



it's about time



Technical Whitepaper

Signal processing and q

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Signal processing and q

Signal processing is the analysis, interpretation and manipulation of signals to reveal important information. Signals of interest include human speech, seismic waves, images of faces or handwriting, brainwaves, radar, traffic counts and many others. This processing reveals information in a signal that can be obscured by non-useful information, commonly called ‘noise’. This noise can be due to the stochastic[1] nature of signals and/or interference from other signals. Traditionally, signal processing has been performed by either an analog process, through the use of a series of electronic circuits, or through the use of dedicated hardware solutions, such as SIP (Systems In Package) or SoC (Systems on a Chip).

The use of Internet of Things[2] (IoT) devices to capture signals is driving the trend towards software-based signal-processing solutions. Software-based solutions are not only cheaper and more widely accessible than their hardware alternatives, but their highly configurable nature is better suited to the modular aspect of IoT sensor setups. The growth of IoT and software-based signal processing has resulted in increased availability of cheaply-processed signal data, enabling more data-driven decision making, particularly in the manufacturing sector [1].

Currently popular software implementations of digital signal-processing techniques can be found in the open source-source libraries of Python (e.g., SciPy, NumPy, PyPlot) and C++ (e.g., SlgPack, Aquila, Liquid-DSP). The convenience of these libraries is offset by the lack of a robust, high-throughput data-capture system, such as kdb+ tick. While it is possible to integrate a kdb+ process with Python or C++, and utilize these signal-processing routines on a kdb+ dataset, it is entirely possible to implement them natively within q.

This whitepaper will explore how statistical signal-processing operations (those which assume that signals are stochastic), can be implemented natively within q to remove noise, extract useful information, and quickly identify anomalies. This integration allows for q/kdb+ to be used as a single platform for the capture, processing, analysis and storage of large volumes of sensor data.

The technical specifications for the machine used to perform the computations described in this paper are as follows:

- CPU: Intel® Core™i7 Quad Core Processor i7-7700 (3.6GHz) 8MB Cache
- RAM: 16GB Corsair 2133MHz SODIMM DDR4 (1 x 16GB)

- OS: Windows 10 64-bit
- kdb+ 3.5 2017.10.11

Basic steps for signal processing

For the purpose of this paper, signal processing is composed of the following steps:

1. Data Capture
2. Spectral Analysis
3. Smoothing
4. Anomaly Detection

This paper will focus on the last three steps listed above, as the first topic of data capture has already been covered extensively in previous Kx whitepapers, including

- [Kdb+tick profiling for throughput optimization](#)¹,
- [Disaster recovery for kdb+tick](#)²
- [Query Routing: a kdb+ framework for a scalable load-balanced](#)³

Moreover, the analysis techniques presented will make use of the following sensor data set, collected by the University of California Irvine, which contains the total power load for a single household over a 4 year period, to a resolution of 1 minute, as illustrated in Figure 1 below.

1. http://code.kx.com/q/wp/kdbtick_profiling_for_throughput_optimization.pdf

2. <http://code.kx.com/q/wp/disaster-recovery/>

3. <http://code.kx.com/q/wp/query-routing/>

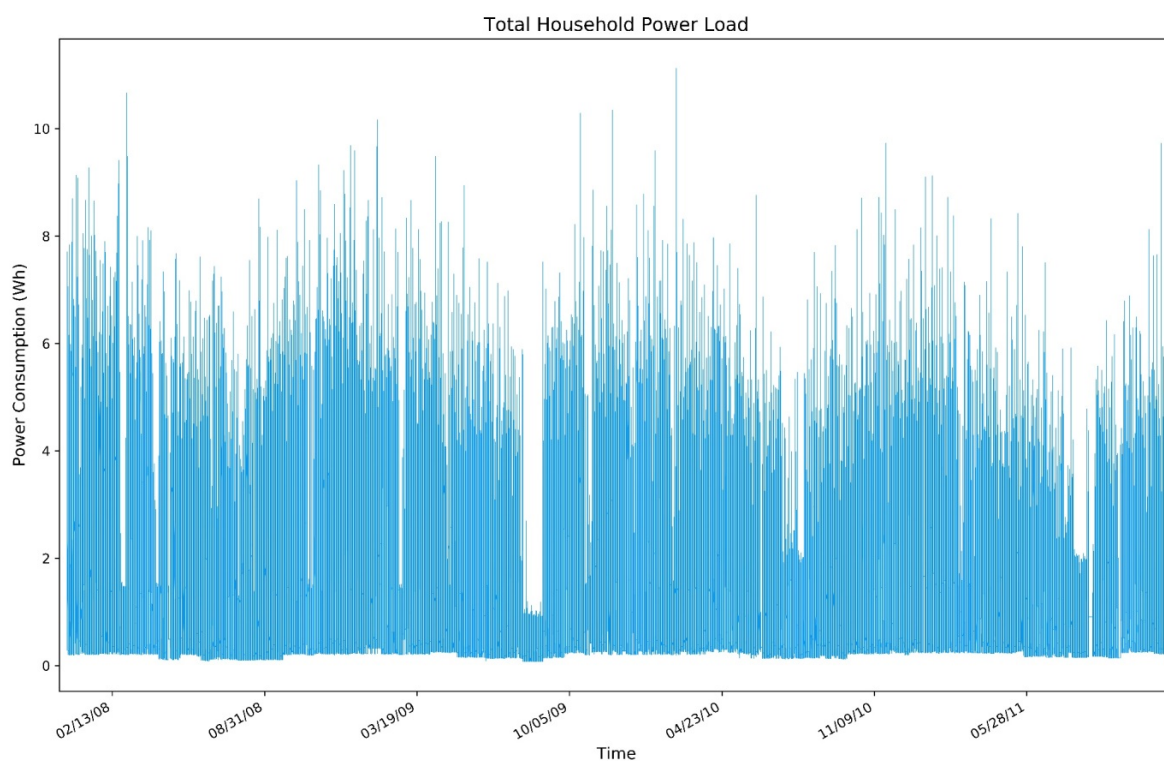


Figure 1: The load data of a single household, in 1-minute increments, from January 2007 to September 2010

Spectral analysis

To understand better the nature of the processing that will be performed on this signal, it will be useful to cover an important signal/wave concept. A fundamental property of any wave (and hence signal) is that of superpositioning[3], which is the combining of different signals (of the same or different frequency), to create a new wave with different amplitude and/or frequency. If simple signals can be combined to make a more complex signal, any complicated signal can be treated as the combination of more fundamental, simpler signals. Spectral analysis is a broad term for the family of transformations that ‘decompose’ a signal from the time domain to the frequency domain[4], revealing these fundamental components that make up a more complex signal.

