# CART Algorithm

## Example

- **Data Set**
- We work on same dataset in ID3. There are 14 instances of golf playing decisions based on **outlook, temperature, humidity** and **wind** factors.
- **Gini index**
- **Gini index is a metric for classification tasks in CART.** It stores sum of squared probabilities of each class. We can formulate it as illustrated below.
- Gini = $1 - \Sigma (Pi)^2$ for i = 1 to number of classes

| Day | Outlook | Temp. | Humidity | Wind | Decision |
|---|---|---|---|---|---|
| 1 | Sunny | Hot | High | Weak | No |
| 2 | Sunny | Hot | High | Strong | No |
| 3 | Overcast | Hot | High | Weak | Yes |
| 4 | Rain | Mild | High | Weak | Yes |
| 5 | Rain | Cool | Normal | Weak | Yes |
| 6 | Rain | Cool | Normal | Strong | No |
| 7 | Overcast | Cool | Normal | Strong | Yes |
| 8 | Sunny | Mild | High | Weak | No |
| 9 | Sunny | Cool | Normal | Weak | Yes |
| 10 | Rain | Mild | Normal | Weak | Yes |
| 11 | Sunny | Mild | Normal | Strong | Yes |
| 12 | Overcast | Mild | High | Strong | Yes |
| 13 | Overcast | Hot | Normal | Weak | Yes |
| 14 | Rain | Mild | High | Strong | No |

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## Outlook

Outlook is a nominal feature. It can be sunny, overcast or rain. we summarize the final decisions for outlook feature.

| Outlook | Yes | No | Number of instances |
|---|---|---|---|
| Sunny | 2 | 3 | 5 |
| Overcast | 4 | 0 | 4 |
| Rain | 3 | 2 | 5 |

Gini(Outlook=Sunny) = $1 - (2/5)^2 - (3/5)^2 = 1 - 0.16 - 0.36 = 0.48$

Gini(Outlook=Overcast) = $1 - (4/4)^2 - (0/4)^2 = 0$

Gini(Outlook=Rain) = $1 - (3/5)^2 - (2/5)^2 = 1 - 0.36 - 0.16 = 0.48$

Then, we will calculate weighted sum of gini indexes for outlook feature.

Gini(Outlook) = $(5/14) \times 0.48 + (4/14) \times 0 + (5/14) \times 0.48 = 0.171 + 0 + 0.171 = 0.342$

## Humidity

Humidity is a binary class feature. It can be high or normal.

| Humidity | Yes | No | Number of instances |
|---|---|---|---|
| High | 3 | 4 | 7 |
| Normal | 6 | 1 | 7 |

Gini(Humidity=High) = $1 - (3/7)^2 - (4/7)^2 = 1 - 0.183 - 0.326 = 0.489$

Gini(Humidity=Normal) = $1 - (6/7)^2 - (1/7)^2 = 1 - 0.734 - 0.02 = 0.244$

Weighted sum for humidity feature will be calculated next

Gini(Humidity) = $(7/14) \times 0.489 + (7/14) \times 0.244 = 0.367$

## Temperature

Similarly, temperature is a nominal feature and it could have 3 different values: Cool, Hot and Mild. Let's summarize decisions for temperature feature.

| Temperature | Yes | No | Number of instances |
|---|---|---|---|
| Hot | 2 | 2 | 4 |
| Cool | 3 | 1 | 4 |
| Mild | 4 | 2 | 6 |

Gini(Temp=Hot) = $1 - (2/4)^2 - (2/4)^2 = 0.5$

Gini(Temp=Cool) = $1 - (3/4)^2 - (1/4)^2 = 1 - 0.5625 - 0.0625 = 0.375$

Gini(Temp=Mild) = $1 - (4/6)^2 - (2/6)^2 = 1 - 0.444 - 0.111 = 0.445$

We'll calculate weighted sum of gini index for temperature feature

Gini(Temp) = $(4/14) \times 0.5 + (4/14) \times 0.375 + (6/14) \times 0.445 = 0.142 + 0.107 + 0.190 = 0.439$

## Wind

Wind is a binary class similar to humidity. It can be weak and strong.

| Wind | Yes | No | Number of instances |
|---|---|---|---|
| Weak | 6 | 2 | 8 |
| Strong | 3 | 3 | 6 |

