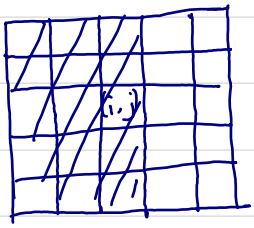
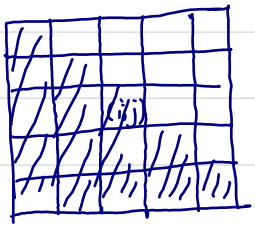
1500010725 张润玉 HW1 理论作业





1. $$\frac{\partial v}{\partial x} = \lambda \cdot \frac{v_{i+1,j} - v_{i-1,j}}{2h} + \frac{1-\lambda}{2}\left(\frac{v_{i+1,j+1} - v_{i-1,j+1}}{2h} + \frac{v_{i+1,j-1} - v_{i-1,j-1}}{2h}\right)$$

$$\frac{\partial v}{\partial y} = \lambda \cdot \frac{v_{i,j+1} - v_{i,j-1}}{2h} + \frac{1-\lambda}{2}\left(\frac{v_{i+1,j+1} - v_{i+1,j-1}}{2h} + \frac{v_{i-1,j+1} - v_{i-1,j-1}}{2h}\right)$$

对于左图 $\frac{\partial v}{\partial x} = \frac{\lambda}{2} + \frac{1-\lambda}{2} = \frac{1}{2}$ $\frac{\partial v}{\partial y} = 0$

对于右图 $\frac{\partial v}{\partial x} = \frac{\lambda}{2} + \frac{1-\lambda}{2}\cdot\frac{1}{2} = \frac{1+\lambda}{4}$ $\frac{\partial v}{\partial y} = \frac{1+\lambda}{4}$

$\Rightarrow \frac{1}{4} = 2\cdot\frac{(1+\lambda)^2}{16}$ 即 $\lambda^2 + 2\lambda - 1 = 0$ $\Rightarrow$ $\lambda = \sqrt{2} - 1$

2. $$(\Delta v)_{ij} = \lambda \frac{v_{i+1,j} + v_{i-1,j} + v_{i,j+1} + v_{i,j-1} - 4v_{i,j}}{h^2}$$

$$+ (1-\lambda)\frac{v_{i+1,j+1} + v_{i-1,j-1} + v_{i-1,j+1} + v_{i+1,j-1} - 4v_{i,j}}{2h^2}$$

对于左图 $(\Delta v)_{ij} = \lambda + (1-\lambda) = 1$

对于右图 $(\Delta v)_{ij} = 2\lambda + \frac{1-\lambda}{2}$

$\Rightarrow 2\lambda + \frac{1-\lambda}{2} = 1$ $\Rightarrow$ $\lambda = \frac{1}{3}$