Study of GHZ states:
On the IBM Quantum Compute Platform

Joel Collazo
Computer Science Department
California State University, Chico
Chico, USA

Matthew Martinez
Computer Science Department
California State University, Chico
Chico, USA

Chirag Dhamange
Computer Science Department
California State University, Chico
Chico, USA

Abstract

In quantum computing, a particular state of interest is the GHZ-state or Greenberg-Horne-Zeilinger state. In the GHZ state, all the qubits are entangled with each other. The project focused on creating n-qubit GHZ, 3 <= n <= 11 states and comparing the results achieved with theoretical values. The experiment results showed that as the number of quantum bits in our GHZ circuit increased the reliability of expected outcomes decreased. Two methods by which we found to decrease the likelihood of these errors included choosing qubits with less measurement errors, and changing the control and target qubits of CNOT gates while keeping the rest of the circuit the same.

I. INTRODUCTION

The Greenberger–Horne–Zeilinger state is a state where you can set a quantum circuit to show entanglement. This is done rather simply with a Hadamard gate and a few CNOT gates. These are used to make the quantum bits entangle to show the building blocks of quantum computation. Furthermore this can be extended upon unlike the bell state that has an upper limit of bits, the GHZ(Greenberger–Horne–Zeilinger) state can do more than three bits.

A GHZ state of **N** qubits can be expressed as follows:

$$\boxed{|GHZ_N\rangle = \frac{1}{\sqrt{2}}(|0\rangle^{\otimes N} + |1\rangle^{\otimes N})}$$

For example, for N = 3, the GHZ state will be:

$$\boxed{|GHZ_3\rangle = \frac{1}{\sqrt{2}}(|000\rangle + |111\rangle)}$$

One of the key ideas is to entangle the bits so that the Q bits will be reliant on one another. This occurs from just the single Hadamard and two CNOT gates. This subsystem will produce entanglement where the bits are now in a way "aware" of the states of the other bits. In addition this is where Albert Einstein's quote of "...spooky actions at a distance." really comes into play.

When there is a successful run of the GHZ state, the outcome should be either all 1s or all 0s. Since they are entangled there will be a 50% probability for the circuit to choose one of the two possible values. This shows one of the fundamentals of a quantum circuit where we can now use this state to do more complex circuits.

Our work takes a look at the study of GHZ states. We created n-qubit GHZ, 3 <= n <= 11 states and compared the results achieved with theoretical values. Furthermore, there were even different circuits we found that mathematically they do show the same results so we did take a look into those as well. Additionally, there were also some things that we did learn from the Quantum computer that we will go over in the paper.

II. METHODOLOGY

The experimental setup consisted of using IBM's Quantum Computing Platform. These quantum computers have 127 qubits. To run any quantum circuit, there are 4 available systems to choose from:

- ibm_brisbane
- ibm_sherbrooke
- ibm_osaka, and
- ibm_kyoto.

Along with this, the user also needs to provide the number of shots, i.e., the number of times to run the quantum circuit. These specifications are provided for every n-qubit GHZ state that was created, 3 <= n <= 11. Furthermore, jobs were run when the number of jobs queued was expected to be low so as to minimize the amount of measurement inaccuracy received.

Nine individual experiments were performed with the number of qubits ranging from three to eleven in which a Hadamard gate was assigned to the number zero qubit and each subsequent qubit received a CNOT gate with the control for that gate being assigned to the qubit one number below it. Each CNOT gate was placed one time increment down its respective rail in relation to the previous qubit's CNOT gate. Finally, all qubits were measured and the resulting states of the qubit system were recorded into histogram format.

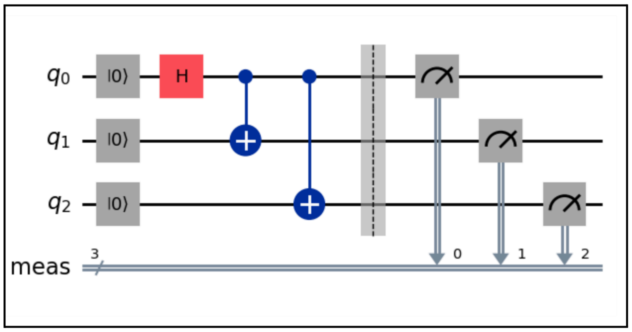

Figure: 3-qubit quantum circuit that creates a 3-qubit GHZ state

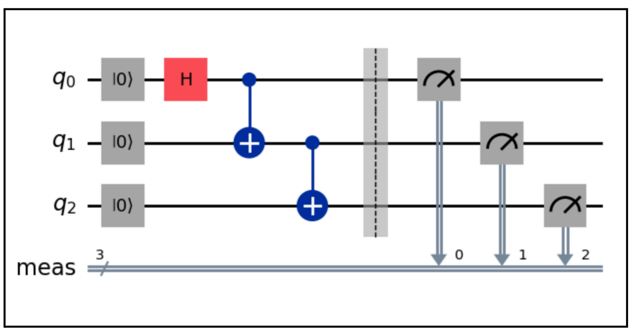

Figure: Another 3-qubit quantum circuit that creates a 3-qubit GHZ state

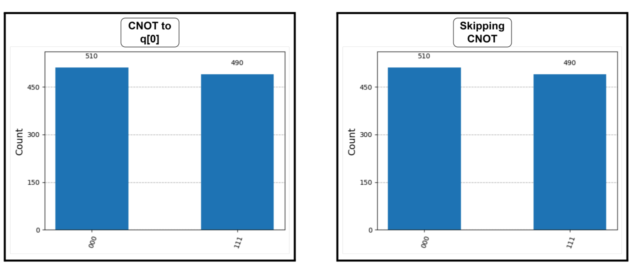

Figure: Histogram of simulations of the above quantum circuits

III. Results

Every quantum circuit for 3 <= n <= 8 has a similar structure. All qubits are initially set to |0>. The Hadamard gate is applied to the top-most or 0^{th} qubit. Then CNOT gates are applied to other qubits such that the target is on the current qubit and the control is on the upper qubit or previous qubit. In the end, the value of every qubit is measured and stored into a classical register that stores either 0s or 1s. The results obtained are as follows:

A. 3 qubit

For the 3-qubit system, the circuit is as shown in Figure 1.

Figure 1: 3-qubit quantum circuit to create a 3-qubit GHZ state

The Circle notation in Figure 3 shows the derivation for the circuit in Figure 2 which is the labeled circuit. It can be seen that the final state has only two values with equal probabilities, i.e., |000> and |111>.

