Solutions to HW5

Note: Most of these solutions were generated by R. D. Yates and D. J. Goodman, the authors of our textbook. I have added comments in italics where I thought more detail was appropriate. I have made corrections where needed. The solution to problem 3.9.2 is my own.

Problem 3.1.1 •

The cumulative distribution function of random variable X is

$$F_X(x) = \begin{cases} 0 & x < -1, \\ (x+1)/2 & -1 \le x < 1, \\ 1 & x \ge 1. \end{cases}$$

(a) What is $P[X > 1/2]$?

(b) What is $P[-1/2 < X \le 3/4]$?

(c) What is $P[|X| \le 1/2]$?

(d) What is the value of a such that $P[X \le a] = 0.8$?

Problem 3.1.1 Solution

The CDF of X is

$$F_X(x) = \begin{cases} 0 & x < -1 \\ (x+1)/2 & -1 \le x < 1 \\ 1 & x \ge 1 \end{cases} \tag{1}$$

Each question can be answered by expressing the requested probability in terms of $F_X(x)$.

(a)

$$P[X > 1/2] = 1 - P[X \le 1/2] = 1 - F_X(1/2) = 1 - 3/4 = 1/4 \tag{2}$$

(b) This is a little trickier than it should be. Being careful, we can write

$$P[-1/2 \le X < 3/4] = P[-1/2 < X \le 3/4] + P[X = -1/2] - P[X = 3/4] \tag{3}$$

Since the CDF of X is a continuous function, the probability that X takes on any specific value is zero. This implies $P[X = 3/4] = 0$ and $P[X = -1/2] = 0$. (If this is not clear at this point, it will become clear in Section 3.6.) Thus,

$$P[-1/2 \le X < 3/4] = P[-1/2 < X \le 3/4] = F_X(3/4) - F_X(-1/2) = 5/8 \tag{4}$$

(c)

$$P[|X| \le 1/2] = P[-1/2 \le X \le 1/2] = P[X \le 1/2] - P[X < -1/2] \tag{5}$$

Note that $P[X \le 1/2] = F_X(1/2) = 3/4$. Since the probability that $P[X = -1/2] = 0$, $P[X < -1/2] = P[X \le 1/2]$. Hence $P[X < -1/2] = F_X(-1/2) = 1/4$. This implies

$$P[|X| \le 1/2] = P[X \le 1/2] - P[X < -1/2] = 3/4 - 1/4 = 1/2 \tag{6}$$

(d) Since $F_X(1) = 1$, we must have $a \le 1$. For $a \le 1$, we need to satisfy

$$P[X \le a] = F_X(a) = \frac{a+1}{2} = 0.8 \tag{7}$$

Thus $a = 0.6$.

Problem 3.1.2 •

The cumulative distribution function of the continuous random variable V is

$$F_V(v) = \begin{cases} 0 & v < -5, \\ c(v+5)^2 & -5 \le v < 7, \\ 1 & v \ge 7. \end{cases}$$

(a) What is c?

(b) What is $P[V > 4]$?

(c) $P[-3 < V \le 0]$?

(d) What is the value of a such that $P[V > a] = 2/3$?

Problem 3.1.2 Solution

The CDF of V was given to be

$$F_V(v) = \begin{cases} 0 & v < -5 \\ c(v+5)^2 & -5 \le v < 7 \\ 1 & v \ge 7 \end{cases} \tag{1}$$

(a) For V to be a continuous random variable, $F_V(v)$ must be a continuous function. This occurs if we choose c such that $F_V(v)$ doesn't have a discontinuity at $v = 7$. We meet this requirement if $c(7+5)^2 = 1$. This implies $c = 1/144$.

(b)

$$P[V > 4] = 1 - P[V \le 4] = 1 - F_V(4) = 1 - 81/144 = 63/144 = 7/16 \tag{2}$$

(c)

$$P[-3 < V \le 0] = F_V(0) - F_V(-3) = 25/144 - 4/144 = 21/144 = 7/48 \tag{3}$$

(d) Since $0 \le F_V(v) \le 1$ and since $F_V(v)$ is a nondecreasing function, it must be that $-5 \le a \le 7$. In this range,

$$P[V > a] = 1 - F_V(a) = 1 - (a+5)^2/144 = 2/3 \tag{4}$$

The unique solution in the range $-5 \le a \le 7$ is $a = 4\sqrt{3} - 5 = 1.928$.