Gini(Wind=Weak) = $1 - (6/8)^2 - (2/8)^2 = 1 - 0.5625 - 0.062 = 0.375$

Gini(Wind=Strong) = $1 - (3/6)^2 - (3/6)^2 = 1 - 0.25 - 0.25 = 0.5$

Gini(Wind) = $(8/14) \times 0.375 + (6/14) \times 0.5 = 0.428$

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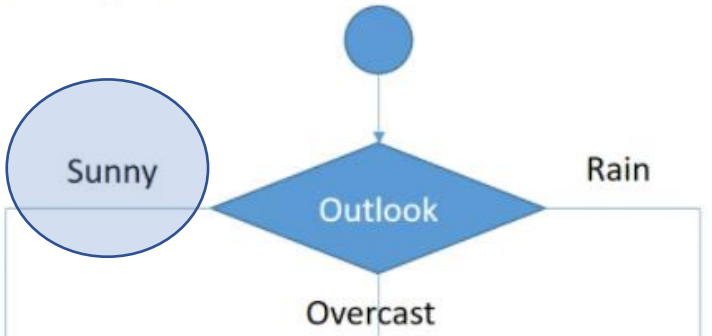
# Time to decide

We've calculated gini index values for each feature. The winner will be outlook feature because its cost is the lowest.

| Feature | Gini index |
|---|---|
| Outlook | 0.342 |
| Temperature | 0.439 |
| Humidity | 0.367 |
| Wind | 0.428 |

We'll put outlook decision at the top of the tree.




| Day | Outlook | Temp. | Humidity | Wind | Decision |
|---|---|---|---|---|---|
| 1 | Sunny | Hot | High | Weak | No |
| 2 | Sunny | Hot | High | Strong | No |
| 8 | Sunny | Mild | High | Weak | No |
| 9 | Sunny | Cool | Normal | Weak | Yes |
| 11 | Sunny | Mild | Normal | Strong | Yes |

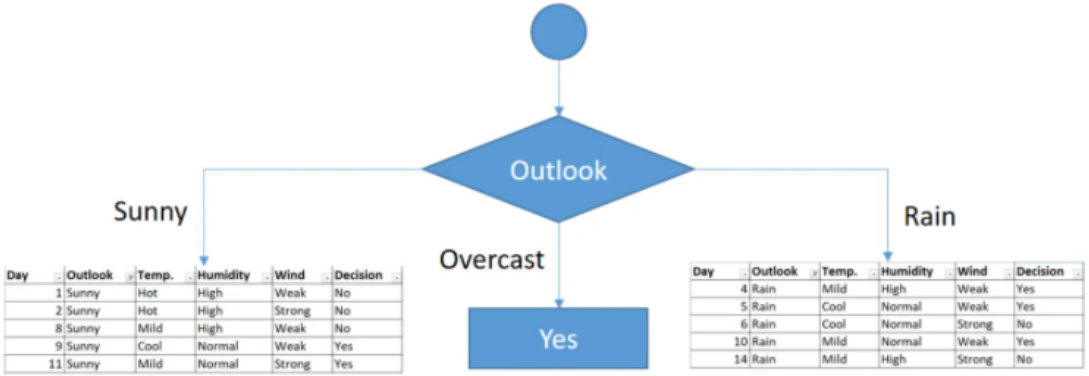
| Day | Outlook | Temp. | Humidity | Wind | Decision |
|---|---|---|---|---|---|
| 4 | Rain | Mild | High | Weak | Yes |
| 5 | Rain | Cool | Normal | Weak | Yes |
| 6 | Rain | Cool | Normal | Strong | No |
| 10 | Rain | Mild | Normal | Weak | Yes |
| 14 | Rain | Mild | High | Strong | No |

| Day | Outlook | Temp. | Humidity | Wind | Decision |
|---|---|---|---|---|---|
| 3 | Overcast | Hot | High | Weak | Yes |
| 7 | Overcast | Cool | Normal | Strong | Yes |
| 12 | Overcast | Mild | High | Strong | Yes |
| 13 | Overcast | Hot | Normal | Weak | Yes |

First decision would be outlook feature

All are Yes

Splitting is occurred at minimum gini index




Tree is over for overcast outlook leaf

| Day | Outlook | Temp. | Humidity | Wind | Decision |
|---|---|---|---|---|---|
| 1 | Sunny | Hot | High | Weak | No |
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| 8 | Sunny | Mild | High | Weak | No |
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## Gini of temperature for sunny outlook

| Temperature | Yes | No | Number of instances |
|---|---|---|---|
| Hot | 0 | 2 | 2 |
| Cool | 1 | 0 | 1 |
| Mild | 1 | 1 | 2 |