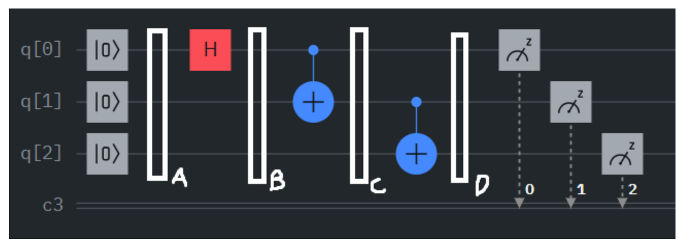

Figure 2: 3-qubit quantum circuit with labeled states to create a 3-qubit GHZ state

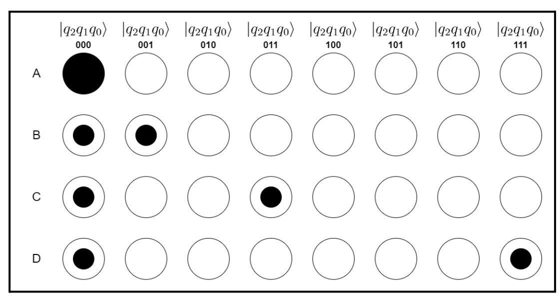

Figure 3: Circle Notation for the 3-qubit quantum circuit that generates a 3-qubit GHZ state

The mathematical derivation is also shown below using Dirac notation below:

i. Initially, all qubits are set to |0>, which is state **A**.

$$\boxed{\begin{array}{c} A = |q_0 q_1 q_2\rangle \\ A = |000\rangle \end{array}}$$

ii. Then Hadamard is applied on q_0, so now q_0 is in superposition of both |0> and |1> states with equal probabilities.

$$\boxed{q_0 = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)}$$

iii. Now, state **B** is calculated as follows:

$$\boxed{\begin{array}{c} B = |q_0 q_1 q_2\rangle \\ B = (\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle))(|0\rangle)(|0\rangle) \\ B = \frac{1}{\sqrt{2}}(|000\rangle + |100\rangle) \end{array}}$$

iv. For state **C**, every time q_0 is |1>, q_1 flips. There is only one possible state where q_0 is |0>, i.e., |100>. Therefore, we have **C** as follows:

$$\boxed{\begin{array}{c} C = |q_0 q_1 q_2\rangle \\ C = \frac{1}{\sqrt{2}}(|000\rangle + |110\rangle) \end{array}}$$

v. Similar to **C**, state **D** is calculated where q_2 flips every time q_1 is |1>. **D** is our final state and its value can be verified from the Circle notation.

$$\boxed{\begin{array}{c} D = |q_0 q_1 q_2\rangle \\ D = \frac{1}{\sqrt{2}}(|000\rangle + |111\rangle) \end{array}}$$

The results obtained for the 3-qubit quantum circuit to create a GHZ state is shown in Figure 4. The circuit ran on the ibm_kyoto quantum computer with 4096 shots. It can be seen that states |000> and |111> have the highest frequencies, $p_{|000>} = 1726 / 4096 = 0.421$ and $p_{|111>} = 1589 / 4096 = 0.388$. These values are pretty close to 50% that is expected, however, when qubits are measured, there exists errors during the measurement. This is one of the many reasons that states |001>, |010> and |110> have some significant frequencies.

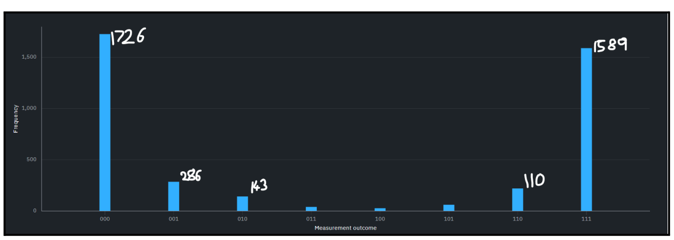

Figure 4: Result of the 3-qubit quantum circuit state upon running on IBM Kyoto Quantum Computer

To mitigate the qubit measurement error, qubits with less measurement errors were chosen and the circuit was formed accordingly. These values are provided in the platform itself for every type of system. The quantum circuit is shown in Figure 5 and this circuit can be called as an optimized version of the circuit in Figure 1. Qubits q[2], q[7] and q[9] had the lowest such values and were chosen to build the circuit. However, the platform automatically chose to use qubits q[7], q[8] and q[9].

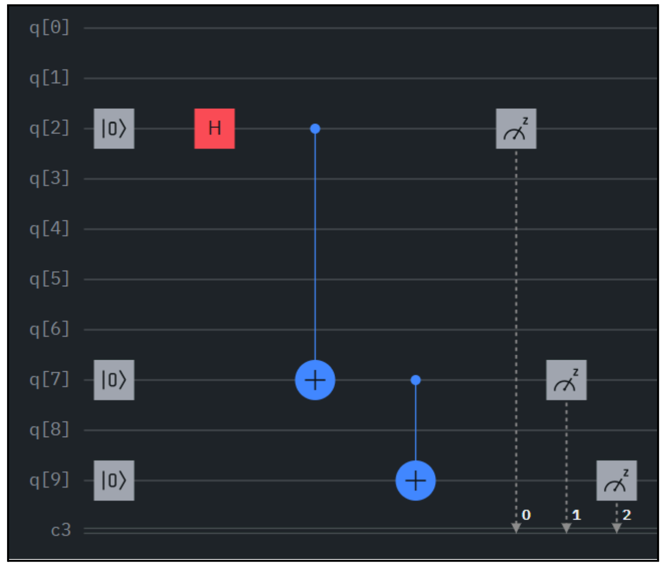

Figure 5: Optimized 3-qubit quantum circuit to create a 3-qubit GHZ state

The results for the circuit in Figure 5 is shown in Figure 6. Clearly, choosing the qubits with low measurement errors gives us higher frequencies (thus, probabilities) for states |000> and |111>. $p_{|000>} = 1991 / 4096 = 0.486$ and $p_{|111>} = 1998 / 4096 = 0.488$. These values are very close to the theoretical probability values of 50%.

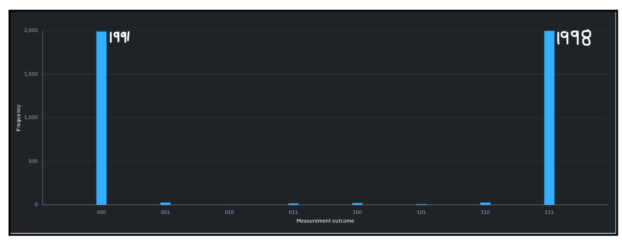

Figure 6: Result of the Optimized 3-qubit quantum circuit upon running on IBM Kyoto Quantum Computer

B. 4-qubit

For the 4-qubit system, the circuit is as shown in Figure 7.

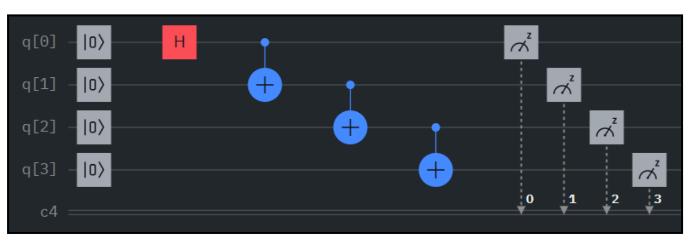

Figure 7: 4-qubit quantum circuit to create a 4-qubit GHZ state

The Circle notation in Figure 9 shows the derivation for the circuit in Figure 8 which is the labeled circuit. It can be seen that the final state has only two values with equal probabilities, i.e., |0000> and |1111>.

