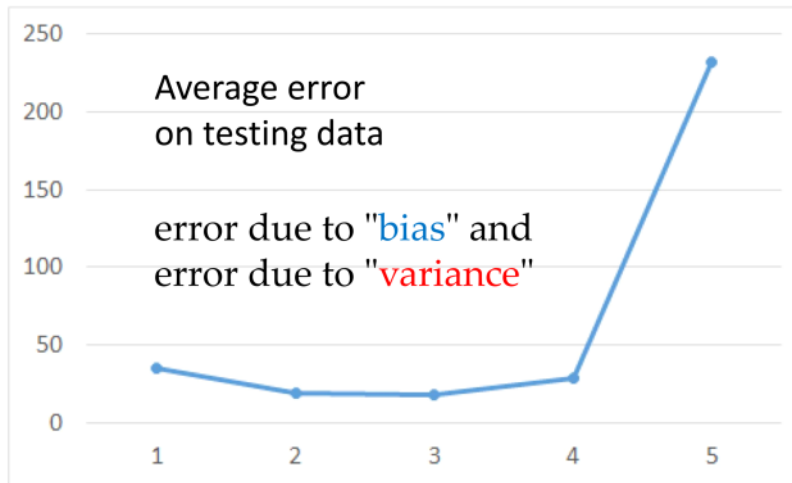


Where does the error
come from?

Review



A more complex model does not always lead to better performance on testing data.

error来源 ← bias
variance

Estimator

进化后的CP值

input

$\hat{y} = \hat{f}(\text{input})$

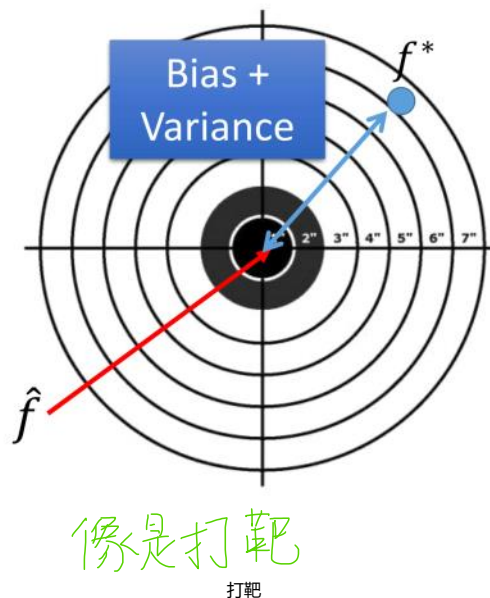
最佳的 \hat{f}

Only Niantic knows \hat{f}

From training data, we find f^*

f^* is an estimator of \hat{f}

估测值



Bias and Variance of Estimator

统计

- Estimate the mean of a variable x
 - assume the mean of x is μ
 - assume the variance of x is σ^2
- Estimator of mean μ
 - Sample N points: $\{x^1, x^2, \dots, x^N\}$

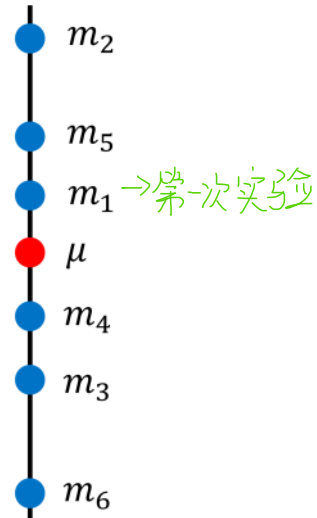
$$m = \frac{1}{N} \sum_n x^n \neq \mu$$

$$E[m] = E\left[\frac{1}{N} \sum_n x^n\right] = \frac{1}{N} \sum_n E[x^n] = \mu$$