Gini(Outlook=Sunny and Temp.=Hot) = $1 - (0/2)^2 - (2/2)^2 = 0$

Gini(Outlook=Sunny and Temp.=Cool) = $1 - (1/1)^2 - (0/1)^2 = 0$

Gini(Outlook=Sunny and Temp.=Mild) = $1 - (1/2)^2 - (1/2)^2 = 1 - 0.25 - 0.25 = 0.5$

Gini(Outlook=Sunny and Temp.) = $(2/5)x0 + (1/5)x0 + (2/5)x0.5 = 0.2$

## Gini of humidity for sunny outlook

| Humidity | Yes | No | Number of instances |
|---|---|---|---|
| High | 0 | 3 | 3 |
| Normal | 2 | 0 | 2 |

Gini(Outlook=Sunny and Humidity=High) = $1 - (0/3)^2 - (3/3)^2 = 0$

Gini(Outlook=Sunny and Humidity=Normal) = $1 - (2/2)^2 - (0/2)^2 = 0$

Gini(Outlook=Sunny and Humidity) = $(3/5)x0 + (2/5)x0 = 0$

## Gini of wind for sunny outlook

| Wind | Yes | No | Number of instances |
|---|---|---|---|
| Weak | 1 | 2 | 3 |
| Strong | 1 | 1 | 2 |

Gini(Outlook=Sunny and Wind=Weak) = $1 - (1/3)^2 - (2/3)^2 = 0.266$

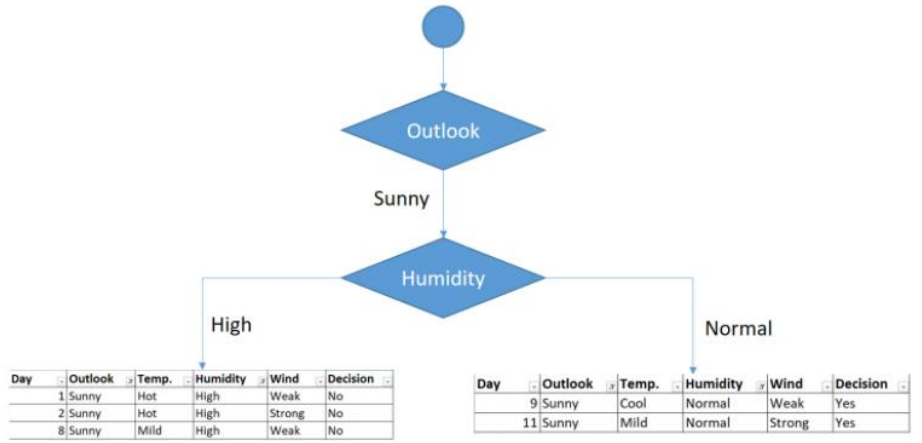
Gini(Outlook=Sunny and Wind=Strong) = $1- (1/2)^2 - (1/2)^2 = 0.2$

Gini(Outlook=Sunny and Wind) = $(3/5)x0.266 + (2/5)x0.2 = 0.466$

## Decision for sunny outlook

We've calculated gini index scores for feature when outlook is sunny. The winner is humidity because it has the lowest value.

| Feature | Gini index |
|---|---|
| Temperature | 0.2 |
| Humidity | 0 |
| Wind | 0.466 |