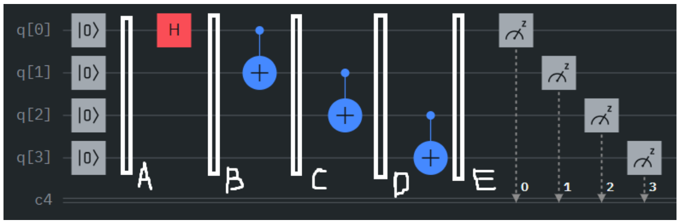

Figure 8: 4-qubit quantum circuit with labeled states to create a 4-qubit GHZ state

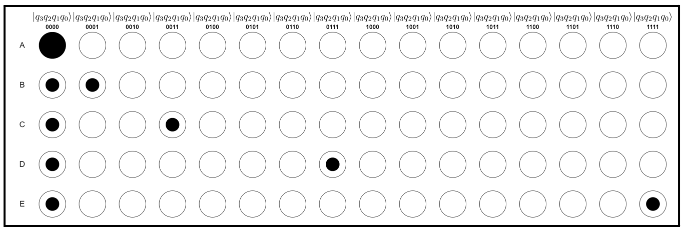

Figure 9: Circle Notation for the 4-qubit quantum circuit that generates a 4-qubit GHZ state

The mathematical derivation is also shown below using Dirac notation below:

i. Initially, all qubits are set to |0>, which is state **A**.

$$\boxed{\begin{gathered}A = |q_0q_1q_2q_3\rangle \\ A = |0000\rangle\end{gathered}}$$

ii. Then Hadamard is applied on q_0, so now q_0 is in superposition of both |0> and |1> states with equal probabilities.

$$\boxed{q_0 = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)}$$

iii. Now, state **B** is calculated as follows:

$$\boxed{\begin{gathered}B = |q_0q_1q_2q_3\rangle \\ B = (\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle))(|0\rangle)(|0\rangle)(|0\rangle) \\ B = \frac{1}{\sqrt{2}}(|0000\rangle + |1000\rangle)\end{gathered}}$$

iv. For state **C**, every time q_0 is |1>, q_1 flips. There is only one possible state where q_0 is |0>, i.e., |1000>. Therefore, we have **C** as follows:

$$\boxed{\begin{gathered}C = |q_0q_1q_2q_3\rangle \\ C = \frac{1}{\sqrt{2}}(|0000\rangle + |1100\rangle)\end{gathered}}$$

v. Similar to **C**, state **D** is calculated where q_2 flips every time q_1 is |1>. **D** is our final state and its value can be verified from the Circle notation.

$$\boxed{\begin{gathered}D = |q_0q_1q_2q_3\rangle \\ D = \frac{1}{\sqrt{2}}(|0000\rangle + |1110\rangle)\end{gathered}}$$

vi. Similar to **C** and **D**, state **E** is calculated where q_3 flips every time q_2 is |1>. **E** is our final state and its value can be verified from the Circle notation.

$$\boxed{\begin{gathered}E = |q_0q_1q_2q_3\rangle \\ E = \frac{1}{\sqrt{2}}(|0000\rangle + |1111\rangle)\end{gathered}}$$

The results obtained for the 4-qubit quantum circuit to create a GHZ state is shown in Figure 10. The circuit ran on the ibm_kyoto quantum computer with 4096 shots. It can be seen that states |0000> and |1111> have the highest frequencies, $p_{|0000>}$ = 1122 / 4096 = 0.274 and $p_{|1111>}$ = 1118 / 4096 = 0.273. These values are very less than 50% which was expected.

Once again, to mitigate the qubit measurement error, qubits with less measurement errors were chosen and the circuit was formed as shown in Figure 11 and this circuit can be called as an optimized version of the circuit in Figure 7. Qubits q[2], q[7], q[9] and q[12] had the lowest such values and were chosen to build the circuit. However, the platform automatically chose to use qubits q[9], q[10], q[11] and q[12].

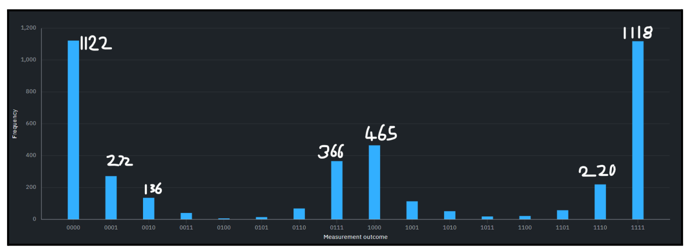

Figure 10: Result of the 4-qubit quantum circuit upon running on IBM Kyoto Quantum Computer

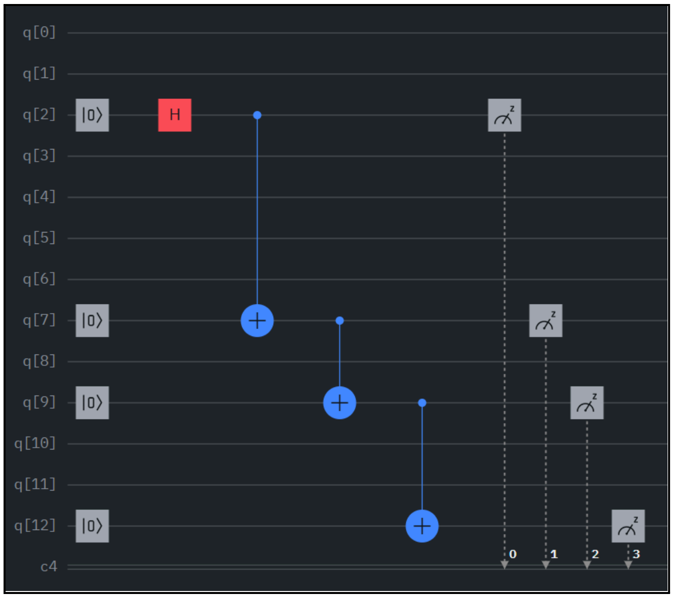

Figure 11: Optimized 4-qubit quantum circuit to create a 4-qubit GHZ state

Figure 12 shows the result of running the circuit in Figure 11 on ibm_kyoto with 4096 shots. The frequencies obtained are pretty close to 50% and much better than the results of the unoptimized 4-qubit circuit. $p_{|0000>}$ = 1862 / 4096 = 0.455 and $p_{|1111>}$ = 1817 / 4096 = 0.444.

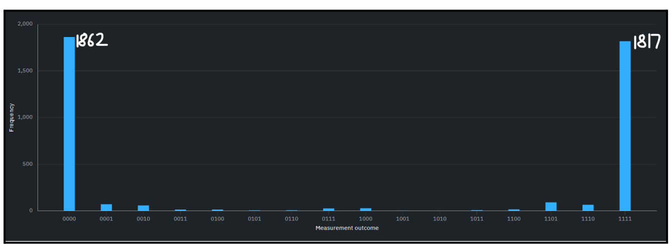

Figure 12: Result of the Optimized 4-qubit quantum circuit upon running on IBM Kyoto Quantum Computer

C. 5-qubit:

For the 5-qubit system, the circuit is as shown in Figure 13.

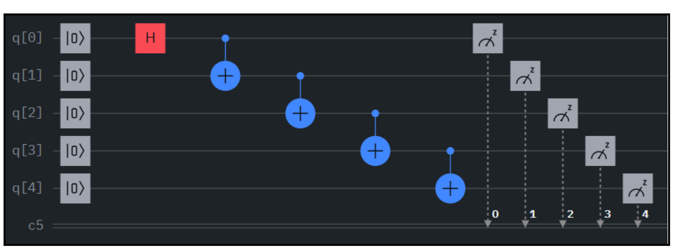

Figure 13: 5-qubit quantum circuit to create a 5-qubit GHZ state

The Circle notation is not shown for this circuit because of a large number of states (32). Instead, only the Dirac notation is used.