Problem 3.2.1 •

The random variable X has probability density function

$$f_X(x) = \begin{cases} cx & 0 \le x \le 2, \\ 0 & \text{otherwise.} \end{cases}$$

Use the PDF to find

(a) the constant c,

(b) $P[0 \le X \le 1]$,

(c) $P[-1/2 \le X \le 1/2]$,

(d) the CDF $F_X(x)$.

Problem 3.2.1 Solution

$$f_X(x) = \begin{cases} cx & 0 \le x \le 2 \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

(a) From the above PDF we can determine the value of c by integrating the PDF and setting it equal to 1.

$$\int_0^2 cx\,dx = 2c = 1 \tag{2}$$

Therefore $c = 1/2$.

(b) $P[0 \le X \le 1] = \int_0^1 \frac{x}{2}\,dx = 1/4$

(c) $P[-1/2 \le X \le 1/2] = \int_0^{1/2} \frac{x}{2}\,dx = 1/16$

(d) The CDF of X is found by integrating the PDF from 0 to x.

$$F_X(x) = \int_0^x f_X(x')\,dx' = \begin{cases} 0 & x < 0 \\ x^2/4 & 0 \le x \le 2 \\ 1 & x > 2 \end{cases} \tag{3}$$

Problem 3.2.2 •

The cumulative distribution function of random variable X is

$$F_X(x) = \begin{cases} 0 & x < -1, \\ (x+1)/2 & -1 \le x < 1, \\ 1 & x \ge 1. \end{cases}$$

Find the PDF $f_X(x)$ of X.

Problem 3.2.2 Solution
From the CDF, we can find the PDF by direct differentiation. The CDF and correponding PDF are

$$F_X(x) = \begin{cases} 0 & x < -1 \\ (x+1)/2 & -1 \le x < 1 \\ 1 & x \ge 1 \end{cases} \qquad f_X(x) = \begin{cases} 1/2 & -1 \le x < 1 \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

Problem 3.3.3 •
Random variable X has CDF

$$F_X(x) = \begin{cases} 0 & x < 0, \\ x/2 & 0 \le x \le 2, \\ 1 & x > 2. \end{cases}$$

(a) What is $E[X]$?

(b) What is $\text{Var}[X]$?

Problem 3.3.3 Solution
The CDF of X is

$$F_X(x) = \begin{cases} 0 & x < 0 \\ x/2 & 0 \le x < 2 \\ 1 & x \ge 2 \end{cases} \tag{1}$$

(a) To find $E[X]$, we first find the PDF by differentiating the above CDF.

$$f_X(x) = \begin{cases} 1/2 & 0 \le x \le 2 \\ 0 & \text{otherwise} \end{cases} \tag{2}$$

The expected value is then

$$E[X] = \int_0^2 \frac{x}{2}\,dx = 1 \tag{3}$$

(b)

$$E\left[X^2\right] = \int_0^2 \frac{x^2}{2}\,dx = 8/6 = 4/3 \tag{4}$$

$$\text{Var}[X] = E\left[X^2\right] - E[X]^2 = 4/3 - 1 = 1/3 \tag{5}$$

Problem 3.3.4 •
The probability density function of random variable Y is

$$f_Y(y) = \begin{cases} y/2 & 0 \le y < 2, \\ 0 & \text{otherwise.} \end{cases}$$

What are $E[Y]$ and $\text{Var}[Y]$?

Problem 3.3.4 Solution

We can find the expected value of X by direct integration of the given PDF.

$$f_Y(y) = \begin{cases} y/2 & 0 \le y \le 2 \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

The expectation is

$$E[Y] = \int_{-\infty}^{\infty} y f_Y(y) dy = \int_0^2 \frac{y^2}{2} dy = 4/3 \tag{2}$$

To find the variance, we first find the second moment

$$E[Y^2] = \int_{-\infty}^{\infty} y^2 f_Y(y) dy = \int_0^2 \frac{y^3}{2} dy = 2. \tag{3}$$

The variance is then $\text{Var}[Y] = E[Y^2] - E[Y]^2 = 2 - (4/3)^2 = 2/9$.

Problem 3.4.2 •

Y is an exponential random variable with variance $\text{Var}[Y] = 25$.

(a) What is the PDF of Y?

(b) What is $E[Y^2]$?

(c) What is $P[Y > 5]$?

Problem 3.4.2 Solution

(a) From Appendix A, we observe that an exponential PDF Y with parameter $\lambda > 0$ has PDF

$$f_Y(y) = \begin{cases} \lambda e^{-\lambda y} & y \ge 0 \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

In addition, the mean and variance of Y are

$$E[Y] = \frac{1}{\lambda} \qquad \text{Var}[Y] = \frac{1}{\lambda^2} \tag{2}$$

Since $\text{Var}[Y] = 25$, we must have $\lambda = 1/5$.