This decomposition is represented as a series of complex[5] vectors, associated with a frequency ‘bin’, whose absolute value is representative of the relative strength of that frequency within the original signal. This is a powerful tool that allows for insight to be gained on the periodic nature of the signal, to troubleshoot the sensor in question, and to guide the subsequent transformations as part of the signal processing.

In general, some of the basic rules to keep in mind when using the frequency decomposition of a signal are:

- Sharp distinct lines indicate the presence of a strong periodic nature;
- Wide peaks indicate there is a periodic nature, and possibly some spectral leakage (which won’t be fully discussed in this paper);
- An overall trend in the frequency distribution indicates aperiodic (and hence of an infinite period) behavior is present.

Consider the following artificial signal in Figure 2, which is a combination of a 10Hz and 20Hz signal together with a constant background Gaussian noise:

```
x:(2*pi*(1%2048)) * til 2048;
y:(5 * sin (20*x))+(5 * sin (10 * x)) + 'float$({first 1?15.0} each x);
```

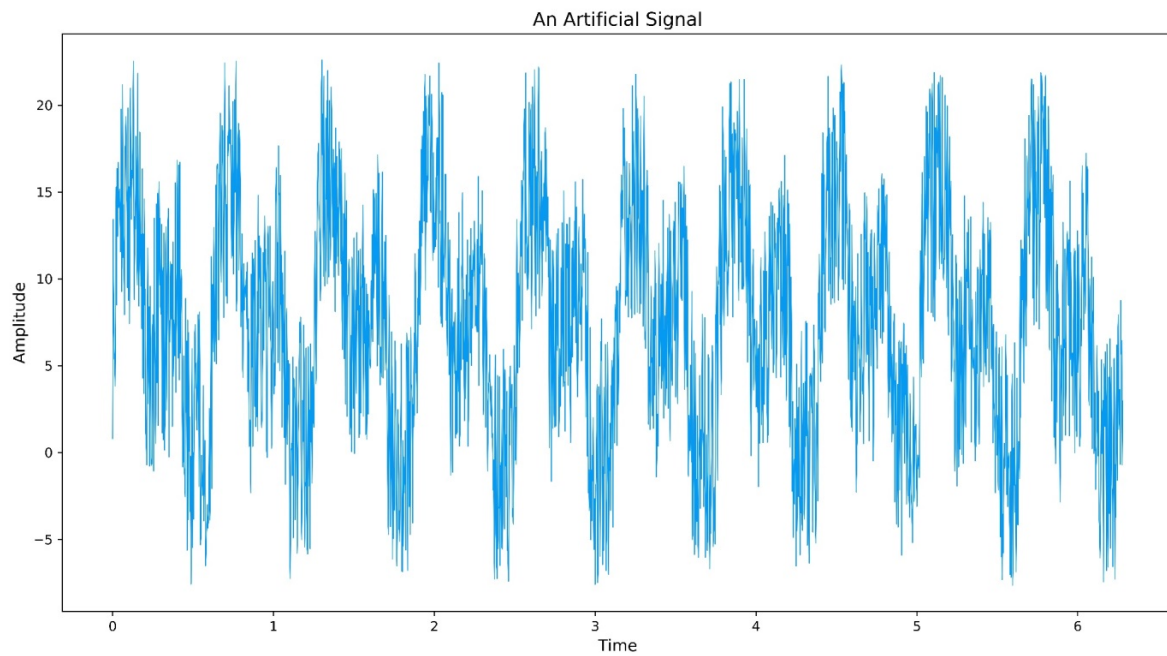


Figure 2: An artificial, noisy signal

This time-series shows a clear periodic nature, but the details are hard to discern from visual inspection. The following graph, Figure 3, shows the frequency distribution of this signal, produced through the application of an un-windowed Fourier Transform, a technique that will be covered in detail below.


```

\d .signal
mult: {[vec1;vec2]
  // Performs the dot product of two vectors
  realOut: ((vec1 0) * vec2 0) - ((vec1 1) * vec2 1);
  imagOut: ((vec1 1) * vec2 0) + ((vec1 0) * vec2 1);
  (realOut;imagOut)};

division: {[vec1;vec2]

  denom: 1/((vec1 1) xexp 2) + ((vec2 1) xexp 2);
  mul: mult[vec1 0;vec1 1;vec2 0;neg vec2 1];
  realOut: (mul 0)*denom;
  imagOut: (mul 1)*denom;
  (realOut;imagOut)};

conj: {[vec]
  (vec 0;neg vec 1)};

mag: {[vec]
  sqrvec: vec xexp 2;
  sqrt (sqrvec 0)+sqrvec 1};

\d .

q).signal.mult[5 -3;9 2]
51 -17
q).signal.mult[(5 2 1;-3 -8 10);(9 8 -4;2 3 6)]
51 40 -64 / Reals
-17 -58 -34 / Imaginary

