Problem 1.
Predicate:
$P(n) = \sum_{i=1}^{n} 2i - 1 = n^2$
Base case ($n = 1$):
$\sum_{i=1}^{1} 2i - 1 = 2 * 1 - 1 = 1 = 1^2$ ✓
Induction Hypothesis:
Assume for purposes of induction that $P(n)$ holds for some $n \in \mathbb{N}$.
Inductive step ($P(n) \implies P(n+1)$):
$\sum_{i=1}^{n+1} 2i - 1 = \sum_{i=1}^{n}(2i-1) + 2(n+1) - 1 =_{IH} n^2 + 2n + 2 - 1 = n^2 + 2n + 1 = (n+1)^2$ ✓
It follows that $P(n)$ holds for all $n \in \mathbb{N}$. $\square$

Problem 2.
1) Find a formula for $\sum_{i=1}^{n} \frac{1}{i(i+1)}$ by computing the first few values:
$n = 1$: $\sum_{i=1}^{1} \frac{1}{i(i+1)} = \frac{1}{1(1+1)} = \frac{1}{2}$
$n = 2$: $\sum_{i=1}^{2} \frac{1}{i(i+1)} = \frac{1}{2} + \frac{1}{2(2+1)} = \frac{1}{2} + \frac{1}{6} = \frac{2}{3}$
$n = 3$: $\sum_{i=1}^{3} \frac{1}{i(i+1)} = \frac{2}{3} + \frac{1}{3(3+1)} = \frac{2}{3} + \frac{1}{12} = \frac{3}{4} \implies$ guess: $\sum_{i=1}^{n} \frac{1}{i(i+1)} = \frac{n}{n+1}$
2) Proof by induction
Predicate:
$P(n) = \sum_{i=1}^{n} \frac{1}{i(i+1)} = \frac{n}{n+1}$
Base case ($n = 1$):
$\sum_{i=1}^{1} \frac{1}{i(i+1)} = \frac{1}{2} = \frac{1}{1+1}$✓
Induction Hypothesis:
Assume for purposes of induction that $P(n)$ holds for some $n \in \mathbb{N}$.
Inductive step ($P(n) \implies P(n+1)$):
$\sum_{i=1}^{n+1} \frac{1}{i(i+1)} = \sum_{i=1}^{n} \frac{1}{i(i+1)} + \frac{1}{(n+1)((n+1)+1)} =_{IH} \frac{n}{n+1} + \frac{1}{(n+1)(n+2)} = \frac{n(n+2)+1}{(n+1)(n+2)} = \frac{(n+1)^2}{(n+1)(n+2)} = \frac{n+1}{n+2}$ ✓
It follows that $P(n)$ holds for all $n \in \mathbb{N}$. $\square$

Problem 3. Note that $\neg(\forall n \in \mathbb{Z} : P(n))$ is $\exists n \in \mathbb{Z} : \neg P(n)$ and $\neg(p \to q)$ is $p \wedge \neg q$
Assume for purposes of contradiction that there is some $n \in \mathbb{Z}$ such that n^2 is even and n is odd. If n is odd, then it is $n = 2k+1$ for some $k \in \mathbb{Z}$. Then $n^2 = (2k+1)^2 = 4k^2 + 4k + 1 = 2(2k^2 + 2k) + 1$. Since $2k^2 + 2k \in \mathbb{Z}$, this means that n^2 is odd. That is a contradiction to the assumption that n^2 is even. So there can be no $n \in \mathbb{Z}$ such that n^2 is even and n is odd.

Problem 4. Since $a = b$ according to (1), we know that $(a - b) = 0$. So the division from (4) to (5) is a division by 0, which is undefined and makes the proof invalid.

Problem 5. To prove that $\max(f(n), g(n)) \in \Theta(f(n) + g(n))$, we need to find positive constants c_1, c_2, n_0 such that $0 \leq c_1(f(n) + g(n)) \leq \max(f(n), g(n)) \leq c_2(f(n) + g(n))$ for all $n > n_0$.
c_1: It is $f(n) \leq \max(f(n), g(n))$ and $g(n) \leq \max(f(n), g(n))$ for all f, g, and n. Therefore, it follows that $f(n) + g(n) \leq 2\max(f(n), g(n))$ and so $\frac{1}{2}(f(n) + g(n)) \leq \max(f(n), g(n))$ for all n. Choose $c_1 = \frac{1}{2}$.
c_2: $f(n)$ and $g(n)$ are asymptotically nonnegative, which means that there is some n_0 such that $f(n) \geq 0$ and $g(n) \geq 0$ for all $n > n_0$. For those sufficiently large n it is $f(n) + g(n) \geq f(n)$ and $f(n) + g(n) \geq g(n)$. Since $\max(f(n), g(n))$ is either $f(n)$ or $g(n)$ for all values of n, it follows from the previous observations that $f(n) + g(n) \geq \max(f(n), g(n))$ for sufficiently large n. Choose $c_2 = 1$ and n_0 s.t. $f(n), g(n) > 0\ \forall n > n_0$. $\square$

Problem 6. For all bases $a, b > 1$, it is $\log_b(x) = \log_a(x)/\log_a(b)$. This means that any logarithm of a value for one base is a constant multiple of the logarithm for another base. Since constant factors do not matter in asymptotic notation, the sets of functions defined by $O(\log_a n)$ and $O(\log_b n)$ are the same, and similarly for Ω, Θ. More formally, if $f(n) \in O(\log_b n)$, there are constants c and n_0 such that $0 \leq f(n) \leq c \log_b n$ for all $n > n_0$. Then it is also $f(n) \leq \frac{c}{\log_a b} \log_a n$ for all $n > n_0$. So we found a new constant $c' := \frac{c}{\log_a b} > 0$ (it is greater than 0 because $a, b > 1$) such that $f(n) \leq c' \log_a n$ for sufficiently large n and so $f(n) \in O(\log_a n)$. The argument works analogously for Ω and therefore also Θ.

Problem 7. $f(n)$ is not asymptotically nonnegative. That is, there is no n_0 such that $f(n) \geq 0$ for all $n > n_0$. Therefore, $f(n)$ does not meet this requirement of all of the asymptotic notations, so $f(n) \notin O(n)$, $f(n) \notin \Omega(n)$ and $f(n) \notin \Theta(n)$.