(b) The expected value of Y is $E[Y] = 1/\lambda = 5$, so

$$E[Y^2] = \text{Var}[Y] + (E[Y])^2 = 50 \tag{3}$$

(c)

$$P[Y > 5] = \int_5^{\infty} f_Y(y)\, dy = -e^{-y/5}\Big|_5^{\infty} = e^{-1} \tag{4}$$

Problem 3.4.3 •
X is an Erlang (n, λ) random variable with parameter $\lambda = 1/3$ and expected value $E[X] = 15$.

(a) What is the value of the parameter n?

(b) What is the PDF of X?

(c) What is $\text{Var}[X]$?

Problem 3.4.3 Solution
From Appendix A, an Erlang random variable X with parameters $\lambda > 0$ *a postive real number* and n `a positive integer` has PDF

$$f_X(x) = \begin{cases} \lambda^n x^{n-1} e^{-\lambda x}/(n-1)! & x \geq 0 \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

In addition, the mean and variance of X are

$$E[X] = \frac{n}{\lambda} \qquad \text{Var}[X] = \frac{n}{\lambda^2} \tag{2}$$

(a) Since $\lambda = 1/3$ and $E[X] = n/\lambda = 15$, we must have $n = 5$.

(b) Substituting the parameters $n = 5$ and $\lambda = 1/3$ into the given PDF, we obtain

$$f_X(x) = \begin{cases} (1/3)^5 x^4 e^{-x/3}/24 & x \geq 0 \\ 0 & \text{otherwise} \end{cases} \tag{3}$$

(c) From above, we know that $\text{Var}[X] = n/\lambda^2 = 45$.

Note: we need not use the definitions in Appendix A to solve these problems. We can obtain the expressions for the expected value and the variance by applying the definitions. This will require using integration by parts and induction, but is not otherwise difficult.

Problem 3.5.1 •
The peak temperature T, as measured in degrees Fahrenheit, on a July day in New Jersey is the Gaussian $(85, 10)$ random variable. What is $P[T > 100]$, $P[T < 60]$, and $P[70 \leq T \leq 100]$?

Problem 3.5.1 Solution
Given that the peak temperature, T, is a Gaussian random variable with mean 85 and standard deviation 10 we can use the fact that $F_T(t) = \Phi((t - \mu_T)/\sigma_T)$ and Table 3.1 on

page 123 to evaluate the following

$$P[T > 100] = 1 - P[T \le 100] = 1 - F_T(100) = 1 - \Phi\left(\frac{100-85}{10}\right)$$
$$= 1 - \Phi(1.5) = 1 - 0.9332 = 0.0668 \tag{1}$$
$$P[T < 60] = \Phi\left(\frac{60-85}{10}\right) = \Phi(-2.5)$$
$$= 1 - \Phi(2.5) = 1 - .9938 = 0.0062 \tag{2}$$
$$P[70 \le T \le 100] = F_T(100) - F_T(70)$$
$$= \Phi(1.5) - \Phi(-1.5) = 2\Phi(1.5) - 1 = .8664 \tag{3}$$

Problem 3.5.3 •

X is a Gaussian random variable with $E[X] = 0$ and $P[|X| \le 10] = 0.1$. What is the standard deviation σ_X?

Problem 3.5.3 Solution

X is a Gaussian random variable with zero mean but unknown variance. We do know, however, that

$$P[|X| \le 10] = 0.1 \tag{1}$$

We can find the variance Var$[X]$ by expanding the above probability in terms of the $\Phi(\cdot)$ function.

$$P[-10 \le X \le 10] = F_X(10) - F_X(-10) = \Phi\left(\frac{10}{\sigma_X}\right) - \left(1 - \Phi\left(\frac{10}{\sigma_X}\right)\right) = 2\Phi\left(\frac{10}{\sigma_X}\right) - 1 \tag{2}$$

This implies $\Phi(10/\sigma_X) = 0.55$. Using Table 3.1 for the Gaussian CDF, we find that $10/\sigma_X \approx 0.125$ or $\sigma_X \approx 80$.

Problem 3.6.2 •

Let X be a random variable with CDF

$$F_X(x) = \begin{cases} 0 & x < -1, \\ x/4 + 1/2 & -1 \le x < 1, \\ 1 & 1 \le x. \end{cases}$$

Sketch the CDF and find

(a) $P[X < -1]$ and $P[X \le -1]$,

(b) $P[X < 0]$ and $P[X \le 0]$,

(c) $P[X > 1]$ and $P[X \ge 1]$.