```

Fast Fourier Transform

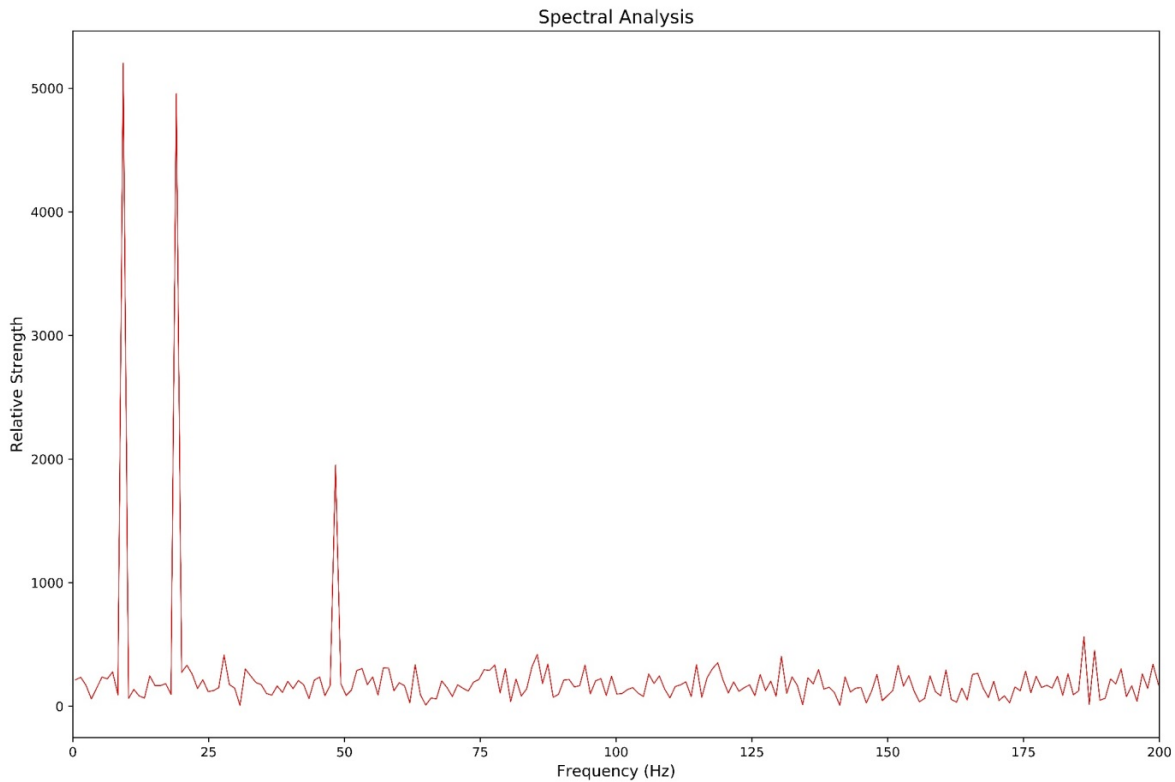


Figure 3: The frequency distribution of the simulated signal shown in Figure 2

The distinct sharp peaks and a low level random ‘static’ in Figure 3 demonstrate the existence of both period trends and some background noise within the signal. Considering that the signal in Figure 2 was to be made up of two different frequencies, why are there clearly three different frequencies (i.e., 10, 20 and 50Hz) present? This is because the original signal contained a low-level 50Hz signal, one that might be observed if a sensor wasn’t correctly shielded from 50Hz main power. This leakage could influence any decisions made from this signal, and may not have been immediately detected without spectral analysis.

In general, spectral analysis is used to provide information to make a better decision on how to process a signal. In the case above, an analyst may decide to implement a high-pass filter to reduce the 50Hz noise, and then implement a moving-mean (or Finite Impulse) filter to reduce the noise.

In the following chapters, an industry-standard technique for spectral analysis, the radix-2 Fast Fourier Transform, is implemented natively within q. This implementation is reliant on the introduction of a framework for complex numbers within q.

Complex numbers and q

All mainstream spectral analysis methods are performed in a complex vector space, due to the increase in computational speed this produces, and the ease at which complex signals (such as RADAR) are handled in a complex vector space. While complex numbers are not natively supported in q, it is relatively straightforward to create a valid complex vector space by representing any complex number as a list of reals and imaginary parts. Consider the following complex number, composed of the real and imaginary parts, 5 and 3, respectively

```
z:(5;3)
```

This means that a complex number series can be represented as

```
z:(5 7 12;3 -9 2)
```

By defining complex numbers as pairs of lists, subtraction and addition can be implemented with the normal + and - operators in q. All that remains to construct a complex vector space in q, is to create operators for complex multiplication, division, conjugation and absolute value.

```
\d .signal
mult: {[vec1;vec2]
  // Performs the dot product of two vectors
  realOut: ((vec1 0) * vec2 0) - (vec1 1) * vec2 1;
  imagOut: ((vec1 1) * vec2 0) + (vec1 0) * vec2 1;
  (realOut;imagOut)};

division: {[vec1;vec2]
  denom: 1%((vec1 1) xexp 2) + ((vec2 1) xexp 2);
  mul: mult[vec1 0;(vec1 1);(vec2 0);neg vec2 1];
  realOut: (mul 0)*denom;
  imagOut: (mul 1)*denom;
  (realOut;imagOut)};

conj: {[vec](vec 0;neg vec 1)};

mag: {[vec]
  sqrvec: vec xexp 2;
  sqrt (sqrvec 0)+sqrvec 1};

\d .
```

```

q).signal.mult[5 -3;9 2]
51 -17
q).signal.mult[(5 2 1;-3 -8 10);(9 8 -4;2 3 6)]
51 40 -64 / Reals
-17 -58 -34 / Imaginary

```

Fast Fourier Transform

The Fourier Transform is a standard transformation to decompose a real or complex signal into its frequency distribution[6]. The Fast Fourier Transform (FFT) is a family of algorithms that allow for the Fourier transform to be computed in an efficient manner, by utilizing symmetries within the computation and removing redundant calculations. This reduces the complexity of the algorithm from n^2 to around $n \times \log(n)$, which scales impressively for large samples.

Traditionally the Fast Fourier Transform and its family of transforms (such as Bluestein Z-Chirp or Rader's), are often packaged up in libraries for languages such as Python or Matlab, or as external libraries such as FFTW (Fastest Fourier Transform in the West). However, with the creation of a complex number structure in q, this family of transformations can be written natively. This reduces dependencies on external libraries and allows for a reduced development time.

The following example shows how the Radix-2 FFT (specifically a Decimation-In-Time, Bit-Reversed Input Radix-2 algorithm) can be implemented in q. The function, `fftrad2`, takes a complex vector of length N (a real signal would have 0s for the imaginary part) and produces a complex vector of length N , representing the frequency decomposition of the signal.

```

\d .signal
// Global Definitions
PI:acos -1; / pi ;
BR:2 sv reverse 2 vs til 256; / Reversed bits in bytes 00-FF
P2:1,prds[7#2],1; / Powers of 2 with outer guard
P256:prds 7#256; / Powers of 256

bitreversal:{{[indices]
  // Applies a bitwise reversal for a list of indices
  ct:ceiling 2 xlog last indices; / Number of significant bits (assumes sorted)
  bt:BR 256 vs indices; / Breakup into bytes and reverse bits
  dv:P2[8-ct mod 8]; / Divisor to shift leading bits down
  bt[0]:bt[0] div dv; / Shift leading bits
  sum bt*count[bt]#1,P256 div dv / Reassemble bytes back to integer
}};

fftrad2:{{[vec]
  // This performs a FFT for samples of power 2 size
  // First, define some constants
  n:count vec 0;
  n2:n div 2;
  indexOrig:til n;
  // Twiddle Factors - precomputed the complex values over the
  //discrete angles, using Euler formula
  angles:{{[n;x]2*PI*x%2n}[n;] til n2;
  .const.twiddle:(cos angles;neg sin angles);
  // Bit-reversed the vector and define it into a namespace so lambdas can access it
  ind:bitreversal[indexOrig];
  .res.vec:'float$vec . (0 1;ind);
  // Precomputing the indices required to implement each temporal phase of the DIT
  // Number of signals
  signalcount:'int$ {2 xexp x} 1+ til 'int$2 xlog n;
  // Number of points in each signal
  signalpoints:reverse signalcount;
  // Define an initial count, this can be used as a
  // backbone to get the even and odd indices by adjustment
  initial:{{[n2;x]2*x xbar til n2}[n2;] peach signalcount div 2;
  evens:{{[n2;x]x + n2#til n2 div count distinct x}[n2;] peach initial;
  odds:evens + signalcount div 2;
  twiddleIndices:{{[n;n2;x]n2#(0.5*x) * til n div x}[n;n2;] peach signalpoints;
  // Butterfly Implementation
  bflyComp:{{[bfInd]
    tmp:mult[.res.vec[;bfInd 1];.const.twiddle[;bfInd 2]];
    .['.res.vec;(0 1;bfInd 1);:;.res.vec[;bfInd 0]-tmp];
    .['.res.vec;(0 1;bfInd 0);+;tmp]];
  bflyComp each flip 'int$(evens;odds;twiddleIndices);
  res.vec}};
\d .

