Problem 1. Prove the following proposition by induction.
$\forall n \in \mathbb{Z}^+$: $\sum_{i=1}^{n} 2i - 1 = n^2$

Problem 2. Find a formula for the following sum and prove its correctness for all $n \in \mathbb{Z}^+$ by induction.
$\sum_{i=1}^{n} \frac{1}{i(i+1)}$

Problem 3. Prove the following proposition by contradiction (clearly state the initial assumption that leads to a contradiction). $\forall n \in \mathbb{Z} : n^2$ is even $\implies$ n is even

Problem 4. Point out the flaw in the following "proof" that 2=1.

$$a = b \tag{1}$$

$$a^2 = ab \tag{2}$$

$$a^2 - b^2 = ab - b^2 \tag{3}$$

$$(a - b)(a + b) = b(a - b) \tag{4}$$

$$a + b = b \tag{5}$$

$$2b = b \tag{6}$$

$$2 = 1 \tag{7}$$

Problem 5. (CLRS, Exercise 3.1-1) Let $f(n)$ and $g(n)$ be asymptotically nonnegative functions. Prove that $\max(f(n), g(n)) = \Theta(f(n) + g(n))$.

Problem 6. Explain why $\Theta(\log_{b_1} n) = \Theta(\log_{b_2} n)$ for any pair of bases $b_1, b_2 > 1$. The same is true for O and Ω. In other words, explain why the base of the logarithm does not matter for asymptotic notation.

Problem 7. Consider the following function $f(n) = n(-1)^n$. Is $f(n) \in O(n)$, $f(n) \in \Omega(n)$, $f(n) \in \Theta(n)$? For each bound, prove why it does or does not hold.