Sub datasets for high and normal humidity

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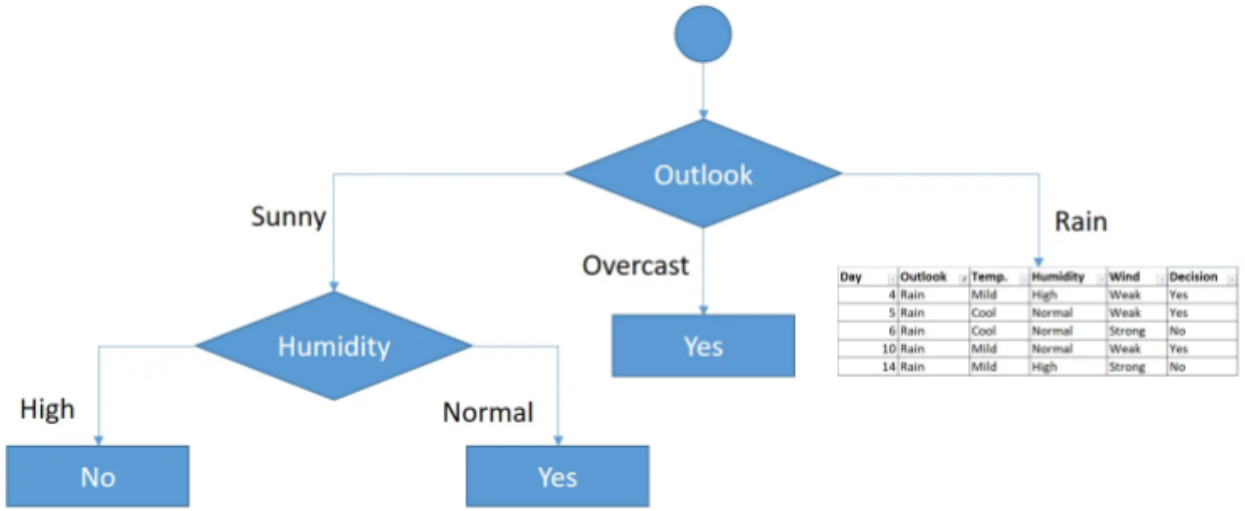


| Day | Outlook | Temp. | Humidity | Wind | Decision |
|---|---|---|---|---|---|
| 4 | Rain | Mild | High | Weak | Yes |
| 5 | Rain | Cool | Normal | Weak | Yes |
| 6 | Rain | Cool | Normal | Strong | No |
| 10 | Rain | Mild | Normal | Weak | Yes |
| 14 | Rain | Mild | High | Strong | No |

Decisions for high and normal humidity

## Rain outlook

| Day | Outlook | Temp. | Humidity | Wind | Decision |
|---|---|---|---|---|---|
| 4 | Rain | Mild | High | Weak | Yes |
| 5 | Rain | Cool | Normal | Weak | Yes |
| 6 | Rain | Cool | Normal | Strong | No |
| 10 | Rain | Mild | Normal | Weak | Yes |
| 14 | Rain | Mild | High | Strong | No |

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## Gini of temprature for rain outlook

| Temperature | Yes | No | Number of instances |
|---|---|---|---|
| Cool | 1 | 1 | 2 |
| Mild | 2 | 1 | 3 |

Gini(Outlook=Rain and Temp.=Cool) = $1 - (1/2)^2 - (1/2)^2 = 0.5$

Gini(Outlook=Rain and Temp.=Mild) = $1 - (2/3)^2 - (1/3)^2 = 0.444$

Gini(Outlook=Rain and Temp.) = $(2/5)x0.5 + (3/5)x0.444 = 0.466$

## Gini of humidity for rain outlook

| Humidity | Yes | No | Number of instances |
|---|---|---|---|
| High | 1 | 1 | 2 |
| Normal | 2 | 1 | 3 |

Gini(Outlook=Rain and Humidity=High) = $1 - (1/2)^2 - (1/2)^2 = 0.5$

Gini(Outlook=Rain and Humidity=Normal) = $1 - (2/3)^2 - (1/3)^2 = 0.444$

Gini(Outlook=Rain and Humidity) = $(2/5)x0.5 + (3/5)x0.444 = 0.466$

## Gini of wind for rain outlook

| Wind | Yes | No | Number of instances |
|---|---|---|---|
| Weak | 3 | 0 | 3 |
| Strong | 0 | 2 | 2 |