Problem 3.6.2 Solution

Here the authors use the notation

$$F_X\left(a^-\right) := \lim_{x\to a^-} F_X\left(a\right) \tag{1}$$

$$F_X\left(a^+\right) := \lim_{x\to a^+} F_X\left(a\right) \tag{2}$$

where a is any value in the range of the CDF.

[As in] the previous problem we find

(a)

$$P\left[X < -1\right] = F_X\left(-1^-\right) = 0 \qquad P\left[X \le -1\right] = F_X\left(-1\right) = 1/4 \tag{3}$$

Here we notice the discontinuity of value 1/4 at $x = -1$.

(b)

$$P\left[X < 0\right] = F_X\left(0^-\right) = 1/2 \qquad P\left[X \le 0\right] = F_X\left(0\right) = 1/2 \tag{4}$$

Since there is no discontinuity at $x = 0$, $F_X(0^-) = F_X(0^+) = F_X(0)$.

(c)

$$P\left[X > 1\right] = 1 - P\left[X \le 1\right] = 1 - F_X\left(1\right) = 0 \tag{5}$$

$$P\left[X \ge 1\right] = 1 - P\left[X < 1\right] = 1 - F_X\left(1^-\right) = 1 - 3/4 = 1/4 \tag{6}$$

Again we notice a discontinuity of size 1/4, here occurring at $x = 1$.

Problem 3.6.3 •

For random variable X of Problem 3.6.2, find

(a) $f_X(x)$

(b) $E[X]$

(c) $\mathrm{Var}[X]$

Problem 3.6.3 Solution

(a) By taking the derivative of the CDF $F_X(x)$ given in Problem 3.6.2, we obtain the PDF

$$f_X\left(x\right) = \begin{cases} \frac{\delta(x+1)}{4} + 1/4 + \frac{\delta(x-1)}{4} & -1 \le x \le 1 \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

The reason for the factor of 1/4 multiplying the impulses can be seen by graphing the CDF and determining the magnitude of the jumps in the CDF that occur at ± 1. (You can calculate this without drawing the graph but it can be helpful to visualize the behavior of the function.)

(b) The first moment of X is

$$E[X] = \int_{-\infty}^{\infty} x f_X(x)\, dx \tag{2}$$

$$= x/4|_{x=-1} + x^2/8\Big|_{-1}^{1} + x/4|_{x=1} = -1/4 + 0 + 1/4 = 0. \tag{3}$$

(c) The second moment of X is

$$E\left[X^2\right] = \int_{-\infty}^{\infty} x^2 f_X(x)\, dx \tag{4}$$

$$= x^2/4\Big|_{x=-1} + x^3/12\Big|_{-1}^{1} + x^2/4\Big|_{x=1} = 1/4 + 1/6 + 1/4 = 2/3. \tag{5}$$

Since $E[X] = 0$, $\text{Var}[X] = E[X^2] = 2/3$.

Problem 3.7.2 •

Let X have an exponential (λ) PDF. Find the CDF and PDF of $Y = \sqrt{X}$. Show that Y is a Rayleigh random variable (see Appendix A.2). Express the Rayleigh parameter a in terms of the exponential parameter λ.

Problem 3.7.2 Solution

Since $Y = \sqrt{X}$, the fact that X is nonegative and that we asume the square root is always positive implies $F_Y(y) = 0$ for $y < 0$. In addition, for $y \geq 0$, we can find the CDF of Y by writing

$$F_Y(y) = P[Y \leq y] = P\left[\sqrt{X} \leq y\right] = P\left[X \leq y^2\right] = F_X\left(y^2\right) \tag{1}$$

For $x \geq 0$, $F_X(x) = 1 - e^{-\lambda x}$. Thus,

$$F_Y(y) = \begin{cases} 1 - e^{-\lambda y^2} & y \geq 0 \\ 0 & \text{otherwise} \end{cases} \tag{2}$$

By taking the derivative with respect to y, it follows that the PDF of Y is

$$f_Y(y) = \begin{cases} 2\lambda y e^{-\lambda y^2} & y \geq 0 \\ 0 & \text{otherwise} \end{cases} \tag{3}$$

In comparing this result to the Rayleigh PDF given in Appendix A, we observe that Y is a Rayleigh (a) random variable with $a = \sqrt{2\lambda}$.

Problem 3.7.3 •

If X has an exponential (λ) PDF, what is the PDF of $W = X^2$?