```

Below is a demonstration of how `.fft.rad2` operates on an example signal shown in Table 1.

Table 1: A 1Hz sine wave, sampled at 4Hz Time (s) 0 0.25 0.5 0.75 Amplitude 0 1 0 -1

```
// Adjusting .signal.fftrad2, to print intermediate results
/ Line 31, .res.vec:0N!\float$vec . (0 1;ind);
/ Line 51, .['.res.vec;(0 1;bfInd 0);++;tmp];0N!.res.vec};

q).signal.fftrad2[(0 1 0 -1;4#0)]
(0 0 1 -1f;0 0 0 0f) // Bitwise reversal
(0 0 0 2f;0 0 0 0f) // Butterfly step 1
(0 1.224647e-016 0 -1.224647e-016;0 -2 0 2f) // Butterfly step 2
0 1.224647e-016 0 -1.224647e-016 // Result is printed
0 -2 0 2
```

Sampling frequency and length

The application of an FFT produces the sequence of complex numbers that are associated with the magnitudes of the frequencies that compose a signal, but not the frequencies themselves. The frequencies that are captured by a spectral analysis are a function of the sampling that was applied to the dataset, and will have the following characteristics,

- The number of distinct frequencies (often called frequency bins) is equal to
- The maximum frequency that can be captured by a Fourier Transform is equal to half the sampling frequency[7]
- Both positive and negative frequencies will result from the transform and will be mirrored about 0Hz

Considering the above characteristics, the frequencies of a spectral analysis are a function of only the number of samples and the sampling frequency. The following function can be used to produce the frequencies of a spectral analysis from the number of samples and sampling frequency, and hence is used to define the x axis.

```
q)xAxis: {[Ns;fs] (neg fs*0.5)+(fs%Ns-1)*til Ns}
```

This function has been applied below to compute the associated frequencies for the sample signal given in Table 1.

```
q)xAxis[4;4] //Calculating the frequency bins for 16 samples, measured at 20Hz
-2 -0.6666667 0.6666667 2
```

```
q)transform:.signal.fftrad2[(0 1 0 -1;4#0)]
q)flip `Frequency`Real`Imaginary!(xAxis[4;4];transform 0;transform 1)
Frequency Real Imaginary
-----
-2          0          0
-0.6666667 1.224647e-016 -2
0.6666667  0          0
2          -1.224647e-016 2
```

Windowing the signal

There is an important assumption that is carried forward from the continuous Fourier Transform to the discrete version, that the data set being operated on is infinitely large. Obviously, this cannot be true for a discrete transform, but it is simulated by operating with a circular topology, i.e., by assuming that the end of the sample connects to the start. This means that there can be discontinuities in the data if the sampled dataset doesn't capture a full period (resulting in different start-end values). These discontinuities result in a phenomenon known as a 'spectral leakage' [8], where results in the frequency domain are spread over adjacent frequency bins

The solution to this problem is to window the signal, which adjusts the overall amplitude of the signal to limit the discontinuities. In general, this is achieved by weighting the signal towards 0 at the start and end of the sampling. For this paper, the commonly used Hanning Window (which finds a middle ground between many different windowing functions), shown in Figure 4, will be applied.

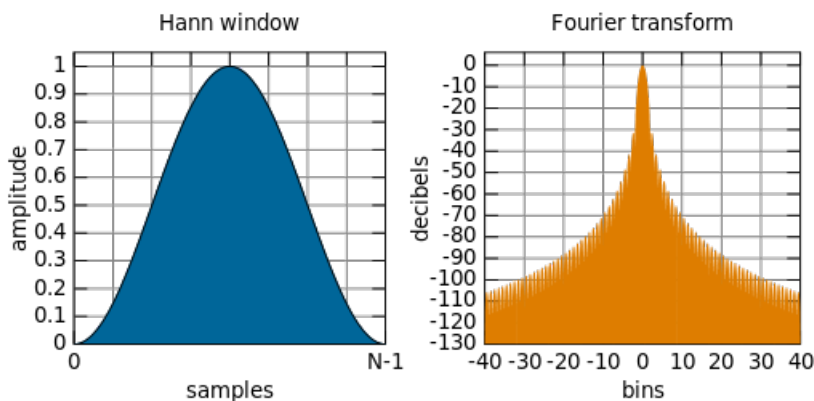


Figure 4: The shape of a Hann Window, which is applied to an input signal to tamp down the signal so that there is greater continuity under the transform. Distributed under CC Public Domain

In the following example, the Hanning weightings are computed and then applied to a complex vector.

```
// The window function will produce a series of weightings
// for a sequence of length nr,
// which would be the count of the complex series
q>window: {[n] {[nr;x](sin(x*acos -1)%nr-1) xexp 2} [n;] each til n}

q)vec:(1 2 1 4;0 2 1 4)
q)n:count first vec
// Applying the window with an in place amendment
q)vec*:(window n>window n);
0 1.5 0.75 5.999039e-032
0 1.5 0.75 5.999039e-032
```

The Fourier Transformation for a real-valued signal

In general, the final step to getting the frequency distribution from the transformed vector series is to get the normalized magnitude of the complex result. The normalized magnitude is computed by taking the absolute value of the real and imaginary components, divided by the number of samples. However, for a real-valued input, where there is symmetry about the central axis[9], the dataset can be halved.

