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Tuning the Frequencies: Robust Training for Sinusoidal Neural Networks

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Abstract

Sinusoidal neural networks have been shown effective as implicit neural representations (INRs) of low-dimensional signals, due to their smoothness and high representation capacity. However, initializing and training them remain empirical tasks which lack on deeper understanding to guide the learning process. To fill this gap, our work introduces a theoretical framework that explains the capacity property of sinusoidal networks and offers robust control mechanisms for initialization and training. Our analysis is based on a novel amplitude-phase expansion of the sinusoidal multilayer perceptron, showing how its layer compositions produce a large number of new frequencies expressed as integer combinations of the input frequencies. This relationship can be directly used to initialize the input neurons, as a form of spectral sampling, and to bound the network's spectrum while training. Our method, referred to as TUNER (TUNing sinusoidal nEtwoRks), greatly improves the stability and convergence of sinusoidal INR training, leading to detailed reconstructions, while preventing overfitting.

1. Introduction

Sinusoidal multilayer perceptrons (MLPs) emerged as powerful implicit neural representations (INRs) for low-dimensional signals [3, 11, 26, 35]. In this context, the INR f should fit the input data $\{x_i, f_i\}$ as close as possible, i.e. $f(x_i) \approx f_i$, without overfitting, thus encoding the signal implicitly in the MLP parameters. Therefore, two major properties are required: (1) f needs **high representation capacity** to fit $\{x_i, f_i\}$; (2) f should have **bandlimit control** to avoid frequencies bypassing the sampling rate.

Training sinusoidal MLPs to satisfy the above properties is challenging, as their initialization and optimization process often lead to undesirable local minima [15]. Recent work made strides towards more effective learning of these models. For example, SIREN [26] proposed an initialization by projecting the input coordinates to a list of sines with frequencies randomly chosen in a range, similar to the Fourier feature mapping (FFM) approach [28]. This way, the model can reach high capacity, but may lead to overfitting with high frequencies resulting in noisy reconstructions. Defining an effective range for the bandlimit initialization remains mostly empirical and often results in noise, as the role of layer composition in generating frequencies is not fully understood. Additionally, uniform initialization may introduce undesired high frequencies and make it harder to model lower ones. More recently, BACON [10] proposes tighter bandlimit control by applying multiplicative filter networks (MFNs) [7] to limit the signal spectrum

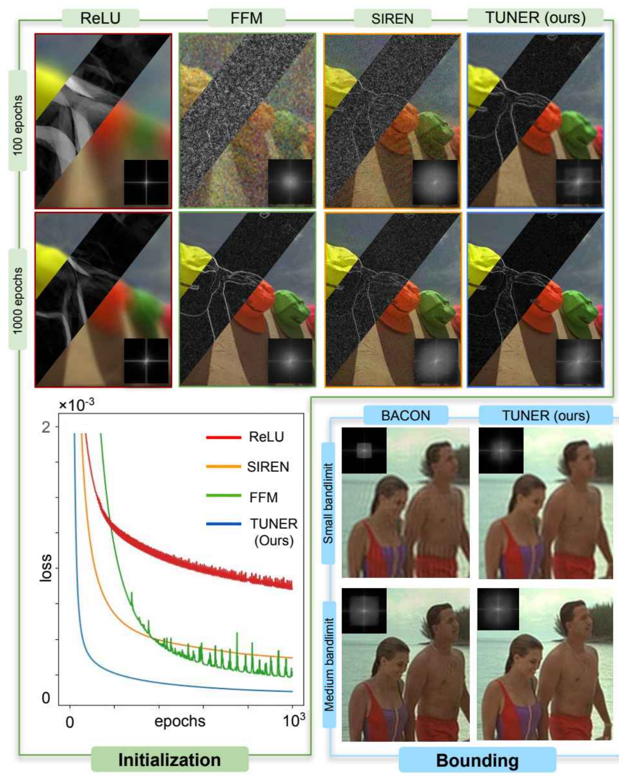

Figure 1. We present **TUNER**, a robust and theoretically grounded training technique for sinusoidal MLPs, overcoming challenges in initialization and enabling bandlimiting control. Our experiments showcase TUNER's strong initialization results against `ReLU`, FFM [28], and SIREN [26] (top), where all models use the same size and training conditions. TUNER achieves both fast and stable convergence (bottom-left) while reconstructing gradients without noise. We also compare with BACON [10] across two bandlimits (bottom-right), enhancing quality and avoiding ringing artifacts.

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