Problem 3.7.3 Solution

Since X is non-negative, $W = X^2$ is also non-negative. Hence for $w < 0$, $f_W(w) = 0$. For $w \geq 0$,

$$\begin{aligned} F_W(w) = P[W \leq w] &= P\left[X^2 \leq w\right] && (1)\\ &= P[X \leq w] && (2)\\ &= 1 - e^{-\lambda\sqrt{w}} && (3)\end{aligned}$$

Taking the derivative with respect to w yields, for $w \geq 0$, $f_W(w) = \lambda e^{-\lambda\sqrt{w}}/(2\sqrt{w})$. The complete expression for the PDF is

$$f_W(w) = \begin{cases} \frac{\lambda e^{-\lambda\sqrt{w}}}{2\sqrt{w}} & w > 0 \\ 0 & \text{otherwise} \end{cases} \tag{4}$$

where we note that we cannot allow $w = 0$ *to be part of the first case when we have* $\sqrt{w}$ *in the denominator.*

Problem 3.8.1 •

X is a uniform random variable with parameters -5 and 5. Given the event $B = \{|X| \leq 3\}$,

(a) Find the conditional PDF, $f_{X|B}(x)$.

(b) Find the conditional expected value, $E[X|B]$.

(c) What is the conditional variance, $\text{Var}[X|B]$?

Problem 3.8.1 Solution

The PDF of X is

$$f_X(x) = \begin{cases} 1/10 & -5 \leq x \leq 5 \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

(a) The event B has probability

$$P[B] = P[-3 \leq X \leq 3] = \int_{-3}^{3} \frac{1}{10}\, dx = \frac{3}{5} \tag{2}$$

From Definition 3.15, the conditional PDF of X given B is

$$f_{X|B}(x) = \begin{cases} f_X(x)/P[B] & x \in B \\ 0 & \text{otherwise} \end{cases} = \begin{cases} 1/6 & |x| \leq 3 \\ 0 & \text{otherwise} \end{cases} \tag{3}$$

(b) Given B, we see that X has a uniform PDF over $[a, b]$ with $a = -3$ and $b = 3$. From Theorem 3.6, the conditional expected value of X is $E[X|B] = (a + b)/2 = 0$.

(c) From Theorem 3.6, the conditional variance of X is $\text{Var}[X|B] = (b-a)^2/12 = 3$. *Of course we do not have to use Theorem 3.6. We can instead use the definitions and write*

$$\text{Var}[X|B] = E\left[X^2|B\right] - (E\left[X|B\right])^2 = \int_{-3}^{3} x^2 \left(\frac{1}{6}\right) dx - 0^2 = \cdots = 3. \qquad (4)$$

Problem 3.8.2 •

Y is an exponential random variable with parameter $\lambda = 0.2$. Given the event $A = \{Y < 2\}$,

(a) What is the conditional PDF, $f_{Y|A}(y)$?

(b) Find the conditional expected value, $E[Y|A]$.

Problem 3.8.2 Solution

From Definition 3.6, the PDF of Y is

$$f_Y(y) = \begin{cases} (1/5)e^{-y/5} & y \geq 0 \\ 0 & \text{otherwise} \end{cases} \qquad (1)$$

(a) The event A has probability

$$P[A] = P[Y < 2] = \int_0^2 (1/5)e^{-y/5}\, dy = -e^{-y/5}\Big|_0^2 = 1 - e^{-2/5} \qquad (2)$$

From Definition 3.15, the conditional PDF of Y given A is

$$\begin{aligned} f_{Y|A}(y) &= \begin{cases} f_Y(y)/P[A] & x \in A \\ 0 & \text{otherwise} \end{cases} && (3) \\ &= \begin{cases} (1/5)e^{-y/5}/(1-e^{-2/5}) & 0 \leq y < 2 \\ 0 & \text{otherwise} \end{cases} && (4) \end{aligned}$$

(b) The conditional expected value of Y given A is

$$E[Y|A] = \int_{-\infty}^{\infty} y f_{Y|A}(y)\, dy = \frac{1/5}{1-e^{-2/5}} \int_0^2 y e^{-y/5}\, dy \qquad (5)$$

Using the integration by parts formula $\int u\, dv = uv - \int v\, du$ with $u = y$ and $dv = e^{-y/5}\, dy$ yields

$$\begin{aligned} E[Y|A] &= \frac{1/5}{1-e^{-2/5}} \left(-5ye^{-y/5}\Big|_0^2 + \int_0^2 5e^{-y/5}\, dy \right) && (6) \\ &= \frac{1/5}{1-e^{-2/5}} \left(-10e^{-2/5} - 25e^{-y/5}\Big|_0^2 \right) && (7) \\ &= \frac{5-7e^{-2/5}}{1-e^{-2/5}} && (8) \end{aligned}$$