\uparrow
m 的期望

很多 m 的期望值为 μ

unbiased



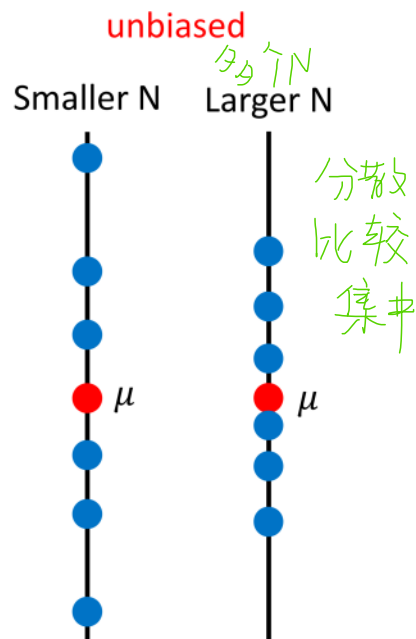
Bias and Variance of Estimator

- Estimate the mean of a variable x
 - assume the mean of x is μ
 - assume the variance of x is σ^2
- Estimator of mean μ
 - Sample N points: $\{x^1, x^2, \dots, x^N\}$

$$m = \frac{1}{N} \sum_n x^n \neq \mu$$

$$\text{Var}[m] = \frac{\sigma^2}{N}$$

Variance depends on the number of samples



Bias and Variance of Estimator

- Estimate the mean of a variable x
 - assume the mean of x is μ
 - assume the variance of x is σ^2
- Estimator of variance σ^2
 - Sample N points: $\{x^1, x^2, \dots, x^N\}$

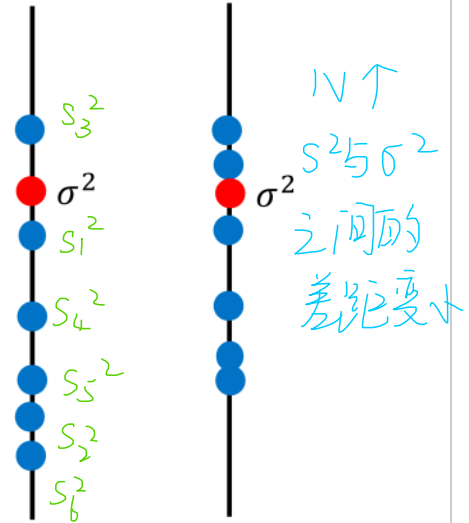
$$m = \frac{1}{N} \sum_n x^n \quad s^2 = \frac{1}{N} \sum_n (x^n - m)^2$$

Biased estimator

$$E[s^2] = \frac{N-1}{N} \sigma^2 \neq \sigma^2$$

s^2 估计 σ^2

Increase N

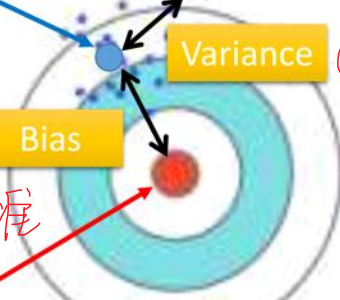
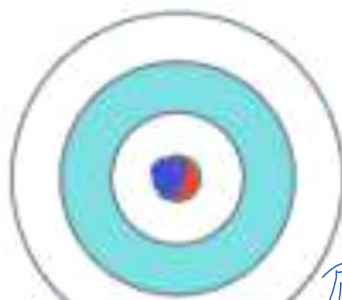


