I. DERIVATION

Here we derive the formula for bubble in paramegnetic state in two-dimensional square lattice with one atom per unit cell, considering one orbital per site.

In general, we can define bubble on Matsubara axis summed over fermionic frequencies as

$$\chi^0(\mathbf{q}, i\omega_m) = -\frac{1}{N}\frac{1}{\beta}\sum_{\mathbf{k},i\nu} G(\mathbf{k}, i\nu_n)G(\mathbf{k}+\mathbf{q}, i\nu_n + i\omega_m). \tag{1}$$

Using bare Green's function and standard Matsubara frequency summation (see documentation for the derivation of RPA susceptibility) we get

$$\begin{aligned} \chi^0(\mathbf{q}, i\omega_m) &= -\frac{1}{N}\frac{1}{\beta}\sum_{\mathbf{k},i\nu_n} \frac{1}{i\nu_n + i\omega_m - E_{\mathbf{k}+\mathbf{q}}}\frac{1}{i\nu_n - E_{\mathbf{k}}} \\ &= -\frac{1}{N}\sum_{\mathbf{k}} \frac{n_F(E_{\mathbf{k}+\mathbf{q}}) - n_F(E_{\mathbf{k}})}{E_{\mathbf{k}+\mathbf{q}} - E_{\mathbf{k}} - i\omega_m}. \end{aligned} \tag{2}$$

In this case, we can easily perform the analytical continuation by substituting $i\omega_m \to \omega + i\delta$ and we obtain bubble on the real axis in the orbital basis

$$\chi^0(\mathbf{q}, \omega) = -\frac{1}{N}\sum_{\mathbf{k}} \frac{n_F(E_{\mathbf{k}+\mathbf{q}}) - n_F(E_{\mathbf{k}})}{E_{\mathbf{k}+\mathbf{q}} - E_{\mathbf{k}} - \omega - i\delta}. \tag{3}$$

Taking into account the fact, that we consider model with one atom per unit cell and one orbital, transformation of bubble from the orbital basis to the physical basis, we are namely interested in component χ^0_{zz}, is very simple

$$\chi^0_{zz} = \frac{1}{2}\left(\chi^0_{\downarrow} + \chi^0_{\uparrow}\right). \tag{4}$$

Since we study paramagnatic state $\chi^0_{\downarrow} = \chi^0_{\uparrow}$.