Problem 3.9.2 •

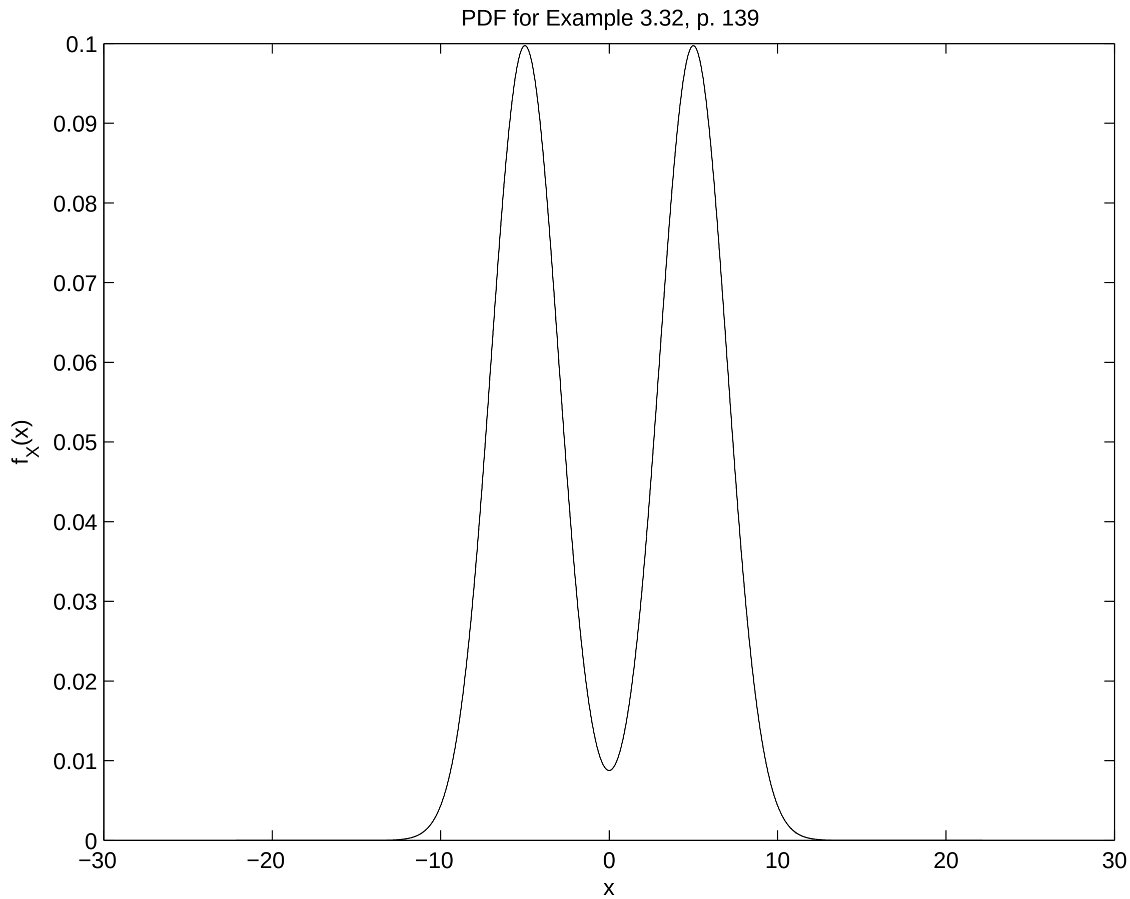
For the modem receiver voltage X with PDF given in Example 3.32, use MATLAB to plot the PDF and CDF of random variable X. Write a MATLAB function `x=modemrv(m)` that produces `m` samples of the modem voltage X.