Low Variance

High Variance

Low Bias

瞄准偏差



做很多次实验, 算 f^* 的期望值

② $E[f^*] = \bar{f}$

④ 瞄准后射出
去有偏移

Bias

Variance

①

bias

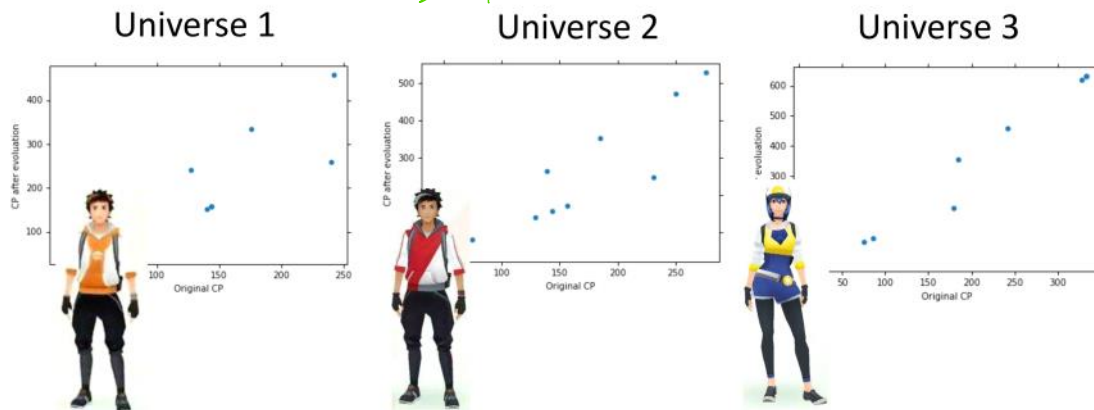
Variance

Variance

Parallel Universes

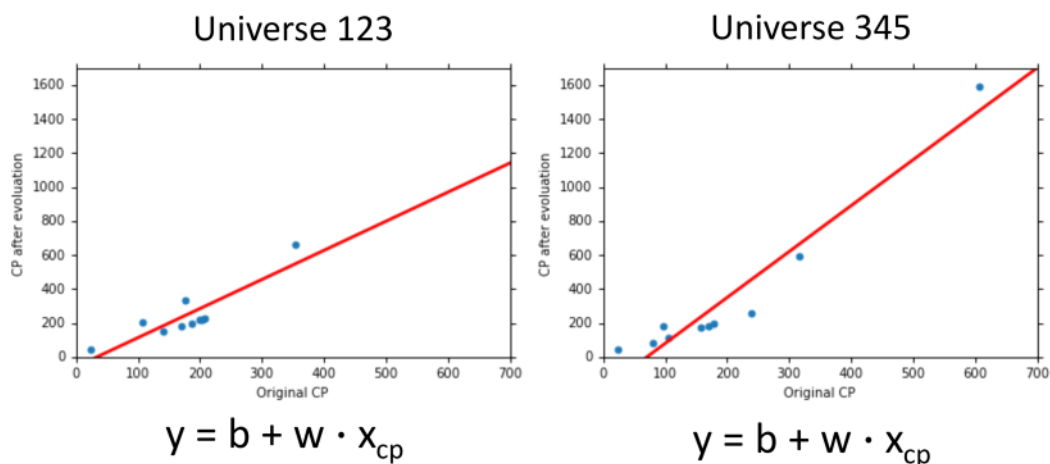
- In all the universes, we are collecting (catching) 10 Pokémon as training data to find f^*

不同宇宙中的数据



Parallel Universes

- In different universes, we use the same model, but obtain different f^*



f^* 的分布

→ 做100次实验, 每次抓10只

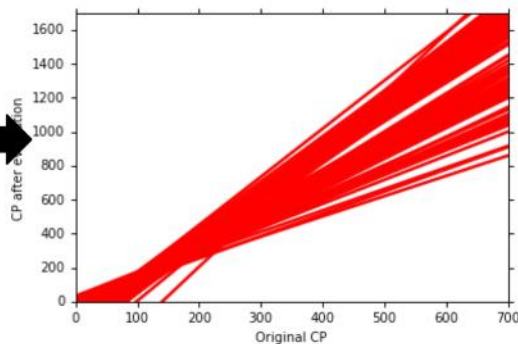
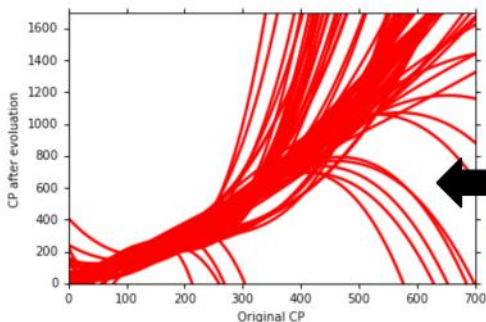
1. 简单模型