Gini(Outlook=Rain and Wind=Weak) = $1 - (3/3)^2 - (0/3)^2 = 0$

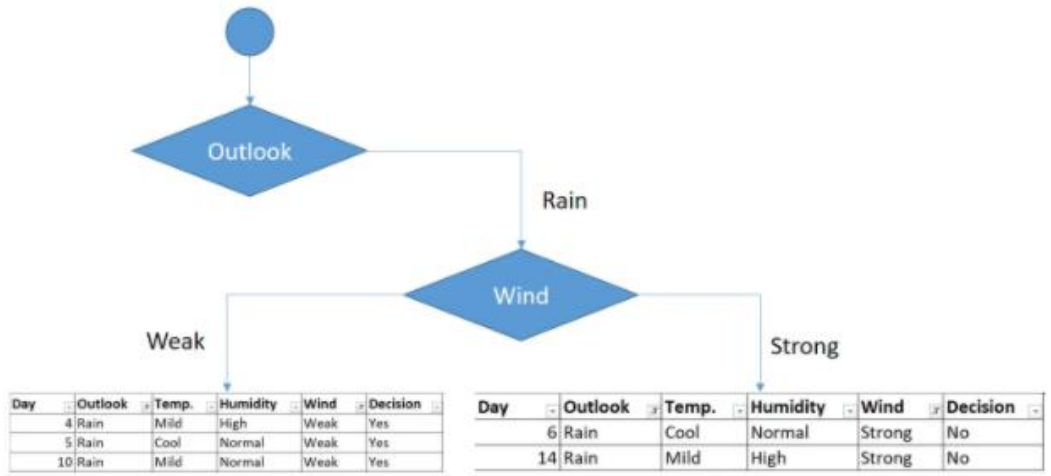
Gini(Outlook=Rain and Wind=Strong) = $1 - (0/2)^2 - (2/2)^2 = 0$

Gini(Outlook=Rain and Wind) = $(3/5)x0 + (2/5)x0 = 0$

## Decision for rain outlook

The winner is wind feature for rain outlook because it has the minimum gini index score in features.

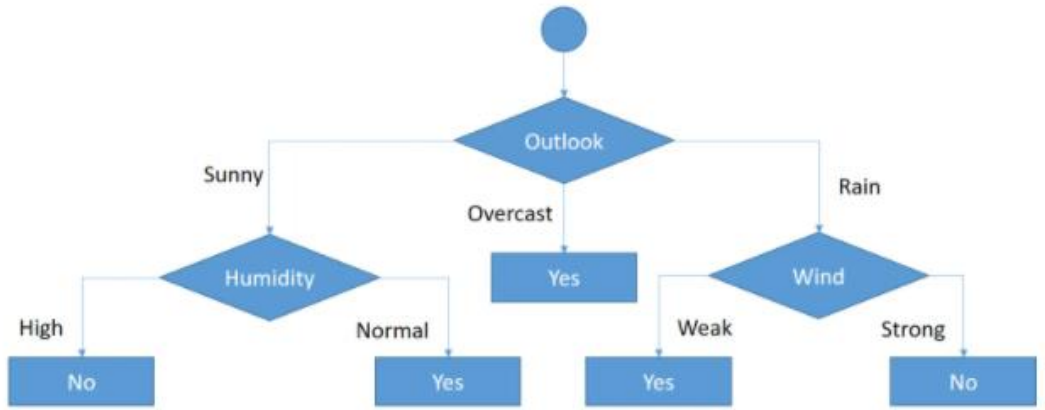
| Feature | Gini index |
|---|---|
| Temperature | 0.466 |
| Humidity | 0.466 |
| Wind | 0 |




| Day | Outlook | Temp. | Humidity | Wind | Decision |
|---|---|---|---|---|---|
| 4 | Rain | Mild | High | Weak | Yes |
| 5 | Rain | Cool | Normal | Weak | Yes |
| 10 | Rain | Mild | Normal | Weak | Yes |

| Day | Outlook | Temp. | Humidity | Wind | Decision |
|---|---|---|---|---|---|
| 6 | Rain | Cool | Normal | Strong | No |
| 14 | Rain | Mild | High | Strong | No |

Sub data sets for weak and strong wind and rain outlook




Final form of the decision tree built by CART algorithm

https://dataaspirant.com/how-decision-tree-algorithm-works/

# Resources/ References


- Introduction to Machine Learning with Python, Andreas C. Müller and Sarah Guido, O'Reilly Media, Inc. October 2016.
- Hands-On Machine Learning with Scikit-Learn, Keras, and TensorFlow, 2nd Edition, Aurélien Géron, O'Reilly Media, September 2019, ISBN: 9781492032649.
- Python Machine Learning - Third Edition, Sebastian Raschka, Vahid Mirjalili, Copyright © 2017 Packt Publishing.
- Discovering Knowledge In Data: An Introduction To Data Exploration, Second Edition, By Daniel Larose And Chantal Larose, John Wiley And Sons, Inc., 2014.
- UCI Repository: http://www.ics.uci.edu/~mlearn/MLRepository.html
- Understanding Autoencoders. (Part I) | by Jelaleddin Sultanov | AI³ | Theory, Practice, Business | Medium
- Statlib: http://lib.stat.cmu.edu
- Some images are used from Google search repository (https://www.google.ie/search) to enhance the level of learning.