Mobile Manipulation Calibration

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1 Introduction

The frames of interest are listed in Table 1.

Table 1: Coordinate frames.

Name	Subscript
World	w
Mobile base	b
Base of arm	a
End effector	e
Tool	t

The pose $\boldsymbol{T}_{wt}$ of an arbitrary tool attached to the end effector can be computed using the sequence of transforms

$$\boldsymbol{T}_{wt} = \boldsymbol{T}_{wb}(\boldsymbol{q}_b)\boldsymbol{T}_{ba}\boldsymbol{T}_{ae}(\boldsymbol{q}_a)\boldsymbol{T}_{et}, \tag{1}$$

where $\boldsymbol{T}_{wb}$ depends on the base configuration $\boldsymbol{q}_b = [x_b, y_b, \theta_b]^T$ and $\boldsymbol{T}_{ae}$ depends on the arm configuration $\boldsymbol{q}_a$.

2 Base Calibration

We need to calibration the Vicon measured base poses $\hat{\boldsymbol{q}}_b$ so that the origin of $\boldsymbol{T}_{wb}$ is correct. We have

$$\boldsymbol{T}_{wb}(\boldsymbol{q}_b) = \begin{bmatrix} \boldsymbol{C}_z(\theta_b) & \boldsymbol{r}_b \\ \boldsymbol{0}^T & 1 \end{bmatrix}, \tag{2}$$

where $\boldsymbol{C}_z(\theta) \in SO(3)$ is a rotation about the z-axis and $\boldsymbol{r}_b = [x_b, y_b, z_b]^T$ with z_b a constant. Our goal in this section is to calibrate the origin (x_b, y_b) so that it is located at the point of rotation of θ_b.

Starting at an arbitrary configuration $\boldsymbol{q}_{b,0}$, we will move the base to a sequence of desired yaw angles $\theta_{b,i}^d$ and obtain the corresponding measured configurations $\hat{\boldsymbol{q}}_{b,i}$. We would like to find an offset $\Delta\boldsymbol{r}_b$ such that

$$\boldsymbol{r}_{b,i}^d = \hat{\boldsymbol{r}}_{b,i} + \Delta\boldsymbol{r}_b \tag{3}$$

is satisfied as closely as possible for each i. We will do so by solving the least squares problem

$$\underset{\Delta\boldsymbol{r}_b}{\operatorname{argmin}} \ \frac{1}{2}\sum_i \|\boldsymbol{r}_{b,0} - \hat{\boldsymbol{r}}_{b,i} - \Delta\boldsymbol{r}_b\|^2, \tag{4}$$

where we've used the fact that $\boldsymbol{r}_{b,i}^d = \boldsymbol{r}_{b,0}$.

3 Arm–End Effector–Tool Calibration

Having calibrated the base pose $\boldsymbol{T}_{wb}$, we now wish to calibrate (static) the transforms between the base and arm $\boldsymbol{T}_{ba}$ and the EE and tool $\boldsymbol{T}_{et}$. We will assume that factory calibration for the arm transform $\boldsymbol{T}_{ae}(\boldsymbol{q}_a)$ is good enough.

To do so, we will collect a sequence of pairs $(\boldsymbol{q}_{a,i}, \hat{\boldsymbol{T}}_{wt,i})$ by moving the arm to the sequence of configurations $\boldsymbol{q}_{a,i}$. We will then solve the (nonlinear) least squares problem[1]

$$\underset{\boldsymbol{T}_{ba},\boldsymbol{T}_{wt}}{\operatorname{argmin}} \ \frac{1}{2}\sum_i \| \boxminus (\boldsymbol{T}_{wt}(\boldsymbol{q}), \hat{\boldsymbol{T}}_{wt})\|^2. \tag{5}$$

over the manifold $SE(3)$, where $\boldsymbol{T}_{wt}$ is computed using (1) and

$$\boxminus (\boldsymbol{T}_1, \boldsymbol{T}_2) = \log(\boldsymbol{T}_1^{-1}\boldsymbol{T}_2)^\vee \tag{6}$$

is the error between the poses.

[1] We may have nominal guesses for the transforms, in which case we would actually optimize over (for example) $\Delta\boldsymbol{T}_{ba}$, where $\boldsymbol{T}_{ba} = \Delta\boldsymbol{T}_{ba}\bar{\boldsymbol{T}}_{ba}$ with nominal guess $\bar{\boldsymbol{T}}_{ba}$.