做100次实验, 每次抓10只

f^* in 100 Universes

①

$$y = b + w \cdot x_{cp}$$



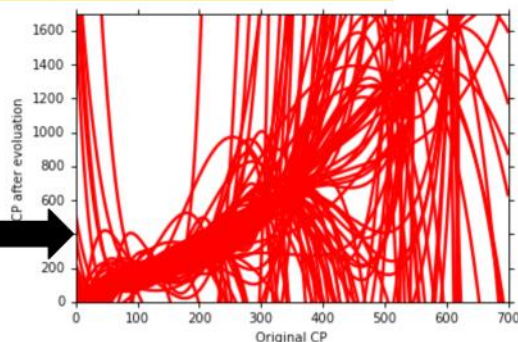
$$y = b + w_1 \cdot x_{cp} + w_2 \cdot (x_{cp})^2 + w_3 \cdot (x_{cp})^3$$

② 换model

再换model

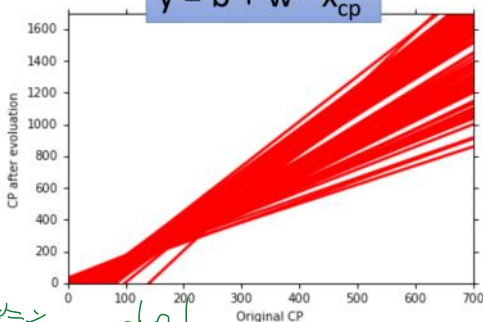
③

$$y = b + w_1 \cdot x_{cp} + w_2 \cdot (x_{cp})^2 + w_3 \cdot (x_{cp})^3 + w_4 \cdot (x_{cp})^4 + w_5 \cdot (x_{cp})^5$$



Variance

$$y = b + w \cdot x_{cp}$$



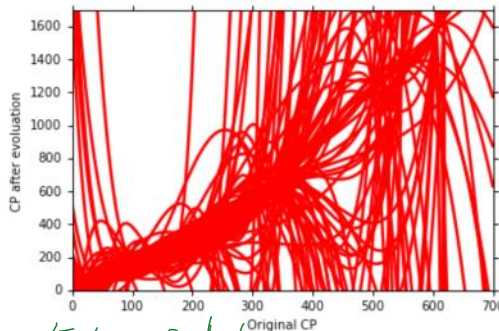
简单model



Small Variance

散布比较窄

$$y = b + w_1 \cdot x_{cp} + w_2 \cdot (x_{cp})^2 + w_3 \cdot (x_{cp})^3 + w_4 \cdot (x_{cp})^4 + w_5 \cdot (x_{cp})^5$$



复杂model



Large Variance

散布比较开

Simpler model is less influenced by the sampled data

不太

会受data的影响

Consider the extreme case $f(x) = c$

受data影响较大

简单model

Bias

$$E[f^*] = \bar{f}$$

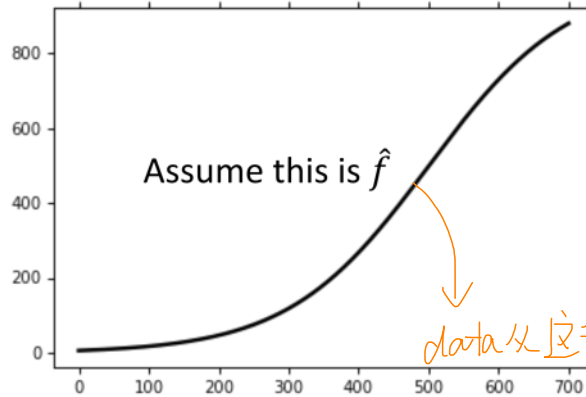
- Bias: If we average all the f^* , is it close to \hat{f} ~ 靶心