Putting it all together

Below, a single function, `spectral`, is defined which performs a spectral analysis (through a Fourier Transform) on a complex vector. This function windows the vector, performs a FFT and scale the results appropriately.


```
spectral: {[vec;fs]

// Get the length of the series, ensure it is a power of 2
// If not, we should abort operation and output a reason
nr:count first vec;
if[not 0f =(2 xlog nr) mod 1;0N! "The input vector is not a power of 2";`error];

// Window the functions, using the Hanning window
wndFactor: {[n] {[nr;x](sin(x*acos -1)%nr-1) xexp 2} [n;] each til n};
vec*:(wndFactor nr;wndFactor nr);

// Compute the fft and get the absolute value of the result
fftRes:.fft.fftrd2 vec;
mag:.cplx.mag fftRes;

// Scale an x axis
xAxis: {[Ns;fs] (neg fs*0.5)+(fs%Ns-1)*til Ns}[nr;fs];

// Table these results
([freq:xAxis;magnitude:mag%nr)
];
```

Let's apply this to the dataset shown in Figure 1, from the University of California, loading it from the .txt file and then filling in null values to ensure a constant sampling frequency.

```
q)household:select Time:Date+Time,Power:fills Global_active_power from
("DTF "; enlist ";") 0:`:household_power_consumption.txt
```

This data is then sampled at 2-hour intervals, ensuring the sample size is a power of 2, before performing a spectral analysis on it with the `spectral` function.

```
q)household2hour:select from household where 0=(i mod 120)
q)n:count household2hour
17294
q)cn:`int$2 xexp (count 2 vs n)-1
16384
q)spec:spectral[(cn sublist household2hour`Power;cn#0f);1%120*60]
q)\t spectral[(cn sublist household2hour`Power;cn#0f);1%120*60]
6
```

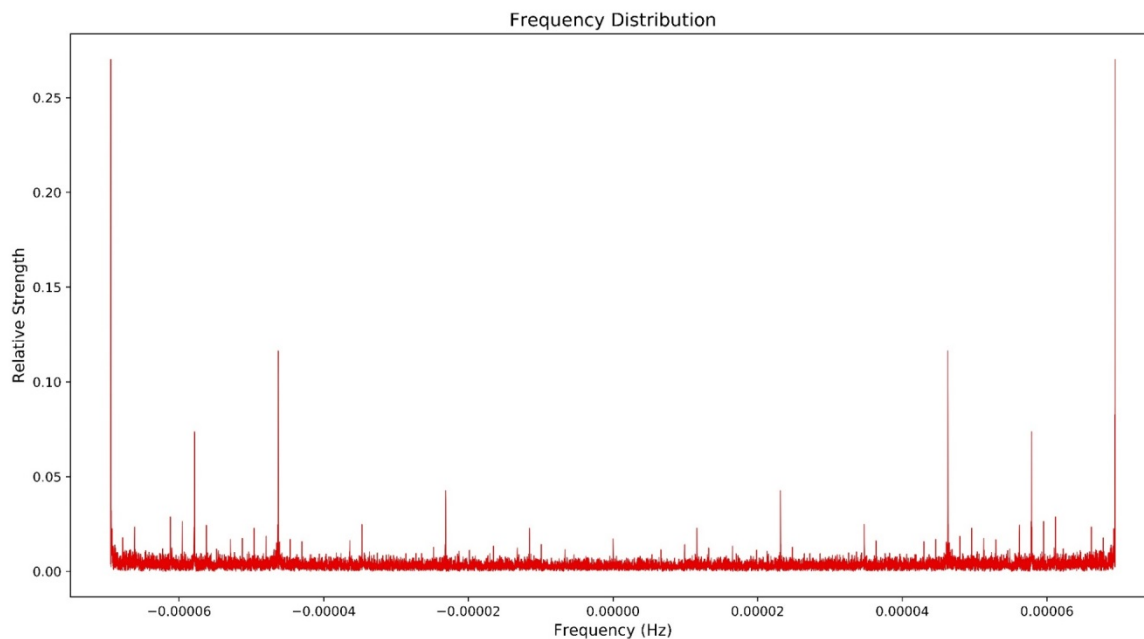


Figure 5: The frequency distribution of the Household Power load, excluding the first result to demonstrate the Hermitian symmetry

The above FFT was formed on 16,384 points and was computed in 6ms. In this case, the frequencies are so small that they are difficult to make inferences from. (It's hard to intuit what a frequency of $1 \mu\text{Hz}$ corresponds to). A nice solution is to remove the symmetry present (taking only the first half), then convert from Frequency (Hz) to Period (hours).

```
specReal:select period:neg (1%freq)%3600, magnitude from (cn div 2) sublist spec
```

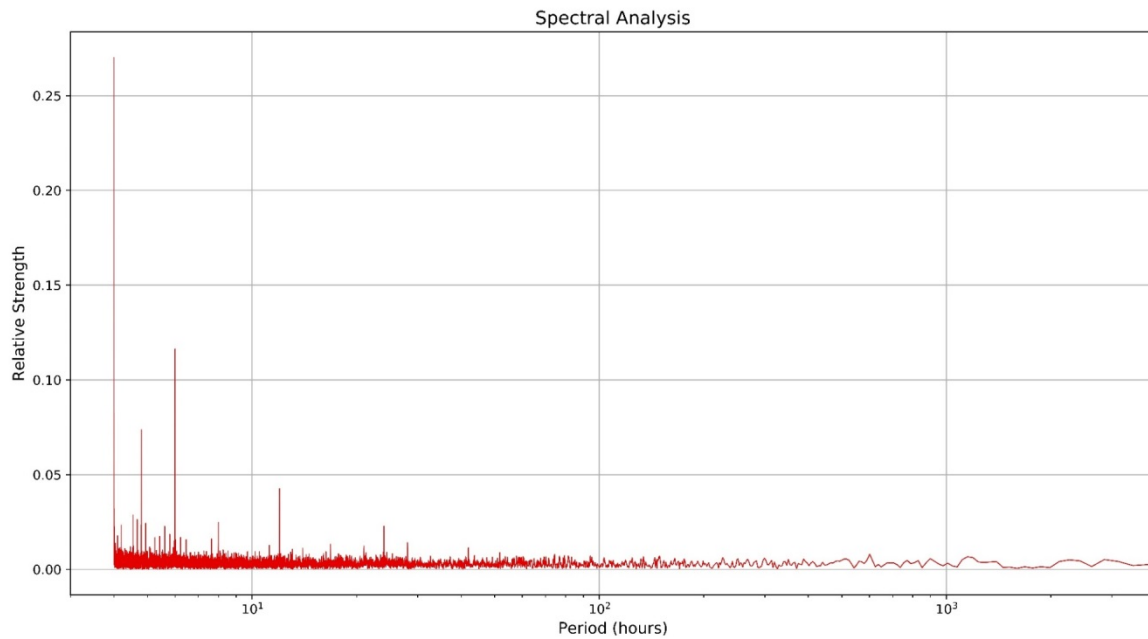


Figure 6: The frequency distribution of the Household Power dataset with the symmetry removed

It can be seen from Figure 6, that the first value is nearly an order of magnitude larger than any other, suggesting that there is a strong high-frequency signal that hasn't been captured. The likely culprit from this would be interference from a 50Hz signal due to mains power, and the existence of others is a strong possibility. After the large initial value, there are distinct 2, 3, 6, 10 and (just below) 12-hour cycles, which could be explained as part of the ebb and flow of day-to-day life, coming home from school and work. This result, with strong sharp lines, definitively shows the existence of periodic trends, along with a constant background noise. This knowledge will be used to further process the signal and obtain more useful information on long-term trends.

