Aufgabe 2

$$h(t) = \begin{cases} \frac{\pi - t}{2} & 0 < t < 2\pi \\ 0 & t = 0,\ 2\pi \end{cases}$$

$$a_0 = \frac{1}{\pi} \int_0^{2\pi} \frac{\pi - t}{2} \cos(0)\, dt = \frac{1}{2\pi} \left[\pi t - \frac{1}{2} t^2\right]_0^{2\pi}$$

$$= \frac{1}{2\pi} \left(2\pi^2 - \frac{1}{2} 4\pi^2\right) = 0$$

$$a_k = \frac{1}{2\pi} \int_0^{2\pi} (\pi - t) \cos(kt)\, dt$$

$$= \frac{1}{2\pi} \int_0^{2\pi} \pi \cos(kt) - \overset{u}{t}\,\overset{v'}{\cos(kt)}\, dt$$

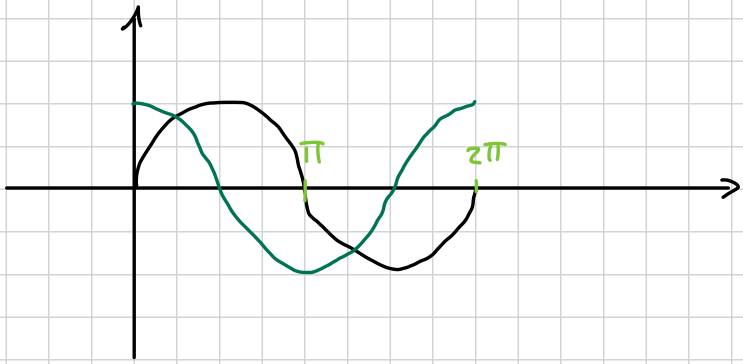
$u' = 1$
$v = \frac{1}{k} \sin(kt)$

$$= \frac{1}{2\pi} \left[\pi \sin(kt) \frac{1}{k} - t \frac{1}{k} \sin(kt)\right]_0^{2\pi} + \int_0^{2\pi} \frac{1}{k} \sin(kt)\, dt$$

$$= \frac{1}{2\pi} \left[\sin(kt) \frac{\pi - t}{k} - \frac{1}{k^2} \cos(kt)\right]_0^{2\pi}$$

$$= \frac{1}{2\pi} \left(0 - \frac{1}{k^2} (1 - 1)\right)$$

$$= 0$$



$$b_k = \frac{1}{2\pi} \int_0^{2\pi} \overset{u}{(\pi - t)} \; \overset{v'}{\sin (kt)} \, dt \qquad u' = 1 \quad v = -\cos(kt)\frac{1}{k}$$

$$= \frac{1}{2\pi} \left[-\frac{\pi}{k} \cos(kt) + \frac{t}{k} \cos(kt) \right]_0^{2\pi} + \int_0^{2\pi} \cos(kt) \frac{1}{k}$$

$$= \frac{1}{2\pi} \left(\underbrace{-\frac{\pi}{k}(1-1)}_{=0} + \frac{2\pi}{k} \cos(k \cdot 2\pi) + \underbrace{\frac{0}{k} \cos(0)}_{=0} \right) + \underbrace{\left[\frac{1}{k^2} \sin(kt) \right]_0^{2\pi}}_{=0}$$

$$= \frac{1}{2\pi} \left(\frac{2\pi}{k} \cdot 1 \right)$$

$$= \frac{1}{k}$$

$$F_n(x) = \frac{1}{1} \sin(x) + \frac{1}{2} \sin(2x) + \frac{1}{3} \sin(3x) + \frac{1}{4} \sin(4x) + \ldots$$