Problem 3.9.2 Solution

I generated the PDF and CDF using the Matlab commands `normpdf` and `normcdf`. I generated a histogram of the random samples from the PDF generated by my Matlab function `modemrv`. The code for generating the required plots is given below.

```
%
% Yates and Goodman 3.9.2 2/14/06 --sk
%
x1 = (normpdf([-30:.01:30],-5,2)+normpdf([-30:0.01:30],5,2))/2;
figure(1)
plot([-30:.01:30],x1)
title('PDF for Example 3.32, p. 139');
xlabel('x');
ylabel('f_X(x)');
print -deps pdf_3_9_2
figure(2)
x2 = (normcdf([-30:.01:30],-5,2)+normcdf([-30:0.01:30],5,2))/2;
plot([-30:.01:30],x2)
title('CDF for Example 3.32, p. 139');
xlabel('x');
ylabel('F_X(x)');
print -deps cdf_3_9_2

%  some checks:

% height of local minimum of PDF at x = 0
disp('height of local minimum of PDF at x = 0')
fx0 = 2*exp(-25/8)/sqrt(32*pi)
% height of local maximum of PDF at x = +/- 5
disp('height of local maximum of PDF at x = +/- 5')
fx5 = (exp(-100/8)+1)/sqrt(32*pi)

x = modemrv(10000);
figure(3)
hist(x,100);
title('Histogram of samples from the PDF of Example 3.32')
xlabel('x (Volts)')
print -deps hist_3_9_2
```

Here is the plot of the PDF.

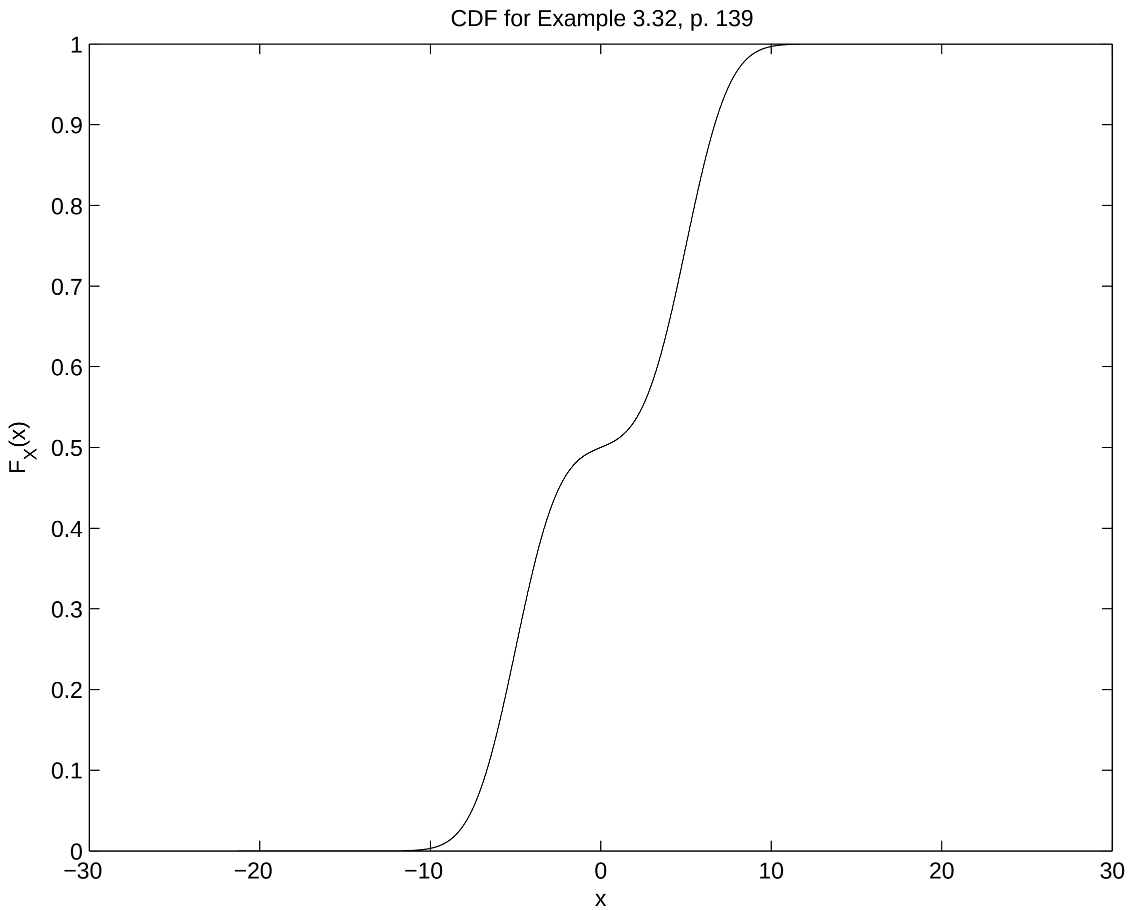


Note that in order to verify that I was using the function `normpdf` correctly I plugged in the values of 0 and 5 to see what the local maxima and minima should be. The results were

```
>> p3_9_2
height of local minimum of PDF at x = 0

fx0 =

    0.0088

height of local maximum of PDF at x = +/- 5

fx5 =

    0.0997
```

which match the values seen in the plot of the PDF.