The mathematical derivation for the circuit in Figure 14 is shown below using Dirac notation below:

i. Initially, all qubits are set to |0>, which is state **A**.

$$\boxed{\begin{array}{c} A = |q_0q_1q_2q_3q_4\rangle \\ A = |00000\rangle \end{array}}$$

ii. Then Hadamard is applied on q_0, so now q_0 is in superposition of both |0> and |1> states with equal probabilities.

$$\boxed{q_0 = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)}$$

iii. Now, state **B** is calculated as follows:

$$\boxed{\begin{array}{c} B = |q_0q_1q_2q_3q_4\rangle \\ B = (\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle))(|0\rangle)(|0\rangle)(|0\rangle)(|0\rangle) \\ B = \frac{1}{\sqrt{2}}(|00000\rangle + |10000\rangle) \end{array}}$$

iv. For state **C**, every time q_0 is |1>, q_1 flips. There is only one possible state where q_0 is |0>, i.e., |10000>. Therefore, we have **C** as follows:

$$\boxed{\begin{array}{c} C = |q_0q_1q_2q_3q_4\rangle \\ C = \frac{1}{\sqrt{2}}(|00000\rangle + |11000\rangle) \end{array}}$$

v. Similar to **C**, state **D** is calculated where q_2 flips every time q_1 is |1>.

$$\boxed{\begin{array}{c} D = |q_0q_1q_2q_3q_4\rangle \\ D = \frac{1}{\sqrt{2}}(|00000\rangle + |11100\rangle) \end{array}}$$

vi. Similarly, q_3 flips every time q_2 is |1>.

$$\boxed{\begin{array}{c} E = |q_0q_1q_2q_3q_4\rangle \\ E = \frac{1}{\sqrt{2}}(|00000\rangle + |11110\rangle) \end{array}}$$

vii. Similarly, q_4 flips every time q_3 is |1>.

$$\boxed{\begin{array}{c} F = |q_0q_1q_2q_3q_4q_5\rangle \\ F = \frac{1}{\sqrt{2}}(|00000\rangle + |11111\rangle) \end{array}}$$

The results obtained for the 5-qubit quantum circuit to create a GHZ state is shown in Figure 15. The circuit ran on the ibm_kyoto quantum computer with 4096 shots. It can be seen that states |00000> and |11111> have the highest frequencies, $p_{|00000>}$ = 1157 / 4096 = 0.282 and $p_{|11111>}$ = 1142 /

4096 = 0.279. These values are very less than 50% which was expected.

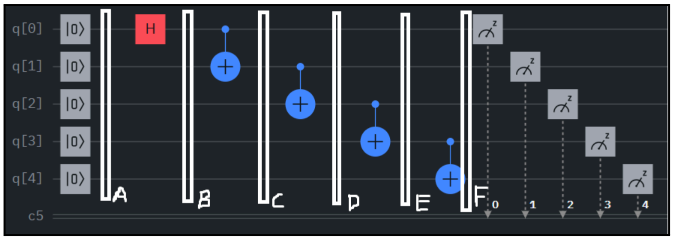

Figure 14: 5-qubit quantum circuit with labeled states to create a 5-qubit GHZ state

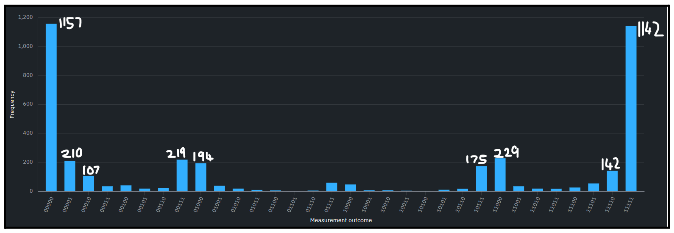

Figure 15: Result of the 5-qubit quantum circuit upon running on IBM Kyoto Quantum Computer

Once again, to mitigate the qubit measurement error, qubits with less measurement errors were chosen and the circuit was formed as shown in Figure 16 and this circuit can be called as an optimized version of the circuit in Figure 13. Qubits q[2], q[7], q[9], q[12] and q[13] had the lowest such values and were chosen to build the circuit. However, the platform automatically chose to use qubits q[9], q[10], q[11], q[12] and q[13].

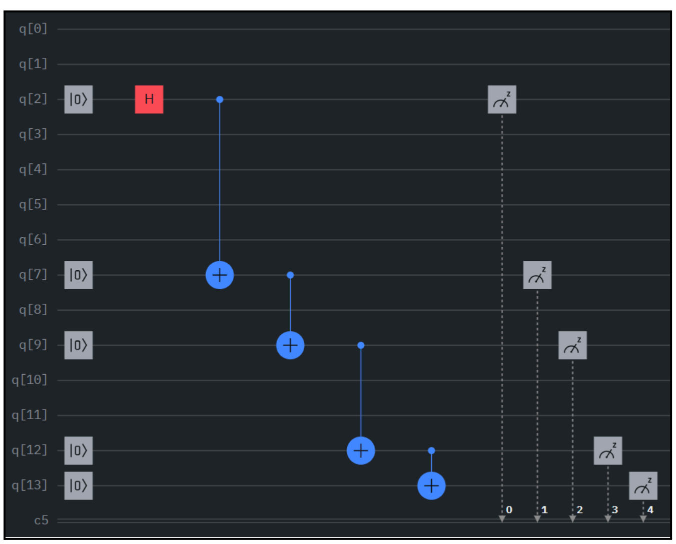

Figure 16: Optimized 5-qubit quantum circuit to create a 5-qubit GHZ state

Figure 17 shows the result of running the circuit in Figure 16 on ibm_kyoto with 4096 shots. The frequencies obtained are pretty close to 50% and much better than the results of the unoptimized 5-qubit circuit. $p_{|00000>} = 1536 / 4096 = 0.375$ and $p_{|11111>} = 1806 / 4096 = 0.441$. It can be seen that there is still state |00010> with significant frequency value.

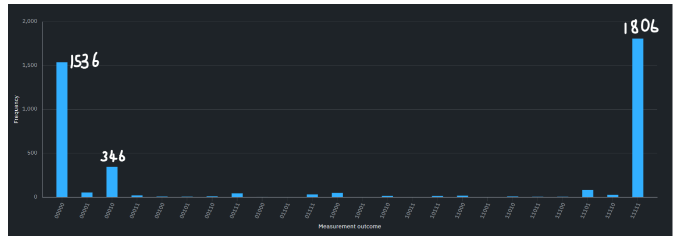

Figure 17: Result of the Optimized 5-qubit quantum circuit upon running on IBM Kyoto Quantum Computer

D. 6 qubit

For the 6-qubit system, the circuit is as shown in Figure 18.

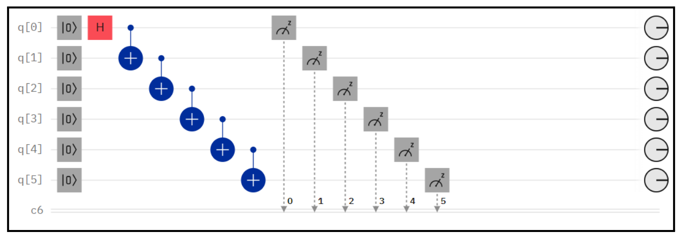

Figure 18: 6-qubit quantum circuit with labeled states to create a 6-qubit GHZ state

The results obtained for the 6-qubit quantum circuit to create a GHZ state is shown in Figure 19. The circuit ran on the ibm_kyoto quantum computer with 4096 shots. It can be seen that states |000000> and |111111> have the highest frequencies. These values are very less than 50% which was expected.

To achieve higher frequencies for these states, an approach was taken where the target and control qubits of the CNOT gates were changed and were arranged as shown in Figure 20. The qubits were kept the same. The results for this quantum circuit are shown in Figure 21. The frequencies of |000000> and |111111> were better and closer to the expected values.