Smoothing

A simple method to see the longer-term trends and remove the daily noise is to build a low-pass filter and reduce the frequencies below a specific threshold. A simple low-pass filter can be constructed as a windowed moving average, which is trivial in q with its inbuilt function `mavg`. All that is needed is to ensure the moving average is centered on the given result, which can be achieved by constructing a function that uses `rotate` to shift the input lists to appropriately center the result. The following implements an N-point centered moving average on a list.

```
\d .signal
movAvg: {[list;N]
  $[0=N mod 2;
    (floor N%2) rotate 2 mavg (N mavg list);
    (floor N%2) rotate N mavg list]};
\d .
```

From the spectral analysis of the Household Power dataset (see Figure 6), it can be seen that the most dominant signals have a period of at most 24 hours. Therefore, this is a good value for the window size for an initial smoothing of the Household Power dataset[10].

```
q)householdMA:select Time, Power: .signal.movAvg[Power;24*60] from household
q)\t householdMA:select Time, Power: .signal.movAvg[Power;24*60] from household
109
```

The following Figure 7 shows the result of the application of a 1,440-point moving average on the raw Household Power data set (see Figure 1), containing over 2 million data points (achieved in just over 100ms).

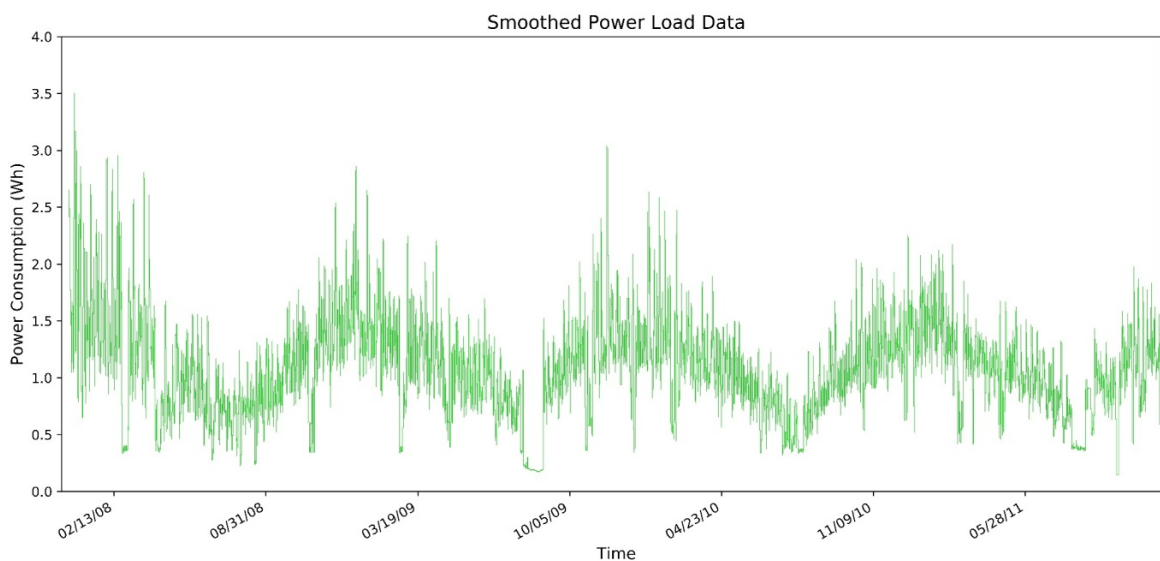


Figure 7: The smoothed dataset: note the now clearly visible yearly trend

Comparison of the smoothed data (Figure 7) and the raw data (Figure 1) demonstrates the clarity achieved by applying a simple moving average to smooth a dataset. This simple process has removed a significant amount of noise and revealed the existence of a year-long, periodic trend.

Application of an FFT to this smoothed dataset can reveal trends which would have been difficult to obtain from the raw data. For this, a lower sampling rate (of 6-hour intervals) will be used to assist in revealing long-term trends[11].

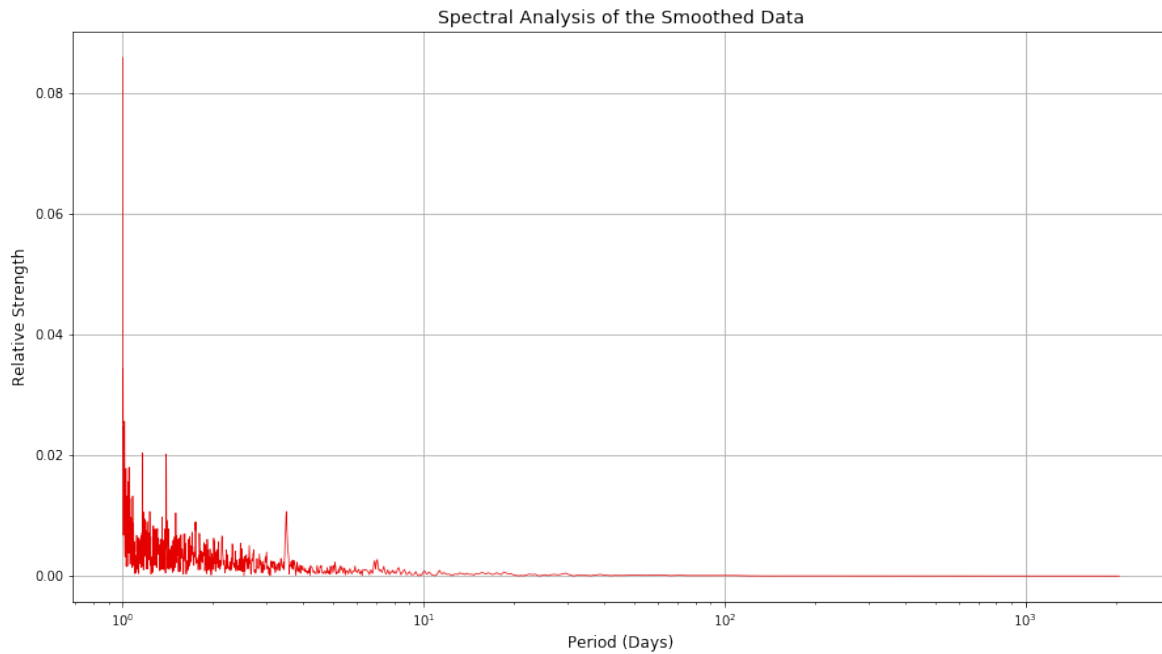


Figure 8: The FFT of the smoothed dataset

From the figure above, it can be seen that whilst there is still some high-frequency noise, there are definite 6- and 12-hour trends, with a definite 2.5-day trend in the power usage. Despite Figure 7 showing a clear year-long periodic trend, this isn't present in the Frequency distribution in Figure 8, probably because the dataset simply does not cover enough years for that trend of that length to be present in a simple spectral analysis.

