M(Y) = t_M^*(Y) = t_{em}(Y) = 8$$

$$t_M^*(C) = \min\{t_M(Y)\} = 8; \quad t_M(C) = t_M^*(C) - 1 = 7$$

$$t_M^*(F) = \min\{t_M(C)\} = 7; \quad t_M(F) = t_M^*(F) - 5 = 2$$

$$t_M^*(B) = \min\{t_M(C)\} = 7; \quad t_M(B) = t_M^*(B) - 4 = 3$$

$$t_M^*(E) = \min\{t_M(B), t_M(F)\} = 2; \quad t_M(E) = t_M^*(E) - 2 = 0$$

$$t_M^*(D) = \min\{t_M(B)\} = 3; \quad t_M(D) = t_M^*(D) - 3 = 0$$

$$t_M^*(A) = \min\{t_M(B)\} = 3; \quad t_M(A) = t_M^*(A) - 1 = 2$$

$$t_M^*(X) = \min\{t_M(A), t_M(D), t_M(E)\} = 0 = t_M(X)$$

Critical activity:  $t_{em}(U) = t_M(U)$

↳ C, F, B, E, D (and x, y - fictive auxiliary activities)