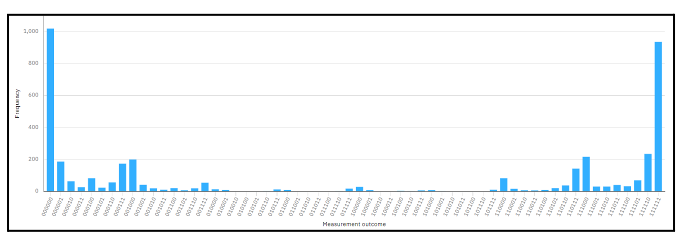

Figure 19: Result of the 6-qubit quantum circuit upon running on IBM Kyoto Quantum Computer

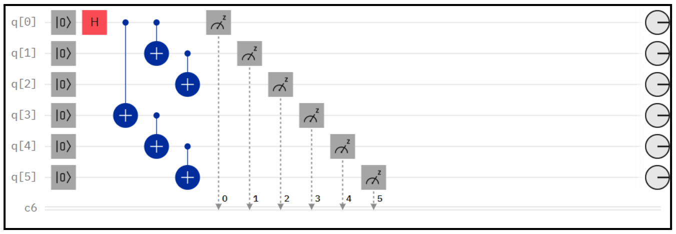

Figure 20: Optimized 6-qubit quantum circuit to create a 6-qubit GHZ state

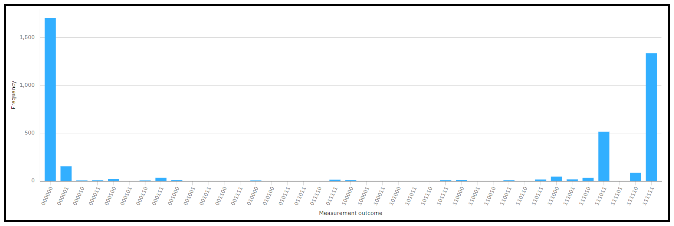

Figure 21: Result of the Optimized 6-qubit quantum circuit upon running on IBM Kyoto Quantum Computer

E. 7 qubit

For the 7-qubit system, the circuit is as shown in Figure 22.

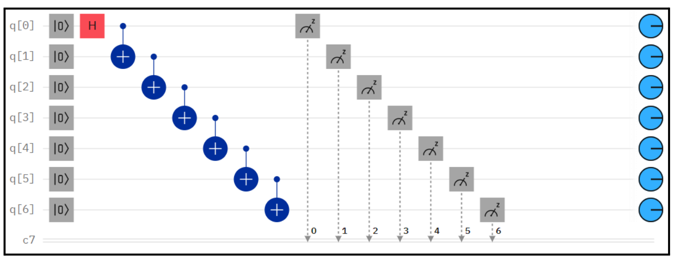

Figure 22: 7-qubit quantum circuit with labeled states to create a 7-qubit GHZ state

The results obtained for the 7-qubit quantum circuit to create a GHZ state is shown in Figure 24. The circuit ran on the ibm_kyoto quantum computer with 4096 shots. It can be seen that states |0000000> and |1111111> have the highest frequencies. These values are very less than 50% which was expected.

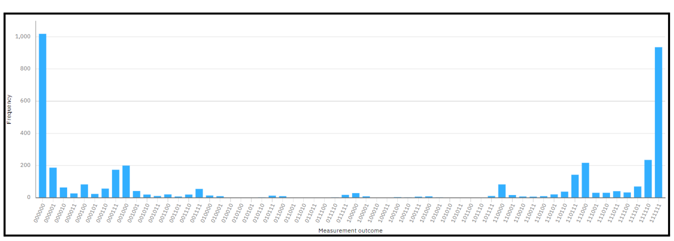

Figure 23: Result of the 7-qubit quantum circuit upon running on IBM Kyoto Quantum Computer

To achieve higher frequencies for these states, the target and control qubits of the CNOT gates were changed and were arranged as shown in Figure 24. The qubits were kept the same. The results for this quantum circuit are shown in Figure 25. The frequencies of |0000000> and |1111111> were better and closer to the expected values.

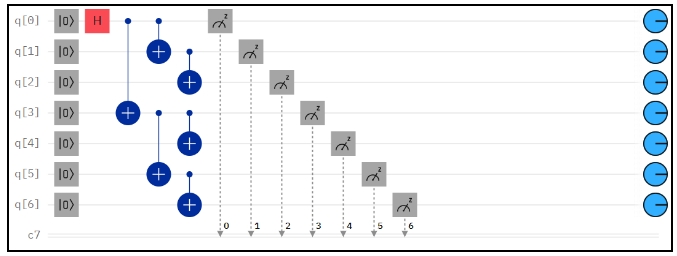

Figure 24: Optimized 7-qubit quantum circuit to create a 7-qubit GHZ state

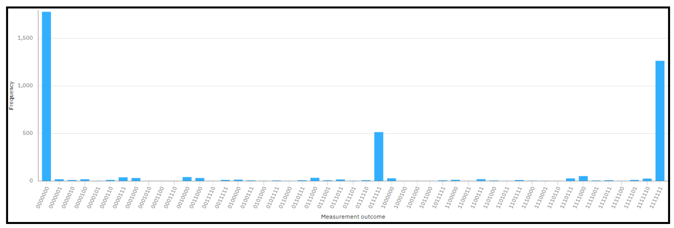

Figure 25: Result of the Optimized 7-qubit quantum circuit upon running on IBM Kyoto Quantum Computer

F. 8 qubit

For the 8-qubit system, the circuit is as shown in Figure 26.

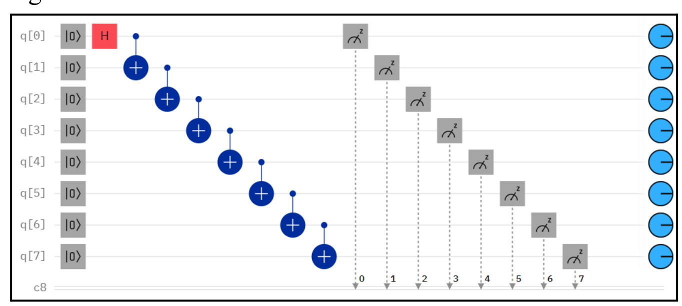

Figure 26: 8-qubit quantum circuit with labeled states to create a 8-qubit GHZ state

The results obtained for the 8-qubit quantum circuit to create a GHZ state is shown in Figure 27. The circuit ran on the ibm_kyoto quantum computer with 4096 shots. It can be seen that states other than |00000000> and |11111111> have the highest frequencies and that the distribution is abnormal. This is not expected and this means that the quantum circuit in Figure 26 is not a good circuit to create an 8-qubit quantum circuit.

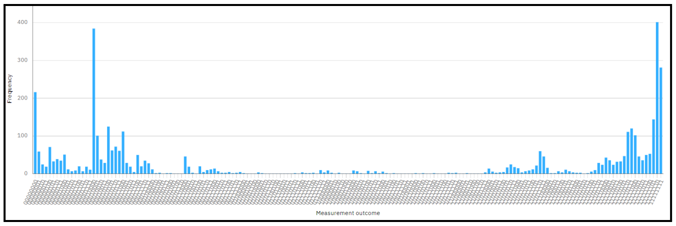

Figure 27: Result of the 8-qubit quantum circuit upon running on IBM Kyoto Quantum Computer

To achieve highest frequencies only for states |00000000> and |11111111>, the target and control qubits of the CNOT gates were changed and were arranged as shown in Figure 28. The qubits were kept the same. The results for this quantum circuit are shown in Figure 29. The distribution is very much expected and the frequencies of |00000000> and |11111111> were better and closer to the expected values.