It can be concluded from the results in this chapter that there are bi-weekly, weekly and yearly periodic trends hidden within the raw data, as well as the 5, 6, 12 and 24-hour trends obtained from earlier work. In the context of household power consumption, these results are not unexpected.

Anomaly detection

In this paper, anomaly detection is considered the last step of signal processing, but it can be implemented independently of any other signal processing, serving as an early warning indicator of deviations from expected behavior. Assuming the sensors are behaving stochastically, outliers can be identified by the level of deviation they are experiencing from the expected behavior, i.e., using the mean and standard deviation. Ideally, these deviations should be measured from the local behavior in the signal rather than from the whole series, which allows for periodic behavior to be accounted for. This can be achieved with a windowed moving average (from ‘Smoothing’ above) and a windowed moving standard deviation.

```
\d .signal
movDev: {[list;N]
$[0=N mod 2;
  (floor N%2) rotate 2 mdev N mavg list;
  (floor N%2) rotate N mdev list]};
\d .
```

Assuming the randomness in the signal is Gaussian in nature, an anomaly is present if the current value is more than a sigma-multiple of the standard deviation away from the moving mean. This can be determined effectively within q by using a vector conditional evaluation in a qSQL statement, comparing the actual value with maximum/minimums created with the moving average and deviation. Below this technique is demonstrated on the Household Power dataset, with a 5-point moving window and a sigma level of 2.

```
q)sigma:2;
q)outliers:update outlier:[
  (Power > movAvg[Power;5] + fills sigma * movDev[Power;5]) |
  (Power < movAvg[Power;5] - fills sigma * movDev[Power;5]);
  Power;
  0n] from household;

q)\t update outliers:update outlier:[
  (Power > movAvg[Power;5] + fills sigma * movDev[Power;5]) |
  (Power < movAvg[Power;5] - fills sigma * movDev[Power;5]);
  Power;
  0n] from household;
380
```

It takes approximately 380ms to perform this detection routine on over 2 million data points, and the results can be seen below in Figure 9. Of the approximately 2 million data

points, 5000 were detected to be anomalous, interestingly, setting the sigma level to over 3 drops the anomalies to 0, perhaps due to the Gaussian distribution within the dataset.

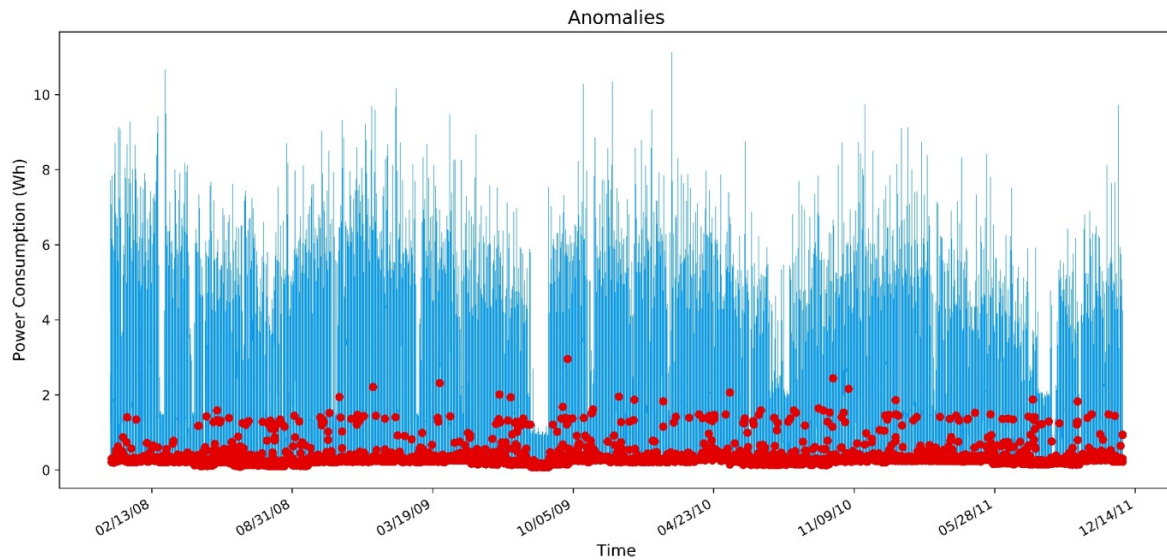


Figure 9: The results of an anomaly detection on the Household Power dataset

Conclusion

In this paper, we have shown the ease with which simple, fast signal processing can be implemented natively within q. All that was required was a basic framework to support complex-number operations to allow for complex-field mathematics within q. With this framework, signal-processing algorithms can easily be implemented natively in q, reducing dependencies on external libraries, and allowing for faster and interactive development. This complex framework was essential to the successful implementation of a Fast Fourier Transform (FFT), a fundamental operation in the processing of many signals

The collection of signal-processing functions that were shown in this paper were able rapidly to process the grid-load dataset obtained from the University of California, Irvine. An FFT was implemented on a sample of 16,000 data points in around 6ms and guided from that spectral analysis, a simple low-pass filter was implemented directly in the time domain, on over 2 million data points, in around 100ms. The resulting data had a clear reduction in noise and revealed the longer-term trends present in the signal. It has also been shown that a simple and robust anomaly-detection routine can be implemented in q, allowing anomalies to be detected in large datasets very rapidly. These smoothing and anomaly detection routines could easily be integrated as real-time operations with other data capture and processing routines.

Clearly, native applications of signal processing, which historically have been the realm of libraries accessed through Python or C++, can be natively integrated into q/kdb+ data systems. This allows for q/kdb+ to be used as a platform for the capture, processing, storage and querying of sensors based signal information.

References

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4. <https://community.plm.automation.siemens.com/t5/Testing-Knowledge-Base/Windows-and-Spectral-Leakage/ta-p/432760>

Appendix

Data sources

The primary dataset, containing sensor readouts for a single households total power load, that was introduced and then transformed throughout this paper, can be found at [uci.edu](https://archive.ics.uci.edu/ml/datasets/Individual+household+electric+power+consumption)⁵.

The code presented in this paper is available at [github.com/callumjbiggs](https://github.com/callumjbiggs/q-signals/blob/master/signal.q)⁶.