Here is the plot of the CDF.

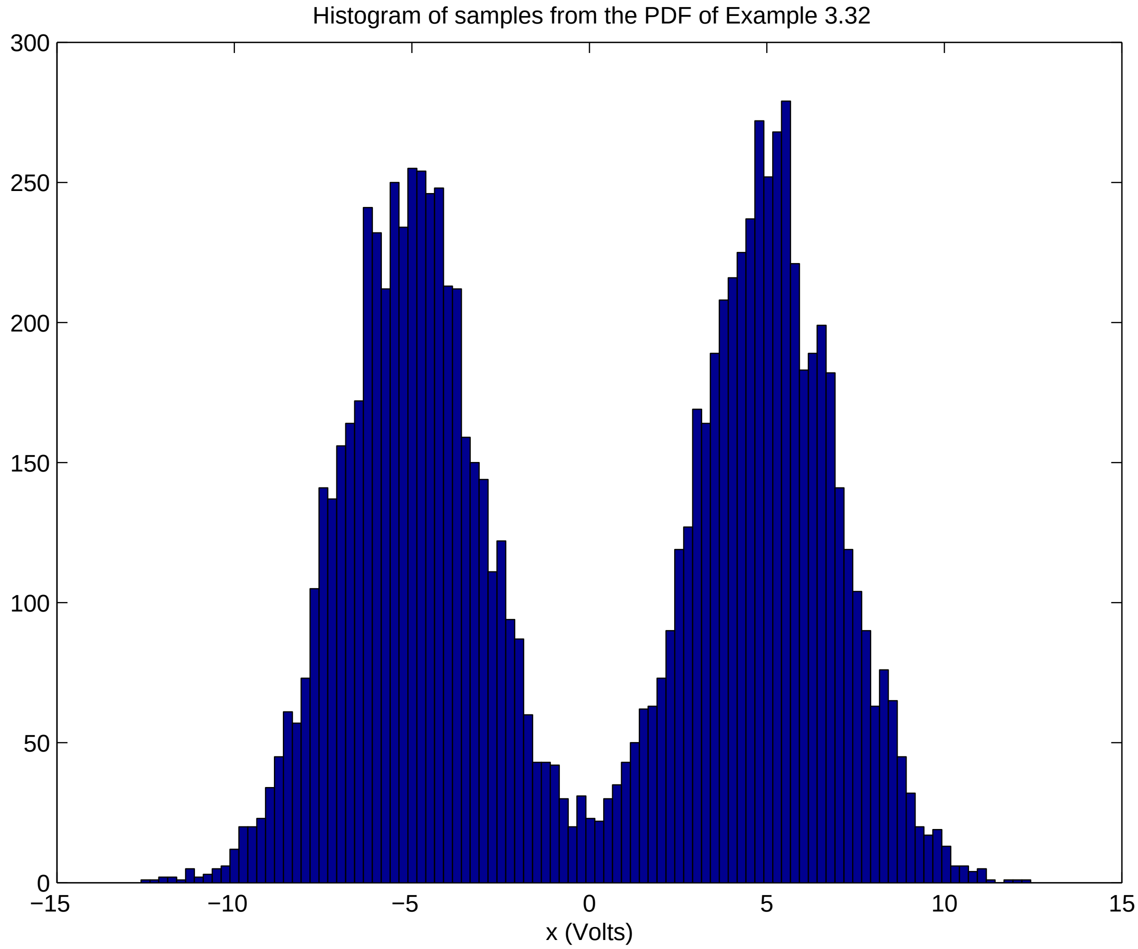


Without doing any calculations, we know that the CDF should have value $F_X(0) = 1/2$ and that $F_X(x)$ should approach 1 as x becomes large. Our plot meets both of these criteria.

Finally, here is the function `modemrv`,

```
function x=modemrv(m);
%Usage: x=modemrv(m)
%generates m samples of X, the modem
%receiver voltage in Exampe 3.32.
%X=+-5 + N where N is Gaussian (0,2)
sb=[-5; 5]; pb=[0.5; 0.5];
b=finiterv(sb,pb,m);
noise=gaussrv(0,2,m);
x=b+noise;
```

and the histogram of the data generated by the function `modemrv`. The shape of the histogram matches the shape of the PDF as it should.


Histogram of samples from the PDF of Example 3.32
300
250
200
150
100
50
0
−15
−10
−5
0
5
10
15
x (Volts)