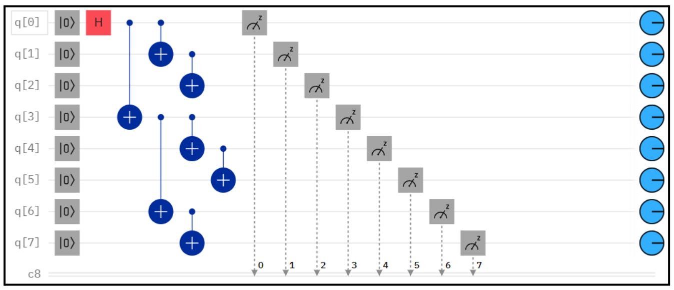

Figure 28: Optimized 8-qubit quantum circuit to create a 8-qubit GHZ state

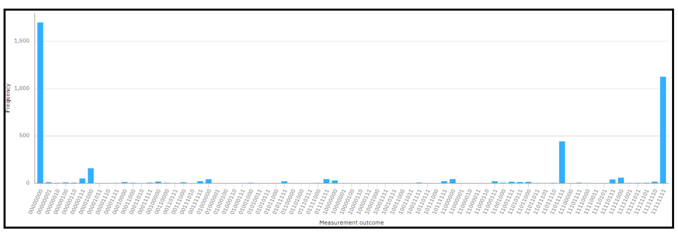

Figure 29: Result of the Optimized 8-qubit quantum circuit upon running on IBM Kyoto Quantum Computer

For quantum circuits with 9 <= n <= 11 qubit GHZ state, the CNOT gates were applied such that all the control bits are on q_0 qubit and the target on q_k, 1 <= k <= n - 1. The rest of the circuit was kept the same.

The results of these circuits, 9 <= n <= 11, is summarized as follows:

The results obtained are shown in the following figures. The circuit ran on the ibm_osaka quantum computer with 4096 shots. It can be seen that states other than all 0s and 1s have higher frequencies and that the distribution is abnormal. This is not expected.

To optimize the circuits, qubits with low measurement errors were chosen. However, the results show that the distribution is abnormal and is not in line with the expectations.

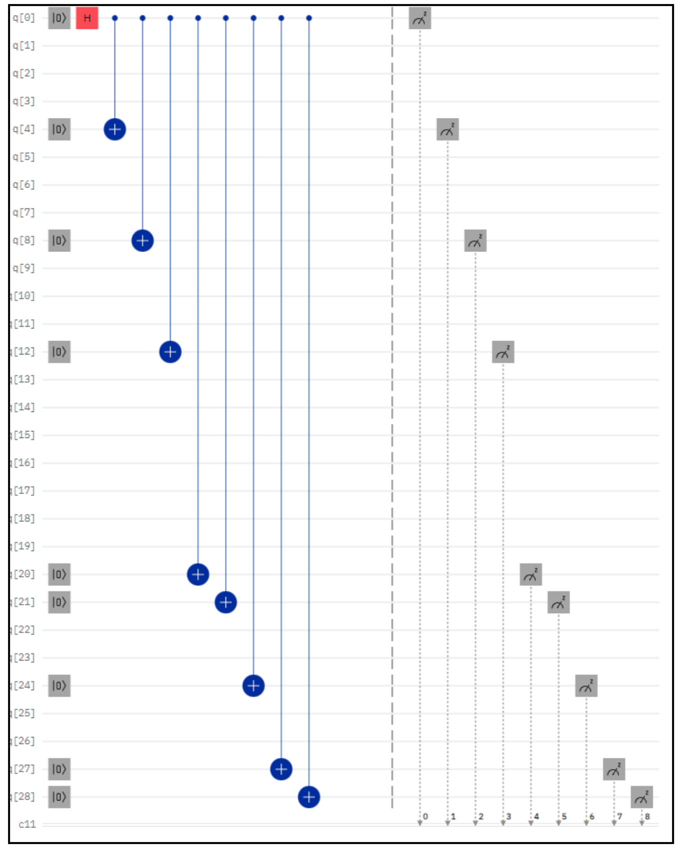

Figure 30: 9-qubit quantum circuit with labeled states to create a 9-qubit GHZ state

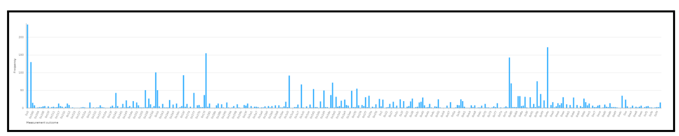

Figure 31: Result of the 9-qubit quantum circuit upon running on IBM Osaka Quantum Computer

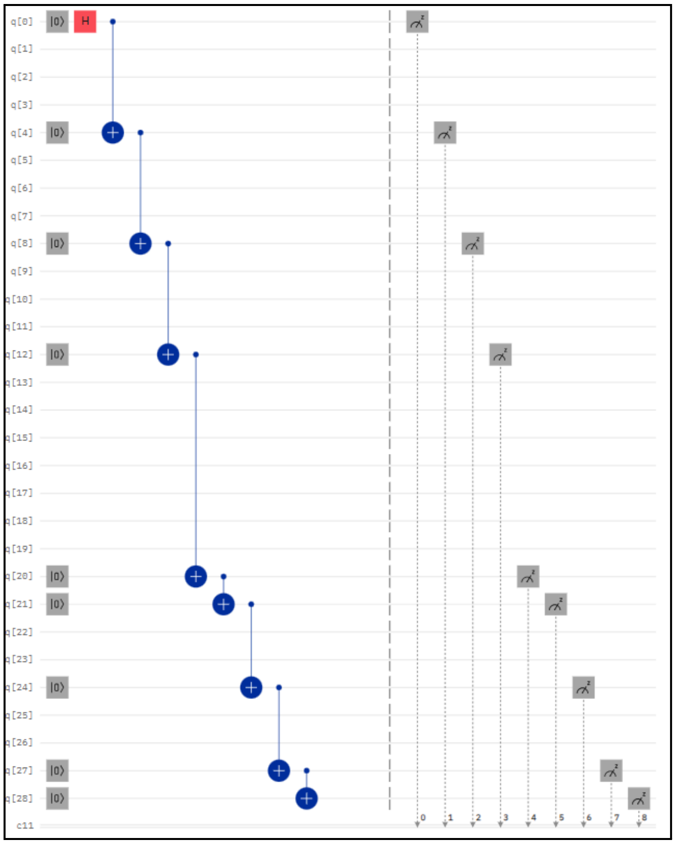

Figure 32: Optimized 9-qubit quantum circuit to create a 9-qubit GHZ state

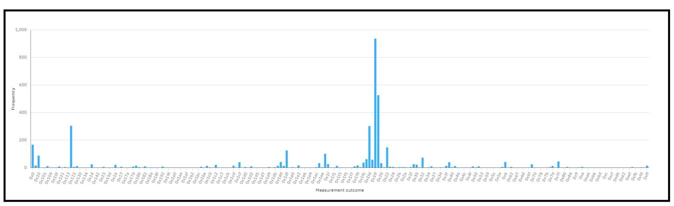

Figure 33: Result of the Optimized 9-qubit quantum circuit upon running on IBM Osaka Quantum Computer