Large Bias



Small Bias
↓
平均值和靶心很接近

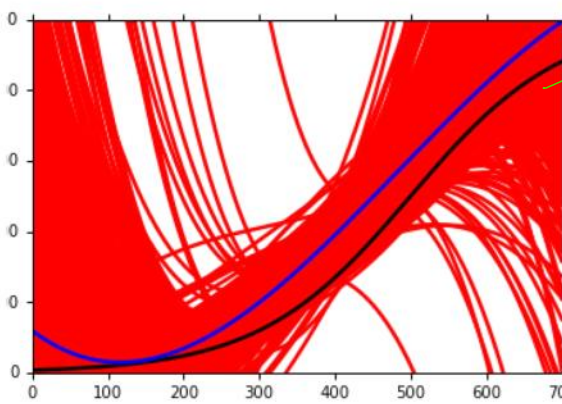


data 从这条线
sample 出来

Black curve: the true function \hat{f}

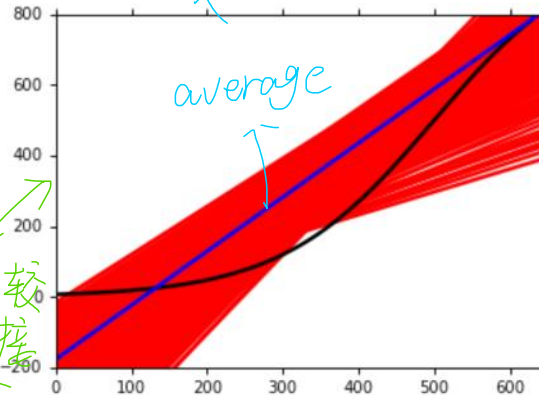
Red curves: 5000 f^* 5000次

Blue curve: the average of 5000 f^*
 $= \bar{f}$

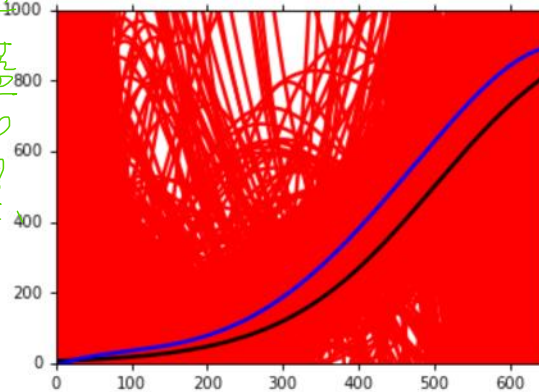


2次

较接近蓝和黑

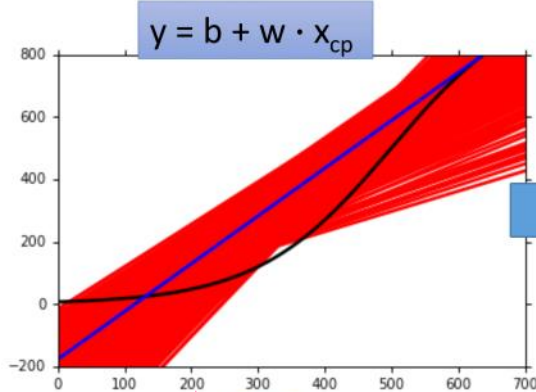


1次
average



3次

Bias

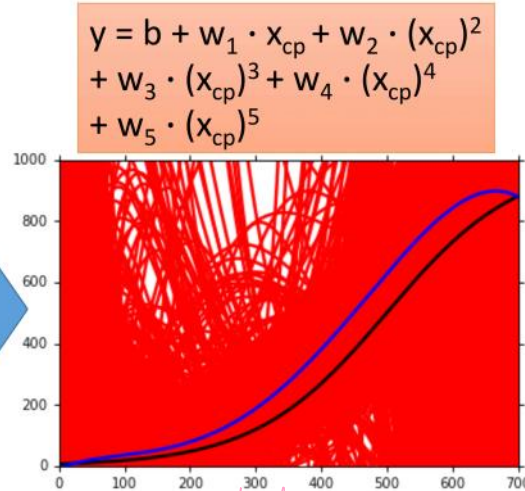


简单 model

model

Large Bias

每次 f 都差不多, 集中某处, 远离靶心



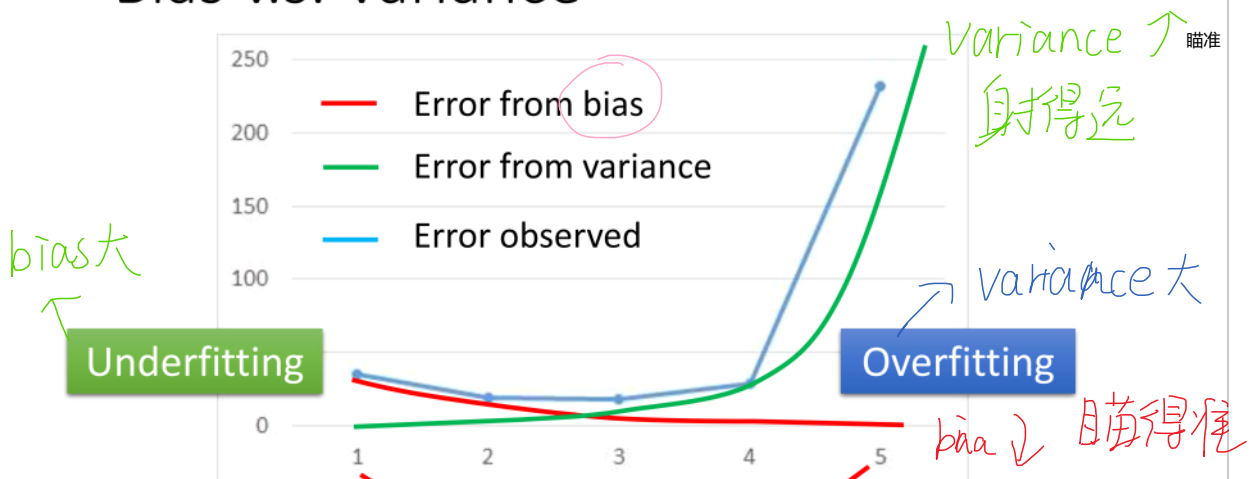
复杂 model

model

Small Bias

每次 f 差别较大, 但在靶心周围

Bias v.s. Variance



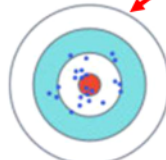
Large Bias
Small Variance

简单



Small Bias
Large Variance

复杂



What to do with large bias?

• Diagnosis:

• If your model cannot even fit the training examples, then you have large bias **Underfitting**

不能fit 和正确 model 有距离

• If you can fit the training data, but large error on testing data, then you probably have large variance **Overfitting**

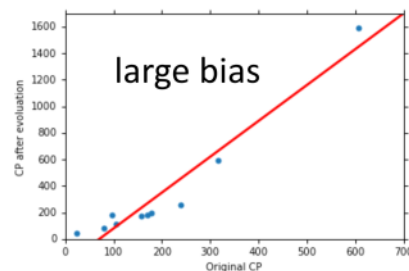
training data 是可以的 但 test data error 大

• For bias, redesign your model:

• Add more features as input

• A more complex model

model 不好

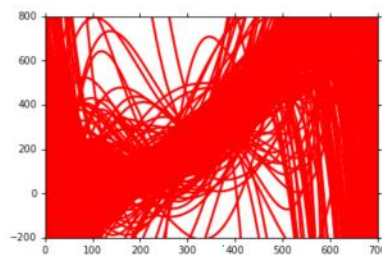


收集更多 data

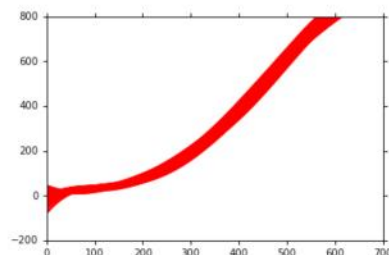
What to do with large variance?

• **More data**

Very effective,
but not always
practical



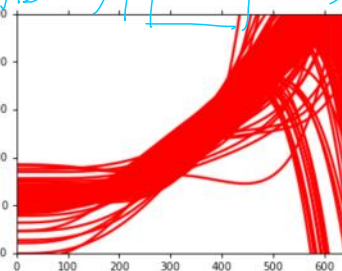
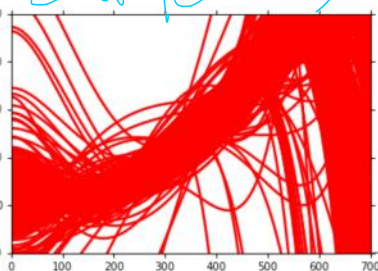
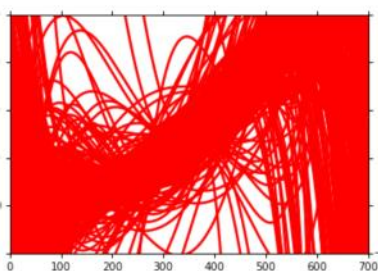
10 examples



