UGANDA MARTYRS UNIVERSITY,
NKOZI

COVID-19 TAKE HOME EXAMINATION

FACULTY OF SCIENCE

DEPARTMENT OF NATURAL SCIENCES

FIRST YEAR END OF SEMESTER ONE FINAL EXAMINATIONS

MTC 1102 : LINEAR ALGEBRA 1

DATE: 17/02/2021

DURATION: 5 HRS

Instructions:

1. Carefully read through **ALL** the questions before attempt.
2. Attempt any five (5) questions.
3. No names should be written anywhere on the answer sheet.
4. Ensure that your **Registration number** is indicated on all pages of the answer booklet.

1. (a) [10 marks] Consider the 3x3 matrix;

$$A = \begin{bmatrix} 0 & 1 & k \\ 2 & k & -6 \\ 2 & 7 & 4 \end{bmatrix}$$

For what values of the constant k is the matrix A invertible?

(b) [10 marks] Consider the 2 x 2 matrix

$$B = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

where a, b, c, d are real numbers and $a \neq 0$. Find all values d such that rank $B = 1$.

2. Consider the following matrix;

$$A = \begin{bmatrix} 1 & 0 & -1 & 1 \\ 2 & 0 & 1 & 7 \\ 1 & 1 & 2 & 7 \\ 3 & 4 & 1 & 5 \end{bmatrix}$$

(a) [10 marks] Find rank A.

(b) [10 marks] Find a basis for matrix A and state it's dimension.

3. Let $A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$.

(a) [8 marks] Write the characteristic equation of A and find all eigenvalues of A.

(b) [3 marks] Find a basis for each eigenspace.

(c) [5 marks] Is A diagonalizable? (Show your working).

(d) 4[marks] Find A^3.

4. Let $T : \mathbb{P}_2 \longrightarrow \mathbb{P}_2$ be defined by $T(p(t)) = p^{'}(t) + p(0)$ for all $p(t)$ in $\mathbb{P}_2$. Here, $p^{'}$ is the first derivative of $p(t)$.

(a) [7 marks] Show that T is a linear transformation.

(b) [6 marks] Find a non-zero vector in the kernel of T.

(c) [7 marks] Find a basis for the kernel of T and the dimension of the kernel of T .

5. Consider the following linear system of equations:

$$x_1 + 2x_3 + 4x_4 = -8$$

$$x_2 - 3x_3 - x_4 = 6$$

$$3x_1 + 4x_2 - 6x_3 + 8x_4 = 0$$

$$-x_2 - 3x_3 + 4x_4 = -6$$

(a) [10 marks] Find the reduced echelon form of the augmented matrix associated with this system.

(b) [5 marks] Write the general solution to the system in parametric vector form.

(c) [5 marks] Write the general solution to the corresponding homogeneous system of equations in parametric vector form.

6. Let $u = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}$, $v = \begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix}$.

(a) [10 marks] Is the set u, v linearly independent or dependent? Justify your answer.

(b) [10 marks] Find all values of h such that $v = \begin{bmatrix} 1 \\ 1 \\ h \end{bmatrix}$, is in Span $\{u, v\}$.

END