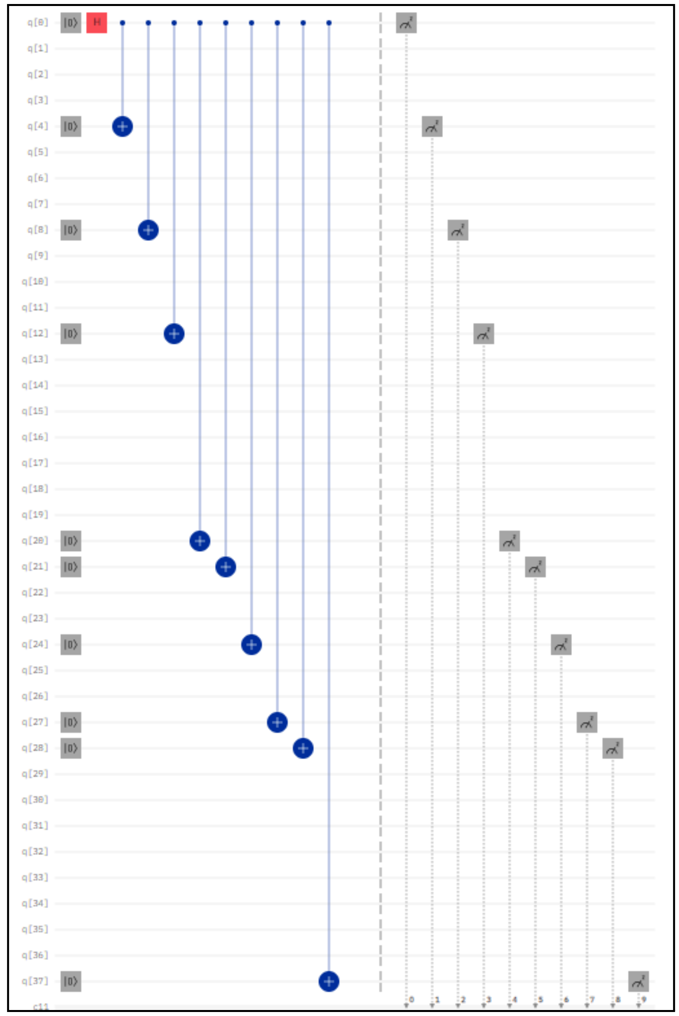

Figure 34: 10-qubit quantum circuit with labeled states to create a 10-qubit GHZ state

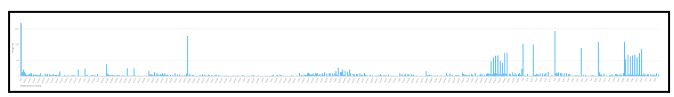

Figure 35: Result of the 10-qubit quantum circuit upon running on IBM Osaka Quantum Computer

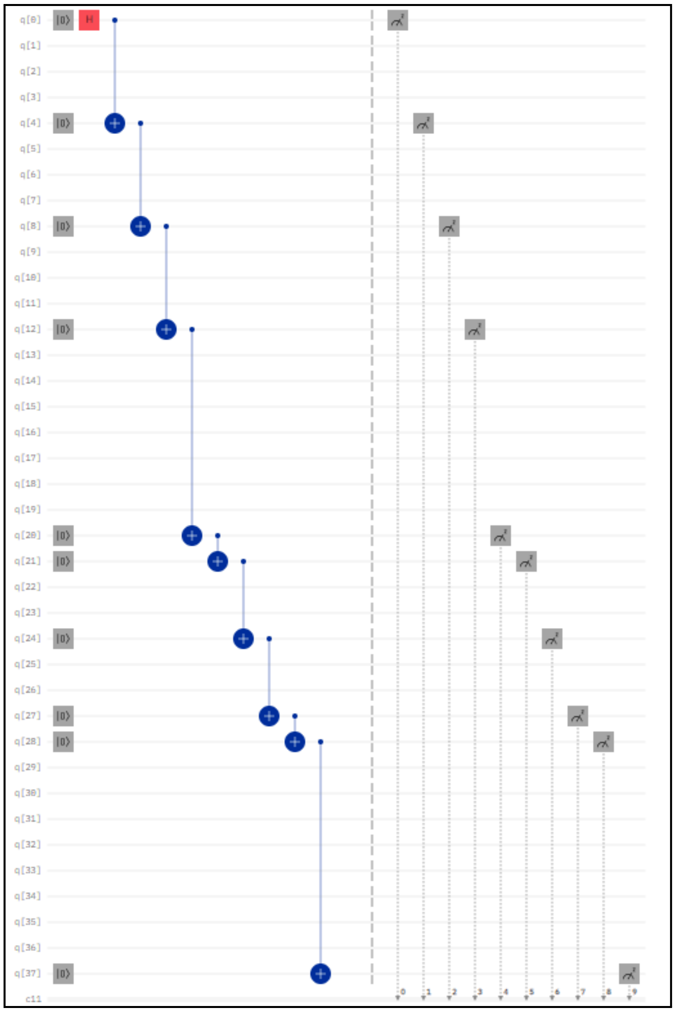

Figure 36: Optimized 10-qubit quantum circuit to create a 10-qubit GHZ state

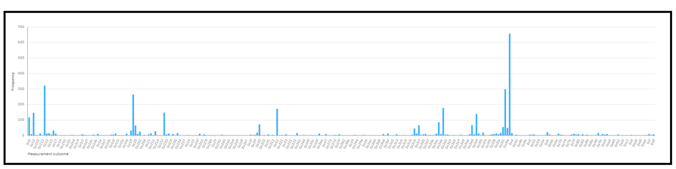

Figure 37: Result of the Optimized 10-qubit quantum circuit upon running on IBM Osaka Quantum Computer

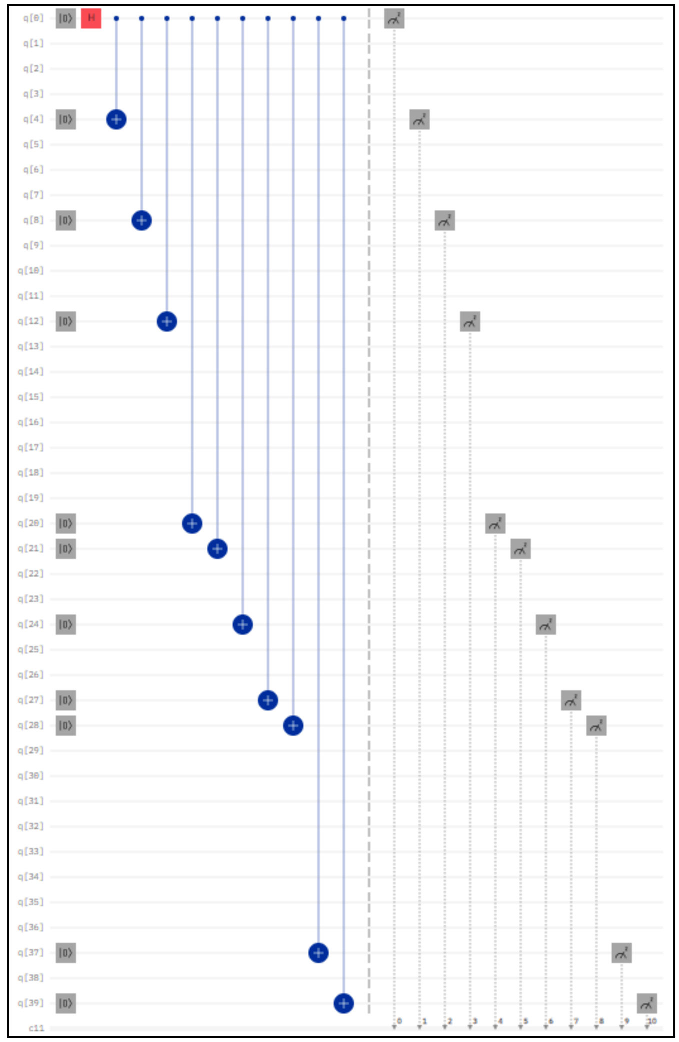

Figure 38: 11-qubit quantum circuit with labeled states to create a 11-qubit GHZ state