100 examples

• **Regularization**

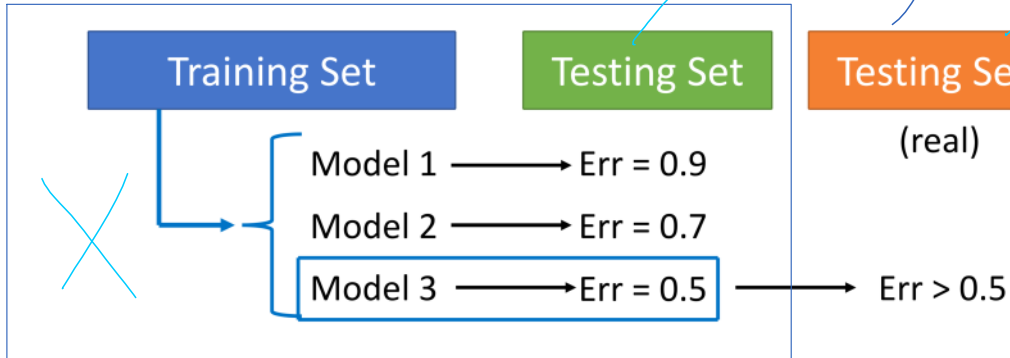
正则化 → 加刀 → 使曲线更平滑



Model Selection

权衡

- There is usually a trade-off between bias and variance.
- Select a model that balances two kinds of error to minimize total error
- What you should NOT do:



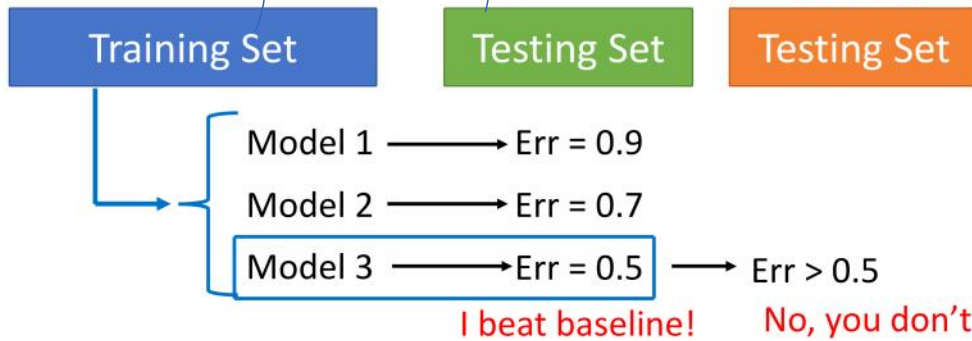
Homework

我有

我有

public

private



What will happen next Friday?

<http://www.chioka.in/how-to-select-your-final-models-in-a-kaggle-competitio/>



Cross Validation

将 training set 分成两组



Using the results of public testing data to tune your model
You are making public set better than private set.

① train model

Model 1	→	Err = 0.9
Model 2	→	Err = 0.7
Model 3	→	Err = 0.5

选 model

Not recommend

看起来比 training 的 err 大

但反映的是 private 的 error

① Training set train model ; Validation set 选 model

② 选出 model 3 将所有 Training set 用于 model 3

★ 担心分坏

N-fold Cross Validation

① 分3种情况用于这些 model

情况1
情况2
情况3

Training Set		
Train	Train	Val
Train	Val	Train
Val	Train	Train

Model 1	Model 2	Model 3
Err = 0.2	Err = 0.4	Err = 0.4
Err = 0.4	Err = 0.5	Err = 0.5
Err = 0.3	Err = 0.6	Err = 0.3
Avg Err = 0.3	Avg Err = 0.5	Avg Err = 0.4

②



↑
少在意 public 上的 error 进而调整 model
说不定 private 比较接近 test

说不定 private v. p...

Reference

- Bishop: Chapter 3.2