Python FFT comparison

In the following, a comparison between two alternative implementations of FFT will be performed to benchmark the native q-based algorithm. Comparisons will be made to a similar algorithm used in `\d .signal`.

```
mult: {[vec1;vec2]
  // Performs the dot product of two vectors
  realOut: ((vec1 0) * vec2 0) - ((vec1 1) * vec2 1);
  imagOut: ((vec1 1) * vec2 0) + ((vec1 0) * vec2 1);
  (realOut;imagOut)};

division: {[vec1;vec2]
  denom: 1/((vec1 1) xexp 2) + ((vec2 1) xexp 2);
  mul: mult[vec1 0;vec1 1;vec2 0;neg vec2 1];
  realOut: (mul 0)*denom;
  imagOut: (mul 1)*denom;
  (realOut;imagOut)};

conj: {[vec](vec 0;neg vec 1)};

mag: {[vec]
  sqrvec: vec xexp 2;
  sqrt (sqrvec 0)+sqrvec 1};

\d .
```

5. <https://archive.ics.uci.edu/ml/datasets/Individual+household+electric+power+consumption>

6. <https://github.com/callumjbiggs/q-signals/blob/master/signal.q>

```

q).signal.mult[5 -3;9 2]
51 -1
q).signal.mult[(5 2 1;-3 -8 10);(9 8 -4;2 3 6)]
51 40 -64 / Reals
-17 -58 -34 / Imaginary

```

Fast Fourier Transform, written natively in Python 3.6, and also against a well refined, and highly-optimized C based library. The library in question will be the [NumPy](#)⁷ library, called via the q fusion with Python – embedPy (details on setting up this environment are available from [Machine Learning section](#)⁸ on code.kx.com).

In order to get an accurate comparison, each method should be tested on a sufficiently large dataset, the largest 2N subset of the previously used Household Power dataset is suitable for this purpose.

```

//Get the largest possible 2N dataset size
q)n:count household;
q)nr:`int$2 xexp (count 2 vs n)-1
1048576
q)comparison:select real:Power, imag:nr#0f from nr#household
q)\t .fft.fftrad2[(comparison`real;comparison`imag)]
1020
q)save `comparison.csv

```

So the q algorithm can determine the Fourier Transform of a million point dataset in about a second, a nice whole number to gauge the comparative performance.

In Python, the native complex-math definitions field can be used to build a similar radix-2 FFT Decimation in Time routine as was implemented above, from a publicly-available GitHub repo github.com/peterhinch⁹.

7. <http://www.numpy.org/>

8. <http://code.kx.com/q/ml/embedpy/>

9. <https://github.com/peterhinch/micropython-fft/blob/master/algorithms.py>

```

import pandas as pd
import numpy as np
import time
import math
from cmath import exp, pi

data = pd.read_csv('c:/q/w64/comparison.csv')
fftinput = data.real.values + 1j * data.imag.values
def ffft(nums, forward=True, scale=False):
    n = len(nums)
    m = int(math.log2(n))
    #n= 2**m #Calculate the number of points
    #Do the bit reversal
    i2 = n >> 1
    j = 0
    for i in range(n-1):
        if i<j: nums[i], nums[j] = nums[j], nums[i]
        k = i2
        while (k <= j):
            j -= k
            k >>= 1
        j+=k
    #Compute the FFT
    c = 0j-1
    l2 = 1
    for l in range(m):
        l1 = l2
        l2 <<= 1
        u = 0j+1
        for j in range(l1):
            for i in range(j, n, l2):
                i1 = i+l1
                t1 = u*nums[i1]
                nums[i1] = nums[i] - t1
                nums[i] += t1
            u *= c
        ci = math.sqrt((1.0 - c.real) / 2.0) # Generate complex roots of unity
        if forward: ci=-ci # for forward transform
        cr = math.sqrt((1.0 + c.real) / 2.0) # phi = -pi/2 -pi/4 -pi/8...
        c = cr + ci*1j#complex(cr,ci)
    # Scaling for forward transform
    if (scale and forward):
        for i in range(n):
            nums[i] /= n
    return nums

t0 = time.time()
ffft(fftinput)
t1 = time.time()
print(t1 - t0)
8.336997509002686

```

From the above computations, it can be seen that the natively-written Python FFT computed the million-point transformation in 8.3 seconds.

And finally using embedPy to access the C-backed NumPy library to perform the FFT, which will likely not use a radix-2 algorithm, but a mixed-radix implementation that allows for multi-threading (which the in-place Decimation-In-Time algorithm fundamentally cannot do).

```
p)import numpy as np;
p)import pandas as pd;
p)import time;
p)data = pd.read_csv('comparison.csv');

// Convert to a complex pair

p)fftinput = data.real.values + 1j * data.imag.values;
p)t0 = time.time();
p)np.abs(np.fft.fft (fftinput));
p)t1 = time.time();
p)print (t1 - t0)
0.04644131660461426
```

Acknowledgments

I would like to acknowledge the Machine Learning Repository hosted by the University of California, Irvine, for providing the real-world data sets used as sample input data sets for the methods presented in this paper. This [repository](http://archive.ics.uci.edu/ml/datasets.html)¹⁰ has an enormous number of freely-available datasets.

10. <http://archive.ics.uci.edu/ml/datasets.html>

Notes

1. A stochastic process is one where at least part of the process is defined by random variables. It can be analyzed but not predicted with certainty, although Autoregressive based models can achieve a modest prediction capability.
2. Internet of Things is the networking and data sharing between devices with embedded electronics, sensors, software and even actuators. Common household examples include devices such as Fitbits, Amazon Echo's and Wi-Fi connected security systems.
3. An everyday example of the superpositioning principle in action is through Noise-Cancelling headphones, which measure the outside soundwave, and then produce an exact opposite wave internally, that nullifies the outside noise.
4. The 'domain' is the variable on which a signal is changing. Humans naturally tend to consider changes in a time-domain, such as how a singer's loudness varies during a song. In this case, the frequency-domain analysis would be to consider how often the singer hit particular notes.
5. Here complex is used in a mathematical sense, each vector contains a real and imaginary part.
6. Technically, a Fourier Transformation decomposes a signal into complex exponentials (called 'cisoids').
7. This is a result of the Nyquist-Shannon sampling theorem.
8. A comprehensive explanation for how sampling discontinuities can result in spectral leakage would require an in-depth discussion of the underlying mathematics of a Fourier Transformation. An excellent discussion of this phenomena (which avoids the heavy math) can be found at [Windows and Spectral Leakage](#)¹¹
9. This is because the result of a real valued FFT is conjugate symmetric, or Hermitian, with the exception of the first value.
10. Designing a filter to smooth the data in this manner is very simplistic and does not account for key filtering criteria such as frequency and impulse response, phase shifting, causality or stability. Application of more rigorous design methods such as a Parks-McClellan algorithm would be beyond the scope and intention of this paper.

11. <https://community.plm.automation.siemens.com/t5/Testing-Knowledge-Base/Windows-and-Spectral-Leakage/ta-p/432760>

11. The smoothing that was performed may have highlighted the longer term trends, but the background noise remained present. It was therefore necessary to decrease the sampling frequency (and therefore reduce the largest frequency captured) in order to better highlight the low frequency/high period trends.