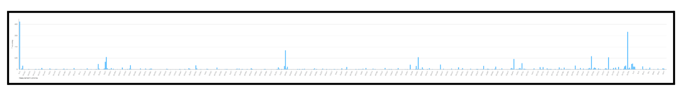

Figure 39: Result of the 11-qubit quantum circuit upon running on IBM Osaka Quantum Computer

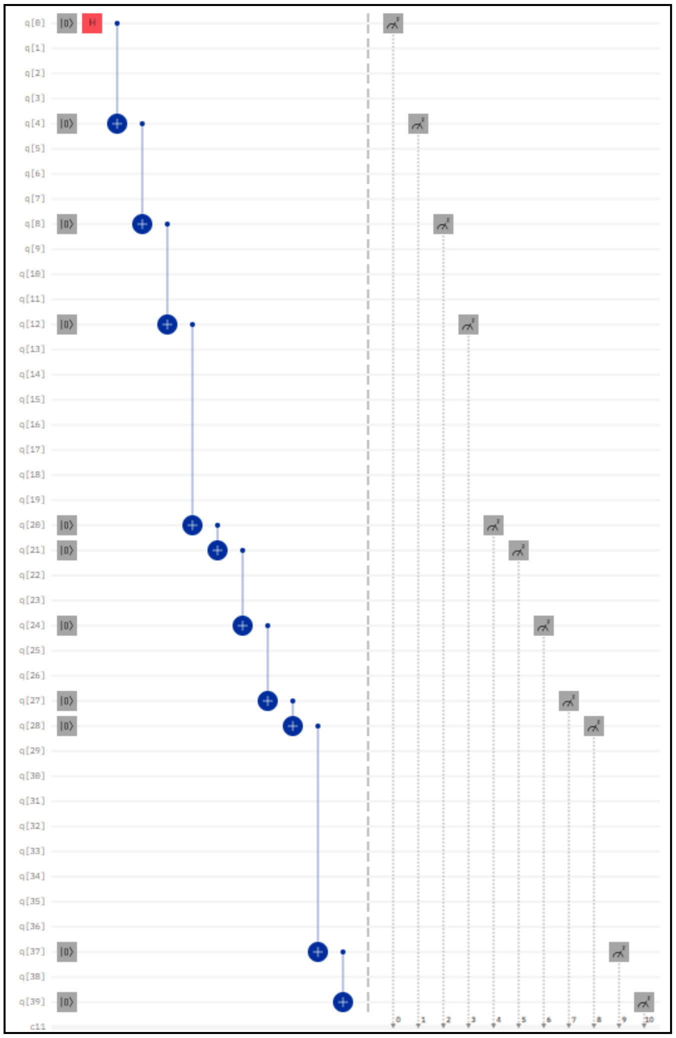

Figure 40: Optimized 11-qubit quantum circuit to create a 11-qubit GHZ state

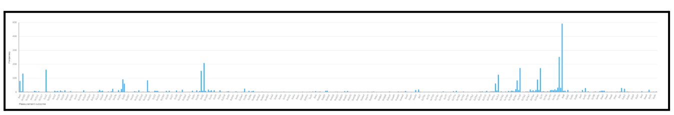

Figure 41: Result of the Optimized 11-qubit quantum circuit upon running on IBM Osaka Quantum Computer

IV. CONCLUSION AND FUTURE WORK

The n-qubit GHZ state, 3 <= n <= 11, was created using IBM's Quantum Compute Platform. For 3 <= n <= 7, the results of these experiments were within the expectations of the theoretical results with some caveats. For 8 <= n <= 12, the results were abnormal. There were significant values of frequency for states other than all 1s and all 0s.

For 3 <= n <= 5 and 9 <= n <= 12, an attempt was made to optimize the quantum circuit by choosing qubits with less measurement error rate. This proved to be effective in improving the probability of the GHZ state for 3 <= n <= 5.

For 9 <= n <= 12, however, there was no improvement in the probability of the GHZ state.

For 6 <= n <= 8, the quantum circuit was optimized by changing the control and target qubits of the CNOT gates and by keeping the qubits the same. There was improvement in the frequency values.

These improvements showed that although the basic and the optimized circuit theoretically give the same set of states, practically the results are very different. Especially, for the higher values of n, the frequency distribution is abnormal and not at par with the theoretical values.

In summary, when creating an n-qubit GHZ state with higher n values, there needs to be careful design of the quantum circuit. Two of those methods were shown in this work that proved to be effective in increasing the frequency values of the expected states. They are:

i. Choosing qubits with less measurement errors, and

ii. Changing the control and target qubits of the CNOT gates while keeping the rest of the circuit the same.

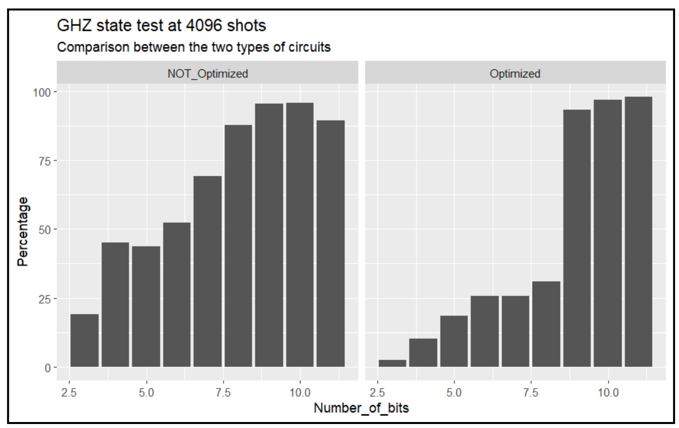

Figure 42: Diagram showing the error rates - fraction of sum of frequencies of states not expected and the total number of shots (4096)

V. REFERENCES

[1] QuantumNerd. "IBM Quantum Computer Online: 5 GHZ(Cat) State Using 3&5 Qubits." YouTube, 20 Oct. 2019, youtu.be/D3EMrqtzOwU?si=x2XfASanjjy4r_mu. Accessed 24 June 2024.

[2] Mooney, Gary J, et al. Generation and Verification of 27-Qubit Greenberger-Horne-Zeilinger States in a Superconducting Quantum Computer. Vol. 5, no. 9, 1 Sept. 2021, pp. 095004–095004, https://doi.org/10.1088/2399-6528/ac1df7.

[3] Popkin, Gabriel. "Einstein's "Spooky Action at a Distance" Spotted in Objects Almost Big Enough to See." Www.science.org, 25 Apr. 2018, www.science.org/content/article/einstein-s-spooky-action-distance-spotted-objects-almost-big-enough-see.

[4] Giles, Martin. “Explainer: What Is a Quantum Computer?” MIT Technology Review, 29 Jan. 2019, www.technologyreview.com/2019/01/29/66141/what-is-quantum-computing/.

[5] Wikipedia Contributors. “Quantum Superposition.” Wikipedia, Wikimedia Foundation, 13 Oct. 2019, en.wikipedia.org/wiki/Quantum_superposition.

[6] Schaffer, Erin. “Intro to Quantum Computing: Qubits, Superposition, & More.” Educative: Interactive Courses for Software Developers, 19 Aug. 2021, www.educative.io/blog/intro-to-quantum-computing.

[7] Wikipedia Contributors. “Greenberger–Horne–Zeilinger State.” Wikipedia, Wikimedia Foundation, 27 Mar. 2024, en.wikipedia.org/wiki/Greenberger%E2%80%93Horne%E2%80%93Zeilinger_state. Accessed 24 June 2024.

[8] QuEra. (2023, January 1). *GHz state*. What is GHZ State. https://www.quera.com/glossary/ghz-state