UGANDA MARTYRS UNIVERSITY,
NKOZI

COVID-19 TAKE HOME EXAMINATION

FACULTY OF SCIENCE

DEPARTMENT OF NATURAL SCIENCES

FIRST YEAR END OF SEMESTER ONE FINAL EXAMINATIONS

MTC 1101: Calculus I

DATE: 17/02/2021

DURATION: 3hrs

Instructions:

1. Carefully read through **ALL** the questions before attempt.
2. Attempt **ALL** the questions.
3. No names should be written anywhere on the answer sheet.
4. Ensure that your **Registration number** is indicated on all pages of the answer booklet.

1. [30 marks] Write True or False (3 marks each). No justification is needed.

(a) If $f(x)$ is concave up on (a, b), then $f(x)$ is increasing on (a, b).

(b) The function $f(x) = \frac{1}{x}$ is continuous at $x = 0$.

(c) $\lim_{x\to 0} \sqrt{x^2 + x^3} sin(\frac{\pi}{x}) = 0$.

(d) The derivative of $f(x) = \frac{2}{x}$ is $f^{'}(x) = 2lnx$.

(e) If $f^{'}(2) = f^{''}(2) = 1$, then $f(2)$ is a local minimum at $x = 2$.

(f) $\int_0^3 cosxsinxdx = \int_0^5 cosxsinxdx + \int_5^3 cosxsinxdx + \int_5^5 cosxsinxdx$

(g) The equation $x^3 - x - 2 = 0$ has a solution in the interval $(1, 2)$.

(h) The function $f(x) = \int_0^x ln(t^3)dt$ is increasing when $x > 1$.

(i) The fundamental theorem of calculus ensures that $\frac{d}{dx} \int_0^x 1dx = 1$.

(j) If $f^{'}(x) = g^{'}(x)$, then $f(x) - g(x)$ is a constant.

2. [4 marks] Compute

$$\lim_{t\to 0}\Big(\frac{1}{t\sqrt{1+t}} - \frac{1}{t}\Big).$$

3. [3 marks] Compute

$$\lim_{t\to\infty} \frac{\sqrt{x^4 + 5x + 2}}{2x^2 - 3}$$

.

4. [4 marks] Compute

$$\lim_{t\to\infty} x^{\frac{3}{2}} sin(\frac{1}{x})$$

5. [4 marks] Find $f^{'}(x)$ if

$$f(x) = \frac{ln(sinx)}{e^{3x}}$$

6. [4 marks] Find the tangent line of $y = e^{cosx}$ at $x = \pi/2$.

7. Let $f(x) = \frac{1}{3}x^3 - x^2 + x + 2$.

(a) [3 marks] Find the critical points. Find the intervals of increase and intervals of decrease.

(b) [2 marks] Find the absolute maximum and absolute minimum of $f(x)$ on the interval $[0, 3]$.

8. [5 marks] The height h of a triangle is increasing at rate of $1cm/min$ while the area of the triangle is increasing at a rate of $2cm^2/min$. At what rate is the base b of the triangle changing when the height is $10cm$ and the area is $100cm^2$.

9. [5 marks] Evaluate the integral:

$$\int_1^5 \frac{1}{x+4} dx$$

.

10. [5 marks] Find the indefinite integral

$$\int \frac{x}{\sqrt{1+x^2}} dx$$

.

11. [5 marks] Compute the definite integral

$$\int_0^1 \frac{1}{(1+\sqrt{x})^3} dx$$

.

12. [5 marks] Compute

$$\int_0^{5\sqrt{2}/2} \left((\sqrt{25-x^2}) - x \right) dx$$

.

13. [5 marks] Find

$$\lim_{t \to 0} \frac{\int_0^x (1 - tan2t)dt}{x}$$

.

14. [5 marks] Find the derivative $f^{'}(x)$ of the function

$$\int_{\sqrt{x}}^{x} \frac{e^t}{t} dt$$

15. [5 marks] The function $f(x)$ satisfies $f^{'}(x) = e^{-4x} + 2x + x$ and $f(0) = 0$. Find $f(x)$.

16. Consider the region enclosed by the curves $y = x^2, y = 2x - 1$, and the x-axis.

(a) [2 marks] Sketch the curves and shade the region.

(b) [4 marks] Find the area of the shaded region in the previous part (a).

END