CS-1203 – Monsoon 2023 – Assignment 3

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October 18, 2023

2. Second Question

Comparison of the time complexities of bubble and insertion sort.
Since for each step, or run through of the algorithm wherein an element is placed in it's proper position, the algorithm must iterate over the entire array. Thus, both algorithms have a worst-case time complexity of $O(n^2)$.
However, bubble sort is still slower than insertion sort, as it performs a large number of unnecessary comparisons and swaps.
Experimental data confirms this theory:
For an array of 10,000 elements,
Time taken by insertion sort: 0.062826s
Time taken by bubble sort: 0.194612s
And for an array of 100,000 elements:
Time taken by insertion sort: 3.486279s
Time taken by bubble sort: 23.113488s
Both algorithms take an order 100 times longer when the size of the array increases by 10 times, and insertion sort is much faster than bubble sort.

3. Third Question

Comparing the time complexities of merge sort, quick sort, and heap sort.
Merge sort involves splitting the array into two parts, and then accessing each element of the split parts to marge them together in the correct order.
This is trivially an O(nlogn) algorithm, from the recurrence relation T(N) = 2T(N/2) * 2N.
Quick sort is fundamentally the same thing, involving splitting an array into elements larger and smaller than a chosen pivot element.
However, it's worst case and average case complexities are different. For it's worse case, the array is split into two highly unequal parts.
Heap sort operates differently, in that it must create a heap from the array first, and then for every element must delete the root node, and maintain the max heap property. The first heap creation is an O(N) operation, while the removal of a node from an heap with n elements is an O(logn) operation.
Therefore, a time complexity analysis yields the expression (for each step of the function) log(n) + log(n-1) + ... + (log1). This is equal to log(n!), which is approximately nlogn.
Thus, heap sort too has a time complexity of O(nlogn).